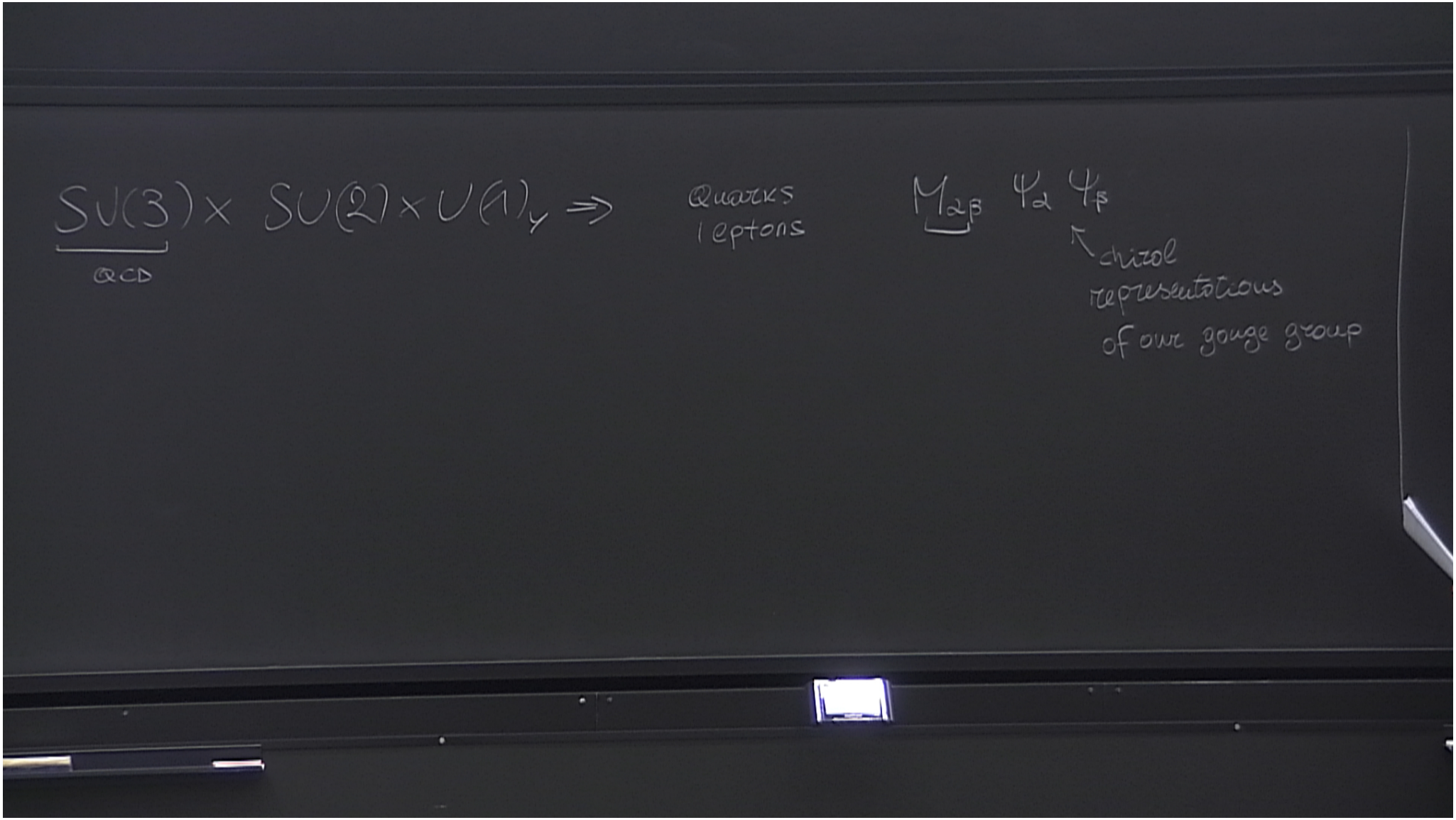
Title: Standard Model-14

Date: Jan 22, 2015 09:00 AM

URL: http://pirsa.org/15010017

Abstract:


SU(3) × SU(2) × U(1)_Y ⇒
QCD
Quarks
leptons
M_αβ ψ_α ψ_β
chiral
representations
of our gauge group

$N_i \to (1, 1, 0)$ (Y)

$i = 1, 2, 3$ flavor

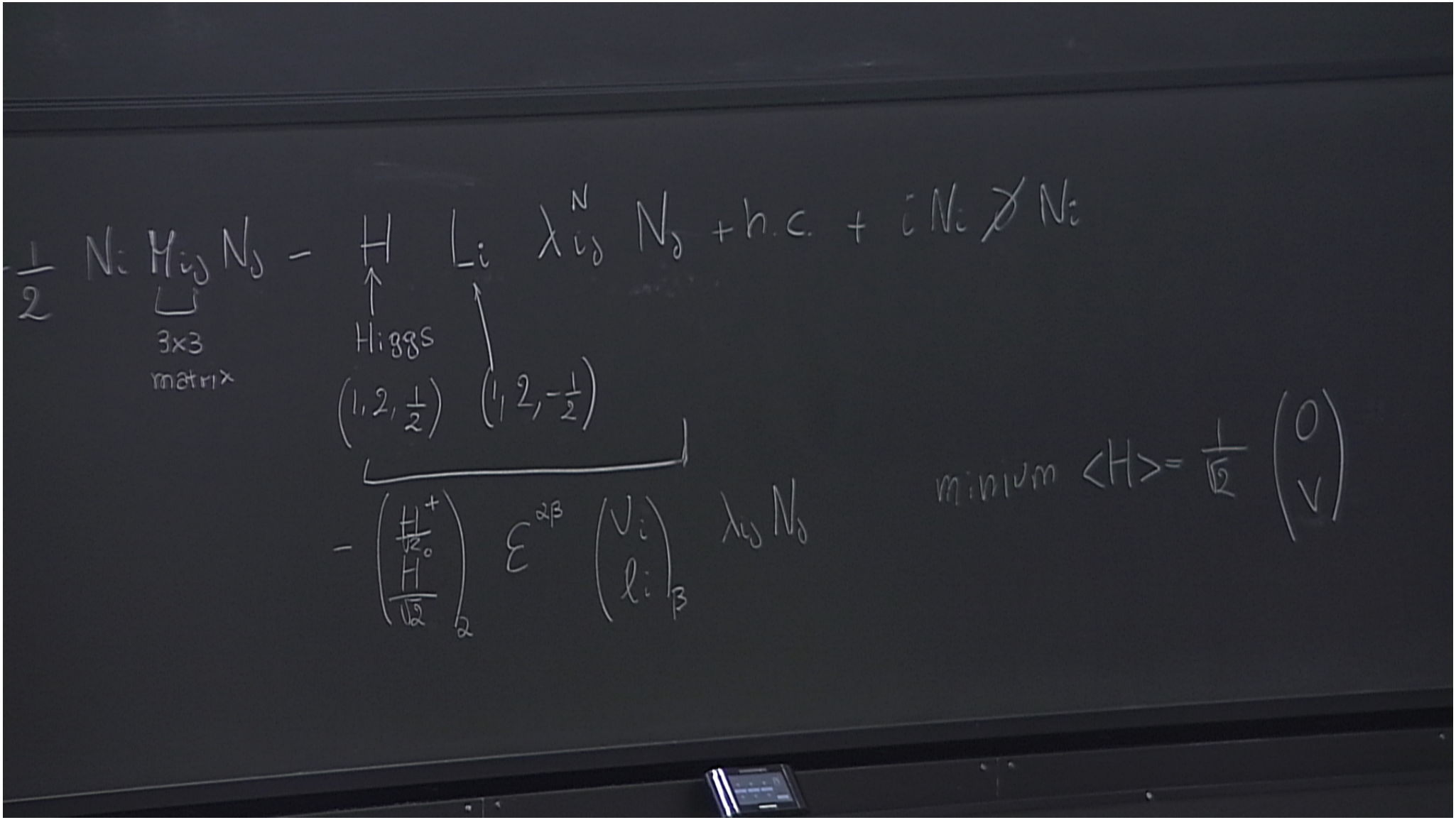
SU(3) SU(2)

$\mathcal{L} = -\frac{1}{2} N_i M_{ij} N_j - \underbrace{\bar{H}}_{\text{Higgs}} L_i \lambda^N_{ij} N_j + h.c. + i N_i \not{\partial} N_i$

$M_{ij}$: 3×3 matrix

$\bar{H}$: $(1, 2, \frac{1}{2})$, $L_i$: $(1, 2, -\frac{1}{2})$

$$- \begin{pmatrix} \frac{H^+}{\sqrt{2}} \\ \frac{H^0}{\sqrt{2}} \end{pmatrix}_\alpha \varepsilon^{\alpha\beta} \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_\beta \lambda_{ij} N_j$$


$-\frac{1}{2} N_i M_{ij} N_j - H L_i \lambda^N_{ij} N_j + h.c. + i N_i \not{\partial} N_i$
$M_{ij}$
3×3
matrix
H
Higgs
$L_i$
$(1,2,\frac{1}{2})$
$(1,2,-\frac{1}{2})$
$-\begin{pmatrix} \frac{H^+}{\sqrt{2}} \\ H^0 \\ \frac{H}{\sqrt{2}} \end{pmatrix}_2 \varepsilon^{\alpha\beta} \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_\beta \lambda_{ij} N_j$
minimum $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

$$-\mathcal{L}_{mass} = -\frac{1}{2}\begin{pmatrix} \nu_i & N_i \end{pmatrix}\left(\begin{array}{c|c} 0 & \lambda_{ij}\frac{v}{\sqrt{2}} \\ \hline \lambda_{ij}^{T}\frac{v}{\sqrt{2}} & M_{ij} \end{array}\right)\begin{pmatrix} \nu_j \\ N_j \end{pmatrix}$$

$5+6$

1) $M_{ij} \ll \lambda_{is} \frac{v}{\sqrt{2}}$

↳ Dirac neutrinos

2) $M_{ij} \gg \lambda_{ij} \frac{v}{\sqrt{2}}$ ⟹ Majorana neutrinos

$$2\,\text{mass} = -\frac{v^2}{4}\, U_i \left[\lambda_{ij} \left(M^{-1}\right)_{jk} \left(\lambda^T\right)_{kl}\right] U_l - \frac{1}{2} N_i M_{ij} N_j + hc$$

Masses $\lesssim 0(1\,eV)$ experimentally

$\Rightarrow v \sim 246$ GeV

1) $M_{ij} \ll \lambda_{ij} \frac{v}{\sqrt{2}} \rightarrow \lambda \sim O(10^{-11} - 10^{-12})$

↳ Dirac neutrinos

2) $M_{ij} \gg \lambda_{ij} \frac{v}{\sqrt{2}}$ ⟹ Majorana neutrinos

$$2\,\text{mass} = -\frac{v^2}{4}\, U_i \left[\lambda_{ij} \left(M^{-1}\right)_{jk} \left(\lambda^T\right)_{kl}\right] U_l - \frac{1}{2} N_i M_{ij} N_j + hc$$

**Masses $\lesssim O(1\,eV)$ experimentally**

$N_i \rightarrow (1,1,0)_Y$

$i = 1,2,3$ flavor

$SU(3)$ $SU(2)$

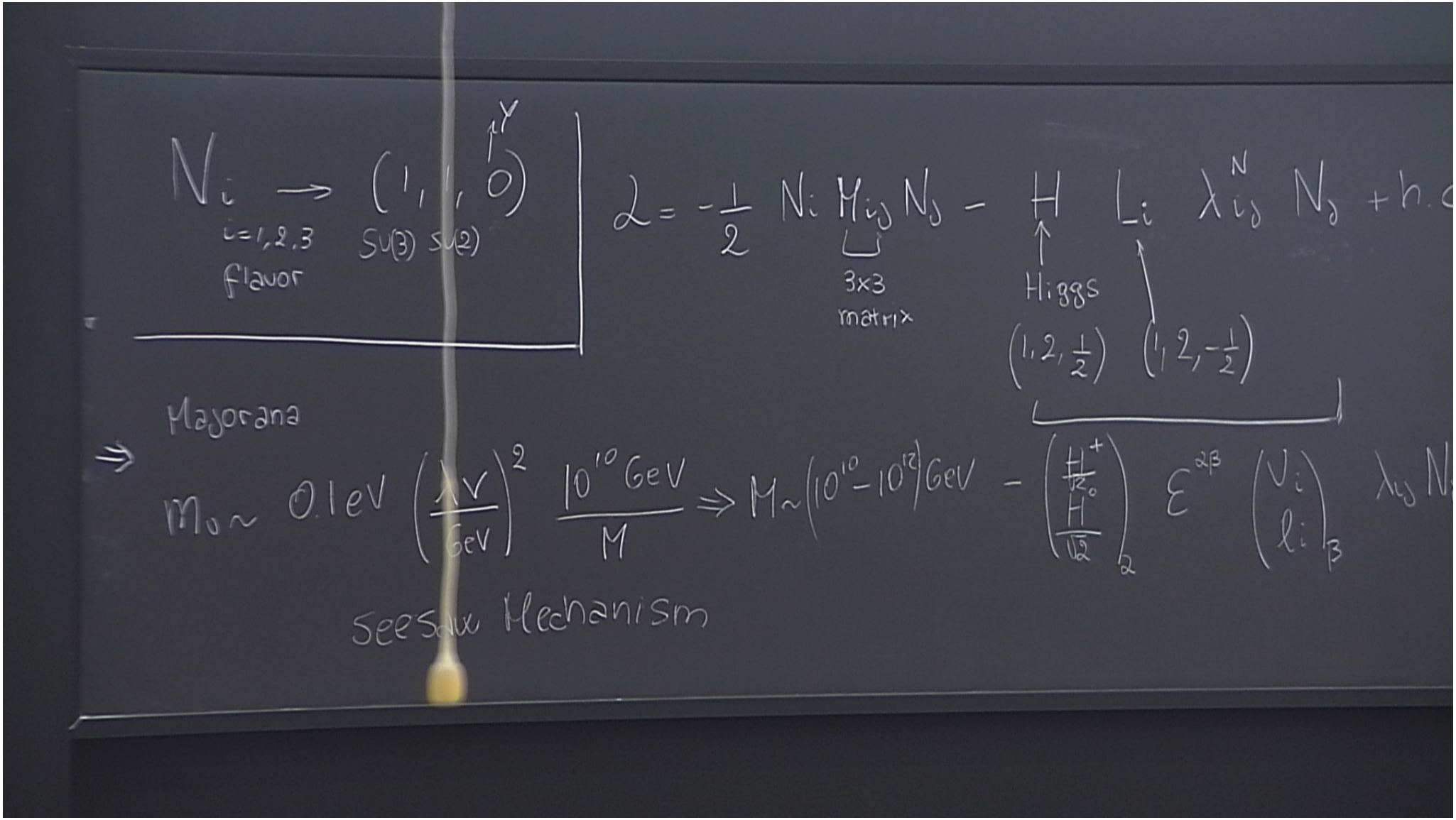
$\mathcal{L} = -\frac{1}{2} N_i M_{ij} N_j - H L_i \lambda^N_{ij} N_j + h.c$

$M_{ij}$: $3\times3$ matrix

$H$: Higgs $\left(1,2,\frac{1}{2}\right)$ $\quad L_i$: $\left(1,2,-\frac{1}{2}\right)$

$\Rightarrow$ Majorana

$m_\nu \sim 0.1\,eV \left(\frac{\lambda v}{GeV}\right)^2 \frac{10^{10}\,GeV}{M} \Rightarrow M \sim (10^{10} - 10^{12})\,GeV$ $\quad -\left(\frac{H^+}{\frac{H^0}{\sqrt{2}}}\right)_2 \varepsilon^{\alpha\beta} \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_\beta \lambda_{ij} N_j$


$N_i \to (1, 1, 0)$
$i = 1,2,3$
flavor
SU(3) SU(2)
$\mu_N$
$\mathcal{L} = -\frac{1}{2} N_i M_{ij} N_j - H L_i \lambda^N_{ij} N_j + h.c$
3x3
matrix
Higgs
$(1,2,\frac{1}{2})$
$(1,2,-\frac{1}{2})$
Majorana
$\Rightarrow$
$m_\nu \sim 0.1\,eV \left(\frac{\lambda v}{GeV}\right)^2 \frac{10^{10}\,GeV}{M} \Rightarrow M \sim (10^{10} - 10^{12})\,GeV$
$- \left(\frac{H^+}{\frac{H}{\sqrt{2}}}\right)_2 \varepsilon^{\alpha\beta} \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_\beta \lambda_{ij} N_j$
See-saw Mechanism

$$\mathcal{L} = -\frac{1}{2} N_i \underbrace{M_{ij}}_{3\times 3 \text{ matrix}} N_j - \underbrace{H}_{\text{Higgs}} L_i \lambda^{N}_{ij} N_j + h.c. + i N_i \not{\partial} N_i$$

$$\underbrace{(1,2,\tfrac{1}{2})\ (1,2,-\tfrac{1}{2})}$$

$$\text{GeV} \Rightarrow M \sim (10^{10} - 10^{12})\,\text{GeV} - \begin{pmatrix} H^{+} \\ \frac{H^{0}}{\sqrt{2}} \\ \frac{H}{\sqrt{2}} \end{pmatrix}_{2} \varepsilon^{\alpha\beta} \begin{pmatrix} \nu_i \\ l_i \end{pmatrix}_{\beta} \lambda_{ij} N_j$$

minimum $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

nism

(*) CKM matrix for quarks

(*) PMNS matrix

↑

Pontecorvo

Maki

Nagawa

Sakata

1. $U^T \left[ \frac{v^2}{4} \lambda_{ij} (M^{-1})_{jk} (\lambda^T)_{ke} \right] U = \text{diagonal}$ :

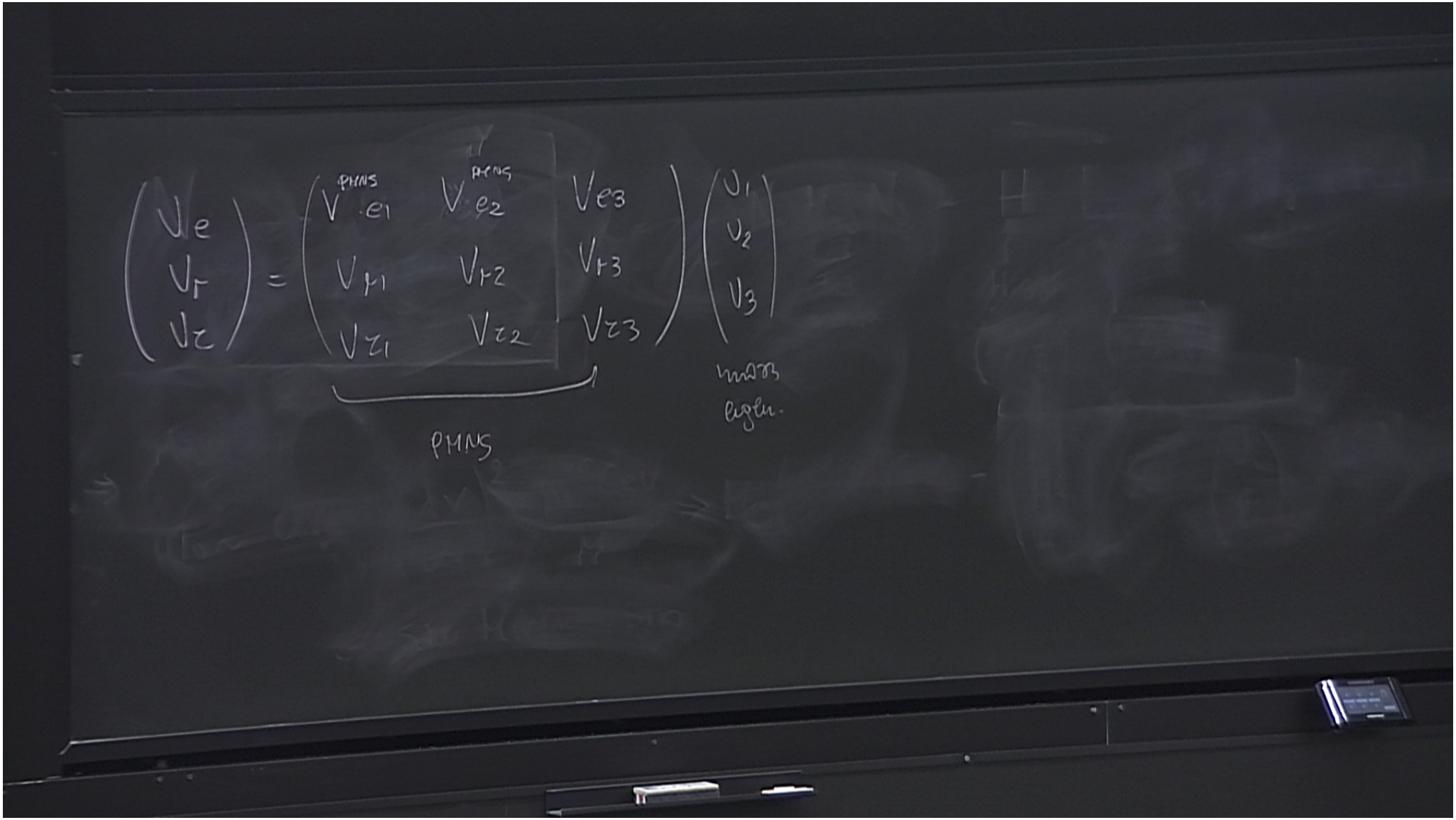
2. $- H L_i (\lambda e)_{ij} e_j^c \Rightarrow - \frac{v}{\sqrt{2}} e_L^i (\lambda e)_{ij} e_j^c$

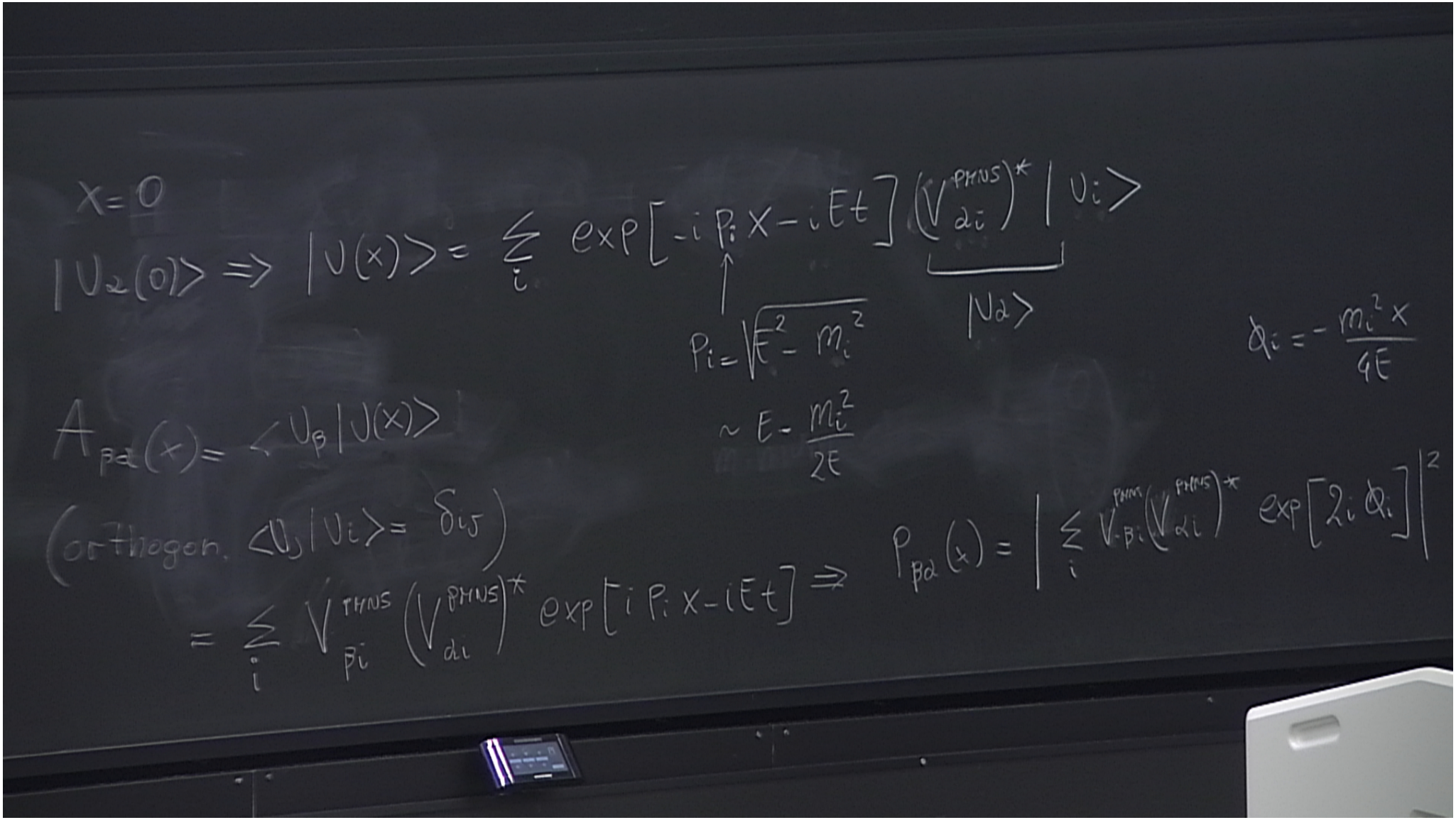
1. $U^T \left[ \frac{v^2}{4} \lambda_{ij} (M^{-1})_{JK} (\lambda^T)_{Ke} \right] U$ = diagonal : neutrinos

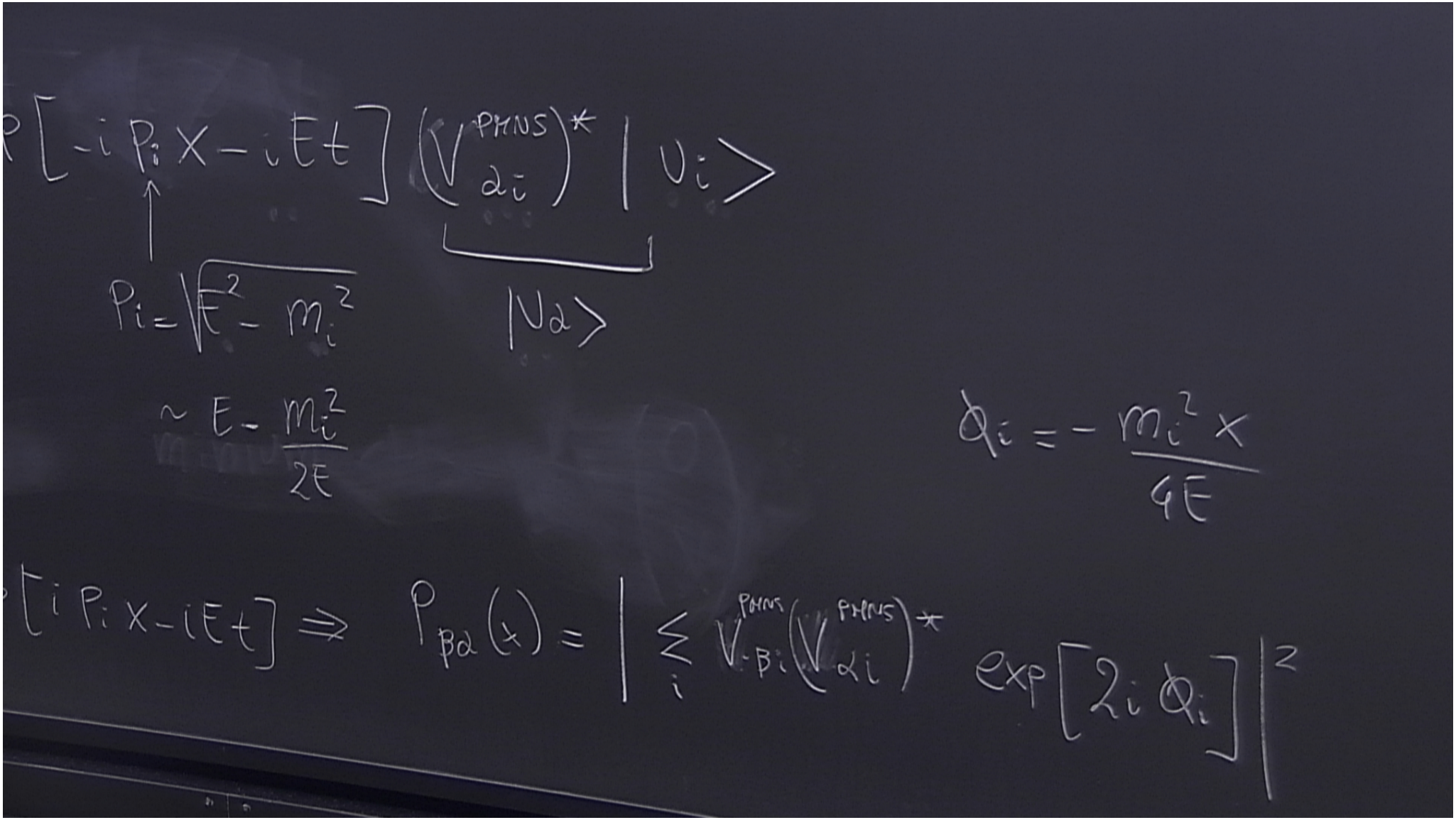
2. $-H L_i (\lambda_e)_{ij} e_J^c \Rightarrow -\frac{v}{\sqrt{2}} e_L^i (\lambda_e)_{ij} e_J^c \Rightarrow V_L : V_L^T (\lambda_e) V_R$ = diagonal

$V_R$

$\Rightarrow \frac{g}{\sqrt{2}} W_\mu^\pm U_L \underbrace{V_{PMNS}} \gamma_\mu e_L$


PMNS
PMNS
PMNS
massa eigen.


x = 0
$|\nu_2(0)\rangle \Rightarrow |\nu(x)\rangle = \sum_i \exp[-i p_i x - i E t] \underbrace{(V^{PMNS}_{\alpha i})^* |\nu_i\rangle}_{|\nu_\alpha\rangle}$
$p_i = \sqrt{E^2 - m_i^2}$
$\sim E - \frac{m_i^2}{2E}$
$\phi_i = -\frac{m_i^2 x}{4E}$
$A_{\beta\alpha}(x) = \langle \nu_\beta | \nu(x) \rangle$
(orthogon. $\langle \nu_j | \nu_i \rangle = \delta_{ij}$)
$= \sum_i V^{PMNS}_{\beta i} (V^{PMNS}_{\alpha i})^* \exp[i p_i x - i E t] \Rightarrow P_{\beta\alpha}(t) = \left| \sum_i V^{PMNS}_{\beta i} (V^{PMNS}_{\alpha i})^* \exp[2i\phi_i] \right|^2$


$\exp[-i p_i x - i E t]\,(V^{PMNS}_{\alpha i})^*\,|\nu_i\rangle$
$p_i = \sqrt{E^2 - m_i^2}$
$|N_\alpha\rangle$
$\sim E - \frac{m_i^2}{2E}$
$\phi_i = -\frac{m_i^2 x}{4E}$
$\exp[i p_i x - i E t] \Rightarrow P_{\beta\alpha}(t) = \left| \sum_i V^{PMNS}_{\beta i} (V^{PMNS}_{\alpha i})^* \exp[2i\phi_i] \right|^2$

$$\phi_i = -\frac{m_i^2 x}{4E}$$

$$P_{\alpha\beta}(x) = \left| \sum_i V_{\beta i}^{PMNS} \left(V_{\alpha i}^{PMNS}\right)^* \exp\left[2i\phi_i\right] \right|^2$$

$$\exp[2i\phi_i] \sim 1 - 2\sin^2(\phi_i) + i\sin(2\phi_i)$$

$$P_{\beta\alpha}(x) = \sum_{ij} J_{ij}^{\beta\alpha} \left(1 - 2\sin^2(\phi_{ij}) + i\sin(2\phi_{ij})\right)$$

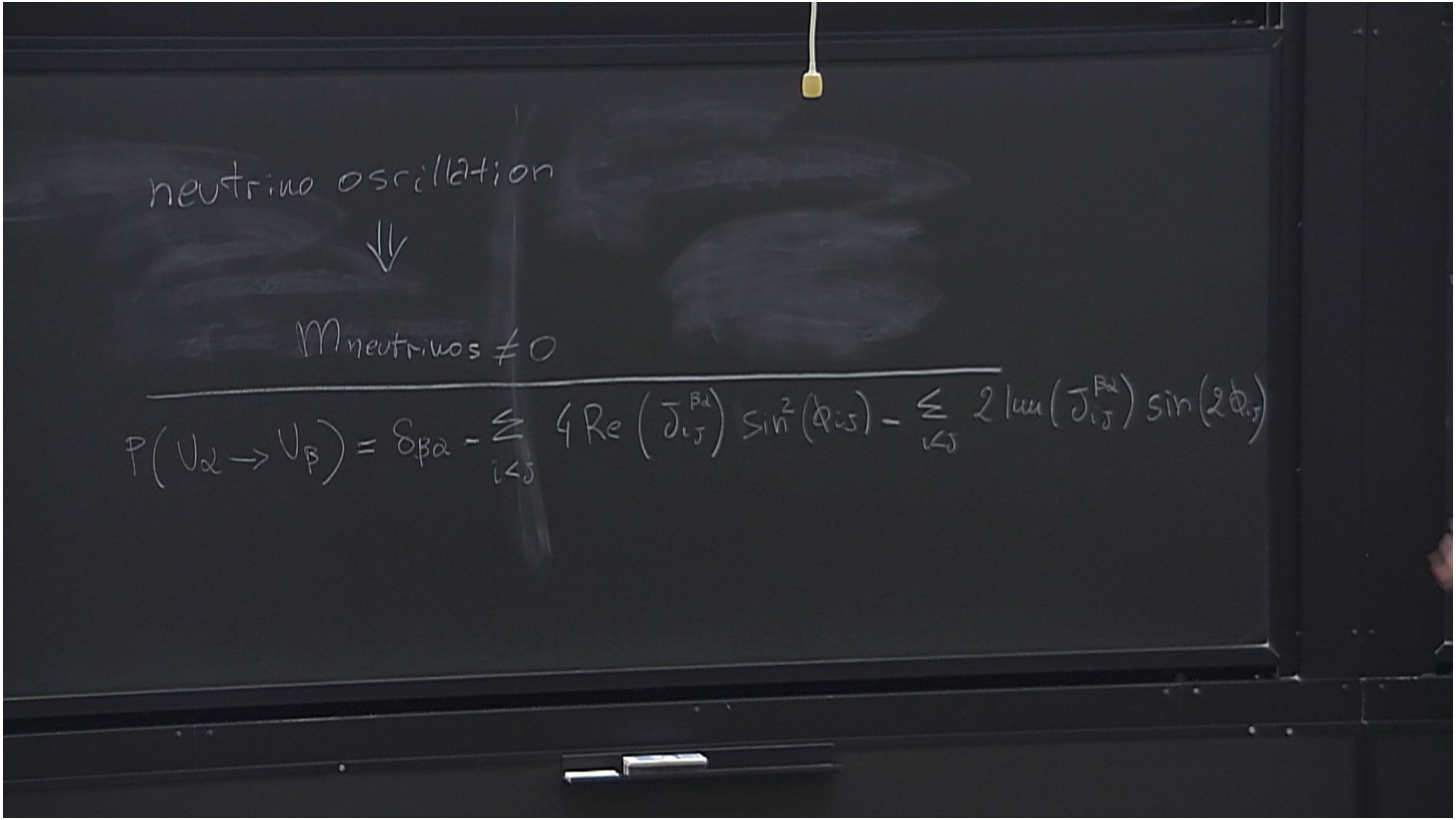
$$\begin{cases} J_{ij}^{\beta\alpha} = \left(V_{\alpha i}^{PMNS}\right)^* V_{\beta i}^{PMNS} \left(V_{\beta j}^{PMNS}\right)^* V_{\alpha j}^{PMNS} \\ \phi_{ij} = \phi_i - \phi_j = \left(m_i^2 - m_j^2\right)\dfrac{x}{4E} \end{cases}$$

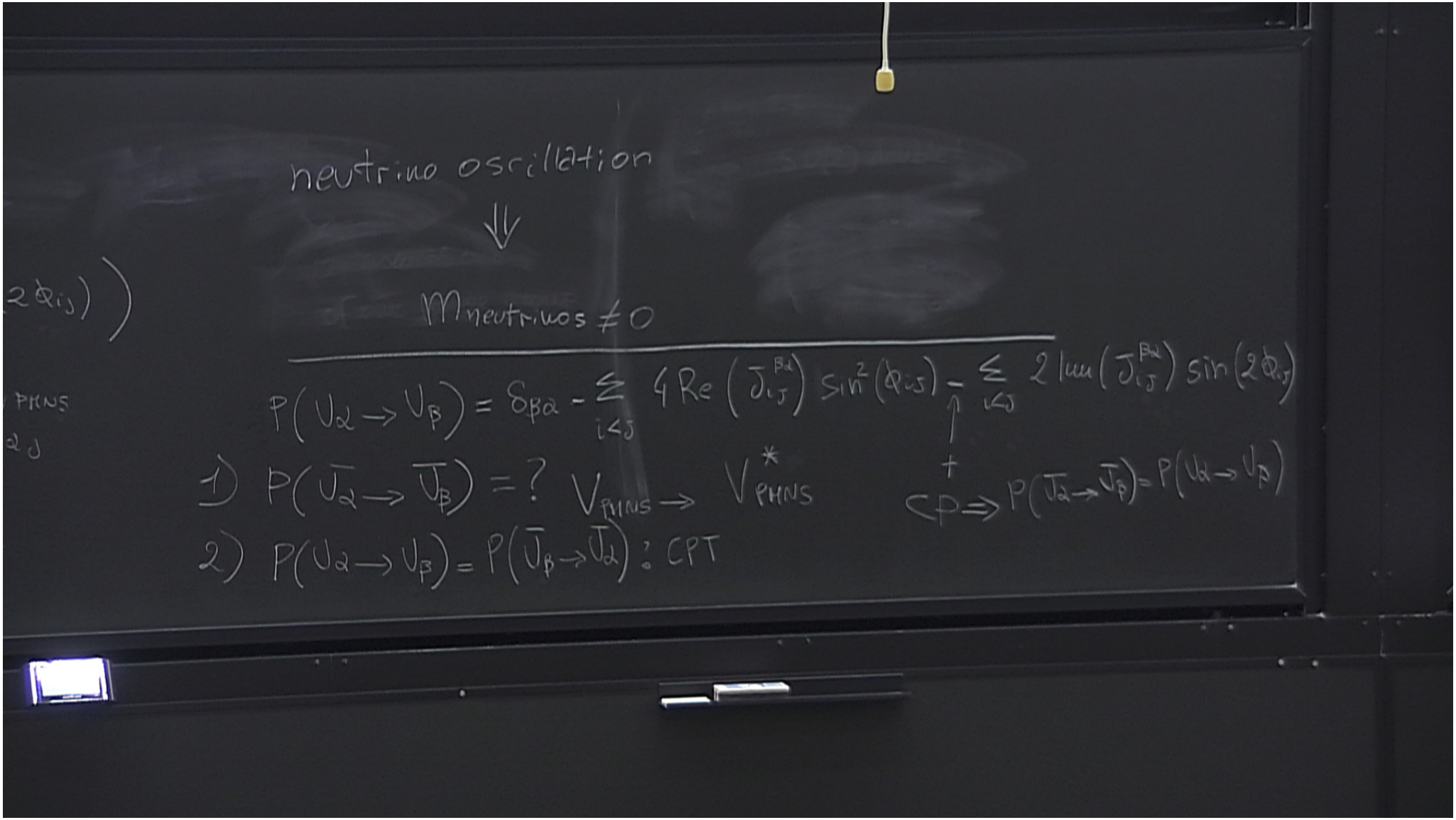
$$\exp[2i\phi_i] \sim 1 - 2\sin^2(\phi_i) + i\sin(2\phi_i)$$

$$P_{\alpha\beta}(x) = \sum_{ij} J_{ij}^{\alpha\beta}\left(1 - 2\sin^2(\phi_{ij}) + i\sin(2\phi_{ij})\right)$$

$$\alpha \neq \beta$$

$$\begin{cases} J_{ij}^{\alpha\beta} = \left(V_{\alpha i}^{PMNS}\right)^* V_{\beta i}^{PMNS} \left(V_{\beta j}^{PMNS}\right)^* V_{\alpha j}^{PMNS} \\ \phi_{ij} = \phi_i - \phi_j = \left(m_i^2 - m_j^2\right)\dfrac{x}{4E} \end{cases}$$


neutrino oscillation
$m_{neutrinos} \neq 0$
$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} - \sum_{i<j} 4\,\mathrm{Re}\left(J_{ij}^{\beta\alpha}\right) \sin^2(\phi_{ij}) - \sum_{i<j} 2\,\mathrm{Im}\left(J_{ij}^{\beta\alpha}\right) \sin(2\phi_{ij})$


neutrino oscillation
$m_{neutrinos} \neq 0$
$P(\nu_\alpha \to \nu_\beta) = \delta_{\beta\alpha} - \sum_{i<j} 4\,\mathrm{Re}\left(J_{ij}^{\beta\alpha}\right) \sin^2(\phi_{ij}) - \sum_{i<j} 2\,\mathrm{Im}\left(J_{ij}^{\beta\alpha}\right) \sin(2\phi_{ij})$
1) $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = ?$ $V_{PMNS} \to V_{PMNS}^{*}$
2) $P(\nu_\alpha \to \nu_\beta) = P(\bar{\nu}_\beta \to \bar{\nu}_\alpha)$ : CPT
$\Leftarrow CP \Rightarrow P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = P(\nu_\alpha \to \nu_\beta)$