

Title: Standard Model-12

Date: Jan 20, 2015 09:00 AM

URL: <http://pirsa.org/15010015>

Abstract:

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + i \bar{\Psi}_3 \not{D} \Psi_3 +$$

↑ SU(3) index, a=1,2,...,8

↑ gluons

3

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + i \bar{\psi}_j \not{D} \psi_j + m \bar{\psi}_j \psi_j$$

$\uparrow$  SU(3) index,  $a=1,2,\dots,8$   
 $\uparrow$  gluons  
 $\downarrow$   $j=1,2,3$   
 $\downarrow$  flavor=1,2,3

QCD is flavor blind

$$\not{D} = \not{D}_\mu \gamma^\mu$$

$\rightarrow$  GAMMA MATRIX

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QCD is flavor blind

1)  $\not{D} = \not{D}_F \gamma^r$

$\hookrightarrow$  GAMMA MATRIX

$$D_F = \partial_F + i g_s A_F^a t^a$$

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + i \bar{\Psi}_j \not{D} \Psi_j + m \bar{\Psi}_j \Psi_j$$

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QCD is flavor blind

1)  $\not{D} = \not{D}_F \gamma^r$

$\hookrightarrow$  GAMMA MATRIX

$$D_F = \partial_F + i \underbrace{\left(\frac{g_s}{2}\right)}_{\text{gluons}} \underbrace{A_F^a}_{a=1,\dots,8} \gamma^a$$

$$\mathcal{L} = -\frac{1}{4} (F_{\mu\nu}^a)^2 + i \bar{\psi}_j \not{D} \psi_j + m \bar{\psi}_j \psi_j$$

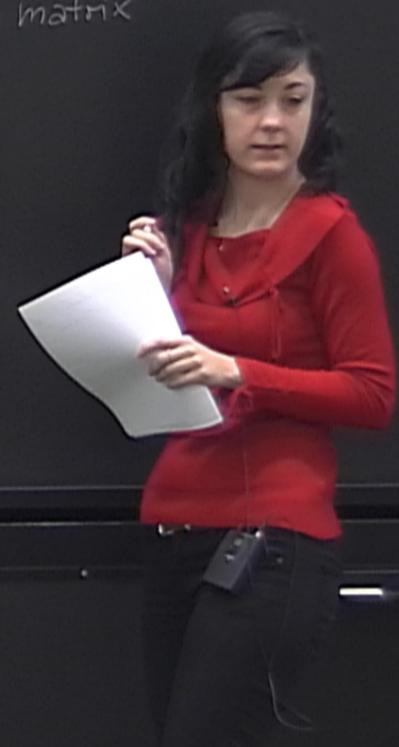
$\uparrow$  SU(3) index,  $a=1,2,\dots,8$   
 $\uparrow$  gluons  
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QCD is flavor blind  
 $t_a =$  Gell-Mann matrix  
 $t_a = \frac{1}{2} \lambda_a$

1)  $\not{D} = \not{D}_F \gamma^\mu$   
 $\hookrightarrow$  GAMMA MATRIX

$$D_F = \partial_F + i \left( \frac{g_s}{2} \right) A_F^a t_a \rightarrow \text{generators of SU(3)}$$

$\underbrace{\hspace{10em}}_{\text{gluons}}$   
 $a=1,\dots,8$



$\bar{\psi}, \psi$

QCD is flavor blind

$t_a$  = Gell-Mann matrix

$$t_a = \frac{1}{2} \lambda_a$$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

→ Normalized traceless matrix

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

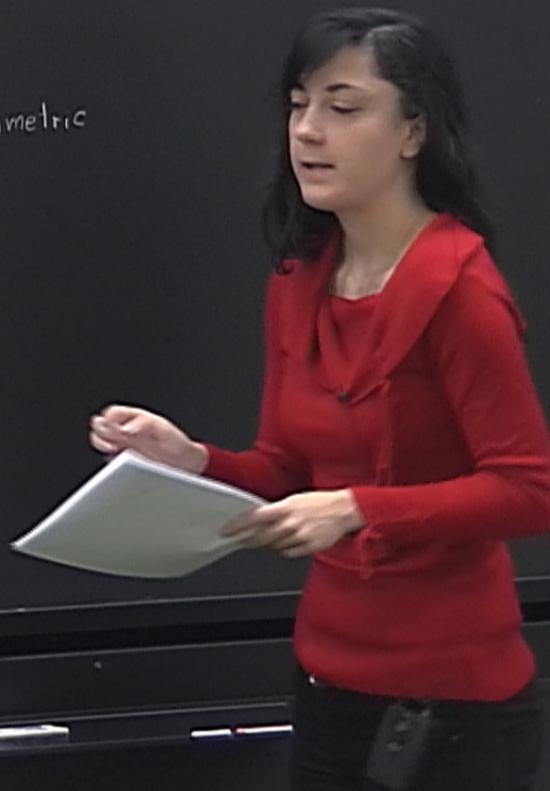
$$\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$[T_a, T_b] = i f_{abc} T_c$$

↑ structure  
constant,  
totally ant. symmetric

$$\{T_a, T_b\} = \frac{1}{3} \delta_{ab} + d_{abc} T_c$$

↑ totally  
symm.



$$[T_a, T_b] = i f_{abc} T_c$$

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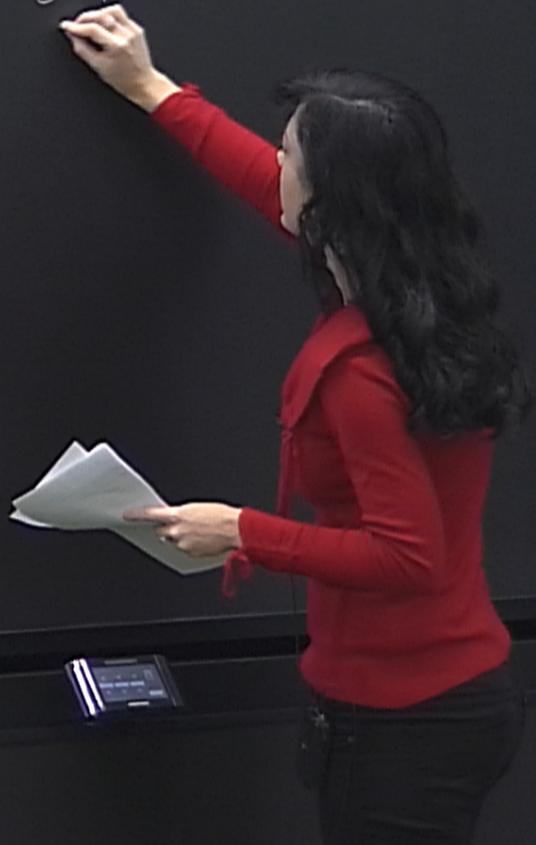
$$\{T_a, T_b\} = \frac{1}{3} \delta_{ab} + d_{abc} T_c$$

↑ totally symmetric

$$(F_{\mu\nu}^a)^2: F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s$$

$abc T_c$   
 structure constant,  
 totally ant. symmetric  
 $\sum_{abc} + dabc T_c$   
 totally symmetric.

$$(F_{\mu\nu}^a)^2: F_{\mu\nu}^a = \underbrace{\partial_\mu A_\nu^a - \partial_\nu A_\mu^a} + g_s f_{abc} A_\mu^b A_\nu^c$$

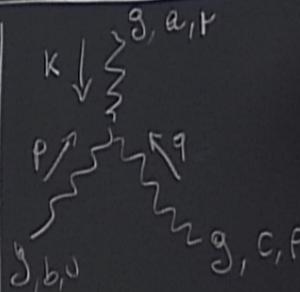


$$(F_{\mu\nu}^a)^2: \quad F_{\mu\nu}^a = \underbrace{\partial_\mu A_\nu^a - \partial_\nu A_\mu^a}_{\text{Abelian}} + \underbrace{g_s f^{abc} A_\mu^b A_\nu^c}_{\text{Non-Abelian}}$$

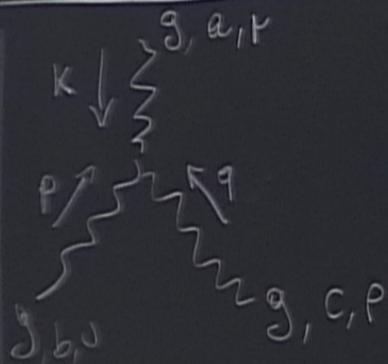
Feynman-rules for QCD

$$= i g_s \gamma_\mu t_a^{ij}$$

$a=1, \dots, 8$   
 $j, \bar{j}=1, 2, 3$



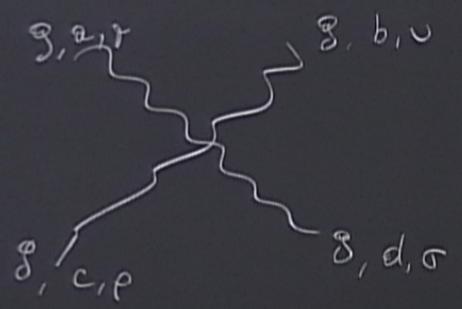
$$+ g_s f_{abc} A_\mu^b A_\nu^c$$



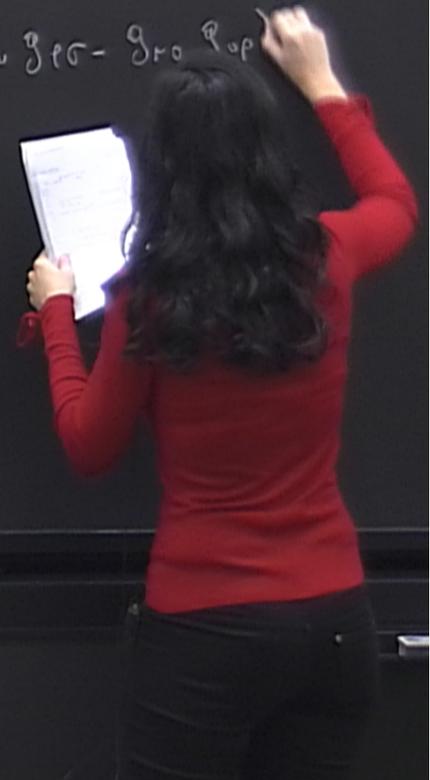
$$= g_s f_{abc} \left[ g_{\mu\nu} (k-p)_\rho + g_{\nu\rho} (p-q)_\mu + g_{\rho\mu} (q-k)_\nu \right]$$

grobuc  
 $\alpha = 1, \dots, 8$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \dots = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \dots \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$



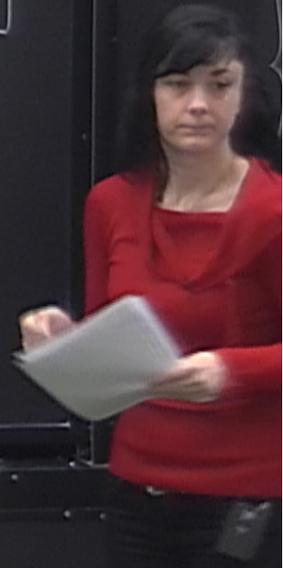
$$= i g_s^2 \left[ f_{abc} f_{cde} (g_{rp} g_{us} - g_{ra} g_{ue}) + f_{ace} f_{bde} (g_{ru} g_{ps} - g_{ro} g_{op}) \right]$$



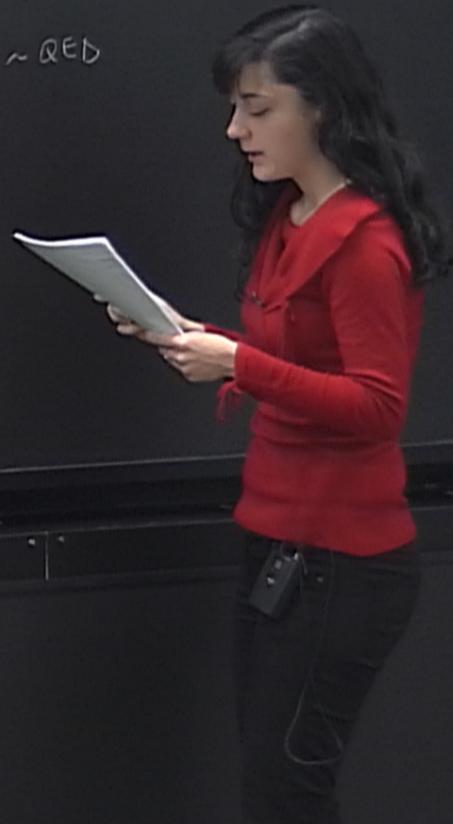
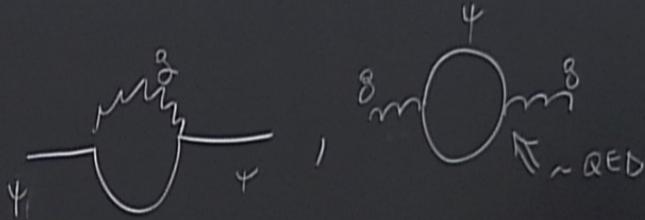
generators of SU(3)  
 $\lambda_1, \dots, \lambda_8$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

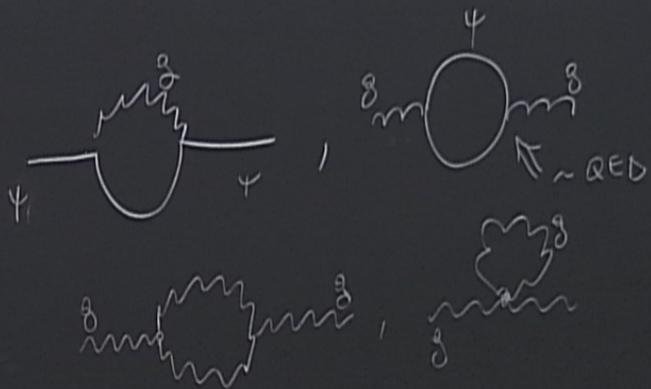
$$= i g_S^2 \left[ f_{abc} f_{cde} (g_{\mu\nu} g_{\sigma\rho} - g_{\nu\sigma} g_{\mu\rho}) + f_{ace} f_{bde} (g_{\mu\nu} g_{\sigma\rho} - g_{\nu\sigma} g_{\mu\rho}) + f_{ade} f_{bce} (g_{\mu\nu} g_{\sigma\rho} - g_{\nu\sigma} g_{\mu\rho}) \right]$$



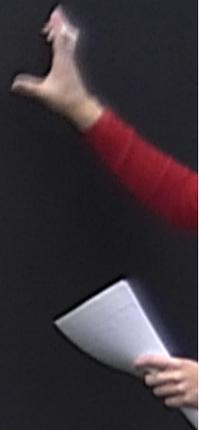
$$\left\{ \begin{aligned}
 A_{\mu\nu}^a &= Z_3^{-1/2} A_{\mu\nu}^a \\
 \psi_{\mu}^{\sigma} &= Z_2^{-1/2} \psi_{\mu}^{\sigma} \\
 g_S^{\mu} &= Z_3^{1/2} g_S^{\mu} \\
 m_{\mu} &= Z_m^{-1} m
 \end{aligned} \right.$$



$$\begin{cases}
 A_{\mu\nu}^a = Z_3^{-1/2} A_{\mu\nu}^a \\
 \psi^a = Z_2^{-1/2} \psi^a \\
 g_s^{\mu} = Z_3^{1/2} g_s^{\mu} \\
 M_{lc} = Z_m^{-1} m
 \end{cases}$$



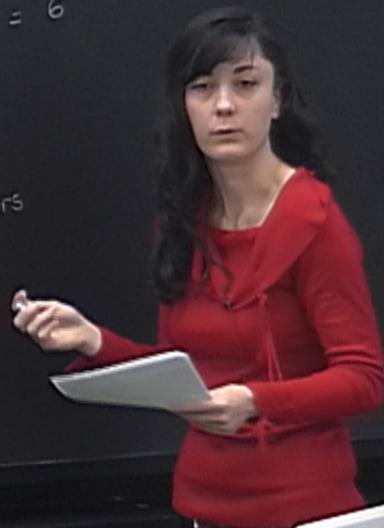
$$\Rightarrow \beta(g_s) = -\frac{g_s^3}{16\pi^2} \left( \frac{11}{3} N - \frac{2}{3} F \right)$$

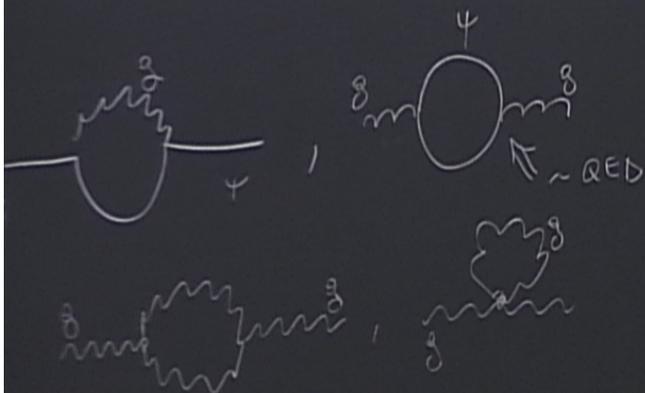




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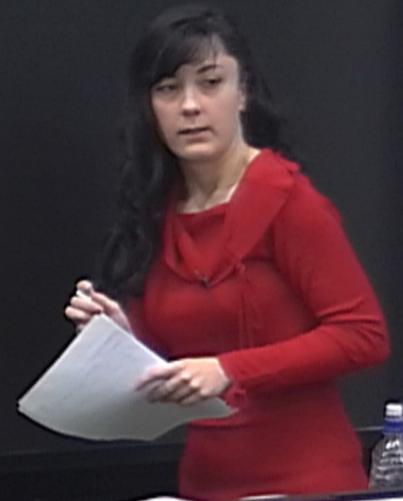
$N = \text{number of colors} = 3$   
 $F = \text{f. n. of flavors} = 6$





$$\Rightarrow \beta(g_s) = -\frac{g_s^3}{16\pi^2} \left( \frac{11}{3} N - \frac{2}{3} F \right) = -\frac{7}{16\pi^2} g_s^3$$

$N = \text{number of colors} = 3$   
 $F = \text{f. n. of flavors} = 6$



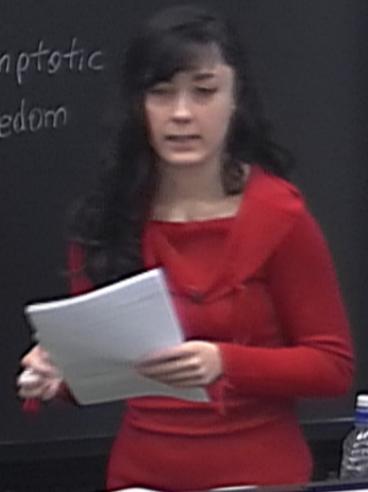
$\Rightarrow \beta(g_s) = -\frac{g_s^3}{16\pi^2} \left( \frac{11}{3} N - \frac{2}{3} F \right) = -\frac{7}{16\pi^2} g_s^3 \Rightarrow \text{asymptotic freedom}$

$N = \text{number of colors} = 3$   
 $F = \text{f. n. of flavors} = 6$



$\Rightarrow \beta(g_s) = -\frac{g_s^3}{16\pi^2} \left( \frac{11}{3} N - \frac{2}{3} F \right) = -\frac{7}{16\pi^2} (g_s^3) \Rightarrow$  asymptotic freedom

$N = \text{number of colors} = 3$   
 $F = \text{f. n. of flavors} = 6$



$\alpha = 1, \dots, 8$

$|100\rangle$

$|i00\rangle$

$|010\rangle$

$\underbrace{sd} \rightarrow \begin{cases} K_0 = \bar{s}d \\ \bar{K}_0 = s\bar{d} \end{cases} \Rightarrow CP?$

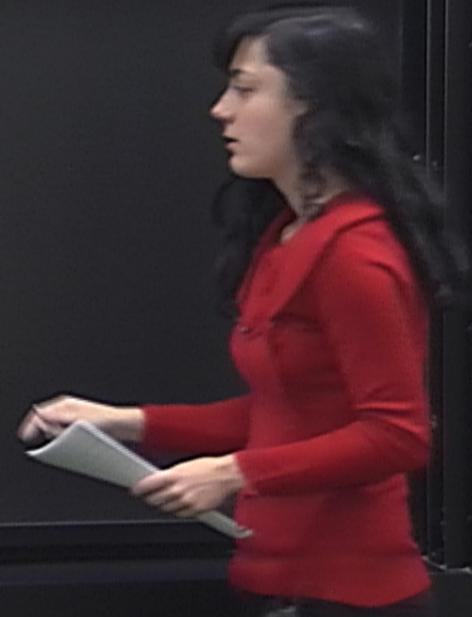
Parity

$$\begin{cases} CP|K_0\rangle = -|\bar{K}_0\rangle \\ CP|\bar{K}_0\rangle = -|K_0\rangle \end{cases} \Rightarrow$$

$$K_1 = \frac{1}{\sqrt{2}} (|K_0\rangle + |\bar{K}_0\rangle)$$

$\bar{s}d$   
 $s\bar{d} \Rightarrow CP?$

$$\begin{cases} CP|K_0\rangle = -|\bar{K}_0\rangle \\ CP|\bar{K}_0\rangle = -|K_0\rangle \end{cases} \Rightarrow \begin{cases} K_1 = \frac{1}{\sqrt{2}}(K_0 - \bar{K}_0) \\ K_2 = \frac{1}{\sqrt{2}}(K_0 + \bar{K}_0) \end{cases} \Rightarrow \begin{cases} CP|K_1\rangle = |K_1\rangle \\ CP|K_2\rangle = -|K_2\rangle \end{cases}$$

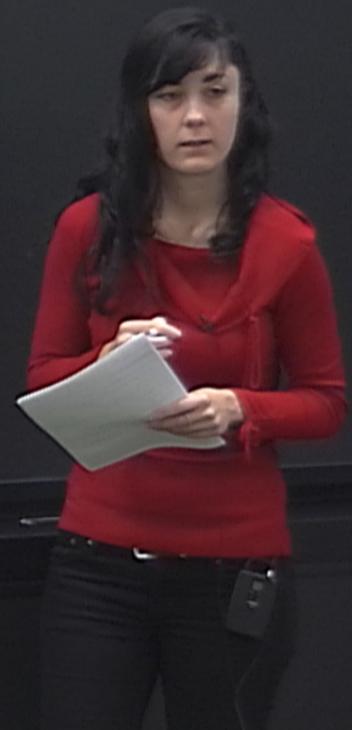


$\bar{s}d$   
 $s\bar{d} \Rightarrow CP?$

$$\begin{cases} CP|K_0\rangle = -|\bar{K}_0\rangle \\ CP|\bar{K}_0\rangle = -|K_0\rangle \end{cases} \Rightarrow \begin{cases} K_1 = \frac{1}{\sqrt{2}}(K_0 - \bar{K}_0) \\ K_2 = \frac{1}{\sqrt{2}}(K_0 + \bar{K}_0) \end{cases} \Rightarrow \begin{cases} CP|K_1\rangle = |K_1\rangle \\ CP|K_2\rangle = -|K_2\rangle \end{cases} \Rightarrow \text{physical?}$$

$$|K_0(t)\rangle = |K_0(0)\rangle \exp[-i H t]$$

$H = M - i \frac{\Gamma}{2}$



$$\begin{aligned}
 & M - i\frac{\Gamma}{2} \\
 & = \hat{H} |\psi(t)\rangle \\
 & \quad \downarrow \hat{H} - i\frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i\frac{\Gamma_{11}}{2} & M_{12} - i\frac{\Gamma_{12}}{2} \\ M_{21} + i\frac{\Gamma_{21}}{2} & M_{22} - i\frac{\Gamma_{22}}{2} \end{pmatrix}
 \end{aligned}$$

$$\text{Herm. : } M_{21} = M_{12}^*, \quad \Gamma_{21} = \Gamma_{12}^*$$

$$\text{CPT : } M_{11} = M_{22} \equiv M, \quad \Gamma_{11} = \Gamma_{22} \equiv \Gamma$$



$$|\psi_0(0)\rangle \exp[-i \hat{H} t]$$

$$\hat{H} = M - i \frac{\Gamma}{2}$$

$$i \frac{d|\psi(t)\rangle}{dt} = \hat{H} |\psi(t)\rangle$$

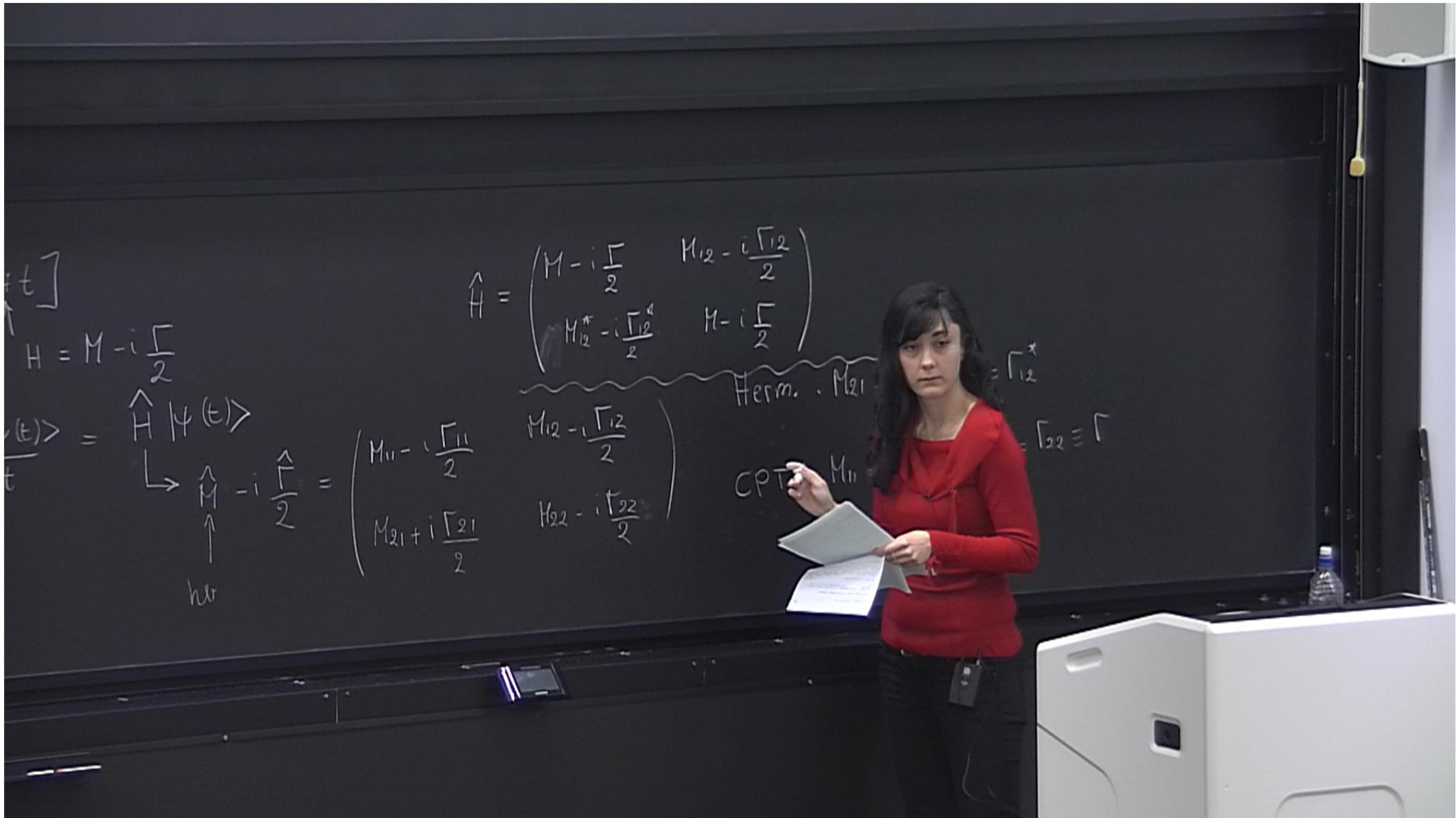
$$\hat{M} - i \frac{\hat{\Gamma}}{2}$$

↑  
Her

$$= \begin{pmatrix} M_{11} - i \frac{\Gamma_{11}}{2} & -i \frac{\Gamma_{12}}{2} \\ M_{21} + i \frac{\Gamma_{21}}{2} & M_{22} - i \frac{\Gamma_{22}}{2} \end{pmatrix}$$

Herm. :  $M_{21} = M_{12}^*$ ,  $\Gamma_{21} = \Gamma_{12}^*$

PT :  $M_{11} = M_{22} \equiv M$ ,  $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$



$$H = M - i \frac{\Gamma}{2}$$

$$\hat{H} = \begin{pmatrix} M - i \frac{\Gamma}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\ M_{12}^* - i \frac{\Gamma_{12}^*}{2} & M - i \frac{\Gamma}{2} \end{pmatrix}$$

Herm.  $M_{21} = M_{12}^*$ ,  $\Gamma_{21} = \Gamma_{12}^*$

$$\hat{H} = \begin{pmatrix} M_{11} - i \frac{\Gamma_{11}}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\ M_{21} + i \frac{\Gamma_{21}}{2} & M_{22} - i \frac{\Gamma_{22}}{2} \end{pmatrix}$$

CPT:  $M_{11} = M_{22} \equiv M$ ,  $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$