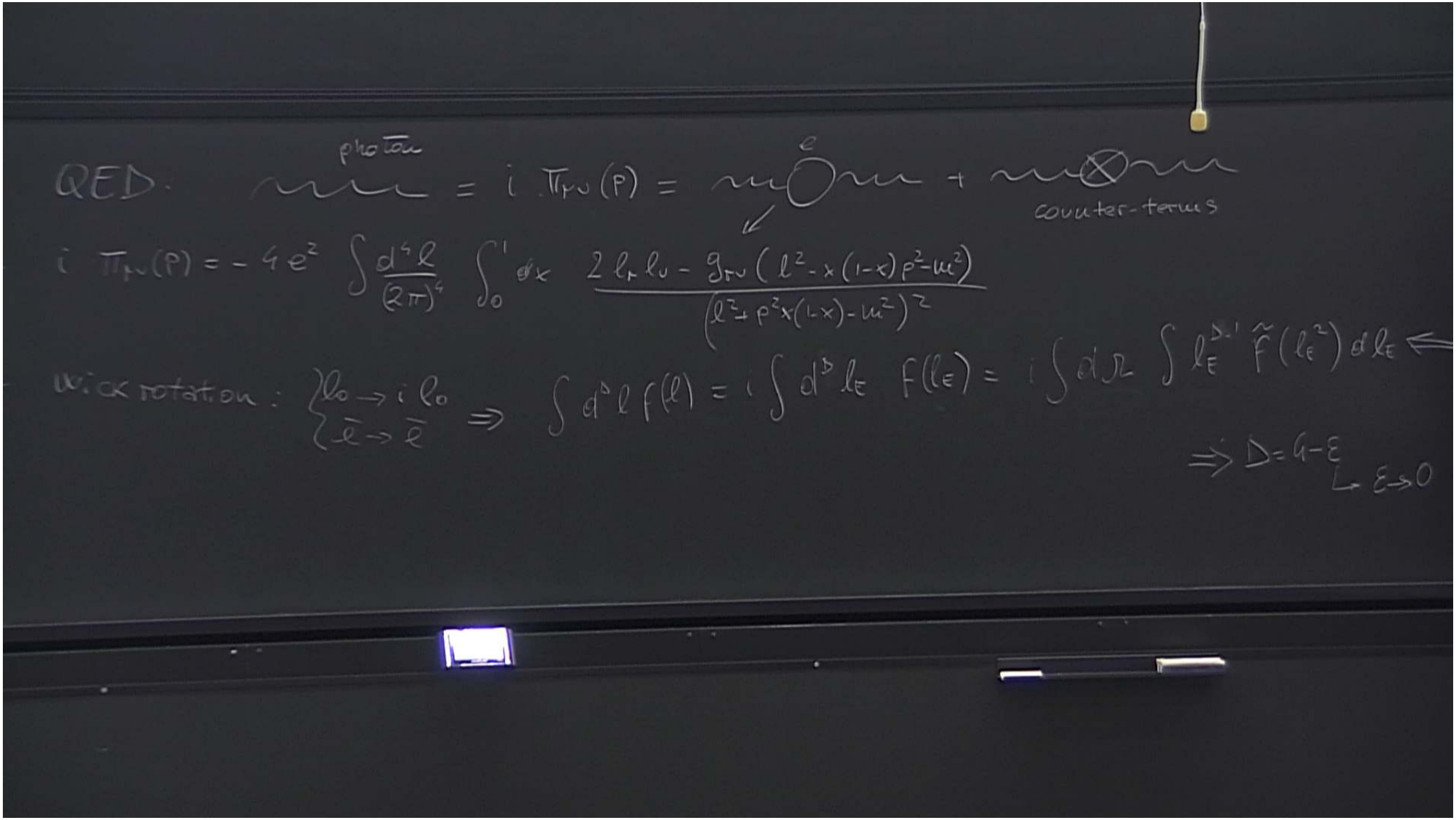
Title: Standard Model-11

Date: Jan 19, 2015  09:00 AM

URL: http://pirsa.org/15010014

Abstract:


QED:
photon
= i Π_μν(p) =
e
+
counter-terms
i Π_μν(p) = −4e² ∫ d⁴ℓ/(2π)⁴ ∫₀¹ dx [2ℓ_μ ℓ_ν − g_μν(ℓ² − x(1−x)p² − m²)] / (ℓ² + p²x(1−x) − m²)²
Wick rotation: ℓ₀ → iℓ₀, ℓ⃗ → ℓ⃗
⇒ ∫ d⁴ℓ f(ℓ) = i ∫ d^D ℓ_E f(ℓ_E) = i ∫ dΩ_D ∫ ℓ_E^{D−1} f̃(ℓ_E²) dℓ_E ⇐
⇒ D = 4 − ε, ε → 0

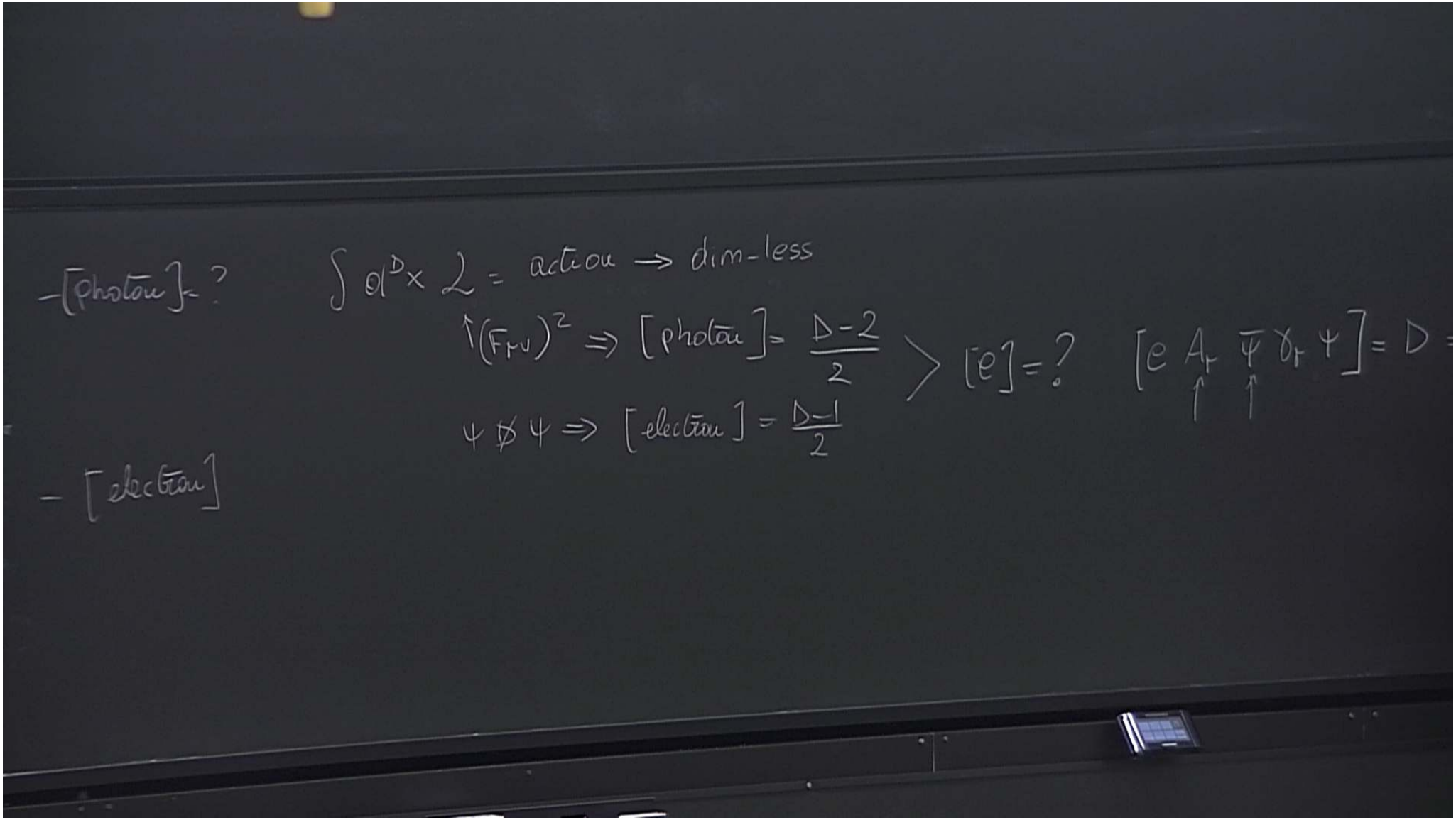
$= i\,\Pi_{\mu\nu}(p) =$ + 

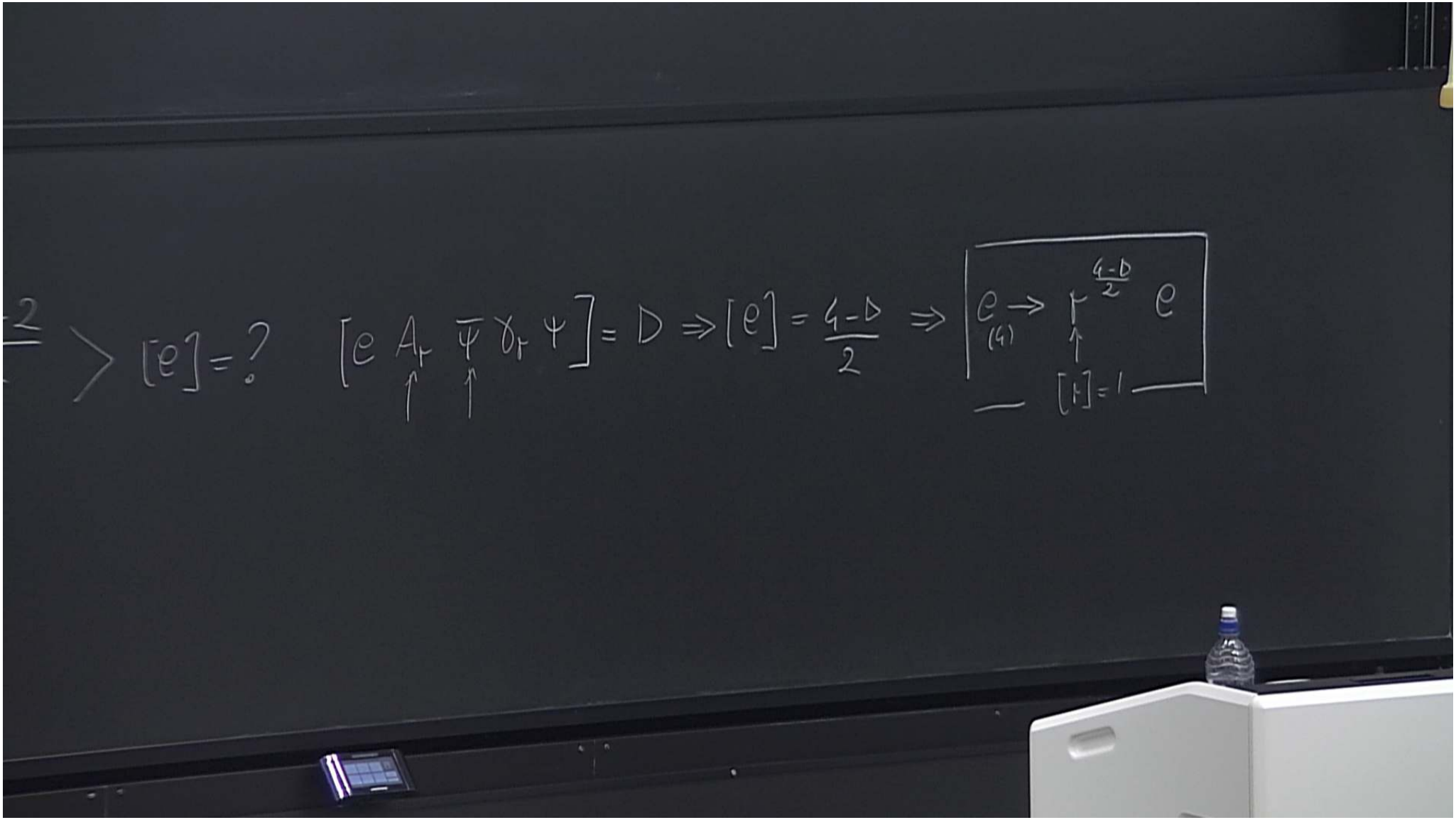
counter-terms

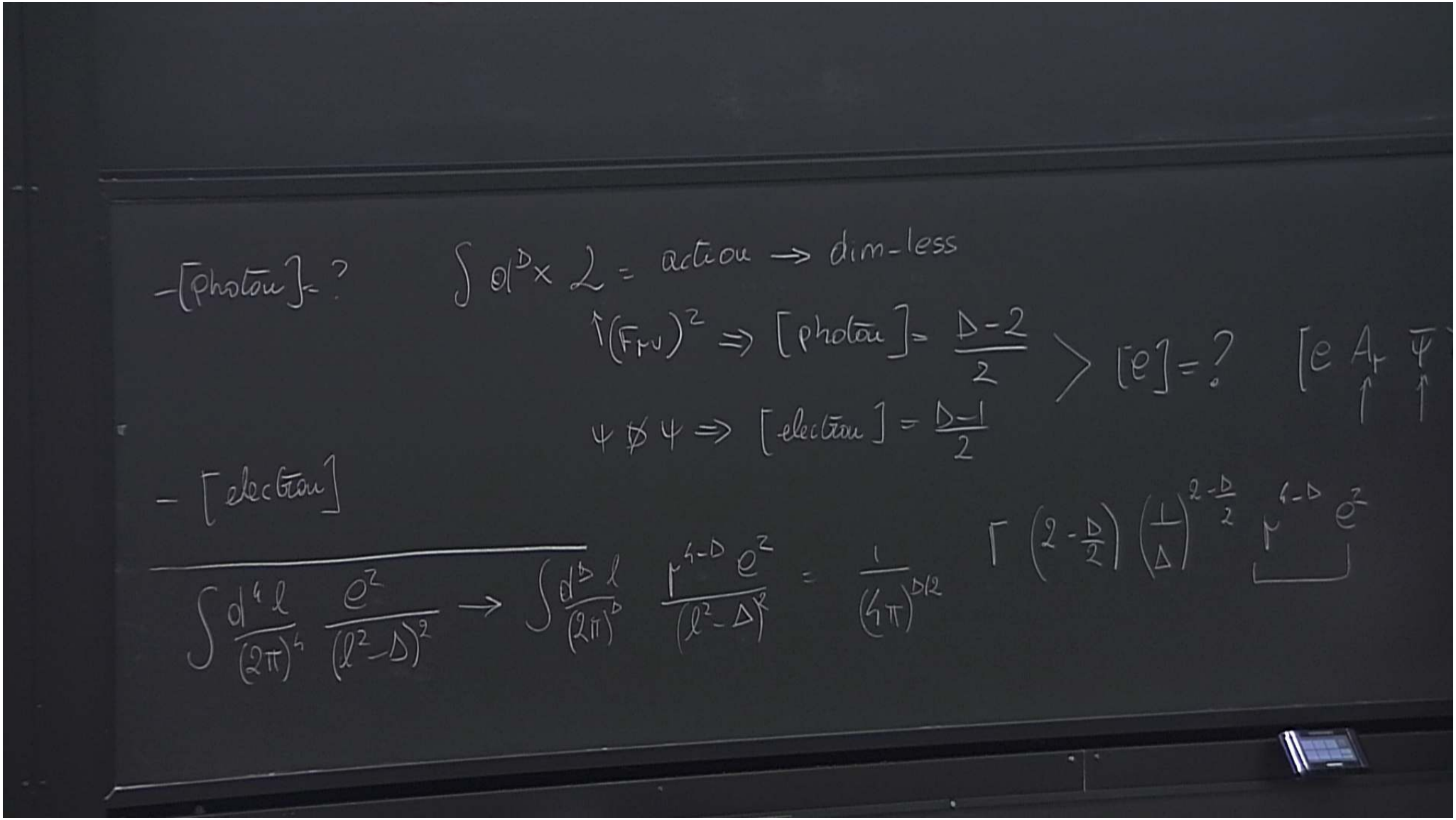
$$\int_0^1 dx \left[ \frac{2\ell_\mu \ell_\nu - g_{\mu\nu}\left(\ell^2 - x(1-x)p^2 - m^2\right)}{\left(\ell^2 + p^2 x(1-x) - m^2\right)^2} \right] \Leftarrow$$

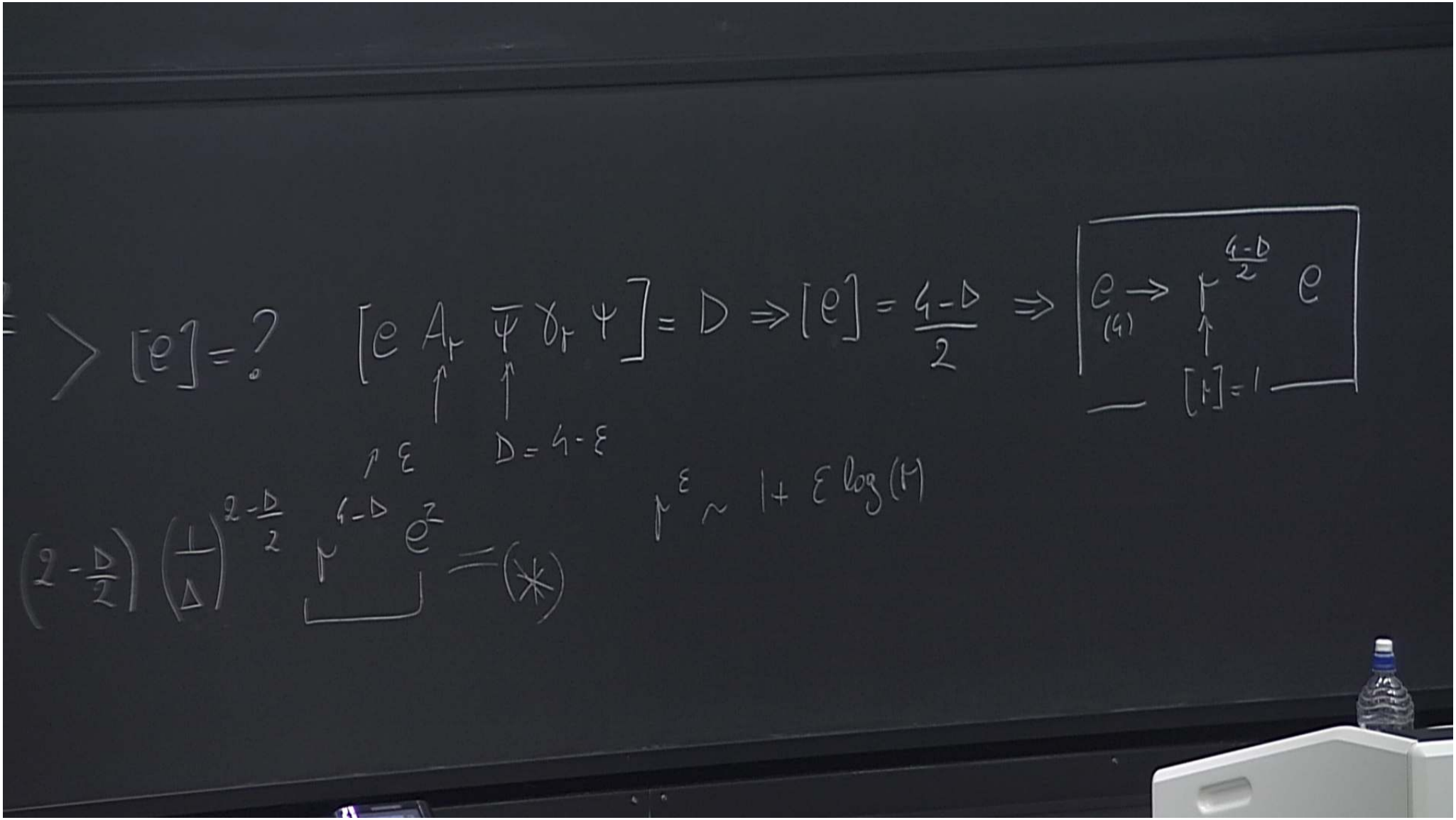
$$\Rightarrow \int d^D\ell\, f(\ell) = i \int d^D\ell_E\, f(\ell_E) = i \int d\Omega \int \ell_E^{D-1}\, \tilde{f}(\ell_E^2)\, d\ell_E \Leftarrow$$

$$\Rightarrow D = 4 - \varepsilon \quad \hookrightarrow \varepsilon \to 0$$


- [photon] = ?
- [electron]
$\int d^D x \, \mathcal{L}$ = action → dim-less
$\uparrow (F_{\mu\nu})^2 \Rightarrow [\text{photon}] = \frac{D-2}{2}$
$\psi \not{\partial} \psi \Rightarrow [\text{electron}] = \frac{D-1}{2}$
> [e] = ?
$[e A_\mu \bar{\psi} \gamma_\mu \psi] = D$


[e] = ?
[e A_μ ψ̄ γ_μ ψ] = D ⇒ [e] = (4-D)/2 ⇒
e_(4) → μ^((4-D)/2) e
[μ] = 1


-[photon]=?
$\int d^D x\, \mathcal{L}$ = action → dim-less
$\uparrow (F_{\mu\nu})^2 \Rightarrow [\text{photon}] = \frac{D-2}{2}$
$\psi \not{\partial} \psi \Rightarrow [\text{electron}] = \frac{D-1}{2}$
[e] = ?
$[e\, A_\mu\, \bar{\psi}\, \psi]$
- [electron]
$\int \frac{d^4 l}{(2\pi)^4} \frac{e^2}{(l^2-\Delta)^2} \to \int \frac{d^D l}{(2\pi)^D} \frac{\mu^{4-D} e^2}{(l^2-\Delta)^2} = \frac{1}{(4\pi)^{D/2}} \Gamma\left(2-\frac{D}{2}\right) \left(\frac{1}{\Delta}\right)^{2-\frac{D}{2}} \mu^{4-D} e^2$


[e] = ?
[e A_μ ψ̄ γ_μ ψ] = D ⇒ [e] = (4-D)/2 ⇒
e → μ^((4-D)/2) e
(4)
[μ] = 1
ε
D = 4 - ε
(2 - D/2) (1/Δ)^(2-D/2) μ^(4-D) e^2 = (*)
μ^ε ~ 1 + ε log(μ)

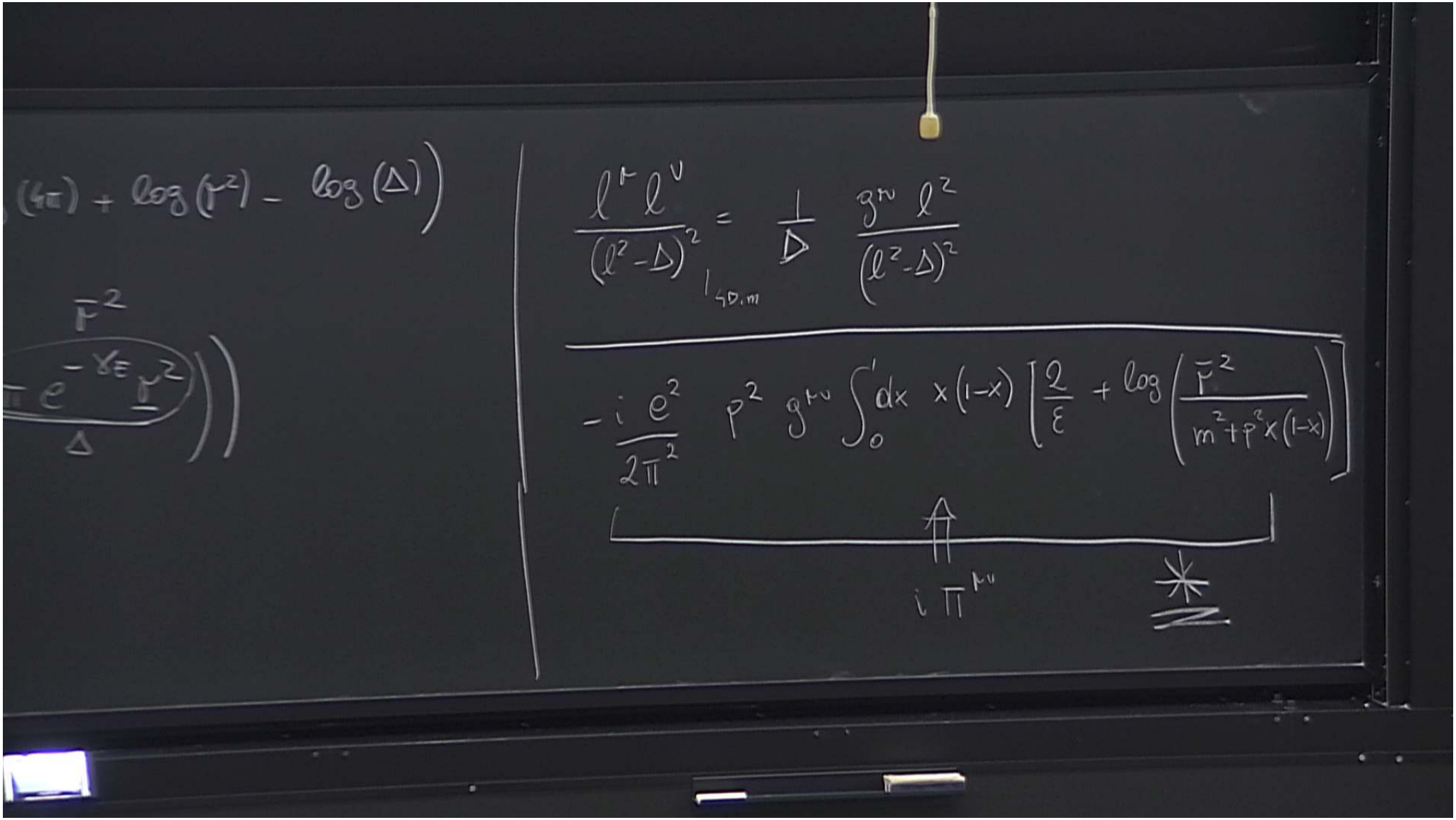
$$(\not{A}) \sim \frac{ie^2}{16\pi^2}\left(\frac{2}{\varepsilon} - \gamma_E + \log(4\pi) + \log(\mu^2) - \log(\Delta)\right)$$

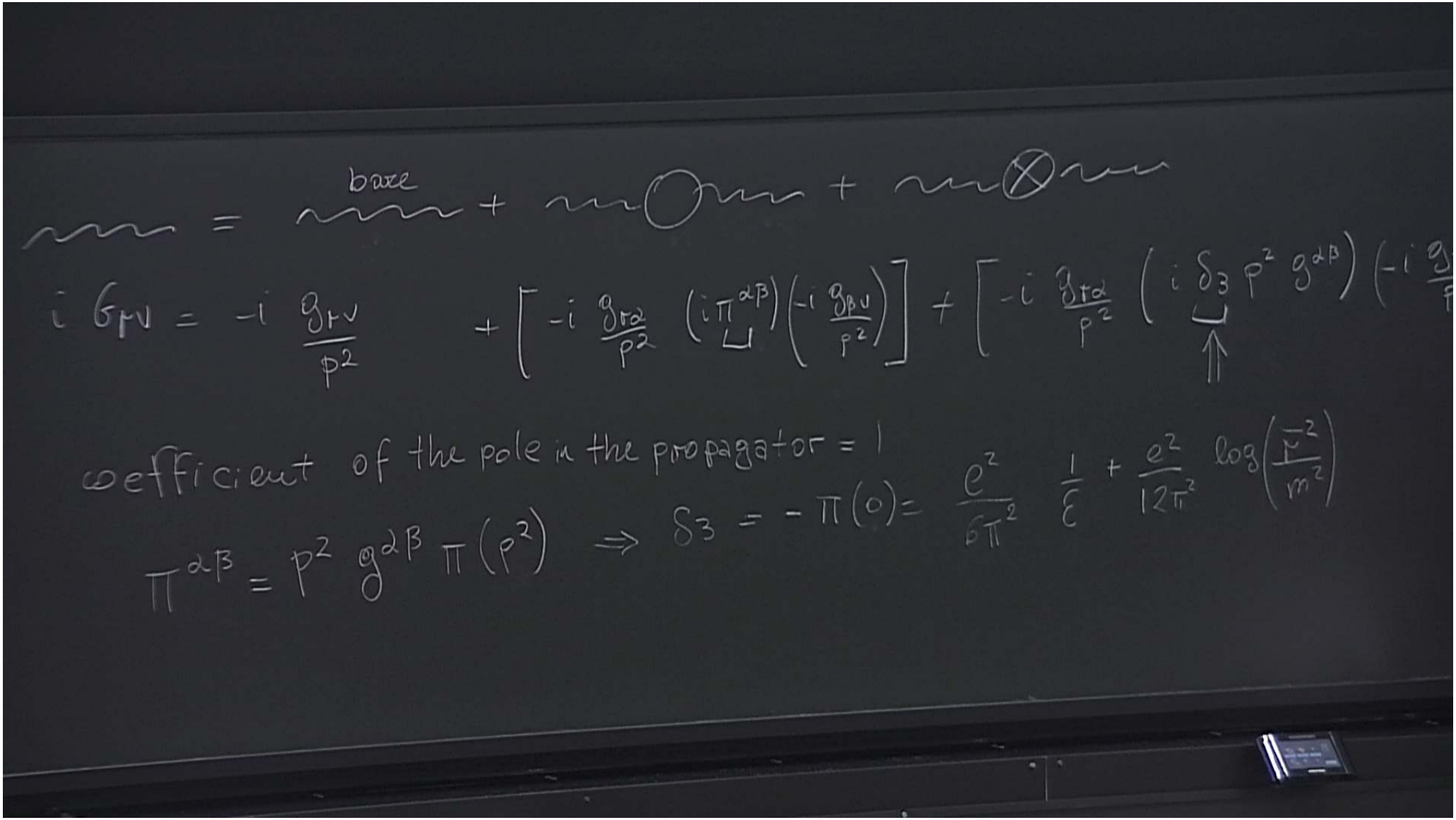
$$= \frac{ie^2}{16\pi^2}\left(\frac{2}{\varepsilon} + \log\left(\frac{4\pi e^{-\gamma_E}\bar{\mu}^2}{\Delta}\right)\right)$$

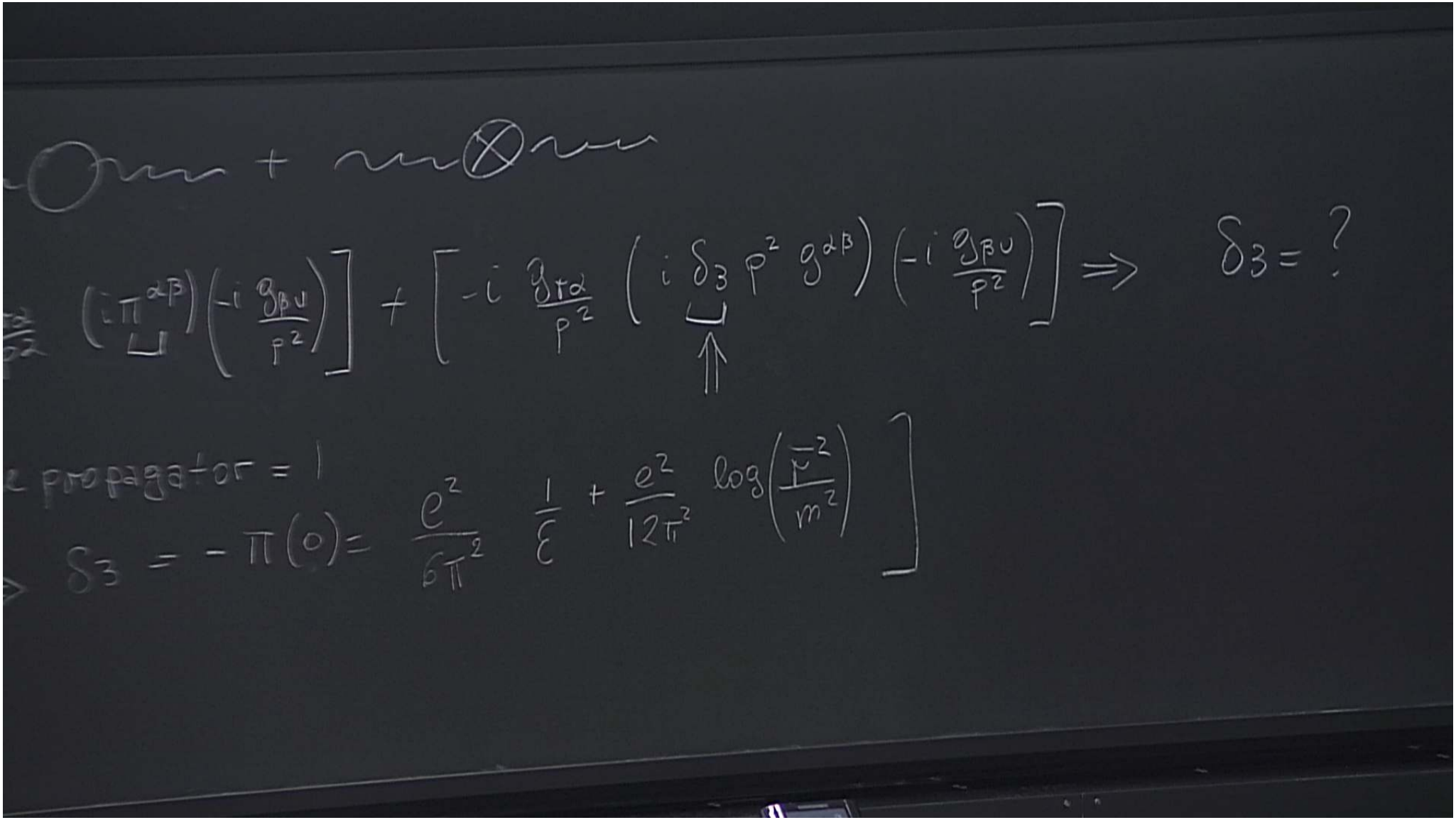
$$\frac{l^\mu l^\nu}{(l^2-\Delta)^2} \Big|_{4D.m} = \frac{1}{D}\,\frac{g^{\mu\nu}\, l^2}{(l^2-\Delta)^2}$$

$$-\frac{i e^2}{2\pi^2}\, p^2 g^{\mu\nu} \int_0^1 dx\; x(1-x)\left[\frac{2}{\varepsilon} + \log\left(\frac{\bar{\mu}^2}{m^2 + p^2 x(1-x)}\right)\right]$$

$$\ast$$


$(4\pi) + \log(\mu^2) - \log(\Delta)\Big)$
$\bar{\mu}^2$
$\pi\, e^{-\gamma_E} \frac{\mu^2}{\Delta}$
$\frac{l^\mu l^\nu}{(l^2-\Delta)^2} = \frac{1}{4D,m}\frac{g^{\mu\nu} l^2}{(l^2-\Delta)^2}$
$-\frac{i e^2}{2\pi^2} p^2 g^{\mu\nu} \int_0^1 dx\, x(1-x)\left[\frac{2}{\varepsilon} + \log\left(\frac{\bar{\mu}^2}{m^2 + p^2 x(1-x)}\right)\right]$
$i\Pi^{\mu\nu}$
$*$


bare
+
+
$i G_{\mu\nu} = -i \frac{g_{\mu\nu}}{p^2} + \left[ -i \frac{g_{\mu\alpha}}{p^2} \left( i \Pi^{\alpha\beta} \right) \left( -i \frac{g_{\beta\nu}}{p^2} \right) \right] + \left[ -i \frac{g_{\mu\alpha}}{p^2} \left( i \delta_3 p^2 g^{\alpha\beta} \right) \left( -i \frac{g}{p} \right. \right.$
coefficient of the pole in the propagator = 1
$\Pi^{\alpha\beta} = p^2 g^{\alpha\beta} \Pi(p^2) \Rightarrow \delta_3 = -\Pi(0) = \frac{e^2}{6\pi^2} \frac{1}{\epsilon} + \frac{e^2}{12\pi^2} \log\left( \frac{\bar{\mu}^2}{m^2} \right)$


+
$\left(i\Pi^{\alpha\beta}\right)\left(-i\frac{g_{\beta\nu}}{p^2}\right)\Big] + \Big[-i\frac{g_{\tau\alpha}}{p^2}\left(i\delta_3 p^2 g^{\alpha\beta}\right)\left(-i\frac{g_{\beta\nu}}{p^2}\right)\Big] \Rightarrow \quad \delta_3 = ?$
propagator = 1
$\delta_3 = -\Pi(0) = \frac{e^2}{6\pi^2}\left[\frac{1}{\varepsilon} + \frac{e^2}{12\pi^2}\log\left(\frac{\bar{\mu}^2}{m^2}\right)\right]$

$$\frac{d\left(Z_3^{1/2}\, r^{-\varepsilon/2}\right)}{d(\log r)} = -\frac{\varepsilon}{2} e_r + \beta(e_r) \qquad \Rightarrow \qquad \beta(e_r) = \frac{e_r}{2Z_3} \frac{dZ_3}{d(\log r)}$$

↑ Beta-function of QED

$$Z_3 = 1 + \sum_{k=1}^{\infty} \frac{g^k}{\varepsilon^k} Z_{3,k}(e_r)$$

$$e_0 \, Z_3^{1/2} Z_2 = e_r \, Z_1 \, r^{\varepsilon/2} \quad , \quad Z_1 = Z_2$$

$$\underbrace{e_r} = \underline{\underline{e_0}} \, Z_3^{1/2} \, r^{-\varepsilon/2} \implies \frac{d e_r}{d(\log r)} = e_0 \, \frac{d \, Z_3^{1/2} \, r^{-\varepsilon/2}}{d(\log r)} = -\frac{\varepsilon}{2} e_r$$

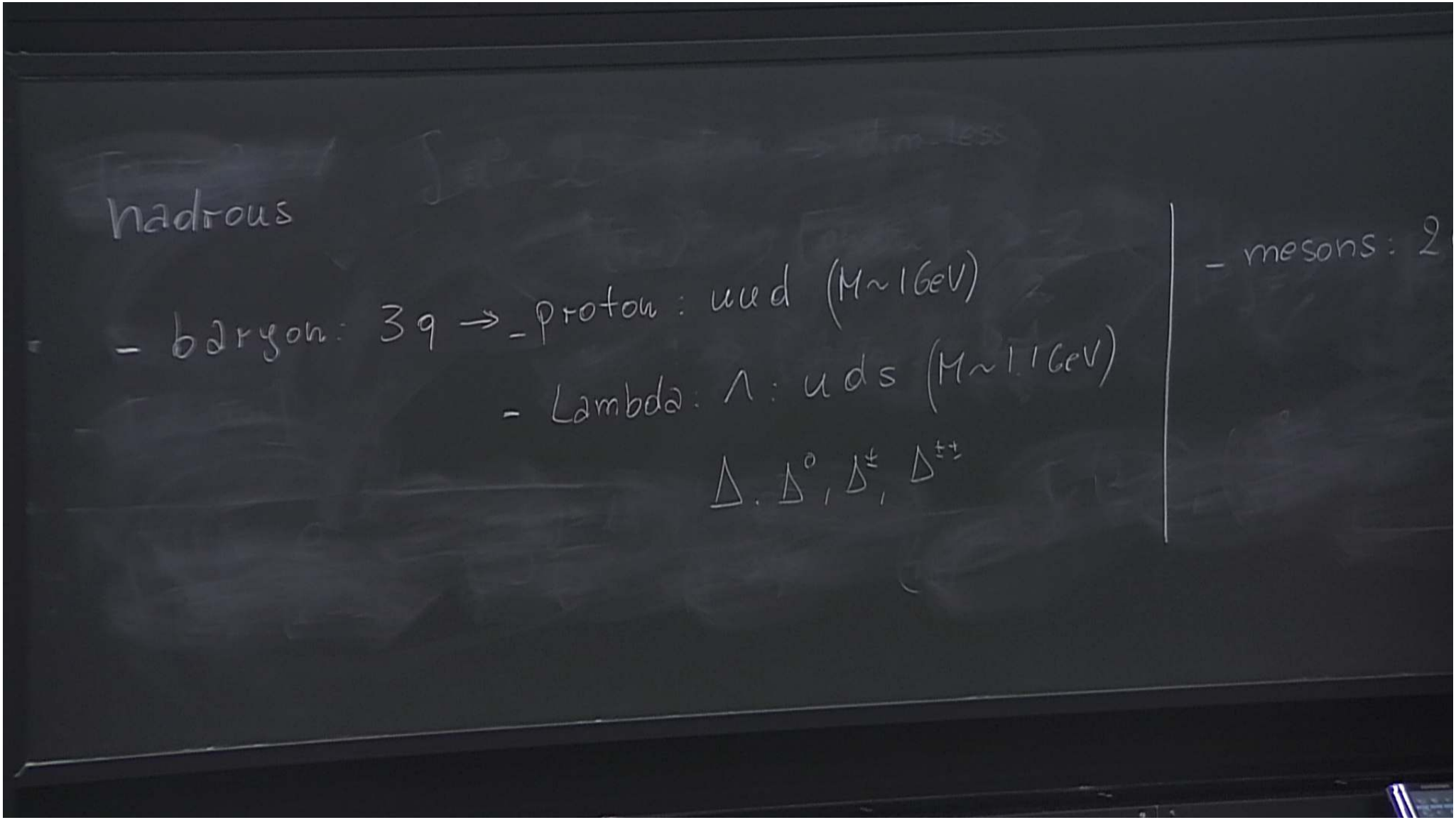
$$Z_3 = 1 + \underline{\underline{\delta_3}}$$

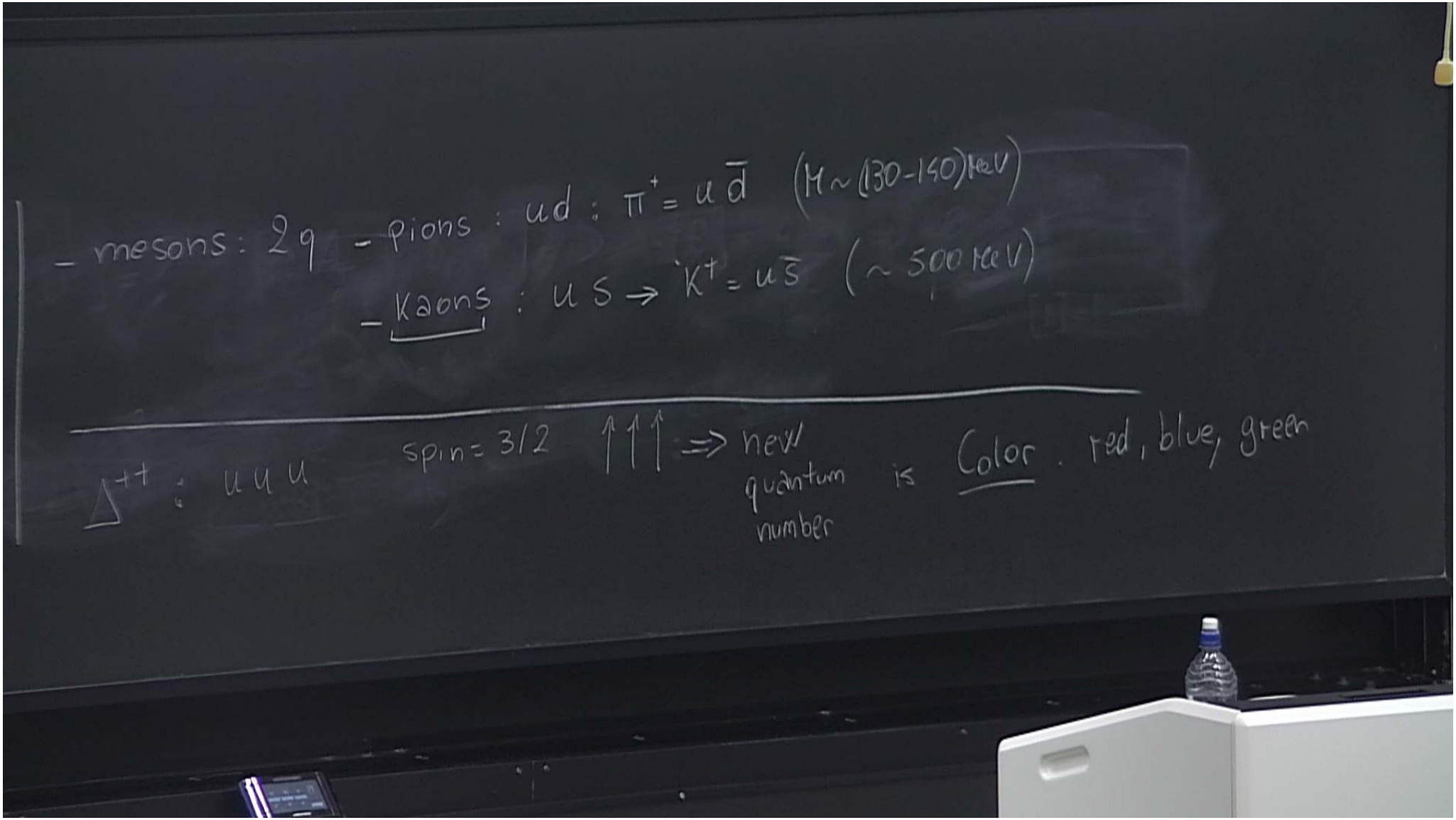
$$Z_3 = 1 + \sum_{k=1}^{\infty} \frac{2^k}{\varepsilon^k} Z$$

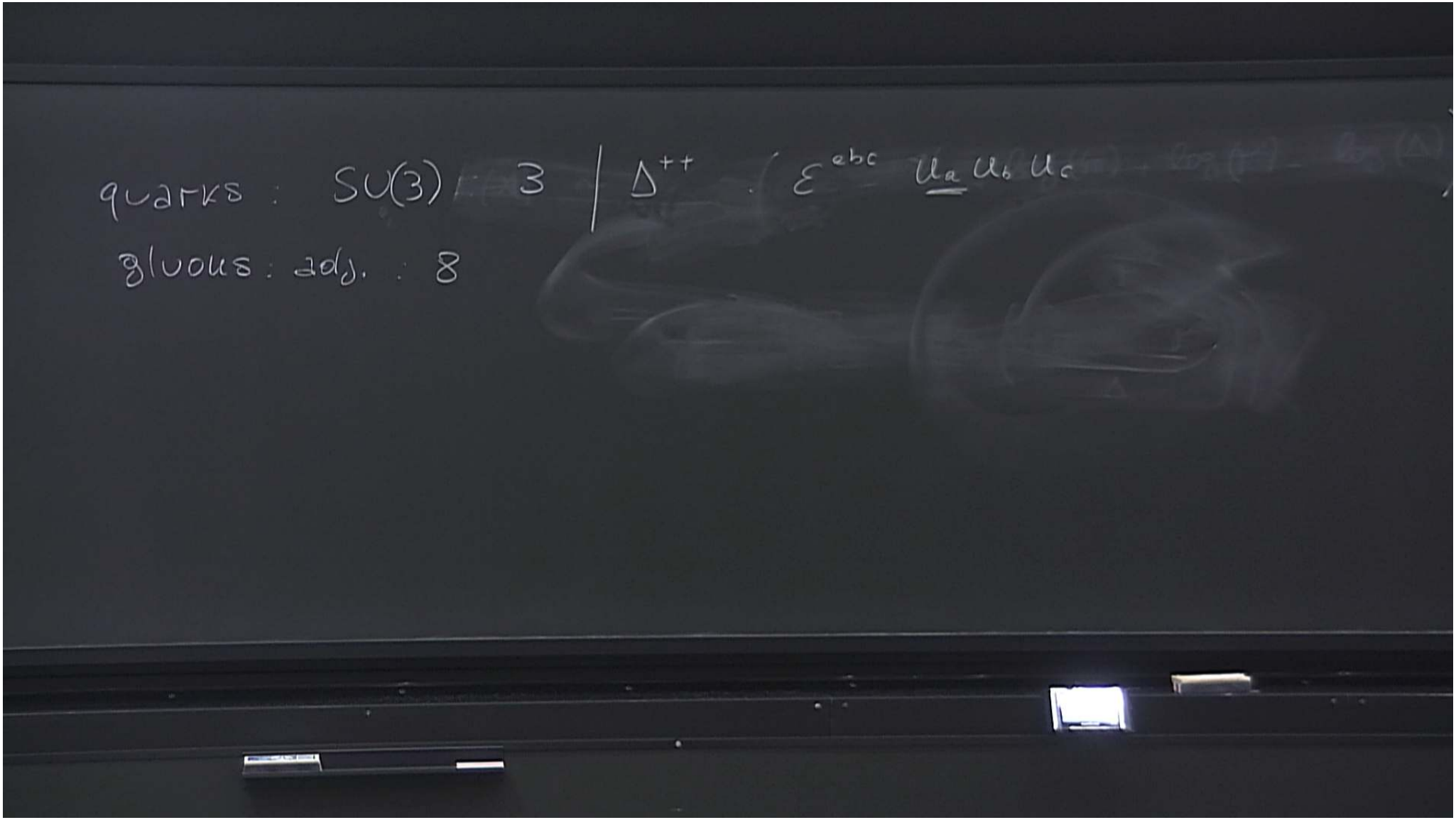
$$\frac{Z_3^{1/2}\,\mu^{-\varepsilon/2}}{(\log \mu)} = -\frac{\varepsilon}{2} e_r + \beta(e_r) \quad \Rightarrow \quad \beta(e_r) = \frac{e_r}{2Z_3}\frac{dZ_3}{d(\log \mu)}$$

↑ Beta-function of QED

$$Z_3 = 1 + \sum_{k=1}^{\infty} \frac{g^k}{\varepsilon^k} Z_{3,k}(e_r) \Rightarrow k=1: \quad \delta_3 \sim \frac{e_r^2}{6\pi^2}\frac{1}{\varepsilon} \Rightarrow \boxed{\beta(e_r) = +\frac{e_r^3}{12\pi^2}}$$


hadrons
- baryon: 3q → - proton: uud (M~1GeV)
- Lambda: Λ: uds (M~1.1GeV)
Δ. Δ°, Δ±, Δ±±
- mesons: 2


- mesons: 2q - pions : ud : $\pi^+ = u\bar{d}$ ($M \sim (130\text{-}140)$ MeV)
- Kaons : us → $K^+ = u\bar{s}$ ($\sim 500$ MeV)
$\Delta^{++}$ : uuu spin = 3/2 ↑↑↑ => new quantum number is Color. red, blue, green


quarks : SU(3) : 3 | $\Delta^{++}$ . $\varepsilon^{abc}\, u_a u_b u_c$
gluons : adj. : 8