Title: Standard Model-7

Date: Jan 15, 2015 09:00 AM

URL: http://pirsa.org/15010010

Abstract:

Today: Marathon Lecture (2hr)

- Review Yesterday (leptons w/ 3 generations)
- Add Quarks.
- full SM lagrangian.
- Add QCD. (Mostly Next Wk.)

fey

leptons: $G_{SM} \equiv SU(2)_L \times U(1)_Y$

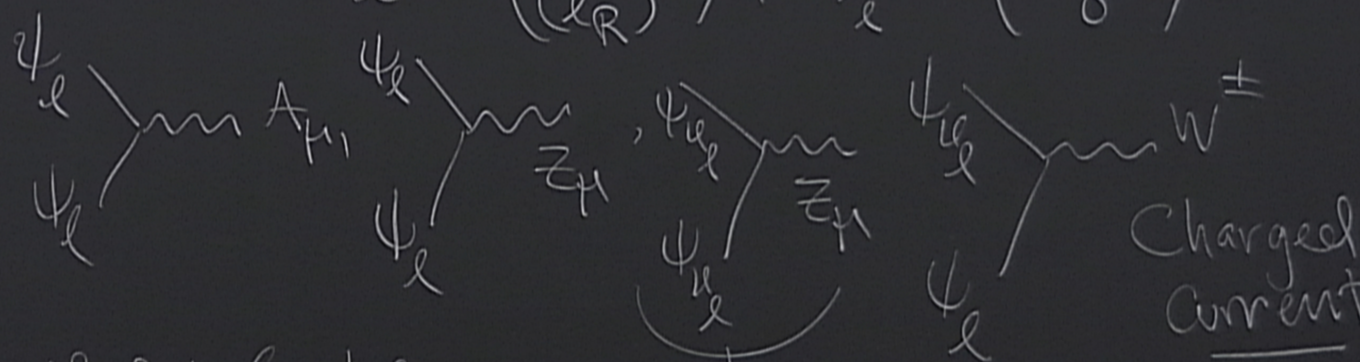
$$(\ell^c_R) \to (\mathbb{1}, +1)$$

$$L_\ell = \begin{pmatrix} \nu_\ell \\ \ell_L \end{pmatrix} \to (2, -1/2)$$

$U(2)_L \times U(1)_Y$

Feynman Rules: for Dirac Spinors

$$\Psi_\ell \equiv \begin{pmatrix} \ell_L \\ (\ell_R^c)^\dagger \end{pmatrix}, \quad \Psi_{\nu_\ell} = \begin{pmatrix} \nu_\ell \\ 0 \end{pmatrix}$$



Neutral Current

Note: ν only couples to W & Z.

Higgs Interactions:

$$\mathcal{L}_{\ell H} = (y_\ell) H^\dagger L_\ell (\ell_R^c) + h.c.$$

$$\xrightarrow[\text{about Min.}]{\text{expand}} \underbrace{\frac{y_\ell v}{\sqrt{2}}}_{m_\ell .} \left(\ell_L \ell_R^c + h.c.\right)$$

We can diagonalize Yuk

We can diagonalize Yukawa Matrix for leptons.

No appearence of Diagonalization Matrices. In other Interactions.

c.

Quarks: (w/ No QCD) $G_{SM} = SU(2)_L \times U(1)_Y$

Introduce: 1^{st} generation $(u_R^c) = (1, -2/3)$

Doublet $SU(2)_L$ $Q \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ $(d_R^c) = (1, +1/3)$

All 2 component Spinors (LH)

Electroweak Interactions of Quarks (

Start w/ u^c_R & d^c_R.

kinetic term $\quad i(u^c_R)^\dagger \bar{\sigma}^\mu D_\mu (u^c_R)$

$\quad i(u^c_R)^\dagger \bar{\sigma}^\mu \partial_\mu ($

∂ on $(u^c_R$ & $d^c_R)$

$$\left[\left(\partial_\mu - \frac{ig}{\sqrt{2}}\cancel{(W^+_\mu T^+ + W^-_\mu T^-)}\right) - \frac{ig}{G_W} Z_\mu\left(T^3 - \hat{Q}\right) - ie\hat{Q}A_\mu\right]$$

$\hat{Q} \equiv T^3 + Y$

Charge Operator!!!

of Quarks (w/ $W^\pm, Z$)

$$\to i(u_R^c)^\dagger \bar\sigma^\mu \partial_\mu (u_R^c) + \frac{g}{c_\theta}(u_R^c)^\dagger \bar\sigma^\mu Z_\mu (u_R^c)\left(+\frac{2}{3} s^2_{\theta_W}\right) + \text{QED}.$$

for (d_R^c) kinetic

$$i(d_R^c)^\dagger \bar\sigma^\mu \partial_\mu (d_R^c) + \frac{g}{c_{\theta_W}}(d_R^c)^\dagger \bar\sigma^\mu Z_\mu (d_R^c)\left(-\frac{1}{3} s^2_{\theta_W}\right) + \text{QED}$$

for Doublet $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \to$ kinetic term

$$iQ^\dagger \bar\sigma^\mu D_\mu Q = iQ^\dagger \bar\sigma^\mu \partial_\mu Q + \frac{g}{\sqrt{2}} Q^\dagger \bar\sigma^\mu \left(W_\mu^+ T^+ + W_\mu^- T^-\right) + \frac{g}{c_\theta} Q^\dagger \bar\sigma^\mu Z_\mu \left(T^3 - \hat{Q}\right) Q$$

of Quarks (w/ $W^\pm, Z$)

$$i(u_R^c)^\dagger \bar\sigma^\mu \partial_\mu (u_R^c) + \frac{g}{C_\theta}(u_R^c)^\dagger \bar\sigma^\mu Z_\mu (u_R^c)\left(+\frac{2}{3}S^2_{\theta_W}\right) + \text{QED}.$$

for (d_R^c) kinetic

$$\boxed{i(d_R^c)^\dagger \bar\sigma^\mu \partial_\mu (d_R^c) + \frac{g}{C_{\theta_W}}(d_R^c)^\dagger \bar\sigma^\mu Z_\mu (d_R^c)\left(-\frac{1}{3}S^2_{\theta_W}\right)}$$

$$W^\pm = \frac{1}{\sqrt{2}}\left(X^1_\mu \mp i X^2_\mu\right)$$

$$T^{\pm} \equiv \frac{1}{2}(\sigma^1 \pm i\sigma^2)$$

$+ \underline{\text{QED}}$

for Doublet $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \longrightarrow$ kinetic term

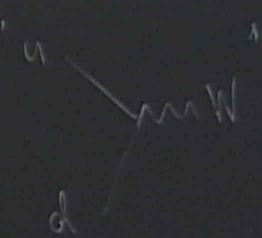
$$iQ^\dagger \bar\sigma^\mu D_\mu Q = iQ^\dagger \bar\sigma^\mu \partial_\mu Q + \frac{g}{\sqrt{2}} Q^\dagger \bar\sigma^\mu \left(W^+_\mu T^+ + W^-_\mu T^-\right) + \frac{g}{C_\theta} Q^\dagger \bar\sigma^\mu Z_\mu (T^3 - \hat{Q}) Q$$

W-Quark Interactions!

$$\mathcal{L} \supset \frac{g}{\sqrt{2}}\left(u_L^\dagger \ d_L^\dagger\right)\bar{\sigma}^\mu\left(W_\mu^+\begin{pmatrix}0&1\\0&0\end{pmatrix}+W_\mu^-\begin{pmatrix}0&0\\1&0\end{pmatrix}\right)\begin{pmatrix}u_L\\d_L\end{pmatrix}$$

$$= \frac{g}{\sqrt{2}}\left(u_L^\dagger\bar{\sigma}^\mu \ \ d_L^\dagger\bar{\sigma}^\mu\right)\left[W_\mu^+\begin{pmatrix}d_L\\0\end{pmatrix}+W_\mu^-\begin{pmatrix}0\\u_L\end{pmatrix}\right]$$

$$= \frac{g}{\sqrt{2}}\left[u_L^\dagger\bar{\sigma}^\mu W_\mu^+ d_L + d_L^\dagger\bar{\sigma}^\mu W_\mu^- u_L\right]$$



Charged Current Interaction for Quarks

like w/ lepto

like w/ leptons: W changes flavor.

Z - Q Interactions. (Doublet Q)

$$\mathcal{L}_{ZQ} = \frac{g}{c_{\theta_W}} \left(u_L^\dagger \ d_L^\dagger\right) \bar{\sigma}^\mu Z_\mu \left(T^3 - \hat{Q}\, s^2_{\theta_W}\right) \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

$$= \frac{g}{c_{\theta_W}} \left(u_L^\dagger \bar{\sigma}^\mu \ \ d_L^\dagger \bar{\sigma}^\mu\right) Z_\mu \begin{pmatrix} \left(+\frac{1}{2} - \frac{2}{3} s^2_{\theta_W}\right) u_L \\ \left(-\frac{1}{2} + \frac{1}{3} s^2_{\theta_W}\right) d_L \end{pmatrix}$$

$$= \frac{g}{c_{\theta_W}} \left[\left(\frac{1}{2} - \frac{2}{3} s^2_{\theta_W}\right) u_L^\dagger \bar{\sigma}^\mu Z_\mu u_L + \left(-\frac{1}{2} + \frac{1}{3} s^2_{\theta_W}\right) d_L^\dagger \bar{\sigma}^\mu Z_\mu d_L \right]$$

"Neutral Current" Interaction

$$\hat{Q} \equiv T^3 + Y$$

$u \to +2/3$

EM.

$d \to -1/3$

Charge Operator !!!

Add Higgs Couplings: Recall $H \to (2, +\frac{1}{2})$ unde

Start by Coupling H w/ down Quarks.

(Recall for leptons we had. $H^\dagger L (e^c_R)$, downs are similar)

$(y_d) H^\dagger Q (d^c_R) \longrightarrow$ Invariant Under G_{SM}.

"down Yukawa" $(2, -\frac{1}{2}), (2, +\frac{1}{6})\ (1, +\frac{1}{3})$

$Y_{H^\dagger} + Y_Q + Y(d^c_R) = 0$

Now Expand $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

...) under $SU(2)_L \times U(1)_Y$, Q has $(2, +\frac{1}{6})$, $(d_R^c) \to (1, +\frac{1}{3})$

$\frac{1}{6} \equiv Y_Q$ $\qquad Y(d_R^c)$

...ilar)

...SSM.

$\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\frac{(y_d)}{\sqrt{2}} (0 \ v) \begin{pmatrix} u_L \\ d_L \end{pmatrix} (d_R^c)$$

$$= \underbrace{\left(\frac{y_d v}{\sqrt{2}}\right)}_{\equiv\, m_d} \left(d_L^\alpha (d_R^c)_\alpha + h.c. \right)$$

$$m_d \cong 4.8 \text{ MeV.}$$

Up Quark?

What's Invariant?

$\cancel{H^\dagger Q (u_R^c)}$ Not Invariant Under $U(1)_Y$

$Y(u_R^c) = -\frac{2}{3}$

Charged Current Interaction for Quarks

$= \frac{g}{c_{\Theta_W}} \Big| \left(\frac{1}{2} - \frac{2}{3} s^2_{\Theta_W}\right) u_L^\dagger \ldots + \left(-\frac{1}{2} + \frac{1}{3} s^2_{\Theta_W}\right) d_L^\dagger \ldots$

How Else Can We Contract doublets of SU(2)?

Consider: $\left(\epsilon^{ij} H_i\right) Q_j$ — doublet SU(2) Indices

let $U \in SU(2)$, Transform Q, H

$$Q \to UQ = U^{jl} Q_l$$

$$H \to UH = U^{im} H_m$$

$$\epsilon^{ij} H_i Q_j \to \epsilon^{ij} \underbrace{\left(U^{im} U^{jl}\right)}_{\equiv \epsilon^{ml}} H_m Q_l$$

(Levi Civita 9/5)

$\epsilon^{ij} H_i Q_j (u_R^c)$

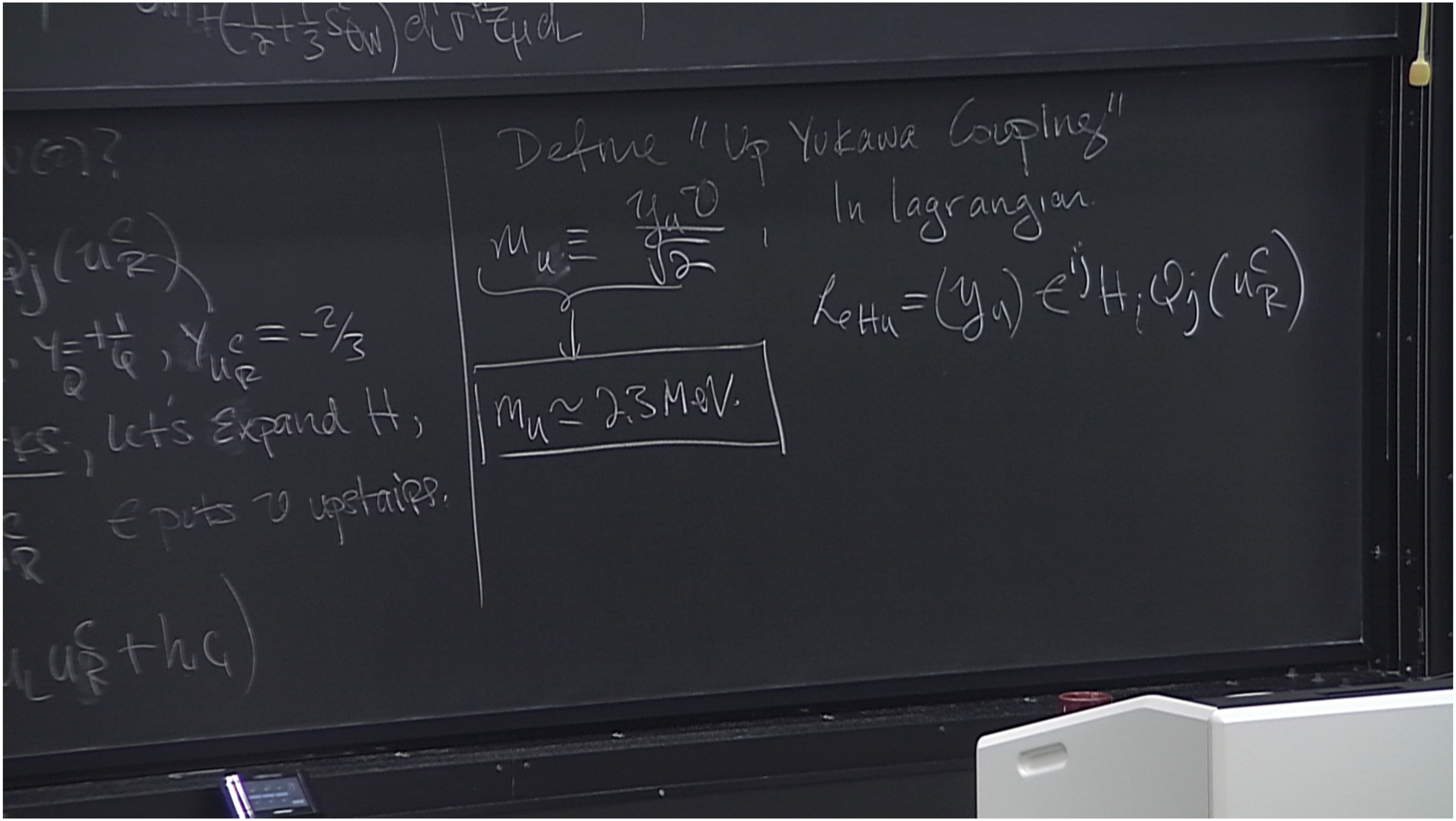
$Y_H = +\frac{1}{2}, \; Y_Q = +\frac{1}{6}, \; Y_{u_R^c} = -2/3$

This Works, Let's Expand H,

ϵ puts v upstairs.

$\underbrace{\epsilon^{ij} H_i Q_j}\, u_R^c$

$= v\left(u_L u_R^c + h.c.\right)$



$Y_Q = +\frac{1}{6}$, $Y_{u^c_R} = -\frac{2}{3}$
let's Expand H,
ϵ puts v upstairs.
Define "Up Yukawa Coupling"
$m_u \equiv \frac{y_u v}{\sqrt{2}}$
$m_u \simeq 2.3$ MeV.
In lagrangian.
$\mathcal{L}_{eHu} = (y_u)\,\epsilon^{ij} H_i Q_j (u^c_R)$