

Title: Standard Model-6

Date: Jan 14, 2015 09:00 AM

URL: <http://pirsa.org/15010009>

Abstract:

Last time: added fermions (2-component), $\begin{pmatrix} e_R^c \\ \chi \end{pmatrix} \rightarrow (1, +1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet
 just Name
 "X"
 $L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \rightarrow (2, -\frac{1}{2})$
 $SU(2)_L \times U(1)_Y$
 All fields are LH spinors (under Lorentz)
 (only Lorentz-inv. objects from LH x LH or RH x RH)
 bilinears.

Last time: added fermions (2-component), $(e_R^c) \rightarrow (1, +1)$ under $(SU(2)_L \times U(1)_Y)$ and
 All fields are LH Spinors (under Lorentz)
 (only Lorentz-inv. objects from LH x LH or RH x RH)

Worked out interactions:

$$i\bar{e}_R^c \not{\partial} e_R^c + g_W^2 \bar{e}_R^c \not{A} e_R^c + e \bar{e}_R^c \not{A} e_R^c$$

(QED)
electric charge

bilinear

Last time: added fermions (2-component). $(e_R^c) \rightarrow (1, +1)$ under $(SU(2)_L \times U(1)_Y)$ and
 just Name "X"
 All fields are LH Spinors (under Lorentz)
 (only Lorentz inv. objects from LH x LH or RH x RH)
 bilinears.

Worked out interactions:

$$i(e_R^c)^\dagger \sigma_\mu \psi_L(e_R^c) + \frac{g}{2} \bar{\psi}_L (T^3 - Q \sin^2 \theta_W) \psi_L + Q A_\mu (e_R^c)_\alpha$$

for doublet "L"

$$i(e_R^c)^\dagger \sigma_\mu \psi_L(e_R^c) + \frac{g}{2} \bar{\psi}_L (T^3 - Q \sin^2 \theta_W) \psi_L + e (e_R^c)^\dagger \sigma_\mu A_\mu (e_R^c)$$

(QED)
electric charge

Last time: added fermions (2-component), $(e_L^c, e_R^c) \rightarrow (1, +1)$ under $(SU(2)_L \times U(1))$
 just Name

All fields are LH Spinors (under Lorentz)

(only Lorentz-inv. objects from LH x LH or RH x RH bilinears.)

Worked Out Interactions:

$$i(e_L^c)^\dagger (\sigma^\mu)^{\alpha\beta} \left(\partial_\mu - i g \frac{\tau^a}{2} W_\mu^a - i e Q A_\mu \right) (e_R^c)_\beta$$

$$- i g \frac{\tau^a}{2} (e_L^c)^\dagger \gamma_\mu (e_L^c) W_\mu^a$$

$$- i e Q A_\mu (e_L^c)^\dagger (e_L^c)$$

for doublet "L" $i \bar{L} \sigma^\mu D_\mu L$

$$= i \bar{e}_L \sigma^\mu D_\mu e_L + i \bar{\nu}_L \sigma^\mu D_\mu \nu_L$$

(QED)
electric charge

$$i(e_L^c)^\dagger \sigma^\mu A_\mu (e_L^c)$$

$$g \bar{e}_L \sigma^\mu W_\mu^3 e_L$$

(2-component), $(e_R^c) \rightarrow (1, +1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet
 (Lorentz) just Name "X" $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \rightarrow (2, -\frac{1}{2})$
 $SU(2)_L \times U(1)_Y$

LH x LH or RH x RH

(QED)
 electric
 charge

$$i(e_P^{\dagger} \overleftrightarrow{\partial}_\mu e_R^c) + \frac{g_{S\Theta}^2}{\Theta} (e_P^{\dagger} \overleftrightarrow{\partial}_\mu e_R^c) + e(e_R^{\dagger} \overleftrightarrow{\partial}_\mu e_P^c)$$

lepton "L" $iL^{\dagger} \overleftrightarrow{\partial}_\mu L$

$$= i\nu_e^{\dagger} \overleftrightarrow{\partial}_\mu \nu_e + ie_L^{\dagger} \overleftrightarrow{\partial}_\mu e_L + ie_e^{\dagger} \overleftrightarrow{\partial}_\mu e_L + \frac{g}{\sqrt{2}} \left(\nu_e^{\dagger} \overleftrightarrow{\partial}_\mu W_\mu^+ e_L + e_L^{\dagger} \overleftrightarrow{\partial}_\mu W_\mu^- \nu_e \right) + \frac{1}{2} \frac{g_{S\Theta}^2}{\Theta} \nu_e^{\dagger} \overleftrightarrow{\partial}_\mu \nu_e + \frac{g_{S\Theta}^2}{\Theta} e_L^{\dagger} \overleftrightarrow{\partial}_\mu e_L$$

(2-component), $(e_R^c) \rightarrow (1, +1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet
 (just Name "X") $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \rightarrow (2, -\frac{1}{2})$
 or Lorentz) $SU(2)_L \times U(1)_Y$

LH x LH or RH x RH)

(QED)
 electric
 charge

$$i(e_P^{\dagger})\overline{\sigma}_\mu(e_P^c) + \frac{g^2}{2} (e_P^{\dagger})\overline{\sigma}_\mu(e_P^c) + e(e_P^{\dagger})\overline{\sigma}_\mu(e_P^c)$$

addit "L" $iL^{\dagger}\overline{\sigma}_\mu L$

$$= i\nu_e^{\dagger}\overline{\sigma}_\mu\nu_e + ie_L^{\dagger}\overline{\sigma}_\mu e_L + ie_e^{\dagger}\overline{\sigma}_\mu e_L + \frac{g}{\sqrt{2}} \left(\nu_e^{\dagger}\overline{\sigma}_\mu W_\mu^+ e_L + e_L^{\dagger}\overline{\sigma}_\mu W_\mu^- \nu_e \right) + \frac{1}{2} \frac{g^2}{\cos^2 \theta} \nu_e^{\dagger}\overline{\sigma}_\mu \nu_e + \frac{g}{\cos^2 \theta} \left(\frac{1}{2} + \frac{Y}{2} \right) e_L^{\dagger}\overline{\sigma}_\mu e_L$$

Coupled these to Higgs Doublet $H \rightarrow (2, +\frac{1}{2})_{SU(2)} \rightarrow U(1)_Y$

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L H^\dagger L(e_R^c)$$

#

Coupled these to Higgs Doublet $H \rightarrow (2, +\frac{1}{2})_{SU(2)} \rightarrow U(1)_Y$

$$\begin{array}{c}
 (y_e) \quad H^\dagger \quad L \quad (e_R^c) \\
 \downarrow \quad \downarrow \quad \downarrow \\
 \# \quad \gamma_H = -\frac{1}{2} \quad \gamma_L = -\frac{1}{2} \quad +1
 \end{array}$$

Coupled these to Higgs Doublet $H \rightarrow (2, +\frac{1}{2})$

$$(y_e) \left[H^\dagger \overset{+1}{\underset{\downarrow}{L}} (e_R^c)^\dagger + H \overset{+1}{\underset{\downarrow}{L}} (e_R^c)^\dagger \right] \overset{SU(2)}{\rightarrow} U(1)_Y$$

$y_H = -\frac{1}{2}$ $y_L = -\frac{1}{2} + 1$

Expand about Minimum. $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\frac{y_e v}{\sqrt{2}} (e_L e_R^c + e_L^c e_R), m_e = \frac{y_e v}{\sqrt{2}} = 0.511 \text{ MeV}$$

Aside (forget this)

Suppose we had chosen " e_R " instead of

Dirac mass for " e "

coupled these to Higgs Doublet $H \rightarrow (2, +\frac{1}{2})$ $U(1)_Y$

$$(y_e) \left[H^\dagger \underset{\substack{\downarrow \\ Y_H = -\frac{1}{2}}} L \underset{\substack{\downarrow \\ Y_L = -\frac{1}{2} + 1}} (e_R^c) + H^\dagger \underset{\substack{\downarrow \\ Y_H = -\frac{1}{2}}} L \underset{\substack{\downarrow \\ Y_L = -\frac{1}{2} + 1}} (e_R^c)^\dagger \right]$$

expand about minimum

$$\frac{y_e v}{\sqrt{2}} (e_L e_R + e_L^\dagger e_R^\dagger) = \frac{y_e v}{\sqrt{2}} = 0.511 \text{ MeV}$$

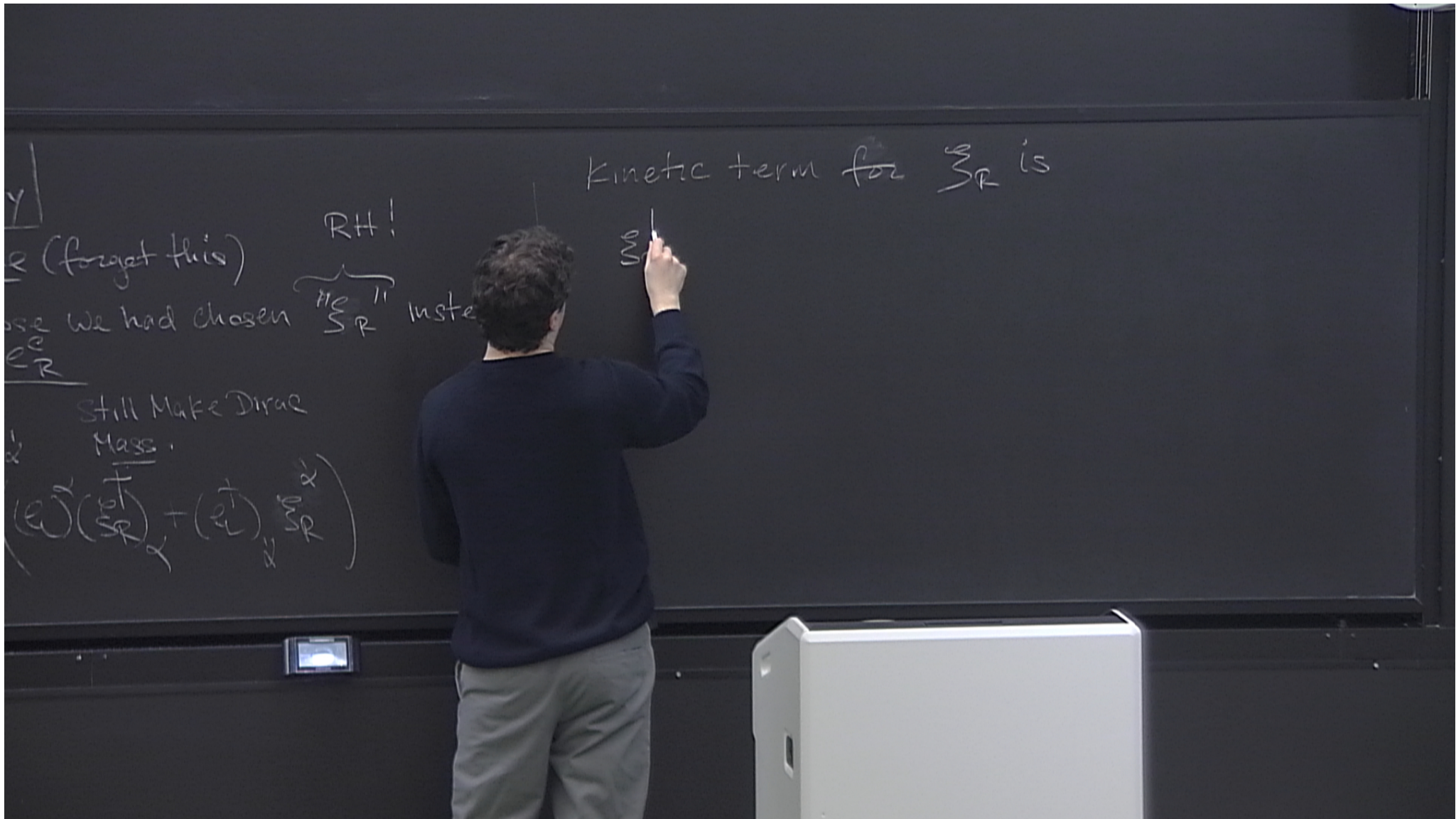
as for e^-

Aside (forget this)

Suppose we had chosen $\overline{\psi}_R$ instead of e_R^c

$(\overline{\psi}_R)_\alpha$ Still Make Dirac Mass.

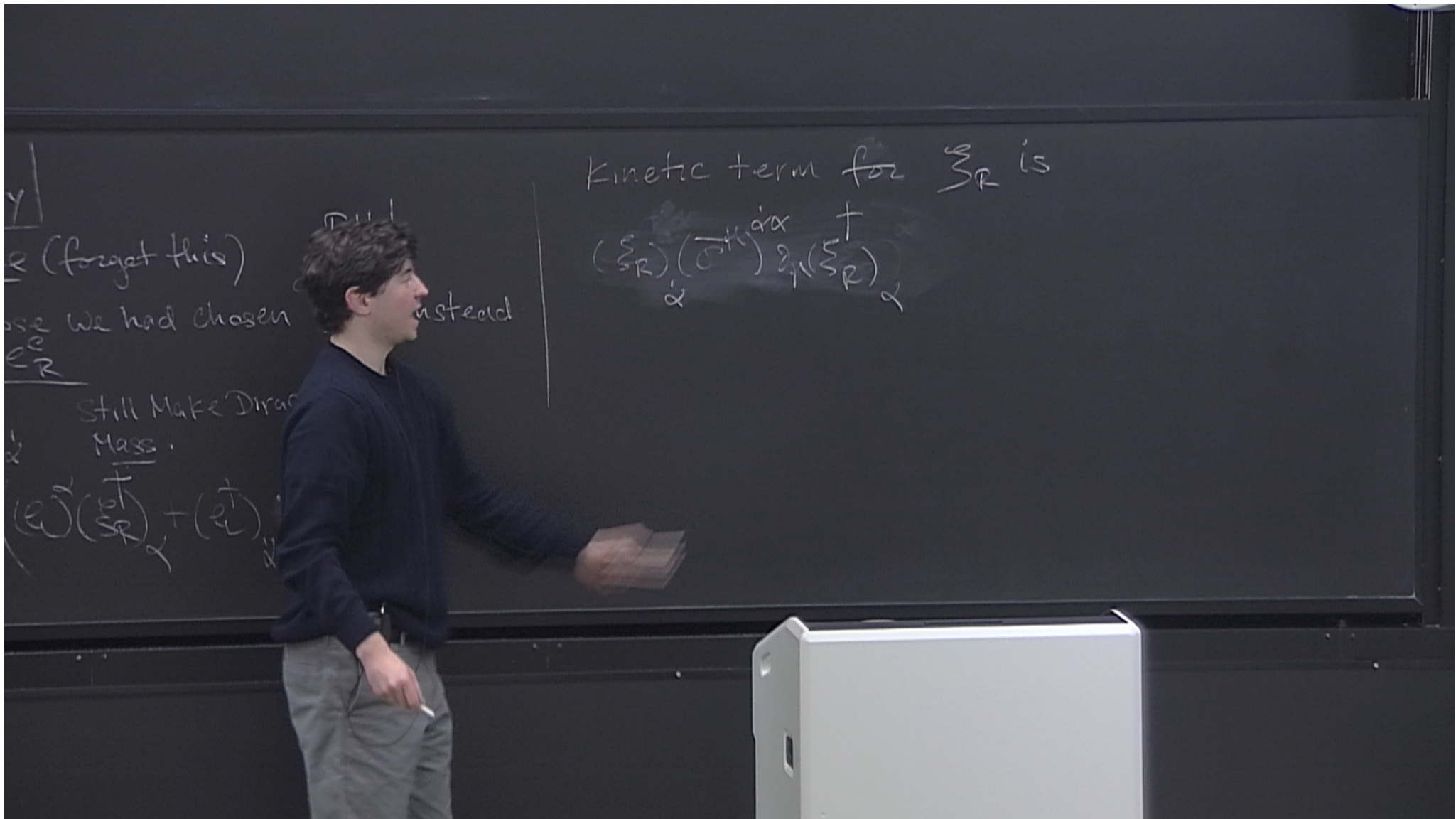
RH!



χ
e (forget this)
RH!
We had chosen $\sum_R$ instead
 $\sum_R$
Still Make Dirac
Mass.
$$\left(\bar{e}_L \right)_\alpha \left(\bar{\nu}_R \right)_\alpha + \left(\bar{e}_L \right)_\alpha \left(\bar{\nu}_R \right)_\alpha$$

kinetic term for $\sum_R$ is

$\sum_R$



Y

e (forget this)

ose we had chosen

e_R

Still Make Dirac

Mass.

$(e)_\alpha (\xi_R)^\alpha + (e_L)^\alpha (\xi_R)_\alpha$

Kinetic term for ξ_R is

$(\xi_R)_\alpha (\sigma^{\mu\nu})_{\alpha\beta} \partial_\mu \xi_R^\beta$

Back to Lecture: Dirac Mass for e^-

$\psi_e \equiv \begin{pmatrix} (e_L)_\alpha \\ (e_R^\dagger)_{\dot{\alpha}} \end{pmatrix}$, Recall: $\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\beta}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\beta} & 0 \end{pmatrix}$, $\sigma^\mu = (\mathbb{1}_{2 \times 2}, \vec{\sigma})$
 $\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$

and: $\bar{\psi}_e \equiv \psi_e^\dagger \gamma^0 = ((e_L)^\dagger, \dots)$

every " ψ "
 will be 4-component
 dirac spinor

structure: Dirac Mass for e^-

$$\begin{pmatrix} (e_L)_\alpha \\ (e_R^c)^\alpha \end{pmatrix}, \text{ Recall: } \gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\dot{\alpha}\alpha} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix}, \quad \sigma^\mu = (\mathbb{1}_{2 \times 2}, \vec{\sigma})$$

$$\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$$

and: $\bar{\psi}_e \equiv \psi_e^\dagger \gamma^0 = ((e_L)_\alpha (e_R^c)^\alpha) \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} = ((e_R^c)^\alpha (e_L)_\alpha)$

So Dirac Mass: $m_e \bar{\psi}_e \psi_e$

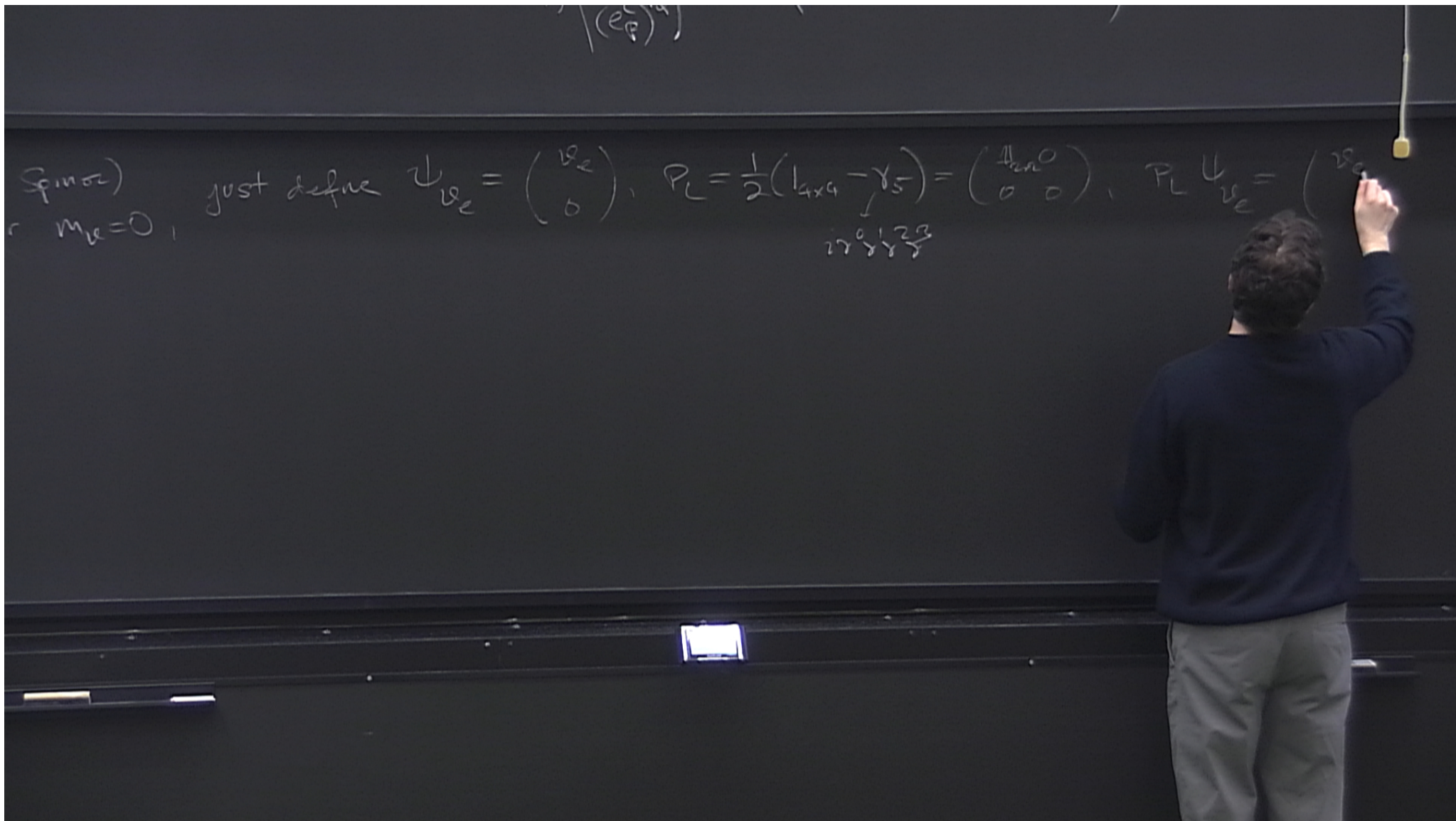
$$= m_e ((e_R^c)^\alpha (e_L)_\alpha) \begin{pmatrix} (e_L)_\alpha \\ (e_R^c)^\alpha \end{pmatrix} = m_e \left((e_R^c)^\alpha (e_L)_\alpha + (e_L)_\alpha (e_R^c)^\alpha \right)$$

for ν_e (2 comp spinor)
no Dirac partner $m_\nu = 0$

$$\alpha) \left(e_{\vec{p}}^{\alpha} \right)$$

for ν_e (2 comp spinor) just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1_{4 \times 4} - \gamma_5)$
 no Dirac partner $m_\nu = 0$

$\gamma_5^2 = 1$



for ν_e (2 comp. spinor)
 no Dirac partner $m_\nu = 0$

Consider: (ν_e)

$$i \gamma^\mu \partial_\mu \psi_{\nu_e} + \frac{g}{\sqrt{2}} (\bar{\psi}_{\nu_e} \gamma^\mu W_\mu^+ P_L \psi_e + \bar{\psi}_e \gamma^\mu W_\mu^- P_L \psi_{\nu_e})$$

$$+ \frac{g}{2} \bar{\psi}_{\nu_e} \gamma^\mu Z_\mu (T^3 P_L - Q_S \Theta) \psi_{\nu_e}$$

Only cares about
 ν_{eL}

just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1 - \gamma_5) =$

$P_R = \frac{1}{2}(1 + \gamma_5) =$

for ν_e (2 comp. spinor)
no Dirac partner $m_\nu = 0$

Now consider (ν_e)

$$i \bar{\psi}_{\nu_e} \partial_\mu \gamma^\mu \psi_{\nu_e} + \frac{g}{\sqrt{2}} \left(\bar{\psi}_{\nu_e} \gamma^\mu W_\mu^+ P_L \psi_e + \bar{\psi}_e \gamma^\mu \right)$$

$$+ \frac{g}{\cos \theta_w} \bar{\psi}_{\nu_e} \gamma^\mu Z_\mu (T^3 P_L - Q \sin^2 \theta) \psi_{\nu_e}$$

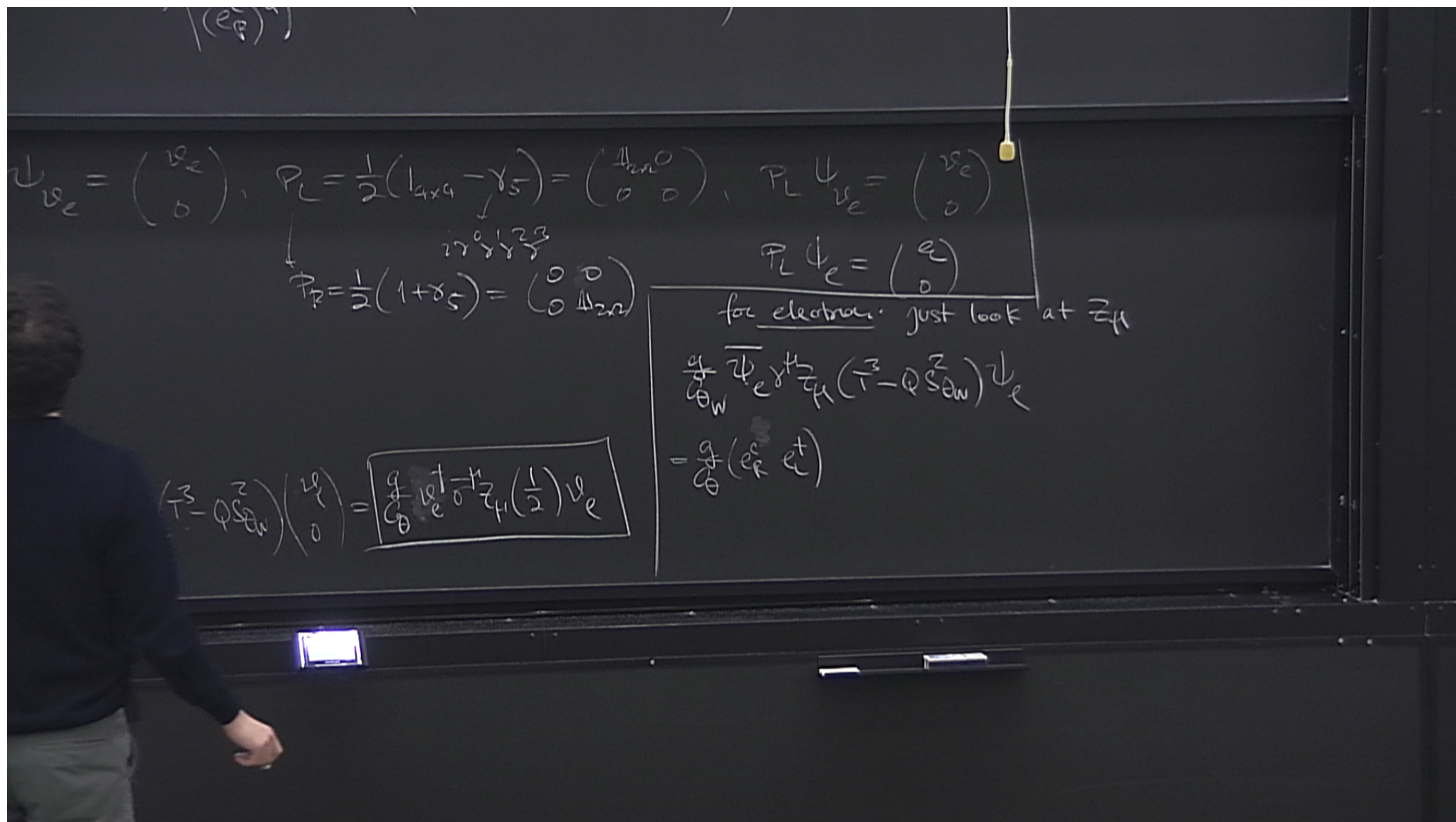
Only cares about
 ν_{eL}

just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1 - \gamma_5) =$

$\begin{matrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \end{matrix}$
 $P_R = \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 0 & 1 \\ \nu_e & 0 \end{pmatrix} \begin{pmatrix} 0 & \sigma^\mu Z_\mu \\ \sigma^\mu Z_\mu & 0 \end{pmatrix} \begin{pmatrix} T^3 - Q \sin^2 \theta_w \\ T^3 - Q \sin^2 \theta_w \end{pmatrix} \begin{pmatrix} \nu_e \\ 0 \end{pmatrix} = \frac{g}{\cos \theta} \nu_e^\dagger \sigma^\mu Z_\mu$$

$$\begin{aligned}
 \psi_{\nu_e} &= \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}, \quad P_L = \frac{1}{2}(1 - \gamma_5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad P_L \psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix} \\
 P_R &= \frac{1}{2}(1 + \gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad P_L \psi_e = \begin{pmatrix} e \\ 0 \end{pmatrix} \\
 \begin{pmatrix} \frac{1}{2} \gamma_5 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 - Q_{SW} \end{pmatrix} \begin{pmatrix} \nu_e \\ 0 \end{pmatrix} &= \begin{pmatrix} \frac{g}{2} \frac{1}{2} \gamma_5 \frac{1}{2} \gamma_5 \left(\frac{1}{2} \right) \nu_e \\ 0 \end{pmatrix}
 \end{aligned}$$



$$(\sigma^{\mu\nu} \gamma_4 e_\mu^c) \bar{S}_0^2 + \frac{g}{g_0} \left(-\frac{1}{2} + \bar{S}_0^2\right) e_c^\dagger \sigma^{\mu\nu} \gamma_4 e_\nu, \text{ use IBP (inside } \int d^4 x \bar{\psi} \psi$$

$$+ \frac{g}{2} \left(-\frac{1}{2} + S_0^2 \right) e_L^\dagger \sigma_3^{\mu\nu} Z_{\mu\nu} e_L, \text{ use IBP (inside } \int d^4 x \ell_{\mu\nu}) \underbrace{\sum \sigma_3^{\mu\nu} Z_{\mu\nu}}_{\downarrow} \chi^\dagger, \underbrace{\sum_i X_i}_{\text{LH weyl.}}$$

$$\frac{g}{2} \bar{\psi}_e \gamma^\mu \gamma_5 (T^3 - Q S_W^2) \psi_e = \left(\frac{g}{2}\right) (e_R^{c\dagger} \gamma^\mu \gamma_5 e_R^c) S_W^2 + \frac{g}{2} \left(-\frac{1}{2} + S_W^2\right) e_L^{c\dagger} \gamma^\mu e_L^c$$

found in Peskin & Schroeder (Srednicki) QFT
Ch 20.

Also

$$\frac{q}{2} \bar{\psi}_e \gamma^\mu \gamma_5 (T^3 - Q S^2) \psi_e = \left(\frac{q}{2}\right) (e_L^{\dagger} \gamma^\mu \gamma_5 e_L) S^2 + \frac{q}{2} \left(-\frac{1}{2} + S^2\right) e_L^{\dagger} \gamma^\mu e_L$$

found in Peskin & Schroeder (Srednicki) QFT
Ch 20.

Also: 2-component spinor QFT

$$\frac{g}{2} \bar{\psi}_e \gamma^\mu \gamma_5 (T^3 - Q S_W^2) \psi_e = \left(\frac{g}{2}\right) (e_R^\dagger \sigma^\mu \gamma_5 e_R) S_W^2 + \frac{g}{2} \left(-\frac{1}{2} + S_W^2\right) e_L^\dagger \sigma^\mu \gamma_5 e_L$$

found in Peskin & Schroeder. (Srednicki) QFT
Ch 20.

Also 2-component spinor QFT.
(Martin, Haber, Dreiner)

$$\frac{g}{2} \bar{\psi}_e \gamma^\mu \gamma_5 (T^3 - Q S_W^2) \psi_e = \left(\frac{g}{2}\right) (e_R^\dagger \sigma^\mu \gamma_5 e_R) S_W^2 + \frac{g}{2} \left(-\frac{1}{2} + S_W^2\right) e_L^\dagger \sigma^\mu \gamma_5 e_L$$

found in Peskin & Schroeder. (Srednicki) QFT
Ch 20.

Also. 2-component spinor QFT.
(Martin, Haber, Dreiner)
arXiv

$$S_{EW}^2 \psi_e = \left(\frac{g}{g_0}\right) (e_R^{\dagger} \overleftrightarrow{\sigma} \frac{1}{2} e_R^e) S_{\theta}^2 + \frac{g}{g_0} \left(-\frac{1}{2} + S_{\theta}^2\right) e_L^{\dagger} \overleftrightarrow{\sigma} \frac{1}{2} e_L, \text{ Use IBP (inside } \int d^4$$

reader. (Srednicki) FT Electroweak Unification $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

not QFT
 Dreiner)

$$S_{EW}^2 \psi_e = \left(\frac{g}{g_0}\right) (e_R^\dagger \sigma^\mu \not{Z}_e e_R) S_\theta^2 + \frac{g}{g_0} \left(-\frac{1}{2} + S_\theta^2\right) e_L^\dagger \sigma^\mu \not{Z}_e e_L, \text{ use IBP (inside } \int d^4$$

order. (S_{EW}) QFT

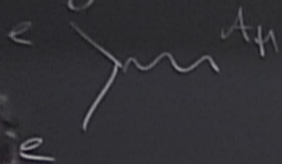
Electroweak Unification $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$

QED $\rightarrow$ "Vectorlike" (photon/fermion democracy)

$W, Z \rightarrow$ Chiral (violate Parity $x^\mu \rightarrow -x^\mu$, $L_{WF} \neq R_{WF}$)

Glashow
W

Feynman Rules: Dirac 4-comp.



Feynman Rules: Dirac 4-comp.

$i\gamma^\mu = -ie\gamma^\mu$ (no v)

only
QED
part.

Feynman Rules: Dirac 4-comp

$$\left. \begin{array}{c} \psi_e \\ \psi_e \end{array} \right\} \text{wavy line} = -ie\gamma^\mu \text{ (no } \gamma^5) \left. \vphantom{\begin{array}{c} \psi_e \\ \psi_e \end{array}} \right\} \begin{array}{l} \text{only} \\ \text{QED} \\ \text{part.} \end{array}$$

$$\left. \begin{array}{c} \psi_e \\ \psi_e \end{array} \right\} \text{wavy line} = \frac{ig}{\cos\theta} (T_L^3 - Q S_\theta)$$

Feynman Rules: Dirac 4-comp

$$\left. \begin{array}{c} \psi_e \text{ } \nearrow \\ \psi_e \text{ } \nwarrow \end{array} \right\} \begin{array}{l} \text{only} \\ \text{QED} \\ \text{FC} \end{array}$$

$$A_\mu = -ie\gamma_\mu \quad (\text{no } \gamma^5)$$

$$\left. \begin{array}{c} \psi_e, \psi_e \text{ } \nearrow \\ \psi_e, \psi_e \text{ } \nwarrow \end{array} \right\} \begin{array}{l} \text{only} \\ \text{Boson to couple} \\ Z = \gamma \end{array}$$

$$Z_\mu = \frac{ig}{\cos\theta} \gamma_\mu (T_L^3 - Q\sin^2\theta)$$

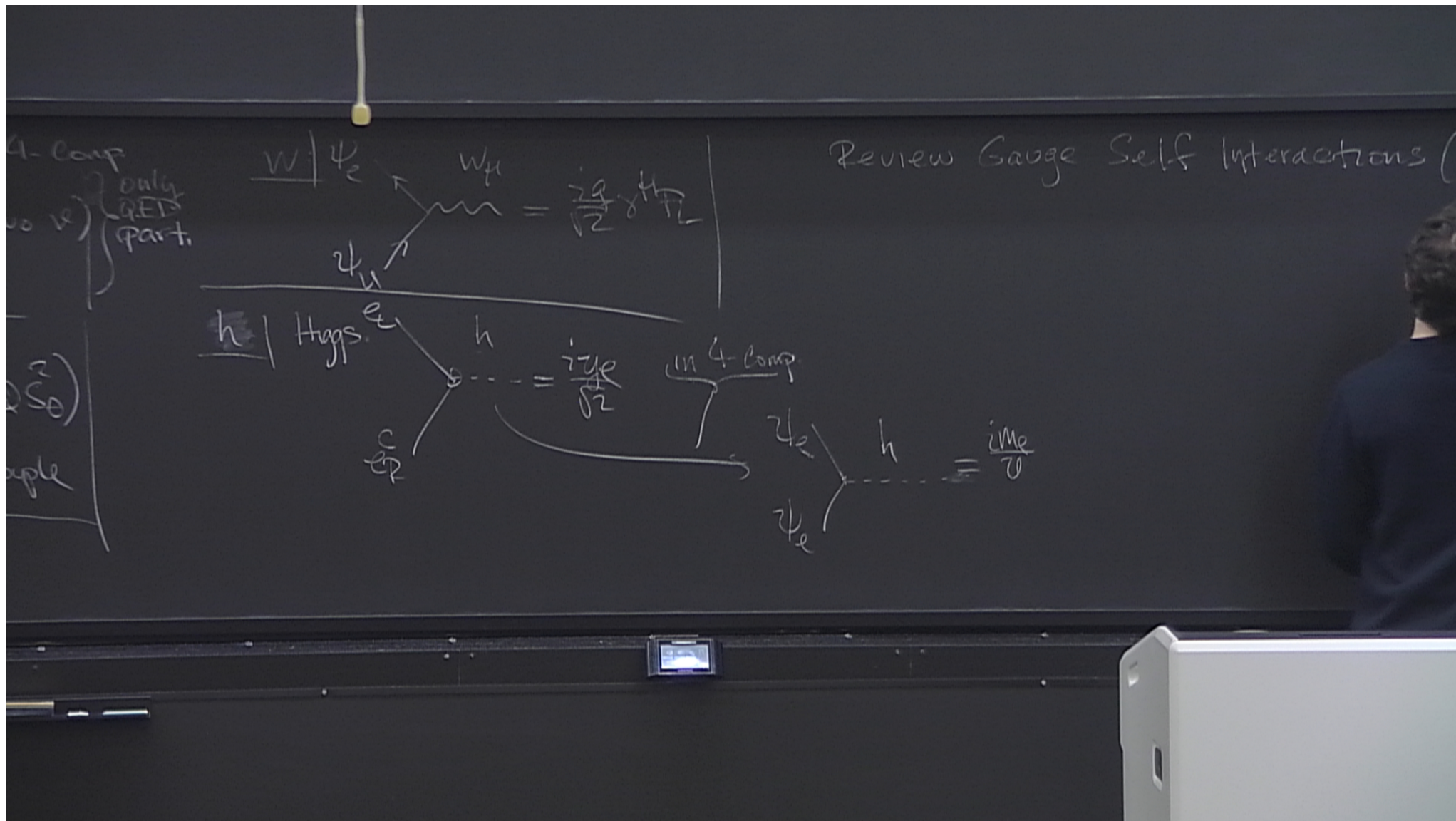
Feynman Rules: Dirac 4-comp

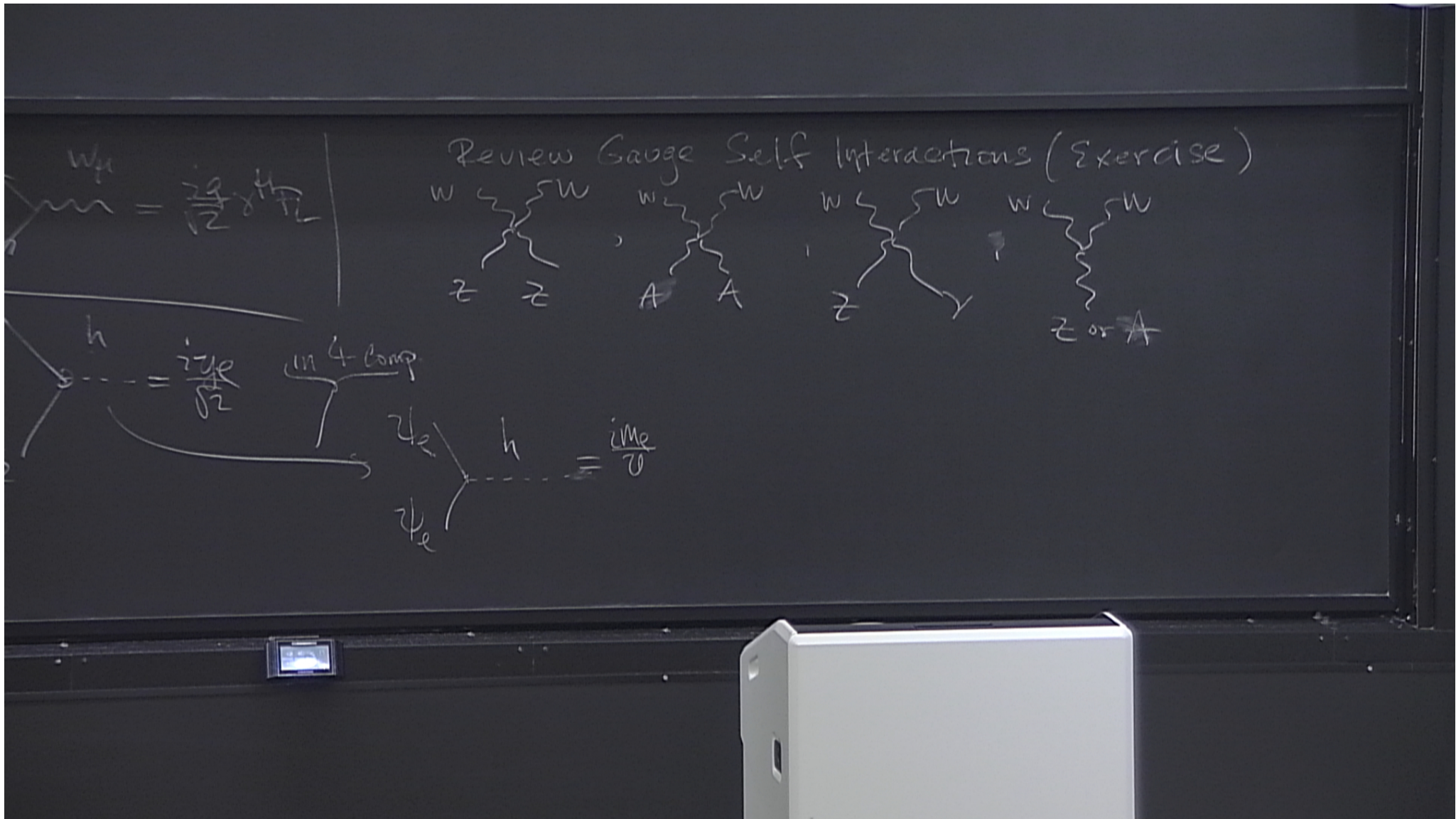
$$\psi_e \quad \psi_e \quad \text{wavy line} \quad A_\mu = -ie\gamma_\mu \quad (\text{no } \gamma^5) \quad \left\{ \begin{array}{l} \text{only} \\ \text{QED} \end{array} \right.$$

$$\psi_e, \psi_e \quad \psi_e, \psi_e \quad \text{wavy line} \quad Z_\mu = \frac{ig}{\cos\theta_W} \gamma_\mu (T_L^3 - QS) \quad \left\{ \begin{array}{l} \text{only Boson to couple} \\ Z \rightarrow \nu \end{array} \right.$$

$$W \mid \psi_e \quad W_\mu \quad \psi_e \quad \text{wavy line} \quad = \frac{ig}{\sqrt{2}} \gamma_\mu T_L$$

$$h \mid \text{Higgs} \quad e \quad h \quad \text{dashed line} \quad = \frac{ig_e}{\sqrt{2}}$$





Other leptons? (μ "muon") (τ "tau") All identical (except Higgs)
 ν_{μ} ν_{τ} ν_e

Other leptons? (μ "muon") (τ "tau") All identical (except Higgs interaction)
 ν_{μ} ν_{τ}

$$(e_L^c) \rightarrow (\mu_R^c) \text{ OR } (\tau_R^c)$$

Same
Rep. under
 $SU(2) \times U(1)$

Other leptons? (μ "muon"), (τ "tau") All identical (except Higgs interaction)
 ν_μ ν_τ

$(e_R^c) \rightarrow (\mu_R^c) \text{ or } (\tau_R^c)$, Add Doublets

Same
Rep. under
 $G_{SM} \equiv SU(2) \times U(1)$

$L_\mu = \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$ or $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$
 all $(2, -1/2)$ under $\underline{G_{SM}}$

all identical (except Higgs interaction)

add Doublets

$$L_e \rightarrow L_\mu = \begin{pmatrix} \nu_\mu \\ e \end{pmatrix} \text{ or } L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

still $(2, -1/2)$ under GSM

Couple to Higgs: (2 comp)

$$\begin{pmatrix} \nu \\ e \end{pmatrix}^\dagger H^\dagger L_\mu (H_\mu^c) + \text{h.c.} \quad (\text{herm. Conj})$$

$$\begin{pmatrix} (-\frac{1}{2} + \frac{e}{2m}) e_L + \\ + \frac{e}{2m} (e_R^c)^\dagger \end{pmatrix}$$

all identical (except Higgs interaction)

all Doublets

$$L_e \rightarrow L_\mu = \begin{pmatrix} \nu_\mu \\ e \end{pmatrix} \text{ or } L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

still $(2, -1/2)$ under GSM

Couple to Higgs: (2 comp)

$$(y_\mu) H^\dagger L_\mu (\psi_R^c) + \text{H.c. (herm. Conj)}$$

$$\rightarrow \text{Expand Minimum} = \left(\frac{y_\mu v}{\sqrt{2}} \right) (H_L (\psi_R^c) + \text{H.c.})$$

$$\boxed{m_\mu \approx 106 \text{ MeV}}$$

all identical (except Higgs interaction)

all Doublets

$$L_e \rightarrow L_\mu = \begin{pmatrix} \nu_\mu \\ e \end{pmatrix} \text{ or } L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

still $(2, -1/2)$ under GSM

Couple to Higgs: (2 comp)

$$(y_\mu) H^\dagger L_\mu (\psi_R^c) + \text{H.c. (herm. Conj)}$$

$$\rightarrow \text{Expand Minimum} = \left(\frac{y_\mu v}{\sqrt{2}} \right) (H_L (\psi_R^c) + \text{H.c.})$$

$$\boxed{m_\mu \approx 106 \text{ MeV}}$$

Same for (τ)

$$m_\tau = \left(\frac{y_\tau v}{\sqrt{2}} \right)$$

all identical (except Higgs interaction)

all Doublets

$$L_e \rightarrow L_\mu = \begin{pmatrix} \nu_\mu \\ e \end{pmatrix} \text{ or } L_\tau = \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

still $(2, -1/2)$ under GSM

Couple to Higgs: (2 comp)

$$(y_{\ell}) H^\dagger L_\ell (\ell_R^c) + \text{H.c. (herm. Conj)}$$

$$\rightarrow \text{Expand Minimum} = \left(\frac{y_{\ell} v}{\sqrt{2}} \right) (H_L (\ell_R^c) + \text{H.c.})$$

$$m_{\ell} \approx 106 \text{ MeV}$$

Same for (τ)

$$m_\tau = \left(\frac{y_\tau v}{\sqrt{2}} \right)$$

each y_ℓ is "Yukawa coupling"

Since μ, e, T have same Quant #s (same Rep. under GSM)

Since μ, e, τ have same Quant #s (same Rep. Under G_{SM})

let y_{ij} be a 3×3 Matrix

$$(y)_{ij} H^\dagger L_i^c (e_R^c)_j$$

generation index.

Since μ, e, τ have same Quant #s (same Rep. under G_{SM})

let y_{ij} be a 3×3 M

1 = e
2 = μ
3 = τ

$$(y)_{ij} H^\dagger L_i^c (e_R^c)^j$$

for leptons (if $m_\nu = 0$) True in SM
Not in Nature!

diagonalize

$$L^i \rightarrow U^{ij} L^j$$

$$(e_R^c)^i \rightarrow V^{ij} (e_R^c)^j$$

Since μ, e, τ have same Quant #s (same Rep. under G_{SM})

let y_{ij} be a 3×3 Matrix

1 = e
2 = μ
3 = τ

$$(y)_{ij}^\dagger H^\dagger L^i (e_R^c)^j$$

for leptons (if $m_R = 0$) True in SM
Not in Nature!

diagonalize

$$L^i \rightarrow U^{ij} L^j$$

$$(e_R^c)^i \rightarrow V^{ij} (e_R^c)^j$$

$$(y)_{ij}^\dagger H^\dagger L^i (e_R^c)^j \rightarrow (y_d)_{ij}^\dagger H^\dagger L^i (e_R^c)^j$$

Since μ, e, τ have same Quant #s (same Rep. under G_{SM})

let y_{ij} be a 3×3 Matrix

1 = e
2 = μ
3 = τ

$$(y)_{ij} H^\dagger L^i (l_R^c)^j$$

generation
index
(flavor)

for leptons (if $m_\nu = 0$) True in SM
Not in Nature!

diagonalize

$$L^i \rightarrow U^{ij} L^j$$

$$(l_R^c)^i \rightarrow V^{ij} (l_R^c)^j$$

U, V disappear from
theory

$$(y)_{ij} H^\dagger L^i (l_R^c)^j \rightarrow (y_d)_{ii} H^\dagger L^i (l_R^c)^i$$

$$\left| \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \right|$$

part #5 (Same Rep. under G_{SM})
 for leptons (if $M_\nu = 0$) True in SM
 Not in Nature!

diagonalize

$$L^i \rightarrow U^{ij} L^j$$

$$(\ell_R^c)^i \rightarrow V^{ij} (\ell_R^c)^j$$

U, V disappear from theory

$$(y)^{ij} H^\dagger L^i (\ell_R^c)^j \rightarrow (y_d)^{ii} H^\dagger L^i (\ell_R^c)^i$$

$$\left(\begin{array}{ccc|ccc} y_e & 0 & 0 & 0 & 0 & 0 \\ 0 & y_\mu & 0 & 0 & 0 & 0 \\ 0 & 0 & y_\tau & 0 & 0 & 0 \end{array} \right)$$

Worry About:

ν_L

Since μ, e, τ have same Quant #s (Same Rep. Under Gsm)

let y_{ij} be a 3×3 Matrix

$$(y)_{ij}^\dagger L^i (e_R^c)^j$$

generation
index
(flavor)

$$(e_R^c)^j$$

for leptons (if $M_\nu = 0$) True in SM
Not in Nature

diagonalize

$$L^i \rightarrow U^{ij} L^j$$

$$(e_R^c)^i \rightarrow V^{ij} (e_R^c)^j$$

U, V disappear
theory

$$(y)_{ij}^\dagger L^i (e_R^c)^j \rightarrow (y_d)_{ii}^\dagger L^i (e_R^c)^i$$

$$\begin{pmatrix} y_d & 0 & 0 \\ 0 & y_d & 0 \\ 0 & 0 & y_d \end{pmatrix}$$

Since μ, e, τ have same Quant #s (Same Rep. under G_{SM})

let y_{ij} be a 3×3 Matrix

1 = e
2 = μ
3 = τ

$(y)_{ij}^\dagger H^\dagger L^i$
action
lex.
(or)

$$\frac{1}{\sqrt{2}} (y)_{ij} \tau (e_L^i e_R^c{}^j)$$

for leptons (if $M_\nu = 0$) True in SM
Not in Nature

diagonalize

$$e_L^i \rightarrow U^{ij} e_L^j$$

$$(e_R^c)^i \rightarrow V^{ij} (e_R^c)^j$$

U, V disappear
theory

$$(y)_{ij}^\dagger H^\dagger L^i (e_R^c)^j \rightarrow (y_d)_{ij}^\dagger H^\dagger L^i (e_R^c)^j$$

$$\left| \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix} \right|$$

5 (Same Rep. Under GSM)
 ms (if $m_{\nu} = 0$) True in SM
 Not in Nature!

$\left. \begin{array}{l} l_R^i \\ l_R^j \\ l_R^k \end{array} \right\} \rightarrow \begin{array}{l} U, V \text{ disappear from} \\ \text{theory} \end{array}$

$$l_R^i \rightarrow (y_d)^{ii} H^\dagger l^i (l_R^i)$$

$$\left| \begin{array}{ccc} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{array} \right|$$

Worry About:

$$(l_e^+)_i \sigma^{\mu\nu} W_\mu (e_L)_j \rightarrow (l_e^+)_i \sigma^{\mu\nu} W_\mu (V^j)_e$$

$$(V_e)_e$$

$$V_e^\dagger V_e = 1$$