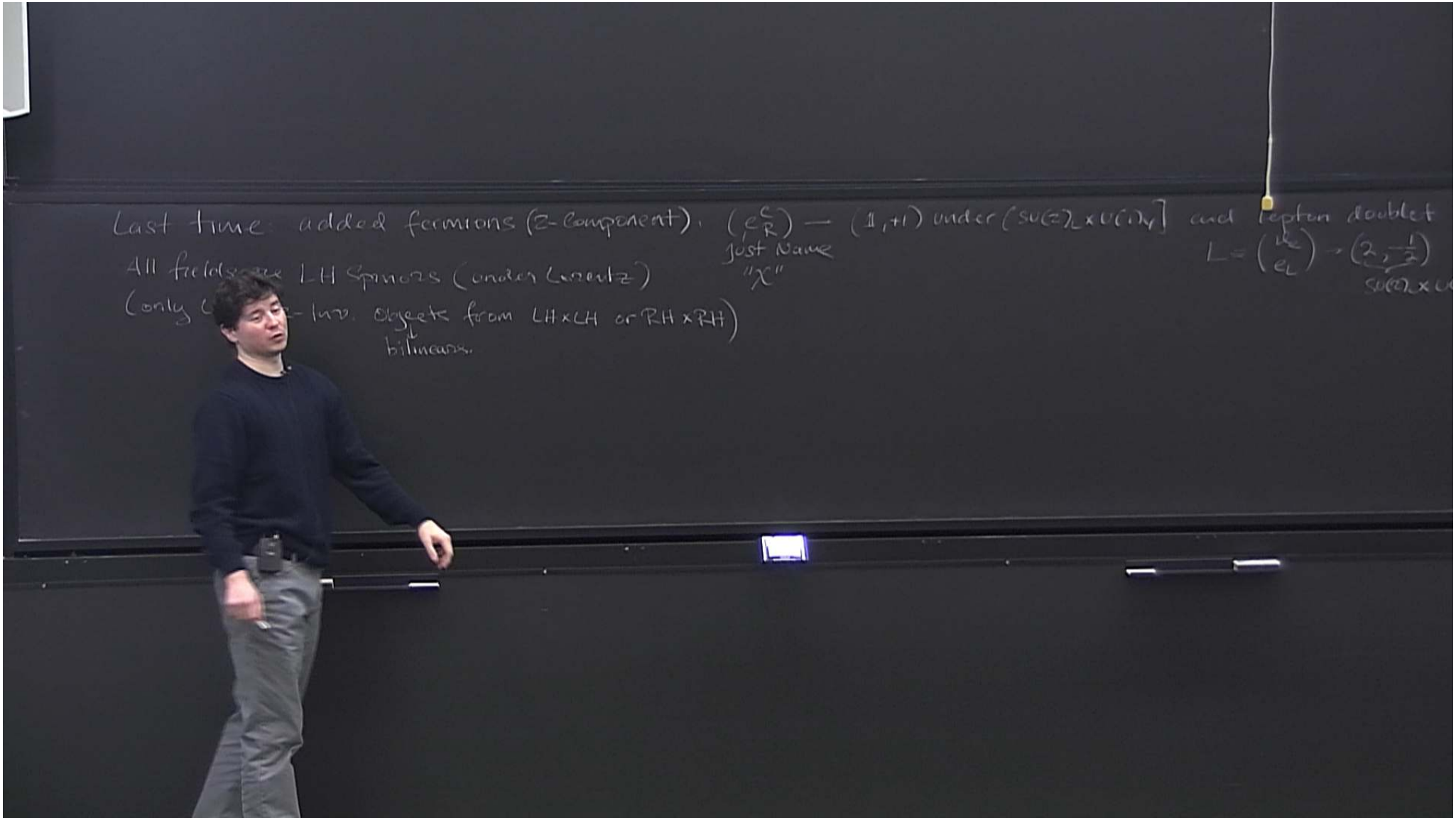
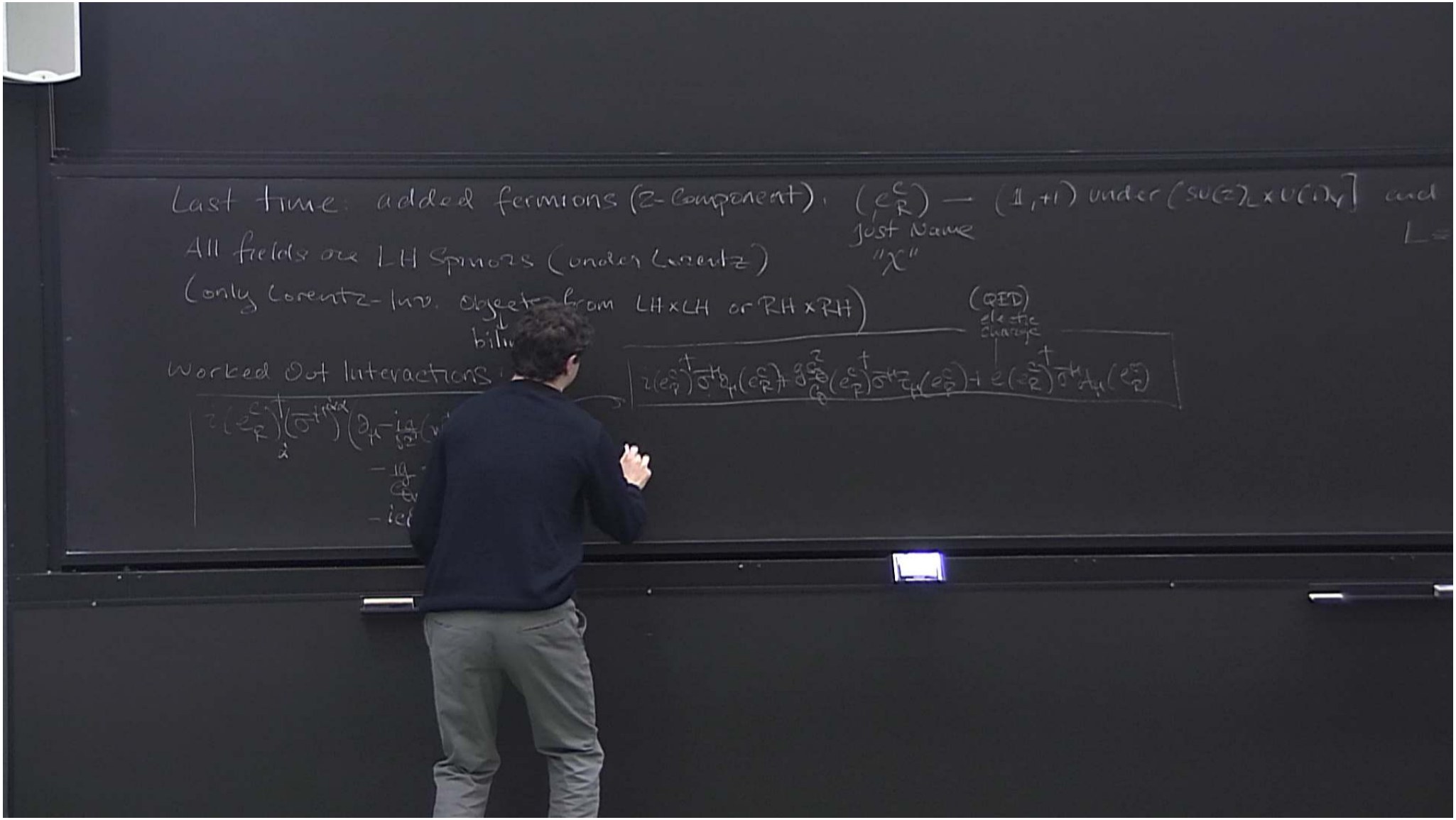
Title: Standard Model-6

Date: Jan 14, 2015 09:00 AM

URL: http://pirsa.org/15010009

Abstract:


Last time: added fermions (2-Component). $(\xi_R^\dagger)$ — $(1,+1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet
just Name "χ"
$L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \to (2, -\frac{1}{2})$
$SU(2) \times U$
All fields are LH Spinors (under Lorentz)
(only
-Inv. Objects from LH×LH or RH×RH)
bilinears.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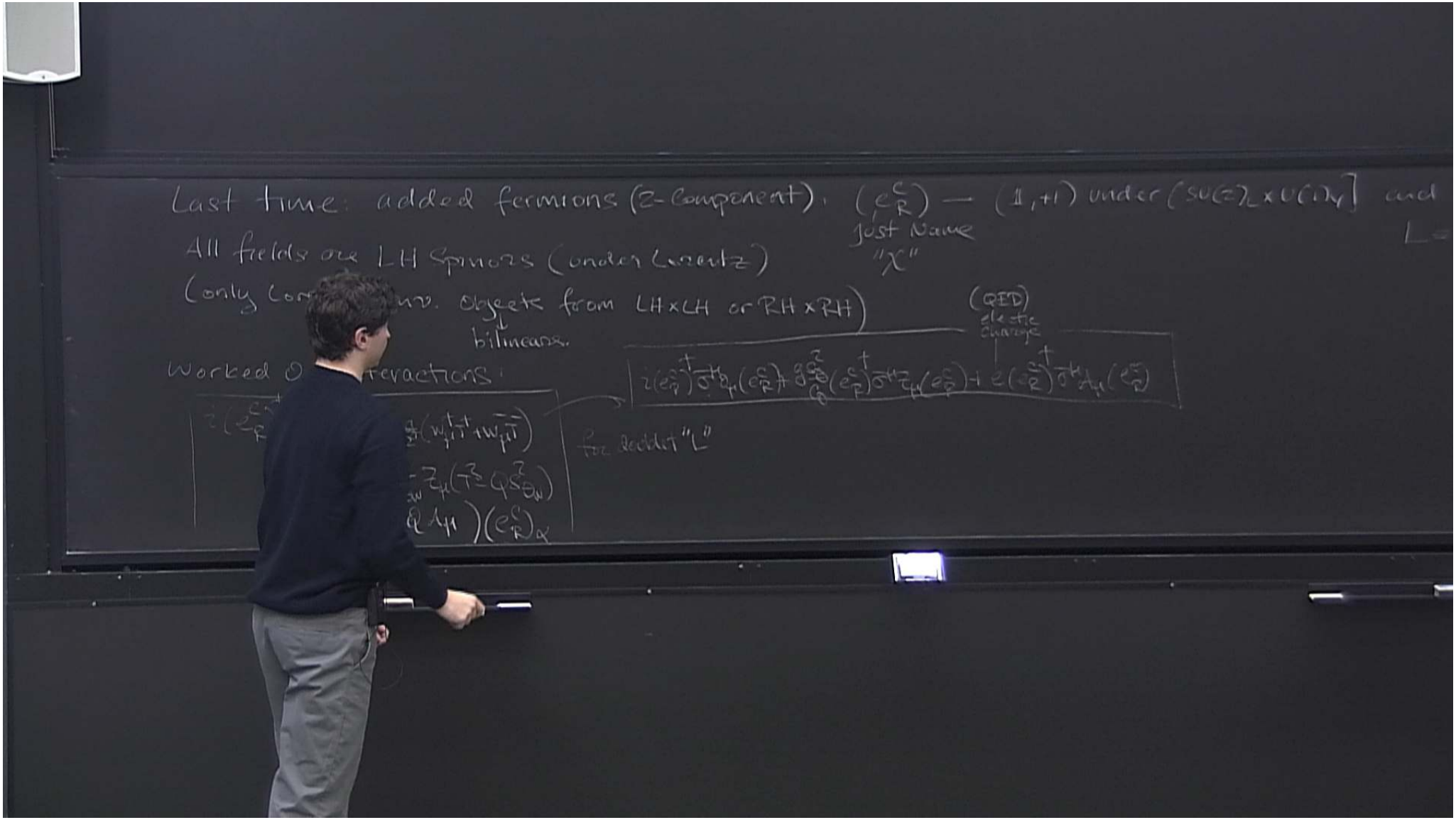
All fields are LH Spinors (under Lorentz)
(only Lorentz-Inv. Objects from LH×LH or RH×RH)
(QED) electric charge
Worked Out Interactions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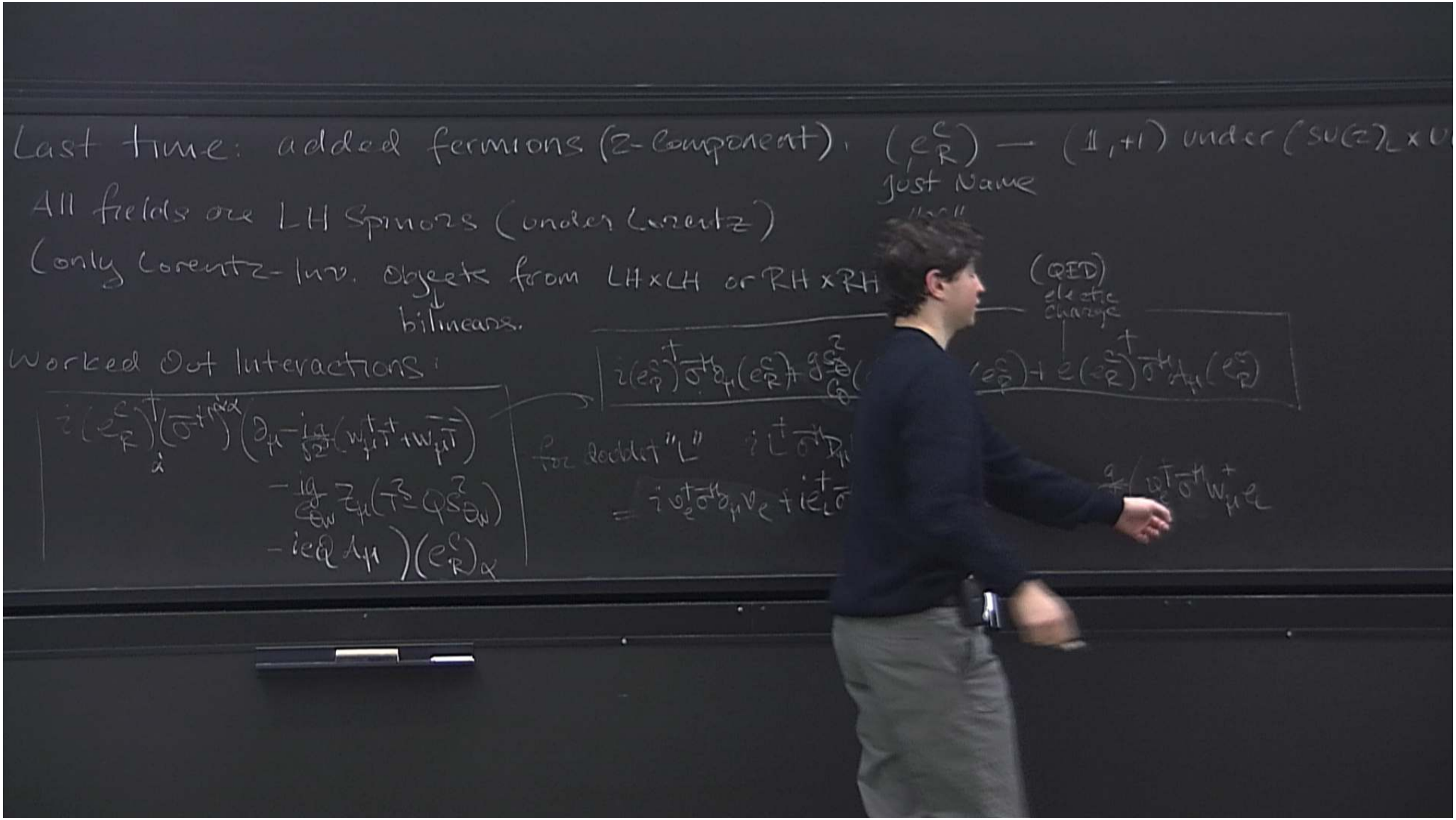
Just Name "χ"
(only
bilinears.
(QED)
electric
charge
Worked
for decided "L"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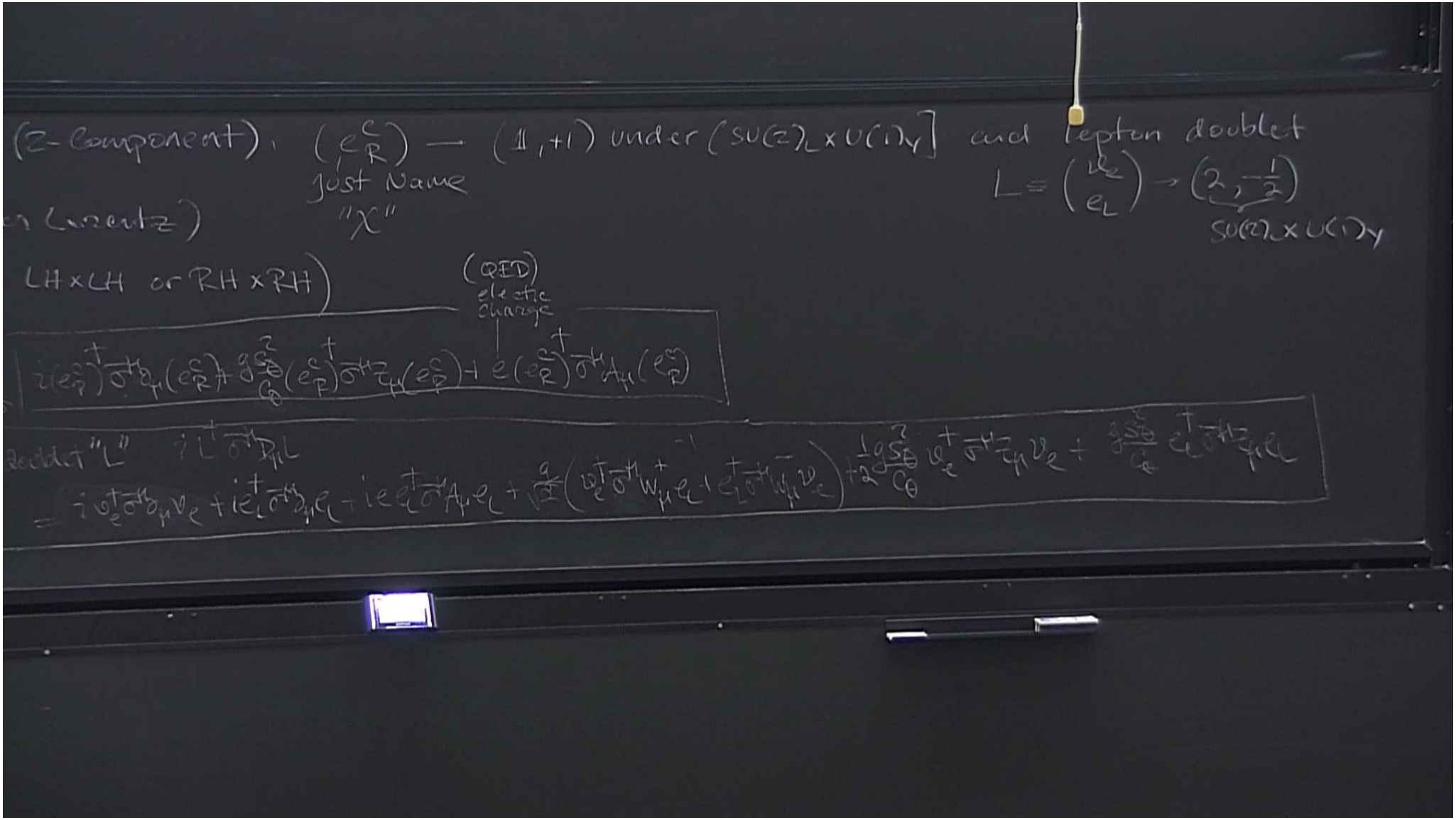
All fields are LH Spinors (under Lorentz)
(only Lorentz-Inv. Objects from LH×LH or RH×RH
bilinears.
Worked Out Interactions:
Just Name
(QED)
electric charge


(2-component): (e^c_R) — $(1,+1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet
$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \to (2, -\frac{1}{2})$
$SU(2)_L \times U(1)_Y$
just Name "χ"
(Lorentz)
LH×LH or RH×RH)
(QED electric charge)
$i(e^c_R)^\dagger \bar{\sigma}^\mu \partial_\mu (e^c_R) + g\frac{s^2_\theta}{c_\theta}(e^c_R)^\dagger \bar{\sigma}^\mu Z_\mu (e^c_R) + e(e^c_R)^\dagger \bar{\sigma}^\mu A_\mu (e^c_R)$
doublet "L" $i L^\dagger \bar{\sigma}^\mu D_\mu L$
$= i\nu_e^\dagger \bar{\sigma}^\mu \partial_\mu \nu_e + i e_L^\dagger \bar{\sigma}^\mu \partial_\mu e_L + i e e_L^\dagger \bar{\sigma}^\mu A_\mu e_L + \frac{g}{\sqrt{2}}\left(\nu_e^\dagger \bar{\sigma}^\mu W^+_\mu e_L + e_L^\dagger \bar{\sigma}^\mu W^-_\mu \nu_e\right) + \frac{1}{2} g \frac{s^2_\theta}{c_\theta} \nu_e^\dagger \bar{\sigma}^\mu Z_\mu \nu_e + g\frac{s^2_\theta}{c_\theta} e_L^\dagger \bar{\sigma}^\mu Z_\mu e_L$

(2-Component): $(e_R^c) \to (1, +1)$ under $[SU(2)_L \times U(1)_Y]$ and lepton doublet $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} \to (2, -\frac{1}{2})$

Just Name "χ"

$SU(2)_L \times U(1)_Y$

(Lorentz)

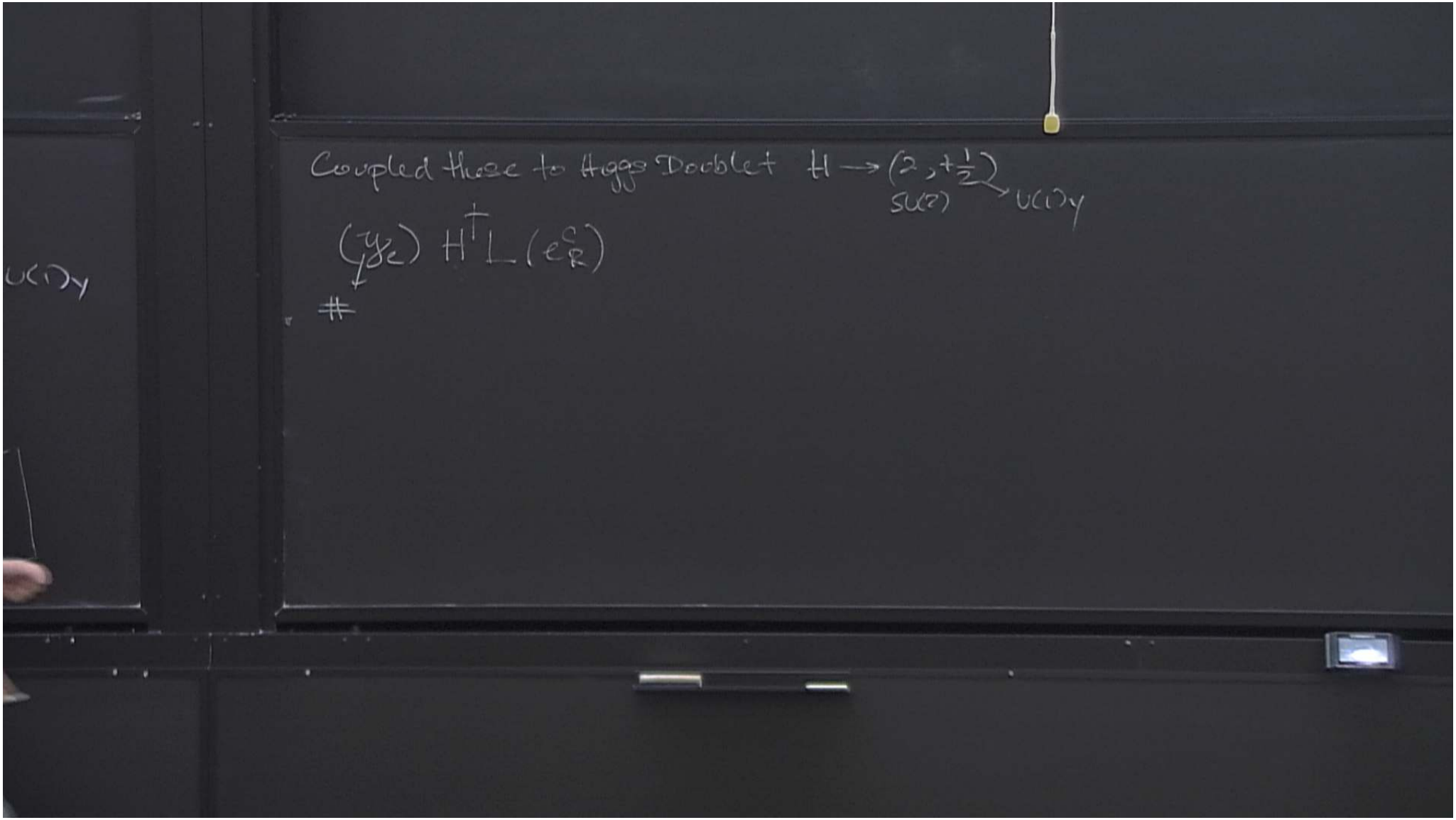
LH×LH or RH×RH)

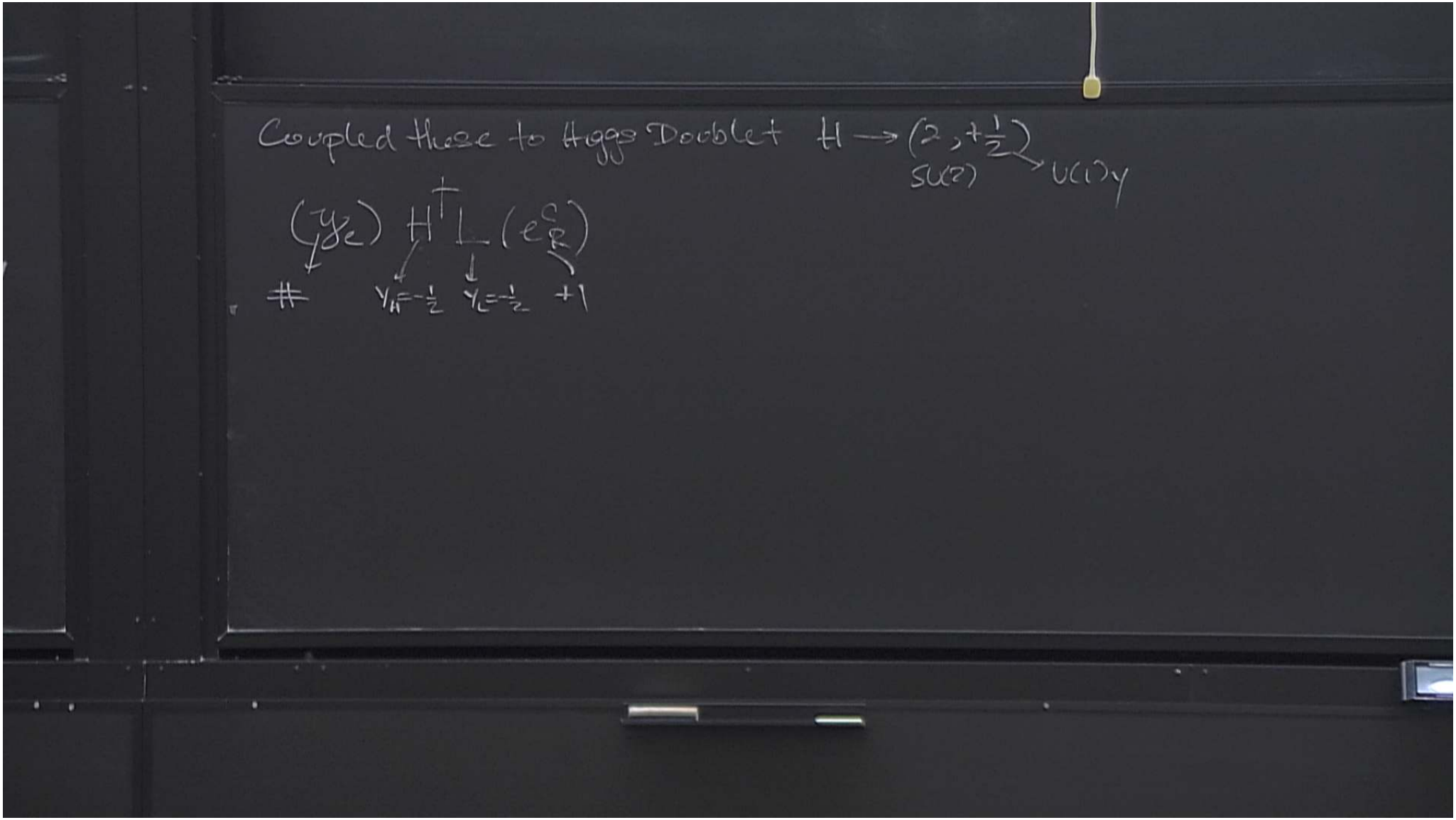
(QED electric charge)

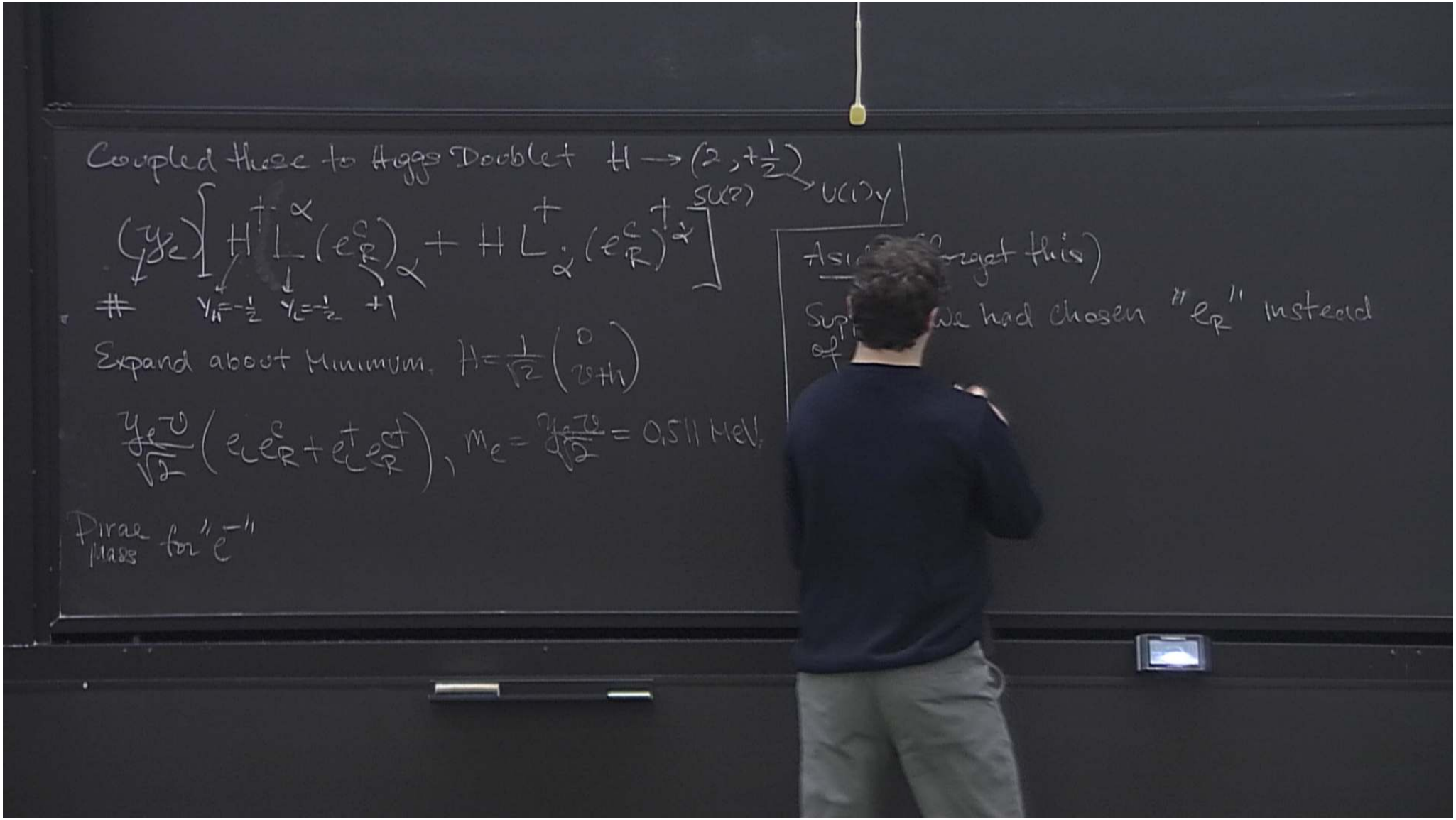
$$i(e_R^c)^\dagger \bar{\sigma}^\mu \partial_\mu (e_R^c) + g \frac{s_\theta^2}{c_\theta} (e_R^c)^\dagger \bar{\sigma}^\mu Z_\mu (e_R^c) + e (e_R^c)^\dagger \bar{\sigma}^\mu A_\mu (e_R^c)$$

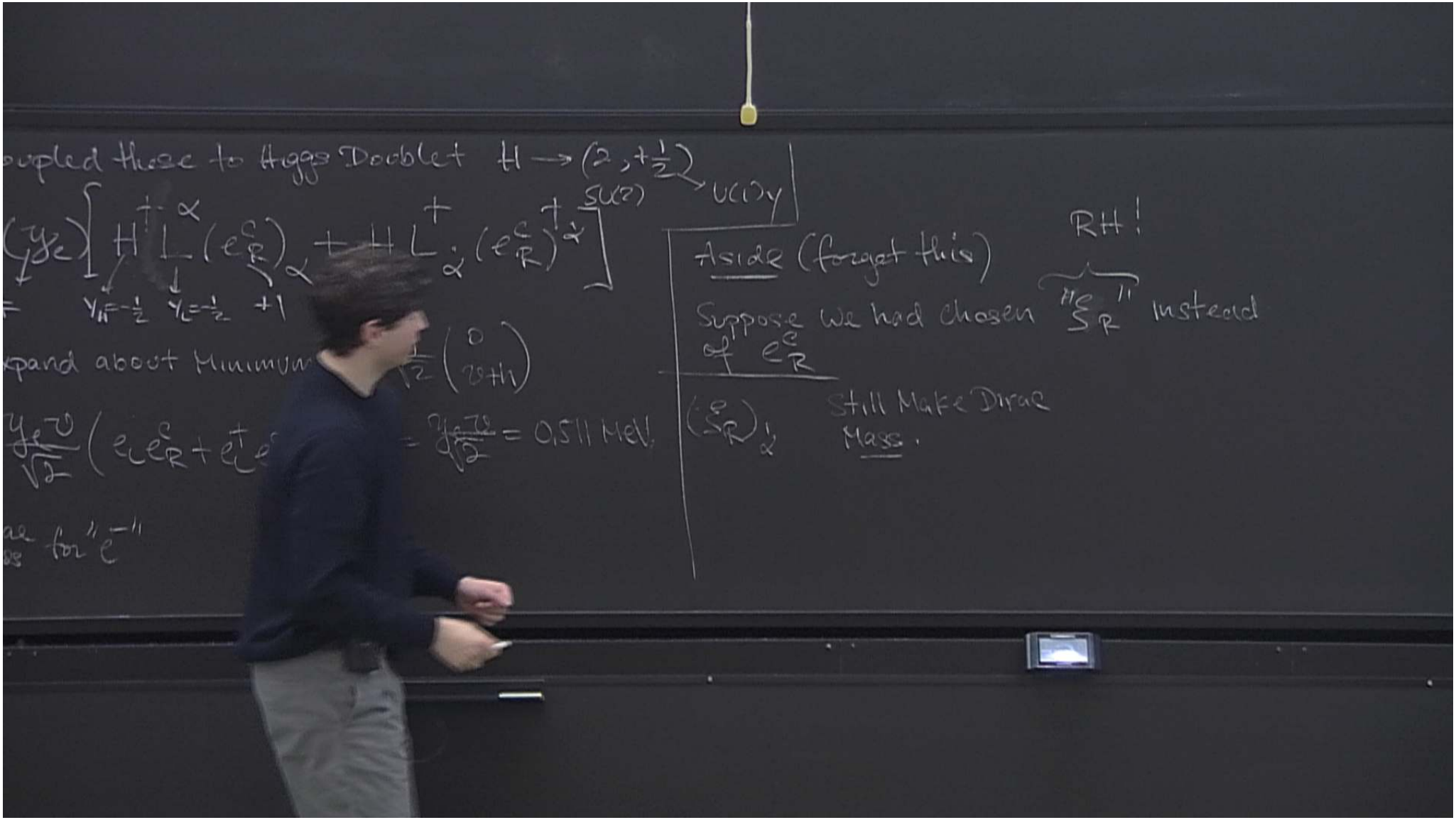
doublet "L" $i L^\dagger \bar{\sigma}^\mu D_\mu L$

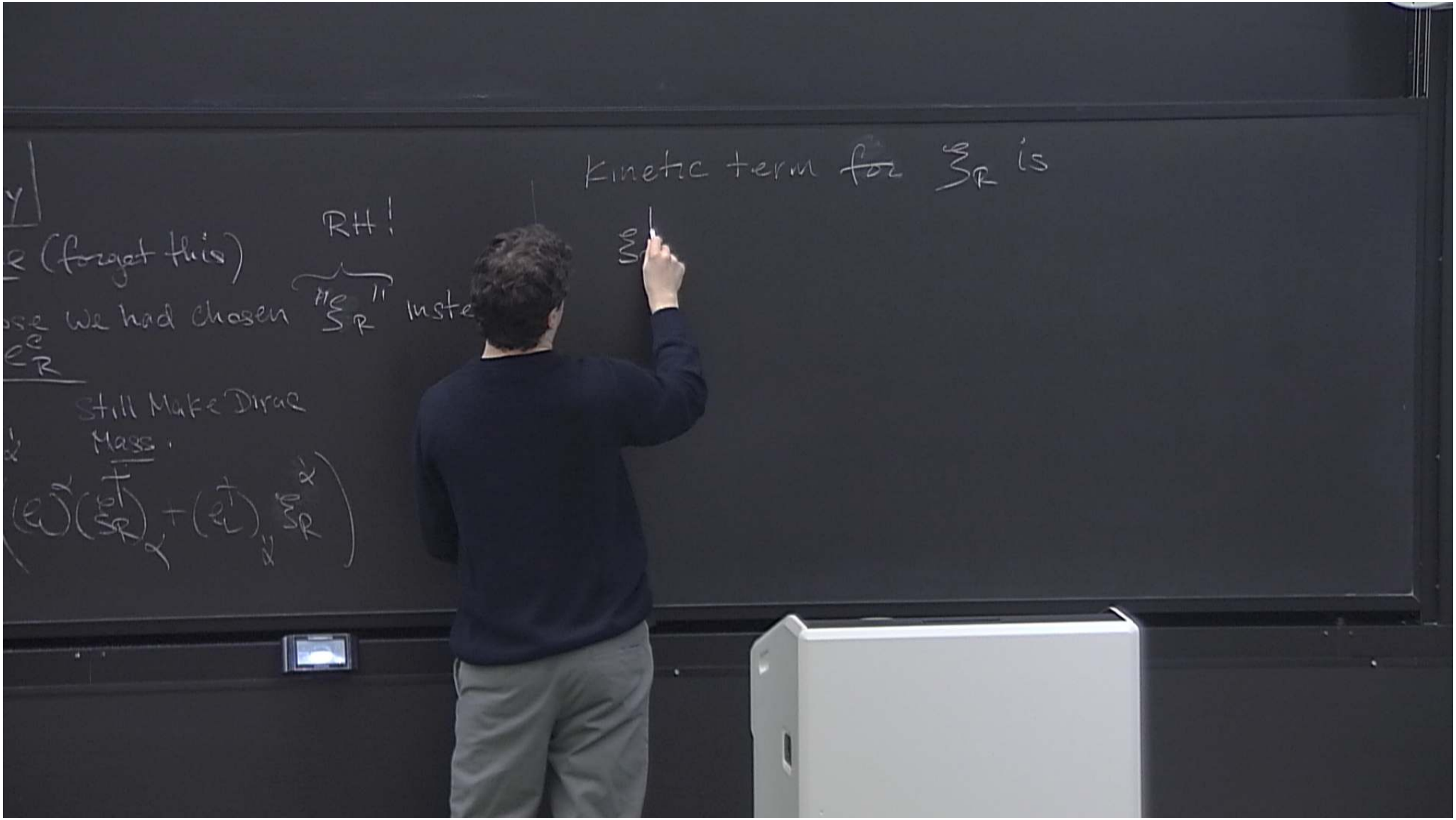
$$= i \nu_e^\dagger \bar{\sigma}^\mu \partial_\mu \nu_e + i e_L^\dagger \bar{\sigma}^\mu \partial_\mu e_L + i e e_L^\dagger \bar{\sigma}^\mu A_\mu e_L + \frac{g}{\sqrt{2}} \left(\nu_e^\dagger \bar{\sigma}^\mu W_\mu^+ e_L + e_L^\dagger \bar{\sigma}^\mu W_\mu^- \nu_e \right) + \frac{1}{2} g \frac{s_\theta^2}{c_\theta} \nu_e^\dagger \bar{\sigma}^\mu Z_\mu \nu_e + \frac{g}{c_\theta} \left(\frac{1}{2} + s_\theta^2 \right) e_L^\dagger \bar{\sigma}^\mu Z_\mu e_L$$

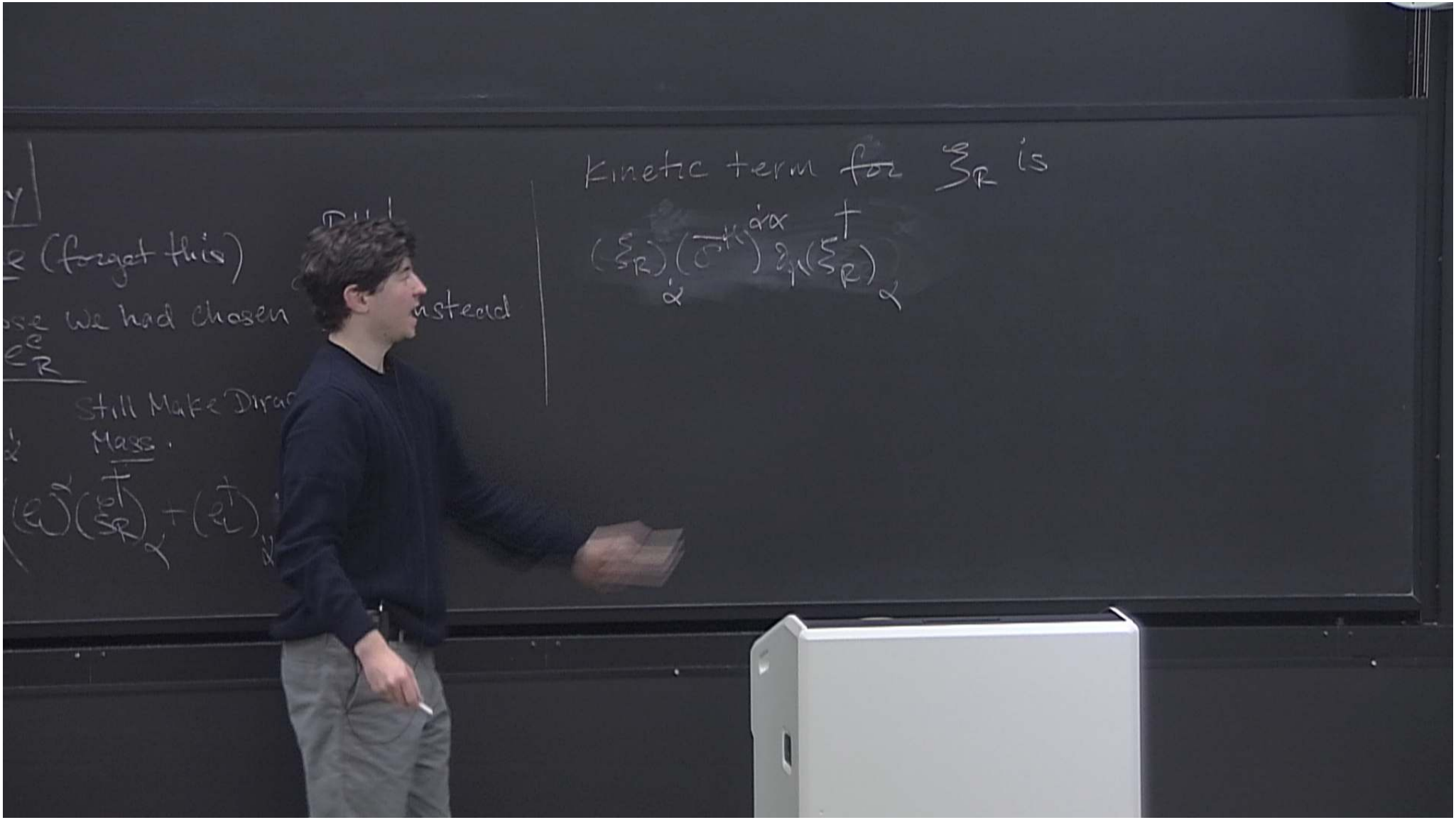

Coupled these to Higgs Doublet $H \to (2, +\frac{1}{2})$
$SU(2)$
$U(1)_Y$
$(y_e)\, H^\dagger L\, (e_R^c)$
#
$U(1)_Y$

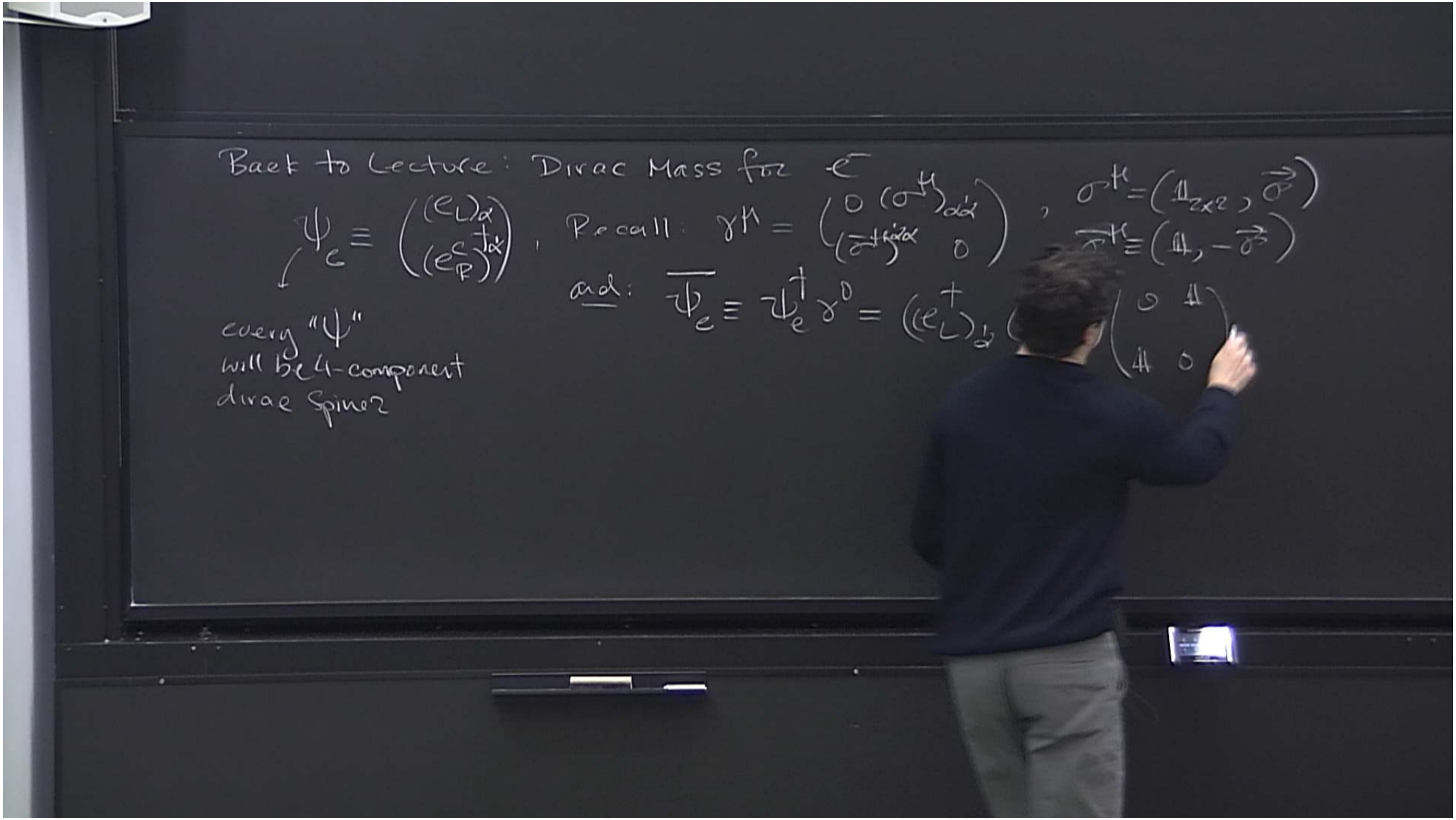

Coupled these to Higgs Doublet $H \to (2, +\frac{1}{2})$
$SU(2)$
$U(1)_Y$
$(y_e)\ H^\dagger L\ (e_R^c)$
#
$Y_H = -\frac{1}{2}$
$Y_L = -\frac{1}{2}$
$+1$


Coupled these to Higgs Doublet $H \to (2, +\frac{1}{2})$
$SU(2)$
$U(1)_Y$
$Y_H = -\frac{1}{2}$
$Y_L = -\frac{1}{2}$
$+1$
Expand about Minimum. $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$
$m_e = \frac{y_e v}{\sqrt{2}} = 0.511\ \text{MeV}$
Dirac mass for "e"


Aside (forget this)
RH!
Still Make Dirac Mass.


Kinetic term for ξ_R is
RH!
(forget this)
Still Make Dirac
Mass.


Kinetic term for ξ_R is
$(\xi_R)_\dot{\alpha}(\bar{\sigma}^\mu)^{\dot{\alpha}\alpha}\partial_\mu(\xi_R^\dagger)_\alpha$
(forget this)
Still Make Dirac
Mass.


Back to Lecture: Dirac Mass for e
$\psi_e \equiv \begin{pmatrix} (e_L)_\alpha \\ (e_R^\dagger)^{\dot\alpha} \end{pmatrix}$, Recall: $\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot\alpha} \\ (\bar\sigma^\mu)^{\dot\alpha\alpha} & 0 \end{pmatrix}$, $\sigma^\mu = (\mathbb{1}_{2\times2}, \vec\sigma)$
$\bar\sigma^\mu \equiv (\mathbb{1}, -\vec\sigma)$
and: $\bar\psi_e \equiv \psi_e^\dagger \gamma^0 = ((e_L^\dagger)_{\dot\alpha}$
$\begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$
every "ψ" will be 4-component dirac spinor

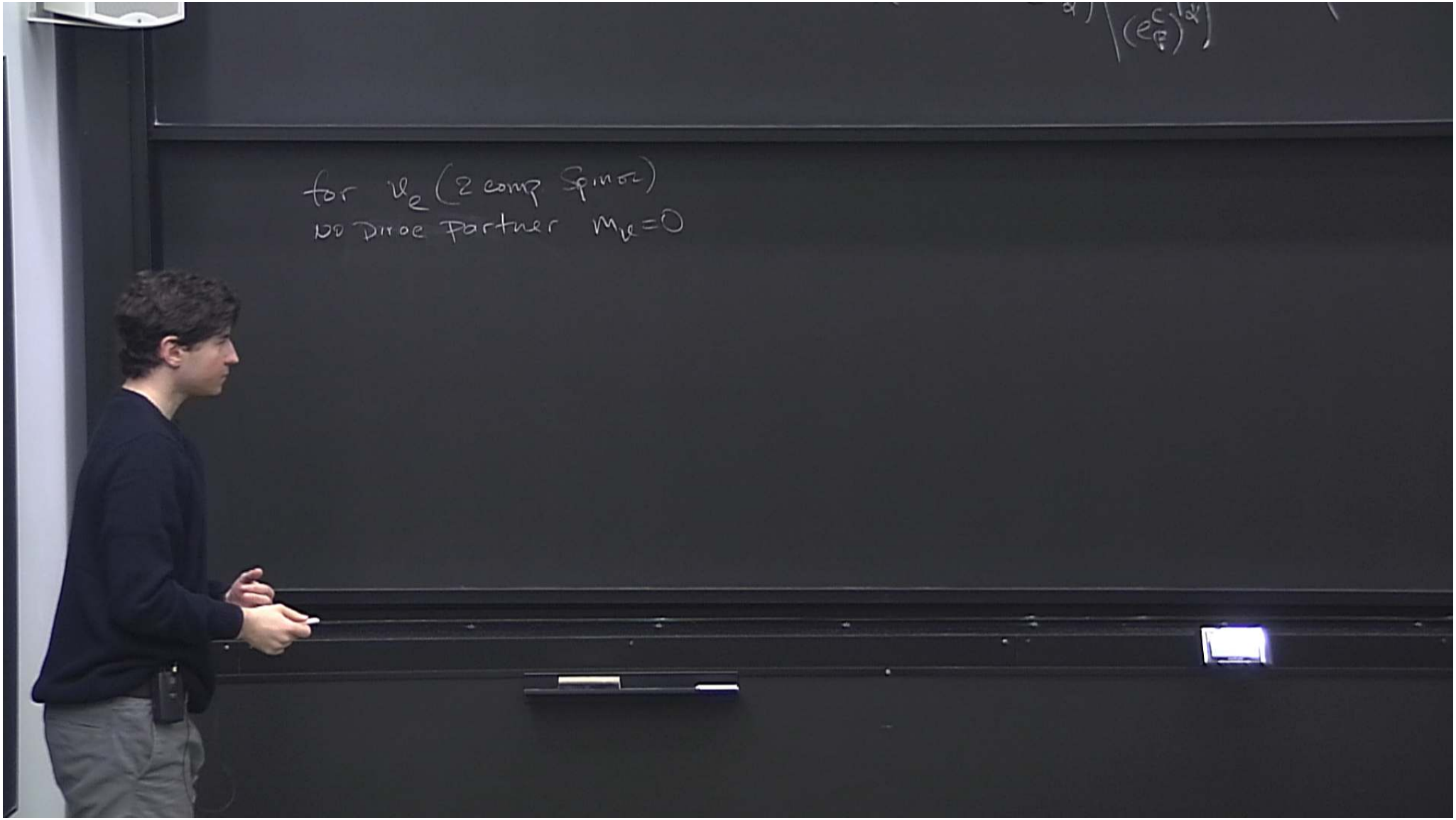
cture: Dirac Mass for e

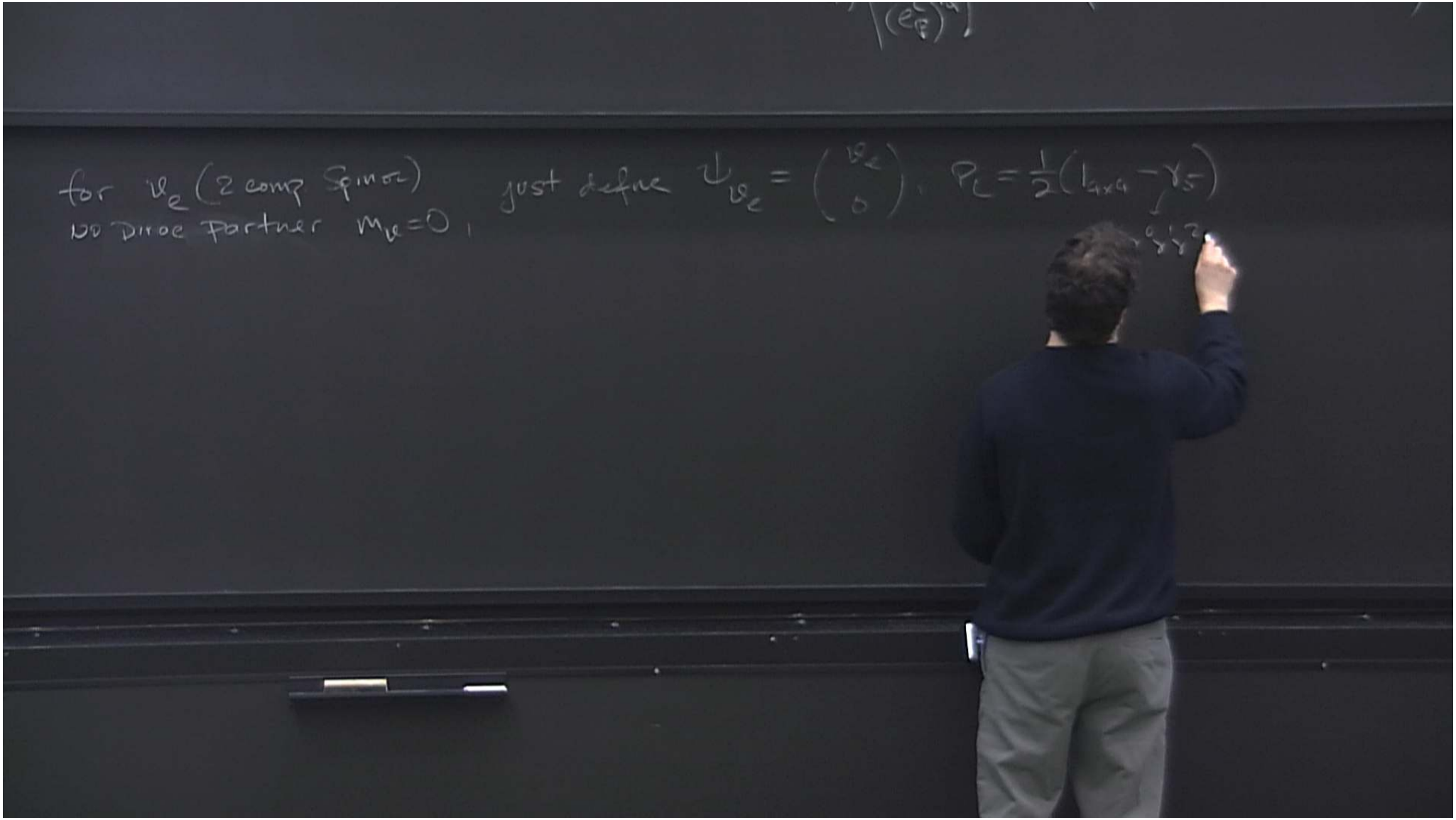
$\begin{pmatrix} (e_L)_\alpha \\ (e_R^c)^{\dagger\dot{\alpha}} \end{pmatrix}$, Recall: $\gamma^\mu = \begin{pmatrix} 0 & (\sigma^\mu)_{\alpha\dot{\alpha}} \\ (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} & 0 \end{pmatrix}$, $\sigma^\mu = (\mathbb{1}_{2\times 2}, \vec{\sigma})$, $\bar{\sigma}^\mu \equiv (\mathbb{1}, -\vec{\sigma})$

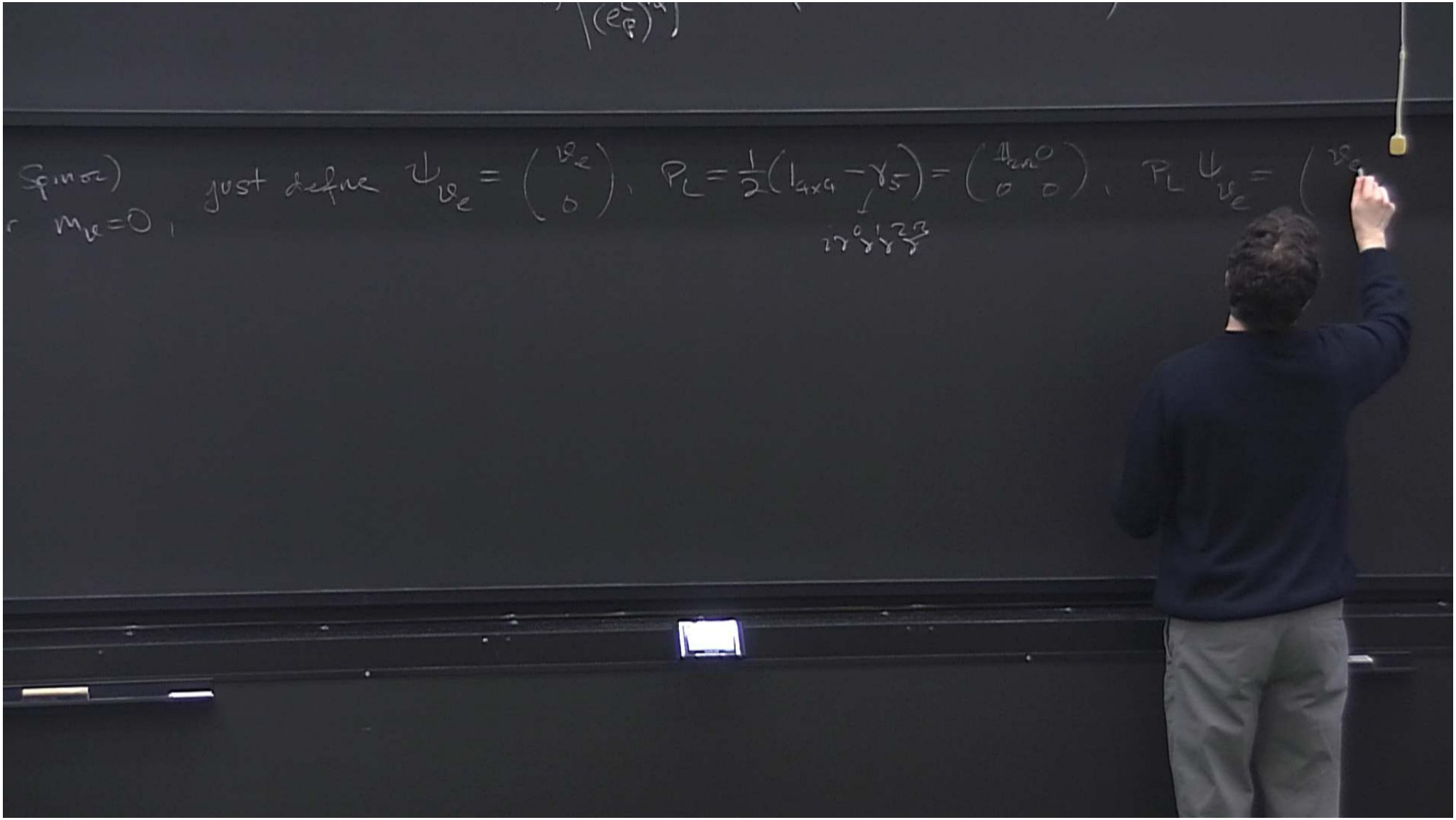
and: $\overline{\psi_e} \equiv \psi_e^\dagger \gamma^0 = \begin{pmatrix} (e_L^\dagger)_{\dot{\alpha}} & (e_R^c)^\alpha \end{pmatrix} \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} = \begin{pmatrix} (e_R^c)^\alpha & (e_L^\dagger)_{\dot{\alpha}} \end{pmatrix}$

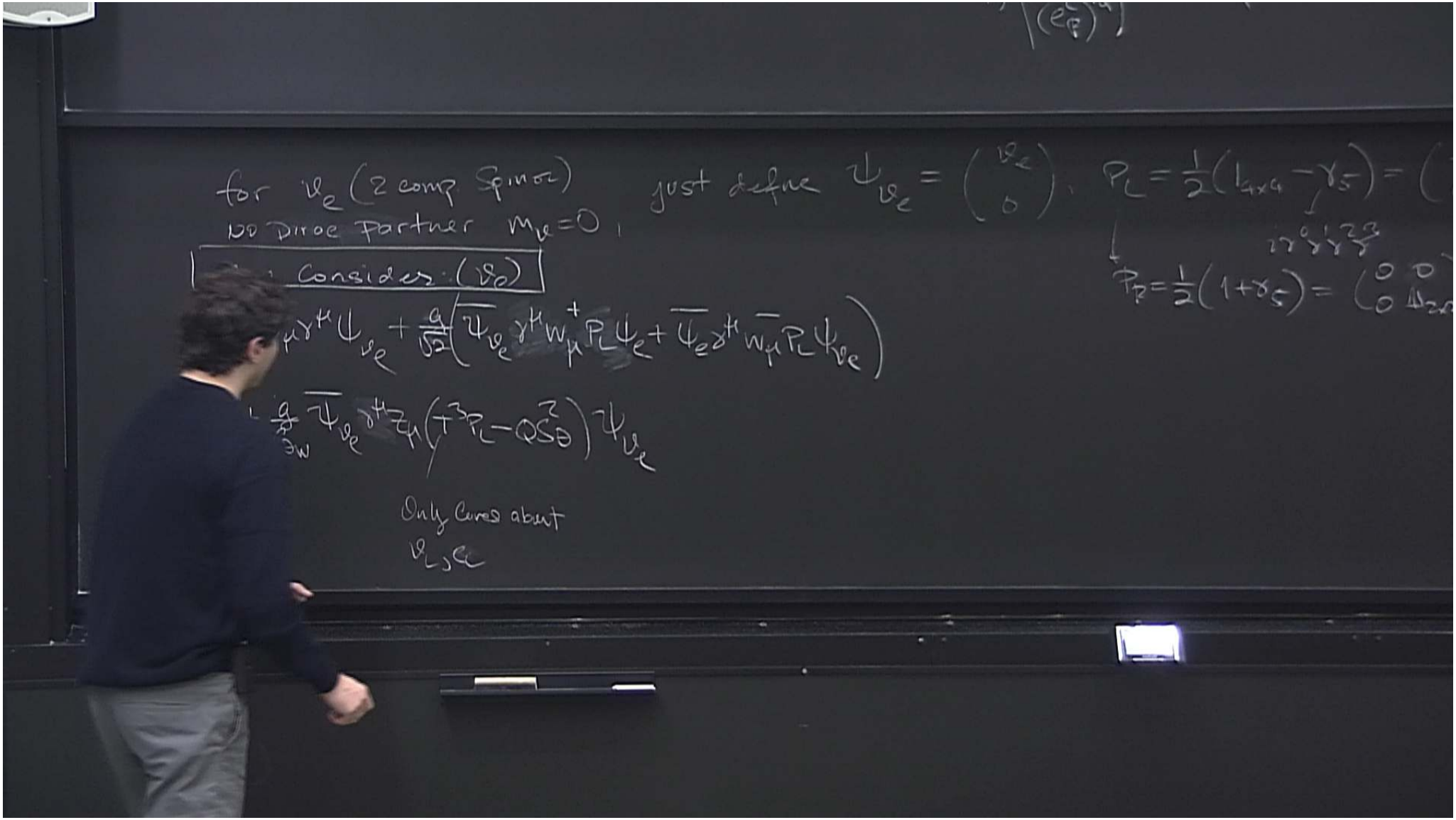
onent So Dirac Mass: $m_e \overline{\psi}_e \psi_e$

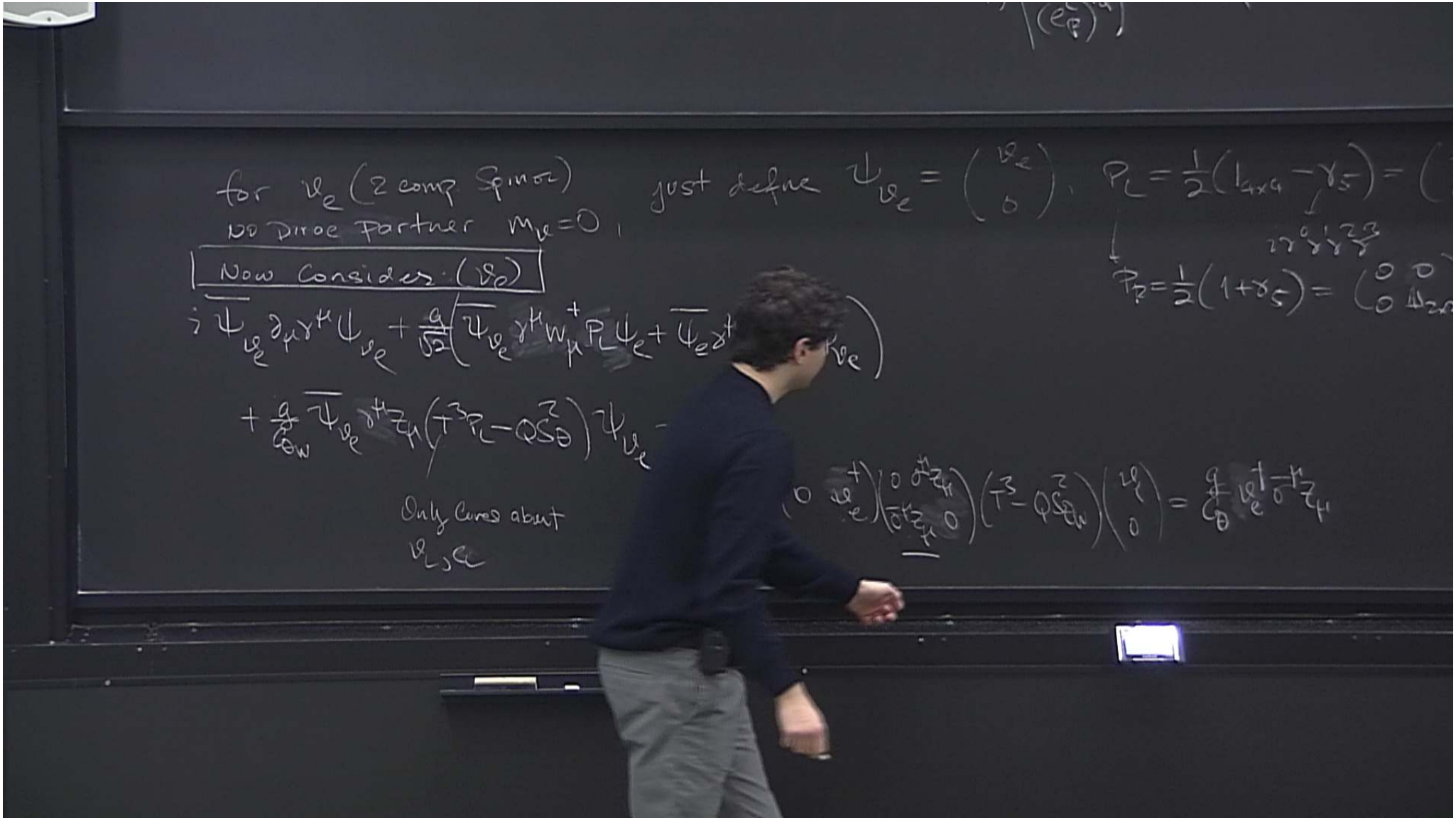
$= m_e \begin{pmatrix} (e_R^c)^\alpha & (e_L^\dagger)_{\dot{\alpha}} \end{pmatrix} \begin{pmatrix} (e_L)_\alpha \\ (e_R^c)^{\dagger\dot{\alpha}} \end{pmatrix} = m_e \left((e_R^c)^\alpha e_{L\alpha} + (e_L^\dagger)_{\dot{\alpha}} (e_R^c)^{\dagger\dot{\alpha}} \right)$

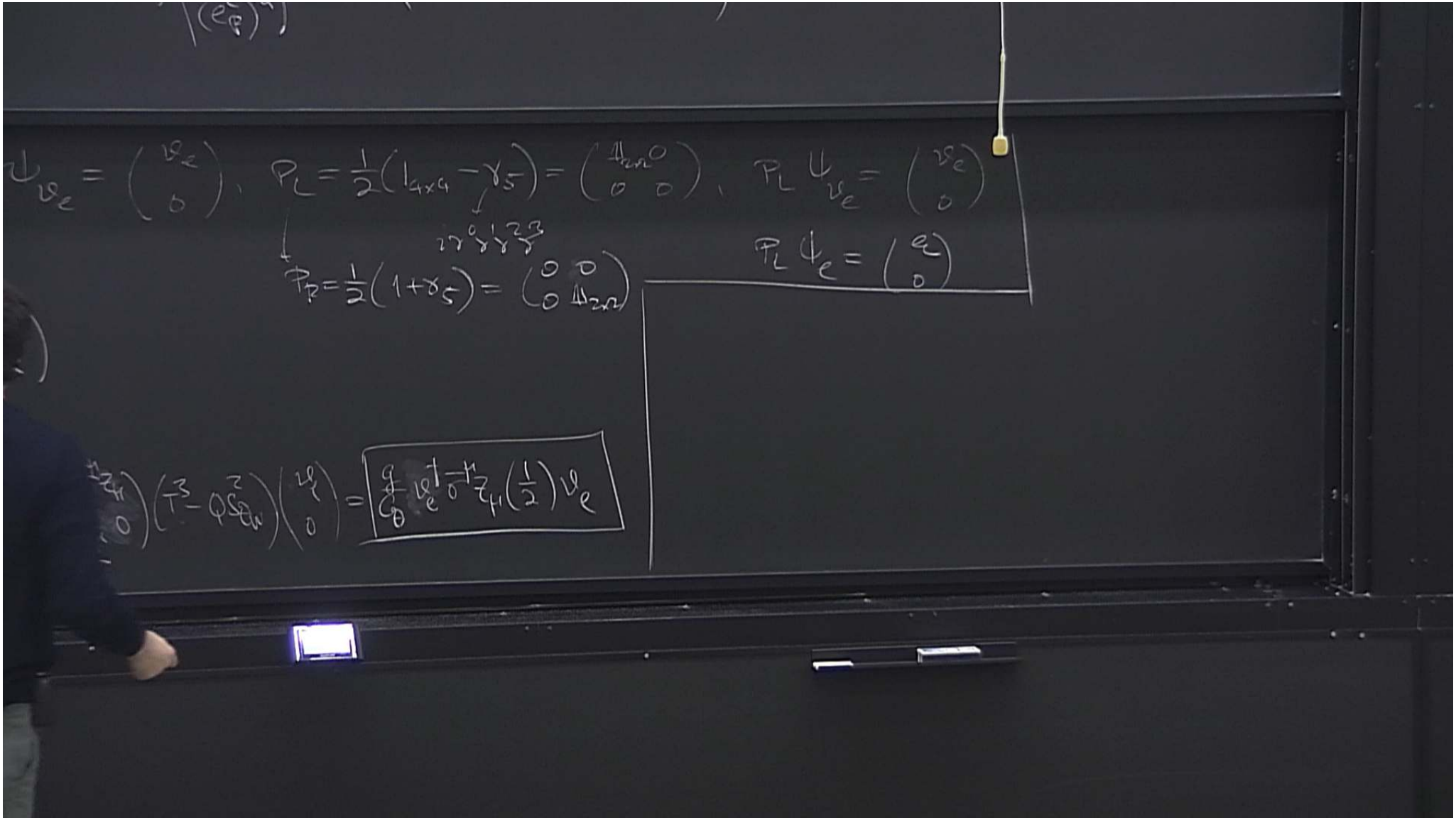

for ν_e (2 comp Spinor)
No Dirac Partner $m_{\nu_e}=0$

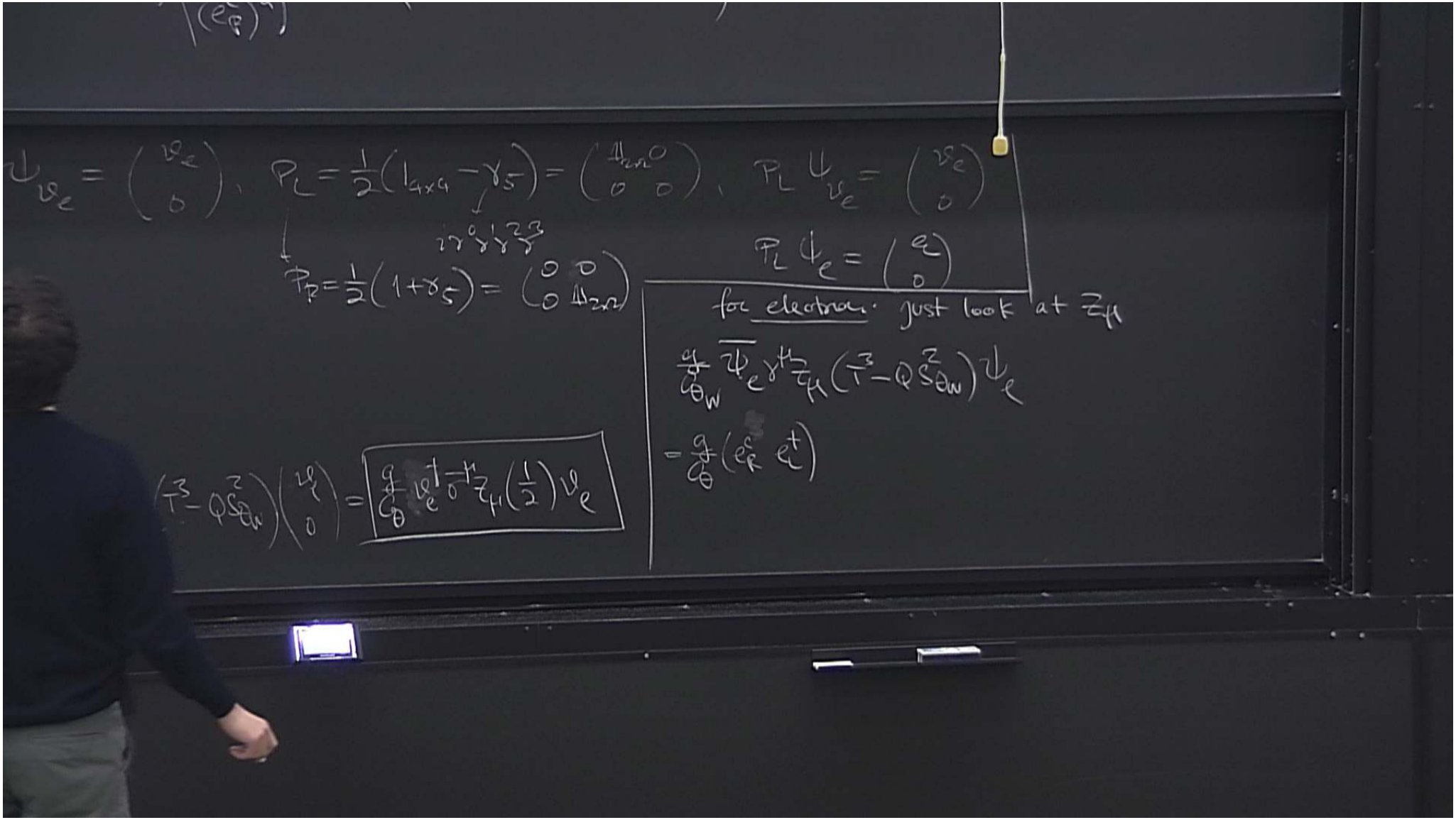

for ν_e (2 comp Spinor)
no Dirac Partner $m_\nu = 0$,
just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1_{4\times4} - \gamma_5)$


Spinor)
just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1_{4\times4} - \gamma_5) = \begin{pmatrix} 1_{2\times2} & 0 \\ 0 & 0 \end{pmatrix}$, $P_L \psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$
$m_\nu = 0$,


for ν_e (2 comp Spinor)
No Dirac Partner $m_\nu = 0$
just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$
$P_L = \frac{1}{2}(1_{4\times4} - \gamma_5)$
$P_R = \frac{1}{2}(1+\gamma_5)$
Consider: (ν_e)
$\frac{g}{\sqrt{2}}\left(\bar{\psi}_{\nu_e}\gamma^\mu W_\mu^+ P_L \psi_e + \bar{\psi}_e \gamma^\mu W_\mu^- P_L \psi_{\nu_e}\right)$
$\frac{g}{2\cos\theta_W}\bar{\psi}_{\nu_e}\gamma^\mu Z_\mu\left(T^3 P_L - Q\sin^2\theta\right)\psi_{\nu_e}$
Only Cares about
ν_L's


for ν_e (2 comp Spinor)
No Dirac Partner $m_\nu = 0$,
just define $\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$
$P_L = \frac{1}{2}(1_{4\times4} - \gamma_5)$
$P_R = \frac{1}{2}(1+\gamma_5)$
Now Consider (ν_e)
$i\bar{\psi}_{\nu_e}\partial_\mu\gamma^\mu\psi_{\nu_e} + \frac{g}{\sqrt{2}}\left(\bar{\psi}_{\nu_e}\gamma^\mu W^+_\mu P_L \psi_e + \bar{\psi}_e \gamma^\mu W^-_\mu P_L \psi_{\nu_e}\right)$
$+ \frac{g}{\cos\theta_W}\bar{\psi}_{\nu_e}\gamma^\mu Z_\mu\left(T^3 P_L - Q\sin^2\theta_W\right)\psi_{\nu_e}$
Only Cares about ν_L's
$(0 \;\; \nu_e^\dagger)\begin{pmatrix} 0 & \bar{\sigma}^\mu Z_\mu \\ \bar{\sigma}^\mu Z_\mu & 0 \end{pmatrix}\left(T^3 - Q\sin^2\theta_W\right)\begin{pmatrix} \nu_e \\ 0 \end{pmatrix} = \frac{g}{\cos\theta_W}\nu_e^\dagger \bar{\sigma}^\mu Z_\mu$


$\psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$, $P_L = \frac{1}{2}(1_{4\times4} - \gamma_5) = \begin{pmatrix} 1_{2\times2} & 0 \\ 0 & 0 \end{pmatrix}$, $P_L \psi_{\nu_e} = \begin{pmatrix} \nu_e \\ 0 \end{pmatrix}$
$P_L \psi_e = \begin{pmatrix} e \\ 0 \end{pmatrix}$
$P_R = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1_{2\times2} \end{pmatrix}$
$\left(T^3 - Q s_W^2\right)\begin{pmatrix} \nu_e \\ 0 \end{pmatrix} = \left(\frac{1}{2} - 0\right)\nu_e = \frac{1}{2}\nu_e$
$Z_\mu \bar{\psi}_e \gamma^\mu Z_\mu \left(\frac{1}{2}\right) \psi_e$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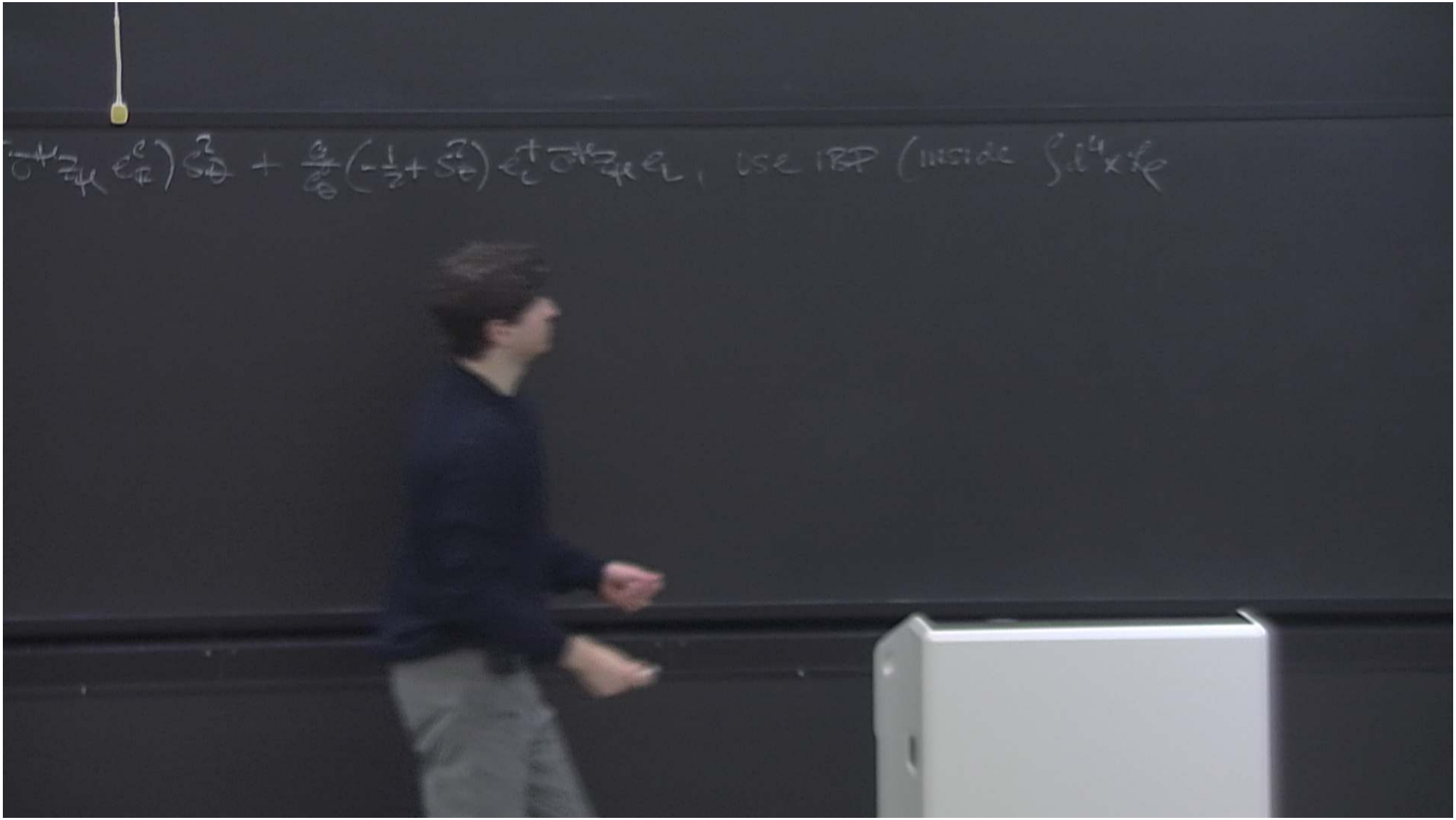
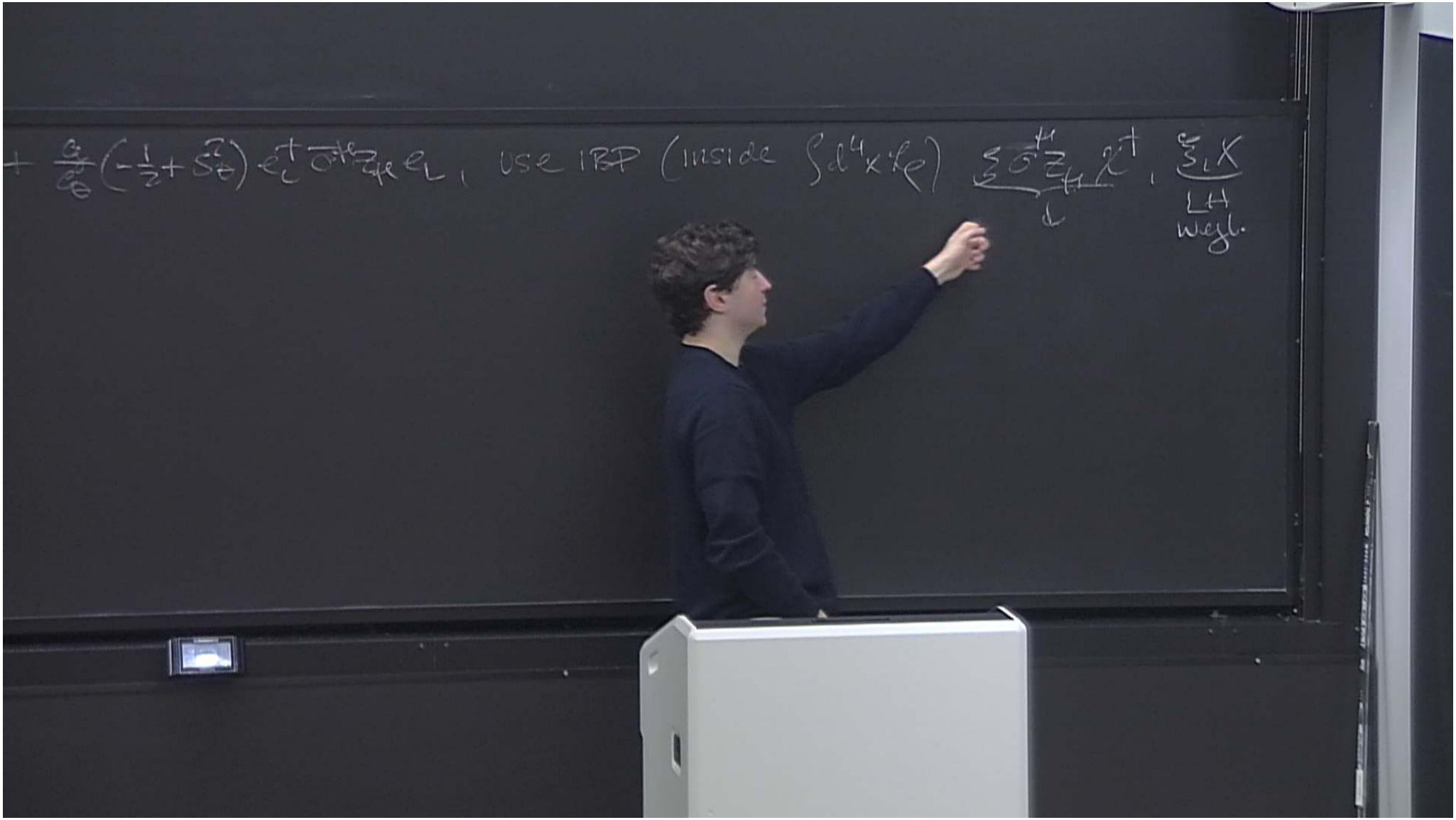
$i\gamma^0\gamma^1\gamma^2\gamma^3$
$P_R = \frac{1}{2}(1+\gamma_5) = \begin{pmatrix} 0 & 0 \\ 0 & 1_{2\times2} \end{pmatrix}$
$P_L \psi_e = \begin{pmatrix} e \\ 0 \end{pmatrix}$
for electron: just look at Z^μ
$\frac{g}{\cos\theta_W} \bar{\psi}_e \gamma^\mu Z_\mu (T^3 - Q\sin^2\theta_W)\psi_e$
$= \frac{g}{\cos\theta_W}(\bar{e}_R \; \bar{e}_L^\dagger)$
$(T^3 - Q\sin^2\theta_W)\begin{pmatrix} \nu_e \\ 0 \end{pmatrix} = \frac{g}{\cos\theta_W} \nu_e^\dagger \bar{\nu}_e^\dagger Z_\mu \left(\frac{1}{2}\right)\psi_e$


use IBP (inside


use IBP (inside ∫d⁴x ℒ)
LH
Weyl.

$$\frac{g}{c_\theta}\,\bar{\psi}_e\,\gamma^\mu Z_\mu\left(T^3 - Q s_{\theta_W}^2\right)\psi_e = \left(\frac{g}{c_\theta}\right)\left(e_R^{c\dagger}\,\gamma^\mu Z_\mu\, e_R^c\right) s_\theta^2 + \frac{g}{c_\theta}\left(-\frac{1}{2} + s_\theta^2\right) e_L^\dagger$$

found in Peskin & Schroeder (Srednicki) QFT
Ch 20.

Also

$$\frac{g}{c_\theta}\bar{\psi}_e \gamma^\mu Z_\mu (T^3 - Q s_{\theta_W}^2)\psi_e = \left(\frac{g}{c_\theta}\right)\left(e_R^c{}^\dagger \bar{\sigma}^\mu Z_\mu e_R^c\right) s_\theta^2 + \frac{g}{c_\theta}\left(-\frac{1}{2} + s_\theta^2\right) \epsilon_L^\dagger$$

found in Peskin & Schroeder (Srednicki) QFT
Ch 20.

Also 2-Component Spinor QFT

$$\frac{g}{c_\theta}\bar{\psi}_e \gamma^\mu z_\mu (T^3 - Q s^2_{\theta w})\psi_e = \left(\frac{g}{c_\theta}\right)\left(e_R^{c\dagger}\bar{\sigma}^\mu z_\mu e_R^c\right) s^2_\theta + \frac{g}{c_\theta}\left(-\frac{1}{2} + s^2_\theta\right) e_L^\dagger \bar{\sigma}^\mu z_\mu e$$

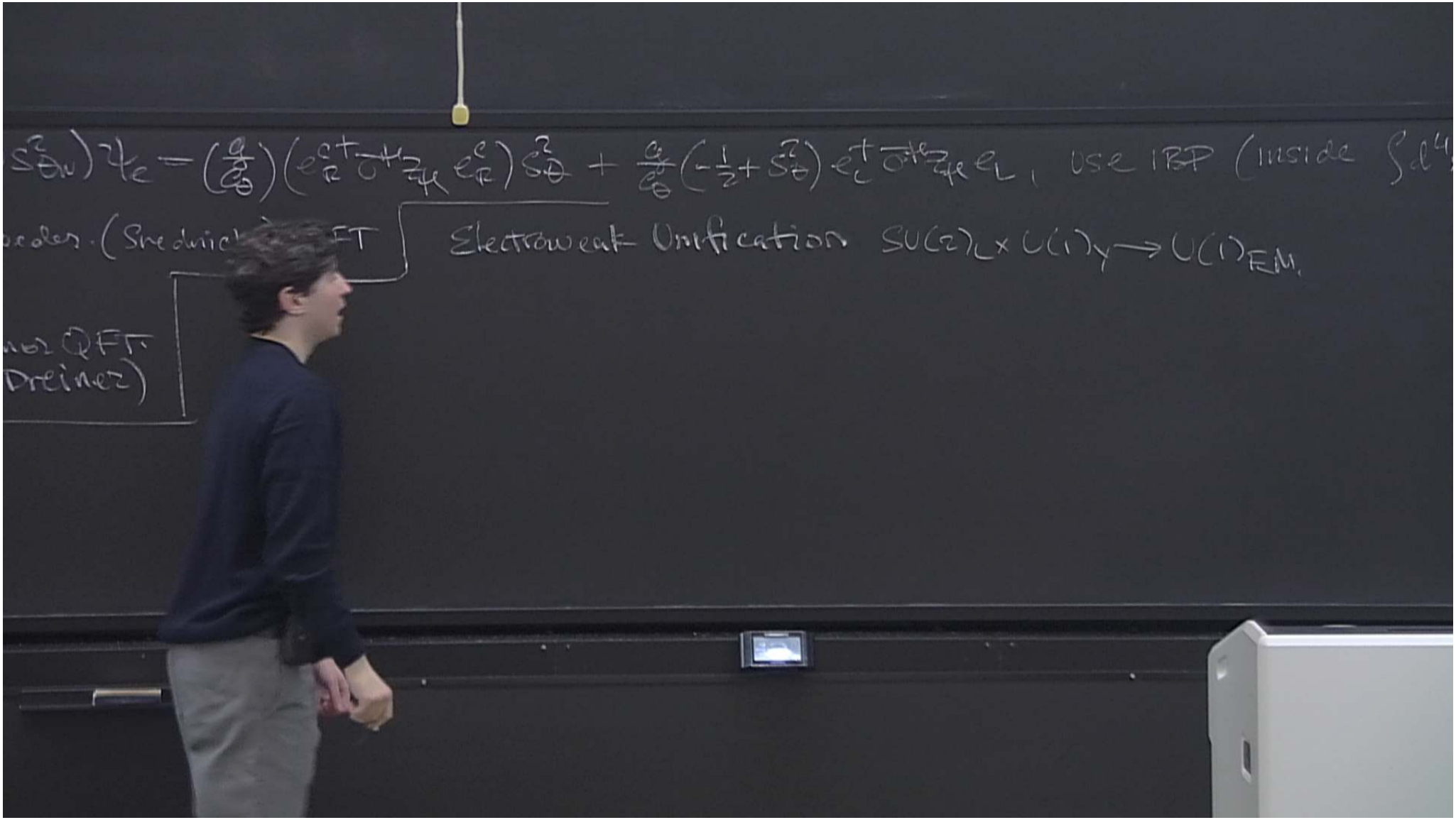
found in Peskin & Schroeder. (Srednicki) QFT
Ch 20.

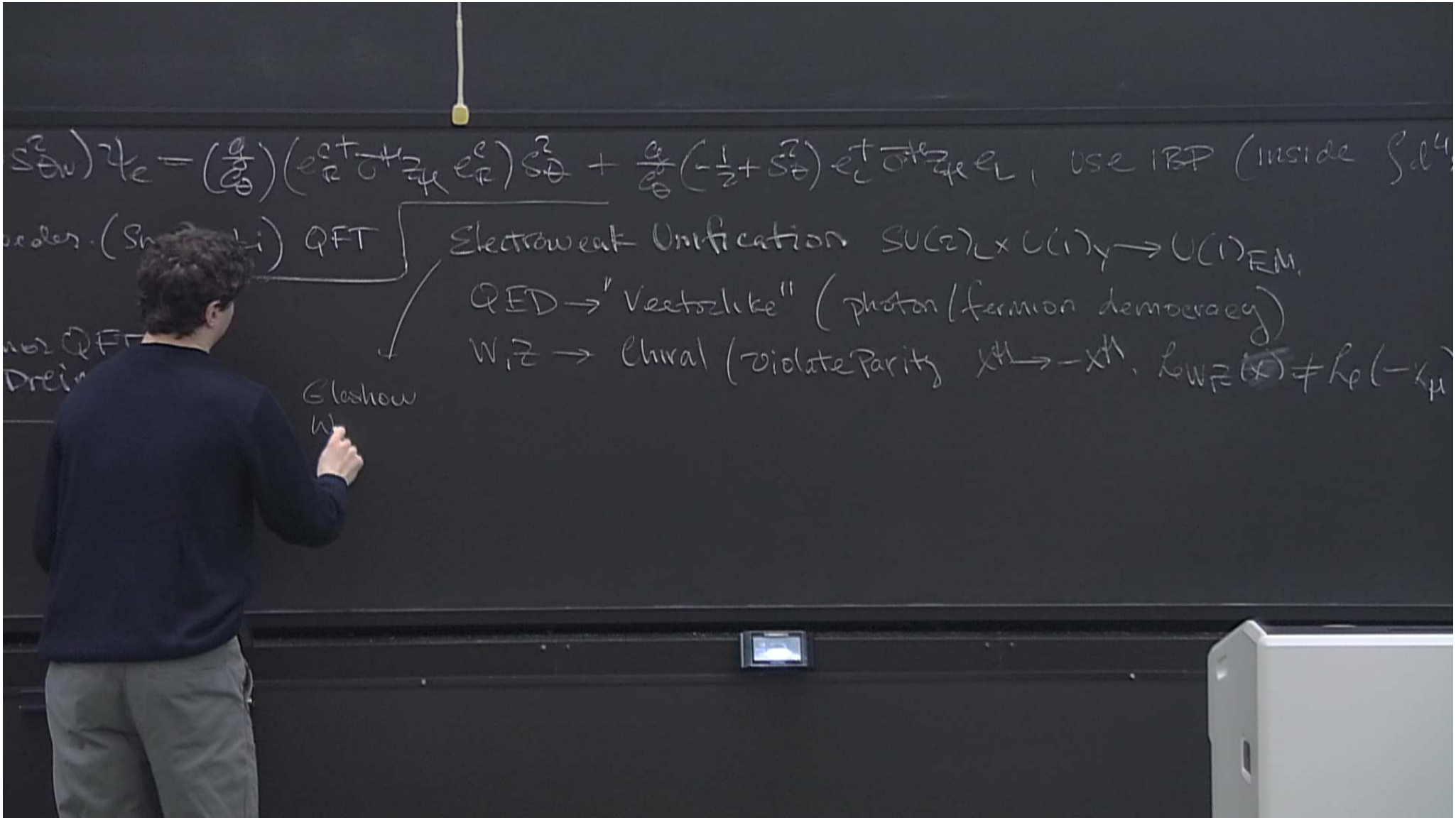
Also 2-Component Spinor QFT.
(Martin, Haber, Dreiner)

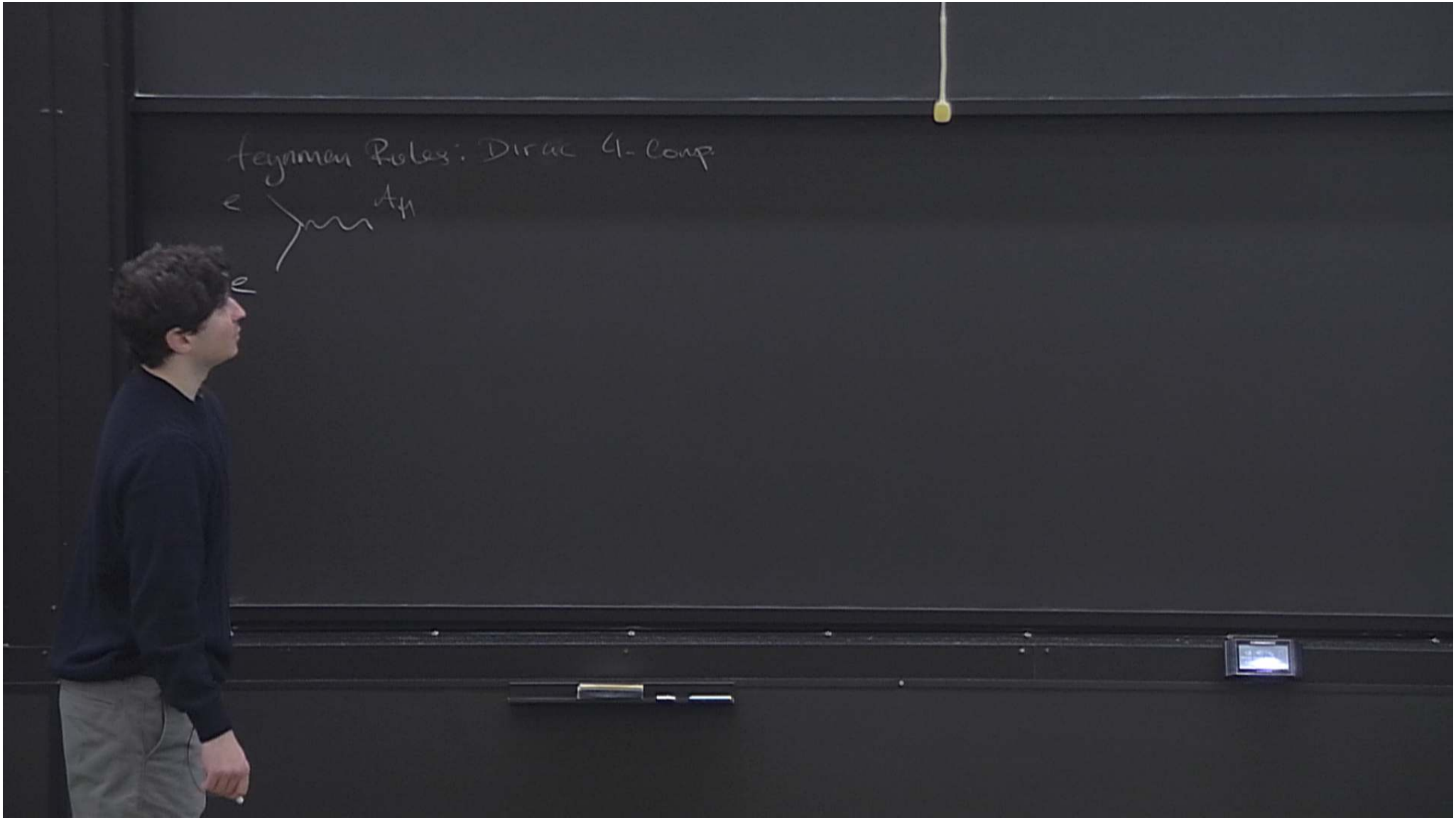
$$\frac{g}{c_\theta}\bar{\psi}_e \gamma^\mu Z_\mu (T^3 - Q s^2_{\theta_W}) \psi_e = \left(\frac{g}{c_\theta}\right)\left(e_R^{c\dagger} \bar{\sigma}^\mu Z_\mu e_R^c\right) s^2_\theta + \frac{g}{c_\theta}\left(-\frac{1}{2} + s^2_\theta\right) e_L^\dagger \bar{\sigma}^\mu Z_\mu e$$

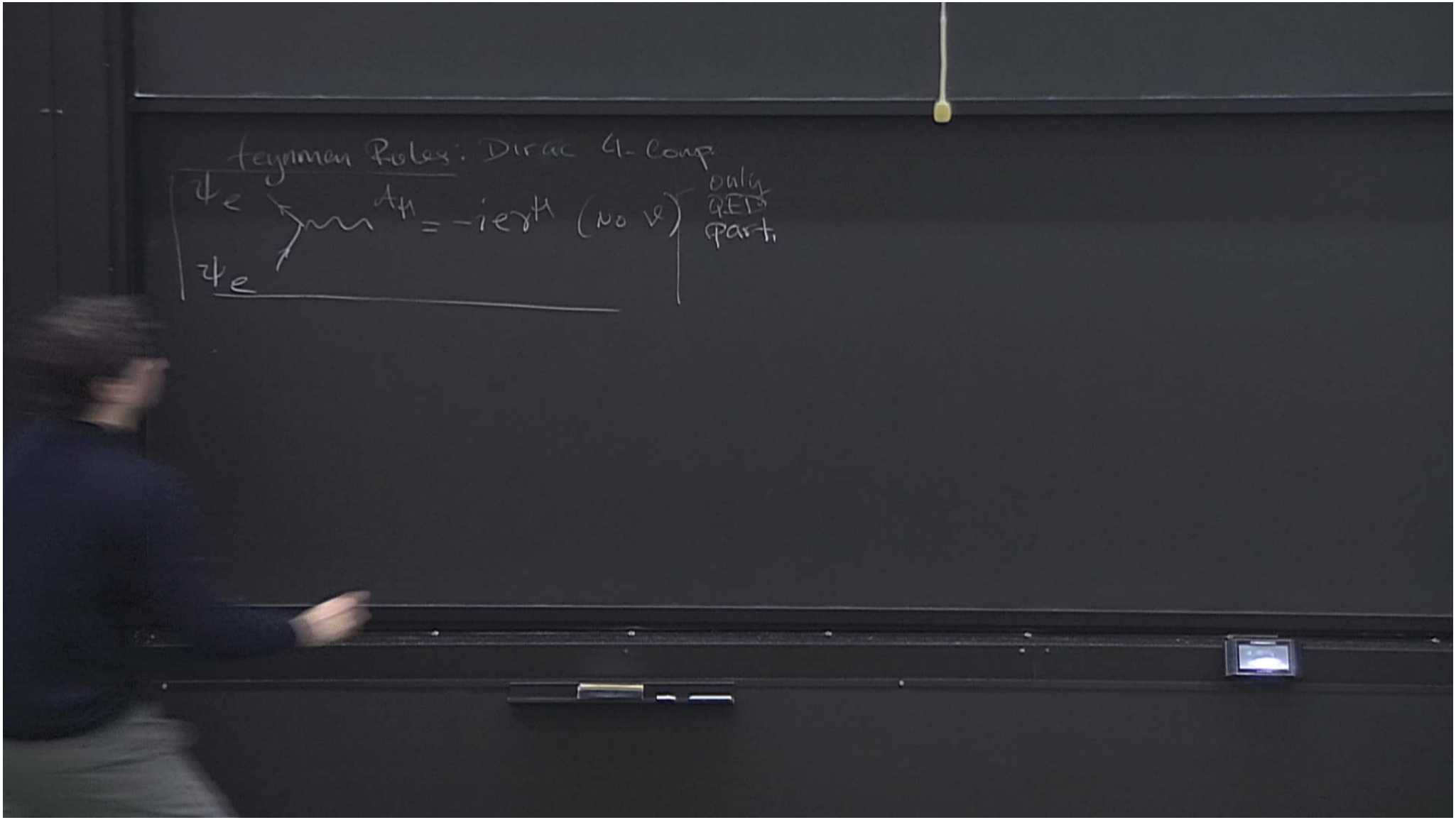
found in Peskin & Schroeder (Srednicki) QFT
Ch 20.

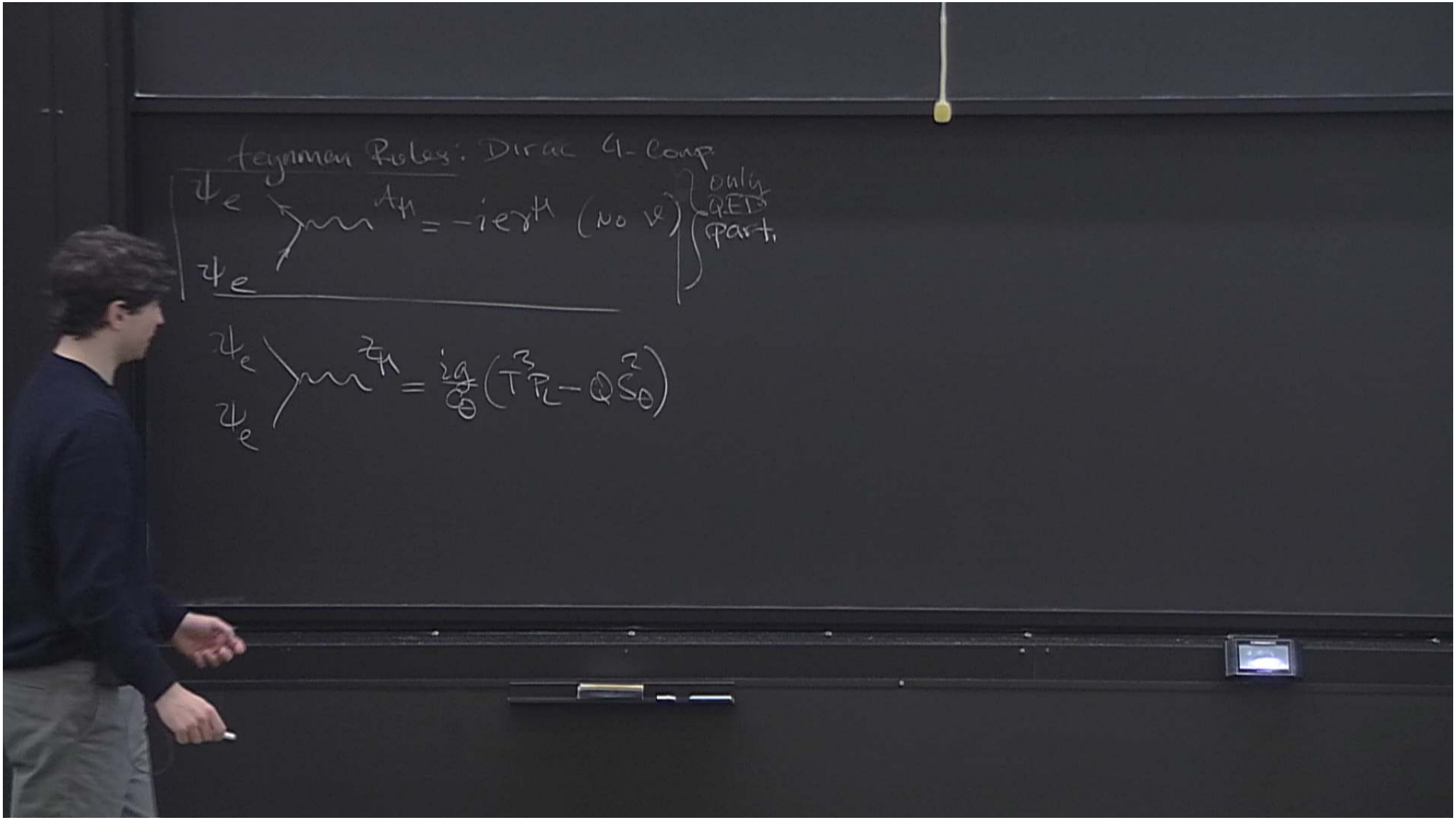
Also 2-Component Spinor QFT.
(Martin, Haber, Dreiner)
arXiv

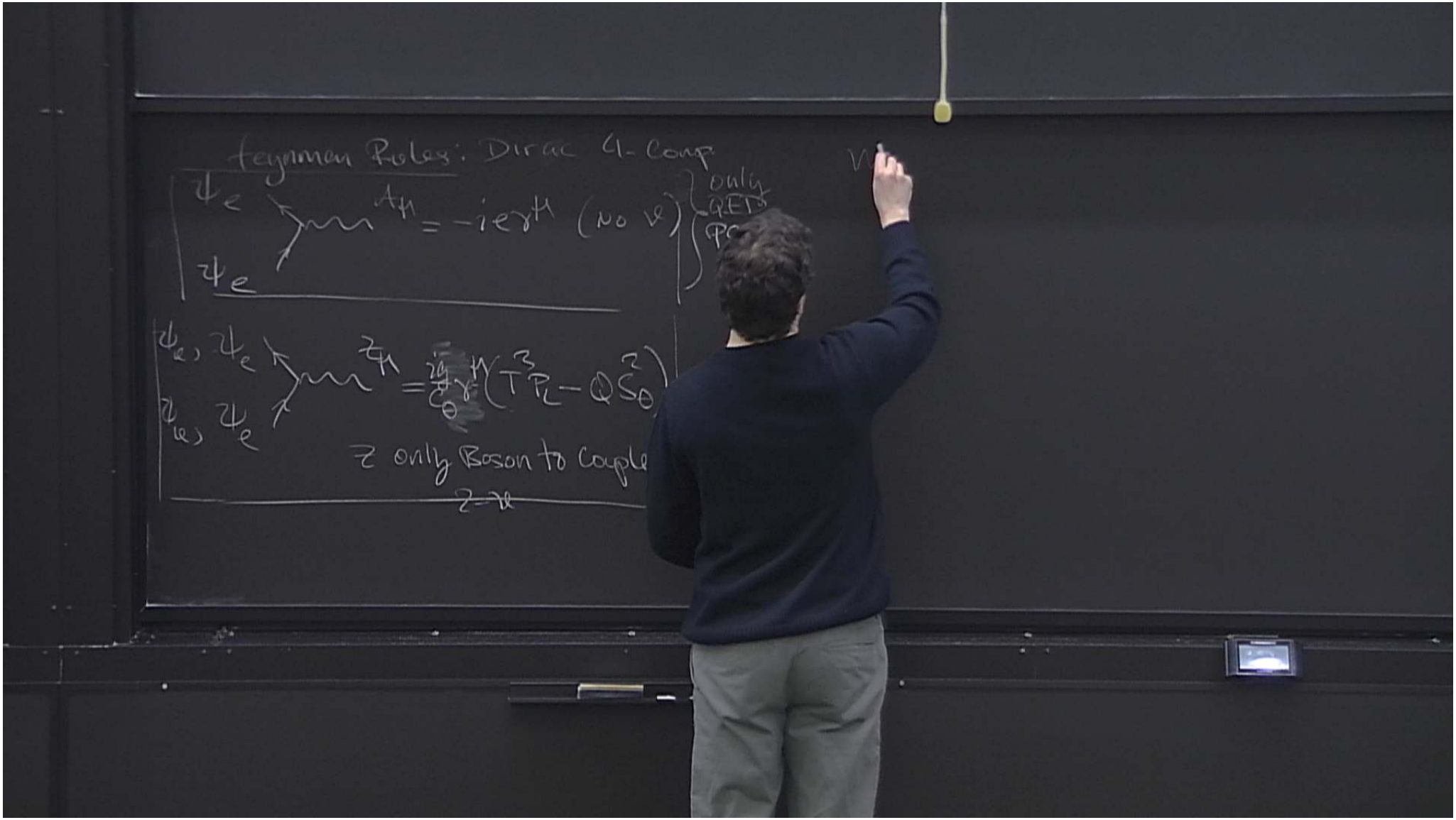

Electroweak Unification $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.

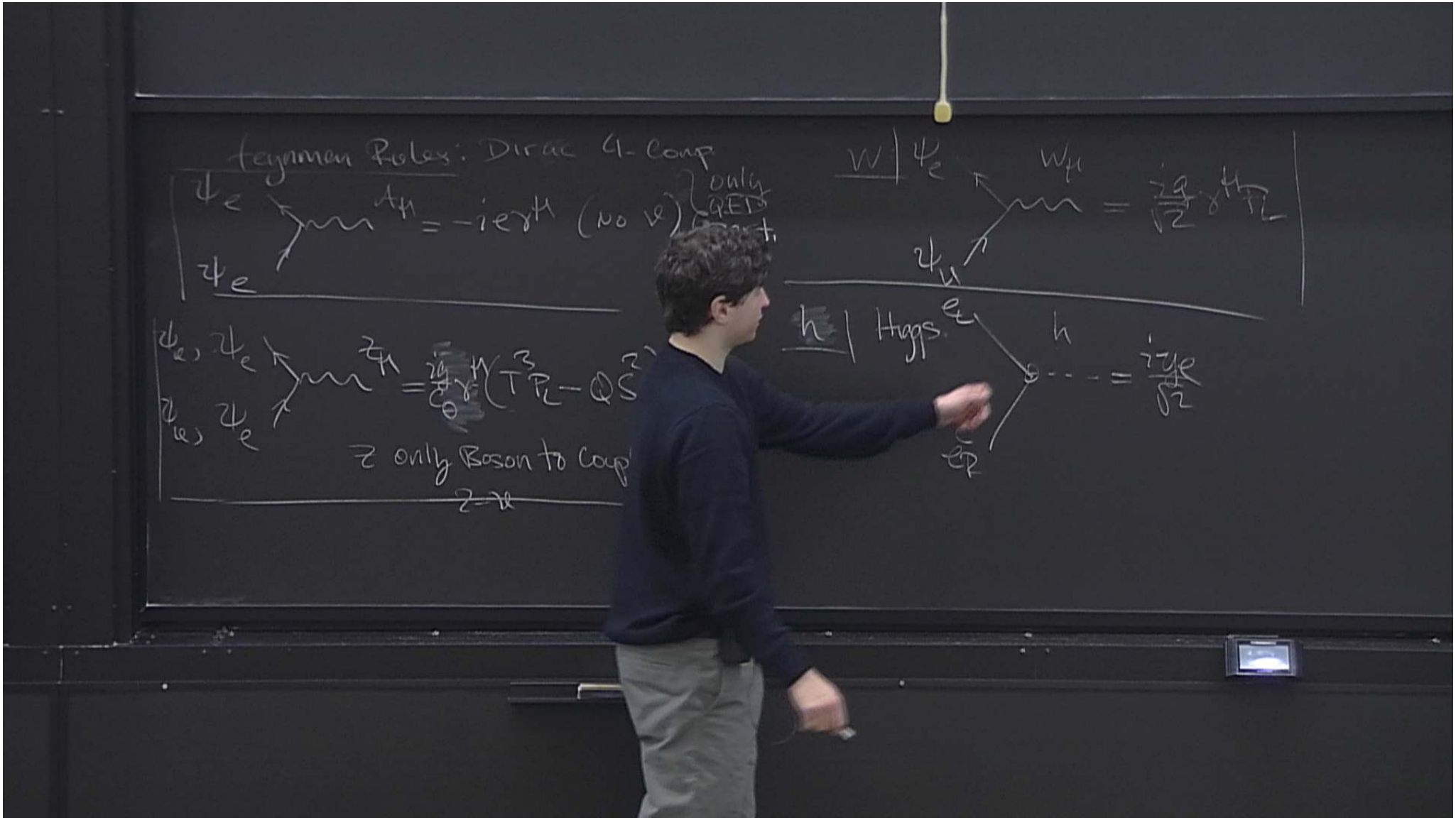

QFT
Electroweak Unification $SU(2)_L \times U(1)_Y \to U(1)_{EM}$.
QED → "Vectorlike" (photon/fermion democracy)
W,Z → Chiral (violate Parity $x^\mu \to -x^\mu$, $\mathcal{L}_{WZ}(x) \neq \mathcal{L}_{WZ}(-x_\mu)$
Glashow

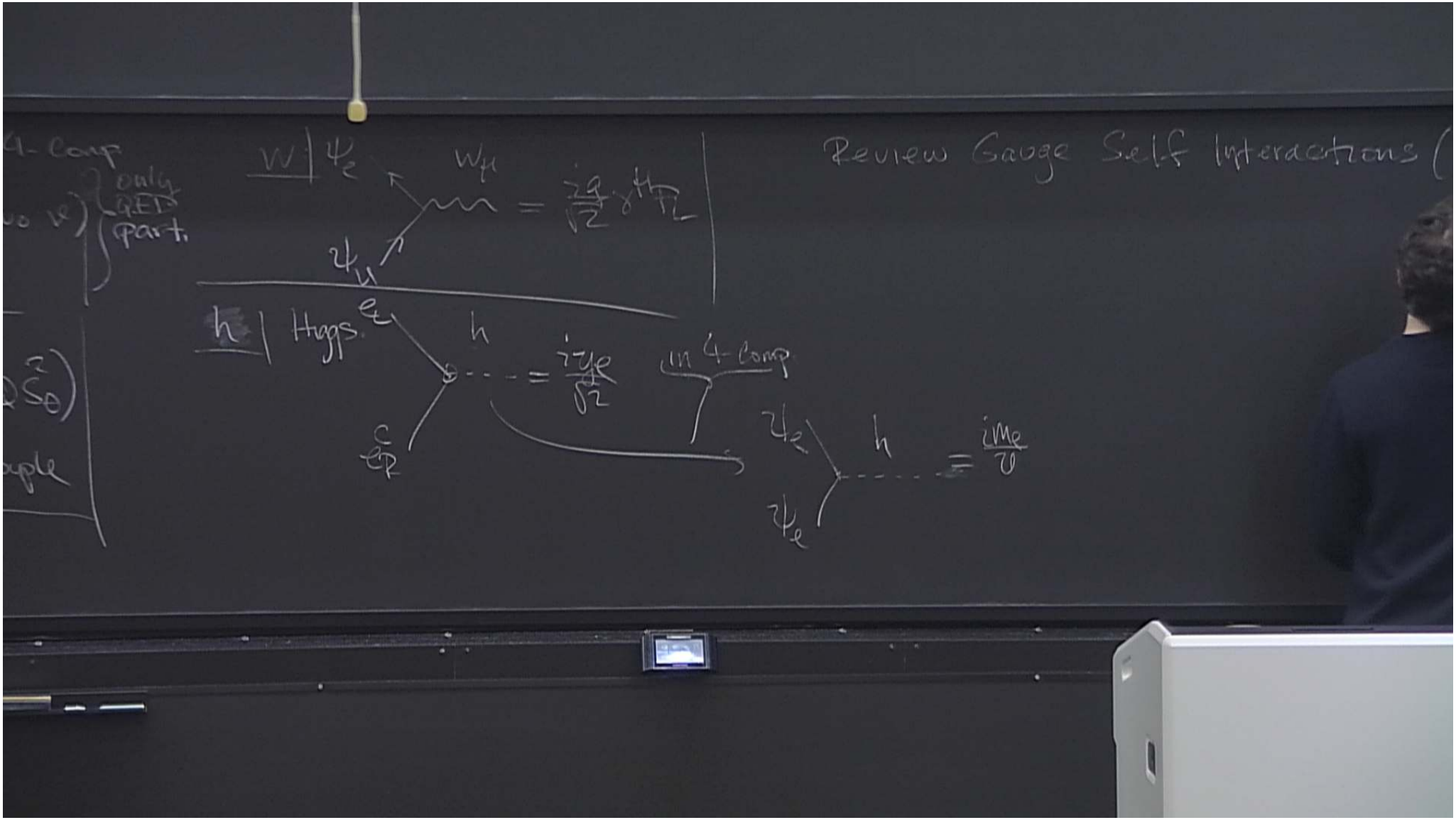

Feynman Rules: Dirac 4-Comp
e
e

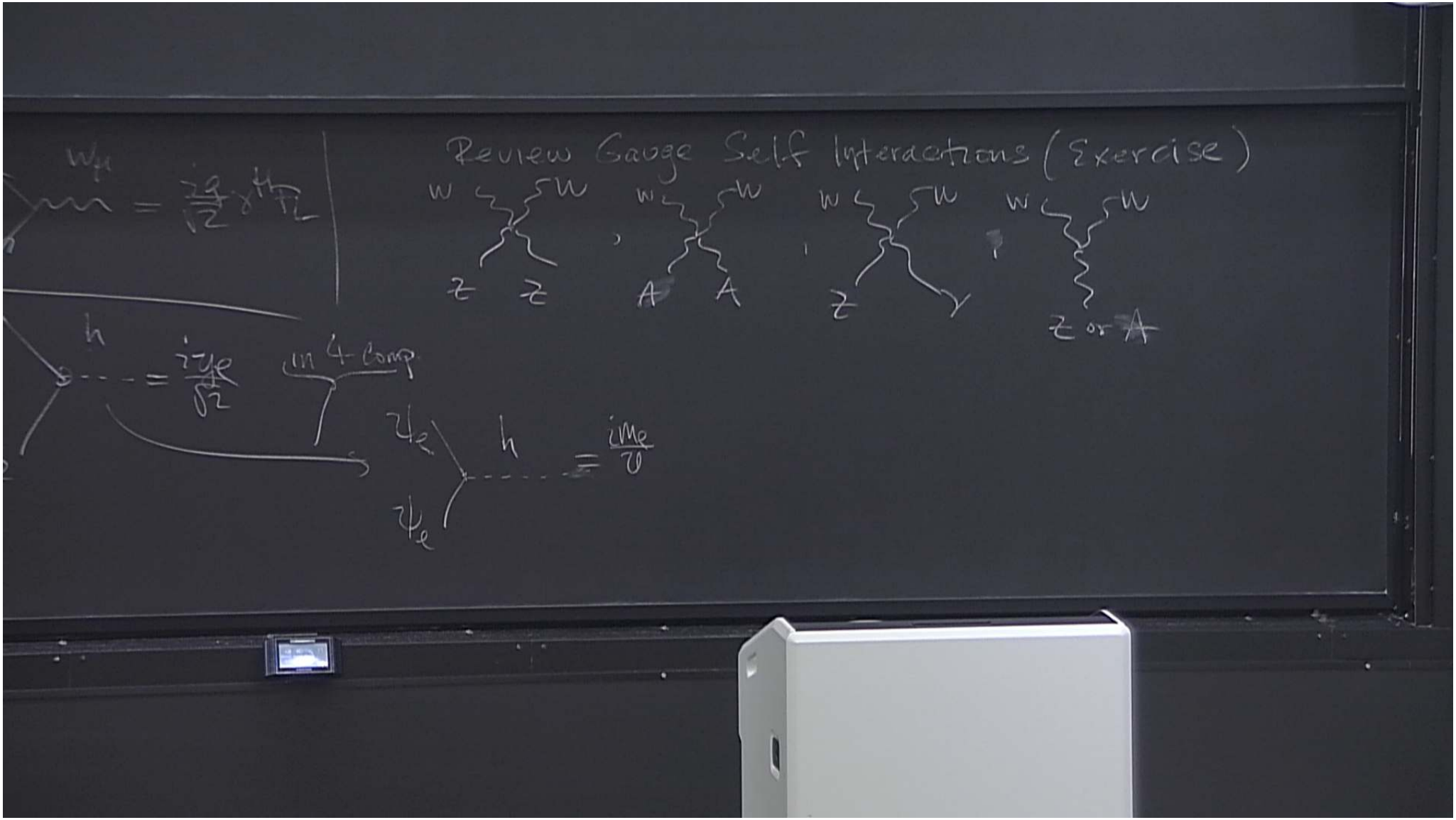

Feynman Rules: Dirac 4-Comp
ψ_e
$A^\mu = -ie\gamma^\mu$ (no v)
ψ_e
only QED part.

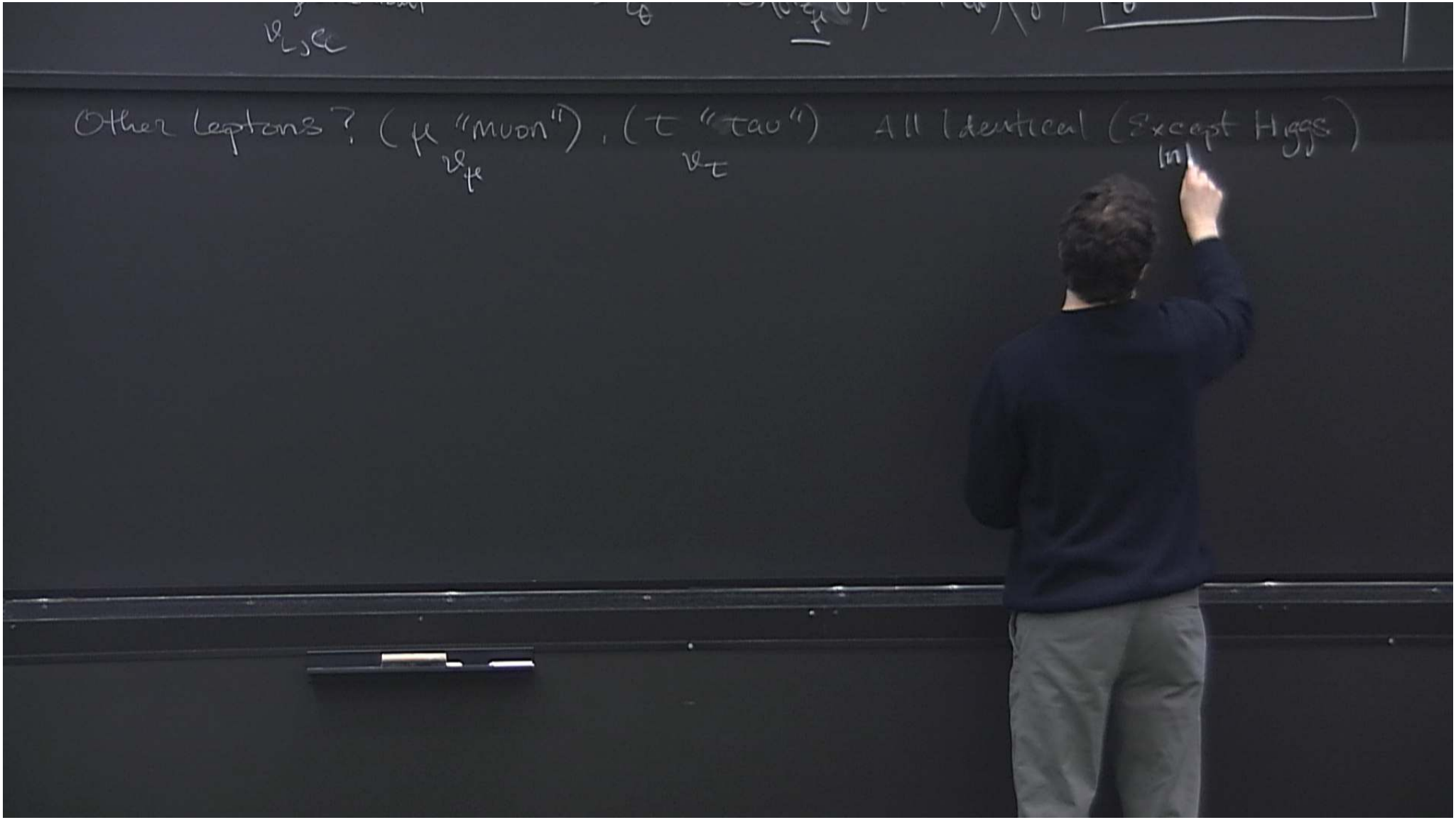

Feynman Rules: Dirac 4-Comp
only QED part
(no V)

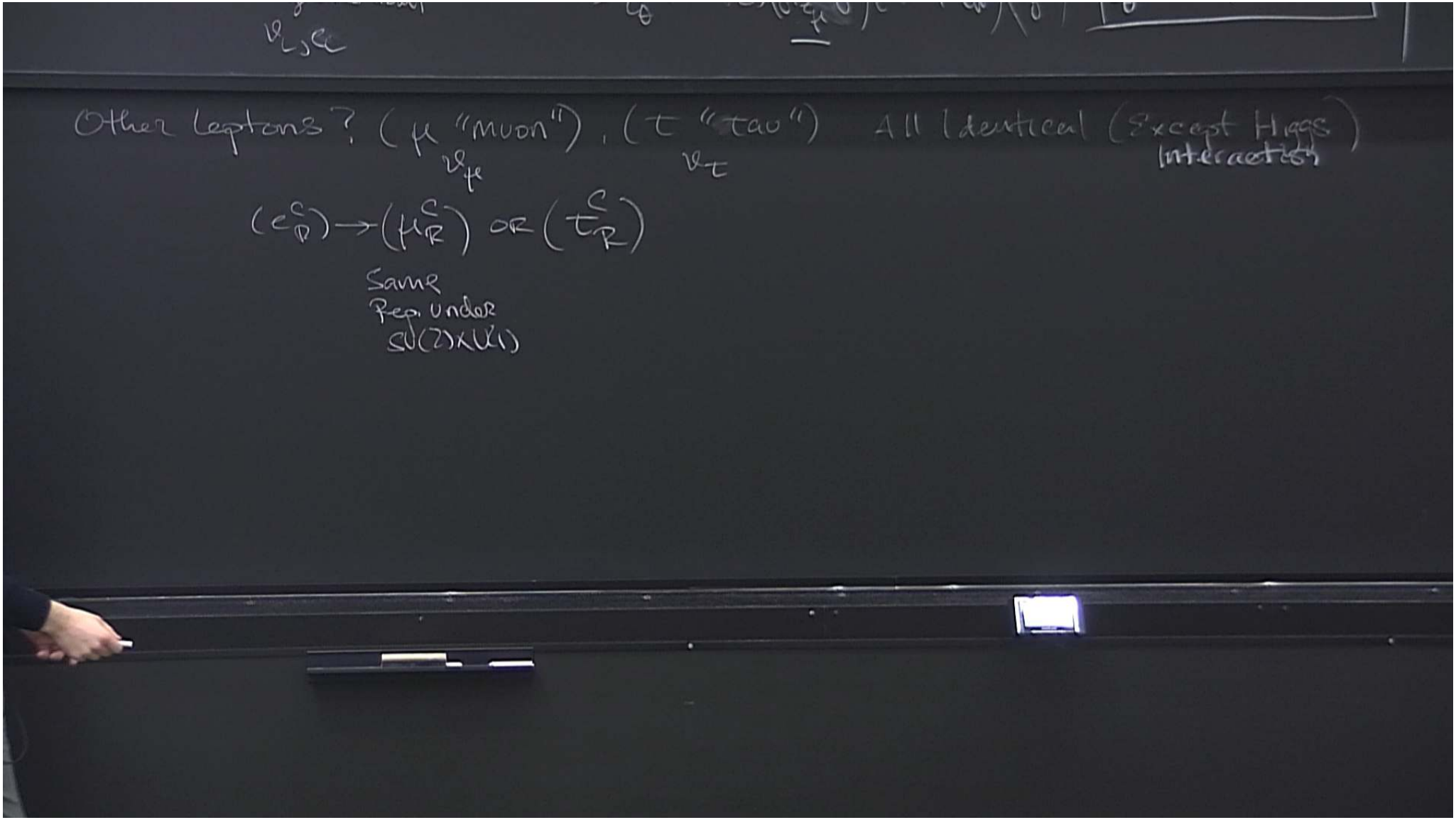

Feynman Rules: Dirac 4-Comp
only QED
Z only Boson to Couple

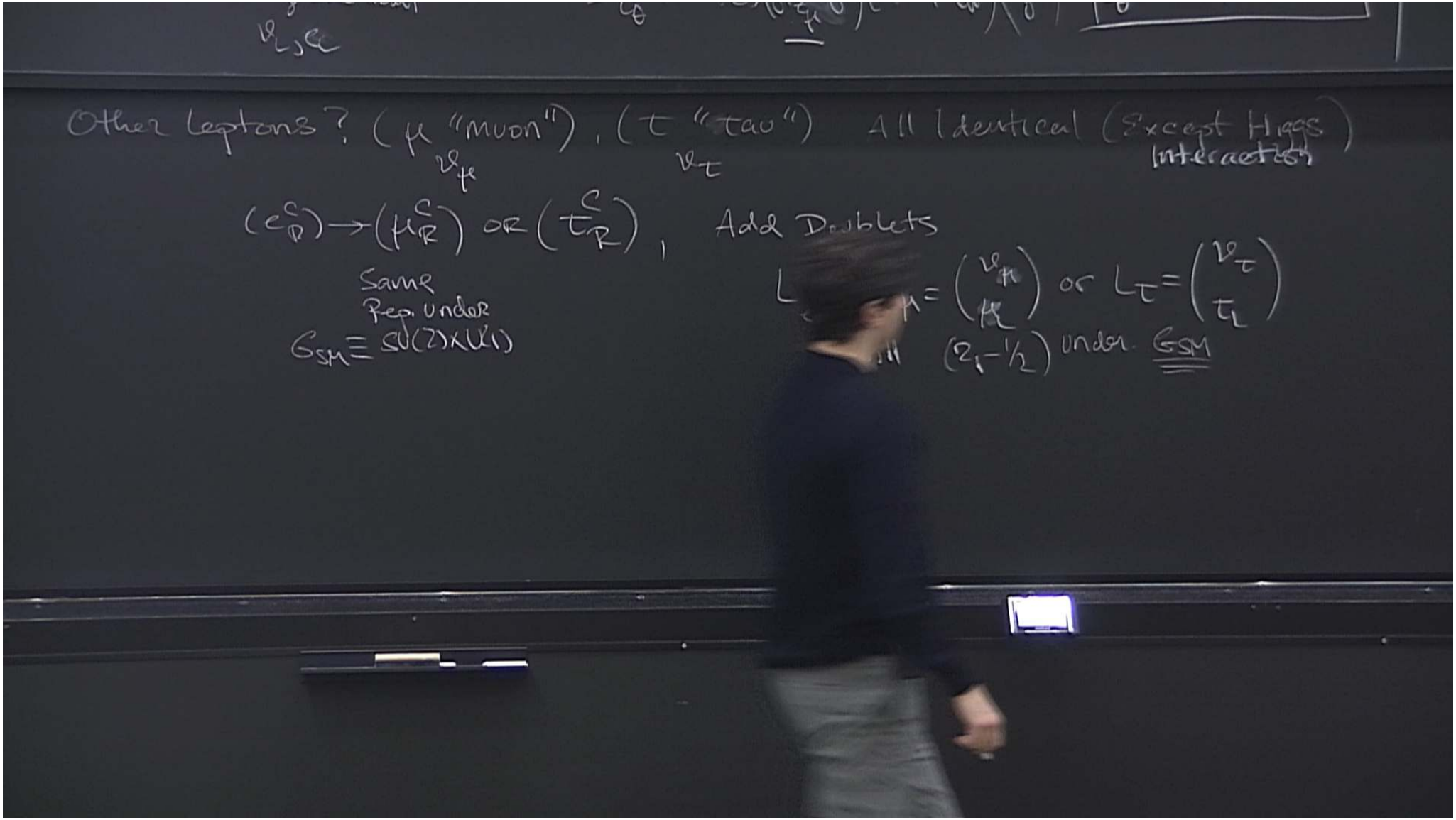

Feynman Rules: Dirac 4-Comp
only
W
h
Higgs
h
only Boson to Coup

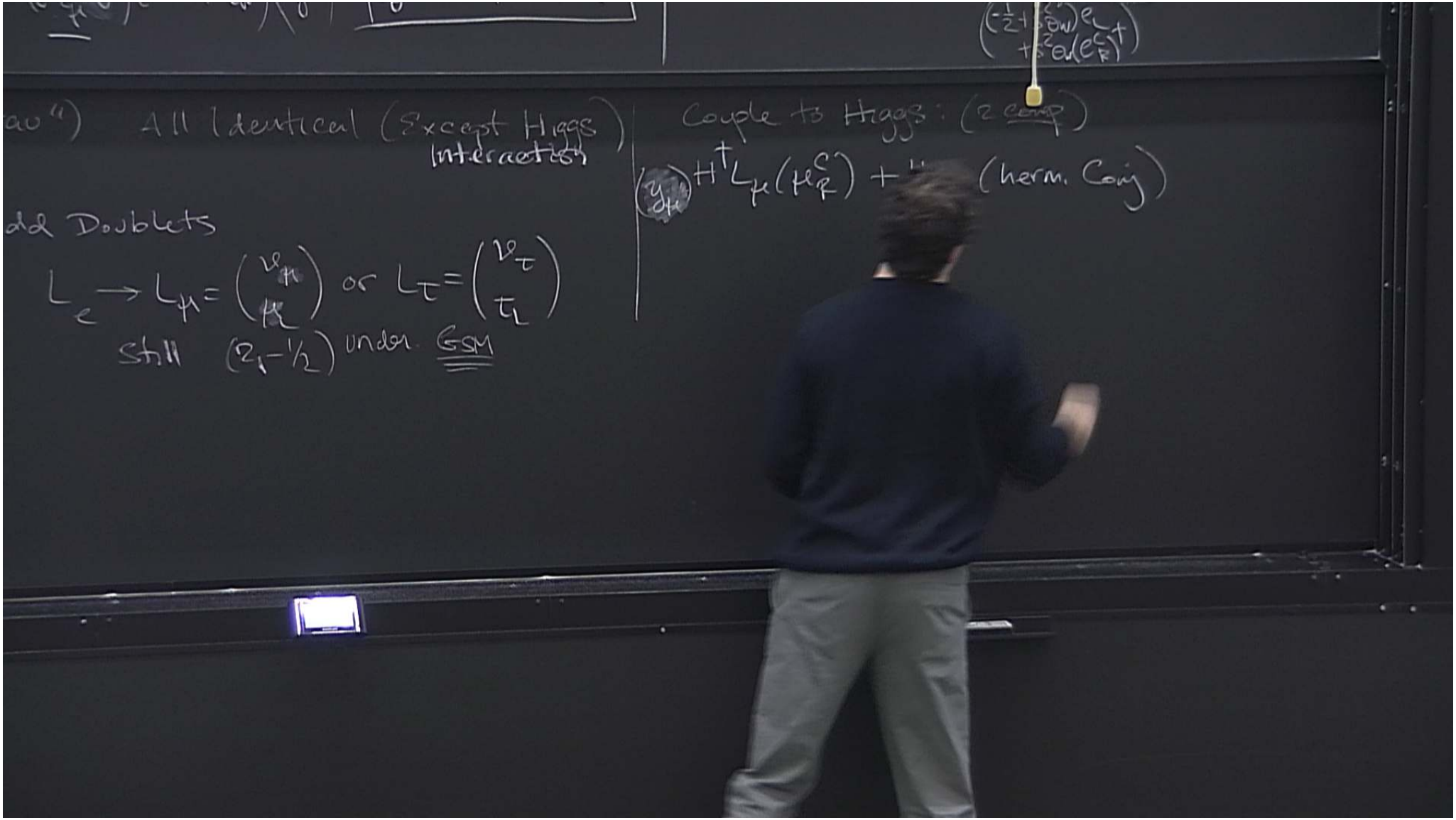

Review Gauge Self Interactions
W
h
Higgs
only QED part
in 4-comp

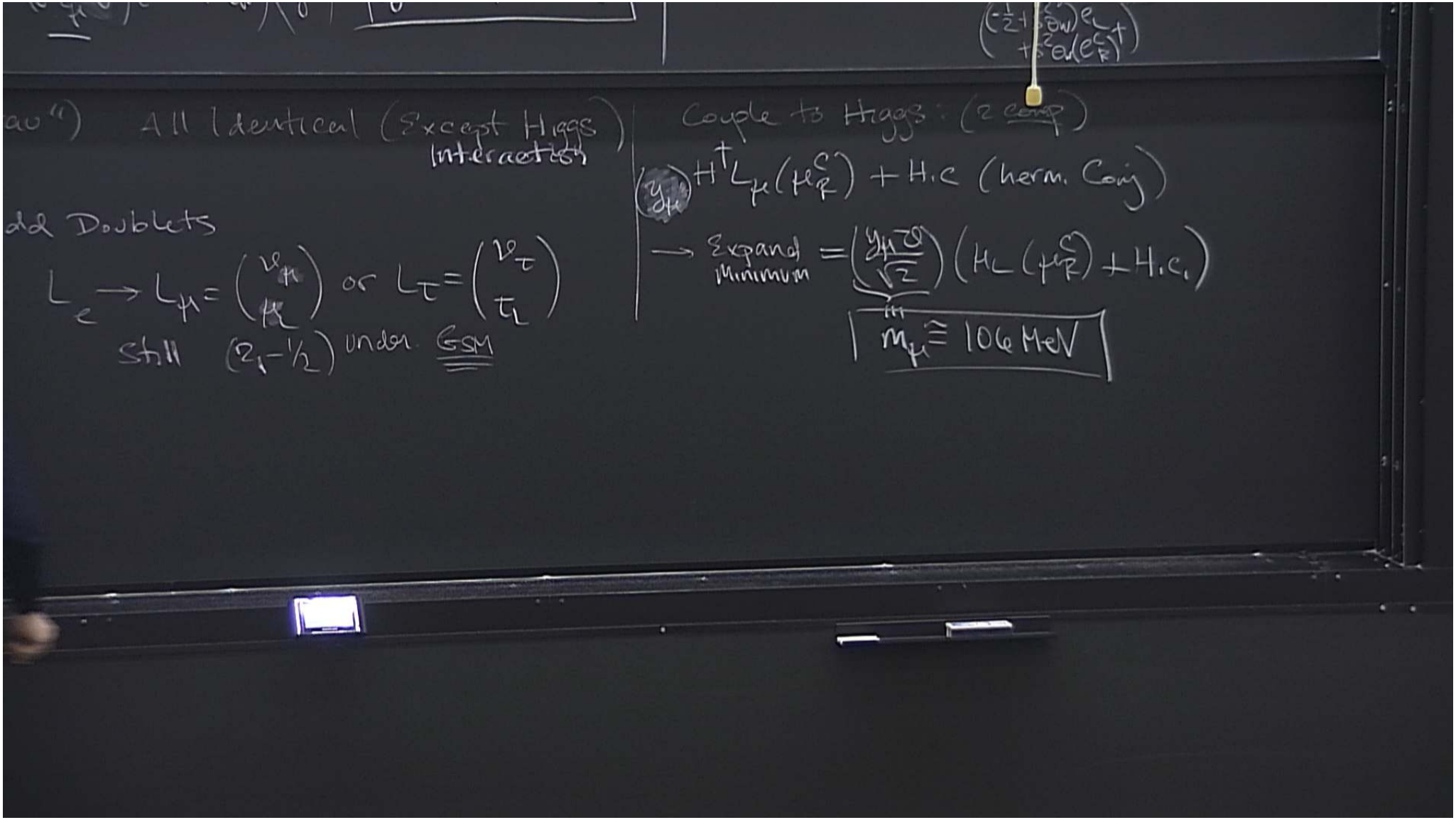

Review Gauge Self Interactions (Exercise)
W W
Z Z
W W
A A
W W
Z
W W
Z or A
h
in 4-Comp
h
Ψe
Ψe

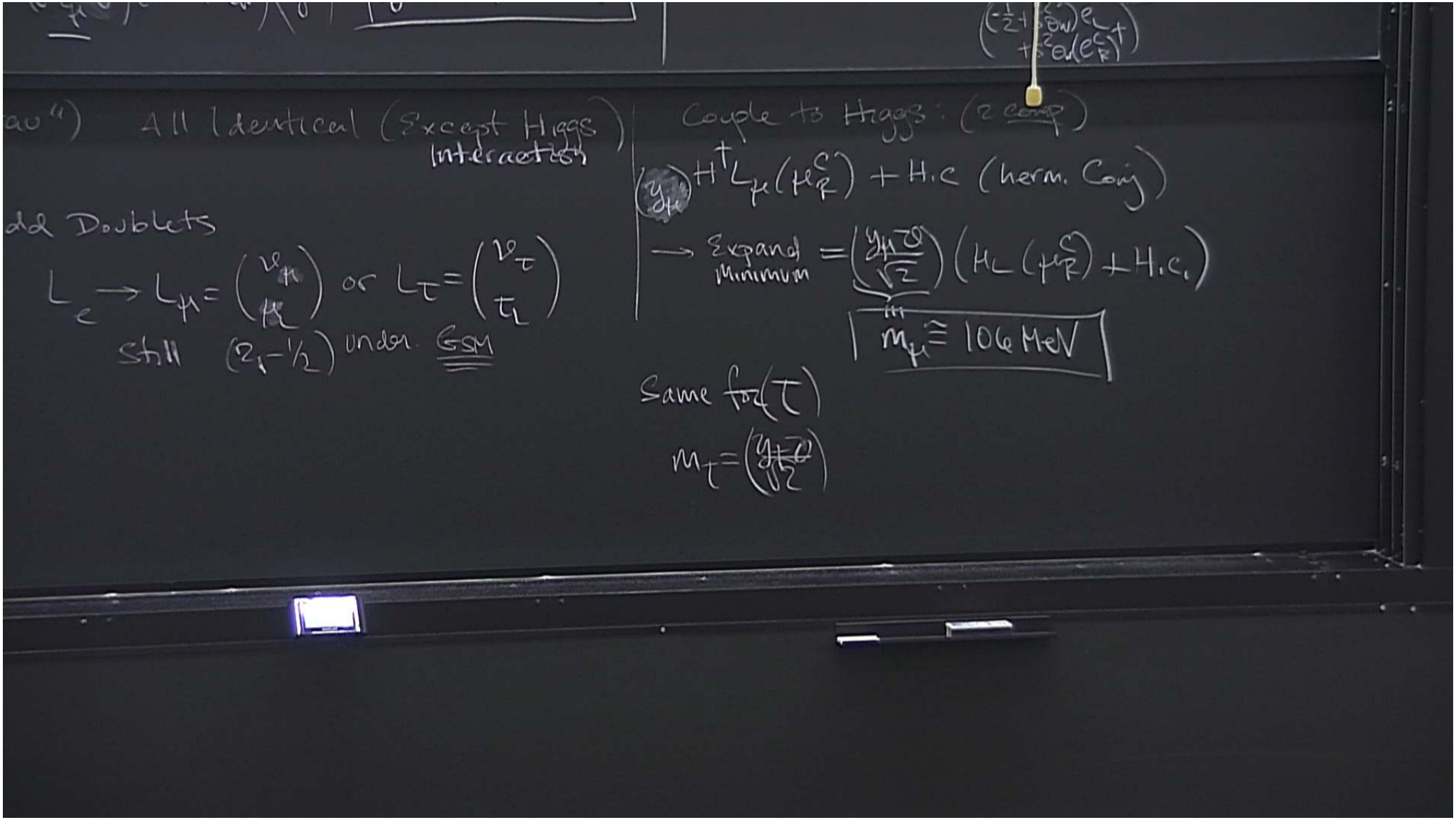

Other Leptons? (μ "muon"), (τ "tau") All identical (except Higgs)

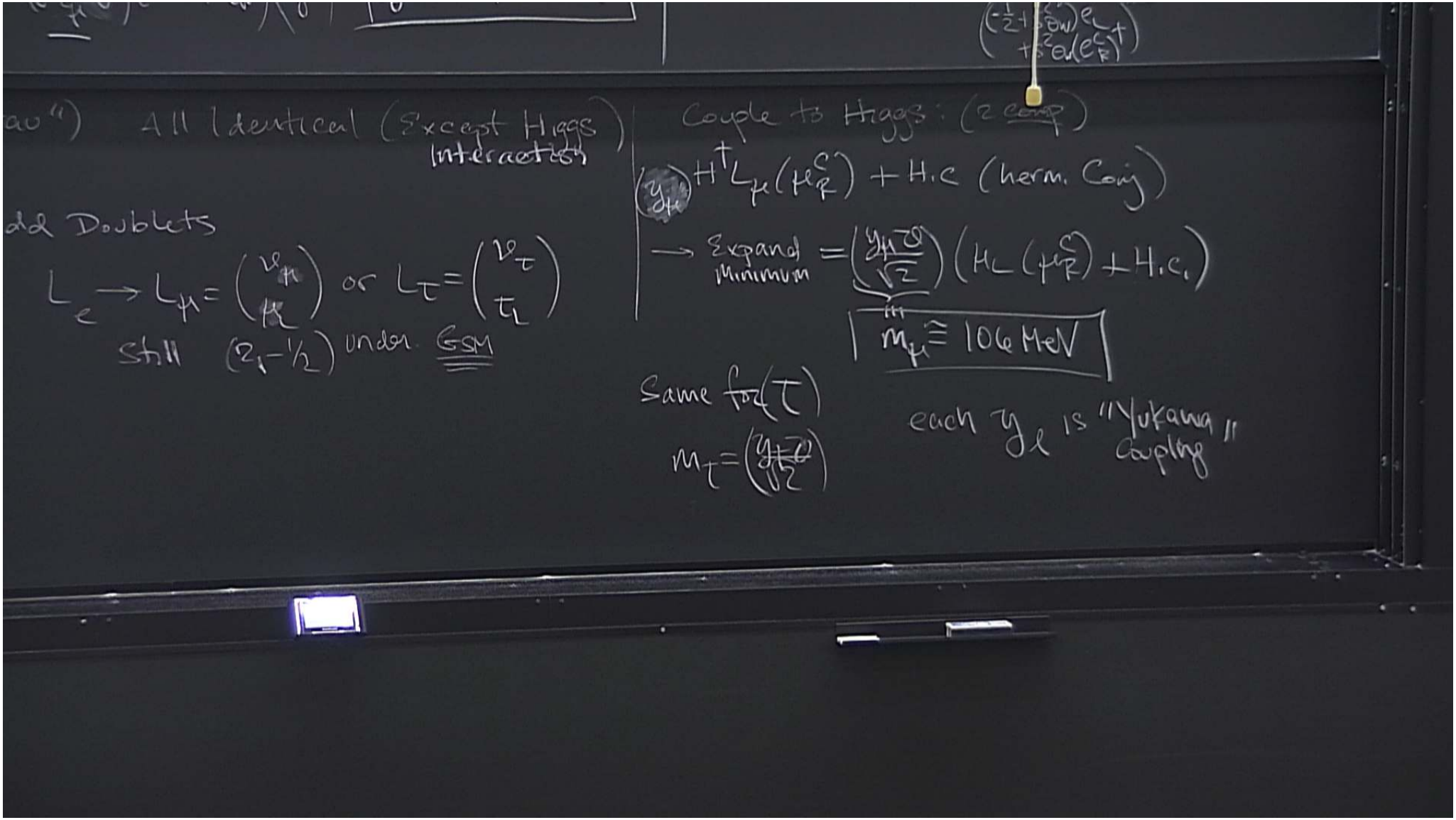

Other Leptons? (μ "muon"), (τ "tau") All identical (except Higgs interactions)
ν_μ
ν_τ
$(e_R^c) \rightarrow (\mu_R^c)$ OR (t_R^c)
Same Rep. under SU(2)xU(1)

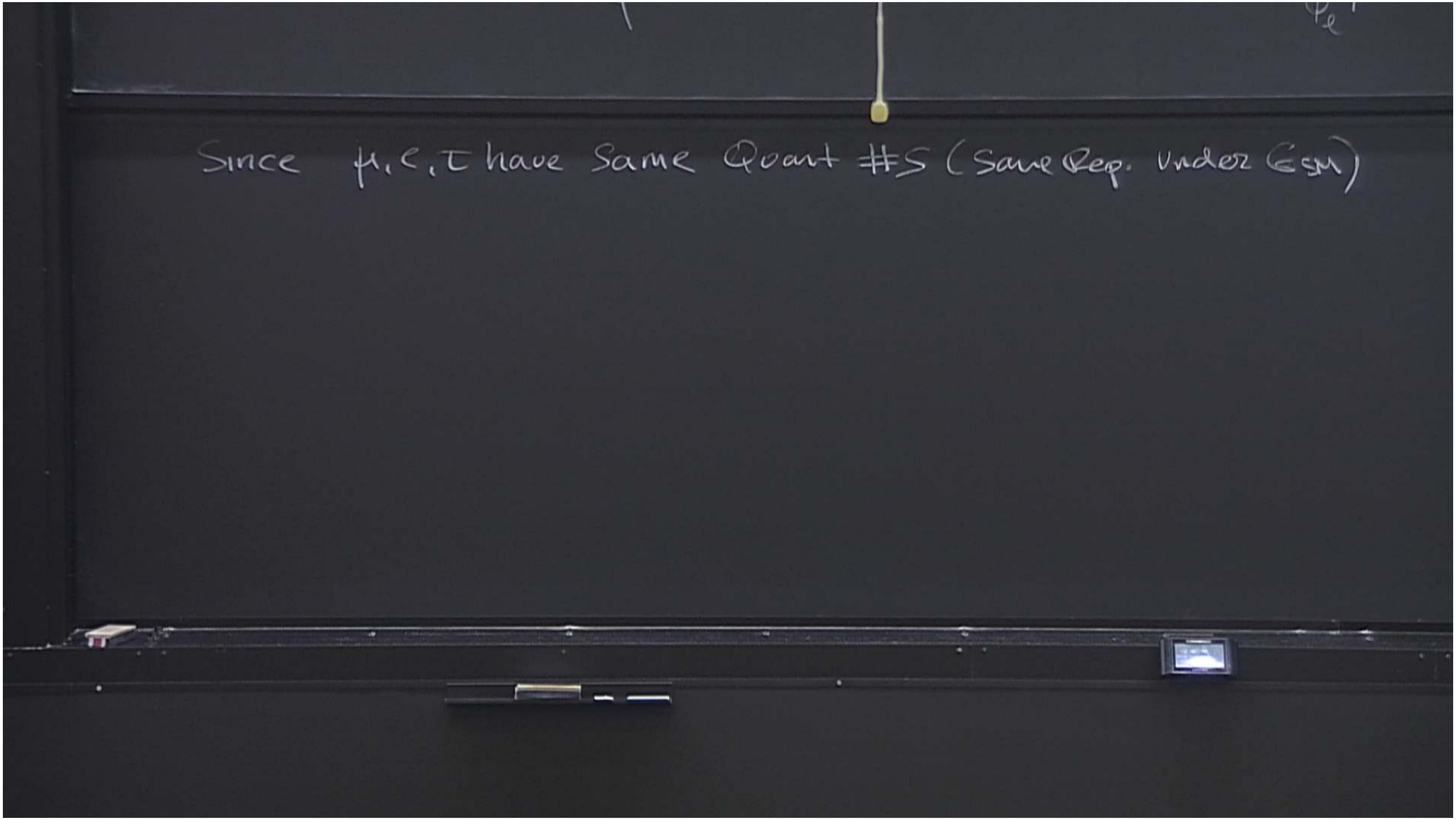

Other Leptons? (μ "muon"), (τ "tau") All Identical (Except Higgs Interactions)
ν_μ
ν_τ
$(e^c_R) \to (\mu^c_R)$ or (τ^c_R), Add Doublets
Same Rep. under
$G_{SM} \equiv SU(2) \times U(1)$
$L_\mu = \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$ or $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$
$(2, -1/2)$ under G_{SM}

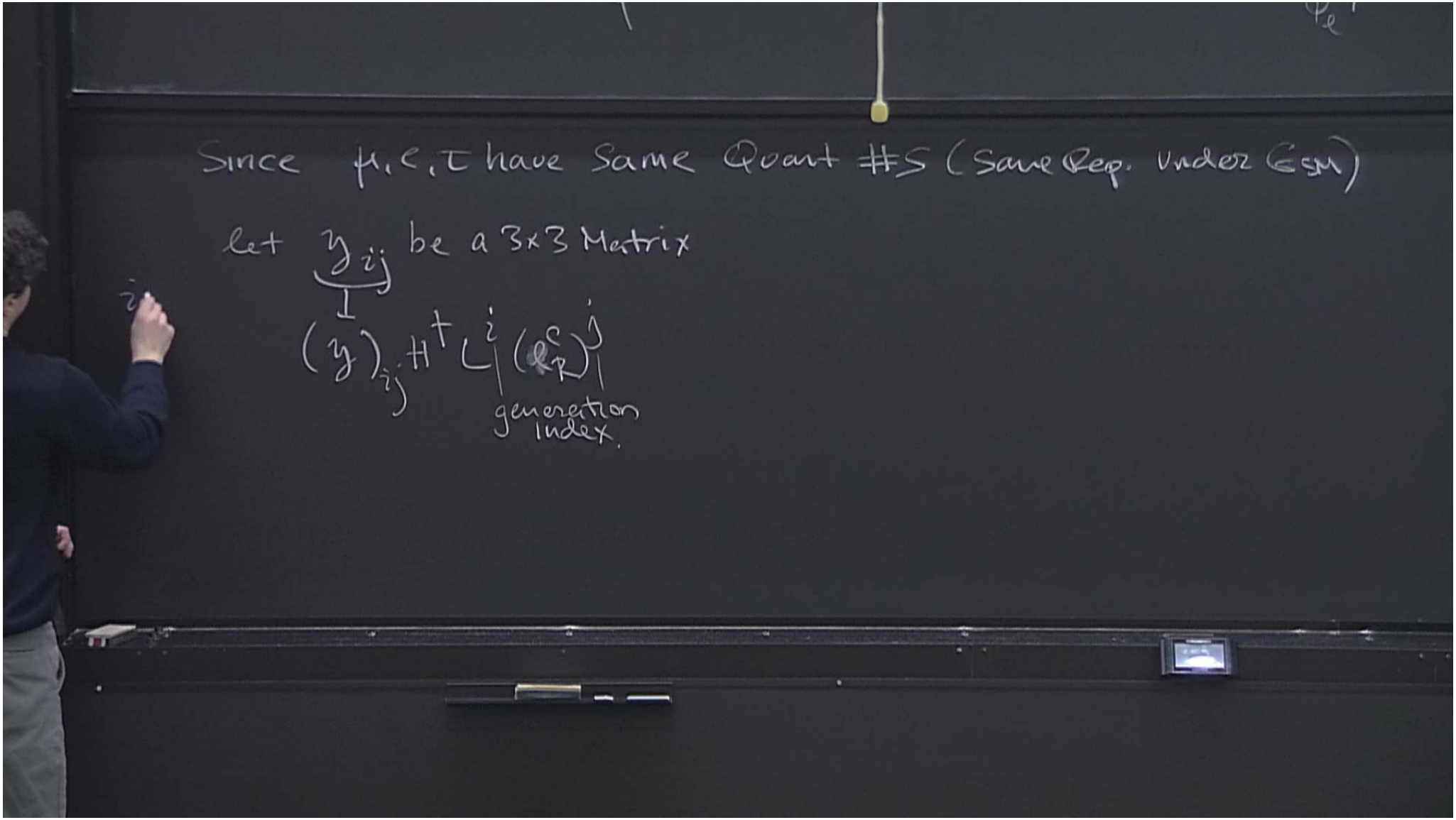

All Identical (Except Higgs Interaction)
Couple to Higgs: (2 comp)
(herm. Conj)
Doublets
or
Still under GSM

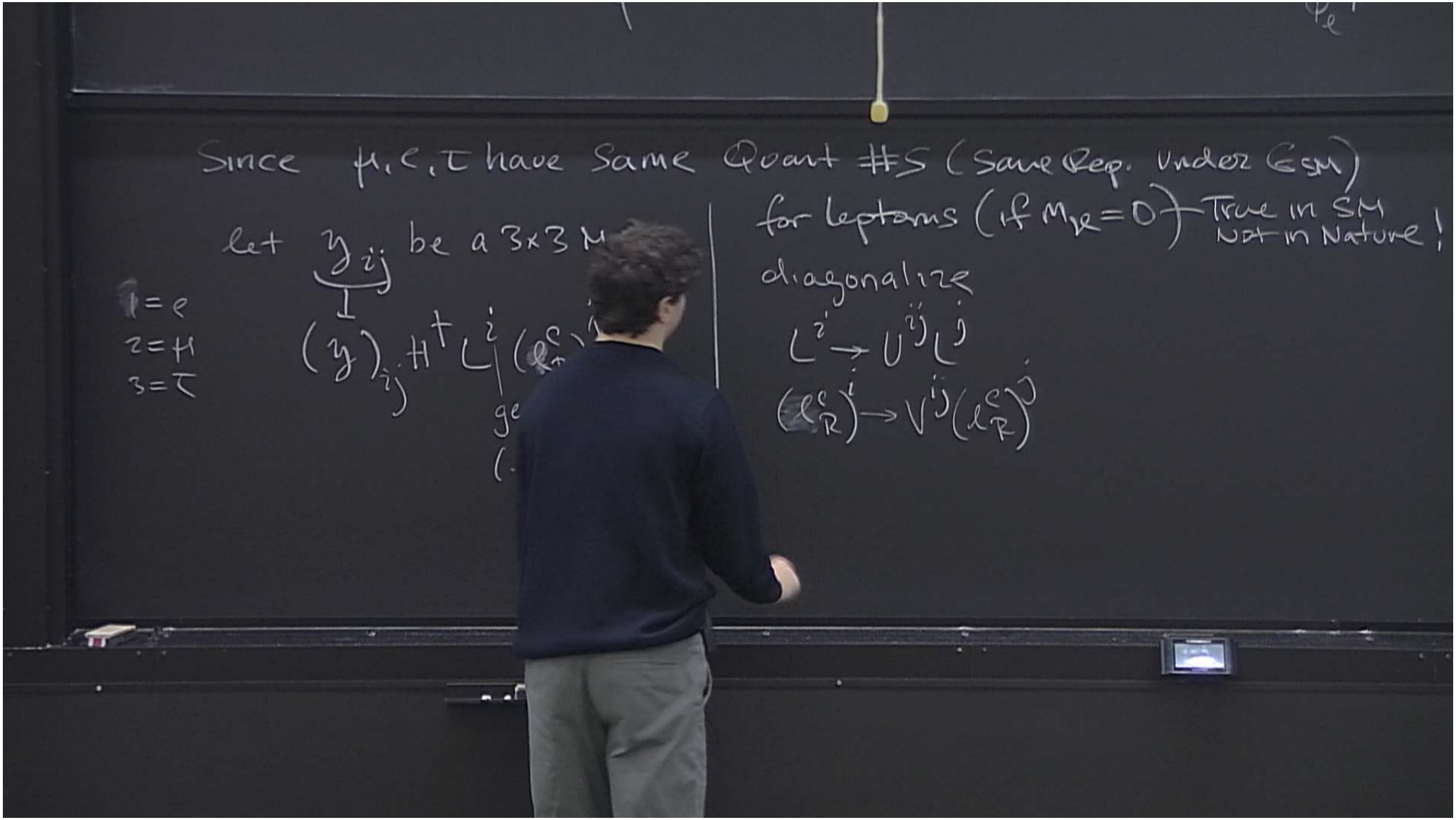

All Identical (Except Higgs Interaction)
Couple to Higgs: (2 comp)
Add Doublets
Still $(2, -1/2)$ under GSM
Expand Minimum
$m_\mu \cong 106$ MeV

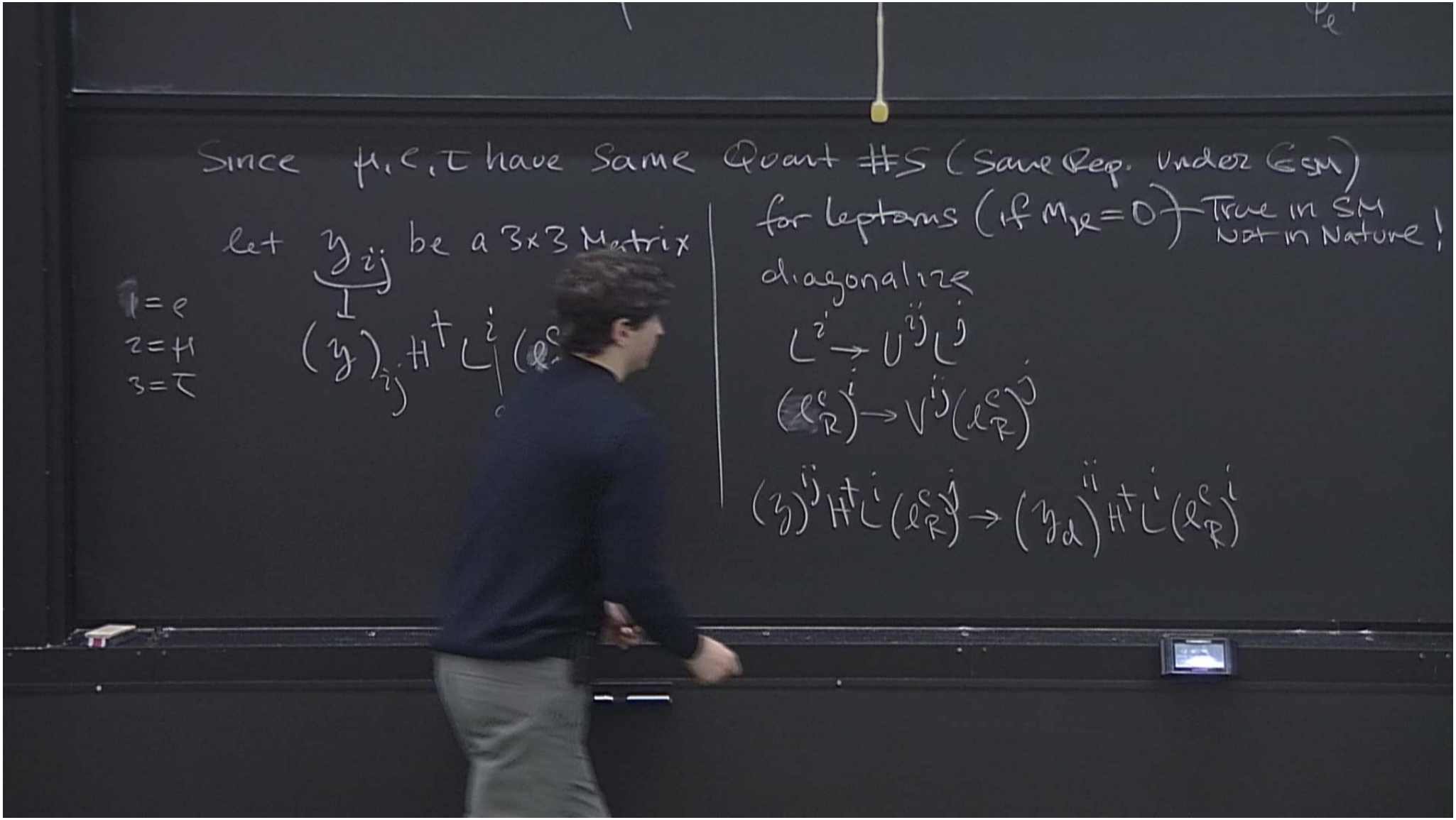

au") All identical (except Higgs interactions)
Couple to Higgs: (2 comp)
Doublets
$L_e \to L_\mu = \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$ or $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$
still $(I_3 = -1/2)$ under GSM
$(y_\mu) H^\dagger L_\mu (\mu_R^c) + H.c$ (herm. Conj)
→ Expand Minimum $= \left(\frac{y_\mu v}{\sqrt{2}}\right)(\mu_L(\mu_R^c) + H.c.)$
$m_\mu \cong 106\ MeV$
Same for τ
$m_\tau = \left(\frac{y_\tau v}{\sqrt{2}}\right)$


au") All identical (except Higgs interaction)
Couple to Higgs: (2 comp)
Doublets
$L_e \to L_\mu = \begin{pmatrix} \nu_\mu \\ \mu_L \end{pmatrix}$ or $L_\tau = \begin{pmatrix} \nu_\tau \\ \tau_L \end{pmatrix}$
still $(2, -1/2)$ under GSM
$(y_\mu) H^\dagger L_\mu (\mu_R^c) + H.c$ (herm. Conj)
→ Expand = $\left(\frac{y_\mu v}{\sqrt{2}}\right)(H_L(\mu_R^c) + H.c.)$
$m_\mu = 106\ MeV$
Same for τ
$m_\tau = \left(\frac{y_\tau v}{\sqrt{2}}\right)$
each y_ℓ is "Yukawa" coupling


Since μ, e, τ have same Quant #s (same Rep. under G_SM)


Since μ, c, t have same Quant #s (same Rep. under G_SM)
let y_{ij} be a 3×3 Matrix
$(y)_{ij} H^\dagger L^i_L (e_R^S)^j_1$
generation index.


Since μ, e, τ have Same Quant #s (Same Rep. Under G_SM)
for leptons (if M_R = 0) — True in SM Not in Nature!
Let y_ij be a 3×3 M
1 = e
2 = H
3 = τ
diagonalize
L^i → U^ij L^j
(l_R^c)^i → V^ij (l_R^c)^j


Since μ, e, τ have Same Quant #s (Same Rep. Under G_SM)
for leptons (if M_ν = 0) — True in SM Not in Nature!
Let y_ij be a 3×3 Matrix
1 = e
2 = μ
3 = τ
(y)_ij H† L^i (ℓ^c
diagonalize
L^i → U^ij L^j
(ℓ^c_R)^i → V^ij (ℓ^c_R)^j
(y)^ij H† L^i (ℓ^c_R)^j → (y_d)^ij H† L^i (ℓ^c_R)^j

Since μ, e, τ have Same Quant #s (Same Rep. Under G_{SM})

for leptons (if $M_R = 0$) — True in SM Not in Nature!

Let y_{ij} be a 3×3 Matrix

$1 = e$
$2 = \mu$
$3 = \tau$

$$(y)_{ij} H^\dagger L^i (\ell_R^c)^j$$

i, j: generation index (flavor)

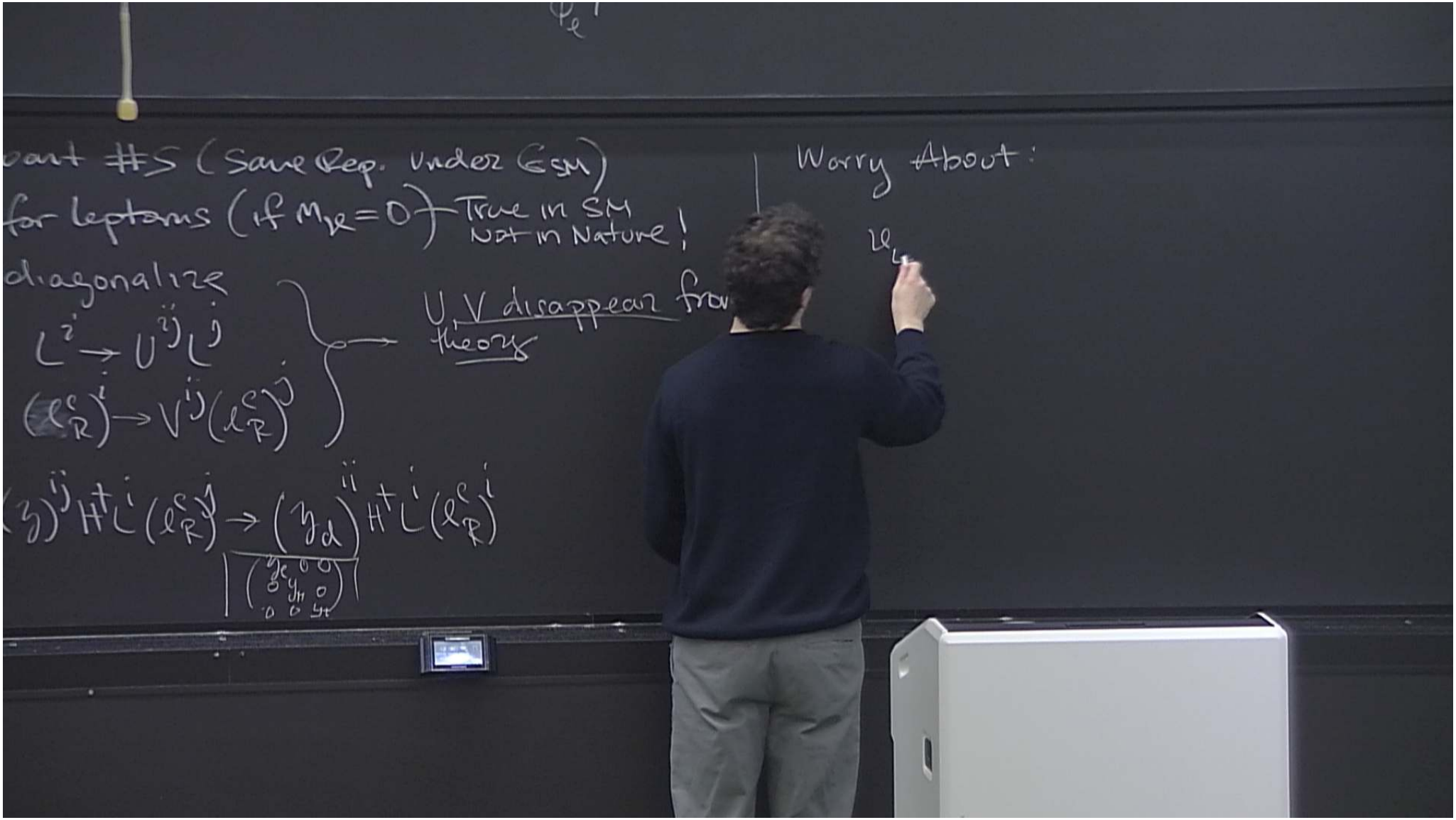
diagonalize

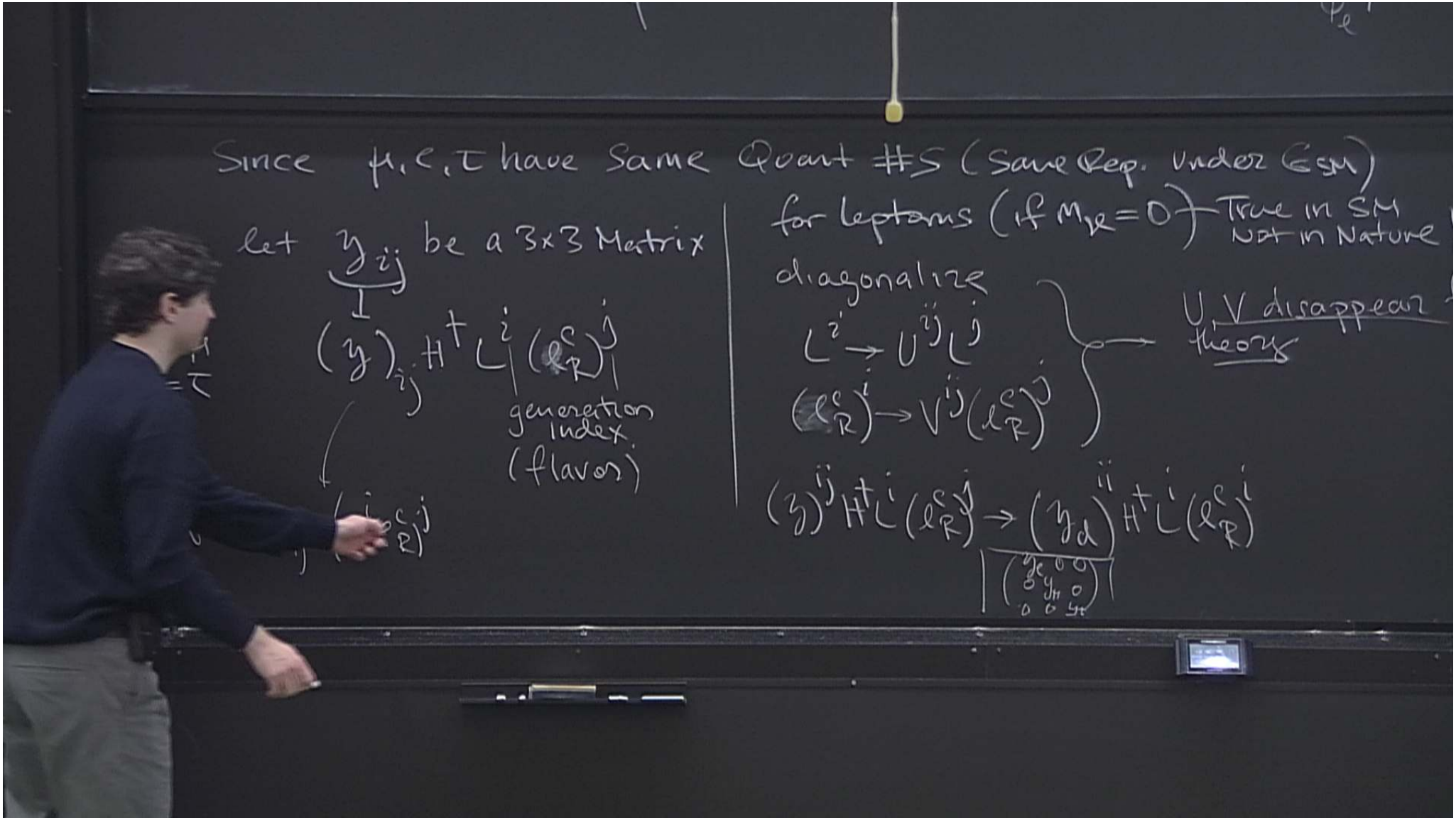
$$L^i \to U^{ij} L^j$$
$$(\ell_R^c)^i \to V^{ij} (\ell_R^c)^j$$

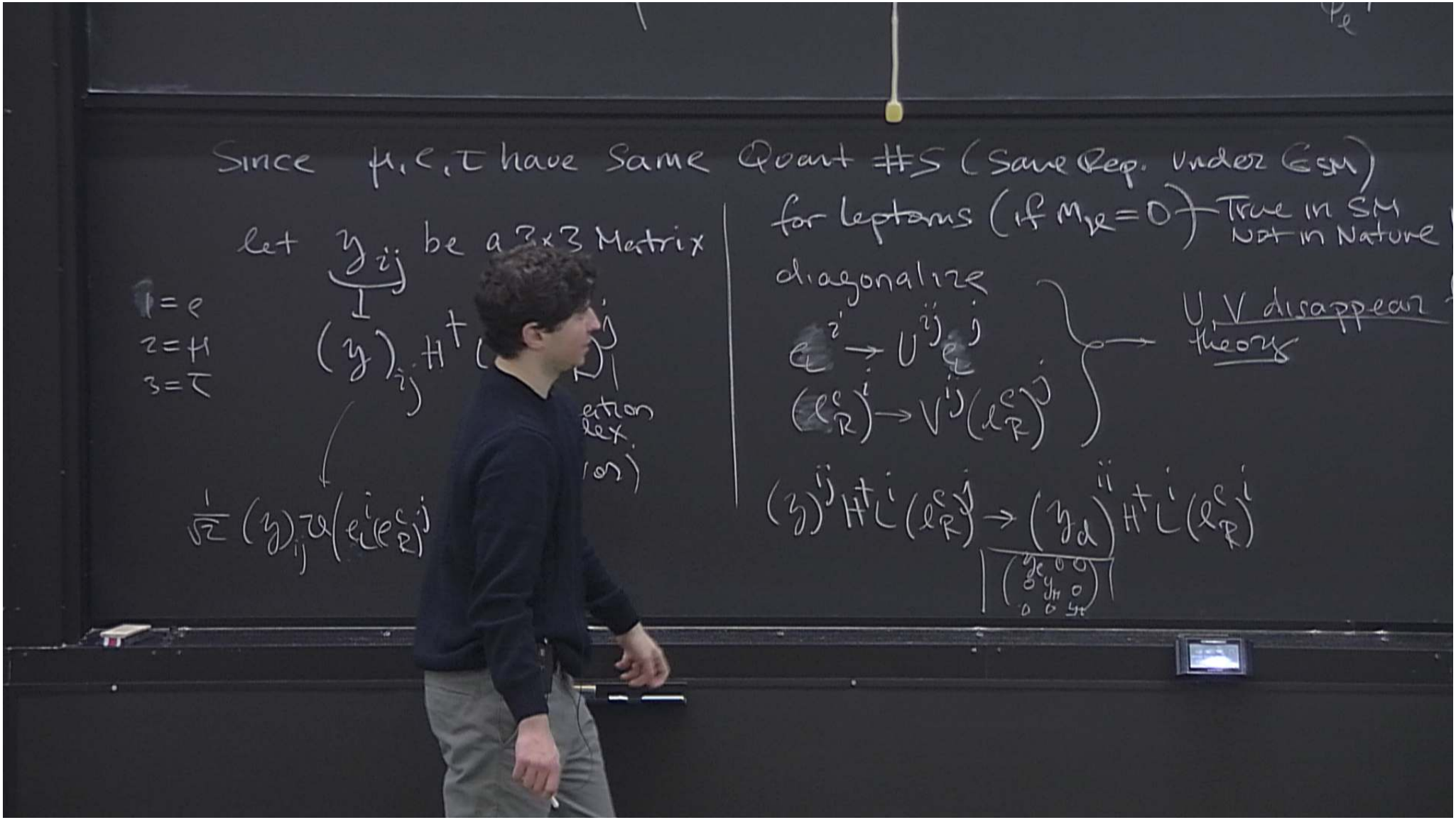
$\Rightarrow$ U, V disappear from theory

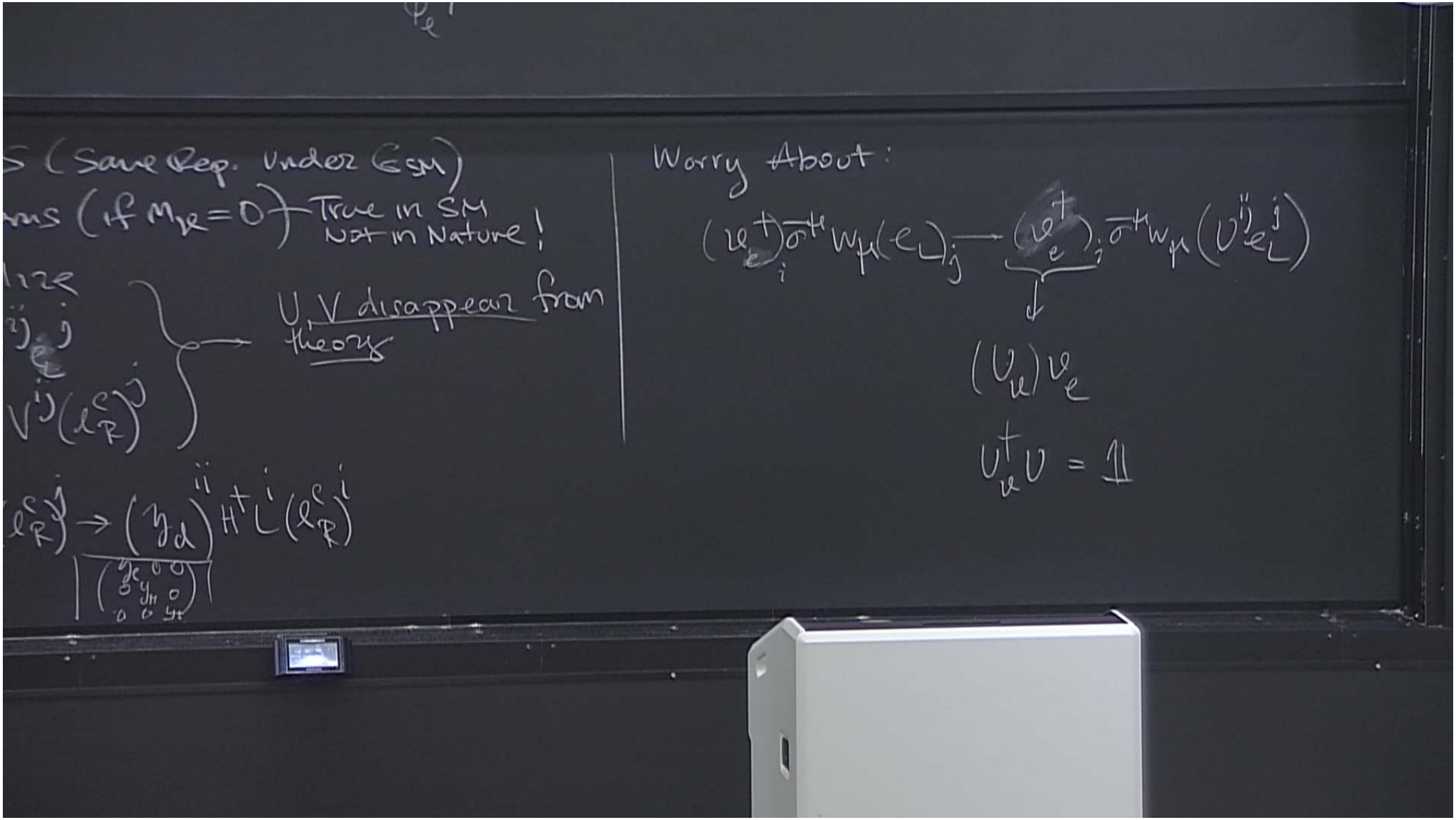
$$(y)^{ij} H^\dagger L^i (\ell_R^c)^j \to (y_d)^{ii} H^\dagger L^i (\ell_R^c)^i$$

$$\begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$


Worry About:
U, V disappear from theory


Since μ, e, τ have same Quant #s (same Rep. under G_SM)
let y_{ij} be a 3×3 Matrix
$(y)_{ij} H^\dagger L^i (\ell_R^c)^j$
generation index (flavor)
for leptons (if $M_\nu = 0$) — True in SM Not in Nature
diagonalize
$L^i \to U^{ij} L^j$
$(\ell_R^c)^i \to V^{ij} (\ell_R^c)^j$
U, V disappear — theory
$(y)^{ij} H^\dagger L^i (\ell_R^c)^j \to (y_d)^{ij} H^\dagger L^i (\ell_R^c)^j$
y_d
y_s
y_m
y_t


Since μ, e, τ have same Quant #s (same Rep. under GSM)
for leptons (if $M_\nu = 0$) — True in SM, Not in Nature
let y_{ij} be a 3×3 Matrix
1 = e
2 = μ
3 = τ
diagonalize
$e_L^i \to U^{ij} e_L^j$
$(e_R^c)^i \to V^{ij} (e_R^c)^j$
U, V disappear — theory
$\frac{1}{\sqrt{2}} (y)_{ij} v (e_L^i e_R^{cj})$
$(y)^{ij} H^\dagger L^i (e_R^c)^j \to (y_d)^{ij} H^\dagger L^i (e_R^c)^j$
y_e 0 0
0 y_μ 0
0 0 y_τ


(Same Rep. Under GSM)
(if $M_\nu = 0$) — True in SM Not in Nature!
U, V disappear from theory
Worry About:
$(U_\nu)\nu_e$
$U_\nu^\dagger U = \mathbb{1}$