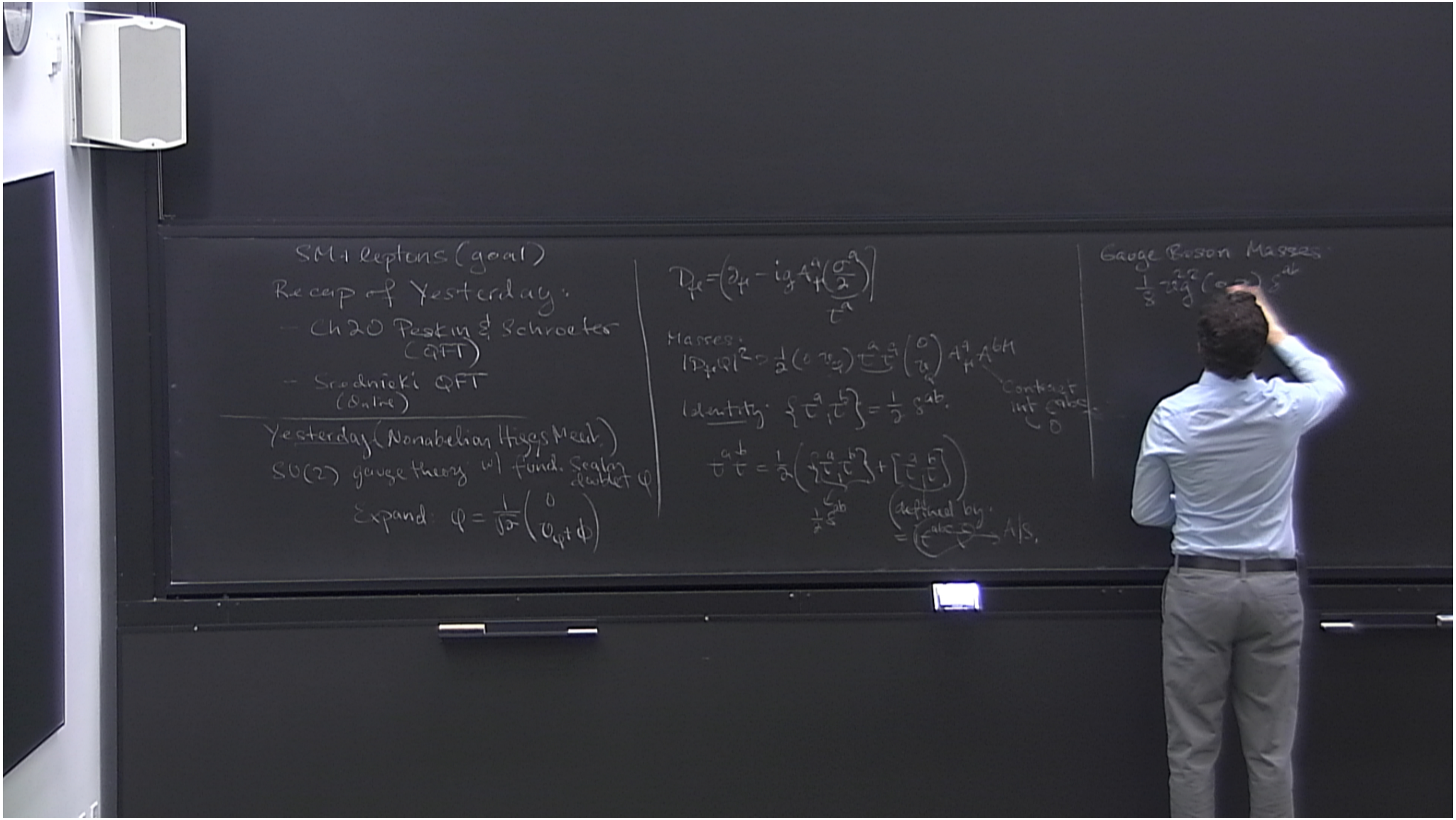
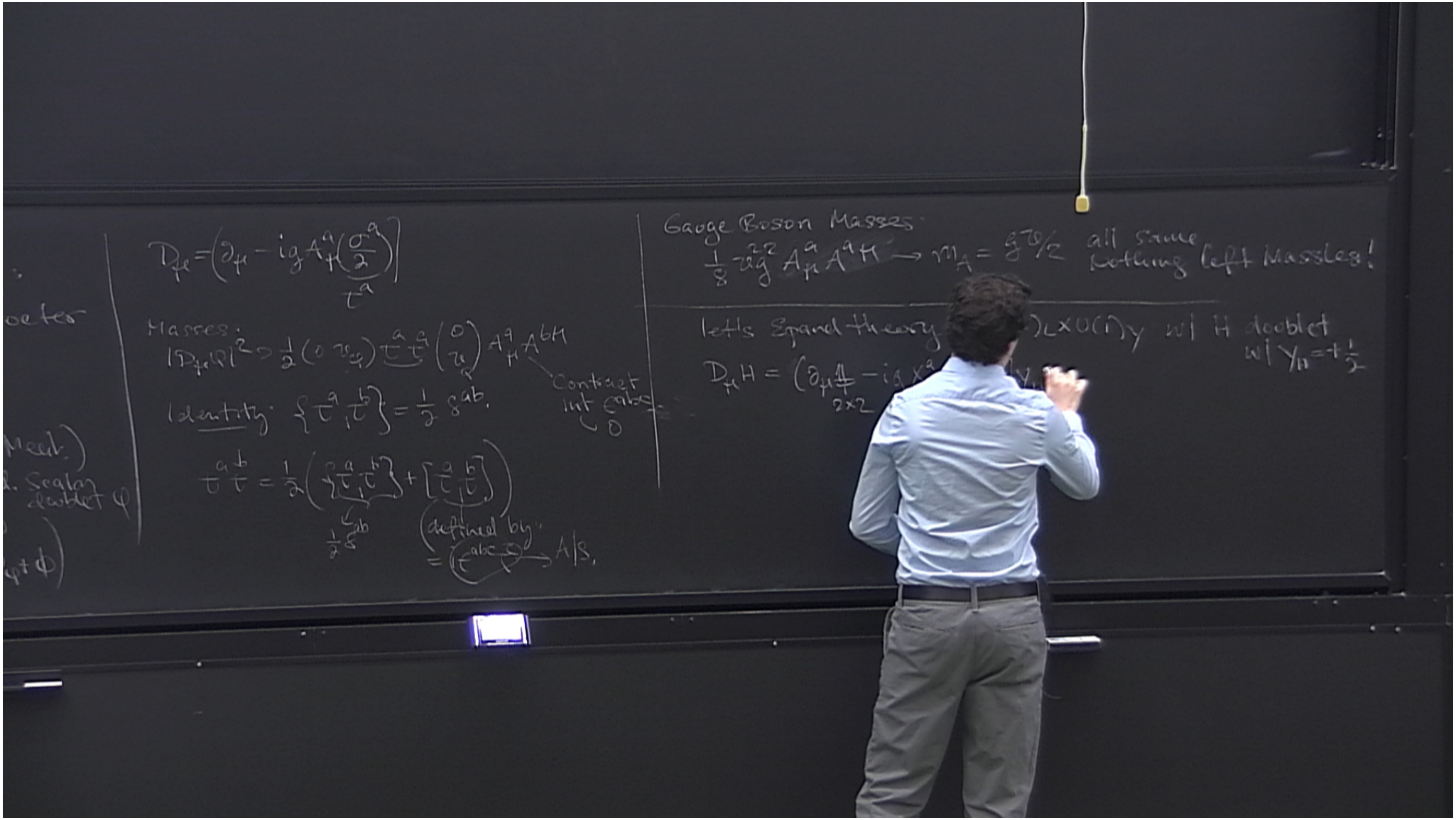
Title: Standard Model-5

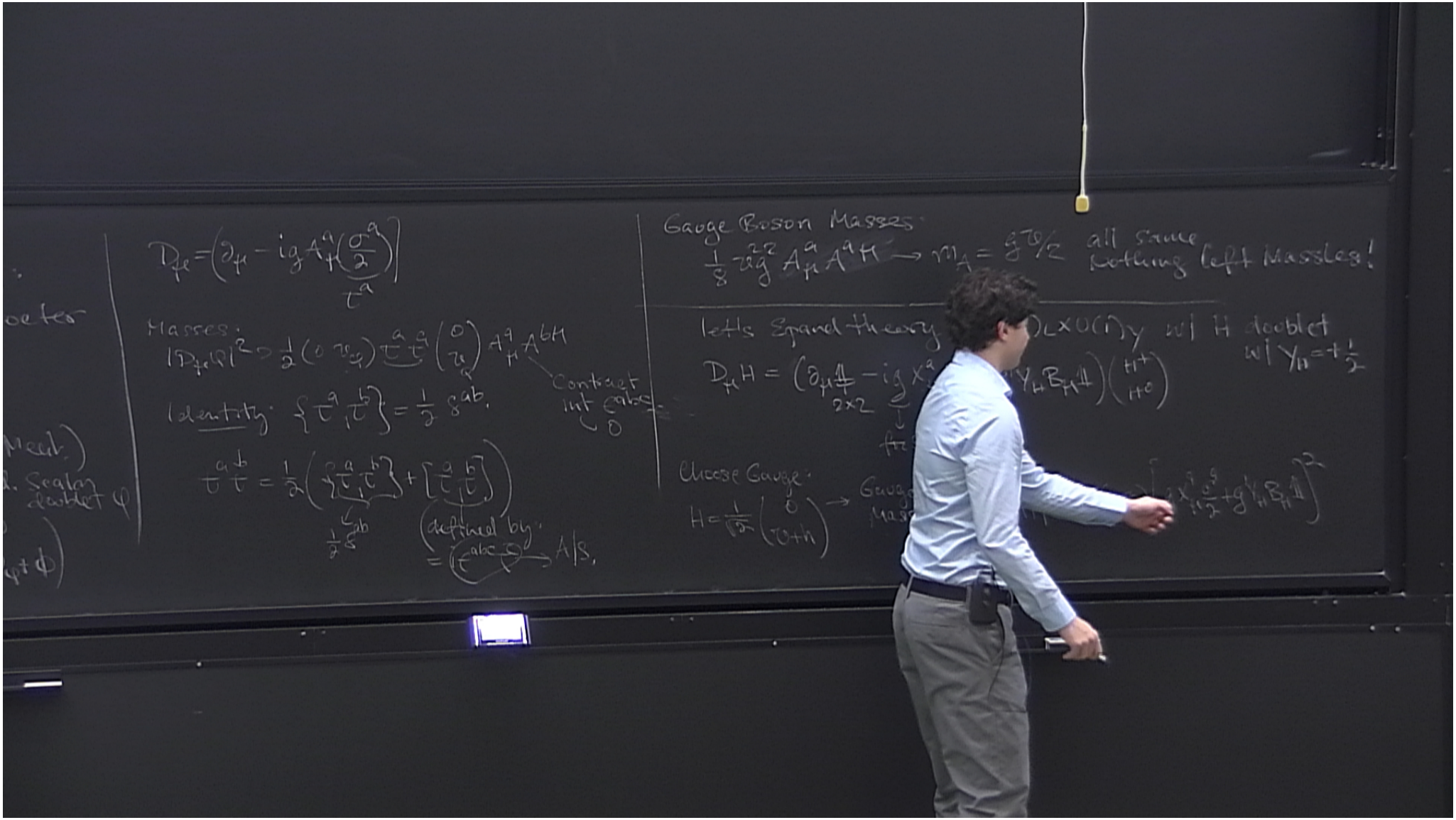
Date: Jan 13, 2015 09:00 AM

URL: http://pirsa.org/15010008

Abstract:


SM+ leptons (goal)
Recap of Yesterday:
- Ch 20 Peskin & Schroeter (QFT)
- Srednicki QFT (Online)
Yesterday (Nonabelian Higgs Mech)
SU(2) gauge theory w/ fund. Scalar doublet φ
Expand
Masses:
Identity:
Gauge Boson Masses:
Contract into zero
defined by
A/S.


Gauge Boson Masses
all since
Masses:
Identity:
Contract
Let's Spand-theory
H doublet
w/ $Y_H = +\frac{1}{2}$
defined by:
A/S.


Gauge Boson Masses:
all since
Nothing left Massless!
Masses:
Identity:
Contract
Scalar
doublet
Let's Spand theory
H doublet
Choose Gauge:

$D_\mu = \left(\partial_\mu - ig A^a_\mu \left(\frac{\sigma^a}{2}\right)\right)$, $\frac{\sigma^a}{2} = t^a$

Masses:

$|D_\mu \Phi|^2 \supset \frac{1}{2}(0\ v_\Phi) t^a t^b \begin{pmatrix} 0 \\ v_\Phi \end{pmatrix} A^a_\mu A^{b\mu}$

Identity: $\{t^a, t^b\} = \frac{1}{2}\delta^{ab}$.

Contract int $\epsilon^{abc} = 0$

$t^a t^b = \frac{1}{2}\left(\{t^a, t^b\} + [t^a, t^b]\right)$

$\{t^a, t^b\} \to \frac{1}{2}\delta^{ab}$

$[t^a, t^b] = (i\epsilon^{abc} t^c)$ defined by.

$= i\epsilon^{abc} \frac{\sigma^c}{2}$ A/S.

Gauge Boson Masses:

$\frac{1}{8} v^2 g^2 A^a_\mu A^{a\mu} H \to m_A = gv/2$ all since nothing left massless!

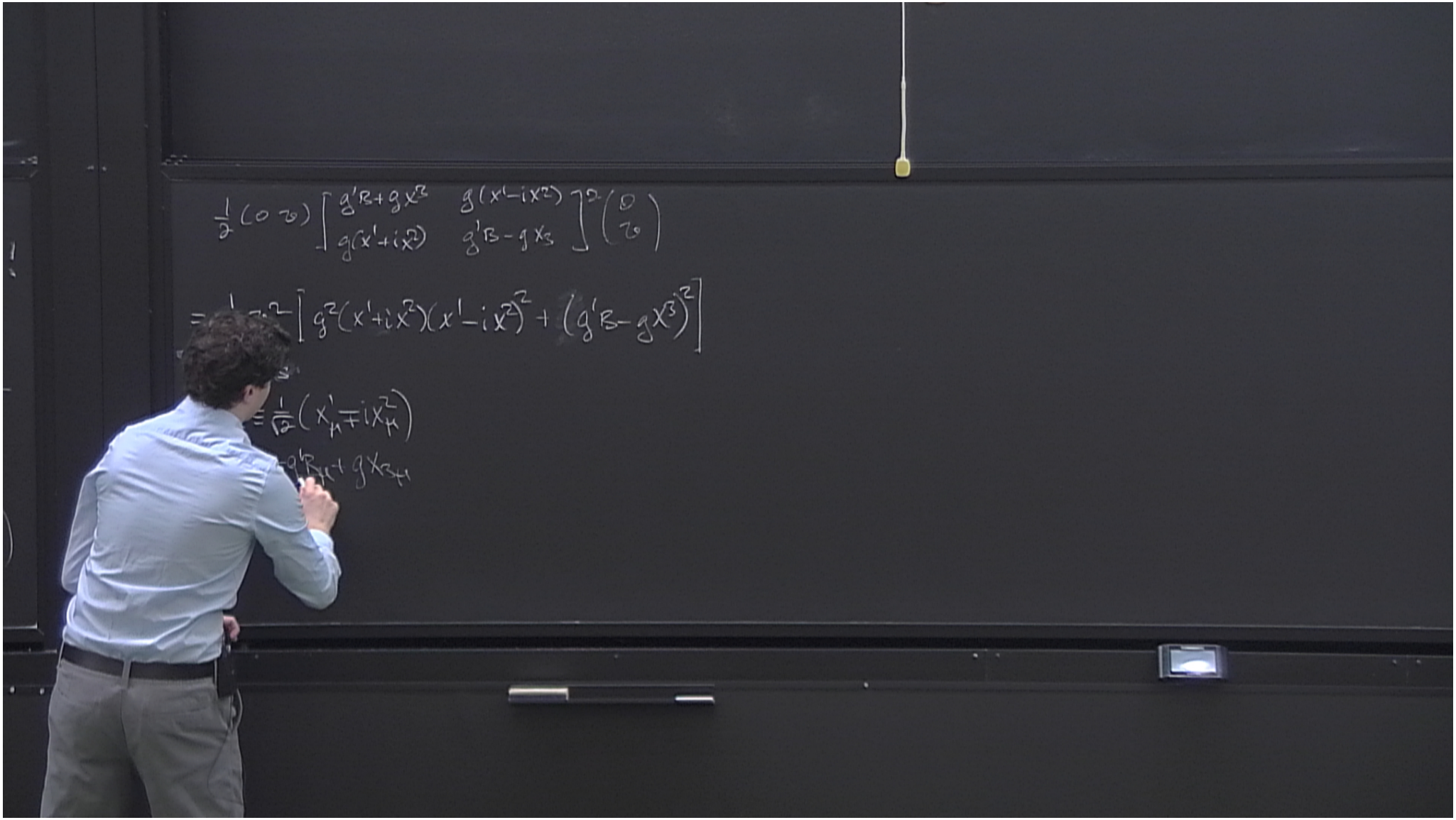
Lets Spand theory $SU(2)_L \times U(1)_Y$ w/ H doublet w/ $Y_H = +\frac{1}{2}$

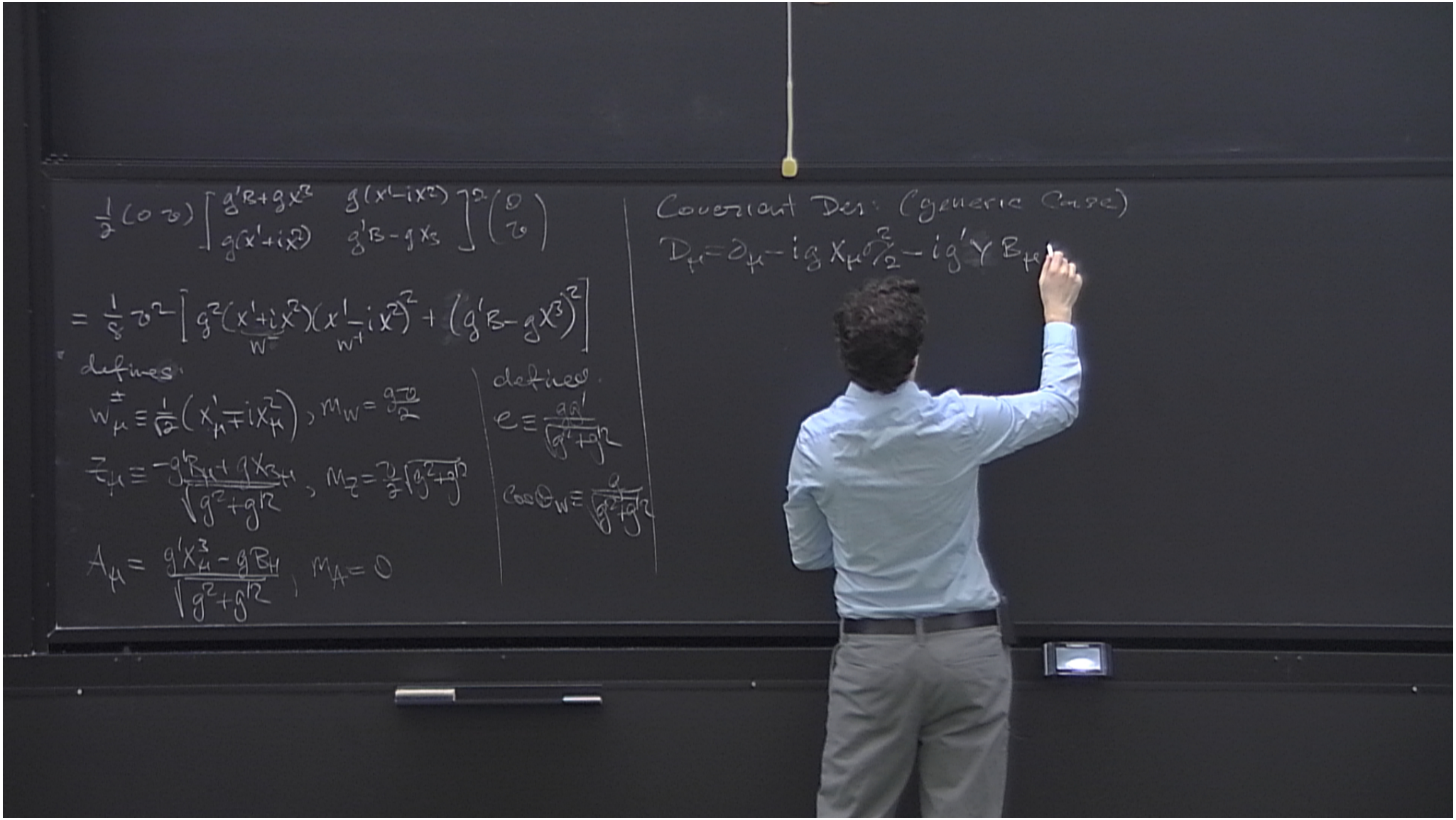
$D_\mu H = \left(\partial_\mu \underbrace{\mathbb{1}}_{2\times 2} - ig X^a_\mu \frac{\sigma^a}{2} - ig' Y_H B_\mu \mathbb{1}\right)\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

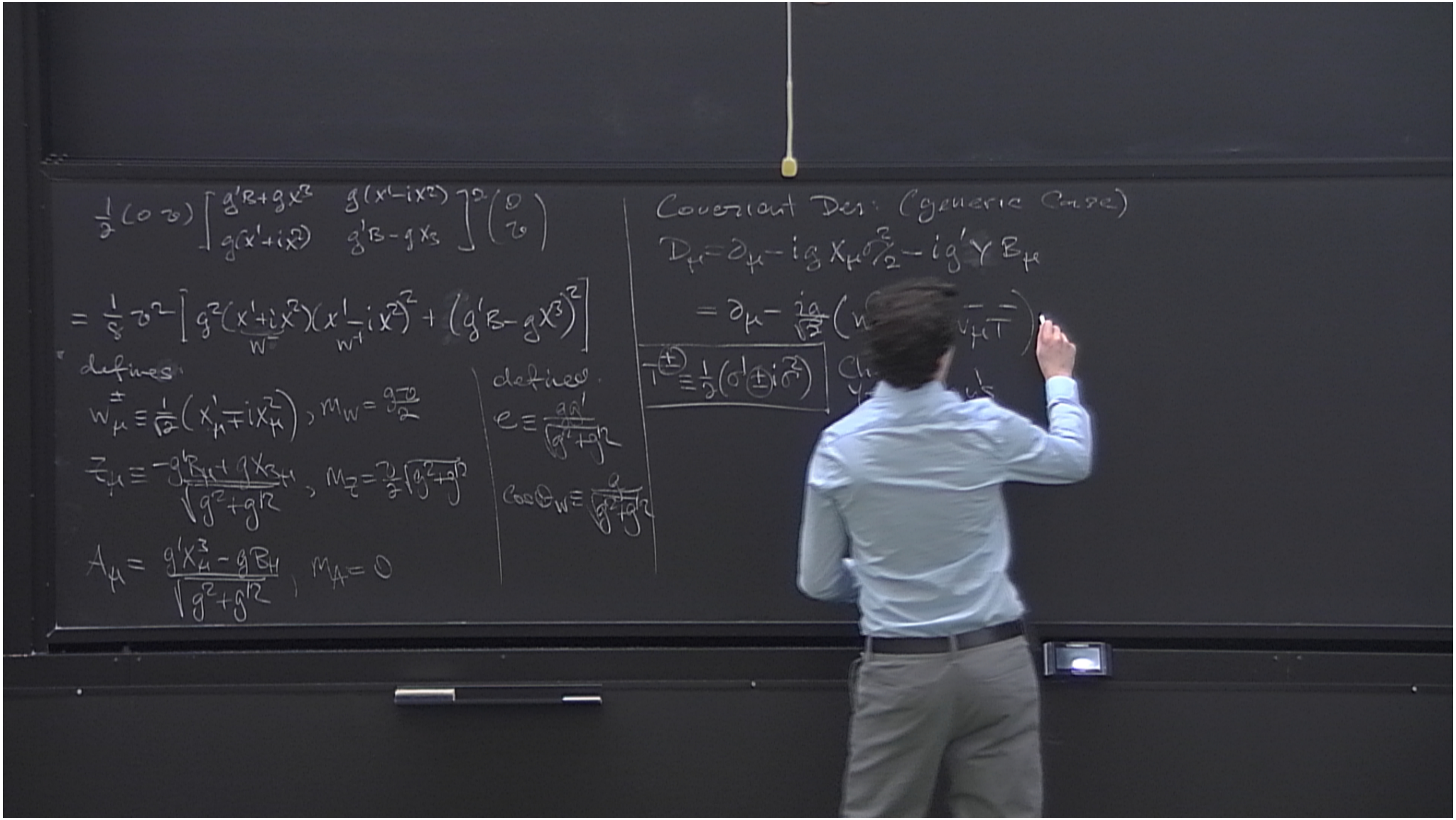
$X \in SU(2)_L$

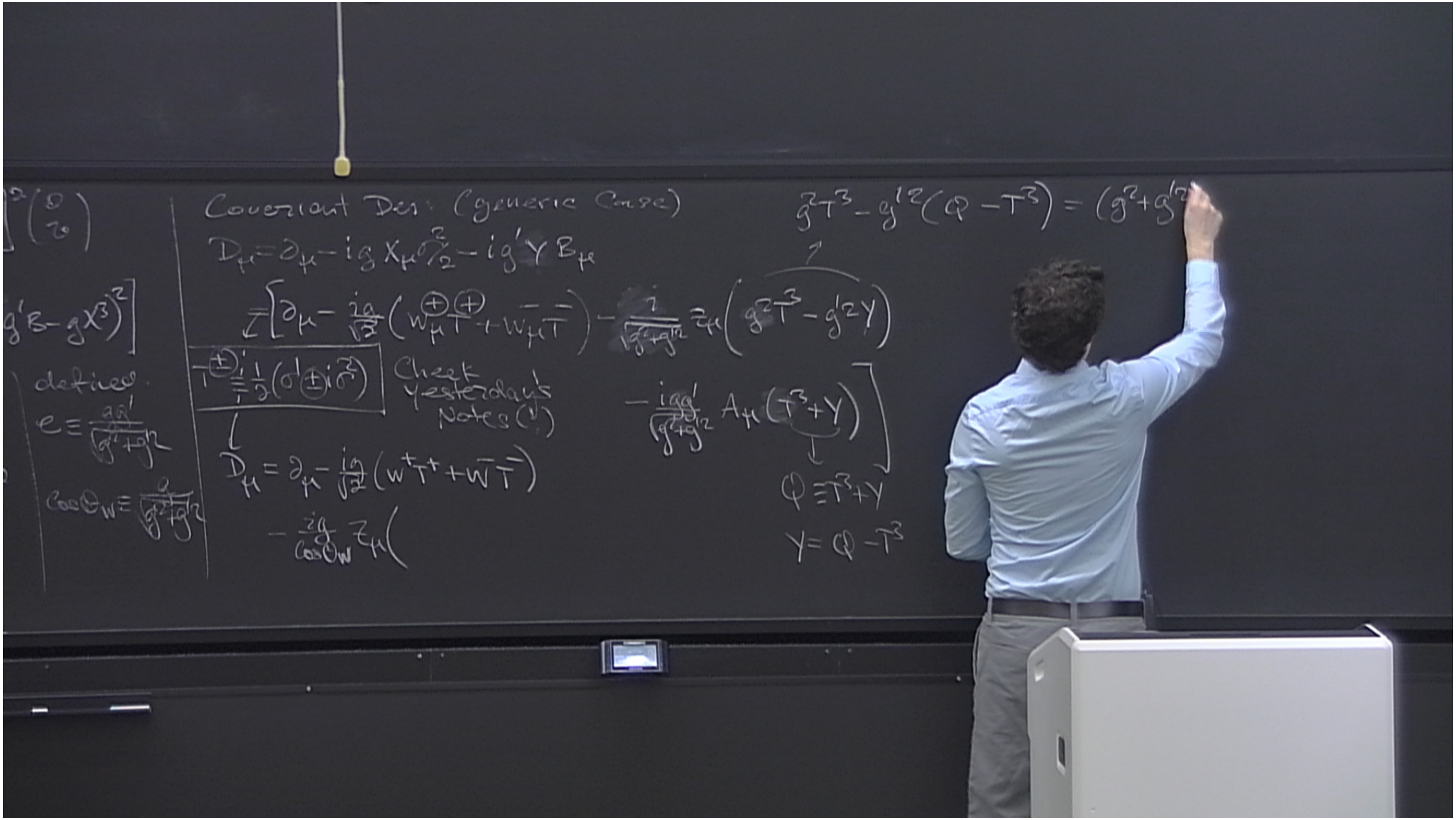
Choose Gauge:

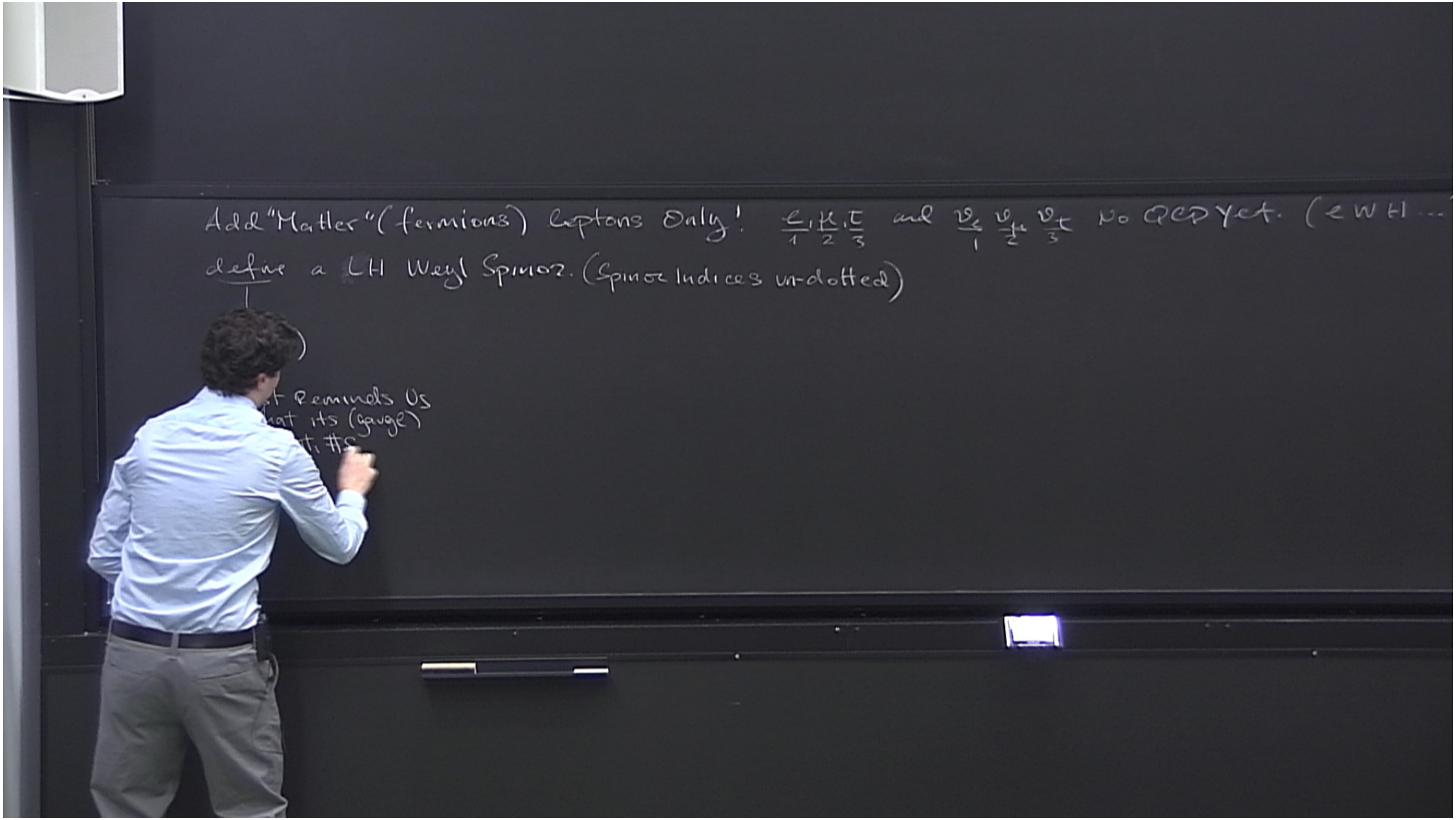
$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ → Gauge Boson Masses: $|D_\mu H|^2 \supset \frac{1}{2}(0\ v)\left[g X^a_\mu \frac{\sigma^a}{2} + g' Y_H B_\mu \mathbb{1}\right]^2 \begin{pmatrix} 0 \\ v \end{pmatrix}$

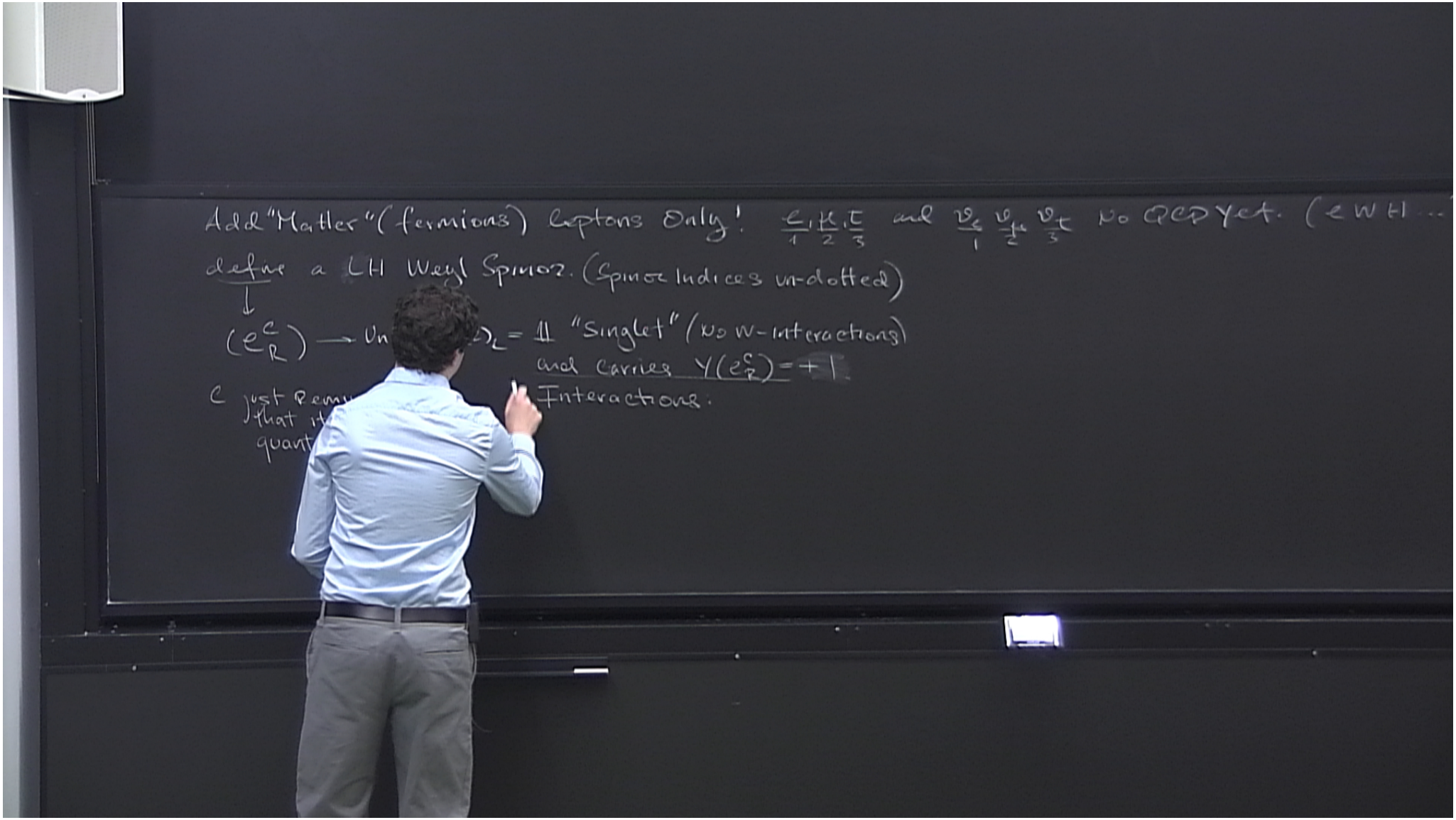

$\frac{1}{2}(0\ v)\begin{bmatrix} g'B+gX^3 & g(X^1-iX^2) \\ g(X^1+iX^2) & g'B-gX_3 \end{bmatrix}^2\begin{pmatrix} 0 \\ v \end{pmatrix}$
$= \frac{1}{2}v^2\left[g^2(X^1+iX^2)(X^1-iX^2)^2+(g'B-gX^3)^2\right]$
$= \frac{1}{\sqrt{2}}(X_\mu^1 \mp iX_\mu^2)$
$gX_{3\mu}$

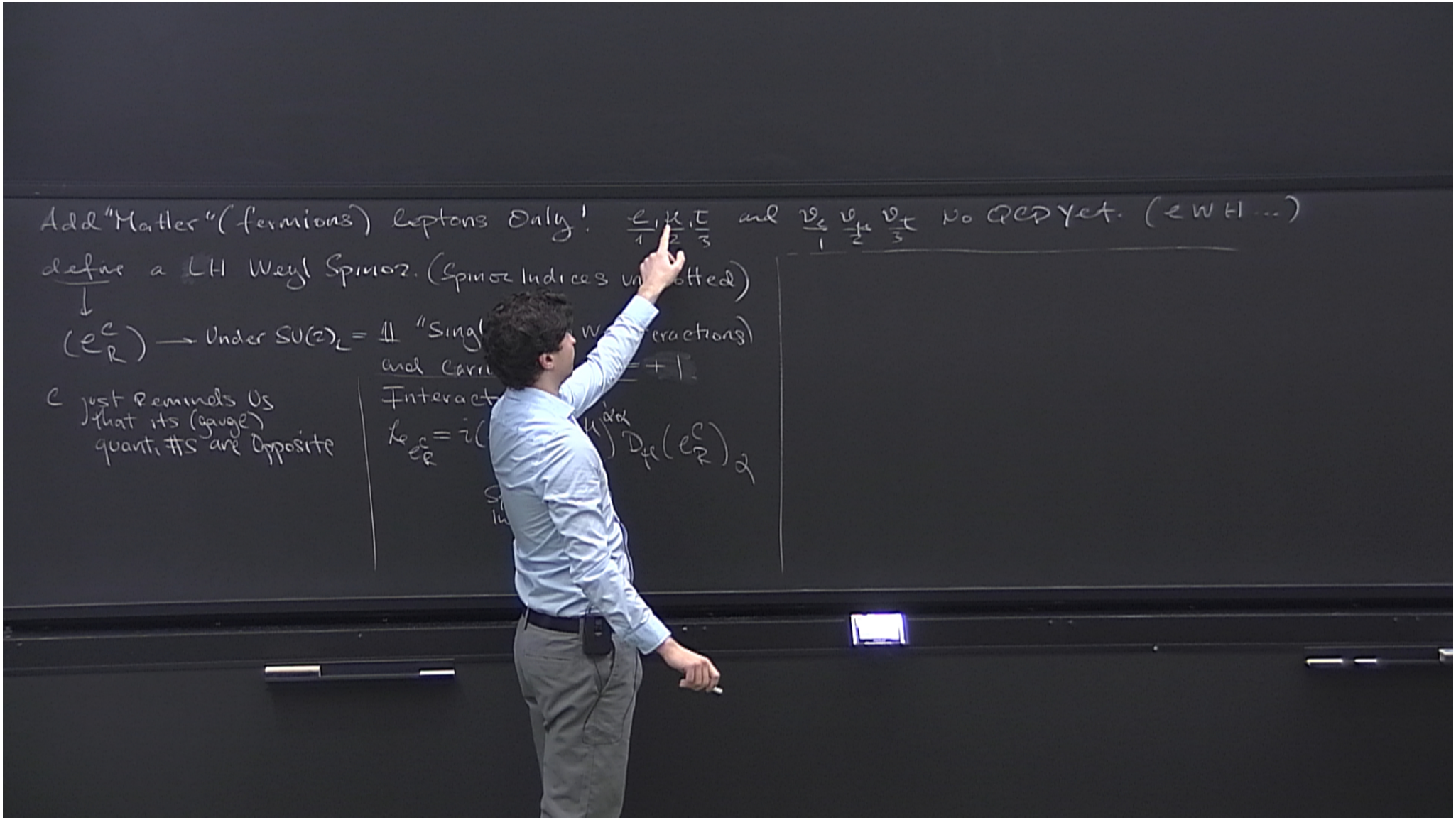

Covariant Der: (generic Case)
defines:
defined:


Covariant Der: (generic case)
defines
defined
Charged


Covariant Der: (generic Case)
$D_\mu = \partial_\mu - ig X_\mu \frac{\sigma^3}{2} - ig' Y B_\mu$
$\mathcal{L}\left[\partial_\mu - \frac{ig}{\sqrt{2}}\left(W_\mu^+ T^+ + W_\mu^- T^-\right) - \frac{i}{\sqrt{g^2+g'^2}} Z_\mu \left(g^2 T^3 - g'^2 Y\right)\right.$
$\left. - \frac{igg'}{\sqrt{g^2+g'^2}} A_\mu (T^3 + Y)\right]$
$T^\pm \equiv \frac{1}{2}(\sigma^1 \pm i\sigma^2)$
Check yesterday's Notes (!)
$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}}\left(W^+ T^+ + W^- T^-\right)$
$- \frac{ig}{\cos\theta_W} Z_\mu ($
$g^2 T^3 - g'^2 (Q - T^3) = (g^2 + g'^2$
$Q \equiv T^3 + Y$
$Y = Q - T^3$
$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}$
$\cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$


Add "Matter" (fermions) Leptons Only! e, μ, τ and ν_e ν_μ ν_τ No QCD yet. (eWH...
define a LH Weyl Spinor. (Spinor Indices undotted)
Reminds Us
its (gauge)


Add "Matter" (fermions) Leptons Only! e, μ, τ and ν_e, ν_μ, ν_τ No QCD yet. (e W H ...
define a LH Weyl Spinor. (Spinor Indices undotted)
"Singlet" (No W-interactions)
and carries Y(e_R^c) = +1
Interactions.


Add "Matter" (fermions) Leptons Only! e, μ, τ and ν_e ν_μ ν_τ No QCD yet. (e W H ...)
define a LH Weyl Spinor. (Spinor Indices
(e^c_R) → Under SU(2)_L =
c just reminds us that its (gauge) quantum #s are Opposite
Interact

ions) Leptons only! e, μ, τ (1, 2, 3) and $\nu_e\ \nu_\mu\ \nu_\tau$ (1, 2, 3) No QCD yet. (EWH ...)

Spinors. (Spinor indices un-dotted)

$(e)_L$ = 𝟙 "Singlet" (no W-interactions) and carries $Y(e_R^c) = +1$.

Interactions.

$$\mathcal{L}_{e_R^c} = i(e_R^c)^\dagger_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha} D_\mu(e_R^c)_\alpha$$

Spinor Index

$$D_\mu(e_R^c) = \left[\partial_\mu - \frac{ig}{\sqrt{2}}(W^+T^+ + W^-T^-) - \frac{ig}{c_W} Z_\mu (T^3 - s_\theta^2 Q) - ieQA_\mu\right](e_R^c)$$

$\frac{ig}{c_W} = \cos\theta_W$; $s_\theta^2 = \sin^2\theta_W$; $Q \equiv T^3 + Y$

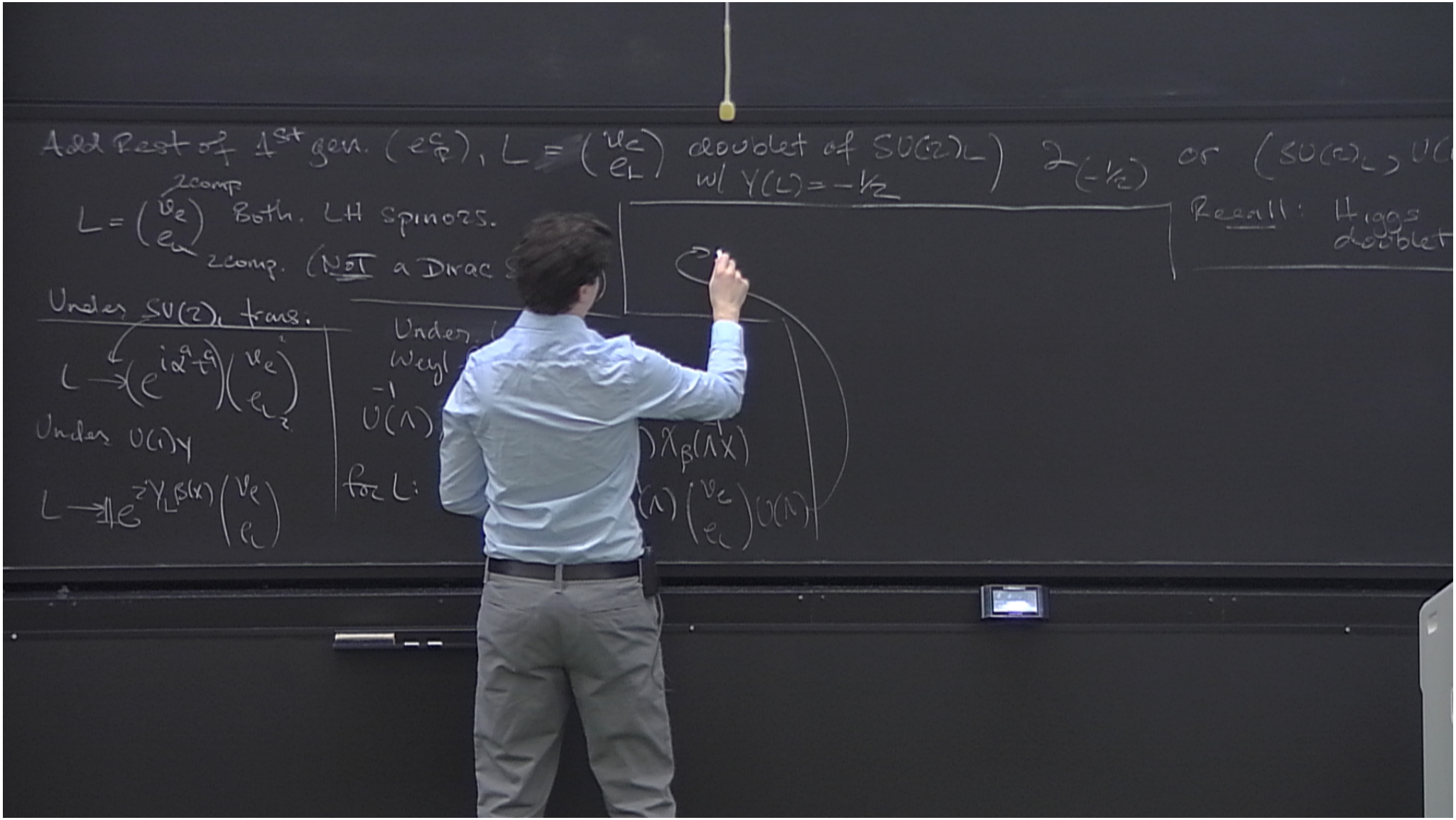
$$Q(e_R^c) = \cancel{T^3(e_R^c)} + Y(e_R^c)$$

$T^3 = 0$; $Y = +1$

$$(D_\mu e_R^c)_\alpha = \left[\partial_\mu - \frac{ig}{c_W} Z_\mu(-s_W^2)(+1) - ieA_\mu\right](e_R^c)_\alpha$$

Kinetic Term: $i(e_R^c)^\dagger_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha} D_\mu(e_R^c)_\alpha$, Recall $\bar\sigma^\mu \equiv (\mathbb{1}_{2\times2}, -\vec\sigma)$

$$= i(e_R^c)^\dagger_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha}\partial_\mu(e_R^c)_\alpha - \frac{gs_W^2}{c_W}(e_R^c)^\dagger_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha} Z_\mu (e_R^c)_\alpha + e(e_R^c)^\dagger_{\dot\alpha}(\bar\sigma^\mu)^{\dot\alpha\alpha} A_\mu (e_R^c)_\alpha$$


Add Rest of 1st gen. (e^c_R), $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ doublet of SU(2)L w/ Y(L) = −½ 2(−½) or (SU(2)L, U(
2 comp
$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ Both. LH spinors.
2 comp. (NOT a Dirac
Recall: Higgs doublet
Under SU(2)L trans.
$L \to (e^{i\alpha^a \tau^a}) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$
Under U(1)Y
$L \to e^{iY_L \beta(x)} \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$
Under
Weyl
$U(\Lambda)$
for L:
$\chi_\beta(\Lambda x)$
$U(\Lambda) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} U(\Lambda)$

Add Rest of 1st gen. (e^c_R), $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ doublet of $SU(2)_L$ w/ $Y(L) = -1/2$ $2_{(-1/2)}$ or $(SU(2)_L, U($

$L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ (2 comp) Both. LH spinors.

2 comp. (NOT a Dirac Spinor)

$= \begin{pmatrix} M^{\beta}_{\alpha}(\nu_e)_{\beta} \\ M^{\beta}_{\alpha}(e_L)_{\beta} \end{pmatrix}$

Recall: Higgs doublet

Under $SU(2)_L$ trans.

$L \to (e^{i\alpha^a \tau^a})\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$

Under $U(1)_Y$

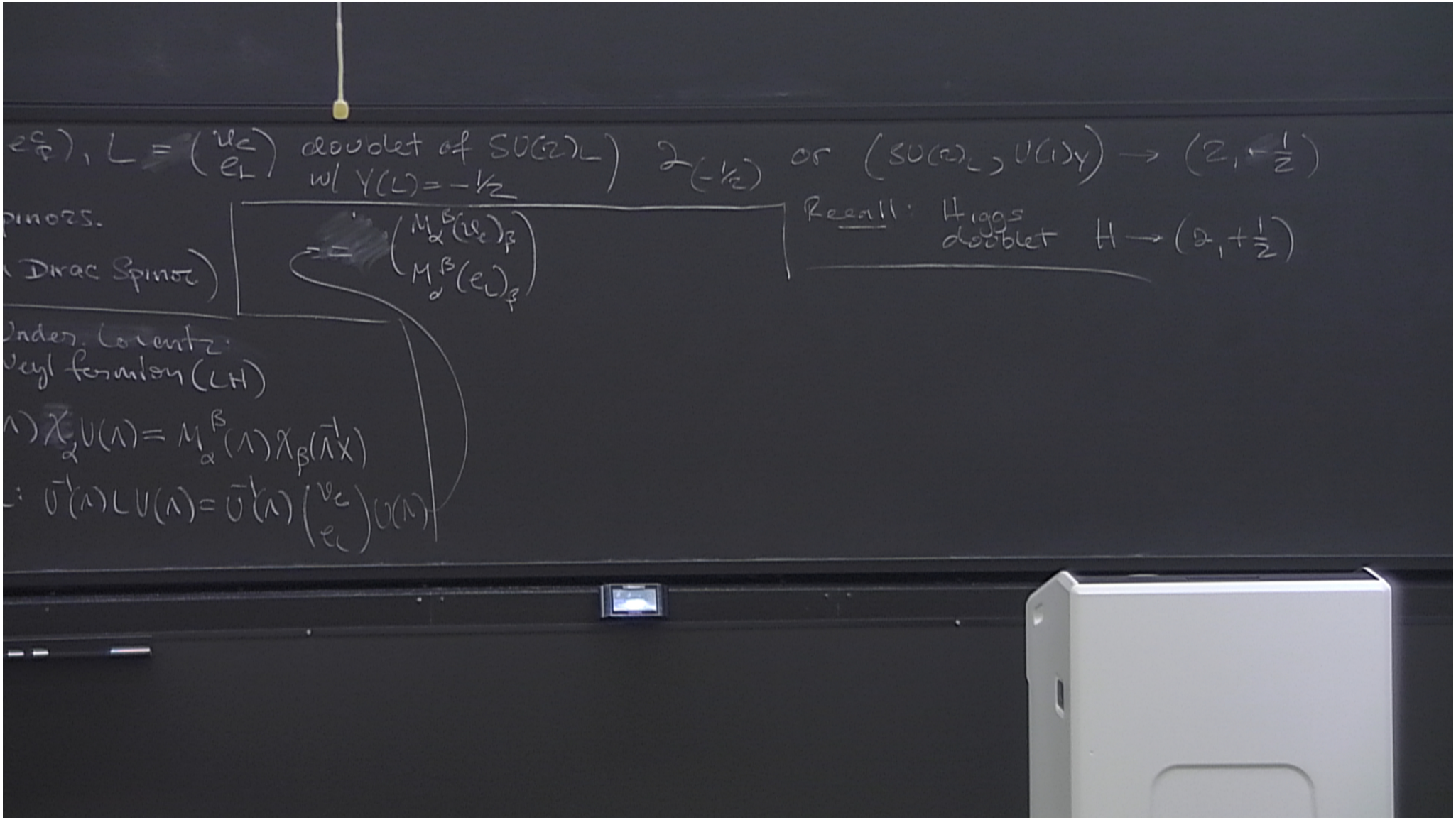
$L \to e^{iY_L \beta(x)}\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$

Under Lorentz

Weyl fermion (LH)

$U^{-1}(\Lambda)\chi_\alpha U(\Lambda) = M^{\beta}_{\alpha}(\Lambda)\chi_\beta(\bar{\Lambda}^{-1}x)$

for L: $\bar{U}^{-1}(\Lambda) L U(\Lambda) = \bar{U}^{-1}(\Lambda)\begin{pmatrix} \nu_e \\ e_L \end{pmatrix} U(\Lambda)$


e_R), $L = \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ doublet of $SU(2)_L$ w/ $Y(L) = -1/2$
$2_{(-1/2)}$ or $(SU(2)_L, U(1)_Y) \to (2, -\frac{1}{2})$
pinors.
Dirac Spinor)
$= \begin{pmatrix} M_\alpha^\beta (\nu_e)_\beta \\ M_\alpha^\beta (e_L)_\beta \end{pmatrix}$
Recall: Higgs doublet $H \to (2, +\frac{1}{2})$
Under Lorentz:
Weyl fermion (LH)
$\Lambda) \chi_\alpha U(\Lambda) = M_\alpha^\beta(\Lambda) \chi_\beta(\bar{\Lambda}^{-1} x)$
$L: \bar{U}^{-1}(\Lambda) L U(\Lambda) = \bar{U}^{-1}(\Lambda) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} U(\Lambda)$

quant. #s are opposite

$\mathcal{L}_{e_R^c} = i(e_R^c)^\dagger_\alpha (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} D_\mu (e_R^c)_{\dot{\alpha}}$

Spinor index

Kinetic Term: $i(e_R^c)^\dagger_\alpha (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} D_\mu (e_R^c)_{\dot{\alpha}}$, Recall $\bar{\sigma}^\mu \equiv (4\times$

$= i(e_R^c)^\dagger_\alpha (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \partial_\mu (e_R^c)_{\dot{\alpha}} - \frac{g s_{\theta_W}^2}{c_{\theta_W}} (e_R^c)^\dagger_\alpha (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} Z_\mu (e_R^c)_{\dot{\alpha}} + e(\quad)^\dagger_\alpha (\bar{\sigma}^$

Interactions of doublet (lepton): Now $SU(2)_L$ is not trivial

for convenience: W-part of $D_\mu \to W_\mu^+ T^+ + W_\mu^- T^-$, $T^+ \equiv \frac{1}{2}(\sigma^1 + i\sigma^2) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ in $SU(2)_L$ space

$T^- = \frac{1}{2}(\sigma^1 - i\sigma^2) = \begin{pmatrix} 0 & \\ 1 & \end{pmatrix}$

$\mathcal{L}_L = i L^\dagger \bar{\sigma}^\mu D_\mu L$ (spinor indices suppressed!)

$= i(\nu_e^\dagger \; e_L^\dagger)\bar{\sigma}^\mu \left(\partial_\mu \mathbb{1} - i\frac{g}{\sqrt{2}}\left(W_\mu^+ \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} + W_\mu^- \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right) - \frac{ig}{c_{\theta_W}} Z_\mu (T^3 - s_{\theta_W}^2 Q) - ieQA_\mu\right)\begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$ $T^3 L$



Under $U(1)_Y$

$L \to e^{iY_L\beta(x)} \begin{pmatrix} \nu_e \\ e_L \end{pmatrix}$

$U(\Lambda)\chi_\alpha U(\Lambda) = \Lambda_\alpha^\beta(\Lambda)\chi_\beta(\Lambda^{-1}x)$

for L: $\bar{U}^\dagger(\Lambda) L U(\Lambda) = \bar{U}^\dagger(\Lambda) \begin{pmatrix} \nu_e \\ e_L \end{pmatrix} U(\Lambda)$

How Do Dirac Spinors Arise? Interact w/ Higgs. B/C all fermions Must Be

fermion Masses — Majorana Mass: $\frac{1}{2}M\left(\chi^\alpha \chi_\alpha + \chi^\dagger_{\dot\alpha} \chi^{\dagger\dot\alpha}\right)$ Mass from Same Spinor!

Problem → Not gauge Invariant

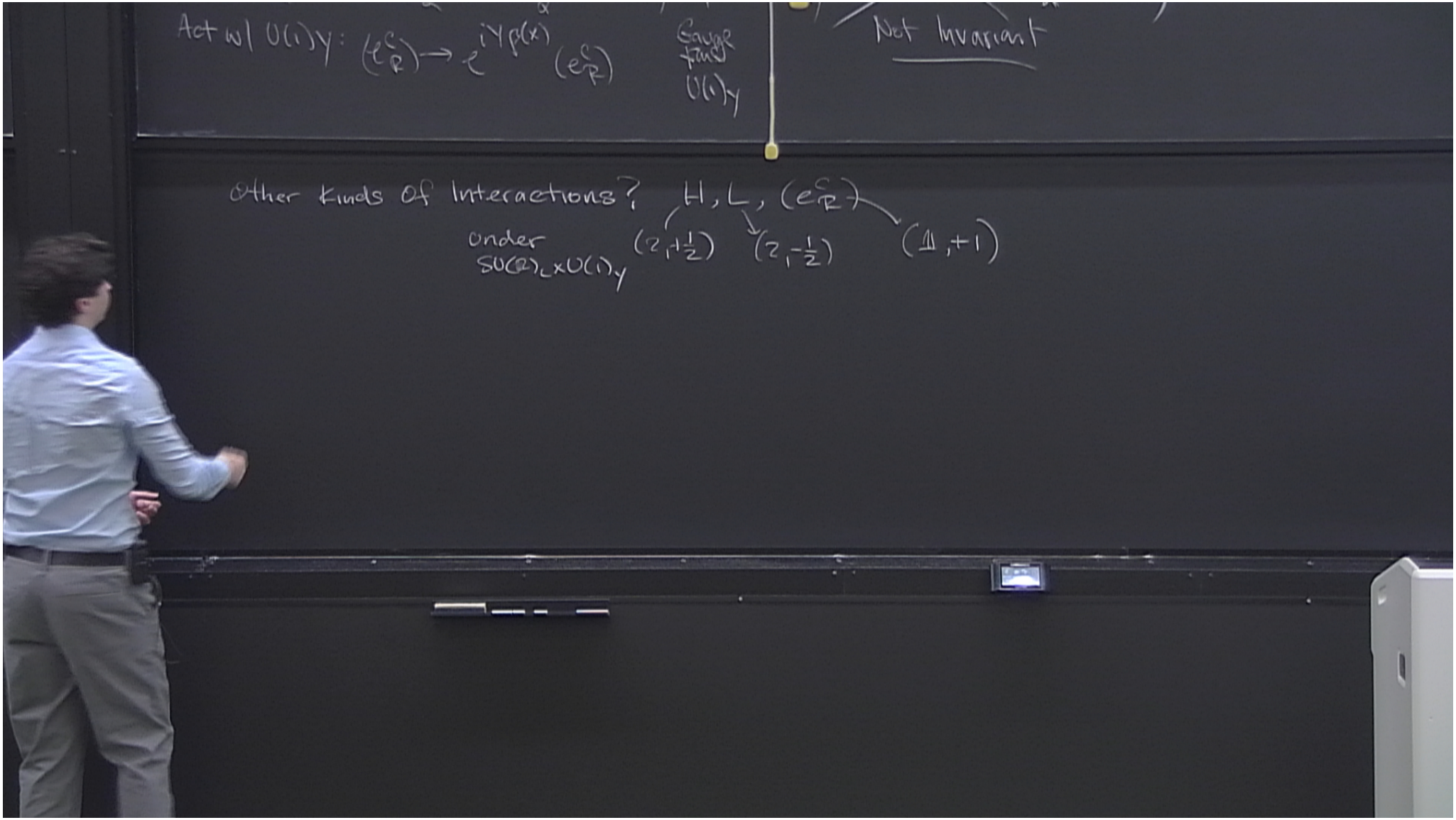
Let's Try Anyway: (e^c_R)

$M\left((e^c_R)^\alpha (e^c_R)_\alpha + (e^c_R)^\dagger_{\dot\alpha} (e^c_R)^{\dagger\dot\alpha}\right) \longrightarrow \frac{M}{2}\left(e^{i2Y\beta(x)} (e^c_R)^\alpha (e^c_R)_\alpha + \text{H.c.}\right)$

Act w/ $U(1)_Y$: $(e^c_R) \to e^{iY\beta(x)} (e^c_R)$

Gauge fixed $U(1)_Y$

Not Invariant


Act w/ U(1)Y: $(e_R^c) \to e^{iY\beta(x)} (e_R^c)$
Gauge fund U(1)Y
Not invariant
Other kinds of interactions? H, L, (e_R^c)
under $SU(2)_L \times U(1)_Y$
$(2, +\frac{1}{2})$
$(2, -\frac{1}{2})$
$(1, +1)$

Act w/ $U(1)_Y$: $(e^c_R) \to e^{iY\beta(x)} (e^c_R)$

Gauge fixed $U(1)_Y$

Not Invariant

Other kinds of Interactions? $H, L, (e^c_R)$ under $SU(2)_L \times U(1)_Y$ $(2, +\frac{1}{2})$ $(2, -\frac{1}{2})$ $(1, +1)$

~~HL~~ Not lorentz inv. (Spinor + Scalar)

$HL(e^c_R) \to (H_i L_j \epsilon^{ij})(e^c_R)$

ok for $SU(2)$ $(U_H)(U_L)\epsilon_{ik}$

Can Contract 2 doublets: in $SU(2)_L$ invariant way

w/ ϵ_{ij} $i, j \to SU(2)$ Indices (fundamental)

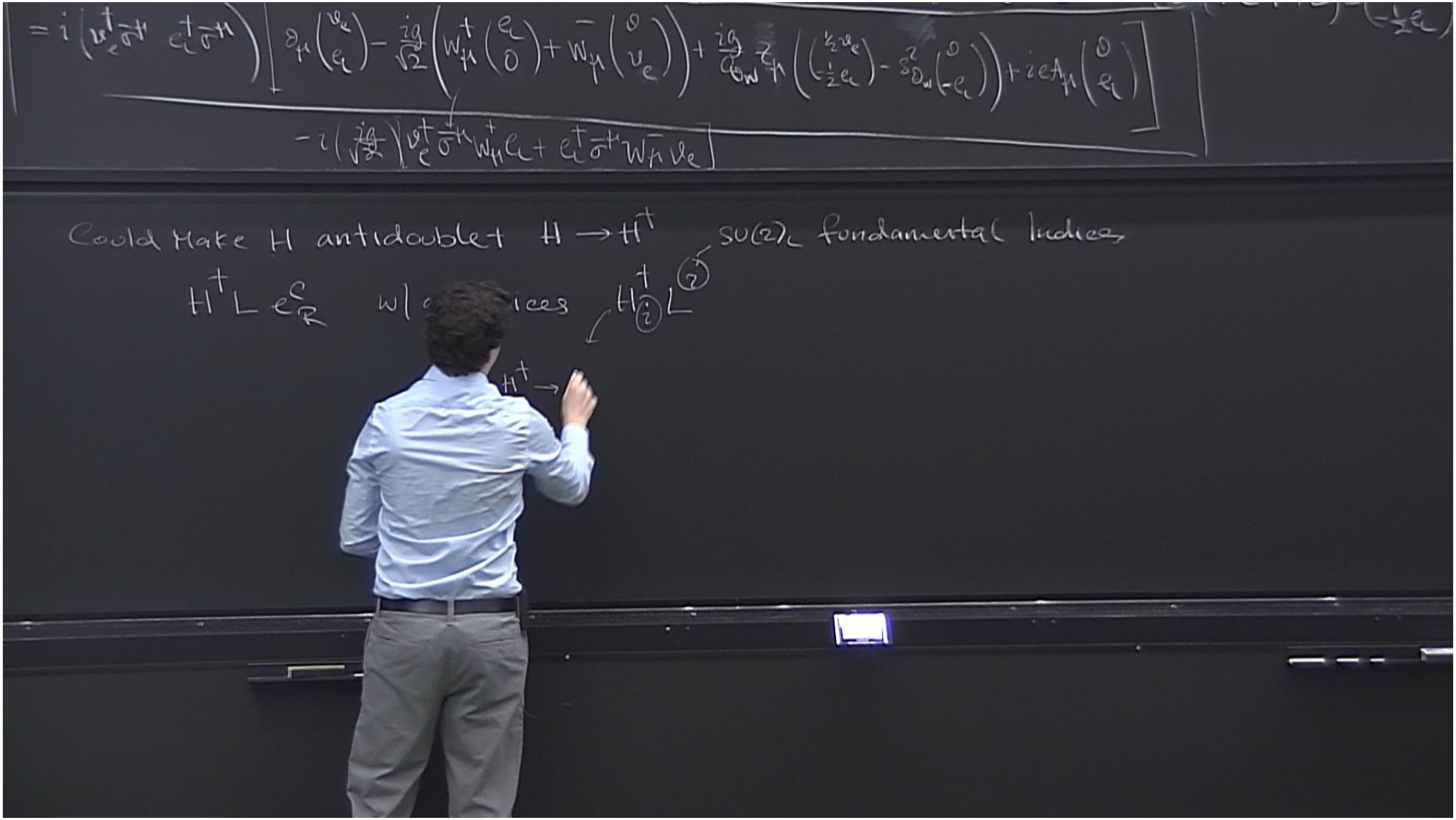
Not ok for $U(1)_Y$

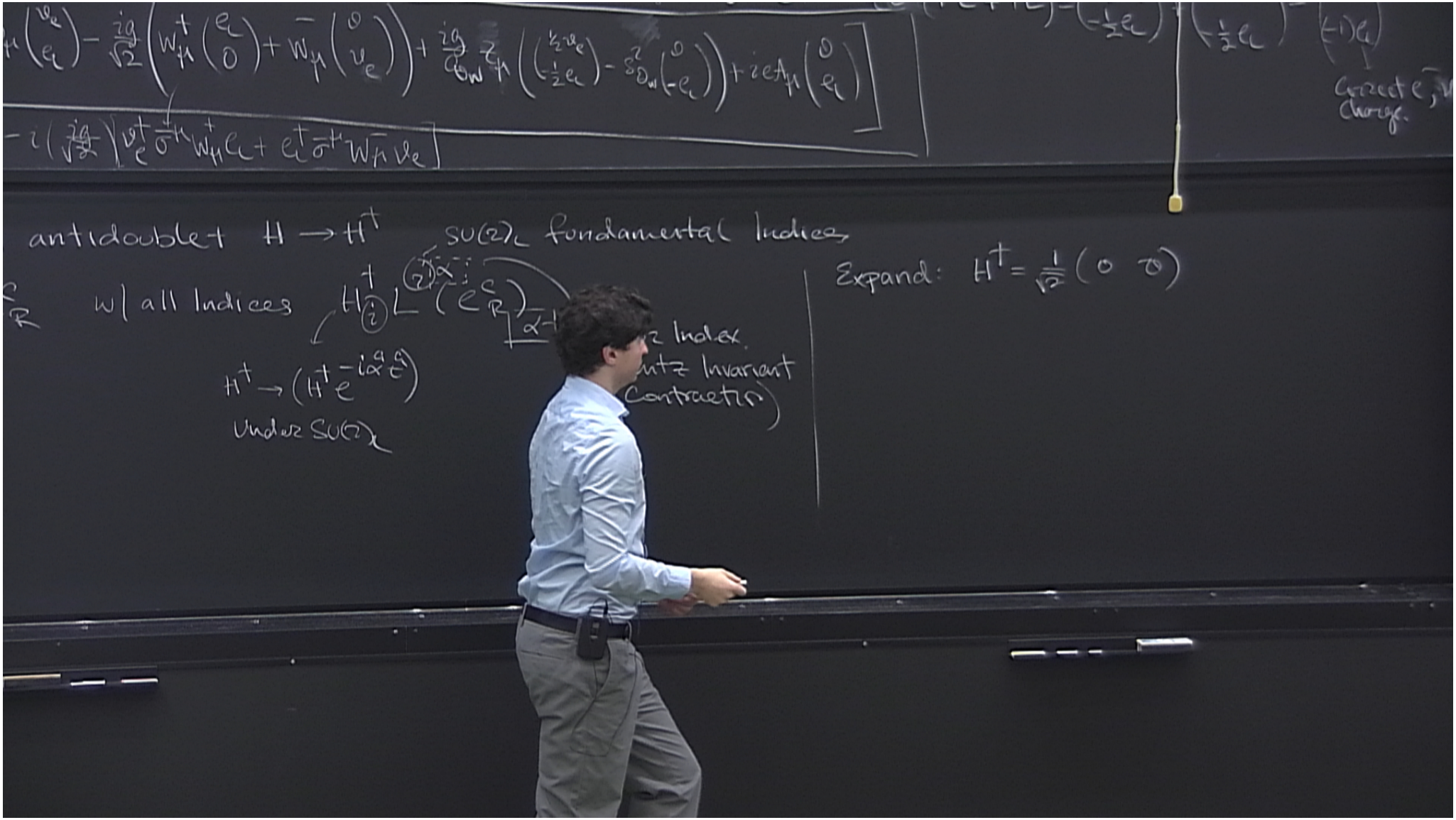
$H \to e^{iY_H \beta} H$

$L \to e^{iY_L \beta} L$

$(e^c_R) \to e^{iY_{e^c_R}\beta}(e^c_R)$

$HL(e^c_R) \to e^{i(+\frac{1}{2})Y_H \beta} e^{i(-\frac{1}{2})Y_L \beta} e^{i(+1)\beta}(e^c_R) \neq$


Could Make H antidoublet $H \to H^\dagger$
SU(2)L fundamental indices
$H^\dagger L e_R^c$ w/
$H^\dagger \to$


antidoublet $H \to H^\dagger$
$SU(2)_L$ fundamental Indices
w/ all Indices
Under $SU(2)_L$
Expand: $H^\dagger = \frac{1}{\sqrt{2}}(0 \;\; v)$