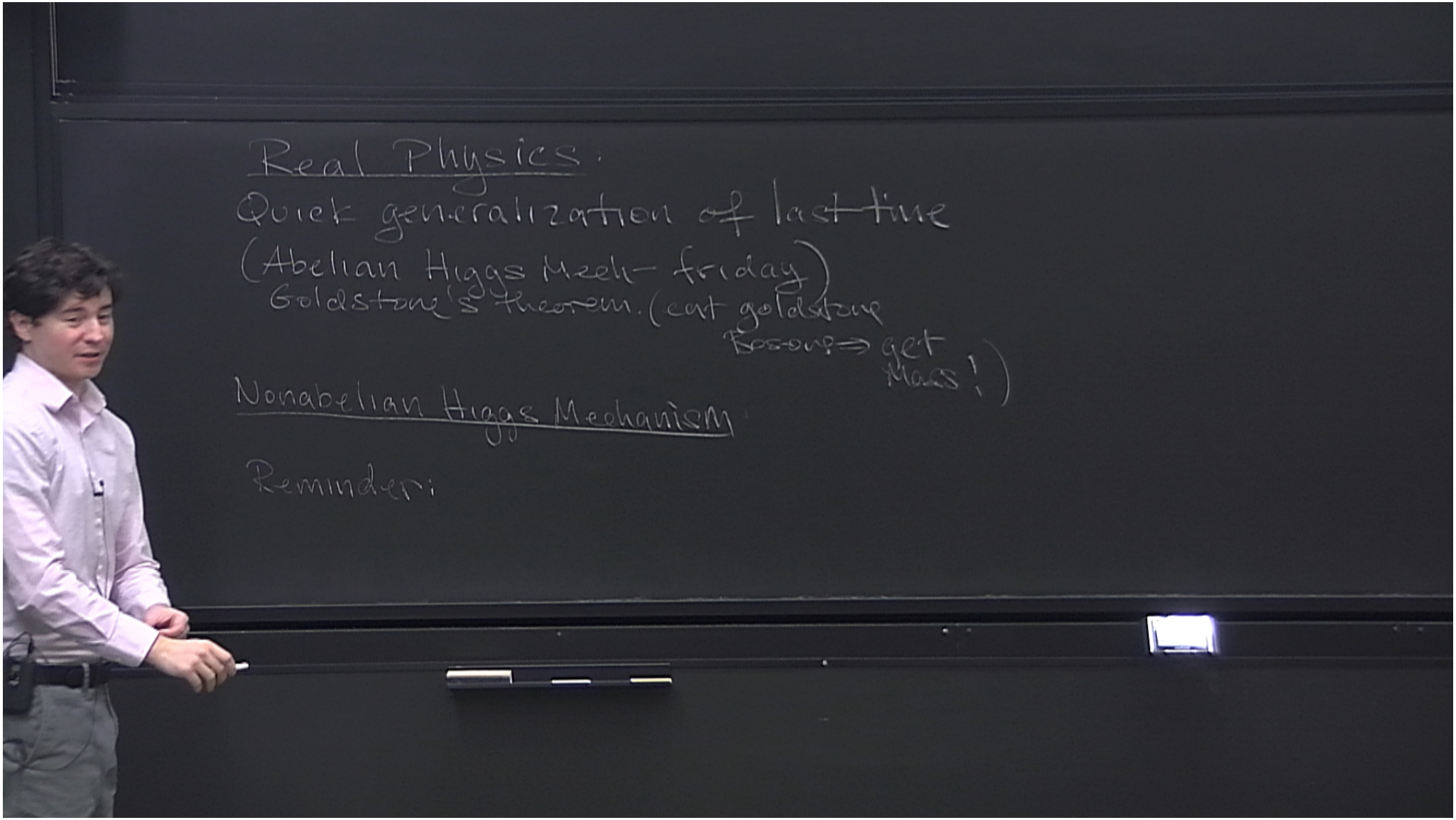
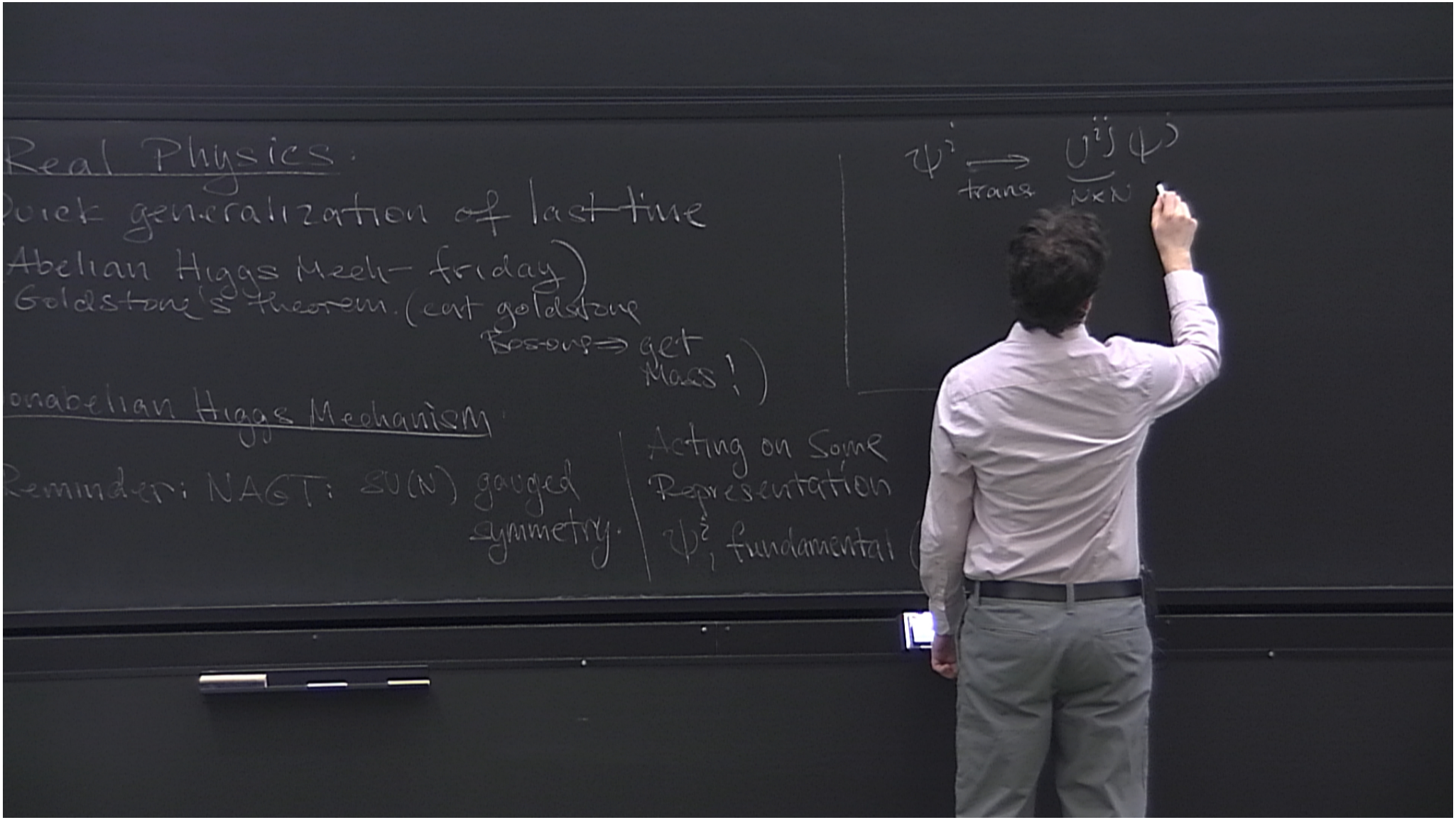
Title: Standard Model-4

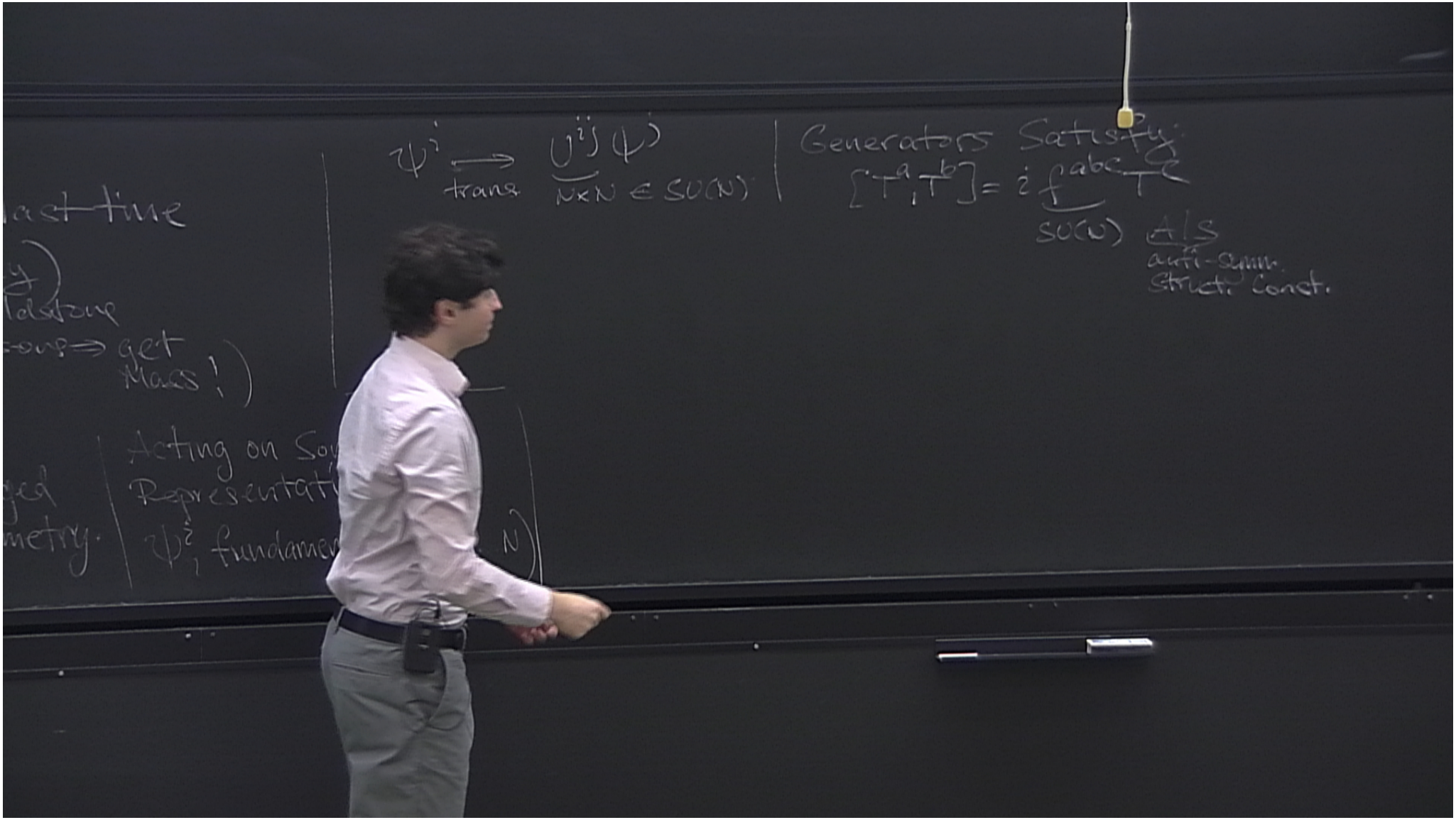
Date: Jan 12, 2015 09:00 AM

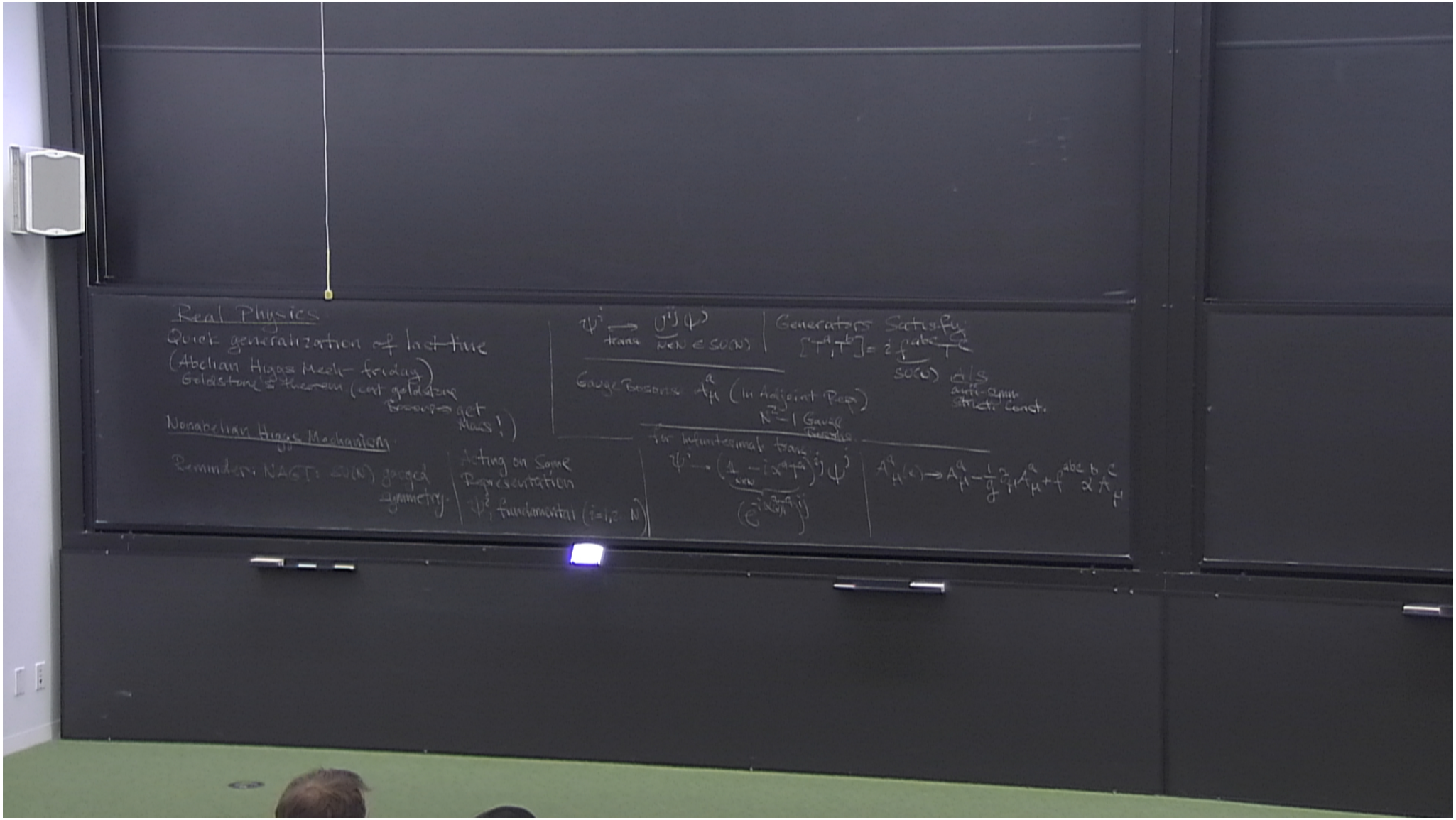
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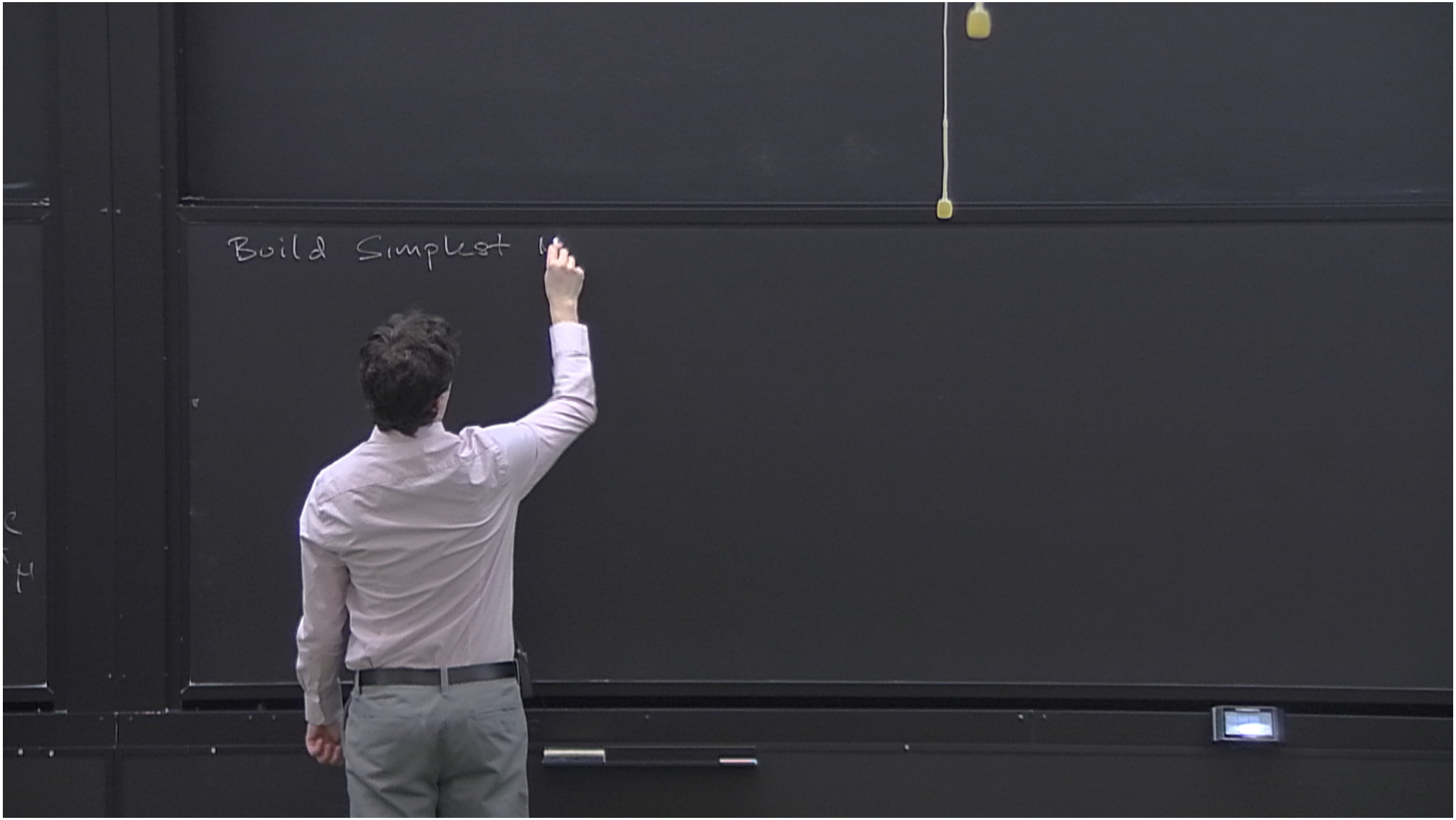
Abstract:

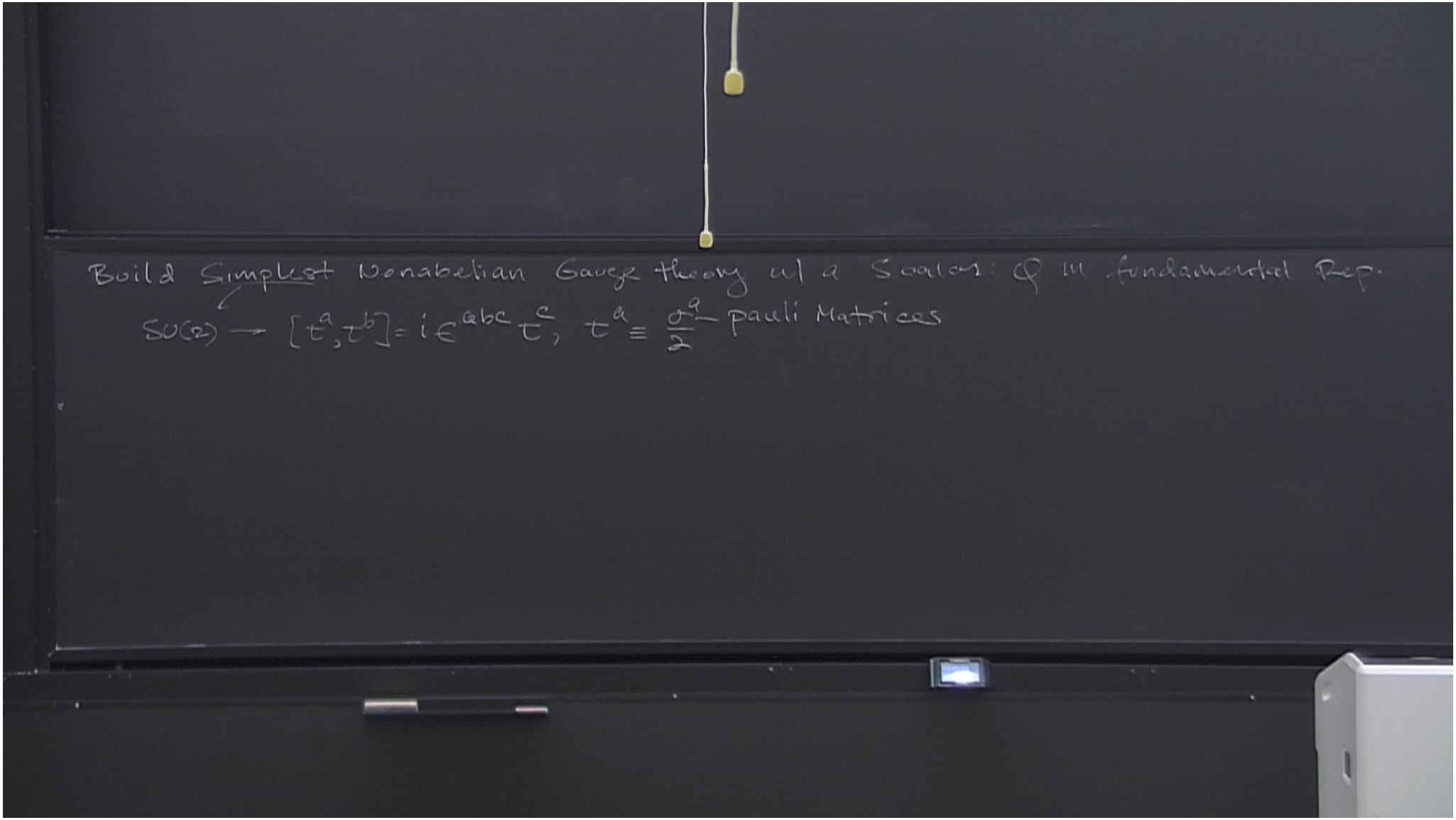

Real Physics:
Quick generalization of last time
(Abelian Higgs Mech- friday)
Goldstone's theorem. (eat goldstone Bosons ⇒ get Mass!)
Nonabelian Higgs Mechanism
Reminder:


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Reminder: NAGT: SU(N) gauged symmetry.
Acting on Some Representation
ψ^i, fundamental
$\psi^i \longrightarrow U^{ij}(\psi^j)$
trans
NxN


$\psi^i \longrightarrow U^{ij}(\psi^j)$
trans
$N \times N \in SU(N)$
Generators Satisfy:
$[T^a, T^b] = i f^{abc} T^c$
$SU(N)$
A/S
anti-Symm.
Struct. Const.
Acting on So
Represent
ψ^i, fundamen
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last time
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ψ^i, fundamental (i=1,2 N)
Generators Satisfy
SU(N)
Gauge Bosons
N^2-1 Gauge
Bosons
For infinitesimal trans
anti-Symm
structure const.


Build Simplest


Build Simplest Nonabelian Gauge theory w/ a Scalar: Φ in fundamental Rep.
SU(2) → $[t^a, t^b] = i\epsilon^{abc} t^c$, $t^a \equiv \frac{\sigma^a}{2}$ pauli Matrices

Build Simplest Nonabelian Gauge theory w/ a Scalar: φ in fundamental Rep.

$SU(2) \rightarrow [t^a, t^b] = i\epsilon^{abc} t^c$, $t^a \equiv \frac{\sigma^a}{2}$ — Pauli Matrices, $N^2-1 = 3$ generators ($N=2$)

Most general Lagrangian: $\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + |D_\mu \varphi|^2 + \mu^2 \varphi^\dagger \varphi - \frac{\lambda}{2} (\varphi^\dagger \varphi)^2$

Field Strength tensor: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

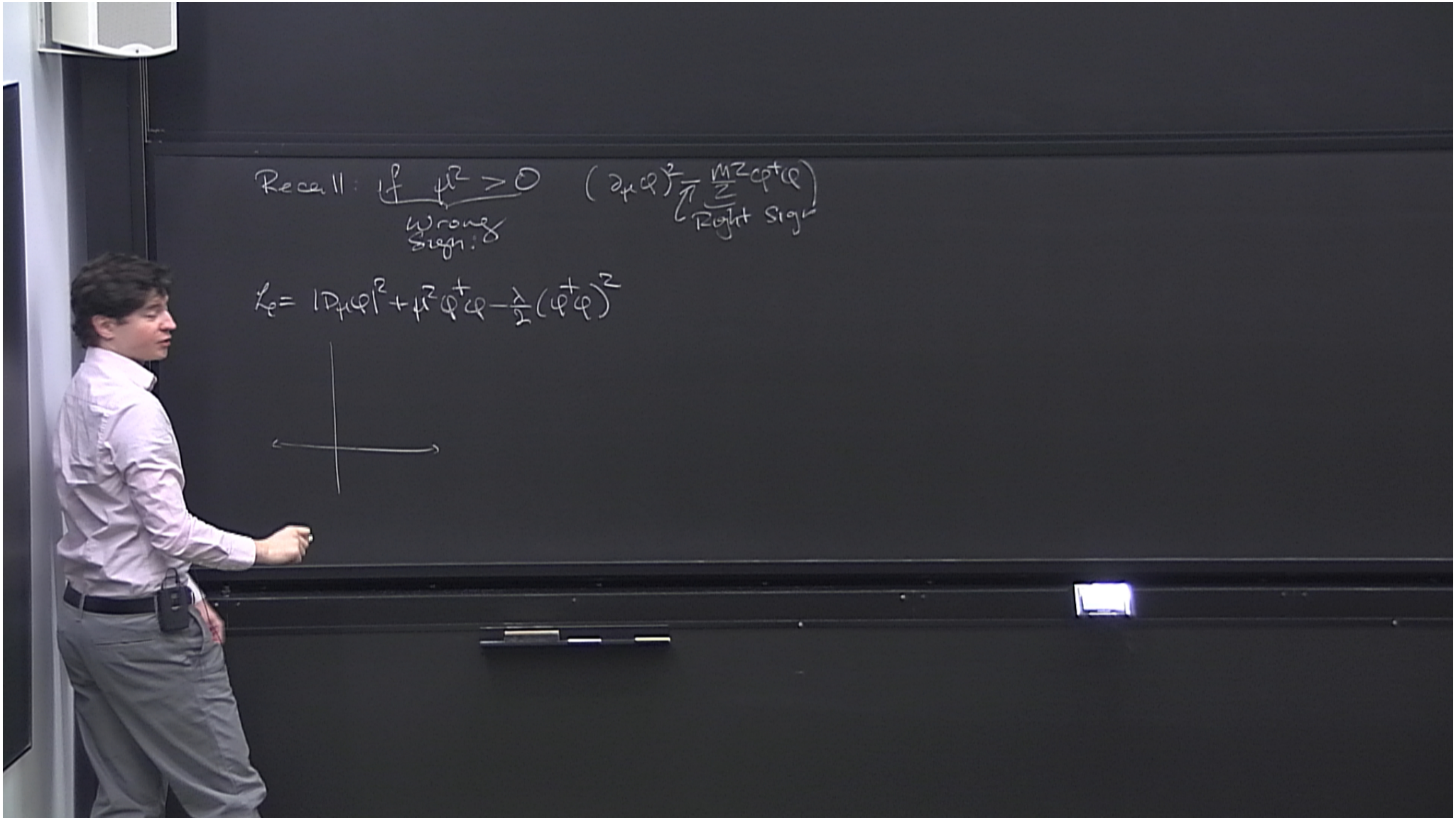
If abelian

If nonabelian: $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc} A^b_\mu A^c_\nu$

$(D_\mu \varphi)^2 = [\partial_\mu \delta^{ij} - ig(A^a_\mu t^{aij})]^2 \varphi^j$

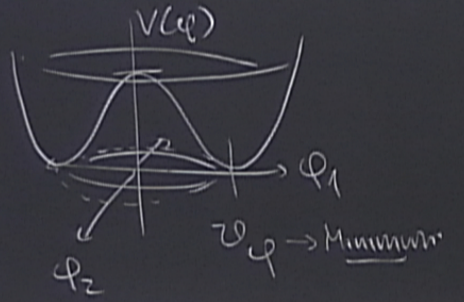
$\varphi = \begin{pmatrix} \varphi_1^R + i\varphi_1^I \\ \varphi_2^R + i\varphi_2^I \end{pmatrix}$

Now choose Mass to be "wrong sign!"


Recall: if $\mu^2 > 0$ $(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^\dagger\varphi$
Wrong sign!
Right sign
$\mathcal{L}_\varphi = |D_\mu\varphi|^2 + \mu^2\varphi^\dagger\varphi - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2$

Recall: If $\mu^2 > 0$ (wrong sign!) $(\partial_\mu \varphi)^2 - \frac{M^2}{2}\varphi^\dagger\varphi$ (Right sign)

$\mathcal{L} = |D_\mu \varphi|^2 + \mu^2 \varphi^\dagger \varphi - \frac{\lambda}{2}(\varphi^\dagger \varphi)^2$. Choose gauge (picking $\alpha^a(x)$)

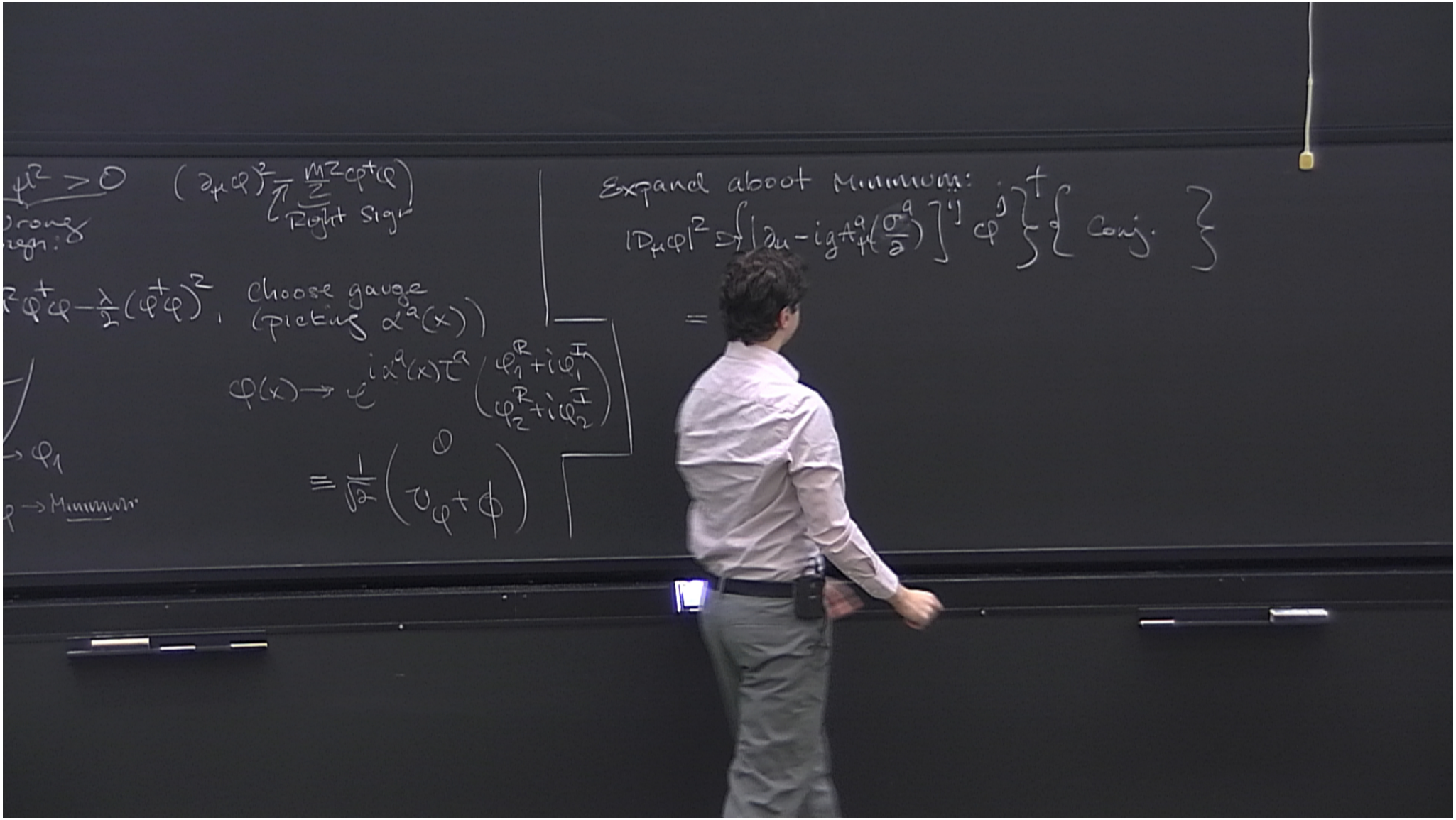


$$\varphi(x) \to e^{i\alpha^a(x)\tau^a} \begin{pmatrix} \varphi_1^R + i\varphi_1^H \\ \varphi_2^R + i\varphi_2^H \end{pmatrix}$$

$$\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon_\varphi + \phi \end{pmatrix}$$

Expand about Minimum:

$$|D_\mu \varphi|^2 \to \left\{ \left[\partial_\mu - ig A_\mu^a \left(\frac{\sigma^a}{2}\right) \right] \varphi^\dagger \right\}^\dagger \left\{ \text{Conj.} \right.$$


$\mu^2 > 0$
$(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^\dagger\varphi$
Right Sign
$\varphi^\dagger\varphi - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2$
Choose gauge (picking $\alpha^a(x)$)
$\varphi(x) \to e^{i\alpha^a(x)\tau^a}\begin{pmatrix}\varphi_1^R + i\varphi_1^H \\ \varphi_2^R + i\varphi_2^H\end{pmatrix}$
$\to \varphi_1$
$\to$ Minimum
$\equiv \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v_\varphi + \phi\end{pmatrix}$
Expand about Minimum:
$|D_\mu\varphi|^2 \to \left\{\left[\partial_\mu - ig\tau^a_\mu\left(\frac{\sigma^a}{2}\right)\right]\varphi^1\right\}^\dagger\left\{\text{Conj.}\right\}$
=

$\mu^2 > 0$ $(\partial_\mu \varphi)^2 - \frac{m^2}{2}\varphi^\dagger\varphi$

Right Sign

$\varphi^\dagger\varphi - \frac{\lambda}{2}(\varphi^\dagger\varphi)^2$. Choose gauge (picking $\alpha^a(x)$)

$\varphi(x) \to e^{i\alpha^a(x)\tau^a} \begin{pmatrix} \varphi_1^R + i\varphi_1^I \\ \varphi_2^R + i\varphi_2^I \end{pmatrix}$

$\to \varphi_1$

$\to$ Minimum

$\equiv \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\varphi + \phi \end{pmatrix}$ Real.

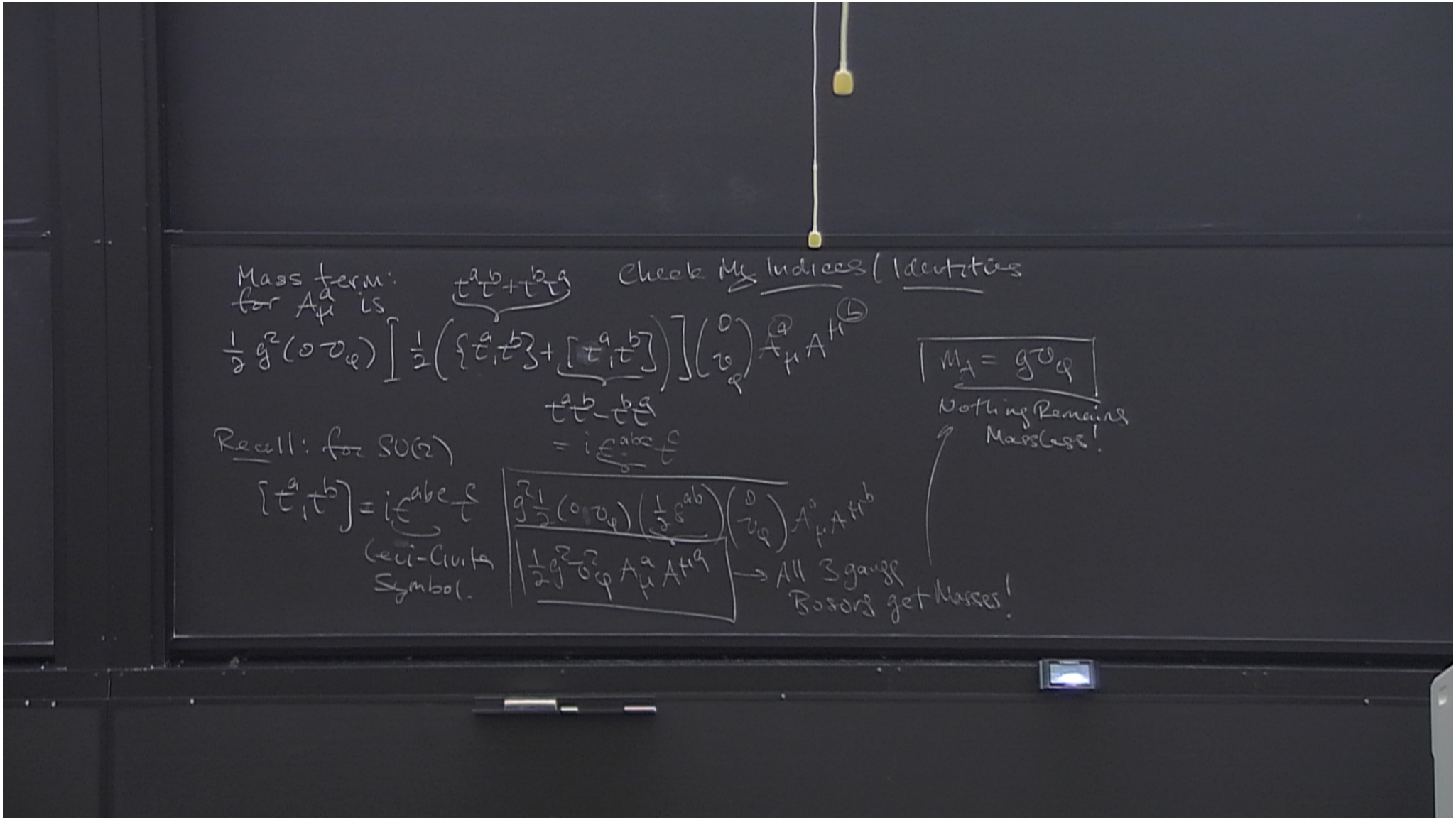
Expand about Minimum:

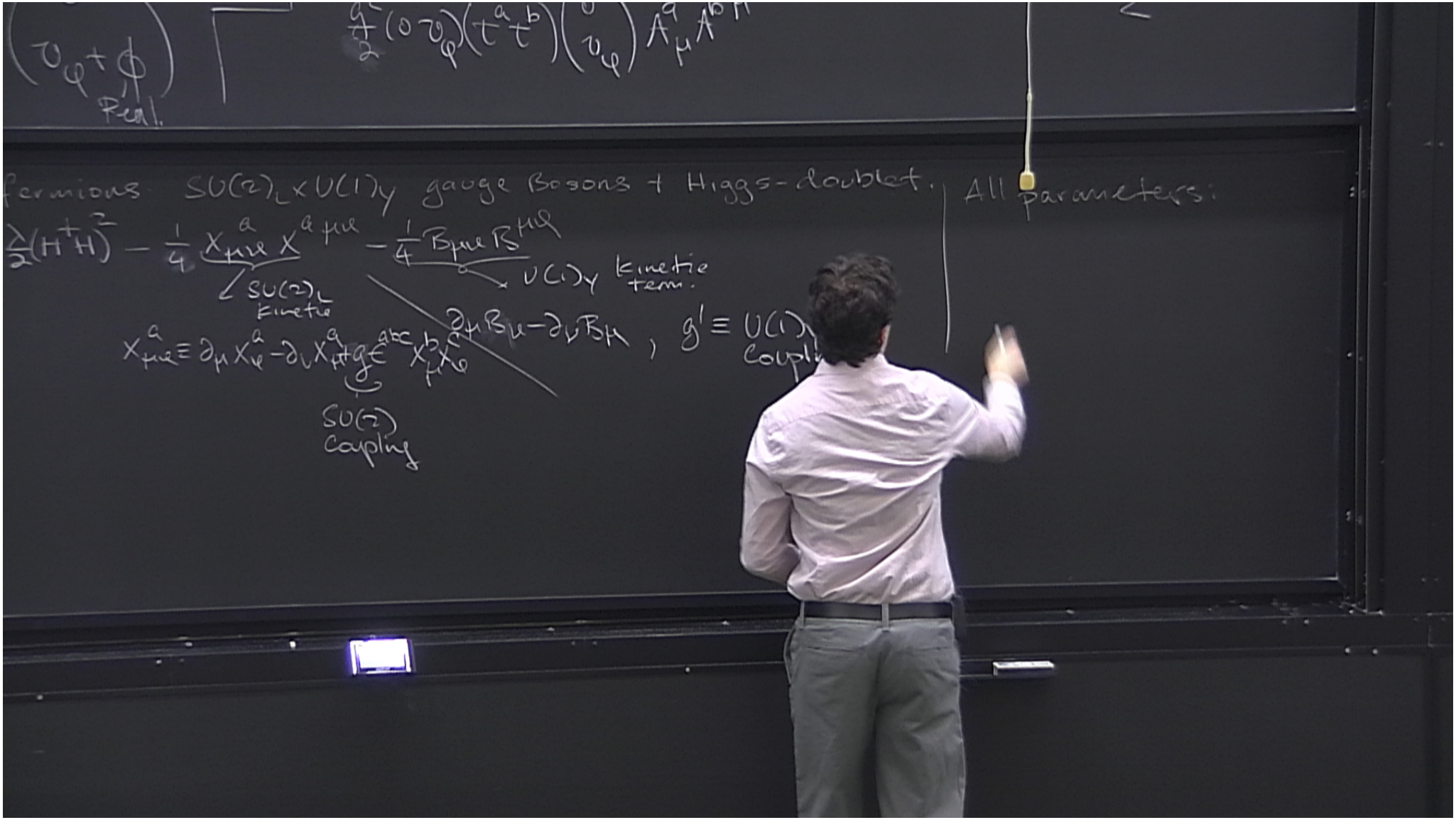
$|D_\mu \varphi|^2 \supset \left\{ \left[\partial_\mu - ig A^a_\mu \left(\frac{\sigma^a}{2}\right) \right] \varphi \right\}^\dagger \{ \text{Conj.} \}$

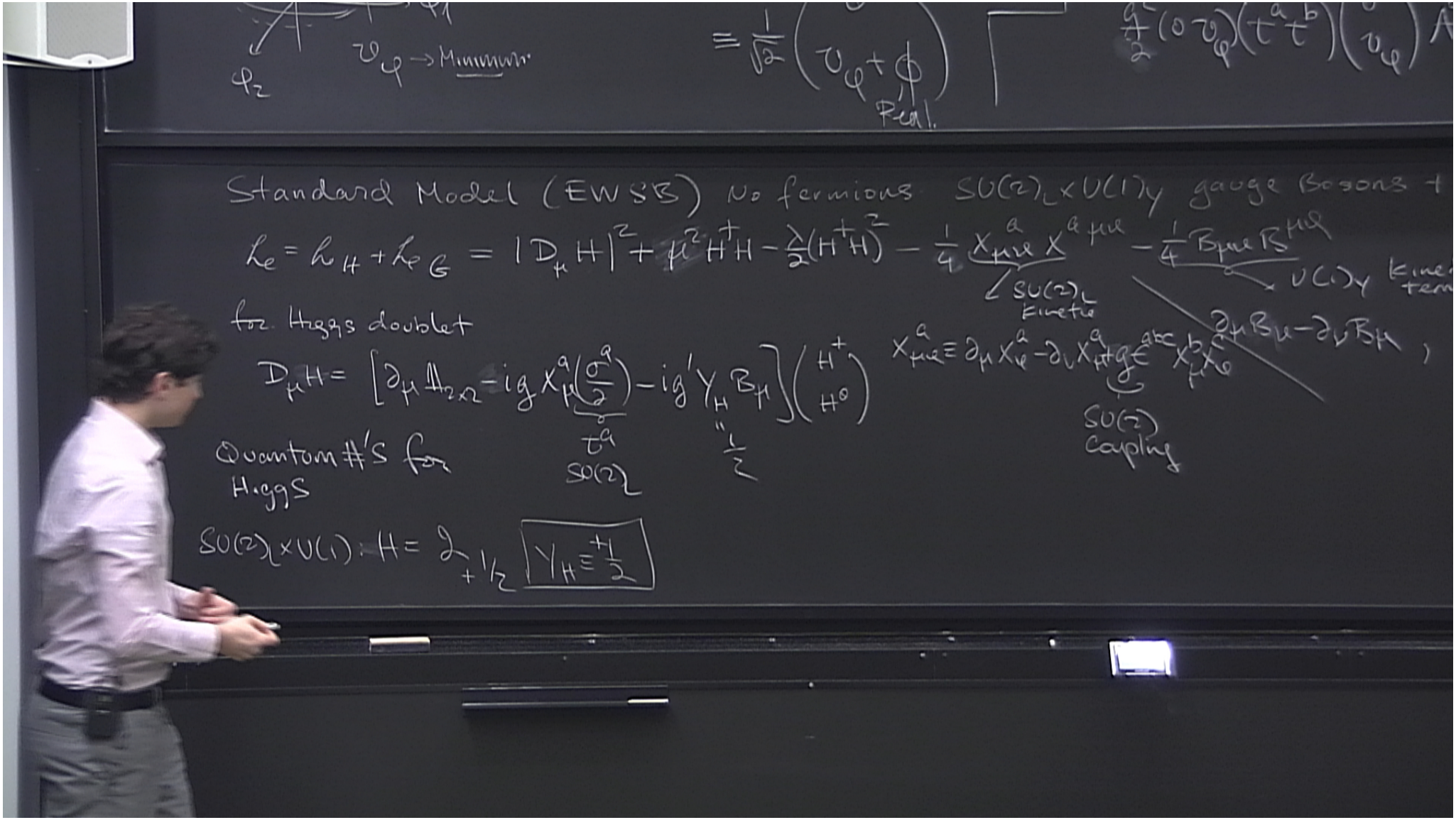
$\to \frac{g^2}{2}(0\ v_\varphi) \left[A^a_\mu \frac{\sigma^a}{2} \right]^\dagger \left[A^{b,\mu} \frac{\sigma^b}{2} \right] \begin{pmatrix} 0 \\ v_\varphi \end{pmatrix} + \cdots$

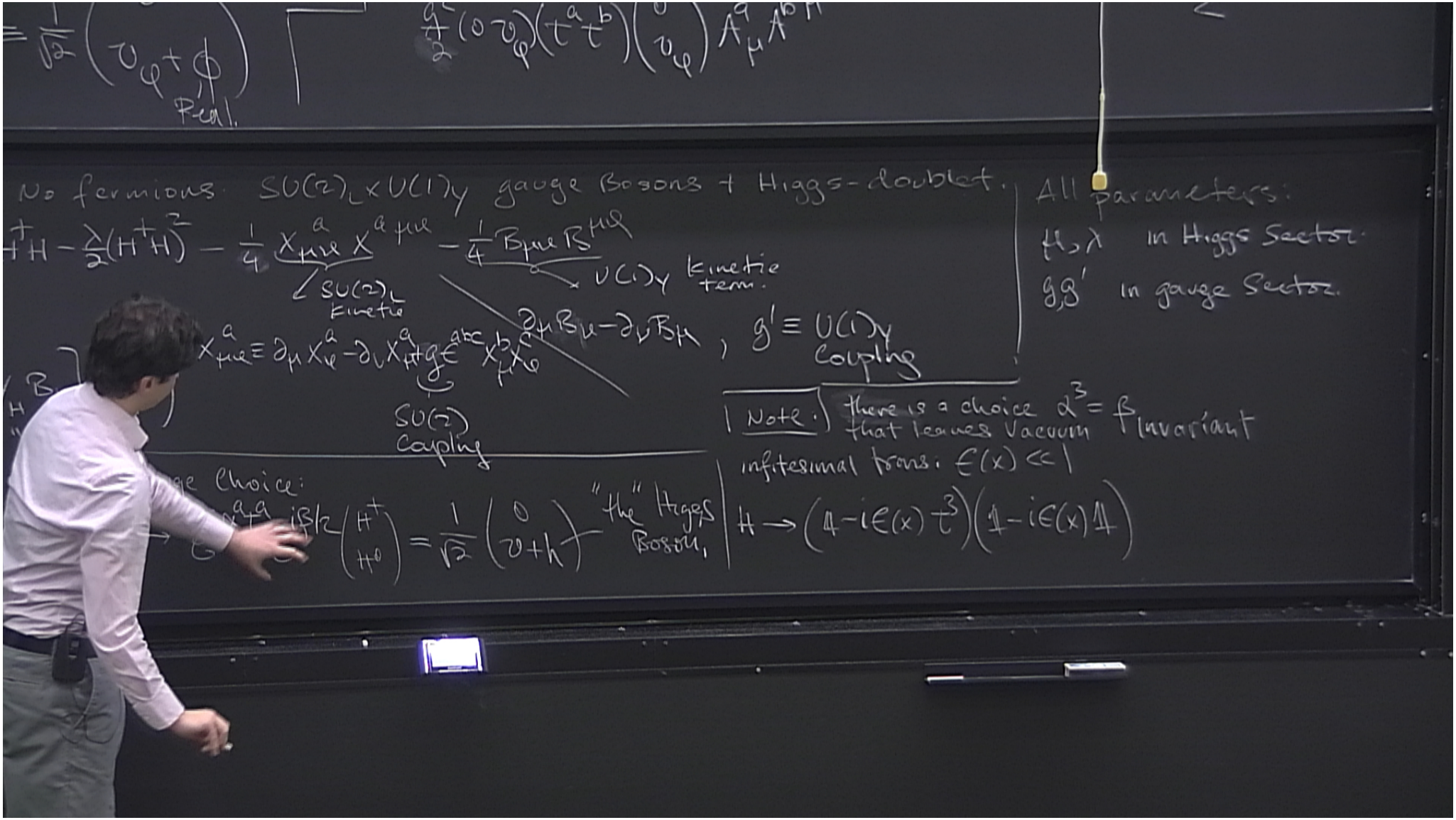
Use Identity for SU(2) $t^a t^b = \frac{1}{2}\{t^a, \tau^b\}$

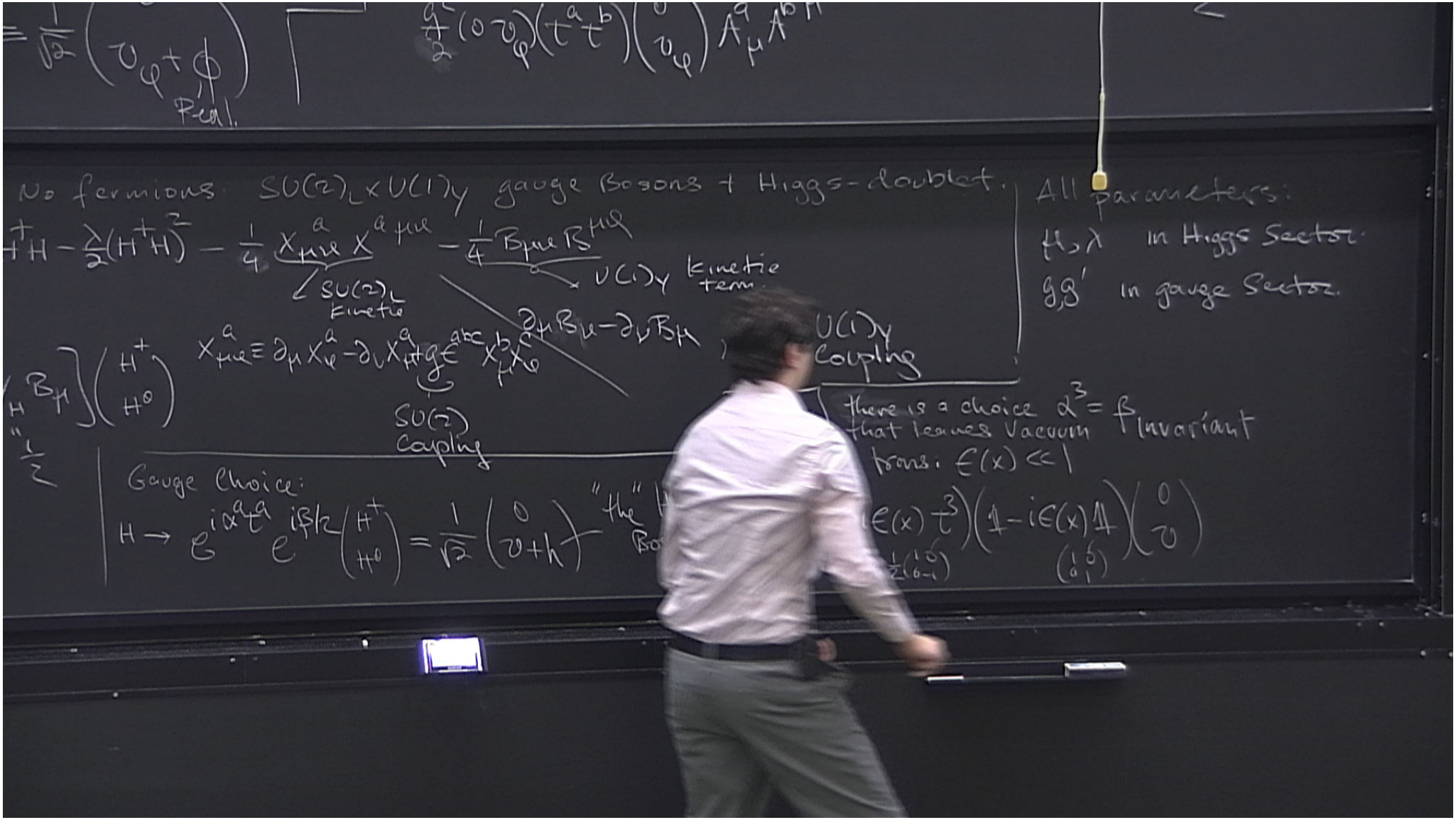
$\frac{g^2}{2}(0\ v_\varphi)(t^a t^b) \begin{pmatrix} 0 \\ v_\varphi \end{pmatrix} A^a_\mu A^{b\,\mu}$

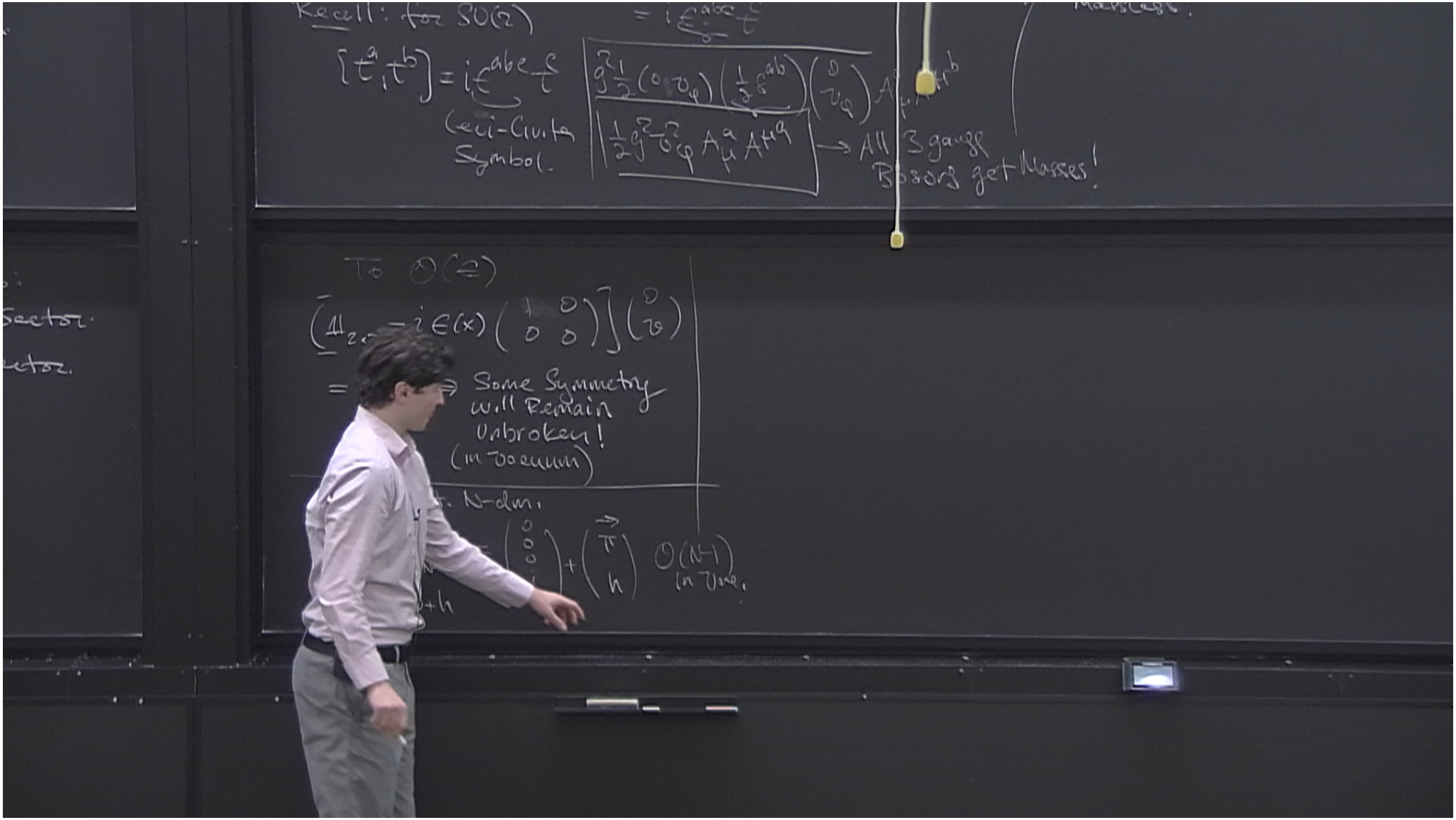

Mass term: for A_μ^a is
$t^a t^b + t^b t^a$
Check My Indices / Identities
$\frac{1}{2} g^2 (0\ v_\varphi) \left[\frac{1}{2} \left(\{t^a, t^b\} + [t^a, t^b] \right) \right] \begin{pmatrix} 0 \\ v_\varphi \end{pmatrix} A^a_\mu A^{\mu b}$
$t^a t^b - t^b t^a = i \epsilon^{abc} t^c$
$M_A = g v_\varphi$
Nothing Remains Massless!
Recall: for SU(2)
$[t^a, t^b] = i \epsilon^{abc} t^c$
Levi-Civita Symbol.
$g^2 \frac{1}{2} (0\ v_\varphi) \left(\frac{1}{2} \delta^{ab} \right) \begin{pmatrix} 0 \\ v_\varphi \end{pmatrix} A^a_\mu A^{\mu b}$
$\frac{1}{2} g^2 v_\varphi^2 A^a_\mu A^{\mu a}$
→ All 3 gauge Bosons get Masses!

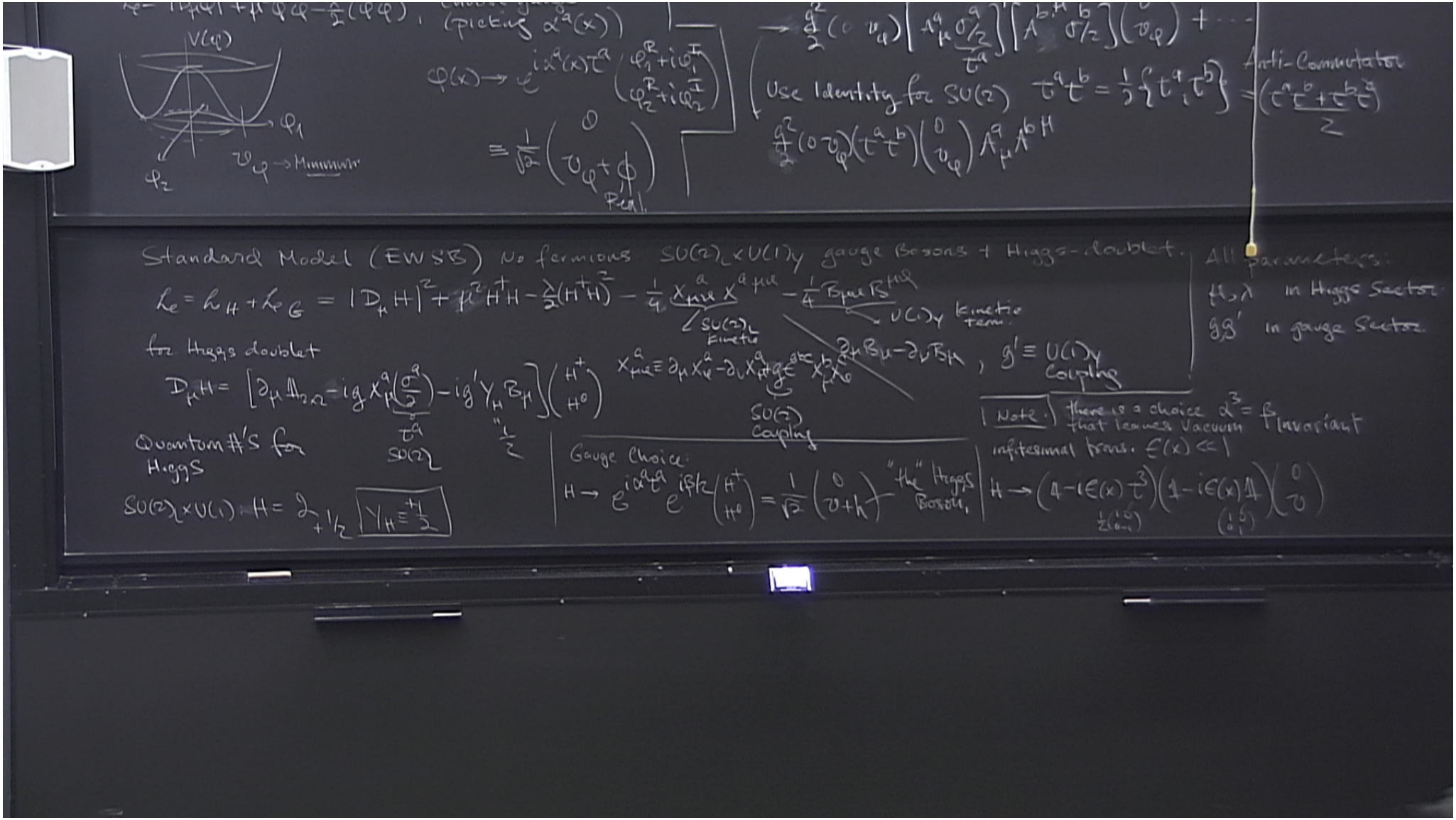

Real.
Fermions $SU(2)_L \times U(1)_Y$ gauge Bosons + Higgs-doublet.
All parameters:
$U(1)_Y$ kinetic term.
$SU(2)_L$ kinetic
$\partial_\mu B_\nu - \partial_\nu B_\mu$,
SU(2) coupling

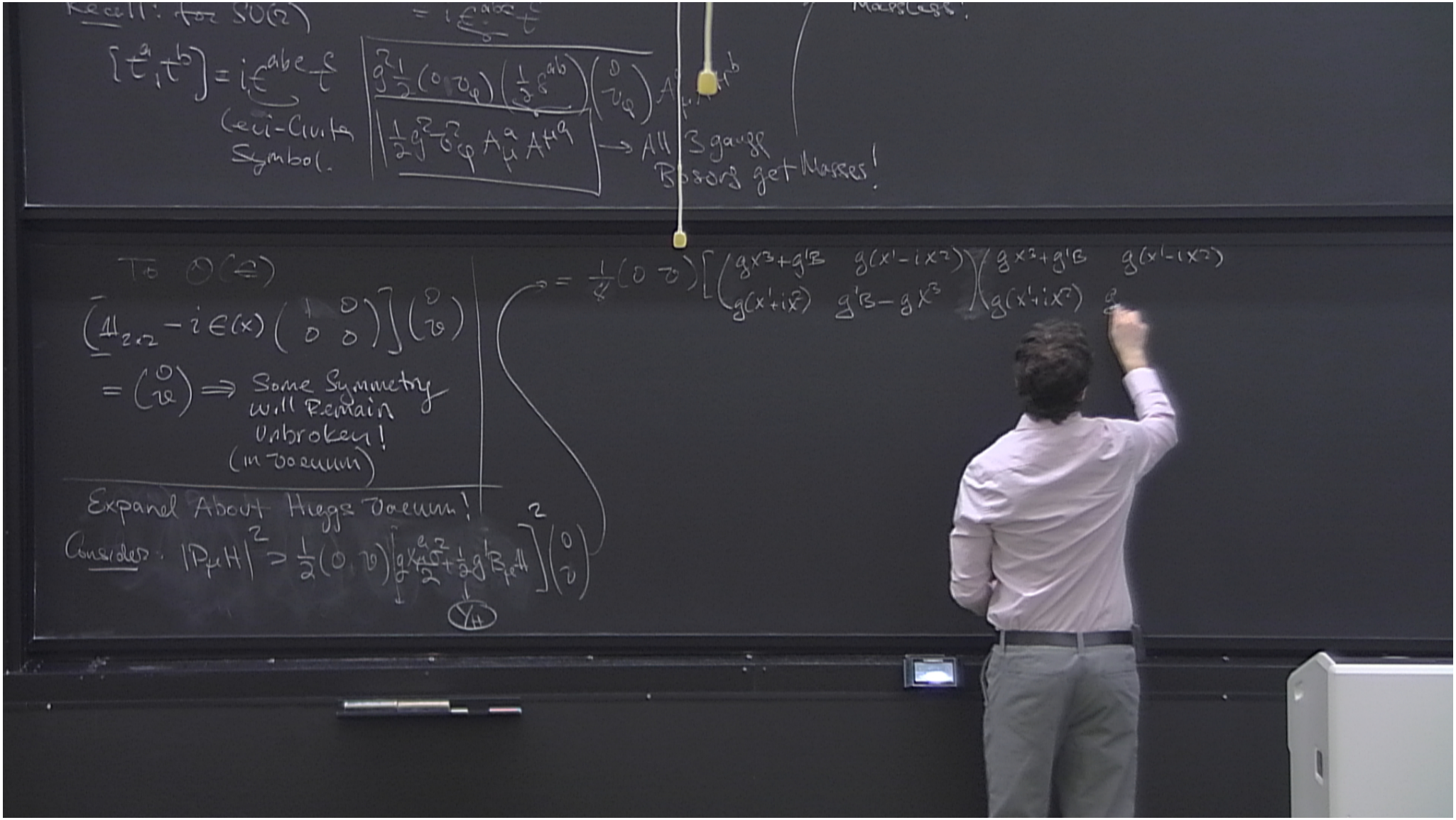

$v_\varphi \to$ Minimum
$\equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v_\varphi + \phi \end{pmatrix}$
Real
Standard Model (EWSB) No fermions $SU(2)_L \times U(1)_Y$ gauge Bosons +
$\mathcal{L}_e = \mathcal{L}_H + \mathcal{L}_{EG} = |D_\mu H|^2 + \mu^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 - \frac{1}{4} X^a_{\mu\nu} X^{a\,\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$
$SU(2)_L$ Kinetic
$U(1)_Y$ Kine term
for Higgs doublet
$D_\mu H = \left[\partial_\mu \mathbb{1}_{2\times 2} - ig X^a_\mu \left(\frac{\sigma^a}{2}\right) - ig' Y_H B_\mu\right]\begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$
t^a
$SU(2)$
$= \frac{1}{2}$
$X^a_{\mu\nu} \equiv \partial_\mu X^a_\nu - \partial_\nu X^a_\mu + g\epsilon^{abc} X^b_\mu X^c_\nu$
$\partial_\mu B_\nu - \partial_\nu B_\mu$)
$SU(2)$ coupling
Quantum #'s for Higgs
$SU(2)_L \times U(1)_Y$: $H = \begin{matrix} 2 \\ +1/2 \end{matrix}$
$Y_H \equiv \frac{+1}{2}$

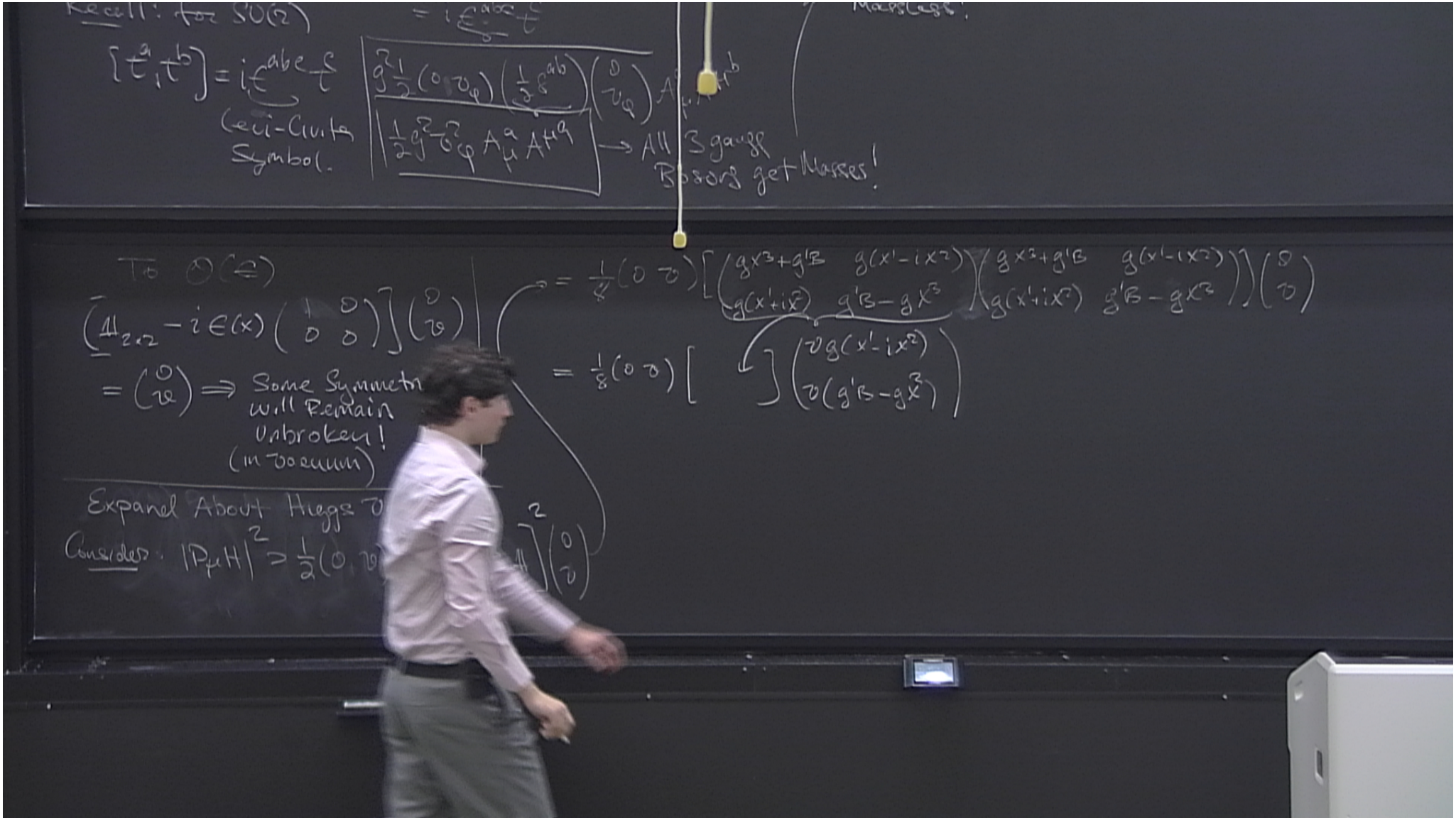

No fermions. $SU(2)_L \times U(1)_Y$ gauge Bosons + Higgs-doublet.
$SU(2)_L$ kinetic
$U(1)_Y$ kinetic term.
$SU(2)$ coupling
$g' \equiv U(1)_Y$ coupling
All parameters:
μ, λ in Higgs Sector.
g, g' in gauge Sector.
Note: there is a choice $\alpha^3 = \beta$ that leaves Vacuum invariant
infitesimal trans. $\epsilon(x) \ll 1$
"the" Higgs Boson,
Real.

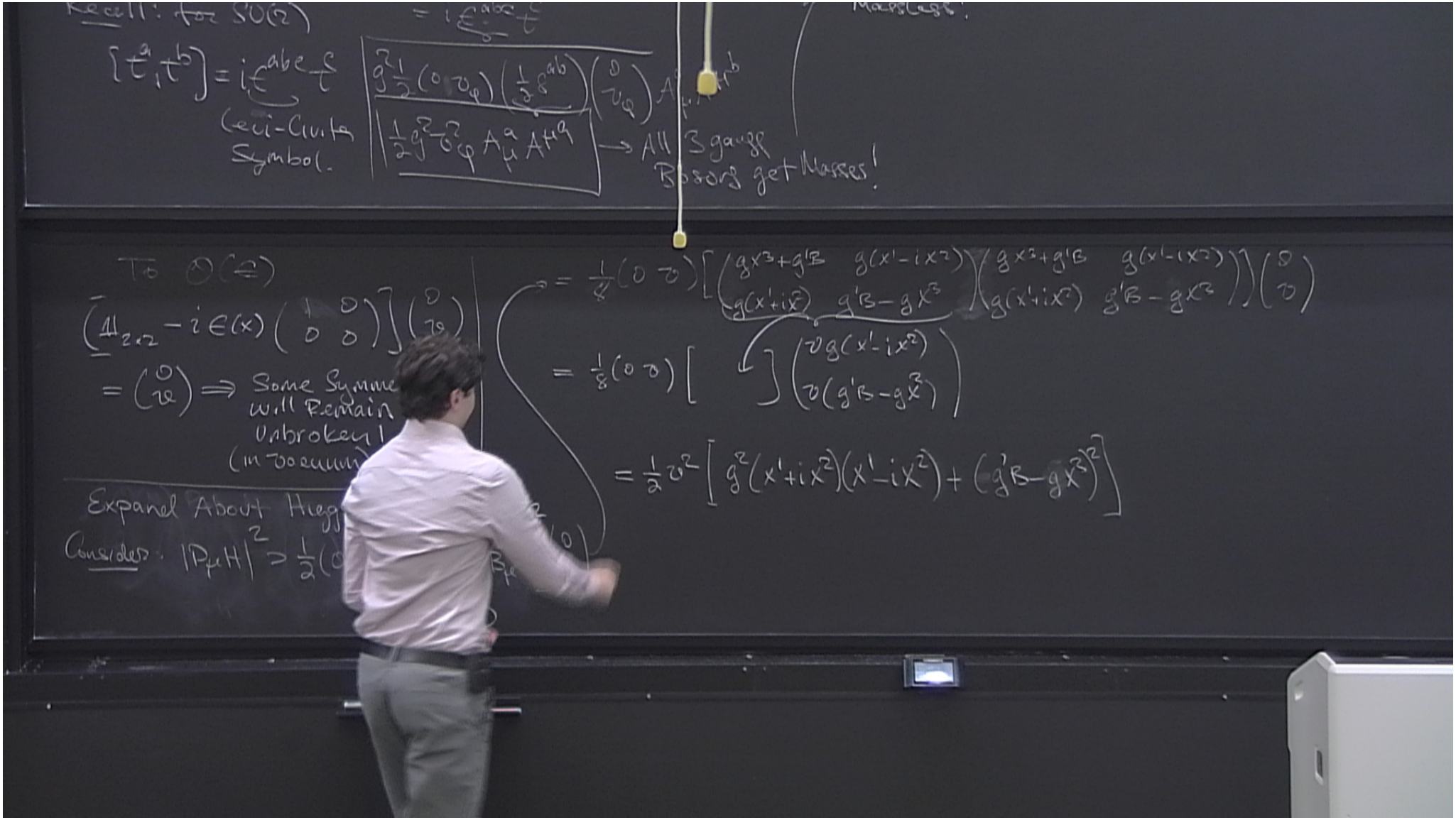

No fermions. $SU(2)_L \times U(1)_Y$ gauge Bosons + Higgs-doublet.
All parameters:
μ, λ in Higgs Sector.
g, g' in gauge Sector.
$SU(2)_L$ kinetic
$U(1)_Y$ kinetic term
$U(1)_Y$ Coupling
$SU(2)$ Coupling
Gauge choice:
there is a choice $\alpha^3 = \beta$ that leaves Vacuum invariant
trans. $\epsilon(x) \ll 1$

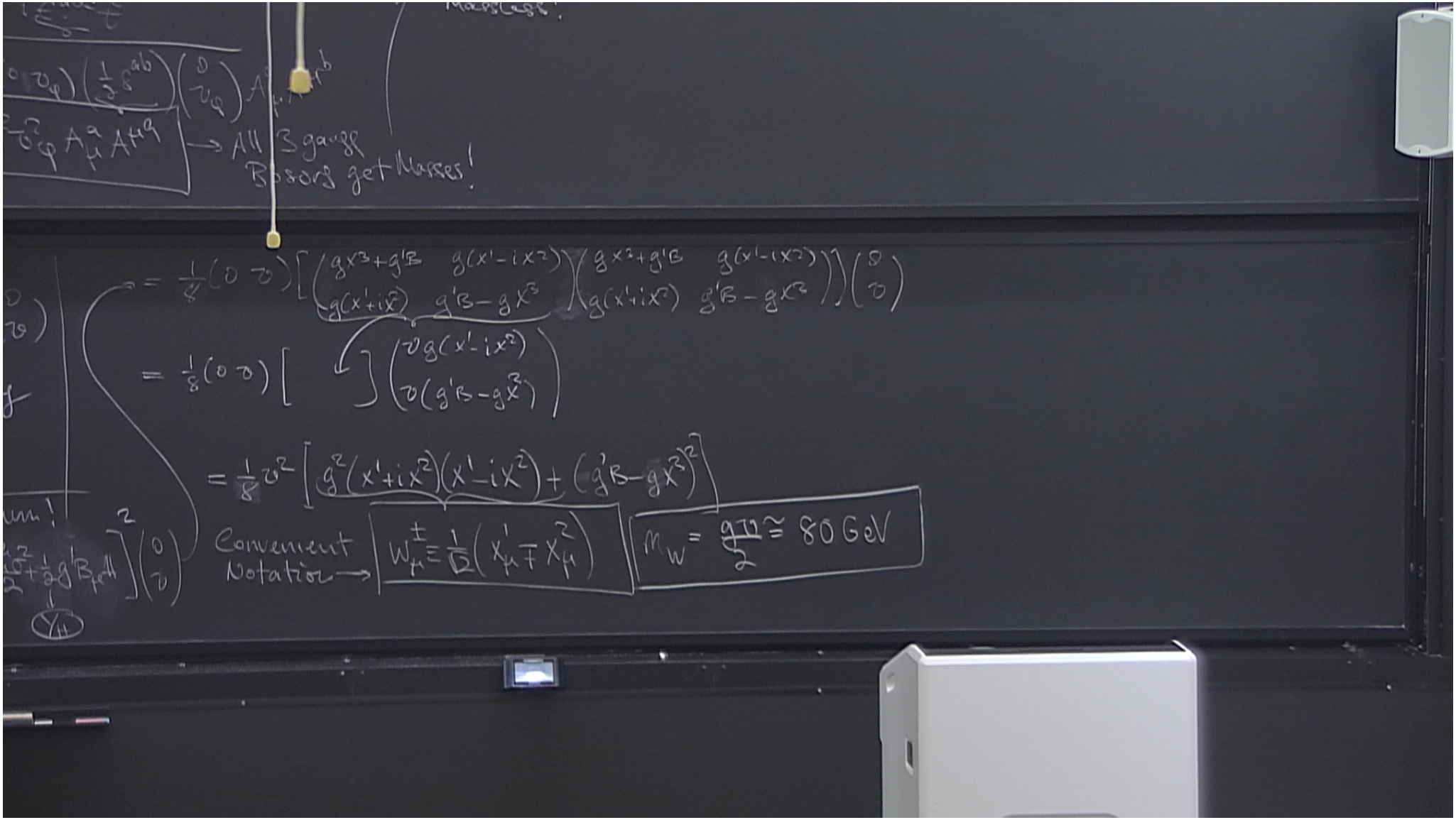

Recall: for SO(2)
$[t^a, t^b] = i\epsilon^{abc} t^c$
Levi-Civita Symbol.
$\frac{1}{2}g^2 v_q^2 A_\mu^a A^{\mu q}$
→ All 3 gauge Bosons get Masses!
To $O(\epsilon)$
Some Symmetry will Remain Unbroken!
(in Theorem)
N-dim.
$O(N-1)$ in Time.

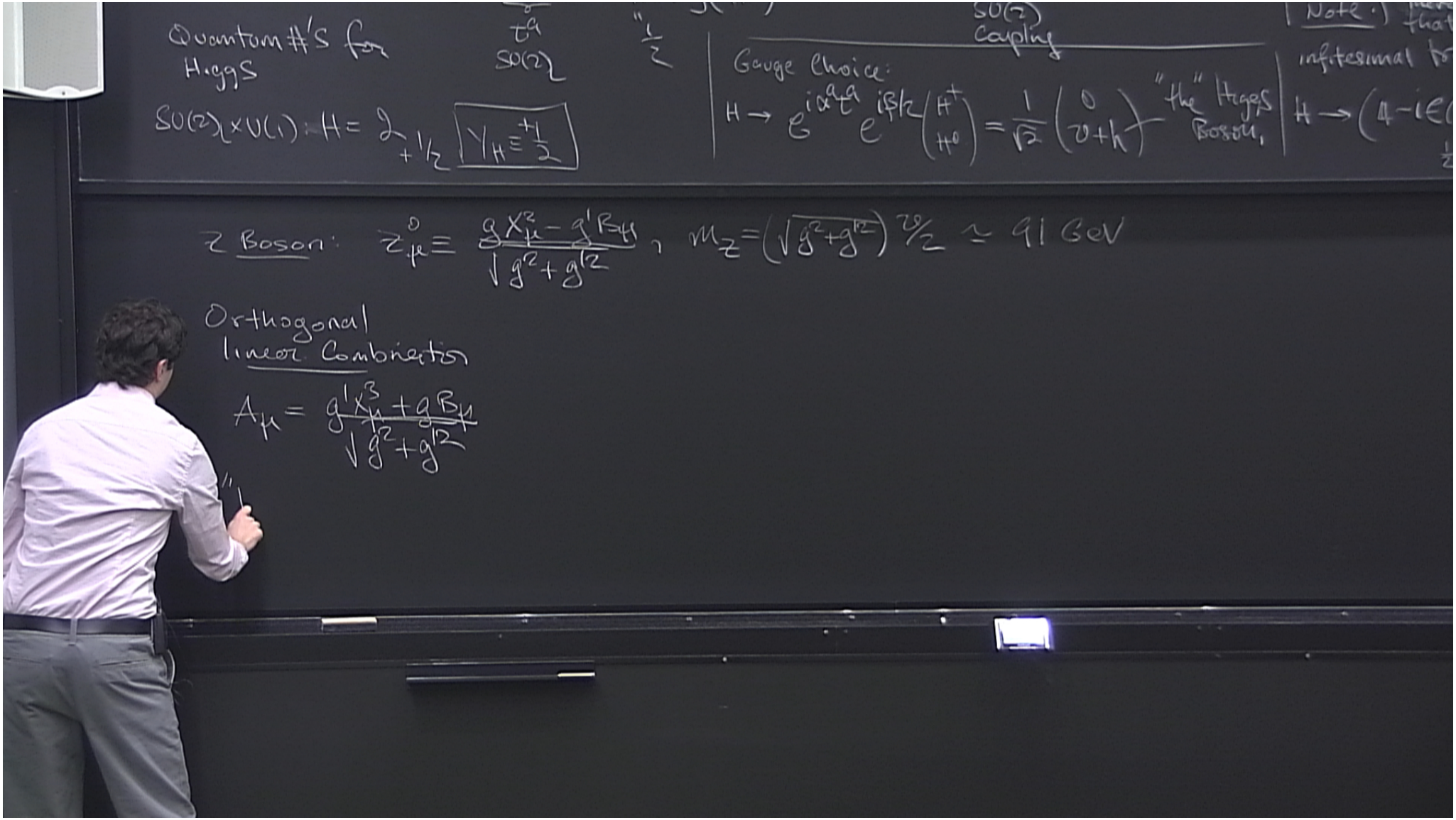

V(φ)
φ1
φ2
v_φ → Minimum
(picking α^a(x))
Use Identity for SU(2)
Anti-Commutator
Standard Model (EWSB) No fermions SU(2)_L × U(1)_Y gauge Bosons + Higgs-doublet.
All parameters:
μ, λ in Higgs Sector
gg' in gauge Sector.
SU(2)_L Kinetic
U(1)_Y kinetic term.
for Higgs doublet
Quantum #'s for Higgs
SU(2)
SU(2) coupling
U(1)_Y Coupling
Gauge Choice:
"the" Higgs Boson
Note: there is a choice α^3 = β that leaves Vacuum invariant
infinitesimal trans. ε(x) ≪ 1

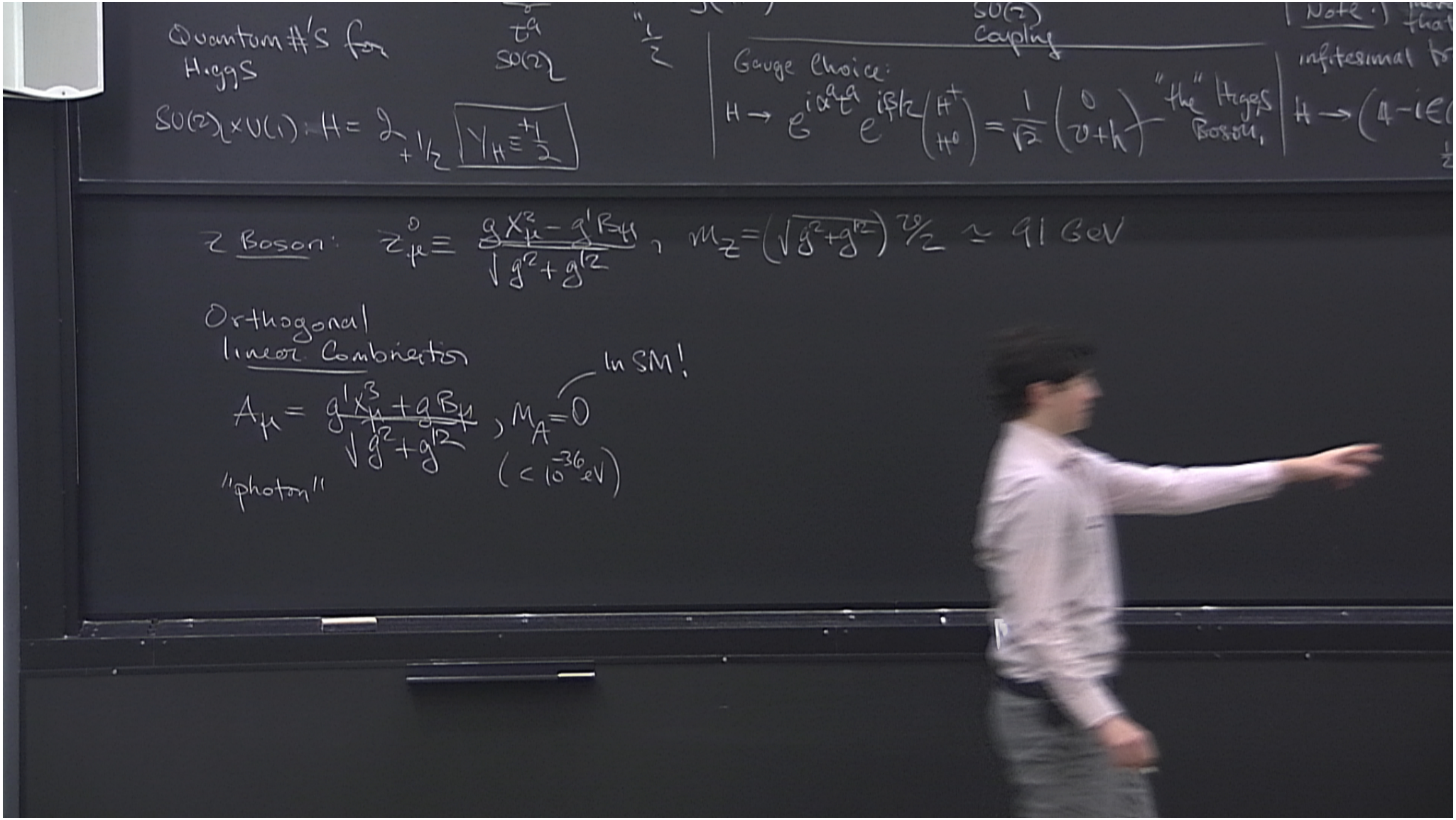

Recall: for SO(2)
$[t^a, t^b] = i\epsilon^{abc} t^c$
Levi-Civita Symbol.
$\frac{g^2}{2}(0\ v_q)\left(\frac{1}{2}\delta^{ab}\right)\begin{pmatrix}0\\v_q\end{pmatrix} A^a_\mu A^{\mu b}$
$\frac{1}{2}g^2 v_q^2 A^a_\mu A^{\mu q}$
→ All 3 gauge Bosons get Masses!
To $O(\epsilon)$
$\left(\mathbb{1}_{2\times2} - i\epsilon(x)\begin{pmatrix}1&0\\0&0\end{pmatrix}\right]\begin{pmatrix}0\\v\end{pmatrix}$
$= \begin{pmatrix}0\\v\end{pmatrix}$ ⇒ Some Symmetry will Remain Unbroken! (in Vacuum)
Expand About Higgs Vacuum!
Consider: $|D_\mu H|^2 = \frac{1}{2}(0\ v)\left[g\vec{x}\cdot\frac{\vec{\sigma}}{2} + \frac{1}{2}g' B_\mu \mathbb{1}\right]^2\begin{pmatrix}0\\v\end{pmatrix}$
Y_H
$= \frac{1}{8}(0\ v)\begin{bmatrix}\begin{pmatrix}gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g'B-gx^3\end{pmatrix}\end{bmatrix}^2$
$\begin{pmatrix}gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g\end{pmatrix}$

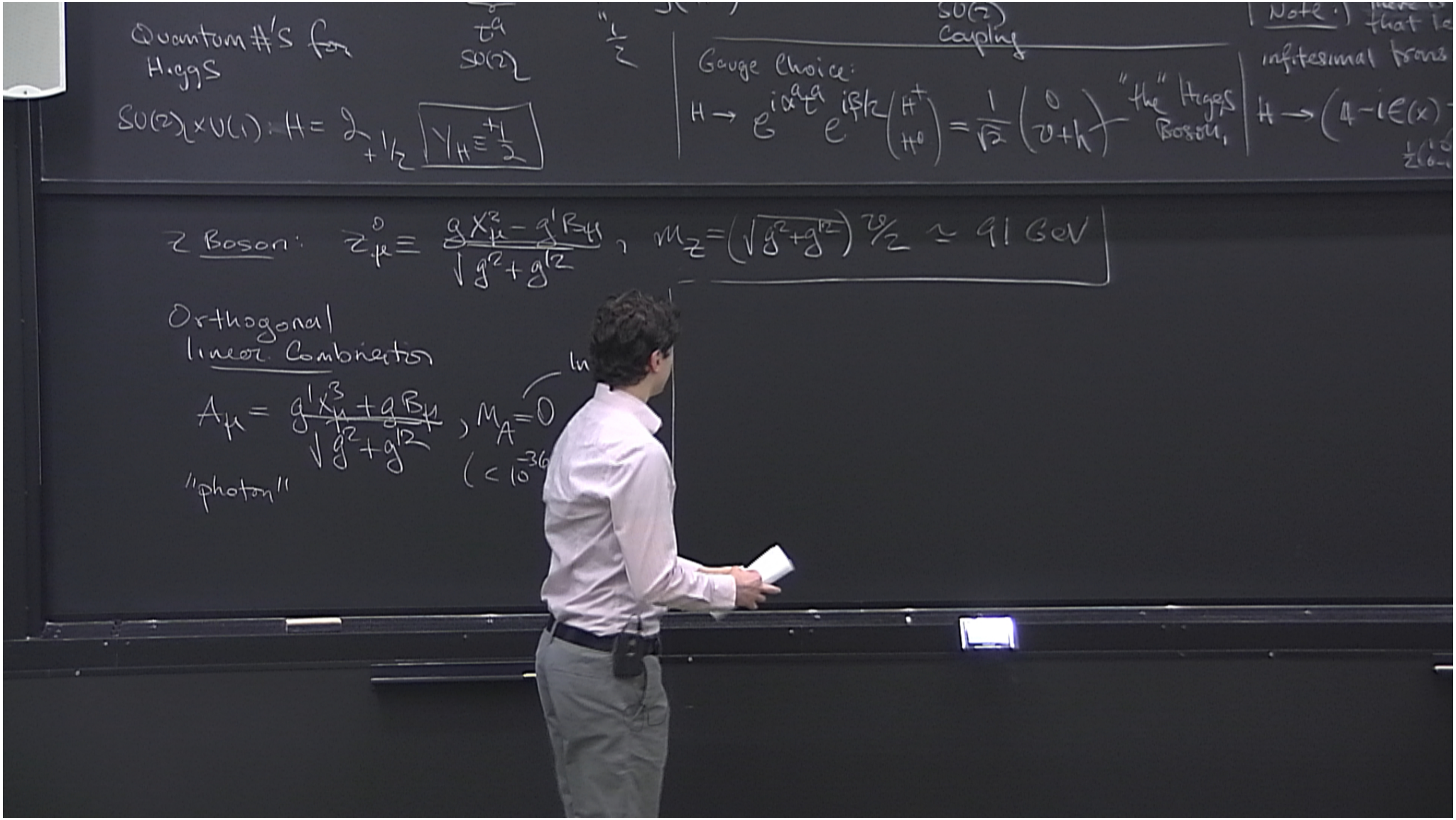

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$\frac{1}{2}g^2 v_q^2 A^a_\mu A^{\mu q}$ → All 3 gauge Bosons get Masses!
To $O(\epsilon)$
$\left(\mathbb{1}_{2\times 2} - i\epsilon(x)\begin{pmatrix}1&0\\0&0\end{pmatrix}\right)\begin{pmatrix}0\\v\end{pmatrix}$
$= \begin{pmatrix}0\\v\end{pmatrix}$ ⇒ Some Symmetry will Remain Unbroken! (in vacuum)
$= \frac{1}{8}(0\ v)\left[\begin{pmatrix} gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g'B-gx^3\end{pmatrix}\begin{pmatrix} gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g'B-gx^3\end{pmatrix}\right]\begin{pmatrix}0\\v\end{pmatrix}$
$= \frac{1}{8}(0\ v)\left[\ \right]\begin{pmatrix} v g(x^1-ix^2)\\ v(g'B-gx^3)\end{pmatrix}$
Expand About Higgs v
Consider: $|D_\mu H|^2 = \frac{1}{2}(0, v)$
$\left[\ \right]^2\begin{pmatrix}0\\v\end{pmatrix}$

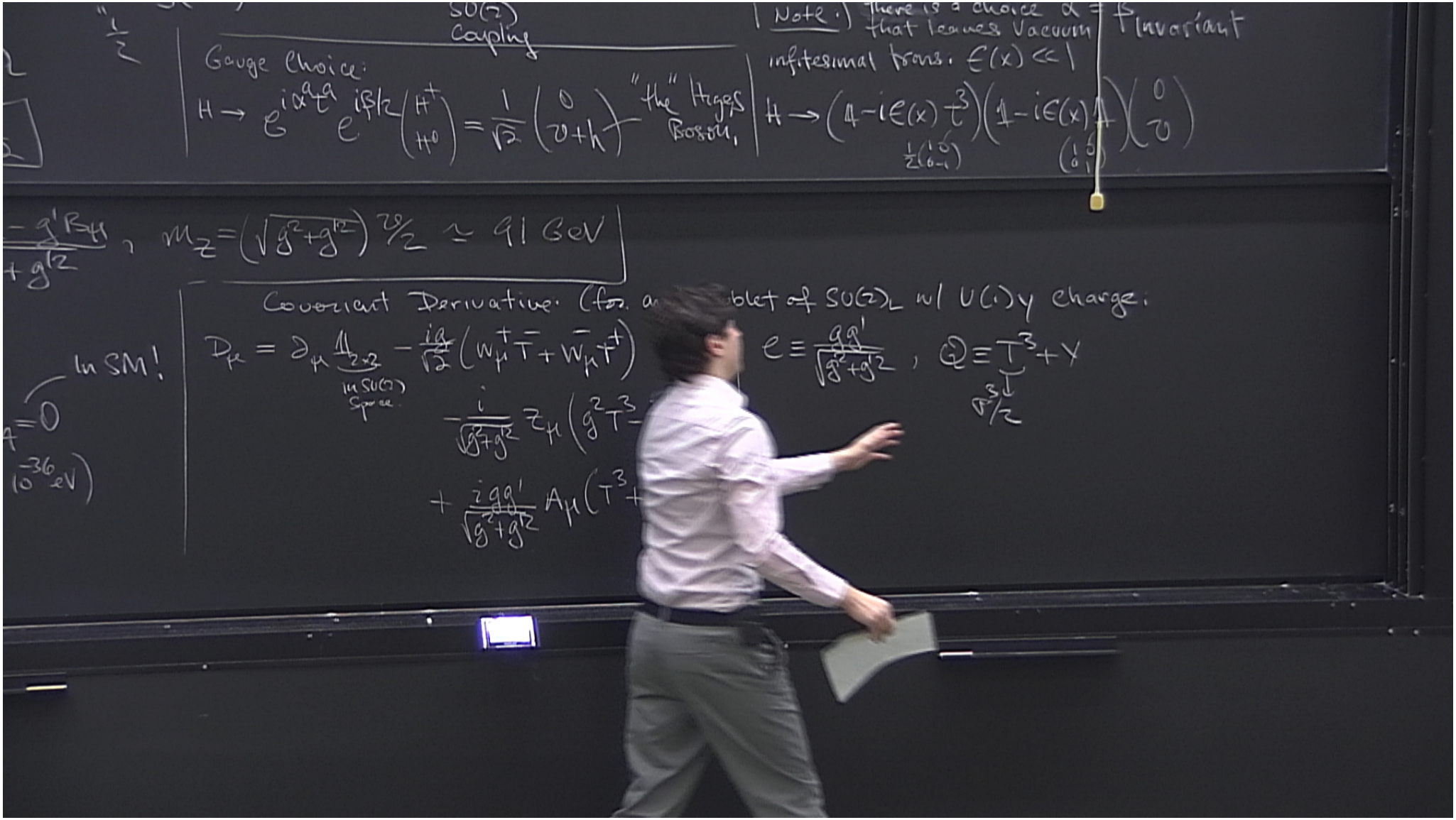

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$\frac{1}{2}g^2 v_q^2 A^q_\mu A^q_\mu$
→ All 3 gauge Bosons get Masses!
To $O(\epsilon)$
$\left(\mathbb{1}_{2\times 2} - i\epsilon(x)\begin{pmatrix}1&0\\0&0\end{pmatrix}\right)\begin{pmatrix}0\\ v\end{pmatrix}$
$= \begin{pmatrix}0\\ v\end{pmatrix}$ ⇒ Some Symme
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Unbroken!
(in vacuum)
Expand About Higg
Consider: $|D_\mu H|^2 = \frac{1}{2}(0$
$= \frac{1}{8}(0\ v)\left[\begin{pmatrix} gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g'B-gx^3\end{pmatrix}\begin{pmatrix} gx^3+g'B & g(x^1-ix^2)\\ g(x^1+ix^2) & g'B-gx^3\end{pmatrix}\right]\begin{pmatrix}0\\ v\end{pmatrix}$
$= \frac{1}{8}(0\ v)\left[\quad\right]\begin{pmatrix} vg(x^1-ix^2)\\ v(g'B-gx^3)\end{pmatrix}$
$= \frac{1}{2}v^2\left[g^2(x^1+ix^2)(x^1-ix^2) + (g'B-gx^3)^2\right]$

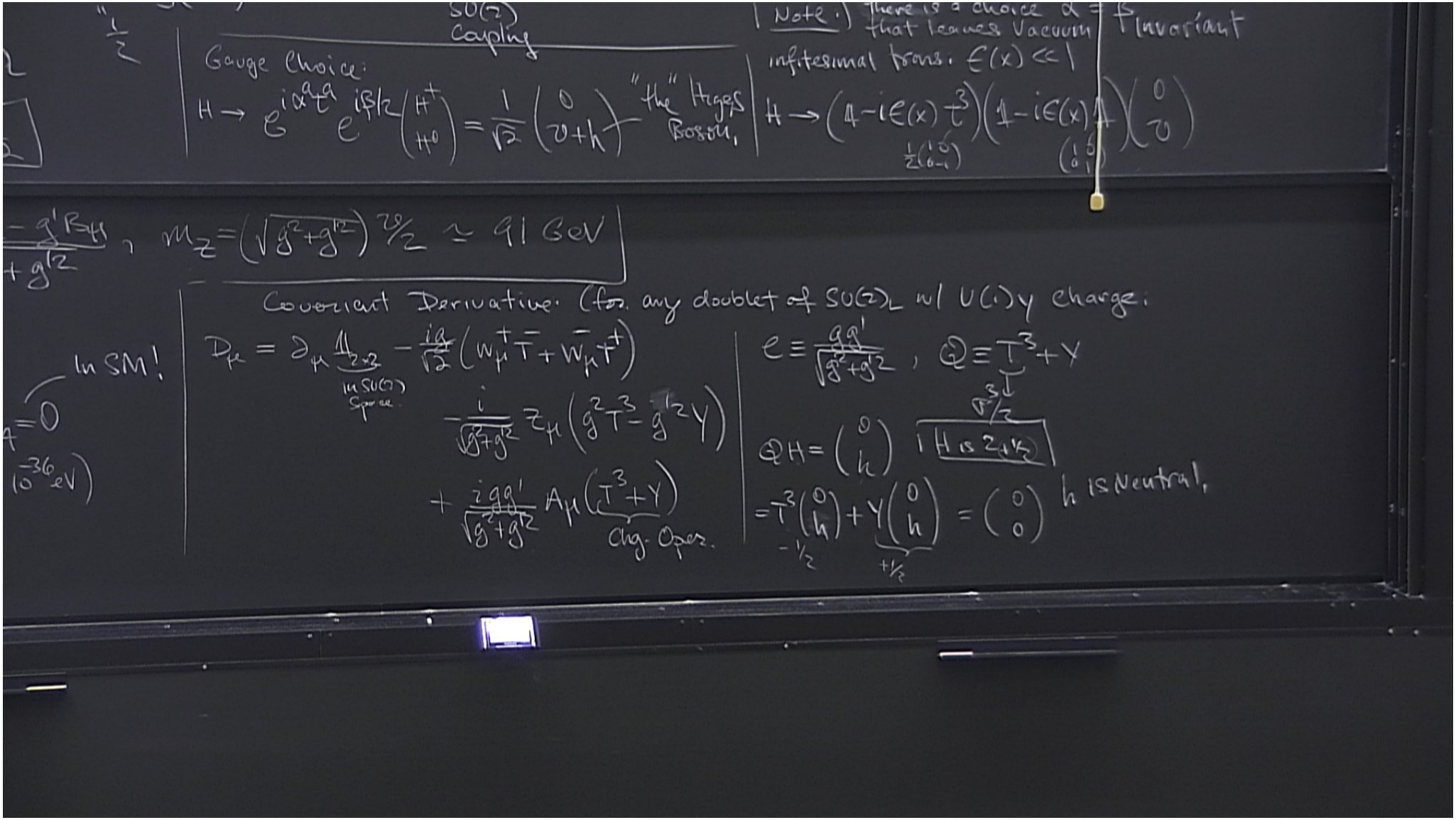

→ All 3 gauge Bosons get Masses!
Convenient Notation →
$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}(X_\mu^1 \mp X_\mu^2)$
$m_W = \frac{gv}{2} \simeq 80\,\text{GeV}$


Quantum #'s for Higgs
$SO(2)\times U(1)$: $H = 2_{+1/2}$ $\boxed{Y_H = +\frac{1}{2}}$
t^a SO(2)
$\frac{1}{2}$
SO(2) coupling
Gauge Choice:
$H \to e^{i\alpha^a t^a} e^{i\beta/2} \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+h \end{pmatrix}$ "the" Higgs Boson,
Note:
infitesimal
$H \to (4 - i\epsilon$
Z Boson: $Z^0_\mu = \frac{g X^3_\mu - g' B_\mu}{\sqrt{g^2+g'^2}}$, $m_Z = (\sqrt{g^2+g'^2})\, v/2 \simeq 91$ GeV
Orthogonal linear Combination
$A_\mu = \frac{g' X^3_\mu + g B_\mu}{\sqrt{g^2+g'^2}}$


Quantum #'s for Higgs
SO(2)
SO(2) coupling
Gauge Choice:
"the" Higgs Boson,
Note:
infitesimal
Z Boson:
Orthogonal linear Combination
In SM!
"photon"


Quantum #'s for Higgs
SO(2)×U(1): H = 2 +1/2
$Y_H \equiv \frac{+1}{2}$
t^a
SO(2)
"1/2"
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"the" Higgs Boson,
Note:
infitesimal trans
$H \to (4 - i\epsilon(x)$
Z Boson: $Z^0_\mu \equiv \frac{g X^3_\mu - g' B_\mu}{\sqrt{g^2+g'^2}}$, $M_Z = (\sqrt{g^2+g'^2})\, v/2 \simeq 91$ GeV
Orthogonal linear Combination
$A_\mu = \frac{g' X^3_\mu + g B_\mu}{\sqrt{g^2+g'^2}}$, $M_A = 0$
"photon"


SO(2) coupling
Note:) there is a choice $\alpha = 0$ that leaves Vacuum invariant
Gauge choice:
$H \to e^{i\alpha^a t^a} e^{i\beta/2}\begin{pmatrix} H^+ \\ H^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ "the" Higgs Boson,
infitesimal trans: $\epsilon(x) \ll 1$
$H \to \left(\mathbb{1} - i\epsilon(x)\tfrac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right)\left(\mathbb{1} - i\epsilon(x)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$
$m_Z = (\sqrt{g^2+g'^2})\, v/2 \simeq 91\,\text{GeV}$
Covariant Derivative: (for a doublet of SU(2)$_L$ w/ U(1)$_Y$ charge:
$D_\mu = \partial_\mu \mathbb{1}_{2\times 2} - \frac{ig}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-)$
in SU(2) Space
$e \equiv \frac{gg'}{\sqrt{g^2+g'^2}}$, $Q \equiv T^3 + Y$
$T^3/2$
In SM!
$= 0$
$(10^{-36}\,\text{eV})$


SO(2) coupling
Gauge choice:
"the" Higgs Boson,
Note:) there is a choice of ϕ that leaves Vacuum invariant
infitesimal trans: ε(x) ≪ 1
Covariant Derivative: (for any doublet of SO(2)L w/ U(1)Y charge:
In SM!
Q H = (0, h)
h is Neutral,
H is 2+½

will Remain Unbroken! (in Vacuum)

Expand About Higgs Vacuum!

Consider: $|D_\mu H|^2 = \frac{1}{2}(0\ v)\left[g\frac{\sigma^a}{2}X^a_\mu + \frac{1}{2}g' B_\mu \mathbb{1}\right]^2 \begin{pmatrix}0\\ v\end{pmatrix}$ (Y_H)

$= \frac{1}{8}v^2\left[g^2(X^1+iX^2)(X^1-iX^2) + (g'B - gX^3)^2\right]$

Convenient Notation → $W^\pm_\mu \equiv \frac{1}{\sqrt{2}}(X^1_\mu \mp X^2_\mu)$ $\quad m_W = \frac{gv}{2} =$

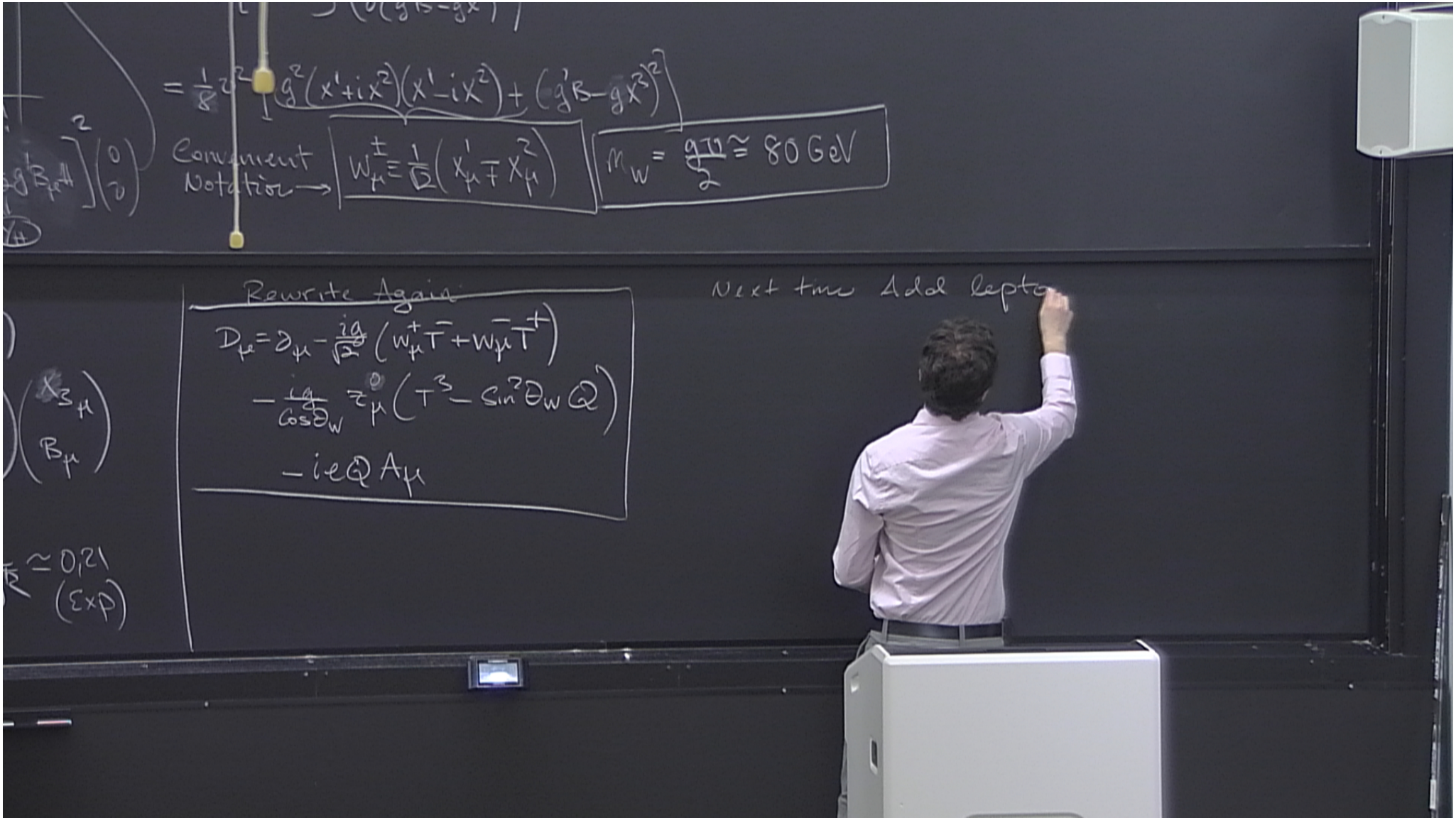
$T^\pm = \frac{1}{2}(\sigma^1 + i\sigma^2) \equiv \sigma^\pm$

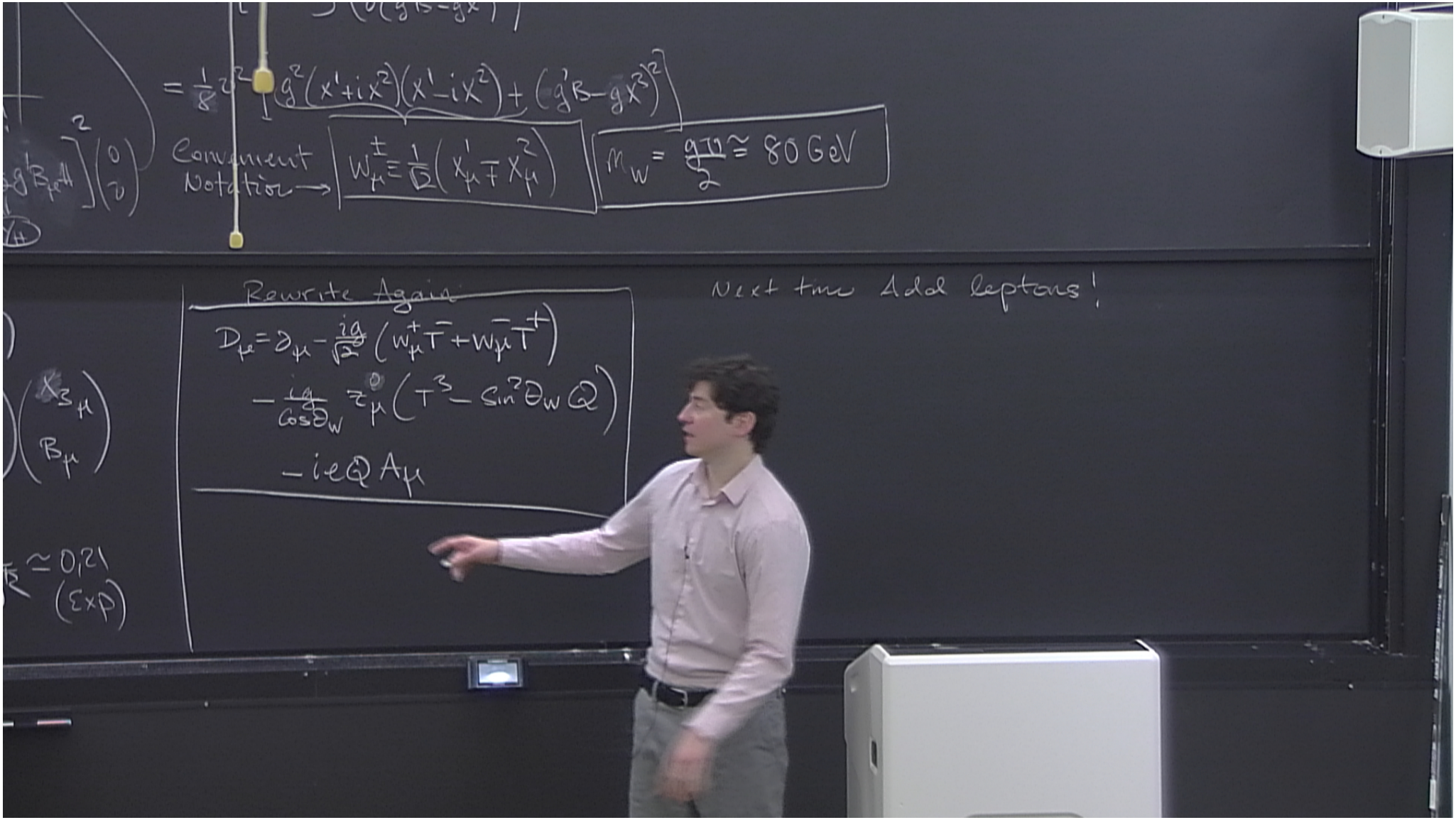
Also Let:

$$\begin{pmatrix} Z^0_\mu \\ A^0_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix}\begin{pmatrix} X^3_\mu \\ B_\mu \end{pmatrix}$$

Weak Mixing Angle:

$\cos\theta_W \equiv \frac{g}{\sqrt{g^2+g'^2}}$, $\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} \simeq 0.21$ (Exp)


$= \frac{1}{8} v^2 \left[g^2 (x^1 + i x^2)(x^1 - i x^2) + (g' B - g x^3)^2 \right]$
Convenient Notation →
$W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (x_\mu^1 \mp x_\mu^2)$
$m_W = \frac{g v}{2} \simeq 80\ \text{GeV}$
Rewrite Again
$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} (W_\mu^+ T^- + W_\mu^- T^+)$
$- \frac{ig}{\cos\theta_W} Z_\mu^0 (T^3 - \sin^2\theta_W Q)$
$- ieQ A_\mu$
Next time Add lepto
$\simeq 0,21$
(Exp)


$= \frac{1}{8} v^2 \left[g^2 (x^1 + i x^2)(x^1 - i x^2) + (g' B - g x^3)^2 \right]$
Convenient Notation →
$W_\mu^\pm \equiv \frac{1}{\sqrt{2}} (x_\mu^1 \mp x_\mu^2)$
$m_W = \frac{g v}{2} \simeq 80\ \text{GeV}$
Rewrite Again
$D_\mu = \partial_\mu - \frac{ig}{\sqrt{2}} (W_\mu^+ T^- + W_\mu^- T^+)$
$- \frac{ig}{\cos\theta_W} Z_\mu^0 (T^3 - \sin^2\theta_W Q)$
$- ieQ A_\mu$
Next time Adal leptons!