Title: Standard Model-3

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Abstract:

Machinery (yesterday: 2-Component Spinors)

Spontaneous Symmetry Breaking ($SU(2)_L \times U(1)_Y \to U($

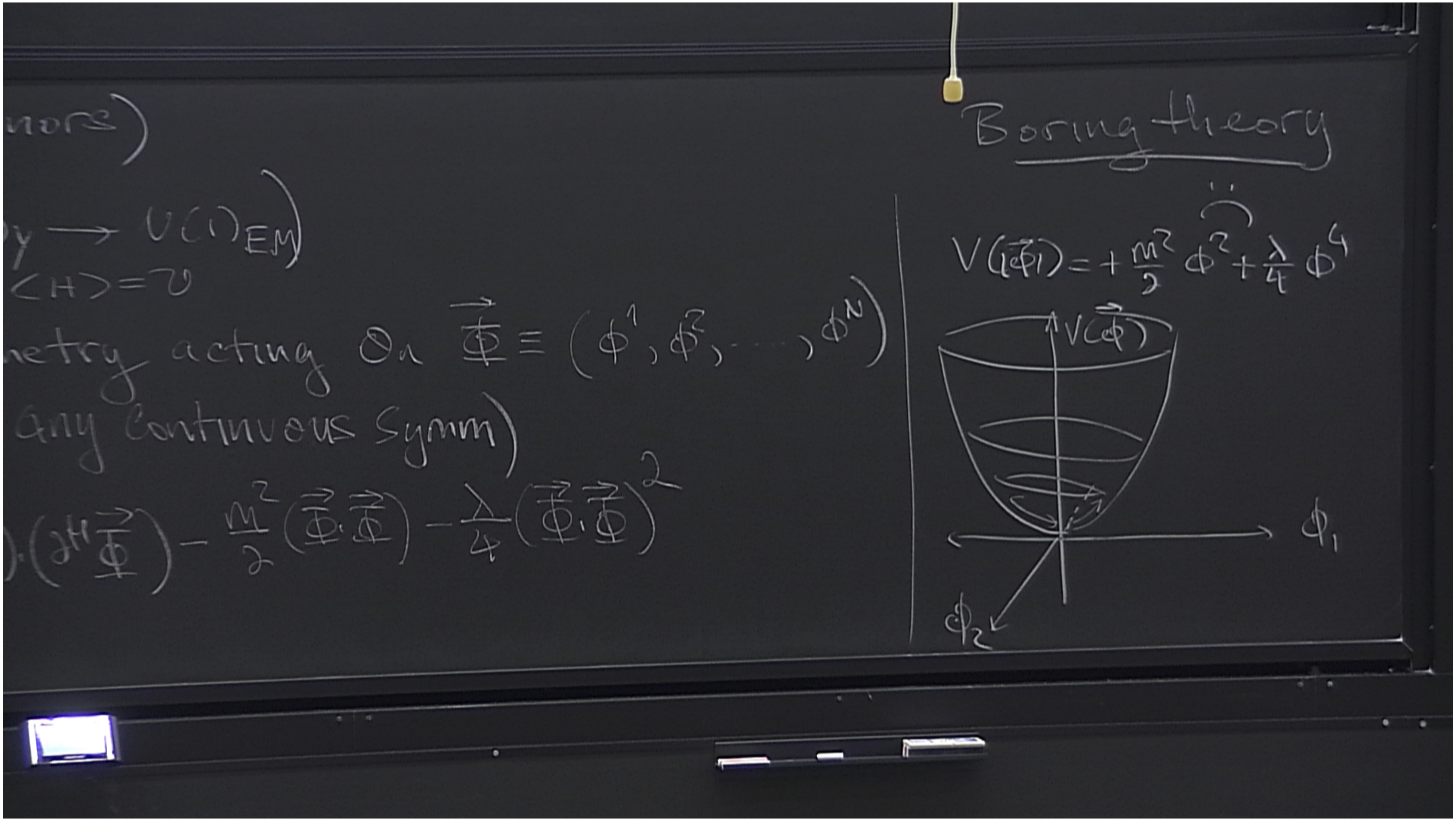
$\langle H \rangle = v$

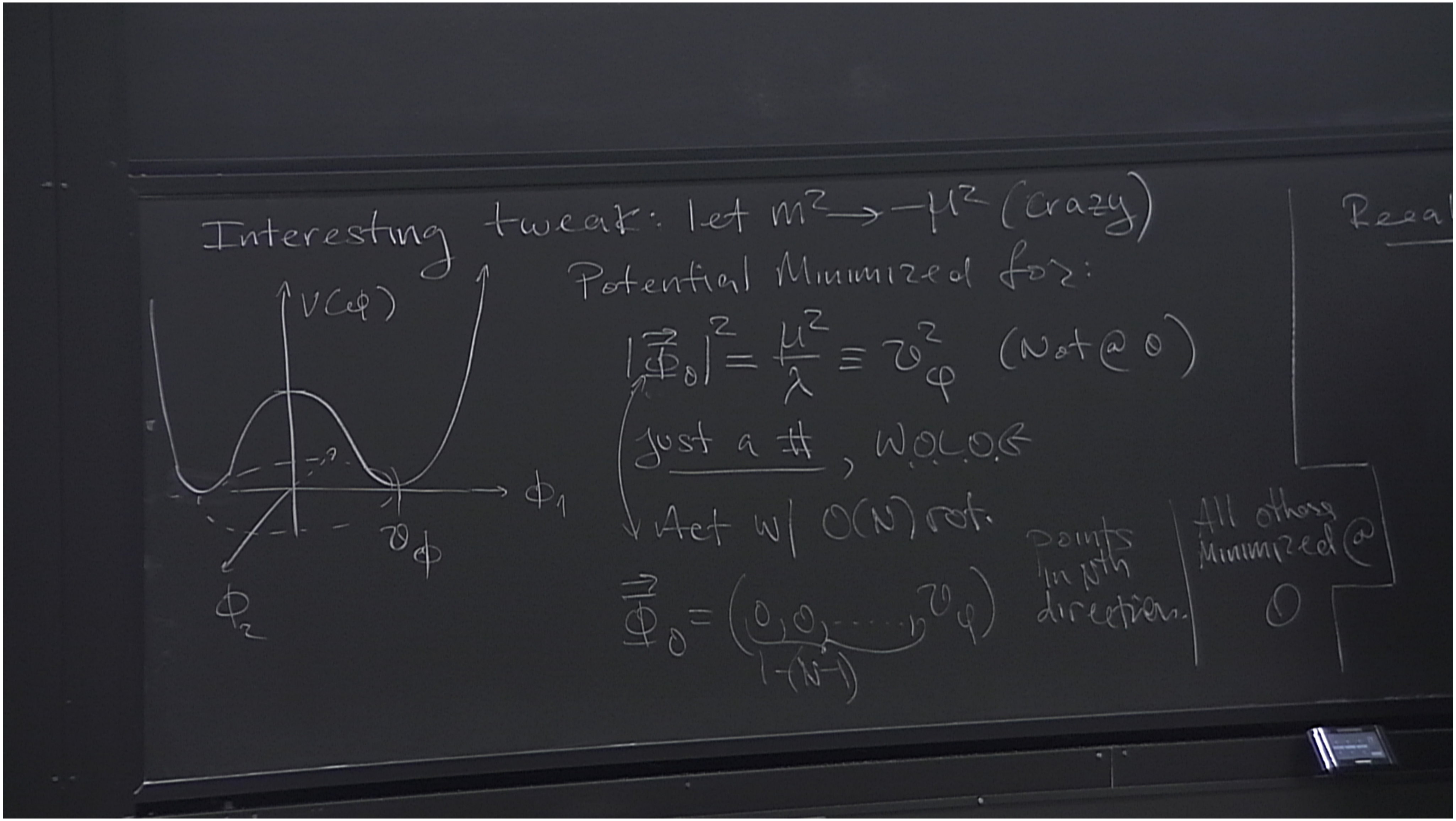
Start w/ familiar Example: O(N) global symmetry a

Rot. grp in N-dim. (any cont

$$[T^\alpha, T^\beta] = i \underbrace{\epsilon^{\alpha\beta\gamma}}_{\text{Levi Civita}} T^\gamma$$

$$\mathcal{L}_{O(N)} = \frac{1}{2}(\partial_\mu \vec{\Phi}) \cdot (\partial^\mu \vec{\Phi}) -$$


nors)
y ⟶ U(1)EM
⟨H⟩ = v
netry acting on Φ ≡ (φ¹, φ², ..., φᴺ)
any Continuous Symm)
·(∂^μ Φ) − m²/2 (Φ·Φ) − λ/4 (Φ·Φ)²
Boring theory
V(|Φ|) = + m²/2 φ² + λ/4 φ⁴
V(φ)
φ1
φ2


Interesting tweak: let $m^2 \to -\mu^2$ (crazy)
$V(\phi)$
ϕ_1
ϕ_2
v_ϕ
Potential Minimized for:
$|\vec{\Phi}_0|^2 = \frac{\mu^2}{\lambda} \equiv v_\varphi^2$ (Not @ 0)
just a #, W.O.L.O.G
Act w/ O(N) rot.
$\vec{\Phi}_0 = (\underbrace{0, 0, \ldots\ldots}_{1-(N-1)}, v_\varphi)$
points in Nth direction.
All others Minimized @ 0
Reca

$\to -\mu^2$ (crazy)

...inimized for:

$\equiv v_\varphi^2$ (Not @ 0)

, W.O.L.O.G

O(N) rot.

$v_\varphi)$

points in Nth direction.

All others Minimized @ 0

Recall: O(N), $\frac{N(N-1)}{2}$ Generators.

N dof (for Rots) $\to$ (N-1) dof. In the Vacuum State.

Let's Expand about Minimum, Rewrite $\vec{\Phi}$

always call "v" VEV

$$\vec{\Phi} \equiv \big(\underbrace{\pi^1(x), \pi^2(x), \dots \pi^{N-1}(x)}_{O(N-1) \text{ symm for } \pi\text{'s}},\ v_\varphi + \sigma(x)\big)$$

$$\mathcal{L}_\varphi = \frac{1}{2}(\partial_\mu \vec{\pi})\cdot(\partial^\mu \vec{\pi}) + \frac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) - \frac{1}{2}\underbrace{(2\mu^2)}_{\text{(mass for } \sigma)}\sigma^2 - \sqrt{\lambda}\,\mu\sigma^3 - \sqrt{\lambda}\,\mu(\vec{\pi}\cdot\vec{\pi})\sigma - \frac{\lambda}{4}\sigma^4 - \frac{\lambda}{2}(\vec{\pi}\cdot\vec{\pi})\sigma^2 - \frac{\lambda}{4}(\vec{\pi}\cdot\vec{\pi})^2$$

1) all $O(N-1)$ Symm.

$\Rightarrow$ Vac. obscures full $O(N)$ symm of Original theory

2) all $\vec{\pi}$ are massless! $\longrightarrow$ Goldstone Bosons, Nambu

3) σ has mass (2μ)

…ways call "v" VEV

$\sqrt{\lambda}\,\mu(\vec{\pi}\cdot\vec{\pi})\sigma - \frac{\lambda}{4}\sigma^4$

$)\sigma^2 - \frac{\lambda}{4}(\vec{\pi}\cdot\vec{\pi})^2$

$\longrightarrow$ Goldstone Bosons, Nambu

Accounting

Started w/ $O(N)$ symm $\frac{N(N-1)}{2}$ gen.

Ended w/ (after expanding about vacuum) $O(N-1)$ w/ $\frac{(N-1)(N-2)}{2}$ gens.

Difference: $N-1 \equiv \frac{N(N-1)}{2} - (N-1)(N-2)/2$

$\downarrow$

Massless fields,

Φ gets a VEV and "Breaks" $O(N) \longrightarrow O(N-1)$

$\Phi_0 = (\underbrace{0, 0, \ldots}_{1-(N-1)}, \phi)$ direction. ①

General Goldstone-Nambu Theorem:
for every Broken Continuous Symmetry $\Rightarrow$ Massless fields (# of Br

$\mathcal{L} = T(\vec{\Phi}) - V(\vec{\Phi})$, let Φ^a transform Under Some G
Not Necessarily gauged!

Also let $(\Phi^a)_0$ be vector of #s that Minimize the

$$\boxed{\rightarrow \left(\frac{\partial V}{\partial \phi^a}\right)\Bigg|_{\phi^a = \phi^a_0} = 0}$$

Now we can
Taylor Expand $V(\phi)$

Theorem:
Symmetry $\Rightarrow$ Massless fields (# of "Broken" generators)

Φ^a transform Under Some G
Not Necessarily gauged!

$(\Phi^a)_0$ be Vector of #s that Minimize the potential $V(\phi)$

$$\Big|\Big| = 0$$

$$\phi^a \equiv \phi^a_0$$

Now we Can
Taylor Expand $V(\phi)$

$$\text{global}\begin{cases} \Phi^a(x) \to R^{ab}\,\Phi^b(x) \quad \text{global trans.},\ R \to \text{Constants} \\ (\partial_\mu \phi^a)(\partial^\mu \phi^a) \end{cases}$$

if gauged

$$\Phi^a \to R^{ab}(x)\,\Phi^b(x)$$

$$\partial_\mu \to D_\mu \equiv \partial_\mu - igA_\mu, \quad A_\mu \to A_\mu - \frac{1}{g}\partial_\mu f(x)$$

$\Phi_0 = (\underbrace{0, 0, \ldots}_{1-(N-1)}, \phi)$ direction.

General Goldstone-Nambu Theorem: Massless fields (#) for every Broken Continuous Symmetry

$\mathcal{L} = T(\vec{\Phi}) - V(|\vec{\Phi}|)$, let Φ^a transform Under Some G

Not Necessarily gauged!

Also let $(\Phi^a)_0$ be vector of #s that Minim

at Min

$\vec{\Phi} = (\phi_0^1, \phi_0^2, \ldots \phi_0^N)$

$|\vec{\phi}| \equiv \#$

$$\boxed{\rightarrow \left(\frac{\partial V}{\partial \phi^a}\right)\Bigg|_{\phi^a \equiv \phi_0^a} = 0}$$

Now we Can Taylor Expand $V(\phi)$

symm of original theory

$$V(\phi) = V(\phi_0) + \frac{1}{2}(\phi^a - \phi_0^a)(\phi^b - \phi_0^b)\underbrace{\left(\frac{\partial^2 V}{\partial\phi^a \partial\phi^b}\right)}_{= M^2_{ab}}\Bigg|_{\substack{\phi^a = \phi_0^a \\ \phi^b = \phi_0^b}} + \cdots$$

Now Consider: transformations of ϕ^a

$$\phi^a \to \phi^a + \alpha\,\Delta^a(\vec{\phi})$$

α ↕ Infitesimal.

Assume Kinetic term and potential are separately Invariant.

Expand in α

$$V(\phi^a) = V\big(\phi^a + \alpha\,\Delta^a(\phi)\big) = V(\phi^a) + \underbrace{\alpha\,\Delta^a(\phi)\frac{\partial V}{\partial\phi^a}}_{=0} + \cdots$$

$|\phi| \equiv \#$

$\phi^a \equiv \phi^a_0$

Taylor expand

$\Delta^a(\phi)\frac{\partial V}{\partial \phi^a} = 0$, Now take derivative

$$\left(\frac{\partial \Delta^a}{\partial \phi^b}\right)\cancel{\left(\frac{\partial V}{\partial \phi^a}\right)}^{\,0}\Bigg|_{\phi_0} + \underbrace{\Delta^a(\phi)\,\frac{\partial^2 V}{\partial \phi^a \partial \phi^b}}_{} = 0$$

Zero always, 2 possibilities:

Eval @ Φ_0 (Minimum)

1. – if $\Delta^a(\vec{\Phi}_0) = 0$

trans. doesnt affect Vacuum State.

2. $\Delta^a(\vec{\phi}_0) \neq 0$

$$\Rightarrow \frac{\partial^2 V}{\partial \phi^a \partial \phi^b}\Bigg|_{\phi_0} = 0$$

Is Eigenvector of M^2_{ab} w/ Eigenvalue 0

$|\phi| \equiv \#$

$\phi^a \equiv \phi^a_0$

Taylor expand

$\Delta^a(\phi)\frac{\partial V}{\partial \phi^a} = 0$

Now take derivative

$\left(\frac{\partial \Delta^a}{\partial \phi^b}\right)\left(\cancel{\frac{\partial V}{\partial \phi^a}}\right)\Bigg|_{\phi_0} + \underbrace{\Delta^a(\phi)\frac{\partial^2 V}{\partial\phi^a\partial\phi^b}}_{\text{Zero always,}} = 0$

Eval @ Φ_0 (Minimum)

2 possibilities:

(1.) – if $\Delta^a(\vec{\Phi}_0) = 0$

trans. doesnt affect Vacuum State.

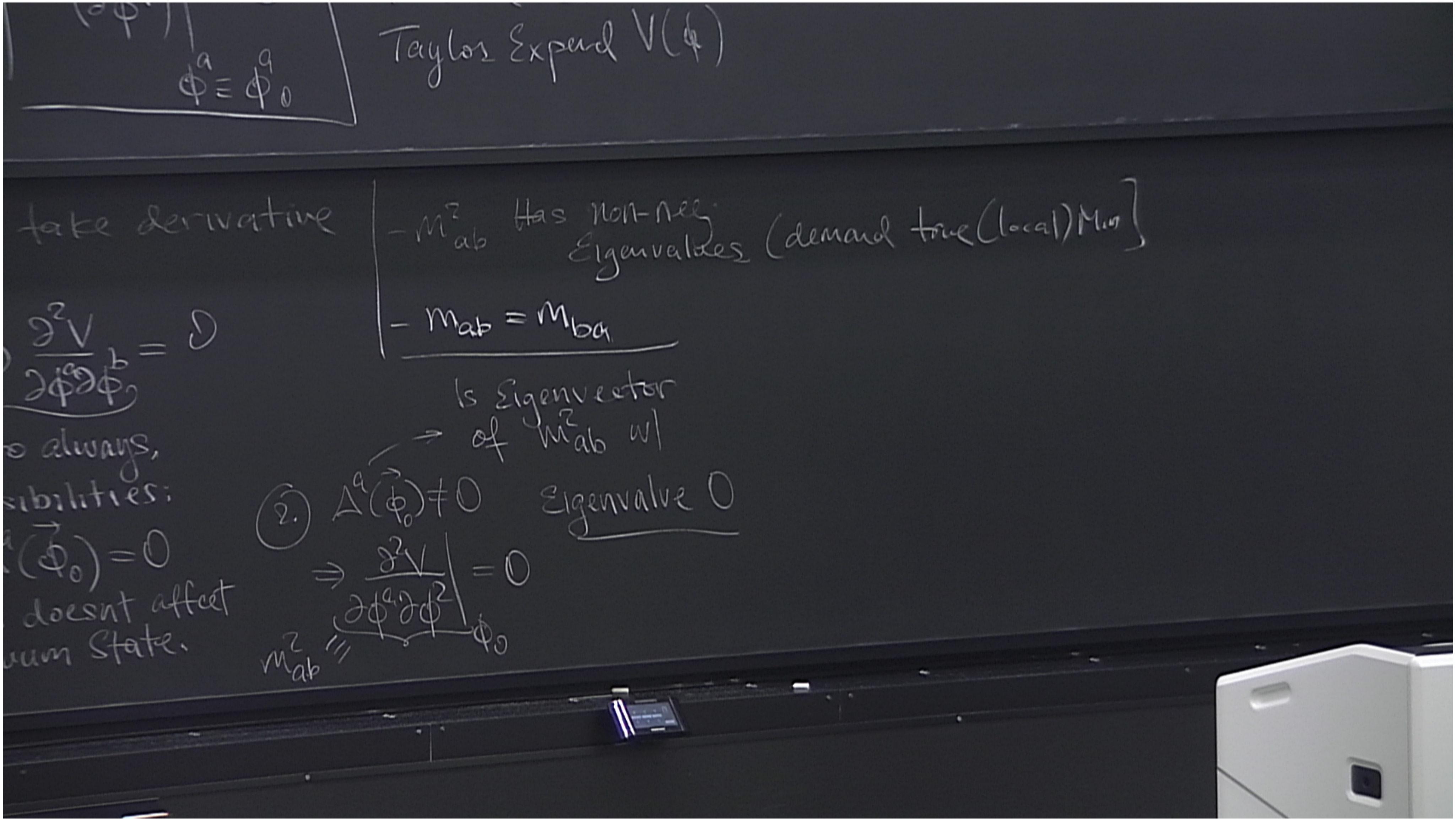
(2.) $\Delta^a(\vec{\phi}_0) \neq 0$

$\Rightarrow m^2_{ab} \equiv \underbrace{\frac{\partial^2 V}{\partial\phi^a\partial\phi^2}}\Bigg|_{\phi_0} = 0$

– M^2_{ab} Has non-neg. Eigenvalues

– $m_{ab} = m_{ba}$

Is Eigenvector of M^2_{ab} w/ Eigenvalue 0


Taylor Expand V(φ)
φ^a = φ^a_0
take derivative
- m²_ab Has non-neg Eigenvalues (demand true (local) Min)
- m_ab = m_ba
Is Eigenvector of m²_ab w/ Eigenvalue 0
o always,
sibilities:
(φ_0) = 0
doesnt affect
uum State.
2. Δ^a(φ_0) ≠ 0
∂²V/∂φ^a∂φ² |_φ0 = 0
m²_ab

Symm of Original theory | 3) ... Mass (2μ)

If you Break 'Continuous' Symm $\rightarrow$ Massless DoF., In

for a Gauged Symm. transformations depend on x

$U(1)$: w/ complex Scalar $\phi(x)$

$$\mathcal{L}_e = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + |D_\mu\phi|^2 + \mu^2\phi^*\phi + \frac{\lambda}{2}(\phi^*\phi)^2$$, No Mass for A

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Separately gauge Invariant.

Unique to Abelian $F_{\mu\nu}$

$D_\mu \equiv \partial_\mu - igA_\mu$

$U(1)$ Coupling Constant,

Under $U(1)$:

$\phi(x) \longrightarrow e^{i\alpha}$

$A_\mu \longrightarrow A_\mu -$

..., In practice, Only true for 'global' Symm'

on x

Mass for A_μ (Not gauge Invariant)

...er $U(1)$:

$\phi(x) \to e^{i\alpha(x)} \phi(x)$

$A_\mu \to A_\mu - \frac{1}{g} \partial_\mu \alpha(x)$

Mass for Spin 1: $m_A^2 \cancel{A_\mu A^\mu}$

Nobel Prize 1999 ('t Hooft, Veltman)

Gauge theories are Renormalizable;

Spin 1 w/ Explicit Mass terms are Not!

$|\vec{\phi}| \equiv \#$

$\phi^a \equiv \phi^a_0$

Taylor Expand $V(\phi)$

propagator: for Spin 1

if Massive (explicitly)

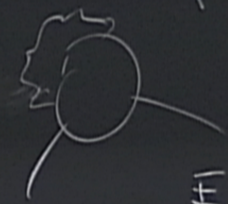
$$\frac{g_{\mu\nu} - \frac{q_\mu q_\nu}{M_A^2}}{q^2 - M_A^2}$$

for Gauge field:

gauge trans param.

$$\frac{g_{\mu\nu} - (\xi - 1)\frac{q_\mu q_\nu}{q^2}}{q^2}$$

loop $\rightarrow \int d^4 q$



More propagators $\Rightarrow$ Better.

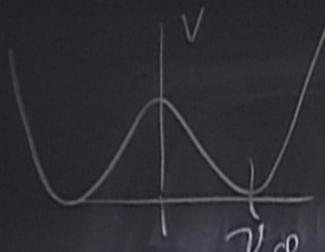
$\xrightarrow{E}$

longitudnal polariz $\sim E$ $\rightarrow$ Violate Unitarity!

$|\vec{\phi}| \equiv \#$

$\phi^a \equiv \phi^a_0$

Taylor Expand $V(\phi)$



Expand about v_φ.

$$\langle \phi \rangle = v_\varphi = \mu/\sqrt{\lambda}$$

Re Im

$$\phi(x) = v_\varphi + \frac{1}{\sqrt{2}}\left(\varphi(x) + i\xi(x)\right)$$

Expand potential

$$V(\phi) = -\frac{1}{2\lambda}\mu^4 + \frac{1}{2}(2\mu^2)\varphi^2 + \cdots O(\varphi^3, \xi^3)$$

Real part gets Mass $(2\mu^2) \equiv m_\varphi^2$ | $m_\xi = 0$

kinetic term:

$$|D_\mu \phi|^2 = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\xi)^2 + \sqrt{2} g v_\varphi A_\mu \partial^\mu \xi + \underbrace{g^2 v_\varphi^2}_{} A_\mu A^\mu + \cdots$$

$$m_A^2 = 2g^2 v_\varphi^2$$

Extra dof?

ξ particle!

Not physical Here!

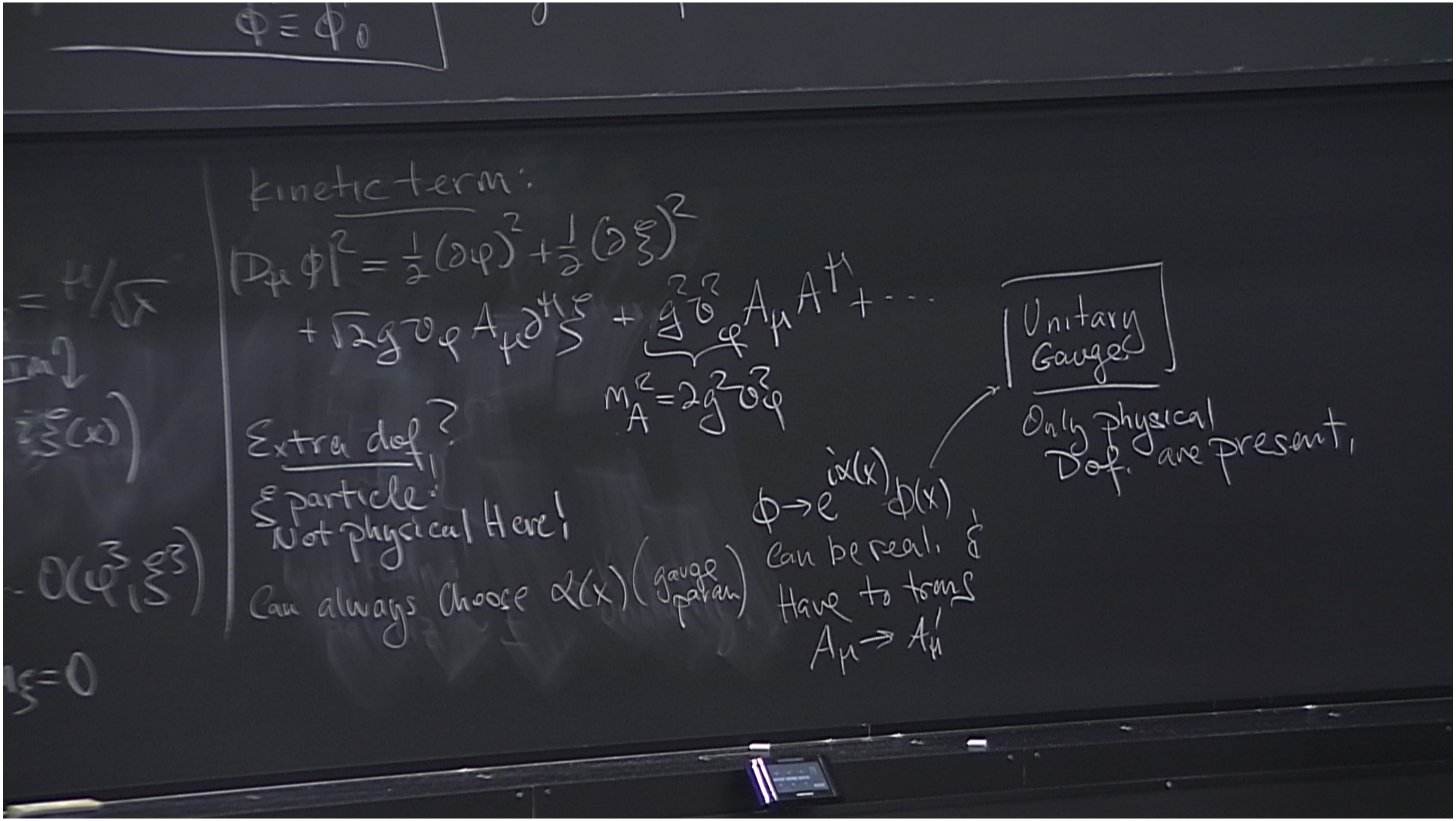
Can always Choose $\alpha(x)$ (gauge param)

$\phi \to e^{i\alpha(x)}\phi(x)$

can be real. &

Have to trans

$A_\mu \to A'_\mu$


$\Phi \equiv \Phi_0$
kinetic term:
$|D_\mu \phi|^2 = \frac{1}{2}(\partial\varphi)^2 + \frac{1}{2}(\partial\xi)^2$
$+ \sqrt{2} g v_\varphi A_\mu \partial^\mu \xi + \underbrace{g^2 v_\varphi^2}_{} A_\mu A^\mu + \cdots$
$M_A^2 = 2 g^2 v_\varphi^2$
Unitary Gauge
Only physical Dof. are present,
Extra dof?
ξ particle:
Not physical Here!
Can always Choose $\alpha(x)$ (gauge param)
$\phi \to e^{i\alpha(x)} \phi(x)$
can be real. &
Have to trans
$A_\mu \to A'_\mu$
$= \mu/\sqrt{\lambda}$
$\xi(x)$
$O(\varphi^3, \xi^3)$
$\xi = 0$