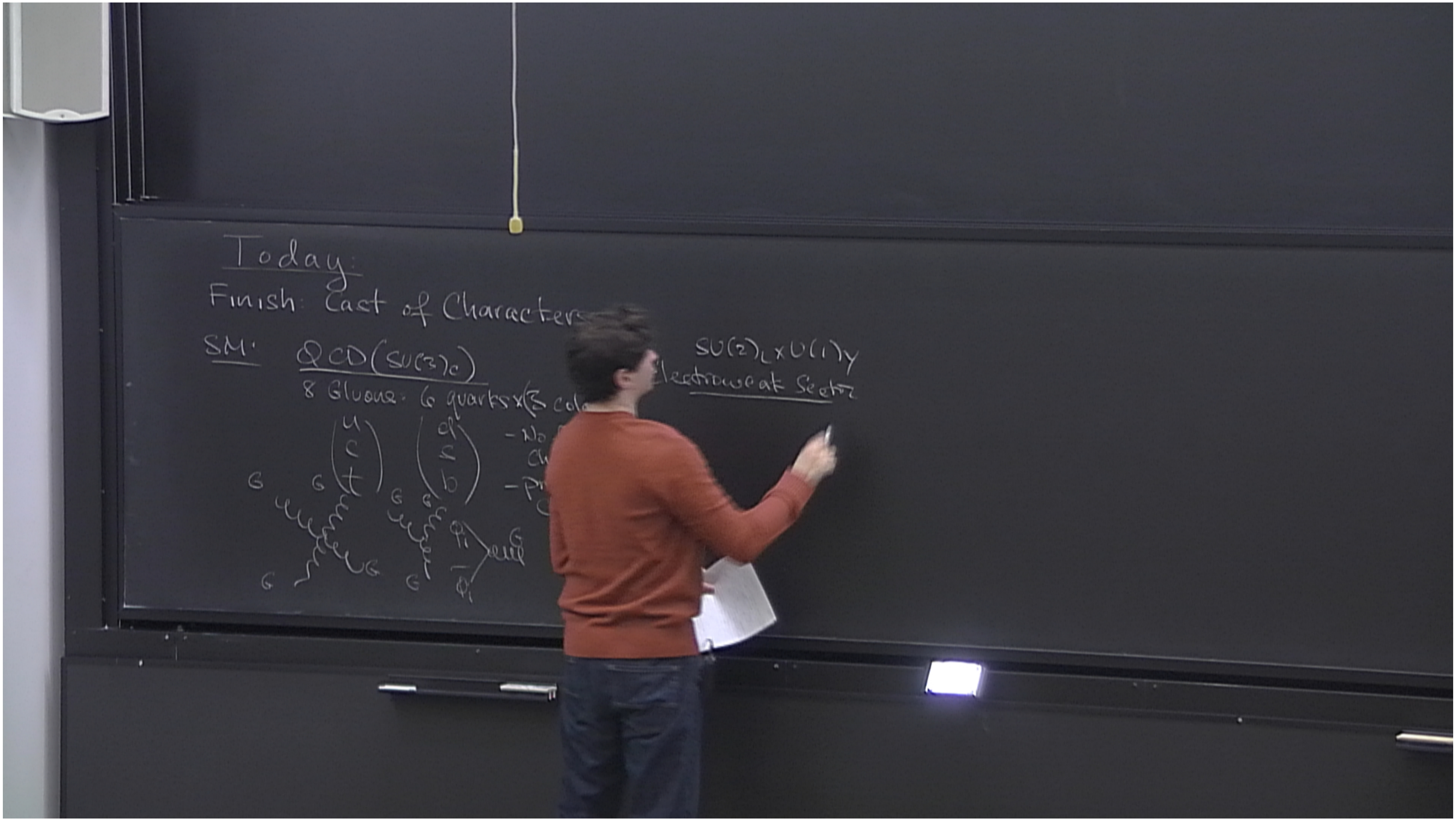
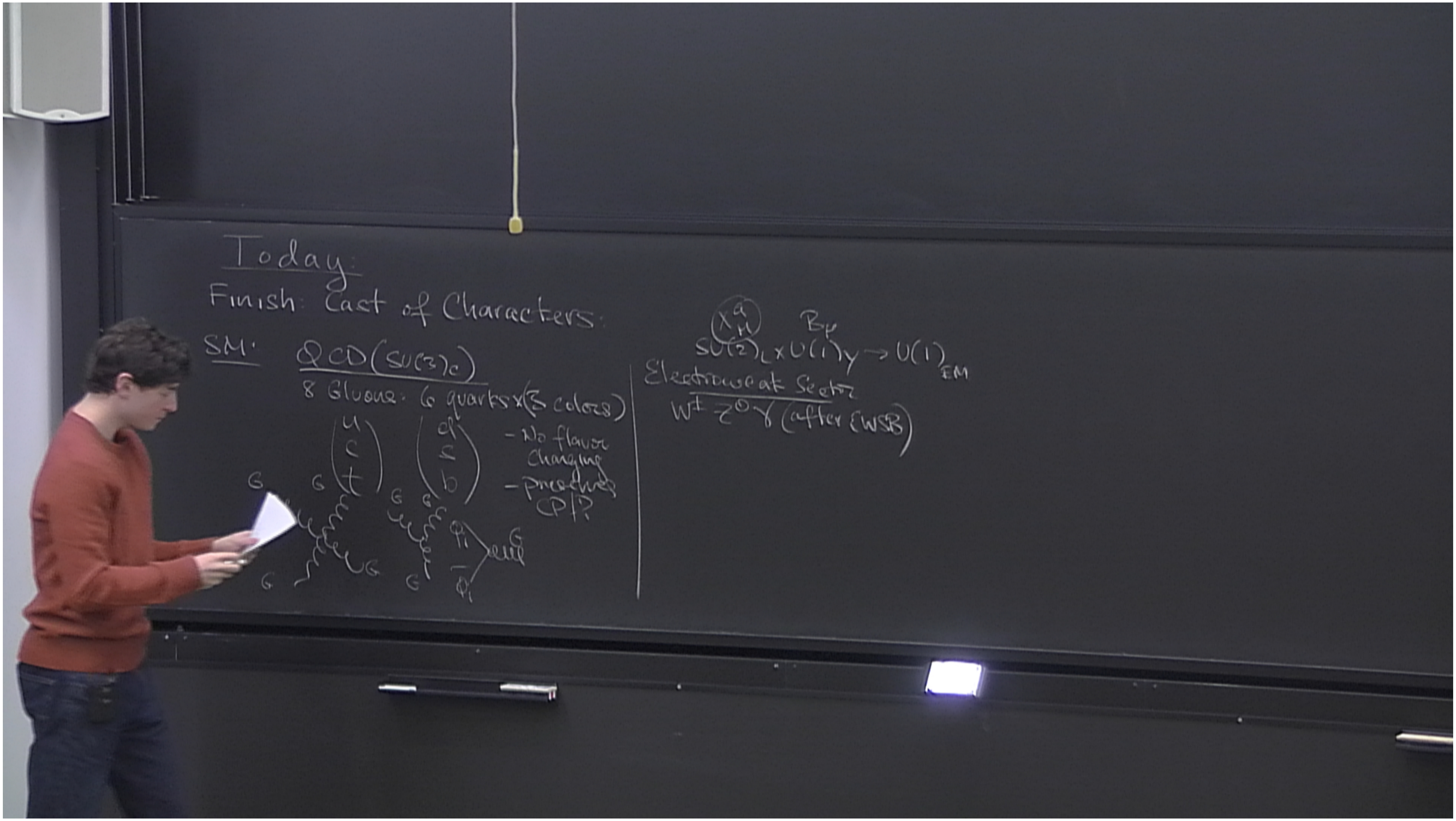
Title: Standard Model-2

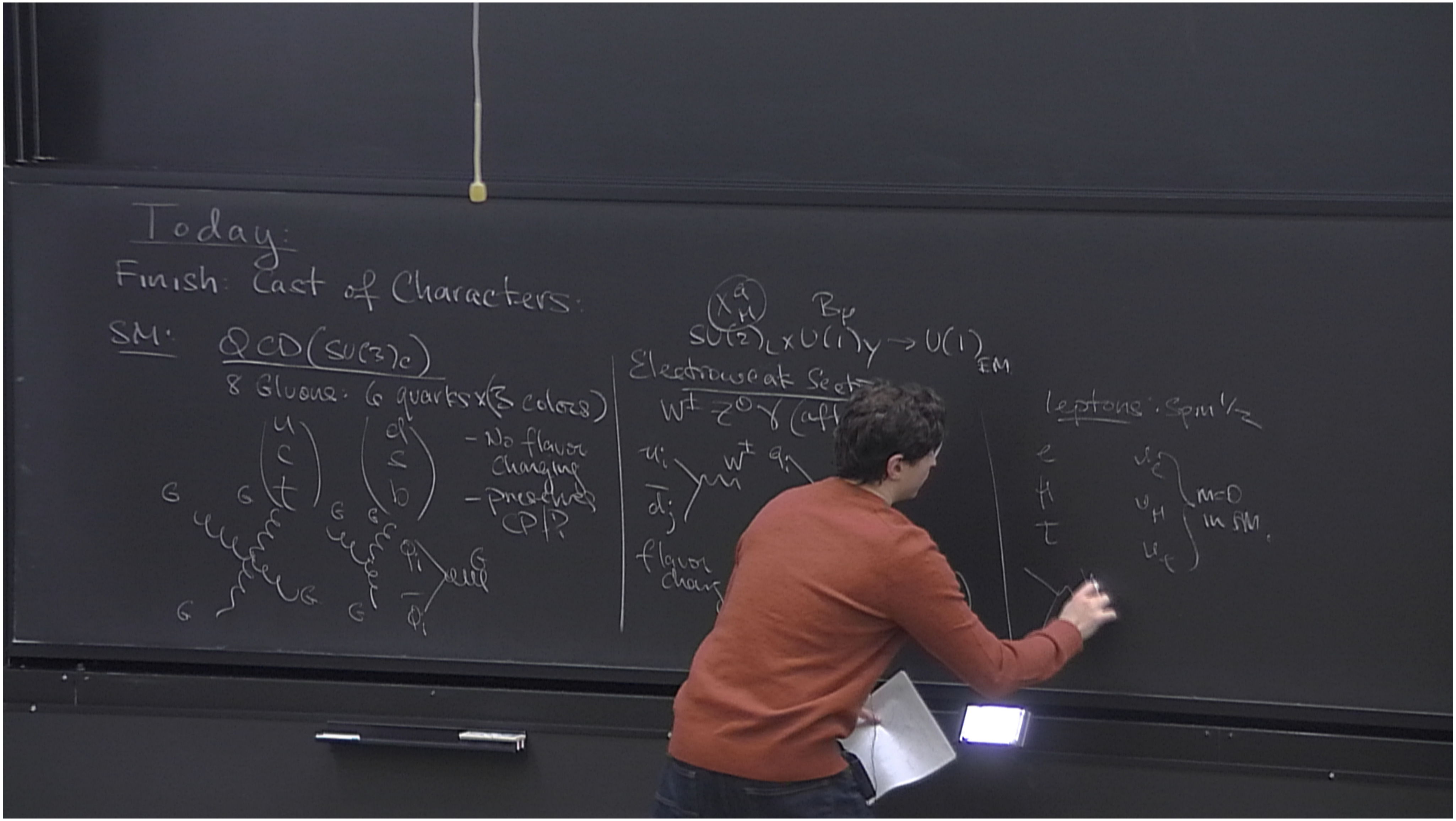
Date: Jan 08, 2015 09:00 AM

URL: http://pirsa.org/15010003

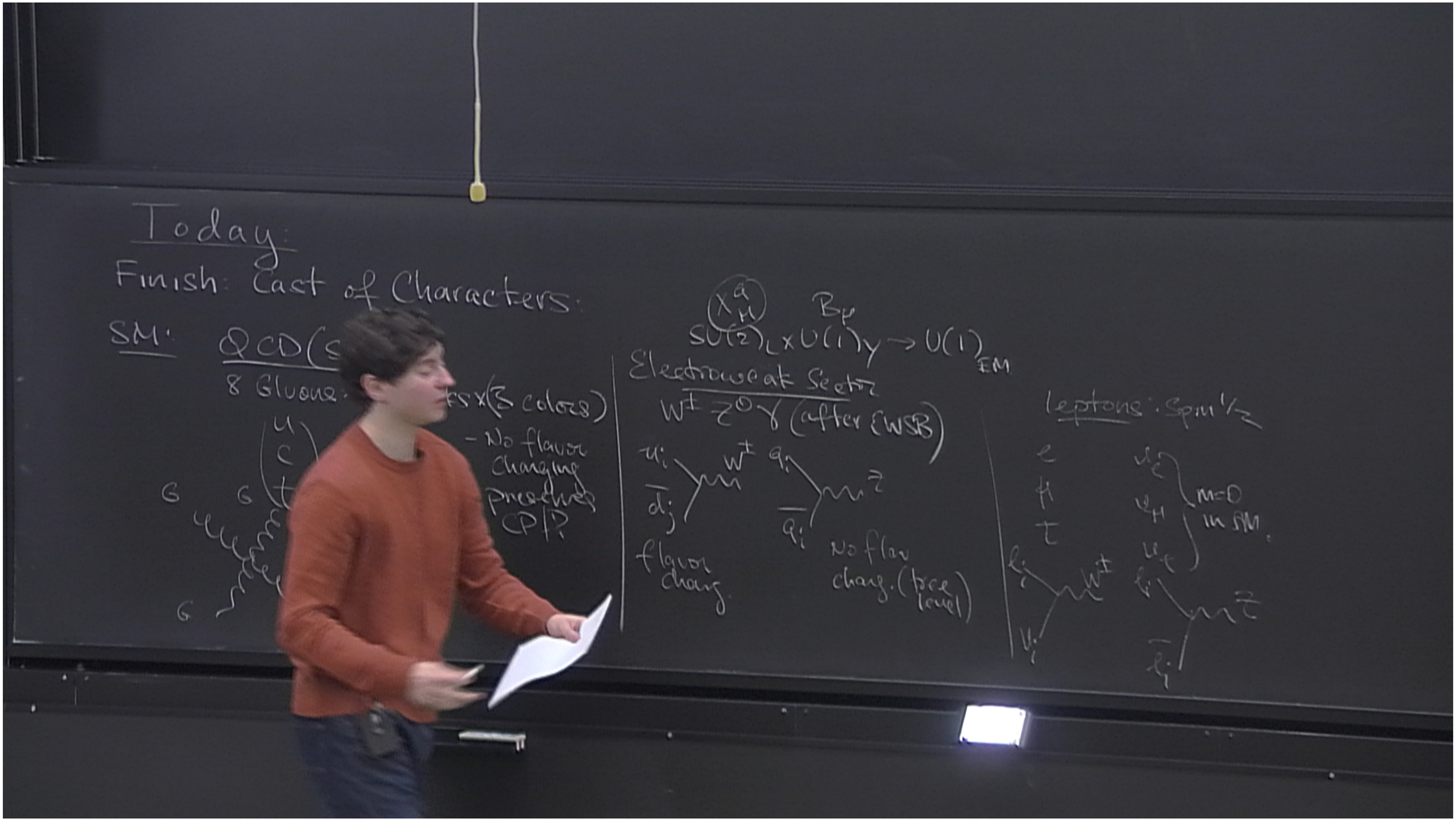
Abstract:


Today:
Finish: Cast of Characters
SM:
QCD (SU(3)c)
8 Gluons
6 quarks x (3 col
SU(2)L x U(1)Y
Electroweak Sector


Today:
Finish: Cast of Characters:
SM:
QCD (SU(3)c)
8 Gluons. 6 quarks x (3 colors)
u c t
d s b
- No flavor changing
- preserves CP!?
SU(2)L x U(1)Y → U(1)EM
Electroweak Sector
W± Z0 γ (after EWSB)



Today:
Finish: Cast of Characters:
SM:
QCD(SU(3)c)
8 Gluons: 6 quarks x (3 colors)
- No flavor changing
- preserves CP|P?
Electroweak Sect
SU(2)L x U(1)Y → U(1)EM
leptons: Spin 1/2
m=0 in SM.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Finish: Cast of Characters:
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8 Gluons
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preserves CP/P?
SU(2)L x U(1)Y → U(1)EM
Electroweak Sector
W± Z0 γ (after EWSB)
flavor chang.
No flav chang. (tree level)
leptons: Spin 1/2
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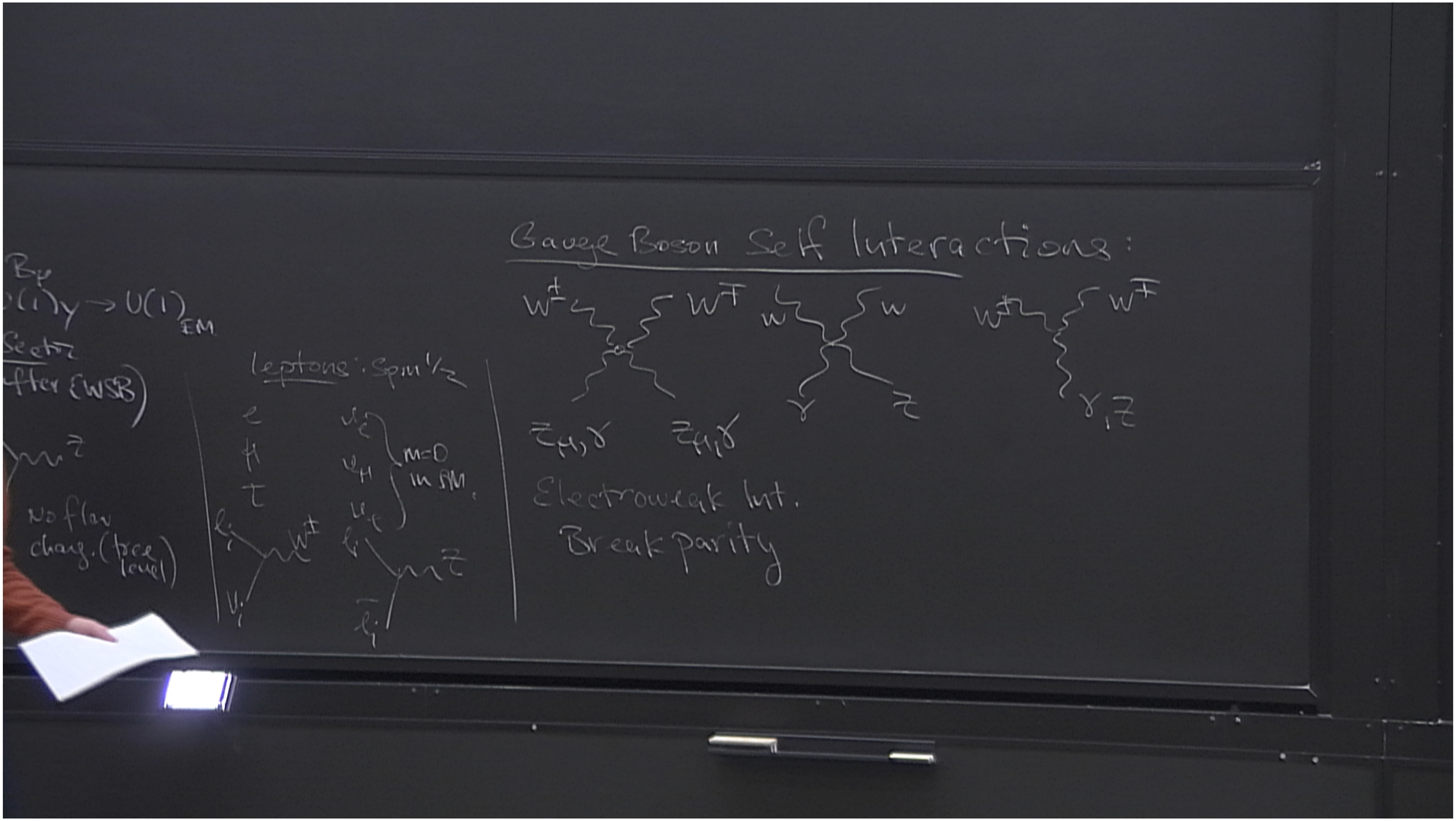
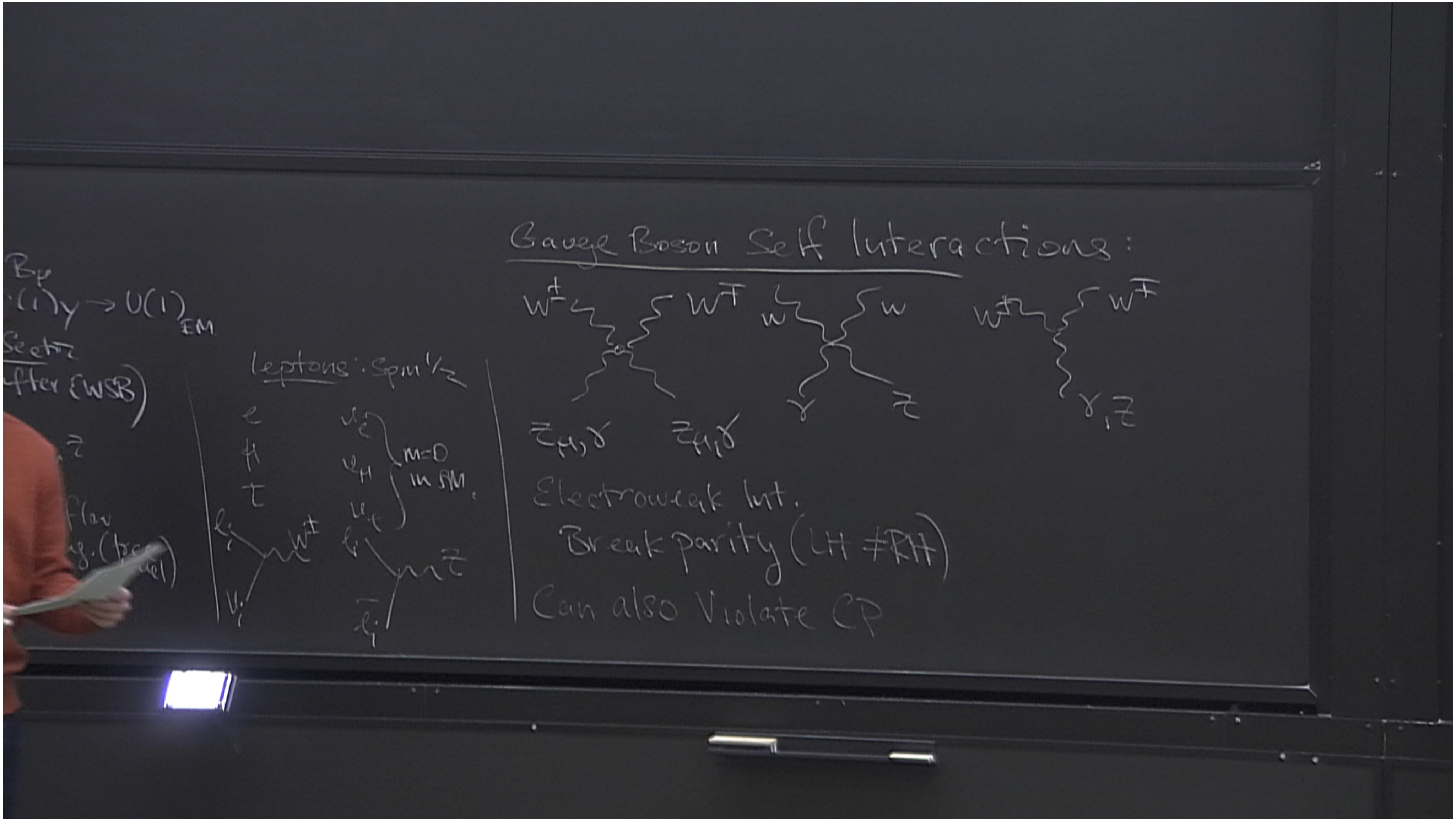
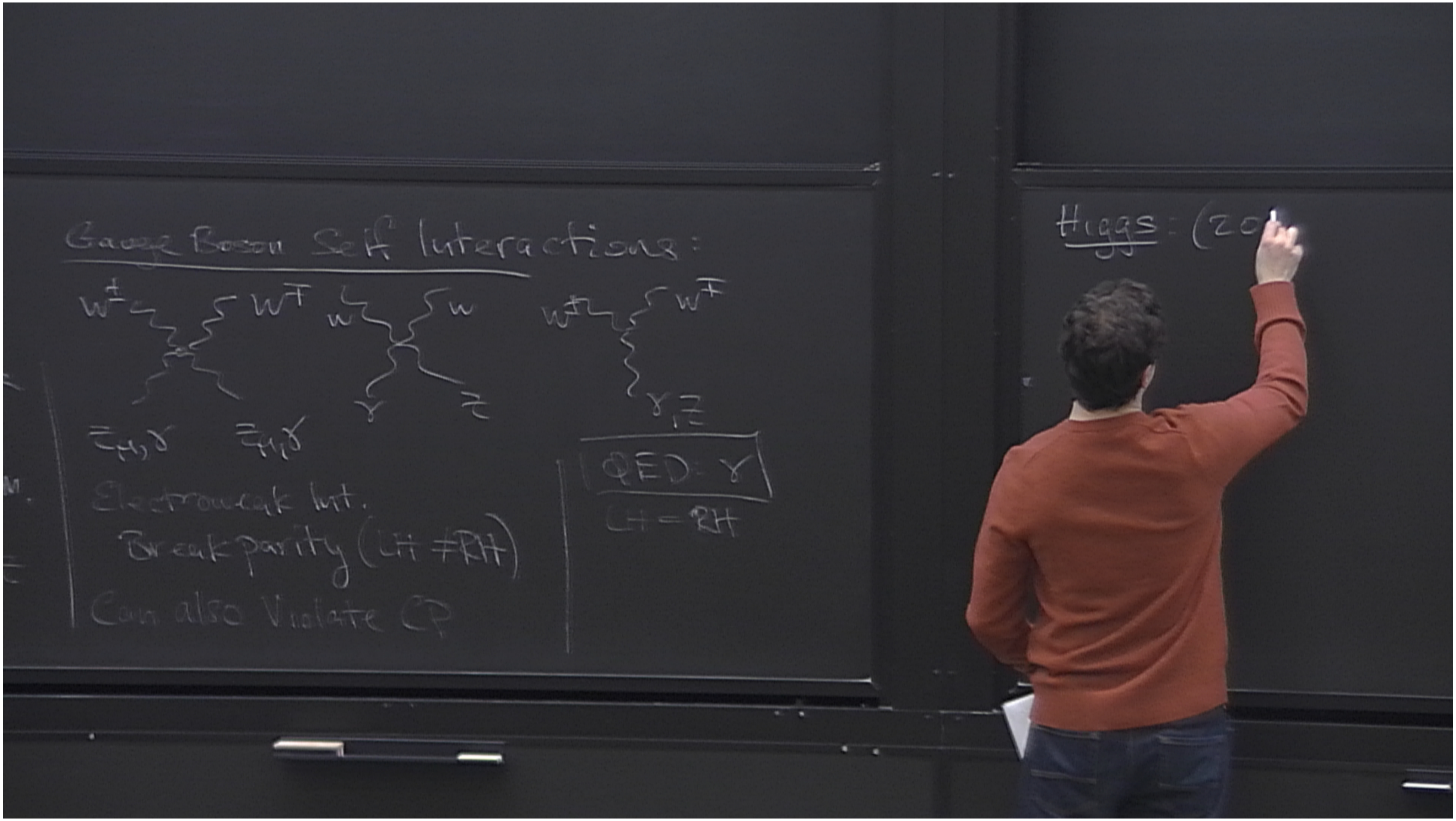
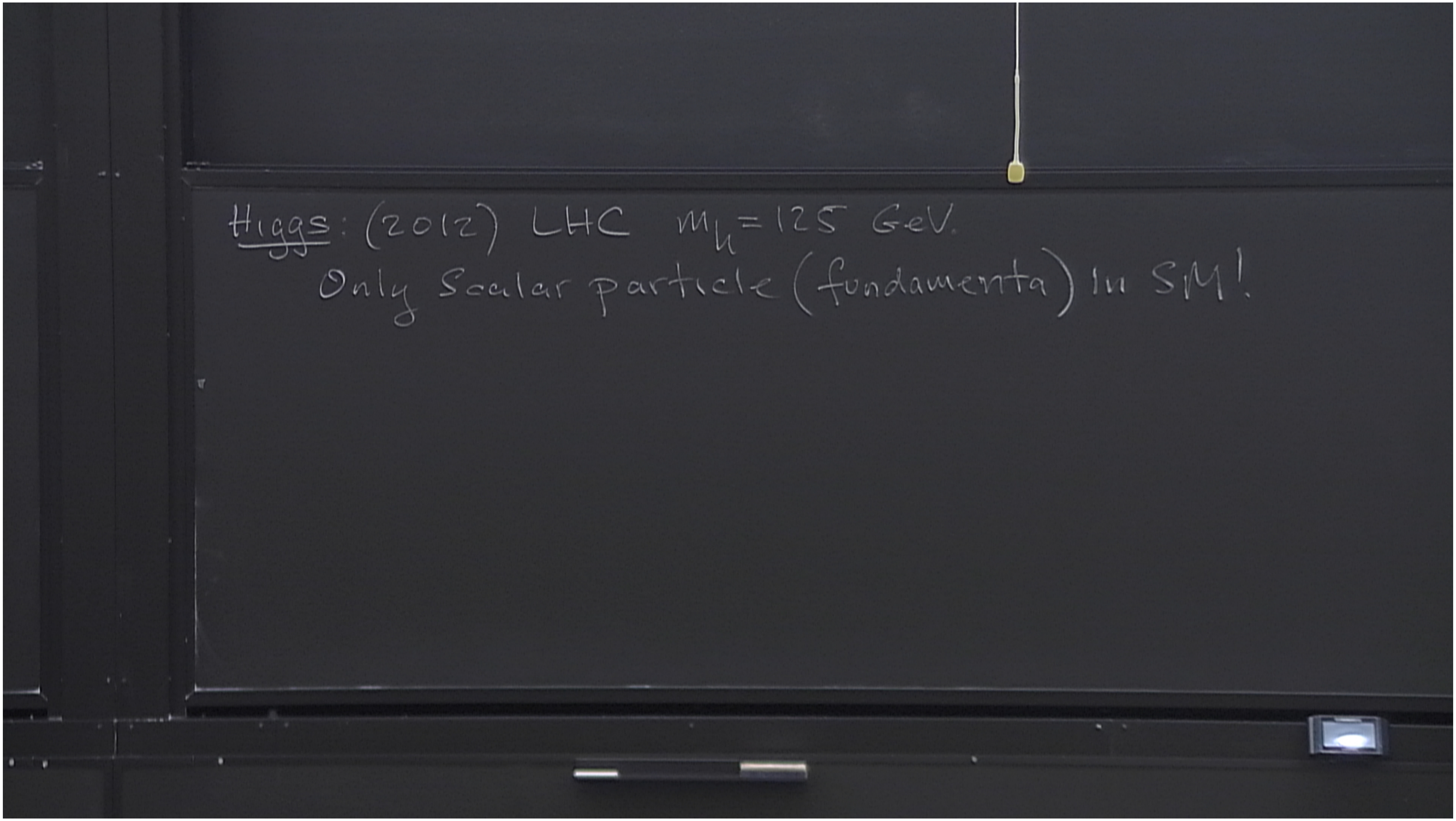
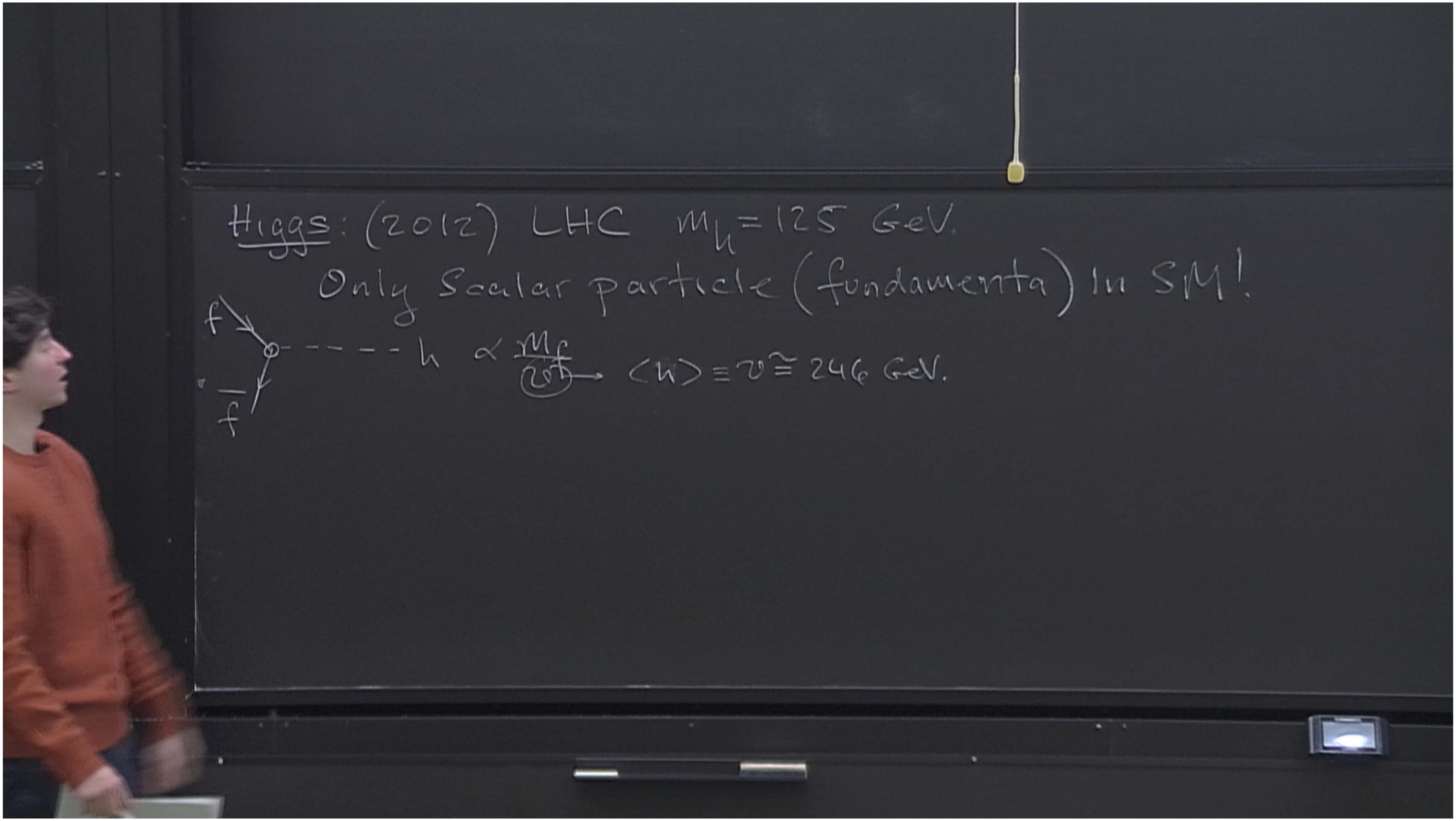
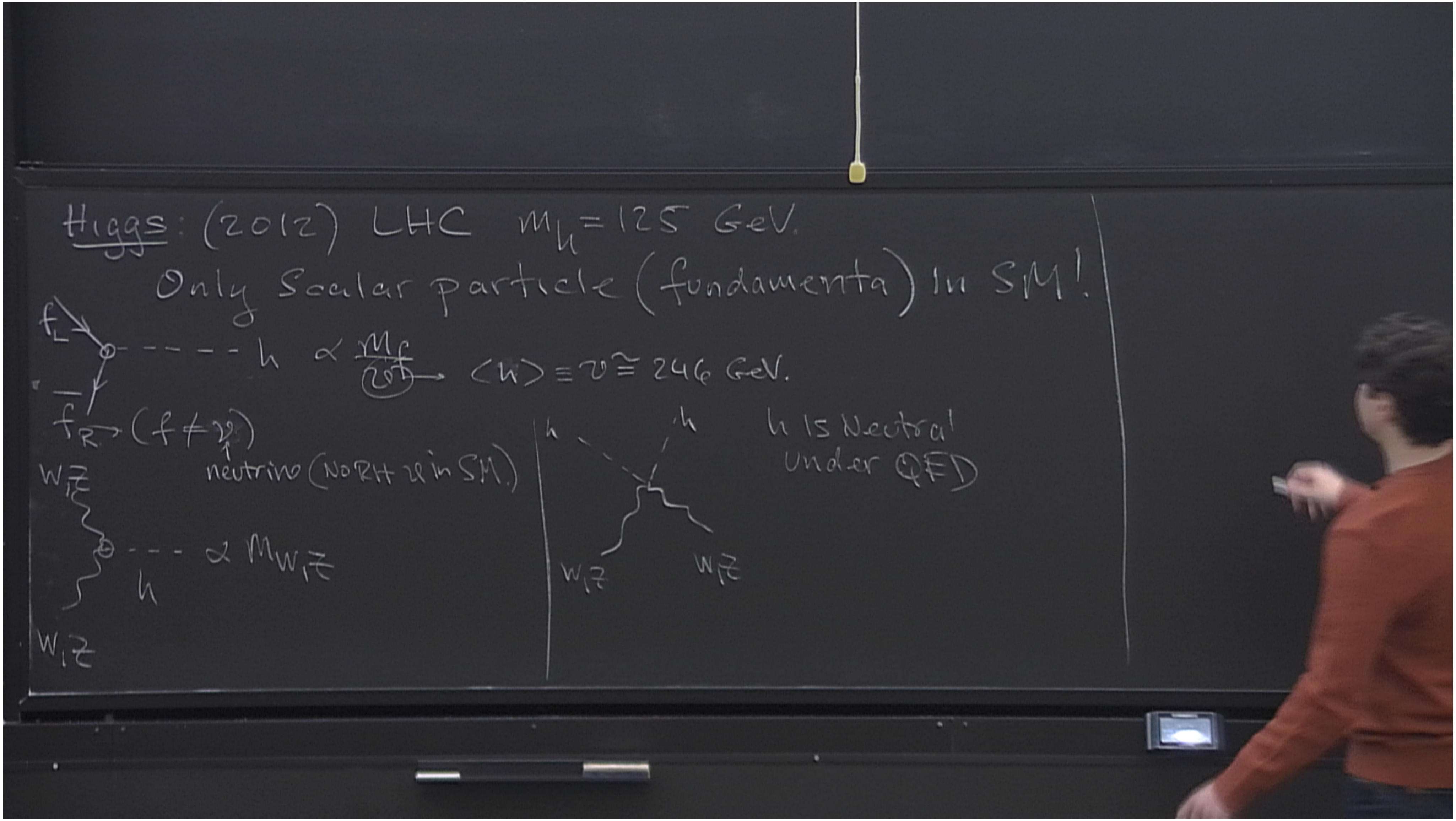

Gauge Boson Self Interactions:
leptons: Spin 1/2
m=0 in SM
No flav chang. (tree level)
Electroweak Int.
Break parity

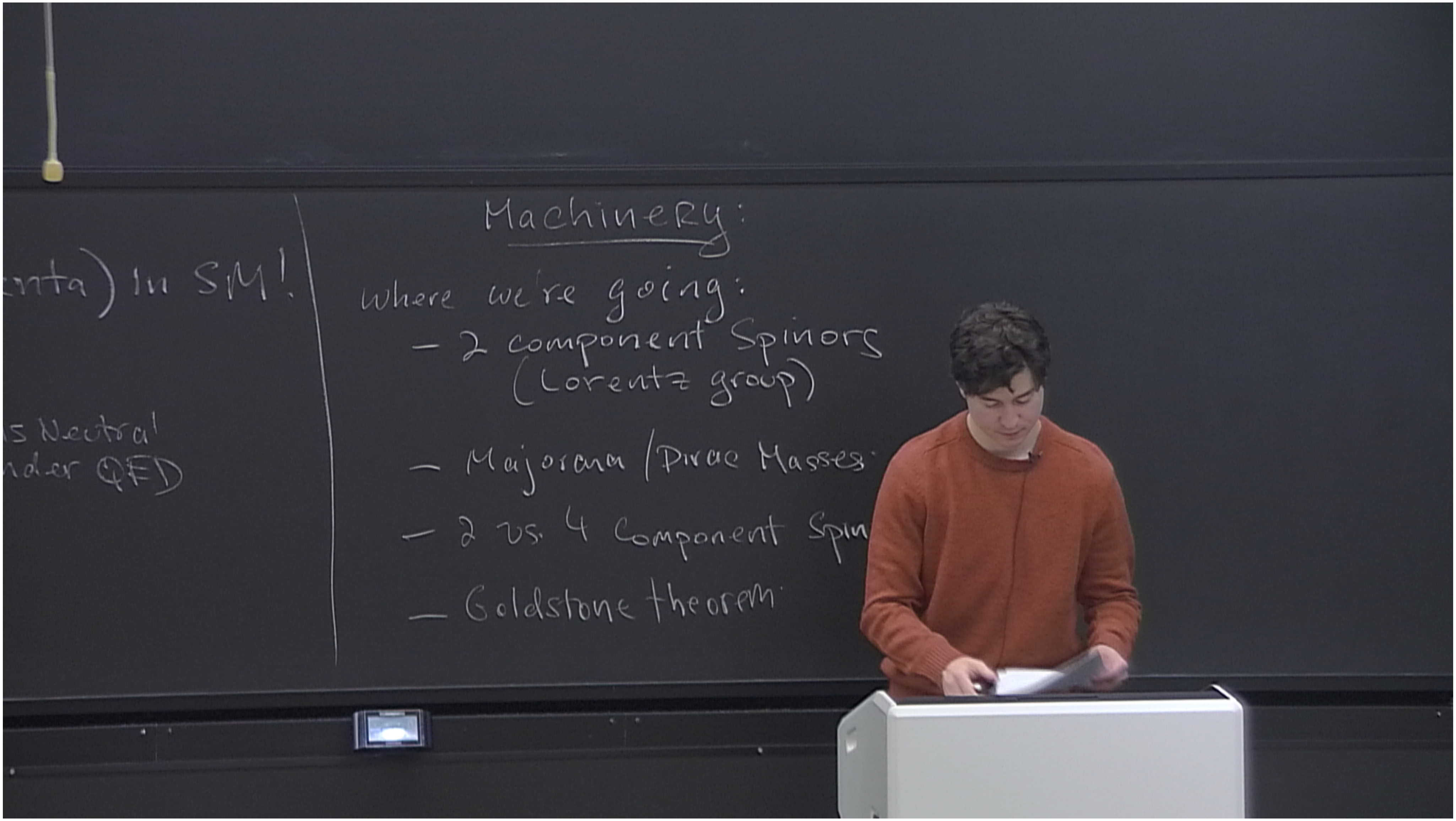

Gauge Boson Self Interactions:
leptons: Spin 1/2
m=0 in SM.
Electroweak Int.
Break parity (LH ≠ RH)
Can also Violate CP


Gauge Boson Self Interactions:
Electroweak Int.
Break parity (LH ≠ RH)
Can also Violate CP
QED: γ
LH = RH
Higgs: (20

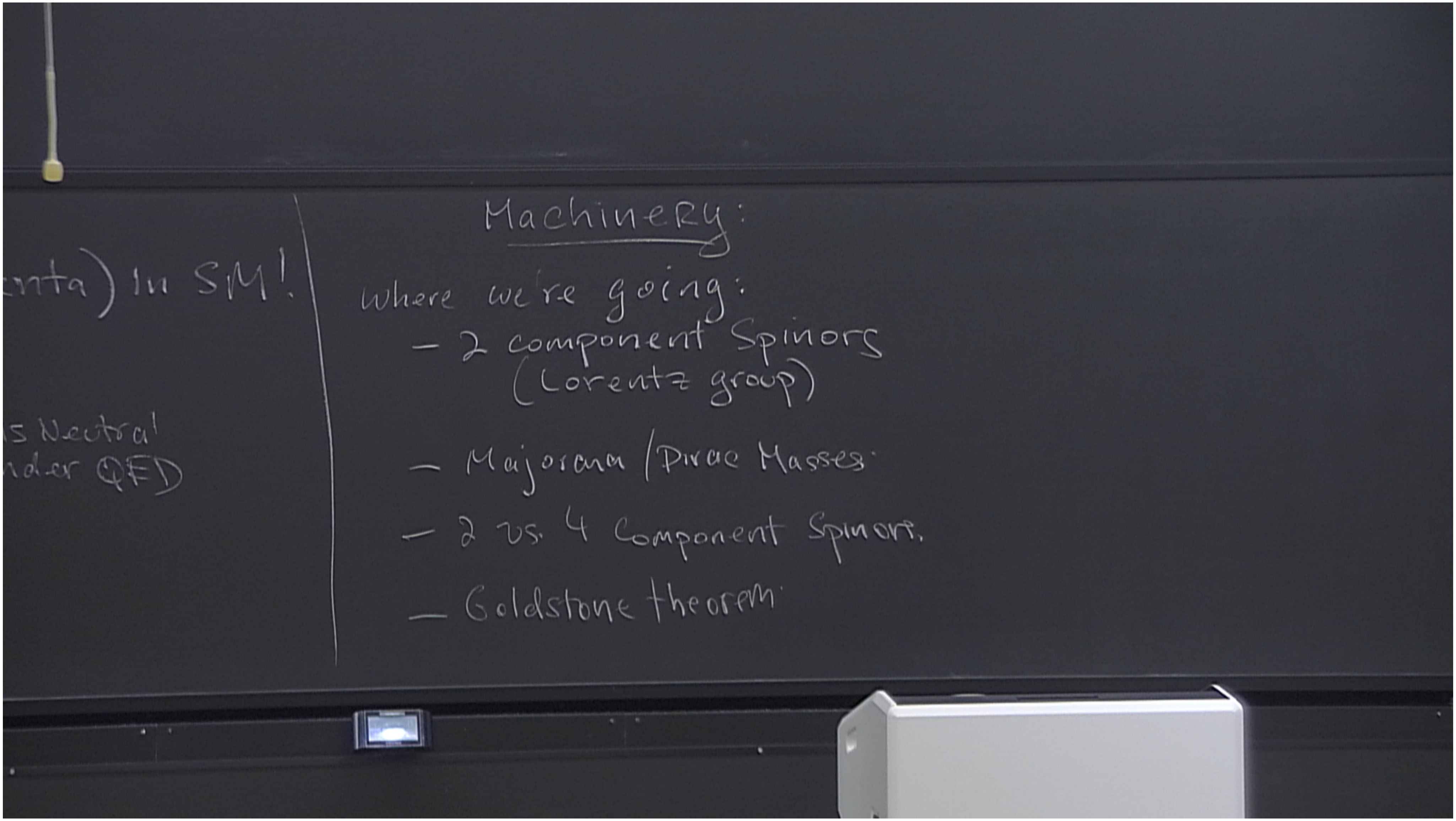

Higgs: (2012) LHC $m_h = 125$ GeV.
Only Scalar particle (fundamenta) in SM!


Higgs: (2012) LHC m_h = 125 GeV.
Only Scalar particle (fundamenta) in SM!
f
h
f̄
⟨h⟩ ≡ v ≅ 246 GeV.


Higgs: (2012) LHC m_h = 125 GeV.
Only Scalar particle (fundamenta) in SM!
f_L
h ∝ m_f / (2v) → ⟨h⟩ ≡ v ≅ 246 GeV.
f_R (f ≠ ν)
neutrino (No RH ν in SM)
h is Neutral
Under QED
W,Z
∝ M_W,Z
h
W,Z



in SM!
Neutral
QED
Machinery:
Where we're going:
– 2 component Spinors (Lorentz group)
– Majorana / Dirac Masses
– 2 vs. 4 Component Spin
– Goldstone theorem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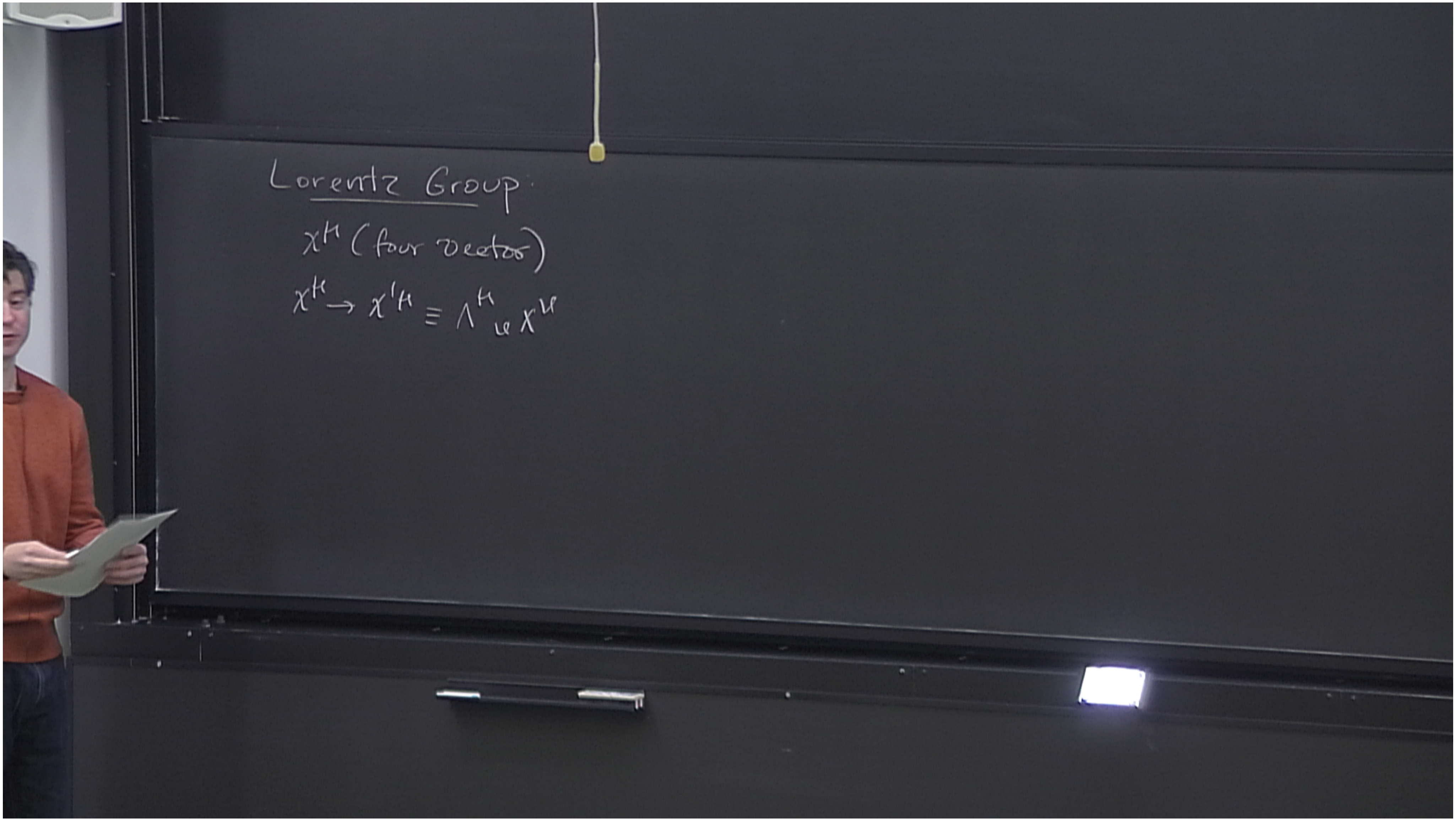
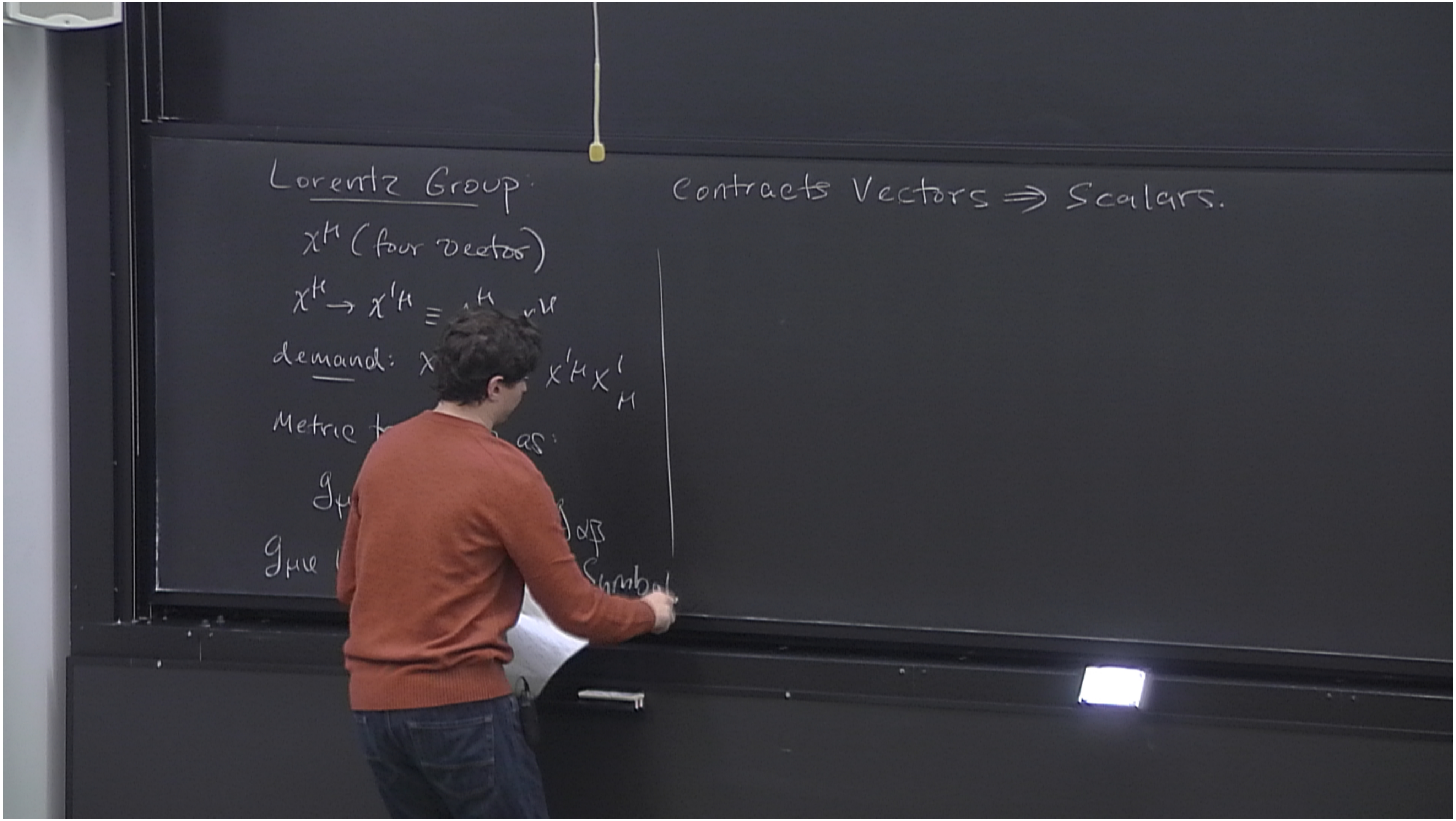
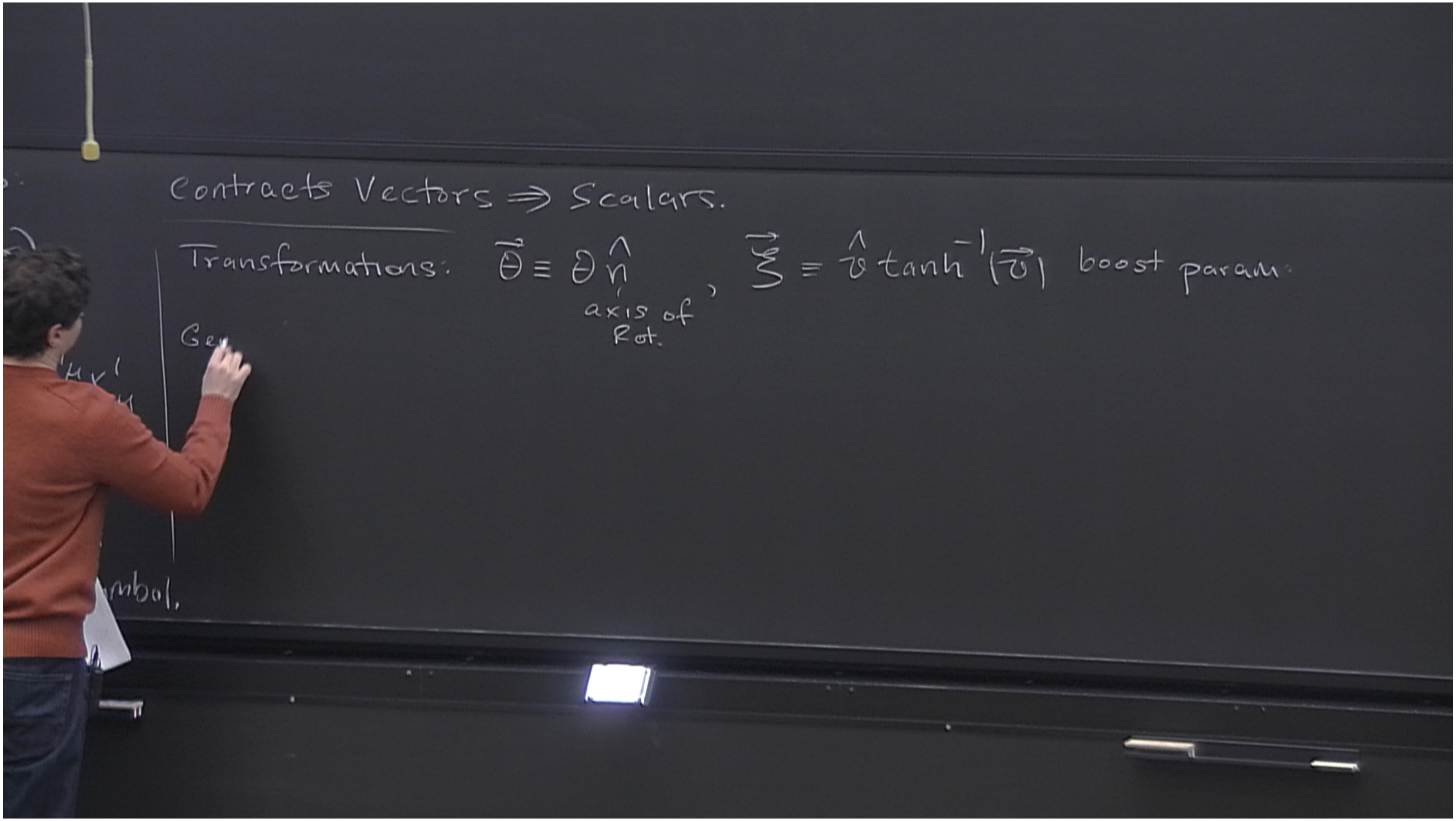
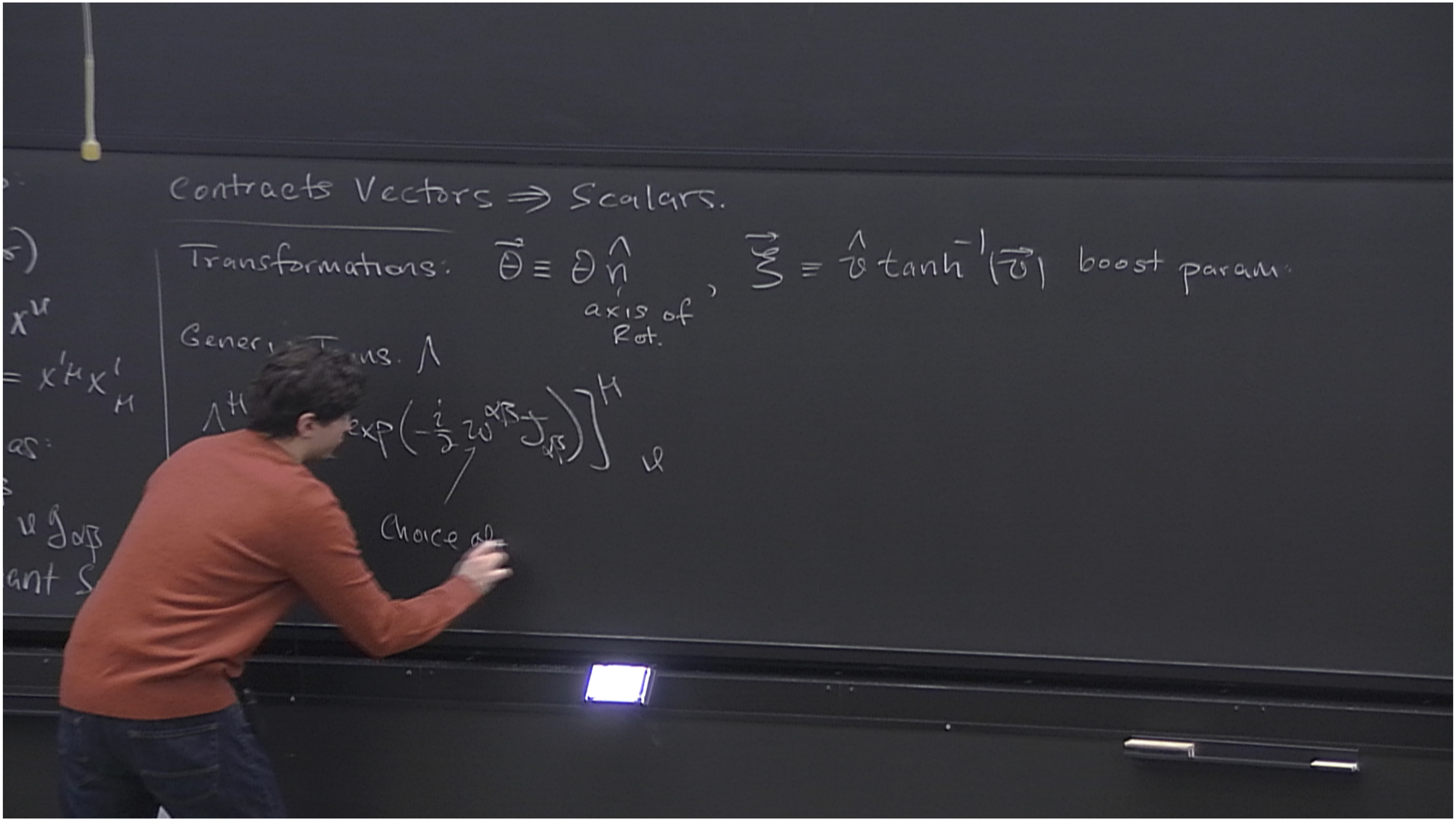
...s Neutral
...nder QED


Lorentz Group:
x^μ (four vector)
$x^\mu \to x'^\mu \equiv \Lambda^\mu{}_\nu x^\nu$


Lorentz Group:
x^μ (four vector)
demand:
Contracts Vectors ⇒ Scalars.


Contracts Vectors ⇒ Scalars.
Transformations: $\vec{\Theta} \equiv \theta \hat{n}$, $\vec{\xi} = \hat{v} \tanh^{-1}|\vec{v}|$ boost param.
axis of Rot.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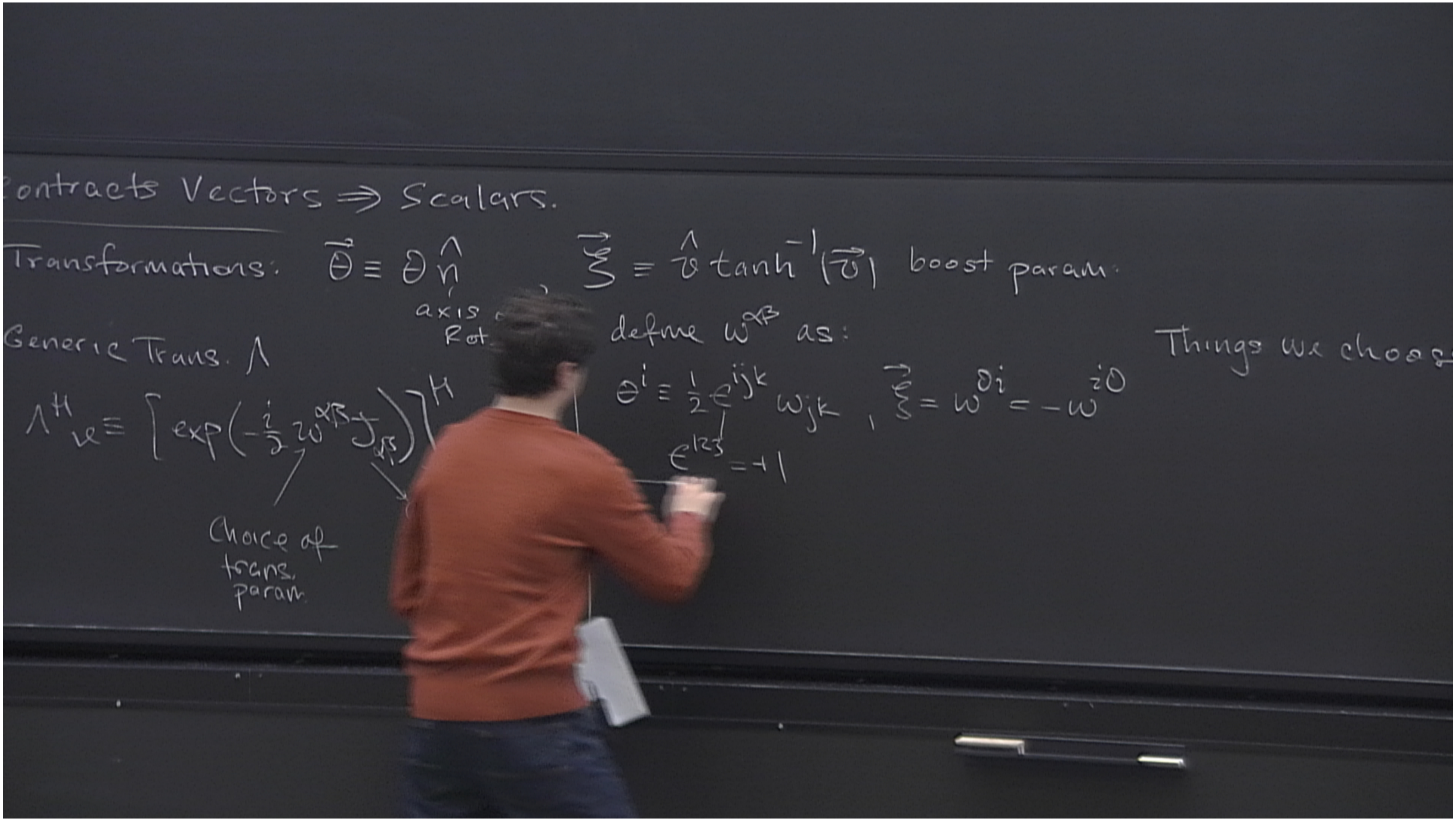
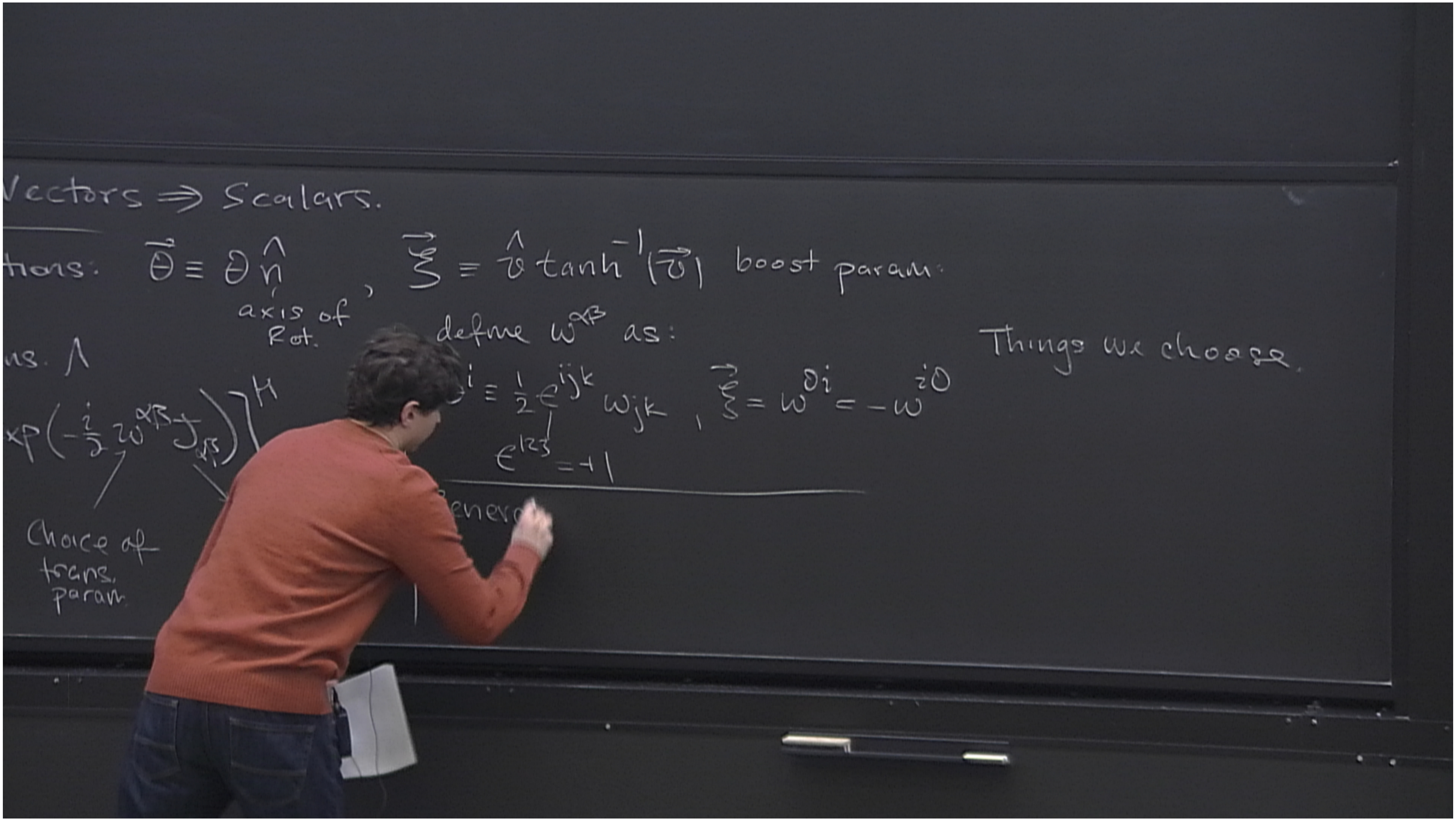
Transformations: $\vec{\theta} \equiv \theta \hat{n}$, $\vec{\xi} = \hat{v} \tanh^{-1}|\vec{v}|$ boost param.
axis of Rot.
Gener Trans. Λ
Λ^{μ} exp$\left(-\frac{i}{2}\omega^{\alpha\beta}J_{\alpha\beta}\right)]^{\mu}{}_{\nu}$
Choice of


Contracts Vectors ⇒ Scalars.
Transformations: $\vec{\Theta} \equiv \theta \hat{n}$ axis Rot.
$\vec{\xi} = \hat{v} \tanh^{-1}|\vec{v}|$ boost param.
Generic Trans. Λ
$\Lambda^{\mu}{}_{\nu} \equiv \left[\exp\left(-\frac{i}{2}\omega^{\alpha\beta} J_{\alpha\beta}\right)\right]^{\mu}$
Choice of trans. param.
define $\omega^{\alpha\beta}$ as:
$\theta^{i} \equiv \frac{1}{2}\epsilon^{ijk}\omega_{jk}$, $\vec{\xi} = \omega^{0i} = -\omega^{i0}$
$\epsilon^{123} = +1$
Things we choose


Vectors ⇒ Scalars.
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Things we choose.
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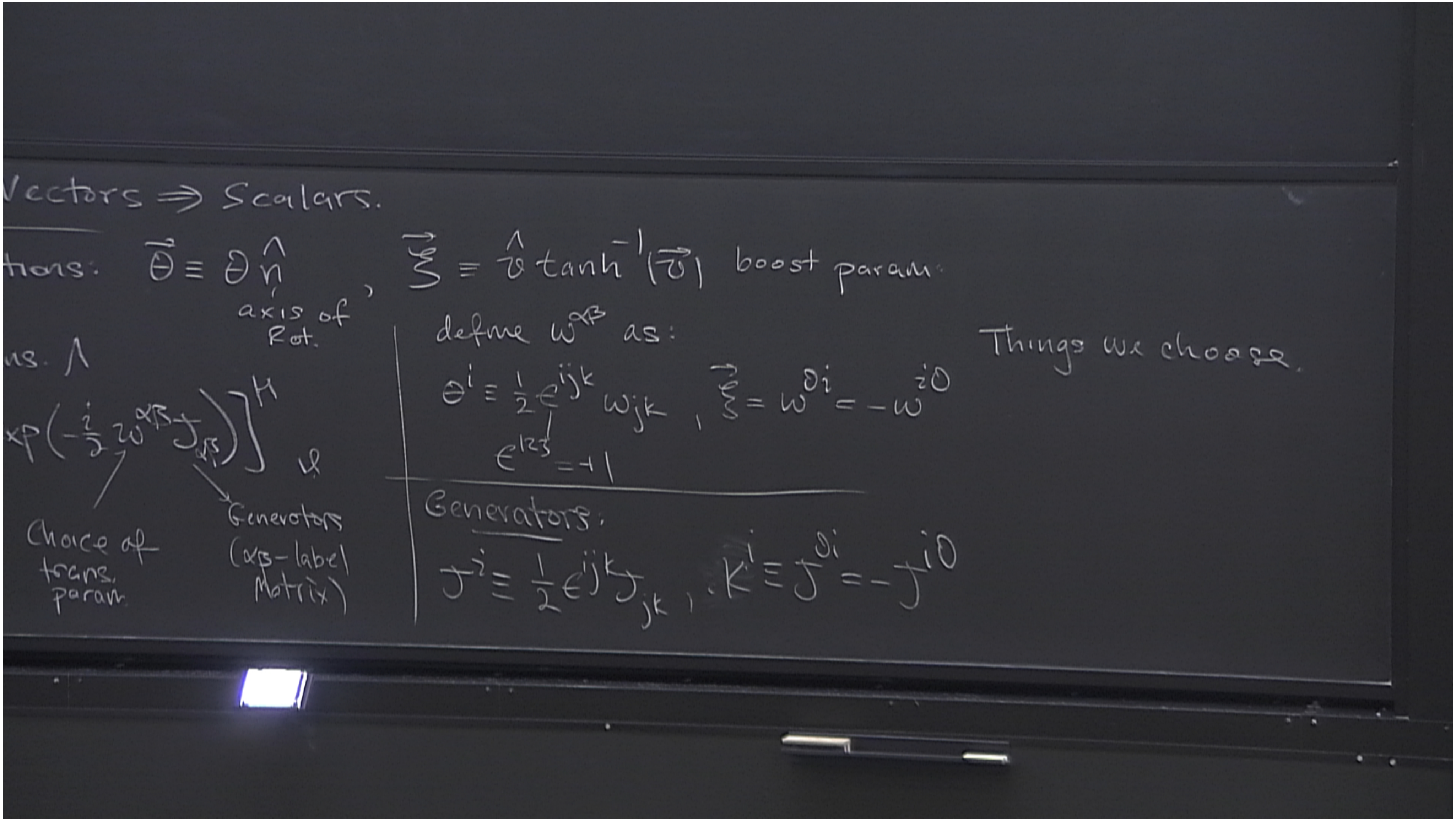
Choice of trans. param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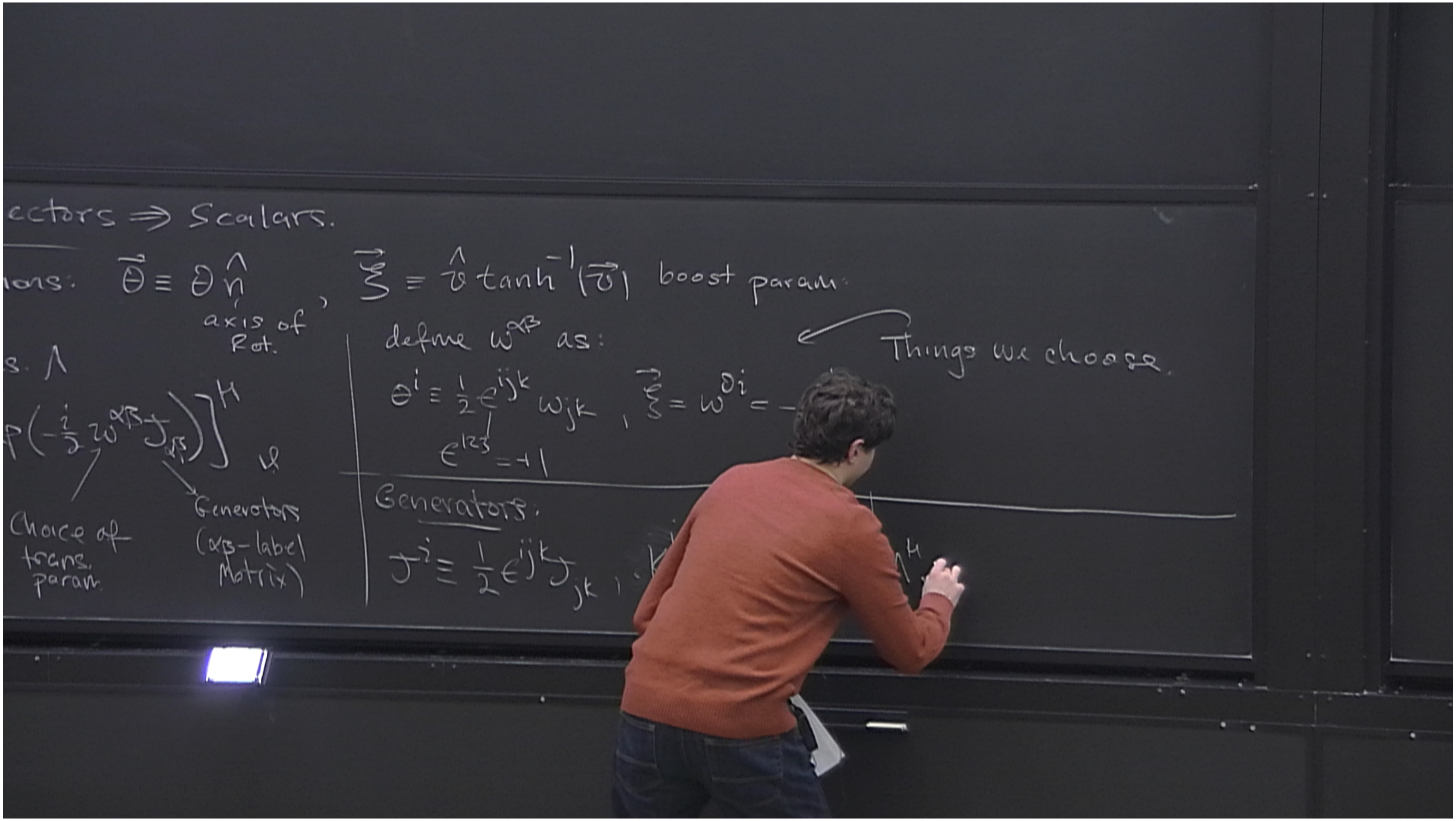
$\epsilon^{123} = +1$
$\exp\left(-\frac{i}{2}\omega^{\alpha\beta}J_{\alpha\beta}\right)\Big]^N$
Choice of trans. param
Generators (αβ-label Matrix)
Generators:
$J^i \equiv \frac{1}{2}\epsilon^{ijk}J_{jk}$, $K^i \equiv J^{0i} = -J^{i0}$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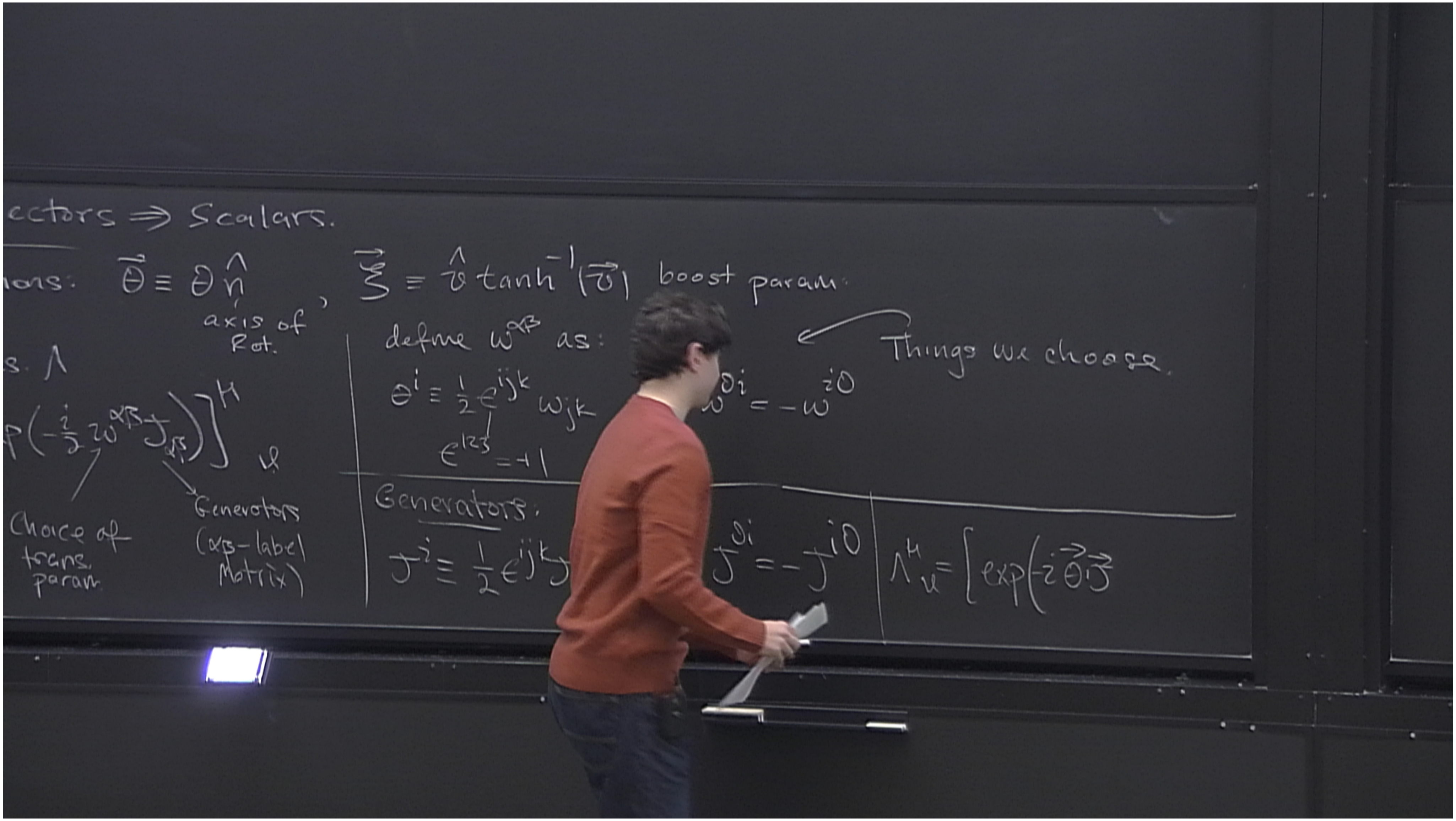
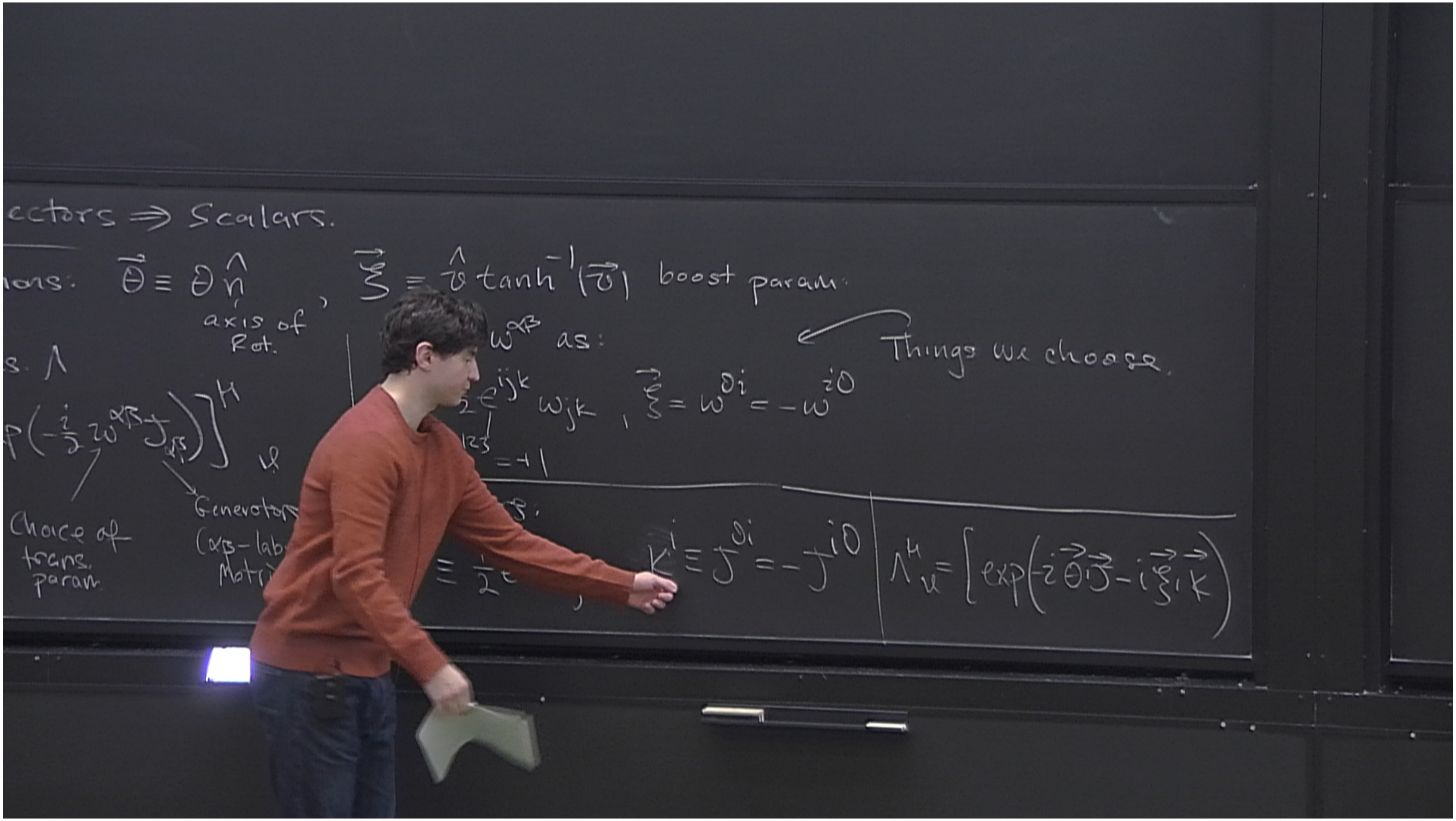
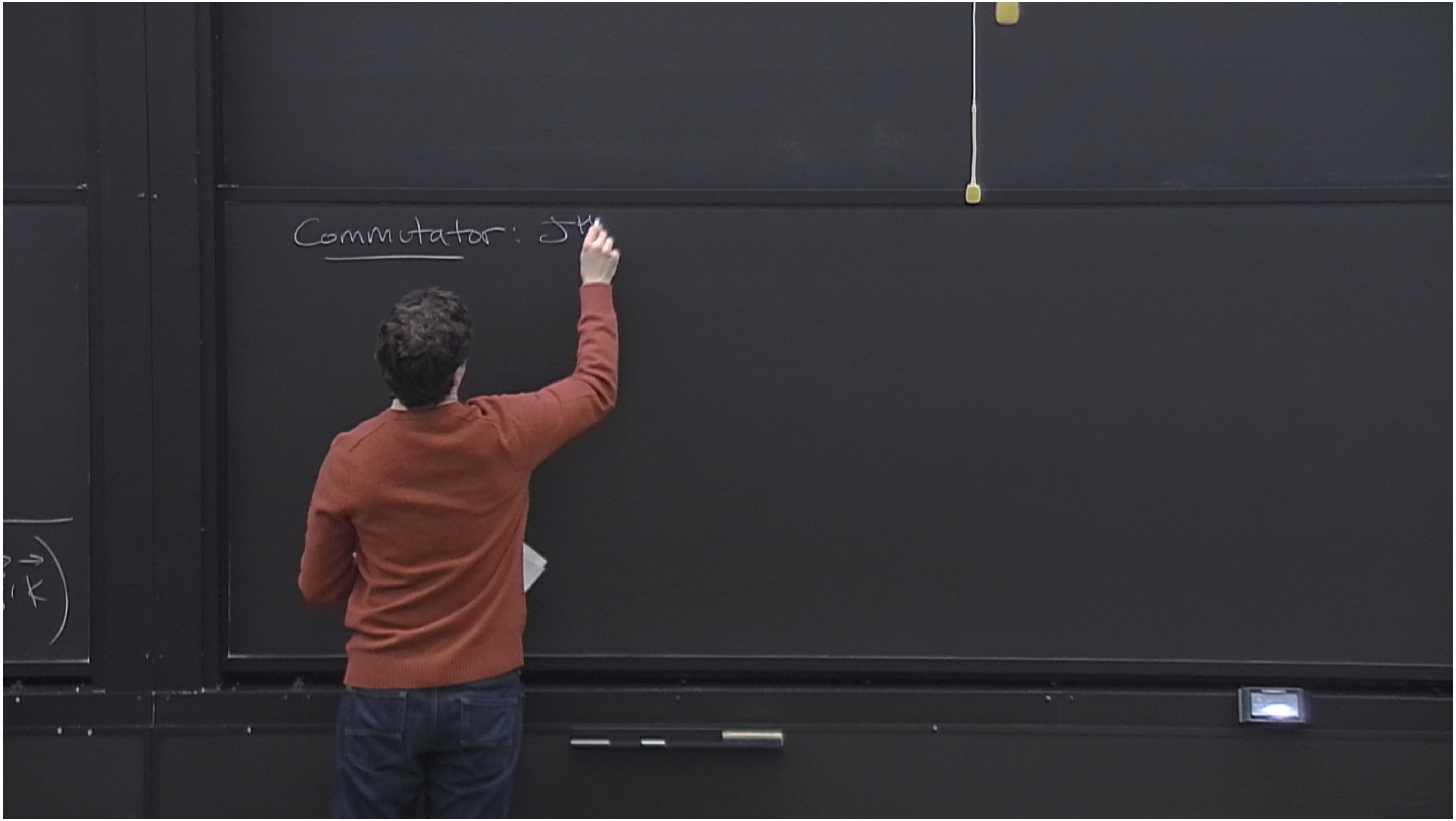
Things we choose.
$\Theta^i \equiv \frac{1}{2}\epsilon^{ijk}\omega_{jk}$, $\vec{\xi} = \omega^{0i}$
$\epsilon^{123} = +1$
Choice of trans. param.
Generators ($\alpha\beta$-label Matrix)
Generators:
$J^i \equiv \frac{1}{2}\epsilon^{ijk} J_{jk}$


ectors ⇒ Scalars.
boost param.
axis of Rot.
define ω^αβ as:
Things we choose.
Choice of trans. param
Generators (αβ-label Matrix)
Generators:

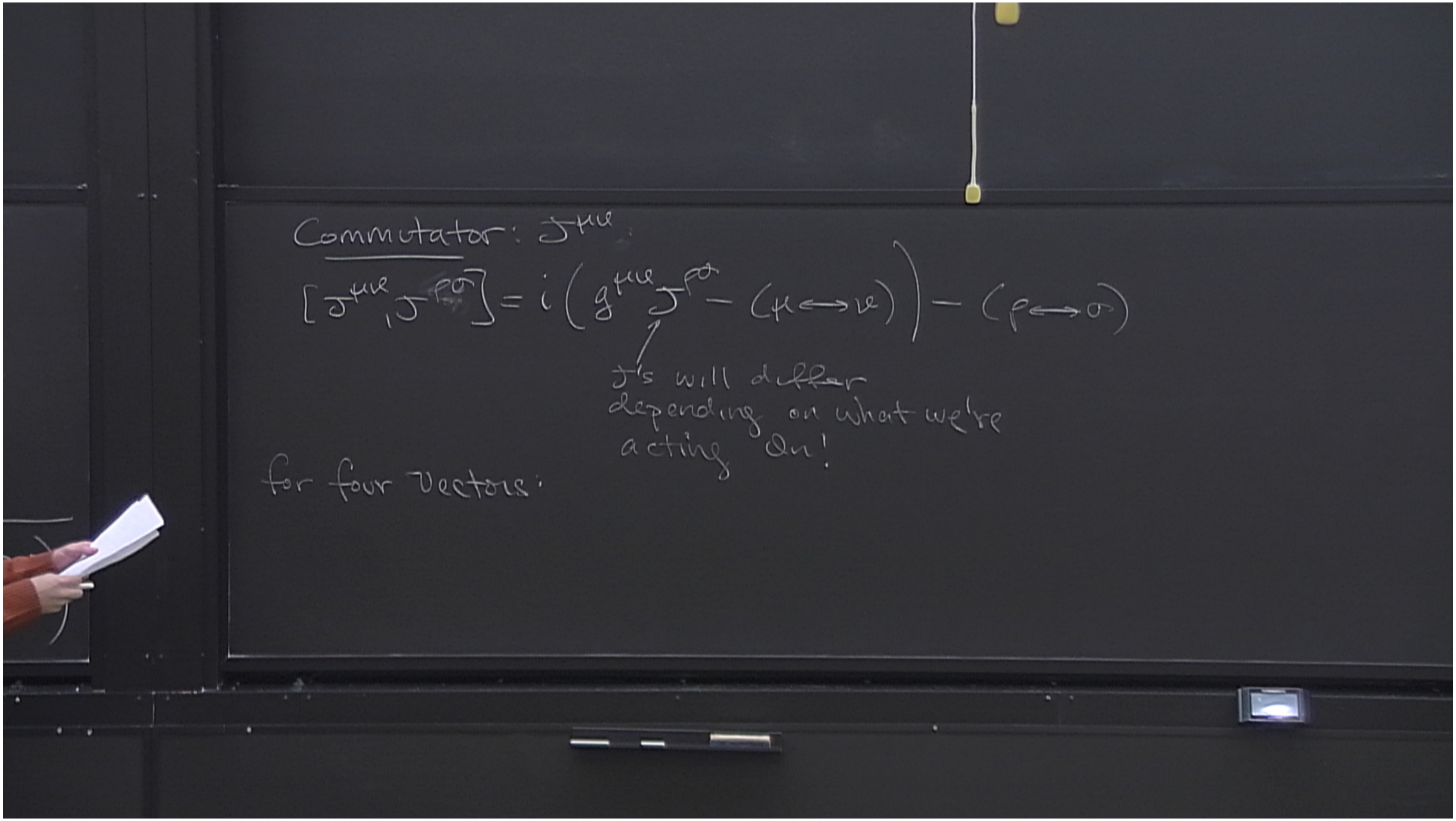

ectors ⇒ Scalars.
boost param.
axis of Rot.
Things we choose.
Choice of trans. param.


Commutator:

Commutator: $J^{\mu\nu}$

$$[J^{\mu\nu}, J^{\rho\sigma}] = i\left(g^{\mu\nu} J^{\rho\sigma} - (\mu \leftrightarrow \nu)\right) - (\rho \leftrightarrow \sigma)$$

J's will differ depending on what we're acting on!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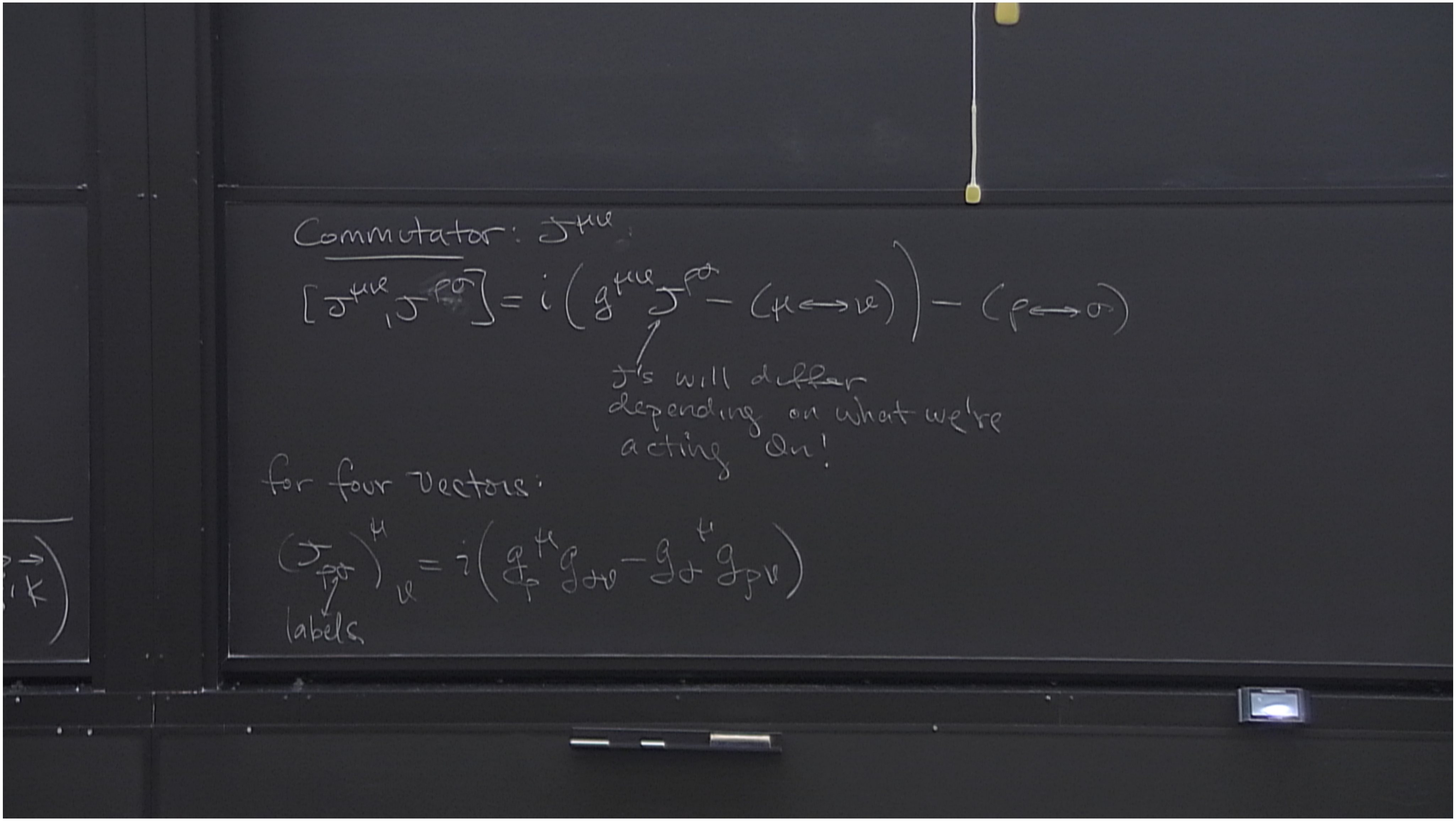
J's will differ depending on what we're acting on!
for four vectors: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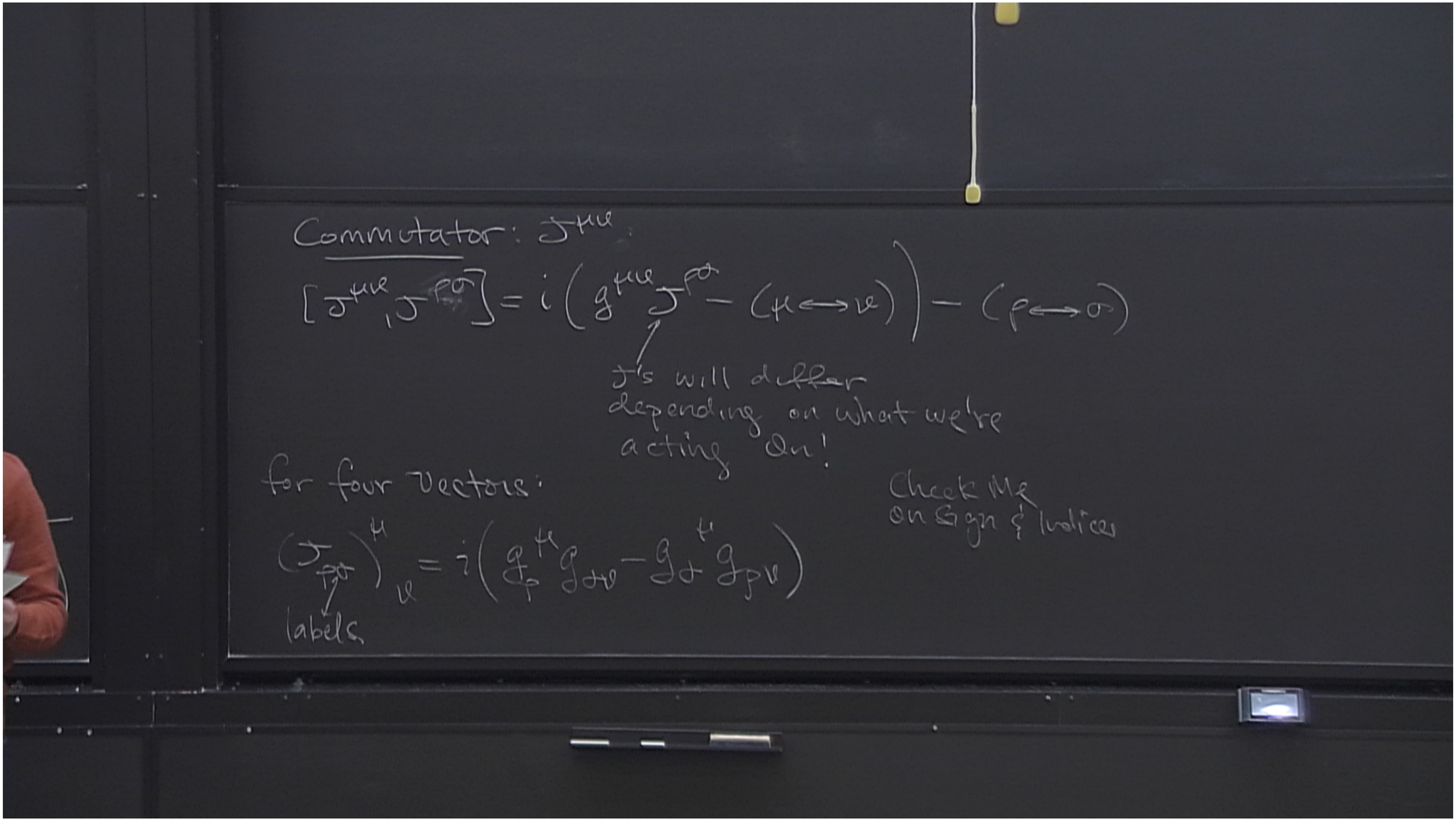
J's will differ depending on what we're acting on!
for four vectors:
$(J_{\rho\sigma})^{\mu}{}_{\nu} = i\left(g_{\rho}^{\mu} g_{\sigma\nu} - g_{\sigma}^{\mu} g_{\rho\nu}\right)$
labels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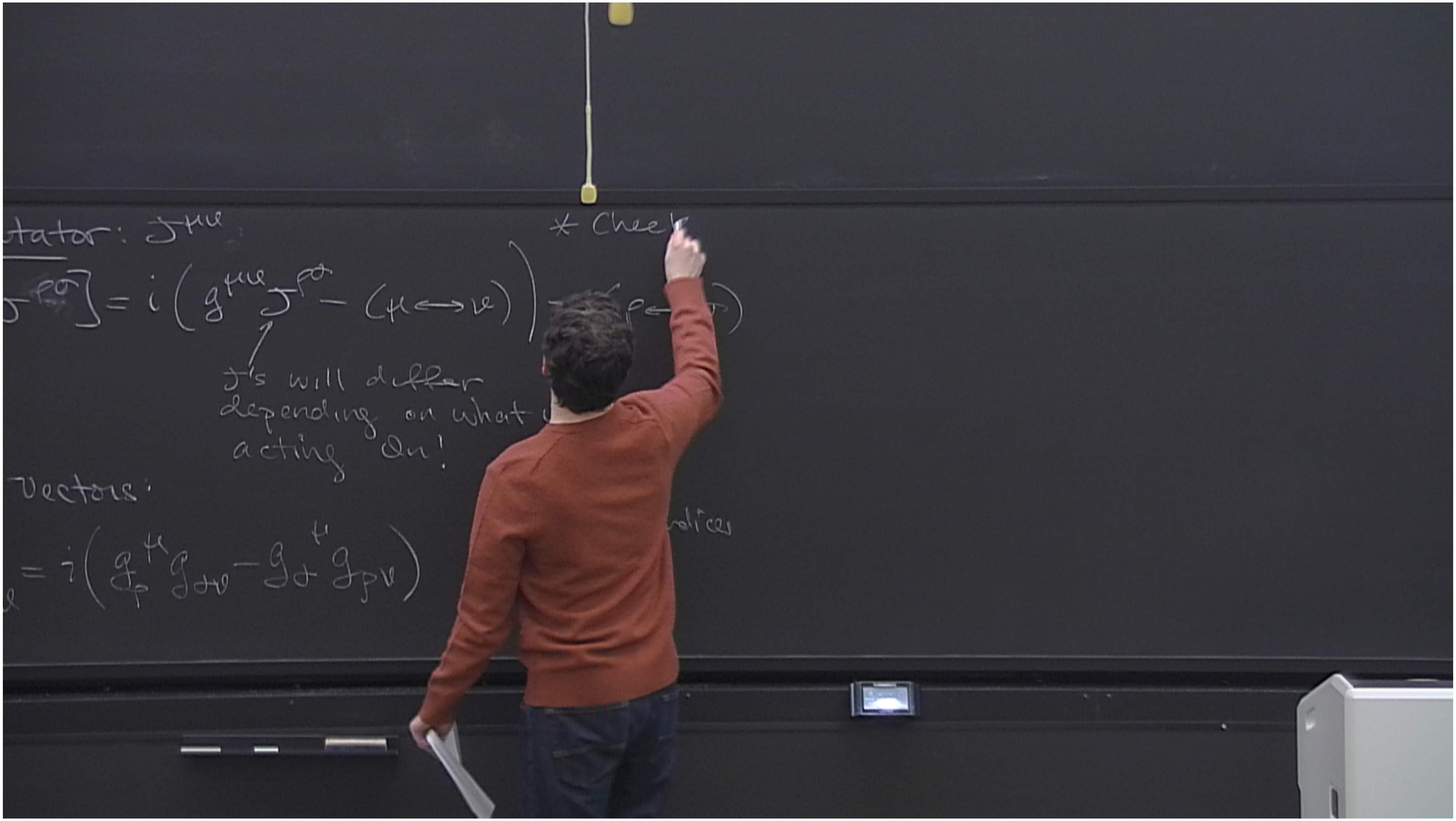
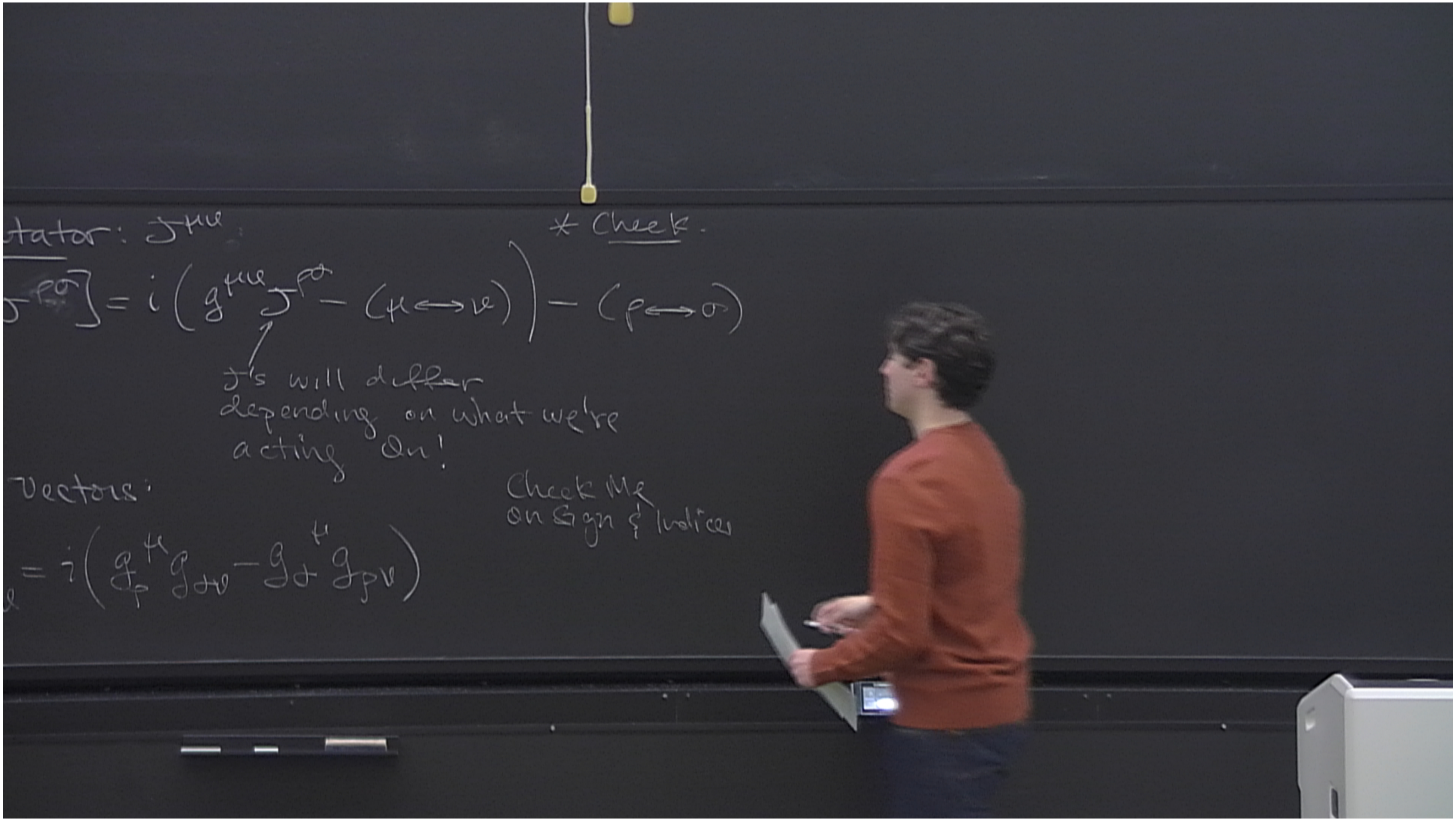
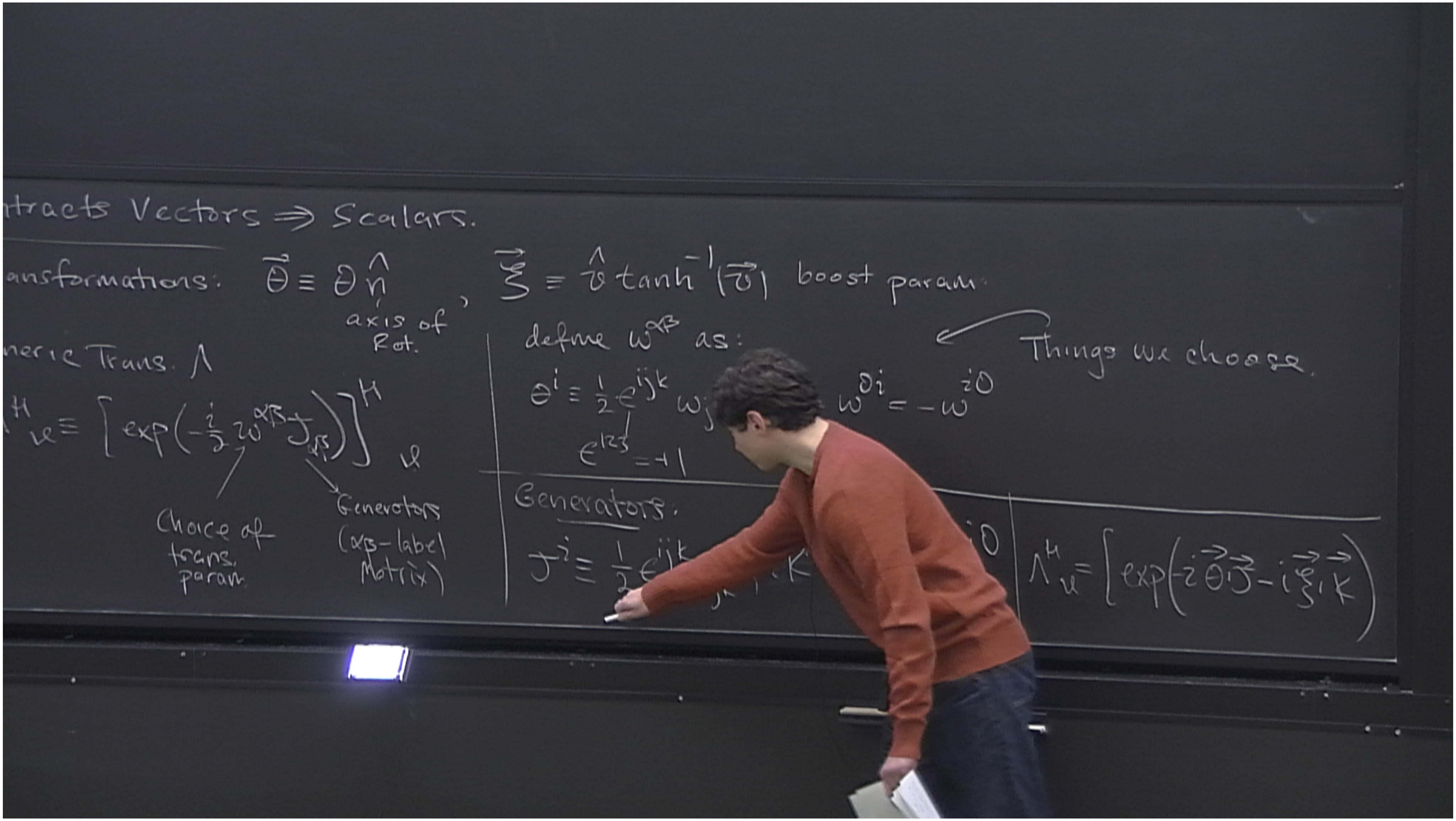
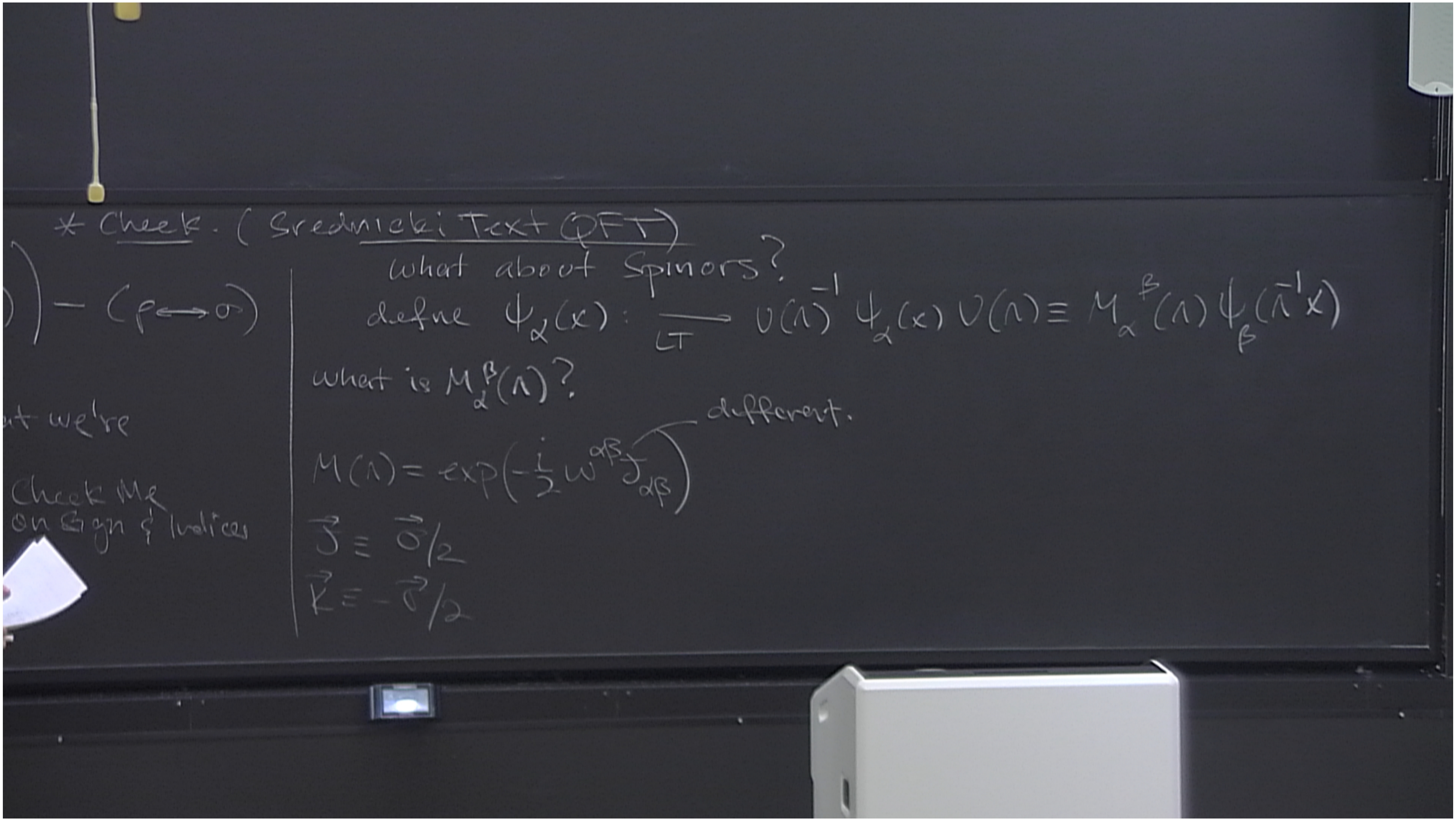
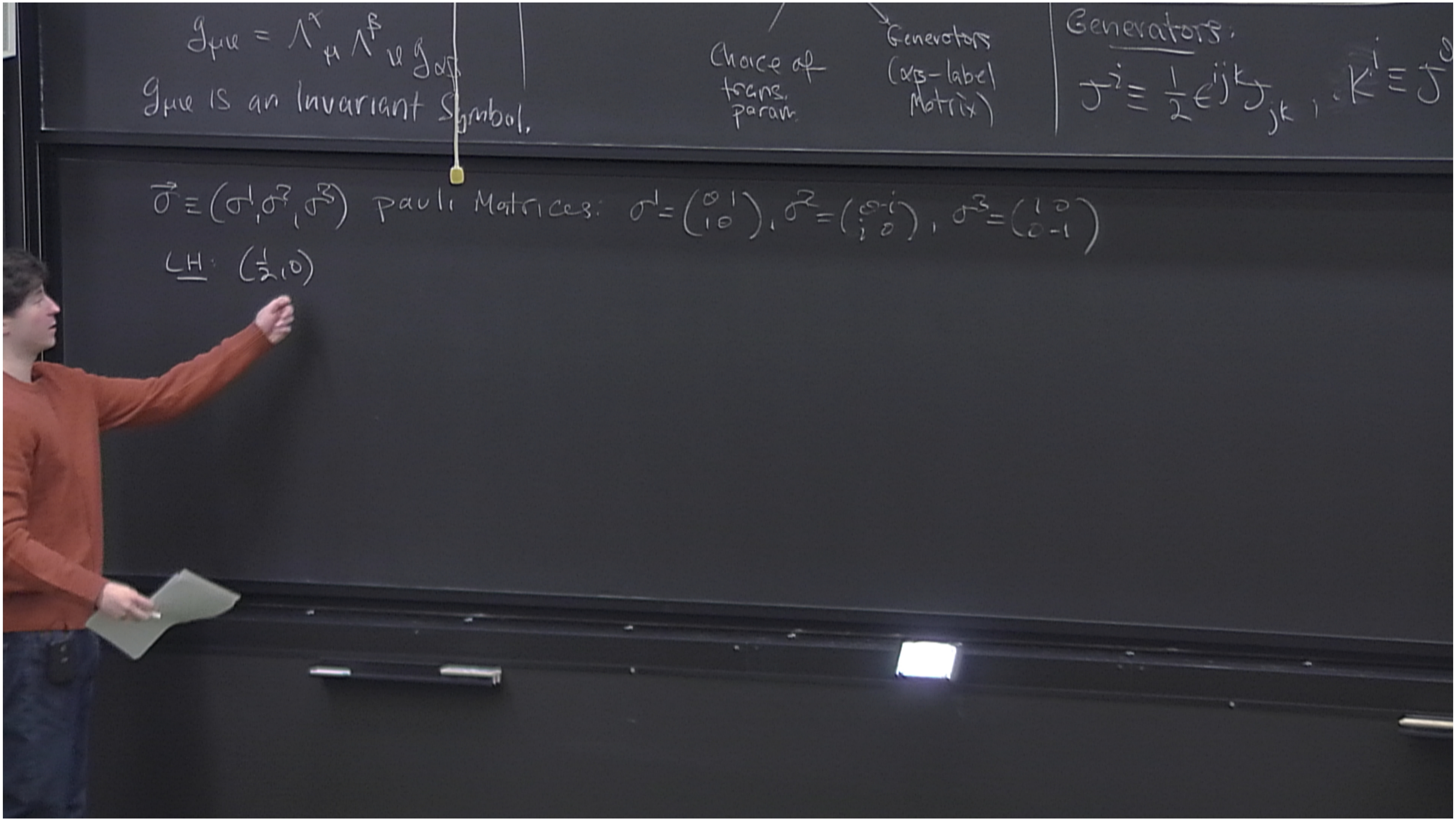
labels
Check Me on Sign & Indices

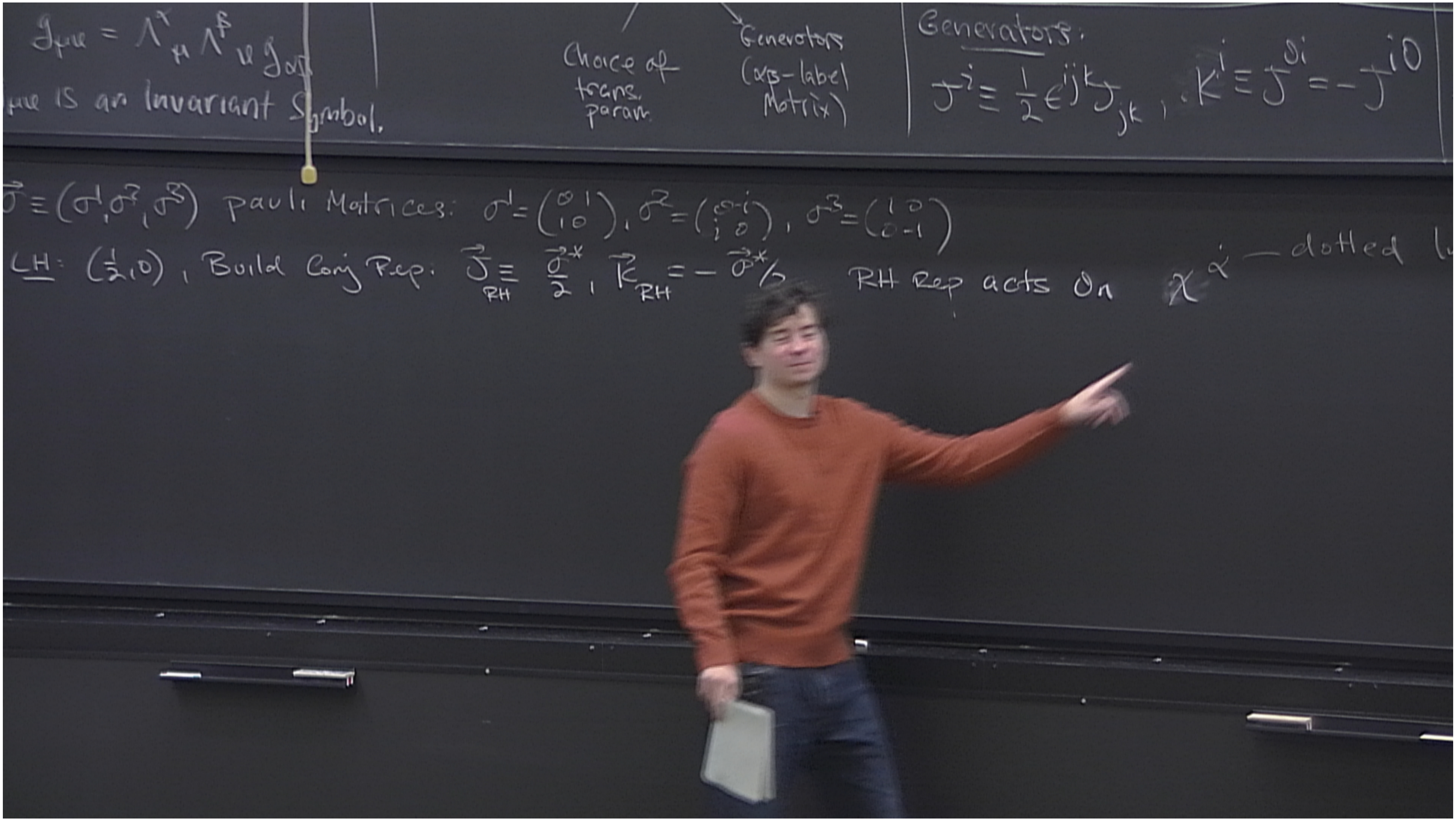

tator: J^μν
J's will differ
depending on what
acting On!
vectors:
* Check

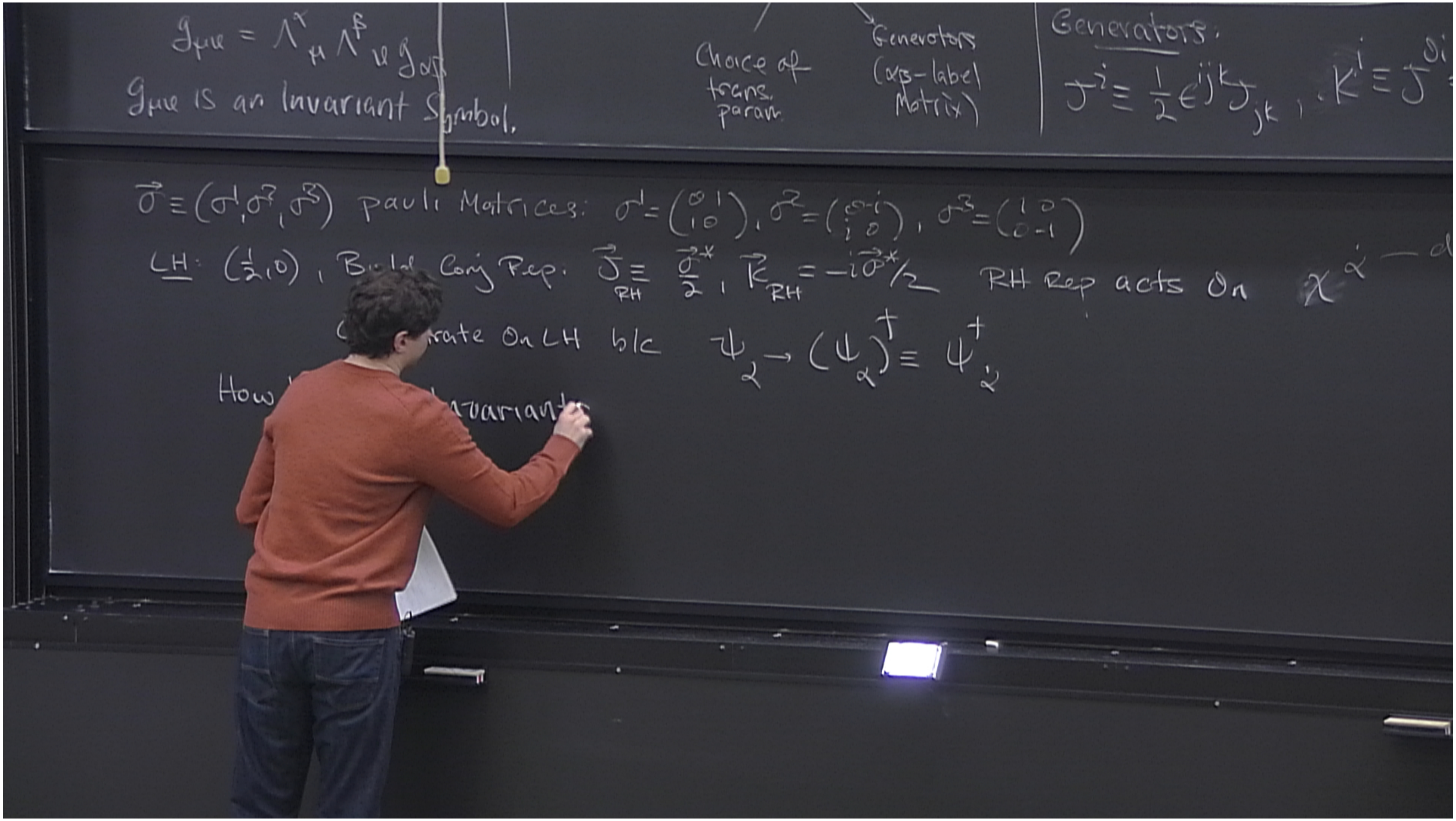

* Check.
J's will differ
depending on what we're
acting on!
Vectors:
Check Me
on Sign & Indices

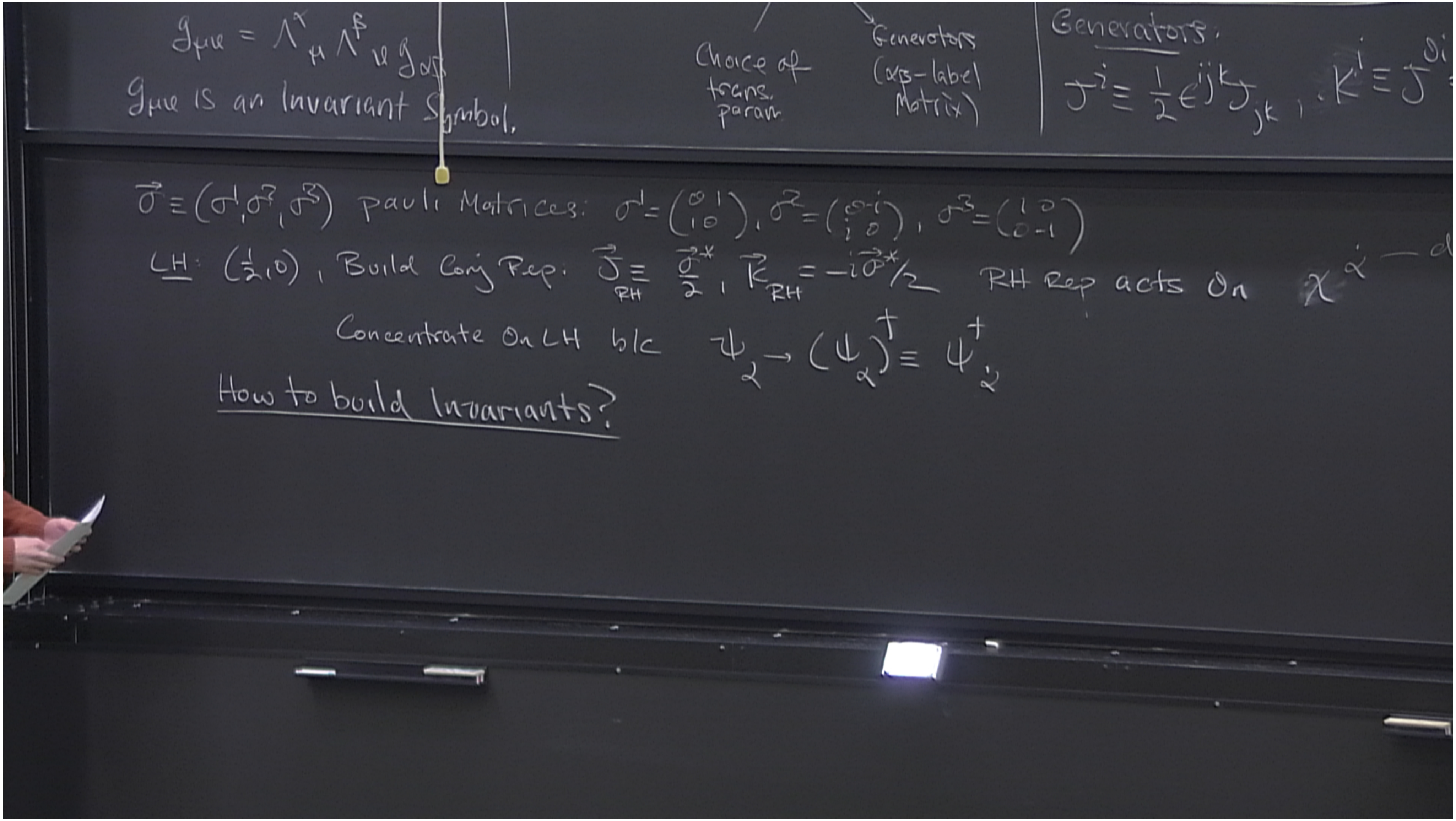

ntracts Vectors ⇒ Scalars.
ansformations: Θ ≡ θ n̂ , ξ = v̂ tanh⁻¹|v| boost param.
axis of Rot.
neric Trans. Λ
Λ^μ_ν ≡ [exp(−i/2 ω^{αβ} J_{αβ})]^μ_ν
Choice of trans. param
Generators (αβ-label Matrix)
define ω^{αβ} as:
Things we choose.
θ^i ≡ ½ ε^{ijk} ω_{jk}
ε^{123} = +1
ω^{0i} = −ω^{i0}
Generators:
Λ^μ_ν = [exp(−i θ·J − i ξ·K)


* Check. (Srednicki Text (QFT))
what about Spinors?
define $\psi_\alpha(x) \xrightarrow[LT]{} U(\Lambda)^{-1}\psi_\alpha(x)U(\Lambda) \equiv M_\alpha{}^\beta(\Lambda)\psi_\beta(\Lambda^{-1}x)$
what is $M_\alpha{}^\beta(\Lambda)$?
different.
$M(\Lambda) = \exp\left(-\frac{i}{2}\omega^{\alpha\beta}J_{\alpha\beta}\right)$
$\vec{J} \equiv \vec{\sigma}/2$
$\vec{K} \equiv -\vec{\sigma}/2$
$) - (\rho \leftrightarrow \sigma)$
at we're
Check Me
on Sign & Indices


gμν = Λ^α_μ Λ^β_ν g_αβ
gμν is an Invariant Symbol.
Choice of trans. param.
Generators (αβ-label Matrix)
Generators:
pauli Matrices:
LH: (1/2, 0)


Choice of trans. param.
Generators (αβ-label Matrix)
Generators:
is an invariant Symbol.
pauli Matrices:
LH: (½,0), Build Conj Rep:
RH Rep acts On
— dotted


gμν = Λ^α_μ Λ^β_ν g_αβ
gμν is an Invariant Symbol.
Choice of trans. param
Generators (αβ-label Matrix)
Generators:
J^i ≡ 1/2 ε^ijk J_jk , K^i ≡ J^0i
σ ≡ (σ¹, σ², σ³) pauli Matrices: σ¹ = (0 1 / 1 0), σ² = (0 -i / i 0), σ³ = (1 0 / 0 -1)
LH: (1/2, 0), Build Conj Rep. J_RH ≡ σ*/2, K_RH = -iσ*/2 RH Rep acts on χ^α̇
rate On LH b/c ψ_α → (ψ_α)† ≡ ψ†_α̇
How
nvariant


$g_{\mu\nu} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta}$
$g_{\mu\nu}$ is an Invariant Symbol.
Choice of trans. param
Generators ($\alpha\beta$-label Matrix)
Generators:
$J^i \equiv \frac{1}{2}\epsilon^{ijk} J_{jk}$, $K^i \equiv J^{0i}$
$\vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$ pauli Matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
LH: $(\frac{1}{2}, 0)$, Build Conj Rep: $\vec{J}_{RH} \equiv \frac{\vec{\sigma}^*}{2}$, $\vec{K}_{RH} = -i\vec{\sigma}^*/2$ RH Rep acts on $\chi^{\dot{\alpha}}$
Concentrate On LH b/c $\psi_\alpha \rightarrow (\psi_\alpha)^\dagger \equiv \psi^\dagger_{\dot{\alpha}}$
How to build Invariants?

$g_{\mu\nu} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta}$

$g_{\mu\nu}$ is an Invariant Symbol.

Choice of trans. param

Generators (αβ-label Matrix)

Generators: $J^{i} \equiv \frac{1}{2}\epsilon^{ijk}J_{jk}$, $K^{i} \equiv J^{0i}$

$\vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$ pauli Matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

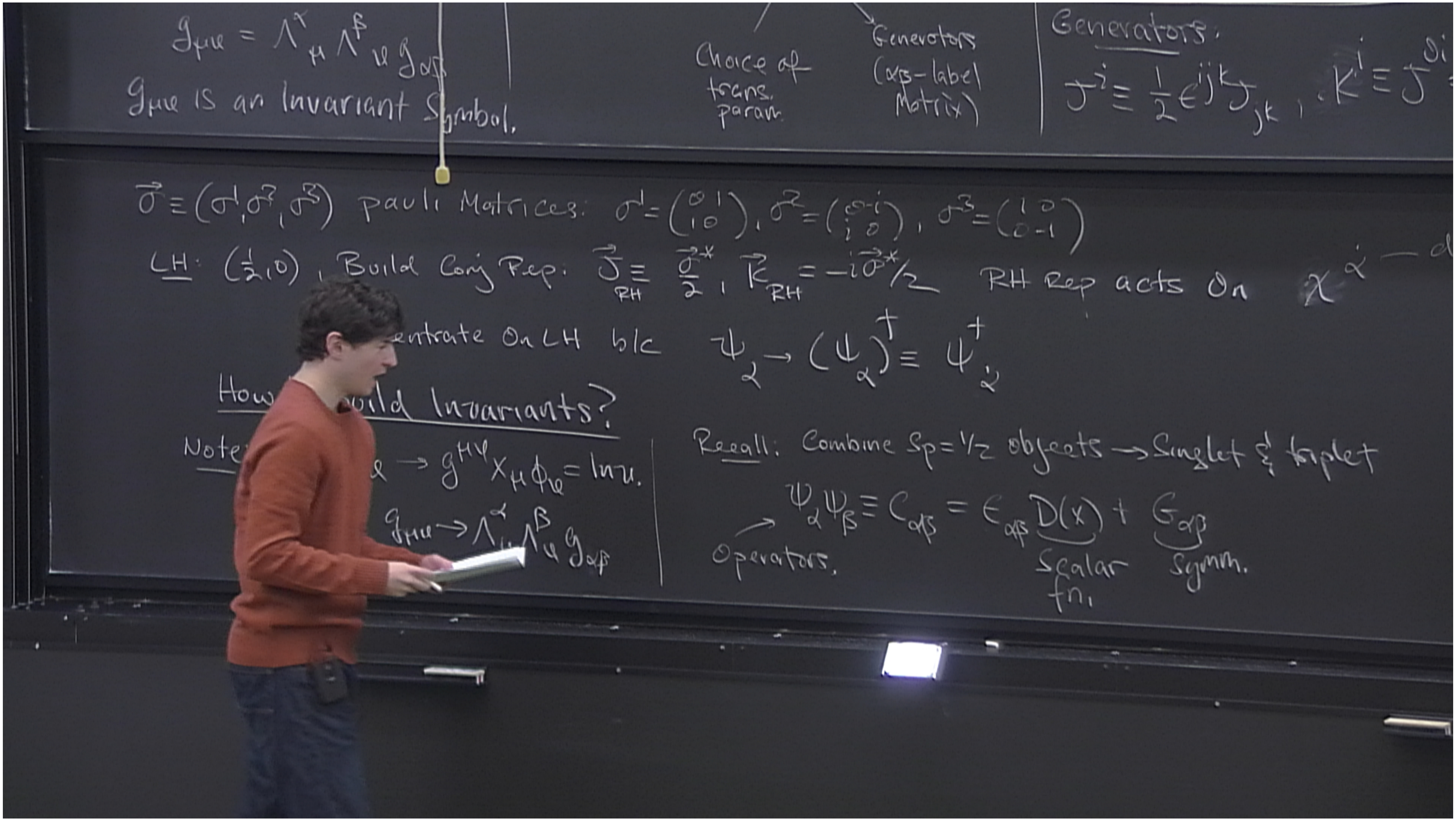
LH: $(\frac{1}{2}, 0)$, Build Conj Rep: $\vec{J}_{RH} \equiv \frac{\vec{\sigma}^*}{2}$, $\vec{K}_{RH} = -i\vec{\sigma}^*/2$ RH Rep acts on $\chi^{\dot{\alpha}}$ —

Concentrate on LH b/c $\psi_{\alpha} \to (\psi_{\alpha})^{\dagger} \equiv \psi^{\dagger}_{\dot{\alpha}}$

How to build Invariants?

Note: $X_{\mu} \phi_{\nu} \to g^{\mu\nu} X_{\mu} \phi_{\nu} =$ Inv.

$g_{\mu\nu} \to \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta}$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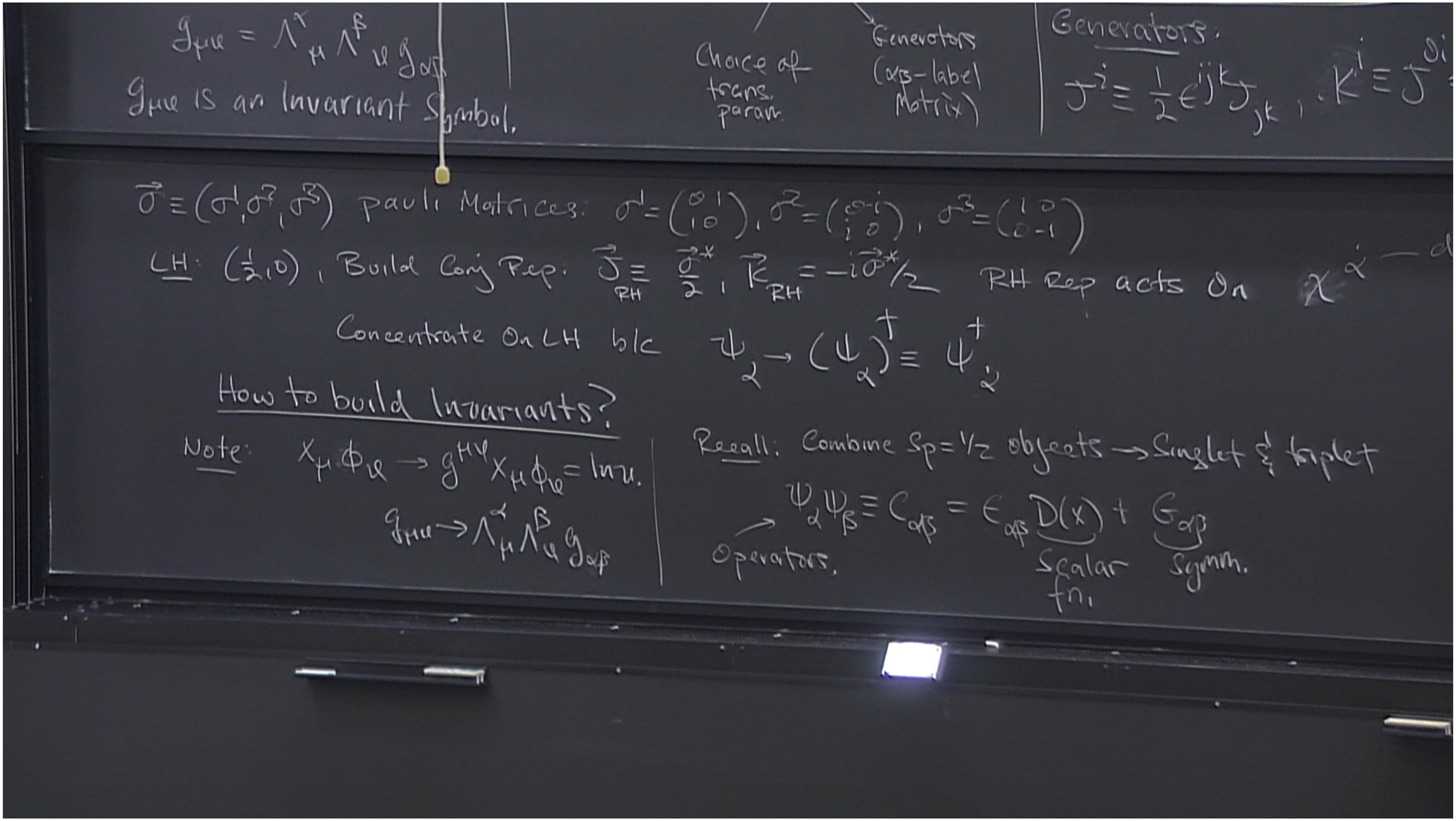
RH Rep acts On $\chi^{\dot{\alpha}}$
entrate On LH b/c $\psi_\alpha \to (\psi_\alpha)^\dagger \equiv \psi^\dagger_{\dot{\alpha}}$
build Invariants?
Recall: Combine Sp = ½ objects → Singlet & triplet
$\psi_\alpha \psi_\beta \equiv C_{\alpha\beta} = \epsilon_{\alpha\beta} D(x) + G_{\alpha\beta}$
Operators.
Scalar fn.
Symm.
$g^{\mu\nu} x_\mu \phi_\nu$ = Inv.
$g_{\mu\nu} \to \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu g_{\alpha\beta}$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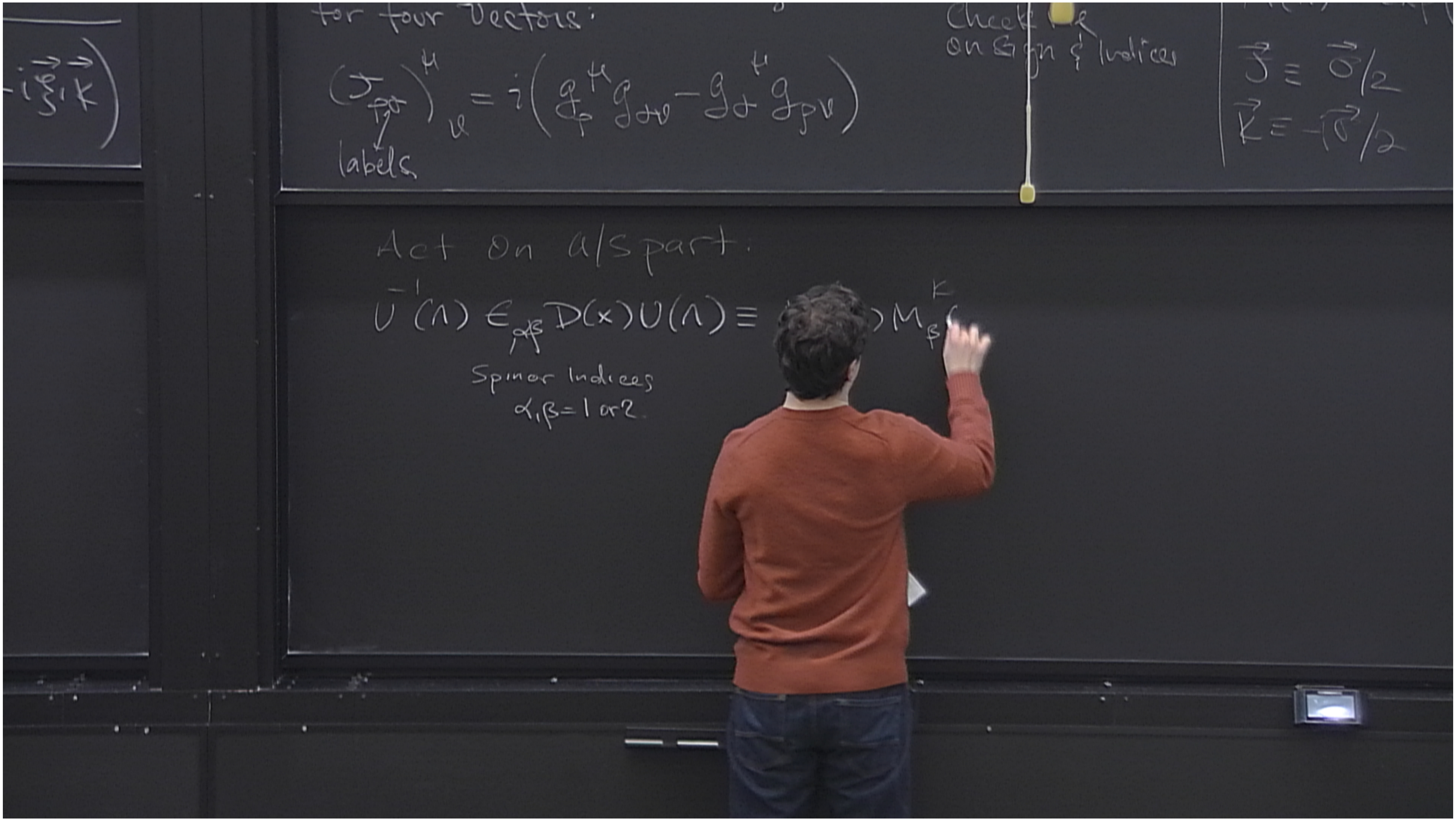
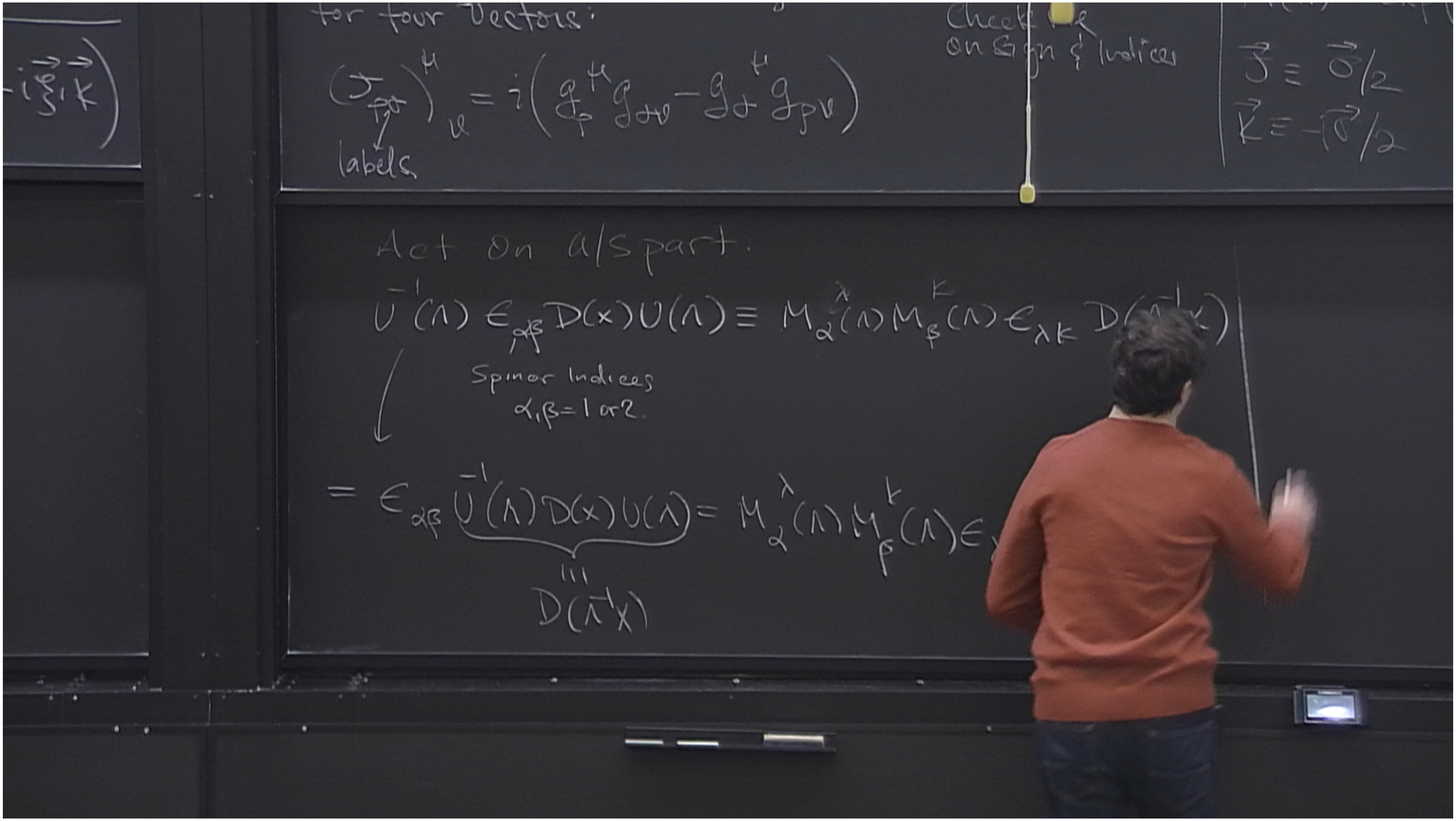
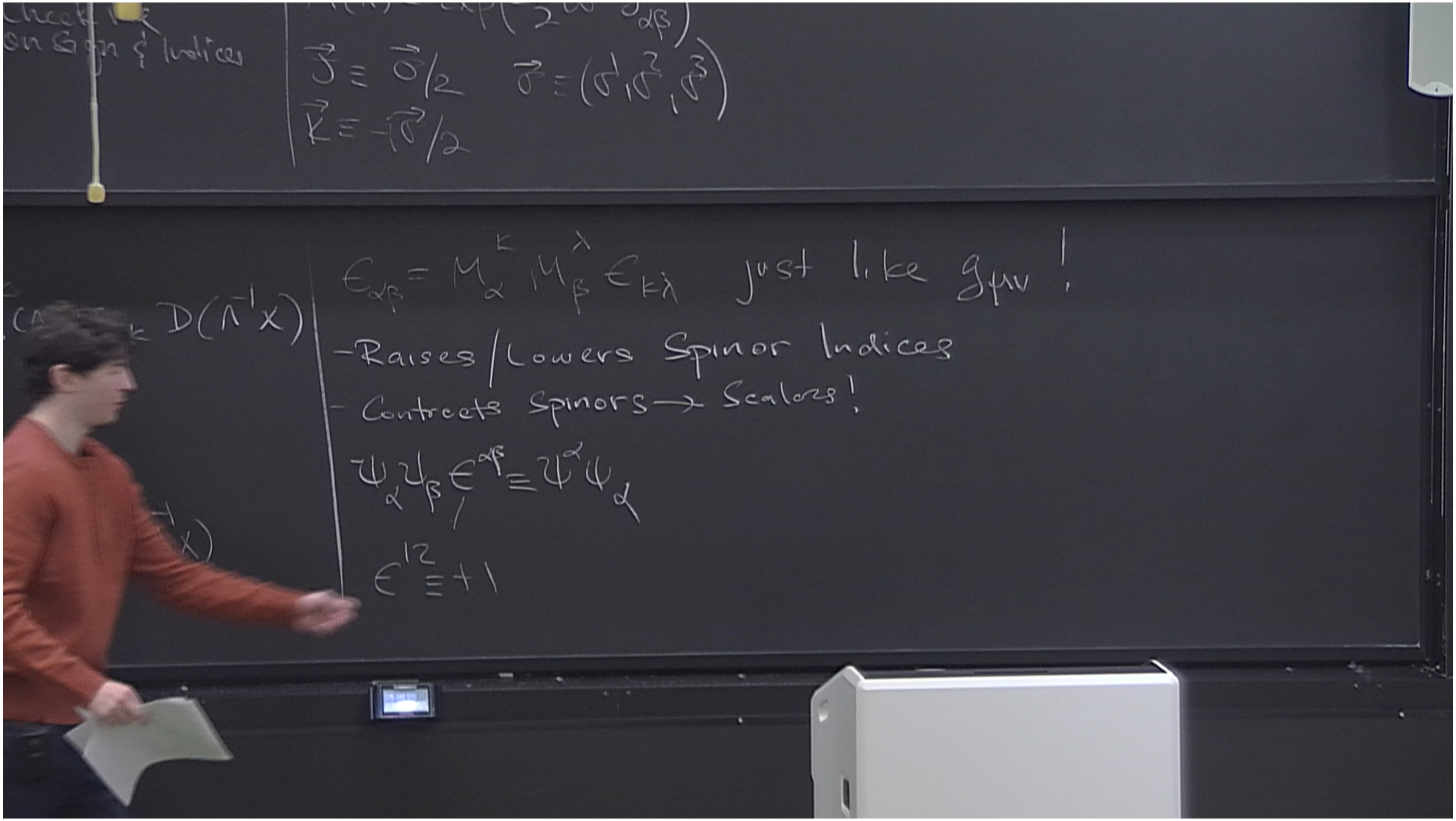
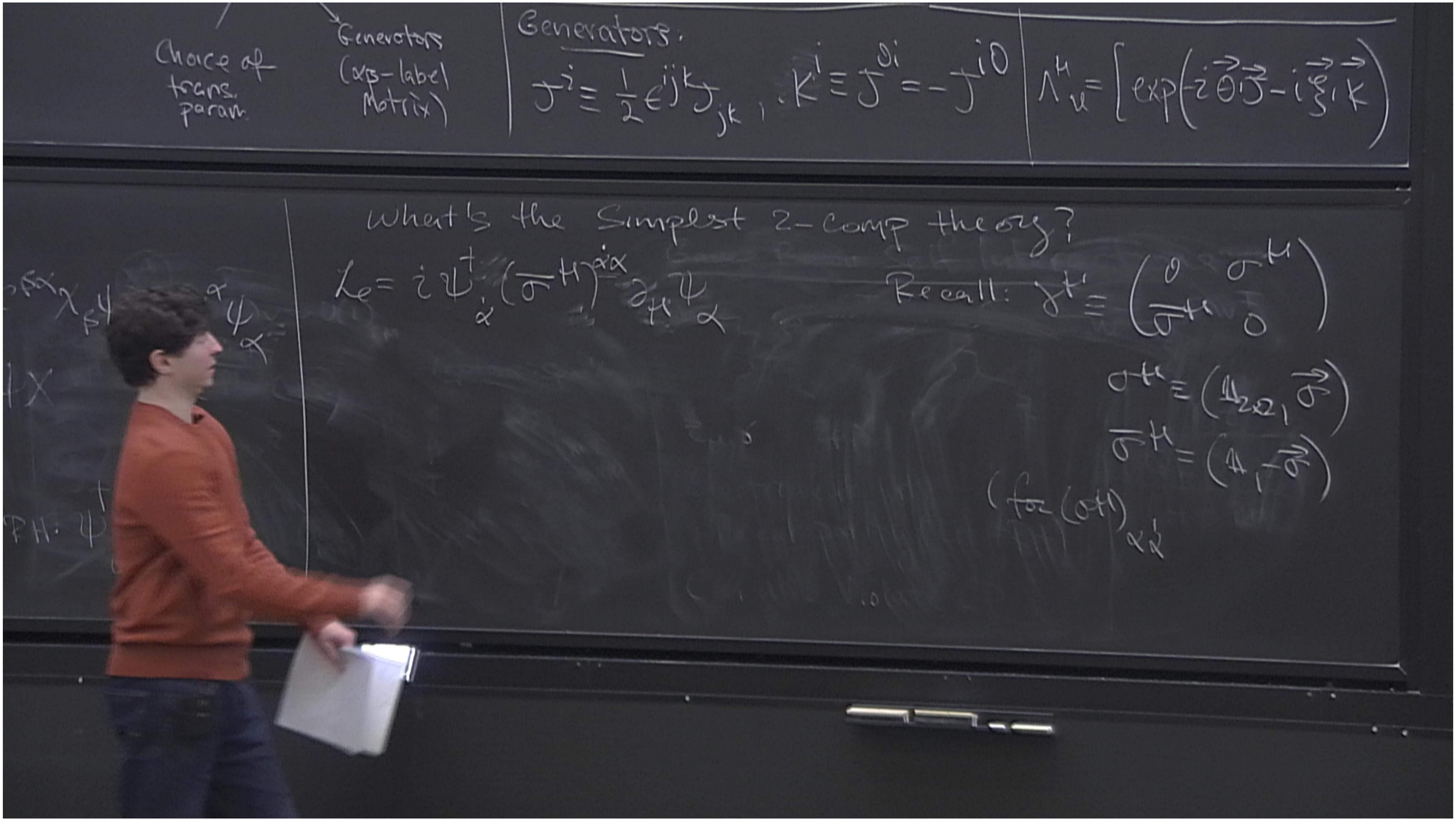
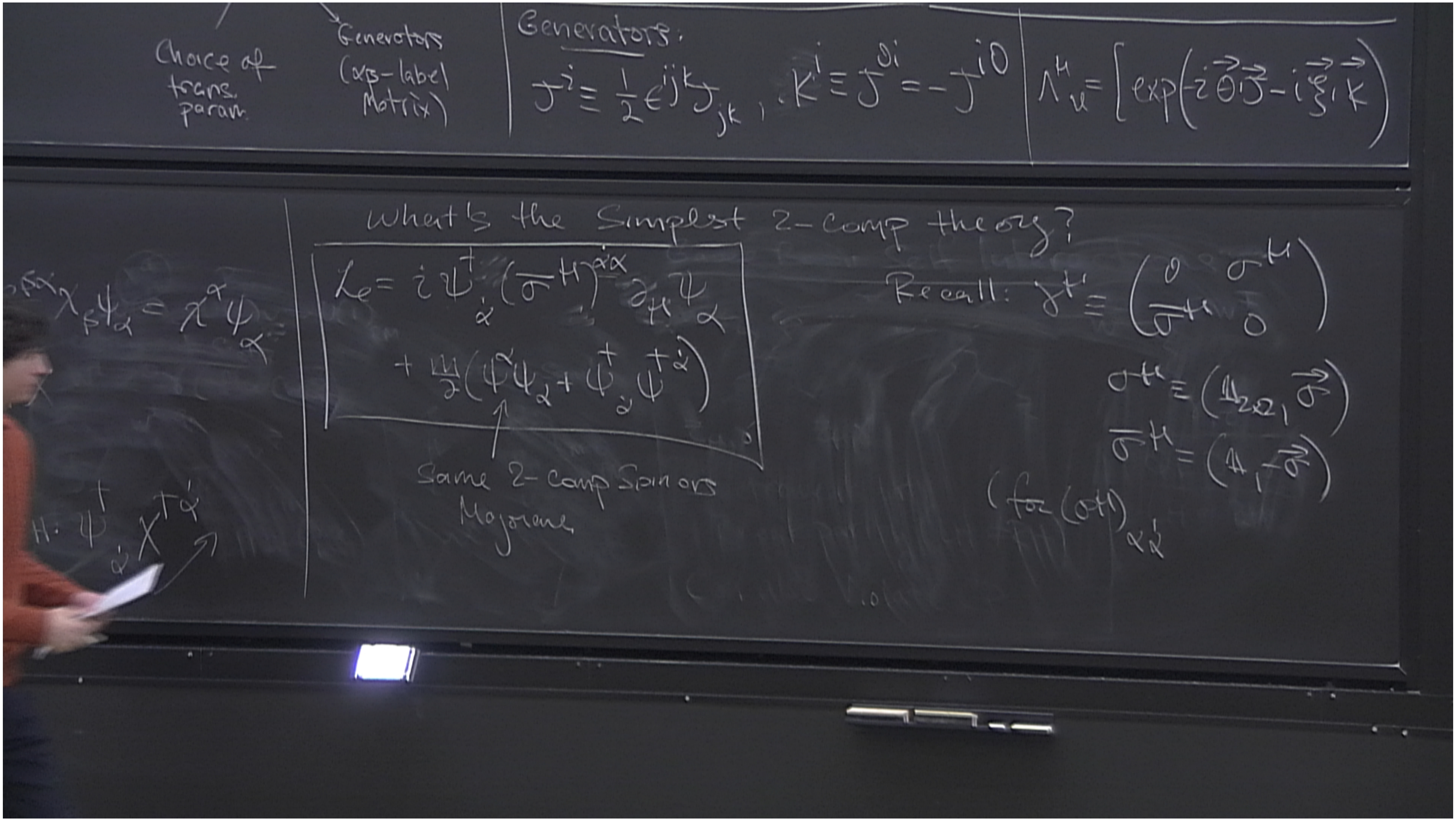
Generators:
$J^i \equiv \frac{1}{2}\epsilon^{ijk} J_{jk}$, $K^i \equiv J^{0i}$
$\vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$ pauli Matrices: $\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
LH: $(\frac{1}{2}, 0)$, Build Conj Rep: $\vec{J}_{RH} \equiv \frac{\vec{\sigma}^*}{2}$, $\vec{K}_{RH} = -i\vec{\sigma}^*/2$ RH Rep acts on $\chi^{\dot{\alpha}}$
Concentrate on LH b/c $\psi_\alpha \to (\psi_\alpha)^\dagger \equiv \psi^\dagger_{\dot{\alpha}}$
How to build Invariants?
Note: $X_\mu \Phi_\nu \to g^{\mu\nu} X_\mu \phi_\nu$ = Inv.
$g_{\mu\nu} \to \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta}$
Recall: Combine Sp = 1/2 objects → Singlet & triplet
$\psi_\alpha \psi_\beta \equiv C_{\alpha\beta} = \epsilon_{\alpha\beta} D(x) + G_{\alpha\beta}$
Operators.
Scalar fn.
Symm.

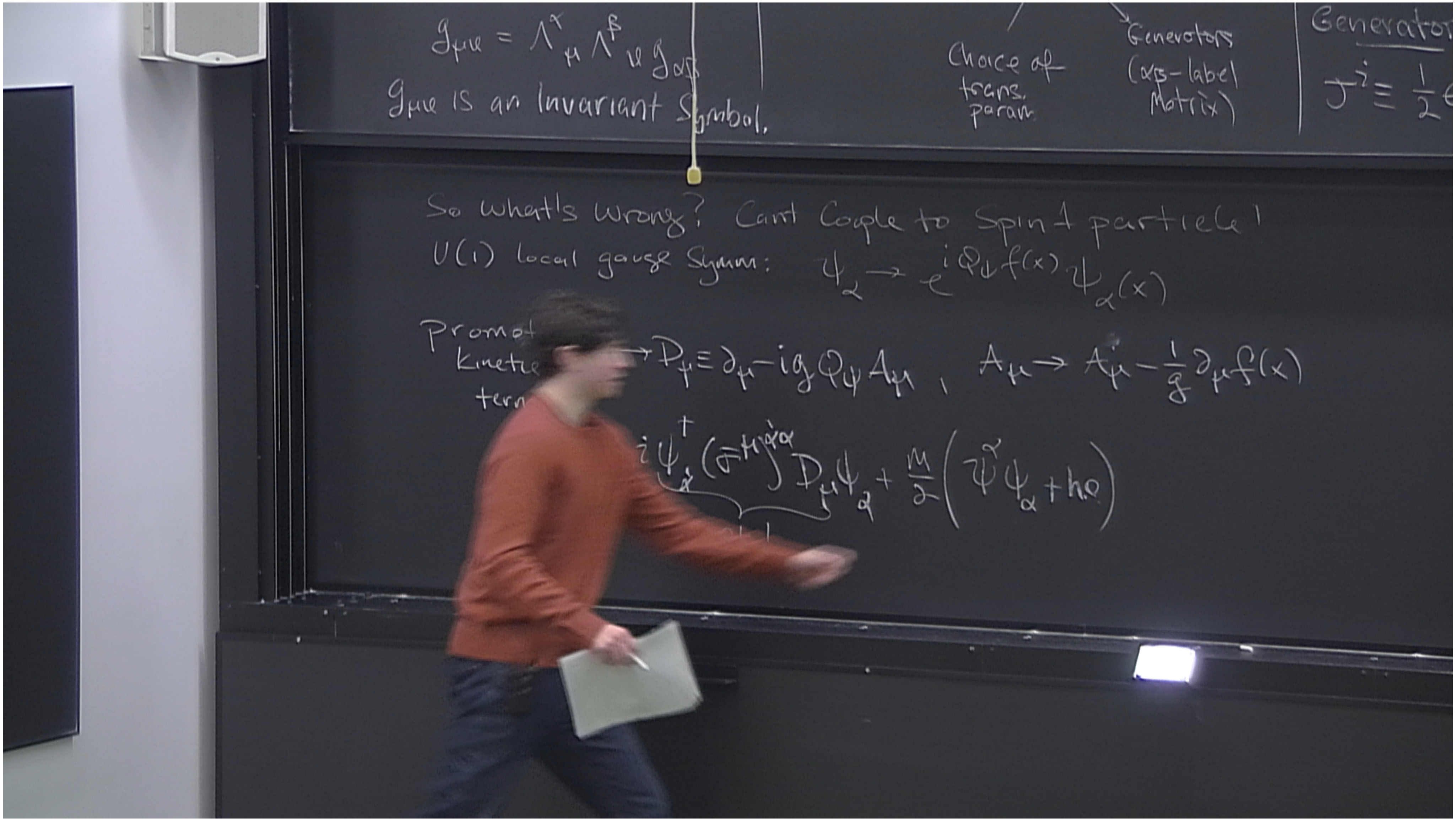

for four Vectors:
(J_ρσ)^μ_ν = i(g_ρ^μ g_σν − g_σ^μ g_ρν)
labels
Check me on Sign & Indices
J ≡ σ/2
K ≡ −iσ/2
Act on a/s part:
U^-1(Λ) ε_αβ D(x) U(Λ) ≡
M_β^κ
Spinor Indices
α,β = 1 or 2

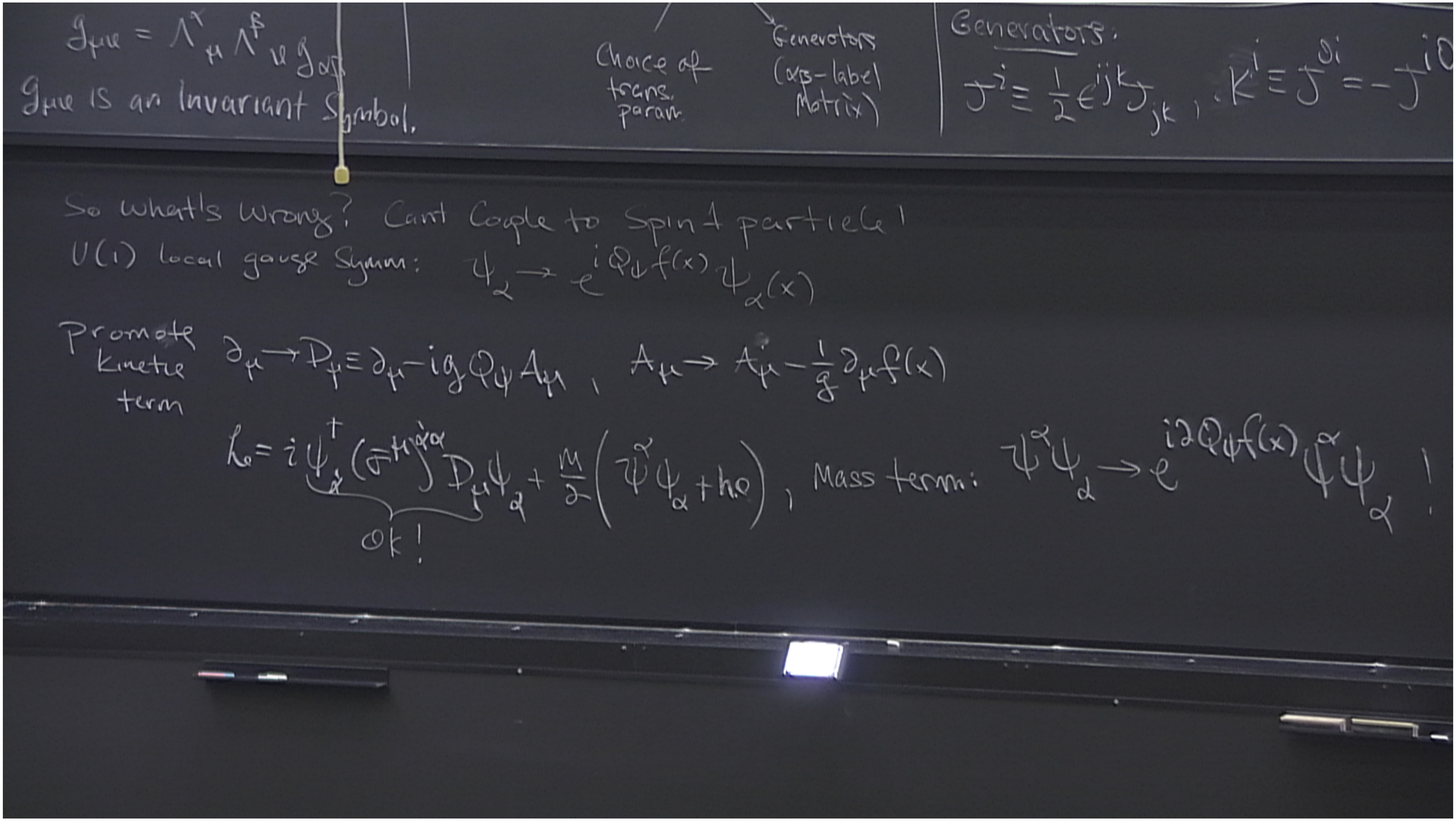

for four Vectors:
(J_ρσ)^μ_ν = i(g_ρ^μ g_σν − g_σ^μ g_ρν)
labels
Check me on Sign & Indices
J ≡ σ/2
K ≡ −iσ/2
Act on a/s part:
U⁻¹(Λ) ε_αβ D(x) U(Λ) ≡ M_α^λ(Λ) M_β^κ(Λ) ε_λκ D(Λ⁻¹x)
Spinor Indices
α, β = 1 or 2
= ε_αβ U⁻¹(Λ) D(x) U(Λ) = M_α^λ(Λ) M_β^κ(Λ) ε
D(Λ⁻¹x)


on Sign & Indices
just like
-Raises/Lowers Spinor Indices
- Contracts Spinors → Scalars!


Choice of trans. param
Generators (αβ-label Matrix)
Generators:
What's the Simplest 2-comp theory?
Recall:


Choice of trans. param
Generators (αβ-label Matrix)
Generators:
What's the Simplest 2-Comp theory?
Recall:
Same 2-Comp Spinors
Majorana


gμν IS an Invariant Symbol.
Choice of trans. param.
Generators (αβ-label Matrix)
Generator
So what's wrong? Can't Couple to Spin ½ particle!
U(1) local gauge Symm:
Promot
Kinetic
term



$g_{\mu\nu} = \Lambda^{\alpha}{}_{\mu} \Lambda^{\beta}{}_{\nu} g_{\alpha\beta}$
$g_{\mu\nu}$ is an invariant symbol.
Choice of trans. param.
Generators ($\alpha\beta$-label matrix)
Generators:
$J^i \equiv \frac{1}{2}\epsilon^{ijk} J_{jk}$, $K^i \equiv J^{0i} = -J^{i0}$
So what's wrong? Can't couple to spin ½ particle!
U(1) local gauge symm: $\psi_\alpha \to e^{iQ_\psi f(x)} \psi_\alpha(x)$
Promote kinetic term
$\partial_\mu \to D_\mu \equiv \partial_\mu - igQ_\psi A_\mu$, $A_\mu \to A_\mu - \frac{1}{g}\partial_\mu f(x)$
$\mathcal{L} = i\psi^\dagger_{\dot\alpha} (\bar\sigma^\mu)^{\dot\alpha\alpha} D_\mu \psi_\alpha + \frac{m}{2}(\psi^\alpha \psi_\alpha + h.c.)$, Mass term: $\psi^\alpha\psi_\alpha \to e^{i2Q_\psi f(x)} \psi^\alpha\psi_\alpha$!
ok!

for four vectors:

$$(\mathcal{J}_{\rho\sigma})^{\mu}{}_{\nu} = i\left(g_{\rho}^{\mu} g_{\sigma\nu} - g_{\sigma}^{\mu} g_{\rho\nu}\right)$$

labels

Check me on sign & indices

$\vec{J} \equiv \vec{\sigma}/2 \quad \vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$

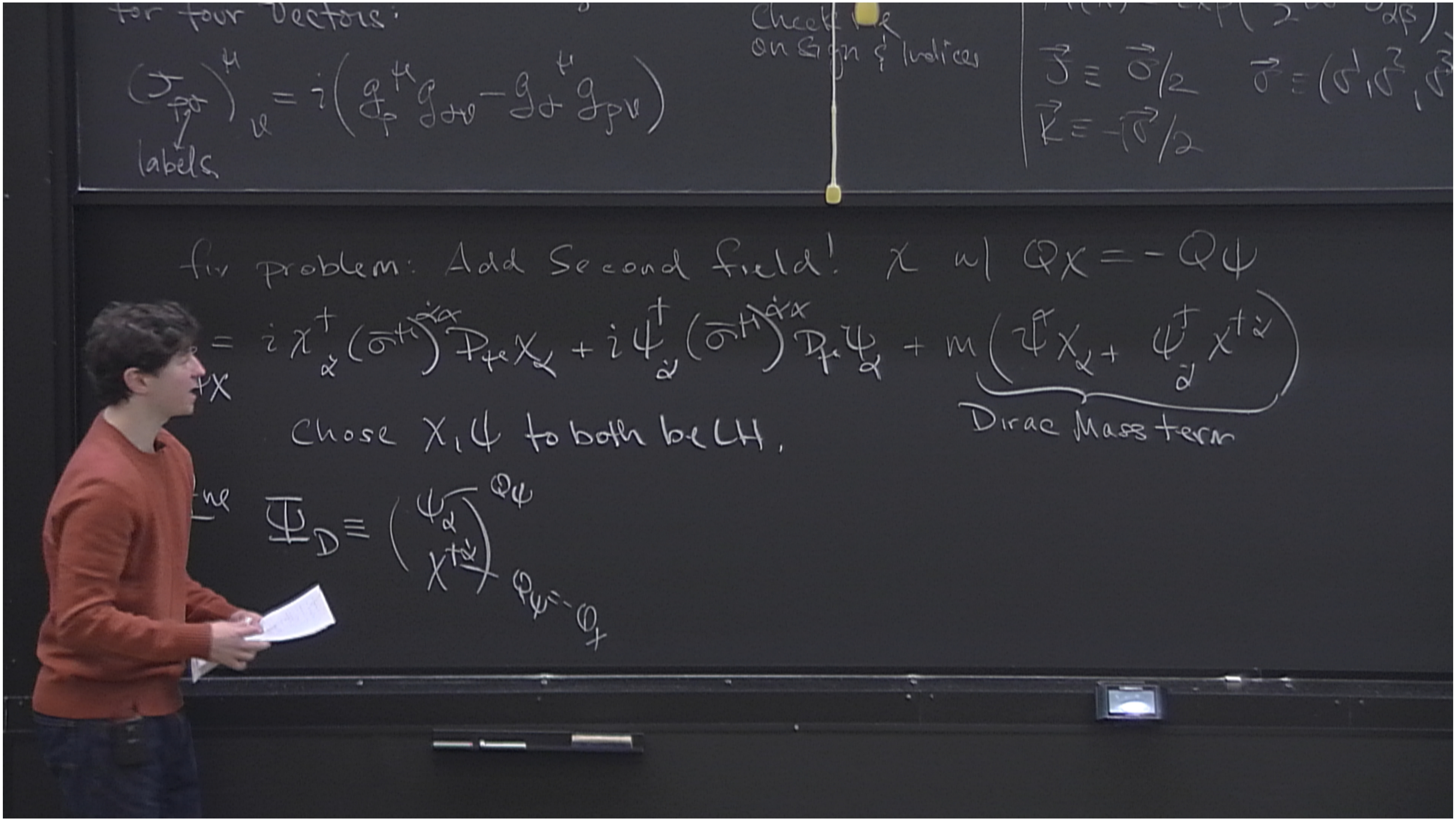
$\vec{K} \equiv -i\vec{\sigma}/2$

fix problem: Add Second field! χ w/ $Q_\chi = -Q_\psi$

$$\mathcal{L}_{\psi\chi} = i\chi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \chi_\alpha + i\psi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \psi_\alpha + m\left(\psi^{\alpha}\chi_{\alpha} + \psi^{\dagger}_{\dot\alpha}\chi^{\dagger\dot\alpha}\right)$$

Dirac Mass term

Chose χ, ψ to both be LH.


for four vectors:
labels
Check me on sign & Indices
fix problem: Add second field! χ w/ Qχ = -Qψ
Dirac Mass term
Chose χ, ψ to both be LH.
$\Psi_D \equiv \begin{pmatrix} \psi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}$

for four vectors:

$$(\mathcal{J}_{\rho\sigma})^{\mu}{}_{\nu} = i\left(g^{\mu}_{\rho}\, g_{\sigma\nu} - g^{\mu}_{\sigma}\, g_{\rho\nu}\right)$$

$\rho\sigma$ ← labels

Check me on sign & indices

$\vec{J} \equiv \vec{\sigma}/2 \qquad \vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$

$\vec{K} \equiv -i\vec{\sigma}/2$

fix problem: Add second field! χ w/ $Q_\chi = -Q_\psi$

$$\mathcal{L}_{\psi\chi} = i\chi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \chi_\alpha + i\psi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \psi_\alpha + m\underbrace{\left(\psi^{\alpha}\chi_\alpha + \psi^{\dagger}_{\dot\alpha}\chi^{\dagger\dot\alpha}\right)}_{\text{Dirac Mass term}}$$

Chose χ, ψ to both be LH.

define $\Psi_D \equiv \begin{pmatrix} \psi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}$ $\quad Q_\psi$, $\quad Q_\psi = -Q_\chi$

define $\bar\Psi_D \equiv \Psi_D^{\dagger}\gamma^0 \longrightarrow i\bar\Psi_D \gamma^\mu D_\mu \Psi_D + m\bar\Psi_D\Psi_D$

$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_{2\times2} \\ \mathbb{1}_{2\times2} & 0 \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$

for four vectors:

$(\mathcal{J}_{\rho\sigma})^{\mu}{}_{\nu} = i\left(g_{\rho}{}^{\mu} g_{\sigma\nu} - g_{\sigma}{}^{\mu} g_{\rho\nu}\right)$

labels

Check me on sign & indices

$\vec{J} \equiv \vec{\sigma}/2 \qquad \vec{\sigma} \equiv (\sigma^1, \sigma^2, \sigma^3)$

$\vec{K} \equiv -i\vec{\sigma}/2$

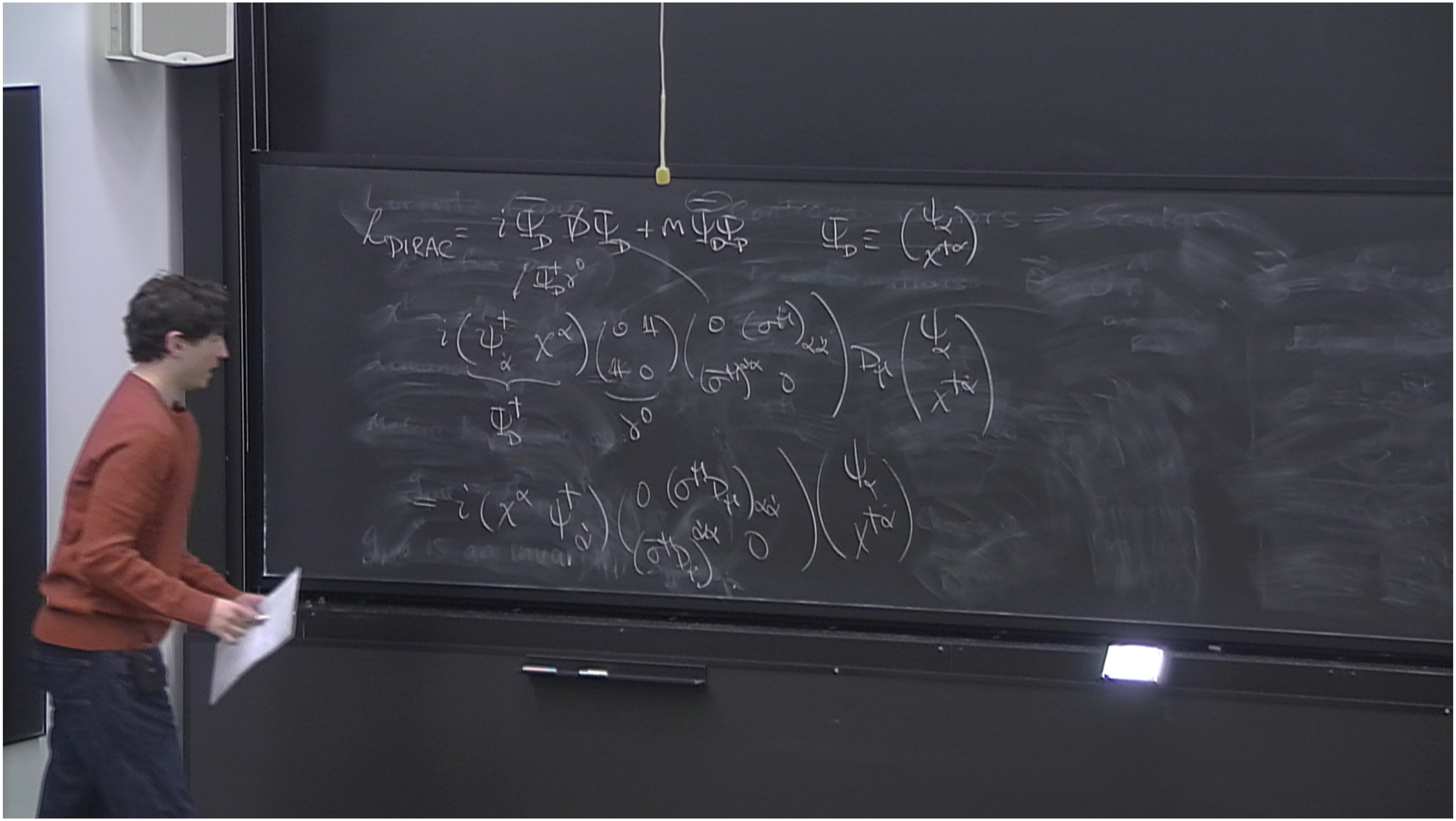
fix problem: Add second field! χ w/ $Q_\chi = -Q_\psi$

$$\mathcal{L}_{\psi\chi} = i\chi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \chi_\alpha + i\psi^{\dagger}_{\dot\alpha}(\bar\sigma^{\mu})^{\dot\alpha\alpha} D_\mu \psi_\alpha + m\underbrace{\left(\psi^{\alpha}\chi_{\alpha} + \psi^{\dagger}_{\dot\alpha}\chi^{\dagger\dot\alpha}\right)}_{\text{Dirac Mass term}}$$

Chose χ, ψ to both be LH.

define $\Psi_D \equiv \begin{pmatrix} \psi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}$ Q_ψ, $Q_\psi = -Q_\chi$

define $\bar\Psi_D \equiv \Psi_D^{\dagger}\gamma^0$, $\gamma^0 = \begin{pmatrix} 0 & \mathbb{1}_{2\times2} \\ \mathbb{1}_{2\times2} & 0 \end{pmatrix}$ $\longrightarrow i\bar\Psi_D \gamma^\mu D_\mu \Psi_D + m\bar\Psi_D\Psi_D$, $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix}$


L_DIRAC = i Ψ̄_D ∂̸ Ψ_D + m Ψ̄_D Ψ_D
Ψ_D ≡ (ψ_α, χ^{†α̇})