Title: PSI Student Seminars

Date: Dec 04, 2014 09:00 AM

URL: http://pirsa.org/14120055

Abstract:


Fun with
Black Holes
Mae Hwee Teo
PSI Presentation
Dec 4, 2014

Road Map

① History: BH Mechanics

↓

BH Thermodynamics

② Go to AdS space

↓

give BH a charge → see PHASE TRANSITION

"Four laws of Black Hole Mechanics" (1973)

0. The **surface gravity κ** is constant over the event horizon.

$$\kappa = g = \frac{GM}{r^2} = \frac{GM}{(2M)^2} = \frac{G}{4M}$$

1. $\delta M = \frac{\kappa}{8\pi G}\delta A + \ldots$

   $(\delta E = T\delta S + \ldots)$

2. Event horizon area never decreases: $\delta A \geq 0$ $(\delta S \geq 0)$

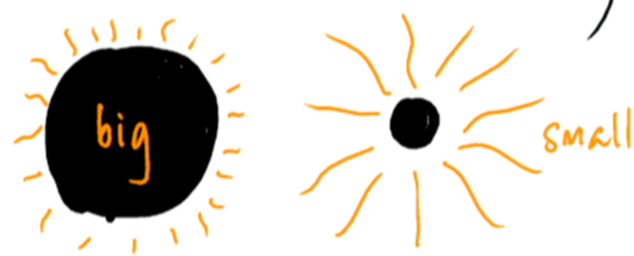
3. It's impossible to reduce the **surface gravity κ** to zero in a finite number of steps.

BH Mechanics → BH Thermodynamics

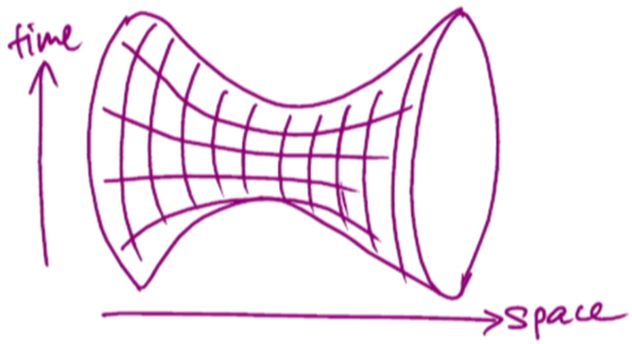
Hawking Radiation (1973, many proofs since)

$T = \frac{\kappa}{2\pi}$ (note $T \propto \frac{1}{M}$ since $\kappa = g = \frac{GM}{r^2} = \frac{GM}{(2M)^2} = \frac{G}{4M}$)

$S = \frac{A}{4}$



Let's go to AdS space ... ($\Lambda < 0$)



$$G^{\mu}{}_{\nu} = -\Lambda \delta^{\mu}{}_{\nu} = 8\pi G \, T^{\mu}{}_{\nu}$$

$$\hookrightarrow \begin{bmatrix} -\rho & \\ & P \end{bmatrix}$$

Positive pressure

$$\Rightarrow P = -\frac{\Lambda}{8\pi} > 0$$

New 1st Law:

$$dM = TdS + \underline{V}\,dP + \ldots \quad (2009)$$

generalized interpretation ↕

Enthalpy $(H = E + PV)$

$P = -\frac{\Lambda}{8\pi}$ ← AdS!

$V = \left(\frac{\partial M}{\partial P}\right)_S = \frac{4}{3}\pi r_+^3$!!!

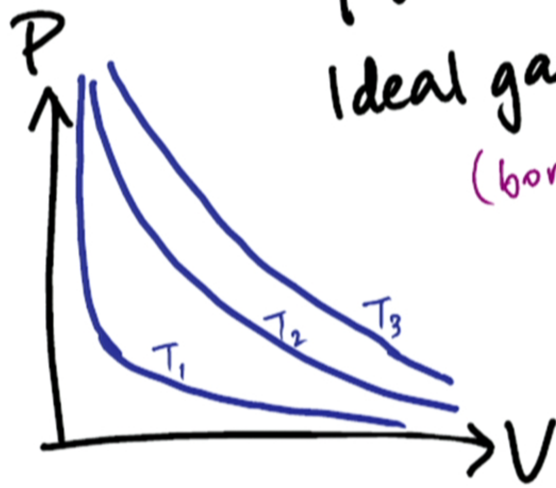
r_+: event horizon radius

Schwarzschild-AdS black hole:

$PV \propto T$

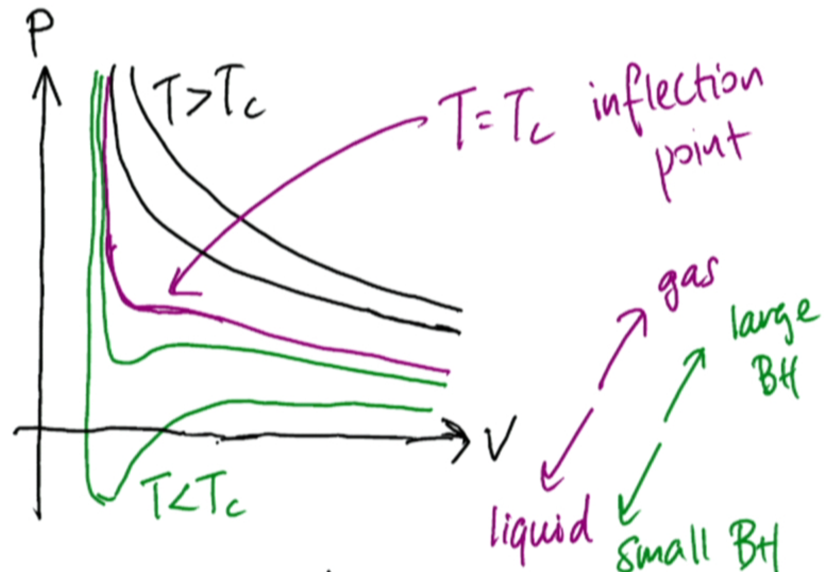
Ideal gas law

(boring...)



Give BH a charge $Q \neq 0$ (Reissner-Nordstrom-AdS BH)

First order Phase Transition!

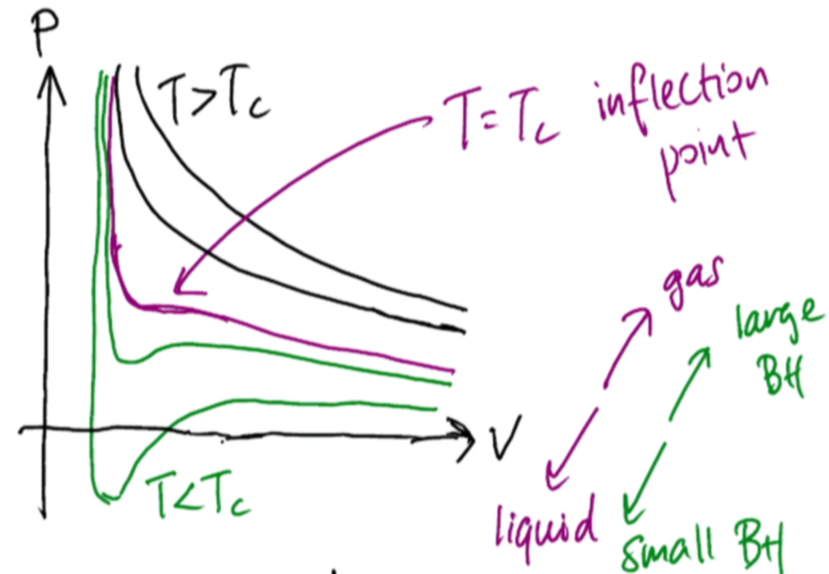


Van der Waals fluid

$$\left(P+\frac{a}{v^2}\right)(v-b)=T$$

Give BH a charge $Q \neq 0$ (Reissner-Nordstrom-AdS BH)

First order Phase Transition!



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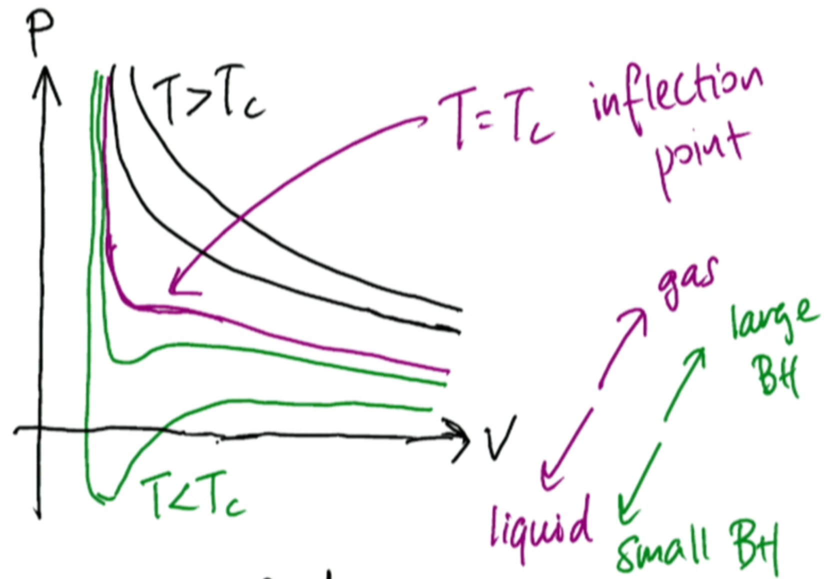

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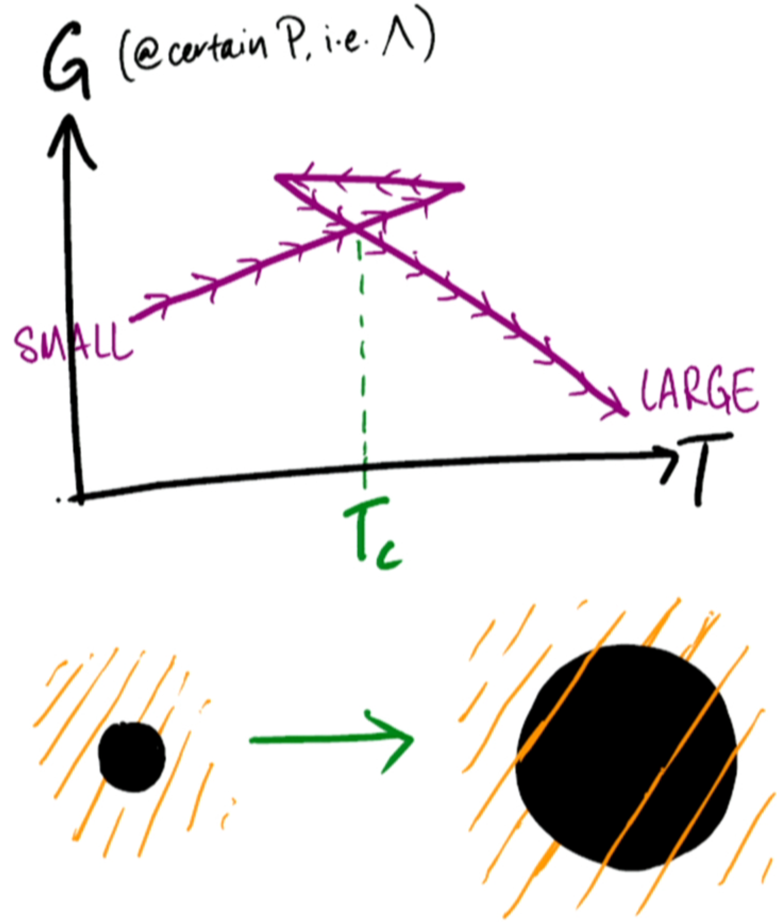


G (@ certain P, i.e. Λ)
SMALL
LARGE
T
T_c

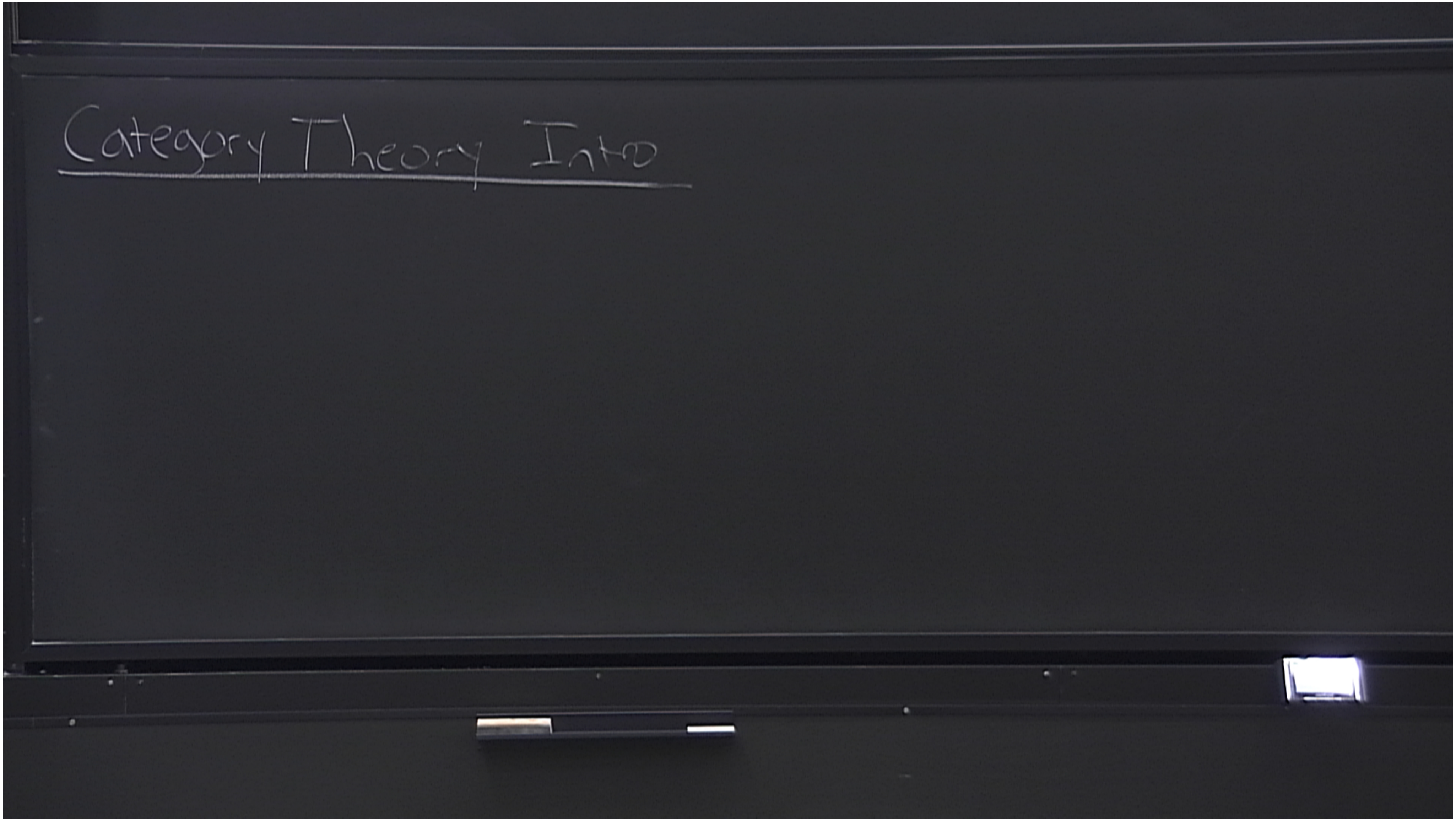
REFERENCES

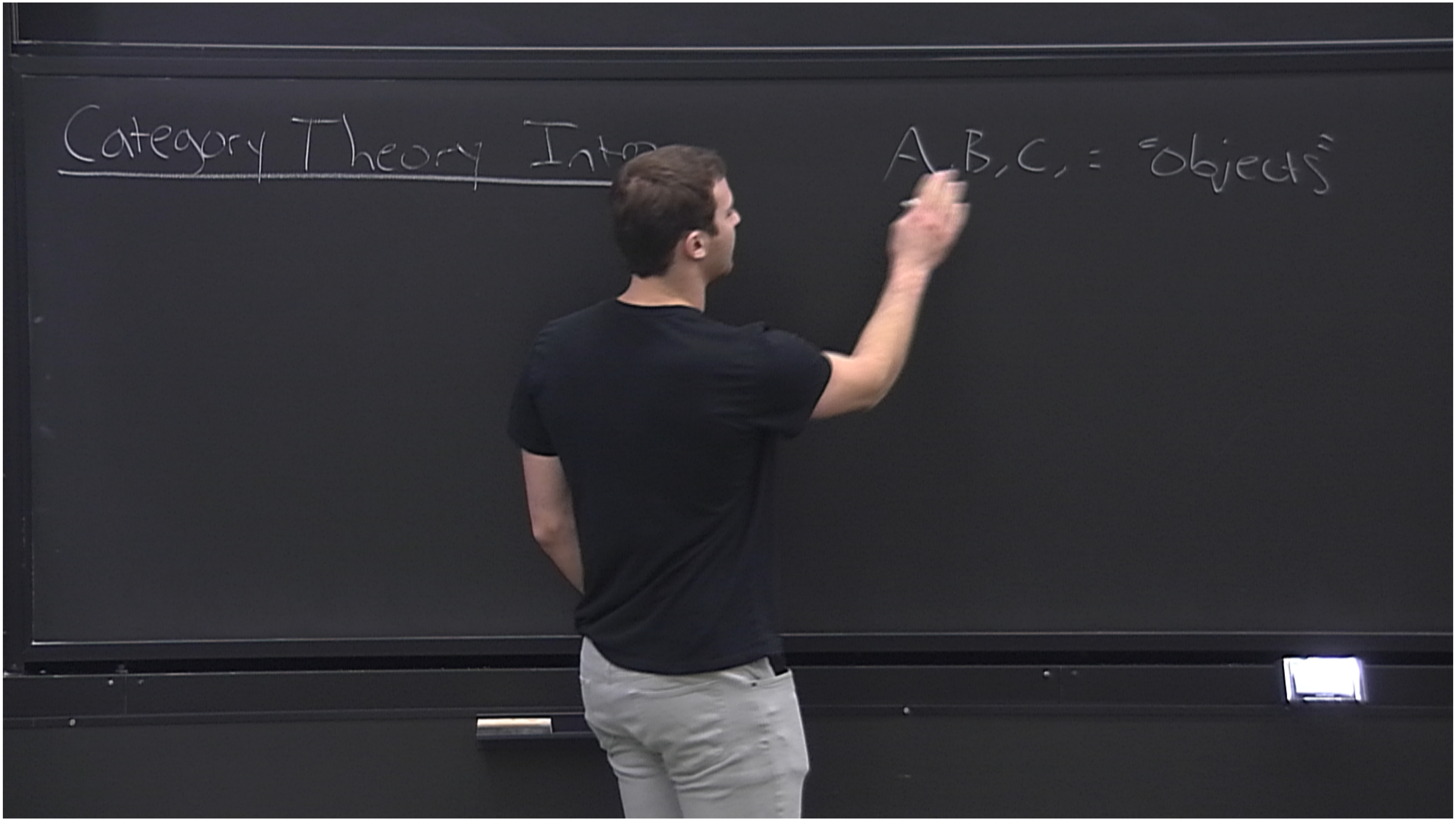
D. Kubizňák, R. B. Mann, "Black Hole Chemistry", arXiv: 1404.2126

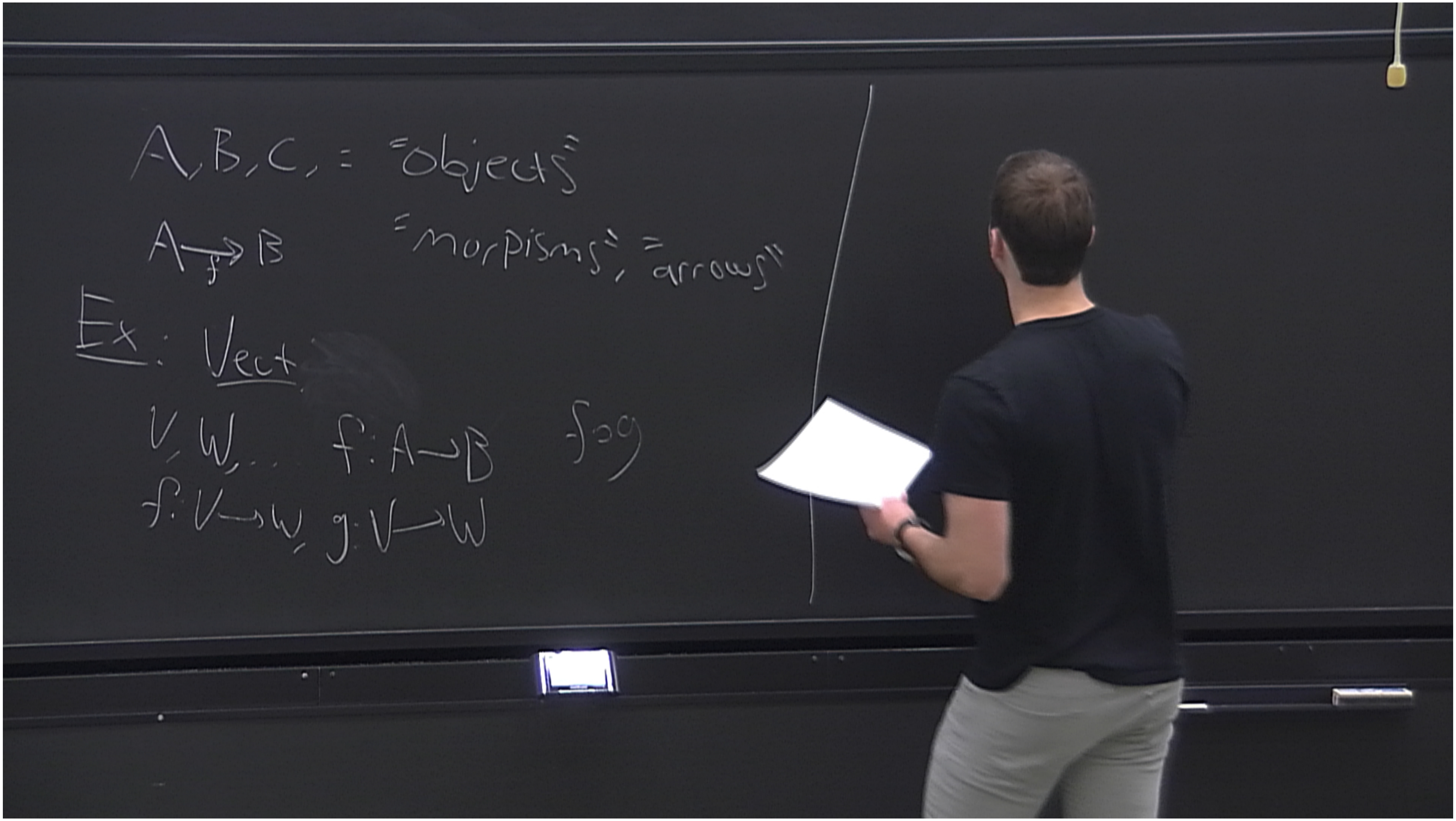
J. D. Bekenstein, "Black-hole thermodynamics," Physics Today, 24-31 (Jan. 1980)

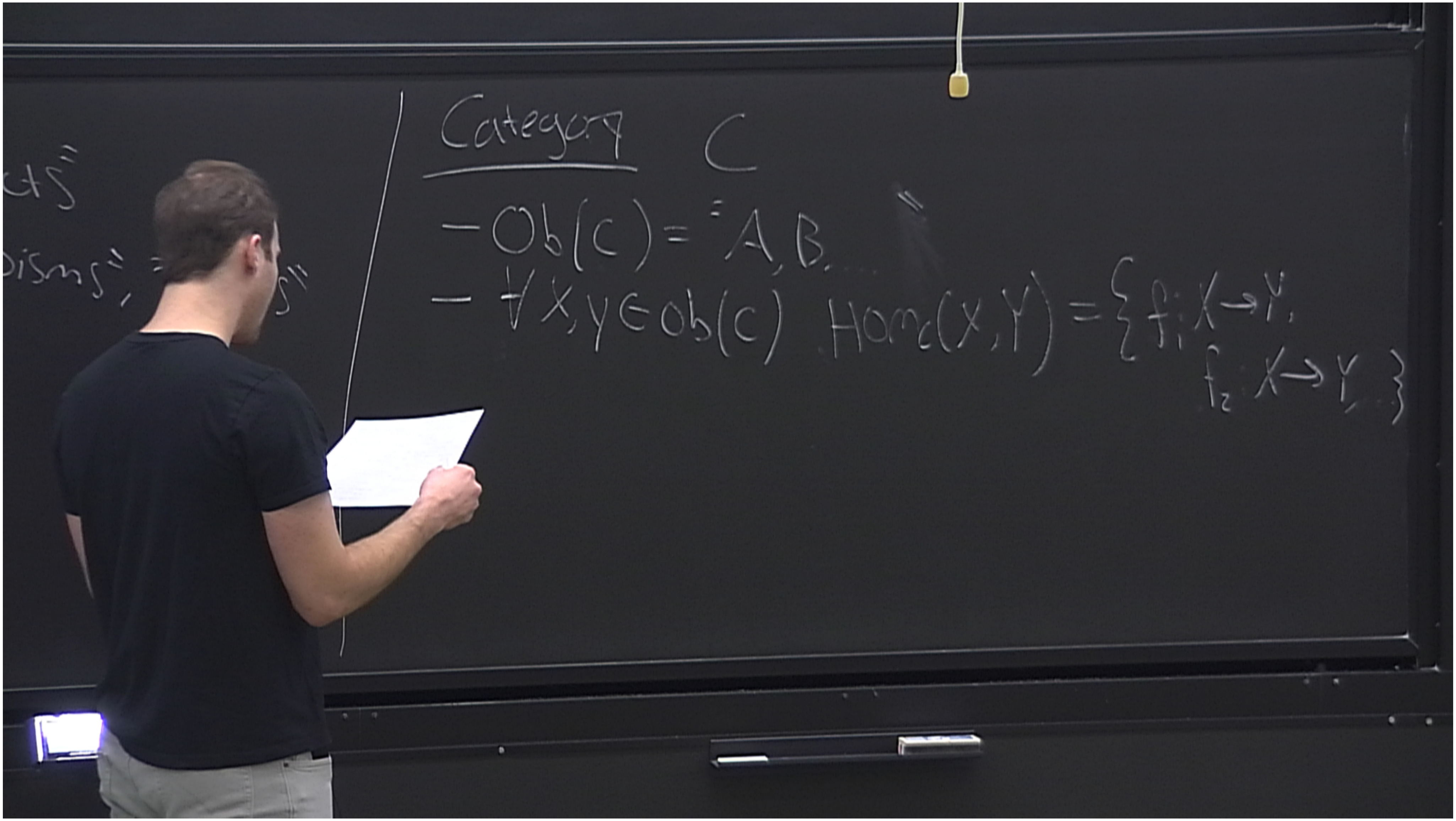
Image credits: Wikipedia

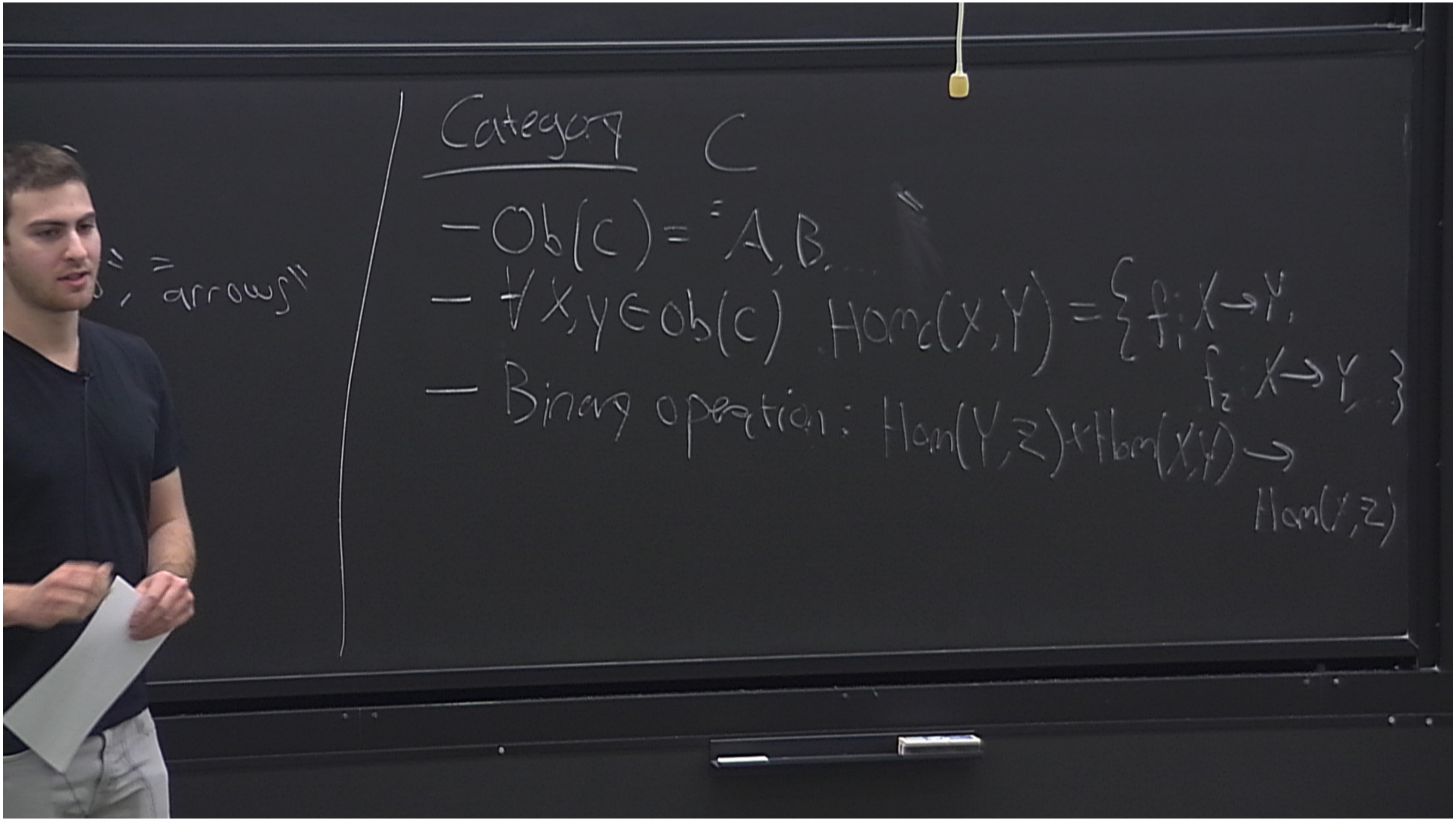
THANK YOU!

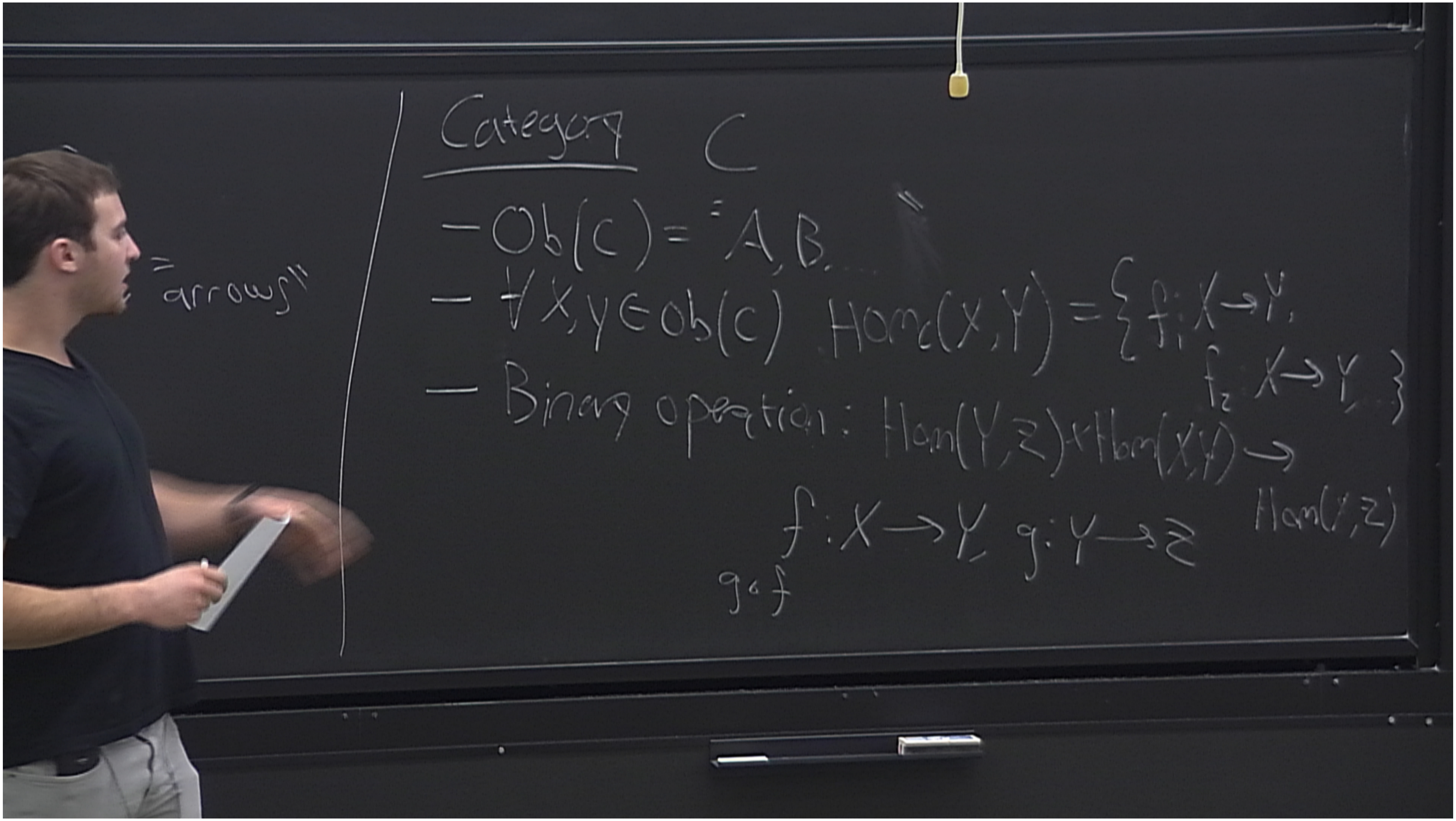

Category Theory Intro


Category Theory
A, B, C, = "objects"


A, B, C, = "objects"
A → B "morpisms", "arrows"
f
Ex: Vect
V, W, ... f: A → B f∘g
f: V → W, g: V → W


Category C
— Ob(C) = "A, B, ...
— ∀ X, Y ∈ Ob(C) Hom_C(X, Y) = { f1: X → Y,
f2: X → Y, ... }


"arrows"
Category C
— Ob(C) = "A, B, ...
— ∀ X, Y ∈ Ob(C) Hom_C(X,Y) = { f_1: X → Y, f_2: X → Y, ... }
— Binary operation: Hom(Y,Z) × Hom(X,Y) → Hom(X,Z)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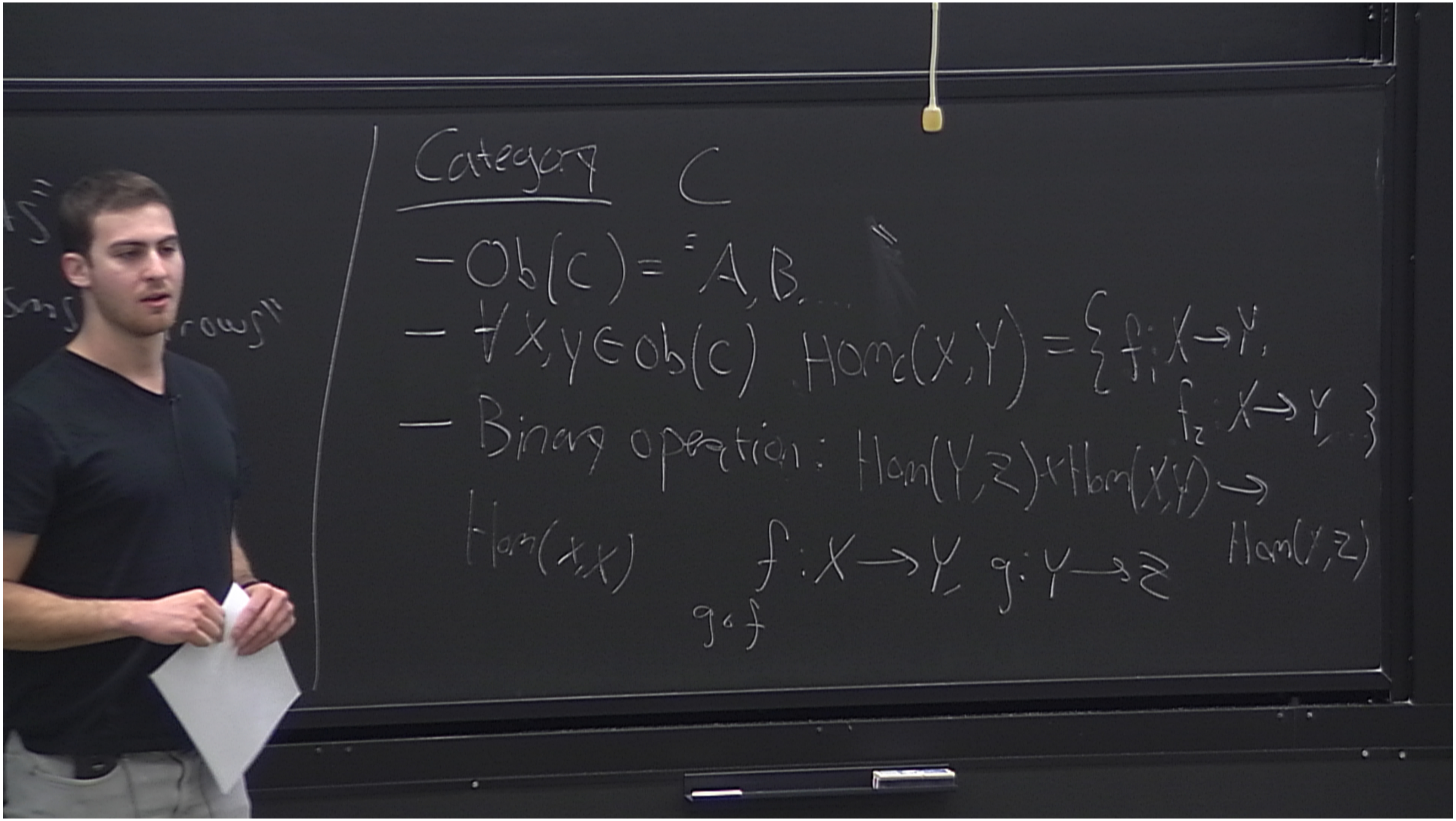
— Binary operation: Hom(Y,Z) × Hom(X,Y) → Hom(Y,Z)
f : X → Y, g : Y → Z
g∘f


Category C
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— Binary operation: Hom(Y,Z) × Hom(X,Y) → Hom(X,Z)
Hom(X,X)
f : X → Y, g : Y → Z
g ∘ f

Ex: Cat G

$ob(G) = \sigma$

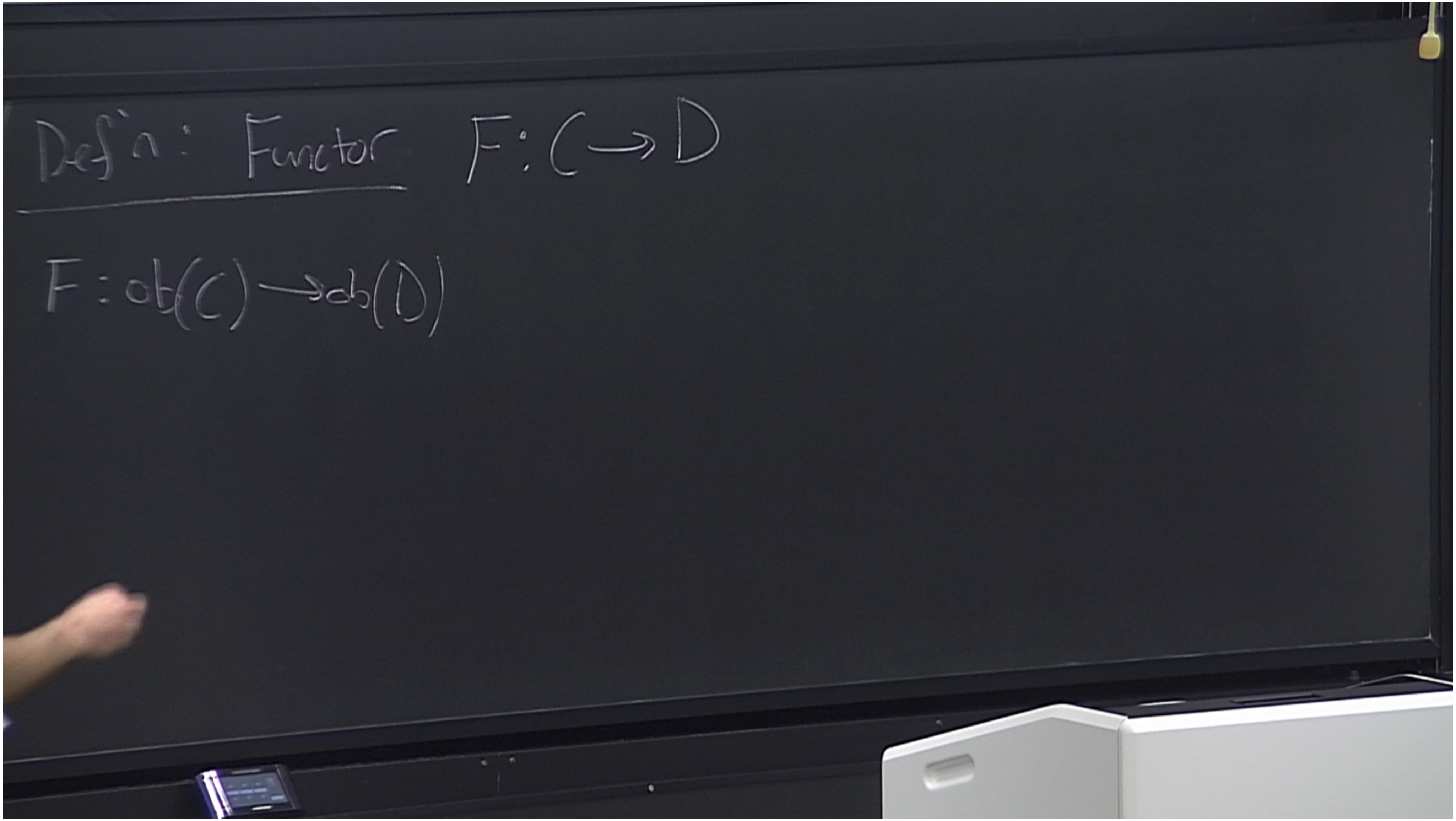
$Hom(\sigma, \sigma)$ isomorphisms

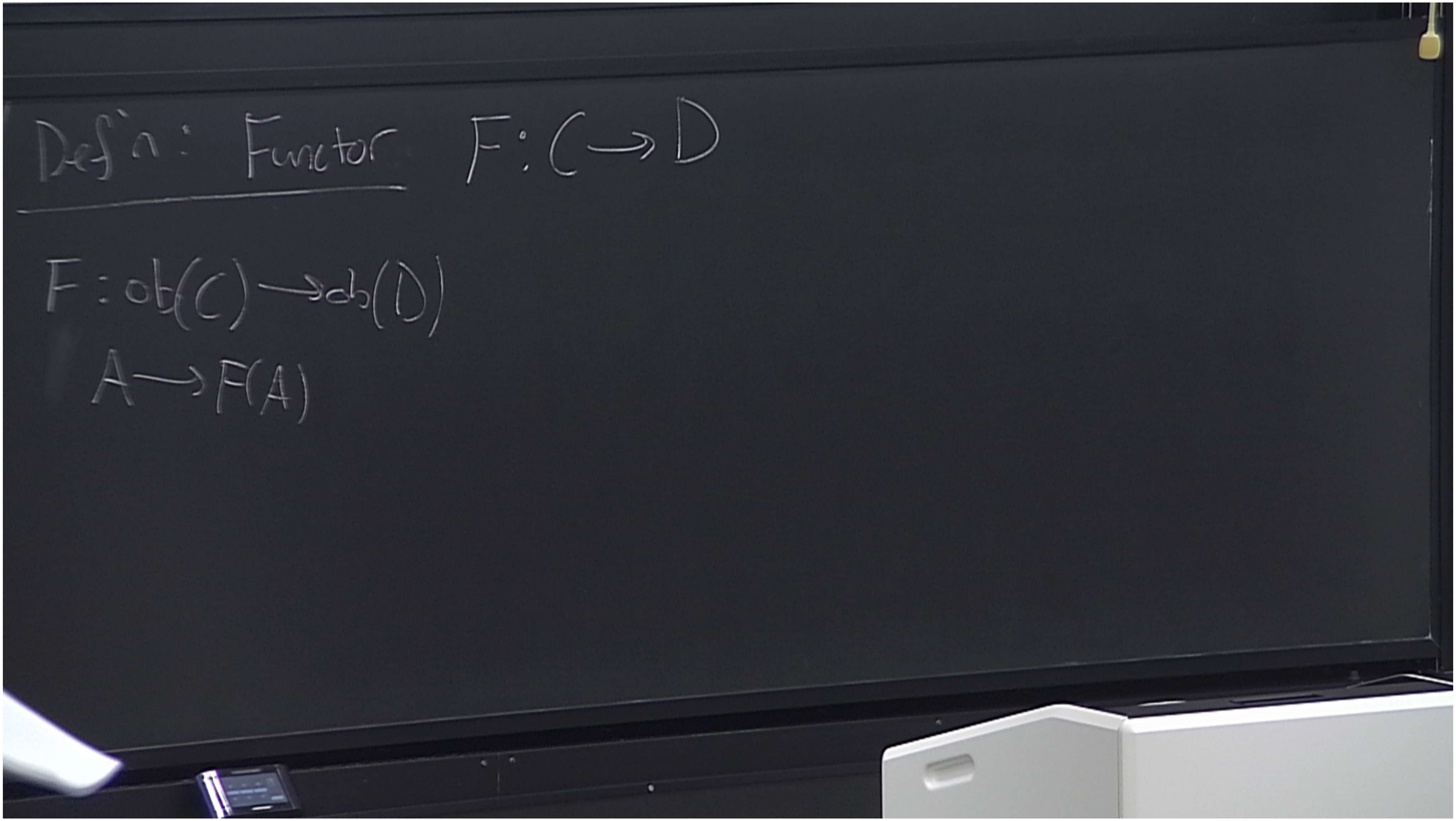
$g_1 : \sigma \to \sigma$

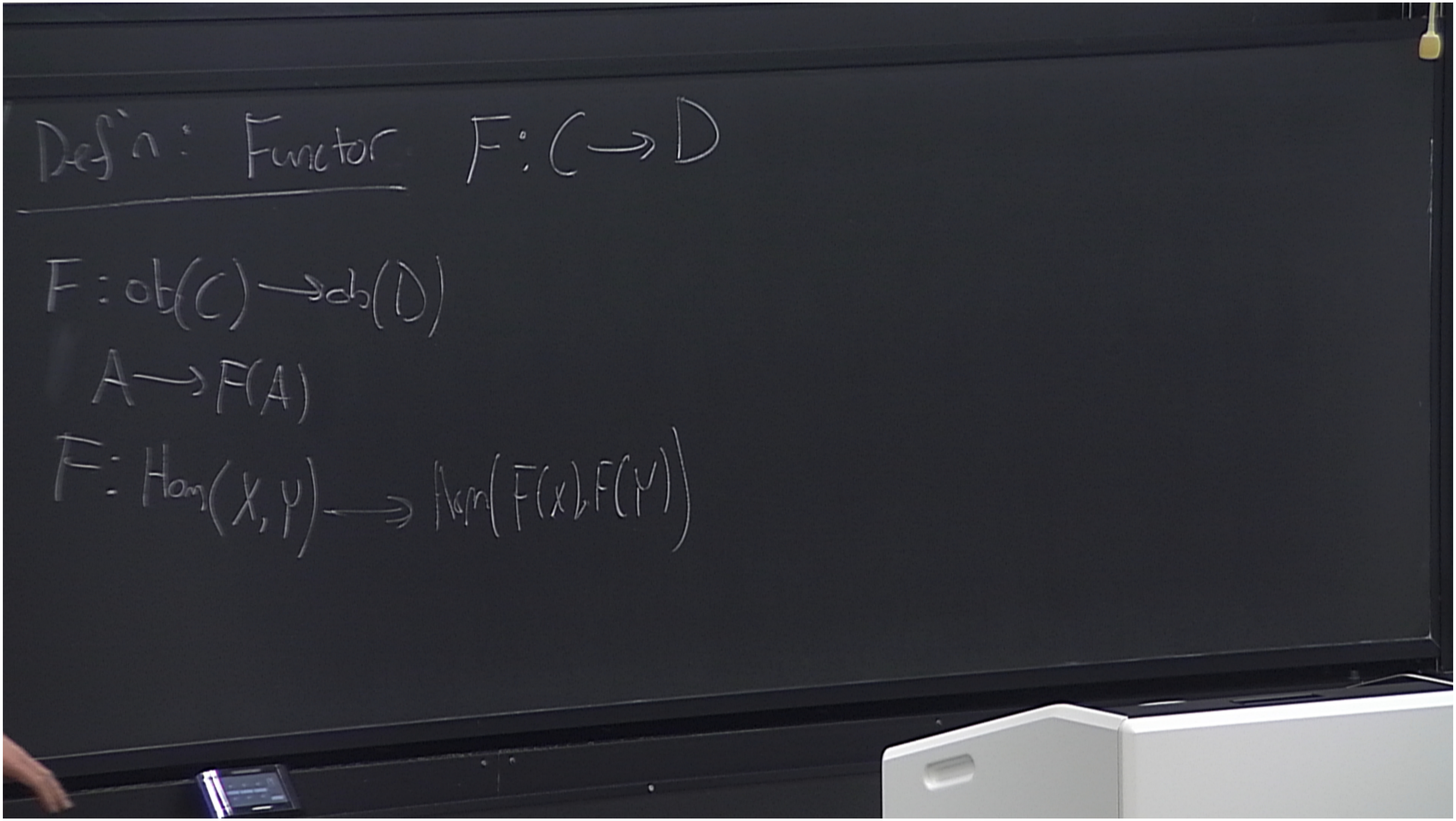
$g_2 : \sigma \to \sigma$

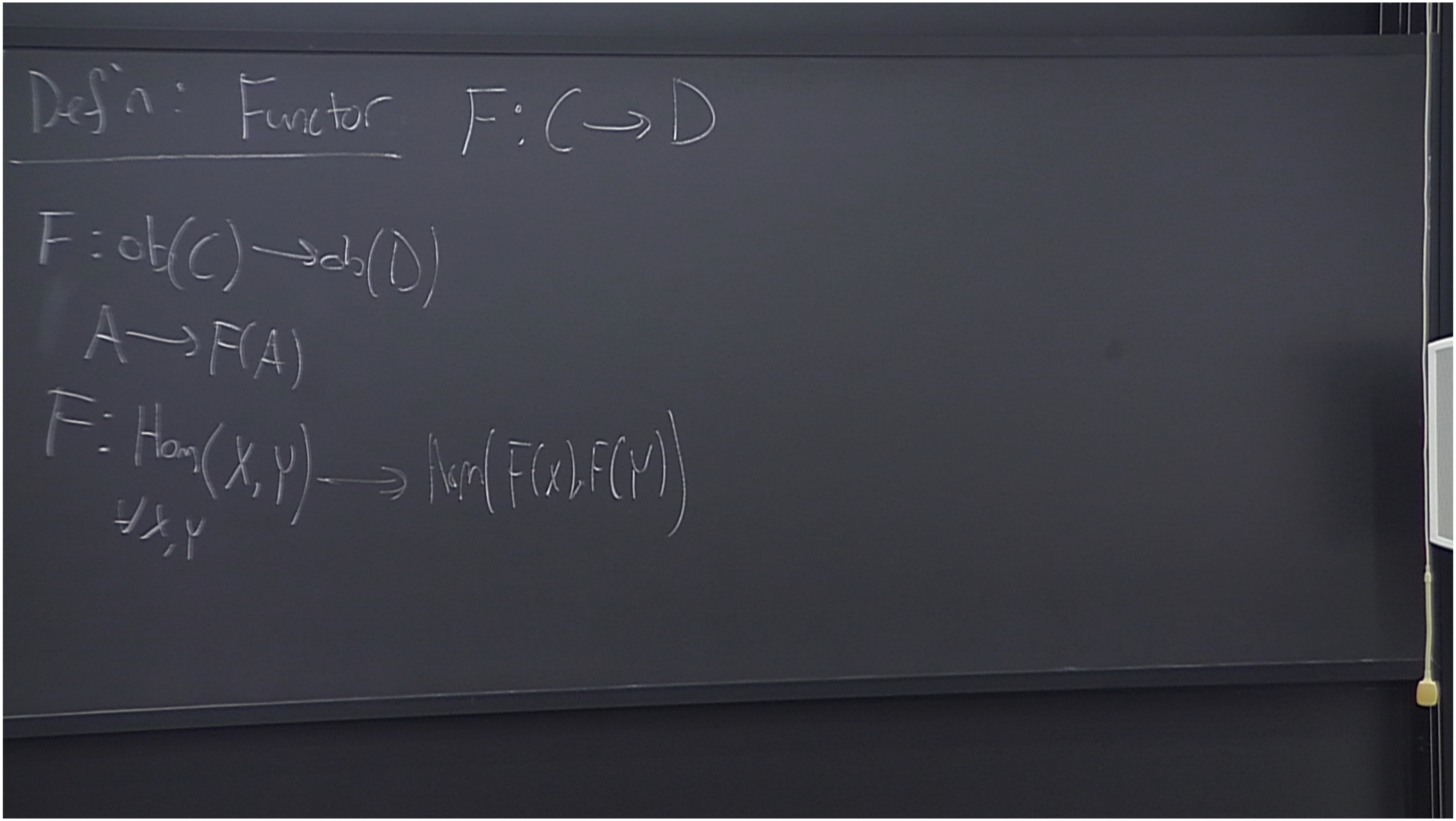
$g_1^{-1} : \sigma \to \sigma$

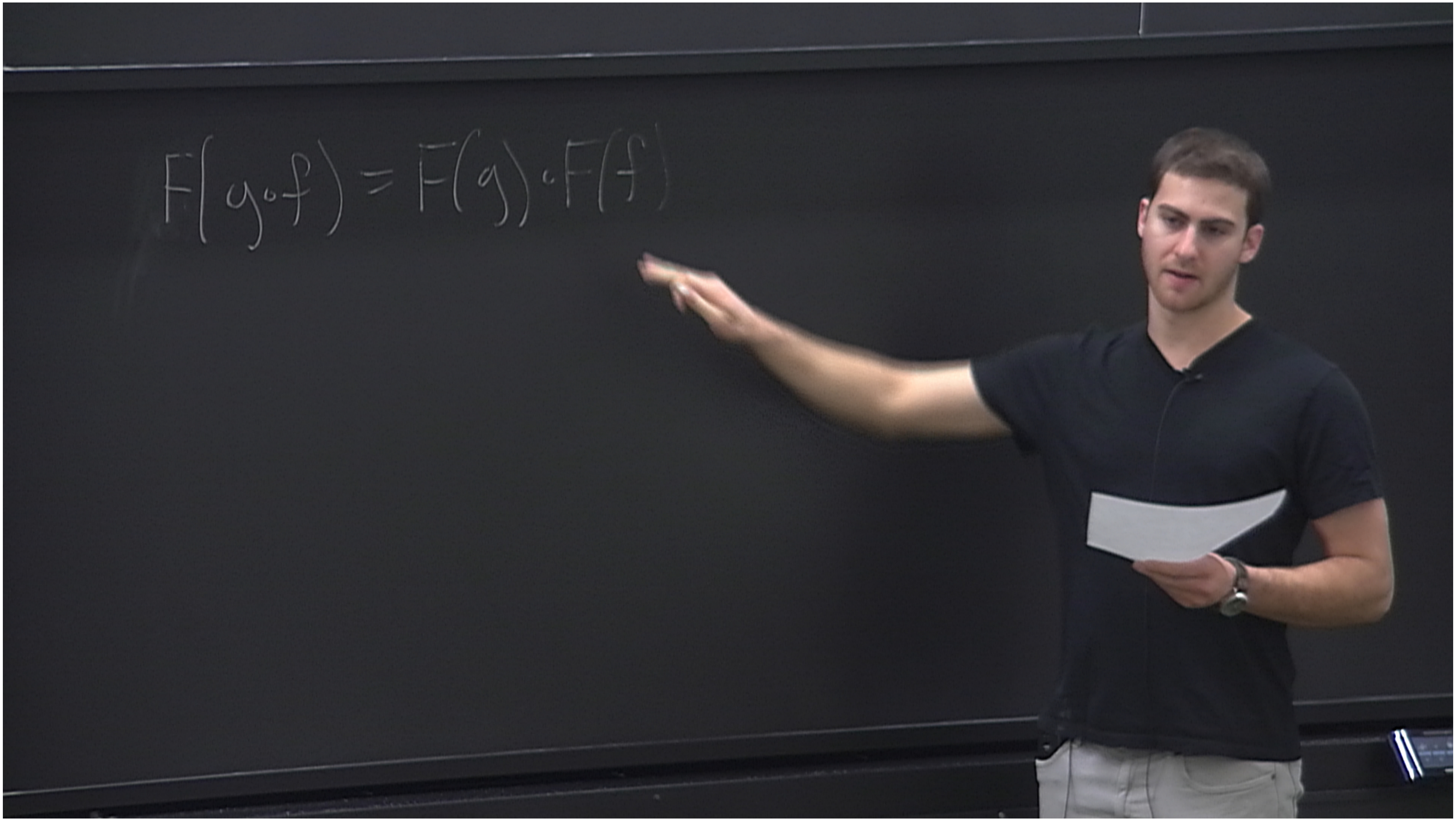
$g_1 \circ g_1^{-1} = \mathbb{1}$

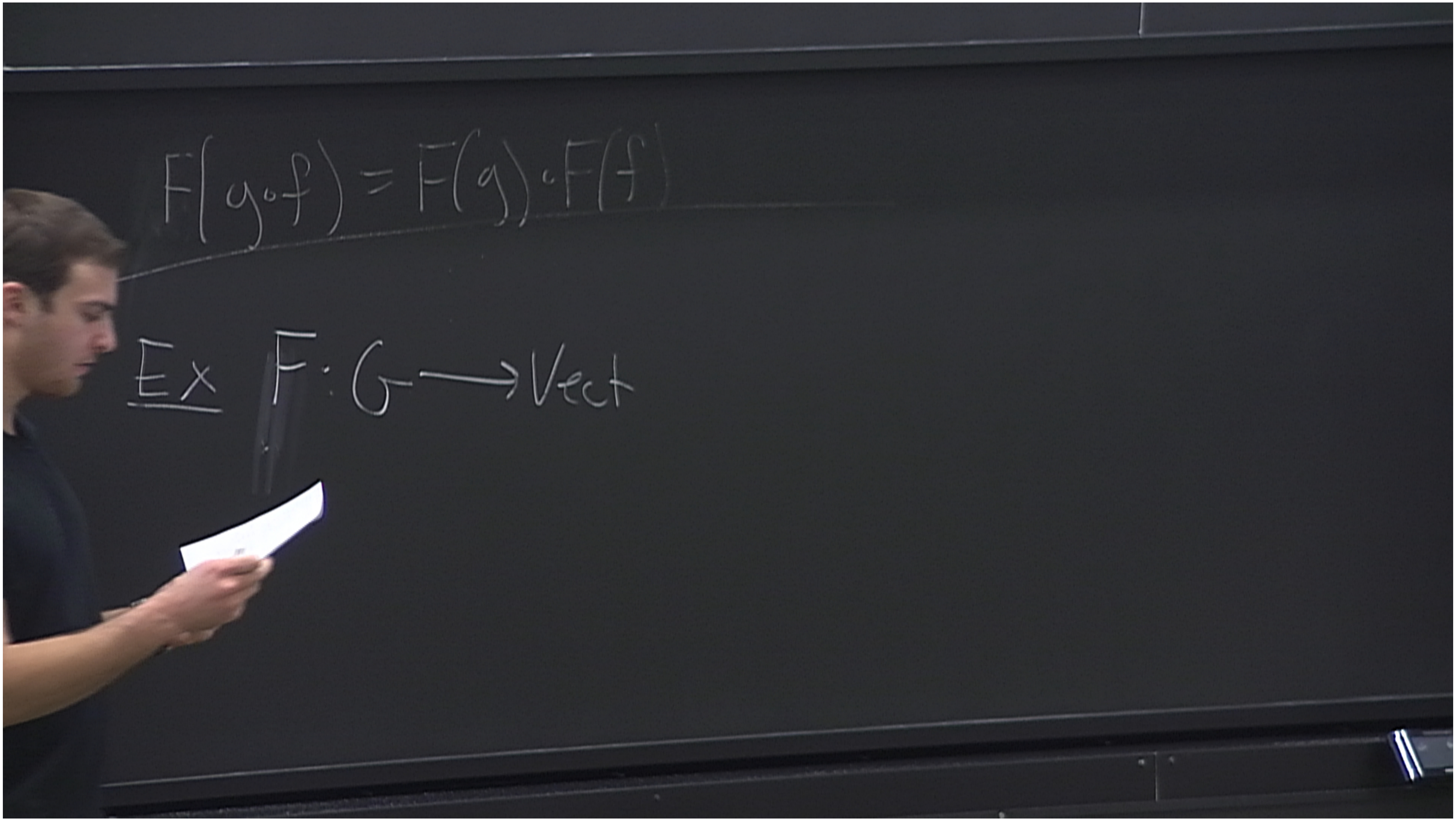

Def'n: Functor F: C → D
F : ob(C) → ob(D)

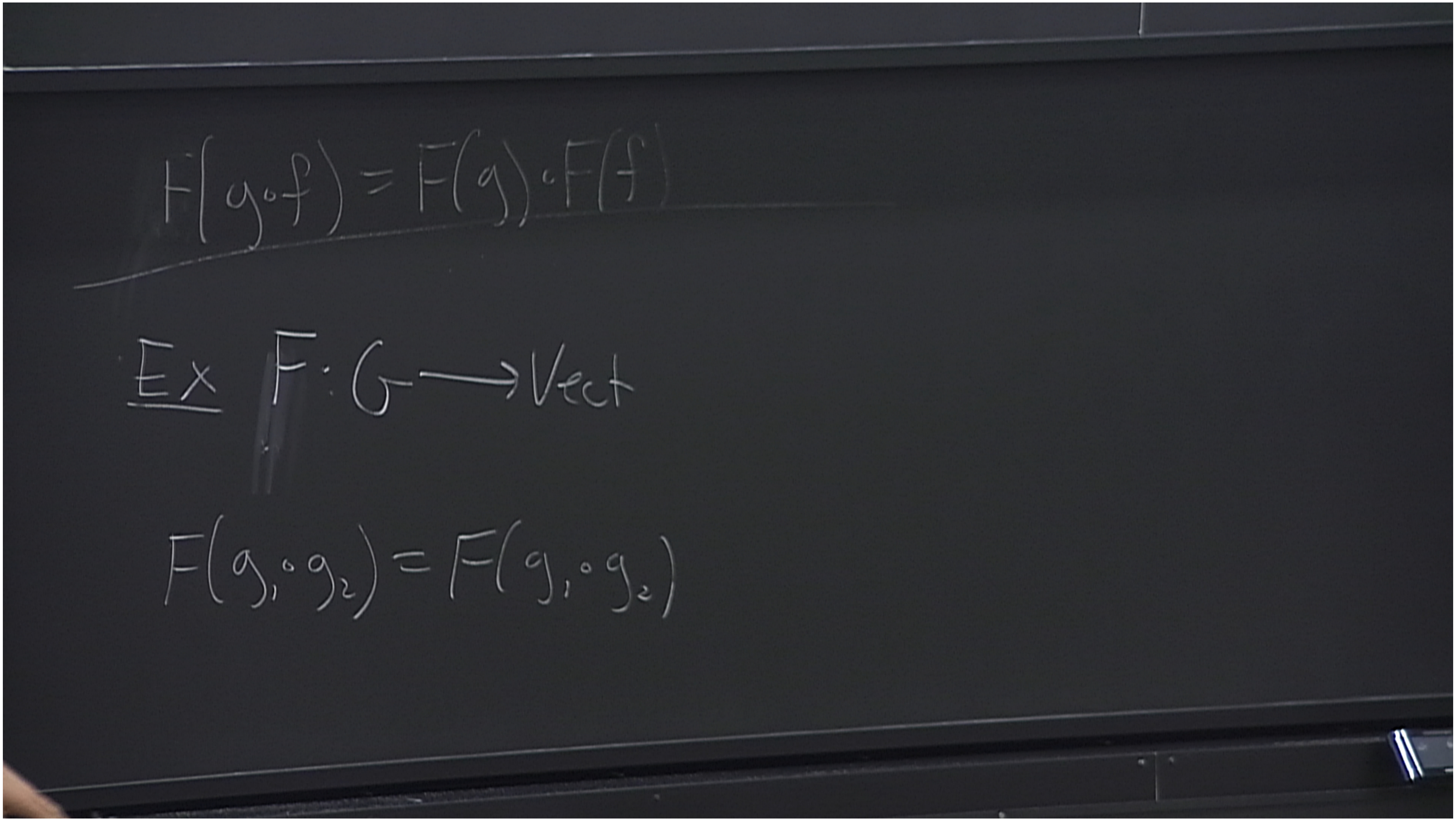

Def'n: Functor F: C → D
F: ob(C) → ob(D)
A → F(A)

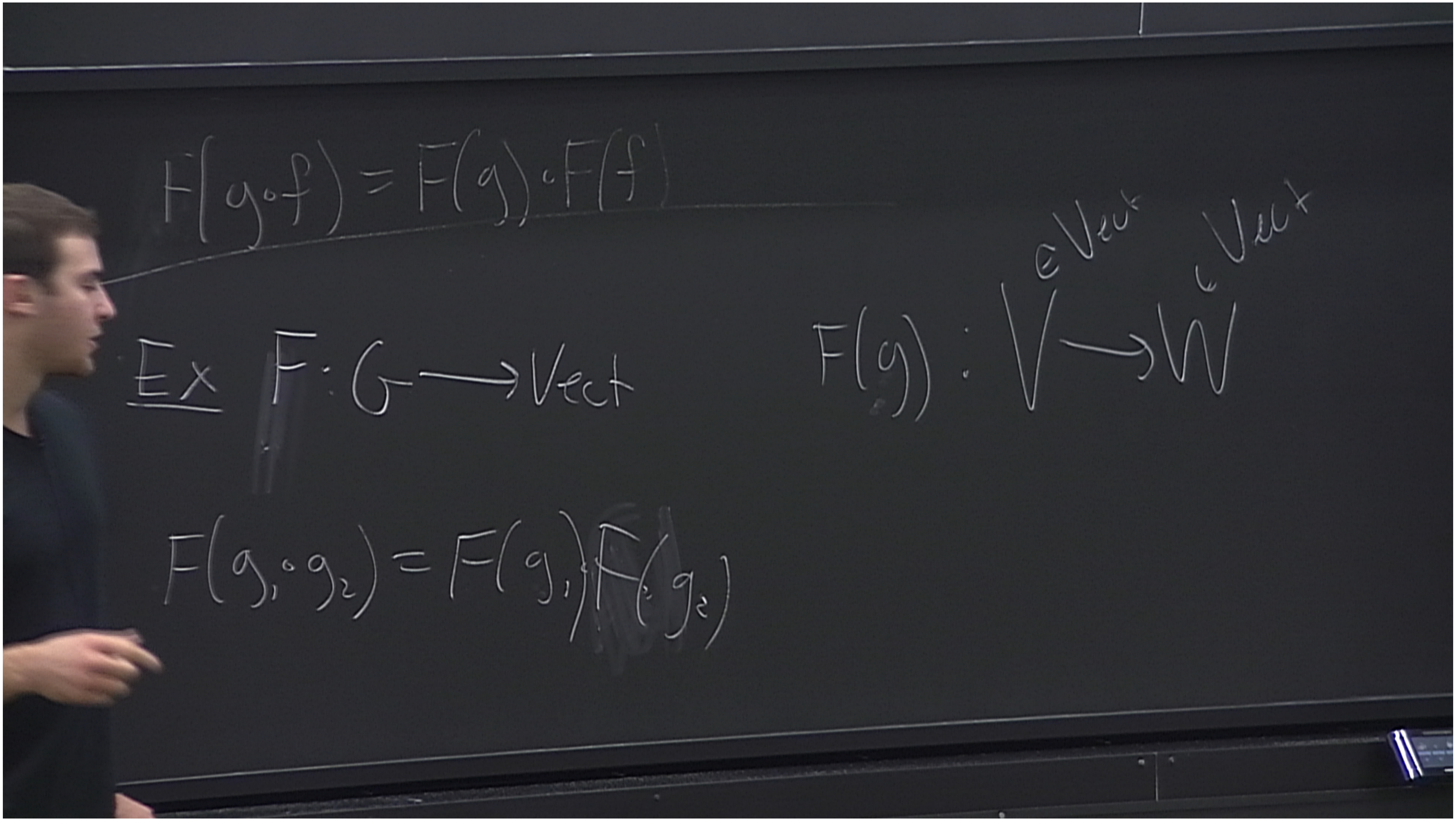

Def'n: Functor $F: C \to D$
$F: ob(C) \to ob(D)$
$A \to F(A)$
$F: Hom(X,Y) \to Hom(F(X), F(Y))$

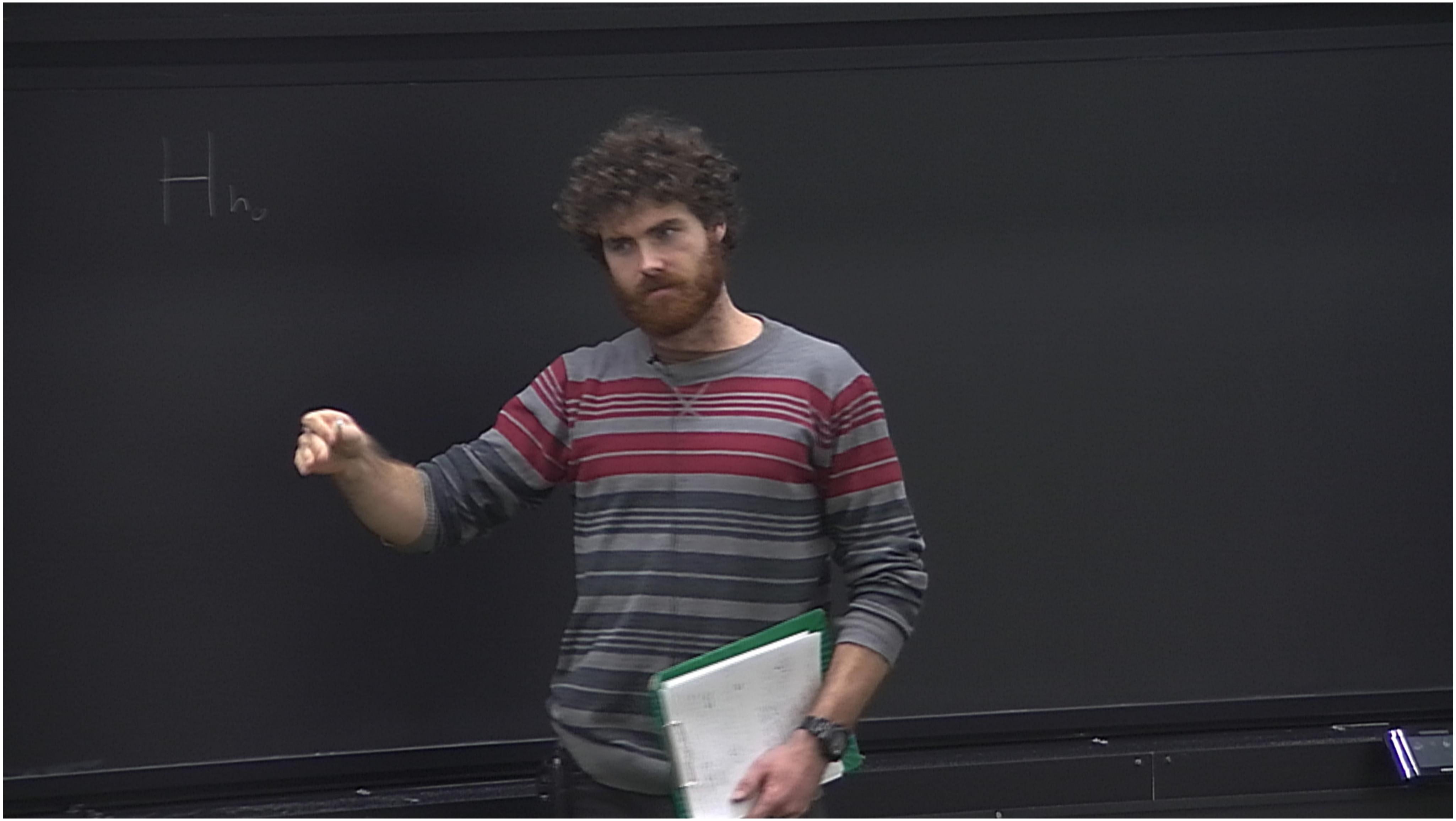

Def'n: Functor $F: C \to D$
$F: ob(C) \to ob(D)$
$A \to F(A)$
$F: Hom(X, Y) \to Hom(F(X), F(Y))$
$\forall X, Y$

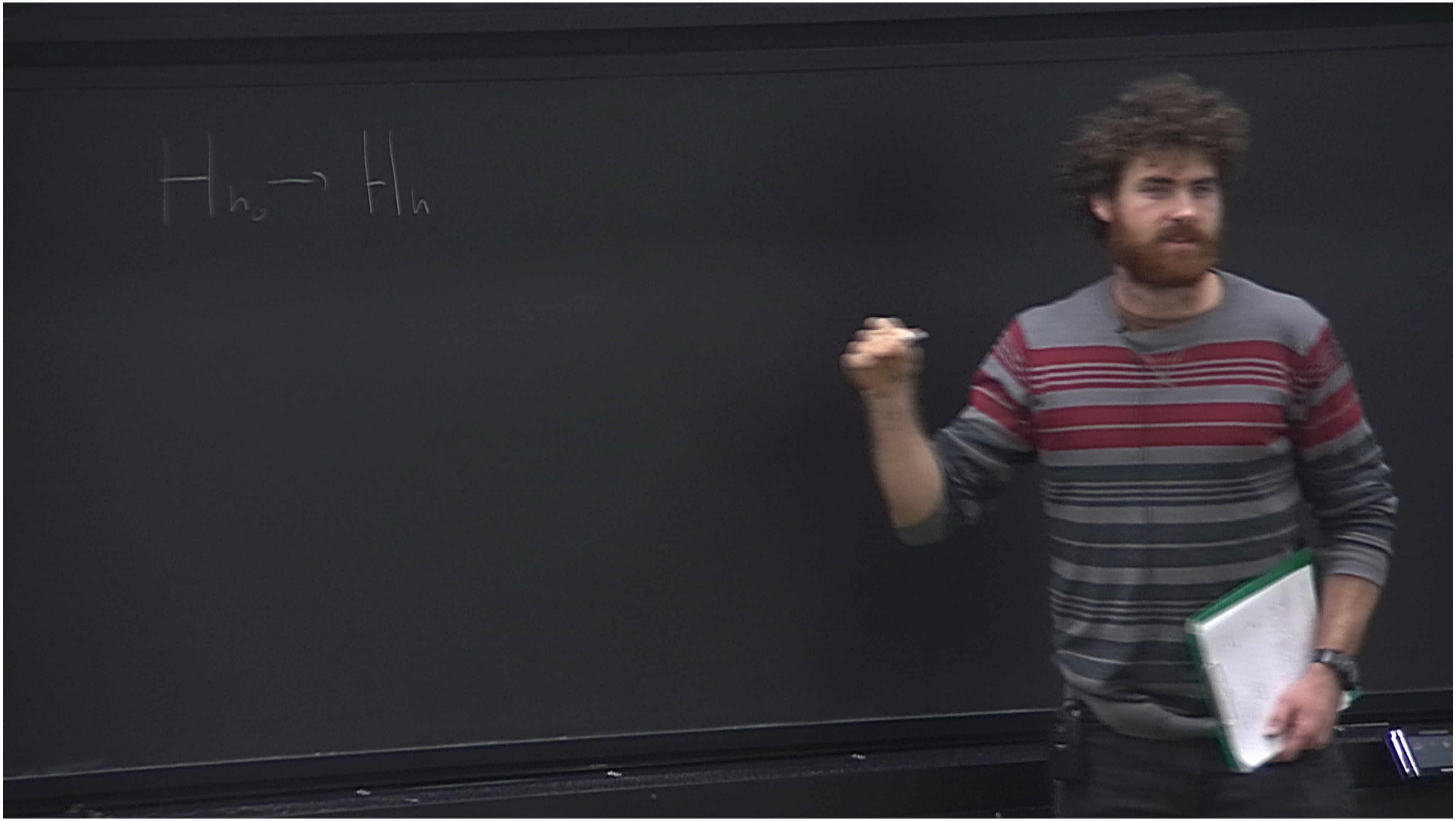

F(g∘f) = F(g)∘F(f)


F(g∘f) = F(g)∘F(f)
Ex F : G ⟶ Vect

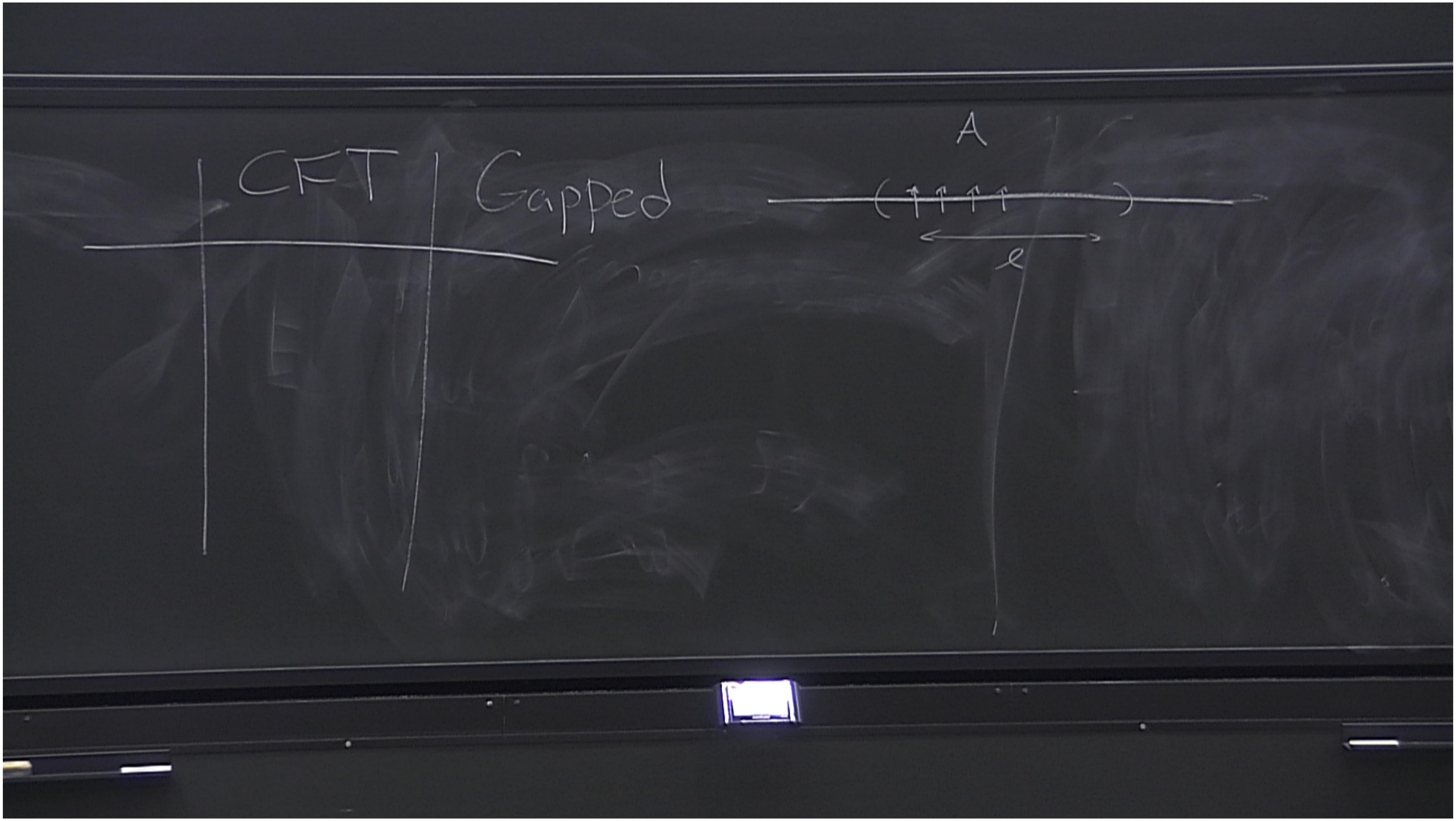

F(g∘f) = F(g)∘F(f)
Ex F : G → Vect
F(g₁∘g₂) = F(g₁∘g₂)

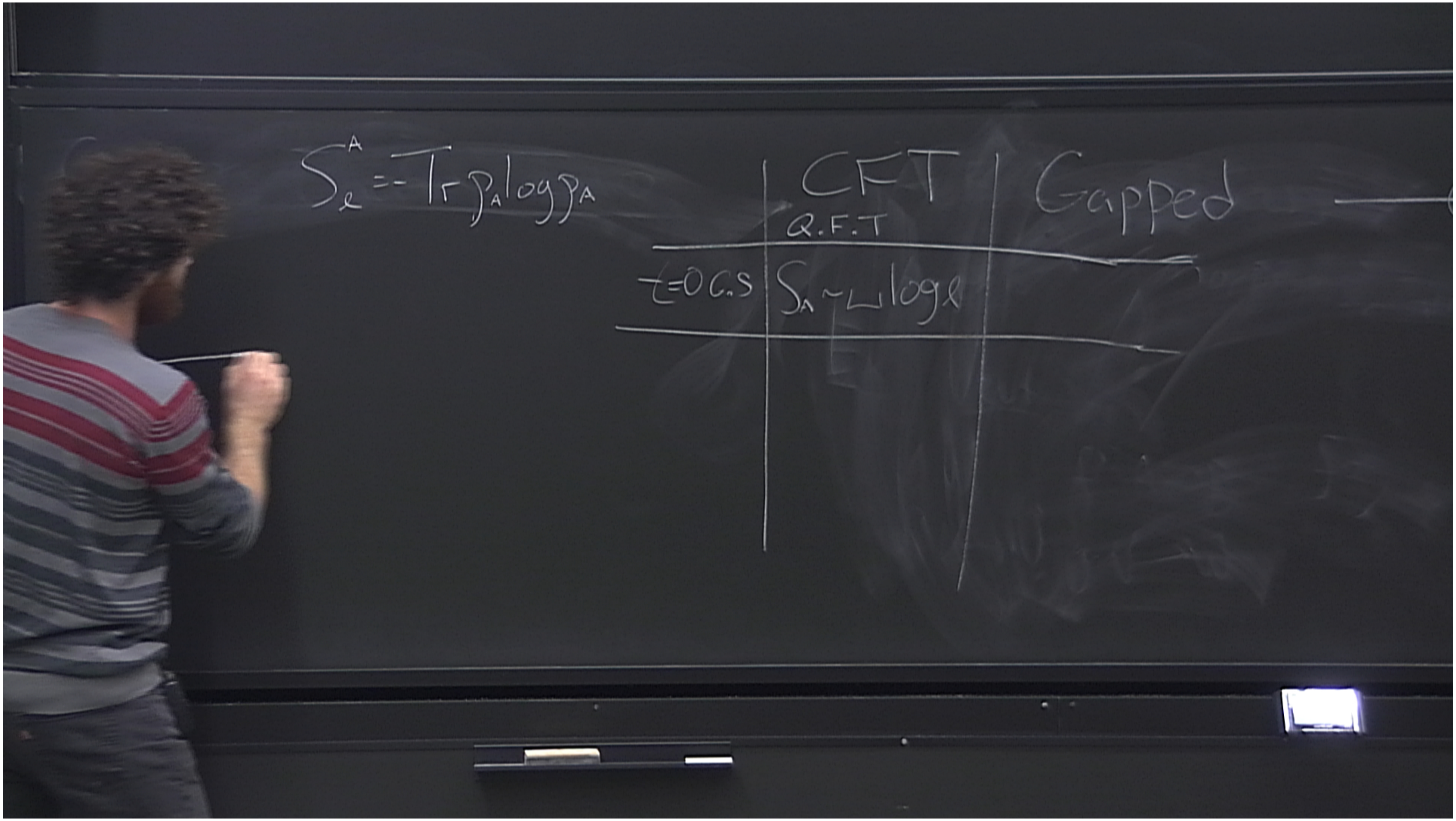

F(g∘f) = F(g)∘F(f)
Ex F: G → Vect
F(g): V → W
∈ Vect
∈ Vect
F(g1∘g2) = F(g1)∘F(g2)

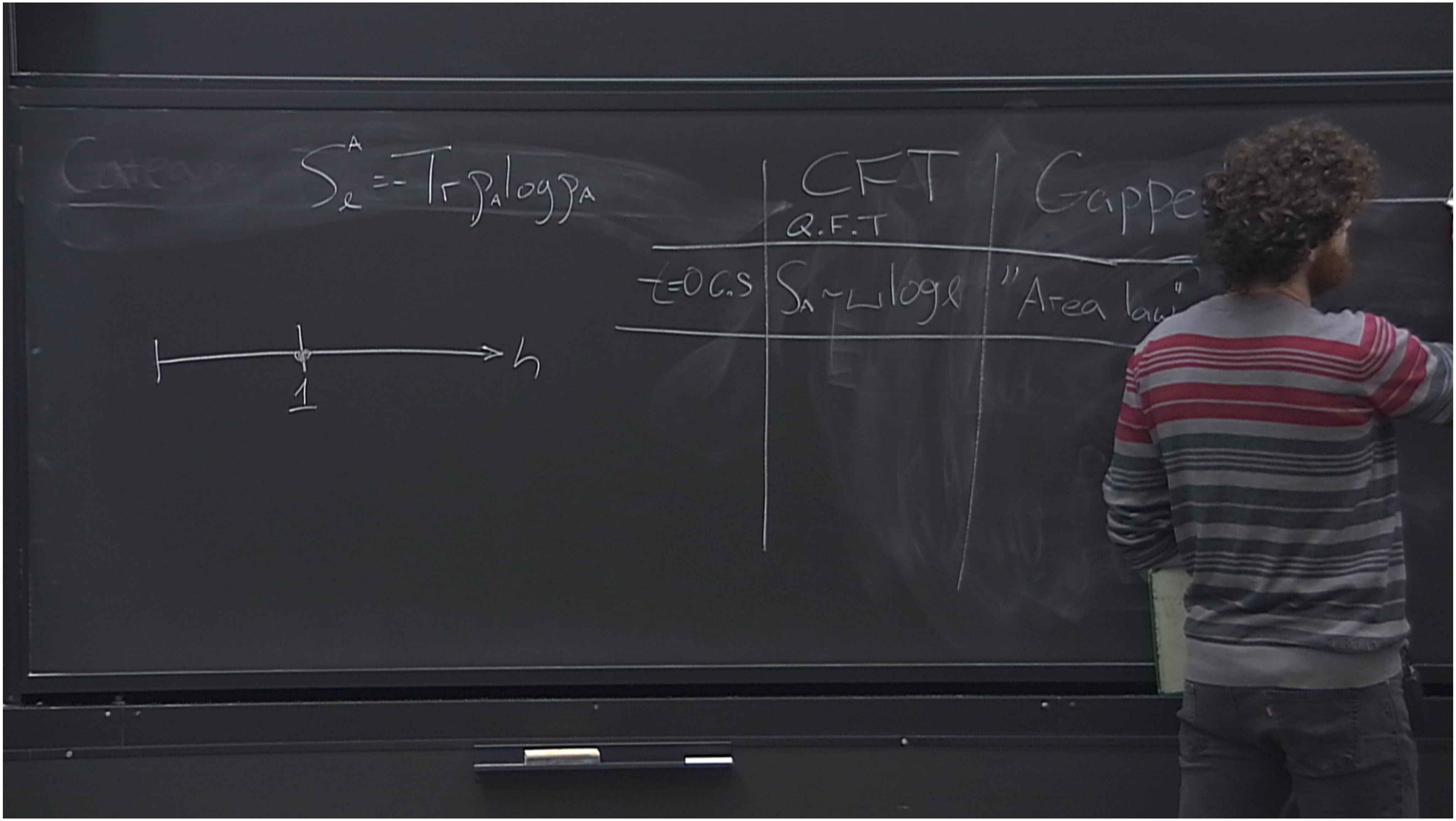

H_{h_0}



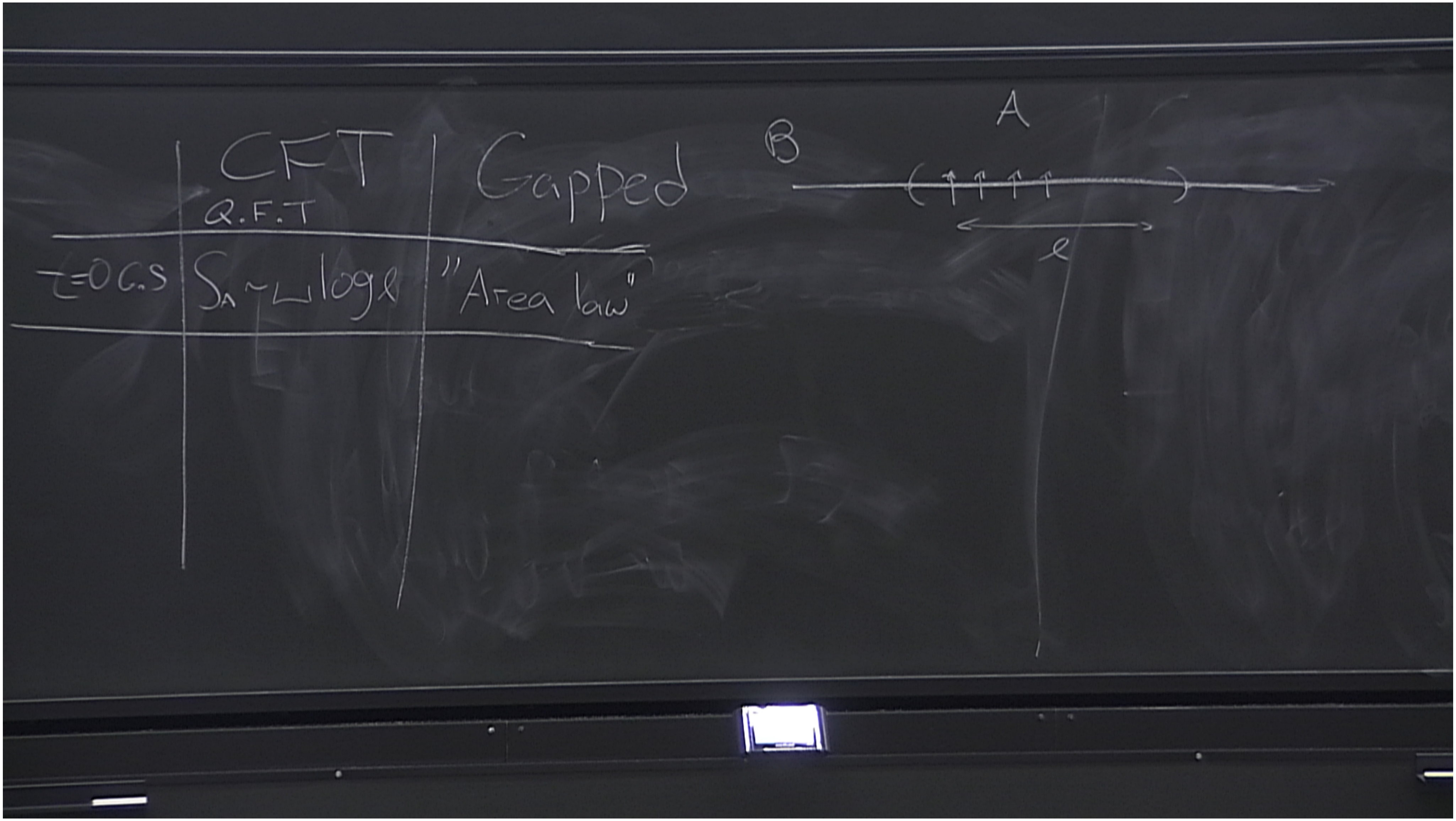
$H_{h_0} \to H_h$

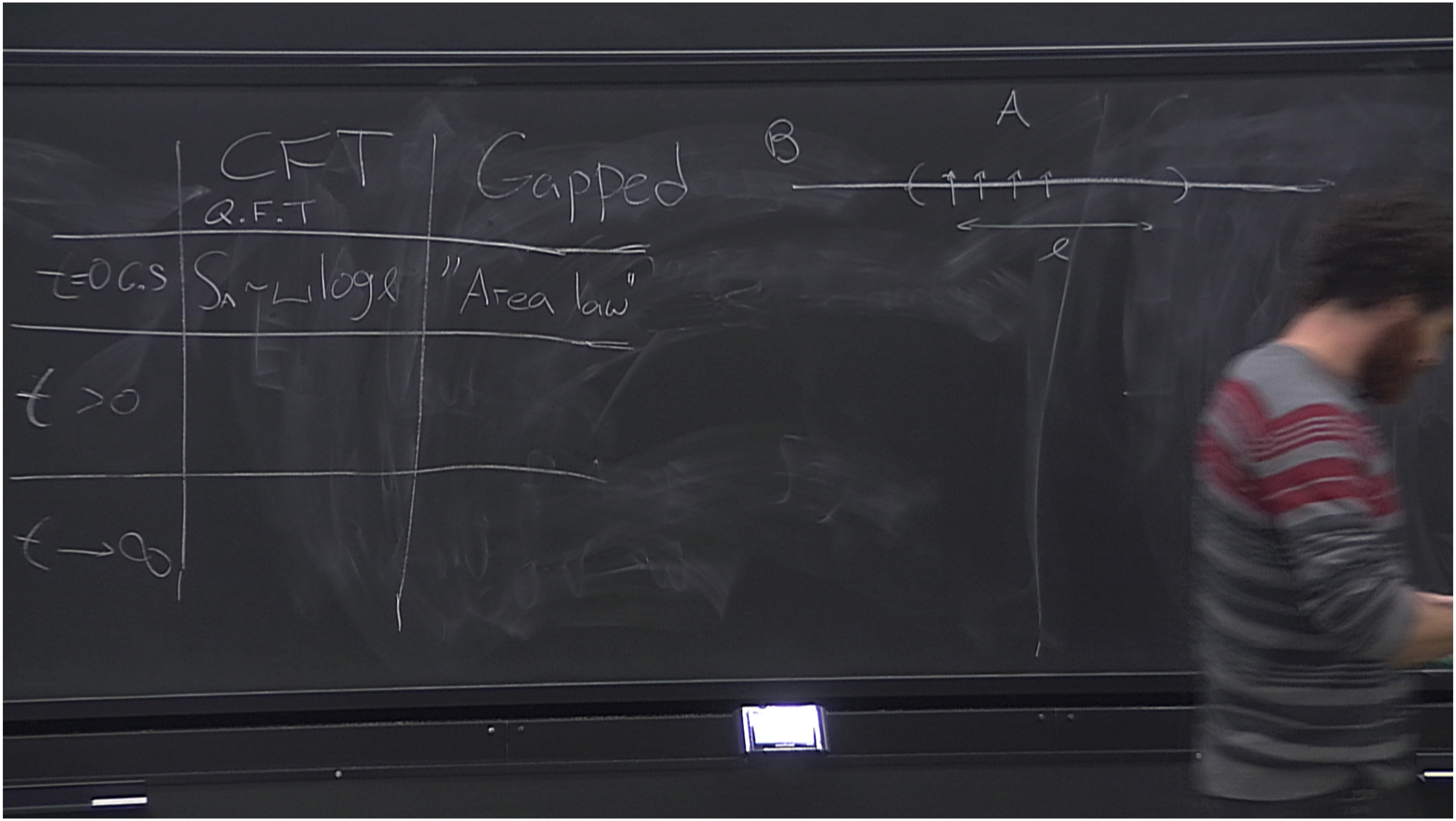

CFT
Gapped
A
ℓ

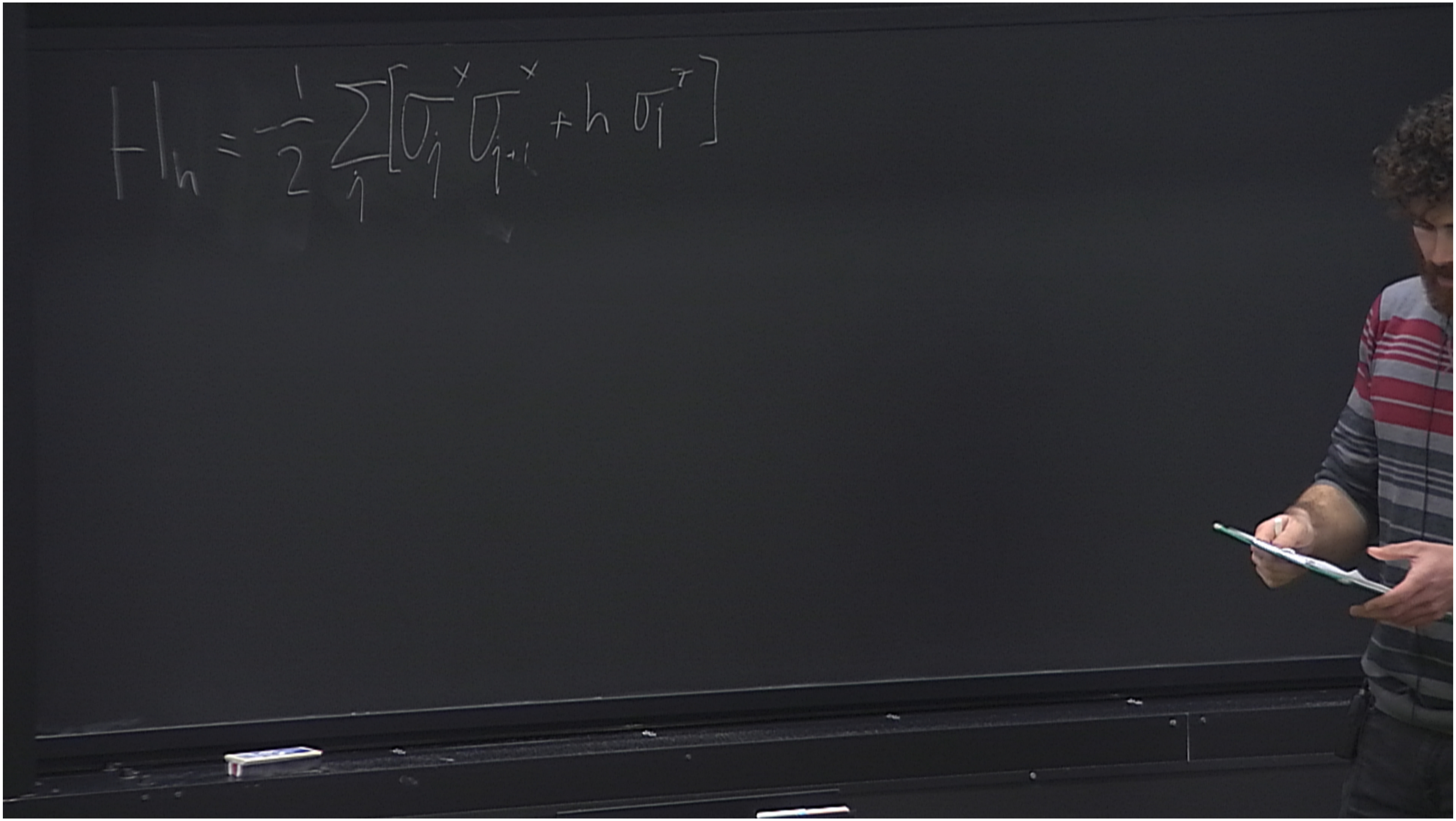

$S^A_\ell = -\mathrm{Tr}\,\rho_A \log \rho_A$
CFT
Q.F.T
Gapped
t=0 G.S
$S_A \sim \log \ell$

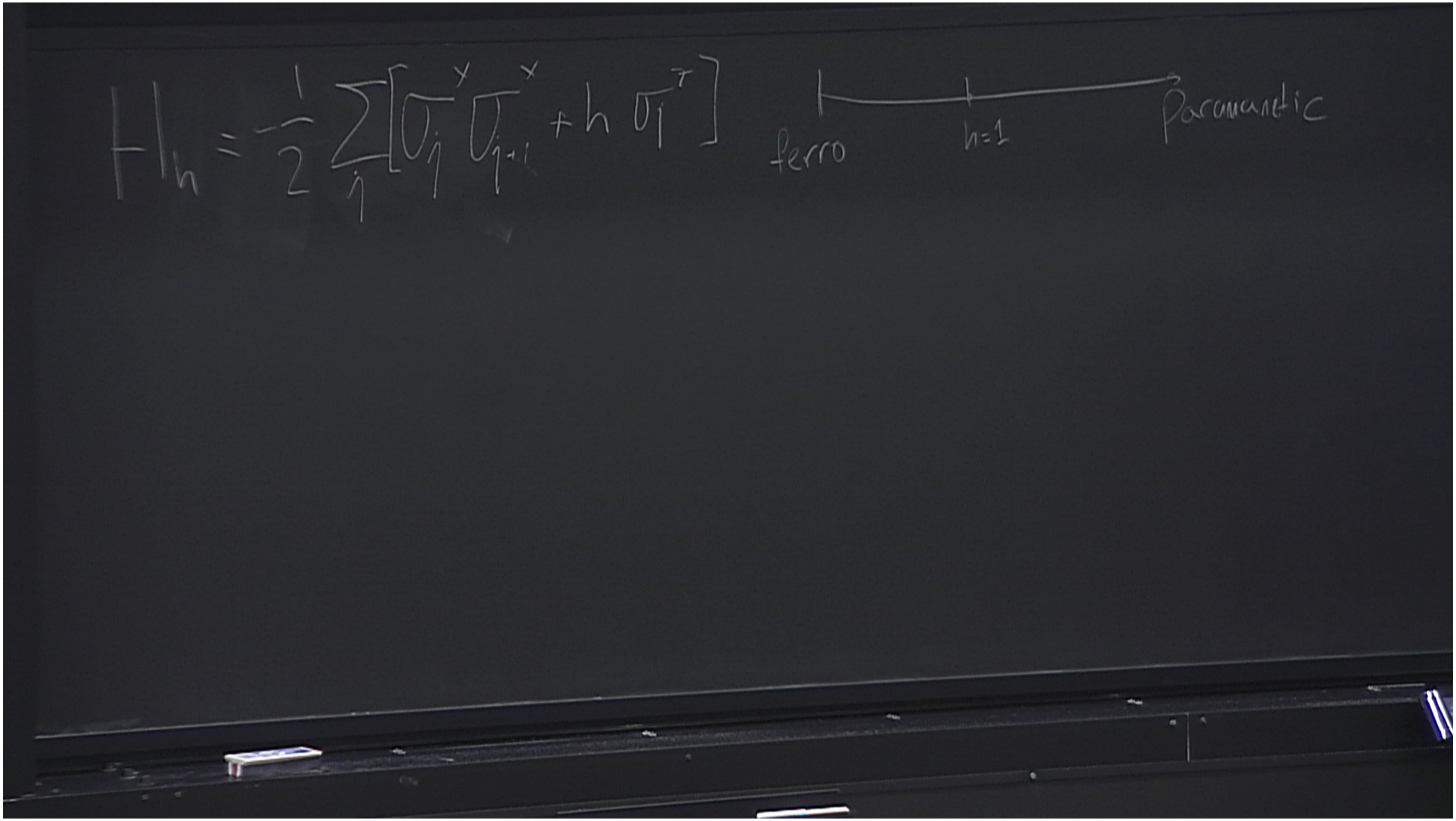


$S_\ell^A = -\mathrm{Tr}\,\rho_A \log \rho_A$
CFT
Q.F.T
Gappe
t=0 G.S
$S_A \sim \log \ell$
"Area law"
1
h

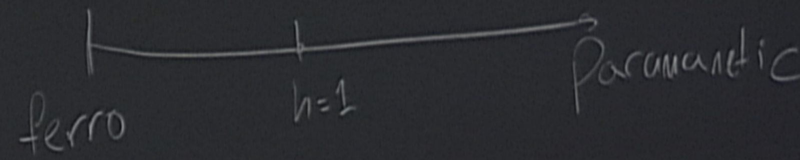

CFT
Q.F.T
Gapped
B
A
ℓ
t=0 G.S
S_A ~ L log ℓ
"Area law"


CFT
Q.F.T
Gapped
B
A
ℓ
t=0 G.S
"Area law"
t>0
t→∞


$H_h = -\frac{1}{2} \sum_i [\sigma_i^x \sigma_{i+1}^x + h \sigma_i^z]$


$H_h = -\frac{1}{2}\sum_i [\sigma_i^x \sigma_{i+1}^x + h\,\sigma_i^z]$
ferro
h=1
Paramanetic

$$H_h = -\frac{1}{2}\sum_j \left[\sigma_j^x \sigma_{j+1}^x + h\,\sigma_j^z\right]$$



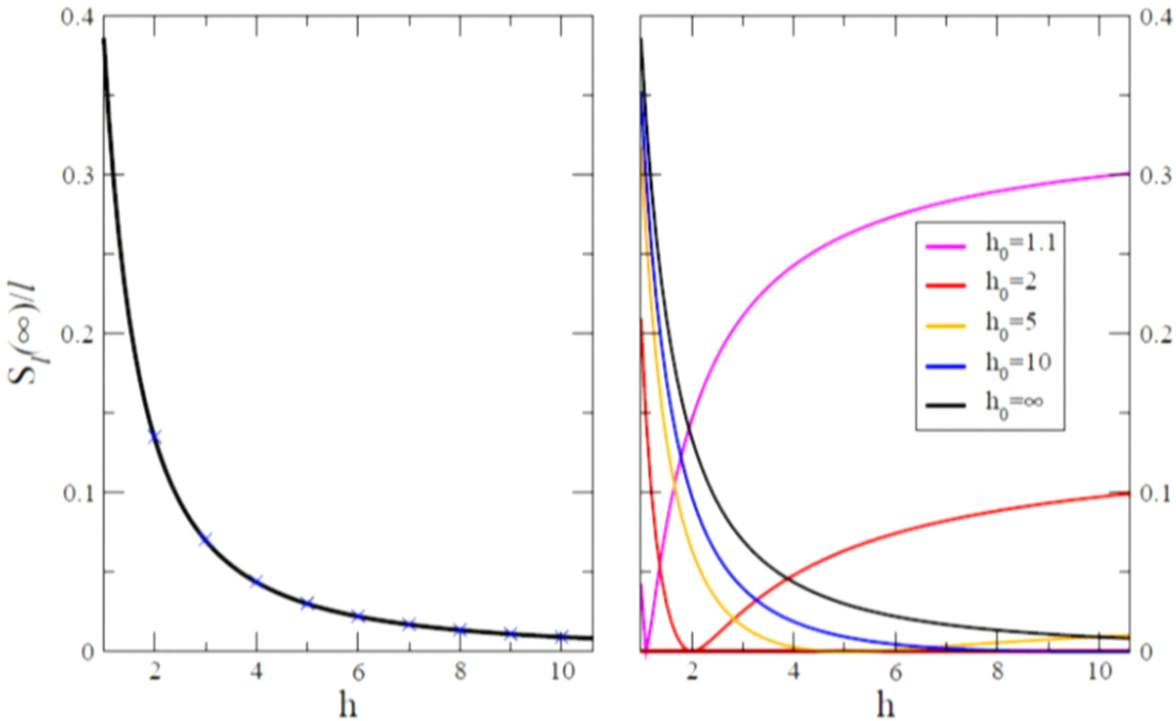
$$|\psi_0\rangle \qquad H_{h_0}|\psi_0\rangle = 0$$

$$H_{h_0} \longrightarrow H_h$$

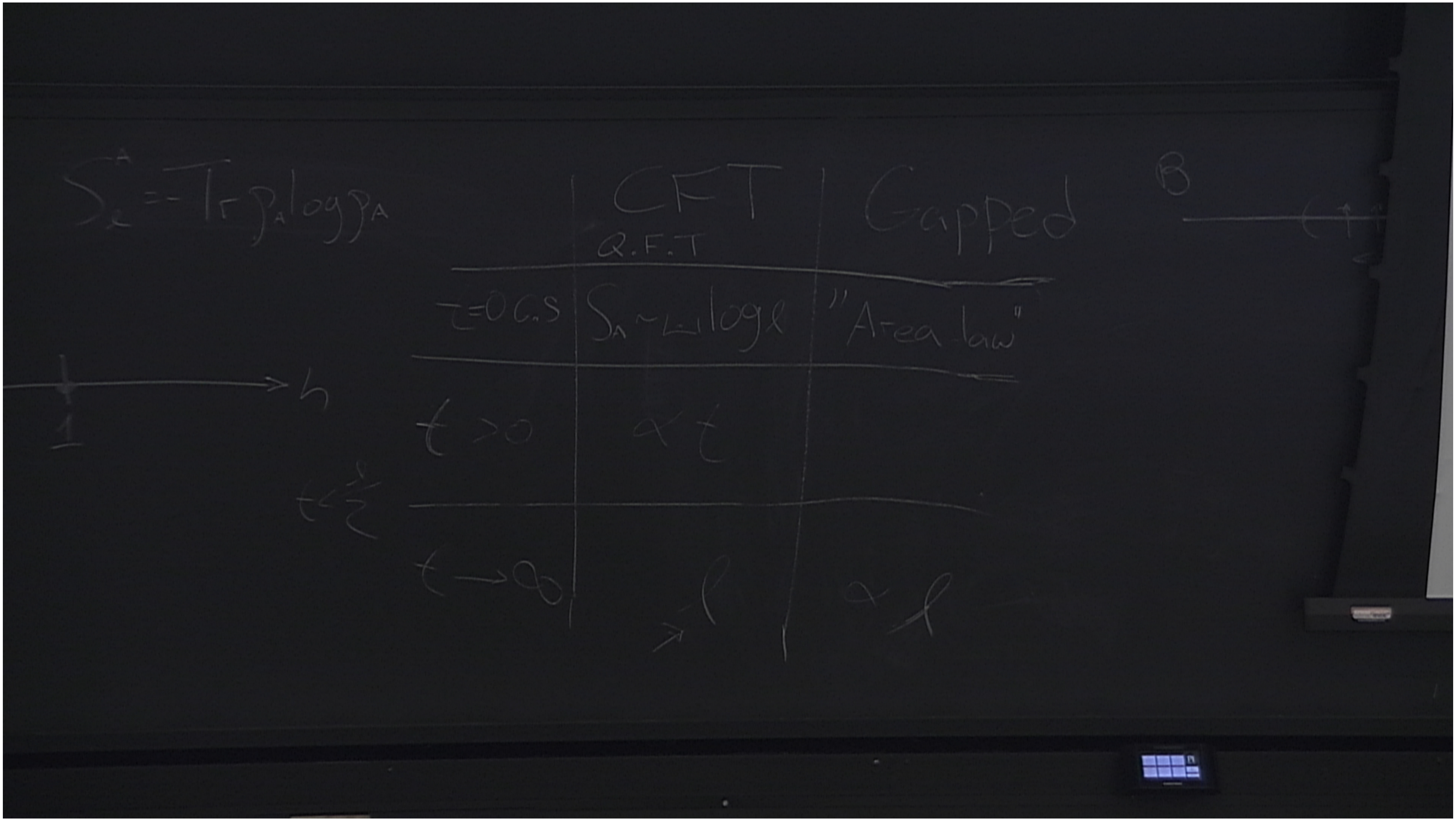
t-->inf

$$S_\ell(\infty) = \lim_{\epsilon\to 0^+} \frac{\ell}{2\pi}\frac{1}{4\pi i}\int_0^{2\pi} d\varphi \oint_C d\lambda\, e(1+\epsilon,\lambda)\frac{2\lambda}{\lambda^2 - \frac{(1-\cos\varphi(h+h_0)+hh_0)^2}{(h^2+1-2h\cos\varphi)(h_0^2+1-2h_0\cos\varphi)}}$$

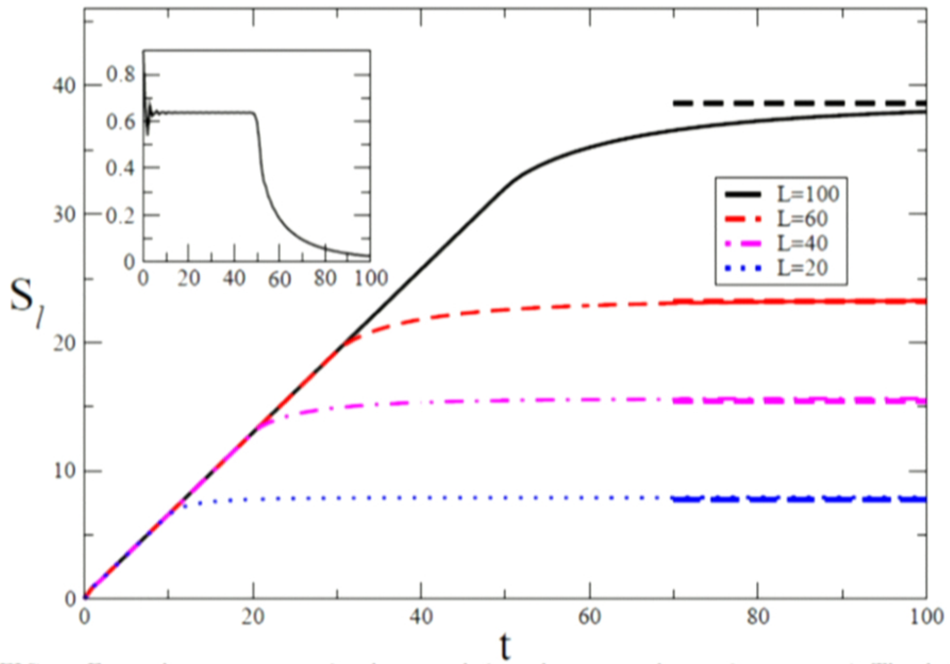
$$= \frac{\ell}{2\pi}\int_0^{2\pi} d\varphi H\left(\frac{1-\cos\varphi(h+h_0)+hh_0}{\sqrt{(h^2+1-2h\cos\varphi)(h_0^2+1-2h_0\cos\varphi)}}\right).$$



John Cardy, Pasquale Calabrese, “Evolution of Entanglement Entropy in One-Dimentional Syatems”

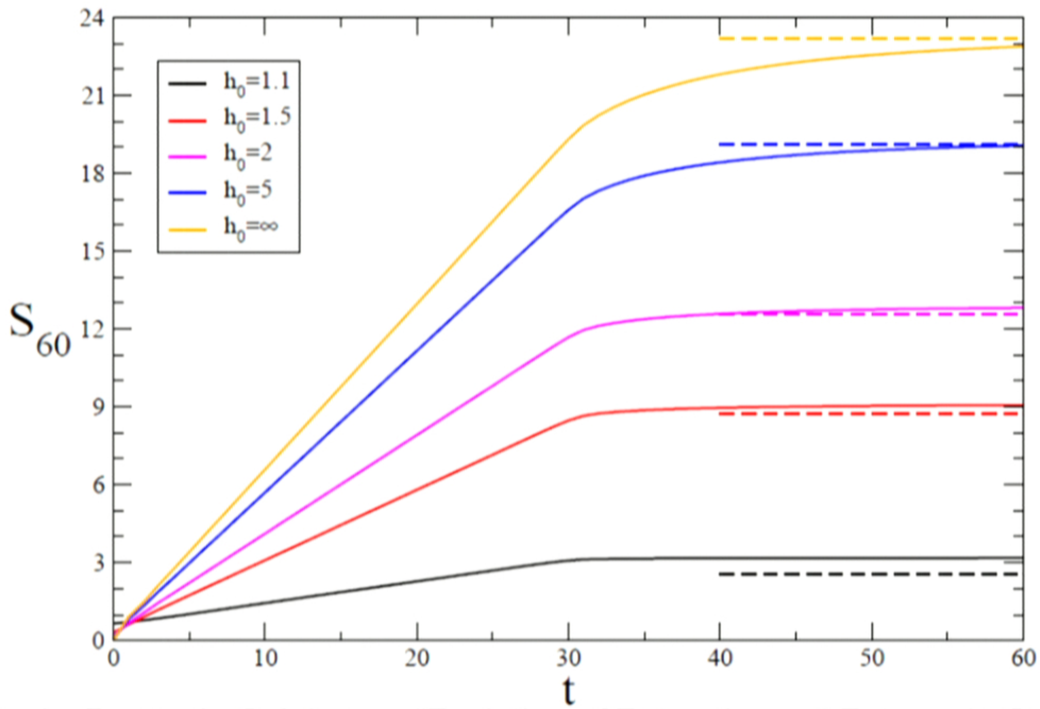

S = -Tr ρA log ρA
CFT
Q.F.T
Gapped
t=0 G.S
SA ~ log ℓ
"Area law"
t>0
∝ t
t→∞
ℓ
∝ ℓ

h0=inf , h=1



John Cardy, Pasquale Calabrese, “Evolution of Entanglement Entropy in One-Dimentional Syatems”

h0>1 , h=1



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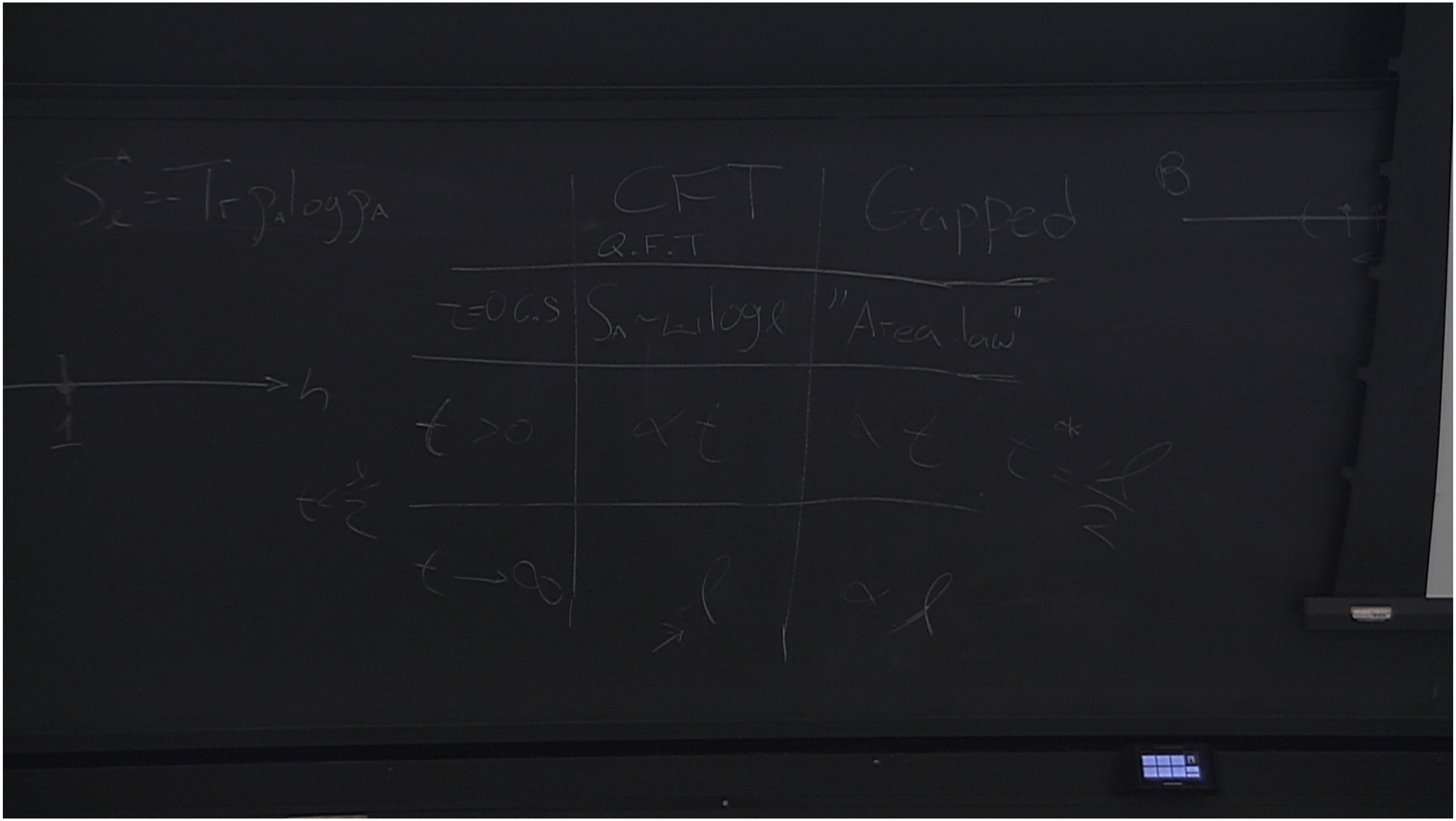
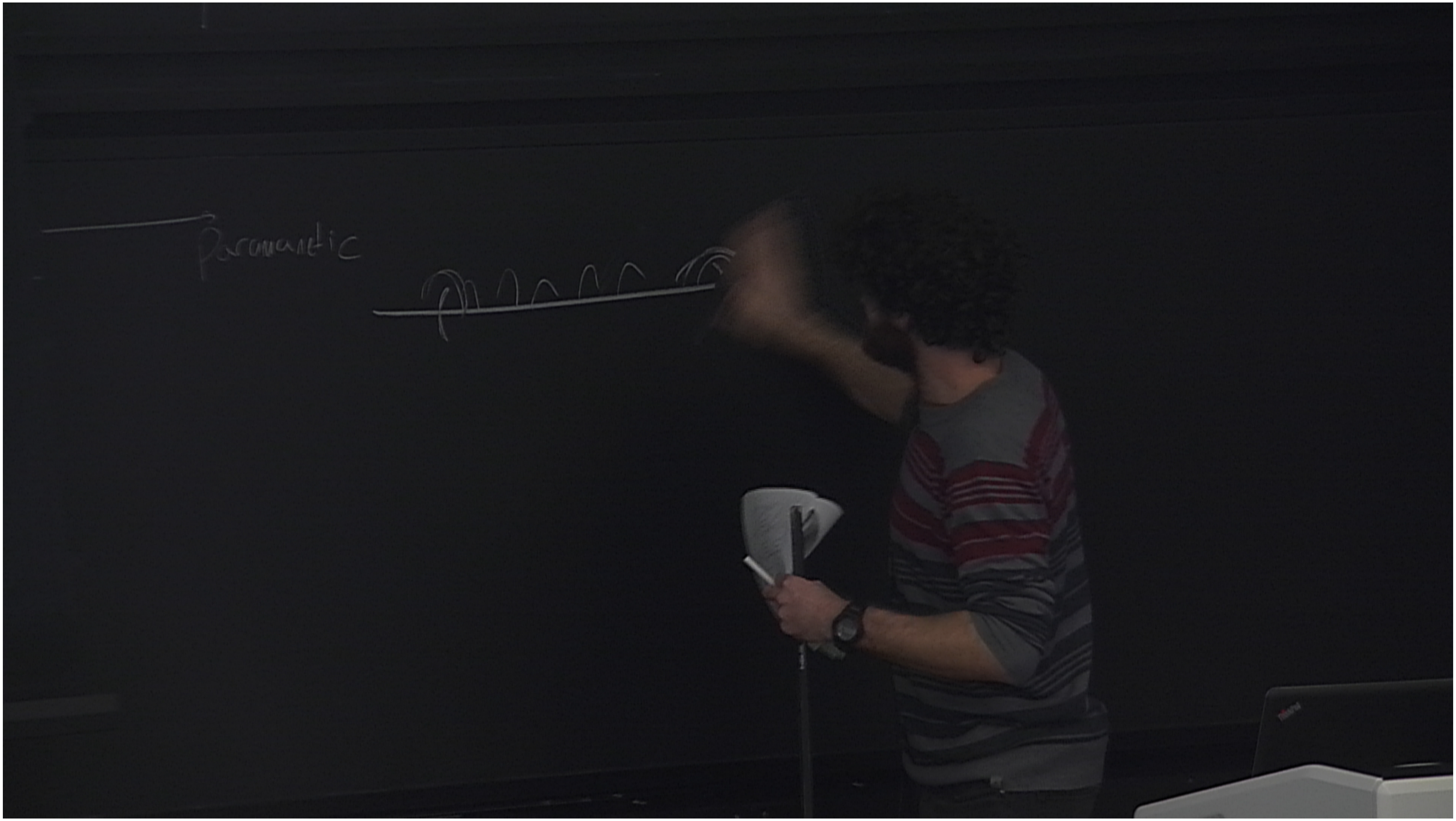
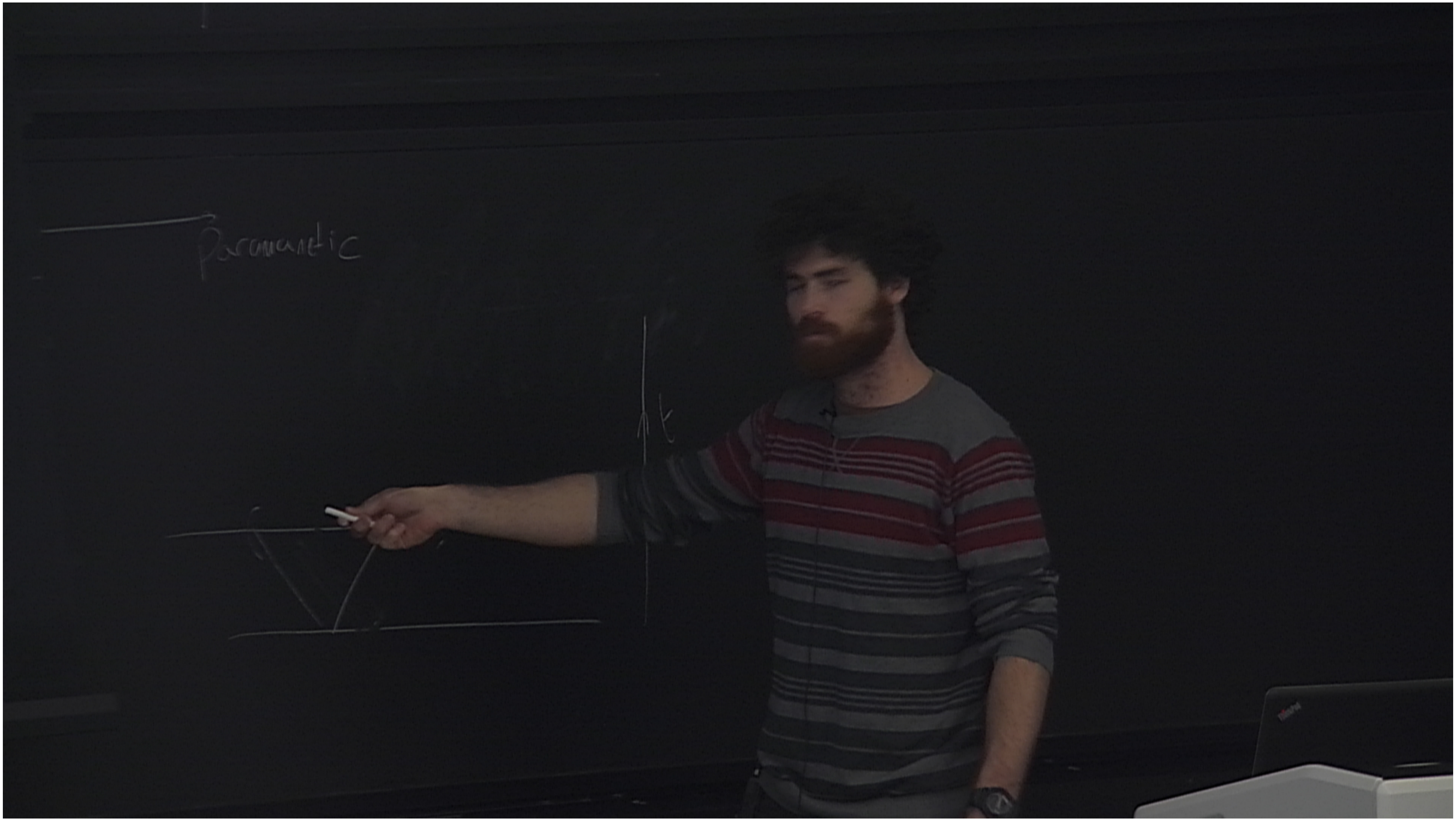
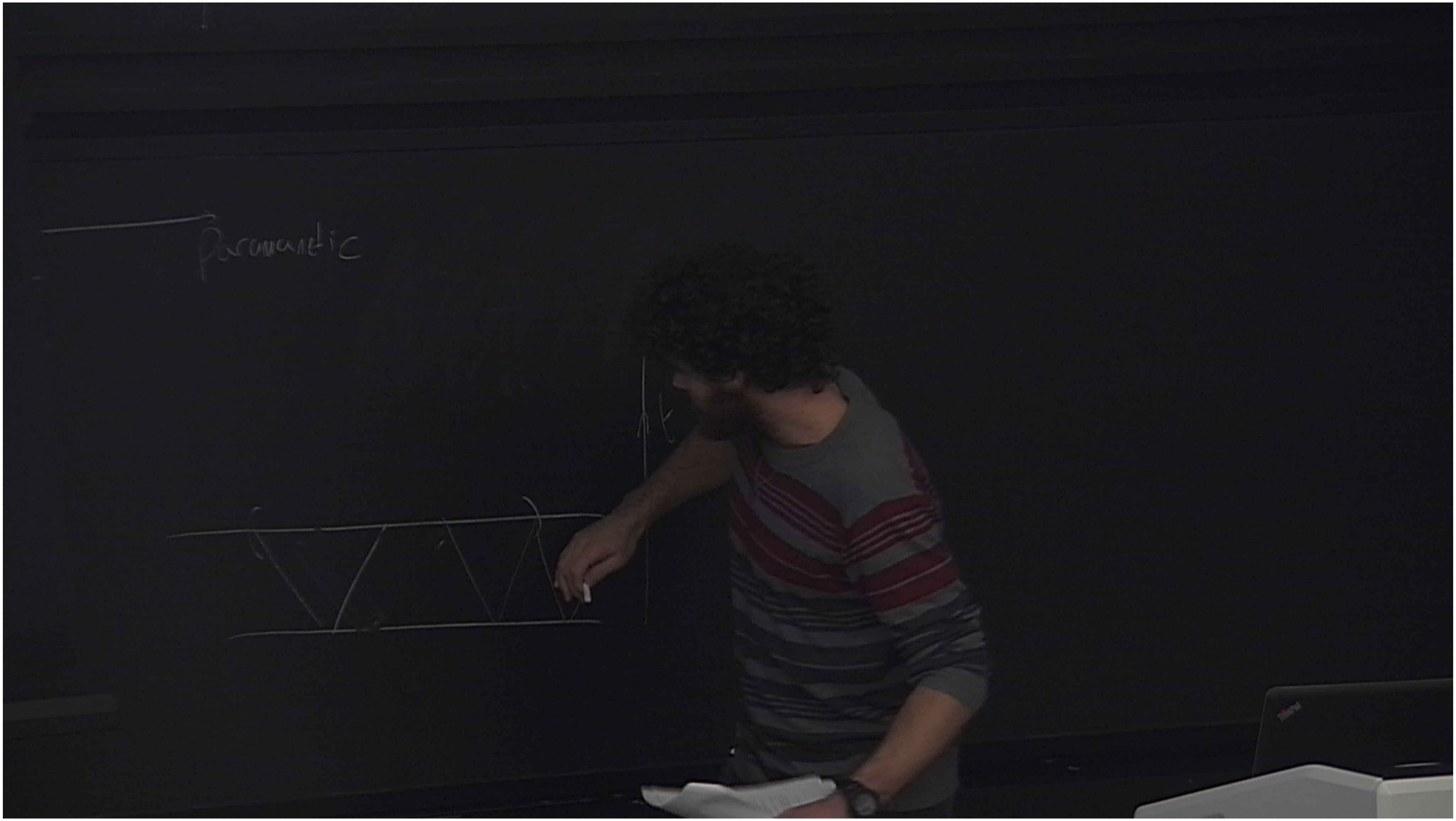
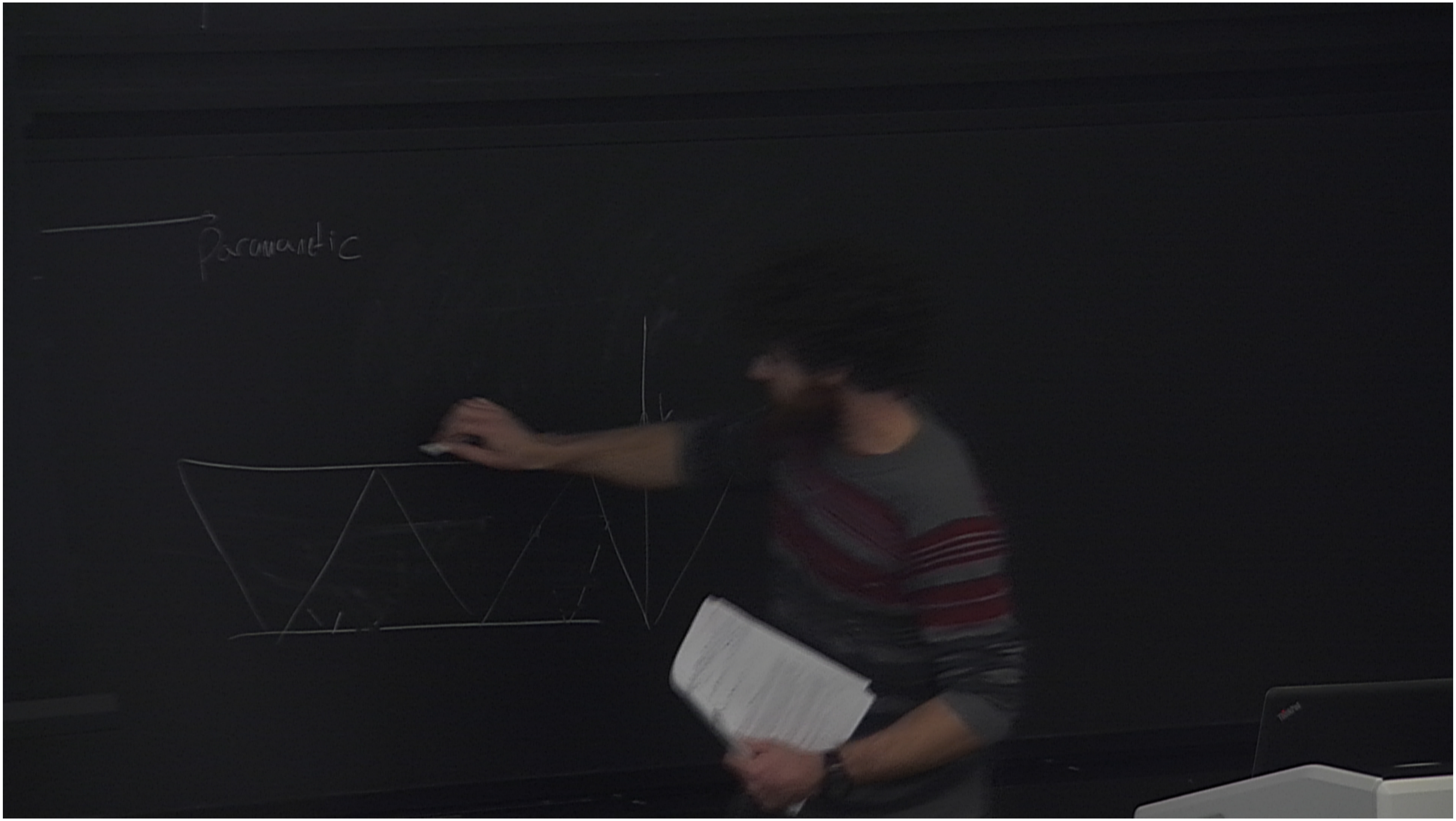
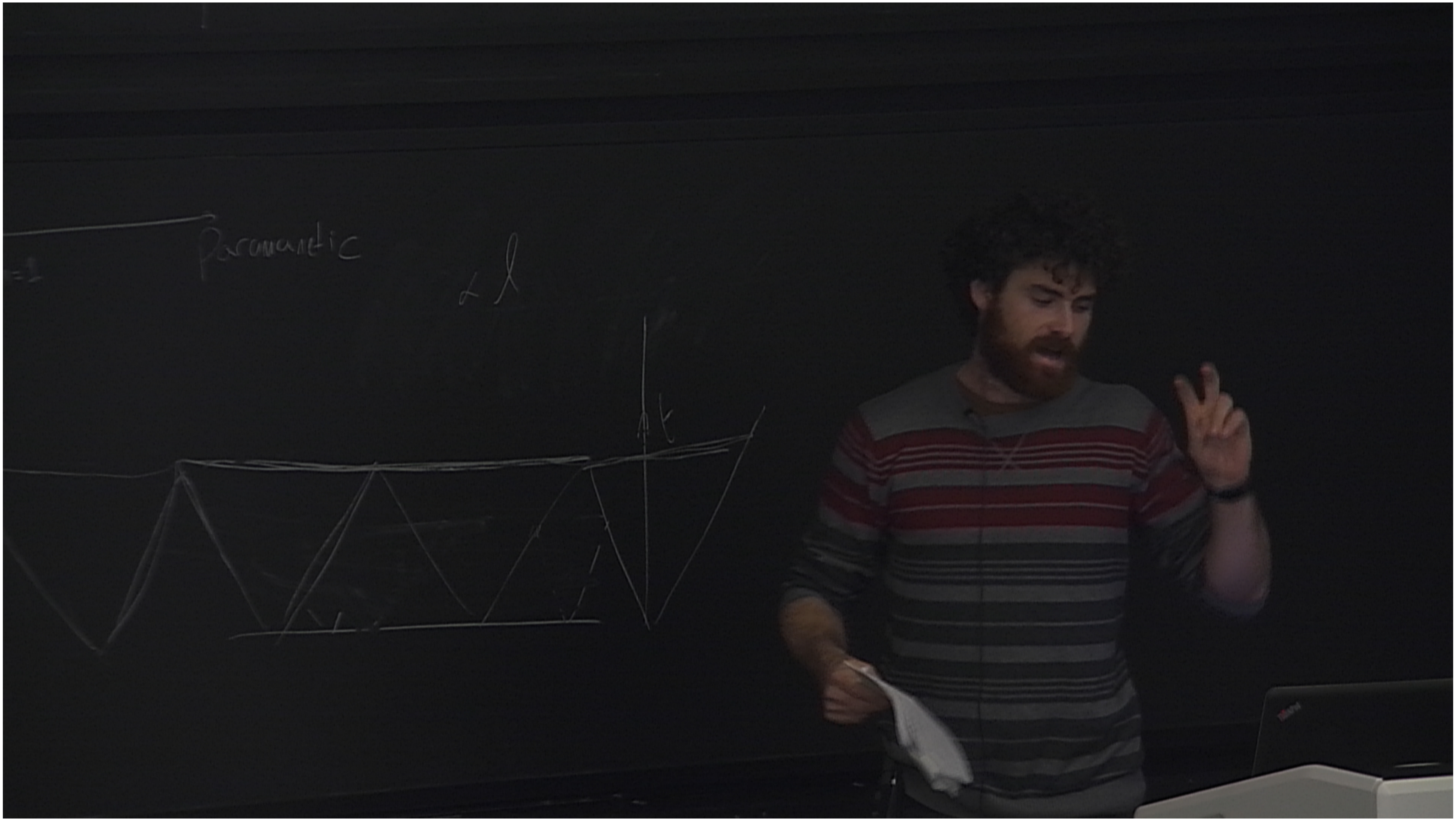

S_A = -Tr ρ_A log ρ_A
CFT
Q.F.T
Gapped
t=0 G.S
"Area law"
t > 0
t → ∞

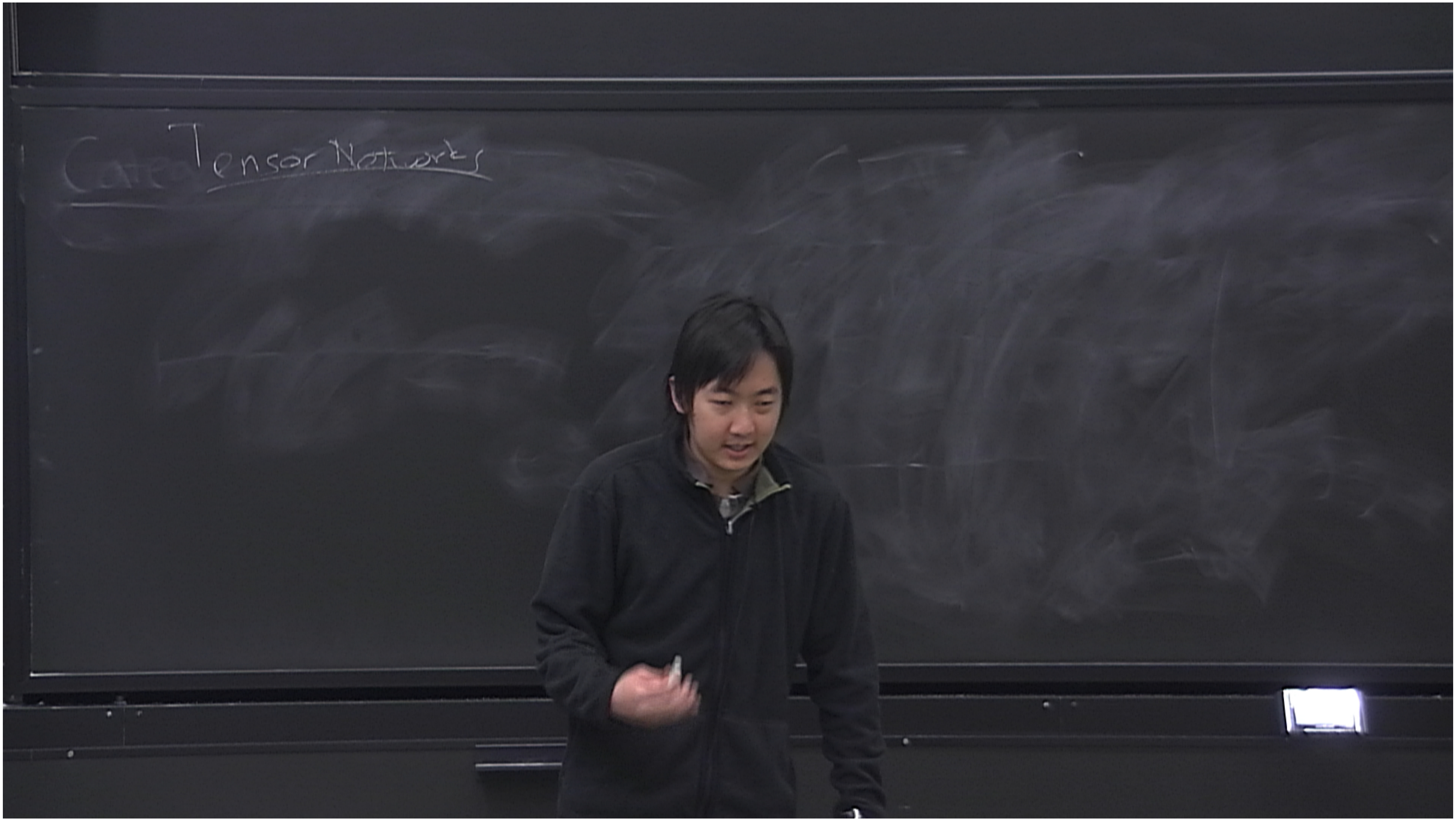

Paramagnetic

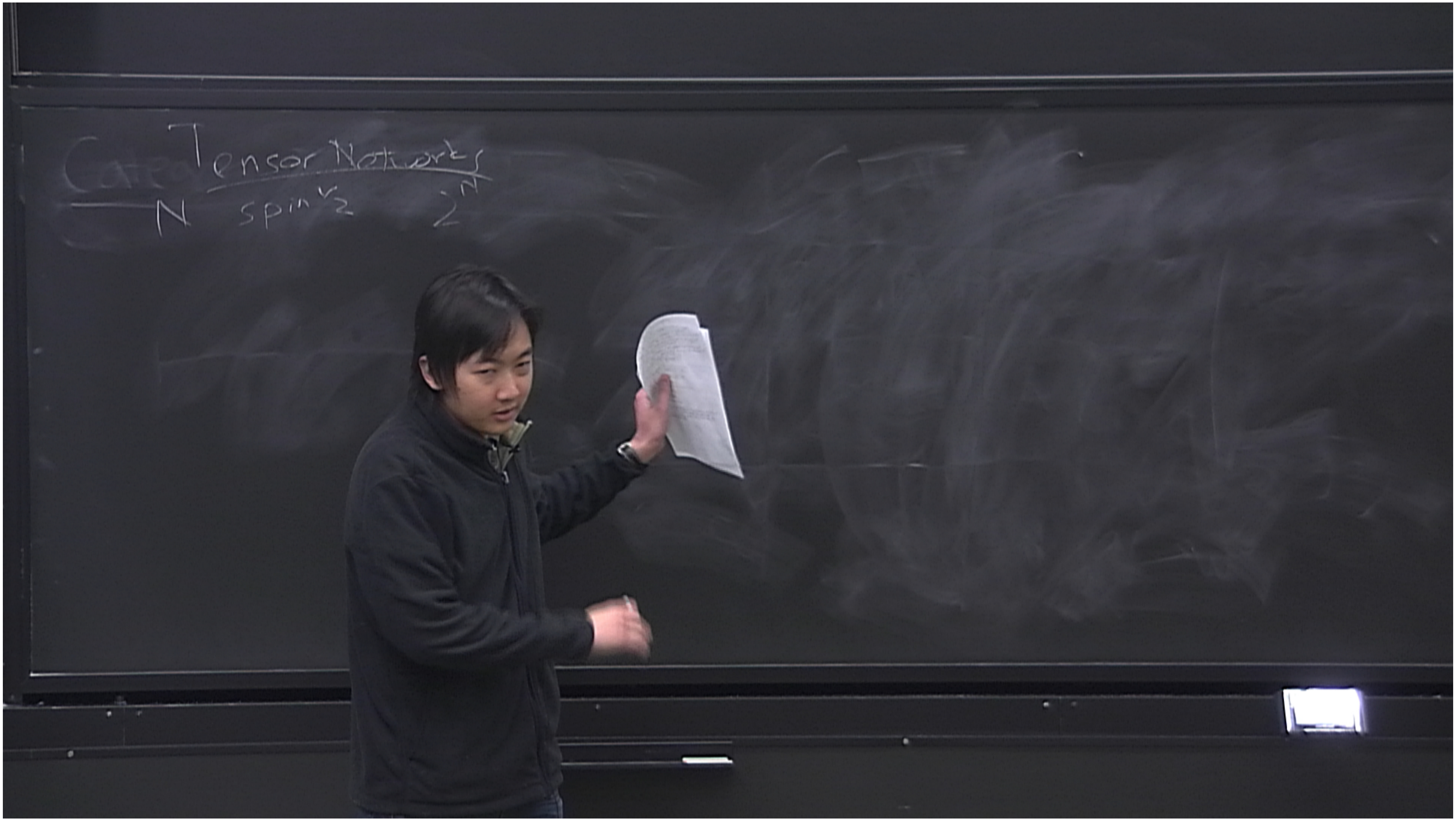

Paramagnetic

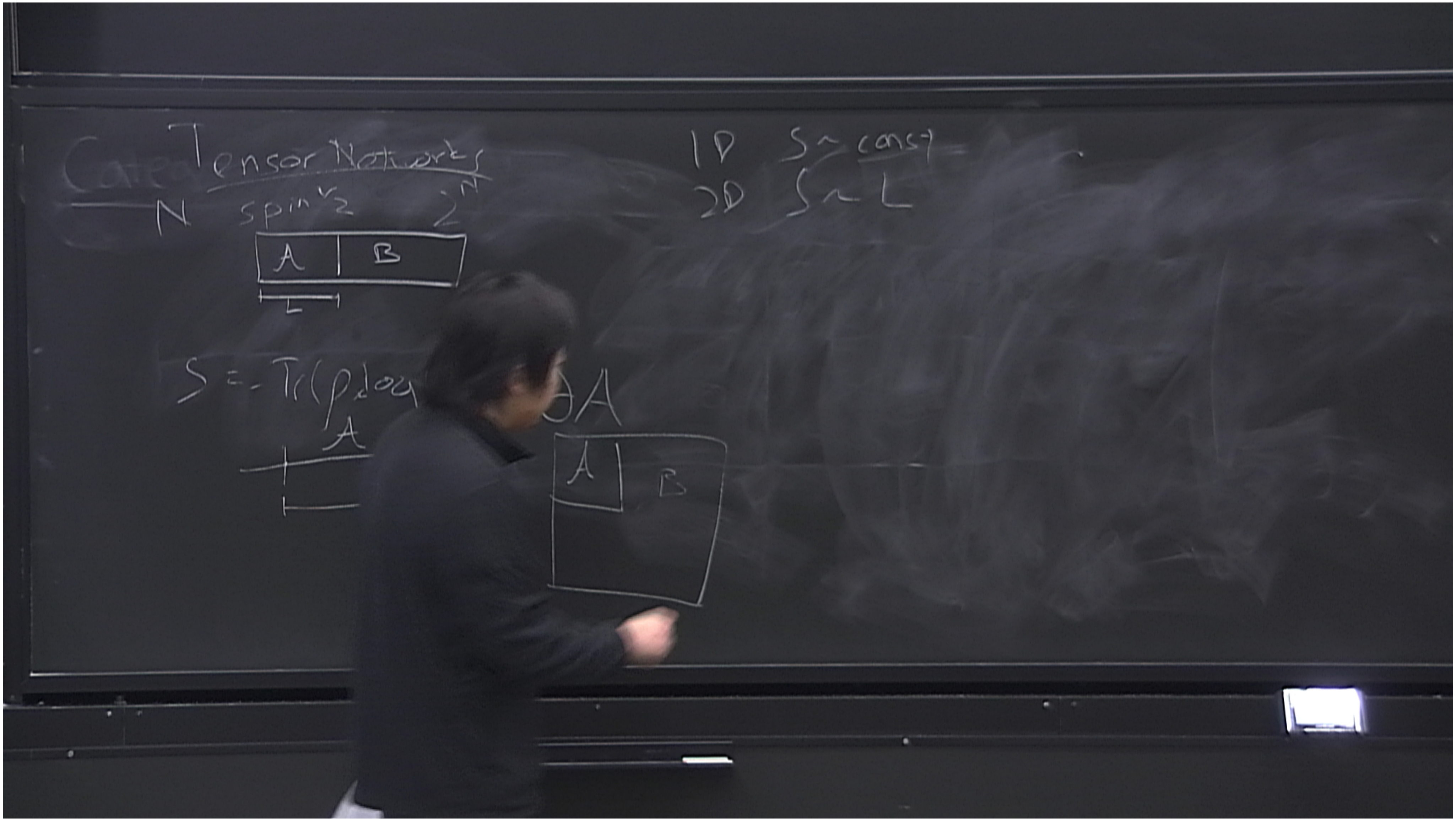

Paramagnetic
t


Paramagnetic

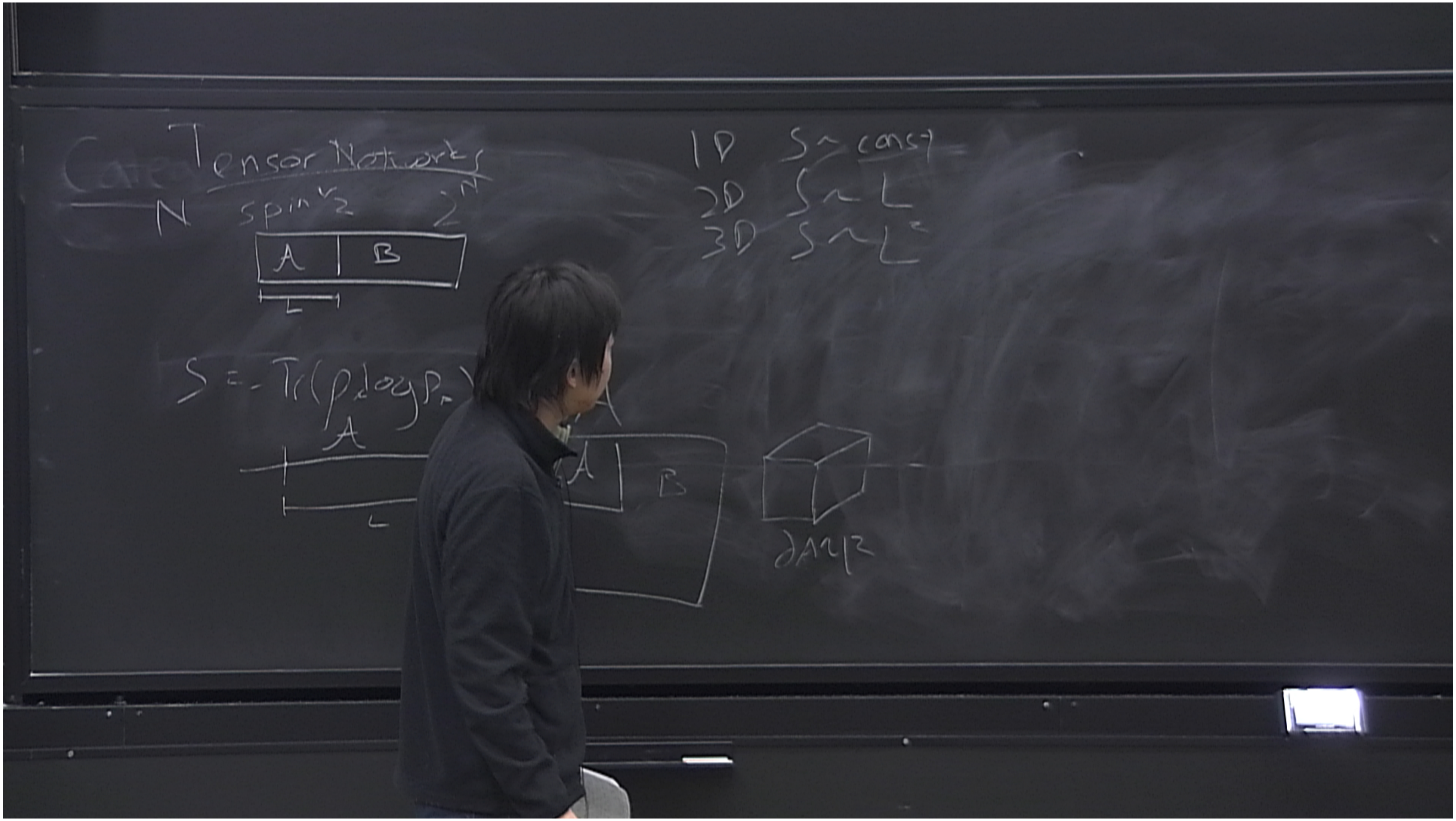

Paramagnetic

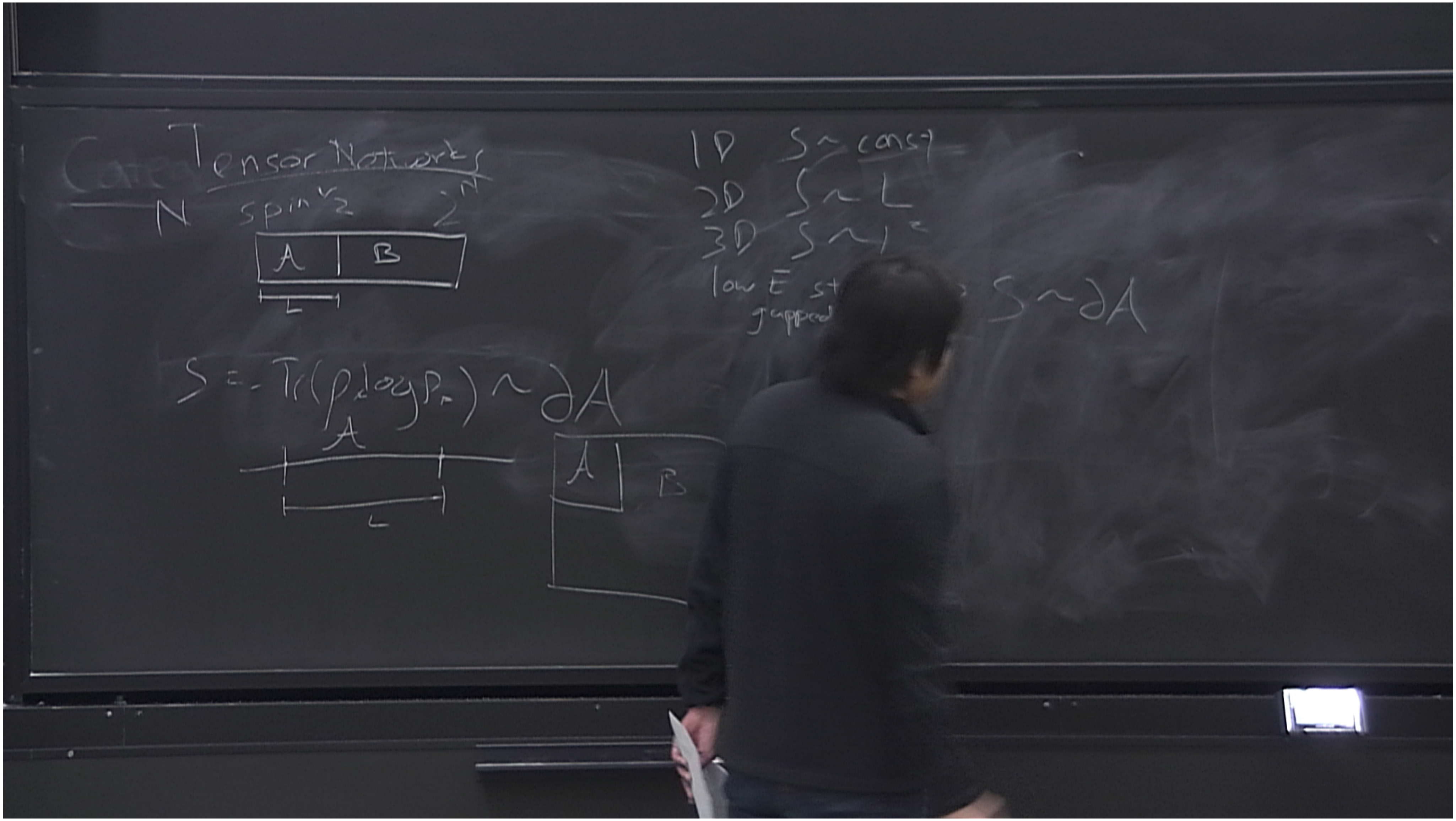

Tensor Networks

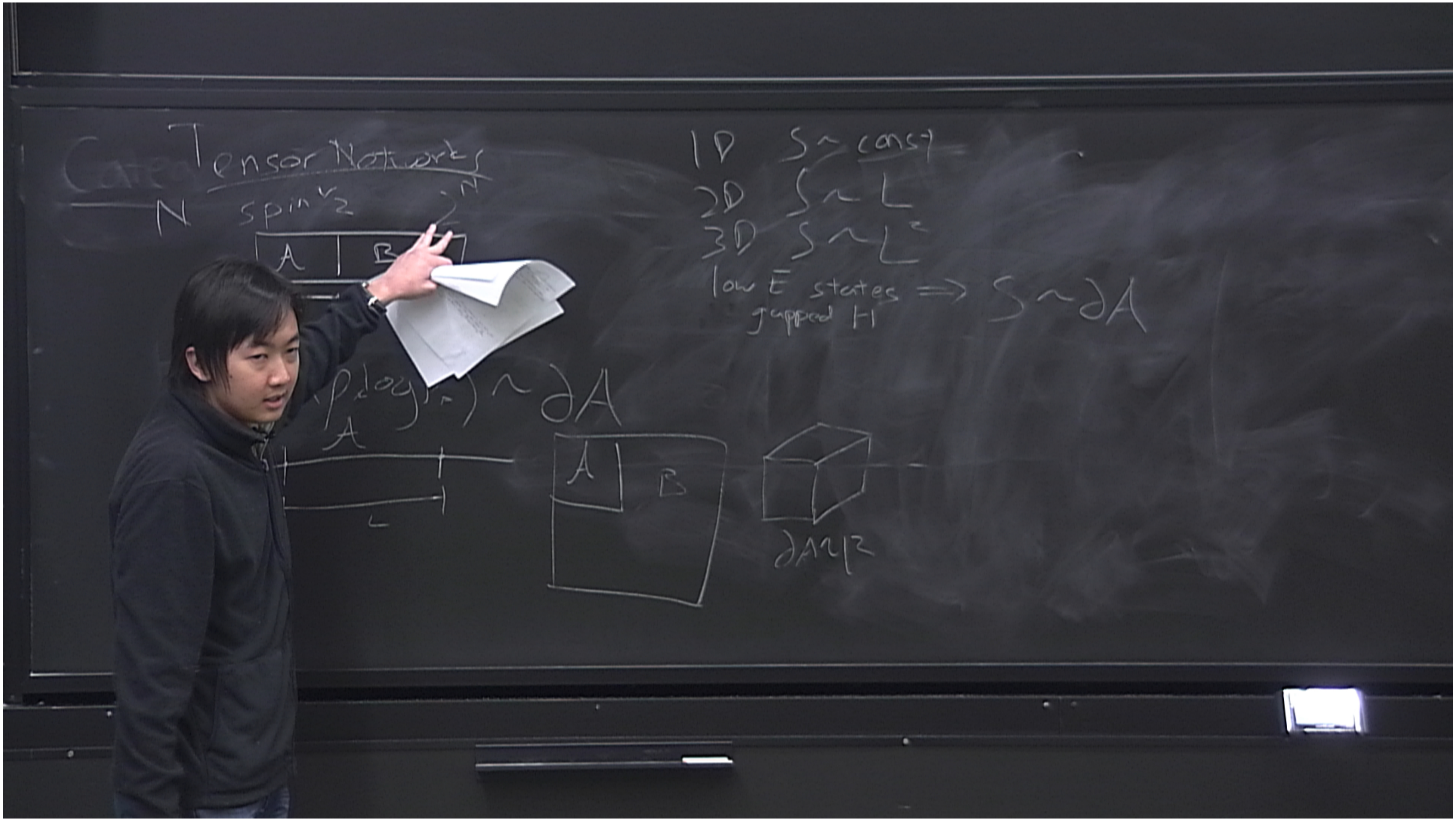

Tensor Networks
N spin ½ 2^N

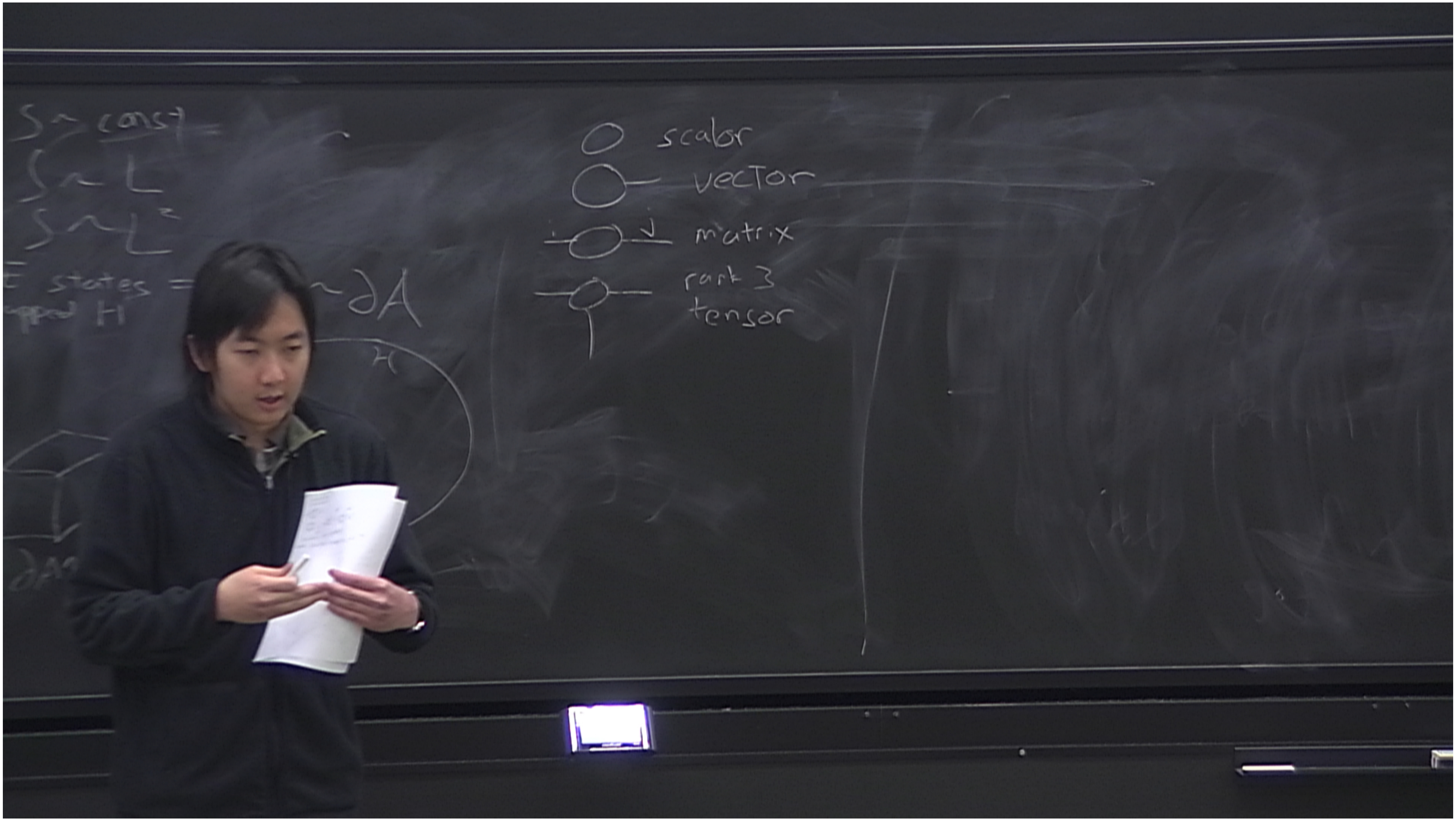


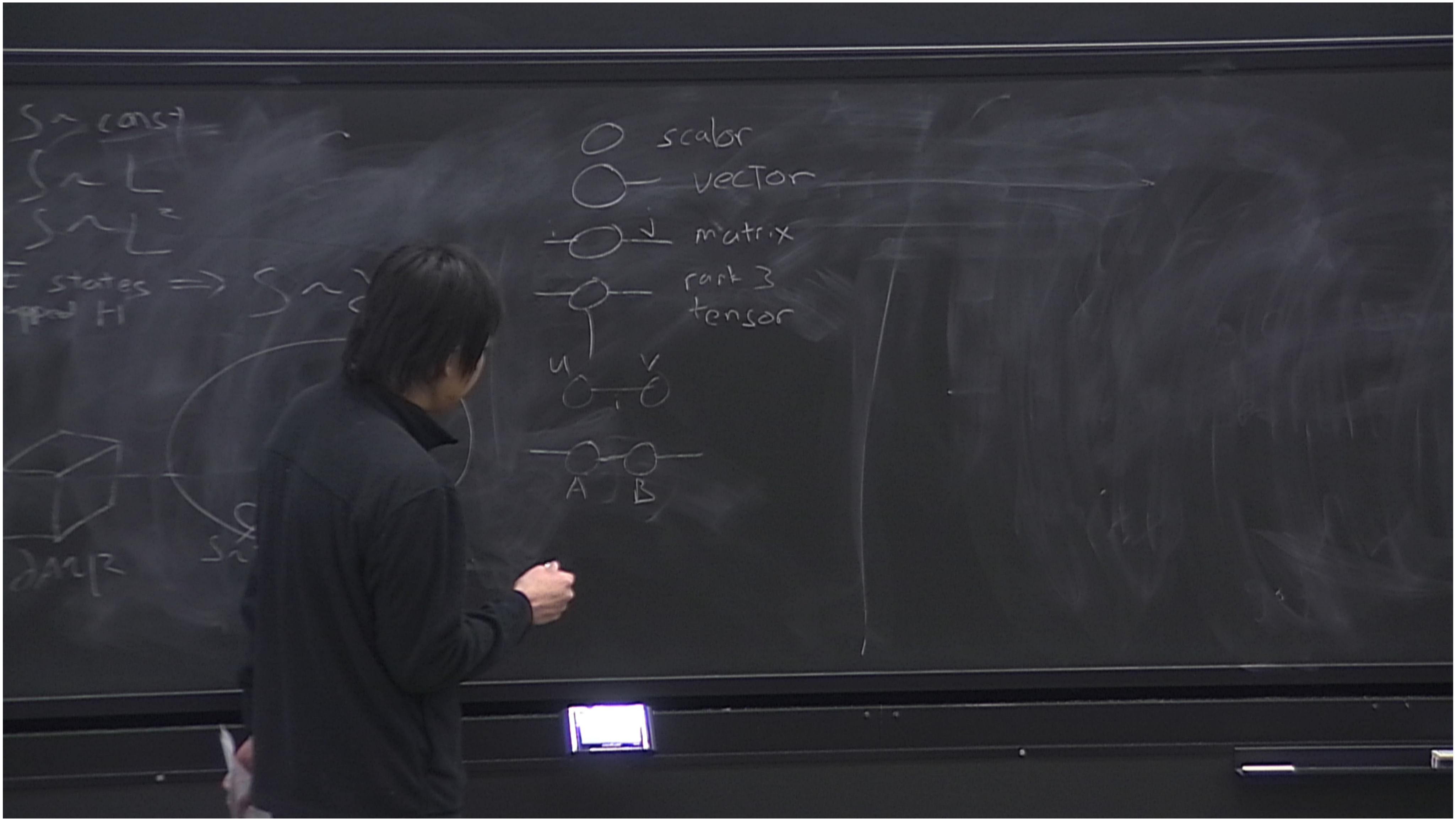
Tensor Networks
N spin ½ 2^N
A B
L
1D S ~ const
2D S ~ L
S = - Tr(ρ_A log ρ_A)
A
A B


Tensor Networks
N spin ½ 2^N
A B
L
S = - Tr(ρ_A log ρ_A)
A
L
1D S ~ const
2D S ~ L
3D S ~ L²
A B
∂A ~ L²


Tensor Networks
N spin ½ 2^N
A
B
L
1D S ~ const
2D S ~ L
S ~ ∂A
S = - Tr(ρA log ρA) ~ ∂A


Tensor Networks
N spin ½
A B
1D S ~ const
2D S ~ L
3D S ~ L²
low E states ⟹ S ~ ∂A
gapped H
∂A
A B
L


scalar
vector
matrix
rank 3
tensor



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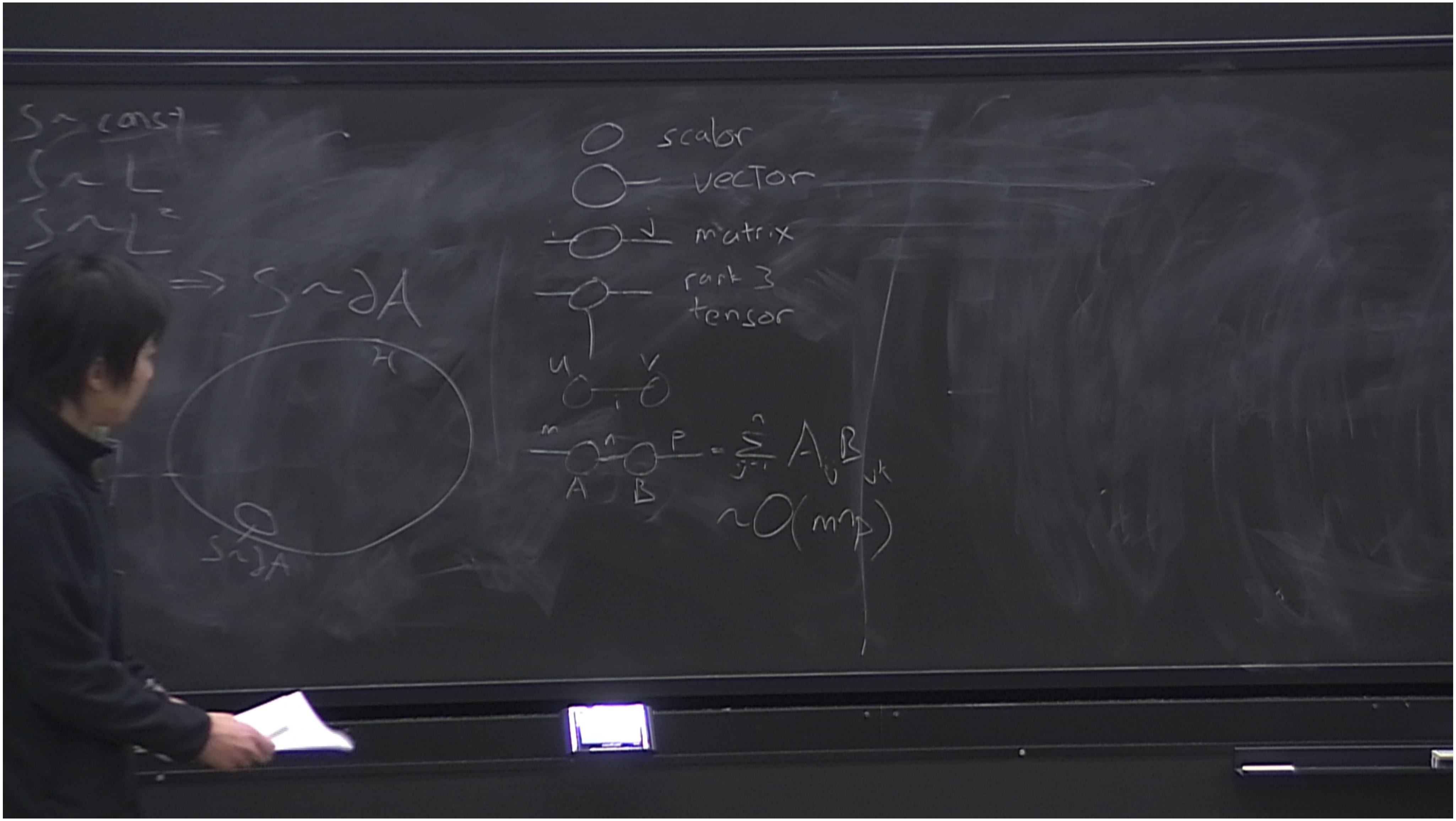
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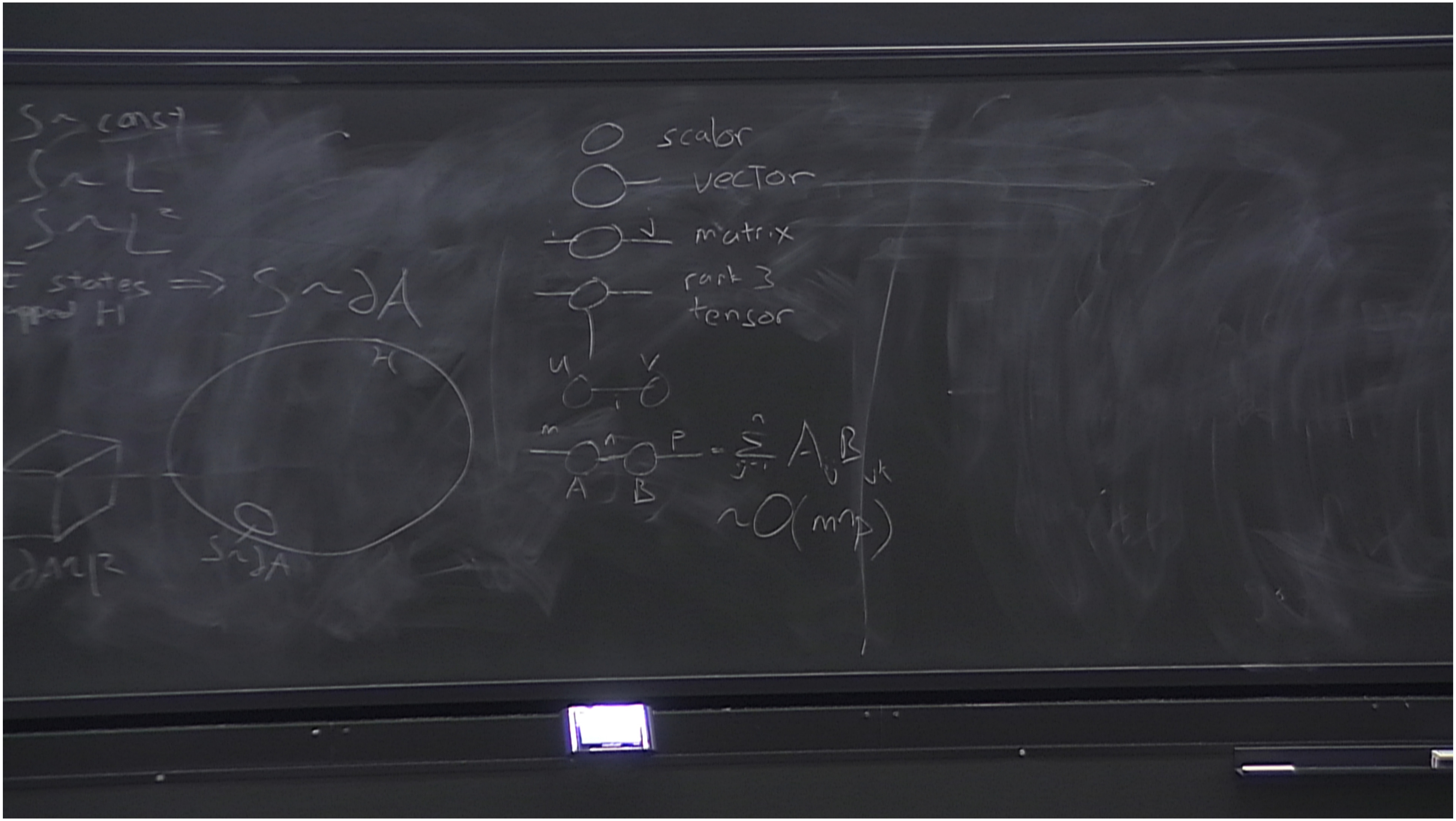

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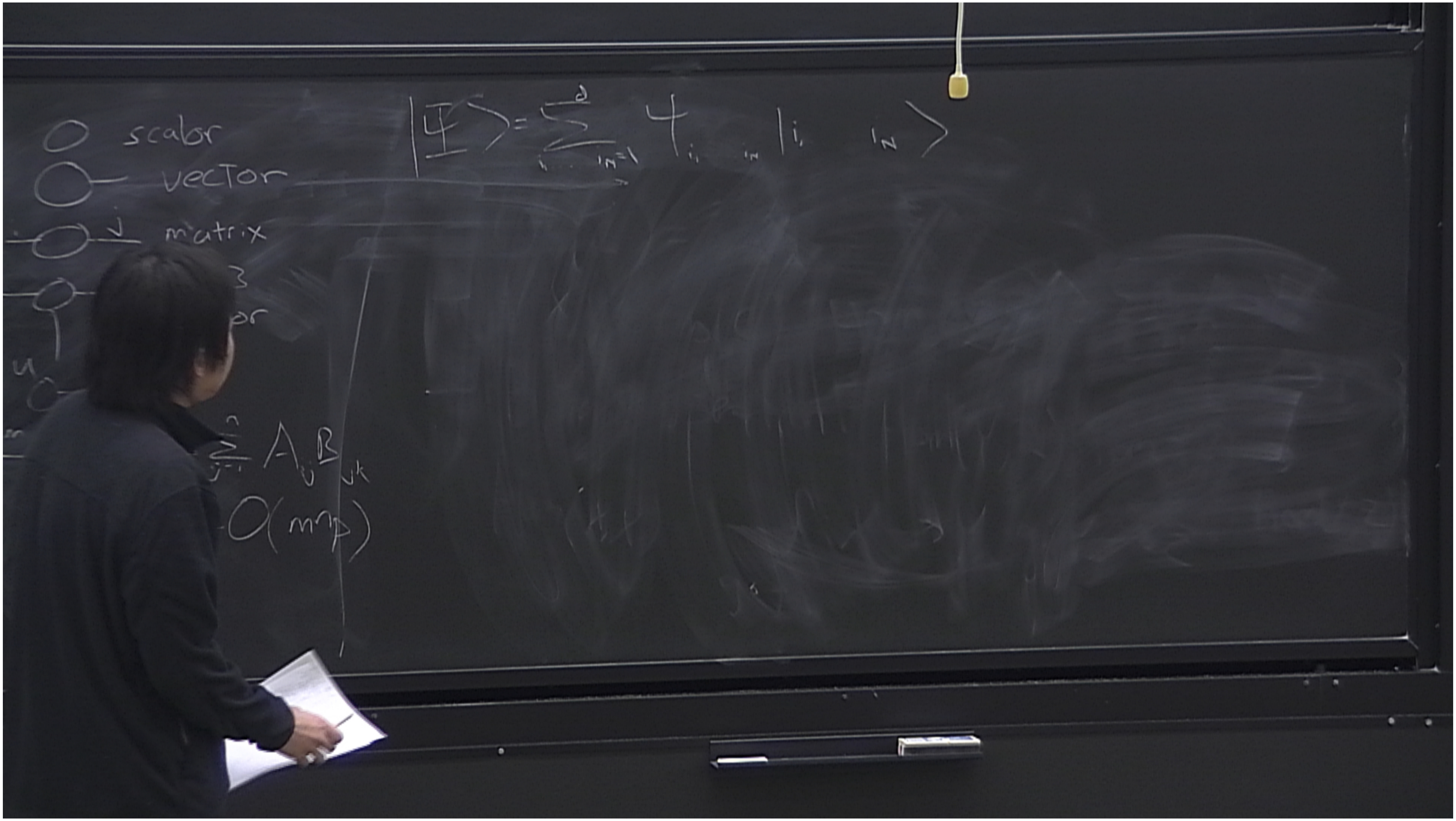
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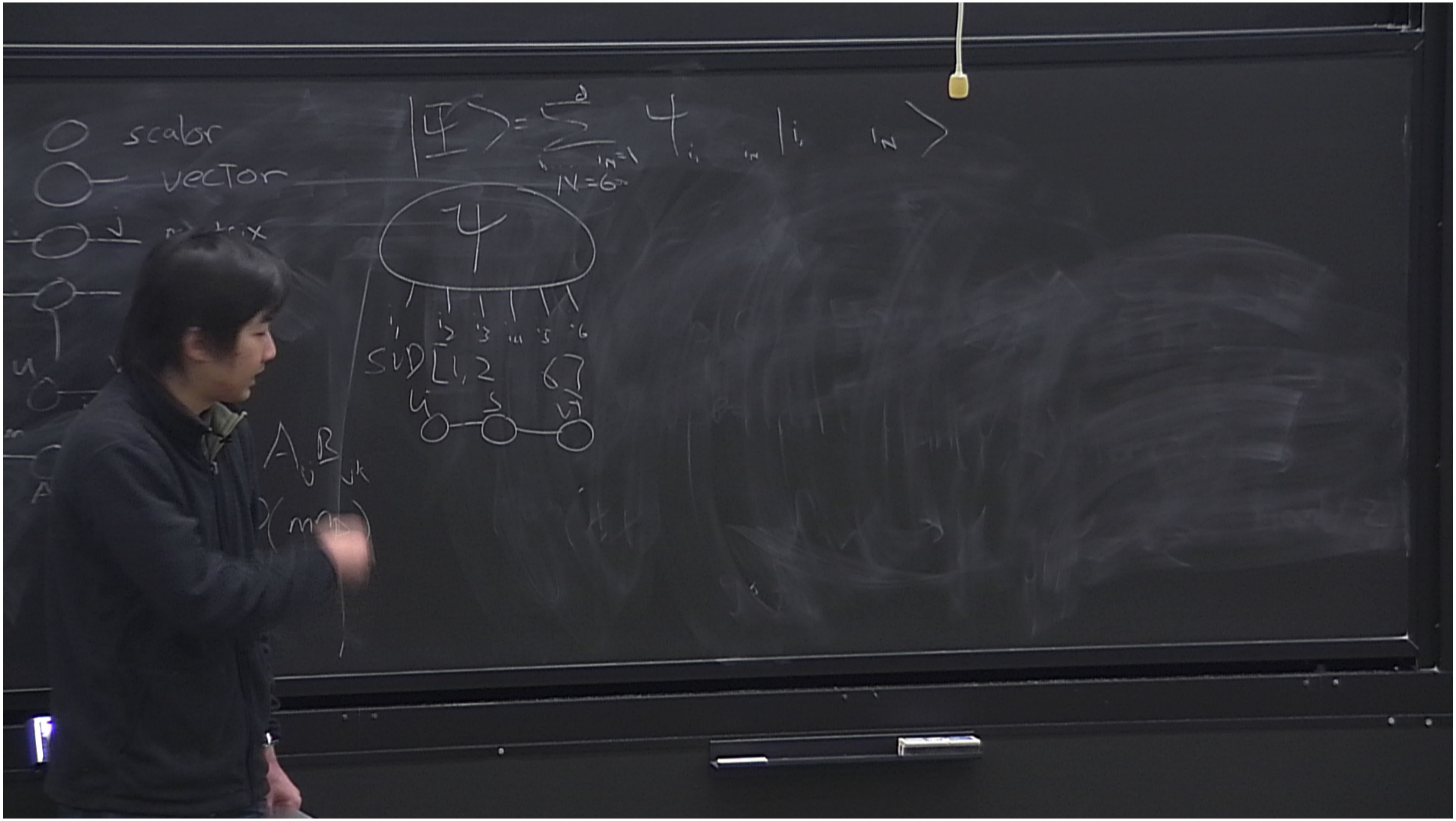
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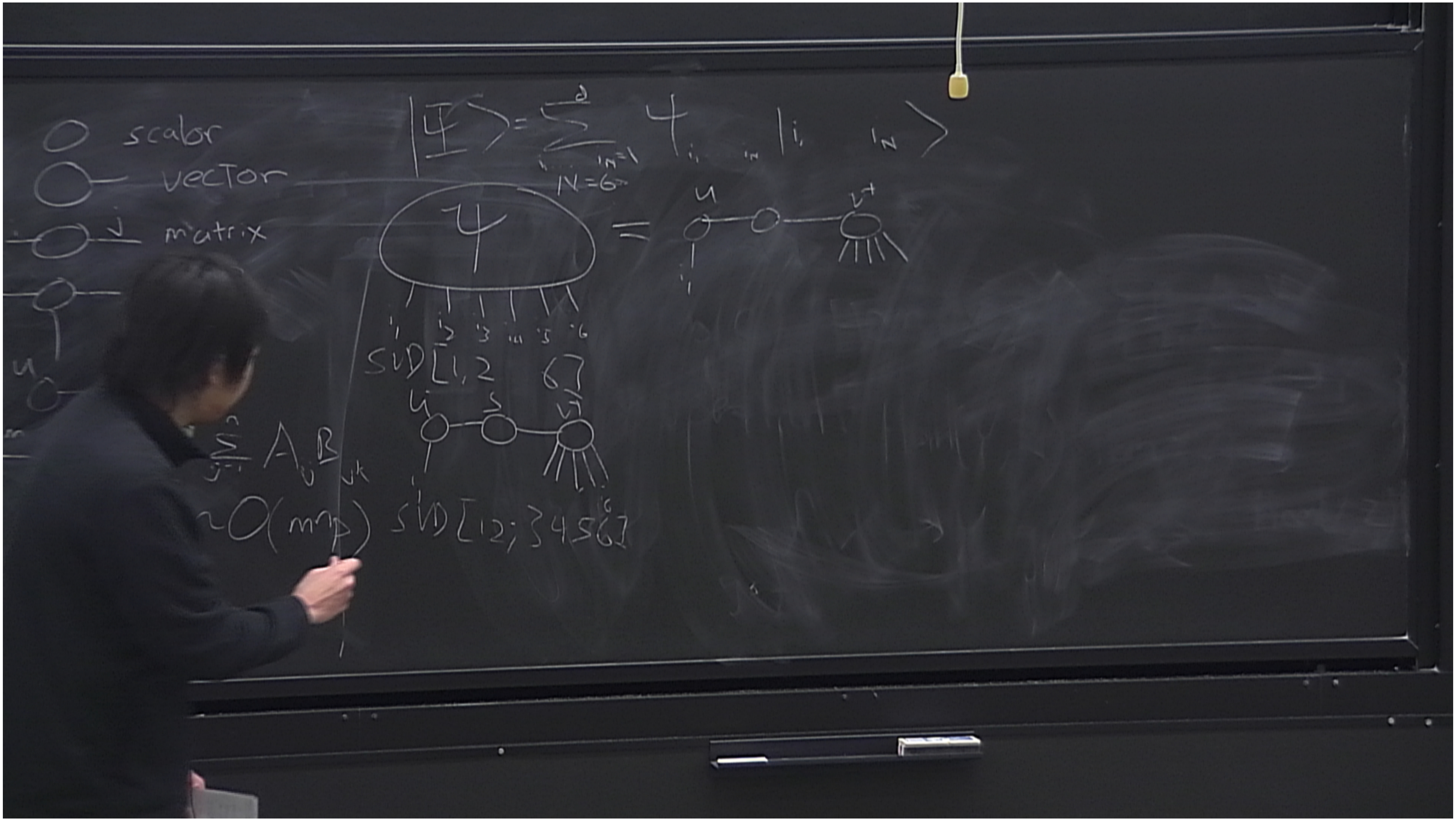
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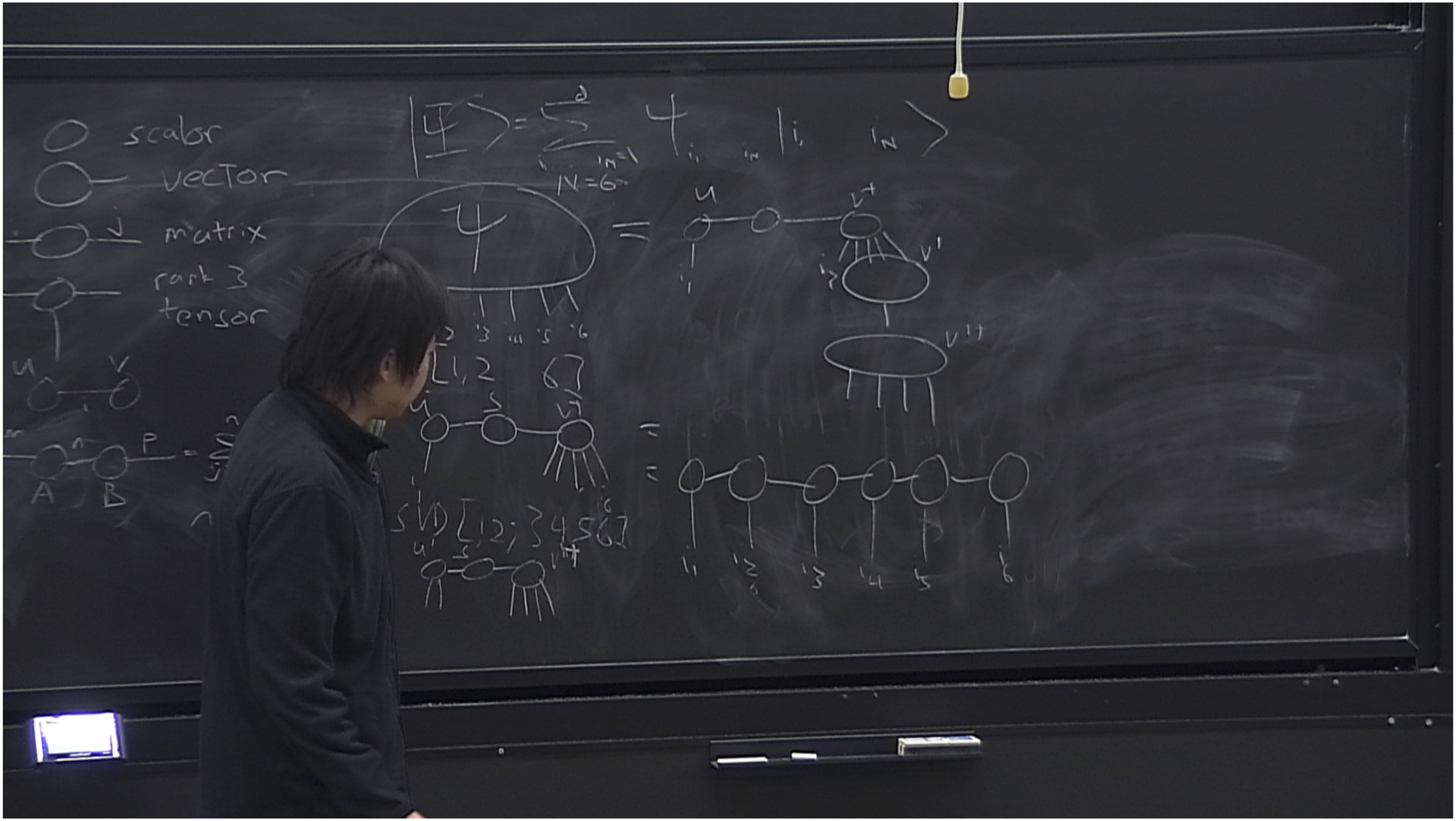
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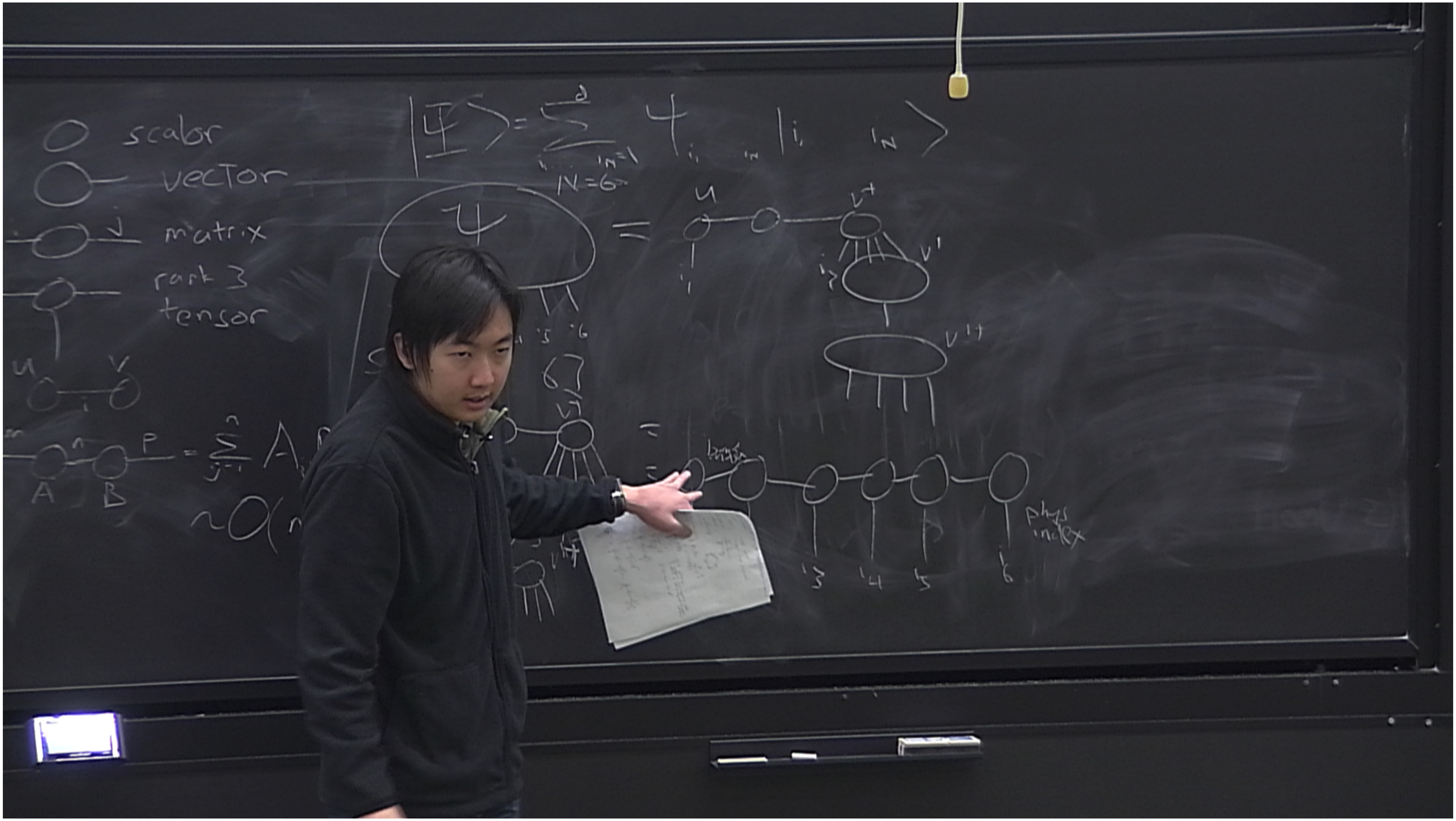
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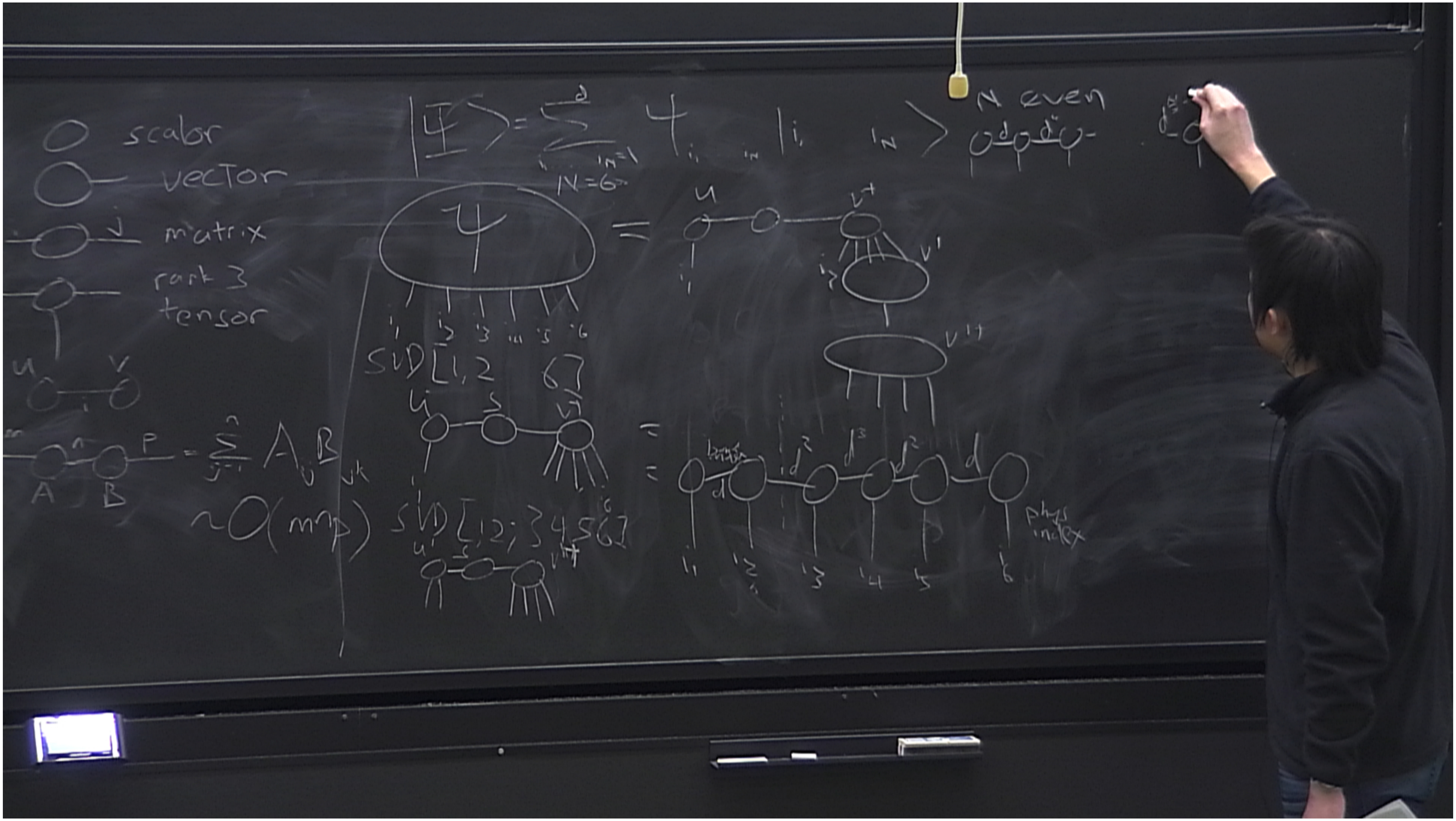

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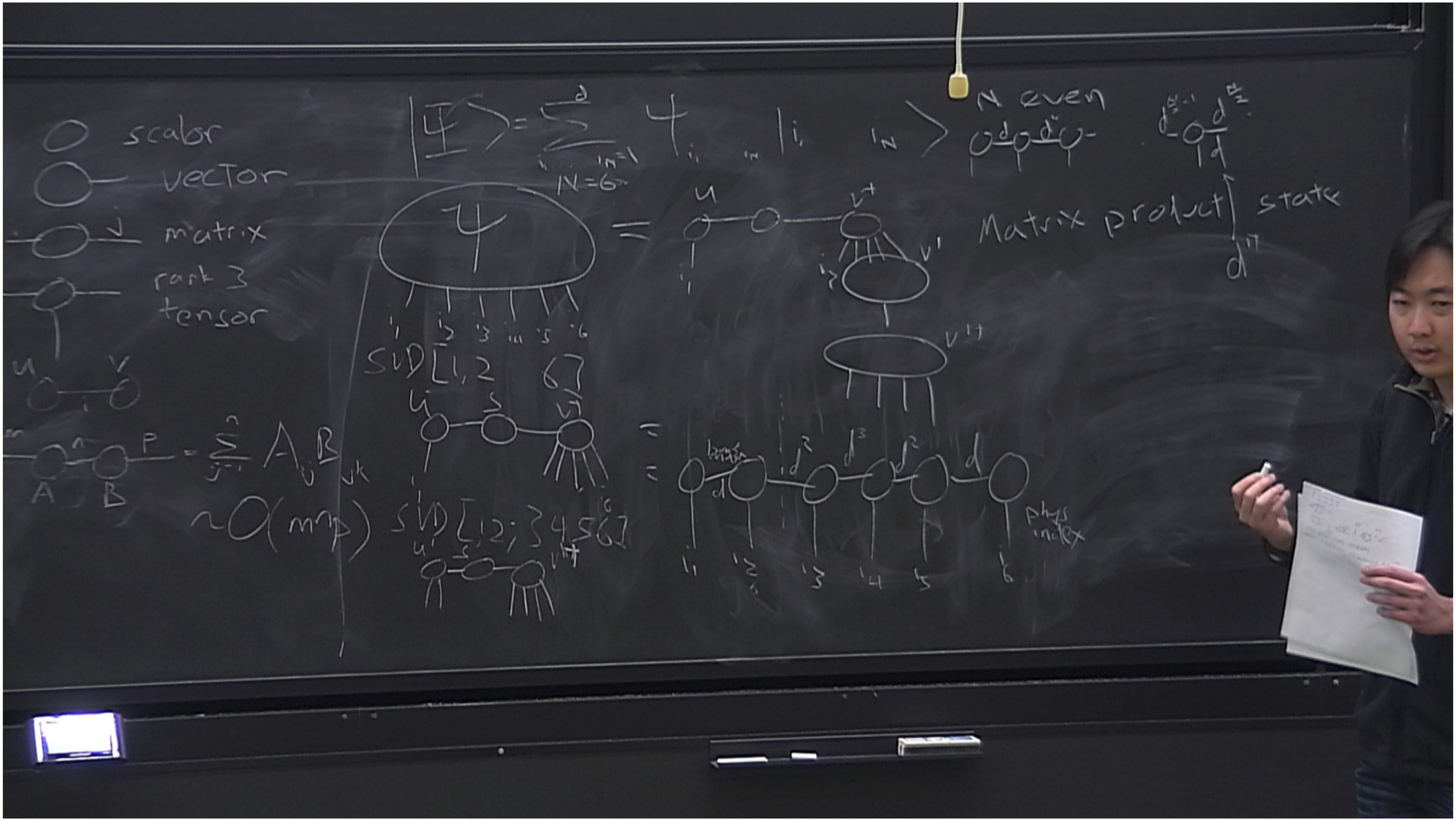
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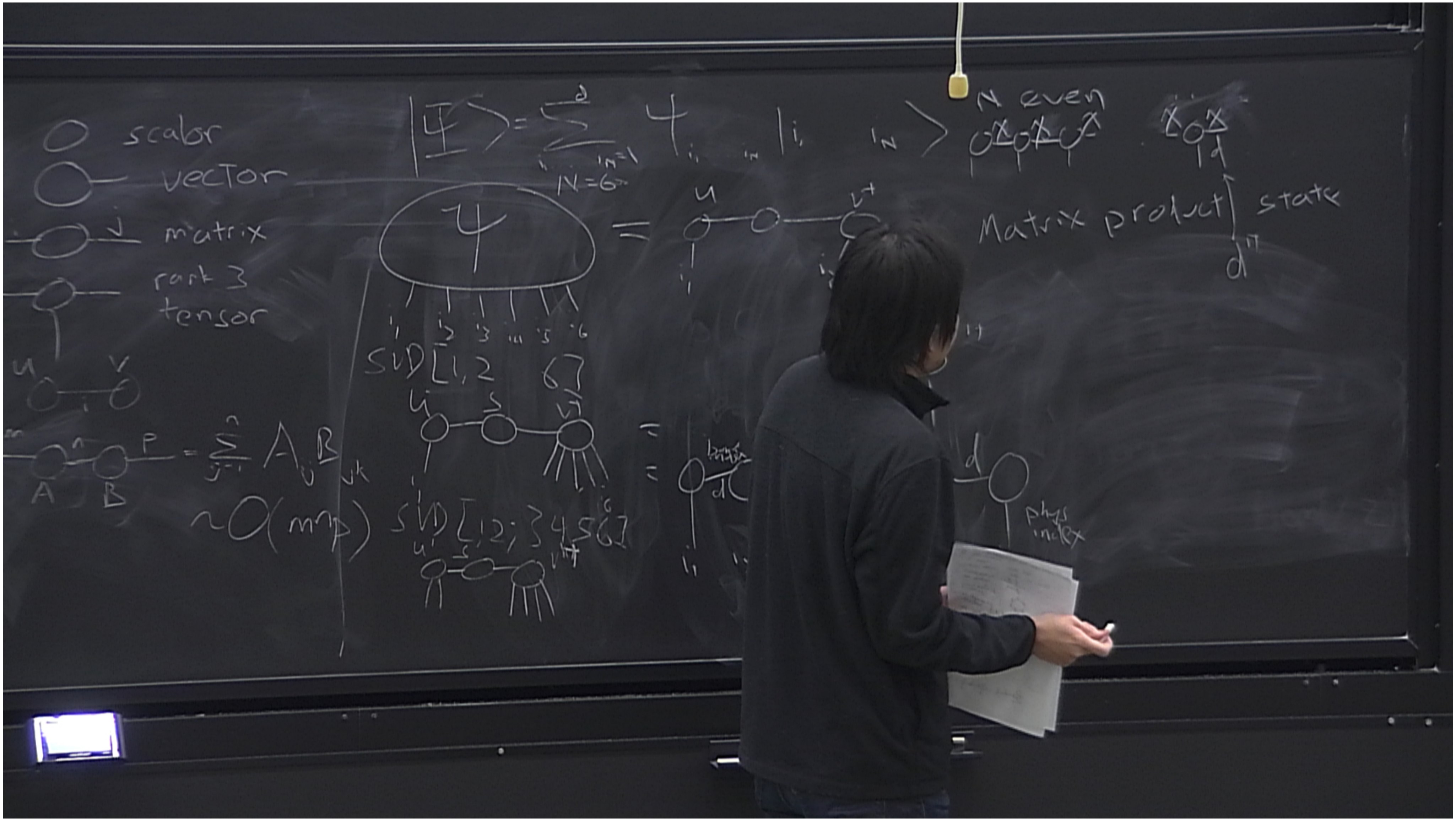

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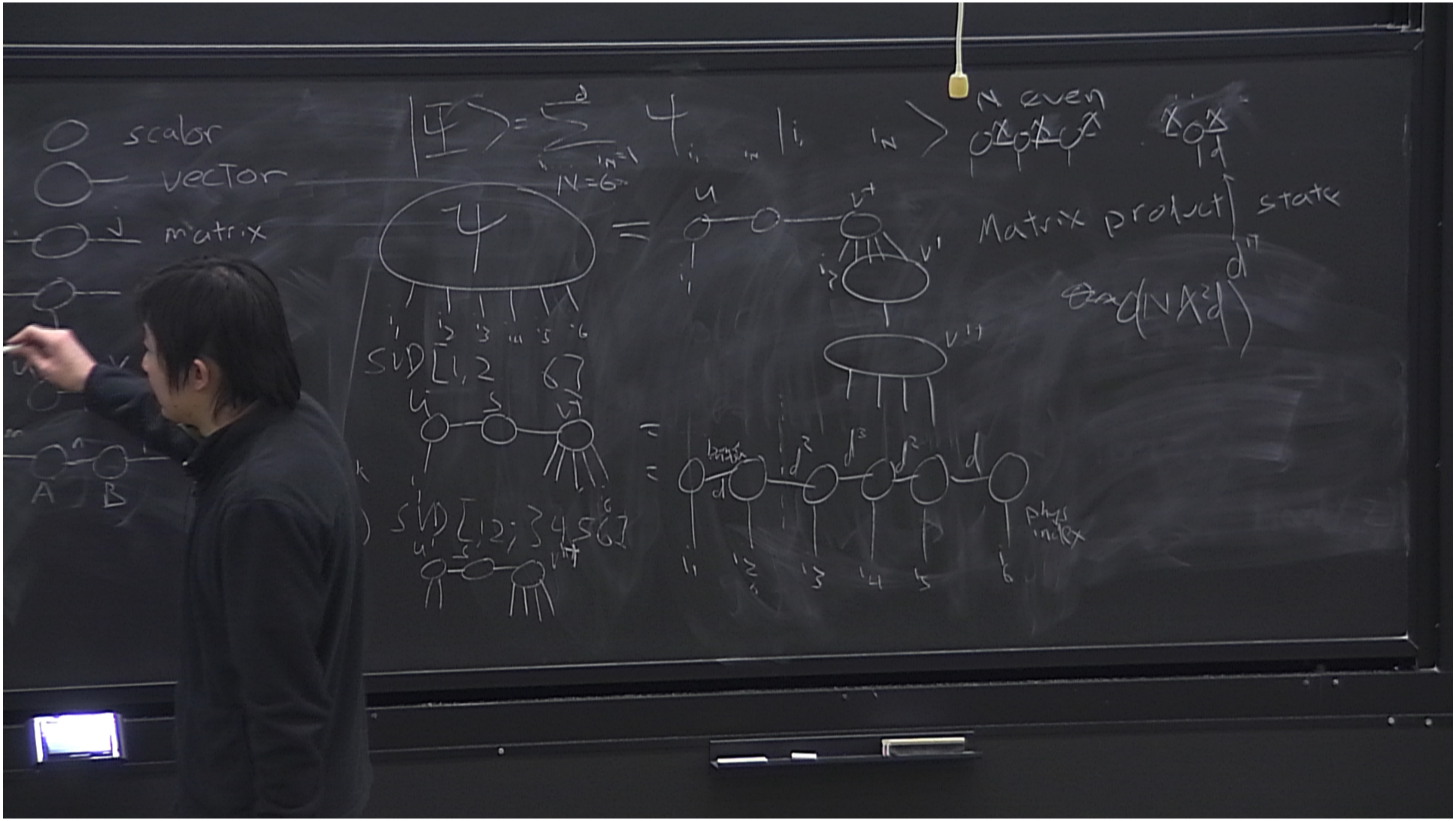
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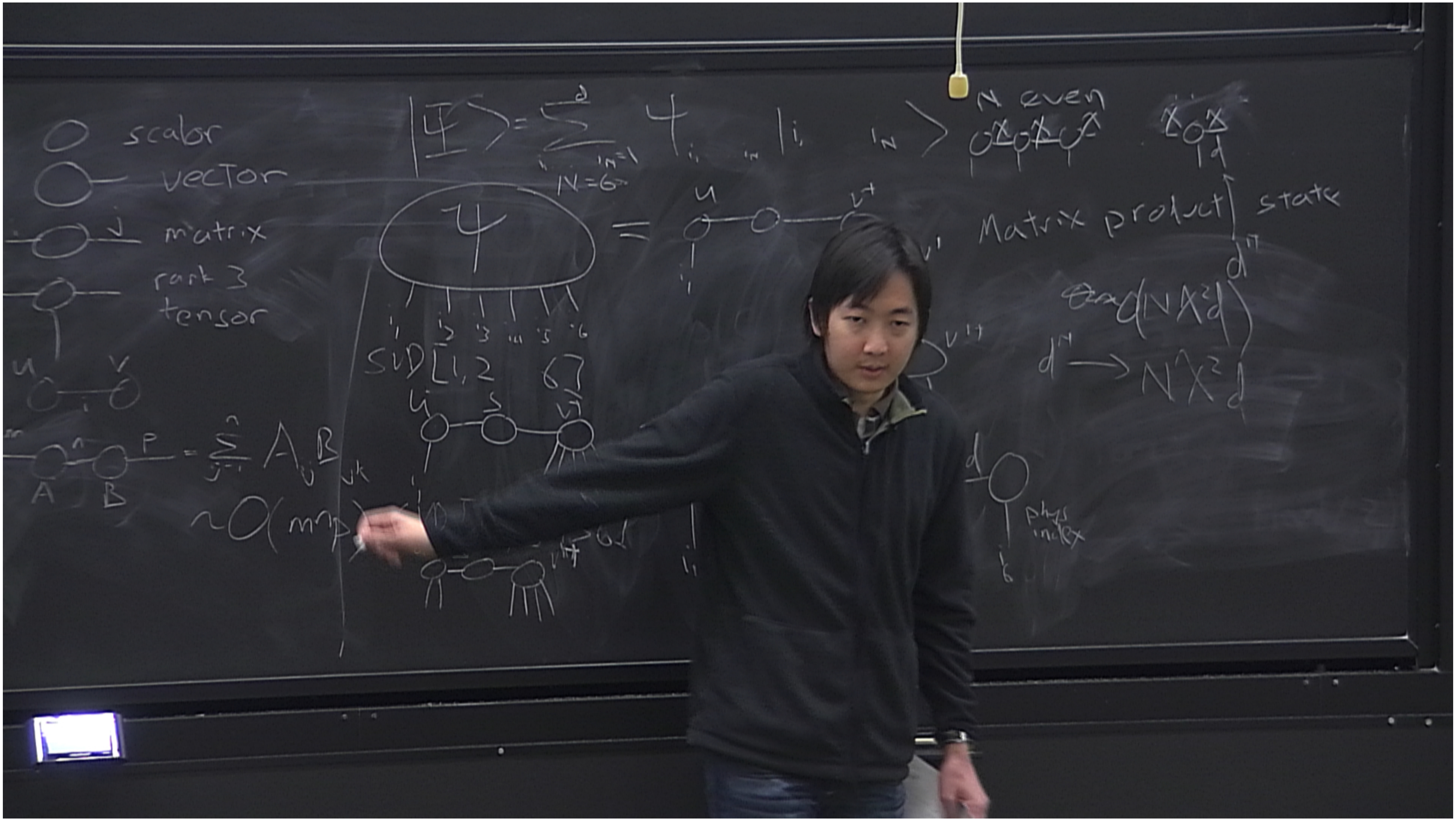
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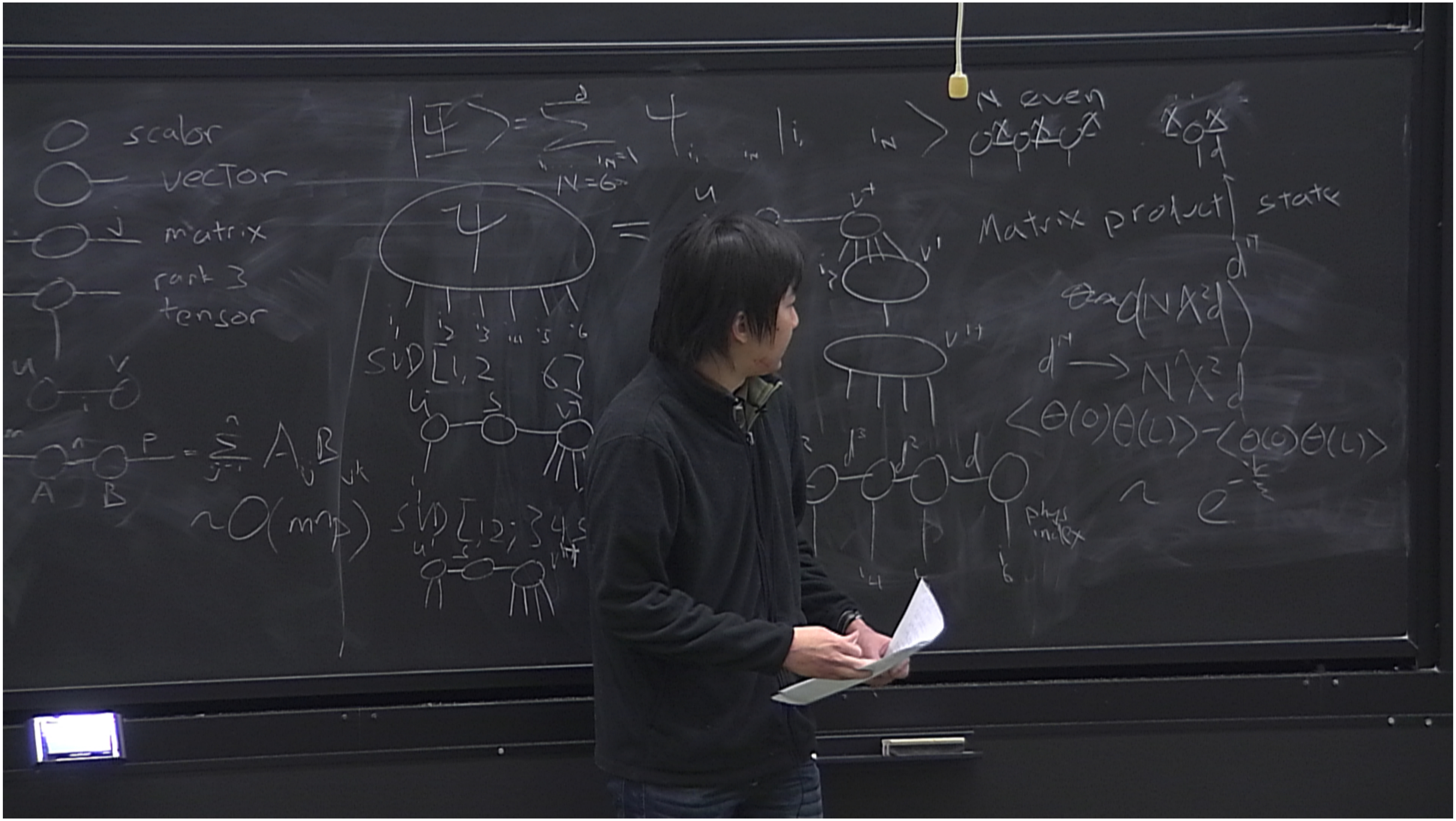
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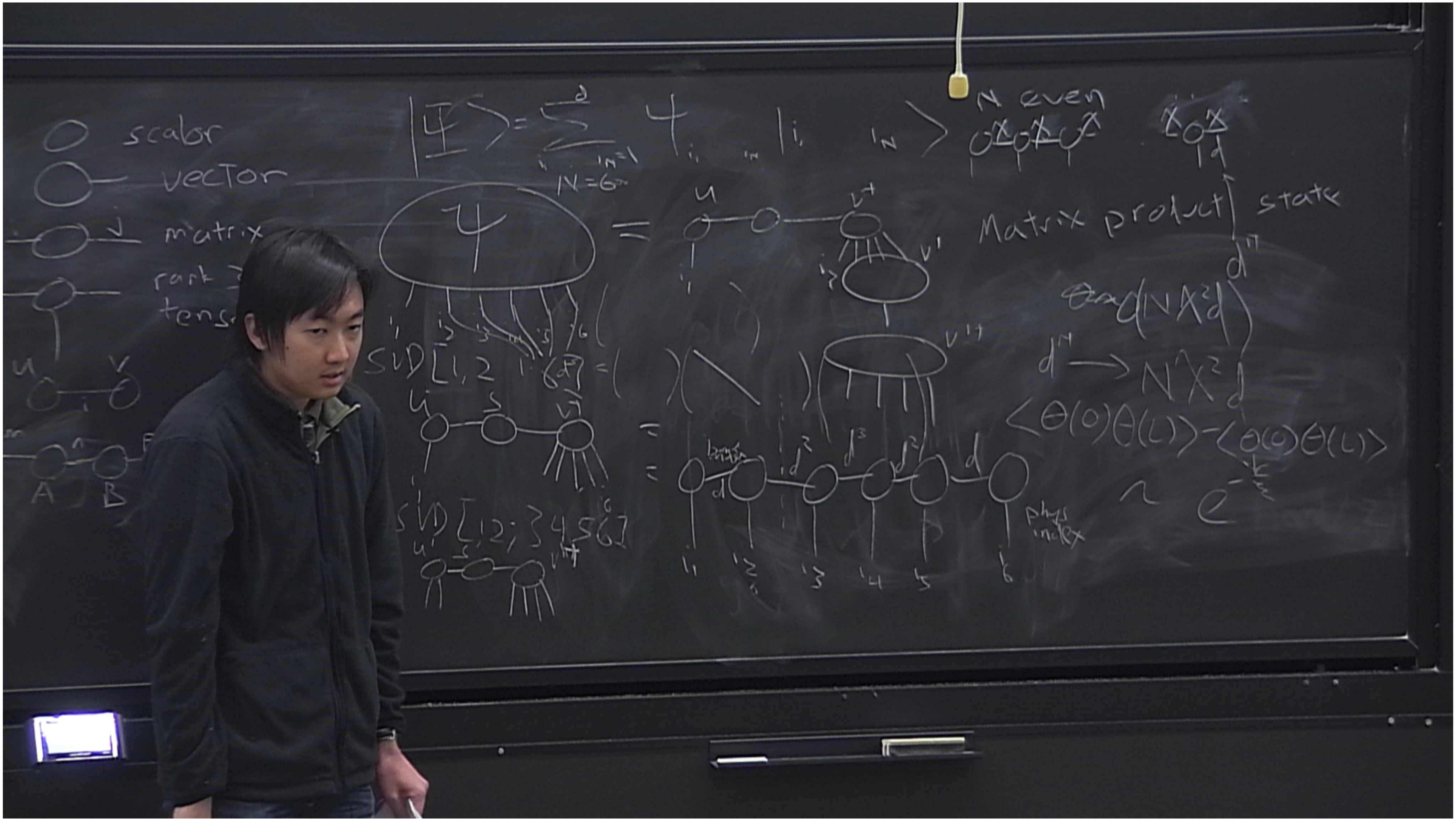
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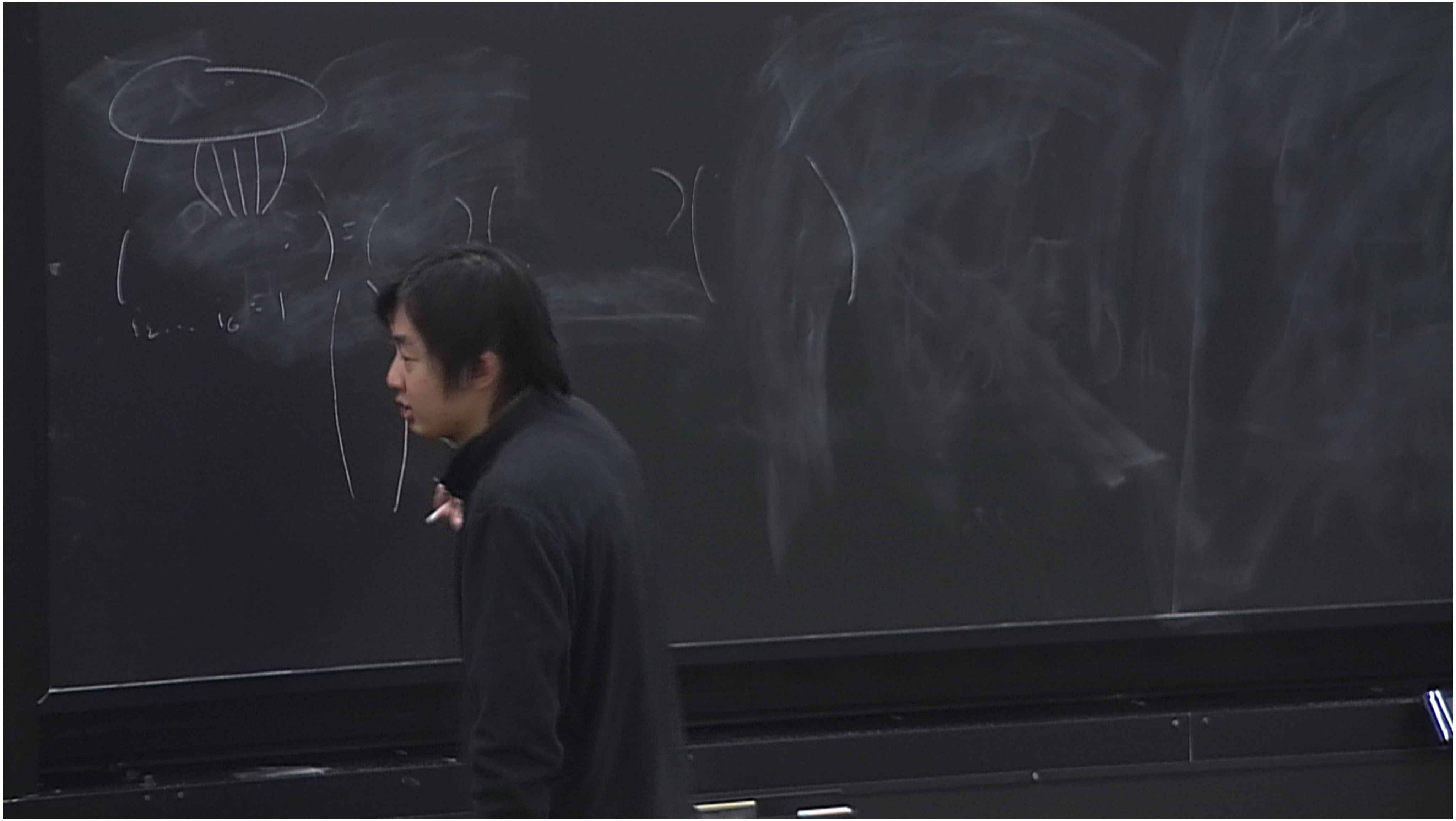
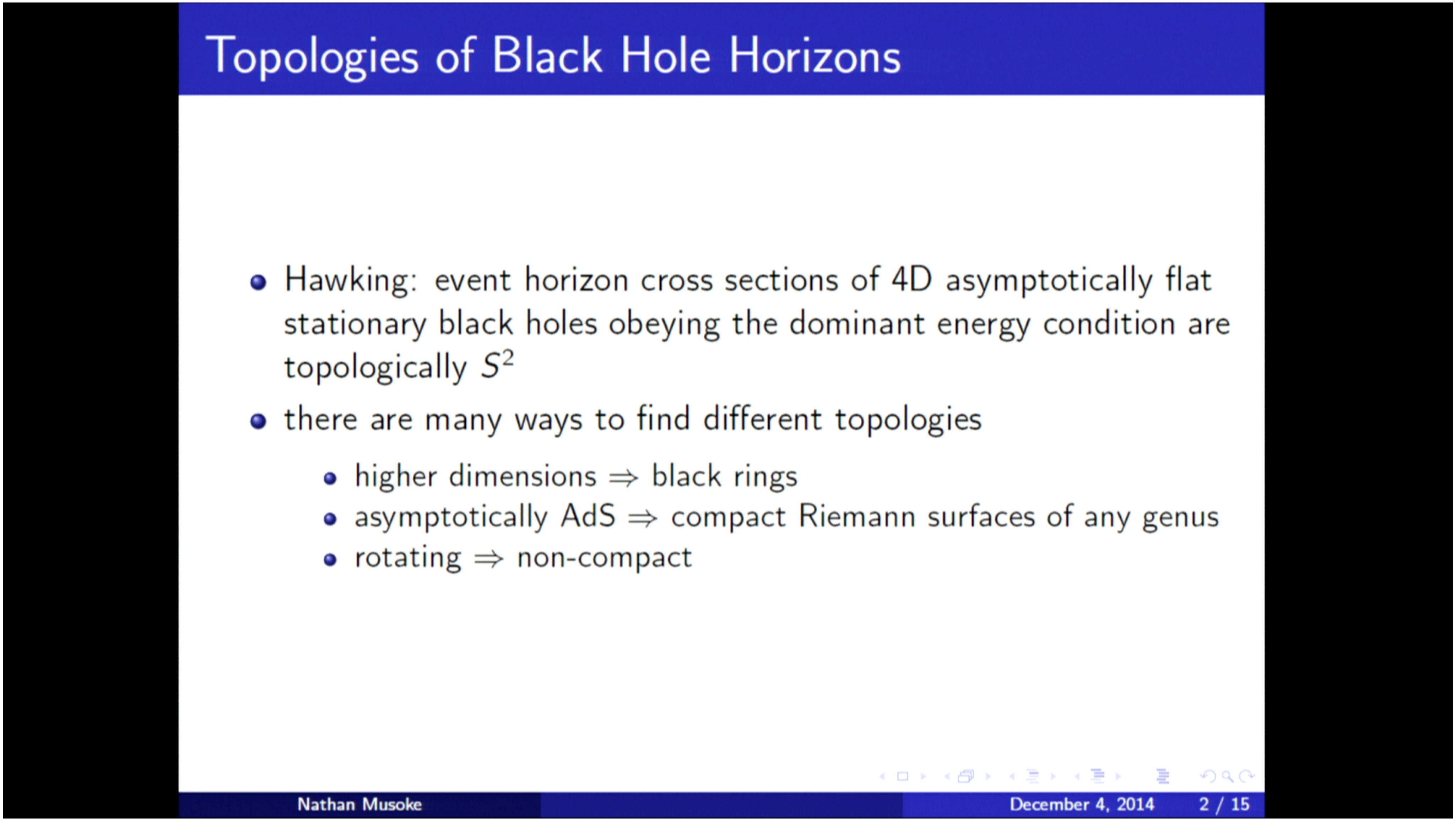


Non-compact Horizons and the Reverse Isoperimetric Inequality

Nathan Musoke

December 4, 2014


Topologies of Black Hole Horizons
Hawking: event horizon cross sections of 4D asymptotically flat stationary black holes obeying the dominant energy condition are topologically S^2
there are many ways to find different topologies
higher dimensions ⇒ black rings
asymptotically AdS ⇒ compact Riemann surfaces of any genus
rotating ⇒ non-compact
Nathan Musoke
December 4, 2014
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Black Hole Thermodynamics

$$\boxed{S = \frac{A}{4}}$$

$$P = -\frac{\Lambda}{8\pi}$$

$$dM = T\, dS + V\, dP$$

$$V = \left(\frac{\partial M}{\partial P}\right)_S$$

The Isoperimetric Equality

- we know that for a given volume, spheres minimize surface area
- there is a general inequality for plane curves

$$\frac{4\pi A}{L^2} \leq 1$$

- more generally, in d-dimensions

$$\mathcal{R} := \left(\frac{(d-1)V}{A_{d-2}}\right)^{1/(d-1)} \cdot \left(\frac{A_{d-2}}{A}\right)^{1/(d-2)} \leq 1$$

The Reverse Isoperimetric Equality

- it has been conjectured that the opposite holds for AdS black holes:

$$\mathcal{R} \geq 1$$

- it is obeyed for the black holes which have been checked
- saturated for Schwarzschild-AdS
  - Schwarzschild-AdS are "maximally entropic"

Kerr-Newman-AdS Metric

$$ds^2 = -\frac{\Delta_a}{\Sigma_a}\left[dt - \frac{a\sin^2\theta}{\Xi}d\phi\right]^2 + \frac{\Sigma_a}{\Delta_a}dr^2 + \frac{\Sigma_a}{S}d\theta^2$$
$$+ \frac{S\sin^2\theta}{\Sigma_a}\left[a\,dt - \frac{r^2+a^2}{\Xi}d\phi\right]^2 ,$$
$$\mathcal{A} = -\frac{qr}{\Sigma_a}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right) ,$$

where

$$\Sigma_a = r^2 + a^2\cos^2\theta\,, \quad \boxed{\Xi = 1 - \frac{a^2}{l^2}}, \quad S = 1 - \frac{a^2}{l^2}\cos^2\theta\,,$$
$$\Delta_a = (r^2 + a^2)\left(1 + \frac{r^2}{l^2}\right) - 2mr + q^2$$

Kerr-Newman-AdS Metric

- this metric satisfies the Einstein-Maxwell-AdS equations

$$G_{\mu\nu} - \frac{6}{l^2} g_{\mu\nu} = T_{\mu\nu} = F_{\mu\alpha} F^{\alpha}_{\ \nu} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}$$

- the volume and area are

$$V = \frac{2\pi}{3} \frac{(r_h^2 + a^2)(2r_h^2 l^2 + a^2 l^2 - r_h^2 a^2) + l^2 q^2 a^2}{l^2 \Xi^2 r_h}$$

$$A = \frac{4\pi (r_h^2 + a^2)}{\Xi}$$

- these satisfy the reverse isoperimetric inequality

Ultraspinning Limit

- one can construct a new black hole from this
- want to look at $a \to l$, but there are Ξ in denominators
- make the substitution $\psi = \phi/\Xi$
- compactify the coordinate $\psi \sim \psi + \mu$

Kerr-Newman-AdS Metric

$$ds^2 = -\frac{\Delta_a}{\Sigma_a}\left[dt - \frac{a\sin^2\theta}{\Xi}d\phi\right]^2 + \frac{\Sigma_a}{\Delta_a}dr^2 + \frac{\Sigma_a}{S}d\theta^2 + \frac{S\sin^2\theta}{\Sigma_a}\left[adt - \frac{r^2+a^2}{\Xi}d\phi\right]^2 ,$$

$$\mathcal{A} = -\frac{qr}{\Sigma_a}\left(dt - \frac{a\sin^2\theta}{\Xi}d\phi\right) ,$$

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Ultraspinning Limit

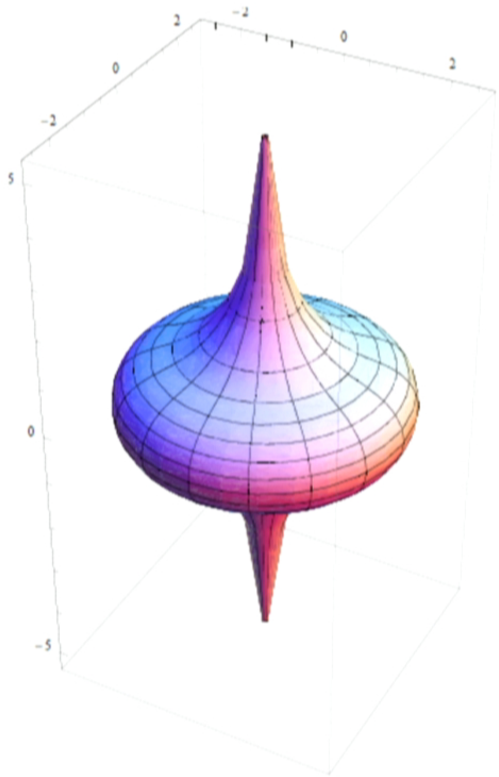
- the usual thermodynamic equations are satisfied
- the area and volume are

$$A = 2\mu(l^2 + r_+^2)$$
$$V = \frac{2}{3}\mu r_+(l^2 + r_+^2)$$

- the reverse isoperimetric inequality is violated:

$$\mathcal{R} = \left(\frac{r_+ A}{2\mu}\right)^{1/3} \left(\frac{2\mu}{A}\right)^{1/2} = \left(\frac{r_+^2}{r_+^2 + l_+^2}\right)^{1/6} < 1 \qquad (1)$$

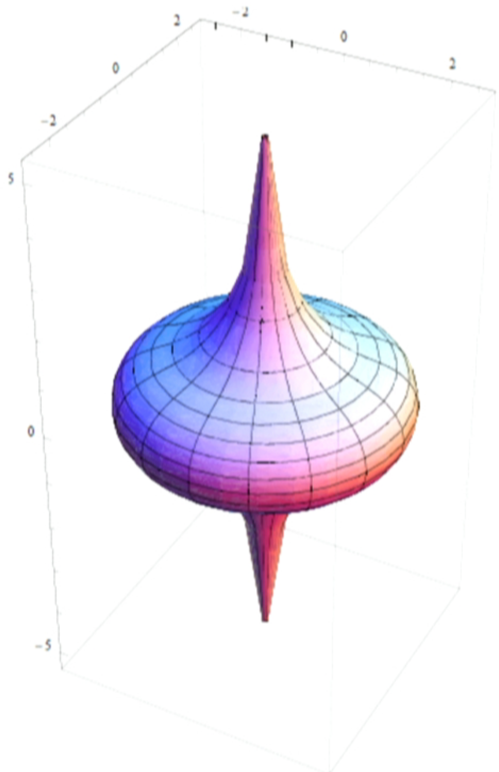
Ultraspinning Limit



(Klemm arXiv:1402.3107)

- however, the horizon is no longer compact

Ultraspinning Limit



(Klemm arXiv:1402.3107)

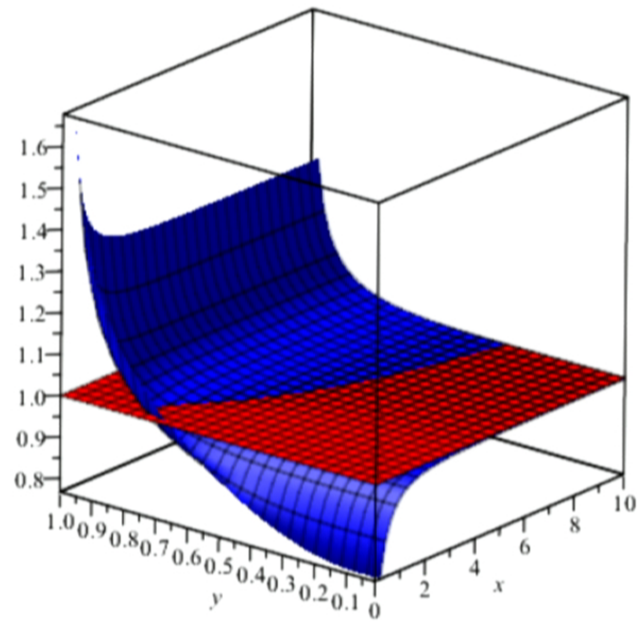
- however, the horizon is no longer compact
- modify the reverse isoperimetric conjecture to require compact horizons

Superentropic Block Hole in 5D

- start with general rotating charged black hole in 5D
  - 2 rotation parameters a and b
- want to look at $a, b \to l$, but again have Ξ_a, Ξ_b in denominators
- again make a coordinate change $\varphi = \phi_R / \Xi_a$

Superentropic Block Hole in 5D

- the reverse isoperimetric inequality is not satisfied for some values of the parameters q, b

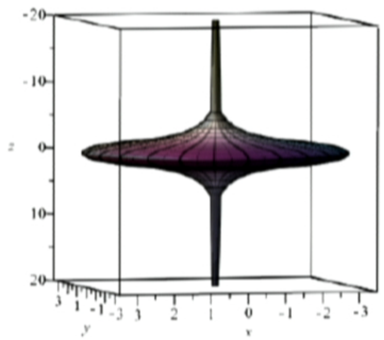


$q = 0$, $x = r/l$, $y = b/l$,

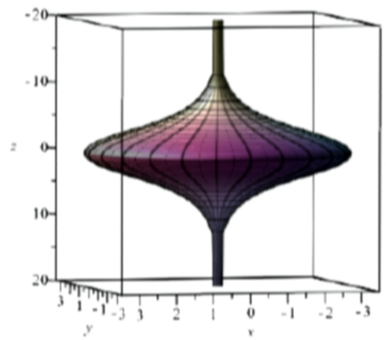
(plot by Robie)

Superentropic Block Hole in 5D

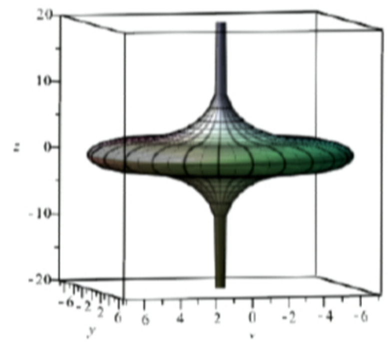
- there are non-compact horizons again



$b = 0,$
$q = 0$



$b = 0.8,$
$q = 0$



$b = 0.8,$
$q = 155$

(plots by Robie)

Future Directions

- perform similar analysis for d-dimensional black holes
- work towards a proof of the Reverse Isoperimetric Inequality

Bibliography I

D. Klemm.
Four-dimensional black holes with unusual horizons
arXiv:1401:3107

M. Cvetic, G.W. Gibbons, D. Kubiznak and C.N. Pope
Black Hole Enthalpy and an Entropy Inequality for the Thermodynamic Volume
arXiv:1012.2888

R. A. Hennigar, R. B. Mann. and D. Kubiznak
Super-Entropic Black Holes
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R. A. Hennigar, R. B. Mann, D. Kubiznak, and N. Musoke.
Super-Entropic Black Holes and Reverse Isoperimetric Inequality
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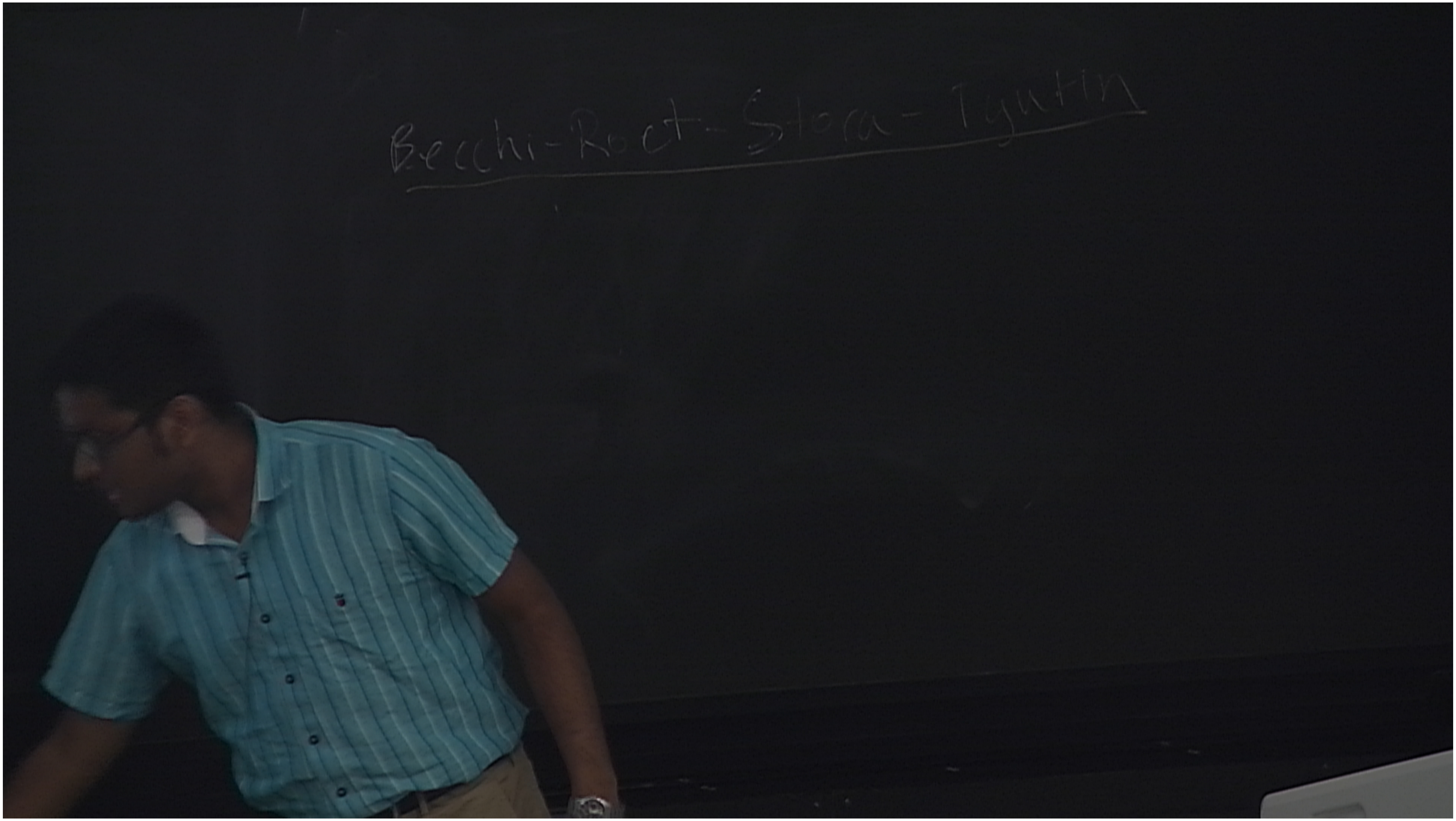
A First look Into BRST Invariance

Vasudev Shyam

Perimeter Scholars International

Student Presentations


Becchi-Roct-Stora-Tyutin

Outline

1. The Quartet Mechanism
   - Harmonic Oscillator Toy Model
   - The Slightly Bigger Picture
2. What is it good for?
   - Gauge Theories

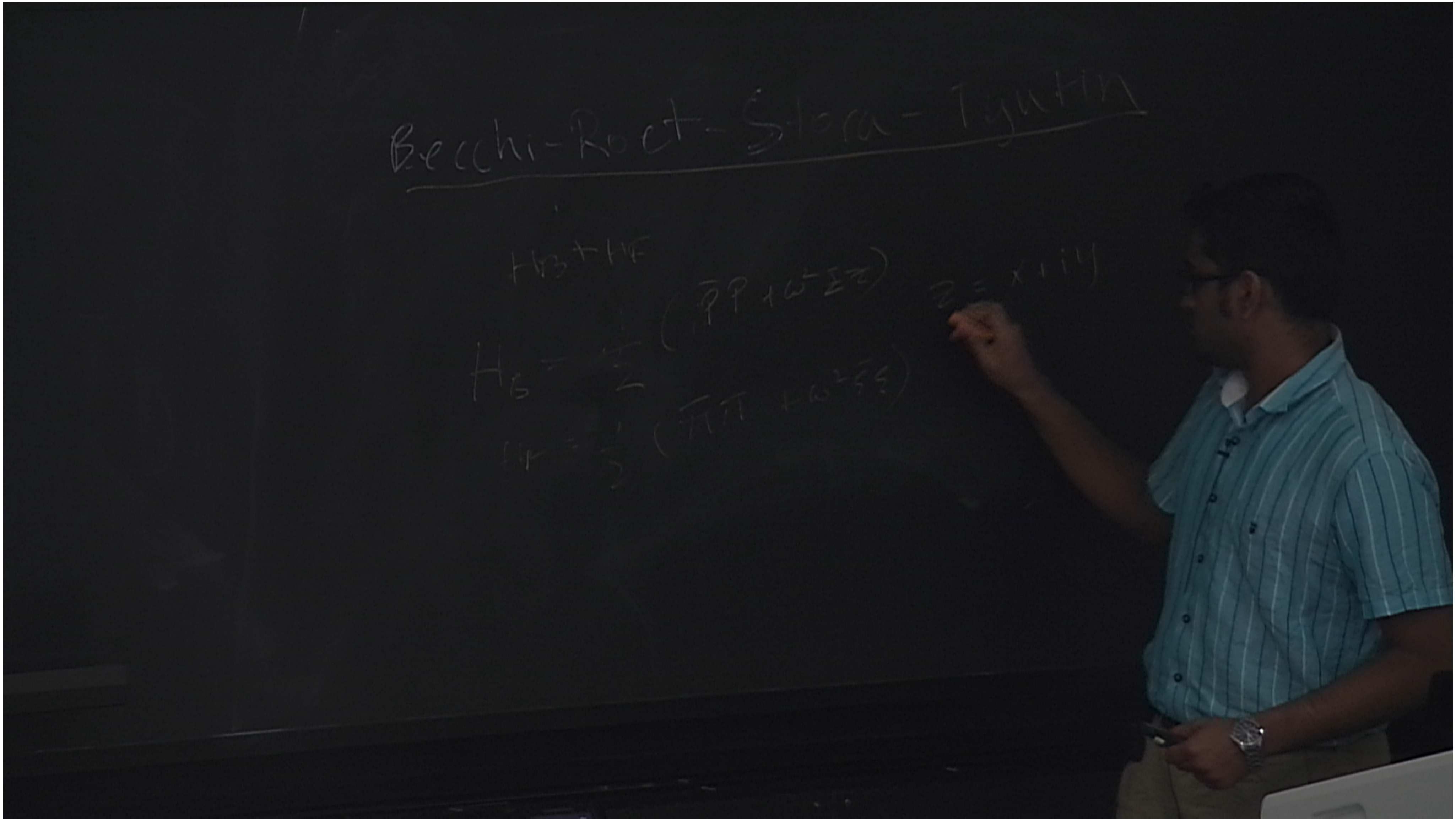
Meet the Super Oscillator

- The Super-Oscillator Hamiltonian:

$$H = \omega \left(\bar{A}^{\dagger}\bar{A} + A^{\dagger}A + \bar{a}^{\dagger}\bar{a} + a^{\dagger}a \right) \tag{1}$$

- The BRST charge mixes Bosonic and fermionic degrees of freedom:

$$Q = i(\bar{A}^{\dagger}\bar{a} - a^{\dagger}A) \tag{2}$$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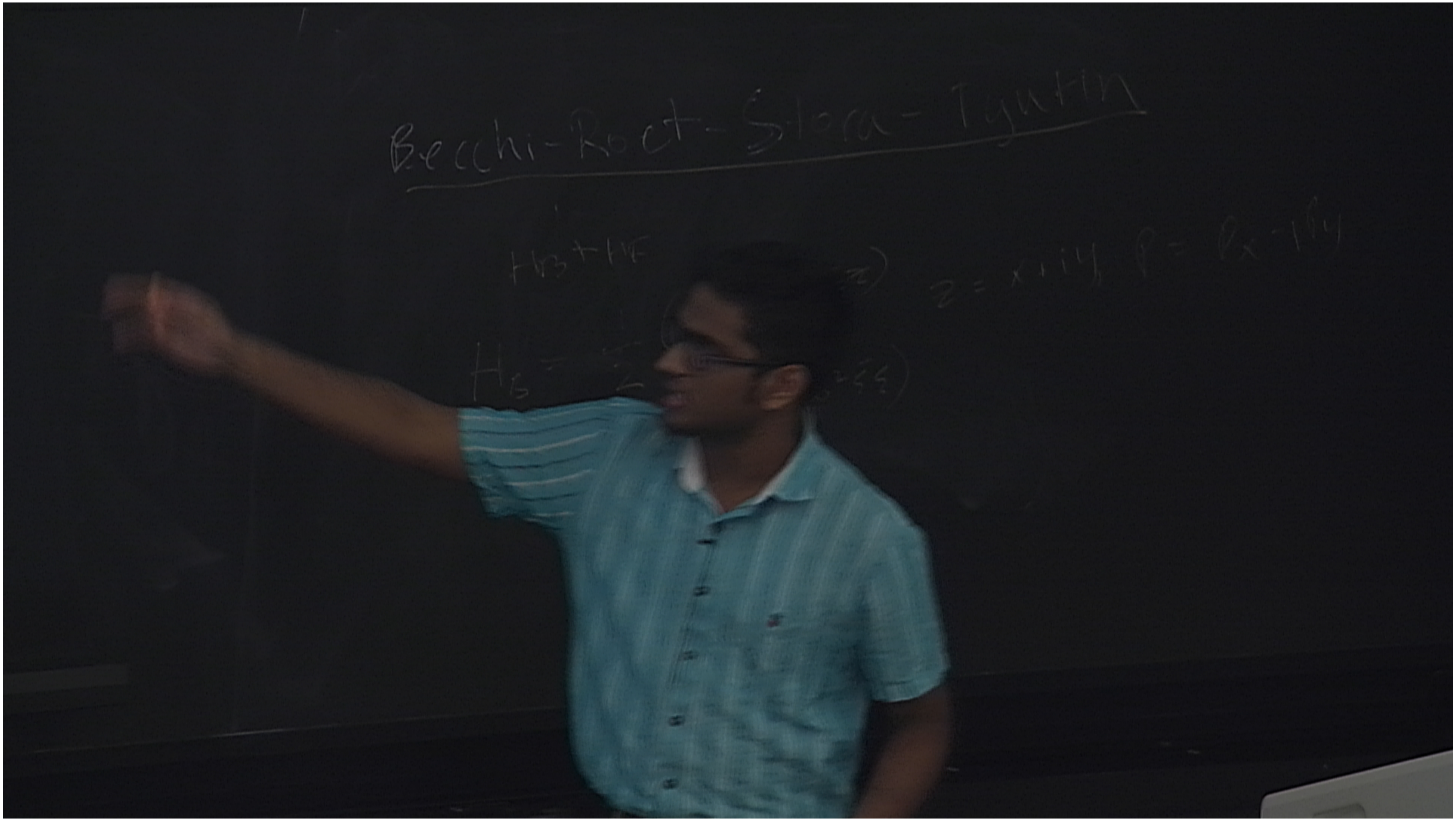
Becchi-Rouet-Stora-Tyutin

What Makes It Super

- The Charge is nilpotent: $Q^2 = 0$ and furthermore

$$\left\{Q, Q^{\dagger}\right\} = \frac{H}{\omega} \tag{3}$$

- Thus the physical Hilbert Space consisting of states that are invariant under the action of both Q and $Q^{\dagger}$ is one dimensional and consists of but the Fock vacuum $|0\rangle$

Outline

1. The Quartet Mechanism
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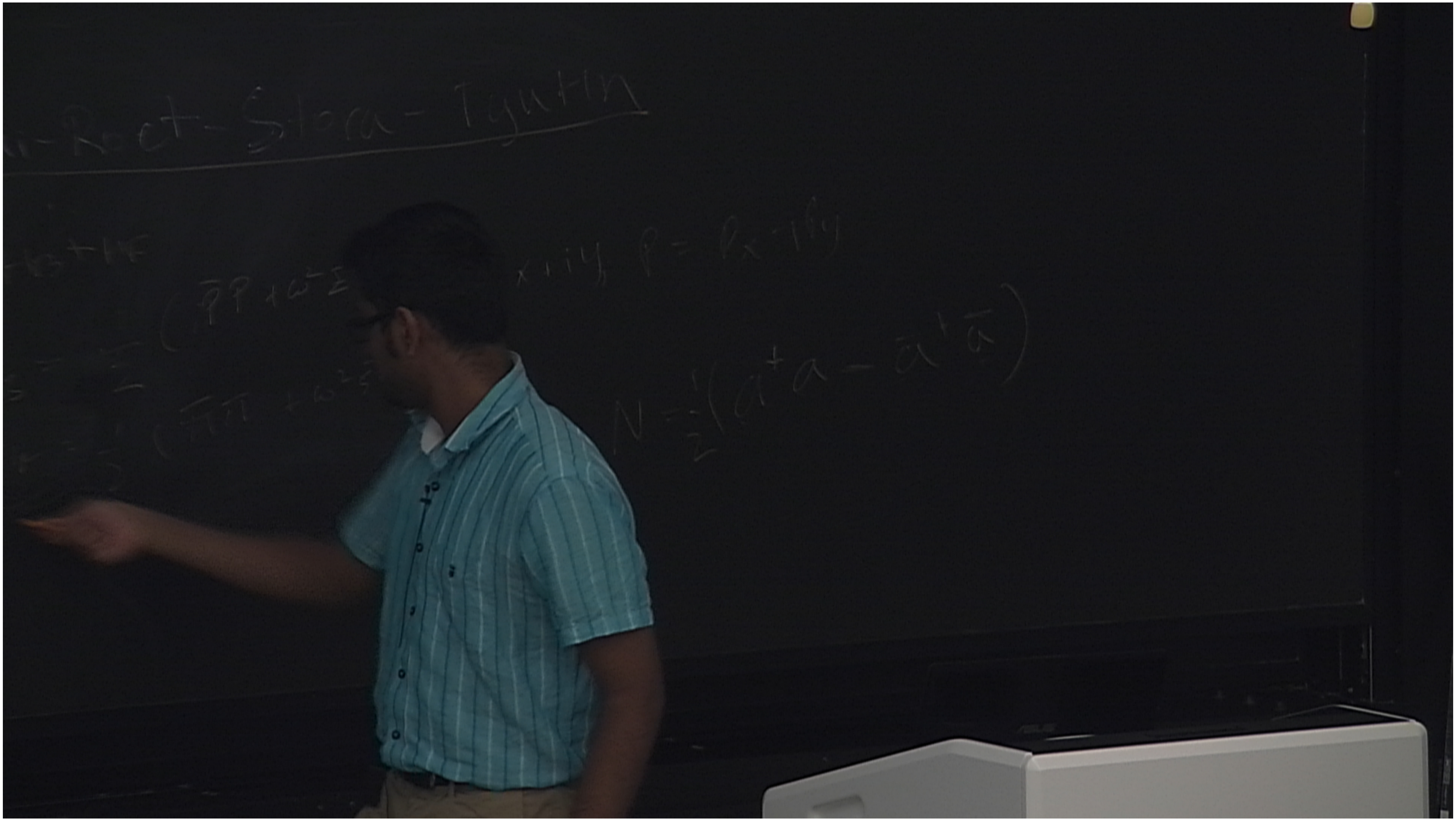
   - Gauge Theories

How Should The Space of Physical States be Identified?

- BRST invariance would have us choose $\ker Q$, but due to the charge's nilpotency, $\mathrm{im} Q \subset \ker Q$, similarly for $Q^\dagger$
- The solution lies in voiding $\mathrm{im} Q$ of physical content and identifying as the space of physical states

$$\mathcal{H}_{Phys} = \ker Q / \mathrm{im} Q \tag{4}$$

- Introduce a Ghost Number operator: N_{gh} s.t. $[N_{gh}, H] = 0$, $\{N_{gh}, Q\} = Q$ and hence leaves $\ker Q$ invaraint, and introduces a grading on it. The eigenvalue 0 subspace of N_{gh} is $\mathcal{H}_{Phys}$



Some Caveats

- Although the Fock space of the Super Oscialltor is a Cartesian product of fermionic and bosonic Fock spces, $\ker Q$ and $\mathcal{H}_{phys}$ are not Hilbert spaces, but are indefinite metric spaces with a pseudonorm $(f'|f) = \langle f|J|f\rangle$ where J is a Hermitian oerator chosen such that $Q^{\dagger}J = JQ$
- Q is thus pseudo-hermitian and states with higher ghost number may have negative pseudo-norm.

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Outline

What The Gauge Fixed Action is Invariant Under

- BRST transformations: $\delta L = 0$

$$L = L_{YM} + \frac{1}{g_{YM}^2}\mathrm{Tr}(-\bar{C}(\partial^\mu(D_\mu C)) - \frac{1}{2}\xi^{-1}(\partial^\mu A_\mu)^2) \quad (5)$$

- The BRST operator's action on the various fields is given as follows

$$\delta A = \epsilon D_\mu C$$

$$\delta C = -\frac{i}{2}\epsilon\{C, C\}$$

$$\delta\bar{C} = \epsilon\xi^{-1}(\partial^\mu A_\mu)$$

$$\delta\xi^{-1}(\partial^\mu A_\mu) = 0$$

Slight Demystification

- Much like before, a BRST charge Q s.t. $\delta = \epsilon Q$ can be introduced and the space of physical states can be identified with $\mathrm{ker}Q/\mathrm{im}Q$.
- Diagrammatics showing cancellations of longitudinal modes etc. to ensure gauge invariance as done in class are basically encapsulated in the fact that the S-matrix commutes with Q, i.e.

$$[S, Q] = 0 \tag{6}$$

and the restriction of the Optical Theorem to states in $\mathcal{H}_{Phys}$,

$$\langle\phi|S^\dagger S|\psi\rangle = \langle\phi|\psi\rangle, \tag{7}$$

$|\phi\rangle, |\psi\rangle \in \mathcal{H}_{Phys}$

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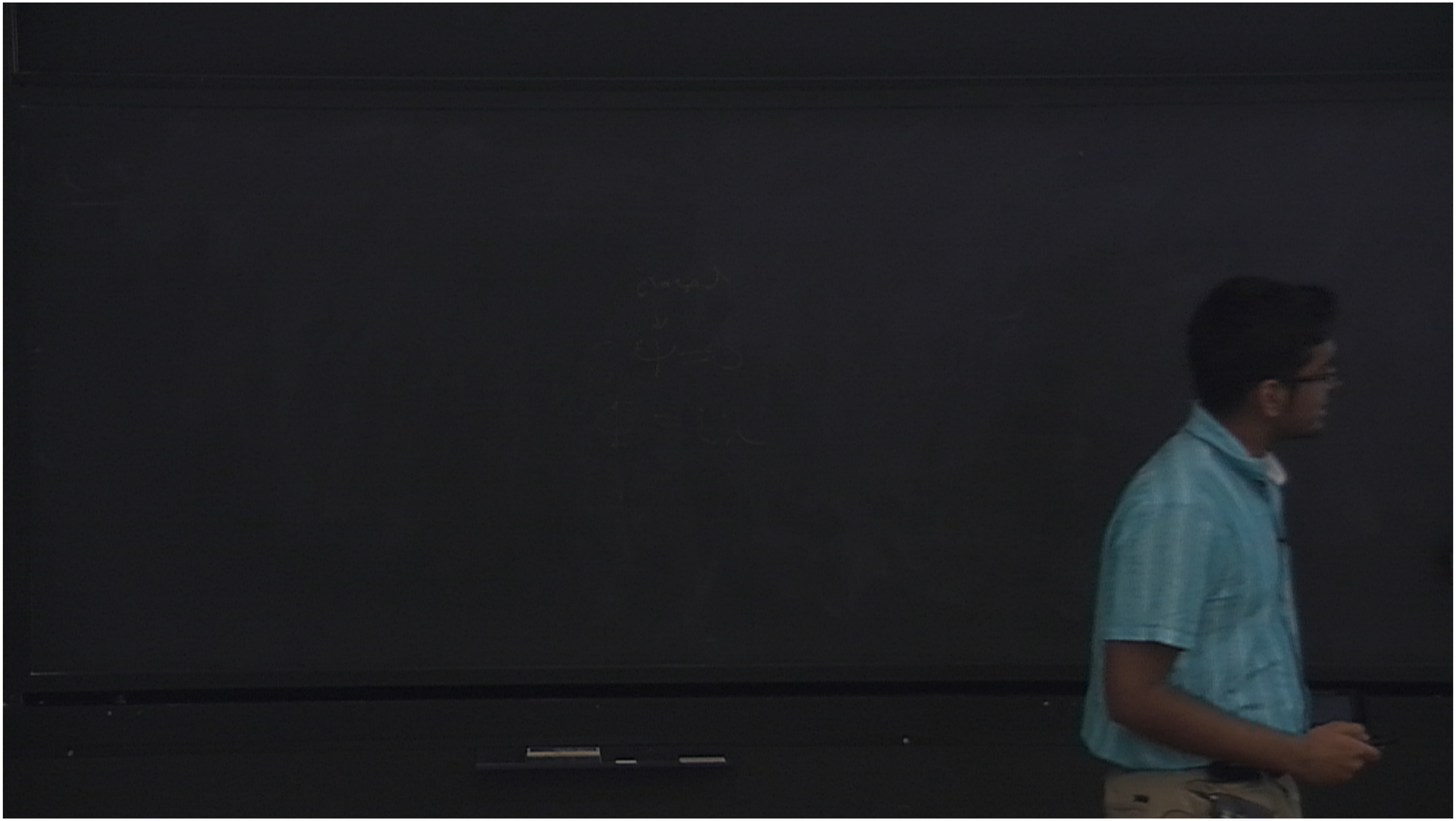

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$$\delta\xi^{-1}(\partial^\mu A_\mu) = 0$$

For Further Reading I

Marc Henneaux and Claudio Teitelboim
Quantization of Gauge Systems.

Click on Tools to convert files to PDF.

Using the Action in Classical Theories of Gravity

Perseas Christodoulidis

December 04, 2014

Perimeter Institute for Theoretical Physics

According to the new rules you are not allowed to sleep

1 / 19

A Puzzle from Galaxies

A Puzzle from Galaxies

- Observations of the orbiting velocity of stars show that they do not follow the "Keplerian Law" $v(r) \propto 1/\sqrt{r}$, as they should!
- Instead we observe that their velocity is constant sufficiently far from the center.

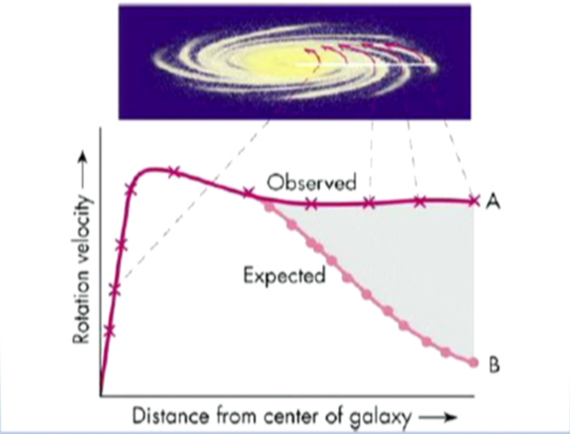

Figure: Rotational Curve (source: www.philica.com)

Some actual data

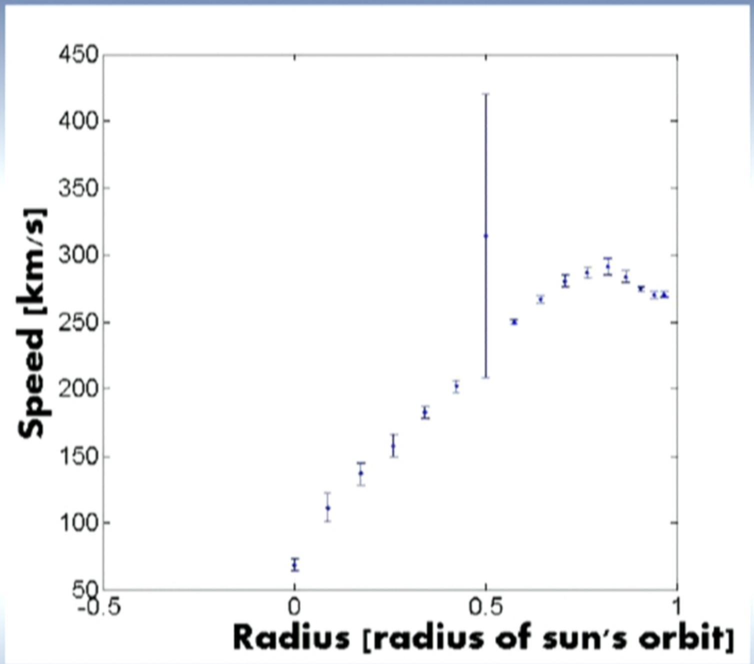

Figure: Rotational Curve of Milky Way (Illan et. al.)

Overview

1. Very short Historical Introduction

2. Main Body
   - How to Construct an Action
   - Deriving Einstein Equations from a Variational Principle
   - The rotational curve problem revisited

3. Conclusions

A Glimpse into the Past

- Lagrange first derived his equations from the principle of virtual works and D'Alembert's principle.

$$\Sigma_i (F - m_i a) \cdot \delta r = 0 \qquad (1)$$

- Soon he realized that his equations were independant of coordinate system. Newton's 2nd law is not.
- Later Hamilton derived Euler-Lagrange equations from a variational principle.

Einstein Rules for S.R.

1. If you are talking about relativity you should use coordinate-invariant objects because the action is an invariant quantity

Einstein Rules for S.R.

1. If you are talking about relativity you should use coordinate-invariant objects because the action is an invariant quantity
2. An invariant is the length of a curve (or the proper time $d\tau = \sqrt{-ds^2}$ for metric signature $\eta_{\mu\nu} = (-1, 1, 1, 1)$)

Einstein Rules for S.R.

1. If you are talking about relativity you should use coordinate-invariant objects because the action is an invariant quantity
2. An invariant is the length of a curve (or the proper time $d\tau = \sqrt{-ds^2}$ for metric signature $\eta_{\mu\nu} = (-1, 1, 1, 1)$)
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3. So one possible action in S.R. would be: $S \propto \tau$
4. This is the case since you get the correct result in the Newtonian limit

$$L = -mc^2 \int d\tau = -mc^2 \int \sqrt{1 - (\frac{u}{c})^2} dt = -mc^2 + \frac{1}{2} m u^2 + \cdots \quad (2)$$

Upgrade to GR

1. The most important tensor is the Riemann tensor $\boldsymbol{R}$. We can form scalar quantities by contracting it to other tensors. For instance:
   $R^{\mu\nu\kappa\lambda}R_{\mu\nu\kappa\lambda}\ ,\quad R^{\mu\nu\kappa\lambda}R_{\mu\nu}R_{\kappa\lambda}\ ,\quad R^{\mu\nu}R_{\mu\nu}\ ,\quad R$

Main Body

Upgrade to GR

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4. What L?

We vary the Einstein-Hilbert Action: $S = \int d^4x\sqrt{-g}R$, w.r.t. the elements of the metric tensor $g_{\mu\nu}$

$$\delta S = \int d^4x \delta(\sqrt{-g}R) = \int d^4x(\delta\sqrt{-g}R + \sqrt{-g}\delta R) \qquad (3)$$

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We have to calculate 3 variations:

1. $\delta(\sqrt{-g})$
2. $R_{\mu\nu}\delta g^{\mu\nu}$
3. $\delta R_{\mu\nu} g^{\mu\nu}$

1. For the first one we use the identity $det(e^{\boldsymbol{A}}) = e^{Tr(\boldsymbol{A})}$ or

$$\frac{1}{det(\boldsymbol{A})}\frac{d\boldsymbol{A}}{dx} = Tr(\boldsymbol{A}^{-1}\frac{d\boldsymbol{A}}{dx}))$$

so in our case with $det(\boldsymbol{A}) = g$ the result is

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2. Use the relation $\delta(g^{\mu\nu}g_{\nu\kappa}) = \delta(\delta^{\nu}_{\kappa})$ so

$$\delta g^{\mu\nu} = -g^{\mu\kappa}g^{\nu\lambda}\delta g_{\kappa\lambda} \qquad (5)$$

- $\delta R_{\mu\nu} = (\delta\Gamma^{\lambda}_{\mu\nu})_{,\lambda} - (\delta\Gamma^{\lambda}_{\mu\lambda})_{,\nu} + \delta\Gamma^{\lambda}_{\mu\nu}\Gamma^{\kappa}_{\lambda\kappa} + \Gamma^{\lambda}_{\mu\nu}\delta\Gamma^{\kappa}_{\lambda\kappa} - \delta\Gamma^{\kappa}_{\mu\lambda}\Gamma^{\lambda}_{\nu\kappa} - \Gamma^{\kappa}_{\mu\lambda}\delta\Gamma^{\lambda}_{\nu\kappa}$
- We go to a local inertial frame where at this point $g_{\mu\nu}(P_0) = \eta_{\mu\nu}$ and all Γ symbols vanish.

$$\delta R_{\mu\nu}(P_0) = (\delta\Gamma^{\lambda}_{\mu\nu})_{,\lambda}(P_0) - (\delta\Gamma^{\lambda}_{\mu\lambda})_{,\nu}(P_0) \qquad (6)$$

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- The Γ symbols are not tensors but their (small) variations are!
- Therefore we have an equation of tensors and in general

$$\delta R_{\mu\nu} = (\delta\Gamma^{\lambda}_{\mu\nu})_{;\lambda} - (\delta\Gamma^{\lambda}_{\mu\lambda})_{;\nu} \tag{7}$$

1. Using these substitutions and the assumption that the variation of the Christoffel symbols vanish sufficiently quickly at infinity we recover the Einstein Equations for the vacuum:

$$\boldsymbol{R} = 0$$

2. Now if we add another term in the lagrangian L_M and define $T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(L_M\sqrt{-g})}{\delta g_{\mu\nu}}$ then we recover the Einstein Equations in the presence of matter:

$$\boldsymbol{R} - \frac{1}{2}R\boldsymbol{g} = \frac{8\pi G}{c^4}\boldsymbol{T} \quad (8)$$

So what have we accomplished?

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- This procedure gives us a straight way of generalizing our theory of gravity.

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- By changing the Lagrangian.

Dark matter? -No thanks

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Both have problems!

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Both have problems!

- what is the nature of the hidden mass? Not baryonic. Can we give a solid theoretical description?
- what the modified gravity law predicts?

Generalizing the gravitational law

- If a modified law exists then what is the area that it starts to diverge from the previous theory?

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- We know that on earth scales Newton's law is sufficient. For greater scales (solar system) GR is more accurate.
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Generalizing the gravitational law

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- We know that on earth scales Newton's law is sufficient. For greater scales (solar system) GR is more accurate.
- The natural step is to assume the existence of a modified law in:
  - large scales (galaxy or even the Universe)
  - bigger curvatures (R)

Modified theories

Modified theories

- MOND (MOdified Newtonian Dynamics). Proposed by Mordehai Milgrom. The new dynamical law is:

$$F = ma \cdot g(\frac{a}{a_0}), \tag{9}$$

with $g \rightarrow 1$ for $a >> 1$.

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- Tensor-Vector-Scalar Gravity (TeVeS) by Jacob Bekenstein. Relativistic generalization of MOND.

- f(R) gravity
- Scalar–tensor–vector gravity, by John Moffat (PI)
- Gauss-Bonnet gravity: $S = S_{HE} + S_{GB}$, where in D dimensions

$$S_{GB} = \int d^D x \sqrt{-g} (R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda}) \qquad (10)$$

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Afterword

- ΛCDM model in cosmology seems to be in greater agreement with observations, while the vast majority of MOG do not. However nature of dark energy yet undetermined.

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- AdS - CFT correspondance.
- Not yet clear which of these actions give physically accepted solutions.
- For instance in the Gauss-Bonnet case the answer is:

OOuups ran out of time!!!

According to the new rules you are not allowed to sleep

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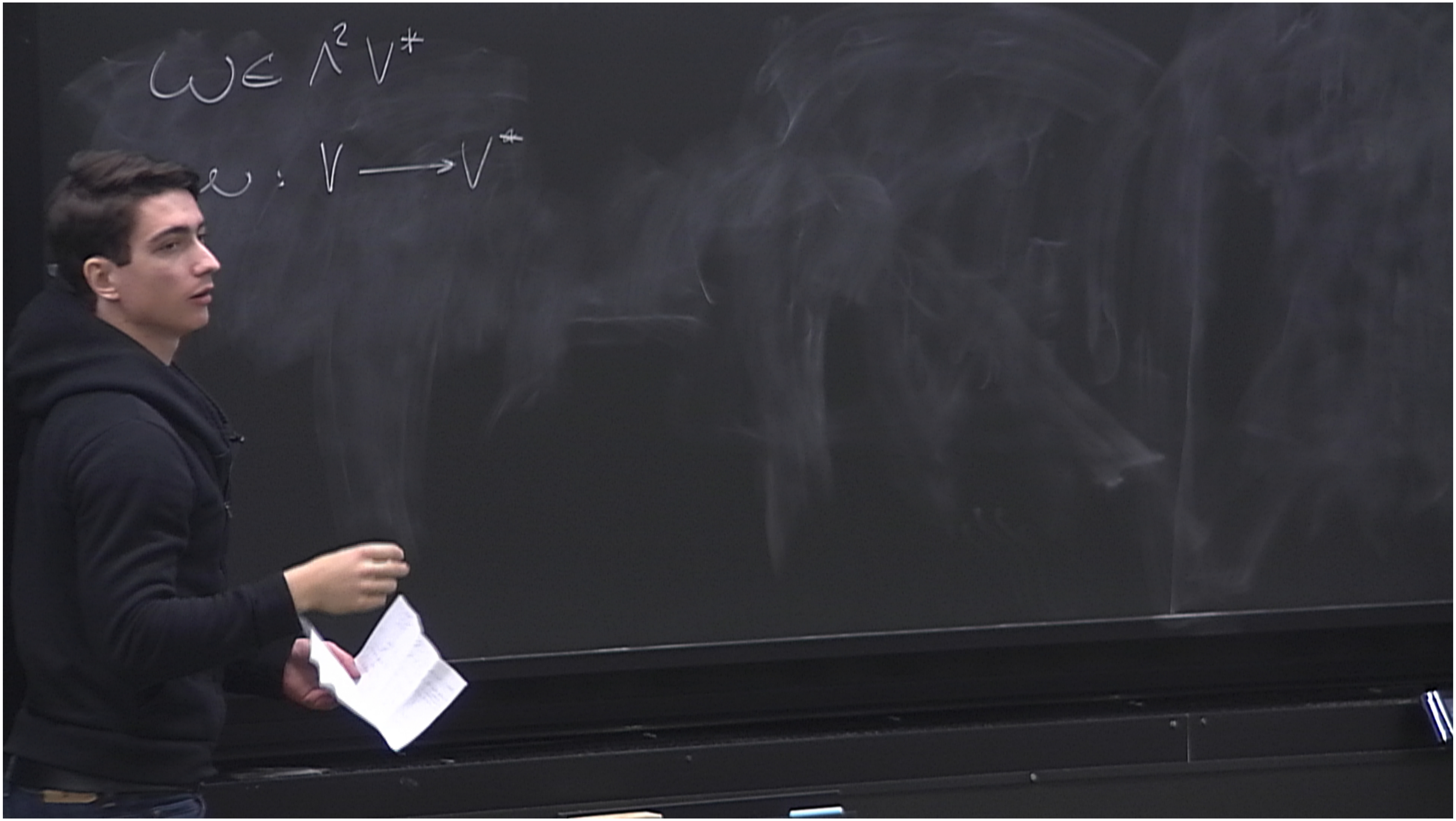
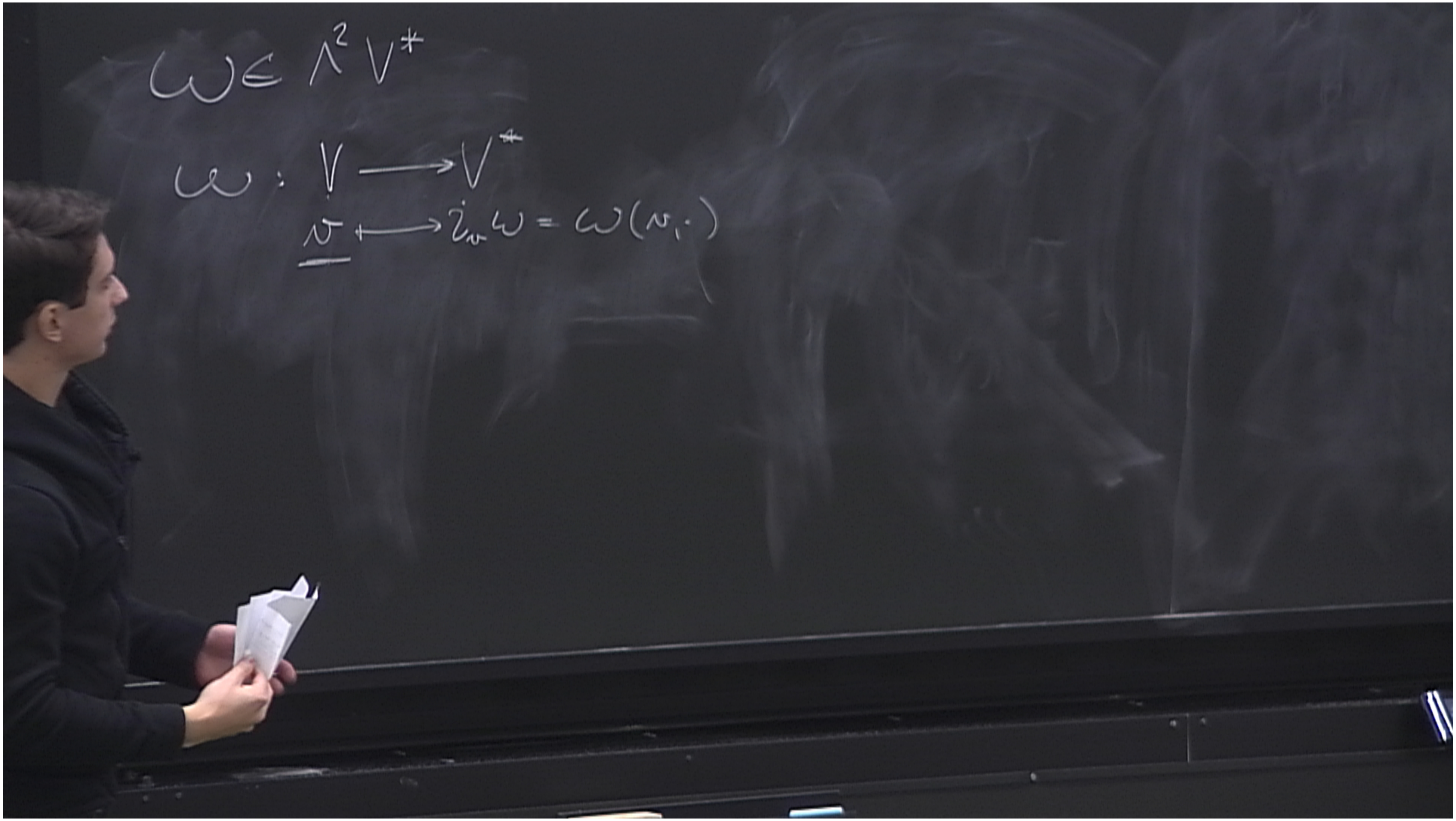
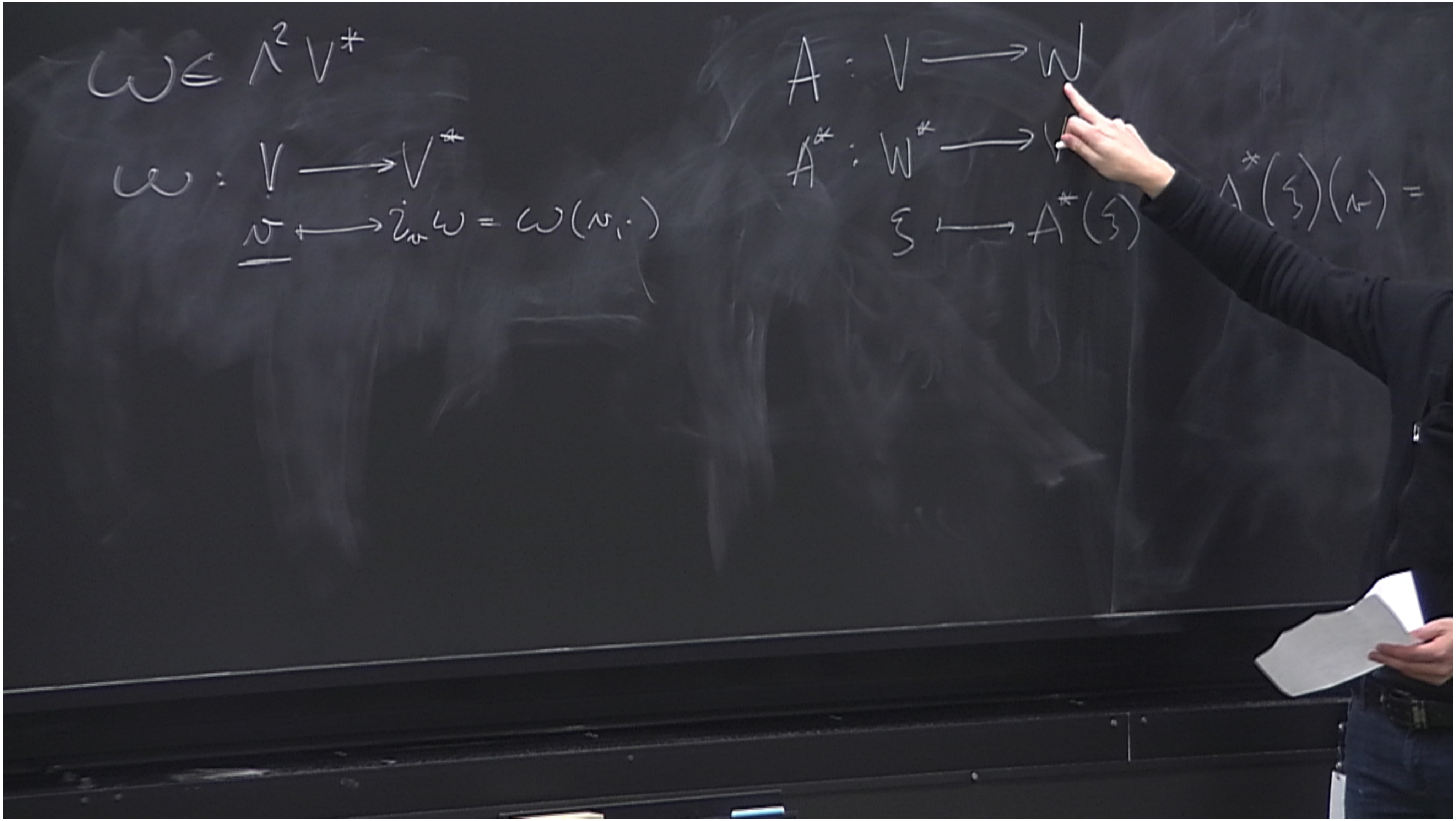
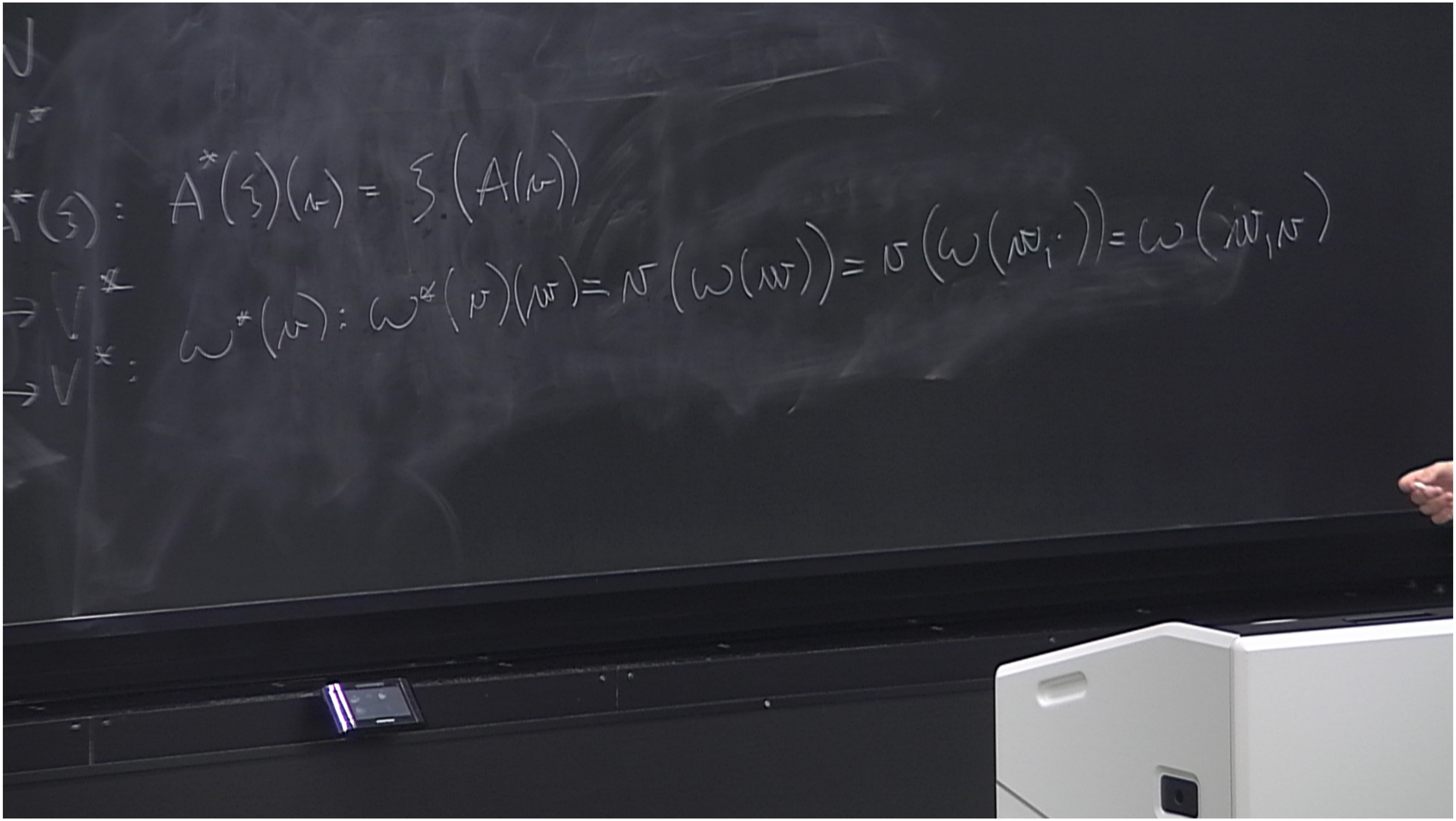
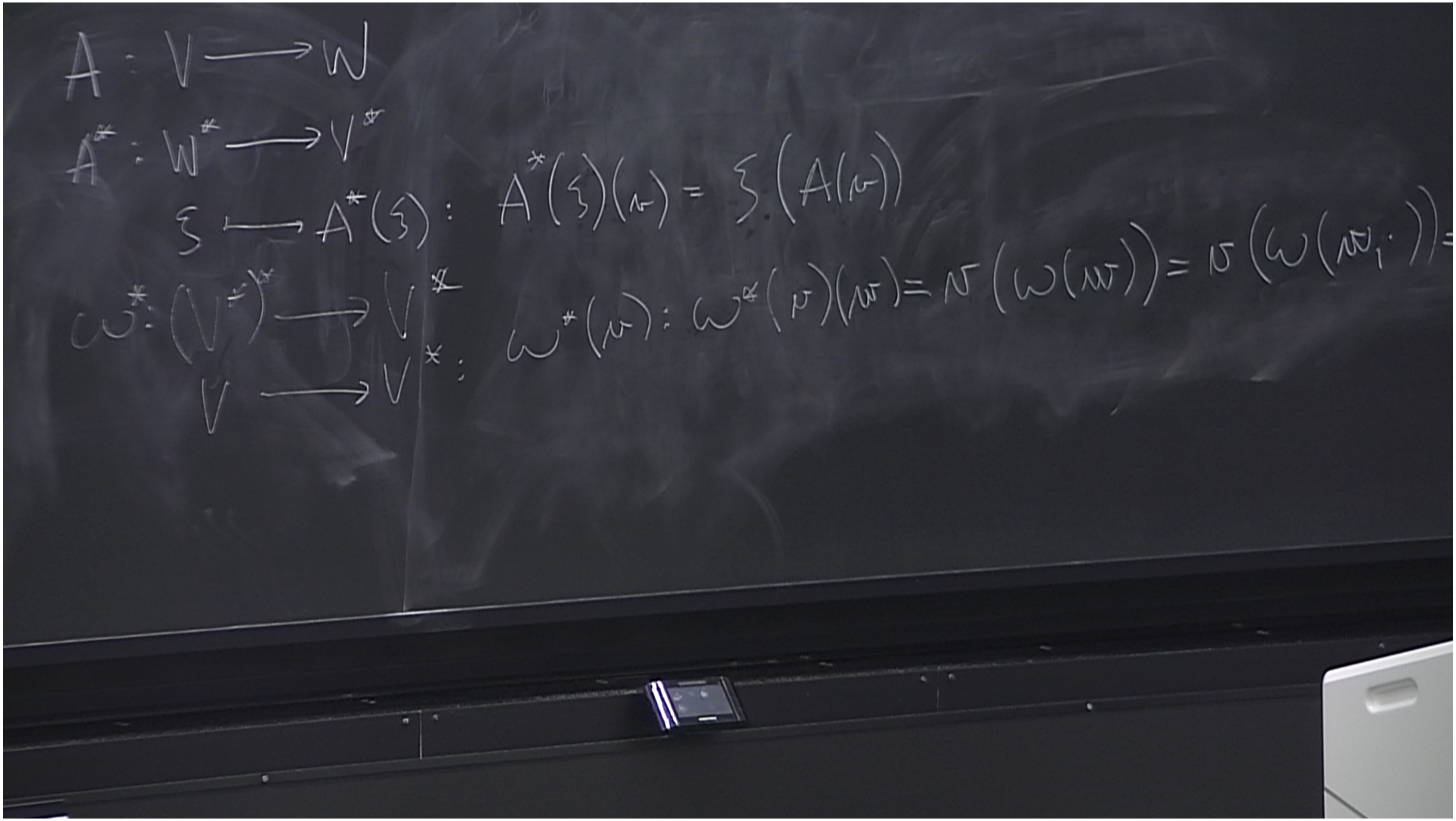
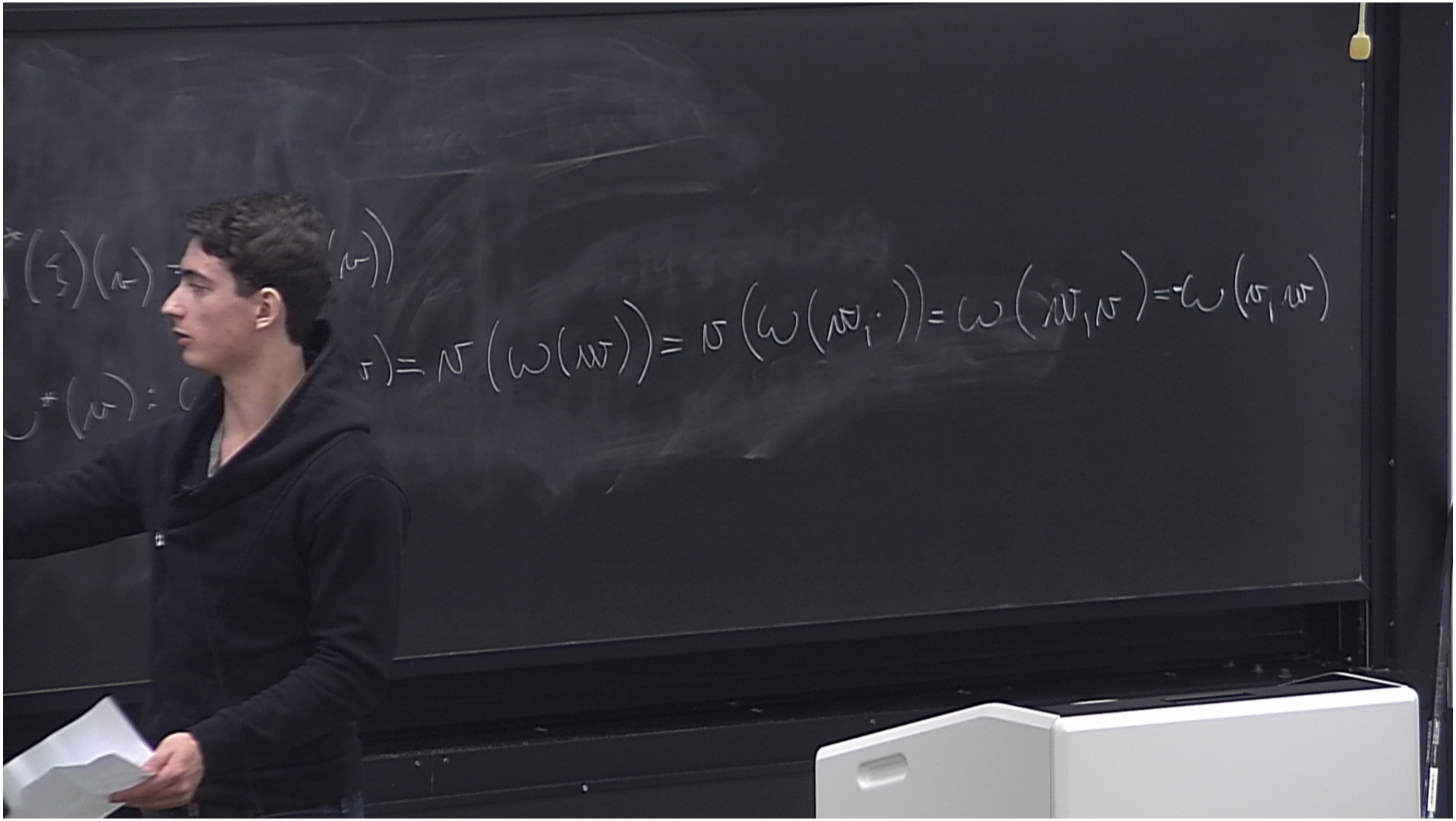

$\omega \in \Lambda^2 V^*$
$\omega : V \longrightarrow V^*$

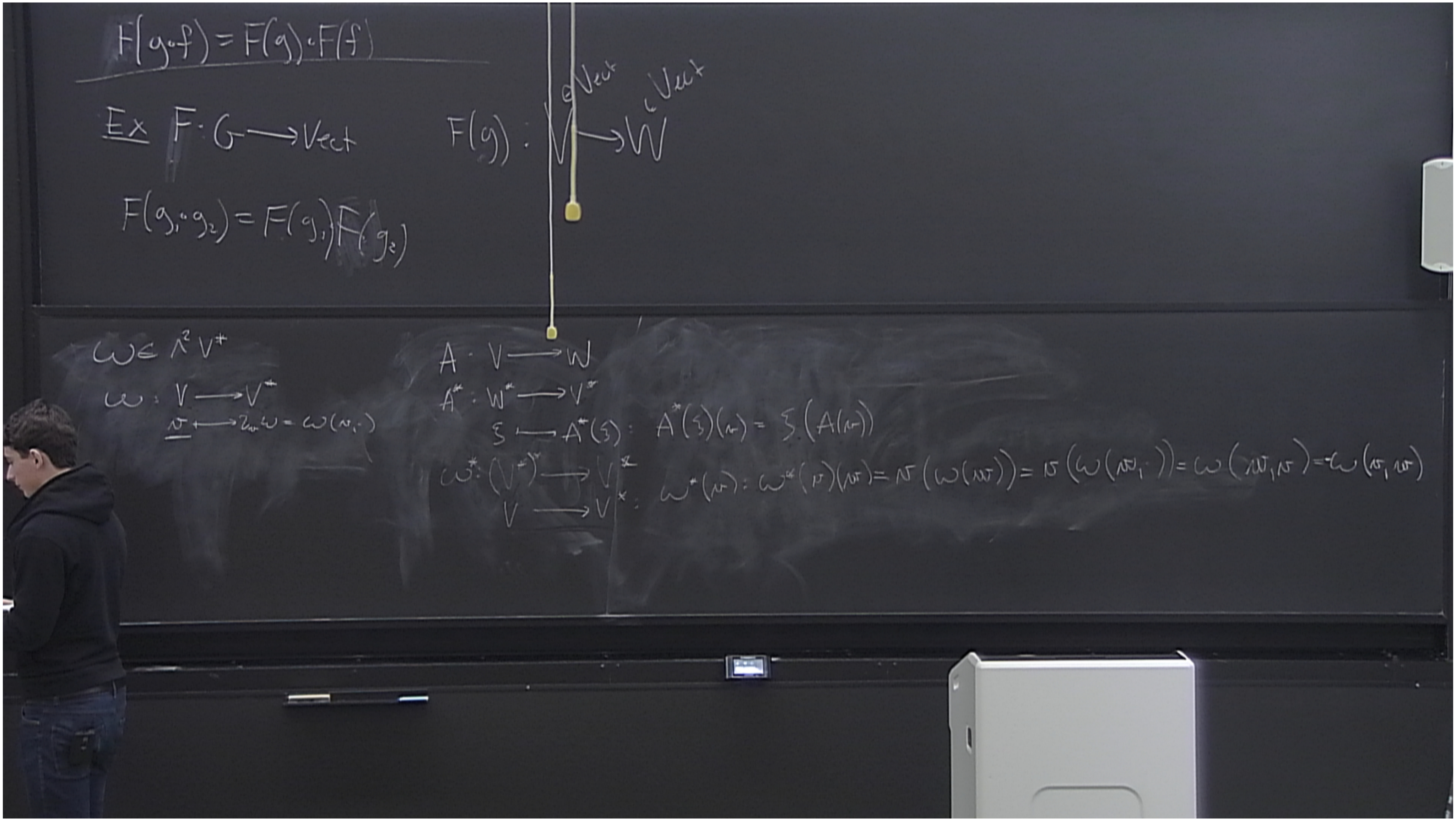

ω ∈ Λ² V*
ω : V ⟶ V*
v ⟼ i_v ω = ω(v, ·)


ω ∈ Λ²V*
ω : V → V*
v ↦ i_v ω = ω(v,·)
A : V → W
A* : W* → V*
ξ ↦ A*(ξ)
A*(ξ)(v) =


$A^*(\xi): A^*(\xi)(v) = \xi(A(v))$
$\omega^*(v): \omega^*(v)(w) = v(\omega(w)) = v(\omega(w,\cdot)) = \omega(w,v)$


$A : V \longrightarrow W$
$A^* : W^* \longrightarrow V^*$
$\xi \longmapsto A^*(\xi) : \quad A^*(\xi)(v) = \xi(A(v))$
$\omega^* : (V^*)^* \longrightarrow V^*$
$V \longrightarrow V^{**} ; \quad \omega^*(v) : \omega^*(v)(w) = v(\omega(w)) = v(\omega(w, \cdot)) =$


)= v(ω(w)) = v(ω(w, ·)) = ω(w, v) = -ω(v, w)


$F(g \circ f) = F(g) \circ F(f)$
Ex $F: G \longrightarrow Vect$
$F(g): V \longrightarrow W$
$\in Vect$
$\in Vect$
$F(g_1 \circ g_2) = F(g_1) F(g_2)$
$\omega \in \Lambda^2 V^*$
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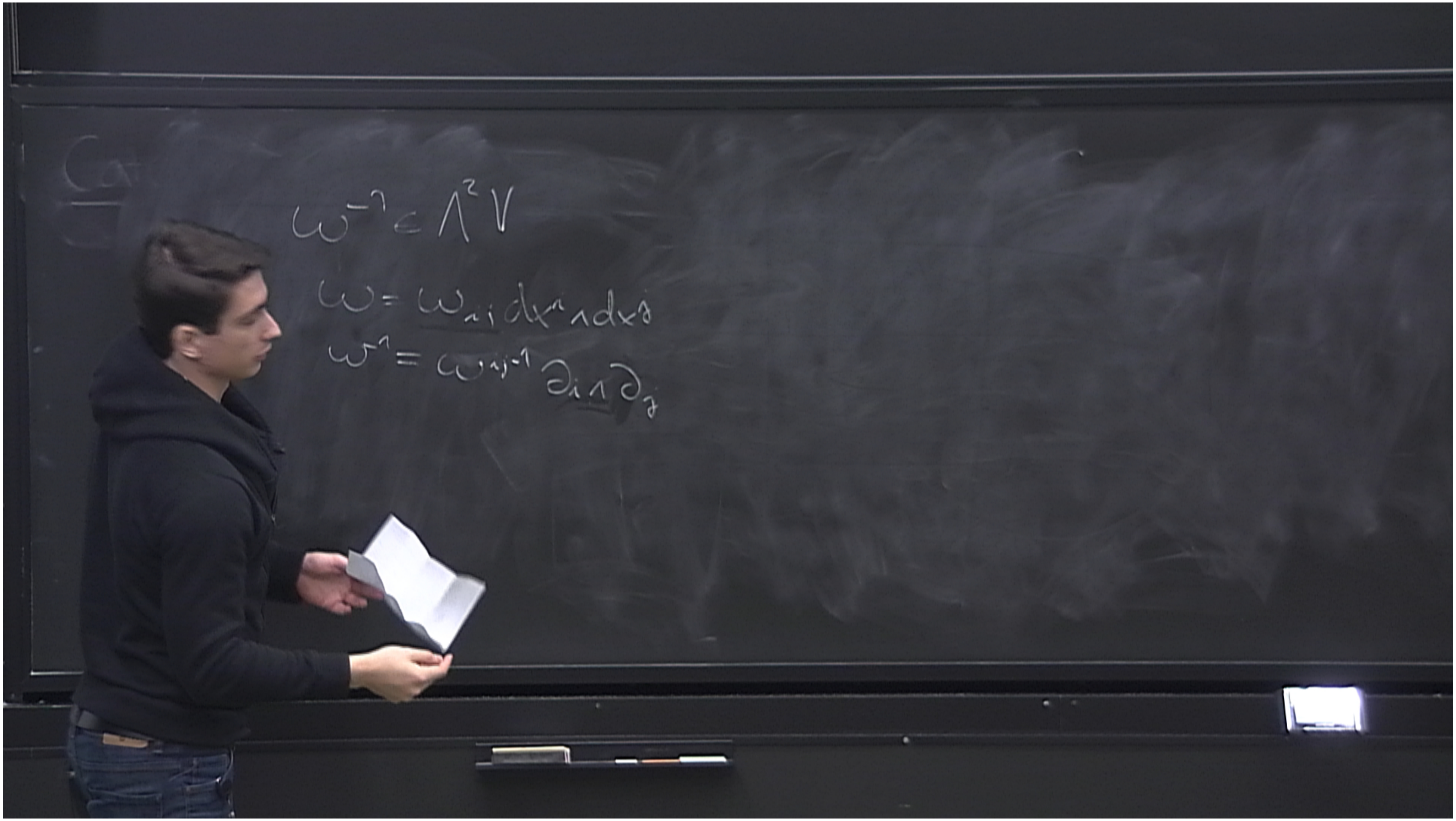
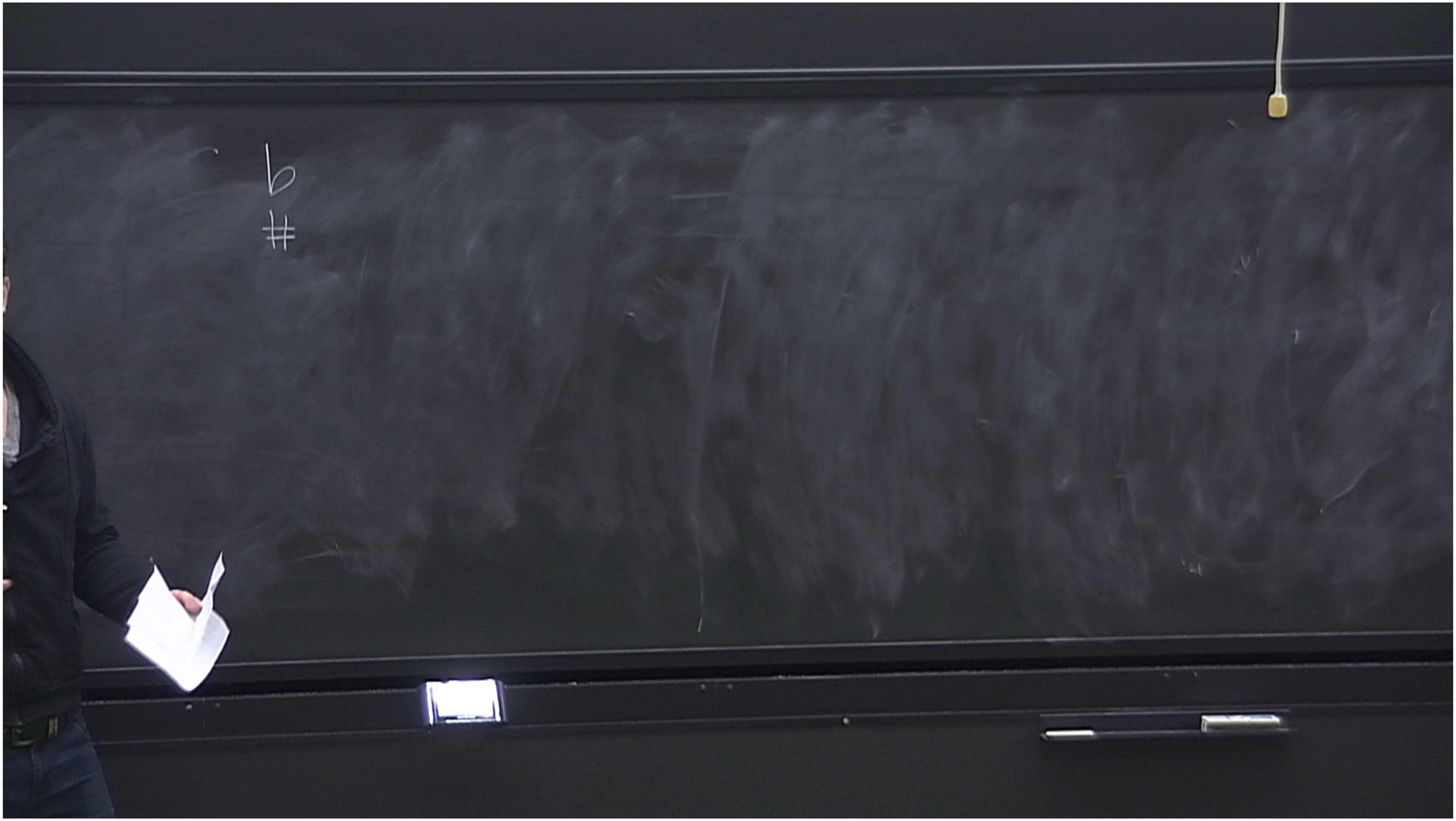
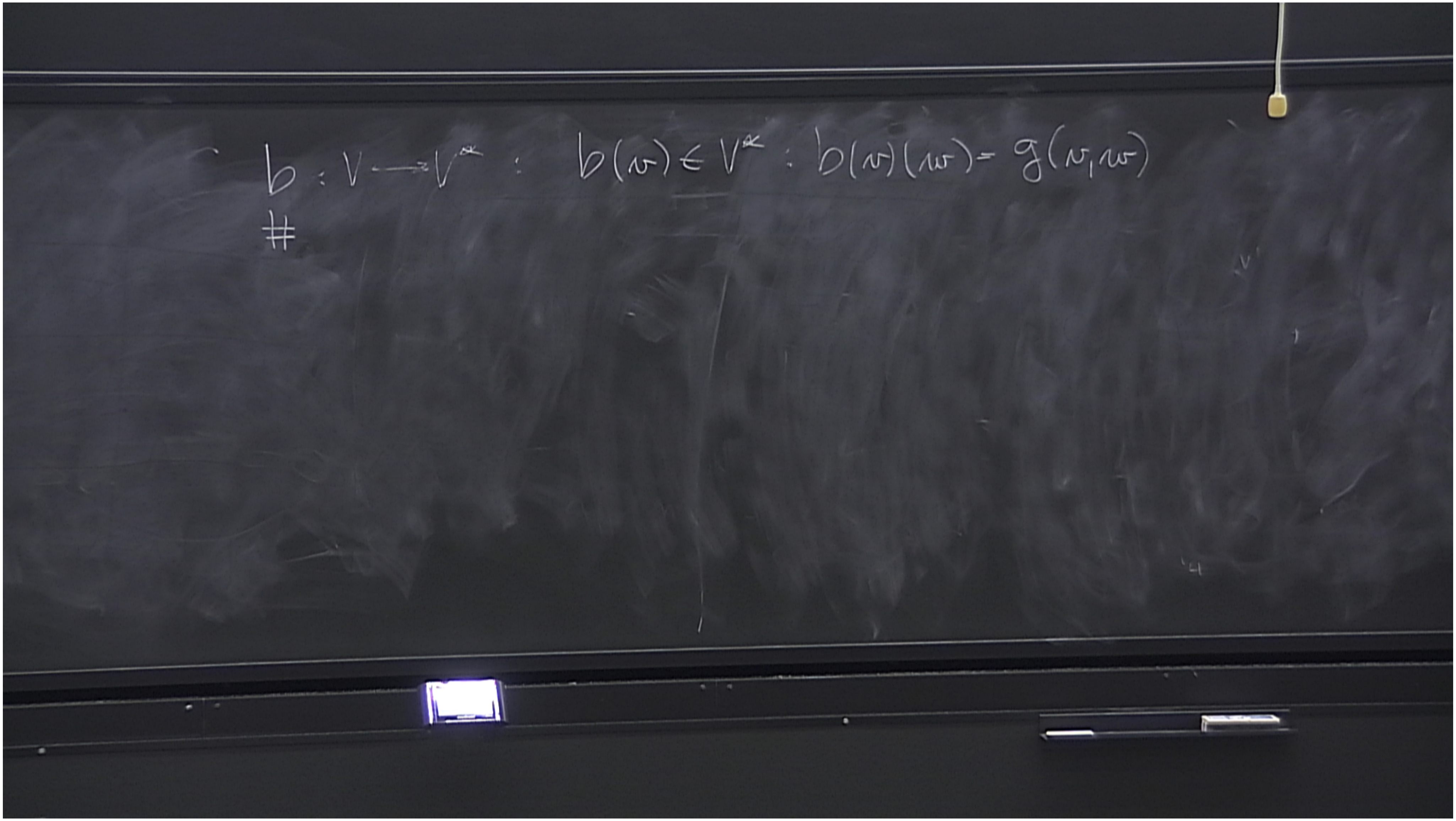
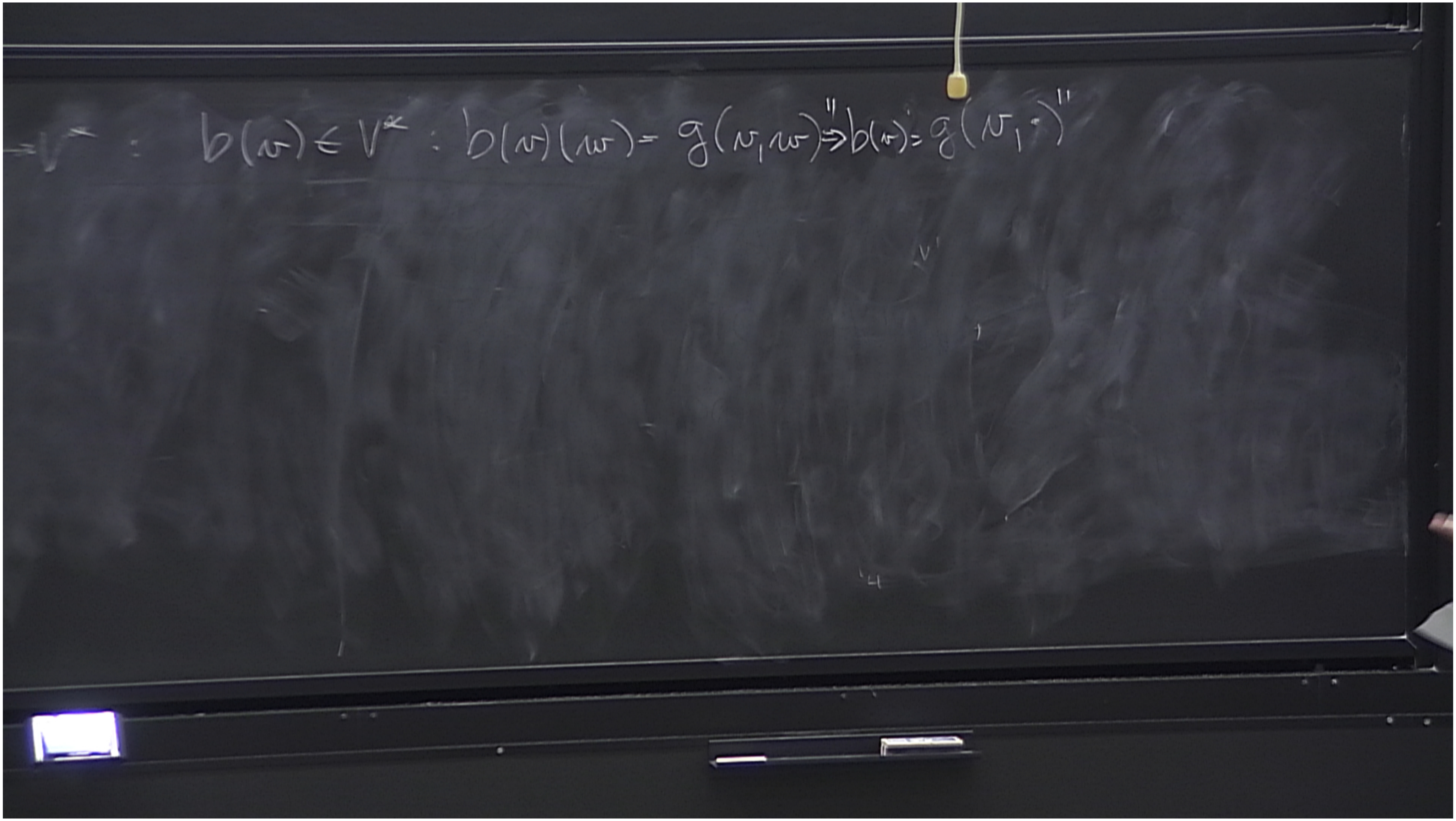
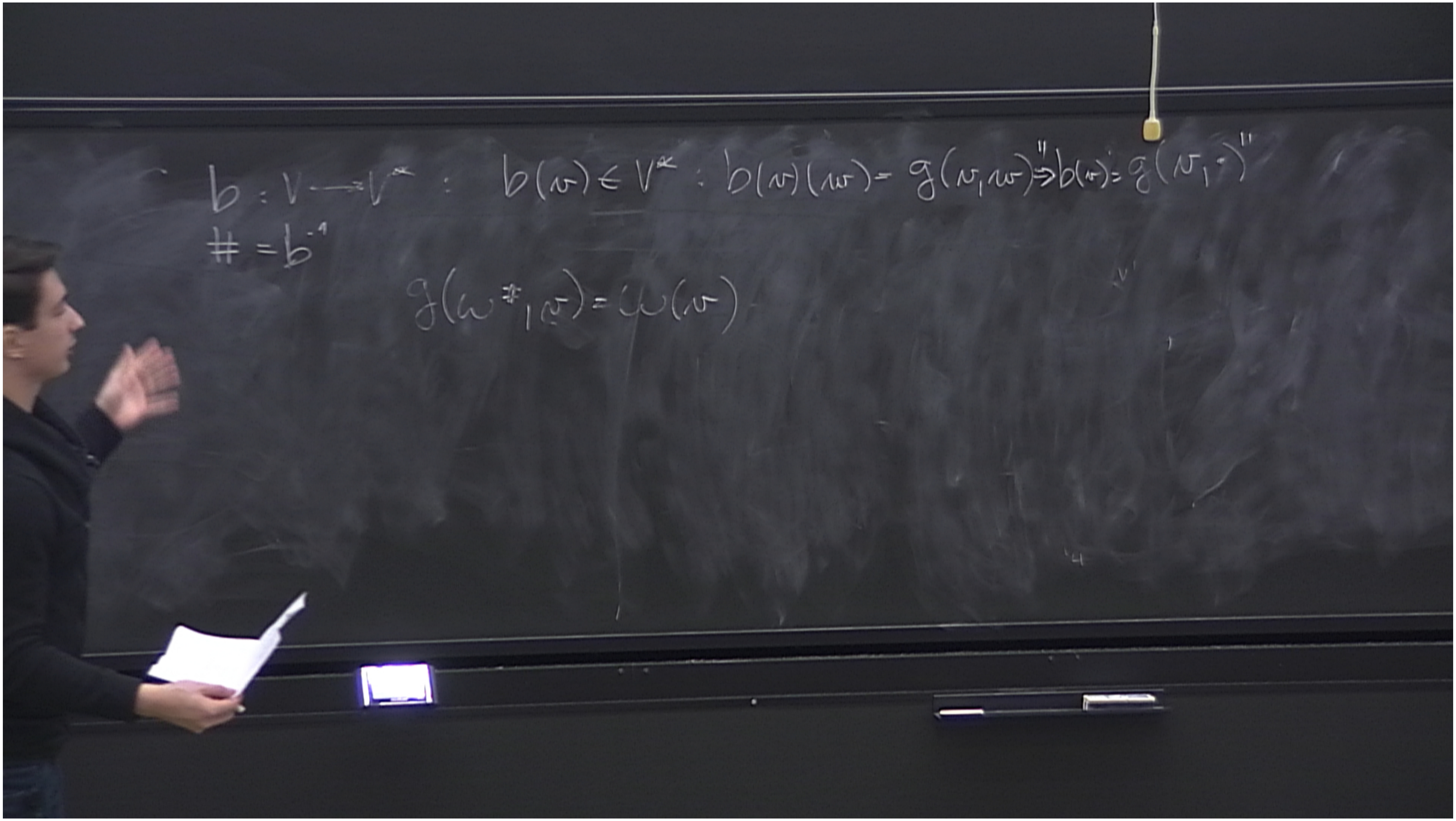

ω⁻¹ ∈ Λ²V
ω = ω_ij dx^i ∧ dx^j
ω⁻¹ = ω_ij⁻¹ ∂_i ∧ ∂_j


b
#


b : V ⟶ V* : b(v) ∈ V* : b(v)(w) = g(v,w)
#


b(v) ∈ V* : b(v)(w) = g(v, w) "⇒ b(v) = g(v, ·)"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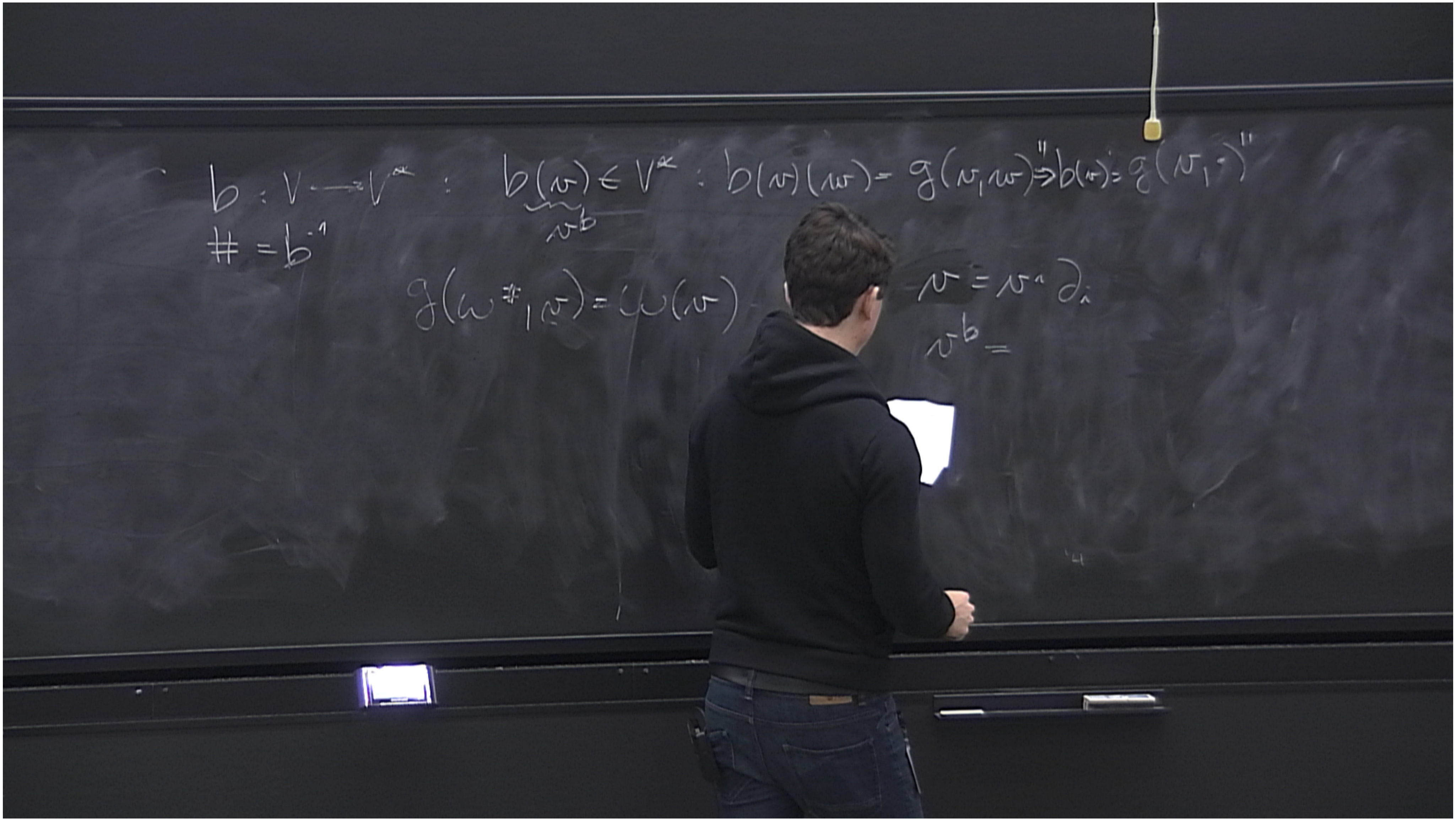
# = b⁻¹
g(ω#, v) = ω(v)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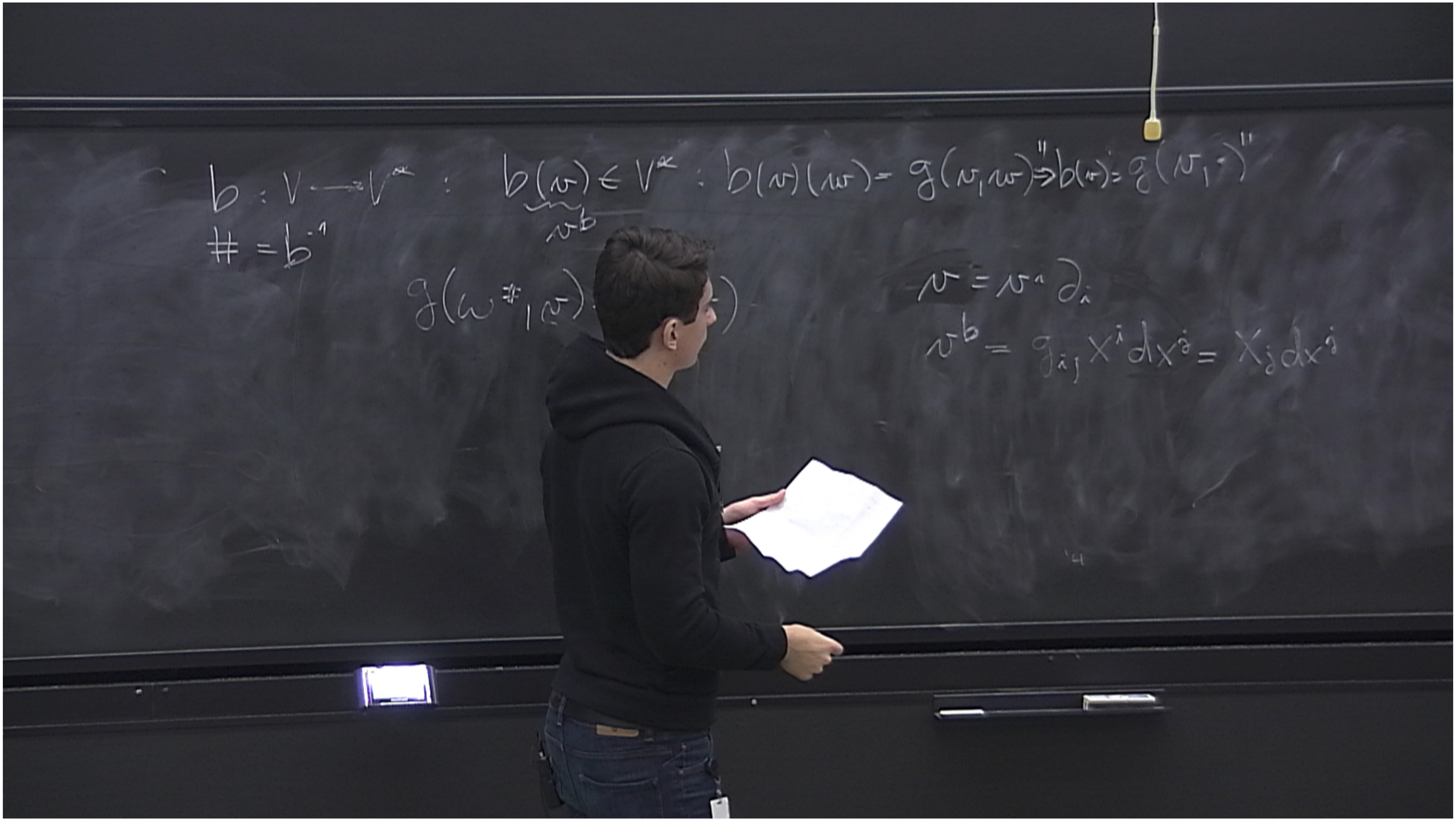
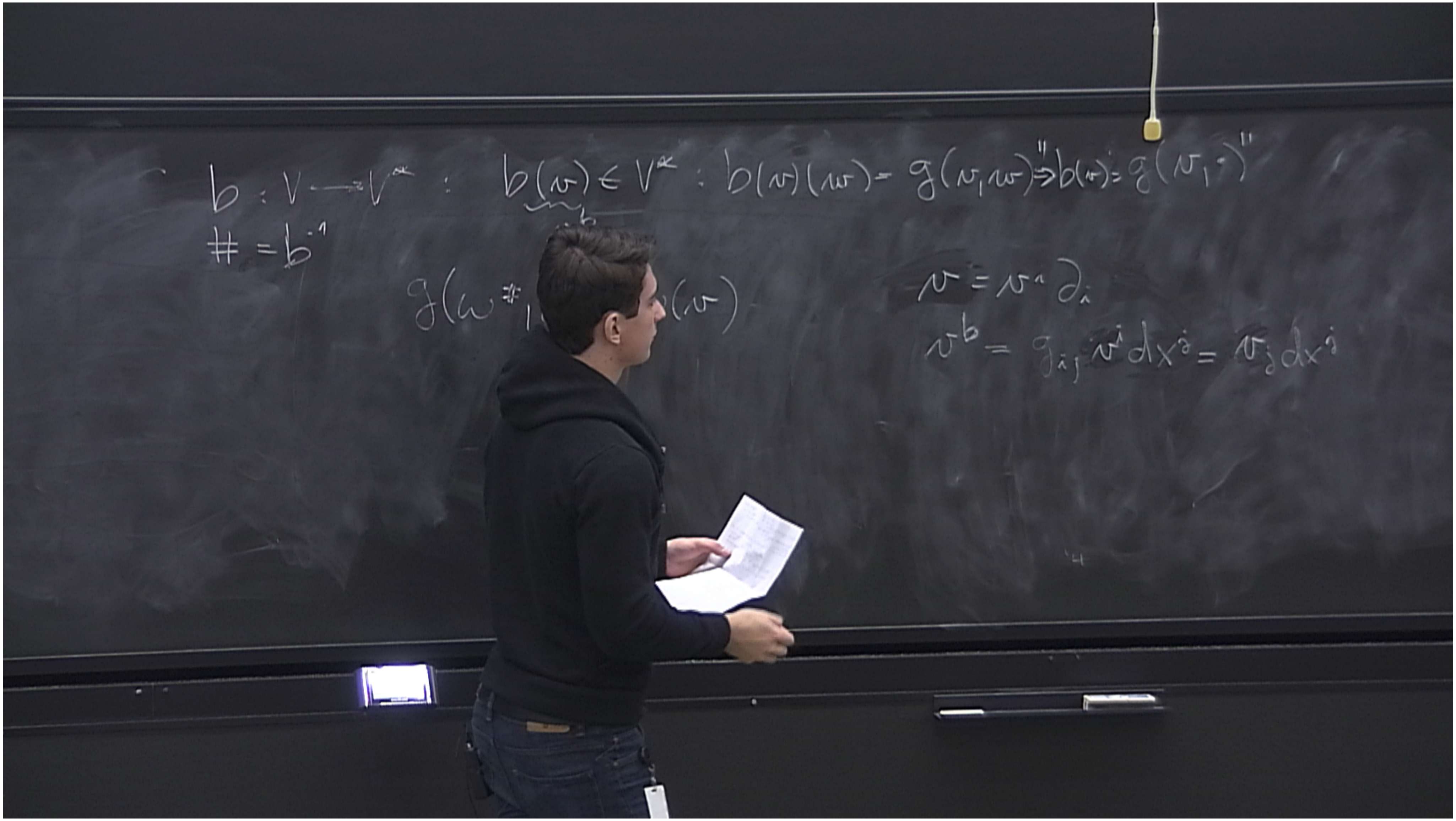
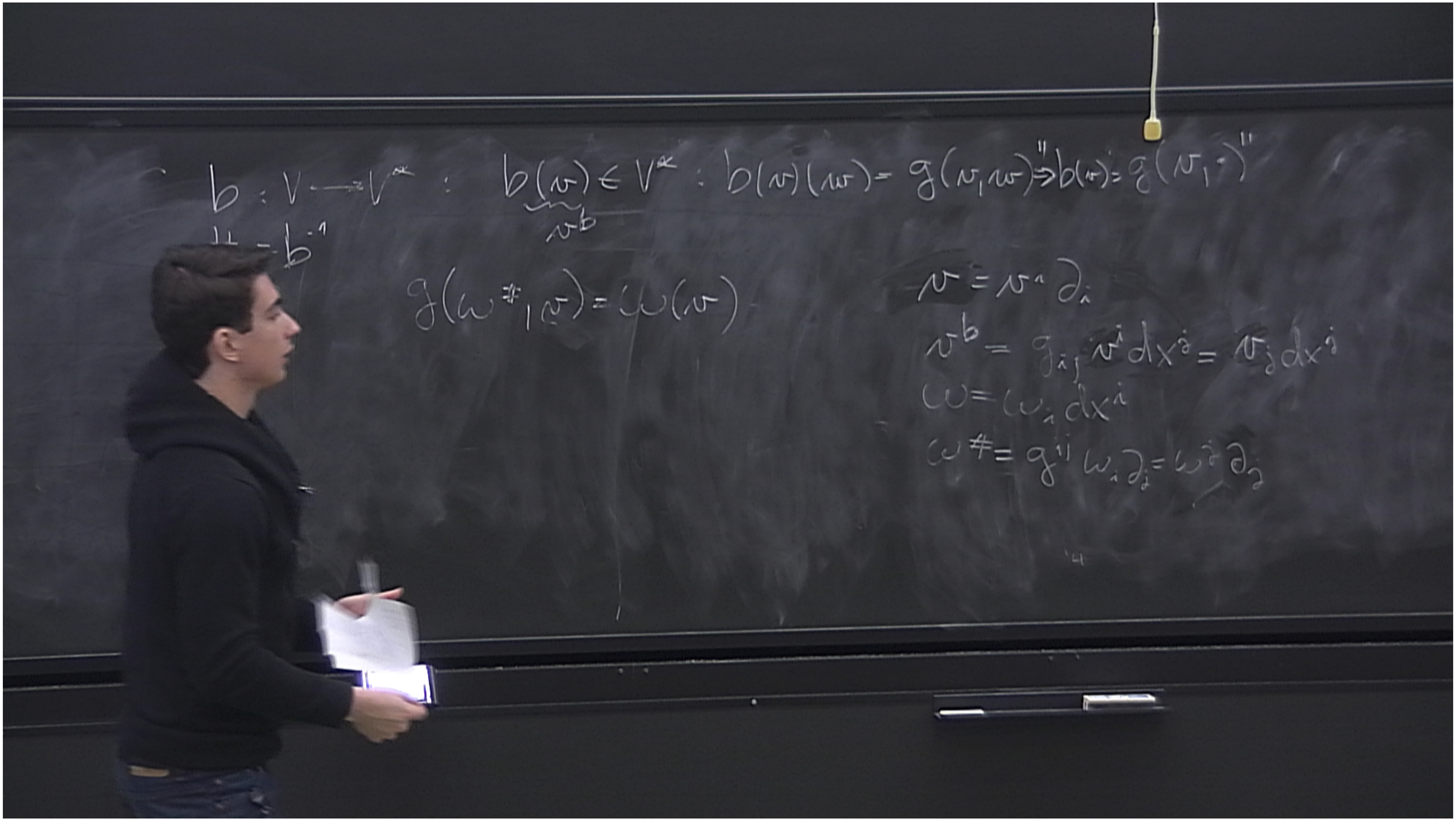
# = b⁻¹
g(ω#, v) = ω(v)
v = vᵃ ∂ₐ
vᵇ =


b : V ⟶ V* : b(v) ∈ V* : b(v)(w) = g(v, w) "⇒ b(v) = g(v, ·)"
v^b
# = b^-1
v = v^i ∂_i
v^b = g_ij X^i dx^j = X_j dx^j


b : V → V* : b(v) ∈ V* : b(v)(w) = g(v, w) "⇒ b(v) = g(v, ·)"
# = b⁻¹
v = vⁱ ∂ᵢ
vᵇ = gᵢⱼ vⁱ dxʲ = vⱼ dxʲ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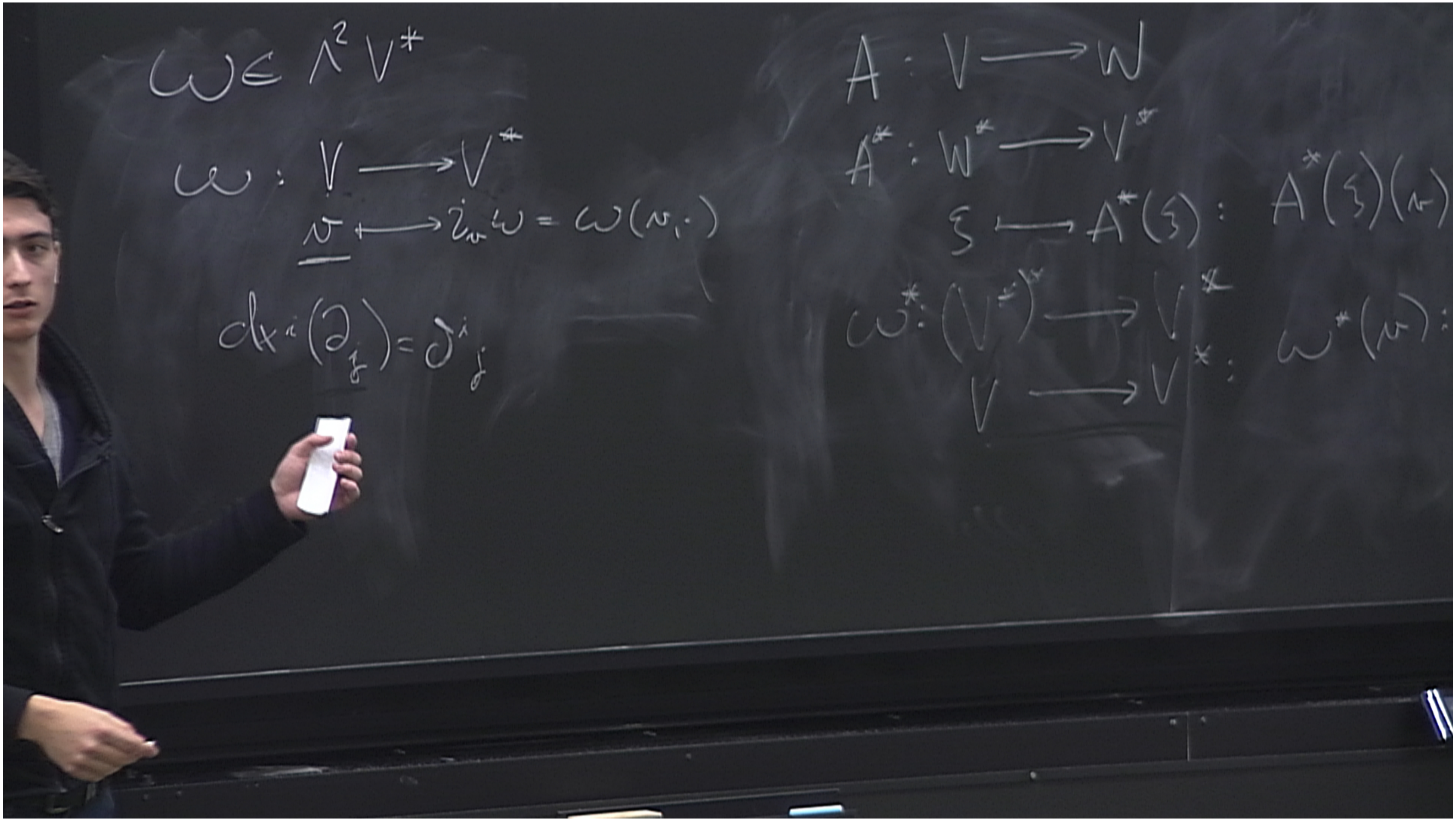
ω = ω_i dx^i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dx^i(∂_j) = δ^i_j
A : V ⟶ W
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ξ ⟼ A*(ξ) :
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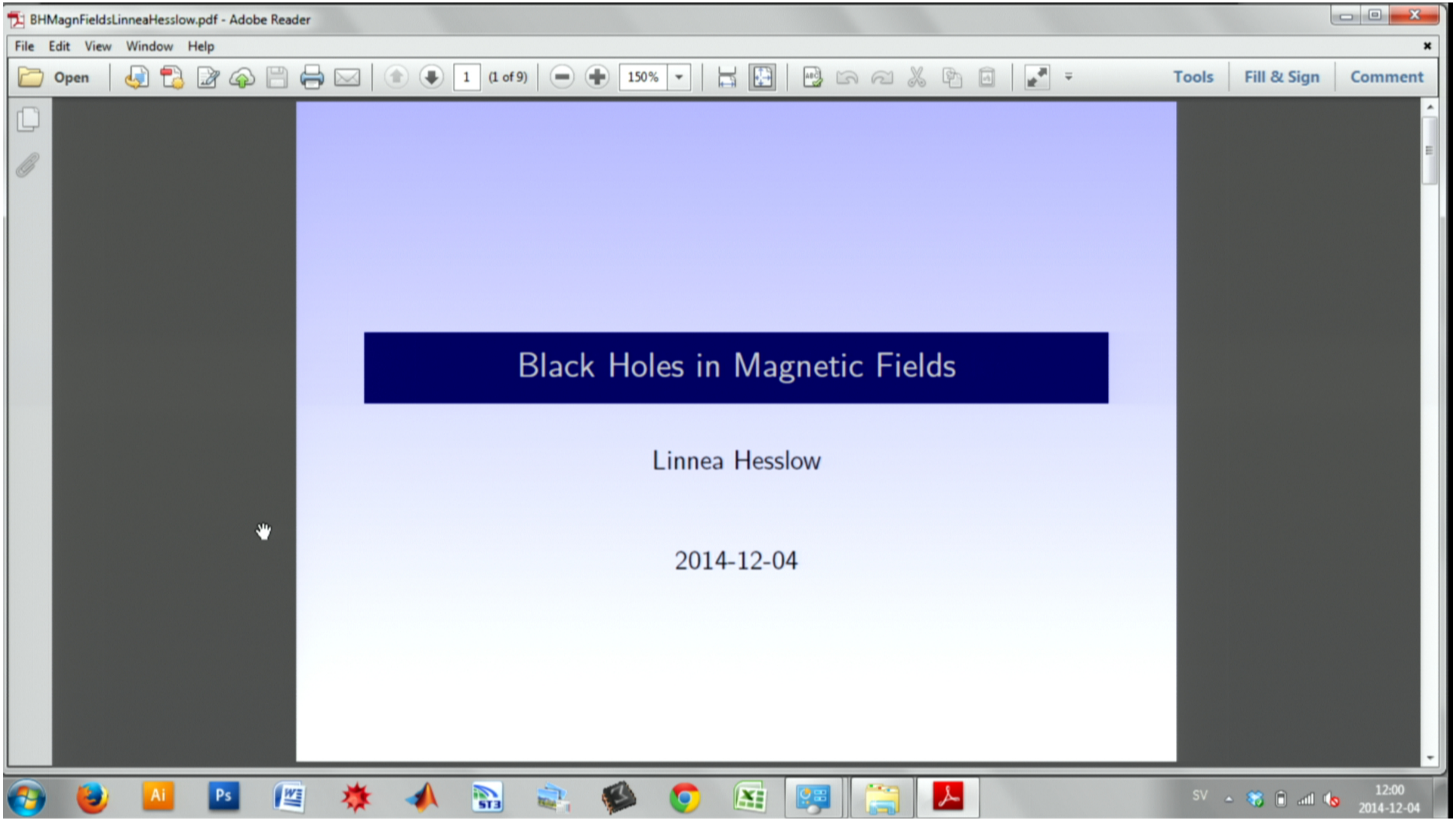
V ⟶ V*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Black Holes in Magnetic Fields
Linnea Hesslow
2014-12-04
SV
12:00
2014-12-04

Outline

- Killing vectors to find solutions for $F_{\mu\nu}$
- Black hole Meissner effect
- Problem explaining jet formation
- Review of two papers: Wald[1] and Bicak et al. [2]

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[1] Wald, R. (1974) "Black hole in uniform magnetic field". *Phys. Review*, vol. 10, nr 6, pp. 1680-84.

[2] J. Bicak, V. Karas and T. Ledvinka, "Black holes and magnetic fields," IAU symp. [IAU Symp. **238**, 139 (2007)] [astro-ph/0610841].

Mathematical Preliminary

- Killing vector - symmetry

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0$$

- Definition of $R_{\lambda\mu\nu\sigma}$:

$$\xi_{\mu;\nu;\sigma} - \xi_{\mu;\sigma;\nu} = -\xi^{\lambda} R_{\lambda\mu\nu\sigma}$$

- Cyclic permutation and Killing equation:

$$\xi_{\mu;\nu;\sigma} = \xi^{\lambda} R_{\lambda\mu\nu\sigma} \Rightarrow \xi^{\mu;\nu}{}_{;\nu} = R^{\mu}{}_{\lambda}\xi^{\lambda}$$

- In vacuum: $R_{\mu\nu} = 0 \Rightarrow$ Solves Maxwell's equations.

Black Holes in Nature

- Schwarzschild black hole: non-rotating, in vacuum
- In astrophysics:
  - Rotating
  - Surrounded by plasma
  - Chargeless
  - Relativistic jets are often emitted at poles
- Model: Kerr black hole and magnetic test field

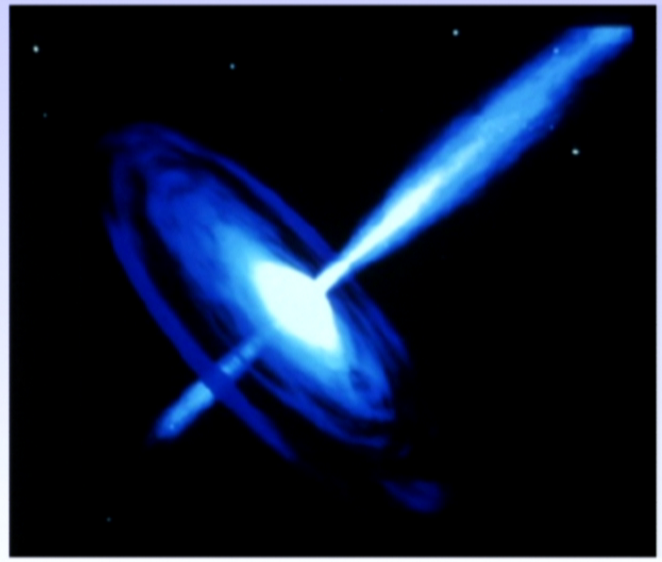


Image: NASA

Kerr Black hole

- Metric:

$$ds^2 = -(1 - \frac{2mr}{\Sigma})dt^2 - (\frac{4marsin^2\theta}{\Sigma})dtd\phi +$$

$$+\frac{(r^2+a^2)^2 - \Delta a^2 sin^2\theta}{\Sigma} sin^2\theta d\phi^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2$$

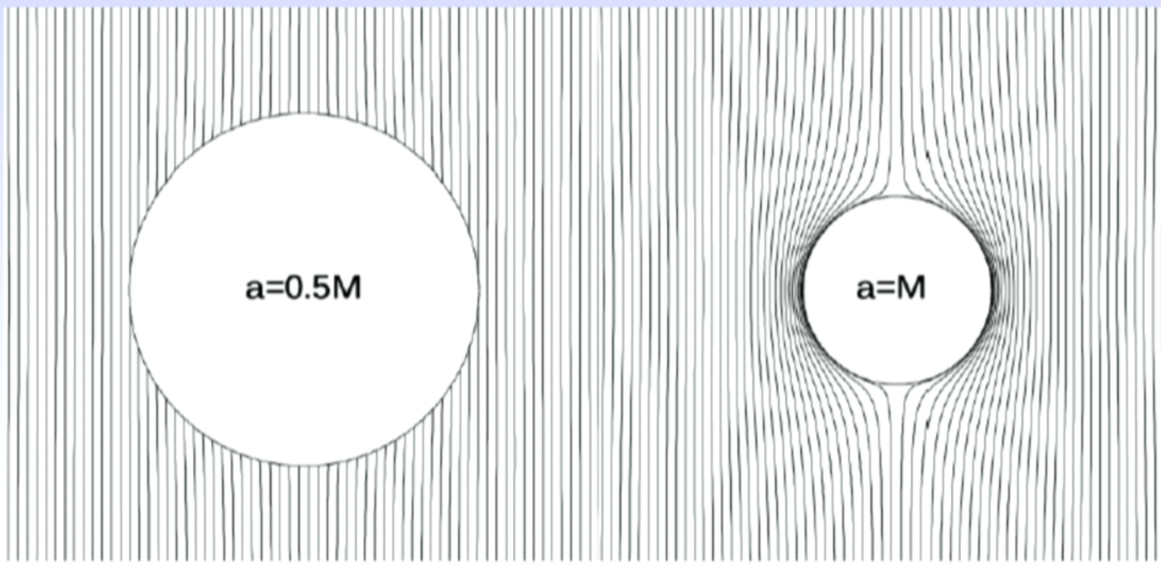
- $\Sigma = r^2 + a^2 cos^2\theta, \Delta = r^2 + a^2 - 2mr$
- Parameter $a = J/M \in [0, M]$ Angular momentum per unit mass. $a = 0 \Rightarrow$ Schwarzschild, $a = M \Rightarrow$ *extremal*

Test field solution

- Keep the metric static and solve Maxwell's equations
- Axisymmetric, stationary, vacuum solution (eg. Kerr)
- Two Killing vectors: timelike and axial
- Superposition of the two $F_{\mu\nu}$ solutions using Killing vectors gives a black hole solution in a uniform magnetic field

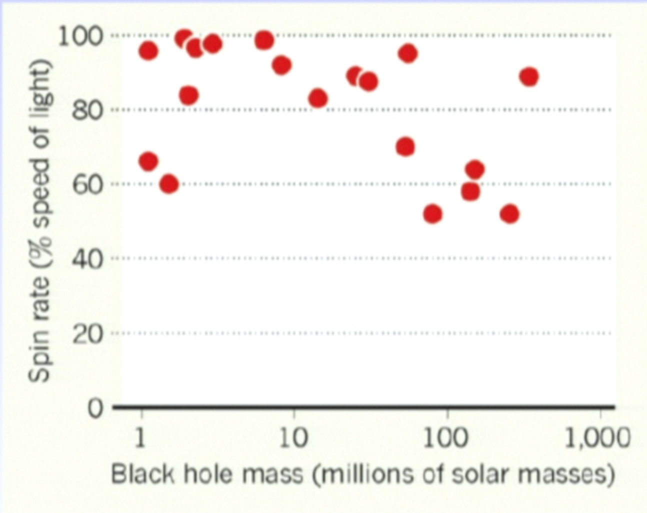
The Meissner effect for black holes

- Use the solution with magn field that is uniform at infinity.
- Extremal black holes quench external magnetic fields



Black holes and jets

- Most observed black holes seem close to extremal[3]
- Jet creation mechanism not understood
- Blandford Znajek mechanism considered the most relevant
- Field must penetrate horizon



[1]E. Samuel Reich (2013) "Spin rate of black holes pinned down" *Nature*, vol 500, Issue 7461.

Open question

Do extremal black holes produce relativistic jets?