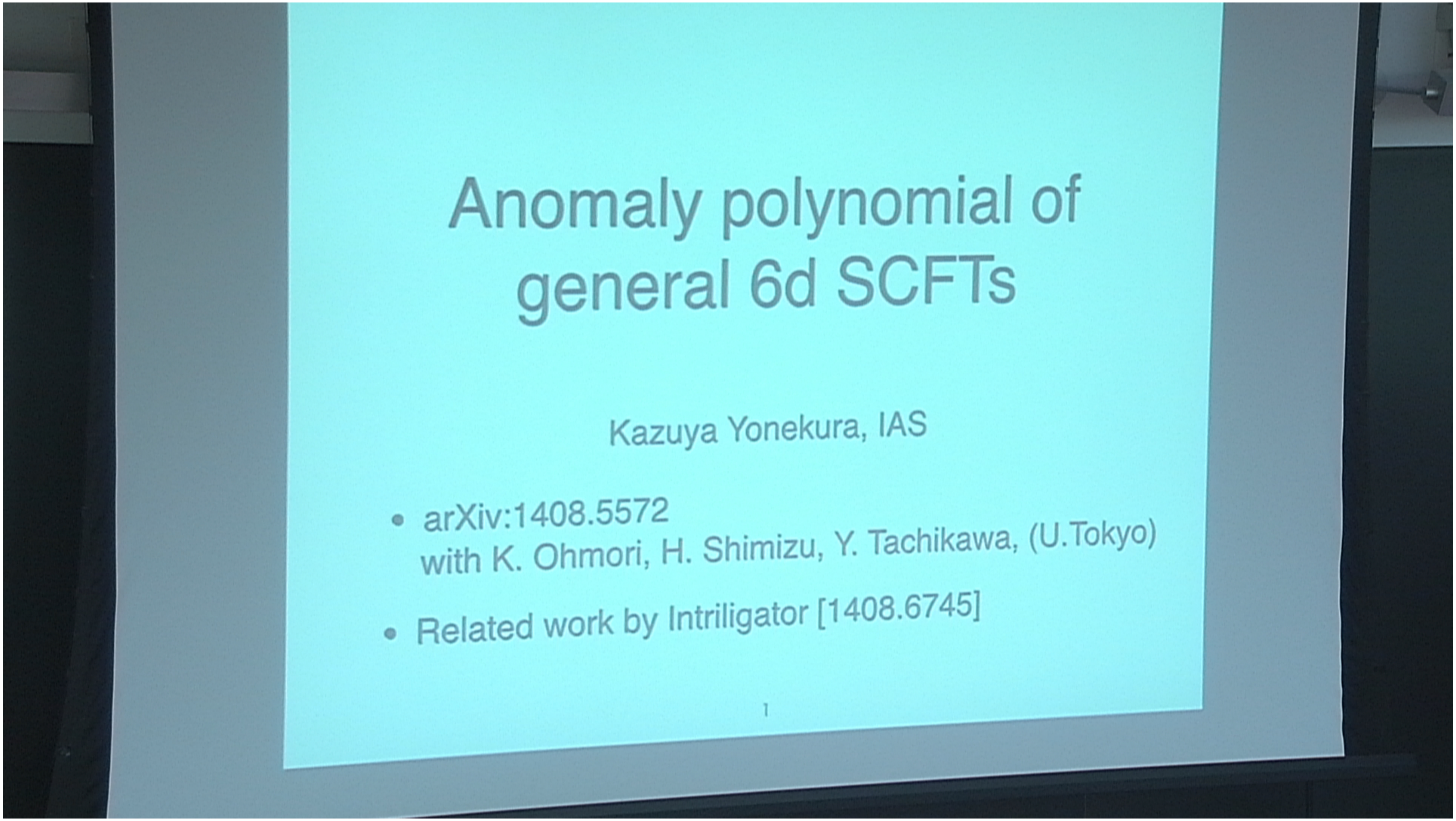
Title: Anomaly polynomial of general 6d SCFTs

Date: Dec 12, 2014 02:30 PM

URL: http://pirsa.org/14120048

Abstract: <p>> I talk about a method to determine the anomaly polynomials of genera 6d N=(2,0) and N=(1,0) SCFTs, in terms of the anomaly matching on their tensor branches. This method is almost purely field theoretical, and can be applied to all known 6d SCFTs. Green-Schwarz mechanism plays the crucial role.</p>


Anomaly polynomial of general 6d SCFTs
Kazuya Yonekura, IAS
• arXiv:1408.5572
with K. Ohmori, H. Shimizu, Y. Tachikawa, (U.Tokyo)
• Related work by Intriligator [1408.6745]
1

Introduction

6d N=(2,0) theories have been very important in recent developments in SUSY gauge theories.
Variety of theories and dualities have been obtained via compactification to lower dimensions.

There are also infinitely many 6d N=(1,0) theories.

- 5 branes at singularities, etc.
- All known theories can be constructed by F-theory

They may be as interesting as N=(2,0) theories!

2

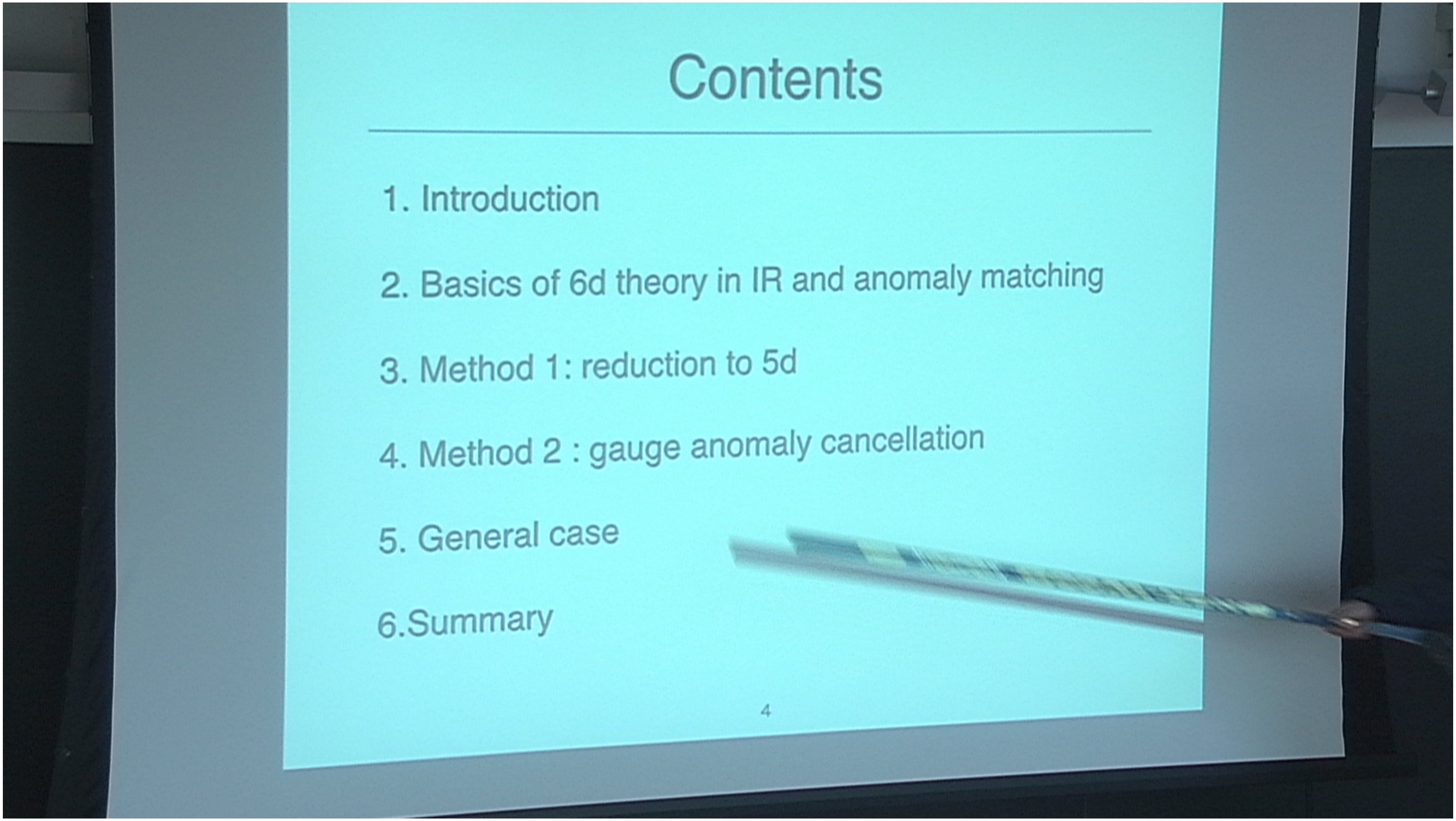
Introduction

However, properties of N=(1,0) theories are not well studied because of the lack of UV Lagrangians.

What we have studied: 't Hooft anomaly
Anomaly polynomials of the theories under background gravity and global symmetry gauge fields.

I would like to explain a general method to compute the anomaly polynomial of any known N=(1,0) theory.

3

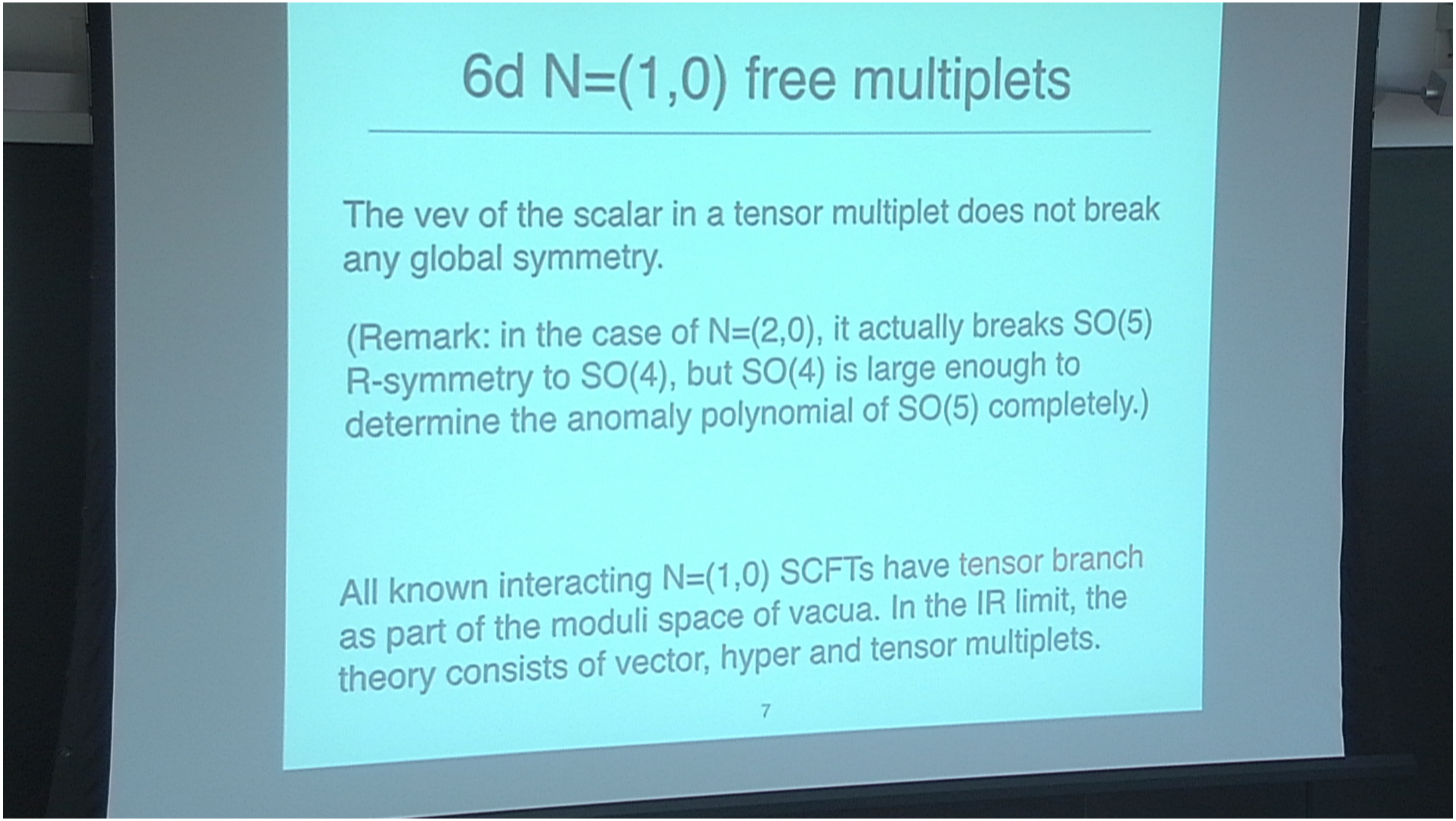

Contents
1. Introduction
2. Basics of 6d theory in IR and anomaly matching
3. Method 1: reduction to 5d
4. Method 2 : gauge anomaly cancellation
5. General case
6.Summary
4

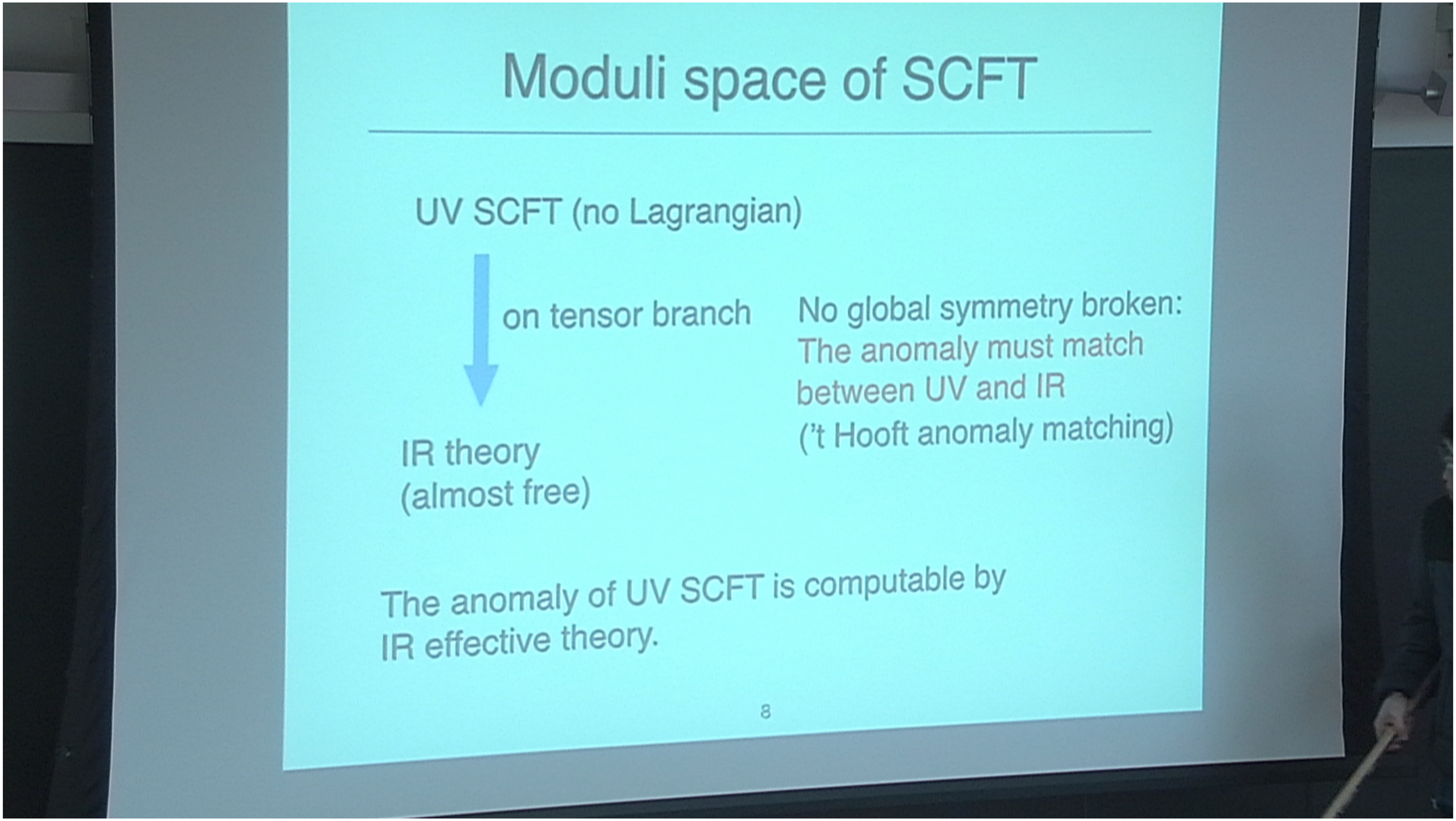
6d N=(1,0) free multiplets

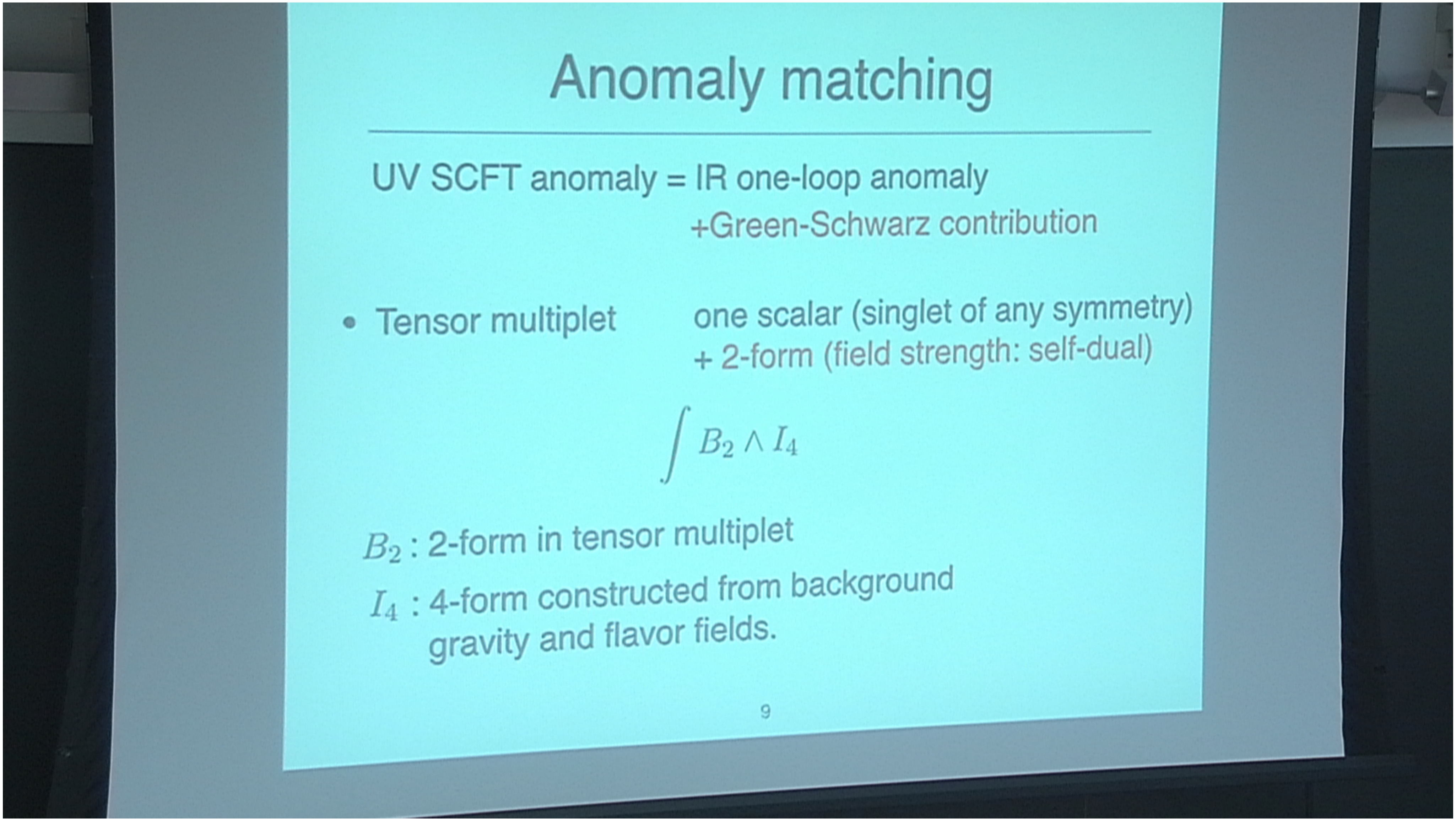
There are three types of multiplets which are free in IR. (Spin $\leq$ 1)

	Bosonic components
• Vector multiplet	gauge fields
• Hyper multiplet	two complex scalars (doublet of SU(2) R-symmetry)
• Tensor multiplet	one scalar (singlet of any symmetry) + two form (field strength: self-dual)

6


6d N=(1,0) free multiplets
The vev of the scalar in a tensor multiplet does not break any global symmetry.
(Remark: in the case of N=(2,0), it actually breaks SO(5) R-symmetry to SO(4), but SO(4) is large enough to determine the anomaly polynomial of SO(5) completely.)
All known interacting N=(1,0) SCFTs have tensor branch as part of the moduli space of vacua. In the IR limit, the theory consists of vector, hyper and tensor multiplets.
7


Moduli space of SCFT
UV SCFT (no Lagrangian)
on tensor branch
IR theory
(almost free)
No global symmetry broken:
The anomaly must match
between UV and IR
('t Hooft anomaly matching)
The anomaly of UV SCFT is computable by
IR effective theory.
8


Anomaly matching
UV SCFT anomaly = IR one-loop anomaly
+Green-Schwarz contribution
• Tensor multiplet
one scalar (singlet of any symmetry)
+ 2-form (field strength: self-dual)
$\int B_2 \wedge I_4$
B_2 : 2-form in tensor multiplet
I_4 : 4-form constructed from background gravity and flavor fields.
9

Anomaly matching

Green-Schwarz contribution to anomaly polynomial:

$(I_4)^2$ (up to coefficient)

Rough explanation (not really a derivation)

E.O.M: $d * H_3 = -I_4$ ⟶ $dH_3 = -I_4$

$$H_3 = dB_2 - I_3^{(0)}; \quad dI_3^{(0)} = I_4$$

Gauge invariance of H_3 requires

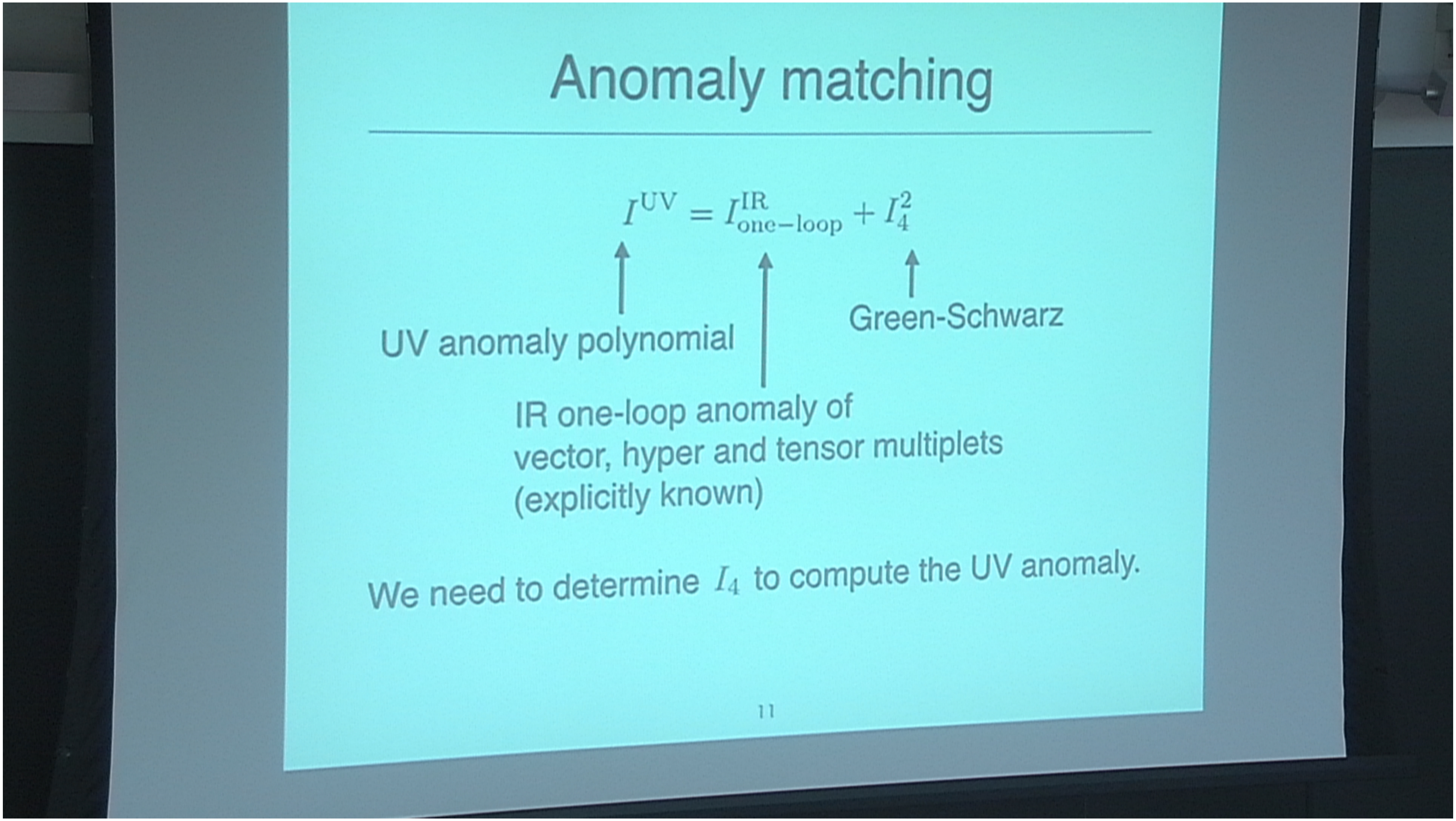
$$\delta_{\text{gauge}} B_2 = I_2^{(1)}; \quad dI_2^{(1)} = \delta_{\text{gauge}} I_3^{(0)}$$

⟶ $\delta_{\text{gauge}} \int B_2 \wedge I_4 = I_2^{(1)} \wedge I_4$

Uplifting to 6+2 dim via the usual descendant eqs.:

$$d(I_2^{(1)} I_4) = \delta_{\text{gauge}}(I_3^{(0)} I_4); \quad d(I_3^{(0)} I_4) = (I_4)^2$$

10


Anomaly matching
$I^{\rm UV} = I^{\rm IR}_{\rm one-loop} + I_4^2$
UV anomaly polynomial
Green-Schwarz
IR one-loop anomaly of
vector, hyper and tensor multiplets
(explicitly known)
We need to determine I_4 to compute the UV anomaly.
11

Contents

12

Reduction to 5d

$$\int B_2 \wedge I_4$$

↓ Compactification on a circle

$$\int A_1 \wedge I_4$$: Chern-Simons in 5d

A_1 : U(1) gauge field of 5d vector multiplet
(tensor mult. in 6d → vector mult. in 5d)

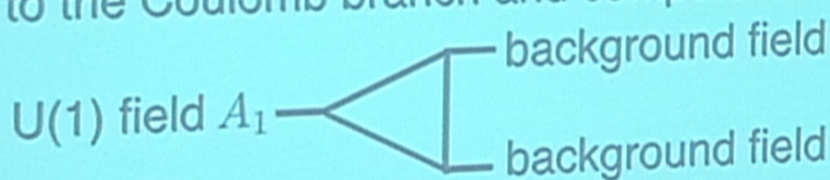
We can determine I_4 by computing the Chern-Simons term of gauge field A_1 with background fields.

Reduction to 5d

Suppose: S^1 compactification of 6d SCFT gives some 5d Super-Yang-Mills theory.

- N=(2,0) theory ⟶ 5d maximal Super-Yang-Mills
- E-string theory (with holonomy) ⟶ Sp(N) with an anti-symmetric and 8 fundamental hypers

Go to the Coulomb branch and compute triangle diagram



This gives the Chern-Simons term $\int A_1 \wedge I_4$

14

Example: U(N) N=(2,0)

U(N) N=(2,0) (N M5-branes) → 5d U(N) SYM (N D4-branes)

Coulomb branch U(N) → U(1)^N

$$\Phi = \mathrm{diag}(\phi_1, \cdots, \phi_N) \quad \text{adjoint scalar}$$

$$\phi_1 > \cdots > \phi_N$$

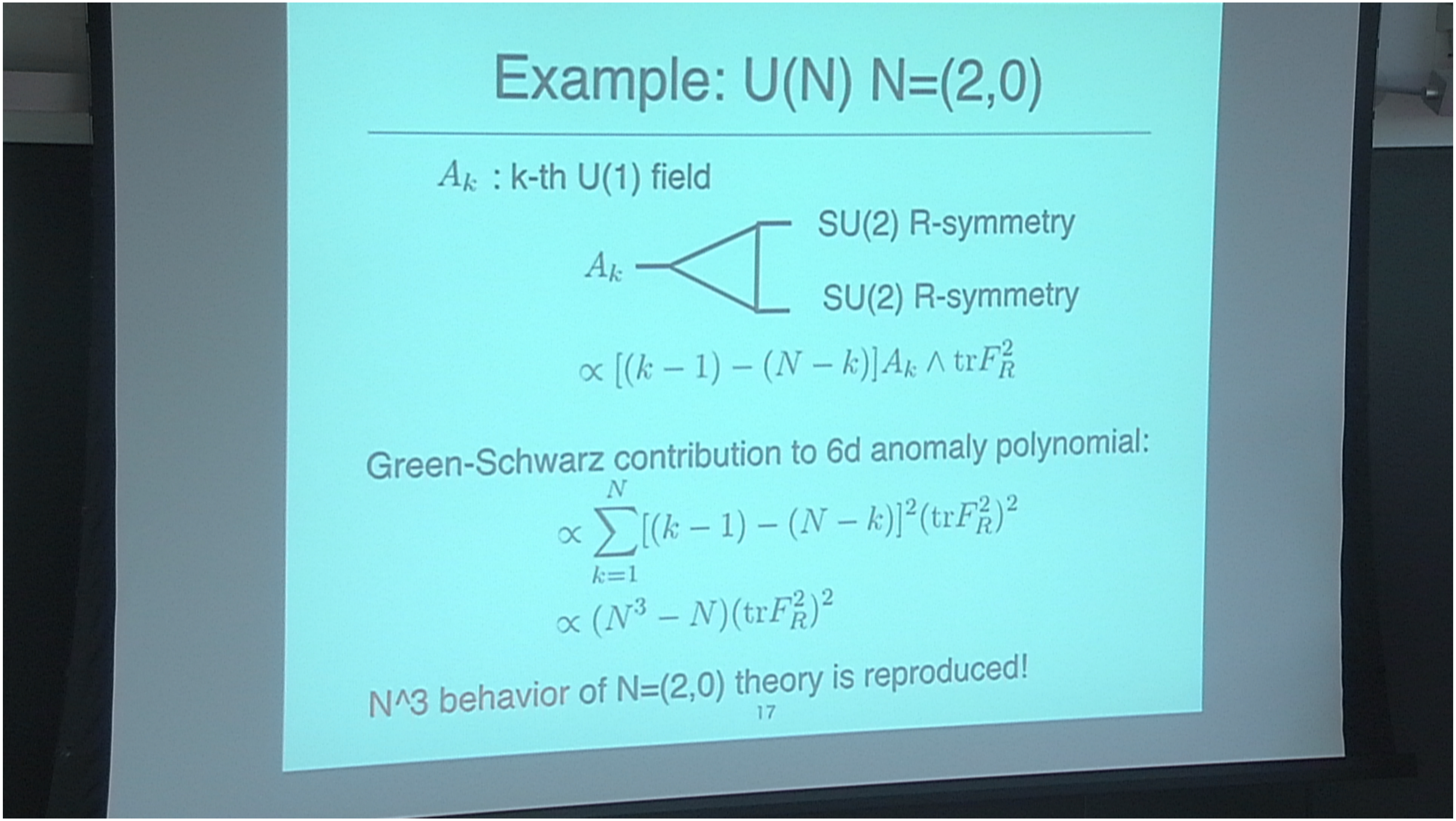
About the k-th U(1) field A_k , there are

- k-1 gauginos of positive charge +1 $(\lambda)_{kj} \ (1 \leq j < k)$
- N-k gauginos of negative charge -1 $(\lambda)_{jk} \ (k < j \leq N)$

with positive masses.

(There are also gauginos with mass and charge flipped.)

16


Example: U(N) N=(2,0)
A_k : k-th U(1) field
A_k
SU(2) R-symmetry
SU(2) R-symmetry
$\propto [(k-1)-(N-k)]A_k \wedge \mathrm{tr}F_R^2$
Green-Schwarz contribution to 6d anomaly polynomial:
$\propto \sum_{k=1}^{N}[(k-1)-(N-k)]^2(\mathrm{tr}F_R^2)^2$
$\propto (N^3-N)(\mathrm{tr}F_R^2)^2$
N^3 behavior of N=(2,0) theory is reproduced!
17

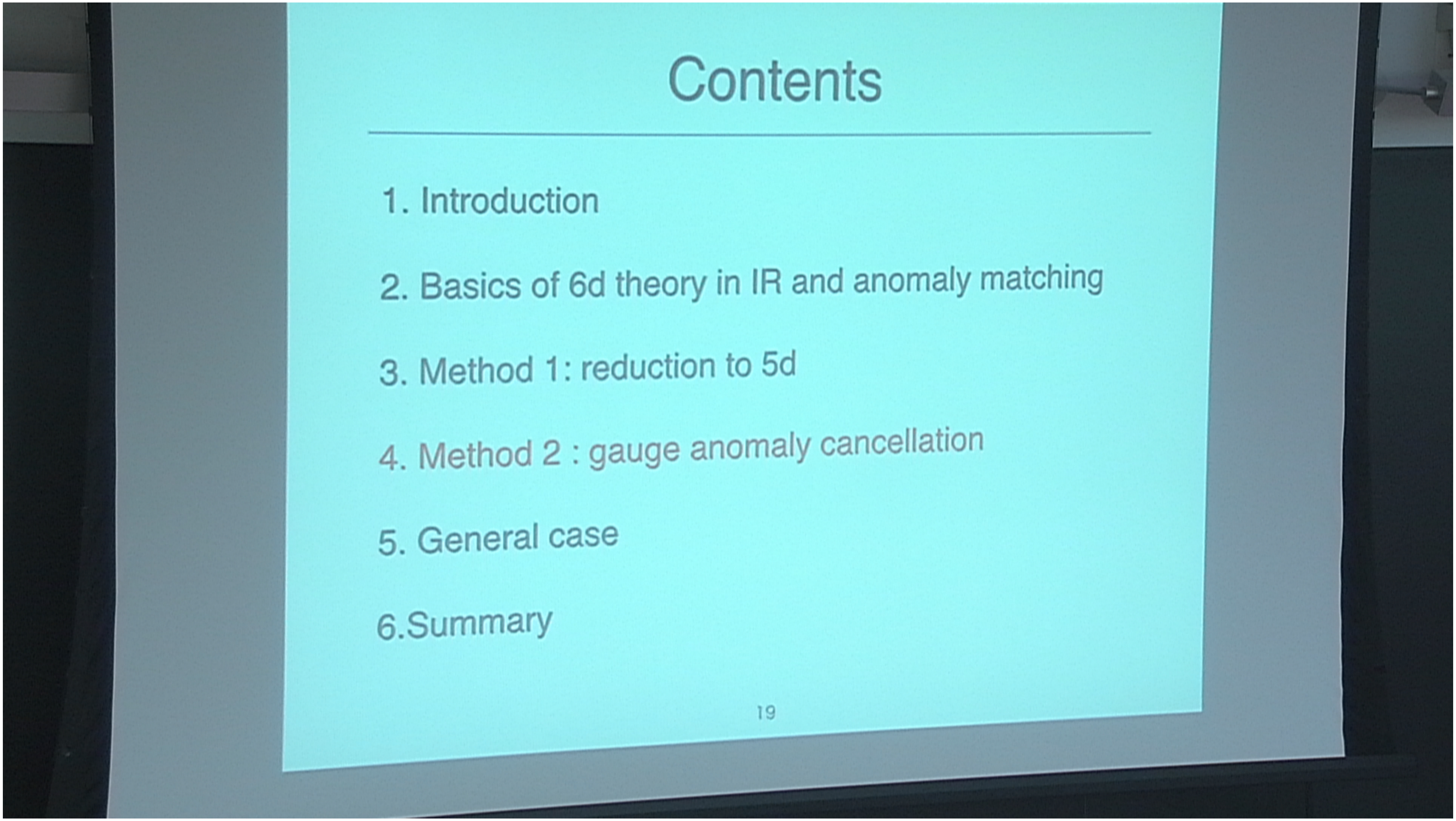
Remark

By this method, we can completely determine anomaly polynomials of N=(2,0) and E-string theories.

- N=(2,0) theory → 5d maximal Super-Yang-Mills
- E-string theory → Sp(N) with an anti-symmetric and 8 fundamental hypers

The results reproduce the known/conjectured ones in the literature.

18


Contents
1. Introduction
2. Basics of 6d theory in IR and anomaly matching
3. Method 1: reduction to 5d
4. Method 2 : gauge anomaly cancellation
5. General case
6.Summary
19

Gauge anomaly cancellation

Fermions (e.g. gauginos) are charged under non-abelian gauge groups and have gauge anomalies.

	fermionic components	
• Vector multiplet	left-handed ⟵	gauge anomaly
• Hyper multiplet	right-handed ⟵	gauge anomaly
• Tensor multiplet	right-handed	

The gauge anomaly must be cancelled by Green-Schwarz contribution.

20

Gauge anomaly cancellation

For simplicity, let's consider the case of single gauge group and single tensor multiplet.

$$I_{\mathrm{one-loop}} \sim -(\mathrm{tr}F_G^2)^2 - 2(\mathrm{tr}F_G^2)X_4 - Y_8$$

F_G : gauge field strength

$X_4,\ Y_8$: a 4-form and an 8-form depending only on background fields.

Remark:
The fact that the first term is a perfect square is required by the Green-Schwarz mechanism to work.
It is a nontrivial constraint on the gauge group G and matter content.

21

Gauge anomaly cancellation

$$I_{\text{one-loop}} = -(\text{tr}F_G^2)^2 - 2(\text{tr}F_G^2)X_4 - Y_8$$

(Here I am using $2 = \pi = \cdots = 1$)

Terms involving F_G must be cancelled by Green-Schwarz

$$(I_4)^2$$

This uniquely determines I_4

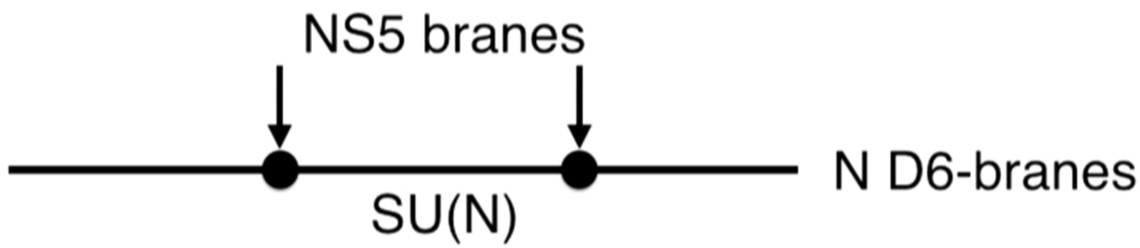
$$I_4 = \text{tr}F_G^2 + X_4$$

So we get

$$I^{\text{UV}} = I_{\text{one-loop}} + (I_4)^2$$
$$= (X_4)^2 - Y_8$$

22

Example: quiver theory in IIA



- SU(N) gauge theory
- 2N=N+N fundamental hypers
- One tensor (distance between two NS5s)

The limit of coincident NS5s gives interacting SCFT.

Generalization to more than two NS5s is straightforward.

Example: quiver theory in IIA

$$I_{\text{gaugino}} = -\text{tr}_{\text{adj}} F_G^4 - \text{tr}_{\text{adj}} F_G^2 \text{tr} F_R^2 - (N^2 - 1)(\text{tr} F_R^2)^2$$

$$I_{\text{hyper}} = 2N \text{tr}_{\text{fund}} F_G^4$$ Only SU(2) R-symm. considered

(Here I am using $2 = \pi = \cdots = 1$)

Group theory relations:

$$\begin{cases} -\text{tr}_{\text{adj}} F_G^4 + 2N \text{tr}_{\text{fund}} F_G^4 = -(\text{tr} F_G^2)^2 \\ \text{tr}_{\text{adj}} F_G^2 = 2N \text{tr} F_G^2 \end{cases}$$

$$I_{\text{one-loop}} = -(\text{tr} F_G^2 + N \text{tr} F_R^2)^2 + (N^2 + O(1))(\text{tr} F_R^2)^2$$

$$I_{\text{one-loop}} + (I_4)^2 = (N^2 + O(1))(\text{tr} F_R^2)^2$$

Remark: Q NS5s give $Q^3 N^2 (\text{tr} F_R^2)^2$ as the leading term.

More general case

$$I_{\text{one-loop}} = -\eta^{IJ}\text{tr}F_I^2\text{tr}F_J^2 - 2\eta^{IJ}X_I\text{tr}F_J^2 - Y$$
$$= -\eta^{IJ}(\text{tr}F_I^2 + X_I)(\text{tr}F_J^2 + X_J) + \eta^{IJ}X_IX_J - Y$$

N_G :number of gauge groups

N_T :number of tensor multiplets

- If $N_G > N_T$, gauge anomaly cannot be cancelled.
- If $N_G = N_T$, anomaly is cancelled in a unique way.

$$I_{\text{one-loop}} + \eta^{IJ}I_II_J = \eta^{IJ}X_IX_J - Y$$

- If $N_G < N_T$, anomaly cancellation does not determine Green-Schwarz contribution uniquely.

General 6d SCFTs

All known 6d SCFTs can be constructed by F-theory on

$$R^6 \times CY_3$$

CY_3 : elliptically fibered Calabi-Yau 3-fold over some base manifold B (complex 2-dimensions).

The base B contains intersecting 2-cycles

General 6d SCFTs

- The size of each cycle = a scalar in a tensor multiplet.
- Some cycles have a simple gauge group G (due to singular fibers) and others have no gauge group at all.
- Shrinking all the cycles: SCFT
  Generic points of the tensor branch: finite size cycles.

More detailed rules are in [Heckman,Morrison,Vafa,2013]

General 6d SCFTs

N_G :number of gauge groups

N_T :number of tensor multiplets = number of cycles

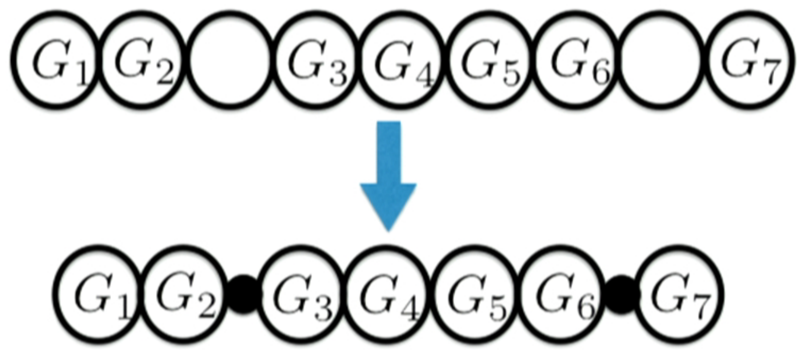
(e.g. in the figure, $N_G = 7$ and $N_T = 9$)

Always $N_G \leq N_T$

We want $N_G = N_T$ so that the anomaly cancellation by Green-Schwarz proceeds in a unique way.

General 6d SCFTs

Shrink the cycles which do not contain gauge groups.



The cycles shrunk ● become E-string theories.

We consider E-string theories as "bifundamental matter".
$N_G = N_T$ is realized in this way.

General method

$$I^{\rm UV} = I_{\rm one-loop} + \eta^{IJ} I_I I_J$$

"One-loop" contains contributions of E-string theory, whose anomaly was already computed.

[Ohmori,Shimizu,Tachikawa,2014], or by the first method of this talk.

Green-Schwarz is uniquely determined by gauge anomaly cancellation.

$I^{\rm UV}$ of any SCFT can be computed in this way!

Contents

Summary

- General method is described to compute anomaly polynomial of any known 6d SCFT.

- Anomaly matching of UV and IR theories on tensor branch and Green-Schwarz contribution of IR theory are the crucial ingredient.

$$I^{\mathrm{UV}} = I^{\mathrm{IR}}_{\mathrm{one-loop}} + I_4^2$$

Future directions

- Compactification of 6d SCFT to 4 or other dimensions

  Anomalies of lower dimensional theories are related to anomalies in 6d.

- 6d central charges, a-theorem, etc.

  In 4d, central charges a, c are related to coefficients of anomaly polynomial in N=1 theories. Similarly in 6d N=(1,0) theories?

Thank you very much!