Title: The N=2 Superconformal Maxi-bootstrap

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Abstract: <p>In this talk I will present some results of an upcoming paper where we study four-dimensional N=2 superconformal field theories using the conformal bootstrap.
We focus on two different four-point functions, involving either the superconformal primary of the flavor current multiplet or the one of the chiral multiplet.
Numerical analysis of the crossing equations yields lower bounds on the allowed central charges, and upper bounds on the dimensions of unprotected operators (for unitary theories).</p>

The $\mathcal{N} = 2$ Superconformal Maxi-Bootstrap

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Based on:
C. Beem, P. Liendo, L. Rastelli and B. van Rees - to appear

Two-step Bootstrap

There is a protected subsector of the superconformal bootstrap equations that is solvable for theories in $d > 2$

- $4d$ SCFTs with $\mathcal{N} \geq 2$
  [Beem ML Liendo Peelaers Rastelli van Rees]
- $6d$ SCFTs with $\mathcal{N} = (2, 0)$
  [Beem Rastelli van Rees]

admit a protected subsector isomorphic to $2d$ chiral algebra

Bootstrap equations split into:

1. Exchange of chiral algebra operators $\Rightarrow$ **Mini-bootstrap**
2. Exchange of **all** operators $\Rightarrow$ **Maxi-bootstrap**

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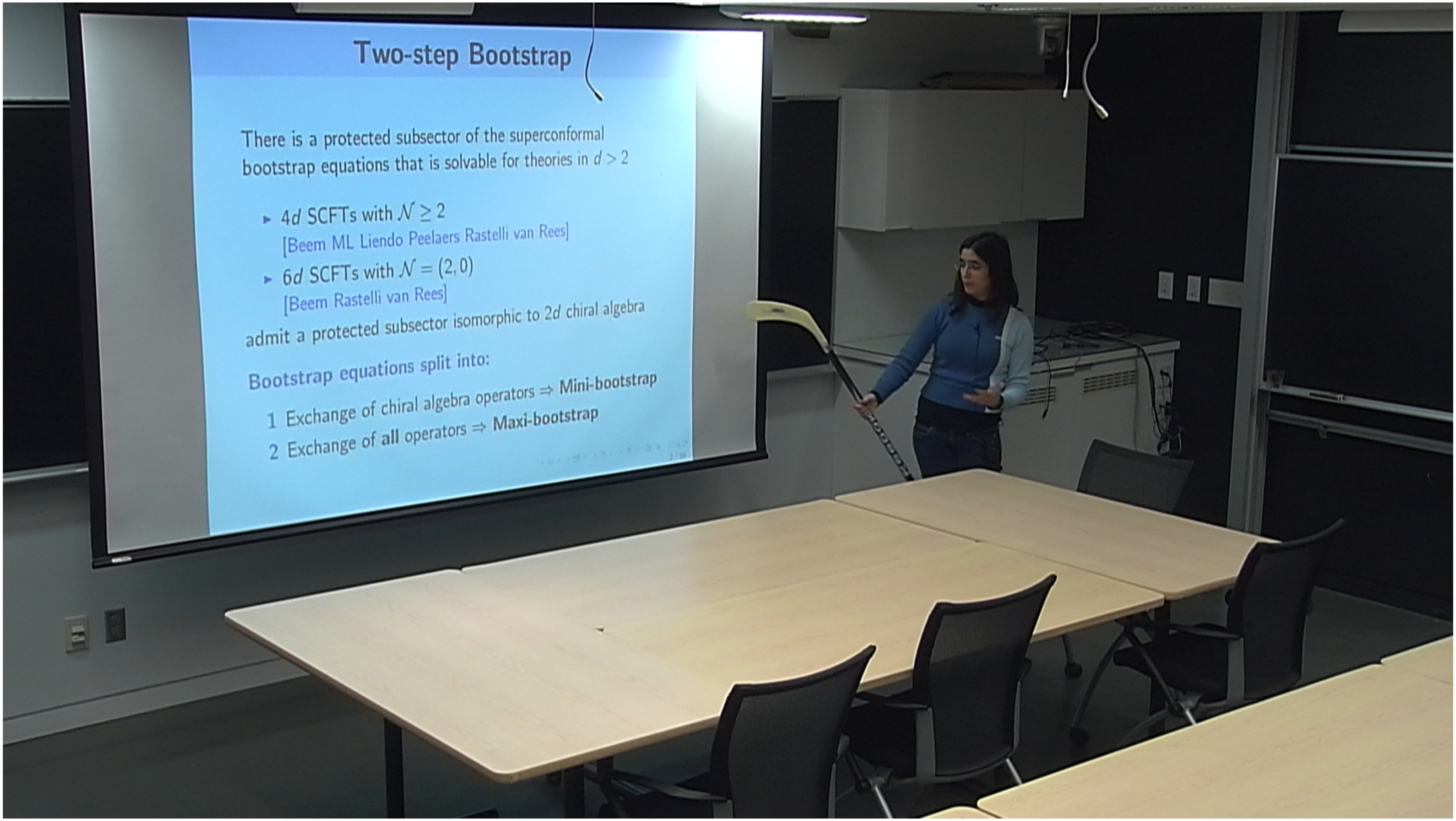
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Outline

Conformal Bootstrap

Conformal field theory defined by

Set of local operators $\{\mathcal{O}_k(x)\}$ and their correlation functions

Operator Product Expansion

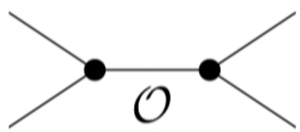
$\mathcal{O}_1(x)\mathcal{O}_2(0) = \sum_k \lambda_{12k} c(x, \partial_x)\mathcal{O}_k(0)$

subject to

- Unitarity
- Associativity of the operator product algebra

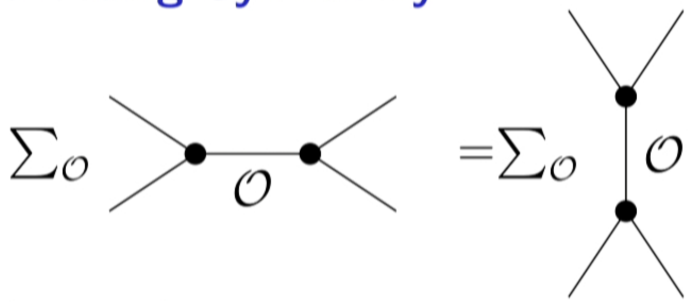
Conformal Block Expansion

$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle = \frac{1}{x_{12}^{2\Delta_\phi} x_{34}^{2\Delta_\phi}} \sum_{\mathcal{O}} \lambda_{\mathcal{O}}^2 g_{\Delta,\ell}(u,v)$

$\sum_{\mathcal{O}}$ 

Conformal Bootstrap

Crossing Symmetry



Sum rule

$$F_{0,0}(u,v) + \sum_{\mathcal{O}} \lambda_{\mathcal{O}}^2 \underbrace{\left(v^{\Delta_\phi} g_{\Delta,\ell}(u,v) - u^{\Delta_\phi} g_{\Delta,\ell}(v,u)\right)}_{F_{\Delta,\ell}} = 0$$

The Superconformal Bootstrap Program

What is the space of consistent SCFTs?

$\rightarrow$ Maximally supersymmetric theories: well known list of theories

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The Superconformal Bootstrap Program

What is the space of consistent SCFTs?

- $\rightarrow$ Maximally supersymmetric theories: well known list of theories
- $\rightarrow$ Have some idea of what is possible in $4d$ $\mathcal{N} = 2$
- $\rightarrow$ Large list of new theories, mostly with no Lagrangian description

The Superconformal Bootstrap Program

The multiplets

- Flavor current multiplet
  $\hookrightarrow$ (Higgs branch chiral ring operators)
- $\mathcal{N} = 2$ Chiral operators
  (Coulomb branch chiral ring)
- Stress tensor

Flavor current multiplet

In the classification of [Dolan Osborn]

$\hat{\mathcal{B}}_R$ multiplet

- Shortening condition: Q^1_α, $\tilde{Q}_{2\,\dot{\alpha}}$
- $\frac{1}{2}$ BPS multiplet
- Lagrangian theory: $qq\ldots\tilde{q}\tilde{q}$
- Higgs branch chiral ring
- Superconformal primary is captured by chiral algebra
- Superconformal blocks known: [Dolan Osborn, Nirschl Osborn, Dolan Gallot Sokatchev]

Flavor current multiplet

$\hat{\mathcal{B}}_1$

- Contains the flavor current of the theory
- Superconformal primary: moment map operator μ
  - $\hookrightarrow$ Dimension 2 scalar
  - $\hookrightarrow$ Triplet of $SU(2)_R$
  - $\hookrightarrow$ Adjoint of flavor group
- In Lagrangian theory: $\mu^{IJ} \sim q^{(I}\tilde{q}^{J)}$
- Flavor current $\sim Q^I_\alpha \tilde{Q}^J_{\dot{\alpha}} \mu^{IJ}$

Flavor current multiplet bootstrap

The SuperOPE [Arutyunov Eden Sokatchev]

$$\hat{\mathcal{B}}_1 \times \hat{\mathcal{B}}_1 \sim \mathbf{1} + \boxed{\hat{\mathcal{B}}_1} + \hat{\mathcal{B}}_2 + $$

$$\downarrow$$

$$\lambda^2 \sim \frac{1}{k}$$

Flavor current multiplet bootstrap

The SuperOPE [Arutyunov Eden Sokatchev]

$$\hat{\mathcal{B}}_1 \times \hat{\mathcal{B}}_1 \sim \mathbf{1} + \boxed{\hat{\mathcal{B}}_1} + \hat{\mathcal{B}}_2 + \boxed{\hat{\mathcal{C}}_{R=0(0,0)}} + \hat{\mathcal{C}}_{R=0(j,j)} + \hat{\mathcal{C}}_{R=1(j,j)}$$

$$\hat{\mathcal{B}}_1 \downarrow \lambda^2 \sim \frac{1}{k} \qquad \hat{\mathcal{C}}_{R=0(0,0)} \downarrow \lambda^2 \sim \frac{1}{c}$$

$$+ \mathcal{A}^{\Delta}_{R=0,r=0,(j,j)}$$

Input

- $\longrightarrow$ Symmetries: Flavor group of the theory
- $\longrightarrow$ Central charges: c and k

$\hat{\mathcal{B}}_1$ Four-point function

omitting flavor structure
$\mu(x_i) = t_i^A \mu^A(x_i)$

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Four-point function [Dolan Osborn]

$$\langle \mu(x_1)\mu(x_2)\mu(x_3)\mu(x_4)\rangle = \frac{t_1 \cdot t_2 t_3 \cdot t_4}{x_{12}^4 x_{34}^4}$$

$$\frac{z(1-\bar{z}w)f(\bar{z}) - \bar{z}(1-zw)f(z)}{z-\bar{z}} + (1-\bar{z}w)(1-zw)\mathcal{G}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{24}^2 x_{13}^2} = z\bar{z}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{24}^2 x_{13}^2} = (1-z)(1-\bar{z})\,, \quad w = \frac{t_1 \cdot t_3 t_2 \cdot t_4}{t_1 \cdot t_2 t_3 \cdot t_4}$$

Two step process

Crossing Equations split

- Crossing equation for $f(z)$
- Crossing equation for $\mathcal{G}(u, v)$ and $f(z)$

Mini-bootstrap: $f(z)$

$\rightarrow$ Crossing equation can be solved exactly

$\rightarrow$ Completely fixed in terms of flavor symmetry central charge k

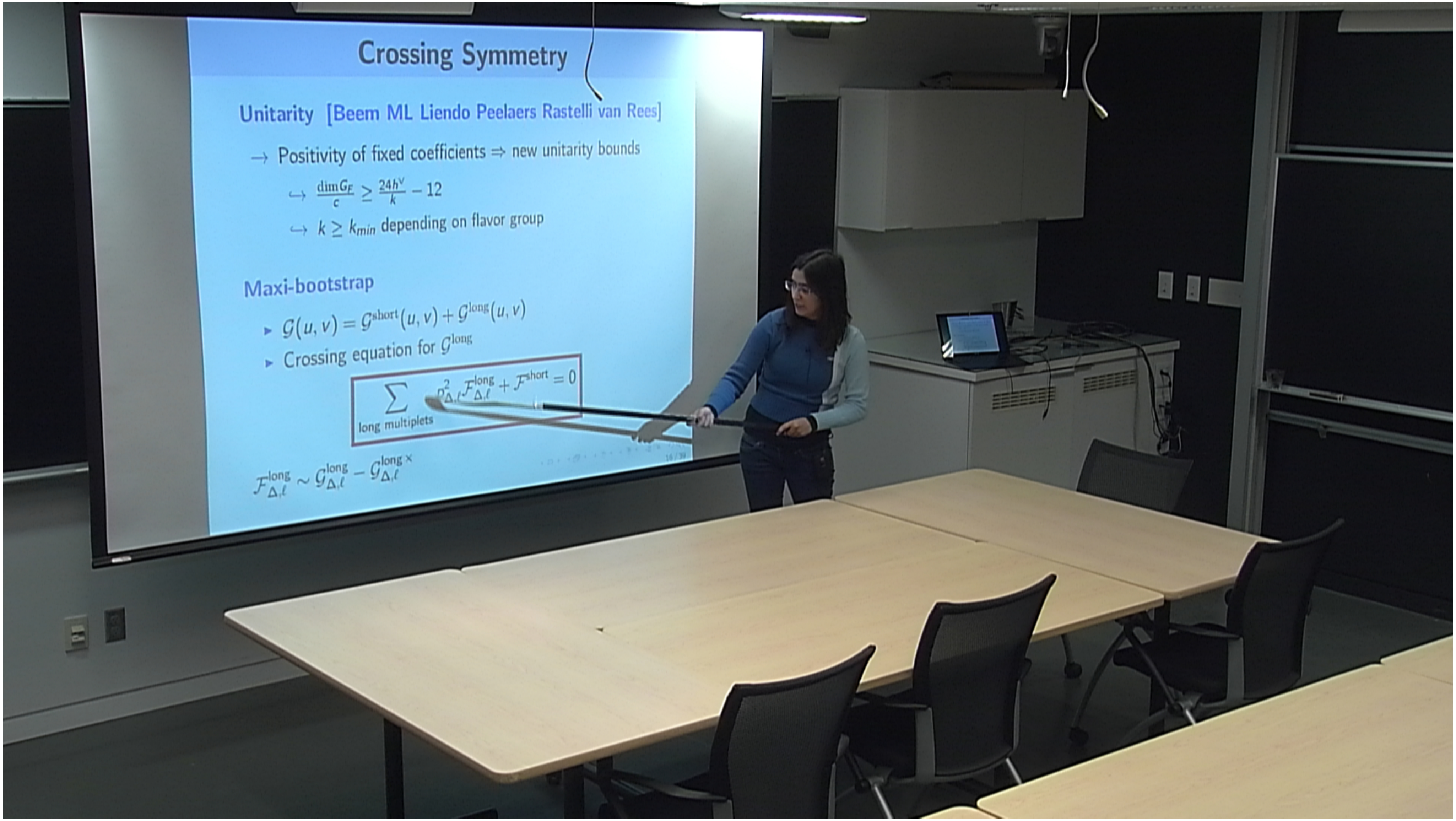
Crossing Symmetry

Unitarity [Beem ML Liendo Peelaers Rastelli van Rees]

→ Positivity of fixed coefficients ⇒ new unitarity bounds

↪ $\frac{\dim G_F}{c} \geq \frac{24h^\vee}{k} - 12$

↪ $k \geq k_{min}$ depending on flavor group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Maxi-bootstrap
$\mathcal{G}(u,v) = \mathcal{G}^{\text{short}}(u,v) + \mathcal{G}^{\text{long}}(u,v)$
Crossing equation for $\mathcal{G}^{\text{long}}$
$\sum_{\text{long multiplets}} \lambda^2_{\Delta,\ell} \mathcal{F}^{\text{long}}_{\Delta,\ell} + \mathcal{F}^{\text{short}} = 0$
$\mathcal{F}^{\text{long}}_{\Delta,\ell} \sim \mathcal{G}^{\text{long}}_{\Delta,\ell} - \mathcal{G}^{\text{long}\times}_{\Delta,\ell}$

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Maxi-bootstrap

- $\mathcal{G}(u,v) = \mathcal{G}^{\text{short}}(u,v) + \mathcal{G}^{\text{long}}(u,v)$
- Crossing equation for $\mathcal{G}^{\text{long}}$

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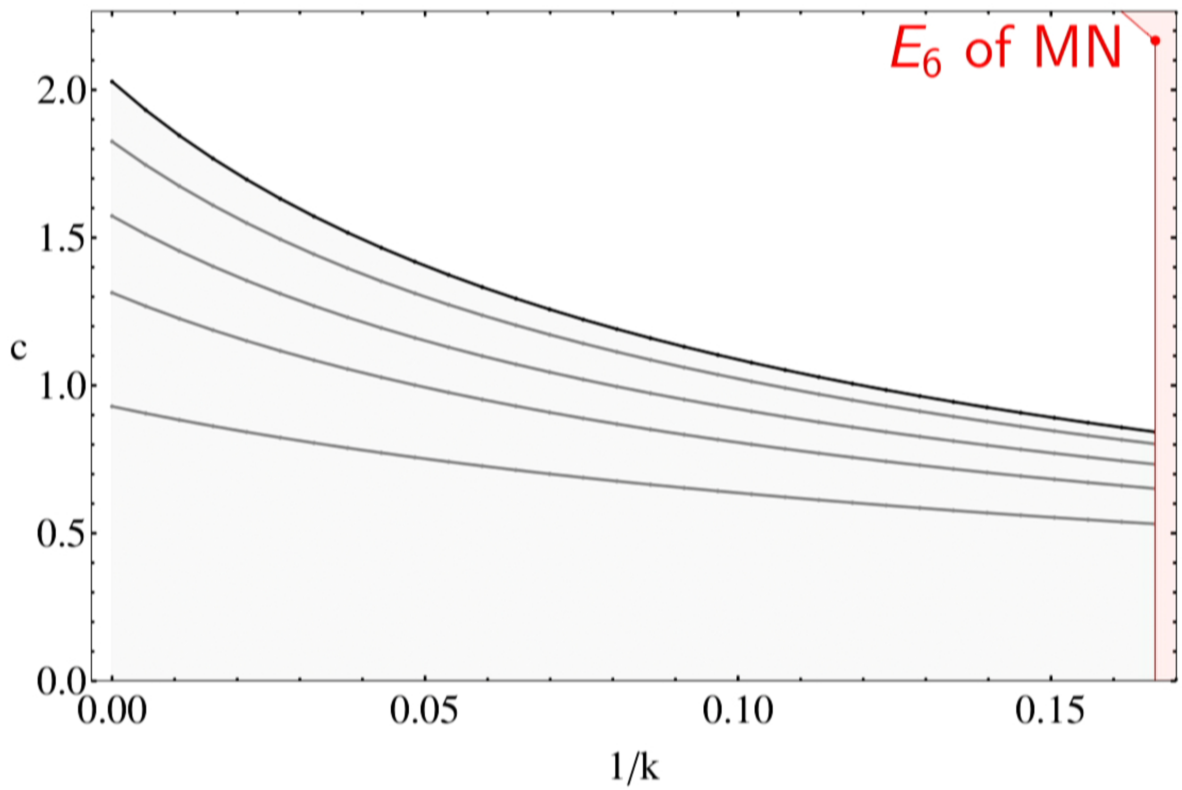
Central charge bounds for E_6 flavor symmetry



$SU(2)$ flavor symmetry

Analytic bounds

$$k \geq \frac{2}{3}$$

$$k \geq \frac{16c}{1+4c}$$

$SU(2)$ flavor symmetry

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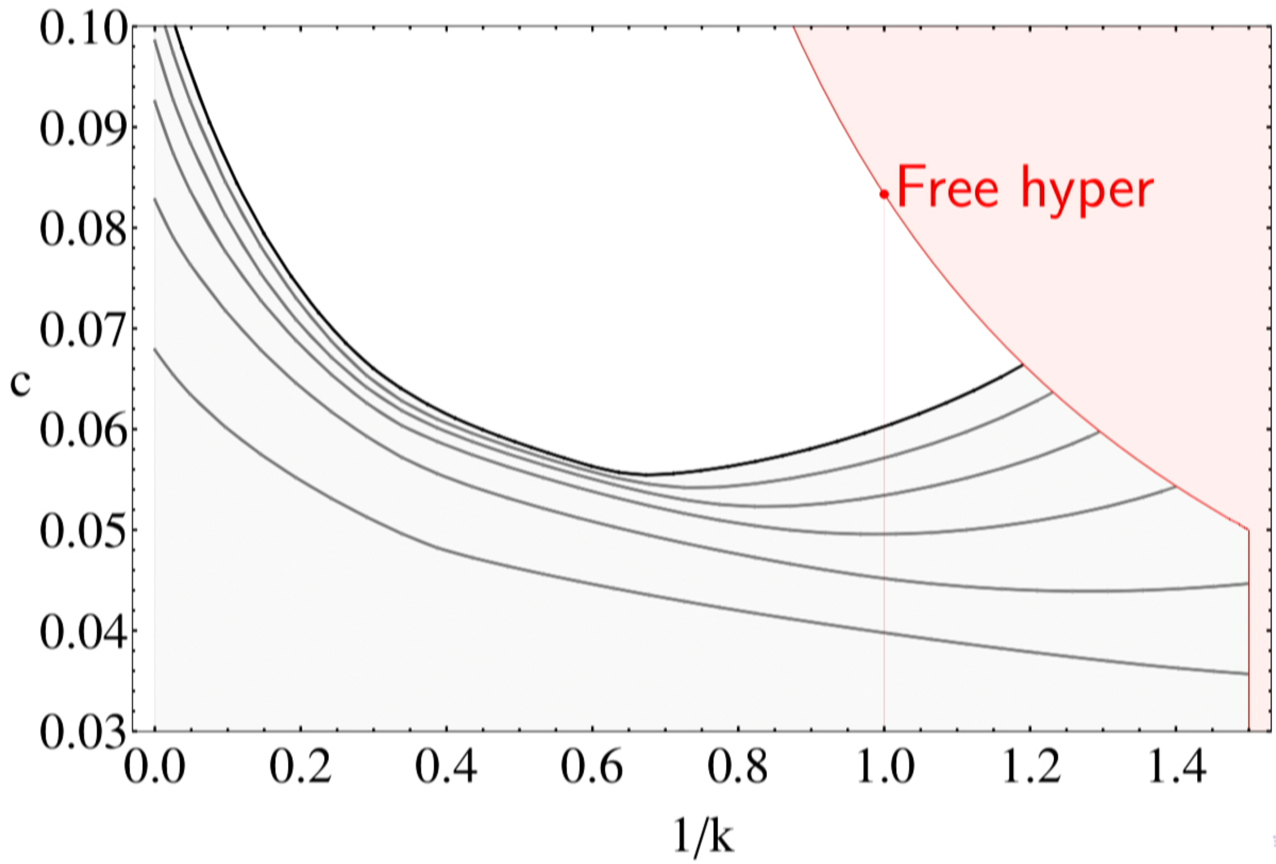
$\rightarrow$ Is there a theory living in the intersection of these two bounds?

$\rightarrow$ Saturation $\longrightarrow$ absence of $\hat{\mathcal{C}}_{1(0,0)}$ and $\hat{\mathcal{B}}_2$ in singlet channel

Central charge bounds for theory with $SU(2)$ flavor symmetry



Space of $\mathcal{N} = 2$ SCFTs

Hofman-Maldacena Bound

$$\frac{1}{2} \underbrace{\leq}_{\text{Saturated by free hyper}} \frac{a}{c} \underbrace{\leq}_{\text{Saturated by free vector}} \frac{5}{4}$$

Shapere Tachikawa sum rule

$$4(2a - c) = \sum_i (2\Delta \underbrace{\mathcal{O}_i}_{\text{Coulomb branch generators}} - 1)$$

$\rightarrow$ Unitarity $\Rightarrow \Delta_{\mathcal{O}_i} \geq 1$

Combining the two: $\Rightarrow c \geq \frac{1}{6} \sum_{\mathcal{O}_i} (2\Delta_{\mathcal{O}_i} - 1) \geq \frac{1}{6}$

Stress tensor multiplet

- For a theory to be consistent must satisfy crossing symmetry for all four-point functions
- Minimum in c plot may be ruled out by a different correlator
- For example: c bounds can be obtained from stress tensor multiplet bootstrap

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Stress tensor multiplet

In the classification of [Dolan Osborn]

$\hat{\mathcal{C}}_{0,(0,0)}$ **multiplet**

- Stress tensor multiplet
- Superconformal blocks not known yet
- Superconformal primary is scalar
  (not captured by chiral algebra)

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- Superconformal blocks not known yet
- Superconformal primary is scalar
  (not captured by chiral algebra)
- $SU(2)_R$ current is captured by chiral algebra
- Sensitive to both c and a anomaly coefficients
  $\hookrightarrow$ Mini-bootstrap $\longrightarrow$ analytic c bounds?

The chiral multiplets

In the classification of [Dolan Osborn]

$\mathcal{E}_r$ multiplet

- $\mathcal{N} = 2$ chiral
- $\frac{1}{2}$ BPS multiplet
- Coulomb branch chiral ring

The chiral multiplets

$\mathcal{E}_{r_0}$

- Superconformal primary
  - $\hookrightarrow$ Scalar operator
  - $\hookrightarrow$ $U(1)_r$ charge r_0
  - $\rightarrow$ Dimension $\Delta = r_0$
- Not in the chiral algebra – just maxi-bootstrap

Coulomb branch

- Believed to be freely generated
- For a freely generated Coulomb branch with $\mathcal{O}_i$ generators of dimension r_i:

  $\rightarrow$ $c \geq \frac{1}{6} \sum_{\mathcal{O}_i} (2r_i - 1)$

The $\langle \mathcal{E}_{r_0} \mathcal{E}_{r_0} \bar{\mathcal{E}}_{-r_0} \bar{\mathcal{E}}_{-r_0} \rangle$ Bootstrap

- Single correlator $\rightarrow$ can only hope to capture completely theories with a single Coulomb branch generator

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  $\hookrightarrow$ $c \geq \frac{1}{6}(2r_0 - 1)$

  $\hookrightarrow$ Can we improve on this bound?

$\mathcal{E}_{r_0} \bar{\mathcal{E}}_{-r_0}$ OPE

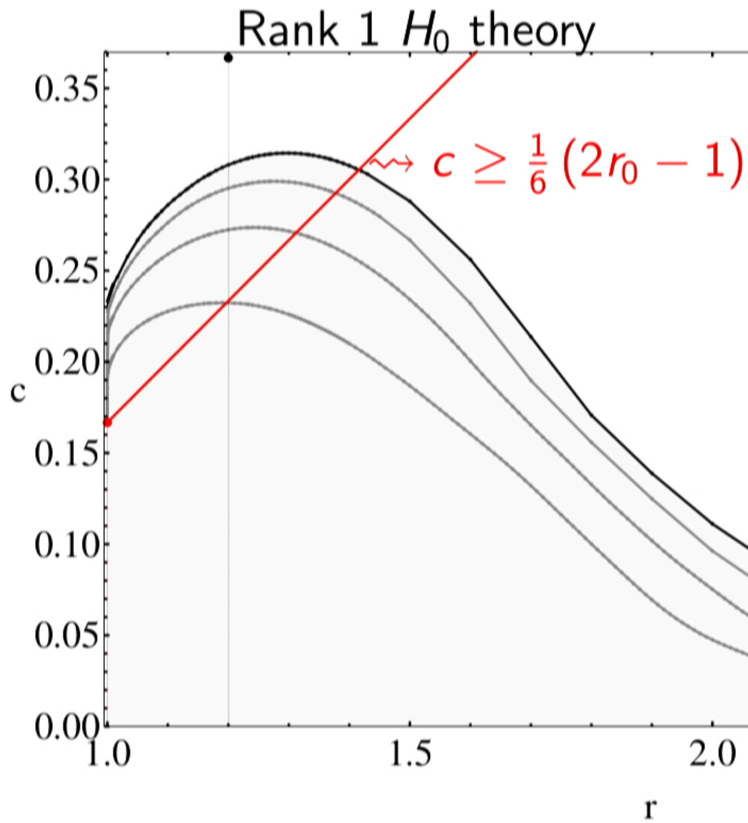
$$\mathcal{E}_{r_0(0,0)} \times \bar{\mathcal{E}}_{-r_0(0,0)} \sim \mathbf{1} + \boxed{\hat{\mathcal{C}}_{0,(0,0)}} + \hat{\mathcal{C}}_{0,(j,j)} + \mathcal{A}^{\Delta}_{0,0,(j,j)}$$

$$\downarrow$$

$$\lambda^2 \sim \frac{1}{c}$$

$\rightarrow$ Superconformal blocks computed in [Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]

Central charge bounds



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$$\downarrow$$

$$\lambda^2 \sim \frac{1}{c}$$

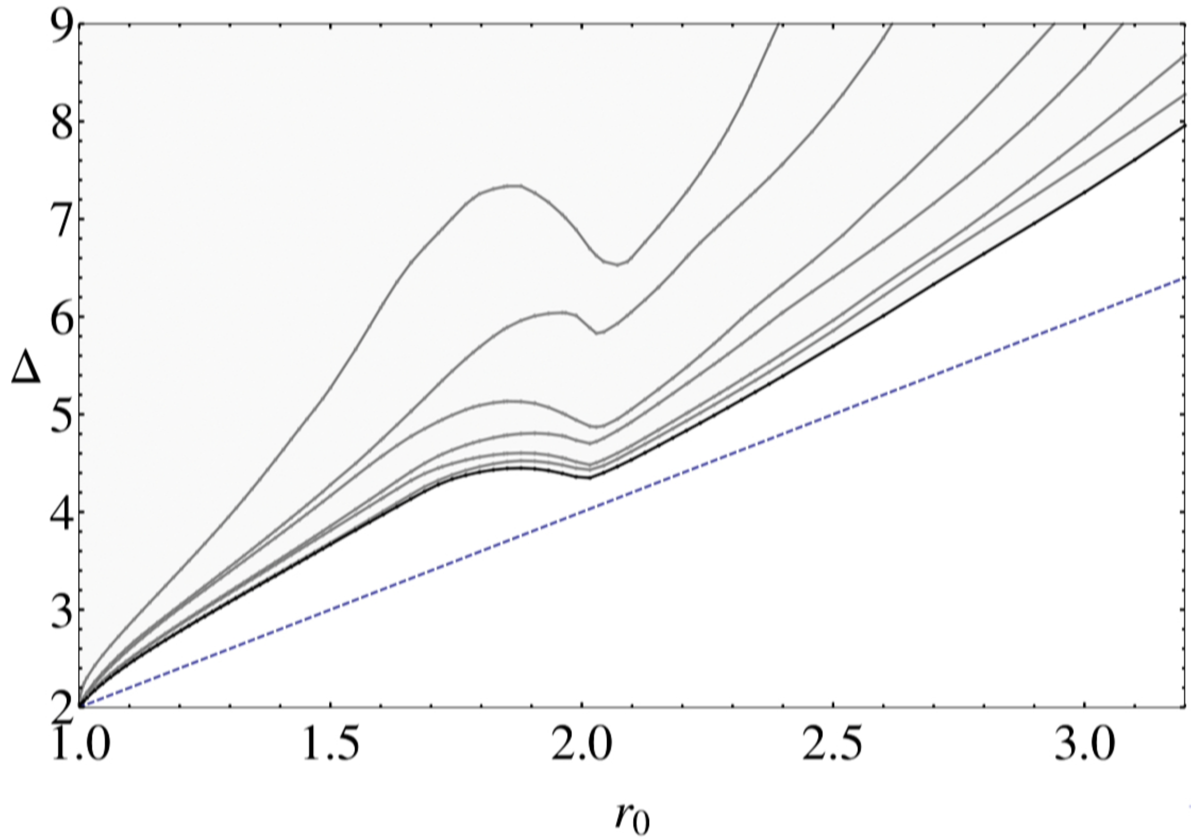
$\rightarrow$ Superconformal blocks computed in [Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]

$\rightarrow$ Two handles: r_0 and c

$\rightarrow$ Will bound $c(r_0)$ and operator dimensions

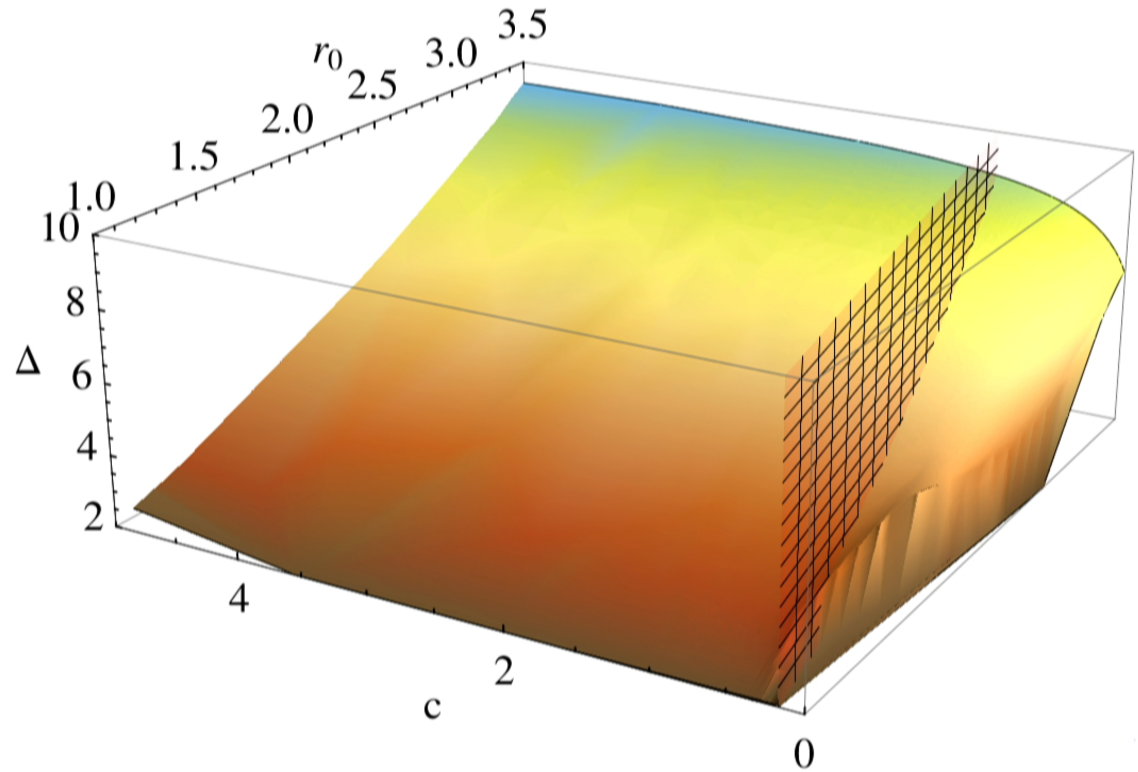
Scalar dimension bound in $\mathcal{E}_{r_0}\bar{\mathcal{E}}_{-r_0}$ channel

Arbitrary central charge



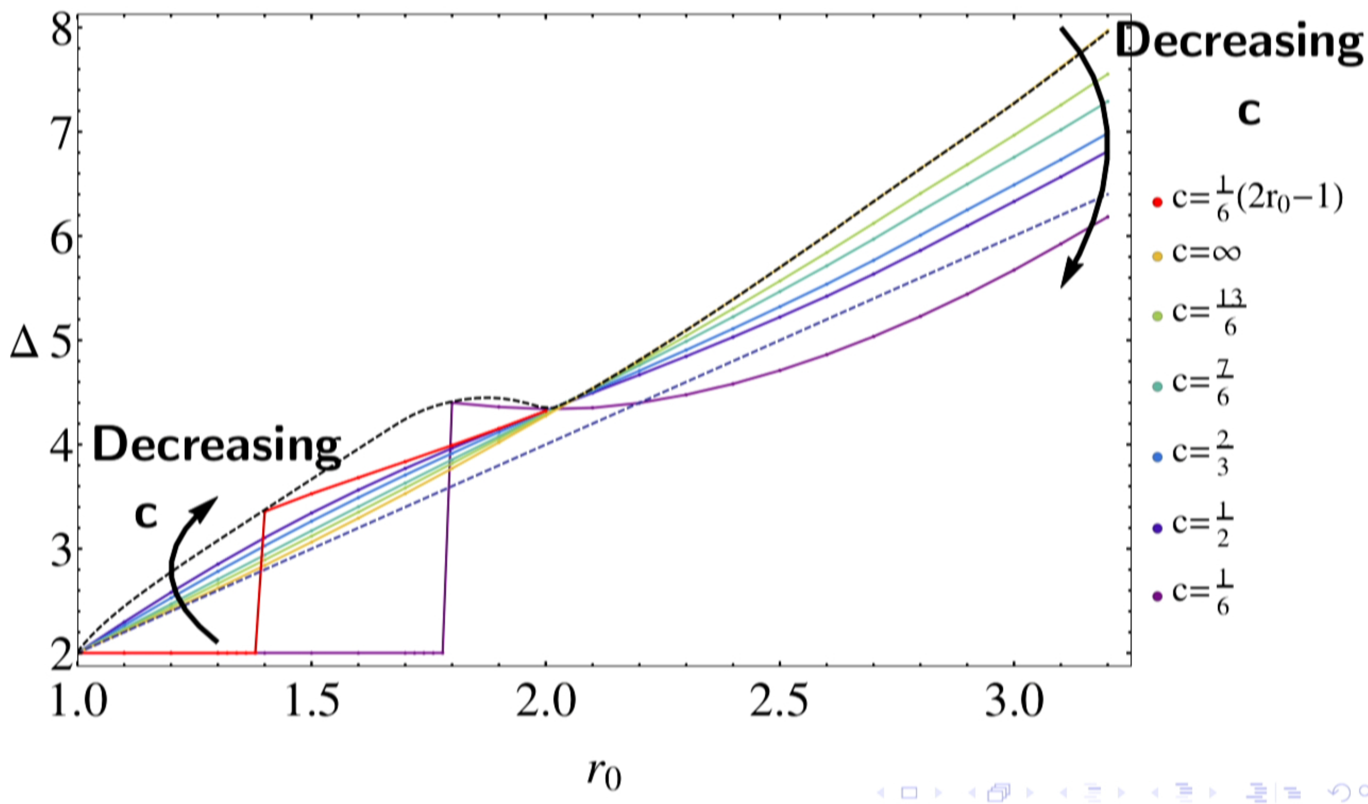
Scalar dimension bound in $\mathcal{E}_{r_0}\bar{\mathcal{E}}_{-r_0}$ channel

Fix central charge



Scalar dimension bound in $\mathcal{E}_{r_0}\bar{\mathcal{E}}_{-r_0}$ channel

Take slices of three-dimensional plot



The $\langle \mathcal{E}_{r_0}\mathcal{E}_{r_0}\bar{\mathcal{E}}_{-r_0}\bar{\mathcal{E}}_{-r_0}\rangle$ Bootstrap

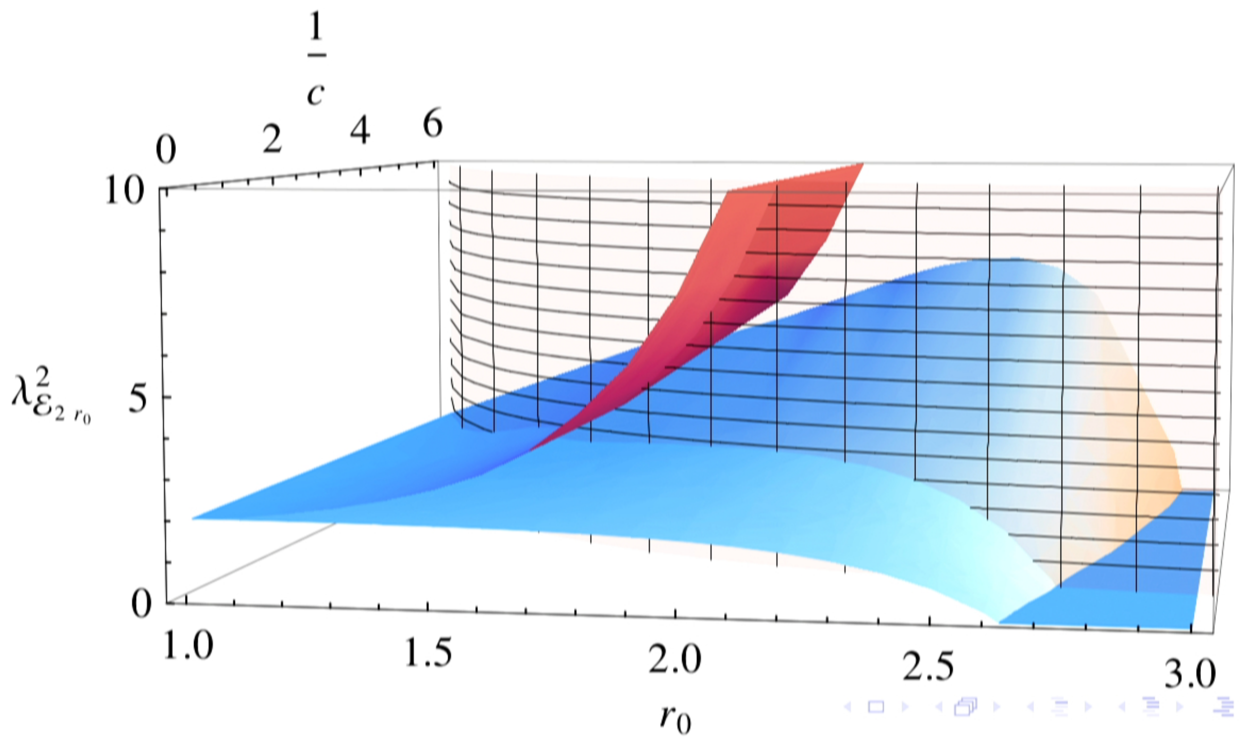
$\mathcal{E}_{r_0}\mathcal{E}_{r_0}$ OPE

$$\mathcal{E}_{r_0(0,0)} \times \mathcal{E}_{r_0(0,0)} \sim \mathcal{A}_{0,2r_0-2,(j,j)} + \boxed{\mathcal{E}_{2r_0}} + \mathcal{C}_{0\,2r_0-1,\,(j,j+1)} + +\mathcal{B}_{1,2r_0-1,(0,0)} + \mathcal{C}_{\frac{1}{2},2r_0-\frac{3}{2},(j,j+\frac{1}{2})}$$

$\rightarrow$ Each multiplet contributes with a single conformal block

Bound on $\mathcal{E}_{2r_0}$ OPE coefficient squared

- Probe relations on the Coulomb branch of the type $\mathcal{E}_{r_0}\mathcal{E}_{r_0} \sim 0$



Outlook

$SU(2)_R$ current

→ Bounds on a and c

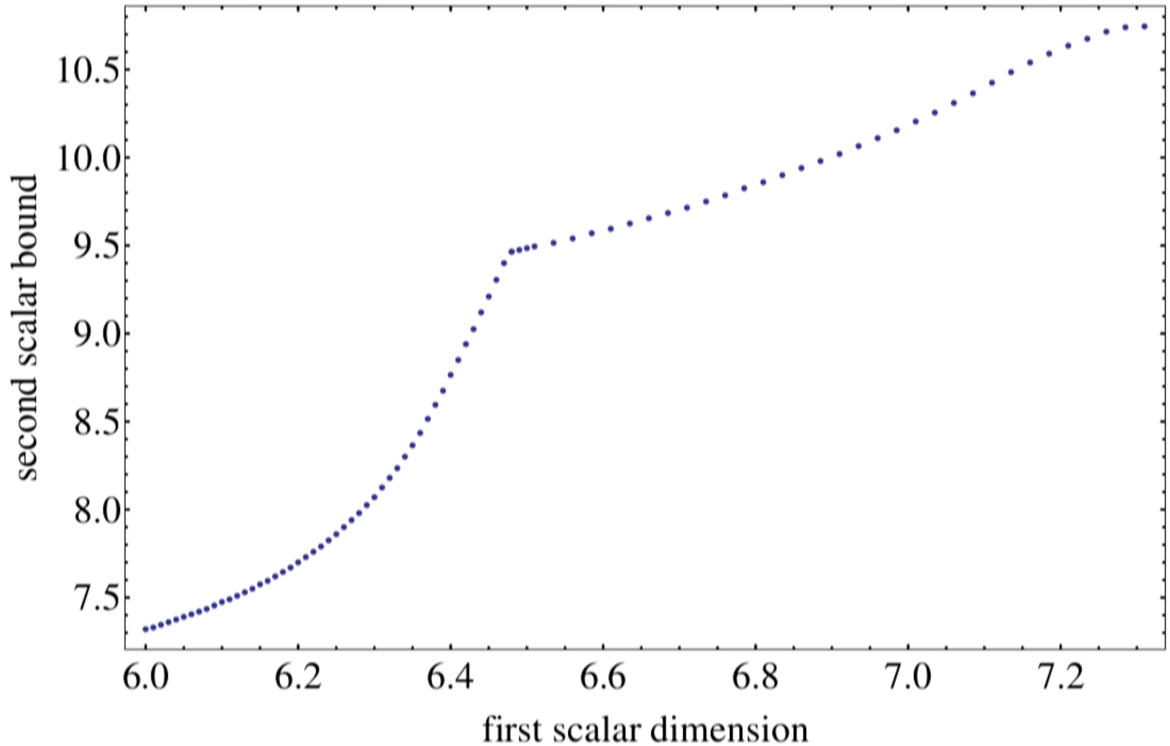
Multiple correlators

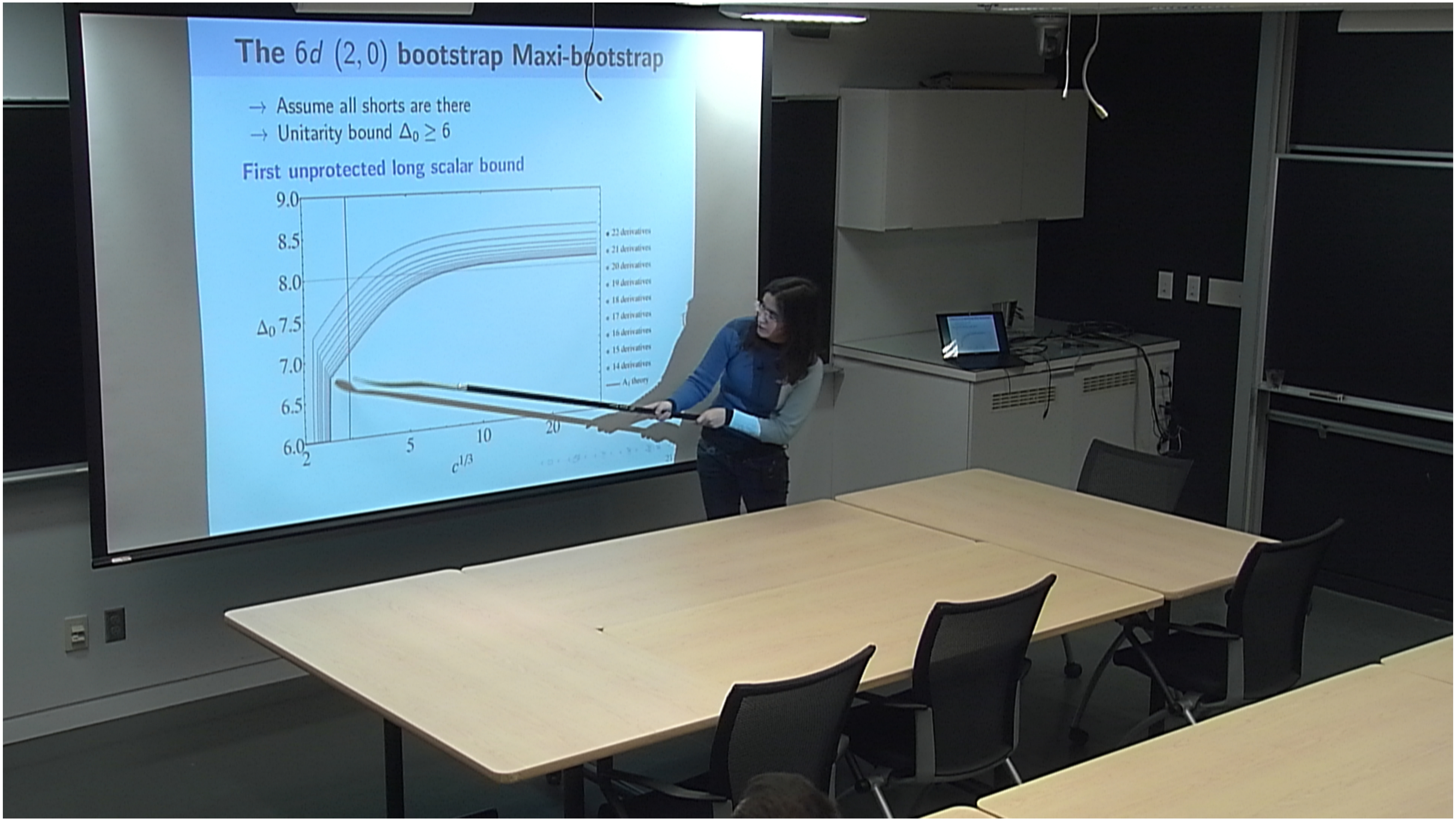
→ Higher rank theories, relations on Coulomb branch

→ Product flavor groups

→ Mixed correlators of operators in the chiral algebra

$6d\ \mathcal{N} = (2,0)$




The 6d (2,0) bootstrap Maxi-bootstrap
→ Assume all shorts are there
→ Unitarity bound $\Delta_0 \geq 6$
First unprotected long scalar bound
9.0
8.5
8.0
Δ_0 7.5
7.0
6.5
6.0
2
5
10
20
$c^{1/3}$
22 derivatives
21 derivatives
20 derivatives
19 derivatives
18 derivatives
17 derivatives
16 derivatives
15 derivatives
14 derivatives

The 6d (2, 0) bootstrap Maxi-bootstrap

Minimum allowed c

→ $c_{min} \to 25$: central charge of A_1 theory (two M5 branes)