Title: Viscous and Thermal Transport in Topological Phases

Date: Dec 12, 2014 01:00 PM

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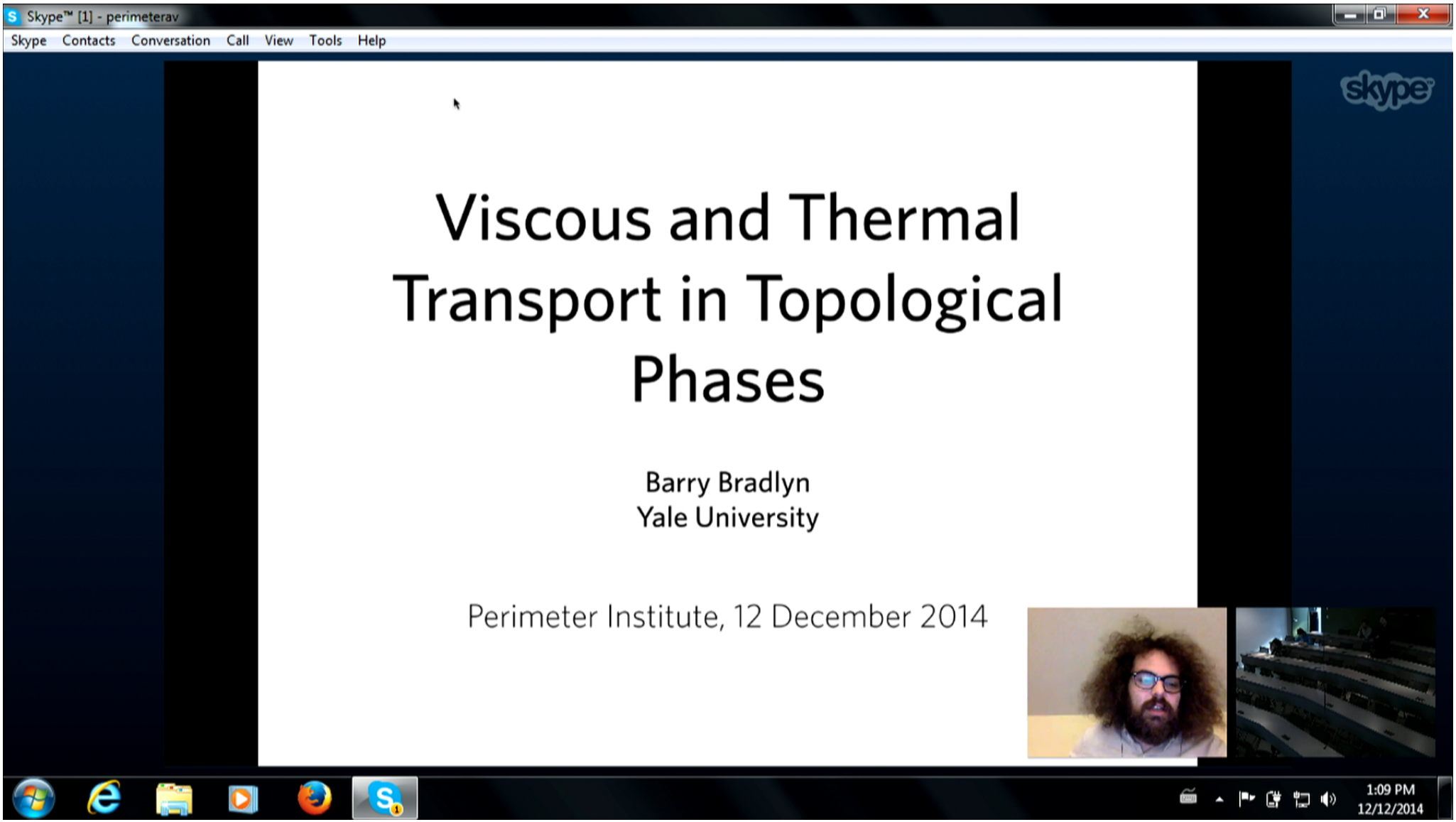
Abstract: One hallmark of topological phases with broken time reversal symmetry is the appearance of quantized non-dissipative transport coefficients, the archetypical example being the quantized Hall conductivity in quantum Hall states. Here I will talk about two other non-dissipative transport coefficients that appear in such systems - the Hall viscosity and the thermal Hall conductivity. In the first part of the talk, I will start by reviewing previous results concerning the Hall viscosity, including its relation to a topological invariant known as the shift. Next, I will show how the Hall viscosity can be computed from a Kubo formula. For Galilean invariant systems, the Kubo formula implies a relationship between the viscosity and conductivity tensors which may have relevance for experiment. In the second part of the talk, I will discuss the thermal Hall conductivity, its relation to the central charge of the edge theory, and in particular the absence of a bulk contribution to the thermal Hall current. I will do this by constructing a low-energy effective theory in a curved non-relativistic background, allowing for torsion. I will show that the bulk contribution to the thermal current takes the form of an "energy magnetization" current, and hence show that it does not contribute to heat transport.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Viscous and Thermal Transport in Topological Phases
Barry Bradlyn
Yale University
Perimeter Institute, 12 December 2014
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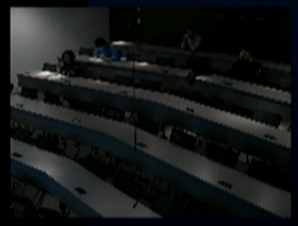
Outline

- Non-dissipative transport coefficients
- Review - quantization of Hall conductivity
- **Hall viscosity**
  - Hall viscosity - what is it?
  - Kubo formula for (Hall) viscosity
  - Viscosity - conductivity relation: possible "experiments"
- **Thermal Hall conductivity**
  - Thermal Hall conductivity and the edge theory
  - Bulk thermoelectric transport
  - Setup - non-relativistic geometry
  - Linear response from low-energy effective theory
- Conclusion



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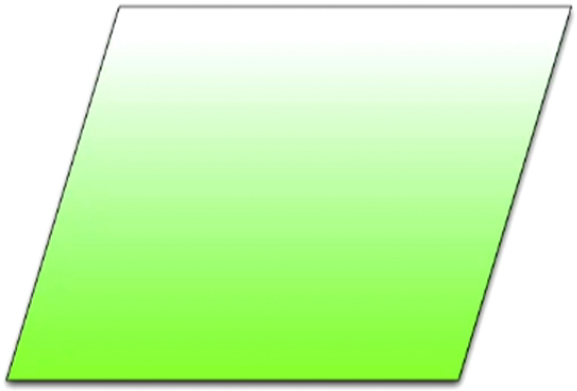
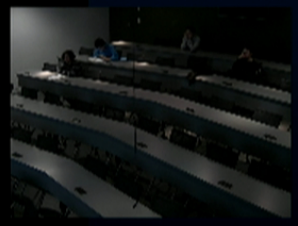
Non-Dissipative Transport

- Topological phase <-> gapped bulk
- At low temperatures, there is no dissipation
- Implication: in 2 dimensions, DC currents are *Hall currents*





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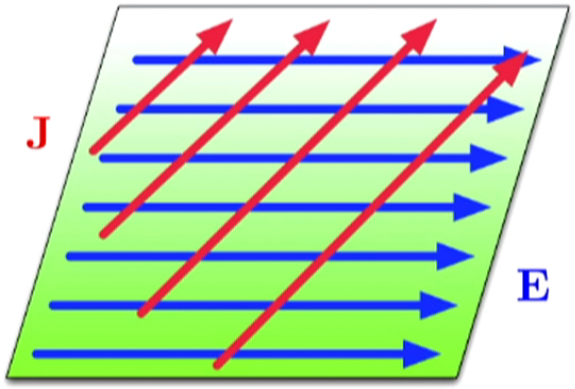
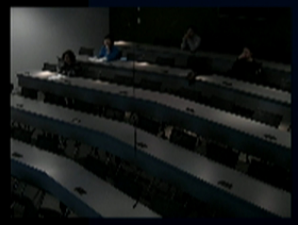


$$P = \mathbf{J} \cdot \mathbf{E} = 0$$



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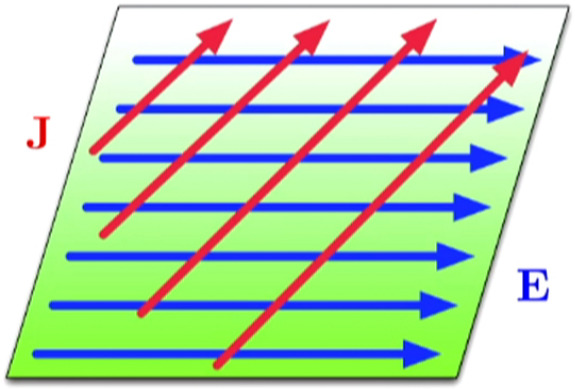

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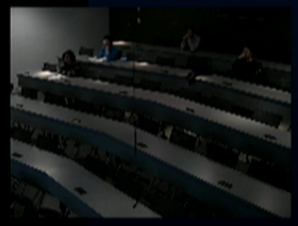

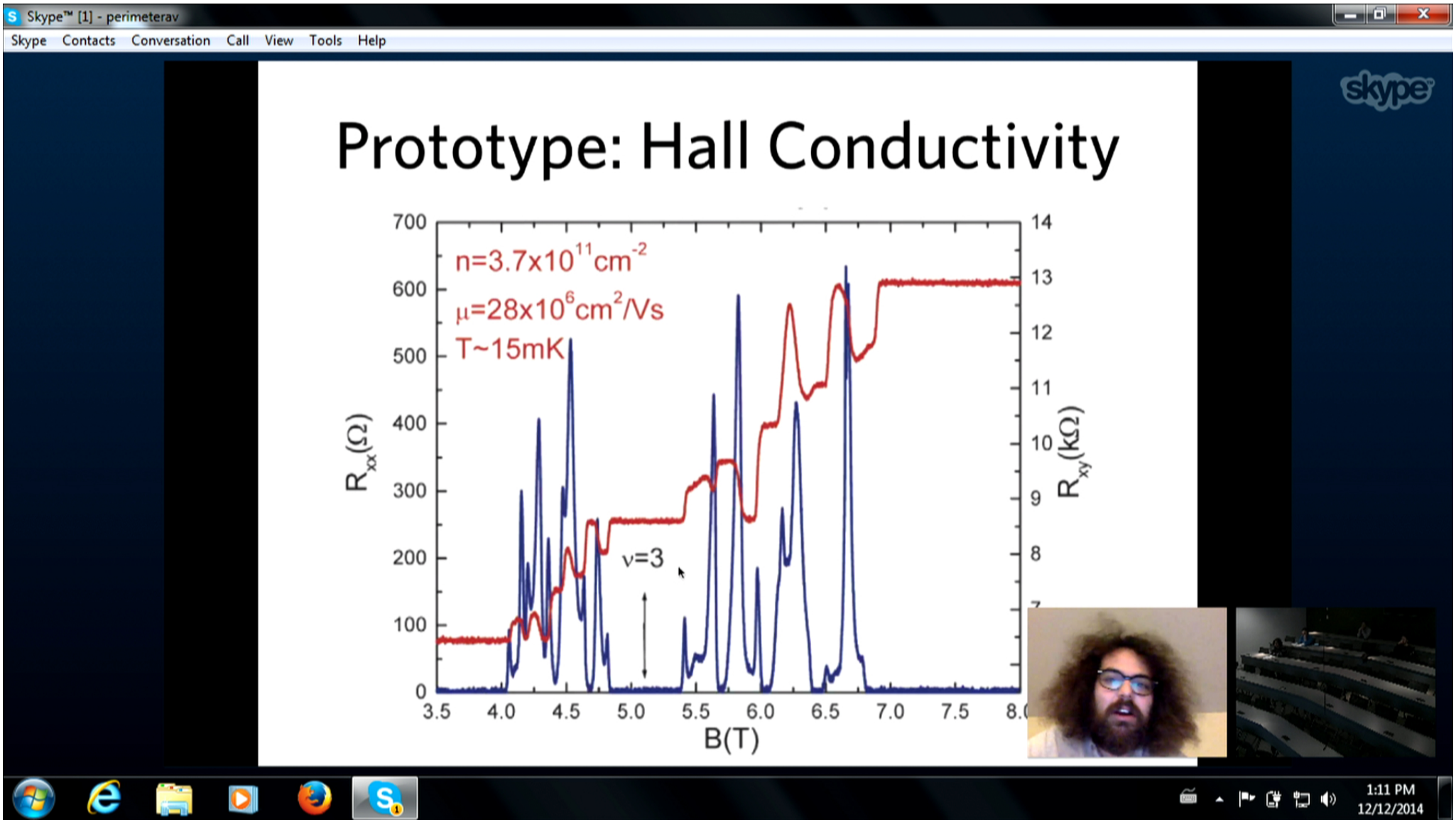
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Prototype: Hall Conductivity
n=3.7x10^11cm^-2
μ=28x10^6cm^2/Vs
T~15mK
ν=3
R_xx(Ω)
R_xy(kΩ)
B(T)
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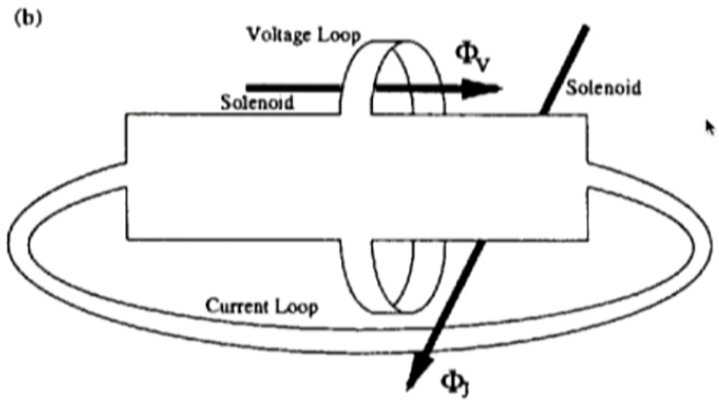
Quantization



$$J^i = \sigma_H \epsilon^{ij} E_j$$

$$\sigma_H = \frac{e^2}{h}\frac{i}{2\pi}\int_0^{h/e} d\Phi_J \int_0^{h/e} d\Phi_V \left\langle \frac{\partial \Psi_0}{\partial \Phi_V} \middle| \frac{\partial \Psi_0}{\partial \Phi_J} \right\rangle - \left\langle \frac{\partial \Psi_0}{\partial \Phi_J} \middle| \frac{\partial \Psi_0}{\partial \Phi_V} \right\rangle$$

$$\sigma_H = \frac{e^2}{h}\nu$$

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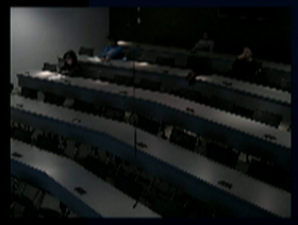
Beyond Hall Conductivity

- What other non-dissipative transport coefficients are interesting?
- Hall Viscosity
- Thermal Hall conductivity
- Both related to topological invariants



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- Non-dissipative transport coefficients
- Review - quantization of Hall conductivity
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  - Hall viscosity - what is it?
  - Kubo formula for (Hall) viscosity
  - Viscosity - conductivity relation: possible "experiments"
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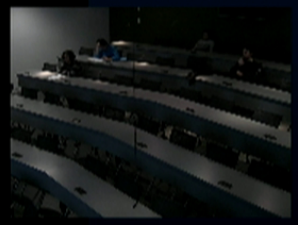

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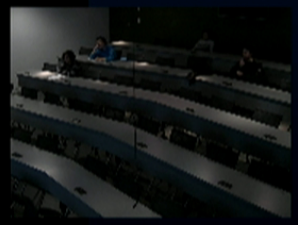
Viscosity

- Viscosity - response of the stress tensor to gradients of the fluid velocity

$$\langle \tau_{ij} \rangle = P\delta_{ij} - \kappa^{-1}\frac{\delta V}{V}\delta_{ij} - \eta_{ijk\ell}\frac{\partial v_\ell}{\partial x_k}$$

- 2 dimensions, rotational invariance:

$$\begin{aligned}\eta_{ijk\ell} =& \eta^{sh}\left(\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk} - \delta_{ij}\delta_{k\ell}\right) \\ &+ \zeta\delta_{ij}\delta_{k\ell} \\ &+ \eta^{H}\left(\delta_{jk}\epsilon_{i\ell} - \delta_{i\ell}\epsilon_{kj}\right)\end{aligned}$$



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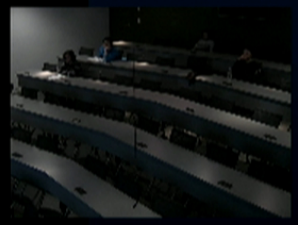
Example: Incompressible Flow



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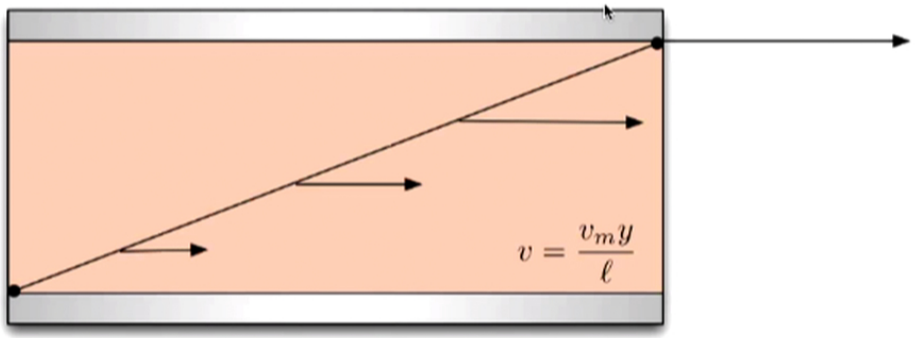
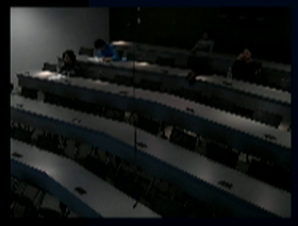
Example: Incompressible Flow





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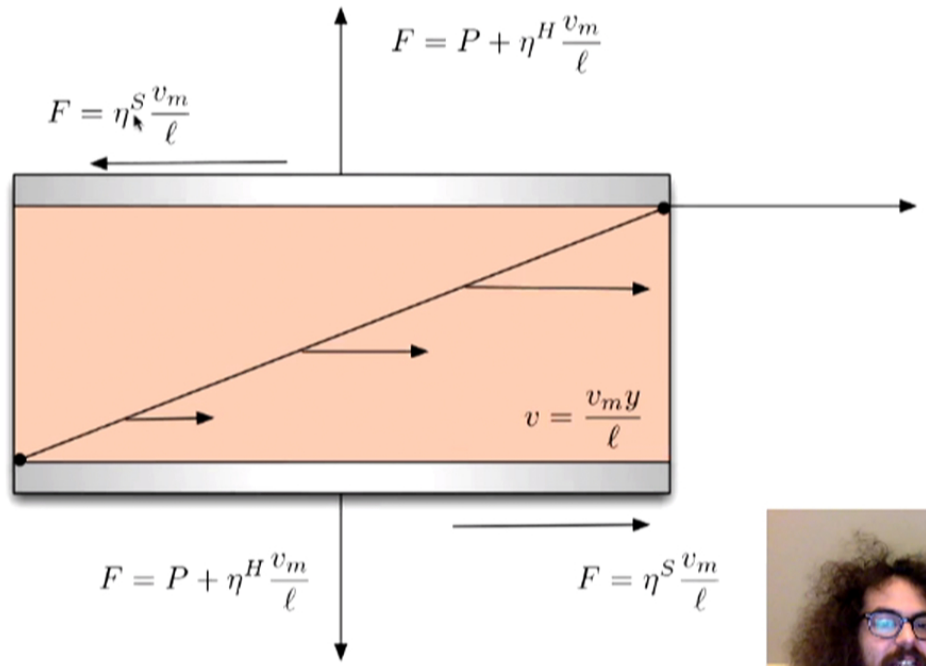
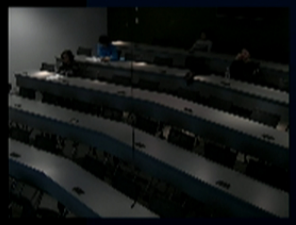
Example: Incompressible Flow





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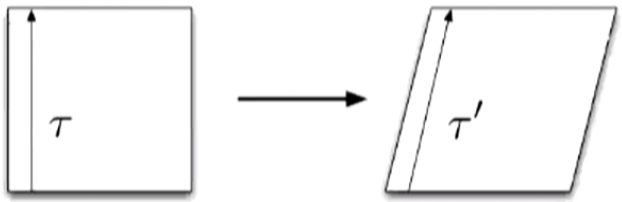
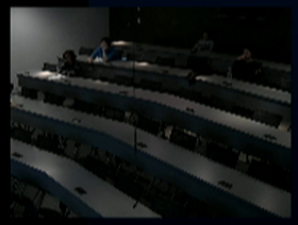
Hall Viscosity for Topological Phases

- First calculations - adiabatic response theory on the torus



- Results:
  - IQH [Avron, Seiler, Zograf, (1995)]: $\eta^H = \frac{\nu}{4}\bar{n}$
  - Conformal Block Trial States [Read (2009), Read&Rezayi (2011)]:

$$\eta^H = \frac{1}{2}\bar{s}\bar{n}$$



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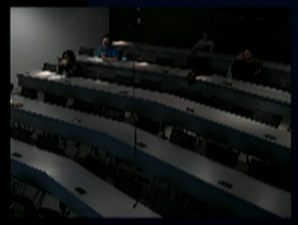
Hall Viscosity and Shift

- Haldane (1983): extra magnetic flux $\mathcal{S}$ needed to stabilize QH states on a sphere

$$N_\phi = \nu^{-1} N ?$$



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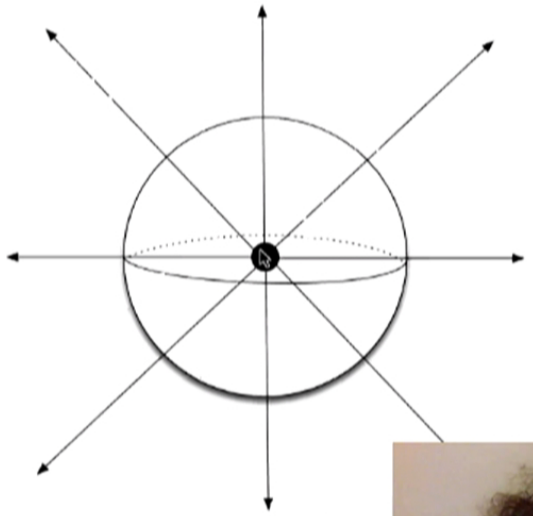
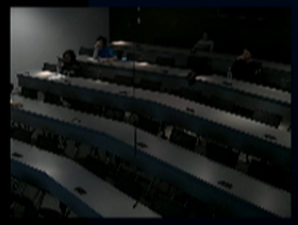
Hall Viscosity and Shift

- Haldane (1983): extra magnetic flux $\mathcal{S}$ needed to stabilize QH states on a sphere
- Wen, Zee (1992): $\mathcal{S} = 2\bar{s}$
- Same spin as in Hall viscosity!
- This implies the quantization of orbital spin to a rational value (with rotational + translational invariance)*
- The ratio $\eta^H/\bar{n}$ is quantized



$$N_\phi = \nu^{-1} N -$$



* See Read, Rezayi 2011 for more quantization arguments

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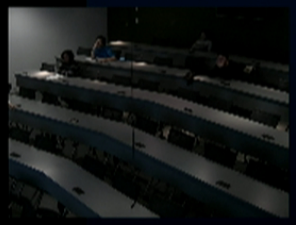
Viscosity from a Kubo Formula

- Goal: make contact with traditional approaches to transport
- What are the difficulties?
  - 1. Contact ("diamagnetic") response terms
    - Correctly accounted for by Luttinger (1964) in zero magnetic field
  - 2. External magnetic field
  - 3. Allow for the possibility of Hall viscosity
- BB, M. Goldstein, N. Read PRB**86** 245309 (2012)



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Setup

- Following Avron et. al. and Read&Rezayi:

$$H = h^{ij}(t) \sum_p \frac{\pi_i^p \pi_j^p}{2m} + \frac{1}{2} \sum_{p \neq q} V(\Lambda^T(t)(\mathbf{x}^p - \mathbf{x}^q))$$

$$h_{ij}(t) = \Lambda_{ik}(t)\Lambda_{jk}(t)$$
$$h^{ij}(t) = \Lambda^{-1}_{ki}(t)\Lambda^{-1}_{kj}(t)$$
$$\Lambda_{ij}(t) = \left(e^{\lambda(t)}\right)_{ij}$$

- Strategy: find a canonical transformation s.t.

$$H \to H_0 + H_1$$

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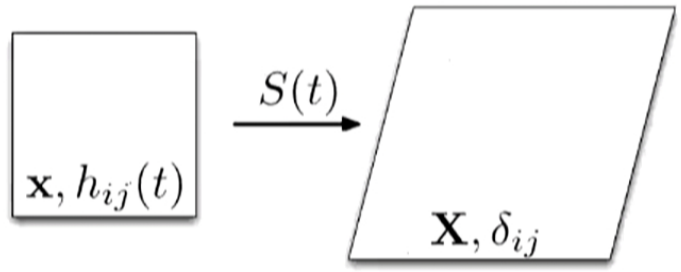
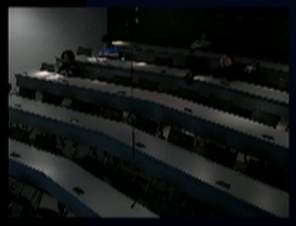
Setup

- Trick - use Λ to define a coordinate transformation



$$S(t) = \exp(-i\lambda_{ij} J_{ij})$$

$$H \to \sum_p \frac{\Pi_i^p \Pi_i^p}{2m} + \frac{1}{2}\sum_{p\neq q} V(\mathbf{X}^p - \mathbf{X}^q) - \frac{\partial \lambda_{ij}}{\partial t}$$



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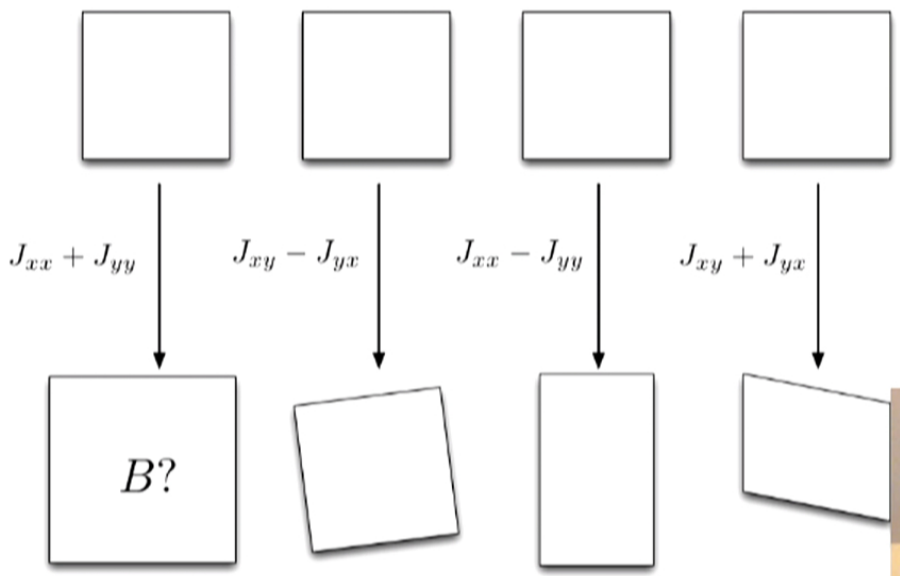
Setup

- J_{ij} are strain generators



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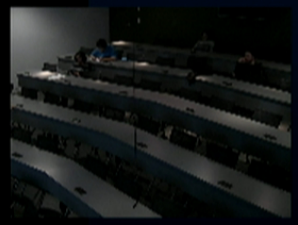
Explicit Form of Strain Generators

- Want to keep the magnetic flux through the system fixed under dilation
- Promote magnetic field strength to an operator:

$$[\mathcal{B}, \Xi] = i$$

$$J_{ij} = \sum_q \left[\mathcal{B}\epsilon_{jk} x_i^q x_k^q - \frac{1}{2}\{x_i^q, \pi_j^q\} \right] + \frac{1}{2}\delta_{ij}\{\mathcal{B}, \Xi\}$$



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Stress Tensor

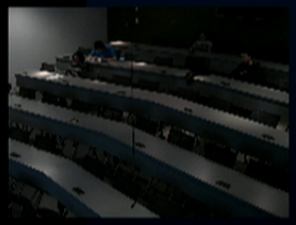
- Momentum continuity equation:

$$-\frac{\mathcal{B}}{m} g_i \epsilon_{ij} = \partial_t g_j + \partial_i \tau_{ij}$$

$$g_i(\mathbf{x}) = \sum_q \{\pi_i^q, \delta(\mathbf{x} - \mathbf{x}^q)\}$$

- Expanding in Fourier components -> Ward Identity

$$T_{ij} = \int d^2x \, \tau_{ij} = -\partial_t J_{ij} = -i \left[H_0, J_{ij}\right]$$



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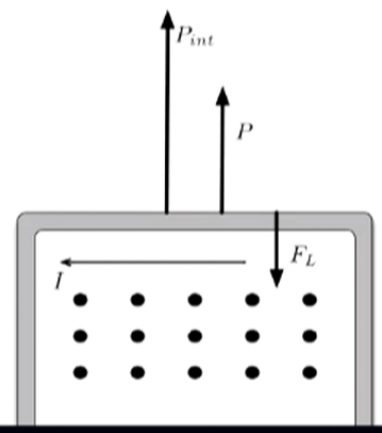
Properties of the Stress Tensor

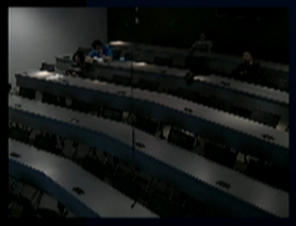
- Isotropic fluid:

$$\langle T_{ij} \rangle_0 = P_{int} V \delta_{ij}$$

- Internal Pressure [Cooper, Halperin, Ruzin (1997)]:

$$P_{int} = -\left(\frac{\partial E}{\partial V}\right)_{\nu,N} = P - mB$$





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Kubo Formula

- Response of an operator A to a perturbation $H_1(t) = f(t)B$ can be written

$$\langle A \rangle = \langle A \rangle_0 - V \int_{-\infty}^{\infty} dt' \, \chi(t-t') f(t')$$

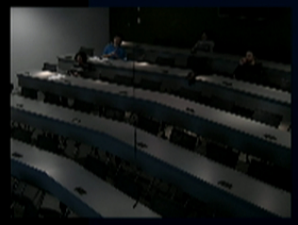
- where the response function $\chi(t-t')$ is

$$\chi(t-t') = -\frac{i}{V} \lim_{\epsilon \to 0^+} \Theta(t-t') \, \langle [A(t), B(t')] \rangle_0 \, e^{-\epsilon(t-t')}$$

- time evolution and averages taken w.r.t. the unperturbed hamiltonian

- Can write this in frequency space as

$$\boxed{\chi(\omega) = -\frac{i}{V} \lim_{\epsilon \to 0^+} \int_0^{\infty} dt \, e^{i\omega^+ t} \, \langle [A(t), B(0)] \rangle_0}$$



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Kubo Formulas for Viscosity

- Adapting this to stress response with $H_1 = -\frac{\partial \lambda_{ij}}{\partial t} J_{ij}$, we have

$$\eta_{ijk\ell} = \frac{1}{V} X_{ijk\ell}(\omega) + \frac{i}{\omega^+}(P_{int} - \kappa_{int}^{-1})\delta_{ij}\delta_{k\ell}$$

- with

$$X_{ijk\ell} = -i \int_0^\infty dt e^{i\omega^+ t} \langle [T_{ij}(t), J_{k\ell}(0)] \rangle_0$$

$$\kappa_{int}^{-1} = -V \left(\frac{\partial P_{int}}{\partial V} \right)_{\nu, N}$$

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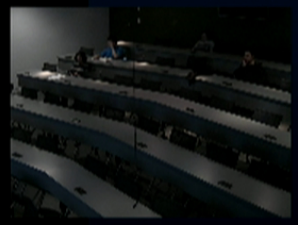
Alternative Forms

- The Ward identity $T_{ij} = -\partial_t J_{ij}$ allows us to integrate by parts:
- Stress-Stress form

$$X_{ijk\ell}(\omega) = \frac{1}{\omega^+}\left(\langle[T_{ij}, J_{k\ell}]\rangle + \int_0^\infty dt e^{i\omega^+ t}\langle[T_{ij}(t), T_{k\ell}(0)]\rangle\right)$$

- Note the contact term!
  - c.f. diamagnetic conductivity



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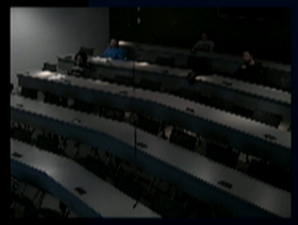
Alternative Forms

- The Ward identity $T_{ij} = -\partial_t J_{ij}$ allows us to integrate by parts:
- Strain-Strain form

$$X_{ijk\ell}(\omega) = -i\langle [J_{ij}, J_{k\ell}]\rangle + \omega^+ \int_0^\infty dt e^{i\omega^+ t} \langle [J_{ij}(t), J_{k\ell}(0)]\rangle$$

- Berry curvature of Read-Rezayi, with correction f degenerate states



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Examples and Remarks

- Free electron gas:

$$\eta^{sh} = \frac{2iE_0}{dV\omega^+}$$

$$\zeta = \eta^H = 0$$

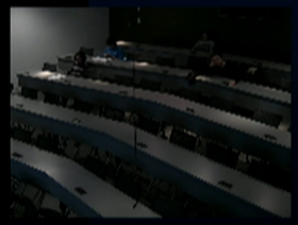
- IQH (non-interacting):

$$\eta^{sh} = \frac{iE_0\omega^+}{V({\omega^+}^2 - 4\omega_c^2)}$$

$$\eta^H = \frac{-2E_0\omega_c}{V({\omega^+}^2 - 4\omega_c^2)}$$

$$\zeta = 0$$

- Laughlin & p+ip, maps on to previous calculations for Hall visco
from strain-strain form



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Viscosity-Conductivity Relation

- Can write *intensive form* for stress response:

$$\chi_{ijk\ell}(\omega) = \frac{1}{\omega^+}\left(\langle[\tau_{ij}(\mathbf{0}), J_{k\ell}]\rangle + \int_0^\infty dt \int d^2x e^{i\omega^+ t}\langle[\tau_{ij}(\mathbf{0},t), \tau_{k\ell}(\mathbf{x},0)]\rangle\right)$$

- Recall Kubo formula for conductivity:

$$\sigma_{ij}(\mathbf{q},\omega) = \frac{i\bar{n}}{m\omega^+}\delta_{ij} + \frac{1}{\omega^+}\int_0^\infty dt \int d^2x e^{i(\omega^+ t - \mathbf{q}\cdot\mathbf{x})}\langle[j_i(\mathbf{x},t), j_j(\mathbf{0},0)]\rangle$$

- Galilean Invariance: $g_i = m j_i$

- Stress continuity equation: $\partial_t g_j + \partial_i \tau_{ij} = \omega_c \epsilon_{ji} g_i$

- Putting it together gives the Ward identity:

$$\boxed{\sigma^{(2)}_{ij}(\mathbf{q},\omega) = -\frac{1}{\bar{n}^2}\sigma^{(0)}_{im}\sigma^{(0)}_{nj} q_k q_\ell \chi_{km\ell n}}$$

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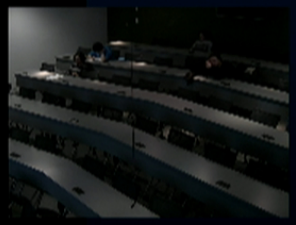
"Experimental" Implications

- $\eta^H(\omega) = \frac{m^2}{2}\frac{\partial^2}{\partial q_x^2}\left\{(\omega^2+\omega_c^2)\sigma^H(\mathbf{q},\omega) - i\omega\omega_c(\sigma_{xx}(\mathbf{q},\omega)+\sigma_{yy}(\mathbf{q},\omega))\right\}\Big|_{\mathbf{q}=\mathbf{0}}$
- All terms on the RHS are in-principle measurable
- Generalizes a similar formula of Hoyos and Son (2011)
- Can be explicitly checked in certain cases
  - IQH systems using conductivity from Chen, et. al (1989)
  - p+ip superfluid using conductivity from Lutchyn et. al (2008)



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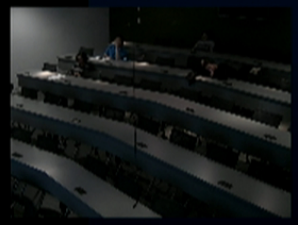
- Non-dissipative transport coefficients
- Review - quantization of Hall conductivity
- Hall viscosity
  - Hall viscosity - what is it?
  - Kubo formula for (Hall) viscosity
  - Viscosity - conductivity relation, possible "experiments"
- **Thermal Hall conductivity**
  - Thermal Hall conductivity and the edge theory
  - Bulk thermoelectric transport
  - Setup - non-relativistic geometry
  - Linear response from low-energy effective theory
- Conclusion



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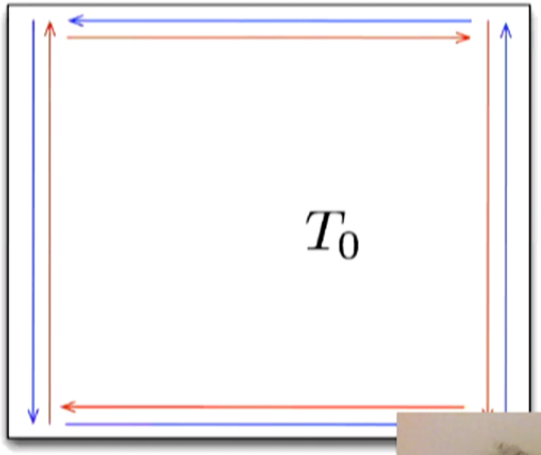
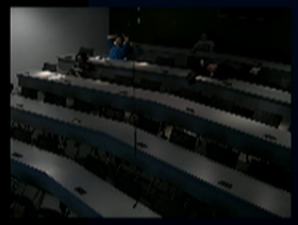
Chiral Edge States and Thermal Hall Conductivity

- Gapped bulk, gapless chiral edge excitations
- velocities $v_{b,a}$ for counter-clockwise modes, $v_{r,a}$ for clockwise modes
- Central charges $c_{b,a}$, $c_{r,a}$





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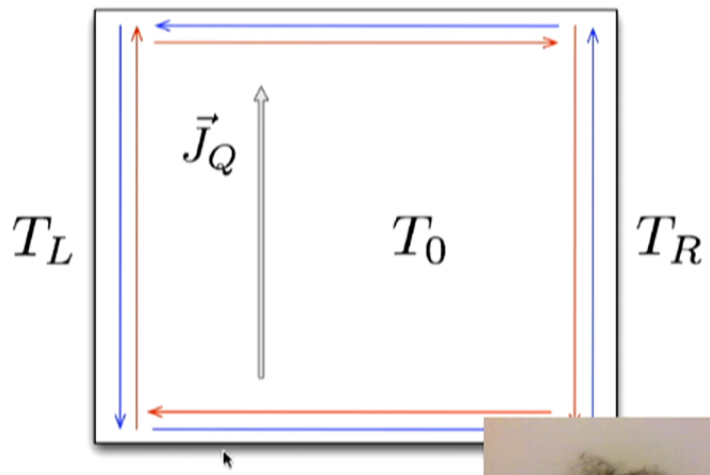
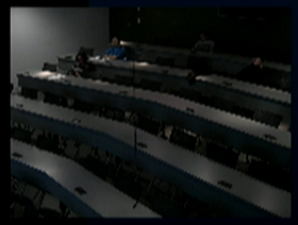
- CFT -> temperature gradient leads to heat current



$$\vec{J}_Q = \vec{J}_{\mathrm{E}} - \mu \vec{J}$$



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Chiral Edge States and Thermal Hall Conductivity

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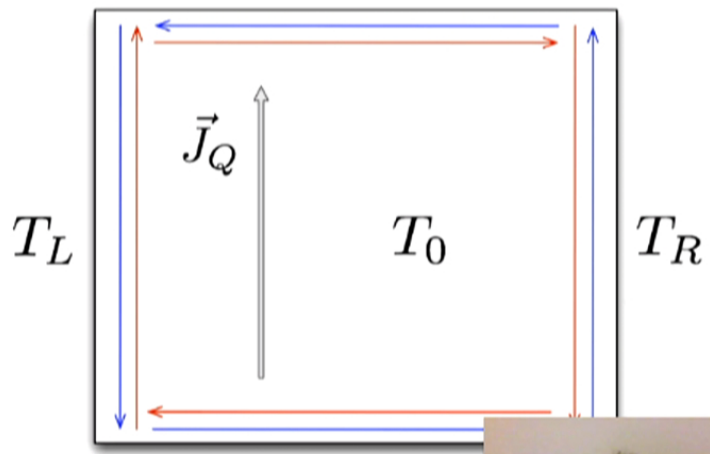
- velocities $v_{b,a}$ for counter-clockwise modes, $v_{r,a}$ for clockwise modes
- Central charges $c_{b,a}$, $c_{r,a}$
- CFT -> temperature gradient leads to heat current

$$J_Q^i = \kappa \epsilon_{ij} \Delta_j T$$

$$\kappa = \frac{\pi T}{6} \sum_a c_{b,a} - c_{r,a}$$



$$\vec{J}_Q = \vec{J}_E - \mu \vec{J}$$

Kane, Fisher (1997);

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Kubo Formulas for Thermoelectric Response

- Luttinger (same paper!) - thermoelectric response <-> response to gradient of "gravitational" perturbation

$$H_1 = \int d^d x \; \psi(x) h_0(x)$$

- Kubo formula for thermal conductance

$$\kappa^{ij} = \frac{1}{T_0} \lim_{\omega \to 0} \int_0^\infty dt e^{i\omega^+ t} \lim_{q \to 0} \frac{1}{q_j} \langle [J_Q^i(q,t), h_0(-q) - \mu\rho(-q)] \rangle$$

- Cooper, Halperin, Ruzin (PRB 1997) - "diathermal" magnetization terms contribute when TRS is broken
  - Proportional to the moment $\int d^2x \left\langle x^i J_Q^j(x) \right\rangle$
- Use of these formulas <u>requires</u> finite size

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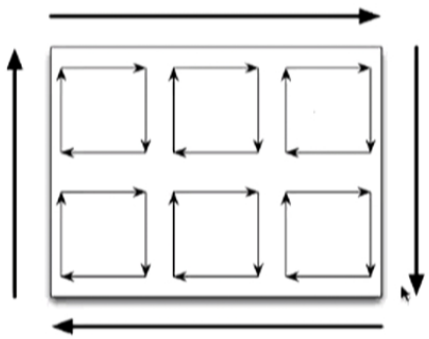
Magnetization Currents and Transport

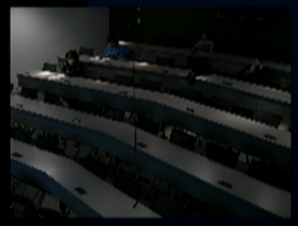
- Classical electrodynamics: equilibrium current

$$\langle \mathbf{j}(\mathbf{r}) \rangle_0 = \nabla \times \mathbf{m}(\mathbf{r})$$

- Homogeneity -> bound *surface* current



- The same is true for energy: $\langle \mathbf{j}^E(\mathbf{r}) \rangle_0 = \nabla \times \mathbf{m}^E(\mathbf{r})$



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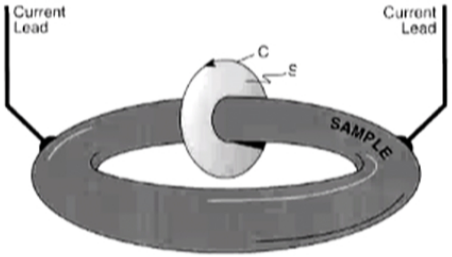
Magnetization Currents and Transport

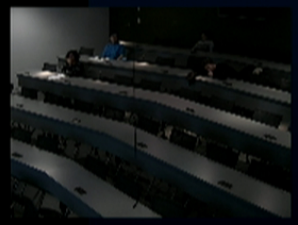
- Electric/ gravitational field -> nonuniform magnetization (CHR)

$$\mathbf{m} \to \mathbf{m}(1+\psi)$$

$$\mathbf{m}^E \to \mathbf{m}^E(1+2\psi) + \phi\mathbf{m}$$

- BUT - corresponding currents do **not** contribute to transport





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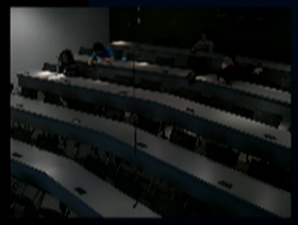
Questions

- Interplay between bulk and edge contributions to thermal Hall conductivity
  - c.f. ordinary Hall effect
- Is there a truly bulk formulation?
- Effective action for thermal response?
- Requires the introduction of perturbations to system geometry
- BB, N. Read arXiv:1407.2911



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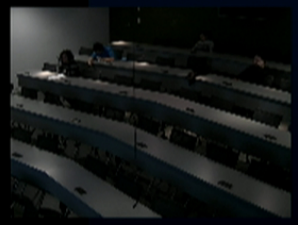
Approach to Thermal Transport

- The microscopic approach is helpful for non-interacting systems [Smrca&Streda (1977), Streda (1983), Sumiyoshi&Fujimoto (2012)]
- Becomes unwieldy for interacting systems
- Hard to separate role of edge effects
- Instead: effective action approach
- generalize Λ_{ij} to e^a_μ which lives in **spacetime**
- captures full thermoelectric & stress response
- What does the induced action tell us based on symmetry



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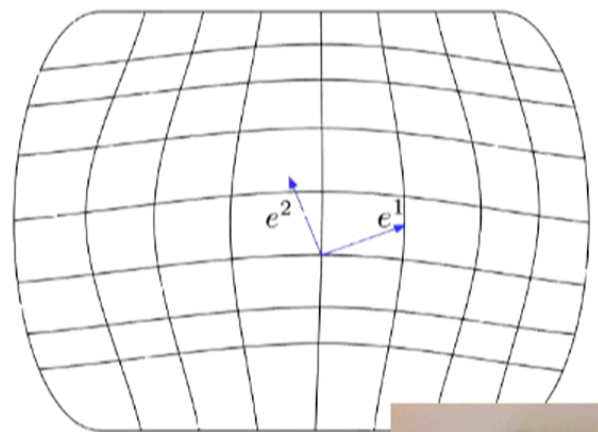
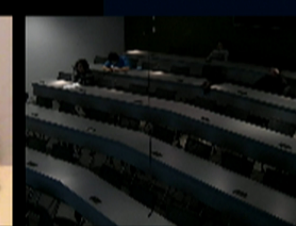
Geometric Perturbations

- Introduce vielbeins e^α_μ - local orthonormal basis of tangent vectors
- Non-relativistic: preferred time direction $\alpha = 0$
  - $$e^0_0 = 1 + \psi$$
- Spin connection ω_μ - tells us how vielbeins rotate from point to point
- Torsion
  - $T^\alpha_{\mu\nu} = \partial_\mu e^\alpha_\nu - \partial_\nu e^\alpha_\mu + \epsilon^{0\alpha\beta}(\omega_\mu e^\beta_\nu - \omega_\nu e^\beta_\mu)$
  - T^0 couples to heat currents
  - $B_G = \frac{1}{2}\epsilon^{\mu\nu\lambda} e^0_\mu T^0_{\nu\lambda}$
  - Spatial part <-> "dislocation density"





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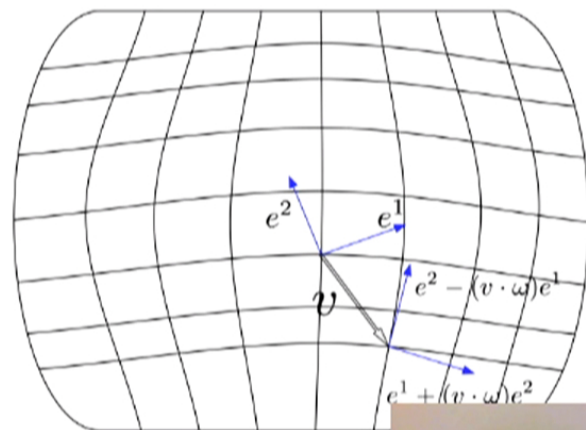
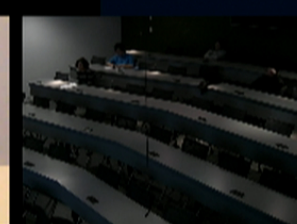


$h_{\mu\nu} = \eta_{ab} e^a_\mu e$

$\sqrt{g} = \det(\mathbf{e})$



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Measurable Currents

- Given an action $S[\psi, e, \omega, A]$ can define

$$J^\mu = \frac{1}{\widehat{\sqrt{g}}} \frac{\delta S}{\delta A_\mu}$$

$$\tau^\mu_\alpha = -\frac{1}{\widehat{\sqrt{g}}} \frac{\delta S}{\delta e^\alpha_\mu}$$

$$J_S{}^\mu{}_a{}^b = \frac{1}{\widehat{\sqrt{g}}} \frac{\delta S}{\delta \omega_\mu{}^a{}_b}$$

- Noether -> these satisfy natural continuity eqs.
- Energy current $J^\mu_E \equiv e^\alpha_0 \tau^\mu_\alpha$
- Stress tensor $\tau^i{}_j \equiv e^\alpha_j \tau^i{}_\alpha$

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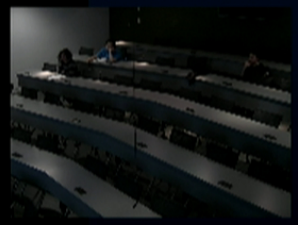
Low Energy Theory

- Gapped system -> Integrate out matter in bulk
- most general induced action to first order in derivatives

$$S^{\text{eff}} = \int d^3x \, \widehat{\sqrt{g}} \, \Big[f(B, B_G) + \gamma(B, B_G) \epsilon^{\mu\nu\lambda} e^a_\mu T^a_{\nu\lambda} + \tilde{\gamma}(B, B_G) \epsilon^{\mu\nu\lambda} \epsilon_{ab} e^a_\mu T^b_{\nu\lambda} + \frac{\nu}{4\pi} \epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \Big]$$



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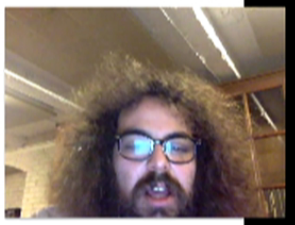

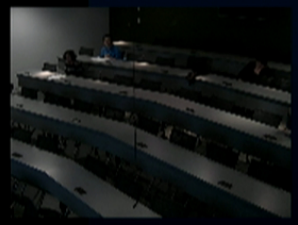

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Linear Response

- Most terms in action are *locally invariant* - integrands are scalars under all symmetries
- => contributions to number, energy current will be pure magnetization - curl of local quantities
- Bulk magnetizations: $$\widehat{m}_{0,b} = -\frac{\partial f}{\partial B}$$ $$\widehat{m}^{E}_{0,b} = -\frac{\partial f}{\partial B_G}$$
- Chern-Simons term is not locally invariant -> orc Hall current is a transport current

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Linear Response

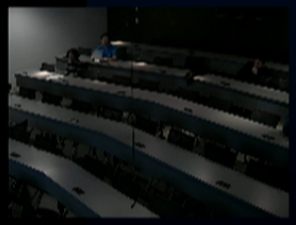
- Apply external electric potential ϕ, gravitational field ψ
- Number current to linear order:

$$\widehat{J}^i = \frac{\delta S^{eff}}{\delta A_i}$$
$$= -\frac{\nu}{2\pi}\epsilon^{ij}\partial_j\phi + \epsilon^{ij}\partial_j\left(\widehat{m}_{0,b}(1+\psi)\right)$$

- Energy current to linear order:

$$\widehat{J}^i_E = -e^\alpha_0 \frac{\delta S^{eff}}{\delta e^\alpha_i}$$
$$= \epsilon^{ij}\partial_j\left(\widehat{m}_{b,0}\phi + \widehat{m}^E_{b,0}(1+2\psi)\right)$$

- All but Hall current are curls of the modified magnetizat



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Punchline

- To linear order, all bulk *transport* number and energy currents vanish except the Hall current
- Thermal Hall current is confined to the edge
- Gravitational Chern-Simons action $S_{GCS} = \frac{c}{48\pi}\int d^3x \epsilon^{\mu\nu\lambda}\omega_\mu \partial_\nu \omega_\lambda$ does not *directly* describe thermal Hall conductivity
  - c.f. Stone (2012) for the relativistic case

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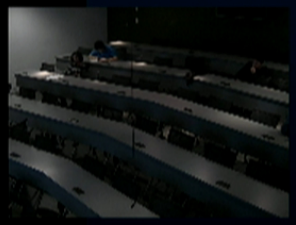
Conclusion

- Hall viscosity <-> shift
- Thermal Hall conductivity <-> central charge
- Kubo formula for viscosity allows for bulk calculation, and relation to conductivity
- Thermal Hall conductivity, on the other hand, is an edge phenomenon
- Further work - what is the *direct* bulk significance of central charge?
- More information:

BB, M. Goldstein, N. Read, PRB**86** 245309 (2012)

BB, N. Read, arXiv:1407.2911



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