Title: The Incompleteness of HRT

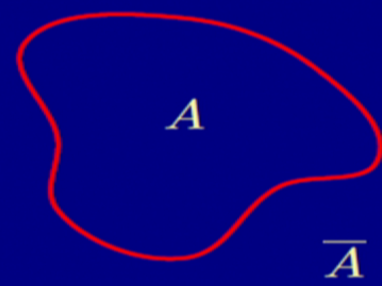
Date: Dec 11, 2014 01:00 PM

URL: http://www.pirsa.org/14120038

Abstract: <p>In AdS/CFT, the HRT prescription relates the entanglement entropy of a region of a CFT to the area of an extremal surface in the dual AdS spacetime. But there exists a class of spacetimes in which the HRT prescription is ill-defined. These spacetimes consist of planar AdS wormholes containing an inflating region. I will introduce these so-called AdS-dS-wormholes, discuss how the HRT prescription fails in them, and suggest possible modifications to remedy the problem.</p>


1 Holographic Entanglement Entropy
2 AdS-dS-Wormholes
3 Completion of HRT
4 Complex Entangling Surfaces

# Entanglement Entropy




- Entanglement entropy of region $A$:

$$\rho_A \equiv \mathrm{Tr}_{\overline{A}}\, \rho; \quad S(A) = -\,\mathrm{Tr}_A(\rho_A \ln \rho_A)$$

- $S(A)$ measures entanglement between $A$ and $\overline{A}$ (nonlocal)
- Very useful (e.g. quantum information, confinement/deconfinement, RG, BH thermo), but hard to calculate
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UCSB

# (H)HRT and Maximin

- Hubeny-Rangamani-Takayanagi proposal (extension of Ryu-Takayanagi):
$$S(A) = \frac{\text{Area}(\Xi)}{4G_N};$$
$\Xi$ is minimal-area extremal surface anchored to $\partial A$
- $\Xi$ must be homologous to $A$ [Headrick & Takayanagi]
- Maximin prescription [Wall]:
  - Foliate spacetime into achronal slices $\Sigma$ containing $\partial A$
  - Calculate area $\min(A, \Sigma)$ on each $\Sigma$
  - Then
$$S(A) = \frac{1}{4G_N} \max_{\Sigma} \min(A, \Sigma)$$
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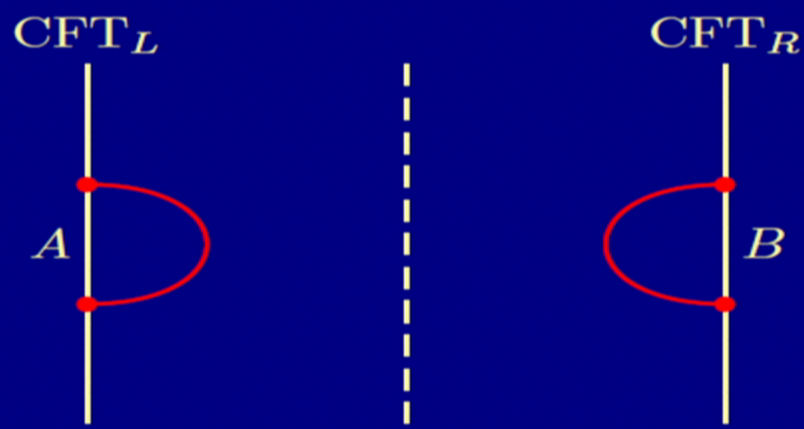
# An Example: Hartman-Maldacena

- Mutual information of regions $A$, $B$ on the opposite boundaries of planar Schwarzschild-AdS:

$$I(A,B) = S(A) + S(B) - S(A \cup B)$$

- If $S(A \cup B)$ comes from disconnected pieces, then $I(A,B) = 0$



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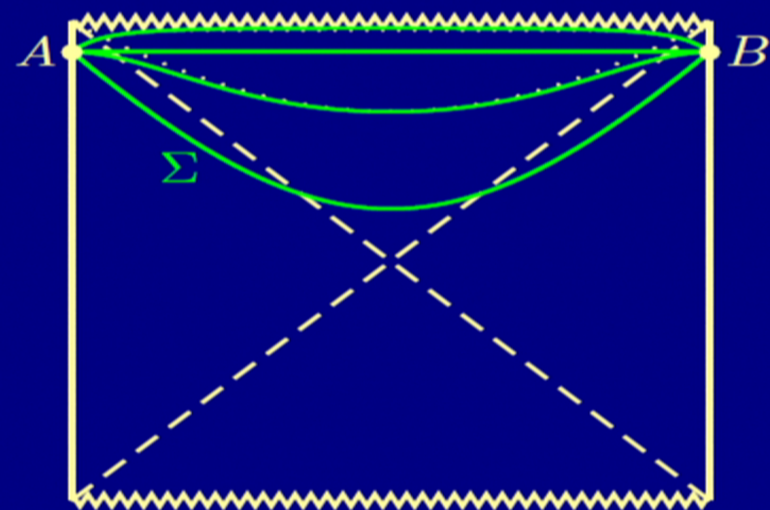

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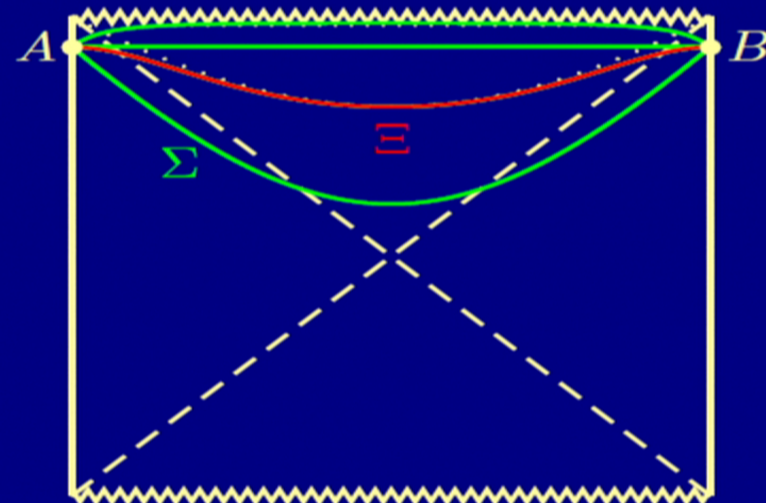

# AdS-dS-Wormholes

- The Hartman-Maldacena scenario is nice for studying properties of entanglement entropy – can we tweak it to probe HHRT?
- Yes - stretch the wormhole and throw an inflating region in the middle:


- Can be explicitly constructed by patching together regions of vacuum spacetimes with constant $\Lambda$ – think of $\Lambda$ as sourced by scalar field potential $V(\phi)$
- Satisfies the NEC
- For now, take planar directions to be compact but large

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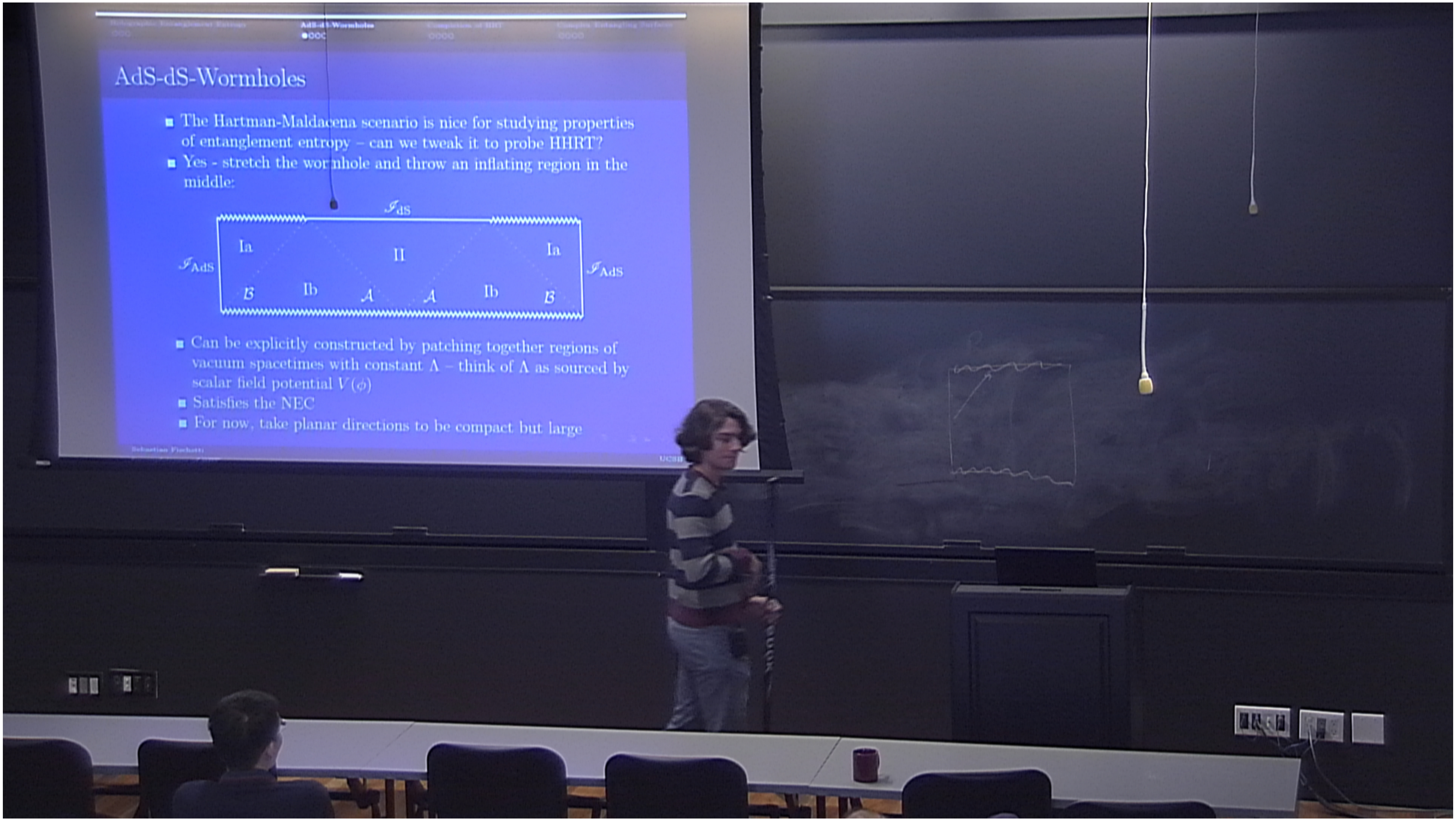
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Ia
II
Ia
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$\mathcal{B}$
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# No Wormhole-Traversing Extremal Surfaces

- What happens to the mutual information between localized regions $A$, $B$?
- Use maximin prescription:

# No Total Entropy Surfaces, Either

- What if $A$, $B$ are taken to be the entire CFTs?
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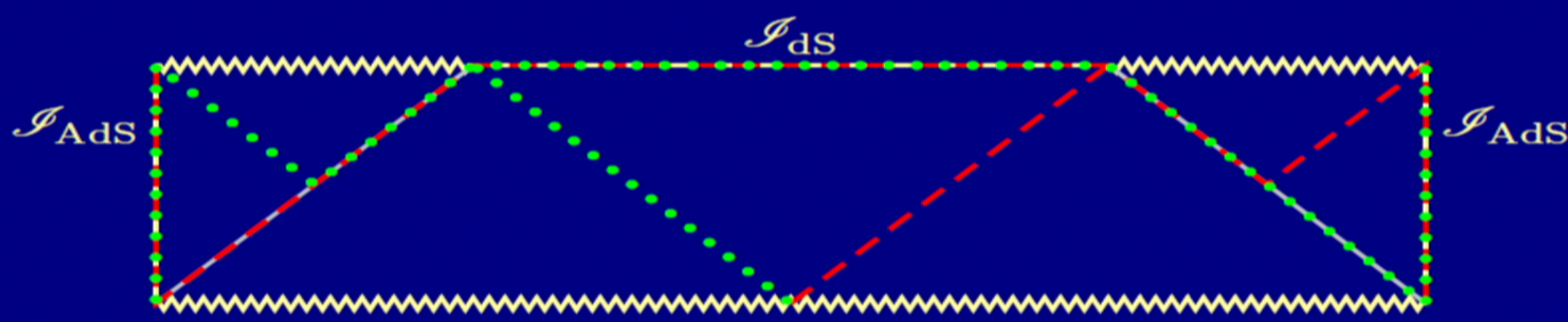

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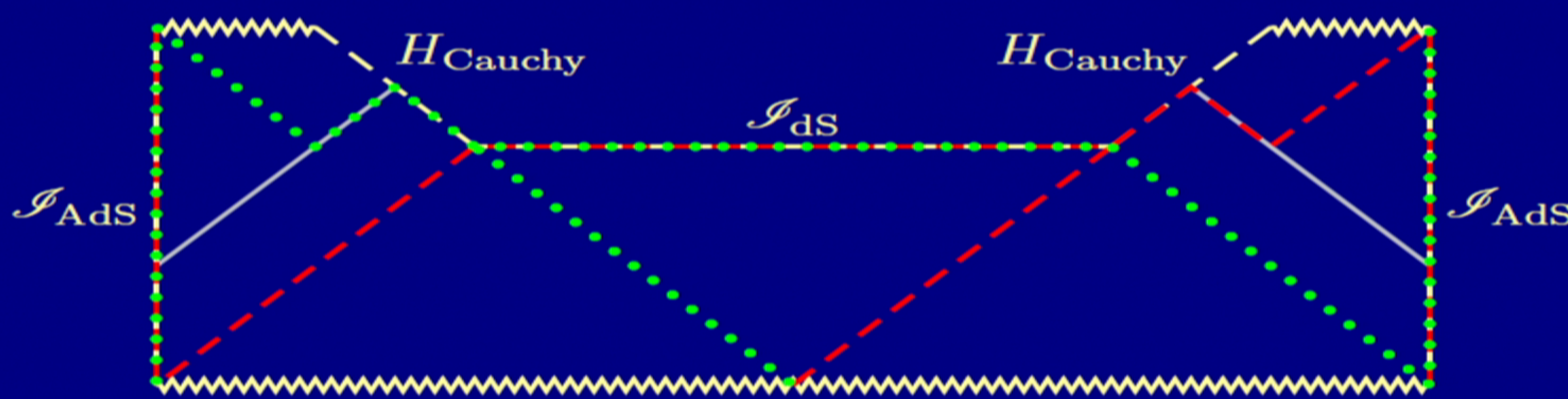


- In general, $\theta_L = 0 = \theta_R$ nowhere in the spacetime
- In such cases, HHRT/maximin entropy of either boundary is ill-defined

**Takeaway Message**

The HHRT and maximin prescriptions cannot calculate the entropy of one boundary of an AdS-dS-wormhole.

So usual picture needs some modification

- Does dual description need more than two CFTs? E.g.:
  - Superselection sectors [Marolf & Wall]
  - Extra degrees of freedom associated with dS region [Freivogel, Hubeny, Maloney, Myers, Rangamani, Shenker]
- Maybe, but not obviously; in [Freivogel et al.] there's a Euclidean section and something like HHRT might hold [Lewkowykz & Maldacena], but would need to investigate further
- Here, let's focus on possible modifications of HHRT/maximin themselves

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# Regulated Wormholes

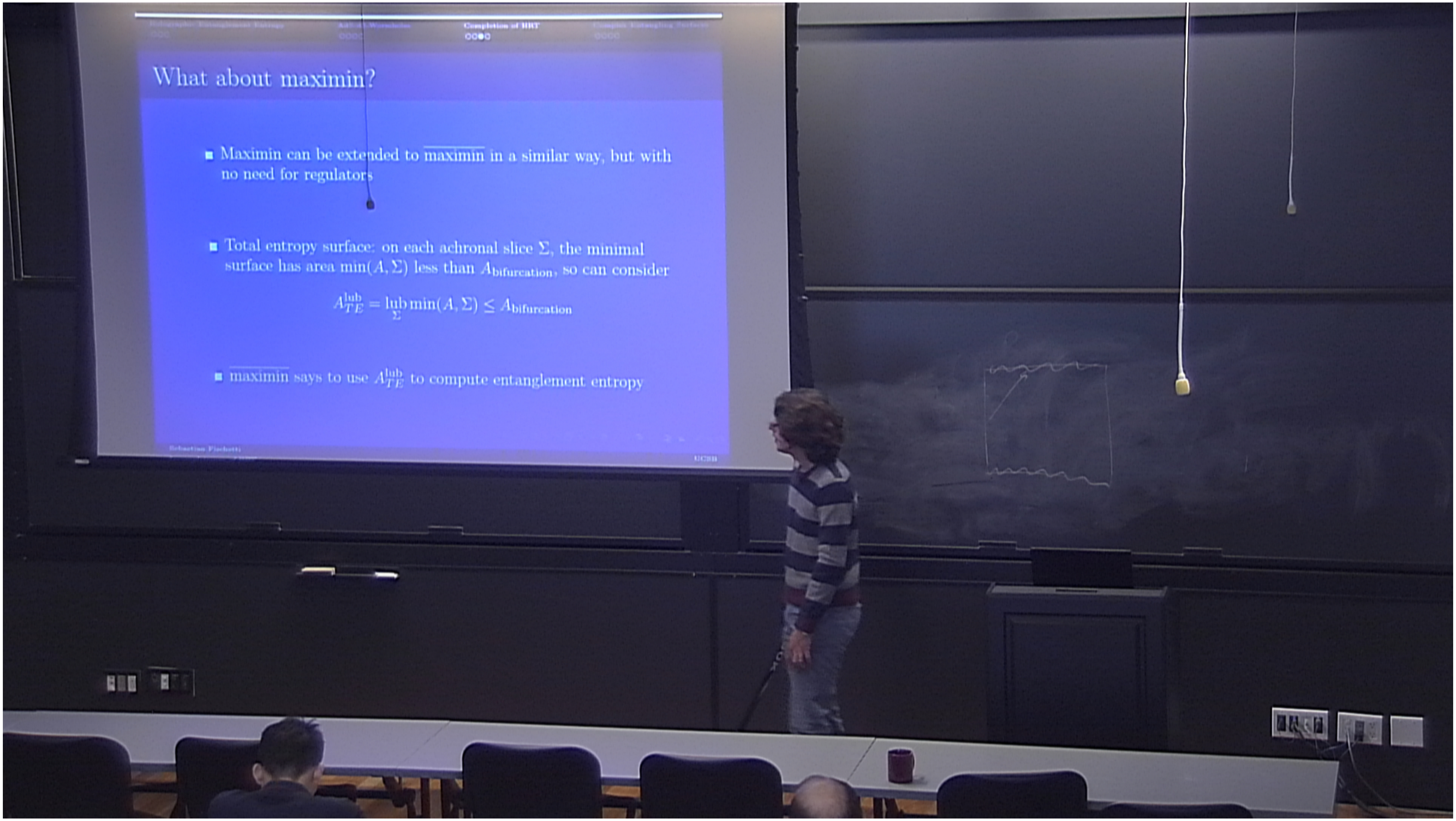
- Regulate $\mathscr{I}_{\text{dS}}$ by cutting off inflation after a finite time $\tau$

## What about maximin?

- Maximin can be extended to $\overline{\text{maximin}}$ in a similar way, but with no need for regulators
- Total entropy surface: on each achronal slice $\Sigma$, the minimal surface has area $\min(A, \Sigma)$ less than $A_{\text{bifurcation}}$, so can consider

$$A_{TE}^{\text{lub}} = \operatorname*{lub}_{\Sigma} \min(A, \Sigma) \leq A_{\text{bifurcation}}$$

- $\overline{\text{maximin}}$ says to use $A_{TE}^{\text{lub}}$ to compute entanglement entropy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# Comments on $\overline{\text{HHRT}}/\overline{\text{maximin}}$

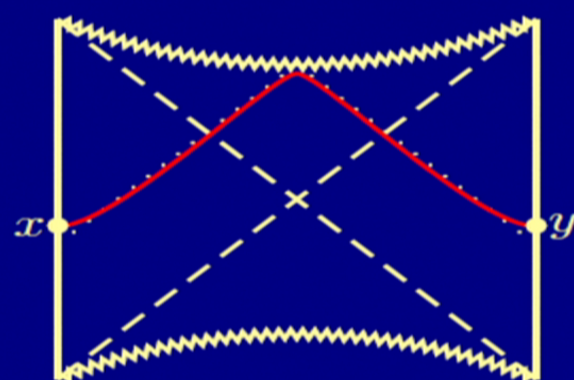
- Limiting procedure of $\overline{\text{HHRT}}$ is conceptually simple, but unclear if it's well-defined
- $\overline{\text{HHRT}}$ is continuous as regulator is removed, whereas HHRT is not
  - E.g. add a Schwarzschild-AdS region with horizon area $A_{\text{new}} > A^{\infty}_{TE}$
  - $\overline{\text{HHRT}}$ also ensures consistency with CFT density of states set by total energy
- Limiting procedure of $\overline{\text{maximin}}$ is well-defined, but unclear whether it always agrees with $\overline{\text{HHRT}}$
- Dual CFT state is pure: $S(\text{CFT}_1 \cup \text{CFT}_2) = 0$
- $I(A, B) = 0$ for localized $A$, $B$ but $I(\text{CFT}_1, \text{CFT}_2) \neq 0$; information is as delocalized as possible
  - Similar to e.g. extremal Reissner-Nordström-AdS [Andrade, SF, Marolf, Ross, Rozali]

# Complex Extremal Surfaces?

- AdS-dS-wormholes are a good testing ground for further modifications of HHRT - is there anything else we can try?
  - Familiar story: geodesic approximation to two-point correlators of heavy fields

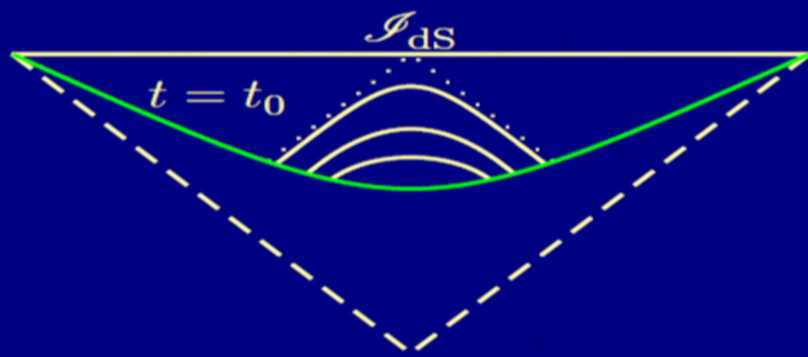
$$\langle \phi(x)\phi(y) \rangle \sim e^{-mL(x,y)}$$

  - Often, complex geodesics are required, e.g. correlator between opposite boundaries of Schwarzschild-AdS [Fidkowski, Hubeny, Kleban, Shenker]:



# Complex Geodesics in $dS_3$

- HHRT also comes from saddle point approximation! [Lewkowycz & Maldacena]
- Including complex surfaces in AdS-dS-wormholes is especially tempting...



# Comments on Complex Extremal Surfaces

- Whether complex extremal surfaces should contribute to HHRT (and if so, how) is unclear

- But it's plausible that they do (and critical for the geodesic approximation to the two-point correlator)

- Unambiguously identifying complex surfaces requires an analytic spacetime – it'd be great to have an analytic model of an AdS-dS-wormhole to explore

# Conclusions/Future Directions

- We constructed AdS-dS-wormholes to test HHRT/maximin – both prescriptions give ill-defined answers when computing the entropy of an entire boundary CFT
- Conformally completed prescriptions:
  - $\overline{\text{HHRT}}$ regulates $\mathscr{I}_{\text{dS}}$ and takes the limit as the regulator is removed
  - $\overline{\text{minimax}}$ replaces the maximization with a least upper bound over all achronal slices $\Sigma$
  - Both behave nicely (and corresponding entangling surfaces can be thought of as living in $\overline{M}$) and CFT states are entangled in most delocalized way possible
  - We don't know if they're equivalent or how well-defined $\overline{\text{HHRT}}$ is – will need to check