Title: Geometric response of FQH states - Andrey Gromov

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Abstract: <p>Two-dimensional interacting electron gas in strong transverse magnetic field forms a collective state -- incompressible electron liquid, known as fractional quantum Hall (FQH) state. FQH states are genuinely new states of matter with long range topological order. Their primary observable characteristics are the absence of dissipation and quantization of the transverse electro-magnetic response known Hall conductance. In addition to quantized electromagnetic response FQH states are characterized by quantized geometric responses such as Hall viscosity and thermal Hall conductance.
I will show how to derive the effective action for various Abelian and non-Abelian FQH states on a curved space. In particular, I will derive the quantized universal responses to the changes in geometry of space. These responses are described by Chern-Simons-type terms. It will be shown that in order to obtain the responses in a self consistent way one has to take into account the framing anomaly of the quantum Chern-Simons(-Witten) theory. This peculiar phenomenon illustrates the failure of a classically topological theory to remain topological at the quantum level.
If time permits I will comment on the coupling of non-relativistic systems to the space-time geometry. Using the appropriate geometry I will write an effective action describing the bulk energy and thermal Hall conductances. From this effective action it will be clear that these response functions are neither universal nor topologically protected.</p>

GEOMETRIC RESPONSE OF FRACTIONAL QUANTUM HALL STATES

Andrey Gromov
Stony Brook University

References:

Phys. Rev. B 90, 014435
arXiv:1403.5809 (To appear in PRL)
arXiv:1407.2908 (To appear in PRL)
with
Alexander G. Abanov

arXiv:1410.6812
with
Gil Young Cho, Yizhi You,
Alexander G. Abanov,
Eduardo Fradkin

PLAN OF THE TALK

I. Introduction

- Fractional quantum Hall effect.
- Flux attachment.

II. Free non-relativistic fermions in curved space

- Perturbative computation of the effective action.
- Disagreement with old results.

III. FQH states and framing anomaly

- Inconsistency of the flux attachment for Laughlin states.
- Framing anomaly.
- K-matrix states, Jain states.
- Non-abelian states

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IV. Thermal Hall effect.

INTRODUCTION I: FQHE

FRACTIONAL QUANTUM HALL EFFECT

- Quantized Hall conductivity

$$\sigma_H = \frac{\nu}{2\pi} = \frac{n}{B} = 1/\rho_{xy}$$

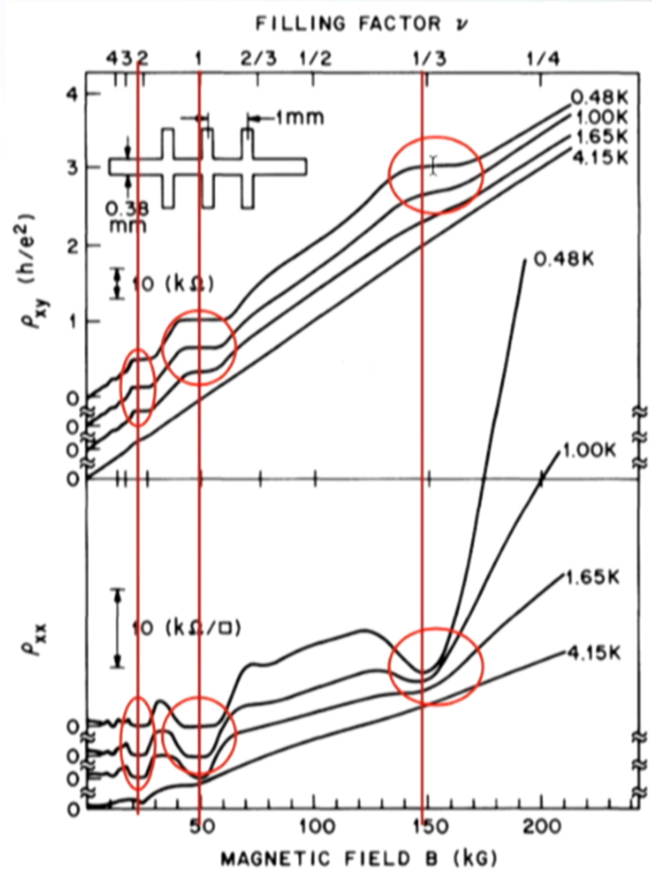
- Number of electrons fixed by magnetic flux

$$N = \nu N_\phi$$

- Vanishing diagonal resistivity

$$\rho_{xx} = 0$$

- No dissipation



Tsui, Stormer, Gossard (1982)

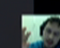
DIFFERENT FQH STATES

Can two different FQH states have the same filling fraction? **Yes.**

- Braiding and fusion of gapped quasiparticles
- Topological degeneracy on higher genus surfaces
- Topological entanglement entropy, entanglement spectrum.

No direct relation to the observables (yet).

Find new measurable, quantized, non-dissipative, universal transport coefficients.

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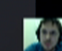
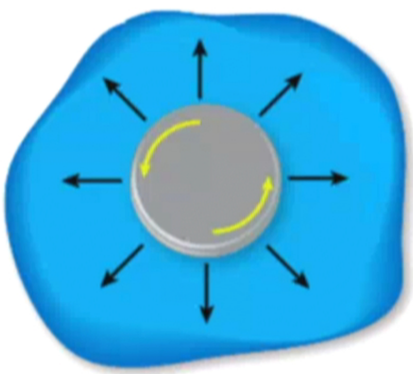


GEOMETRIC RESPONSES I

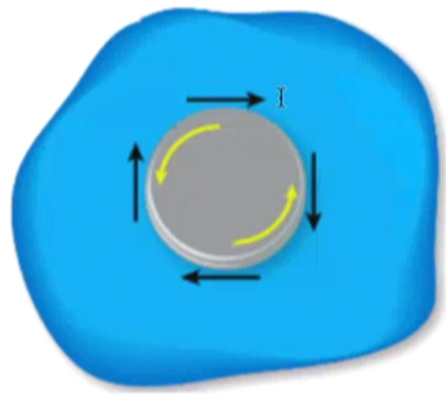
- Is there a viscous analogue of $\langle j_x j_y \rangle \sim \sigma_H$? Avron, Zograf, Seiler (1995)
- Yes! **Hall viscosity** $\langle T_{xx} T_{xy} \rangle \sim \eta_H$

Dissipative viscosity



Stress tensor is a response to external strain (or metric)

Lapa, Hughes (2013)



- General relation $\eta_H = \hbar \frac{\bar{s}}{2} \rho$ $\hbar \bar{s}$ is the average spin density

  Read (2008)

IQHE $\bar{s} = \frac{1}{N}\left(\frac{1}{2} + \frac{3}{2} + \ldots + \frac{2N-1}{2}\right) = \frac{N}{2}$

CFT trial states $\bar{s} = \frac{\nu^{-1}}{2} + h$ $h = 1 - \frac{1}{k}$

GEOMETRIC RESPONSES II

- Non-dissipative edge thermal Hall effect $\langle j_x^Q j_y^Q \rangle \sim K_H$

$j_i^Q = j_i^E - A_0 j_i$ is the thermal current.

thermal Hall conductivity is $K_H = (c_R - c_L)\frac{\pi}{6}k_B^2 T$

Kane-Fisher (1996)

Read-Green (1999)

c_R is the right-moving chiral central charge

K_H does not depend on the velocities of edge states

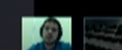
Universal thermal transport can occur only at the edge.

Contrary to e/m Hall effect bulk thermal currents are non-universal.

There are other responses.. Torsion, geometry of defects, etc.

How to compute them in a system of strongly interacting electrons?

Flux attachment!



INTRODUCTION II:
FLUX ATTACHMENT

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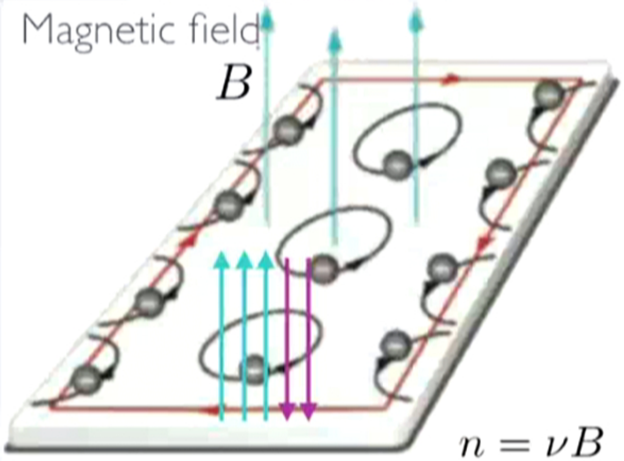
FLUX ATTACHMENT I: INFORMAL

Electrons form an FQH state with filling fraction $\nu = \dfrac{1}{2m+1} \sim \dfrac{n}{B}$



Magnetic field is too strong $\dfrac{n}{B} < 1$

Attach $2p$ fluxes of "statistical" field

Then total magnetic field $B_{tot} = \pm n$

- The composite fermions fill 1 Landau level and form an IQH state.
- **Free** composite fermions can be integrated out.
- *After that* statistical field is integrated out.

Interacting problem can be reduced to free fermions

FLUX ATTACHMENT II: FORMAL

Consider interacting electrons in a curved space

$$S[\psi, A, g] = \int d^2x dt \sqrt{g} \left[i\psi^\dagger D_0 \psi - \frac{g^{ij}}{2m} (D_i \psi)^\dagger D_j \psi \right]$$

$$+ \int d^2x d^2y \sqrt{g(x)} \sqrt{g(y)} \left[\rho(x) V(d(x,y)) \rho(y) \right]$$

Geodesic distance

"Flux attached" action

Statistical field

$$S' = S[\psi, A + a + p\omega, g] + \frac{1}{4\pi} \frac{1}{2p} \int \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

Spin connection (explain later)

Number of attached fluxes of a

Partition functions of S and S' coincide.

Lopez, Fradkin (1991)

Cho, You, Fradkin (2014)

FREE FERMIONS

FREE FERMION ACTION

$$S = \int d^2x dt \sqrt{g} \left[\frac{i}{2} \hbar \psi^\dagger \overleftrightarrow{D_0} \psi - \frac{\hbar^2}{2m} g^{ij} (D_i \psi)^\dagger D_j \psi + \frac{g_s B}{4m} \psi^\dagger \psi \right]$$

time dependent metric

$$D_\mu = \partial_\mu - \frac{i}{\hbar} A_\mu$$

gyromagnetic ratio

- Diffeomorphism invariant
- Weyl invariant
- (locally) Galilean invariant

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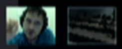
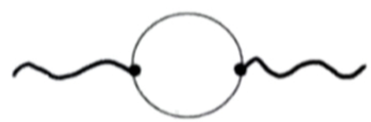


THE CALCULATION

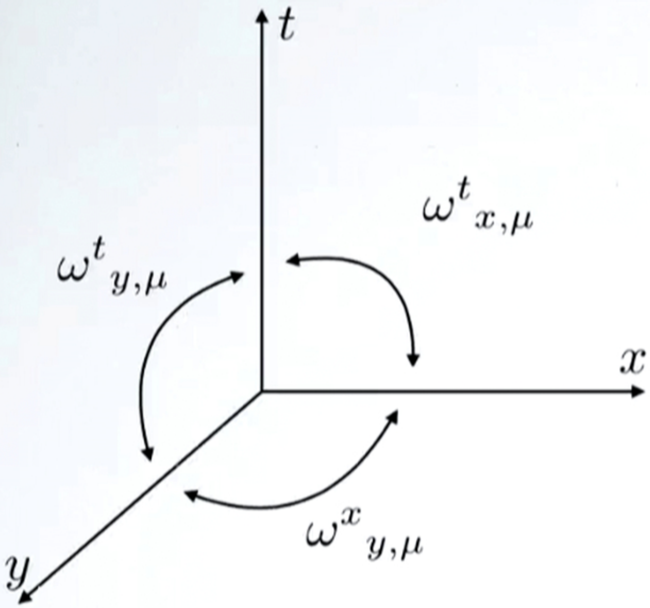
$$e^{\frac{i}{\hbar}S_{eff}[A_\mu, g_{ij}]} = \int D(g^{\frac{1}{4}}\psi)D(g^{\frac{1}{4}}\psi^\dagger)e^{\frac{i}{\hbar}S}$$

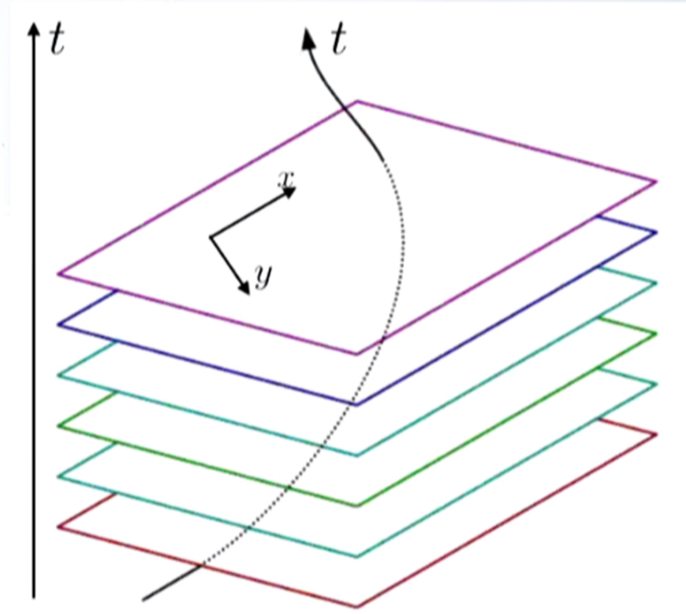
- Background: $A_\mu = \bar{A}_\mu + \delta A_\mu, \quad \partial_1 \bar{A}_2 - \partial_2 \bar{A}_1 = B = \frac{1}{l^2}$

  $g_{ij} = \delta_{ij} + \delta g_{ij}$
- No anomalies
- No divergencies at integer fillings

$S_{eff}[A_\mu, g_{ij}]$ =  +  + ...

SPIN CONNECTION

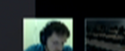




Spin connection gauges rotational symmetry

In non-relativistic physics the notion of time is absolute.

Non-relativistic systems do not couple to $\omega^{t}{}_{i,\mu}$. Only to $\omega^{x}{}_{y,\mu}$



ORBITAL SPIN AND SPIN CONNECTION

Electrons carry electric charge $\mathbf{e}$ $\quad \partial_\mu \to \partial_\mu - ieA_\mu$

Electrons also carry cyclotron spin $\bar{\boldsymbol{s}}$ $\quad eA_\mu \to eA_\mu + \bar{s}\omega_\mu$

The $SO(2)$ spin connection $\quad \omega_i \leftrightarrow -\frac{1}{2}\epsilon^{jk}\partial_j \delta g_{ik} \quad \omega_0 \leftrightarrow -\frac{1}{2}\epsilon^{jk}\delta g_{ij}\delta\dot{g}_{ik}$

Wen, Zee (1992)

$$\frac{\nu}{4\pi}AdA \to \frac{\nu}{4\pi}(A_\mu + \bar{s}\omega_\mu)d(A_\mu + \bar{s}\omega_\mu)$$

Despite the similarities there is no complete analogy between A_μ and ω_μ.

For free electrons there is an additional gravitational Chern-Simons term in the effective action.

EFFECTIVE ACTION FOR FREE FERMIONS

General structure:

Abanov, **AG** (2014)

$$S_{eff} = S^{(1)} + S^{(geom)} + S^{(2)} + \dots$$

$$S^{(geom)} = \int \frac{\nu}{4\pi} (A + \bar{s}\omega)\, d(A + \bar{s}\omega) - \frac{c}{48\pi} \omega d\omega$$

Filling fraction — Orbital spin — "Chiral central charge"

1	$\frac{1}{2}$	1
N	$\frac{1}{2} + \frac{3}{2} + \dots = \frac{N^2}{2}$	N
$\frac{1}{2p+1}$	$\frac{2p+1}{2}$	1

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WHAT DOES IT ALL MEAN?

Flat space

$$\frac{\nu}{4\pi} AdA \longrightarrow \quad n = \frac{\nu}{2\pi} B, \quad N = \nu N_\phi, \quad \sigma_H = \frac{\nu}{2\pi}, \quad \rho_{xx} = 0$$

Curved space

$$\boxed{\frac{\nu}{4\pi}(A + \bar{s}\omega) d (A + \bar{s}\omega) - \frac{c}{48\pi} \omega d\omega}$$

Shift

$$n = \frac{\nu}{2\pi} B + \frac{\nu \bar{s}}{4\pi} R \longrightarrow \quad N = \nu N_\phi + \nu \bar{s} \chi$$

$$\eta_H = \frac{\bar{s}}{2} n - \frac{c}{24} \frac{R}{4\pi}$$

← "Shift" in Hall viscosity

thermal Hall conductance

$$K_H = c \frac{\pi k_B^2 T}{6}$$

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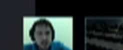

HIGHER ORDER TERMS

$$S^{(2)} = \frac{\nu}{4\pi}\int ml^2E^2 - \frac{\nu}{m}B^2 - \frac{3\nu}{2}l^2B(\partial_i E_i)$$

Static susceptibility

Magnetic susceptibility

gradient correction to σ_H

$$+\frac{2\nu^2-1}{4m}BR + \frac{2\nu^2-1}{6}l^2R(\partial_i E_i) + \frac{\nu(\nu^2-1)}{8m}R^2 + \ldots$$

Curvature acts on particles with orbital spin **almost** like magnetic field on particles with charge

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SUMMARY SO FAR

For free fermions filling 1 Landau level common sense suggests

$$S_{eff} = \int \frac{\nu}{4\pi}(A + \bar{s}\omega)d(A + \bar{s}\omega)$$

Direct computation of the fermionic determinant gives

$$S_{eff} = \int \frac{\nu}{4\pi}(A + \bar{s}\omega)d(A + \bar{s}\omega) - \frac{c}{48\pi}\omega d\omega$$

Is the a general mechanism that produces the last term ?

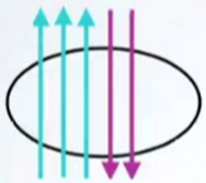
Yes! Framing anomaly.

FQH STATES
AND
FRAMING ANOMALY

INCONSISTENCY OF FLUX ATTACHMENT

Start with the Laughlin state at filling $\nu = \frac{1}{3}$ $n = \frac{1}{3}B$

There are two ways to reduce this state to IQHE via flux attachment



Attach 2 fluxes

$n = B_{tot}$

OR

Attach 4 fluxes

$n = -B_{tot}$

Integrate out composite fermions filling 1 Landau level

$$S_{eff} = \int \pm \frac{1}{4\pi}\left(A + a + \frac{5}{2}\omega\right) d\left(A + a + \frac{5}{2}\omega\right) \mp \frac{1}{48\pi}\omega d\omega - \frac{1}{2p}\frac{1}{4\pi} ada$$

Integrate out statistical gauge field

$$S_{eff} = \int \frac{1}{4\pi}\frac{1}{2m+1}\left(A + \frac{2m+1}{2}\omega\right) d\left(A + \frac{2m+1}{2}\omega\right) \pm \frac{1}{48\pi}\omega d\omega$$

Cho, You, Fradkin (2014)

Witten (1989)

FRAMING ANOMALY

Consider a $U(1)$ Chern-Simons theory on level k

$$S = \frac{k}{4\pi}\int ada$$

Compute the partition function

$$Z = \int Da\ e^{iS}$$

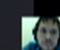
Partition function is not well-defined until gauge fixing term is added

$$S_{gauge} = \int \sqrt{g}\phi D^{\mu}a_{\mu}$$

Well-defined partition function explicitly depends on metric and gives

$$\tilde{Z} = \int DaD\phi e^{iS+iS_{gauge}} = \tau e^{-i\frac{k}{|k|}\frac{1}{96\pi}\int \omega d\omega + \frac{2}{3}\omega^3}$$

This is the **framing anomaly:** while the classical Chern-Simons theory is topological, the quantum theory depends on metric (or framing) of the manifold through the gauge fixing term.



FIXING LAUGHLIN EFFECTIVE ACTION

Before integrating out the composite fermions

$$S_0[\Psi, A + a + p\omega] - \int \frac{2p}{4\pi} bdb - \int \frac{1}{2\pi} adb$$

Integration over the fermions and statistical fields generates

$$S_{eff} = \int \frac{1}{4\pi} \frac{1}{2m+1} \left(A + \frac{2m+1}{2}\omega \right) d \left(A + \frac{2m+1}{2}\omega \right) \pm \frac{1}{48\pi} \omega d\omega$$

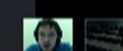
While the framing anomaly generates

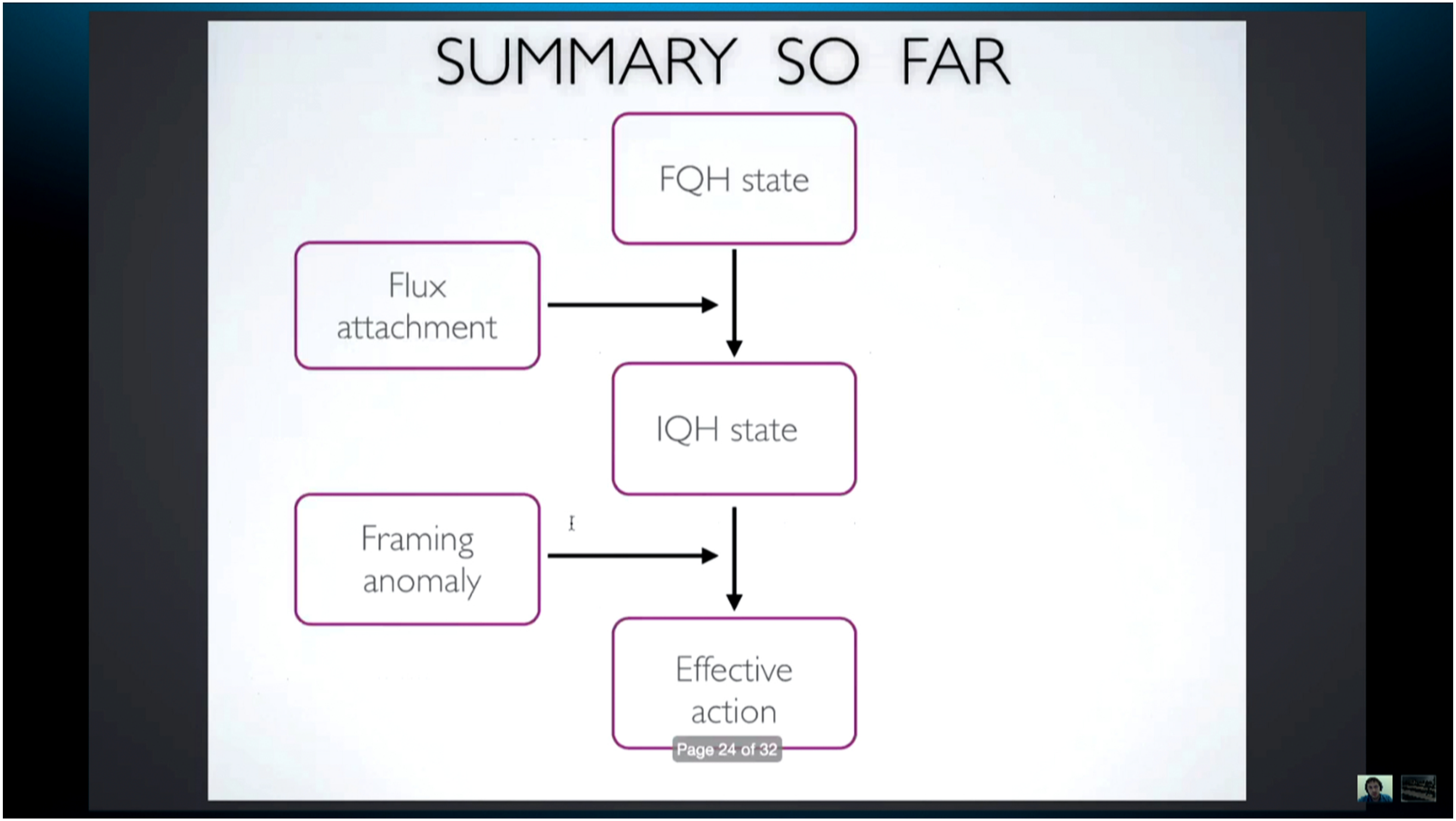
$$S_{anom} = \frac{sgn(K_{\mp})}{48\pi} \int \omega d\omega \quad \text{where} \quad K_{\mp} = \begin{pmatrix} \mp N & -1 \\ -1 & -2p \end{pmatrix}$$

Framing anomaly fixes the problem!

$S_{eff} + S_{anom}$ does not depend on the choice of the flux attachment

and gives $c = 1$




SUMMARY SO FAR
FQH state
Flux attachment
IQH state
Framing anomaly
Effective action
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FIXING WEN-ZEE APPROACH

Starting point is topological effective action fixed by a K-matrix.

$$S = \frac{1}{4\pi}\int K_{IJ} a^I da^J + 2t_I A da^I + 2s_I \omega da^I$$

t_I is charge vector s_I is spin vector

AG, Cho, You, Abanov, Fradkin (2014)

Integrating over statistical fields gives

$$S_{eff} = \frac{1}{4\pi}\int K^{-1}_{IJ}(t_I A + s_I\omega)d(t_J A + s_J \omega) - \frac{\text{sgn}(K)}{48\pi}\omega d\omega$$

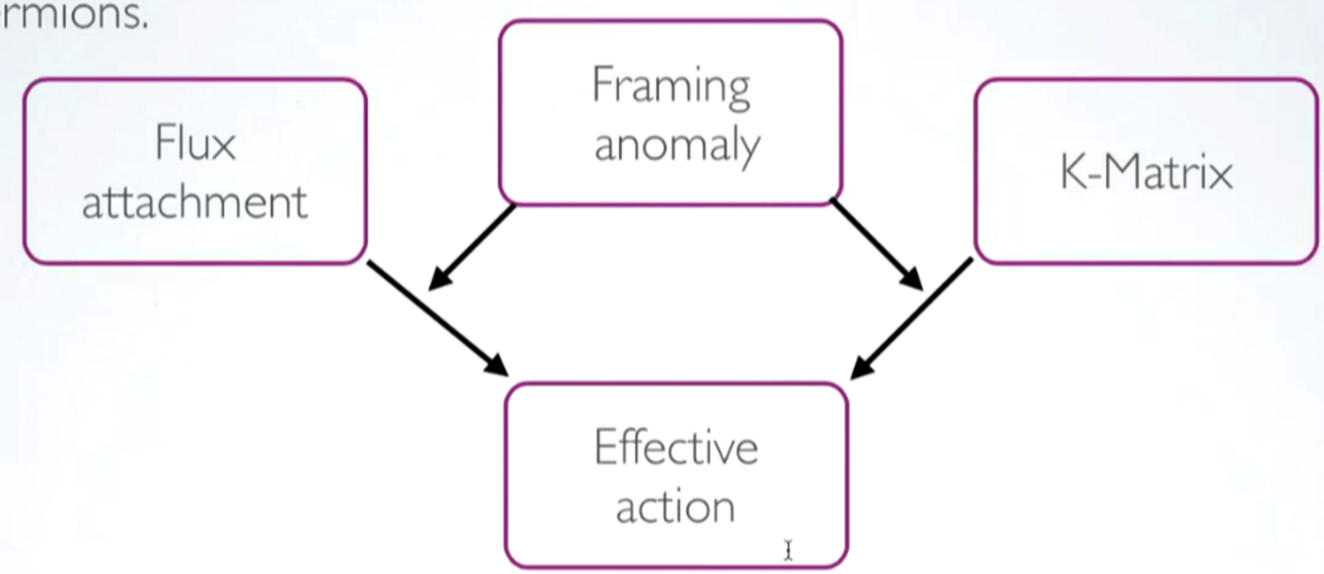
For free fermions filling 1 Landau level

$t = 1$ $s = \frac{1}{2}$ $K = \frac{1}{\nu}$ reproduces the correct effective action

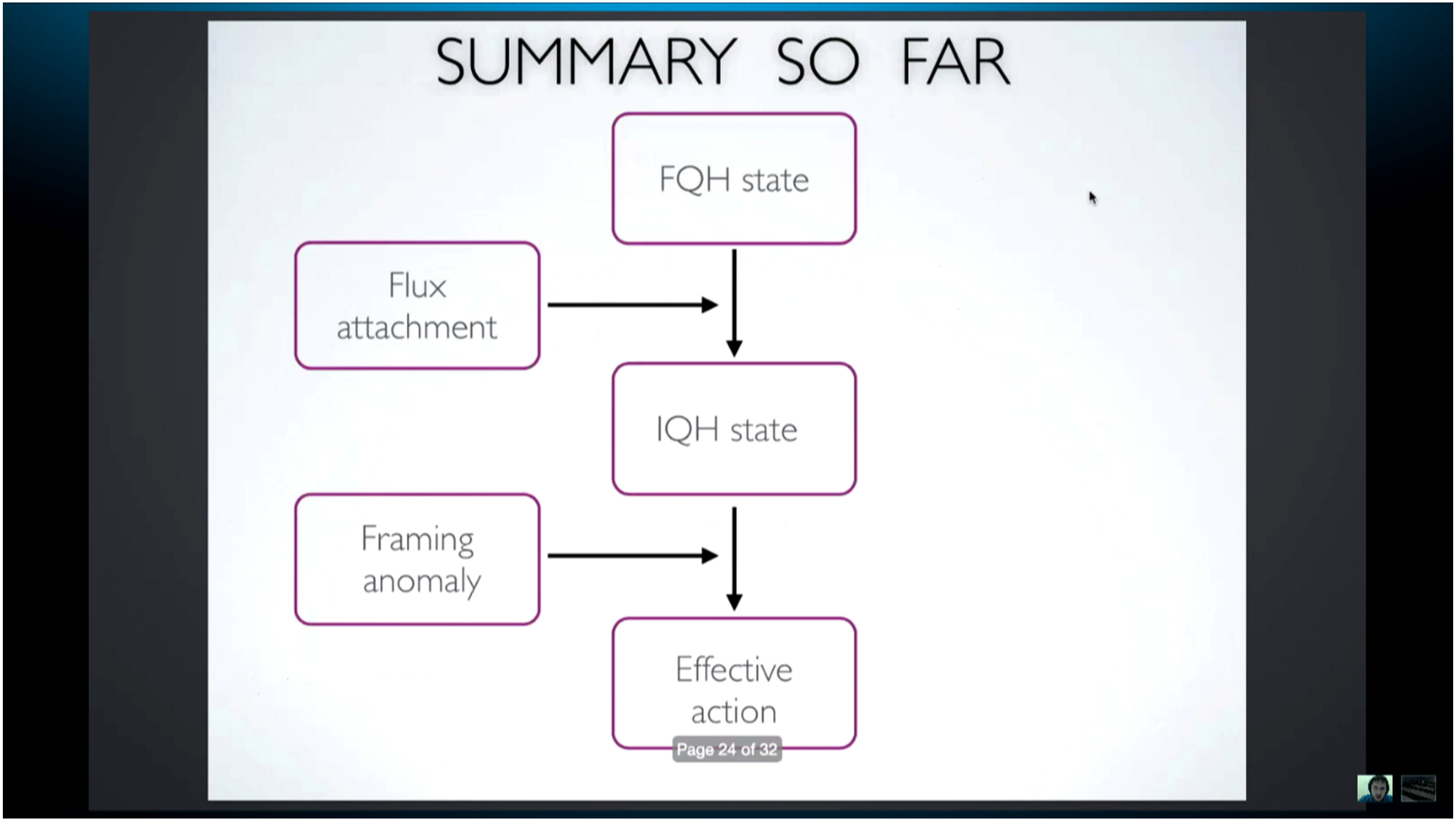
In fact, we have computed the framing anomaly using free fermions as a natural regularization of quantum Chern-Simons theory.

JAIN STATES

Jain states are obtained by filling N Landau levels with composite fermions.



$$c = N\,, \quad \text{for} \quad \nu = \frac{N}{2pN+1} \qquad\qquad c = -(N-2), \quad \text{for} \quad \nu = \frac{N}{2pN-1}$$


SUMMARY SO FAR
FQH state
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READ-REZAYI STATES

Are characterized by $\nu = \frac{k}{Mk+2}$ and $c = \frac{3k}{k+2}$ $\bar{s} = \frac{M+2}{2}$

Low energy effective edge theory is $U(1)^{2k+M}/(U(M) \times Sp(2k))_1$ Barkeshli, Wen (2010)

Bulk effective theory is $U(1)^{2k+M}/(U(M) \times Sp(2k))_1$ Chern-Simons

$$\frac{1}{4\pi}\int \mathrm{Tr}\left[ada + \frac{2}{3}a^3\right] - \frac{1}{4\pi}\int \mathrm{Tr}\Big[bdb + 2(QA + S\omega)db\Big]$$ Moore, Seiberg (1989)

$Q = \frac{1}{kM+2}\mathrm{diag}\left(1_{2k}, k \times 1_M\right)$ Charge matrix $S = \frac{1}{2}1_{2k+M}$ Spin matrix

Charged $U(1)$ sector interacts with external fields.

Non-abelian neutral sector contributes *only* to chiral central charge via the framing anomaly.

THERMAL TRANSPORT AND GEOMETRY

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CONCLUSIONS

- Framing anomaly contributes to observable quantities such as Hall viscosity and thermal Hall conductance.
- Framing anomaly is *necessary* to perform self-consistent flux attachment.
- Framing anomaly is *necessary* to ensure that K-matrix effective action is consistent with the flux attachment.
- Effective action for a topological phase "knows" about the geometry of the manifold through the framing anomaly.
- Effective action for non-abelian FQH states receives a contribution from neutral sector in the form of gCS term with *fractional* coefficient.
- ...