Title: Testing the membrane paradigm with holography

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Abstract: <p>One version of the membrane paradigm states that as far as outside observers are concerned, black holes can be replaced by a dissipative membrane with simple physical properties located at the stretched horizon. We demonstrate that such a membrane paradigm is incomplete in several aspects. We argue that it generically fails to capture the massive quasinormal modes, unless we replace the stretched horizon by the exact event horizon, and illustrate this with a scalar field in a BTZ black hole background. We also consider as a concrete example linearized metric perturbations of a five-dimensional AdS-Schwarzschild black brane and show that a spurious excitation appears in the long-wavelength response that is only removed from the spectrum when the membrane paradigm is replaced by ingoing boundary conditions at the event horizon. We interpret this excitation in terms of an additional Goldstone boson that appears due to symmetry breaking by the classical solution ending on the stretched horizon rather than the event horizon.

Based on arXiv:1405.4243</p>

Testing the membrane paradigm with holography

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Perimeter Institute for Theoretical Physics

1405.4243 [hep-th] with Jan de Boer and Natalia Pinzani-Fokeeva


Introduction

Motivation

The black hole membrane paradigm

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properties of a black hole* can be <u>mimicked</u> from the point of view of an outside observer by a <u>simple dissipative membrane</u> situated <u>at the stretched horizon</u>.

In particular, astrophysical black holes* behave as conductors with resistivity of 377 Ω.

The membrane paradigm faced a recent revival of interest in due to holography:

- fluid-gravity duality Son et al., Buchel & J. Liu, Iqbal & H. Liu, Minwalla et al.

- renormalization group-based understanding of holography Faulkner et al., Son and Nickel

- "holographic" description of non-extremal near-horizon geometries Strominger et al., Skenderis et al., Rangamani et al.

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1/14

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The membrane paradigm - modern formulation

Parikh & Wilczek gr-qc/9712077 Iqbal & Liu 0809.3808

Consider a probe Maxwell field in a generic non-extremal black hole spacetime.

$$S = -\frac{1}{4}\int du\, d^d x \sqrt{-g}\, F_{ab}F^{ab}$$

Focusing on black hole's exterior the variational principle is not well defined unless

$$S = S + \delta S_{surf} \text{ with } \delta S_{surf} = -\int d^d x \sqrt{-h} \left(\frac{\sqrt{-g}}{\sqrt{-h}} F^{u\nu}\right) A_\nu \text{ from the stretched horizon}$$

$$\left(\frac{\sqrt{-g}}{\sqrt{-h}} F^{u\nu}\right) = j^{\nu}_{surf}$$

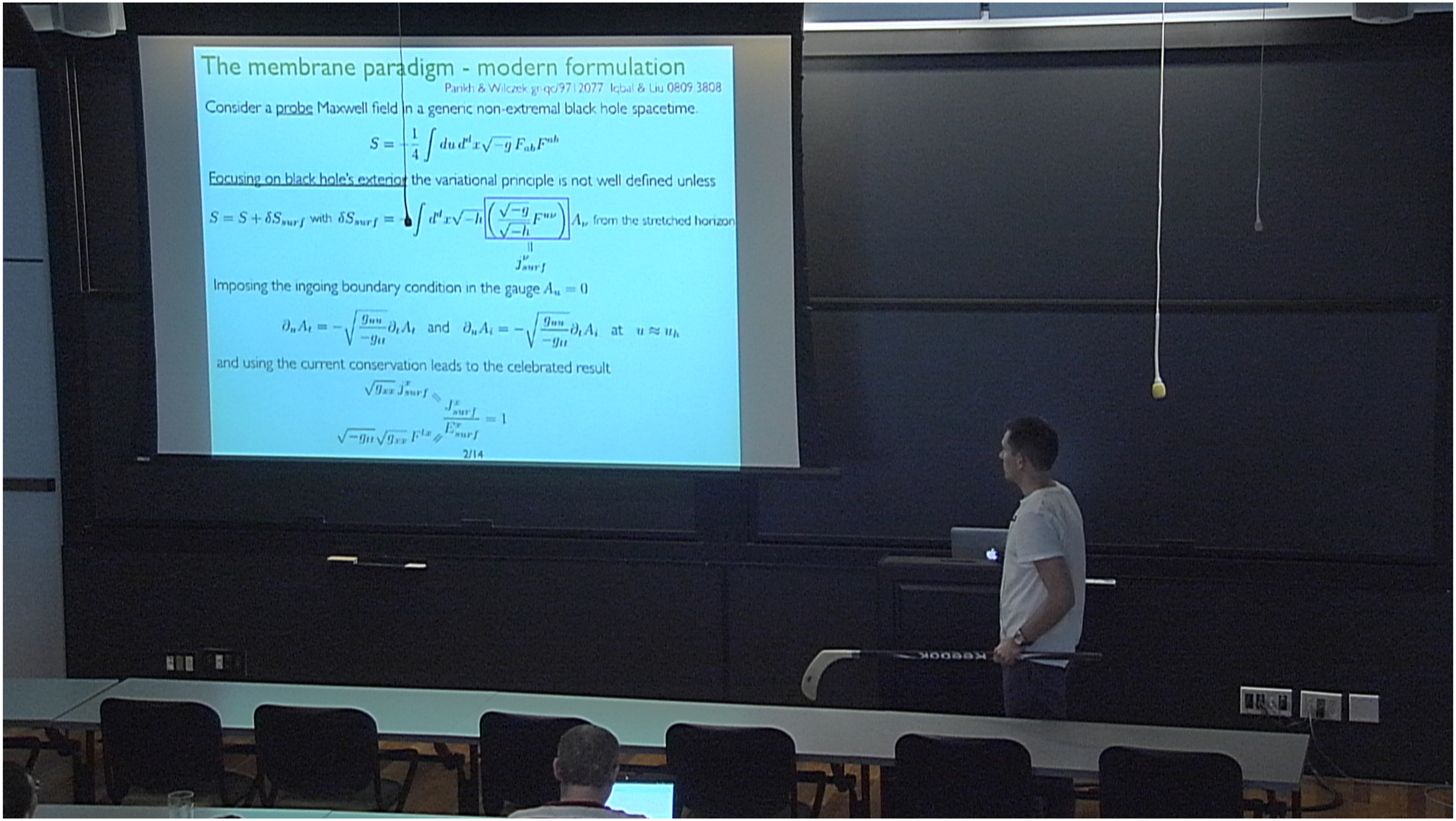
Imposing the ingoing boundary condition in the gauge $A_u = 0$

$$\partial_u A_t = -\sqrt{\frac{g_{uu}}{-g_{tt}}}\partial_t A_t \quad \text{and} \quad \partial_u A_i = -\sqrt{\frac{g_{uu}}{-g_{tt}}}\partial_t A_i \quad \text{at} \quad u \approx u_h$$

and using the current conservation leads to the celebrated result

$$\sqrt{g_{xx}}\, j^x_{surf} = J^x_{surf}, \qquad \sqrt{-g_{tt}}\sqrt{g_{xx}}\, F^{tx} = E^x_{surf}$$

$$\frac{J^x_{surf}}{E^x_{surf}} = 1$$



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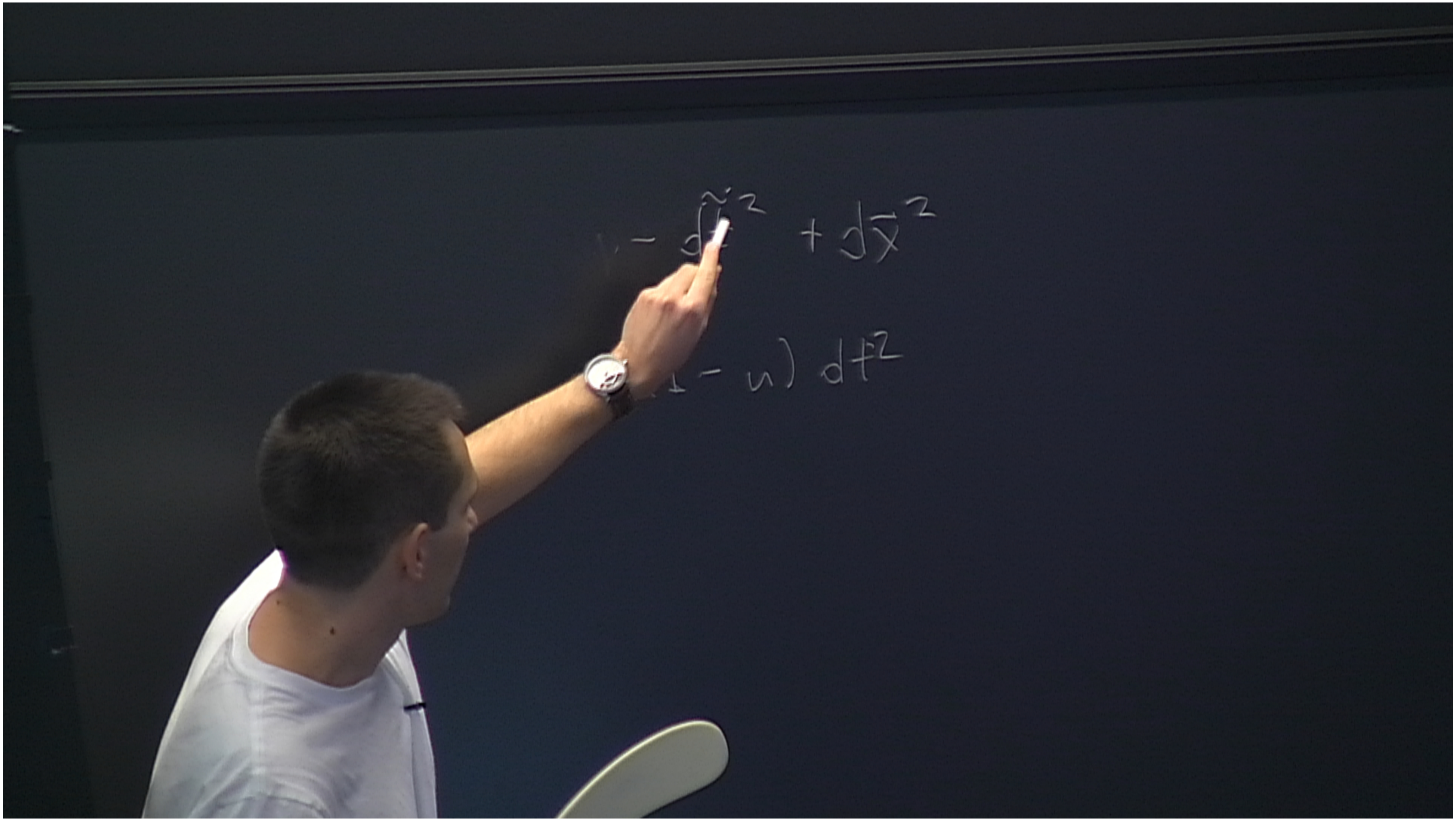

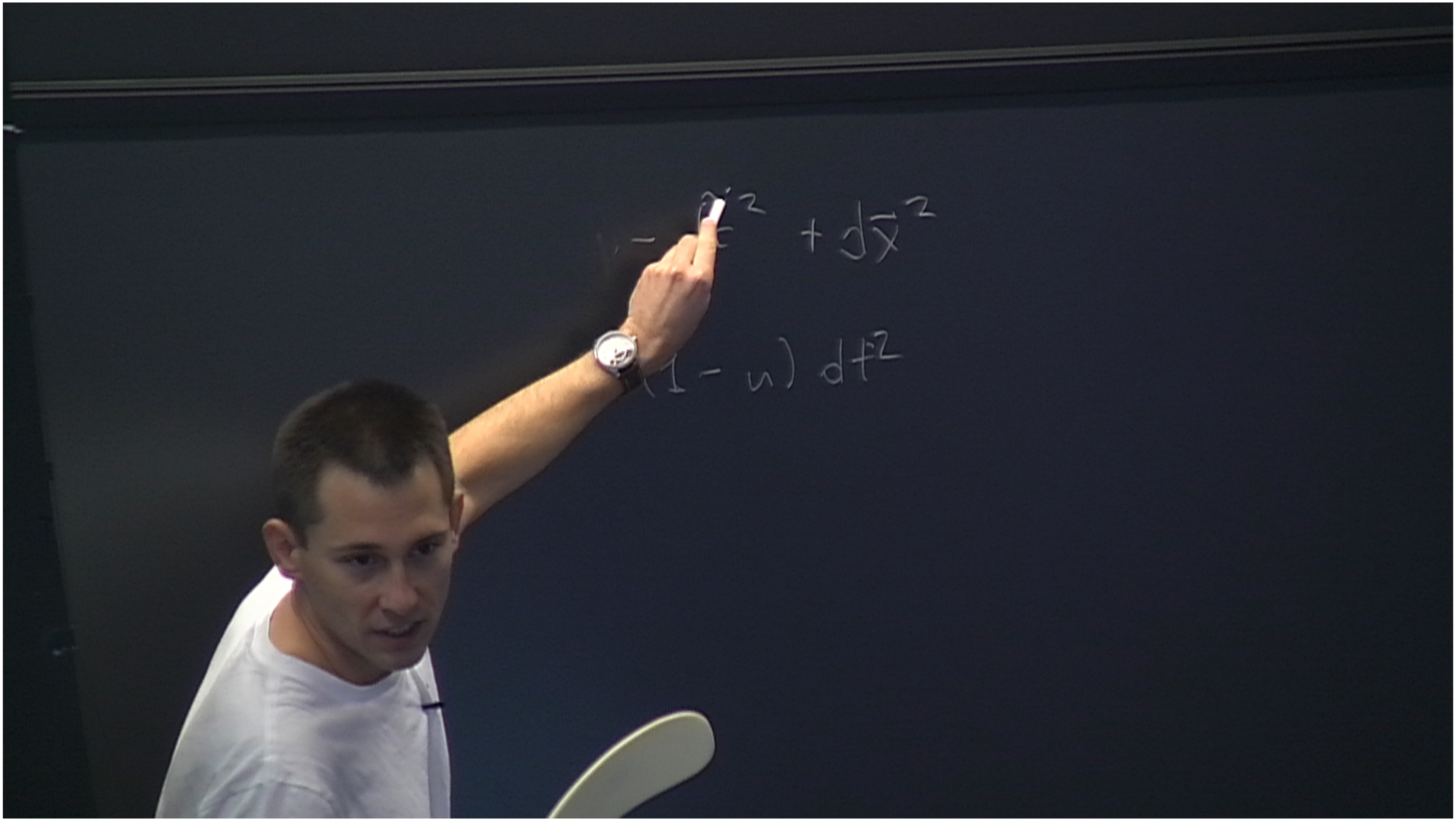
$+ d\vec{x}^2$
$(1 - u)\, dt^2$

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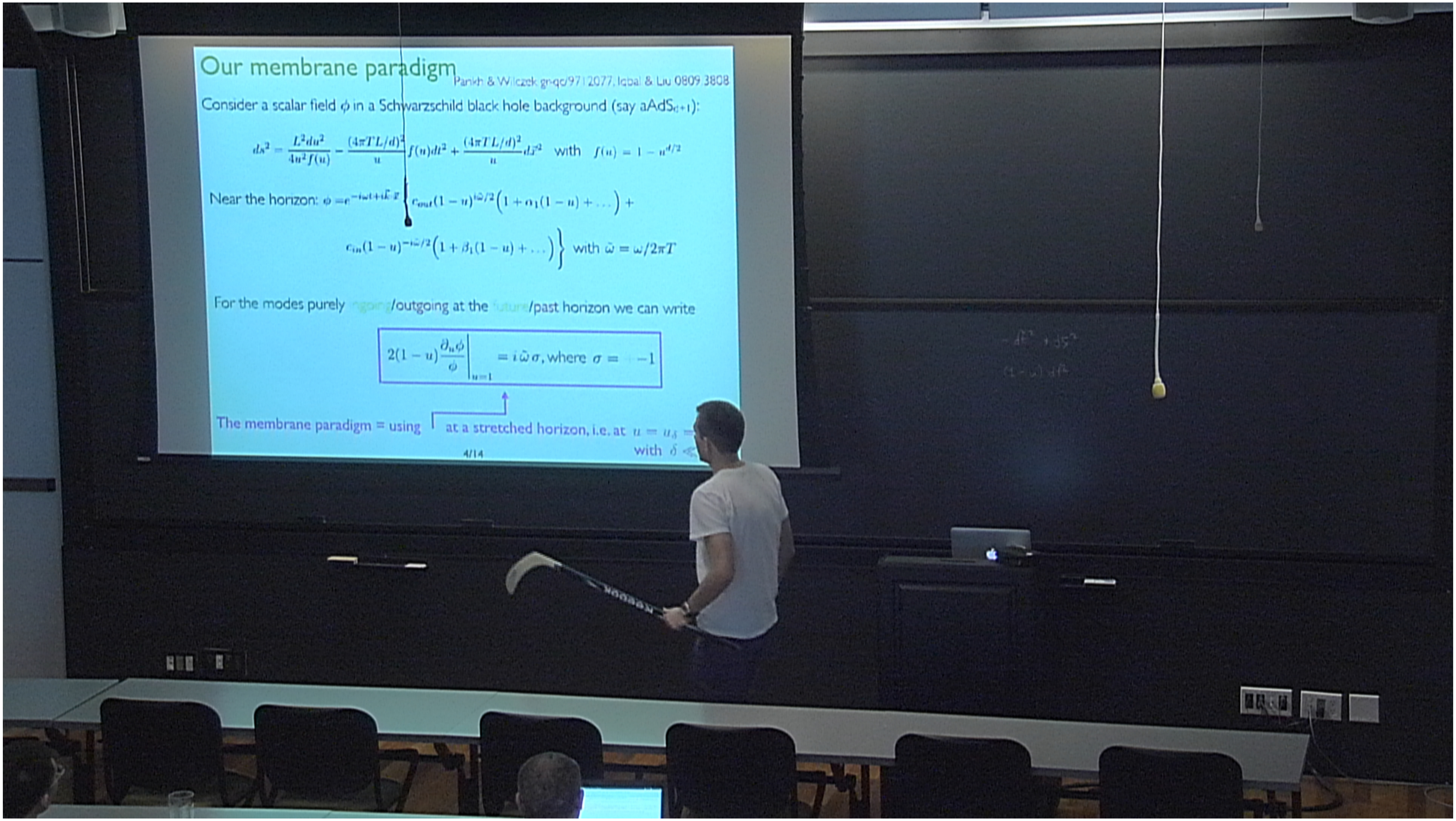
2/14

Goal

I want to use ideas from holography to understand to what extent, and in what sense this membrane paradigm is an accurate statement:

- Does it correctly reproduce correlation functions at infinity?
- Does the membrane live at a stretched or the event horizon?


Our membrane paradigm
Parikh & Wilczek gr-qc/9712077, Iqbal & Liu 0809.3808
Consider a scalar field ϕ in a Schwarzschild black hole background (say aAdS$_{d+1}$):
$ds^2 = \frac{L^2 du^2}{4u^2 f(u)} - \frac{(4\pi TL/d)^2}{u} f(u) dt^2 + \frac{(4\pi TL/d)^2}{u} d\vec{x}^2$ with $f(u) = 1 - u^{d/2}$
Near the horizon: $\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}} \{ c_{out}(1-u)^{i\tilde{\omega}/2} (1 + \alpha_1(1-u) + \dots) +$
$c_{in}(1-u)^{-i\tilde{\omega}/2} (1 + \beta_1(1-u) + \dots) \}$ with $\tilde{\omega} = \omega/2\pi T$
For the modes purely ingoing/outgoing at the future/past horizon we can write
$2(1-u)\frac{\partial_u \phi}{\phi}\Big|_{u=1} = i\tilde{\omega}\sigma$, where $\sigma = +/-1$
The membrane paradigm = using at a stretched horizon, i.e. at $u = u_\delta$
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4/14

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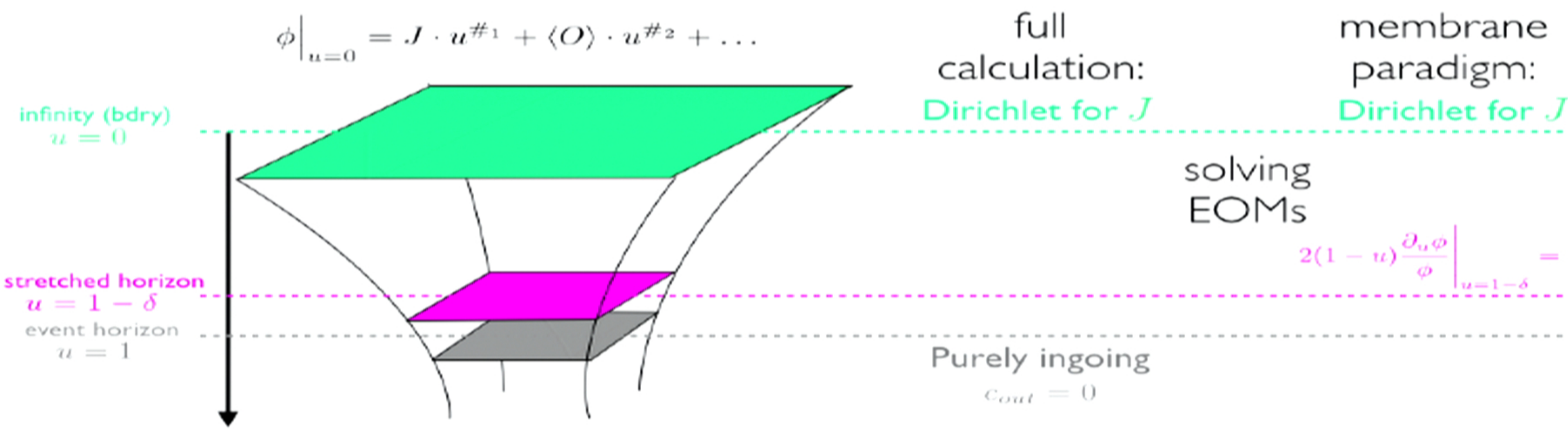
Testing the membrane paradigm with holography

In holography, invariant information resides in correlation functions of a dual theory.

The retarded two-point function is computed by imposing the ingoing bdry condition at the future horizon and taking a ratio of the expectation value $\langle O \rangle$ to a source J.

Son & Starinets hep-th/0205051

Testing the membrane paradigm: imposing $2(1-u)\frac{\partial_u \phi}{\phi}\Big|_{u=1-\delta} = i\tilde{\omega}$ **instead of** $c_{out} = 0$.



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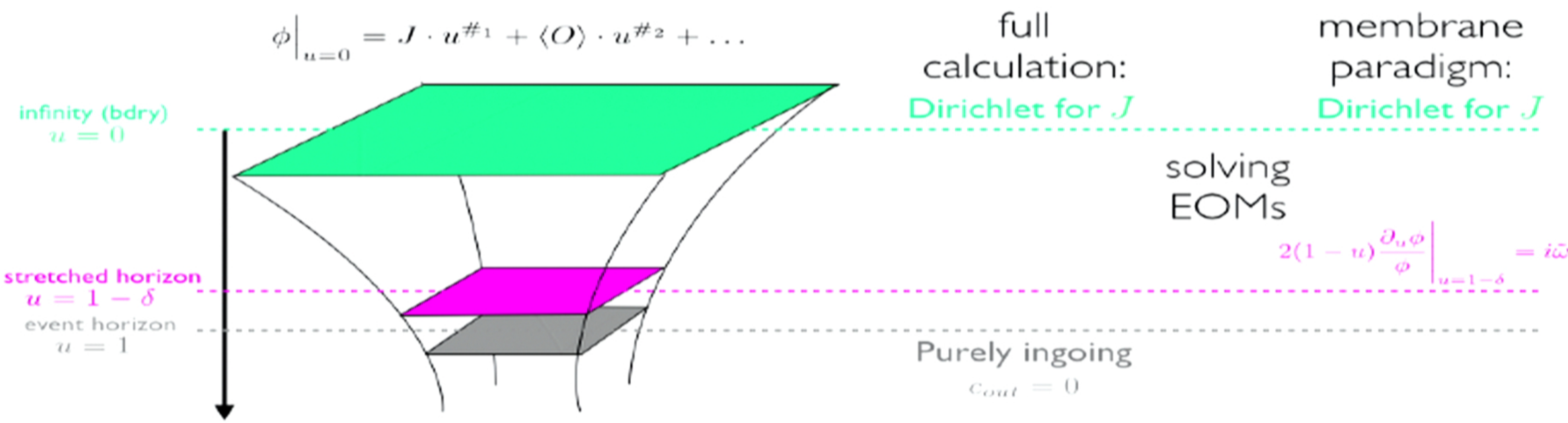

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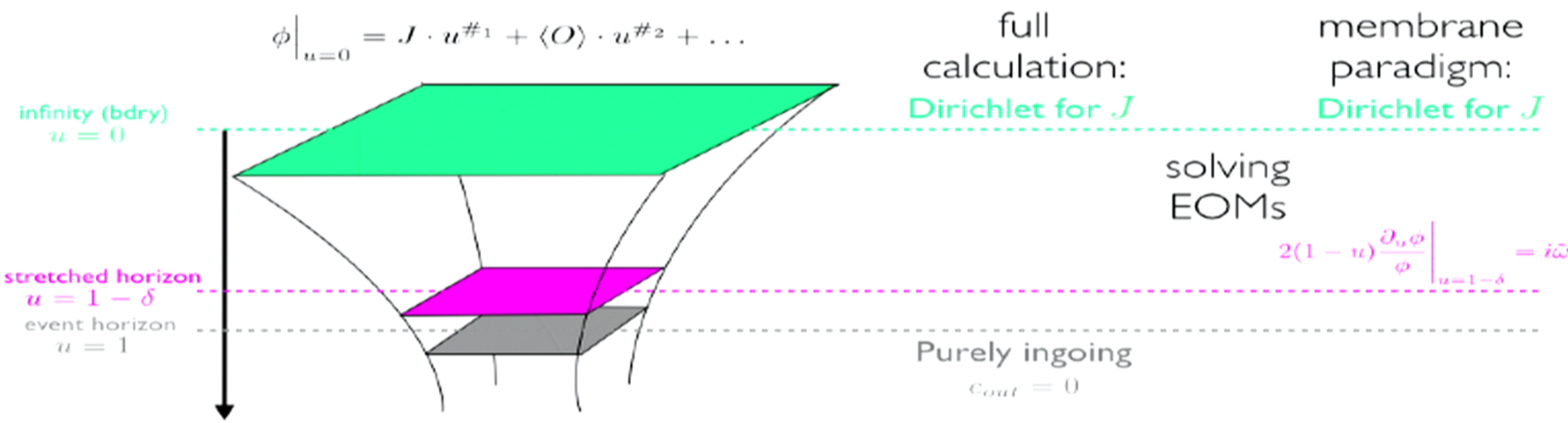

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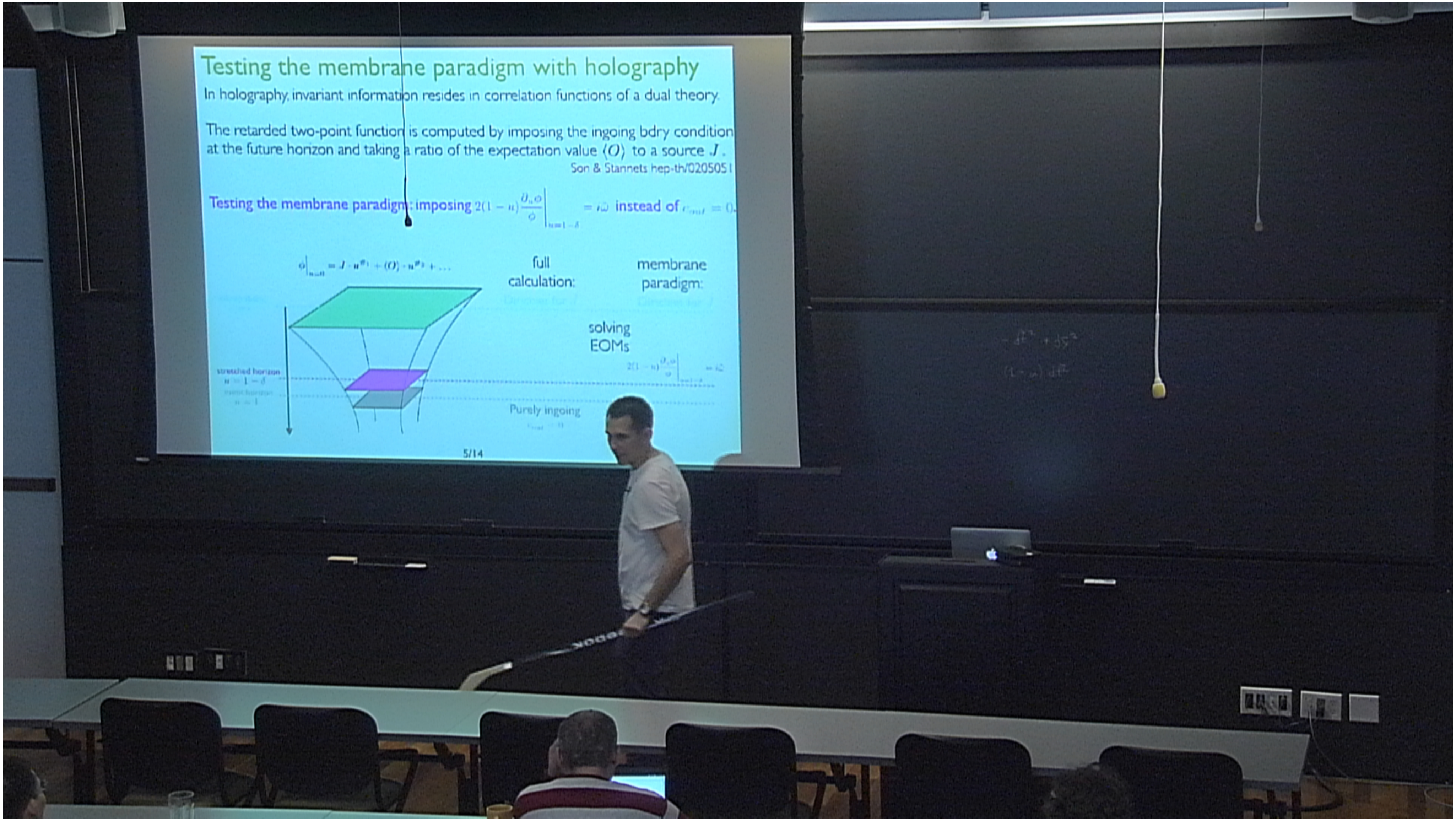
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full calculation:
membrane paradigm:
solving EOMs
stretched horizon
Purely ingoing
5/14

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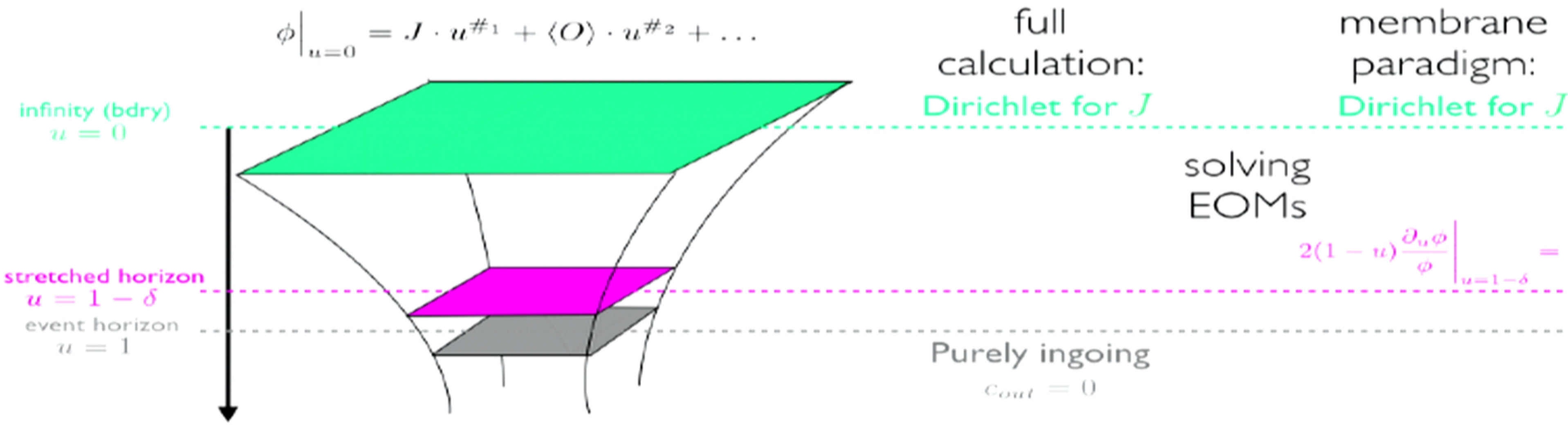

TEST 1:
the membrane paradigm and quasinormal modes

The near-horizon surprise

Let's do the simplest thing possible:

Take $\begin{aligned}\phi =& e^{-i\omega t+i\vec{k}\cdot\vec{x}}\Big\{c_{out}(1-u)^{i\tilde{\omega}/2}\Big(1+\alpha_1(1-u)+\dots\Big)+\\ & c_{in}(1-u)^{-i\tilde{\omega}/2}\Big(1+\beta_1(1-u)+\dots\Big)\Big\}\end{aligned}$, plug into $2(1-u)\frac{\partial_u\phi}{\phi}\Big|_{u=1-\delta}=i\tilde{\omega}$

and solve for $\frac{c_{out}}{c_{in}}$!

The answer in the LO in the near-horizon expansion reads

$$c_{out}/c_{in} = (1-u_\delta)^{1-i\tilde{\omega}} \times \frac{i\beta_1}{\tilde{\omega}}$$

For $\Im(\tilde{\omega}) > -1$ imposing the membrane paradigm leads to $|c_{out}/c_{in}| \ll 1$ (OKAY!)

However, for $\Im(\tilde{\omega}) < -1$ it goes completely wrong: $|c_{out}/c_{in}| \gg 1$!


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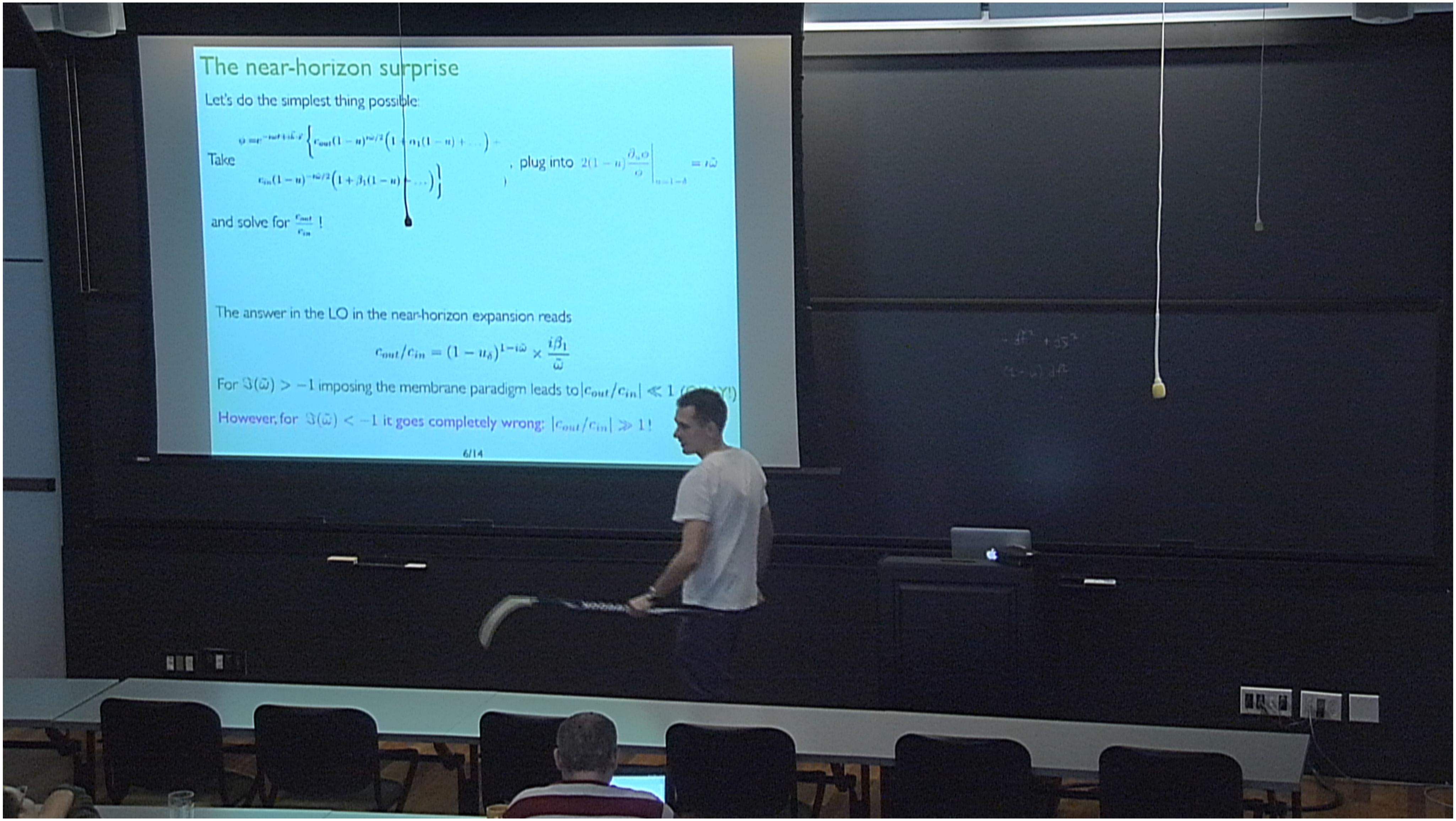
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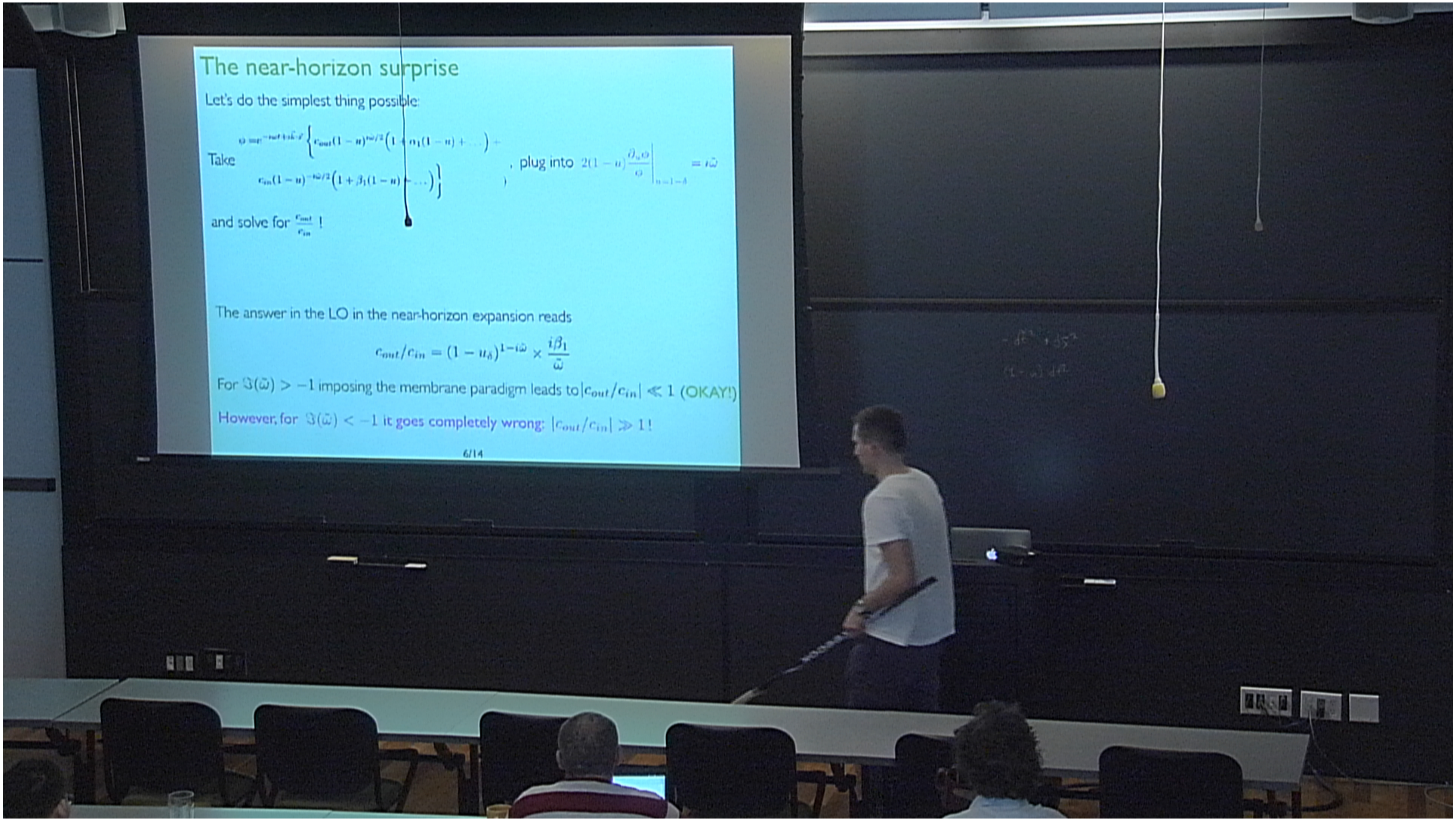
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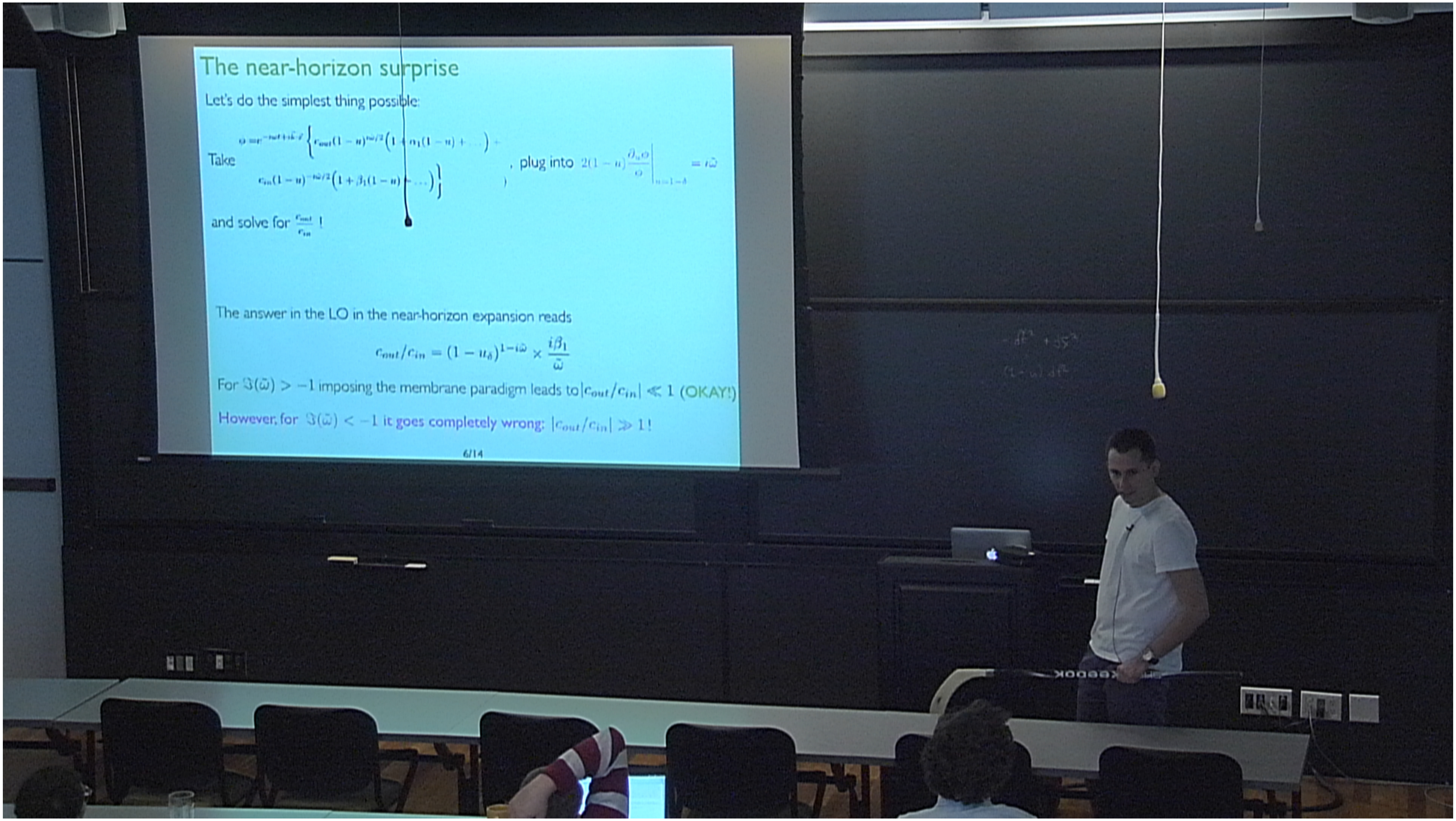
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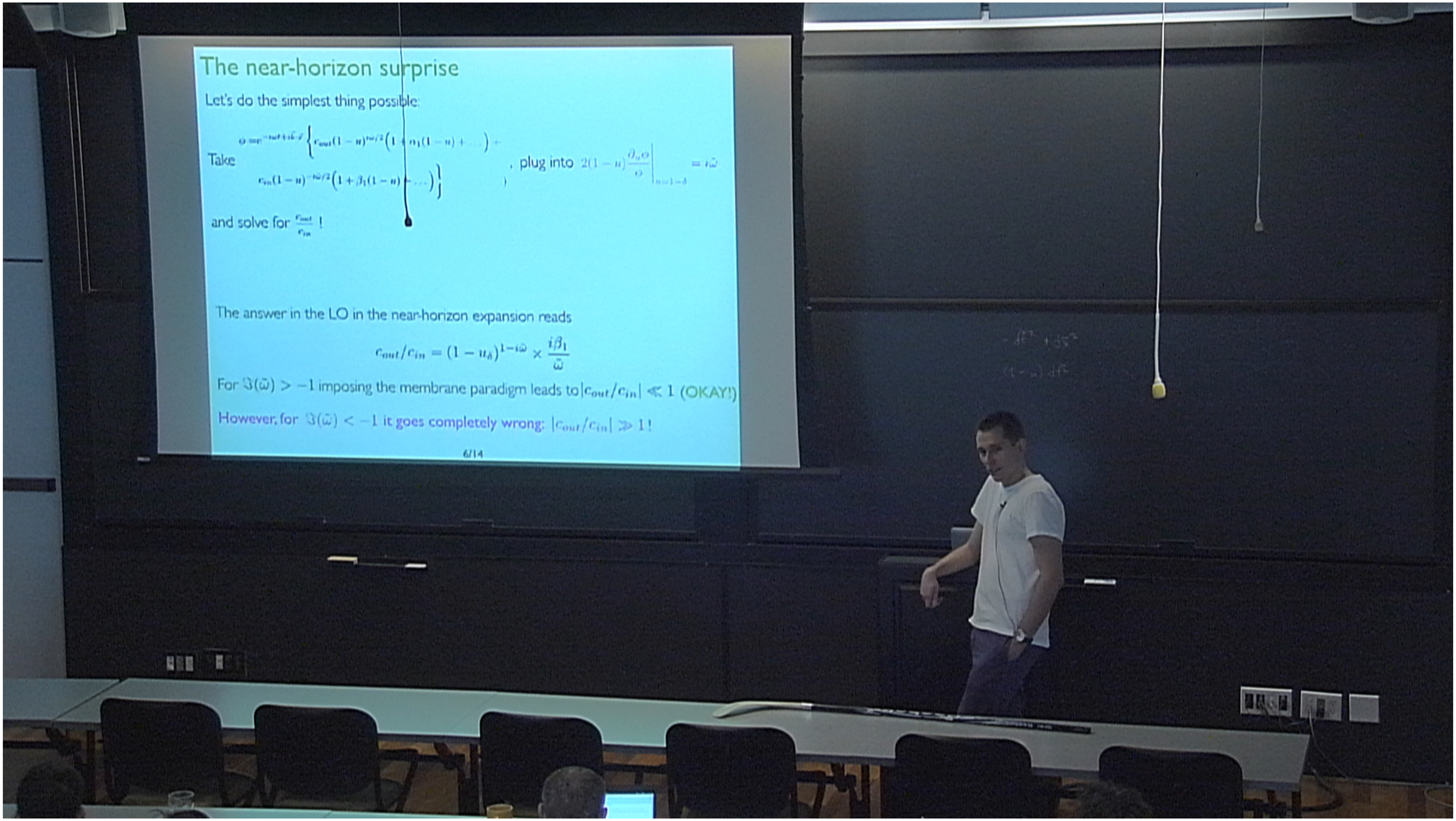
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6/14

The near-horizon surprise

Let's do the simplest thing possible:

Take $\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}} \Big\{ c_{out}(1-u)^{i\tilde{\omega}/2}\Big(1+\alpha_1(1-u)+\ldots\Big) + c_{in}(1-u)^{-i\tilde{\omega}/2}\Big(1+\beta_1(1-u)+\ldots\Big)\Big\}$, plug into $2(1-u)\frac{\partial_u \phi}{\phi}\Big|_{u=1-\delta} = i\tilde{\omega}$

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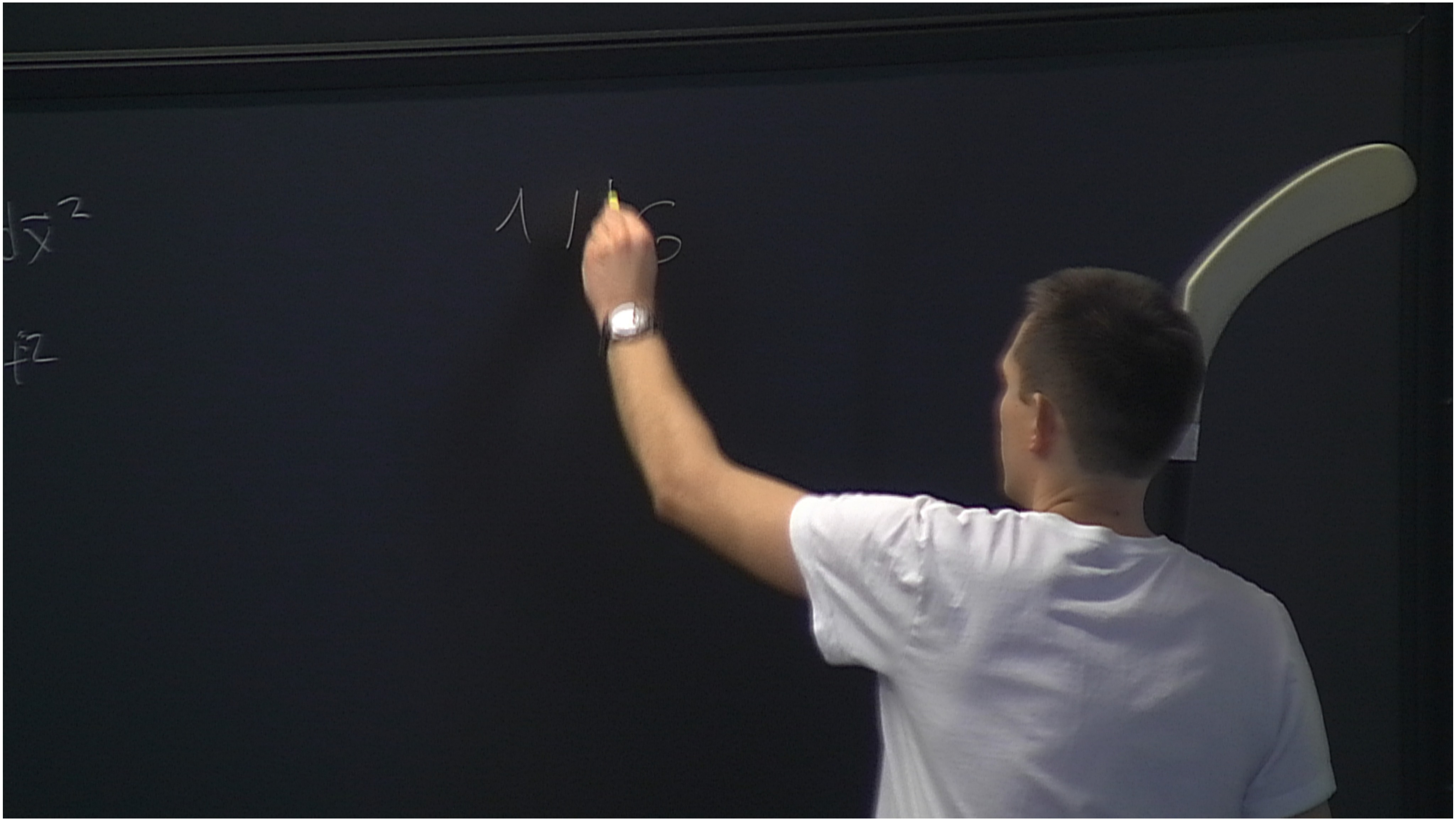
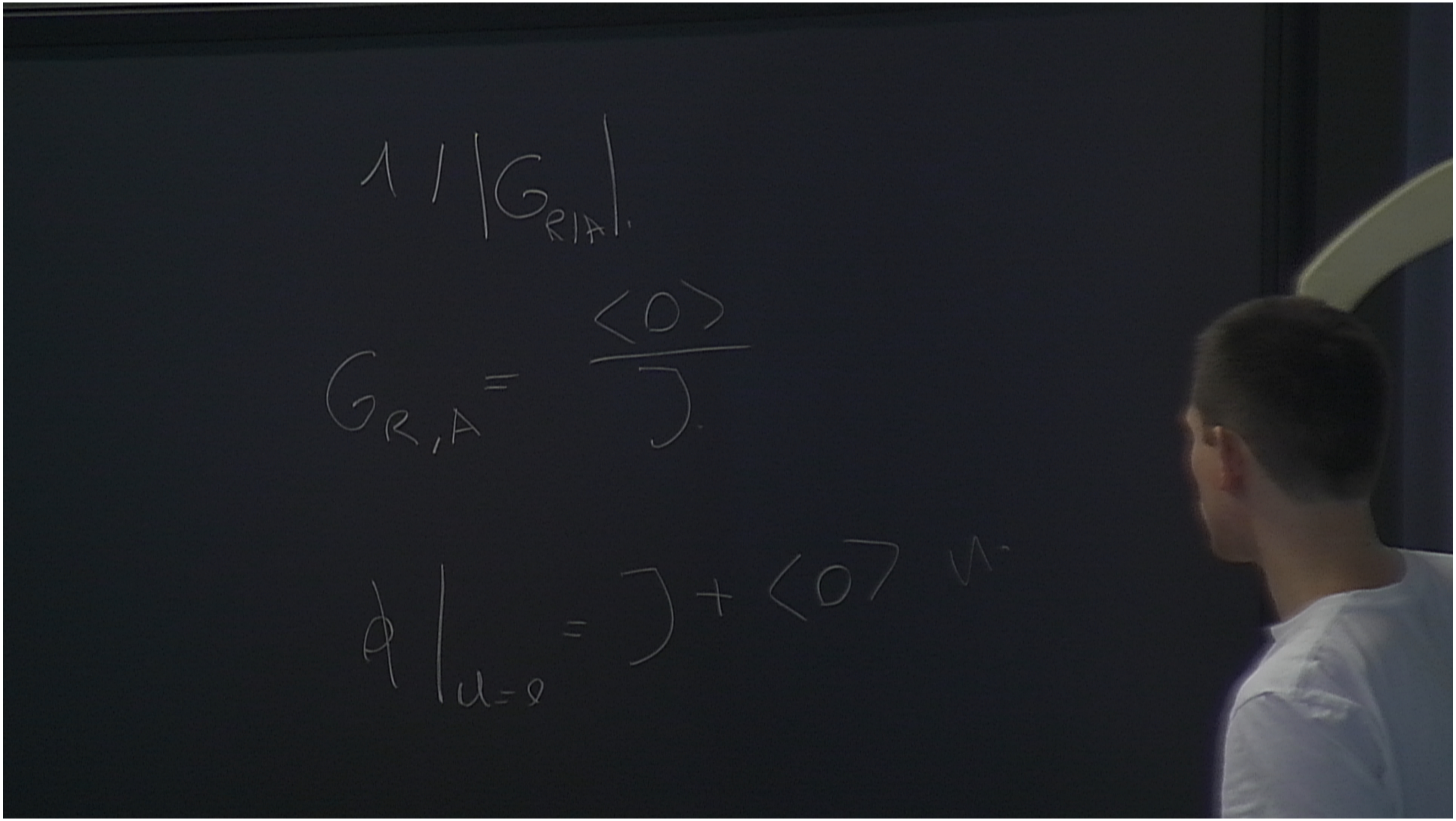
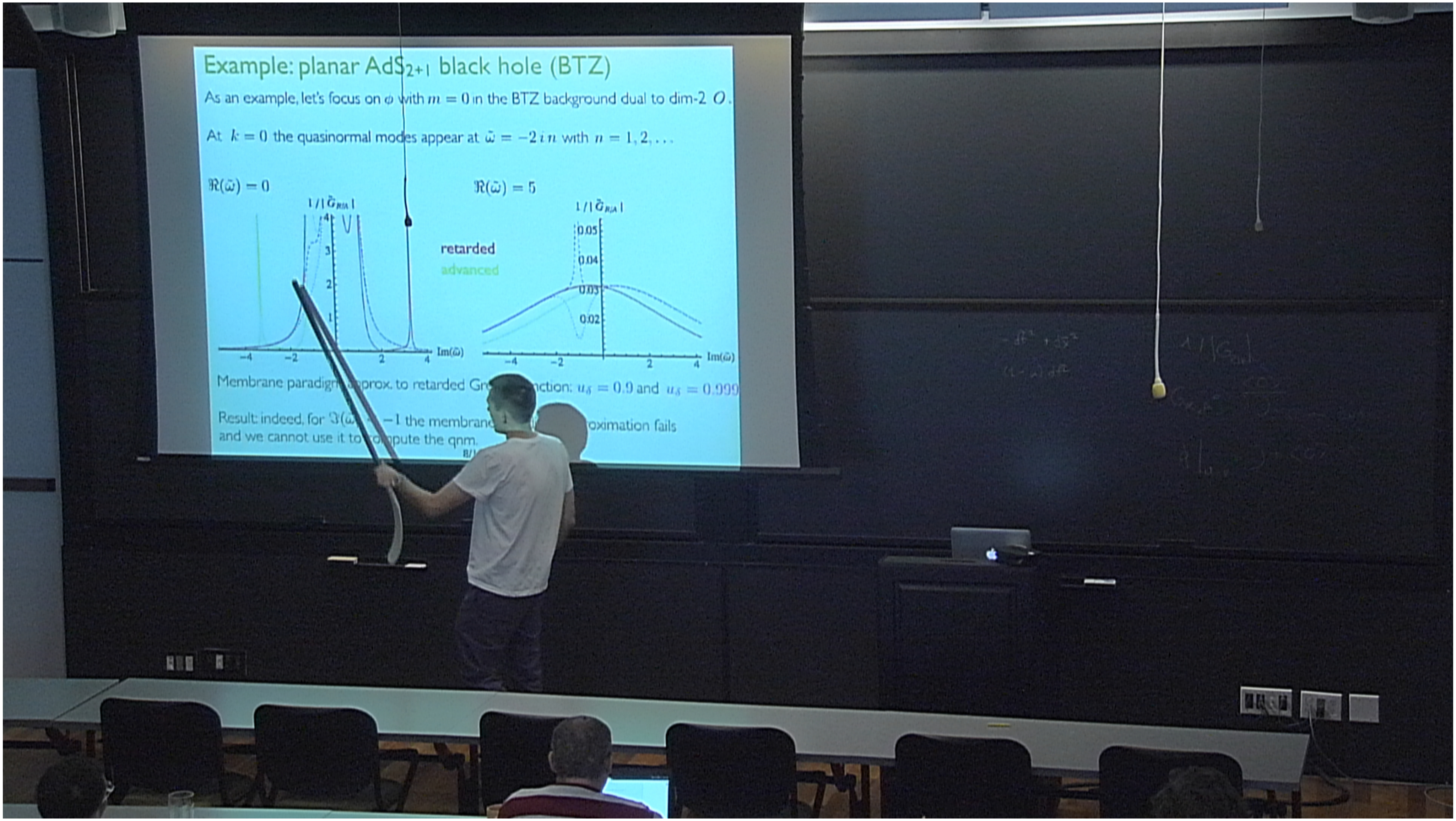



1 / |G_{R|A}|.
G_{R,A} = <O> / J
φ|_{u=0} = J + <O> u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At k = 0 the quasinormal modes appear at ω̃ = −2 i n with n = 1, 2, ...
ℜ(ω̃) = 0
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Membrane paradigm approx. to retarded Green function: u_δ = 0.9 and u_δ = 0.999
Result: indeed, for ℑ(ω̃) < −1 the membrane paradigm approximation fails
and we cannot use it to compute the qnm.

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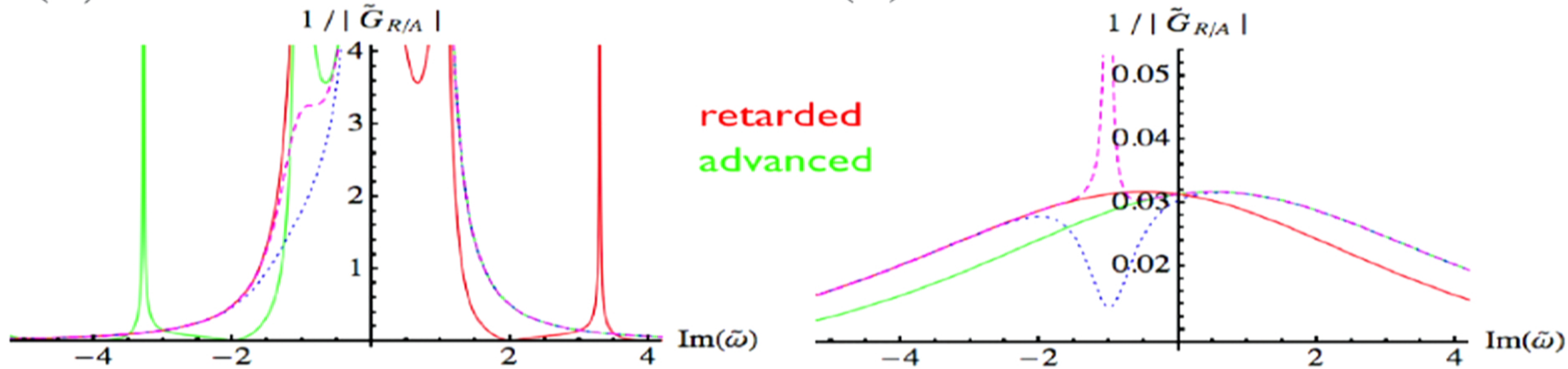

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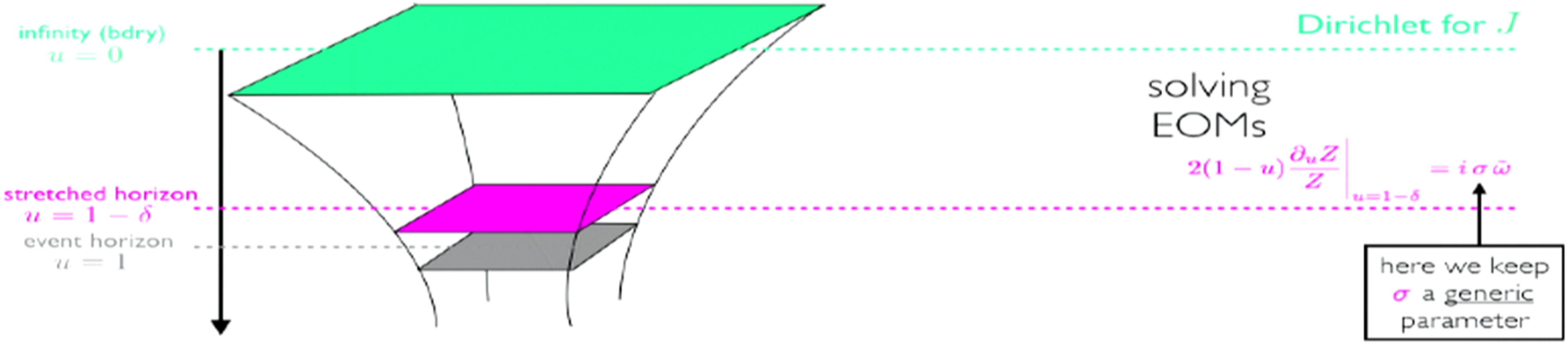
Sound waves and the membrane paradigm

Playing the same game as before, i.e.

$$Z\Big|_{u=0} = J + \langle O \rangle \cdot u^2 + \ldots$$



we find two long-wavelength quasinormal modes*

$$\tilde{\omega}_1\Big|_{\delta\ll 1} \approx \pm\sqrt{\frac{1}{3}}\tilde{k} + \mathcal{O}(\tilde{k}^2) \quad \text{and} \quad \tilde{\omega}_2\Big|_{\delta\ll 1} \approx \pm\sqrt{\frac{2}{3}}\tilde{k} + \mathcal{O}(\tilde{k}^2)$$

$\tilde{\omega}_1$ is the approximation of the sound wave (OKAY!). What's up with $\tilde{\omega}_2$?

A clue: we kept σ arbitrary and the result does not depend on σ .

Sound waves and the membrane paradigm

The fact that we kept σ generic and the result does not depend on σ suggests that the unphysical mode is a property of spacetime ending on the stretched horizon.

If true, we could have hence set, say, Dirichlet bdry condition at u_δ and get the same!*

Let's examine this possibility backtracking a little. We have 4 variables

$$\delta h_{tt}, \;\; \delta h_{tx}, \;\; \delta h_{xx} \;\; \text{and} \;\; \delta h_{aa} = \frac{1}{2}(\delta h_{yy} + \delta h_{zz})$$

and 7 independent equations ($G_{ab} = R_{ab} - \frac{1}{2} R g_{ab} - \frac{6}{L^2} g_{ab}$):

$$G_{tt} = G_{tx} = G_{xx} = G_{yy} = 0 \;\; \text{and} \;\; G_{tu} = G_{xu} = G_{uu} = 0$$

Additional 3 equations $G_{au} = 0$ come from $\frac{\delta S_{EH+CC}}{\delta(\delta h_{au})}$. Let's not set δh_{au} to 0 and examine now perturbations satisfying Dirichlet bdry conditions at $u = 0$ and $u = u_\delta$.

11/14

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Solving the equations in the long-wavelength limit

We can solve $G_{tt} = G_{tx} = G_{xx} = G_{yy} = 0$ for $\delta h_{tt},\ \delta h_{tx},\ \delta h_{xx}$ and δh_{aa}

and the resulting expression is going to depend on δh_{au} through (schematically!)

$$\psi_a \sim \int_0^{u_\delta} du\, \delta h_{au}$$

Evaluating the constraints ($G_{tu} = G_{xu} = G_{uu} = 0$) leads to

$$(\tilde{\omega}^2 - \frac{1}{3}\tilde{k}^2)\psi_x = 0 \quad \text{and} \quad (\tilde{\omega}^2 - \frac{2}{3}\tilde{k}^2)\psi_t = 0 \quad \text{when} \quad 0 \neq \delta \ll 1$$

and

$$(\tilde{\omega}^2 - \frac{1}{3}\tilde{k}^2)\psi_x = 0 \quad \text{when} \quad \delta = 0$$

Conclusion: the additional unphysical mode indeed disappears (as it should) when the stretched horizon is taken to coincide with the event horizon.

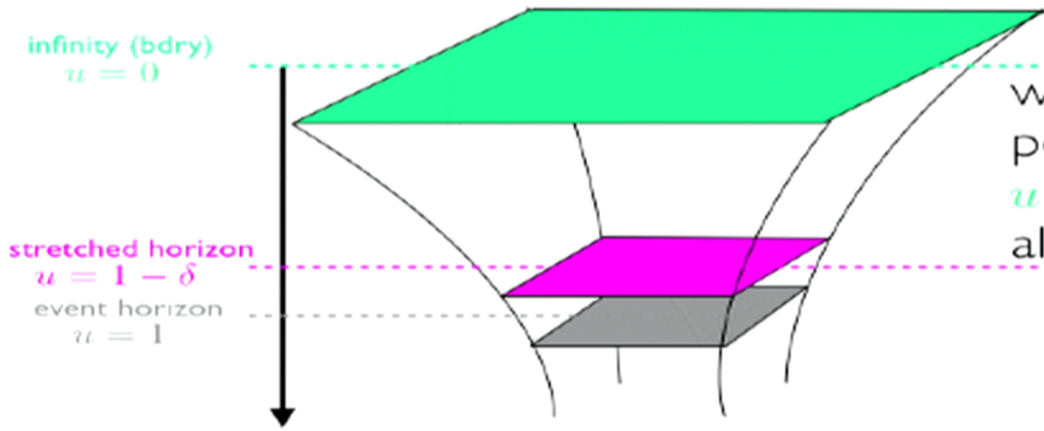
Interpretation

Long-wavelength gapless modes can appear as a consequence of SSB (e.g. pions).

Are hence ψ_t and ψ_x the Goldstone bosons? Yes!

Nickel & Son 1009.3094

A supporting indication is the following:



we cannot use a diffeomorphism to perform 2 independent translations at $u = 0$ and $u = 1 - \delta$ without altering the solution!

We suspect that on the stretched horizon the SB pattern is to diagonal Poincare and it changes when the stretched horizon becomes null (when ψ_t decouples!).

Summary and conclusions

I used ideas from holography to understand to what extent, and in what sense the following membrane paradigm is an accurate statement:

$$2(1-u)\frac{\partial_u Z}{Z}\Bigg|_{u=1-\delta} = i\,\tilde{\omega}$$

Q: Does it correctly reproduce all correlation functions at infinity?

A: It fails for complex frequencies in a way that does not allow to obtain the spectrum of the quasinormal modes!

Q: Does the membrane live at a stretched or the event horizon?

A: Appearance of an additional Goldstone boson on the stretched horizon suggests the membrane is actually the event horizon.

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