Title: Higher Rank Wilson Loops in AdS/CFT: Beyond the Leading Order - Leopoldo A. Pando Zayas

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Abstract:

Introduction

- Wilson Loops (WL) are an important class of gauge invariant non-local operators.

$$W_R(C) = \mathrm{Tr}_R P \exp\left(i \int_C A\right)$$

- The expectation value measures the effective action of an external particle; order parameter for confinement.
- The Lüscher term as quantum corrections to the QCD string (linear potential) $\mapsto$ Quantum Corrections in $\mathcal{N} = 4$.

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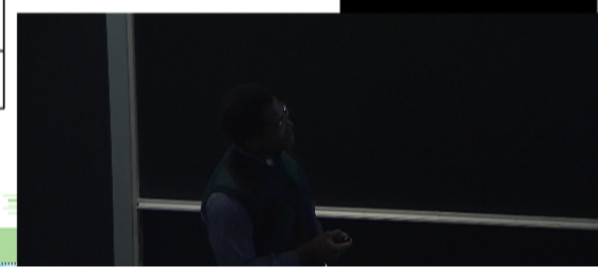
- Identification of the $\mathcal{N} = 4$ WL in fundamental of $SU(N)$ and a string in $AdS_5 \times S^5$ was done early (Maldacena, hep-th/9803002; Rey & Yee, hep-th/9803001).
- Drukker & Fiol (hep-th/0501109) recognized that the D3 captured the leading behavior of multiply wrapped WL.
- The WL in the antisymmetric representation of $SU(N)$ is described by a D5 brane (Yamaguchi, hep-th/0603208).
- A complete dictionary was proposed by Gomis & Passerini (hep-th/0604007) for all half-BPS WL in arbitrary representations.

Configuration	Representation of $SU(N)$
F1	Fundamental
D3	Symmetric
D5	Antisymmetric



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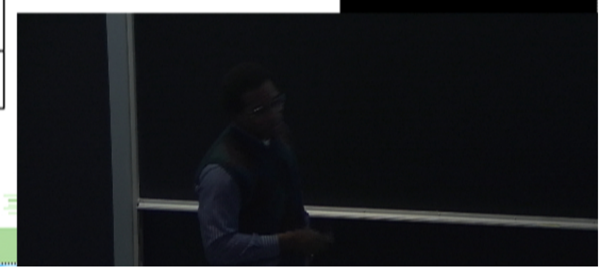
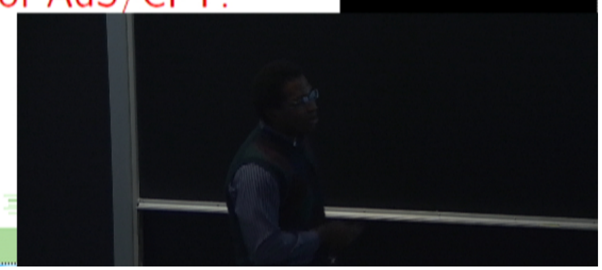


- According to dictionary the WL in the k-symmetric representation of $SU(N)$ is dual to a single D3 brane with k units of fundamental string charge.
- In the probe approximation $k \sim N$ the D3 has $AdS_2 \times S^2$ geometry with k units of flux (Drukker & Fiol, hep-th/0501109).
- Localization: in $\mathcal{N} = 4$ SYM the half-BPS circular WL is captured by a Gaussian Matrix Model (Drukker, Gross; Erickson, Semenoff, Zarembo; Pestun).
- A semi-classical analysis on the D-brane side:
  - We find functional agreement in the higher representations.
  - We find numerical discrepancies.
  - Exact results on the gauge theory side; precision tests of AdS/CFT.



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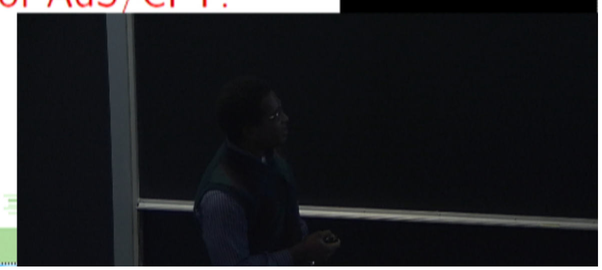

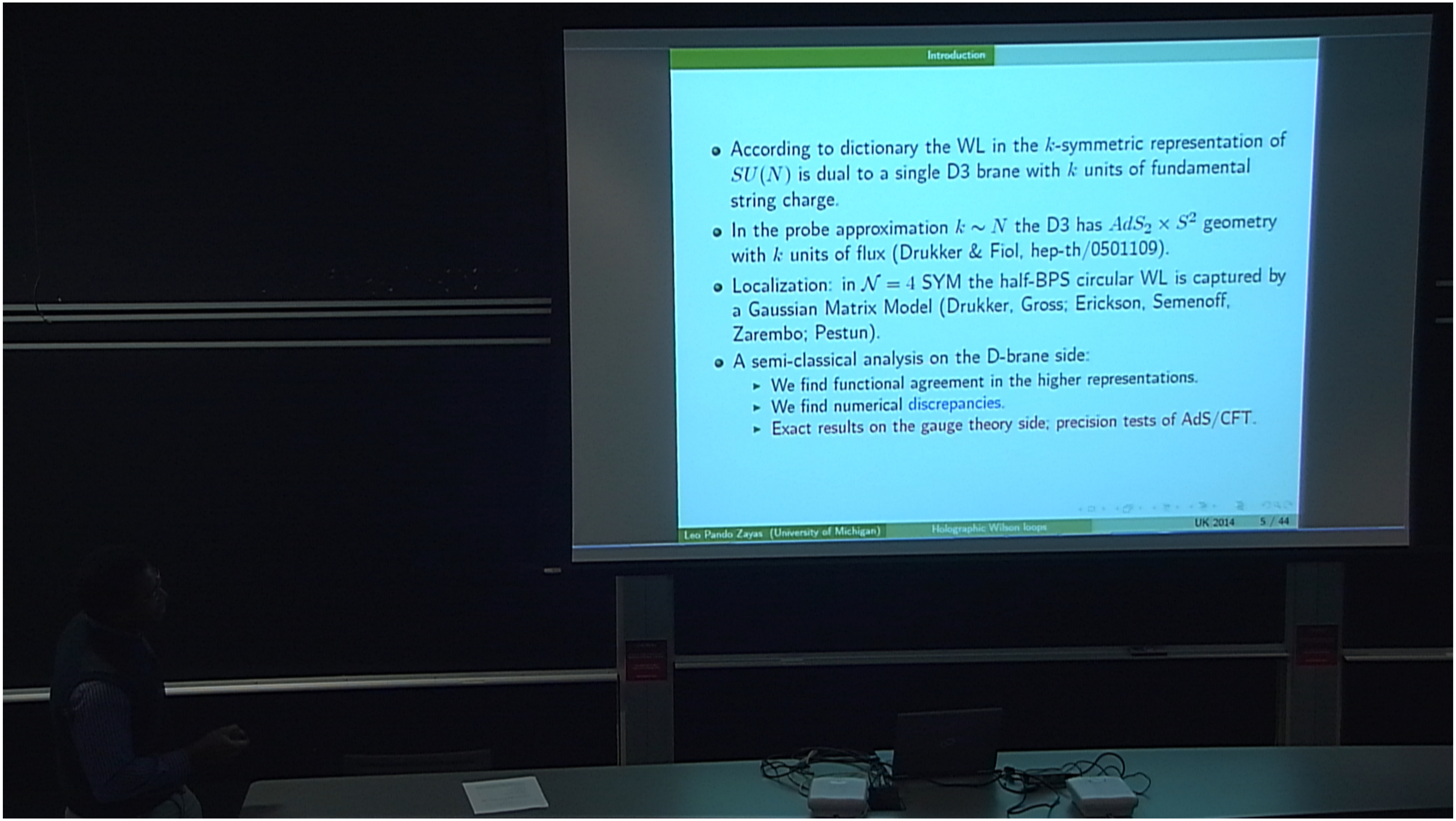
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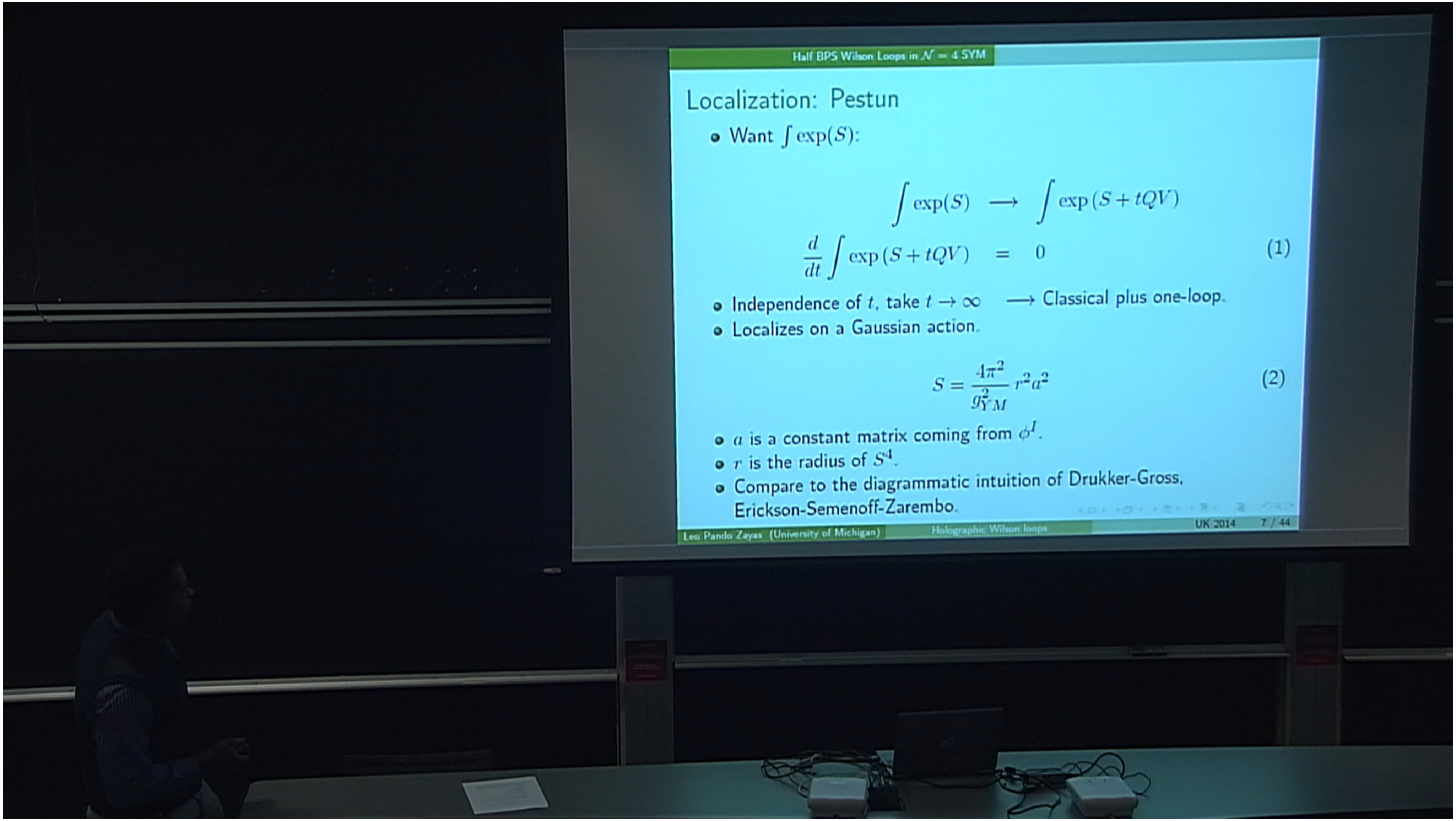
5 / 44


Half BPS Wilson Loops in $\mathcal{N} = 4$ SYM
Localization: Pestun
Want $\int \exp(S)$:
$\int \exp(S) \longrightarrow \int \exp(S + tQV)$
$\frac{d}{dt} \int \exp(S + tQV) = 0$ (1)
Independence of t, take $t \to \infty$ $\longrightarrow$ Classical plus one-loop.
Localizes on a Gaussian action.
$S = \frac{4\pi^2}{g_{YM}^2} r^2 a^2$ (2)
a is a constant matrix coming from ϕ^I.
r is the radius of S^4.
Compare to the diagrammatic intuition of Drukker-Gross, Erickson-Semenoff-Zarembo.
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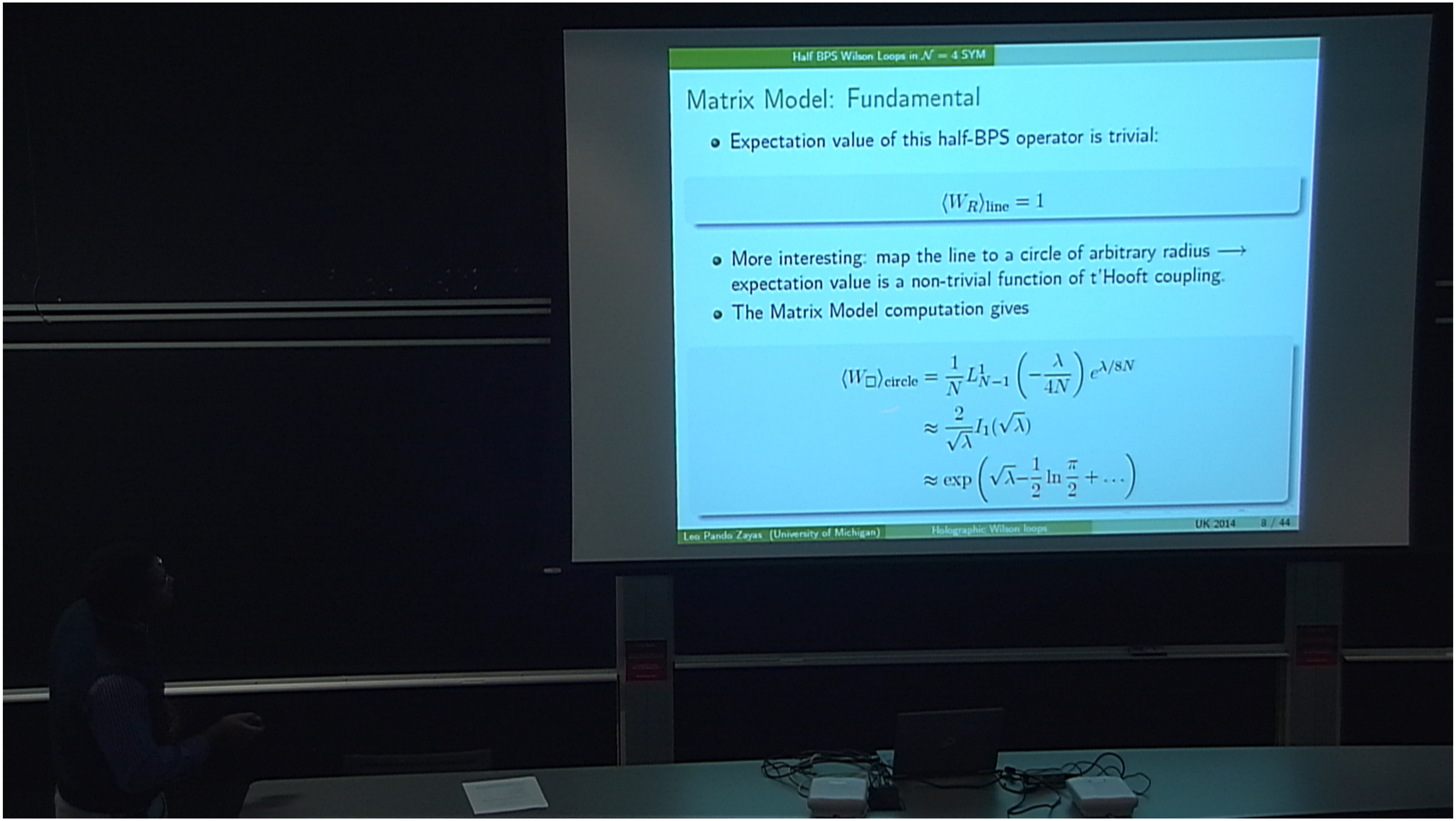
7 / 44


Half BPS Wilson Loops in $\mathcal{N} = 4$ SYM
Matrix Model: Fundamental
Expectation value of this half-BPS operator is trivial:
$\langle W_R \rangle_{\text{line}} = 1$
More interesting: map the line to a circle of arbitrary radius ⟶ expectation value is a non-trivial function of t'Hooft coupling.
The Matrix Model computation gives
$\langle W_\Box \rangle_{\text{circle}} = \frac{1}{N} L^1_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N}$
$\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda})$
$\approx \exp\left(\sqrt{\lambda} - \frac{1}{2}\ln\frac{\pi}{2} + \dots\right)$
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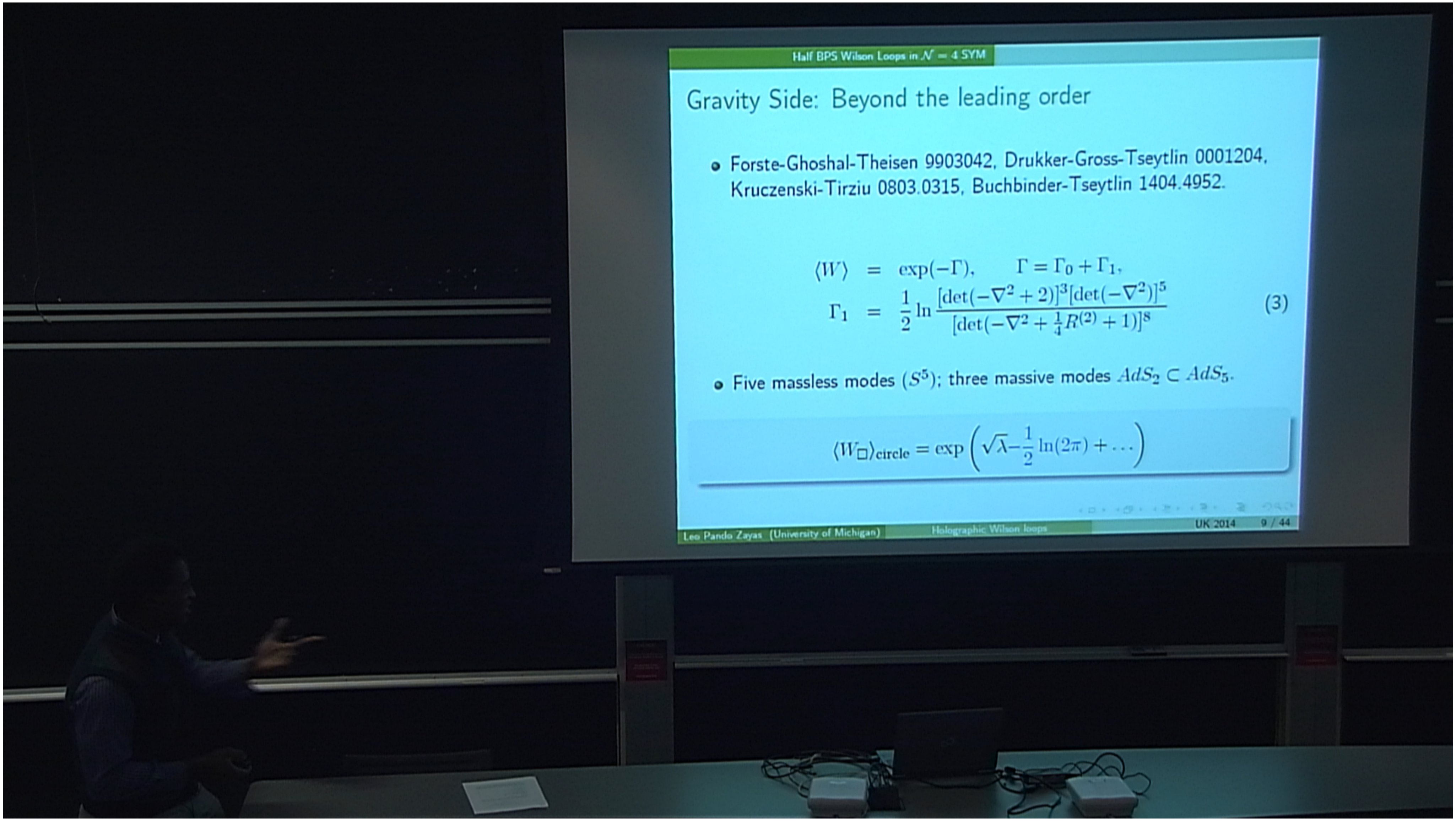
8 / 44


Half BPS Wilson Loops in $\mathcal{N} = 4$ SYM
Gravity Side: Beyond the leading order
Forste-Ghoshal-Theisen 9903042, Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.
$\langle W \rangle = \exp(-\Gamma), \qquad \Gamma = \Gamma_0 + \Gamma_1,$
$\Gamma_1 = \frac{1}{2} \ln \frac{[\det(-\nabla^2 + 2)]^3 [\det(-\nabla^2)]^5}{[\det(-\nabla^2 + \frac{1}{4} R^{(2)} + 1)]^8}$ (3)
Five massless modes (S^5); three massive modes $AdS_2 \subset AdS_5$.
$\langle W_\Box \rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2} \ln(2\pi) + \dots\right)$
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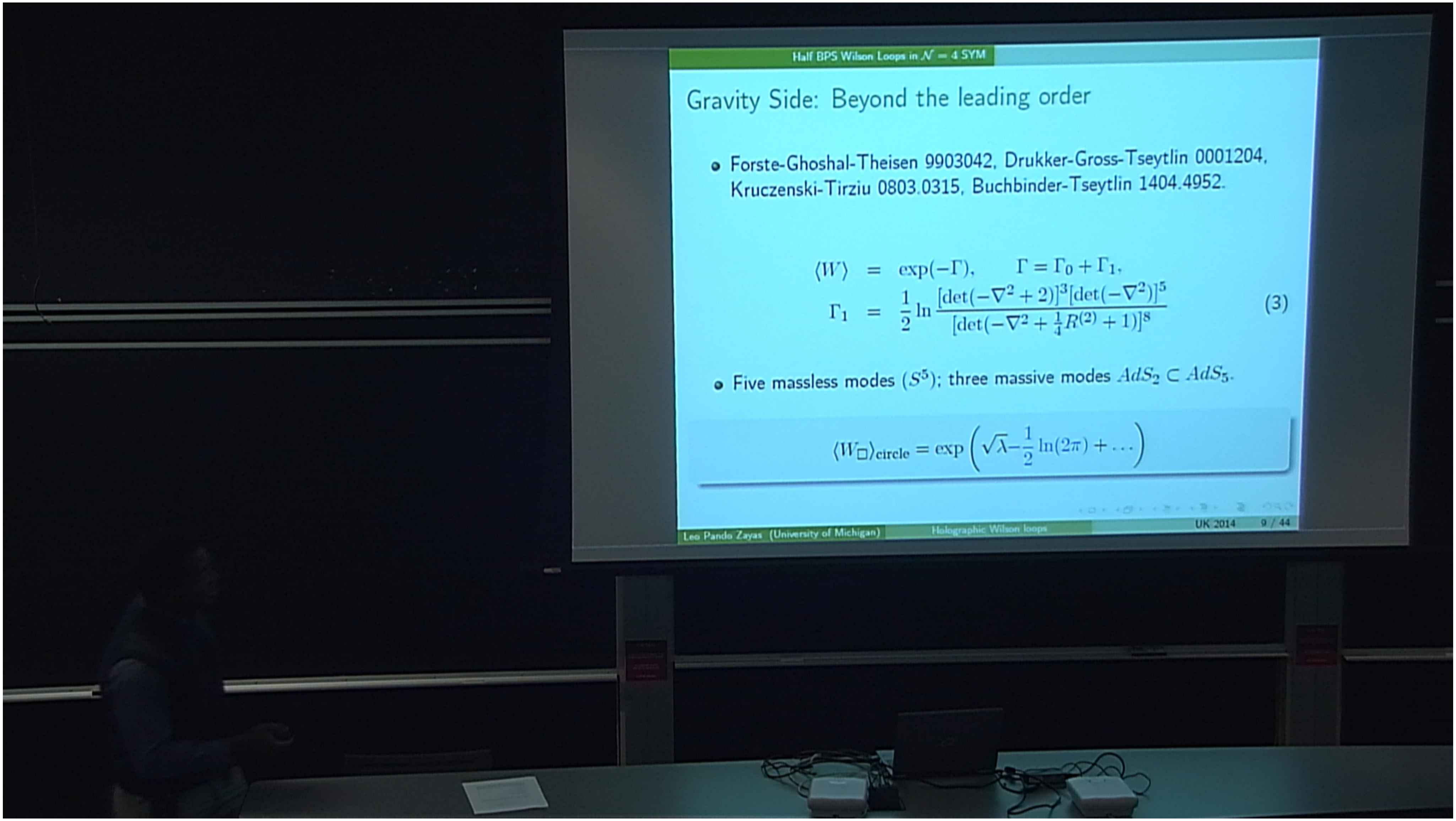
9 / 44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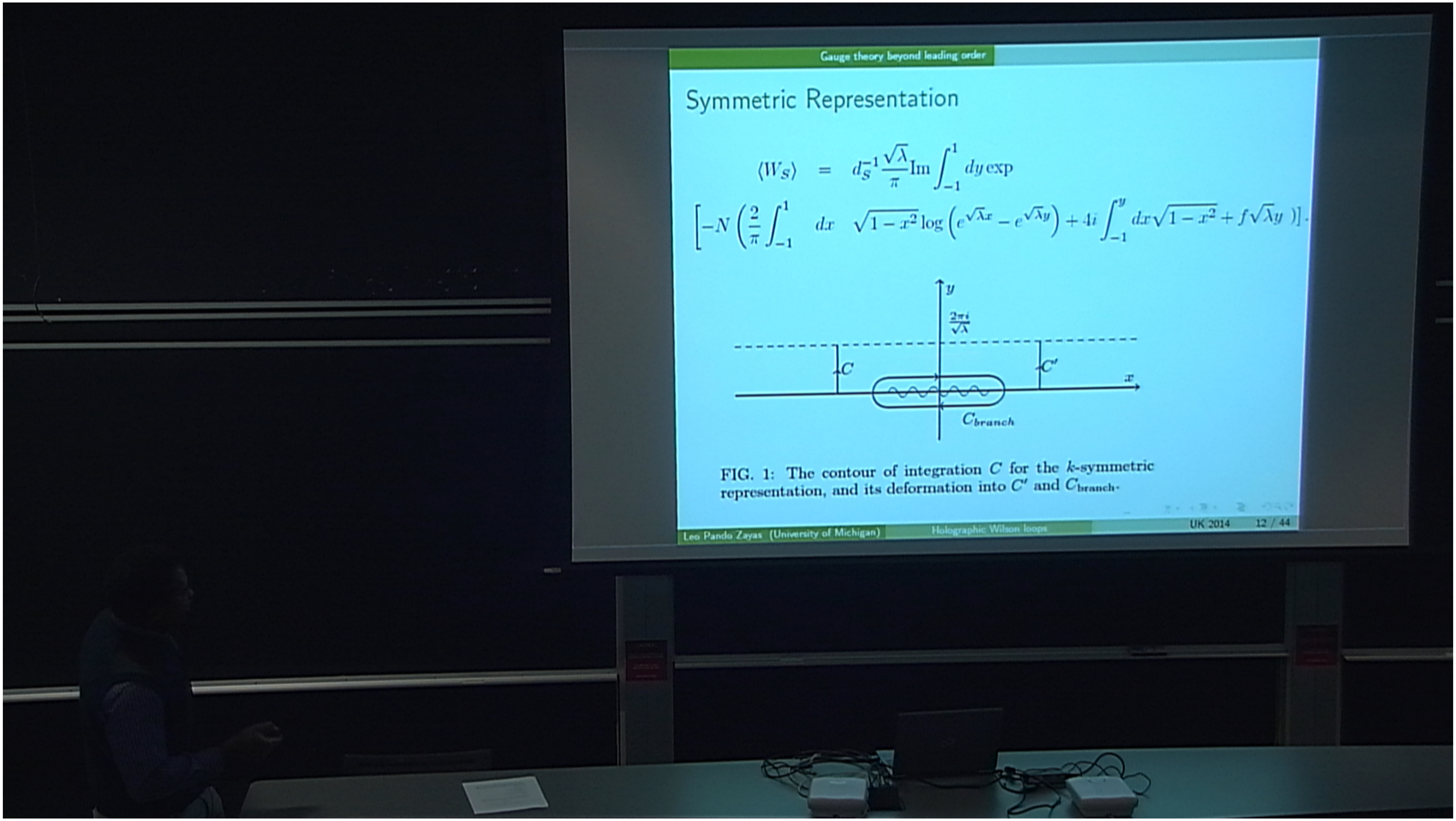
Gauge theory beyond leading order
Symmetric Representation
$\langle W_S \rangle = d_S^{-1} \frac{\sqrt{\lambda}}{\pi} \mathrm{Im} \int_{-1}^{1} dy \exp$
$\left[-N \left(\frac{2}{\pi} \int_{-1}^{1} dx \sqrt{1-x^2} \log \left(e^{\sqrt{\lambda} x} - e^{\sqrt{\lambda} y} \right) + 4i \int_{-1}^{y} dx \sqrt{1-x^2} + f \sqrt{\lambda} y \right) \right].$
y
$\frac{2\pi i}{\sqrt{\lambda}}$
C
C'
x
C_{branch}
FIG. 1: The contour of integration C for the k-symmetric representation, and its deformation into C' and $C_{\mathbf{branch}}$.
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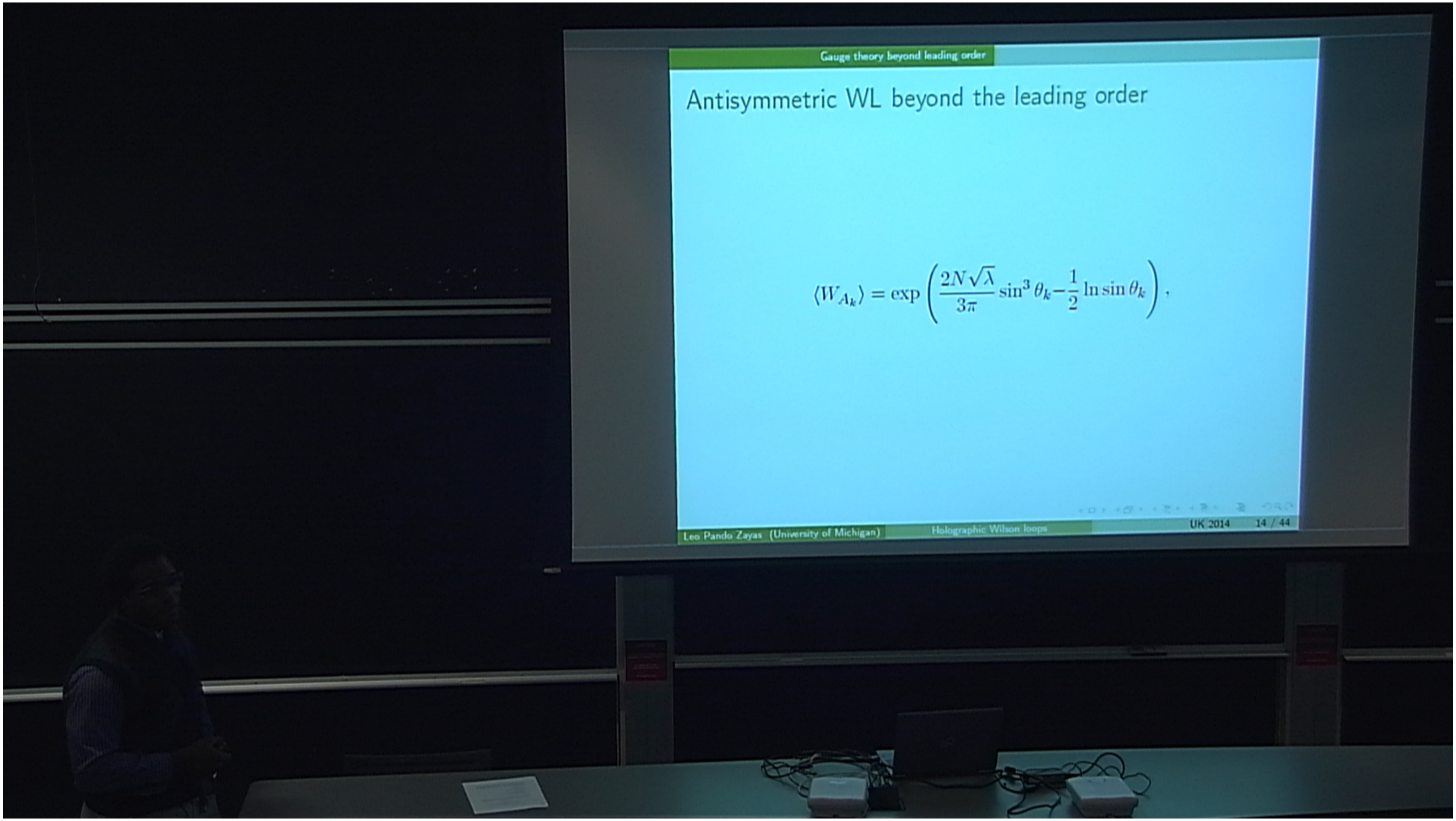
12 / 44


Gauge theory beyond leading order
Antisymmetric WL beyond the leading order
$$\langle W_{A_k}\rangle = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{2}\ln\sin\theta_k\right),$$
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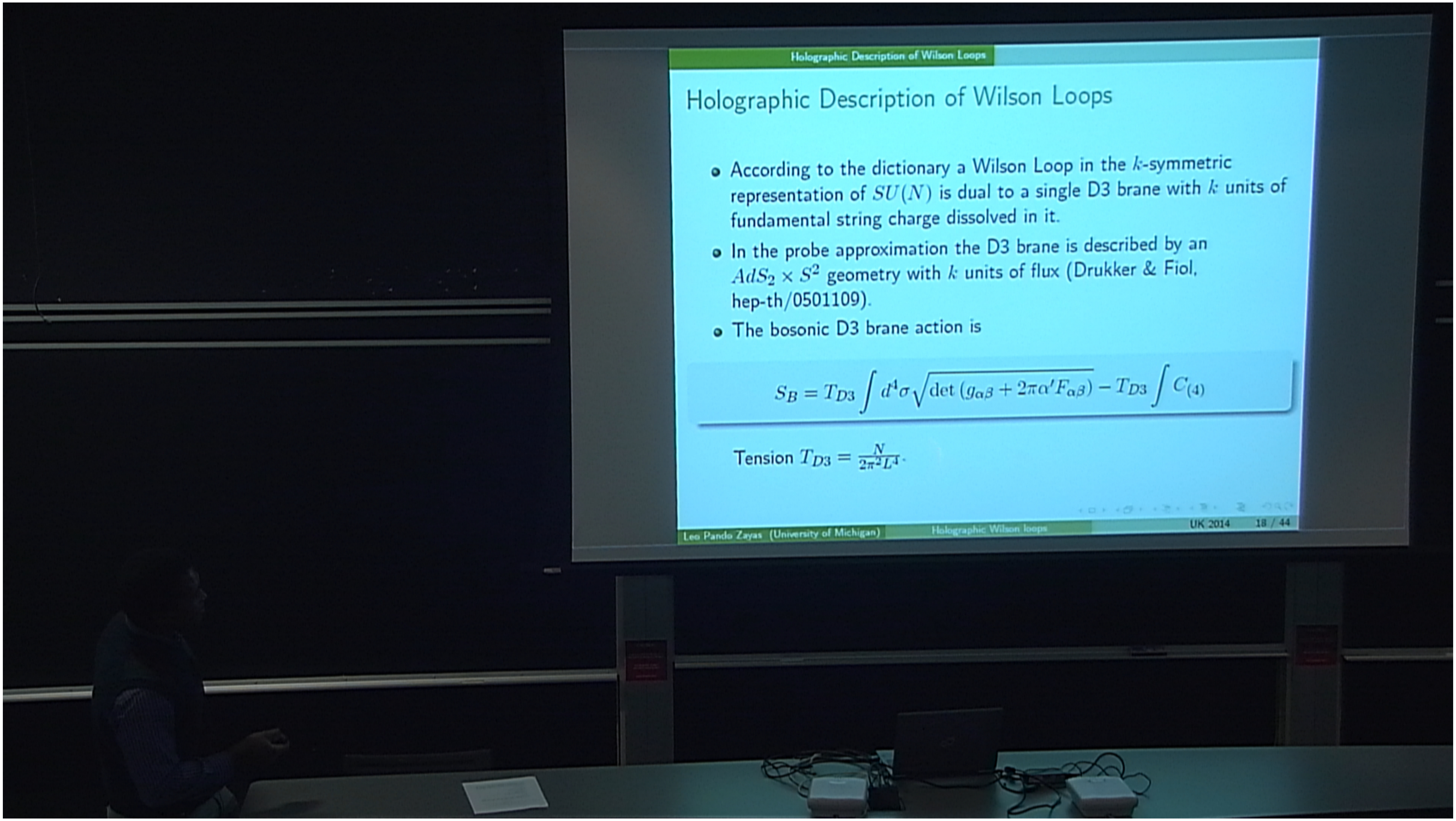
14 / 44



Holographic Description of Wilson Loops
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According to the dictionary a Wilson Loop in the k-symmetric representation of $SU(N)$ is dual to a single D3 brane with k units of fundamental string charge dissolved in it.
In the probe approximation the D3 brane is described by an $AdS_2 \times S^2$ geometry with k units of flux (Drukker & Fiol, hep-th/0501109).
The bosonic D3 brane action is
$S_B = T_{D3} \int d^4\sigma \sqrt{\det\left(g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta}\right)} - T_{D3} \int C_{(4)}$
Tension $T_{D3} = \frac{N}{2\pi^2 L^4}$.
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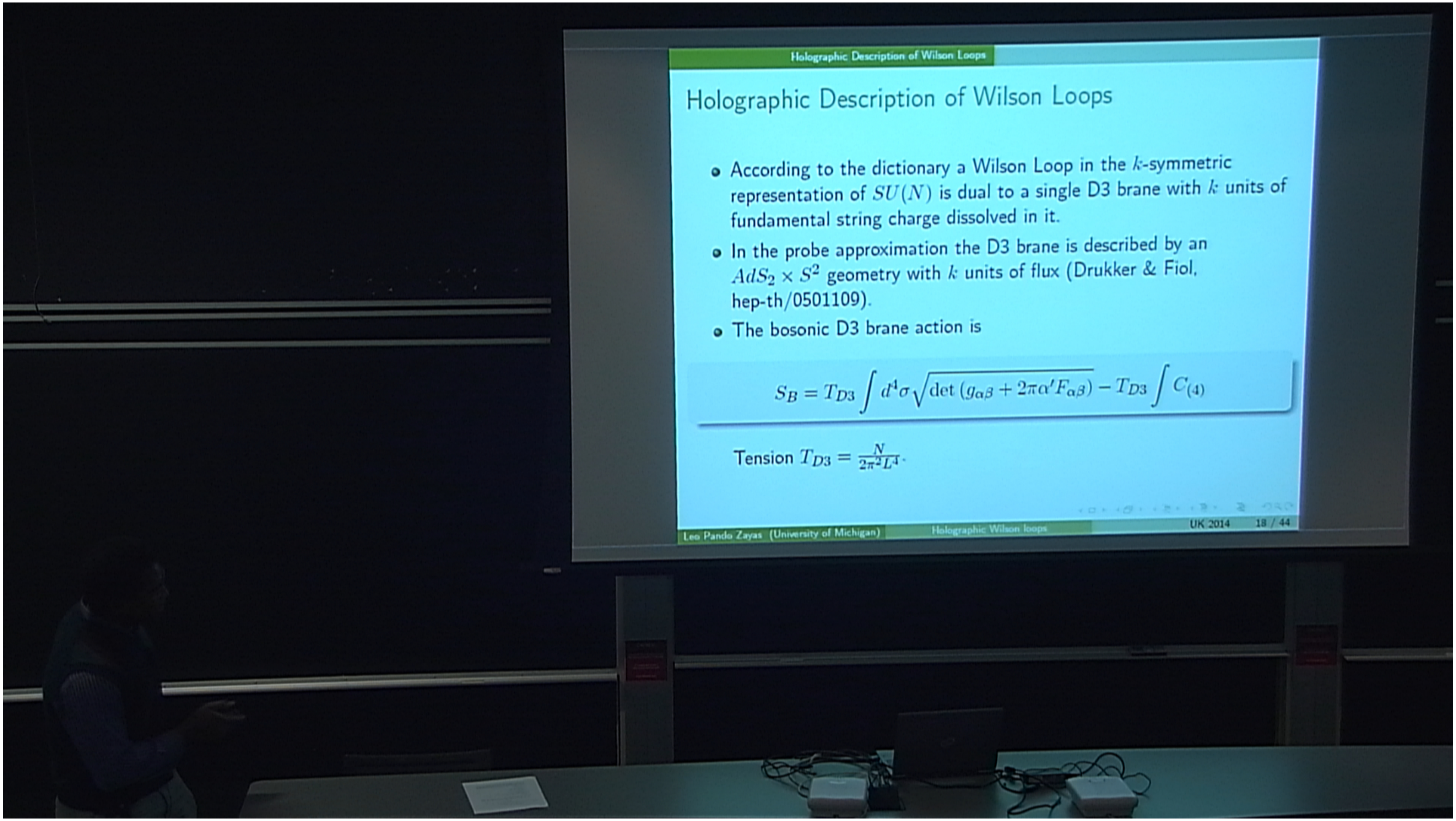
18 / 44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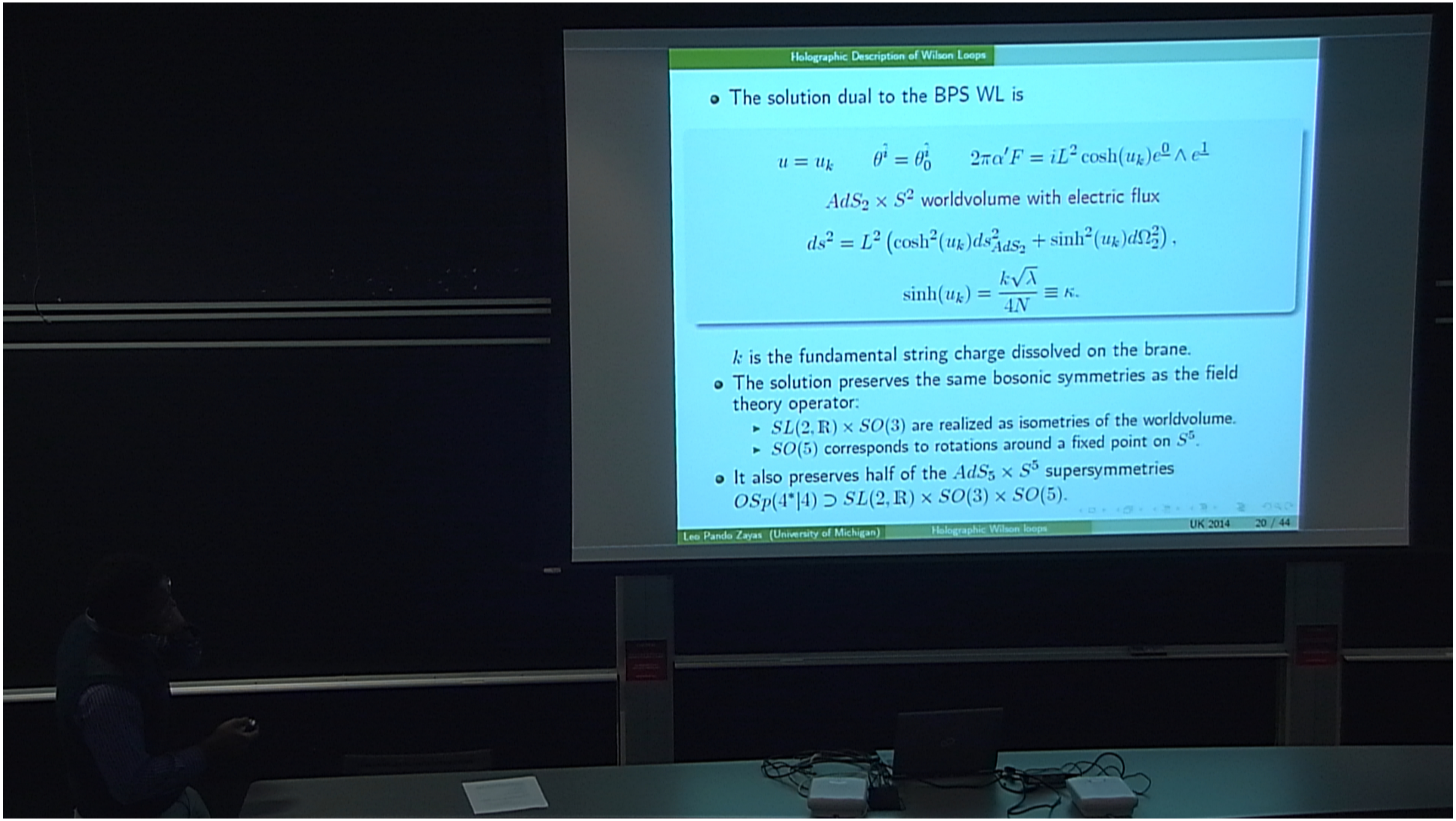
18 / 44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The solution dual to the BPS WL is
$u = u_k \qquad \theta^{\hat{i}} = \theta^{\hat{i}}_0 \qquad 2\pi\alpha' F = iL^2 \cosh(u_k) e^{\underline{0}} \wedge e^{\underline{1}}$
$AdS_2 \times S^2$ worldvolume with electric flux
$ds^2 = L^2 \left(\cosh^2(u_k) ds^2_{AdS_2} + \sinh^2(u_k) d\Omega_2^2\right),$
$\sinh(u_k) = \frac{k\sqrt{\lambda}}{4N} \equiv \kappa.$
k is the fundamental string charge dissolved on the brane.
The solution preserves the same bosonic symmetries as the field theory operator:
$SL(2,\mathbb{R}) \times SO(3)$ are realized as isometries of the worldvolume.
$SO(5)$ corresponds to rotations around a fixed point on S^5.
It also preserves half of the $AdS_5 \times S^5$ supersymmetries $OSp(4^*|4) \supset SL(2,\mathbb{R}) \times SO(3) \times SO(5)$.
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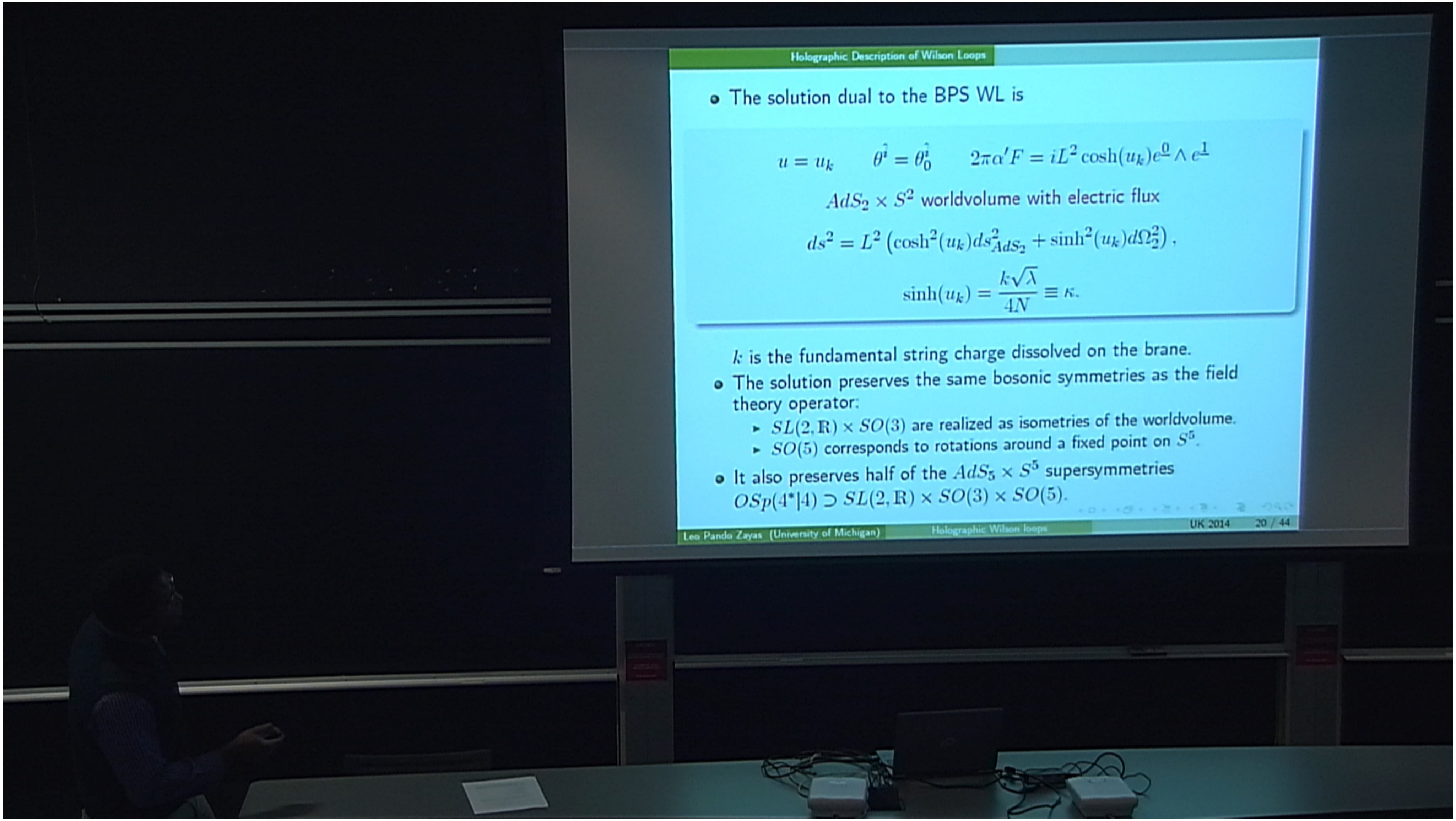
20 / 44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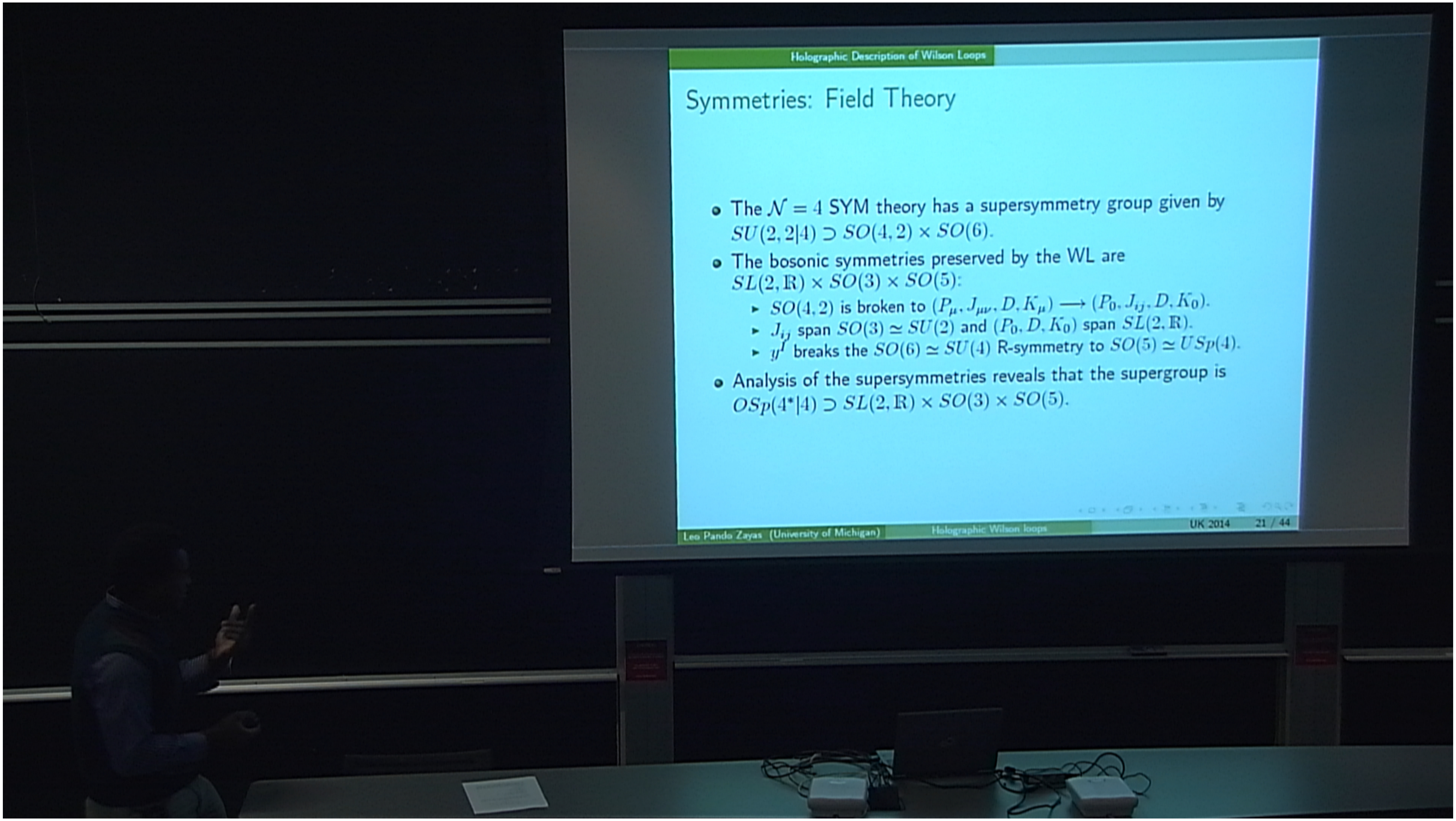
20 / 44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Symmetries: Field Theory
The $\mathcal{N}=4$ SYM theory has a supersymmetry group given by $SU(2,2|4) \supset SO(4,2) \times SO(6)$.
The bosonic symmetries preserved by the WL are $SL(2,\mathbb{R}) \times SO(3) \times SO(5)$:
$SO(4,2)$ is broken to $(P_\mu, J_{\mu\nu}, D, K_\mu) \longrightarrow (P_0, J_{ij}, D, K_0)$.
J_{ij} span $SO(3) \simeq SU(2)$ and (P_0, D, K_0) span $SL(2,\mathbb{R})$.
y^I breaks the $SO(6) \simeq SU(4)$ R-symmetry to $SO(5) \simeq USp(4)$.
Analysis of the supersymmetries reveals that the supergroup is $OSp(4^*|4) \supset SL(2,\mathbb{R}) \times SO(3) \times SO(5)$.
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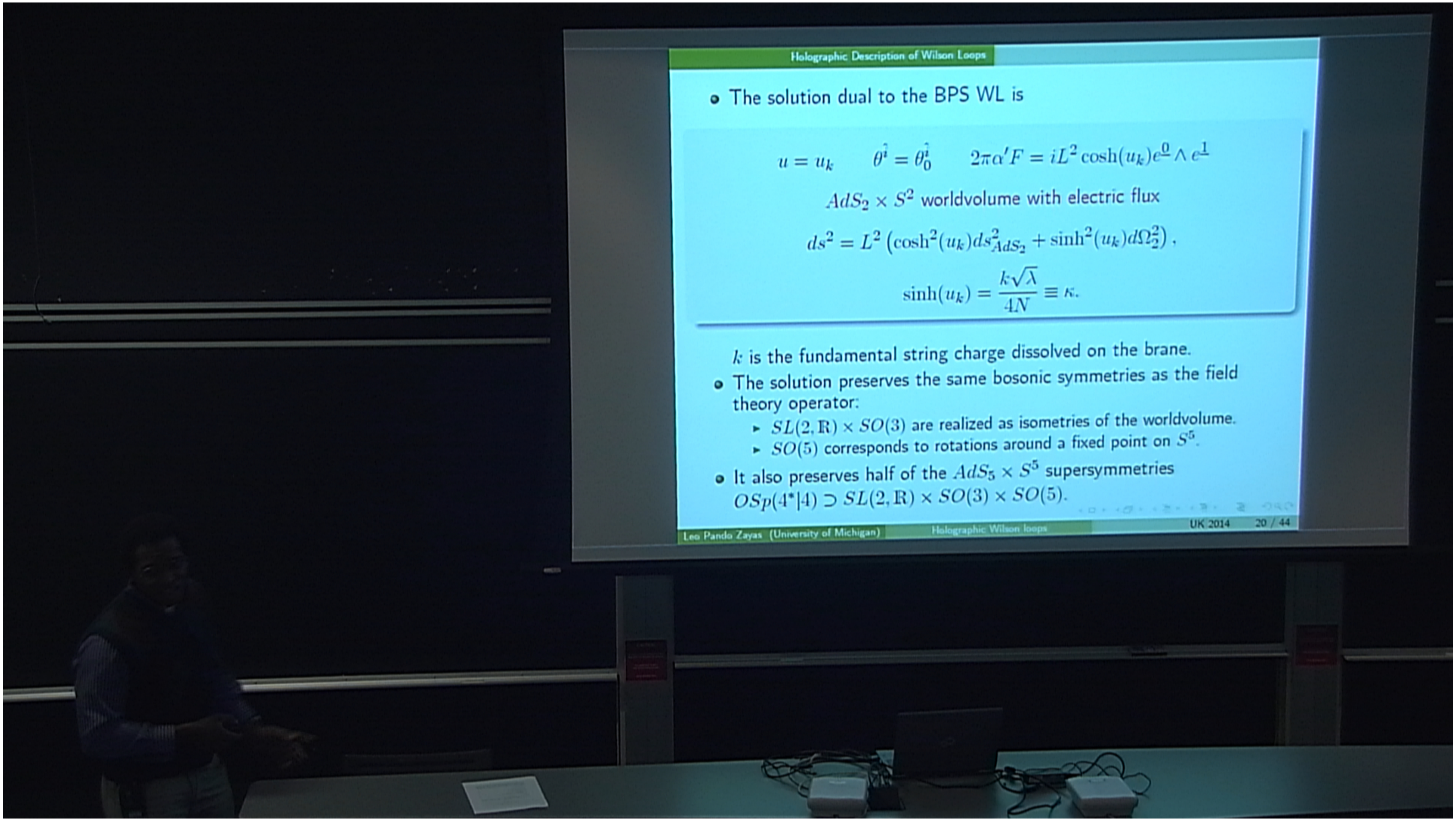
21 / 44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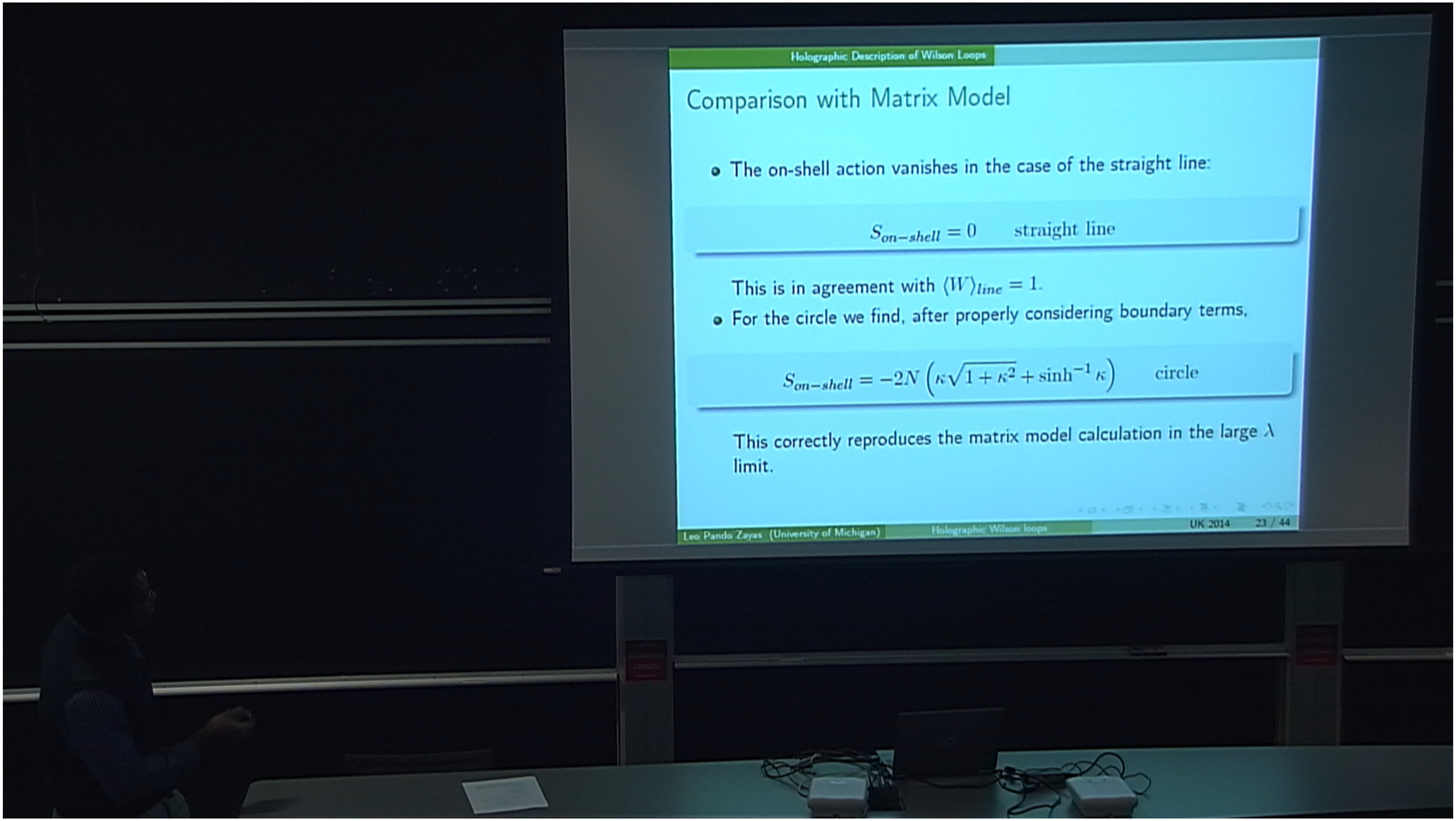
20 / 44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Comparison with Matrix Model
The on-shell action vanishes in the case of the straight line:
$S_{on-shell} = 0 \qquad \text{straight line}$
This is in agreement with $\langle W \rangle_{line} = 1$.
For the circle we find, after properly considering boundary terms,
$S_{on-shell} = -2N\left(\kappa\sqrt{1+\kappa^2} + \sinh^{-1}\kappa\right) \qquad \text{circle}$
This correctly reproduces the matrix model calculation in the large λ limit.
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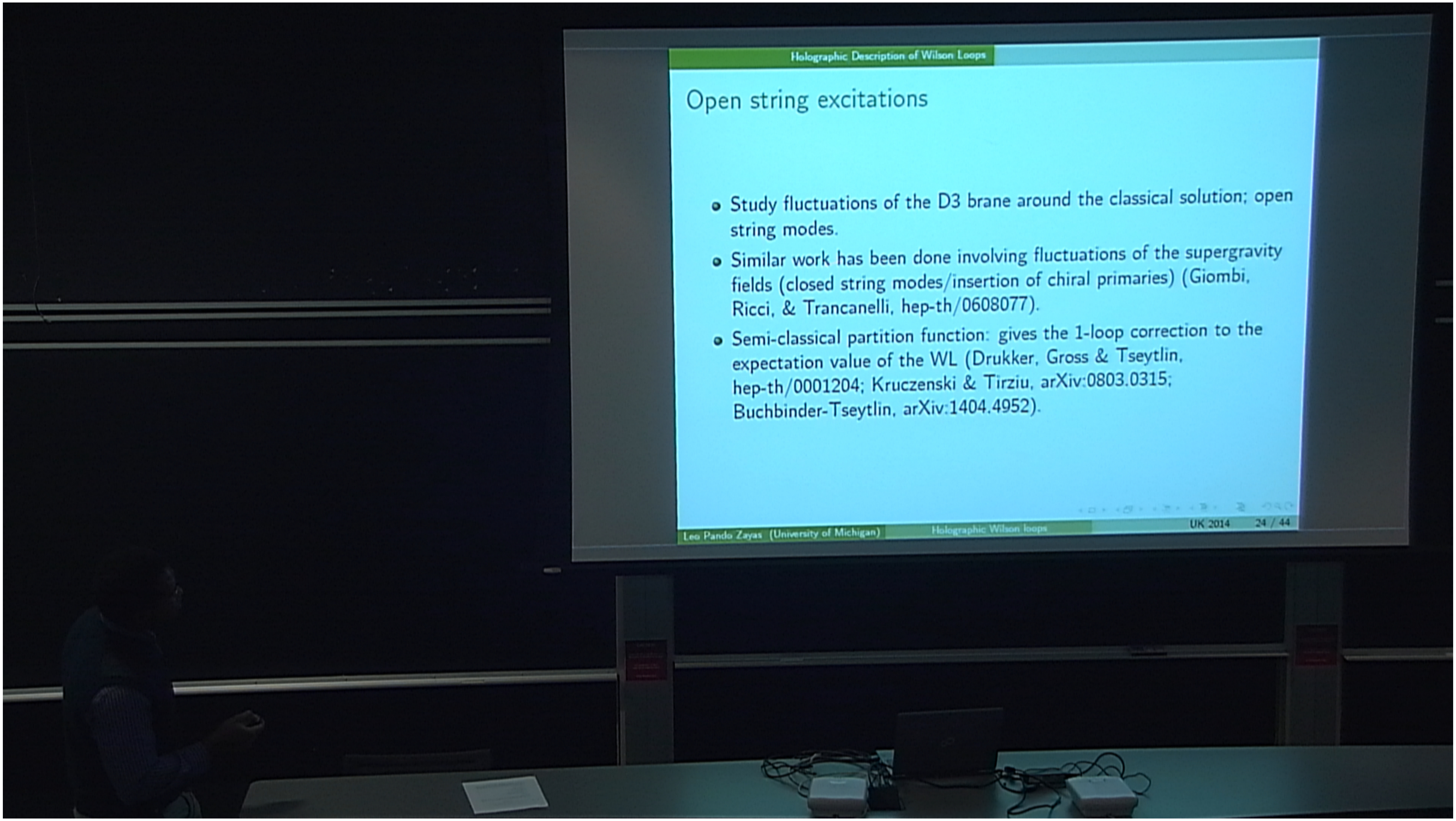
23 / 44


Holographic Description of Wilson Loops
Open string excitations
Study fluctuations of the D3 brane around the classical solution; open string modes.
Similar work has been done involving fluctuations of the supergravity fields (closed string modes/insertion of chiral primaries) (Giombi, Ricci, & Trancanelli, hep-th/0608077).
Semi-classical partition function: gives the 1-loop correction to the expectation value of the WL (Drukker, Gross & Tseytlin, hep-th/0001204; Kruczenski & Tirziu, arXiv:0803.0315; Buchbinder-Tseytlin, arXiv:1404.4952).
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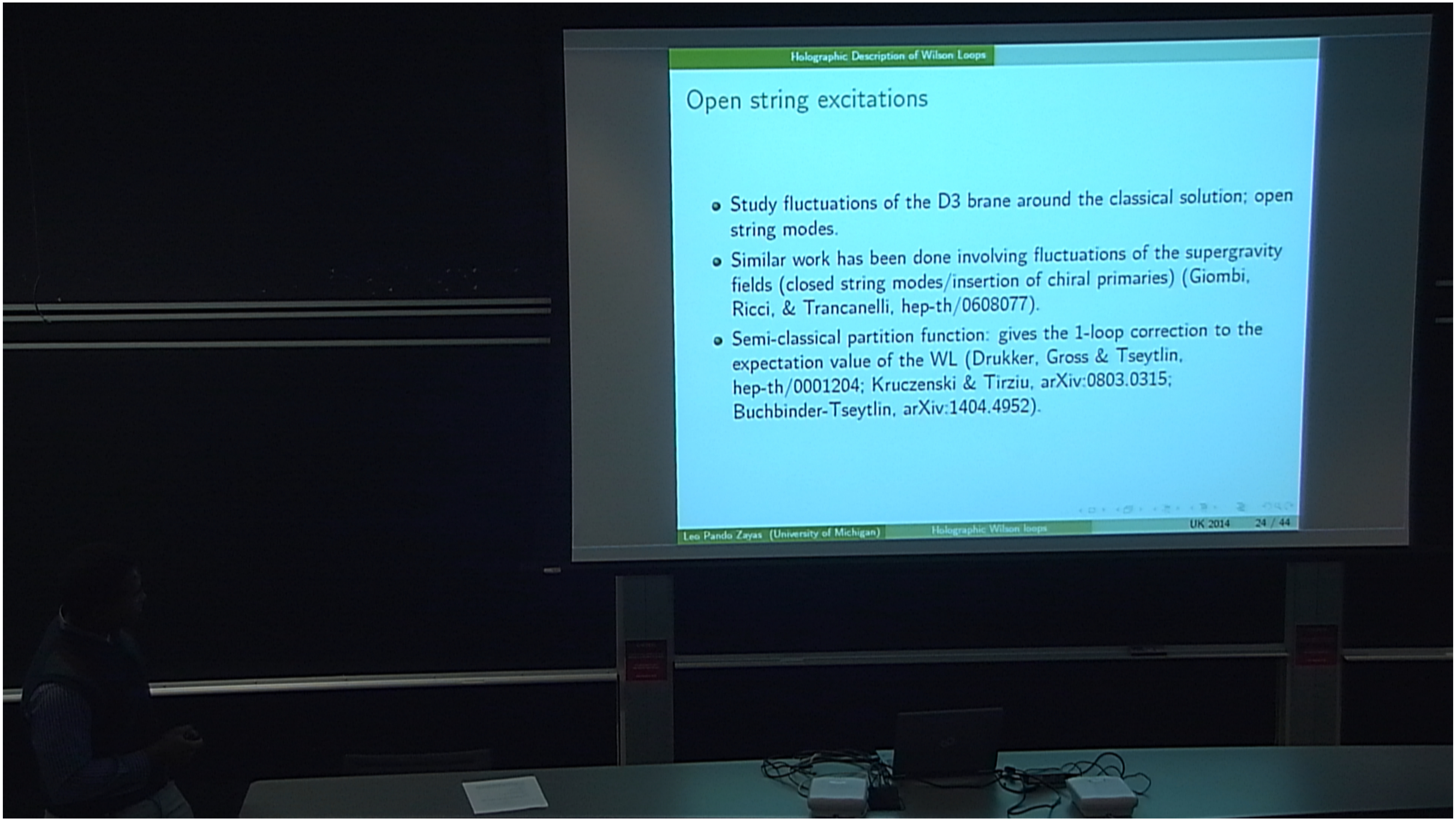
24 / 44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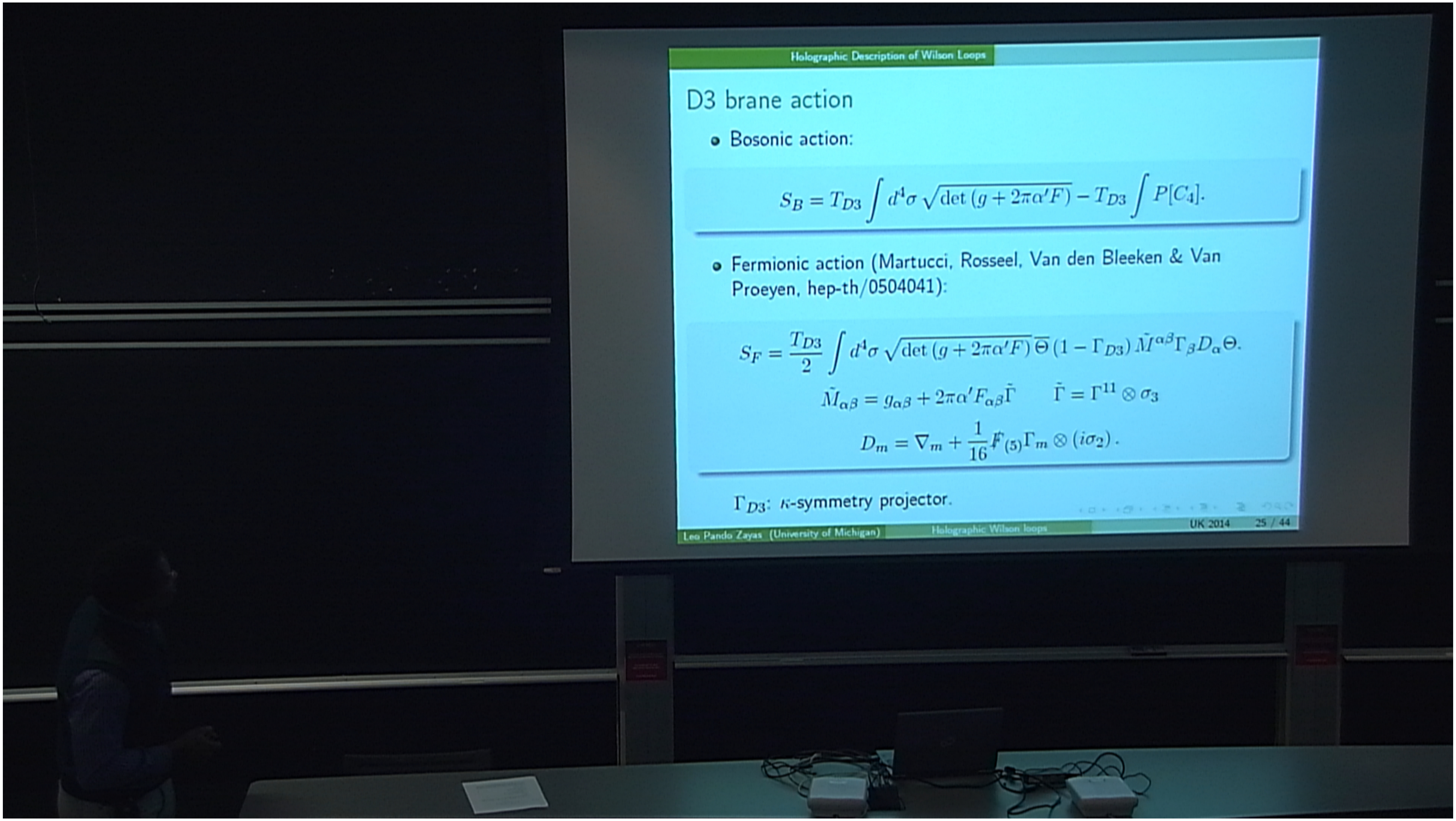
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D3 brane action
Bosonic action:
$S_B = T_{D3} \int d^4\sigma \sqrt{\det(g + 2\pi\alpha' F)} - T_{D3} \int P[C_4].$
Fermionic action (Martucci, Rosseel, Van den Bleeken & Van Proeyen, hep-th/0504041):
$S_F = \frac{T_{D3}}{2} \int d^4\sigma \sqrt{\det(g + 2\pi\alpha' F)}\, \overline{\Theta}\, (1 - \Gamma_{D3})\, \tilde{M}^{\alpha\beta} \Gamma_\beta D_\alpha \Theta.$
$\tilde{M}_{\alpha\beta} = g_{\alpha\beta} + 2\pi\alpha' F_{\alpha\beta} \tilde{\Gamma} \qquad \tilde{\Gamma} = \Gamma^{11} \otimes \sigma_3$
$D_m = \nabla_m + \frac{1}{16} \not{F}_{(5)} \Gamma_m \otimes (i\sigma_2).$
Γ_{D3}: κ-symmetry projector.
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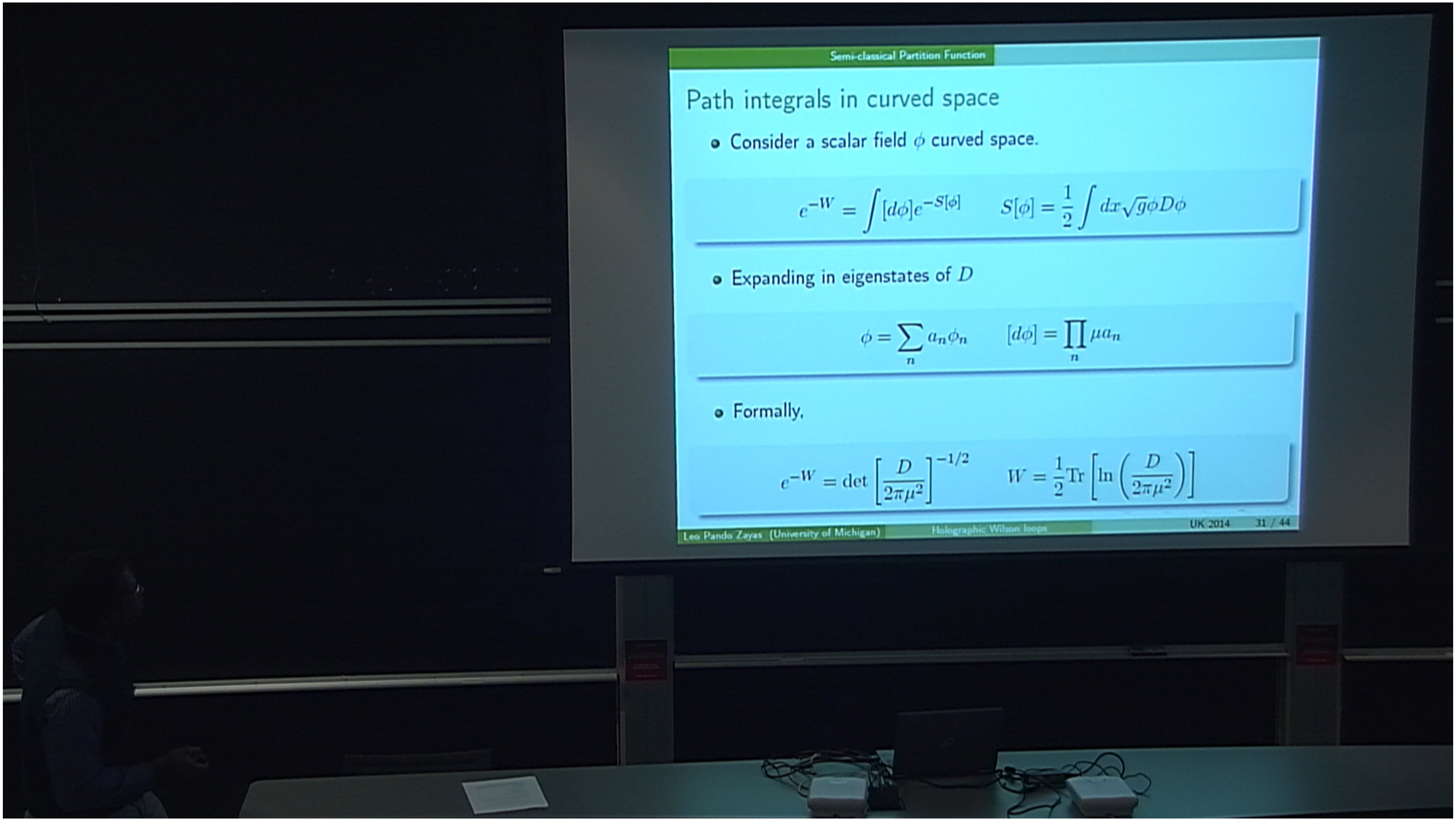
25 / 44


Semi-classical Partition Function
Path integrals in curved space
Consider a scalar field ϕ curved space.
$e^{-W} = \int [d\phi] e^{-S[\phi]} \qquad S[\phi] = \frac{1}{2}\int dx \sqrt{g}\phi D \phi$
Expanding in eigenstates of D
$\phi = \sum_n a_n \phi_n \qquad [d\phi] = \prod_n \mu a_n$
Formally,
$e^{-W} = \det\left[\frac{D}{2\pi\mu^2}\right]^{-1/2} \qquad W = \frac{1}{2}\mathrm{Tr}\left[\ln\left(\frac{D}{2\pi\mu^2}\right)\right]$
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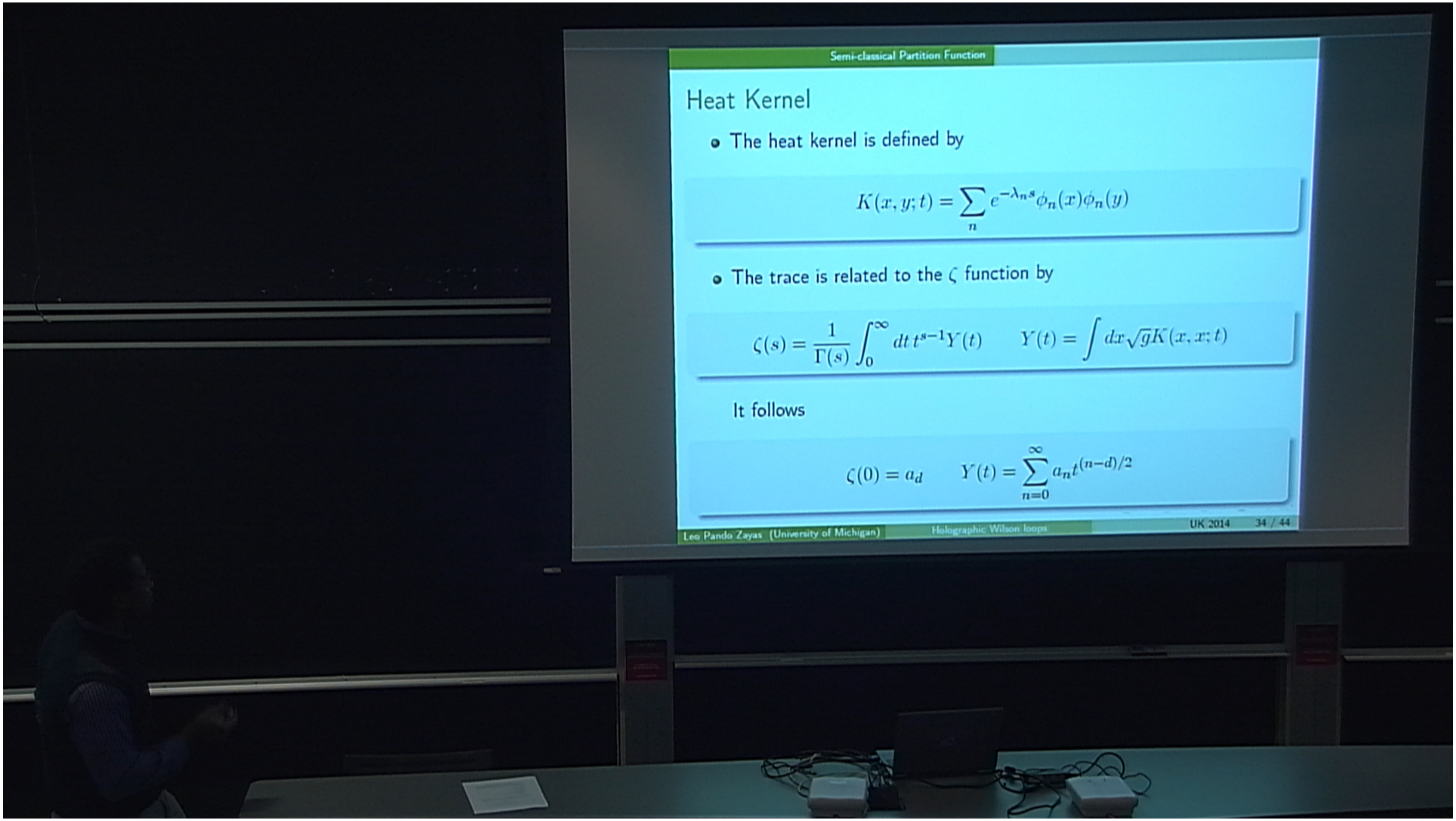
31 / 44


Semi-classical Partition Function
Heat Kernel
The heat kernel is defined by
$K(x,y;t) = \sum_n e^{-\lambda_n s}\phi_n(x)\phi_n(y)$
The trace is related to the ζ function by
$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty dt\, t^{s-1}Y(t) \qquad Y(t) = \int dx\sqrt{g}K(x,x;t)$
It follows
$\zeta(0) = a_d \qquad Y(t) = \sum_{n=0}^{\infty} a_n t^{(n-d)/2}$
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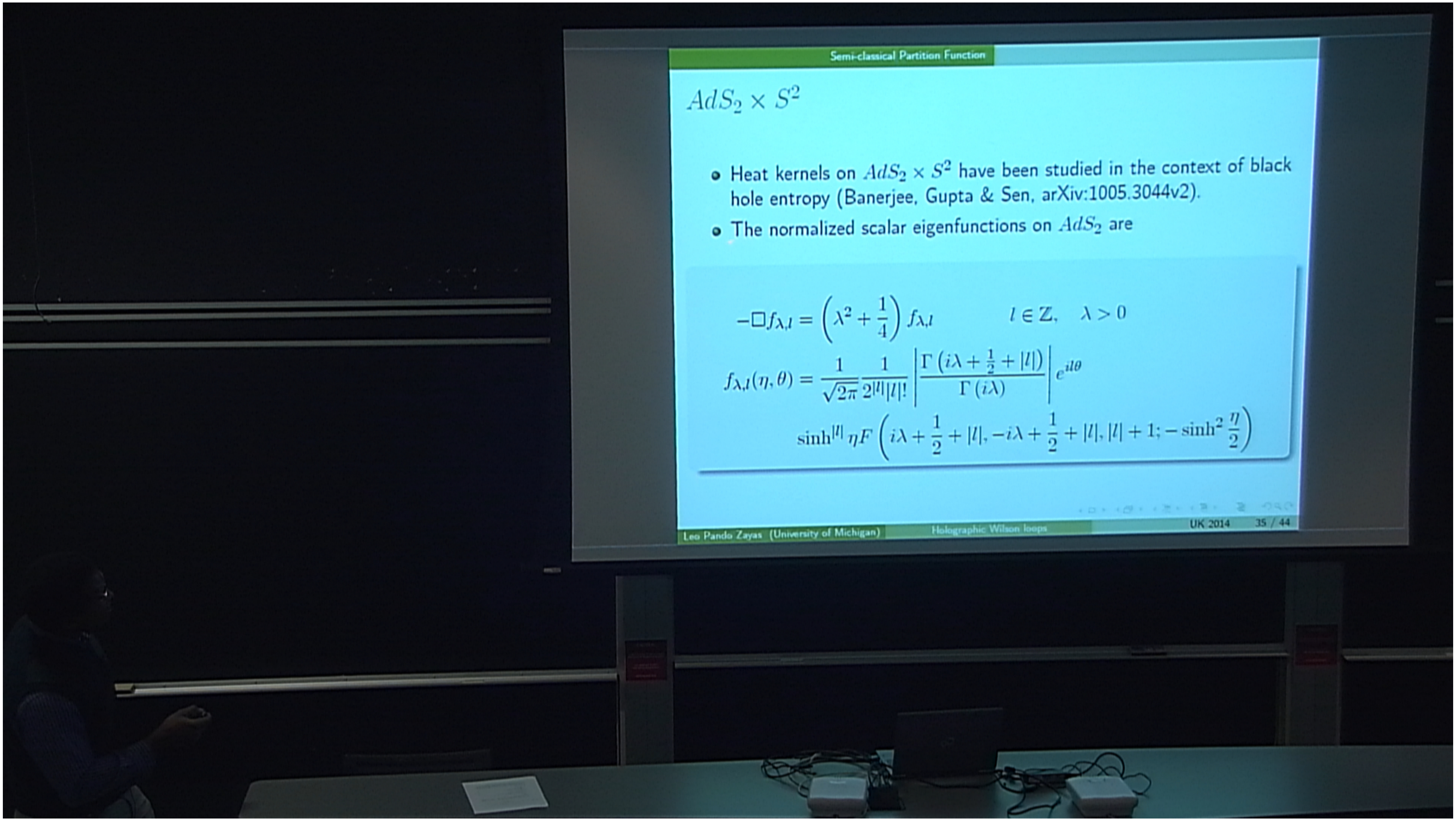
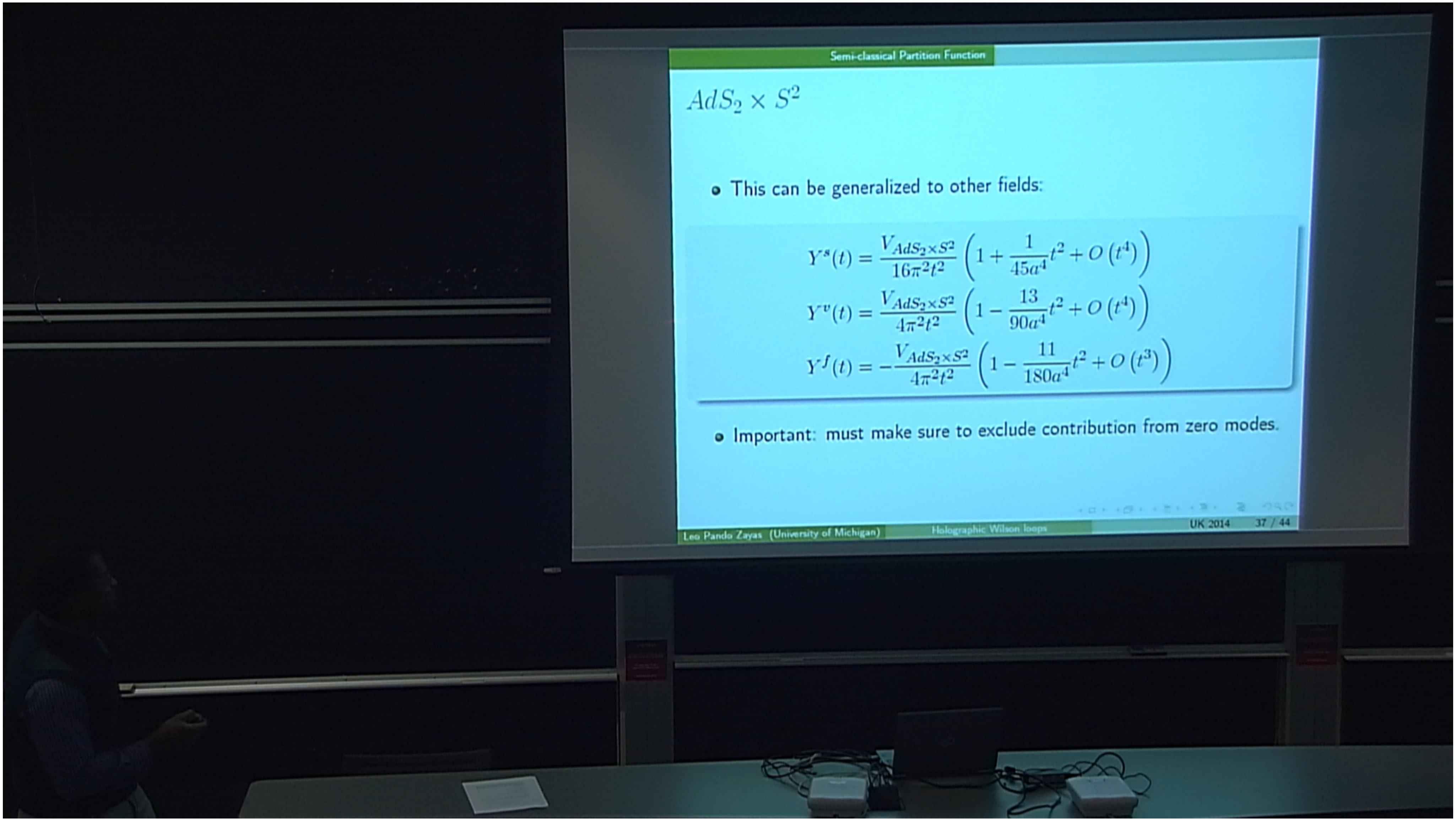
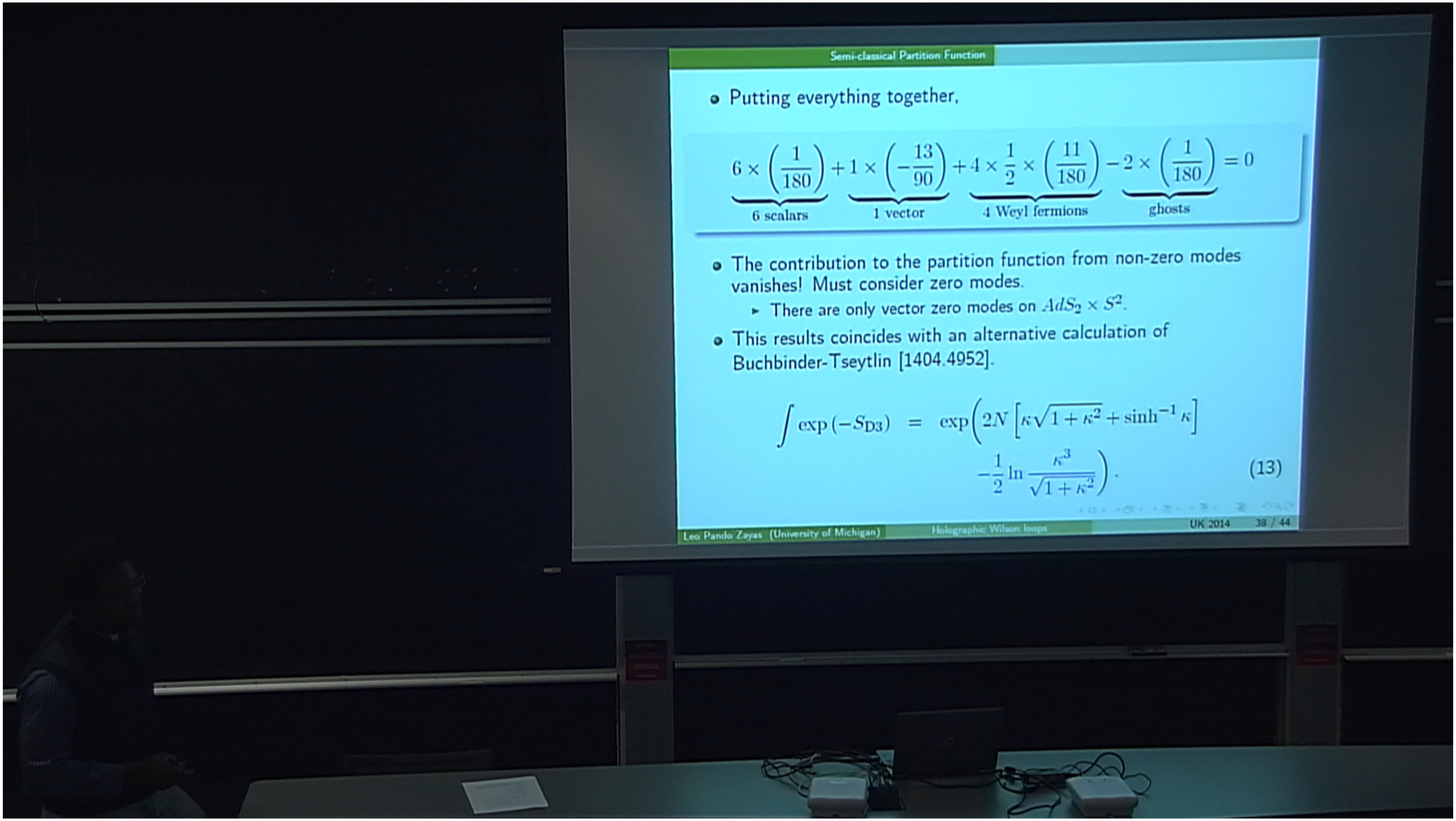
UK 2014
34 / 44


Semi-classical Partition Function
$AdS_2 \times S^2$
Heat kernels on $AdS_2 \times S^2$ have been studied in the context of black hole entropy (Banerjee, Gupta & Sen, arXiv:1005.3044v2).
The normalized scalar eigenfunctions on AdS_2 are
$-\Box f_{\lambda,l} = \left(\lambda^2 + \frac{1}{4}\right) f_{\lambda,l} \qquad l \in \mathbb{Z}, \quad \lambda > 0$
$f_{\lambda,l}(\eta,\theta) = \frac{1}{\sqrt{2\pi}} \frac{1}{2^{|l|}|l|!} \left| \frac{\Gamma\left(i\lambda + \frac{1}{2} + |l|\right)}{\Gamma(i\lambda)} \right| e^{il\theta}$
$\sinh^{|l|} \eta F\left(i\lambda + \frac{1}{2} + |l|, -i\lambda + \frac{1}{2} + |l|, |l| + 1; -\sinh^2 \frac{\eta}{2}\right)$
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
35 / 44


Semi-classical Partition Function
$AdS_2 \times S^2$
This can be generalized to other fields:
$Y^s(t) = \frac{V_{AdS_2\times S^2}}{16\pi^2 t^2}\left(1 + \frac{1}{45a^4}t^2 + O\left(t^4\right)\right)$
$Y^v(t) = \frac{V_{AdS_2\times S^2}}{4\pi^2 t^2}\left(1 - \frac{13}{90a^4}t^2 + O\left(t^4\right)\right)$
$Y^f(t) = -\frac{V_{AdS_2\times S^2}}{4\pi^2 t^2}\left(1 - \frac{11}{180a^4}t^2 + O\left(t^3\right)\right)$
Important: must make sure to exclude contribution from zero modes.
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
37 / 44


Semi-classical Partition Function
Putting everything together,
$\underbrace{6\times\left(\frac{1}{180}\right)}_{\text{6 scalars}}+\underbrace{1\times\left(-\frac{13}{90}\right)}_{\text{1 vector}}+\underbrace{4\times\frac{1}{2}\times\left(\frac{11}{180}\right)}_{\text{4 Weyl fermions}}-\underbrace{2\times\left(\frac{1}{180}\right)}_{\text{ghosts}}=0$
The contribution to the partition function from non-zero modes vanishes! Must consider zero modes.
There are only vector zero modes on $AdS_2\times S^2$.
This results coincides with an alternative calculation of Buchbinder-Tseytlin [1404.4952].
$\int \exp(-S_{D3}) = \exp\left(2N\left[\kappa\sqrt{1+\kappa^2}+\sinh^{-1}\kappa\right] -\frac{1}{2}\ln\frac{\kappa^3}{\sqrt{1+\kappa^2}}\right).$ (13)
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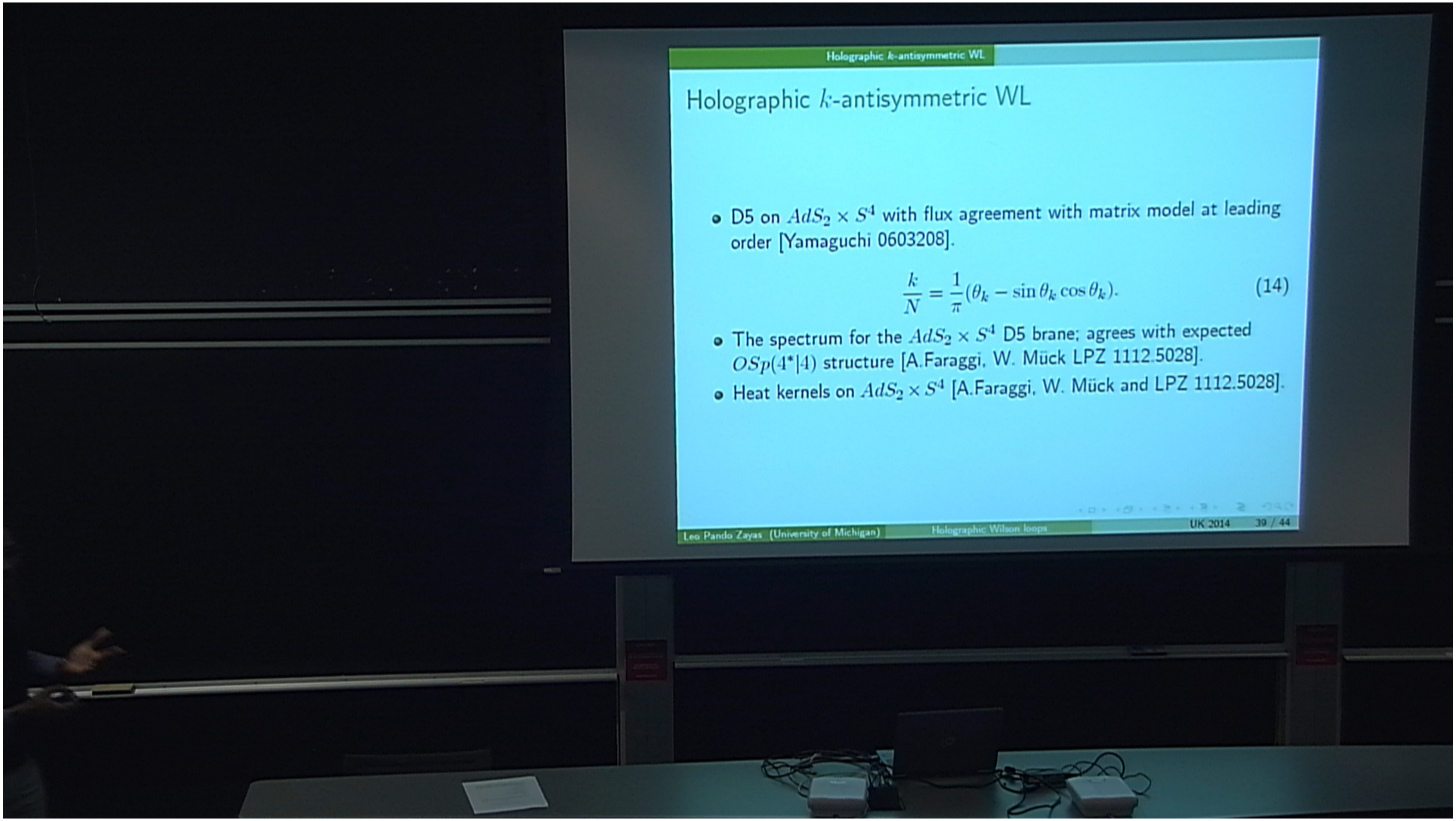
Holographic Wilson loops
UK 2014
38 / 44


Holographic k-antisymmetric WL
Holographic k-antisymmetric WL
D5 on $AdS_2 \times S^4$ with flux agreement with matrix model at leading order [Yamaguchi 0603208].
$\frac{k}{N} = \frac{1}{\pi}(\theta_k - \sin\theta_k \cos\theta_k)$. (14)
The spectrum for the $AdS_2 \times S^4$ D5 brane; agrees with expected $OSp(4^*|4)$ structure [A.Faraggi, W. Mück LPZ 1112.5028].
Heat kernels on $AdS_2 \times S^4$ [A.Faraggi, W. Mück and LPZ 1112.5028].
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
39 / 44

Holographic k-antisymmetric WL

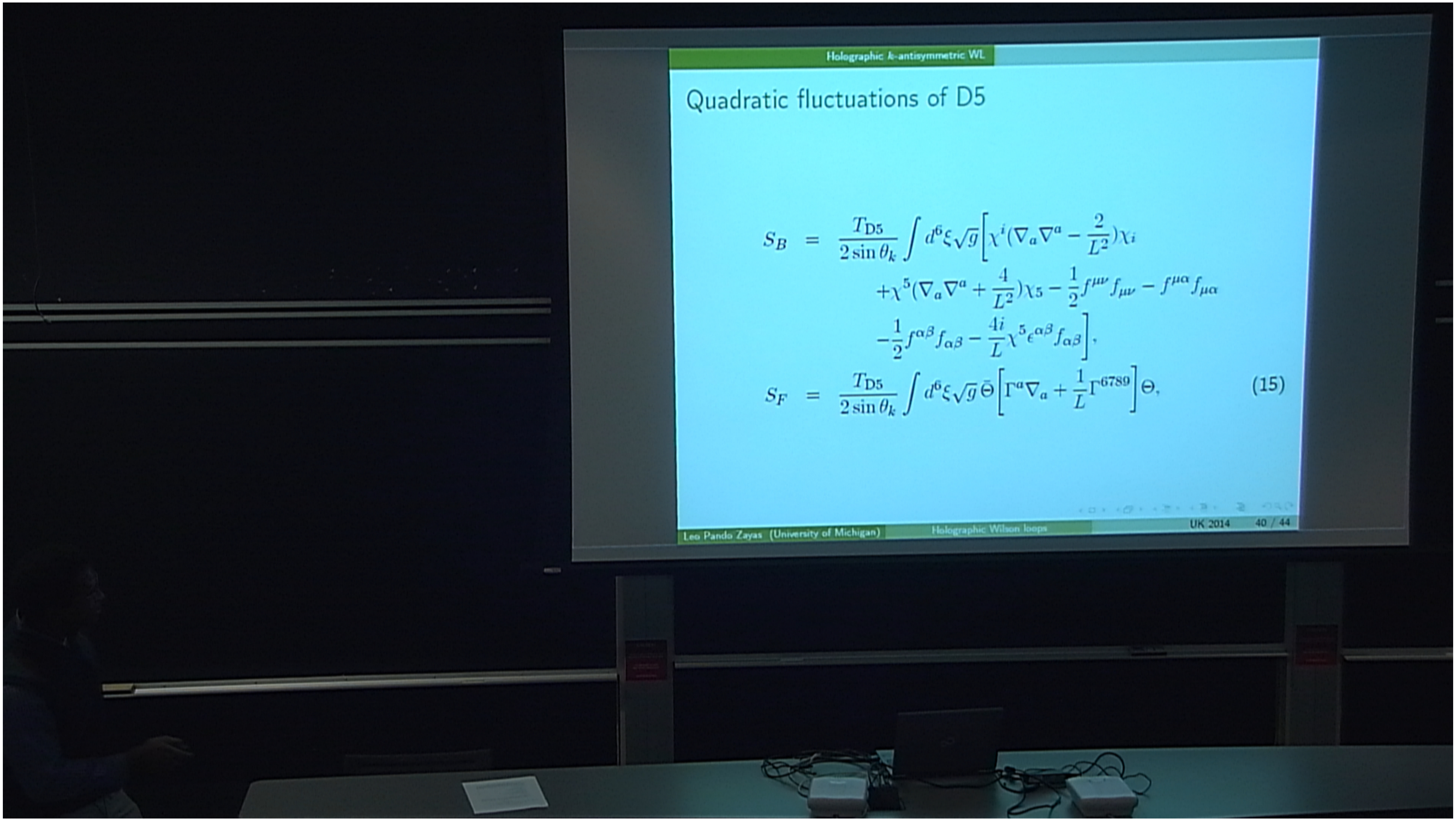
- D5 on $AdS_2 \times S^4$ with flux agreement with matrix model at leading order [Yamaguchi 0603208].

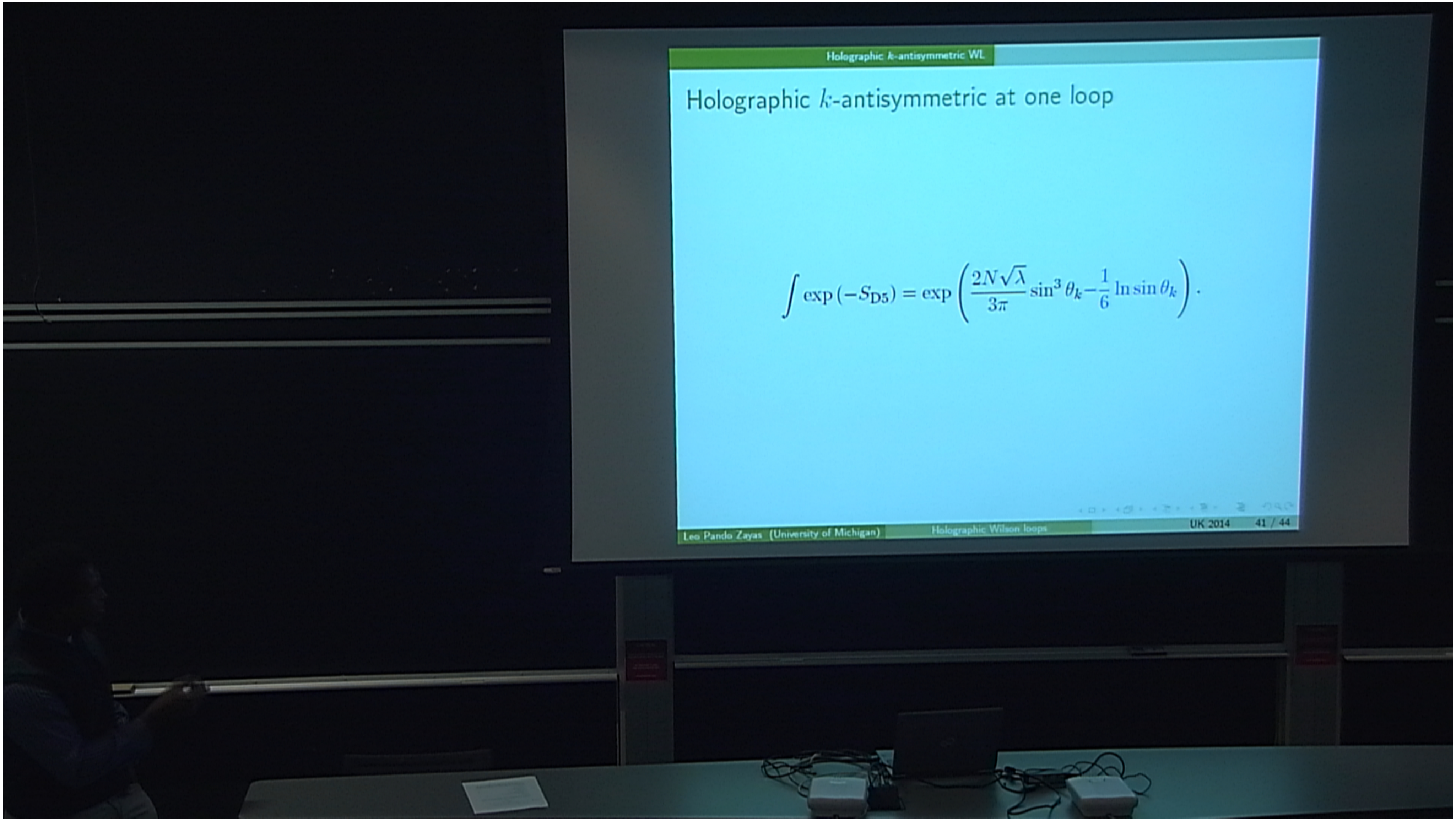
$$\frac{k}{N} = \frac{1}{\pi}(\theta_k - \sin\theta_k \cos\theta_k). \tag{14}$$

- The spectrum for the $AdS_2 \times S^4$ D5 brane; agrees with expected $OSp(4^*|4)$ structure [A.Faraggi, W. Mück LPZ 1112.5028].
- Heat kernels on $AdS_2 \times S^4$ [A.Faraggi, W. Mück and LPZ 1112.5028].

Quadratic fluctuations of D5

$$
\begin{aligned}
S_B &= \frac{T_{\rm D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\Big[\chi^i(\nabla_a\nabla^a - \frac{2}{L^2})\chi_i \\
&\quad +\chi^5(\nabla_a\nabla^a + \frac{4}{L^2})\chi_5 - \frac{1}{2}f^{\mu\nu}f_{\mu\nu} - f^{\mu\alpha}f_{\mu\alpha} \\
&\quad -\frac{1}{2}f^{\alpha\beta}f_{\alpha\beta} - \frac{4i}{L}\chi^5\epsilon^{\alpha\beta}f_{\alpha\beta}\Big], \\
S_F &= \frac{T_{\rm D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\,\bar\Theta\Big[\Gamma^a\nabla_a + \frac{1}{L}\Gamma^{6789}\Big]\Theta, \qquad (15)
\end{aligned}
$$


Holographic k-antisymmetric WL
Quadratic fluctuations of D5
$$S_B = \frac{T_{D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\Big[\chi^i(\nabla_a\nabla^a - \frac{2}{L^2})\chi_i + \chi^5(\nabla_a\nabla^a + \frac{4}{L^2})\chi_5 - \frac{1}{2}f^{\mu\nu}f_{\mu\nu} - f^{\mu\alpha}f_{\mu\alpha} - \frac{1}{2}f^{\alpha\beta}f_{\alpha\beta} - \frac{4i}{L}\chi^5\epsilon^{\alpha\beta}f_{\alpha\beta}\Big],$$
$$S_F = \frac{T_{D5}}{2\sin\theta_k}\int d^6\xi\sqrt{g}\,\bar{\Theta}\Big[\Gamma^a\nabla_a + \frac{1}{L}\Gamma^{6789}\Big]\Theta, \qquad (15)$$
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
40 / 44


Holographic k-antisymmetric WL
Holographic k-antisymmetric at one loop
$$\int \exp(-S_{D5}) = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{6}\ln\sin\theta_k\right).$$
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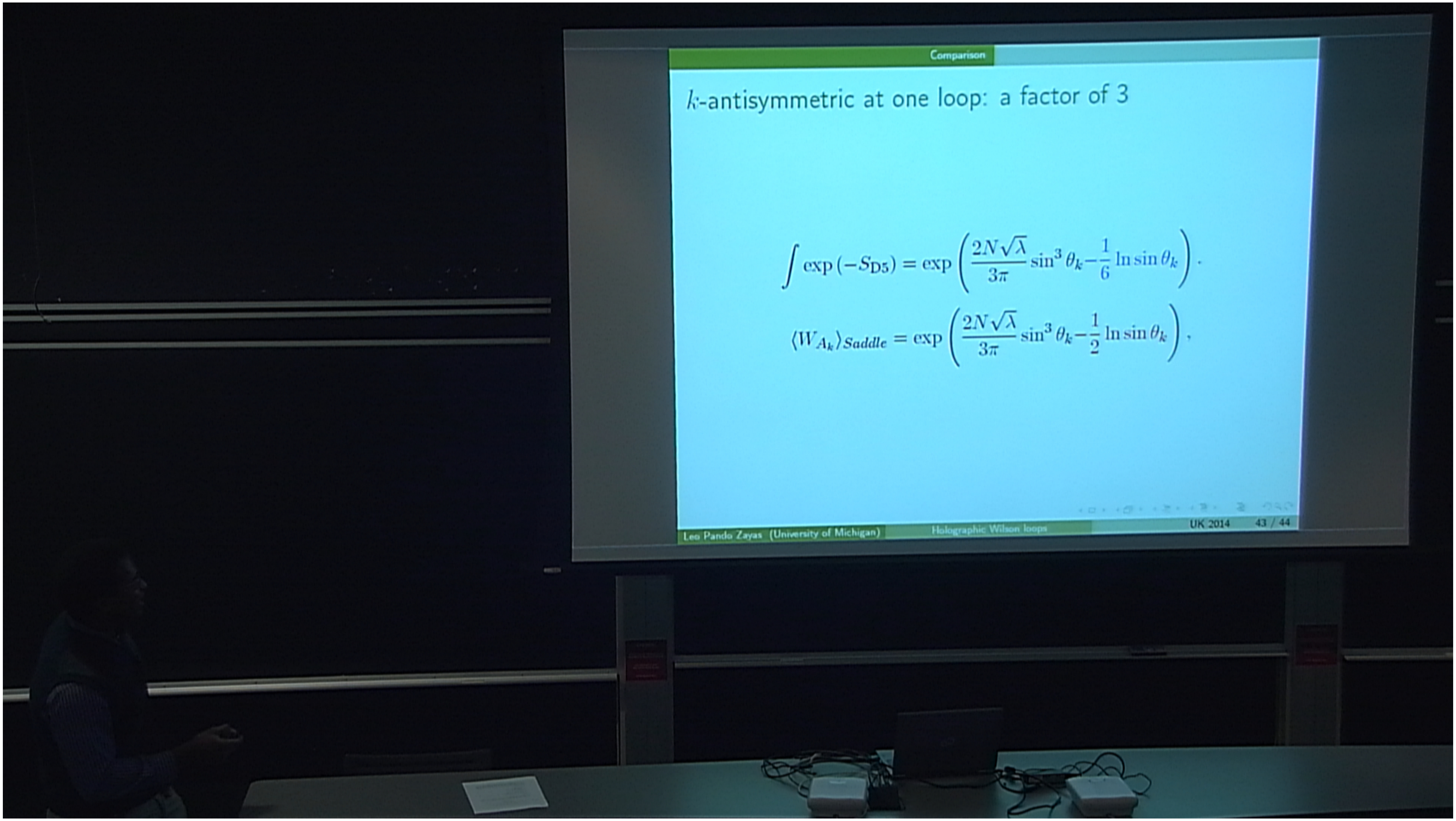
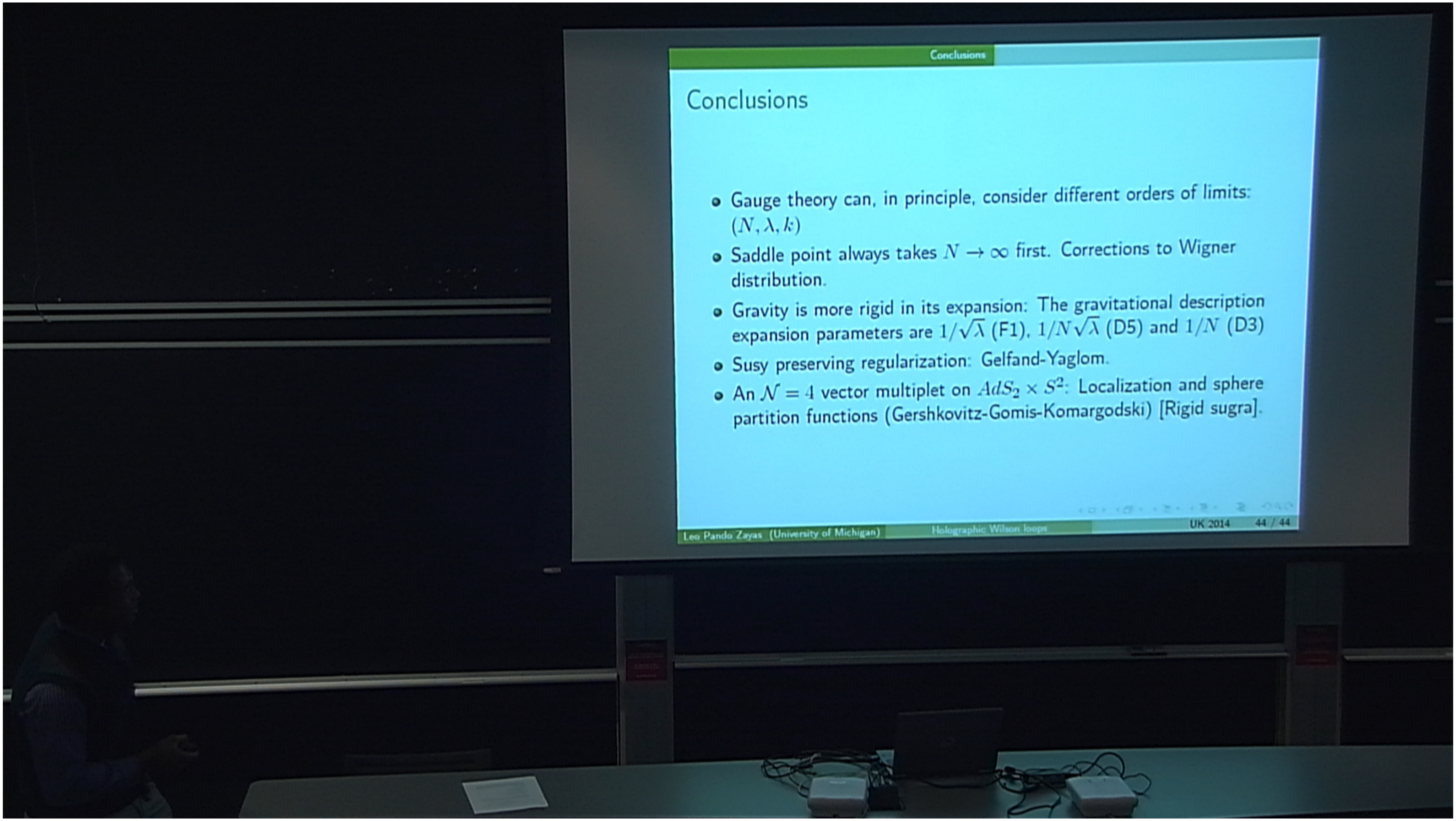
Holographic Wilson loops
UK 2014
41 / 44


Comparison
k-antisymmetric at one loop: a factor of 3
$$\int \exp\left(-S_{\mathrm{D5}}\right) = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{6}\ln\sin\theta_k\right).$$
$$\langle W_{A_k}\rangle_{Saddle} = \exp\left(\frac{2N\sqrt{\lambda}}{3\pi}\sin^3\theta_k - \frac{1}{2}\ln\sin\theta_k\right),$$
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
43 / 44


Conclusions
Conclusions
Gauge theory can, in principle, consider different orders of limits: (N, λ, k)
Saddle point always takes $N \to \infty$ first. Corrections to Wigner distribution.
Gravity is more rigid in its expansion: The gravitational description expansion parameters are $1/\sqrt{\lambda}$ (F1), $1/N\sqrt{\lambda}$ (D5) and $1/N$ (D3)
Susy preserving regularization: Gelfand-Yaglom.
An $\mathcal{N} = 4$ vector multiplet on $AdS_2 \times S^2$: Localization and sphere partition functions (Gershkovitz-Gomis-Komargodski) [Rigid sugra].
Leo Pando Zayas (University of Michigan)
Holographic Wilson loops
UK 2014
44 / 44

Conclusions

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