Title: Consistency relations from inflation to the Large Scale Structure

Date: Nov 27, 2014 11:00 AM

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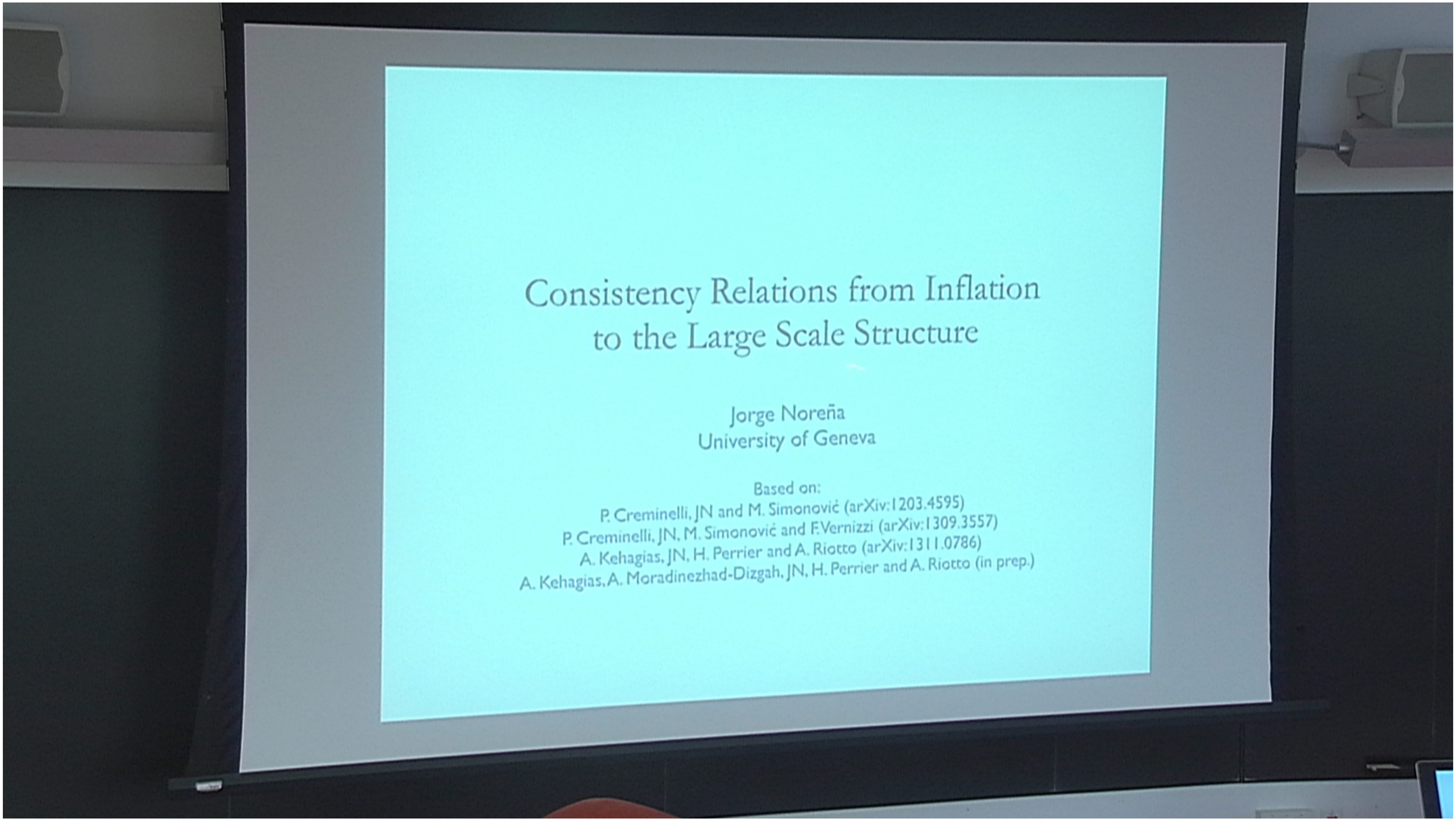
Abstract: <p>According to the Newtonian intuition, a constant gravitational field has no physical effect on a system since it can always be redefined, and a homogeneous gradient of the gravitational field (i.e. a homogeneous gravitational force) is equivalent to an accelerated reference frame. I will show how to extend this intuition to cosmological scales; in the presence of a single clock a constant curvature perturbation and its gradient can be set to zero through a coordinate transformation. This allows one to connect the squeezed limit of an n-point correlation function of the curvature perturbation to an (n+1)-point correlation function in the limit in which one of the momenta is very small (the so-called squeezed limit). These consistency relations are valid from inflation to the LSS. As an example, I will use them to write down a non-perturbative relativistic relation between galaxy number over-density correlation functions.</p>


Consistency Relations from Inflation
to the Large Scale Structure
Jorge Noreña
University of Geneva
Based on:
P. Creminelli, JN and M. Simonović (arXiv:1203.4595)
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A. Kehagias, JN, H. Perrier and A. Riotto (arXiv:1311.0786)
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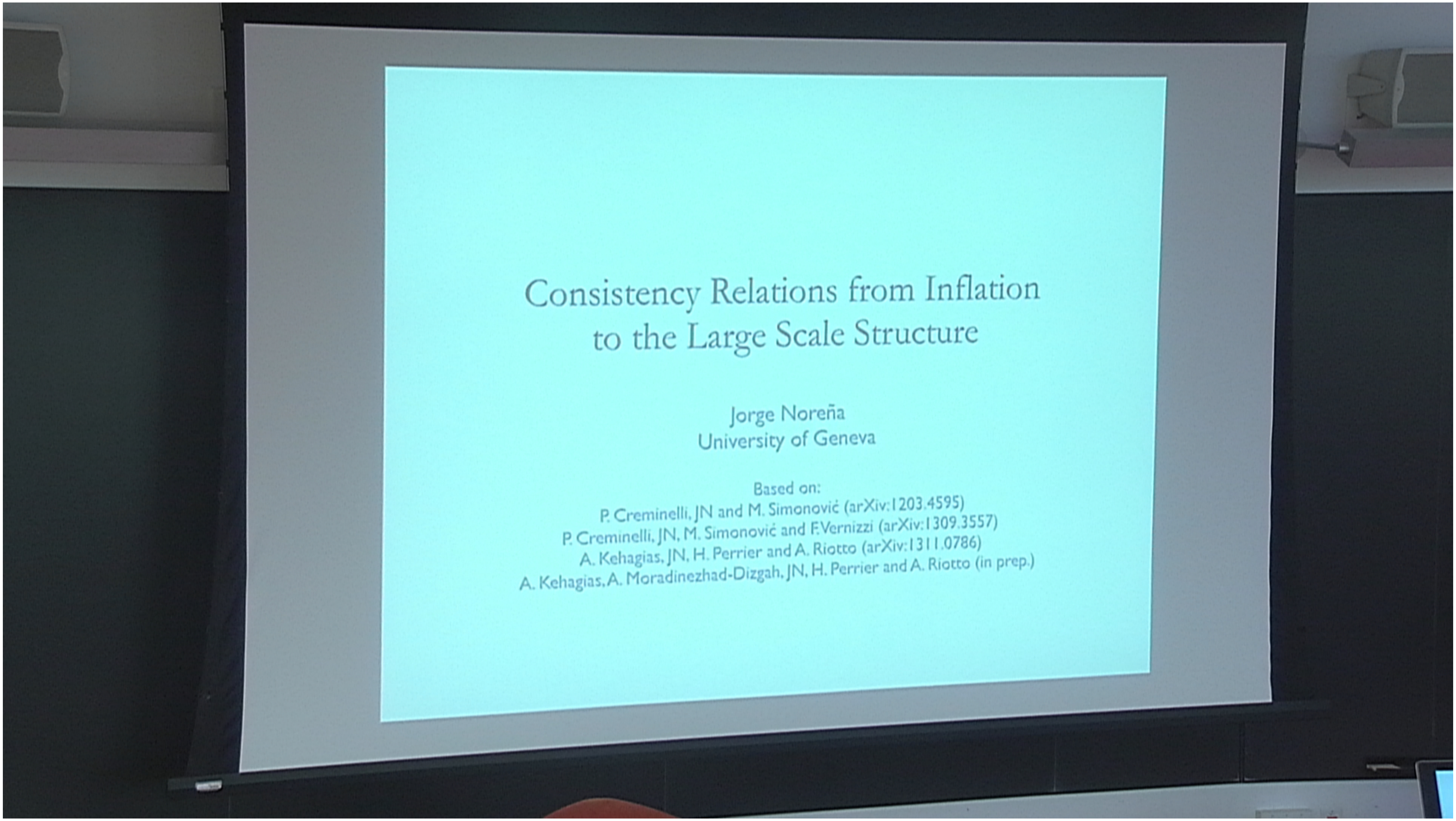
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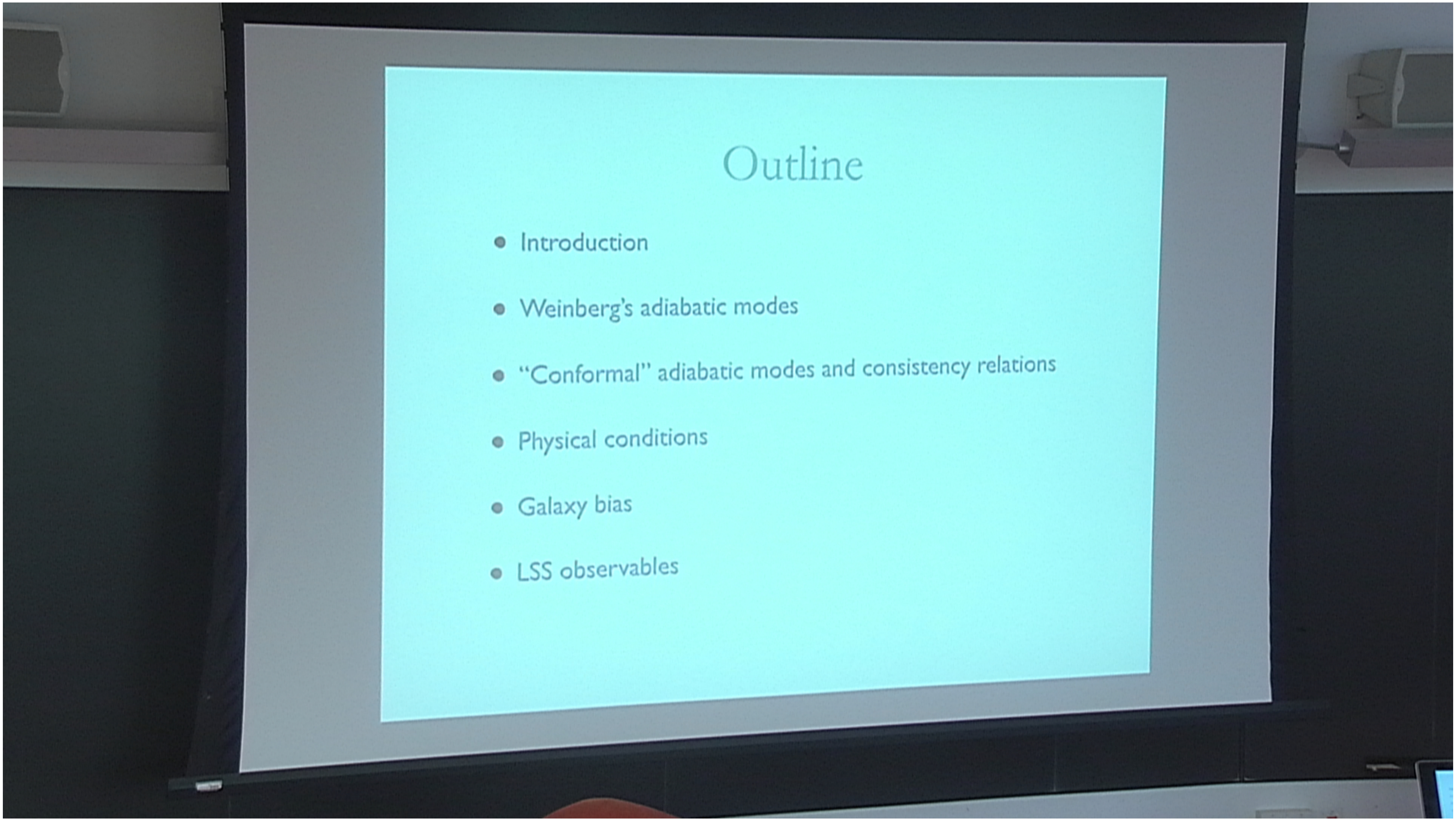
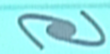

Outline
Introduction
Weinberg's adiabatic modes
"Conformal" adiabatic modes and consistency relations
Physical conditions
Galaxy bias
LSS observables

Introduction

A homogeneus gravitational potential has no physical meaning



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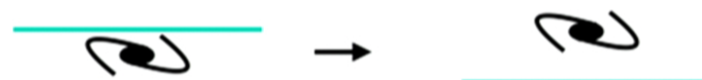

A homogeneus gravitational force can be set to zero by going to a freely falling frame

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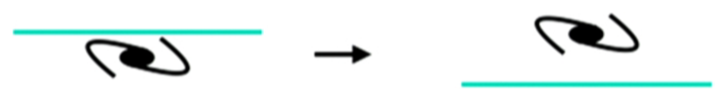

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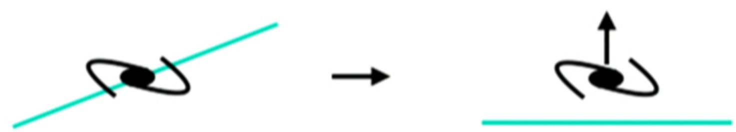

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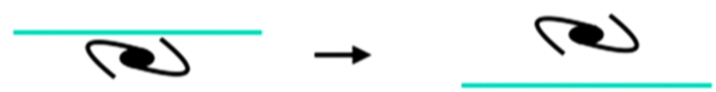

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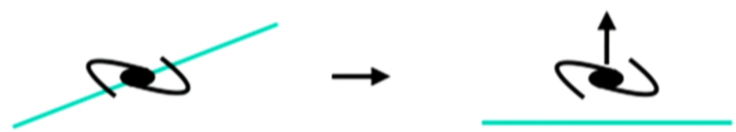

Introduction

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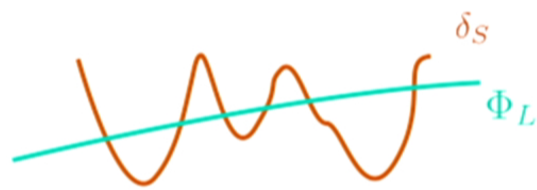

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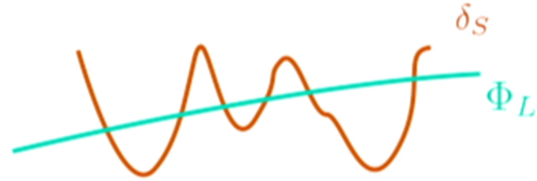

$$\langle \Phi(\mathbf{q})\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle \overset{q\to 0}{=} \langle \Phi(\mathbf{q})\langle\delta(\mathbf{k}_1)\ldots\delta(\mathbf{k}_n)\rangle_L\rangle$$

Indeed, one can use this to write consistency relations for the large scale structure in the Newtonian limit.

A. Kehagias and A. Riotto, 2013

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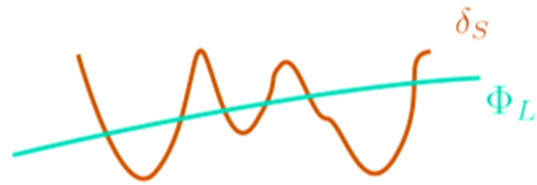

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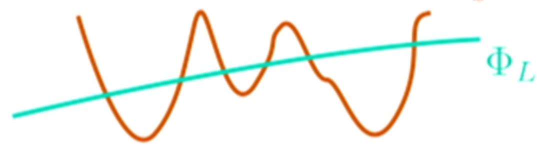

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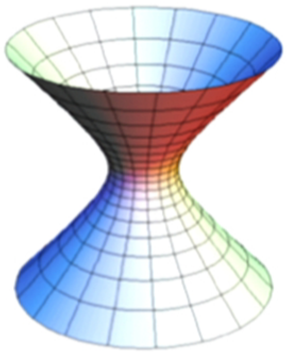
$$ds^2 = -dt^2 + e^{2Ht}dx^2$$



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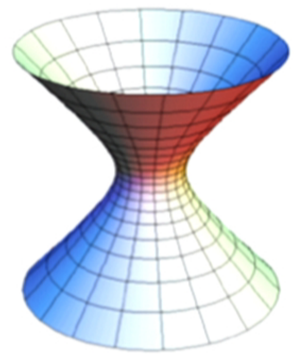

de Sitter has 10 isometries:

- 3 spatial translations
- 3 spatial rotations

- 1 dilation

$$t \to t - H^{-1}\log\lambda\,, \quad x^i \to \lambda x^i$$

- 3 special conformal

$$t \to t - 2H^{-1}\vec{b}\cdot\vec{x}\,,$$
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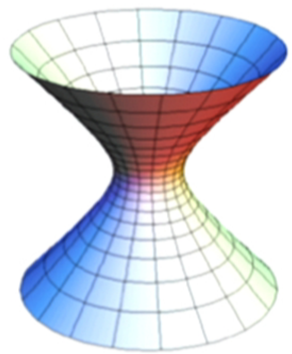

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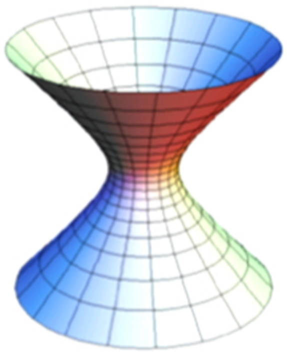

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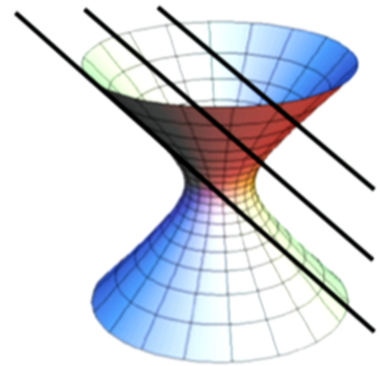

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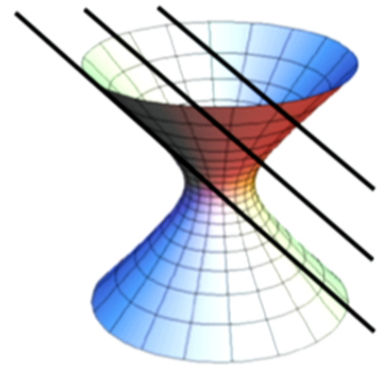

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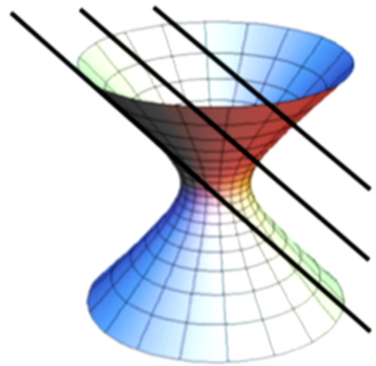

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Weinberg's Adiabatic Modes

Let us begin by writing the unperturbed flat FLRW metric

$$\mathrm{d}s^2 = a^2(\tau)\big[-\mathrm{d}\tau^2 + \mathrm{d}x^2\big]$$

And we make the following coordinate transformation

$$\tau \mapsto \tilde{\tau} = \tau + \epsilon(\tau)$$

$$x^i \mapsto \tilde{x}^i = (1+\lambda)x^i$$

S. Weinberg, 2003

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For inflation, out of the horizon

$$\boxed{\epsilon = 0 \qquad \lambda = \zeta_L}$$

S. Weinberg, 2003

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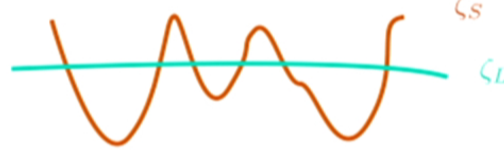
Let us first consider inflation and study perturbations outside of the horizon assuming scale invariance

Let us split perturbations into a long-wavelength piece which is nearly constant and a short-wavelength piece

$$\zeta(x) = \zeta_L + \zeta_s(x)$$



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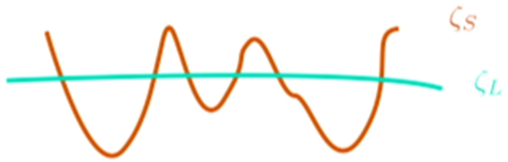

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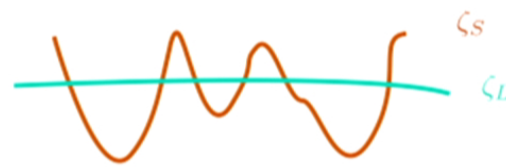

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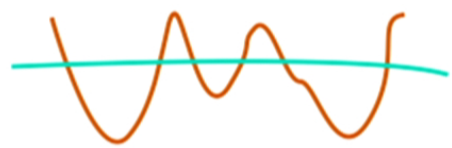

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There is no correlation between long and short modes.

$$\langle \zeta(q)\zeta(k_1)\ldots\zeta(k_n)\rangle \overset{q\to 0}{=} \langle \zeta_L \langle \zeta(k_1)\ldots\zeta(k_n)\rangle_{\zeta_L}\rangle = 0 + \mathcal{O}(q/k) + \mathcal{O}(\epsilon,\eta)$$

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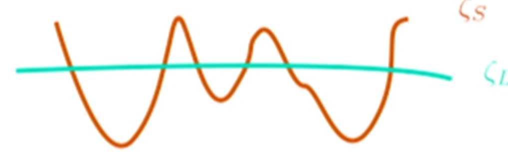

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Additional Adiabatic Modes

Let us do the same to induce a constant gradient

$$\mathrm{d}s^2 = a^2(\tau)\big[-\mathrm{d}\tau^2 + \mathrm{d}x^2\big]$$

And we make the following coordinate transformation

$$\tau \mapsto \tilde{\tau} = \tau + \vec{x}\cdot\vec{\xi}'(\tau)$$

$$x^i \mapsto \tilde{x}^i = x^i + 2\vec{x}\cdot\vec{b}\,x^i - x^2 b^i + \xi^i(\tau)$$

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Let us split perturbations into a long-wavelength piece which is nearly constant and a short-wavelength piece

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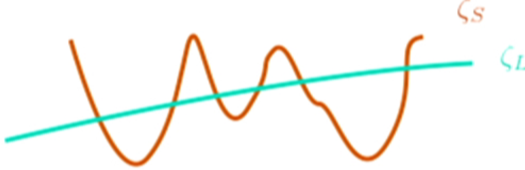

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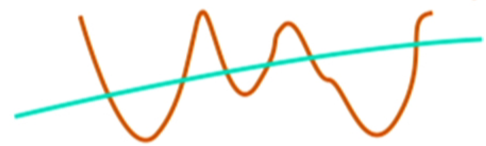

Let's do the derivation step by step

$$\begin{aligned} \langle \zeta(q)\zeta(k_1)\ldots\zeta(k_n)\rangle &\overset{q\to 0}{=} \langle \zeta_L \langle \zeta(k_1)\ldots\zeta(k_n)\rangle_{\partial\zeta_L}\rangle \\ &= \langle \zeta_L \langle \tilde{\zeta}(k_1)\ldots\tilde{\zeta}(k_n)\rangle\rangle \\ &= -\frac{1}{2}P(q)q^i D_i \langle \zeta(k_1)\ldots\zeta(k_n)\rangle\rangle\big(1+\mathcal{O}(q^2/k^2)\big) \end{aligned}$$

$$D_i \equiv \sum_{a=1}^{n} \left(6\partial^i_{k_a} - k^i_a \vec{\partial}^2_{k_a} + 2\vec{k}_a \cdot \vec{\partial}_{k_a} \partial^i_{k_a}\right)$$

Putting everything together

When scale invariance is not assumed:

$$\langle \zeta_{\vec{q}} \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' \overset{q \to 0}{=} -P(q) \left[3(n-1) + \sum_a \vec{k}_a \cdot \vec{\partial}_{k_a} + \frac{1}{2} q^i D_i \right] \langle \zeta_{\vec{k}_1} \ldots \zeta_{\vec{k}_n} \rangle' + \mathcal{O}(q/k)^2$$

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J. Maldacena, 2003

P. Creminelli and M. Zaldarriaga, 2004

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$$\Phi_L = \epsilon' + \mathcal{H}\epsilon + \vec{x}\cdot\vec{\xi}'' + \mathcal{H}\vec{x}\cdot\vec{\xi}'$$

$$\Psi_L = -\lambda - \mathcal{H}\epsilon - \vec{x}\cdot\vec{b} - \mathcal{H}\vec{x}\cdot\vec{\xi}'$$

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This is Einstein's elevator.

Large Scale Structure

Solving for the physical conditions gives a rather complicated coordinate transformation. That in Einstein-de Sitter looks like

$$\tilde{\eta} = \eta\Big(1 + \frac{1}{3}\Phi_L + \frac{1}{3}\partial_i\Phi_L x^i\Big)\ ,$$
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One can make the same construction as before for the dark matter overdensity

$$\langle\Phi_{\vec{q}}\,\delta_{\vec{k}_1}(\eta_1)\cdots\delta_{\vec{k}_n}(\eta_n)\rangle'_{q\to 0} = P_\Phi(q)\left[3n - 5 + \frac{1}{3}\sum_a\Big(5\vec{k}_a\cdot\vec{\partial}_{k_a} + \eta_a\partial_{\eta_a}\Big)\right.$$
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$$\cancel{\Phi} \qquad \cancel{\vec{v} \sim \nabla\Phi} \qquad \nabla_i \nabla_j \Phi \qquad \cdots$$

Thus we can write generally

$$\delta_g = b_1\delta + b_2\delta^2 + c_{\nabla^2}\nabla^2\delta + c_s s_{ij}s^{ij} + \ldots$$

$$s_{ij} \equiv \partial_i\partial_j\Phi - \frac{\delta_{ij}}{2}\Omega_m\mathcal{H}^2\delta$$

P. McDonald and A. Roy, 2009

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Assume now that different objects have a different large-scale velocity. For example due to a 5th force + screening.

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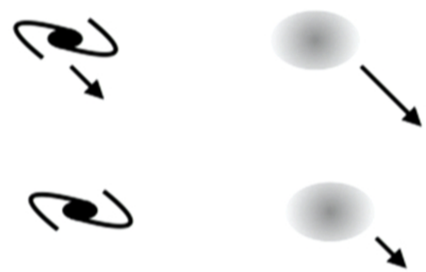

$$\vec{v}'_A + \mathcal{H}\vec{v}_A + (\vec{v}_A \cdot \nabla)\vec{v}_A = -\nabla\Phi$$

$$\vec{v}'_B + \mathcal{H}\vec{v}_B + (\vec{v}_B \cdot \nabla)\vec{v}_B = -\nabla\Phi - \alpha\nabla\varphi$$

$$\alpha \lesssim 10^{-3}$$

P. Creminelli, J. Gleyzes, L. Hui, M. Simonović, F. Vernizzi, 2014

Large Scale Structure observables

We observe the number density of galaxies in a direction $\hat{n}$ and a redshift z

$$N_g(z, \hat{n}) = \bar{N}_g(z)\big(1 + \Delta_g(z, \hat{n})\big)$$

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Redshift space distortions

$$z \mapsto z - (1+\bar{z})\left[\Phi_L + \boxed{\hat{n}\cdot\vec{V}_L} + 2\int_{\tau_o}^{\tau_e} \mathrm{d}\hat{\tau}\,\Phi'_L\right]$$

$$\hat{n} \mapsto \hat{n} + \boxed{2\int_{\tau_o}^{\tau_e} \mathrm{d}\tilde{\tau}\,\partial_i^{\perp}\Phi_L}$$

Lensing

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Thus we can compute the galaxy number over-density in the presence of the long-wavelength mode

$$\Delta_g(\hat{n}, z)|_L = \Delta_g(\hat{\bar{n}}, \bar{z}) + (1 + \Delta_g(\bar{z}, \hat{\bar{n}}))\,(e\delta z + t\delta\mathcal{D}_L + \delta V)$$

$$+ \delta z \frac{\partial}{\partial z}\Delta_g(\hat{\bar{n}}, \bar{z}) + \delta\bar{n} \cdot \frac{\partial}{\partial \hat{n}}\Delta_g(\hat{\bar{n}}, \bar{z}) + \mathcal{O}\left(\frac{\nabla^2\Phi_L}{\mathcal{H}^2}\Delta_g\right)$$

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Contains relativistic corrections that look like $f_{NL} \sim 1$

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$$\langle a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}\rangle \stackrel{l_1\ll l_2,l_3}{=} \mathcal{G}^{m_1m_2m_3}_{l_1l_2l_3}\left[C^{\Delta_g d}_{l_1}(z_1,z_2) + C^{\Delta_g\delta z}_{l_1}(z_1,z_2)\frac{\partial}{\partial z_2}\right.$$

$$\left. + \frac{1}{2}(l_2(l_2+1) - l_1(l_1+1) - l_3(l_3+1))\int_{\tau_o}^{\tau_2} \mathrm{d}\tilde{\tau}\, C^{\Delta_g\Phi}_{l_1}(z_1, z(\tilde{\tau}))\right] C^{\Delta_g\Delta_g}_{l_3}(z_2,z_3)$$

Conclusions

- The curvature perturbation is endowed with a conformal symmetry that allows one to write consistency relations similar to Ward identities.
- The same argument can be made for scales out of the sound horizon but inside the Hubble horizon. This implies that one can write a consistency relation for the LSS.
- This consistency relation is valid at the non-perturbative level in the short modes.
- It can be written in terms of the galaxy number over-density.


THE
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