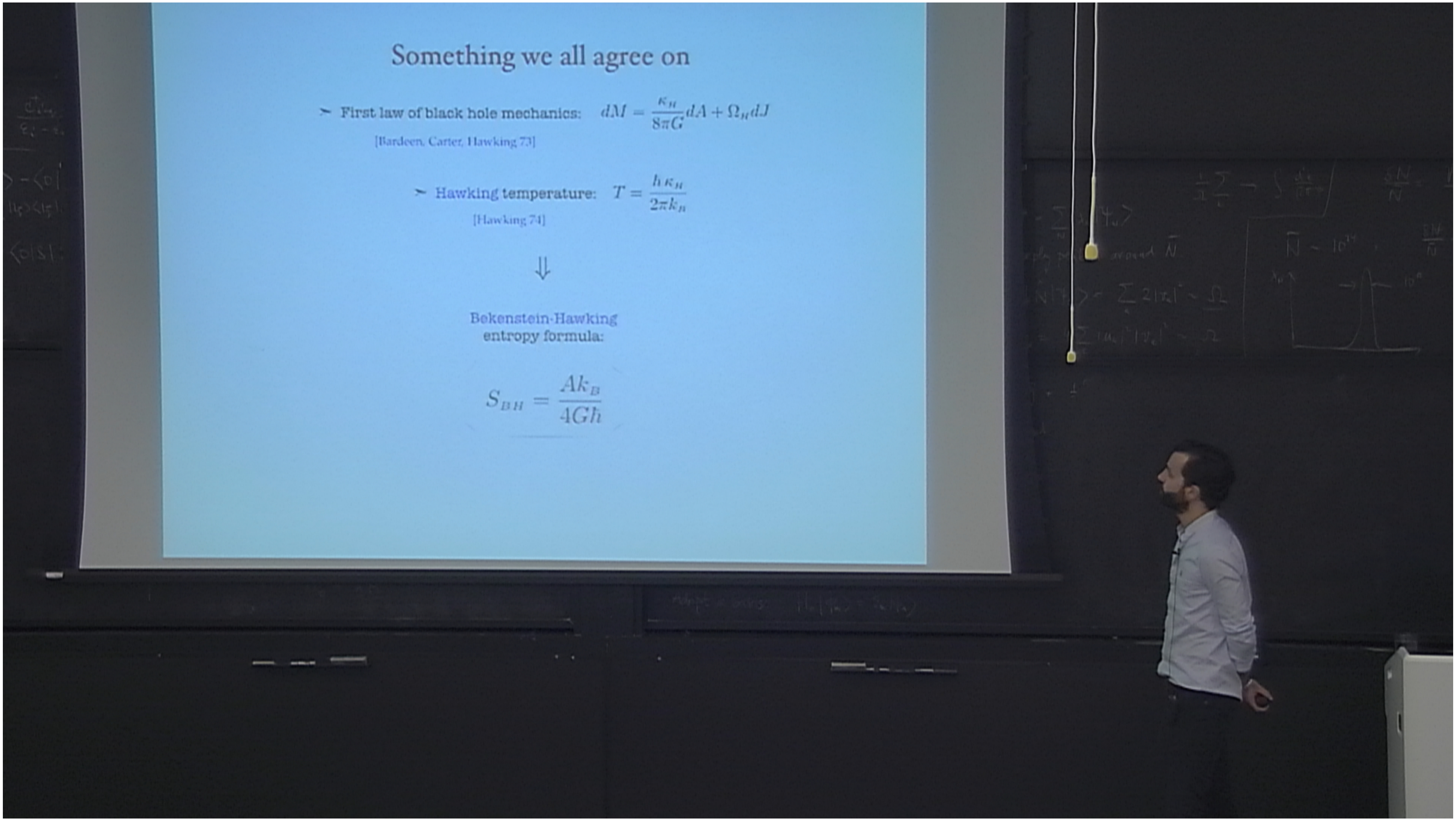
Title: CFT/Gravity Correspondence on the Isolated Horizon

Date: Nov 27, 2014 02:30 PM

URL: http://pirsa.org/14110158

Abstract: <p>A quantum isolated horizon can be modeled by an SU(2) Chern-Simons theory on a punctured 2-sphere. We show how a local 2-dimensional conformal symmetry arises at each puncture inducing an infinite set of new observables localized at the horizon which satisfy a Kac-Moody algebra. By means of the isolated horizon boundary conditions, we represent the gravitational fluxes degrees of freedom in terms of the zero modes of the Kac-Moody algebra defined on the boundary of a punctured disk. In this way, our construction encodes a precise notion of CFT/gravity correspondence. The higher modes in the algebra represent new non-geometric charges which can be represented in terms of free matter field degrees of freedom. When computing the CFT partition function of the system, these new states induce an extra degeneracy factor, representing the density of horizon states at a given energy level, which reproduces the Bekenstein's holographic bound for an imaginary Barbero-Immirzi parameter.</p>


Something we all agree on
First law of black hole mechanics: $dM = \frac{\kappa_H}{8\pi G} dA + \Omega_H dJ$
[Bardeen, Carter, Hawking 73]
Hawking temperature: $T = \frac{\hbar \kappa_H}{2\pi k_B}$
[Hawking 74]
$\Downarrow$
Bekenstein-Hawking entropy formula:
$S_{BH} = \frac{A k_B}{4G\hbar}$

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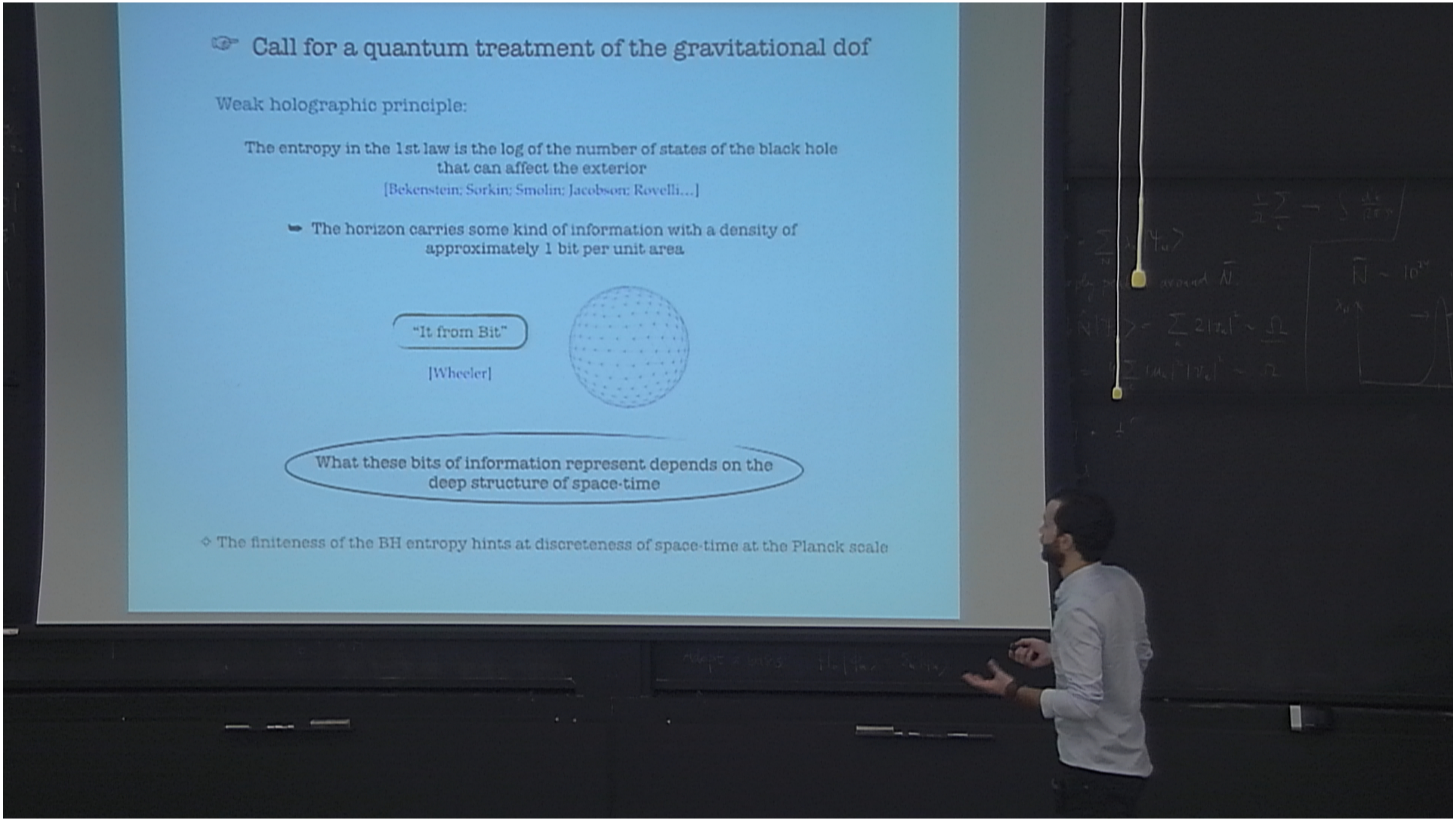
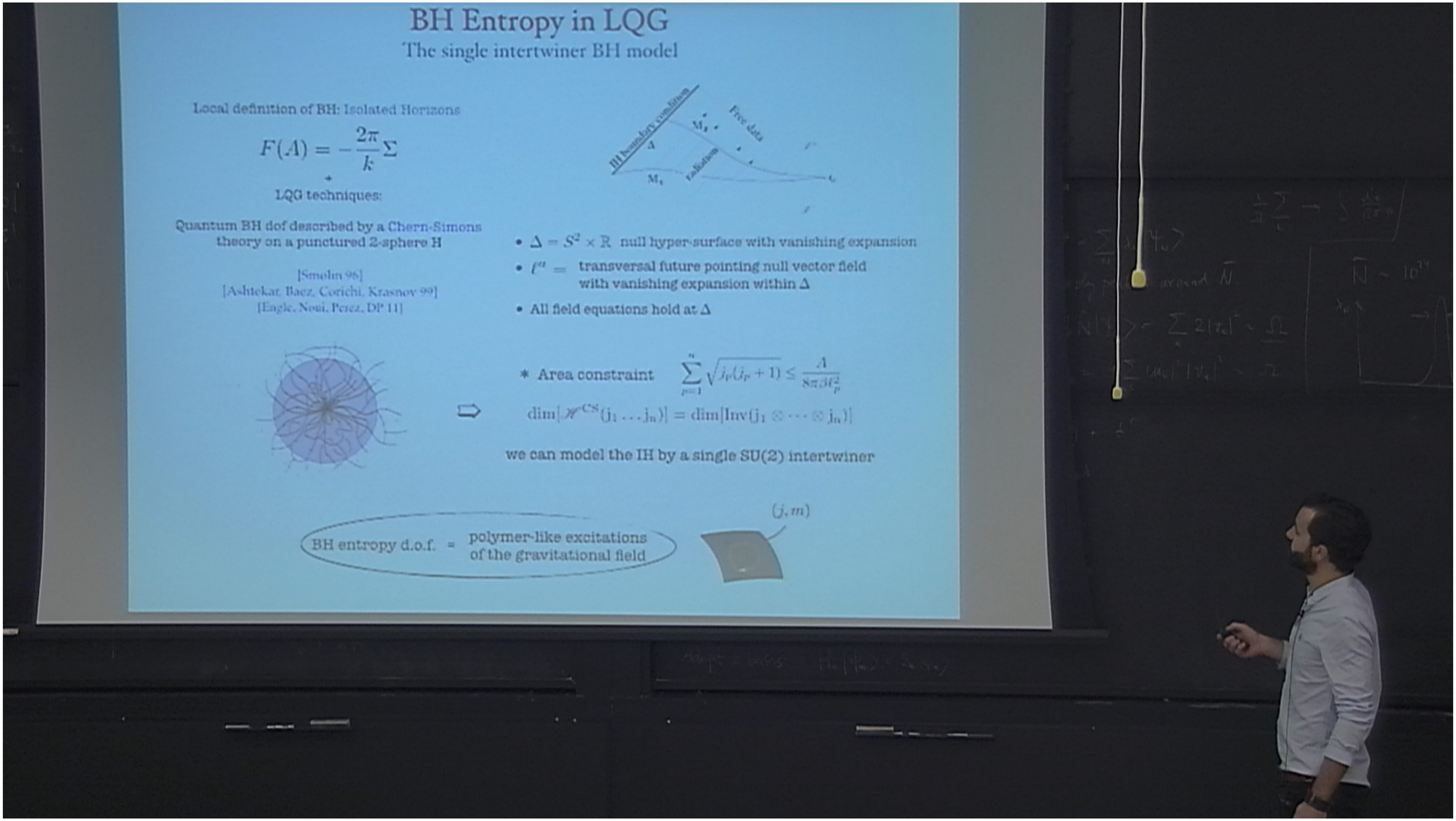

Call for a quantum treatment of the gravitational dof
Weak holographic principle:
The entropy in the 1st law is the log of the number of states of the black hole that can affect the exterior
[Bekenstein; Sorkin; Smolin; Jacobson; Rovelli...]
The horizon carries some kind of information with a density of approximately 1 bit per unit area
"It from Bit"
[Wheeler]
What these bits of information represent depends on the deep structure of space-time
The finiteness of the BH entropy hints at discreteness of space-time at the Planck scale


BH Entropy in LQG
The single intertwiner BH model
Local definition of BH: Isolated Horizons
$F(A) = -\frac{2\pi}{k}\Sigma$
LQG techniques:
Quantum BH dof described by a Chern-Simons theory on a punctured 2-sphere H
[Smolin 96]
[Ashtekar, Baez, Corichi, Krasnov 99]
[Engle, Noui, Perez, DP 11]
BH boundary condition
Free data
radiation
Δ
M_1
• $\Delta = S^2 \times \mathbb{R}$ null hyper-surface with vanishing expansion
• $\ell^a =$ transversal future pointing null vector field with vanishing expansion within Δ
• All field equations hold at Δ
∗ Area constraint $\sum_{p=1}^{n} \sqrt{j_p(j_p+1)} \leq \frac{A}{8\pi\beta\ell_P^2}$
$\dim[\mathcal{H}^{CS}(j_1 \dots j_n)] = \dim[\mathrm{Inv}(j_1 \otimes \cdots \otimes j_n)]$
we can model the IH by a single SU(2) intertwiner
BH entropy d.o.f. = polymer-like excitations of the gravitational field
(j, m)

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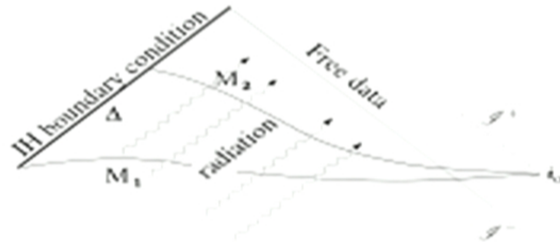

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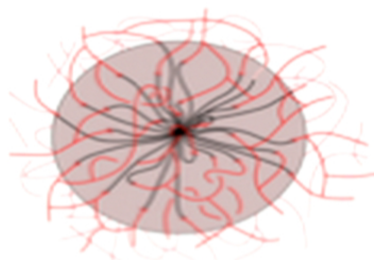

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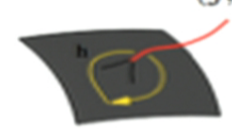

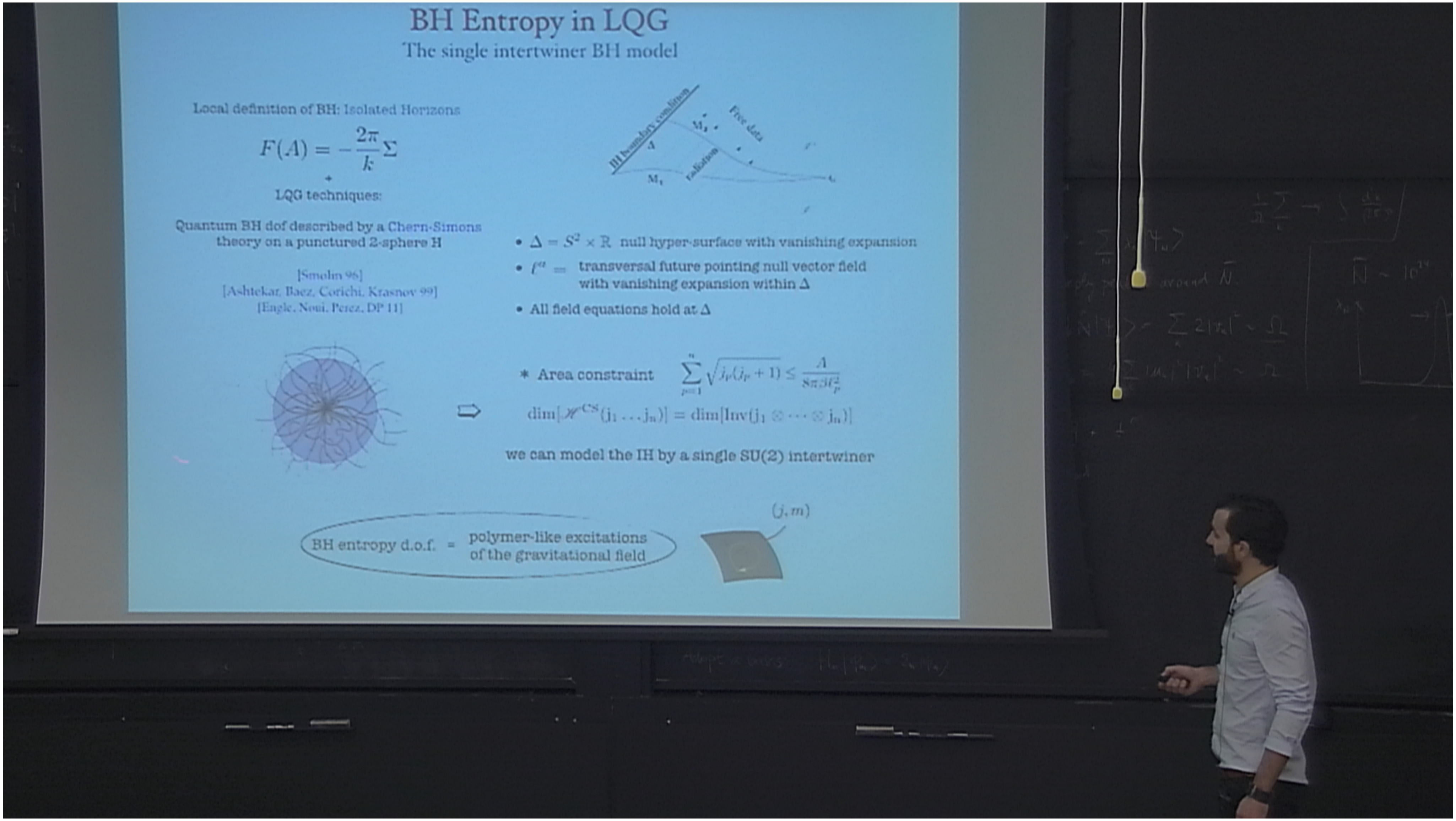
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"The nature of BH entropy is intimately related to the nature of BH temperature.
We cannot understand the one without the other." [Bill Unruh, Loops13]

* Local observer perspective + Unruh temp. by hand [Ghosh, Perez 11; Frodden, Ghosh, Perez 11]
* Analytic continuation to $\gamma = i$ [Frodden, Geiller, Noui, Perez 12]
* KMS-state of a quantum IH: $\beta_{IH} = 2\pi(1-1/k)$ $\Leftrightarrow$ $\gamma = i$ [DP 13] (see also [Bianchi 12])

$$\Rightarrow \quad S = \frac{A_{IH}}{4\ell_P^2} + \mu N$$

quantum hair argued to be associated to a new horizon microscopic observable

(call for a GFT description in order to make sense of it)

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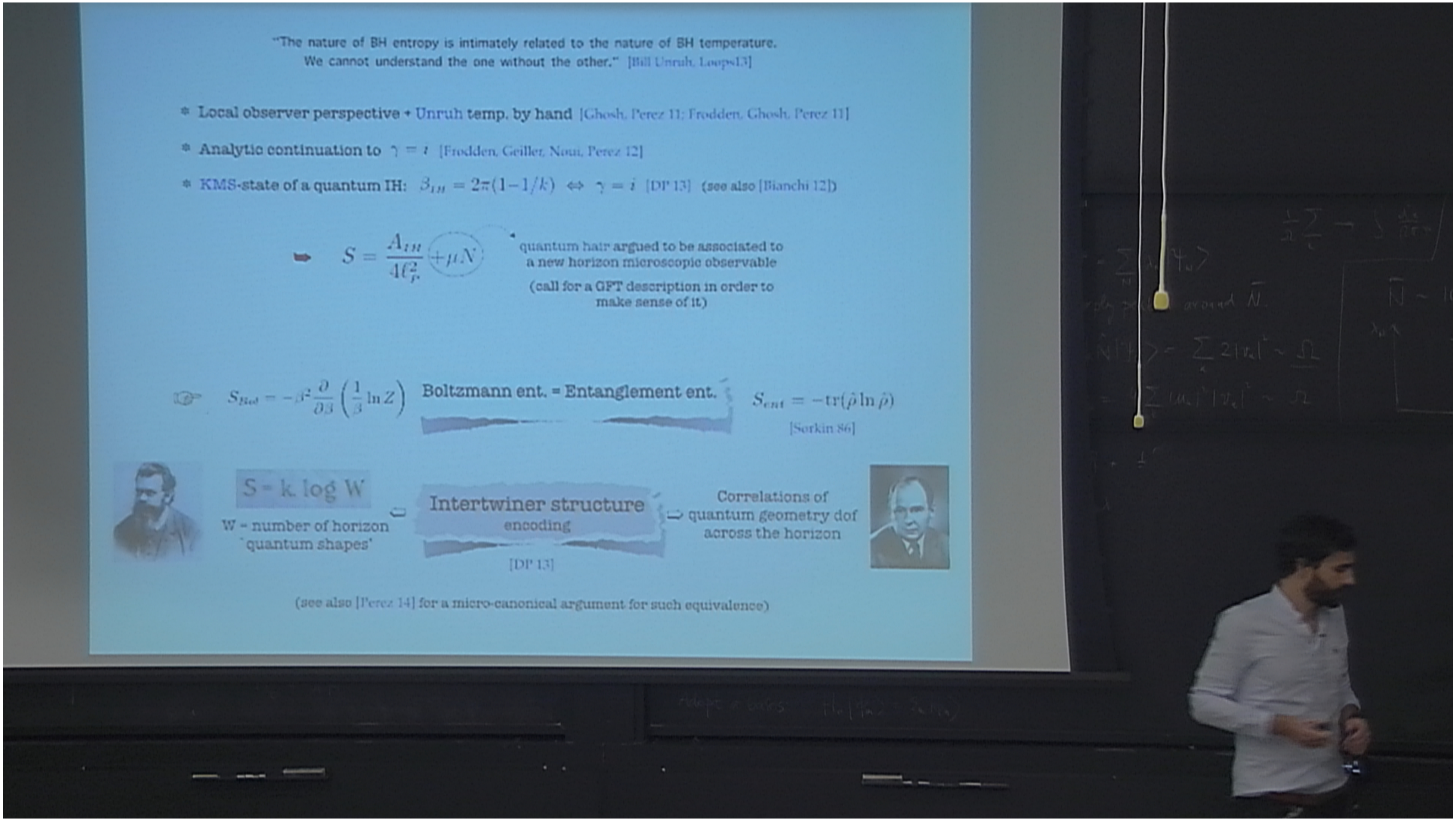
☞ $S_{Bol} = -\beta^2 \frac{\partial}{\partial\beta}\left(\frac{1}{\beta}\ln Z\right)$ **Boltzmann ent. = Entanglement ent.** $S_{ent} = -\mathrm{tr}(\hat\rho \ln \hat\rho)$ [Sorkin 86]

S = k. log W

W = number of horizon 'quantum shapes' $\Leftarrow$ **Intertwiner structure** encoding [DP 13] $\Rightarrow$ Correlations of quantum geometry dof across the horizon

(see also [Perez 14] for a micro-canonical argument for such equivalence)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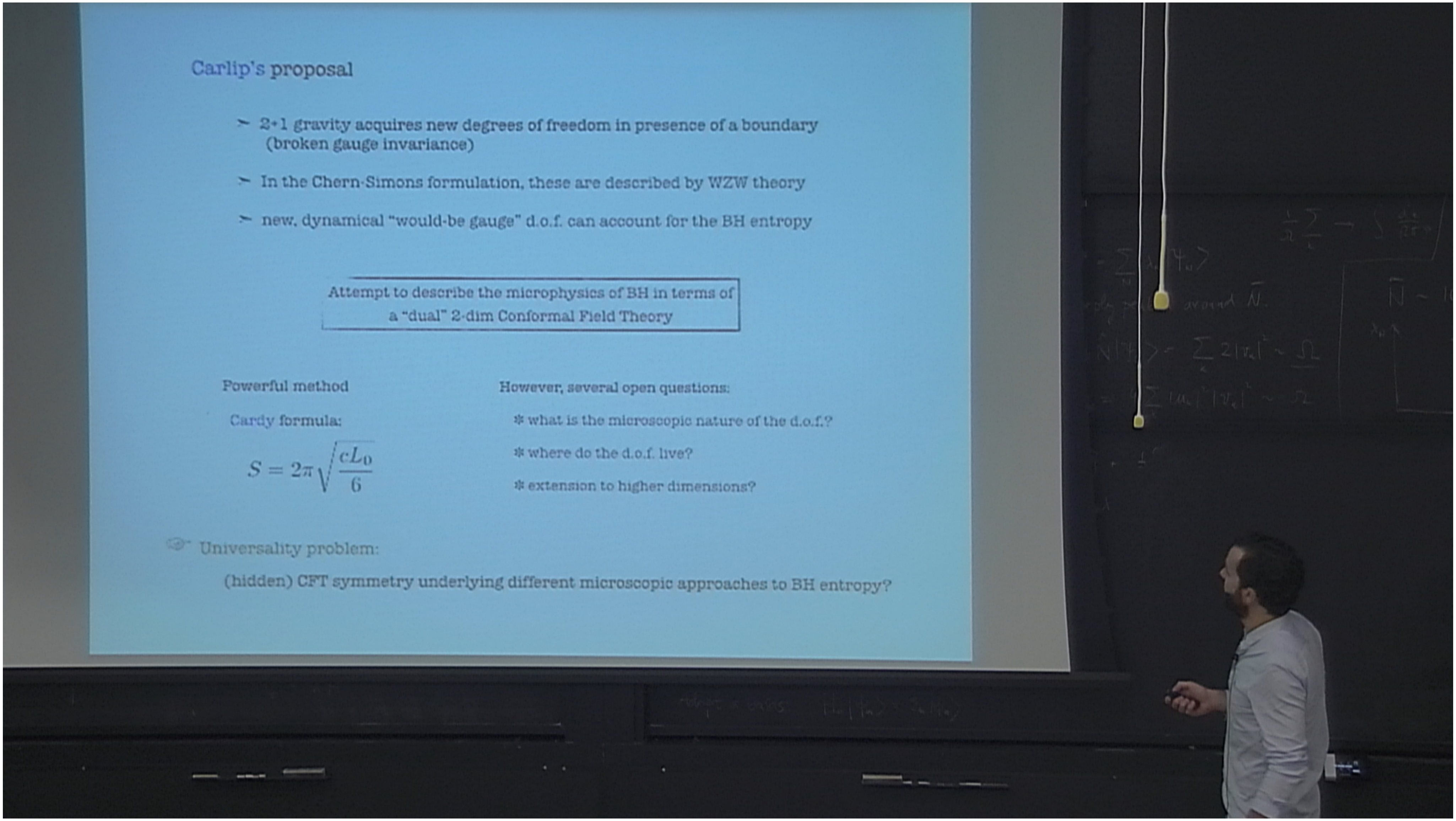
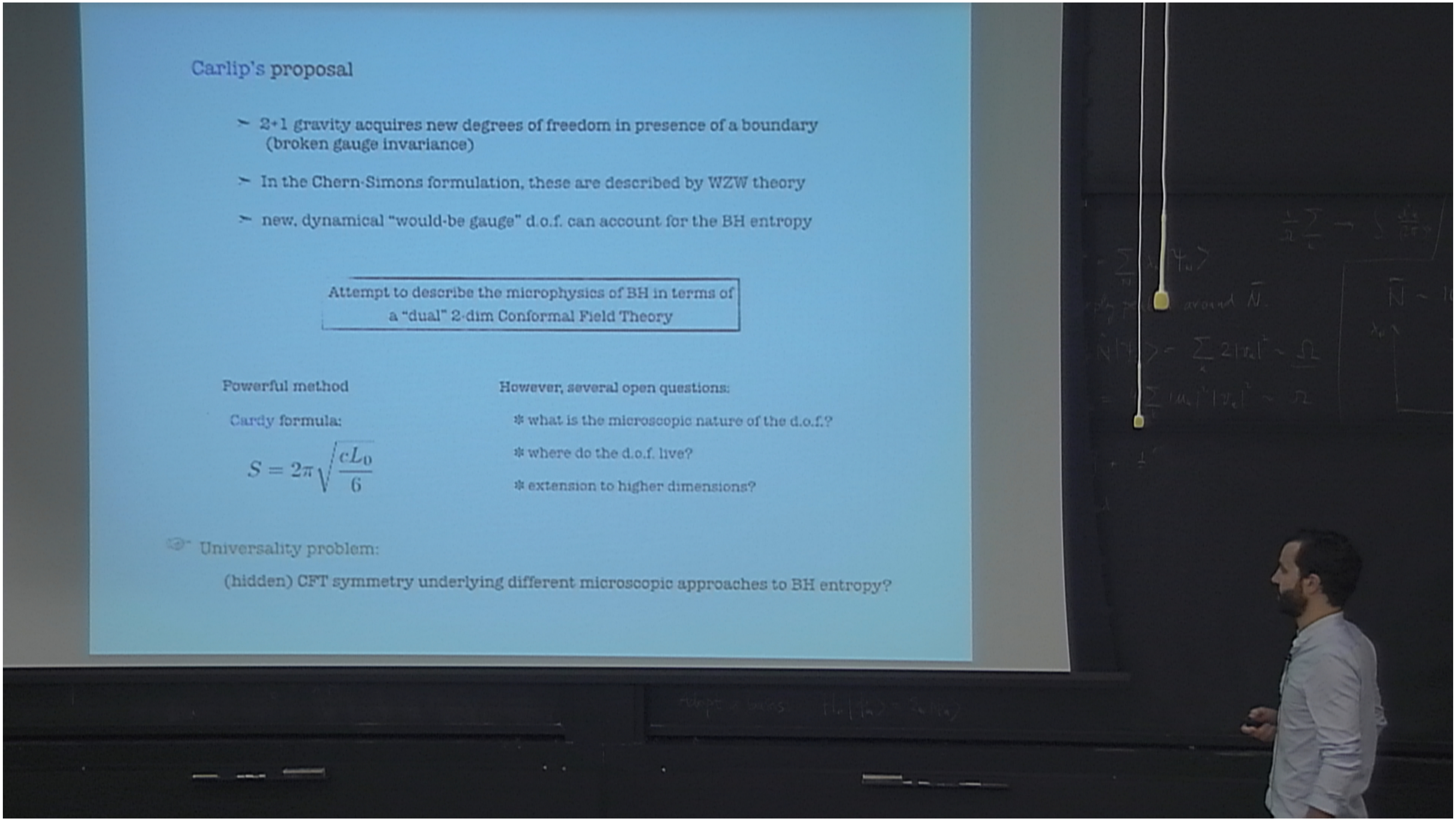

Carlip's proposal
➣ 2+1 gravity acquires new degrees of freedom in presence of a boundary (broken gauge invariance)
➣ In the Chern-Simons formulation, these are described by WZW theory
➣ new, dynamical "would-be gauge" d.o.f. can account for the BH entropy
Attempt to describe the microphysics of BH in terms of a "dual" 2-dim Conformal Field Theory
Powerful method
Cardy formula:
$S = 2\pi\sqrt{\frac{cL_0}{6}}$
However, several open questions:
✻ what is the microscopic nature of the d.o.f.?
✻ where do the d.o.f. live?
✻ extension to higher dimensions?
Universality problem:
(hidden) CFT symmetry underlying different microscopic approaches to BH entropy?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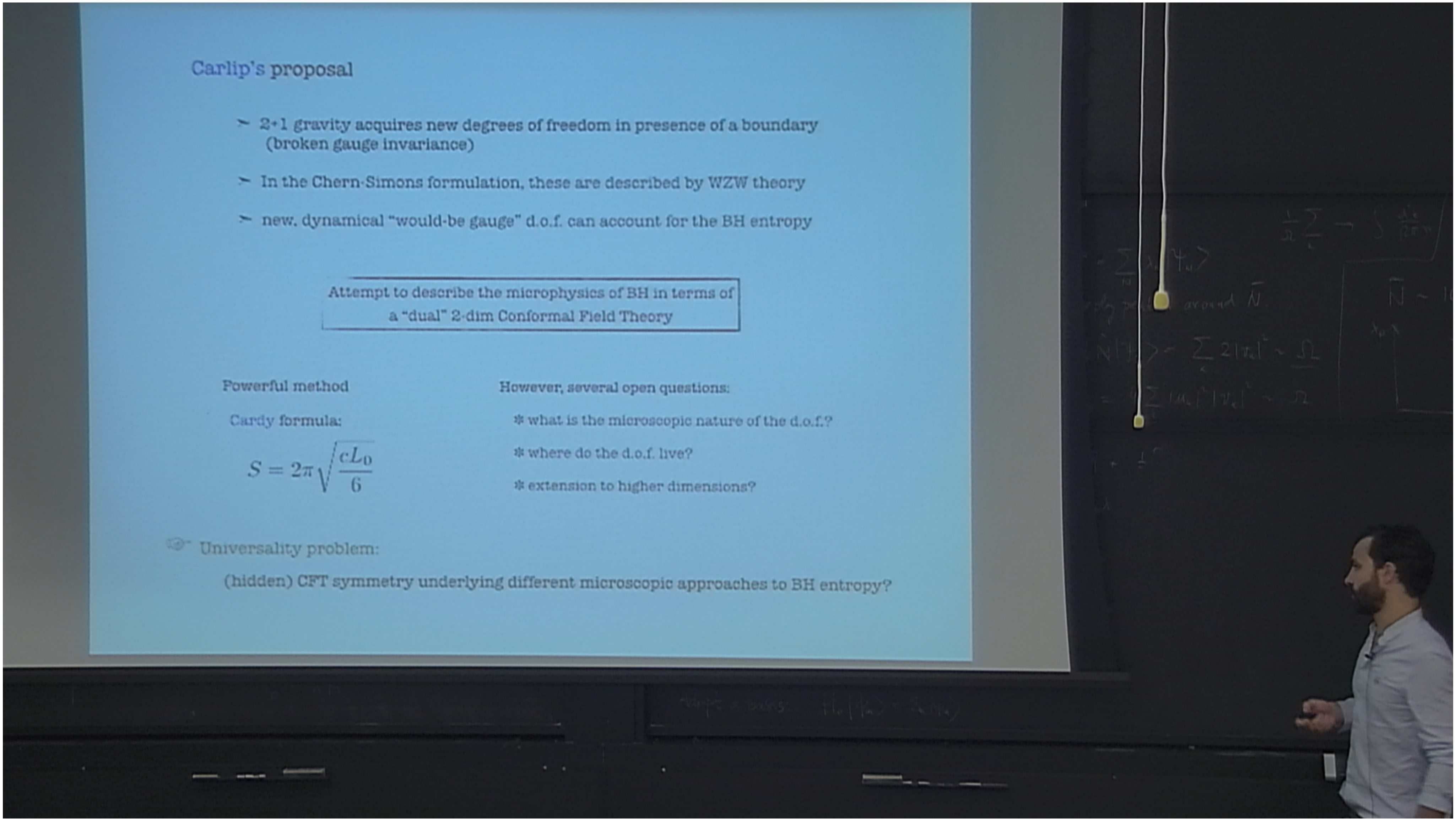
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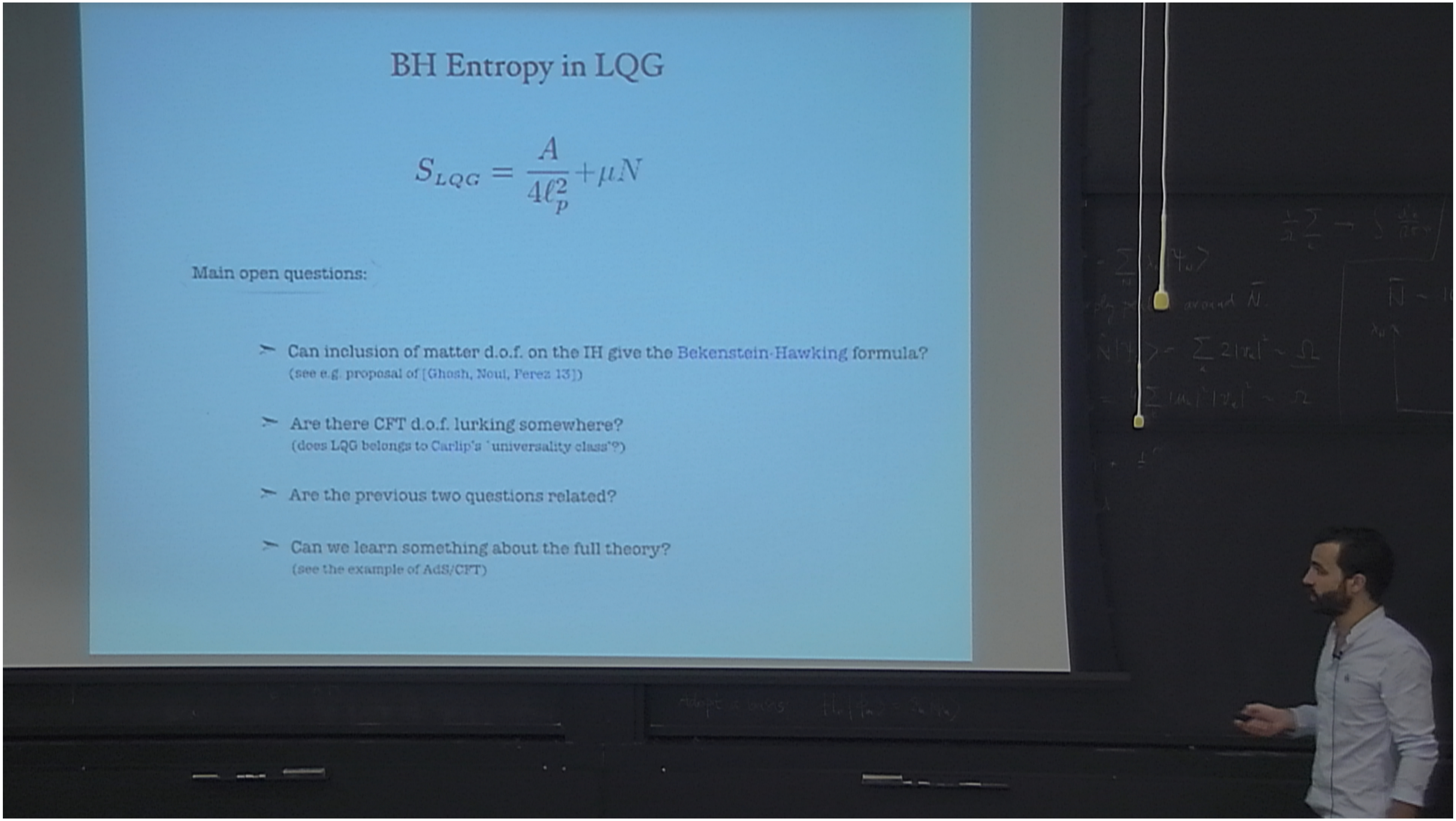
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Can inclusion of matter d.o.f. on the IH give the Bekenstein-Hawking formula?
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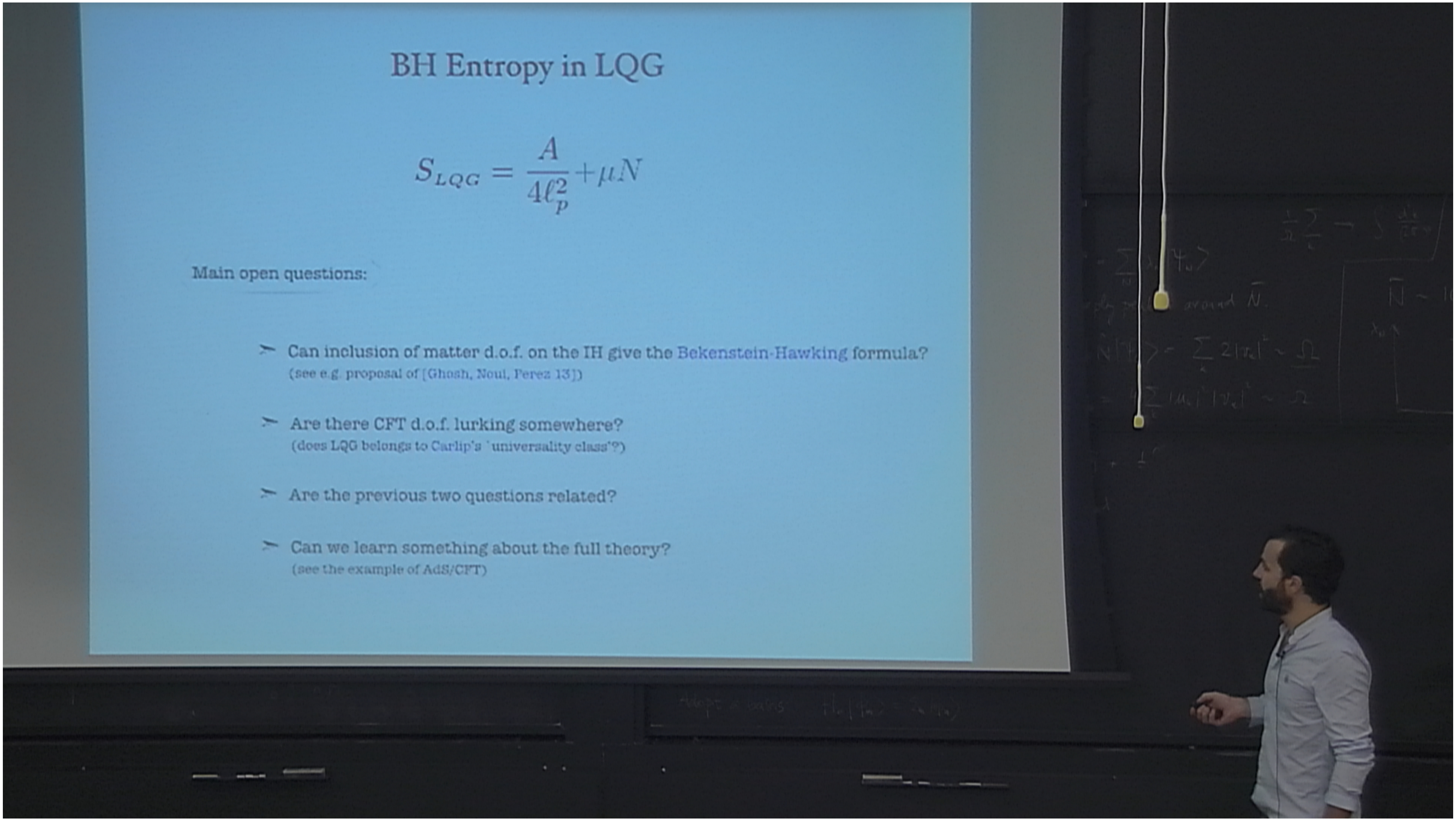
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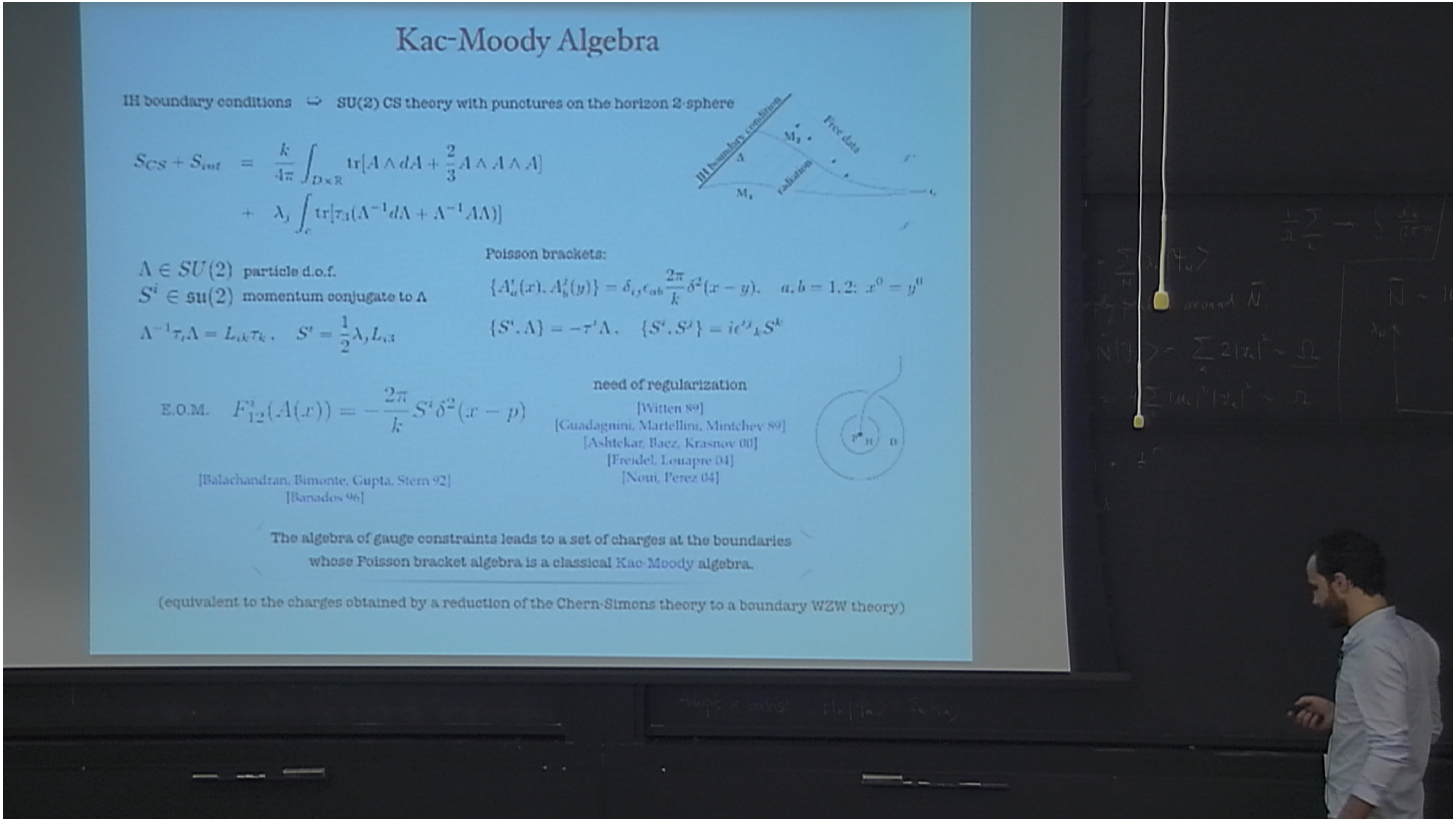
Kac-Moody Algebra
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$S_{CS} + S_{int} = \frac{k}{4\pi}\int_{D\times\mathbb{R}} \mathrm{tr}[A\wedge dA + \frac{2}{3}A\wedge A\wedge A] + \lambda_i \int_c \mathrm{tr}[\tau_3(\Lambda^{-1}d\Lambda + \Lambda^{-1}A\Lambda)]$
$\Lambda \in SU(2)$ particle d.o.f.
$S^i \in \mathfrak{su}(2)$ momentum conjugate to Λ
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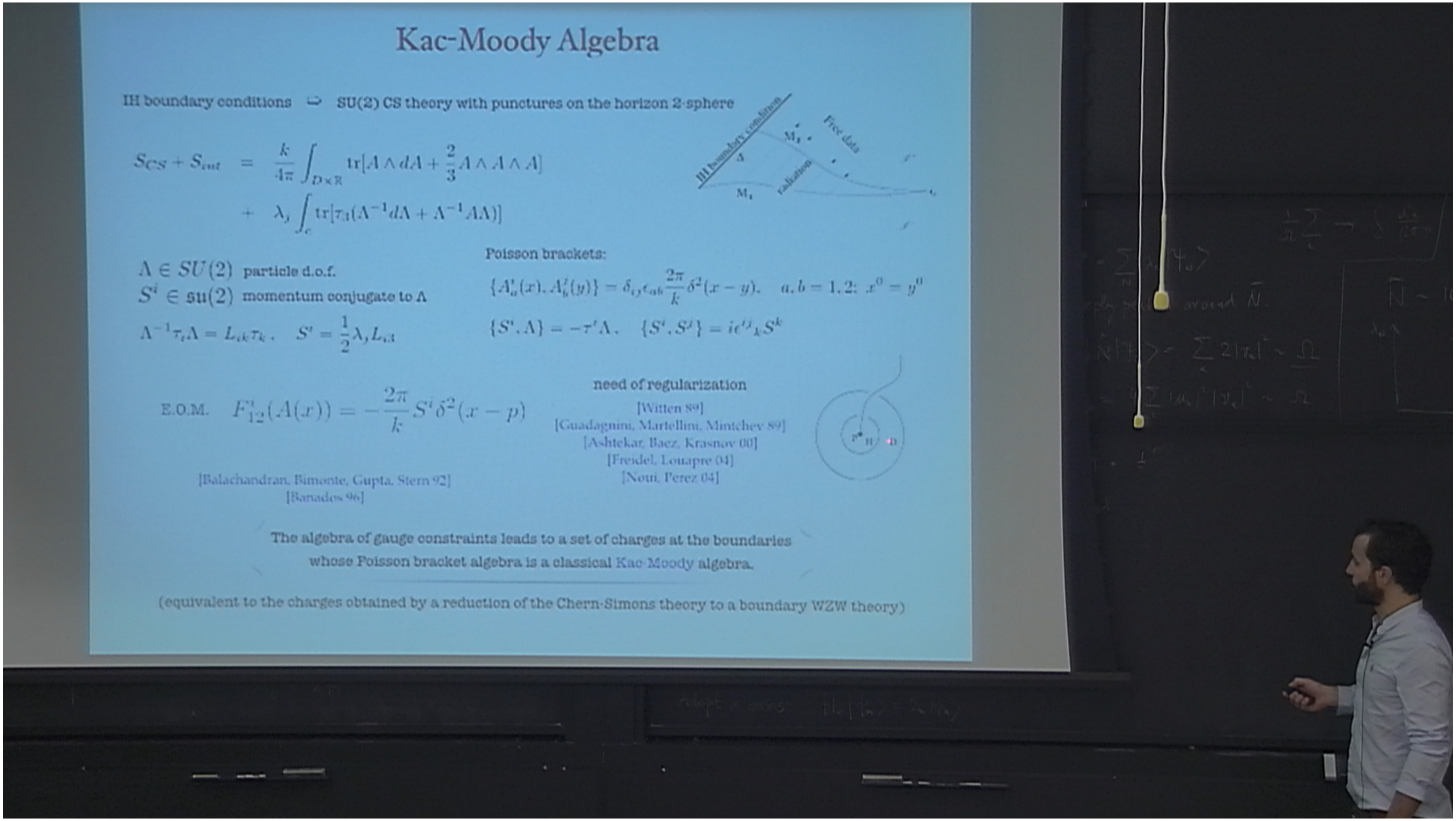
(equivalent to the charges obtained by a reduction of the Chern-Simons theory to a boundary WZW theory)



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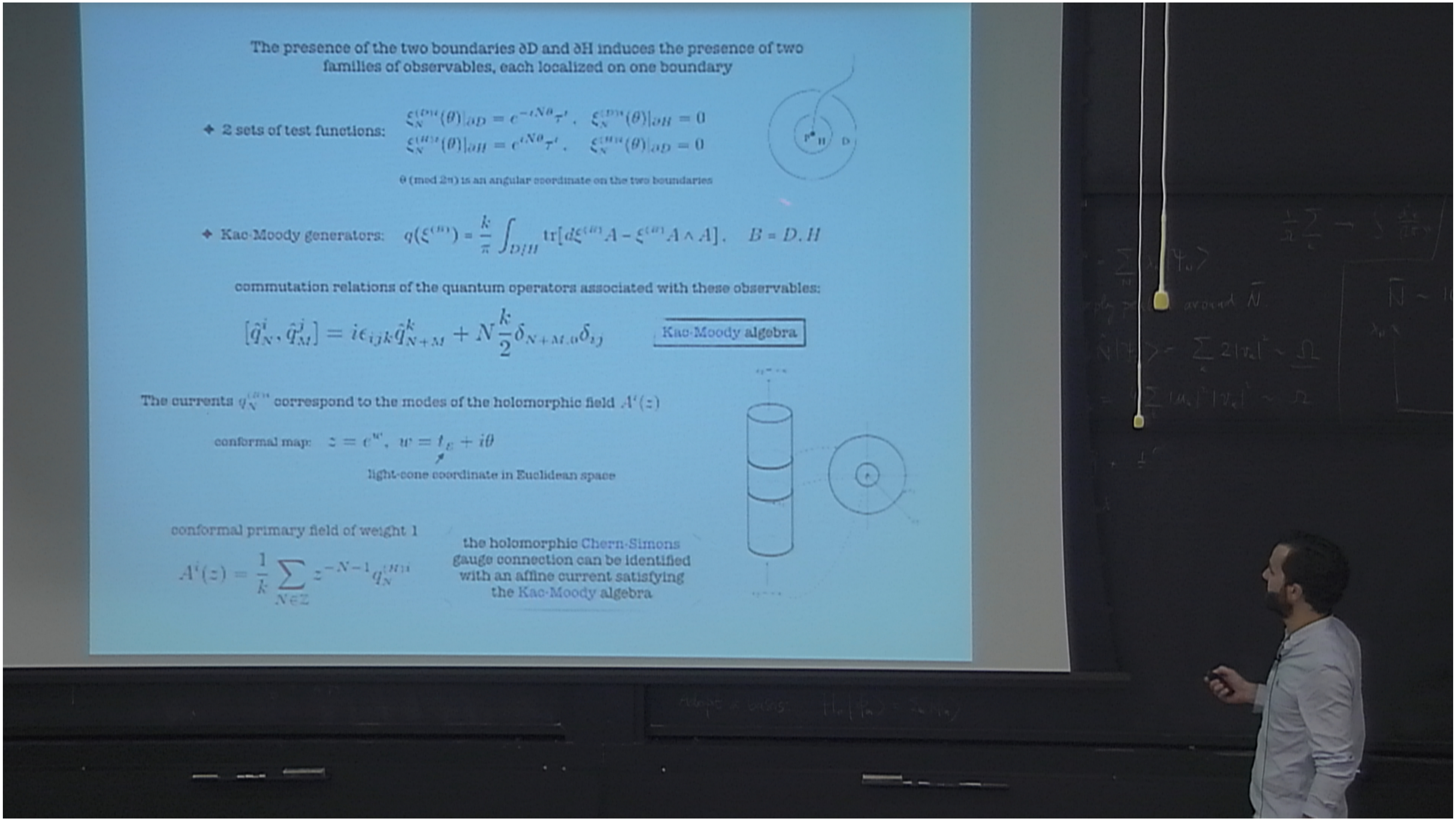

The presence of the two boundaries ∂D and ∂H induces the presence of two families of observables, each localized on one boundary
✦ 2 sets of test functions:
$\xi_N^{(D)i}(\theta)|_{\partial D} = e^{-iN\theta}\tau^i\,, \quad \xi_N^{(D)i}(\theta)|_{\partial H} = 0$
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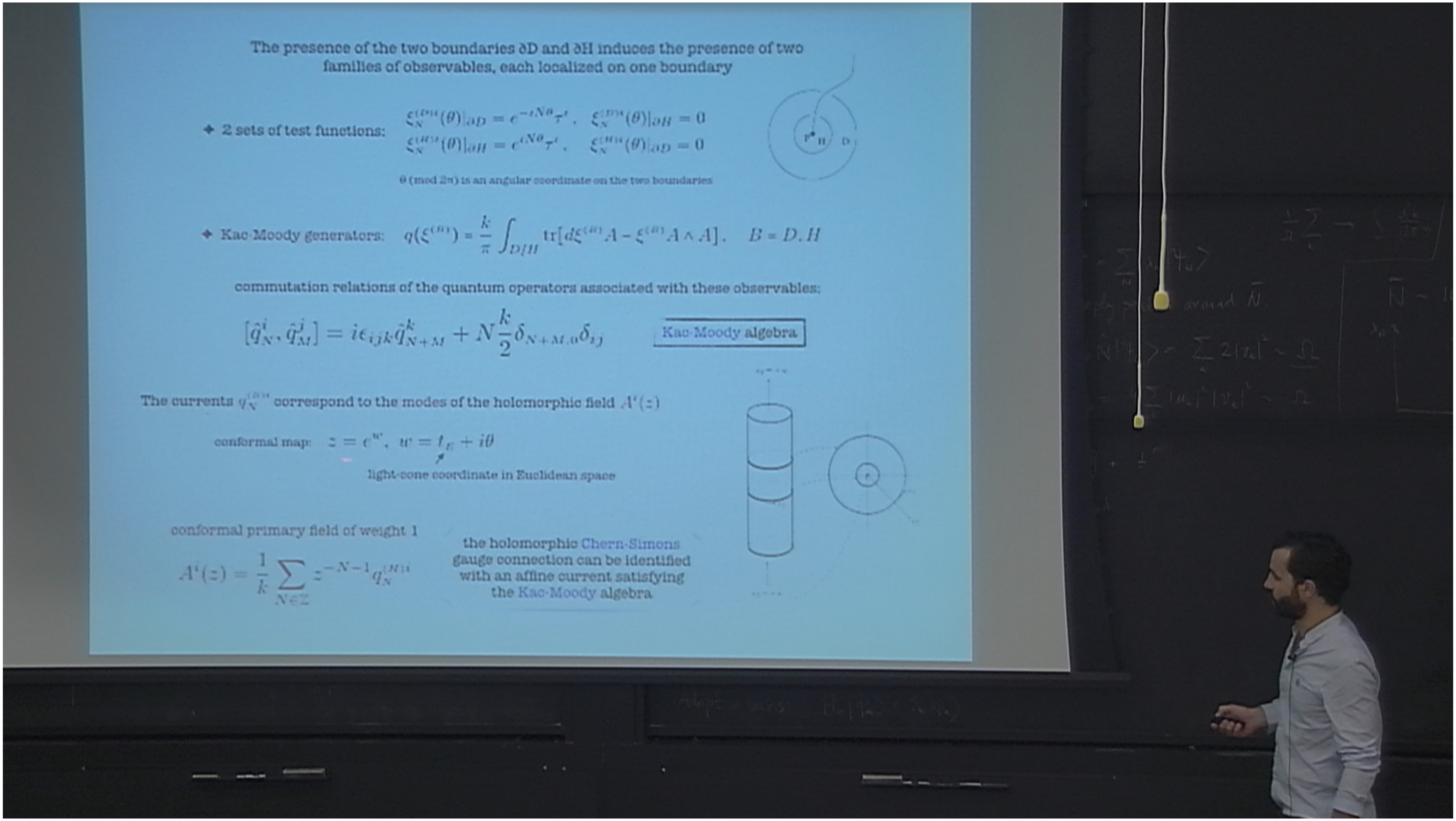
θ (mod 2π) is an angular coordinate on the two boundaries
✦ Kac-Moody generators:
$q(\xi^{(B)}) = \frac{k}{\pi}\int_{D/H} \mathrm{tr}[d\xi^{(B)}A - \xi^{(B)}A\wedge A]\,, \quad B = D, H$
commutation relations of the quantum operators associated with these observables:
$[\hat{q}^i_N, \hat{q}^j_M] = i\epsilon_{ijk}\hat{q}^k_{N+M} + N\frac{k}{2}\delta_{N+M,0}\delta_{ij}$
Kac-Moody algebra
The currents $q_N^{(H)i}$ correspond to the modes of the holomorphic field $A^i(z)$
conformal map: $z = e^w, \; w = t_E + i\theta$
light-cone coordinate in Euclidean space
conformal primary field of weight 1
$A^i(z) = \frac{1}{k}\sum_{N\in\mathbb{Z}} z^{-N-1}q_N^{(H)i}$
the holomorphic Chern-Simons gauge connection can be identified with an affine current satisfying the Kac-Moody algebra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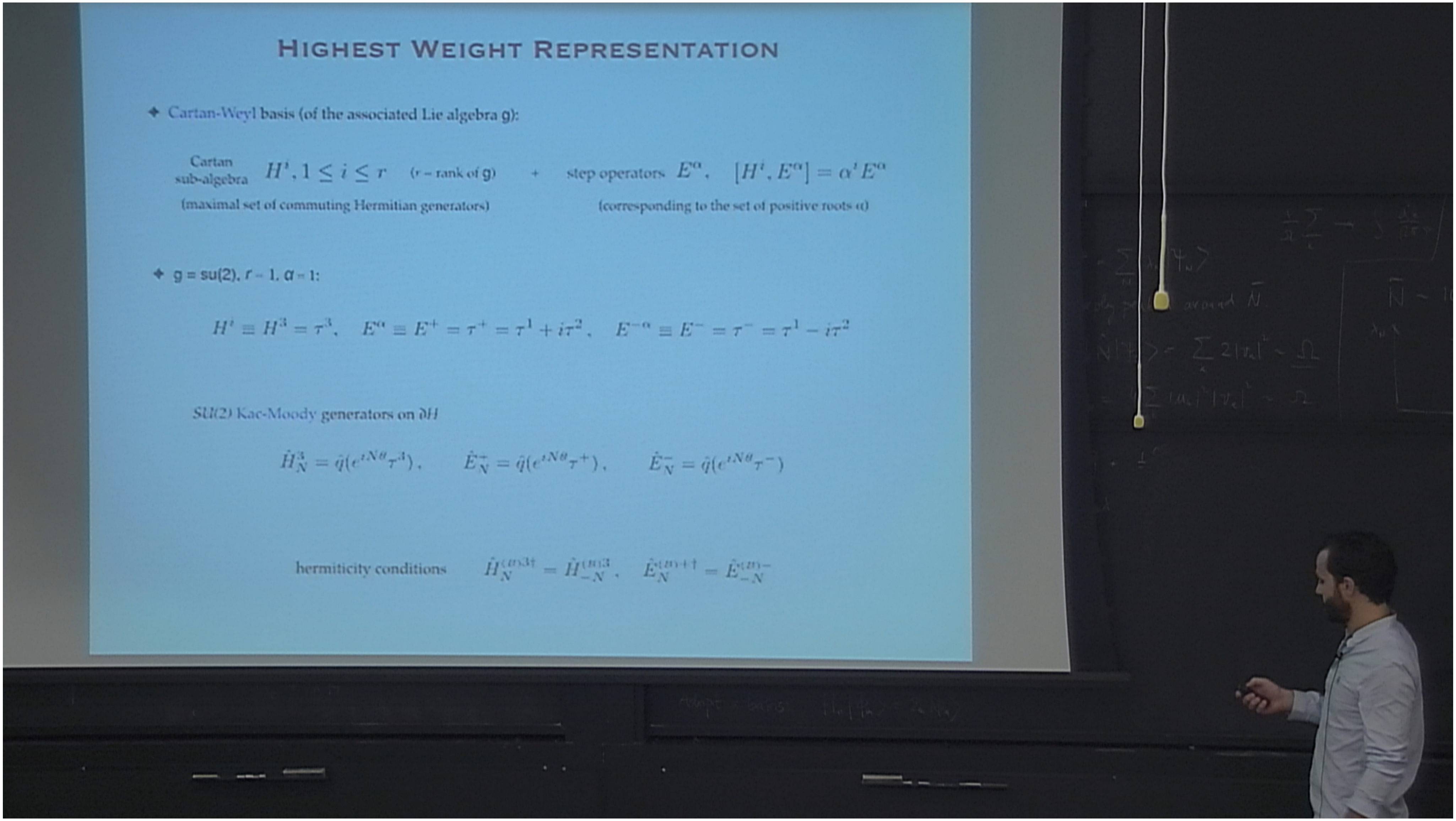
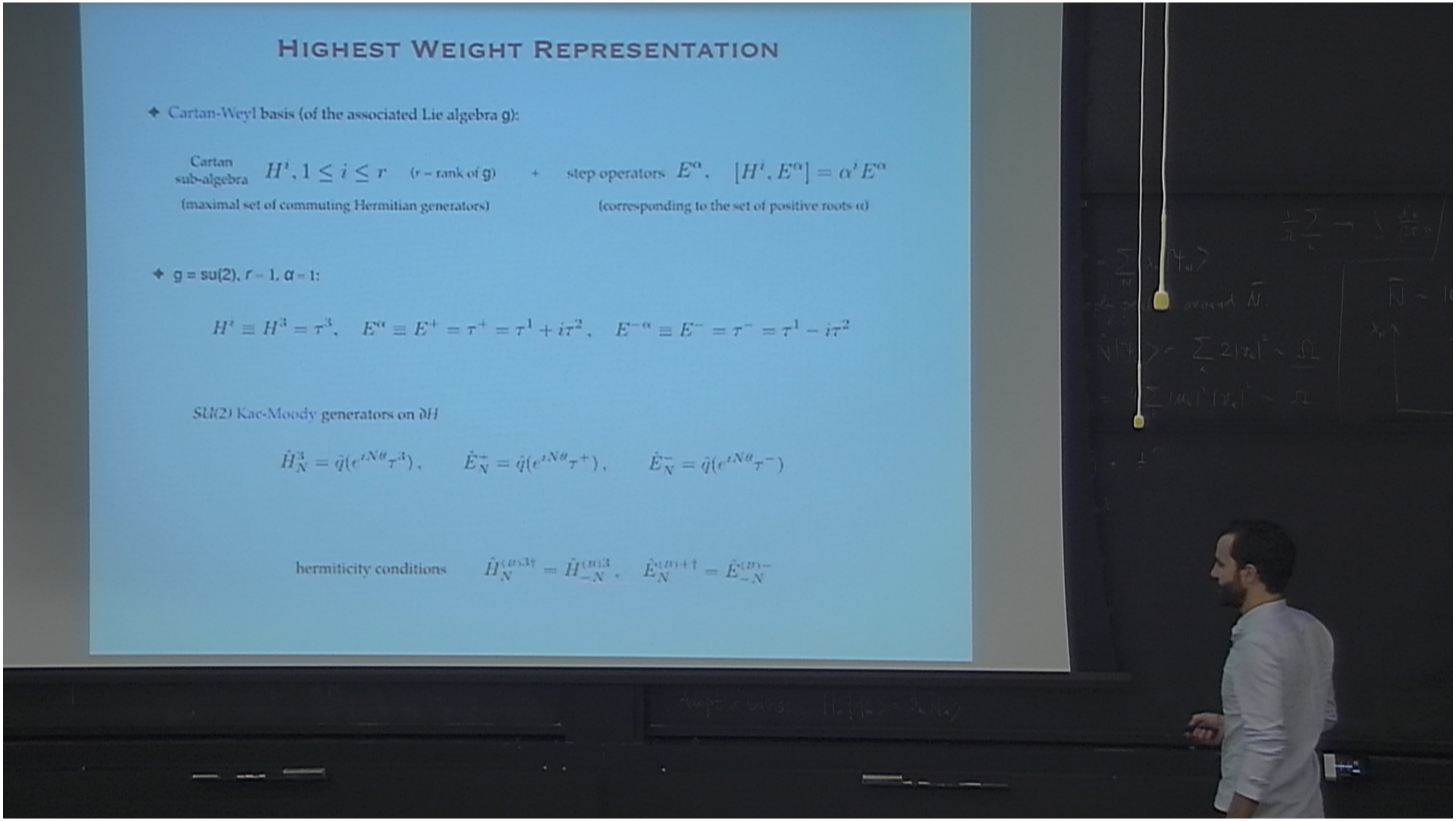
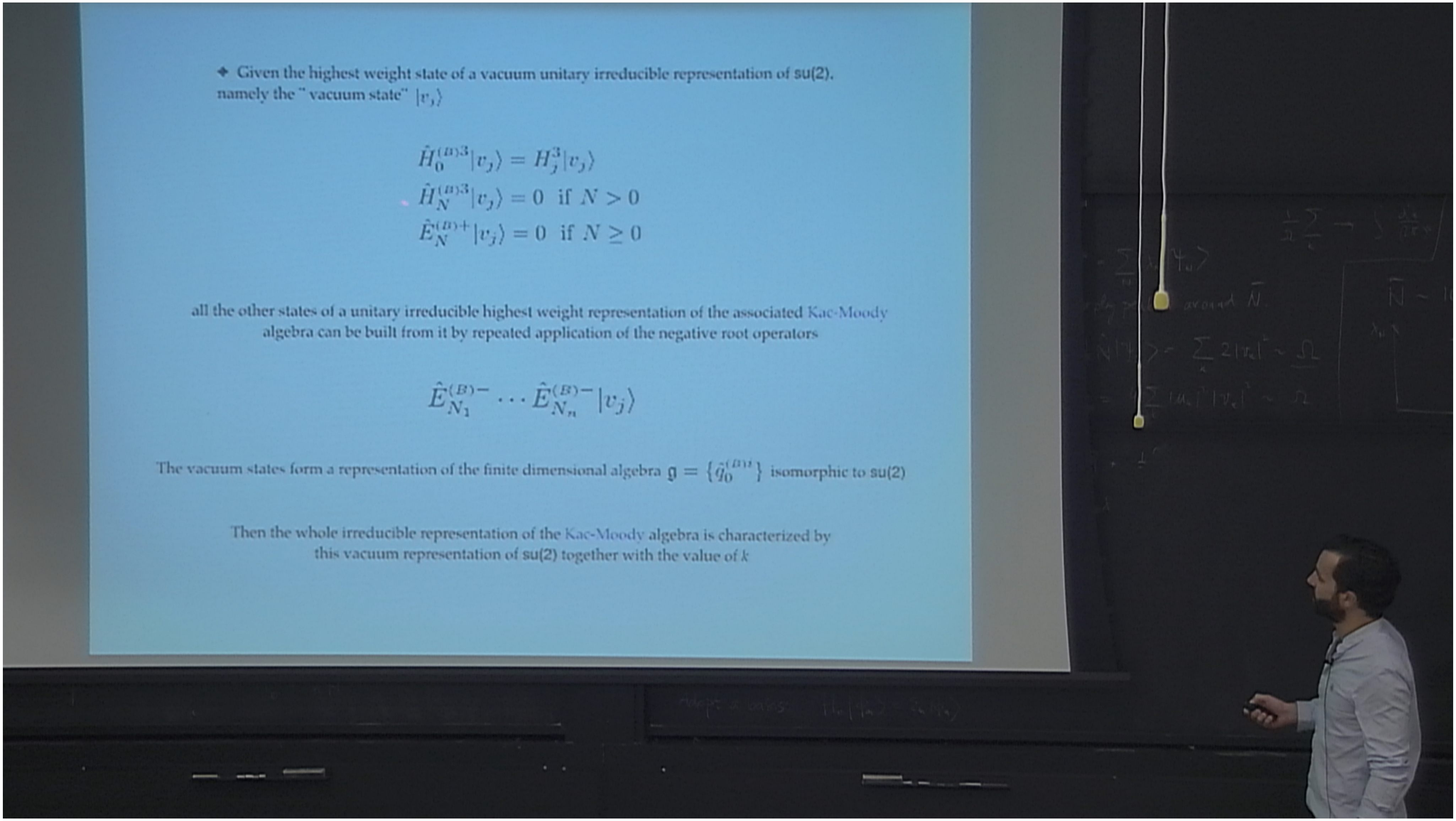
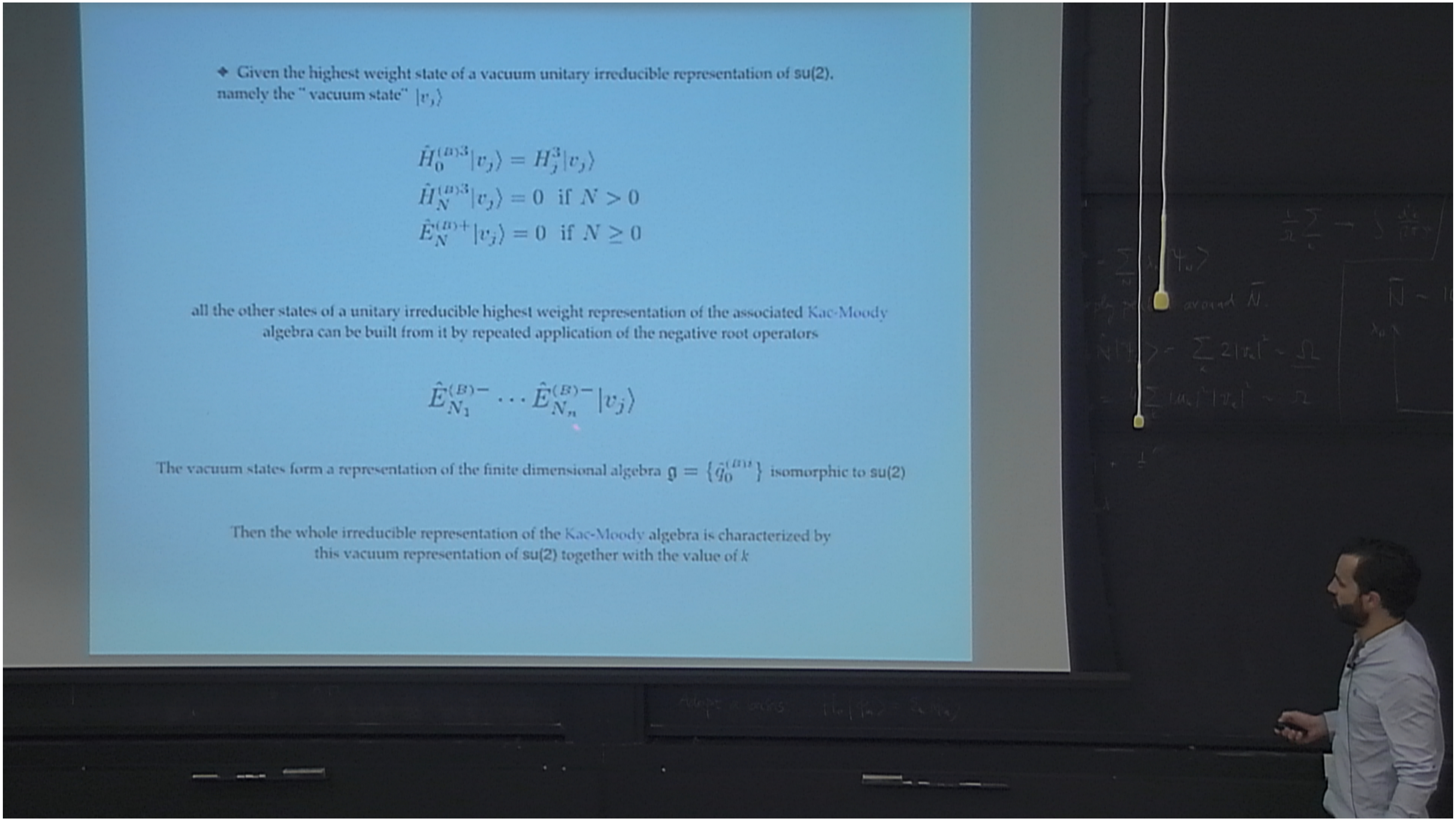

HIGHEST WEIGHT REPRESENTATION
Cartan-Weyl basis (of the associated Lie algebra g):
Cartan sub-algebra $H^i, 1 \leq i \leq r$ (r – rank of g) + step operators E^α, $[H^i, E^\alpha] = \alpha^i E^\alpha$
(maximal set of commuting Hermitian generators)
(corresponding to the set of positive roots α)
g = su(2), $r = 1$, $\alpha = 1$:
$H^i \equiv H^3 = \tau^3$, $E^\alpha \equiv E^+ = \tau^+ = \tau^1 + i\tau^2$, $E^{-\alpha} \equiv E^- = \tau^- = \tau^1 - i\tau^2$
$SU(2)$ Kac-Moody generators on ∂H
$\hat{H}^3_N = \hat{q}(e^{iN\theta}\tau^3)$, $\hat{E}^+_N = \hat{q}(e^{iN\theta}\tau^+)$, $\hat{E}^-_N = \hat{q}(e^{iN\theta}\tau^-)$
hermiticity conditions $\hat{H}^{(B)3\dagger}_N = \hat{H}^{(B)3}_{-N}$, $\hat{E}^{(B)+\dagger}_N = \hat{E}^{(B)-}_{-N}$



HIGHEST WEIGHT REPRESENTATION
Cartan-Weyl basis (of the associated Lie algebra g):
Cartan sub-algebra $H^i, 1 \leq i \leq r$ (r – rank of g) + step operators E^α, $[H^i, E^\alpha] = \alpha^i E^\alpha$
(maximal set of commuting Hermitian generators)
(corresponding to the set of positive roots α)
g = su(2), $r = 1$, $\alpha = 1$:
$H^i \equiv H^3 = \tau^3$, $E^\alpha \equiv E^+ = \tau^+ = \tau^1 + i\tau^2$, $E^{-\alpha} \equiv E^- = \tau^- = \tau^1 - i\tau^2$
$SU(2)$ Kac-Moody generators on ∂H
$\hat{H}^3_N = \hat{q}(e^{iN\theta}\tau^3)$, $\hat{E}^+_N = \hat{q}(e^{iN\theta}\tau^+)$, $\hat{E}^-_N = \hat{q}(e^{iN\theta}\tau^-)$
hermiticity conditions $\hat{H}^{(B)3\dagger}_N = \hat{H}^{(B)3}_{-N}$, $\hat{E}^{(B)+\dagger}_N = \hat{E}^{(B)-}_{-N}$


✦ Given the highest weight state of a vacuum unitary irreducible representation of su(2), namely the " vacuum state" $|v_j\rangle$
$\hat{H}_0^{(B)3}|v_j\rangle = H_j^3|v_j\rangle$
$\hat{H}_N^{(B)3}|v_j\rangle = 0 \text{ if } N > 0$
$\hat{E}_N^{(B)+}|v_j\rangle = 0 \text{ if } N \geq 0$
all the other states of a unitary irreducible highest weight representation of the associated Kac-Moody algebra can be built from it by repeated application of the negative root operators
$\hat{E}_{N_1}^{(B)-} \cdots \hat{E}_{N_n}^{(B)-}|v_j\rangle$
The vacuum states form a representation of the finite dimensional algebra $\mathfrak{g} = \{\hat{g}_0^{(B)a}\}$ isomorphic to su(2)
Then the whole irreducible representation of the Kac-Moody algebra is characterized by this vacuum representation of su(2) together with the value of k



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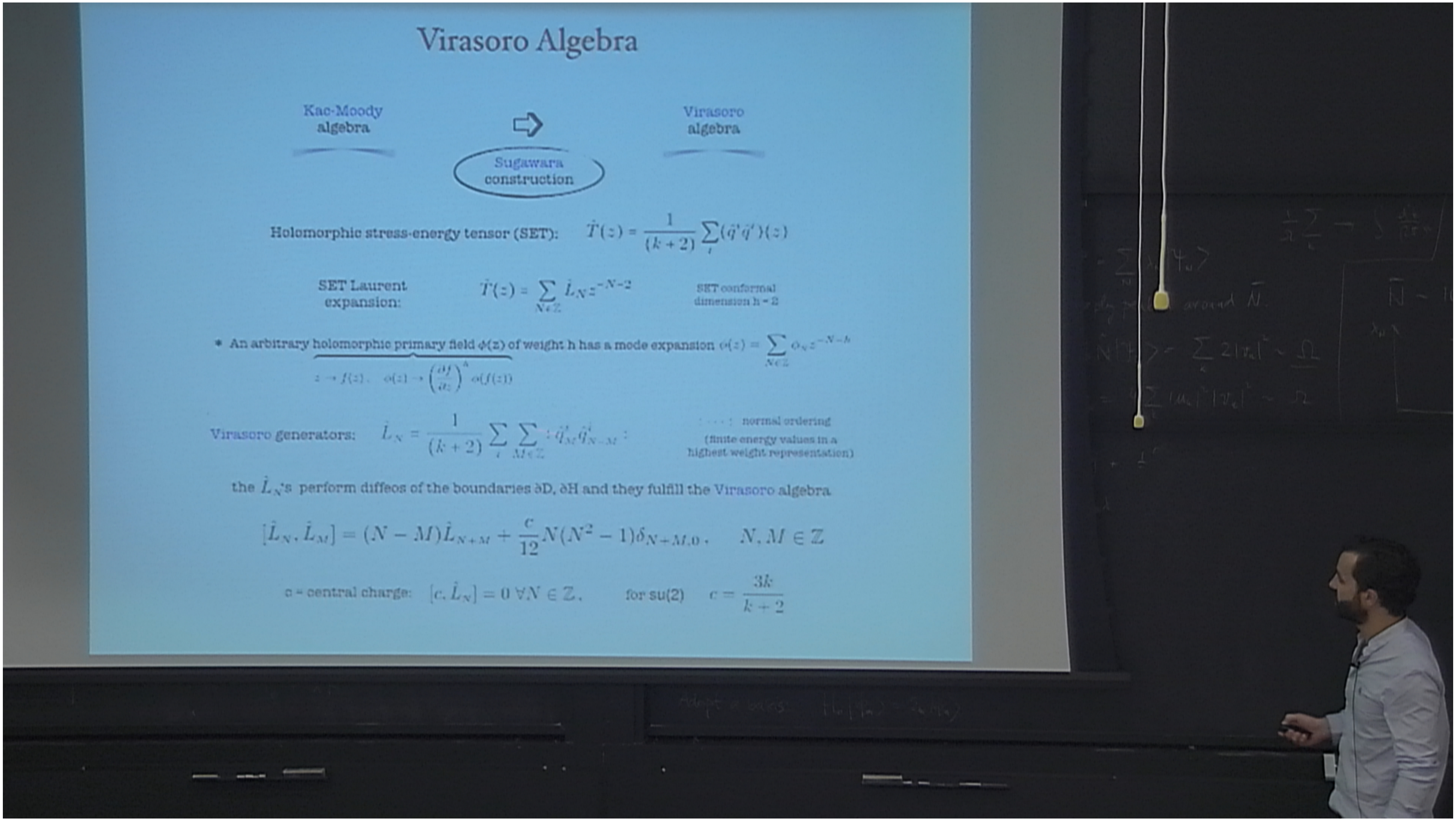
Virasoro Algebra
Kac-Moody algebra
Virasoro algebra
Sugawara construction
Holomorphic stress-energy tensor (SET): $\hat{T}(z) = \frac{1}{(k+2)} \sum_i (\hat{q}^i \hat{q}^i)(z)$
SET Laurent expansion: $\hat{T}(z) = \sum_{N \in \mathbb{Z}} \hat{L}_N z^{-N-2}$
SET conformal dimension h = 2
• An arbitrary holomorphic primary field $\phi(z)$ of weight h has a mode expansion $\phi(z) = \sum_{N \in \mathbb{Z}} \phi_N z^{-N-h}$
$z \to f(z), \quad \phi(z) \to \left(\frac{\partial f}{\partial z}\right)^h \phi(f(z))$
Virasoro generators: $\hat{L}_N = \frac{1}{(k+2)} \sum_i \sum_{M \in \mathbb{Z}} :\hat{q}^i_M \hat{q}^i_{N-M}:$
$: \cdots :$ normal ordering
(finite energy values in a highest weight representation)
the $\hat{L}_N$'s perform diffeos of the boundaries ∂D, ∂H and they fulfill the Virasoro algebra
$[\hat{L}_N, \hat{L}_M] = (N-M)\hat{L}_{N+M} + \frac{c}{12} N(N^2-1)\delta_{N+M,0}, \quad N, M \in \mathbb{Z}$
c = central charge: $[c, \hat{L}_N] = 0 \; \forall N \in \mathbb{Z}$, for su(2) $c = \frac{3k}{k+2}$

✦ Energy operator: $\hat{L}_0 = \frac{1}{(k+2)}\left(\hat{q}^i_0\hat{q}^i_0 + 2\sum_{M>0}\hat{q}^i_{-M}\hat{q}^i_M\right)$

$H \propto \hat{L}_0 + \hat{\bar{L}}_0$ generator of dilations in the z-plane → time translation in the cylinder

- Fields in a CFT can be grouped into families $[\varphi_n]$
  - a single primary field φ_n
  - an infinite set of secondary fields (descendants)

  } Irreps of the conformal group (primary field – highest weight)

In any given highest weight representation the spectrum of $\hat{L}_0$ is bounded from below and there is only one highest weight state $|v_j\rangle$ s.t.

$$\hat{L}_0|v_j\rangle = \Delta_j|v_j\rangle\,, \quad \hat{L}_N|v_j\rangle = 0 \quad N > 0$$

Δ_j: conformal dimension

All the other states in the given highest weight representation (c, Δ_j) can be constructed by repeated application of $\hat{L}_{-N}$, $N > 0$ on $|v_j\rangle$

unitary representations: $c \geq 1$, $\Delta_j \geq 0$ $\qquad c = \frac{3k}{k+2} \underset{\text{large } k}{\rightarrow} 3$ ✓

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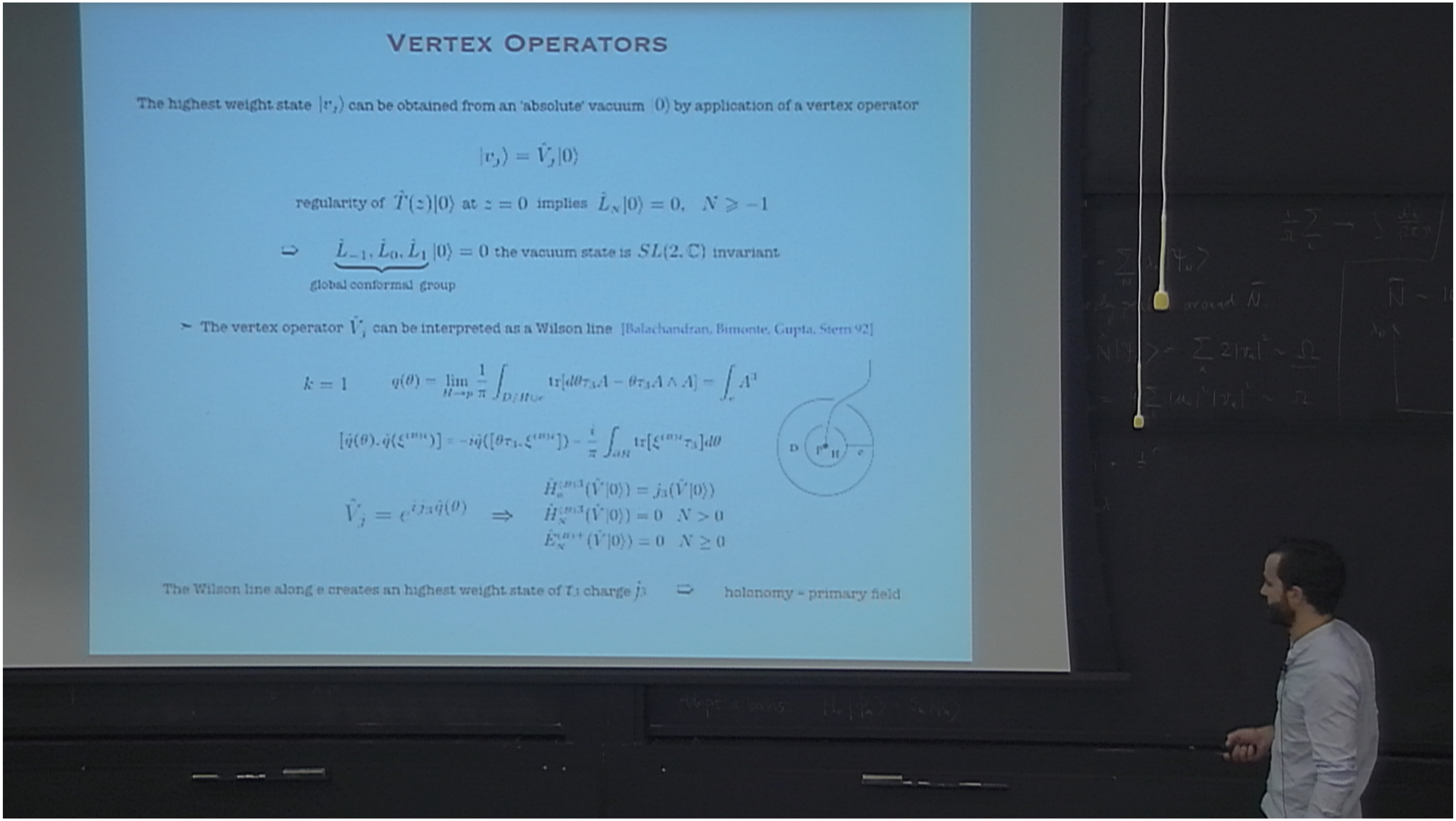
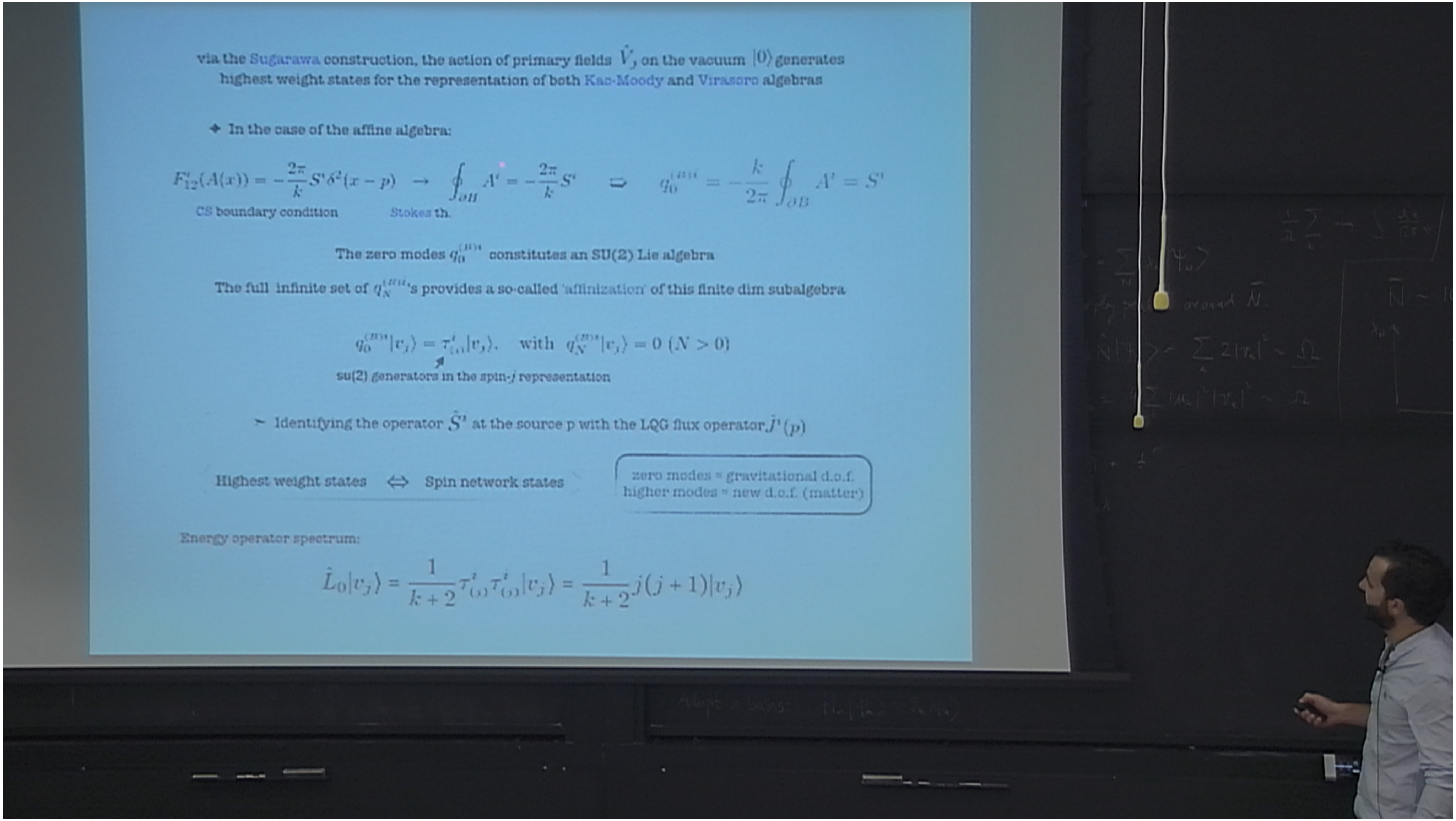
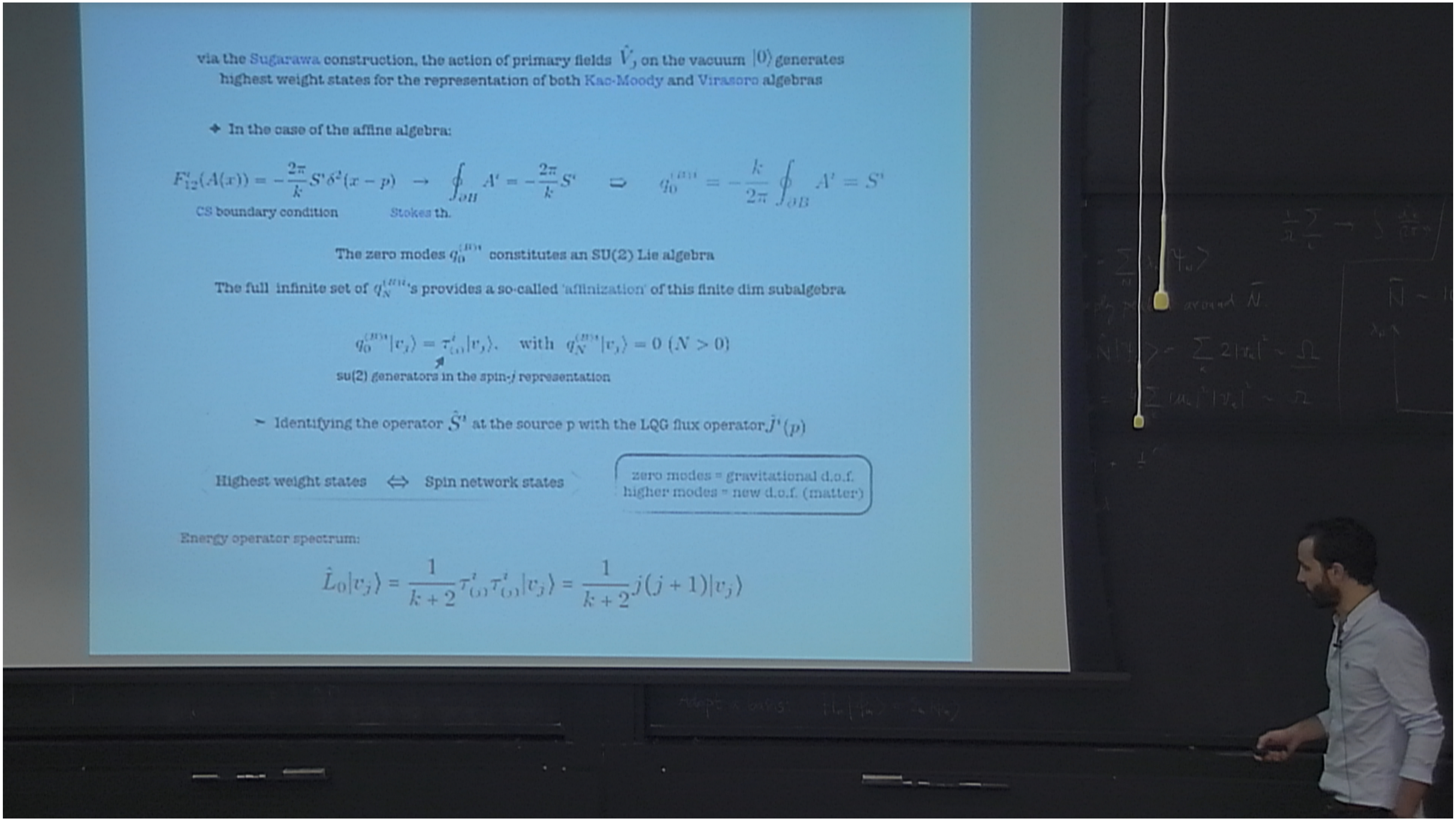

VERTEX OPERATORS
The highest weight state $|v_j\rangle$ can be obtained from an 'absolute' vacuum $|0\rangle$ by application of a vertex operator
$|v_j\rangle = \hat{V}_j|0\rangle$
regularity of $\hat{T}(z)|0\rangle$ at $z=0$ implies $\hat{L}_N|0\rangle = 0, \quad N \geqslant -1$
$\Rightarrow$ $\underbrace{\hat{L}_{-1}, \hat{L}_0, \hat{L}_1}_{\text{global conformal group}}|0\rangle = 0$ the vacuum state is $SL(2,\mathbb{C})$ invariant
➢ The vertex operator $\hat{V}_j$ can be interpreted as a Wilson line [Balachandran, Bimonte, Gupta, Stern 92]
$k=1 \qquad q(\theta) = \lim_{H\to p}\frac{1}{\pi}\int_{D/H\cup e}\mathrm{tr}[d\theta\tau_3 A - \theta\tau_3 A\wedge A] = \int_e A^3$
$[\hat{q}(\theta), \hat{q}(\xi^{(m)})] = -i\hat{q}([\theta\tau_3, \xi^{(m)}]) - \frac{i}{\pi}\int_{\partial H}\mathrm{tr}[\xi^{(m)}\tau_3]d\theta$
$\hat{V}_j = e^{ij_3\hat{q}(\theta)} \quad \Rightarrow \quad \begin{array}{l}\hat{H}_0^{(m)3}(\hat{V}|0\rangle) = j_3(\hat{V}|0\rangle)\\ \hat{H}_N^{(m)3}(\hat{V}|0\rangle) = 0 \quad N>0\\ \hat{E}_N^{(m)+}(\hat{V}|0\rangle) = 0 \quad N\geq 0\end{array}$
D
p
H
e
The Wilson line along e creates an highest weight state of τ_3 charge j_3 $\Rightarrow$ holonomy ~ primary field



via the Sugarawa construction, the action of primary fields $\hat{V}_j$ on the vacuum $|0\rangle$ generates highest weight states for the representation of both Kac-Moody and Virasoro algebras
In the case of the affine algebra:
$F^i_{12}(A(x)) = -\frac{2\pi}{k} S^i \delta^2(x-p) \rightarrow \oint_{\partial H} A^i = -\frac{2\pi}{k} S^i \Rightarrow q_0^{(H)i} = -\frac{k}{2\pi} \oint_{\partial B} A^i = S^i$
CS boundary condition
Stokes th.
The zero modes $q_0^{(H)i}$ constitutes an SU(2) Lie algebra
The full infinite set of $q_N^{(H)i}$'s provides a so-called 'affinization' of this finite dim subalgebra
$q_0^{(H)i}|v_j\rangle = \tau^i_{(j)}|v_j\rangle$, with $q_N^{(H)i}|v_j\rangle = 0 \ (N > 0)$
su(2) generators in the spin-j representation
Identifying the operator $\hat{S}^i$ at the source p with the LQG flux operator $\hat{J}^i(p)$
Highest weight states ⇔ Spin network states
zero modes = gravitational d.o.f.
higher modes = new d.o.f. (matter)
Energy operator spectrum:
$\hat{L}_0|v_j\rangle = \frac{1}{k+2}\tau^i_{(j)}\tau^i_{(j)}|v_j\rangle = \frac{1}{k+2} j(j+1)|v_j\rangle$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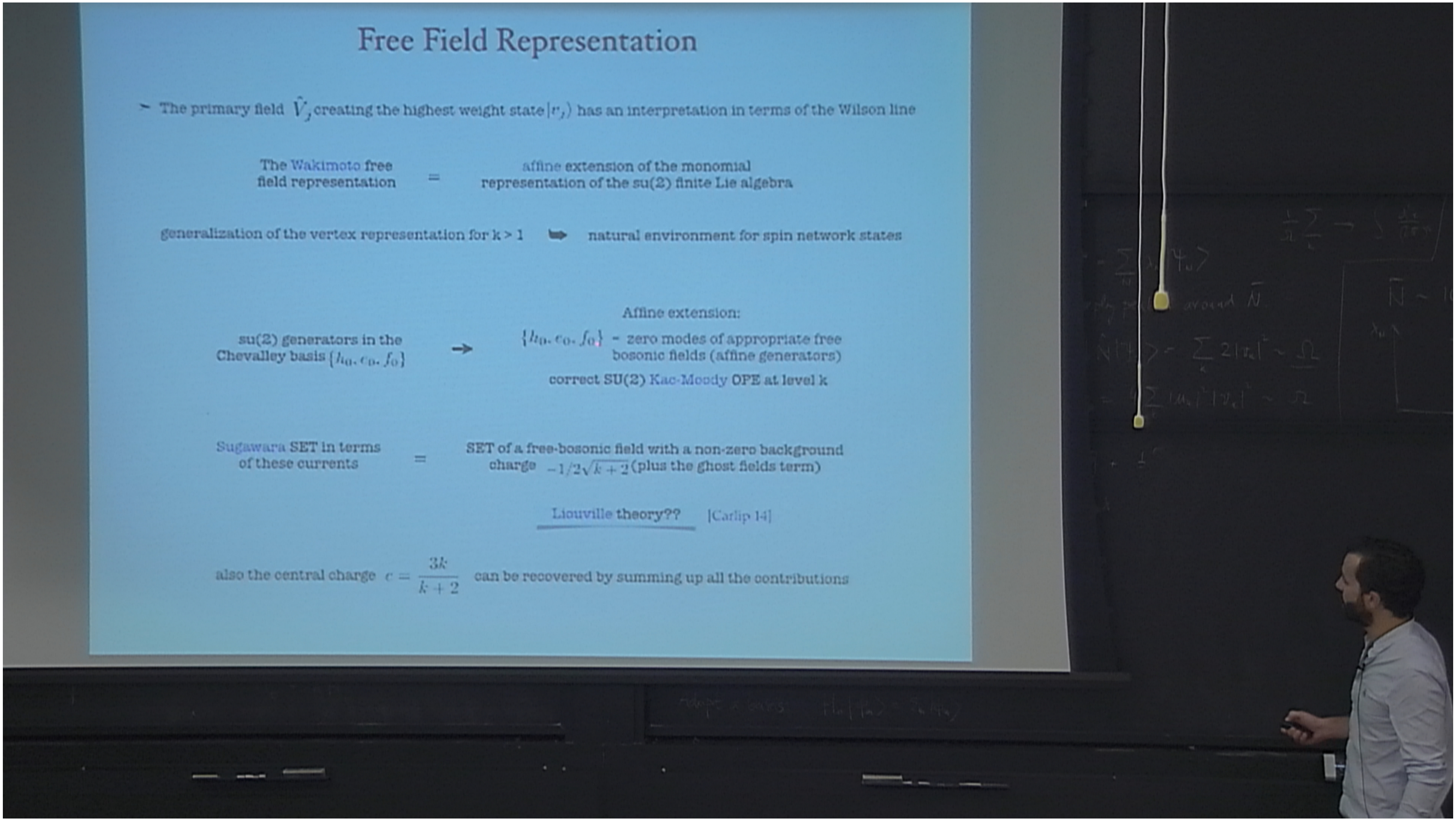
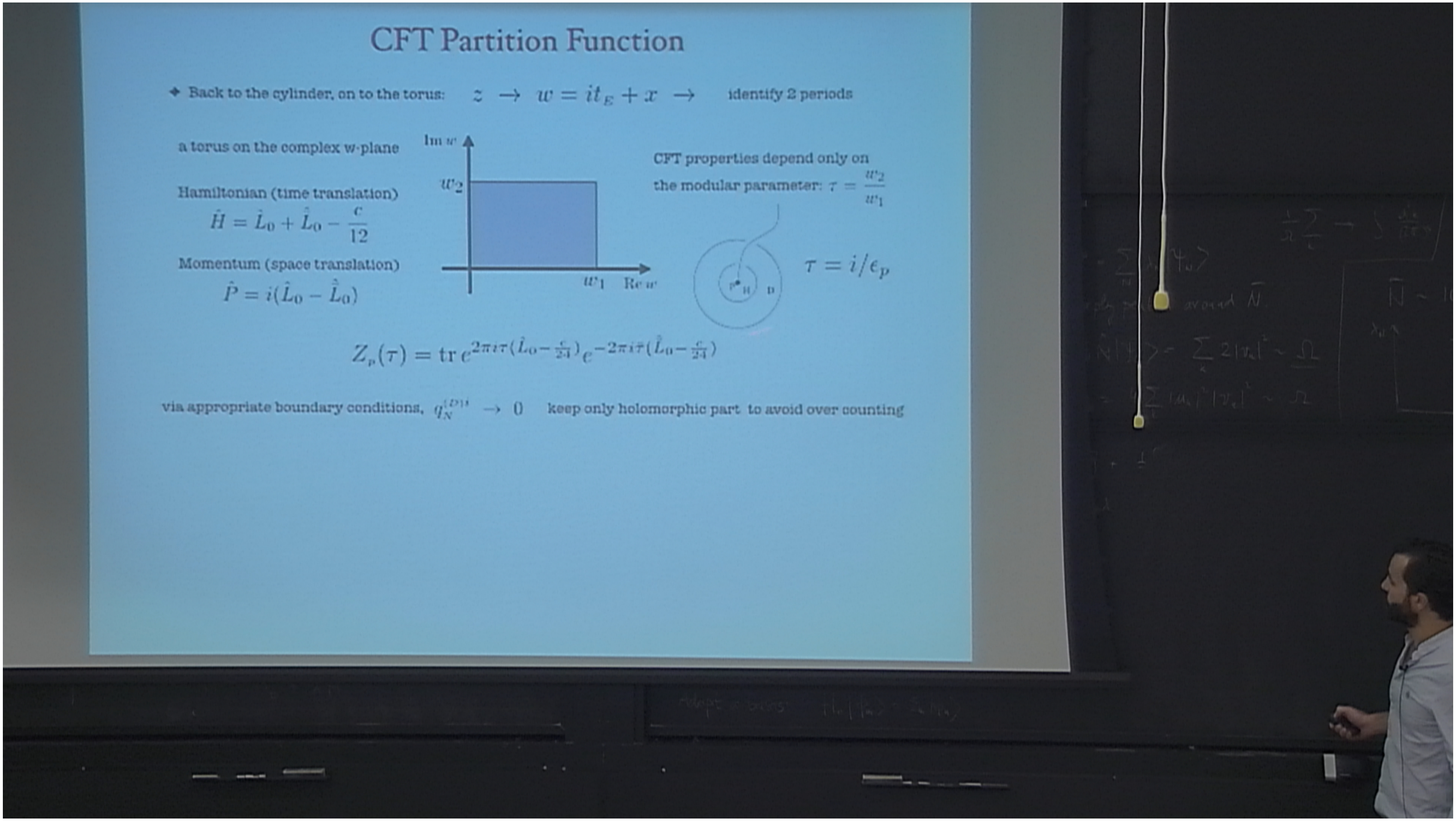
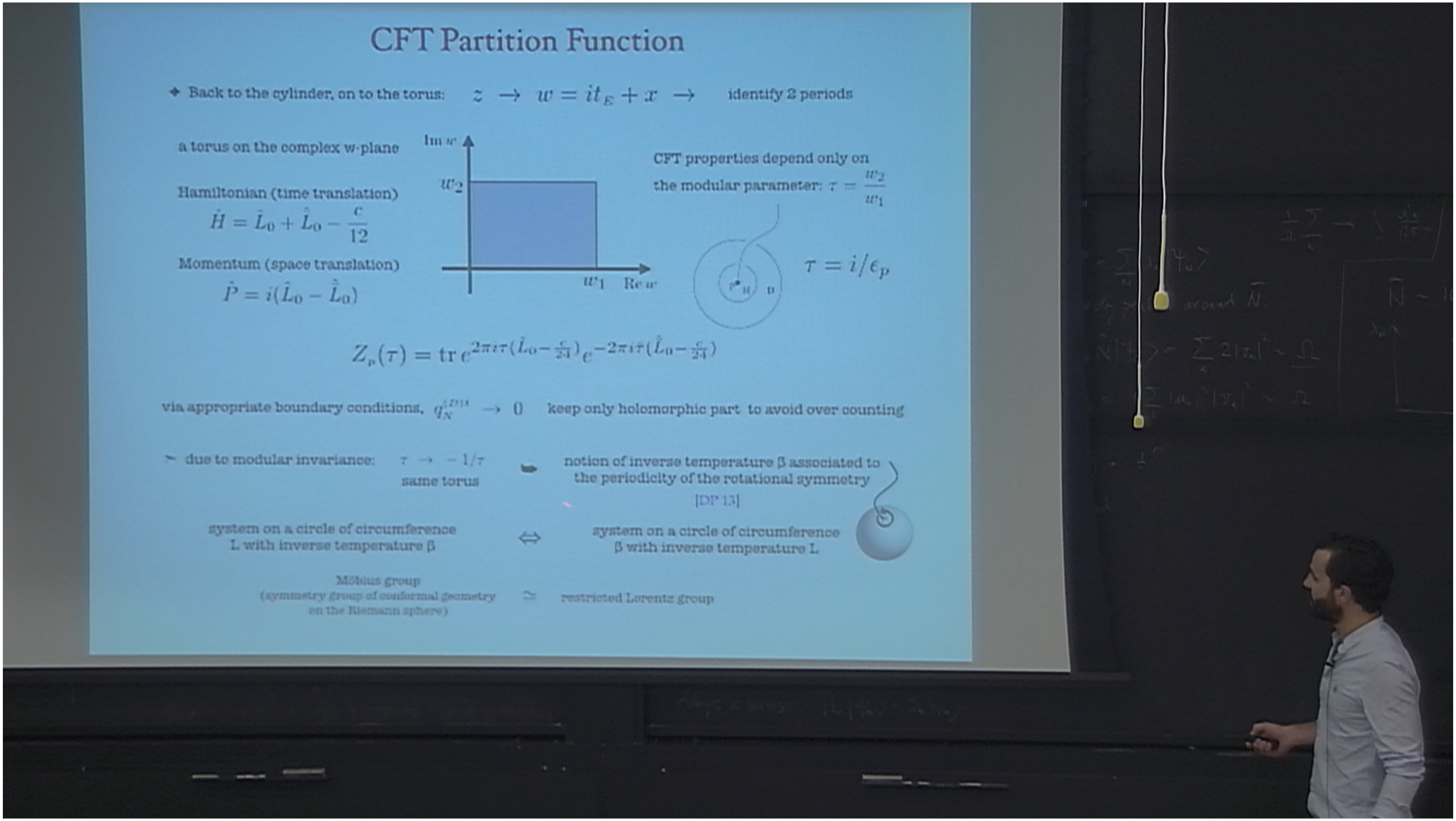


Free Field Representation
The primary field $\hat{V}_j$ creating the highest weight state $|v_j\rangle$ has an interpretation in terms of the Wilson line
The Wakimoto free field representation
=
affine extension of the monomial representation of the su(2) finite Lie algebra
generalization of the vertex representation for k > 1
natural environment for spin network states
Affine extension:
su(2) generators in the Chevalley basis $\{h_0, e_0, f_0\}$
$\{h_0, e_0, f_0\}$ - zero modes of appropriate free bosonic fields (affine generators)
correct SU(2) Kac-Moody OPE at level k
Sugawara SET in terms of these currents
=
SET of a free-bosonic field with a non-zero background charge $-1/2\sqrt{k+2}$ (plus the ghost fields term)
Liouville theory??
[Carlip 14]
also the central charge $c = \dfrac{3k}{k+2}$ can be recovered by summing up all the contributions



CFT Partition Function
Back to the cylinder, on to the torus: $z \rightarrow w = it_E + x \rightarrow$ identify 2 periods
a torus on the complex w-plane
Hamiltonian (time translation)
$\hat{H} = \hat{L}_0 + \hat{\bar{L}}_0 - \frac{c}{12}$
Momentum (space translation)
$\hat{P} = i(\hat{L}_0 - \hat{\bar{L}}_0)$
Im w
w_2
w_1
Re w
CFT properties depend only on
the modular parameter: $\tau = \frac{w_2}{w_1}$
$\tau = i/\epsilon_p$
$Z_p(\tau) = \mathrm{tr}\, e^{2\pi i \tau (\hat{L}_0 - \frac{c}{24})} e^{-2\pi i \bar{\tau} (\hat{\bar{L}}_0 - \frac{c}{24})}$
via appropriate boundary conditions, $q_N^{(D)i} \rightarrow 0$ keep only holomorphic part to avoid over counting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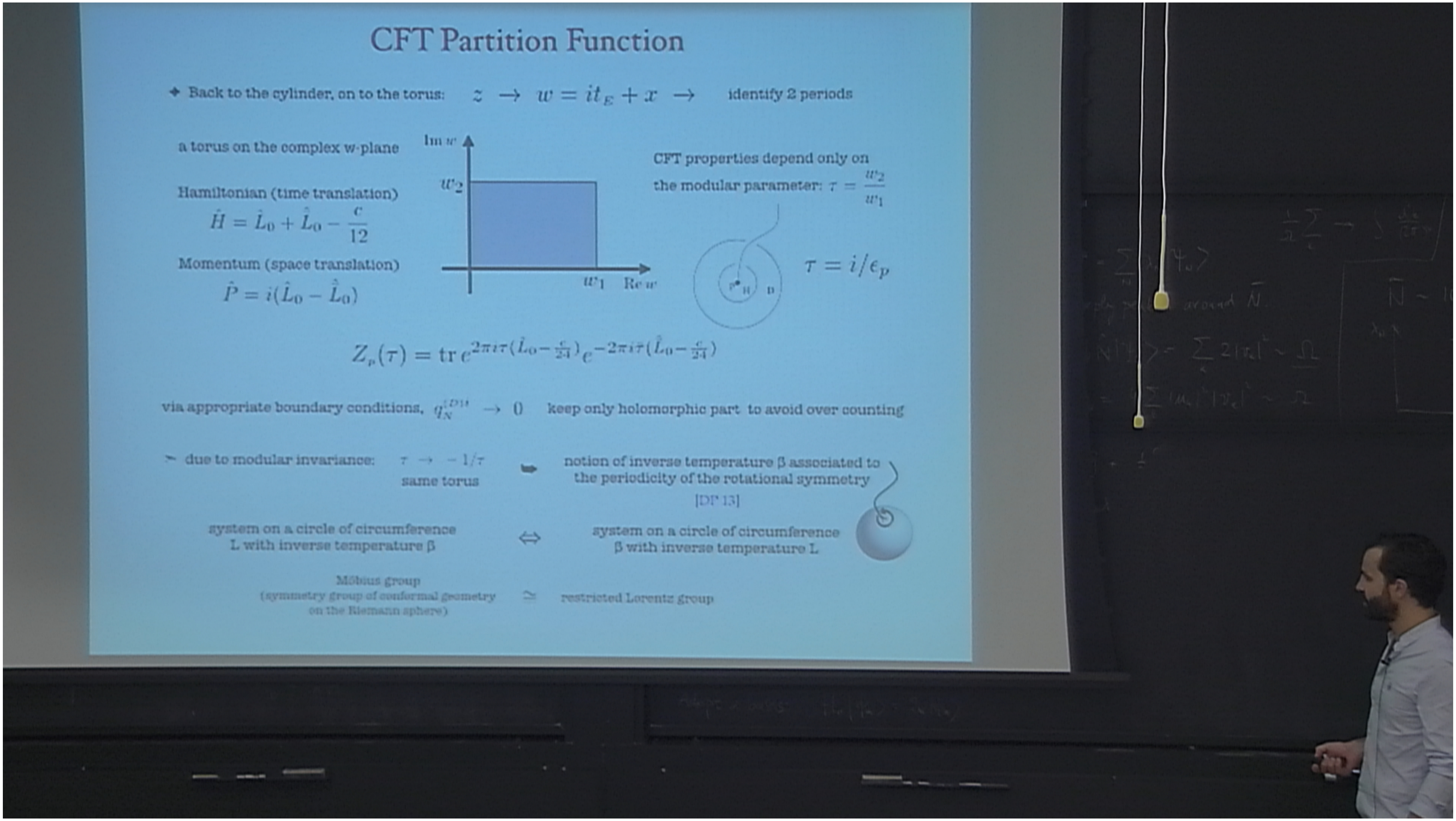
due to modular invariance: $\tau \rightarrow -1/\tau$
same torus
notion of inverse temperature β associated to
the periodicity of the rotational symmetry
[DP 13]
system on a circle of circumference
L with inverse temperature β
⇔
system on a circle of circumference
β with inverse temperature L
Möbius group
(symmetry group of conformal geometry
on the Riemann sphere)
≃
restricted Lorentz group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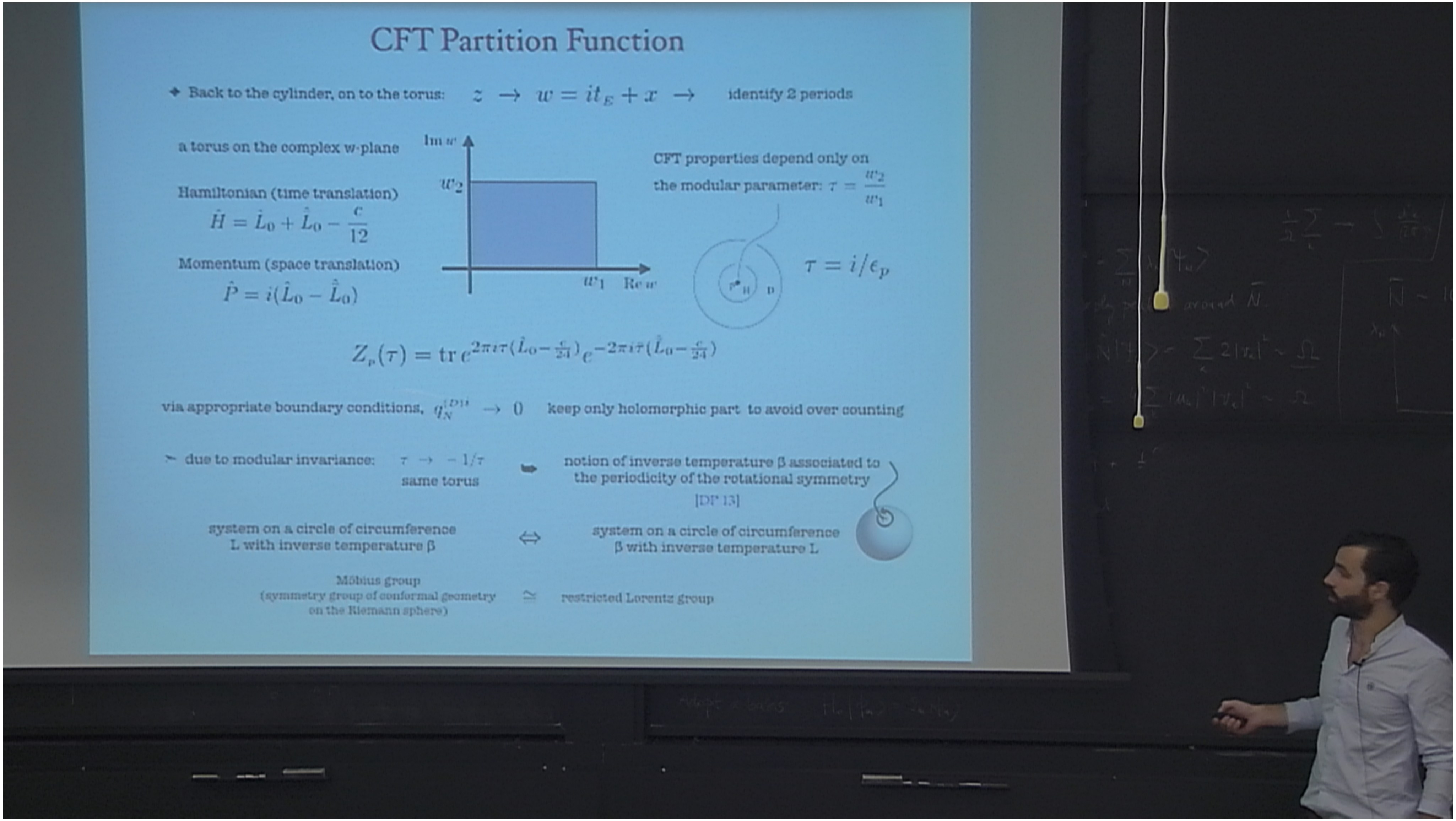
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CFT Partition Function

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a torus on the complex w-plane

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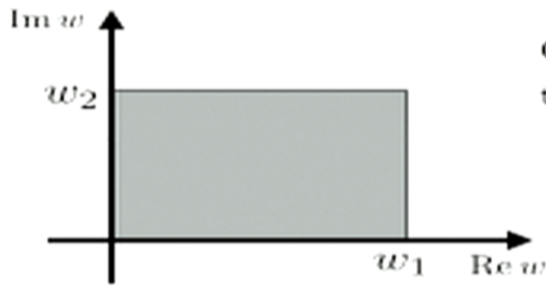

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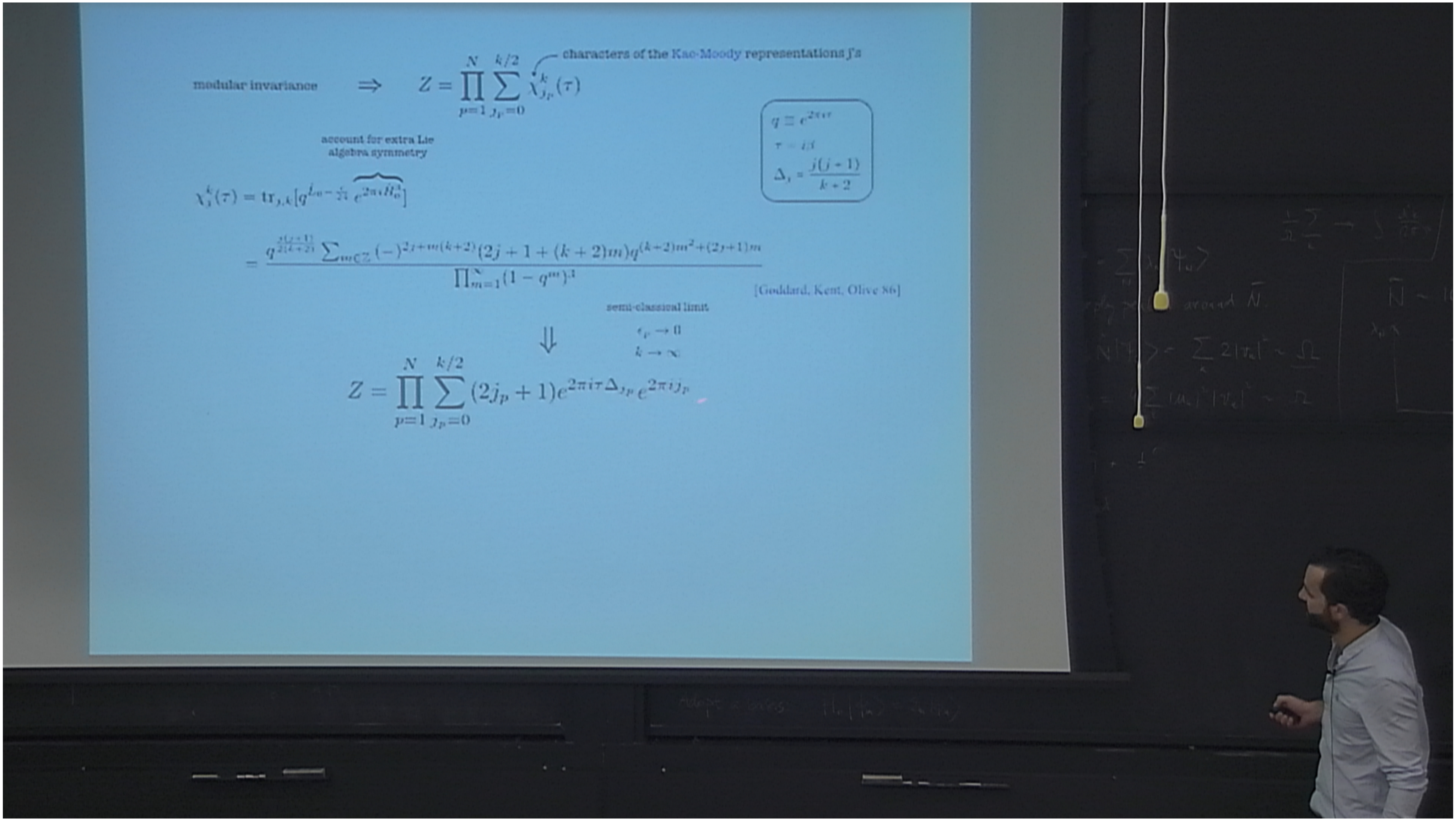
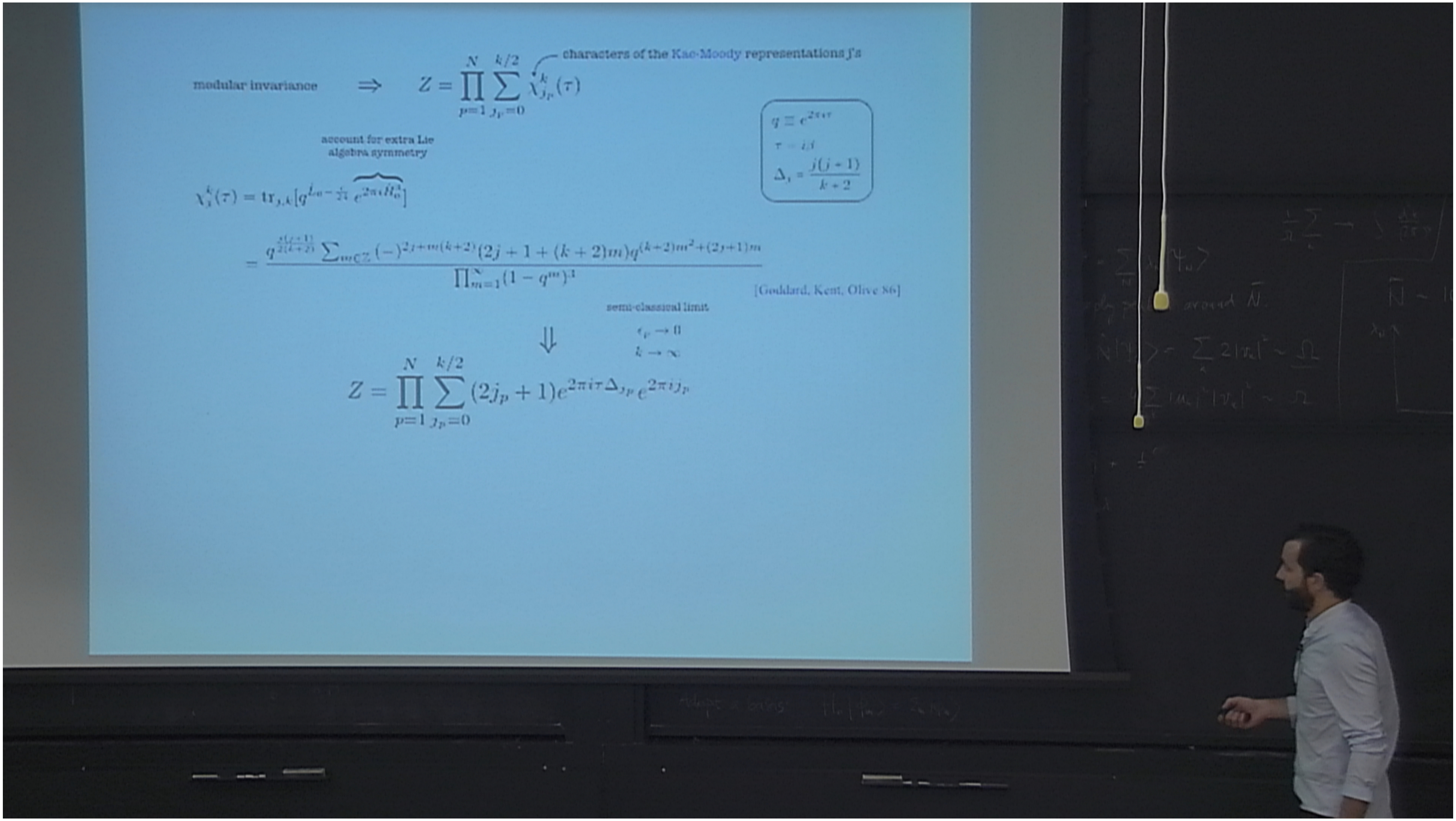
➢ due to modular invariance: $\tau \;\rightarrow\; -1/\tau$ same torus ➥ notion of inverse temperature β associated to the periodicity of the rotational symmetry [DP 13]

system on a circle of circumference L with inverse temperature β $\Leftrightarrow$ system on a circle of circumference β with inverse temperature L

Möbius group (symmetry group of conformal geometry on the Riemann sphere) $\cong$ restricted Lorentz group



modular invariance
$\Rightarrow \quad Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} \chi^k_{j_p}(\tau)$
characters of the Kac-Moody representations j's
$q \equiv e^{2\pi i \tau}$
$\Delta_j = \frac{j(j+1)}{k+2}$
account for extra Lie algebra symmetry
$\chi^k_j(\tau) = \mathrm{tr}_{j,k}[q^{L_0 - \frac{c}{24}} \overbrace{e^{2\pi i H_0^3}}]$
$= \frac{q^{\frac{j(j+1)}{2(k+2)}} \sum_{m\in\mathbb{Z}} (-)^{2j+m(k+2)}(2j+1+(k+2)m)q^{(k+2)m^2+(2j+1)m}}{\prod_{m=1}^{\infty}(1-q^m)^3}$
[Goddard, Kent, Olive 86]
semi-classical limit
$\Downarrow$
$k \to \infty$
$Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} (2j_p+1) e^{2\pi i \tau \Delta_{j_p}} e^{2\pi i j_p}$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characters of the Kac-Moody representations j's
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$\tau = i\beta$
$\Delta_j = \frac{j(j+1)}{k+2}$
account for extra Lie algebra symmetry
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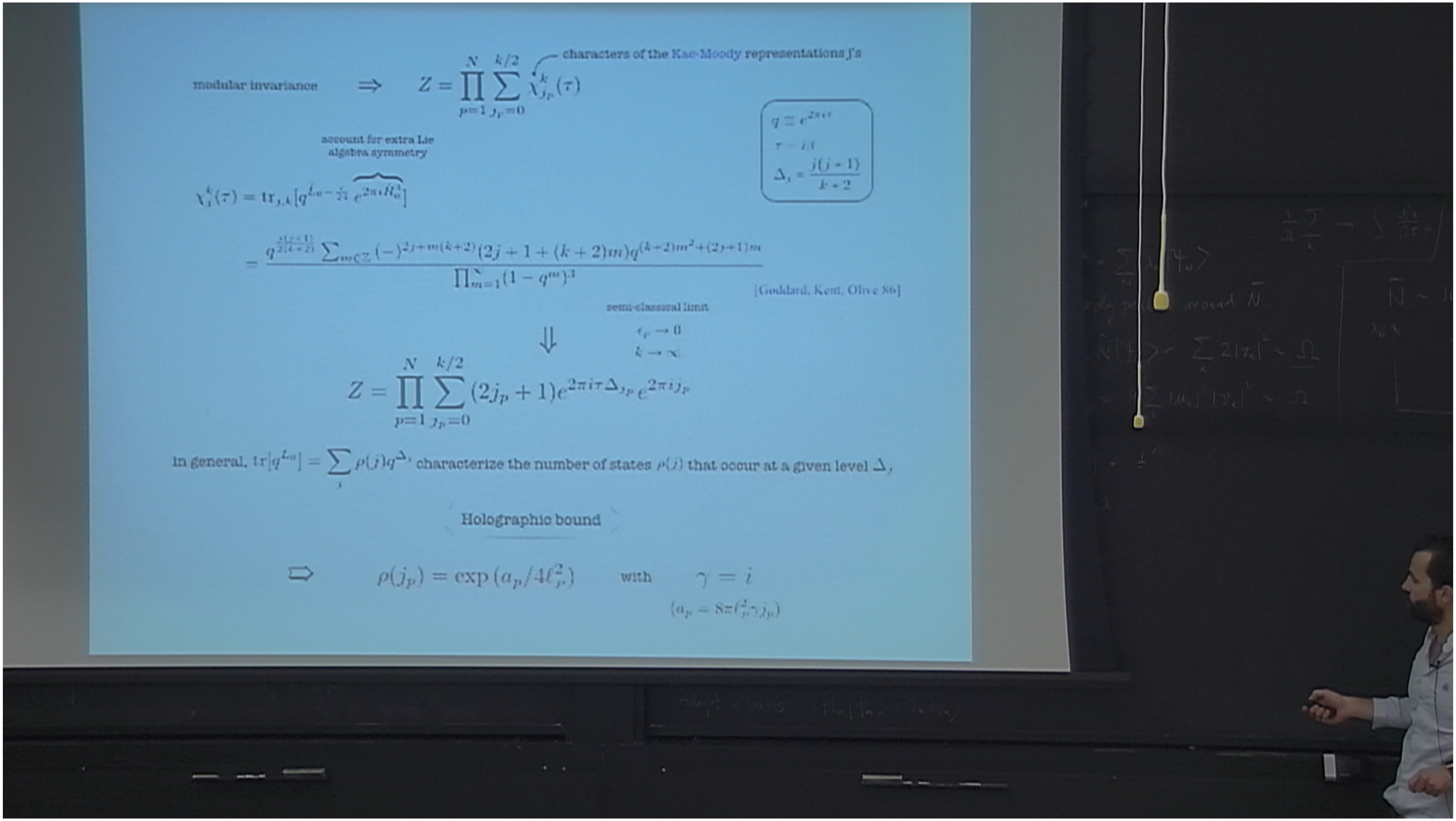
[Goddard, Kent, Olive 86]
semi-classical limit
$\Downarrow$
$\epsilon_p \to 0$
$k \to \infty$
$Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} (2j_p+1) e^{2\pi i \tau \Delta_{j_p}} e^{2\pi i j_p}$



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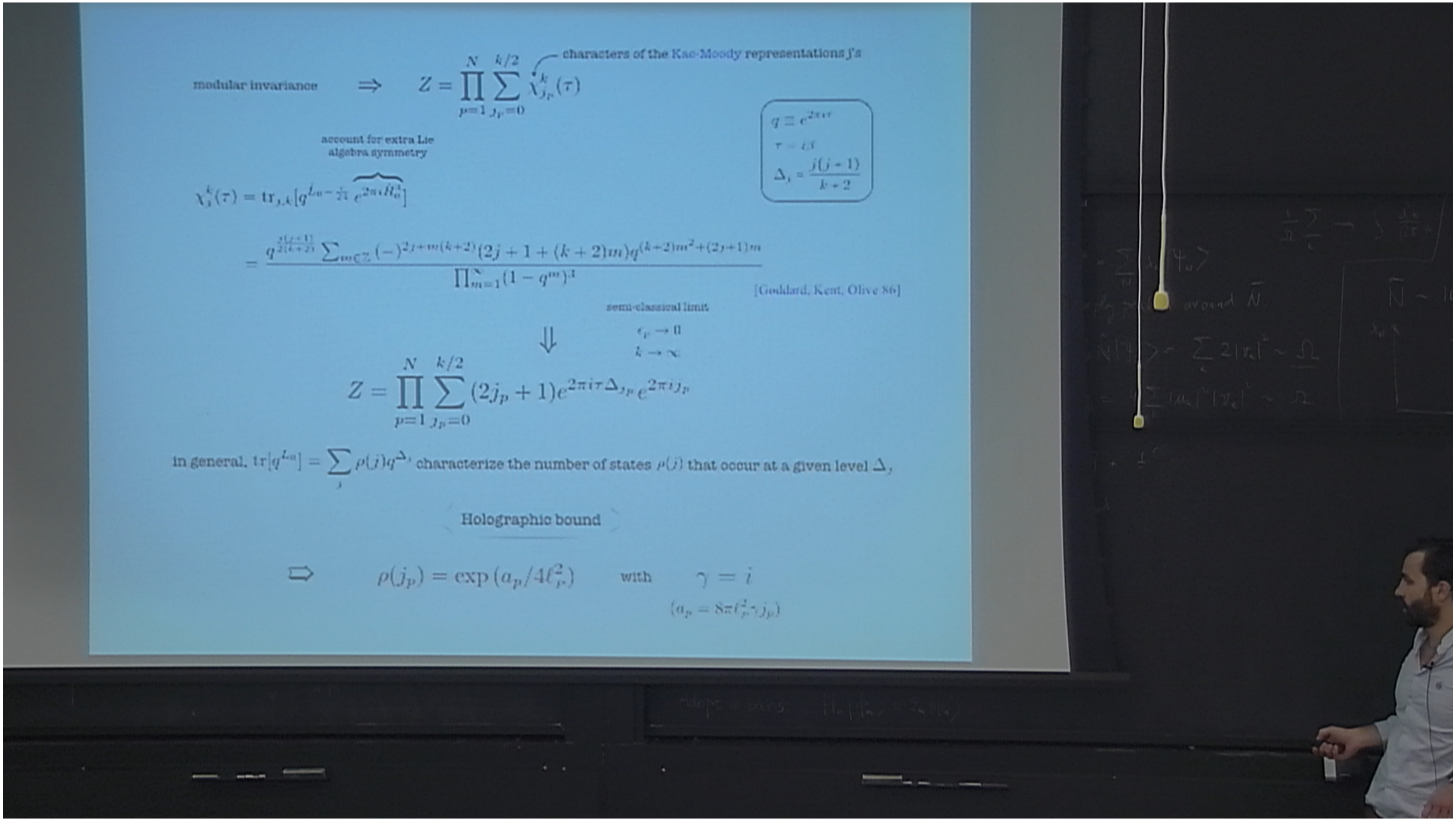
semi-classical limit
$k \to \infty$
$Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} (2j_p + 1) e^{2\pi i \tau \Delta_{j_p}} e^{2\pi i j_p}$
characterize the number of states $\rho(j)$ that occur at a given level Δ_j
Holographic bound
$\rho(j_p) = \exp(a_p/4\ell_p^2)$
with $\gamma = i$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$q \equiv e^{2\pi i \tau}$
$\tau = i\beta$
$\Delta_j = \frac{j(j+1)}{k+2}$
$\chi^k_j(\tau) = \mathrm{tr}_{j,k}[q^{L_0 - \frac{c}{24}} \overbrace{e^{2\pi i H^3_0}}]$
$= \frac{q^{\frac{j(j+1)}{2(k+2)}} \sum_{m\in\mathbb{Z}} (-)^{2j+m(k+2)}(2j+1+(k+2)m)q^{(k+2)m^2+(2j+1)m}}{\prod_{m=1}^{\infty}(1-q^m)^3}$
[Goddard, Kent, Olive 86]
semi-classical limit
$\epsilon_p \to 0$
$k \to \infty$
$\Downarrow$
$Z = \prod_{p=1}^{N} \sum_{j_p=0}^{k/2} (2j_p+1)e^{2\pi i\tau\Delta_{j_p}} e^{2\pi i j_p}$
in general, $\mathrm{tr}[q^{L_0}] = \sum_j \rho(j) q^{\Delta_j}$ characterize the number of states $\rho(j)$ that occur at a given level Δ_j
Holographic bound
$\Rightarrow$ $\rho(j_p) = \exp(a_p/4\ell_P^2)$ with $\gamma = i$
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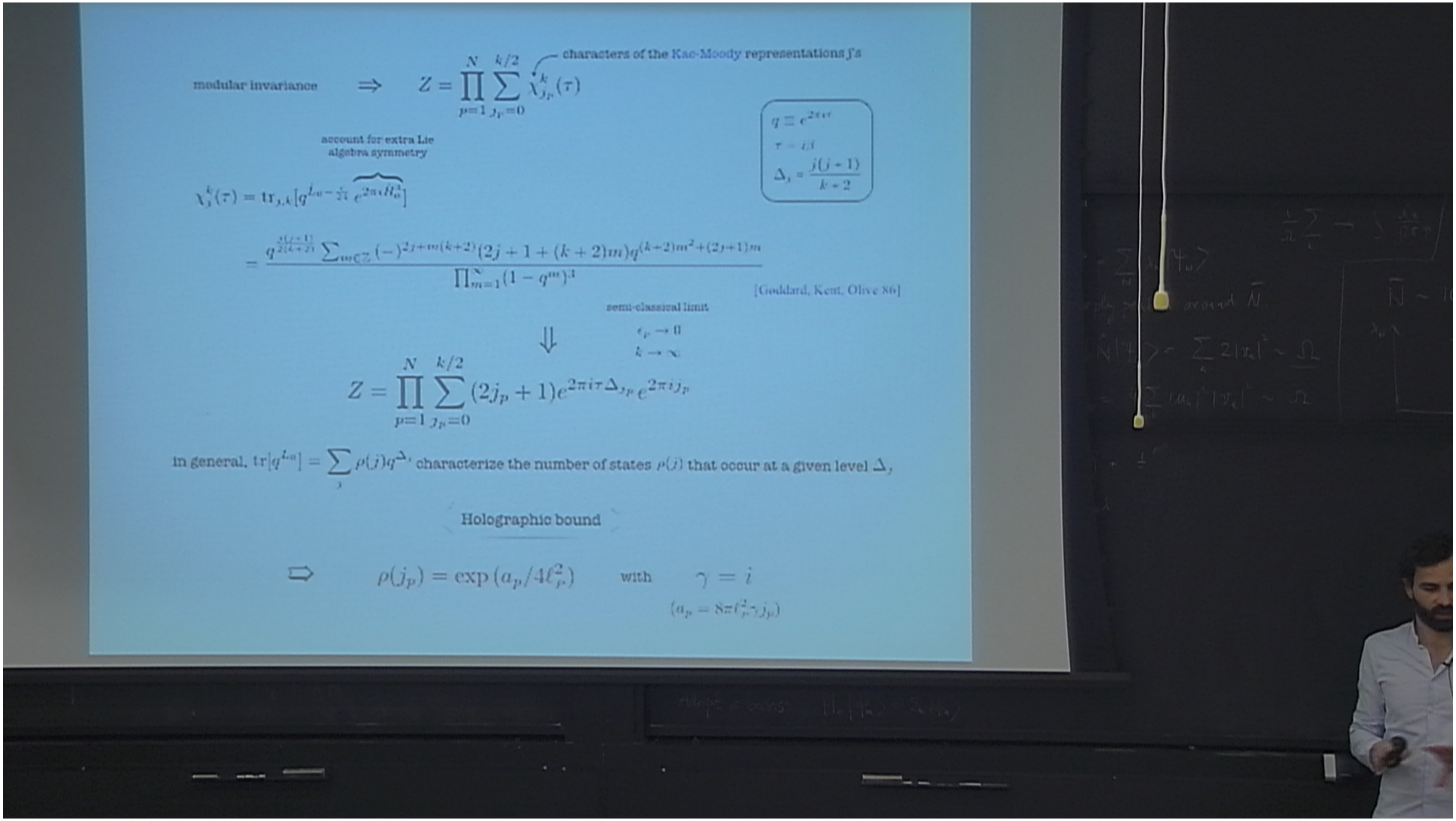

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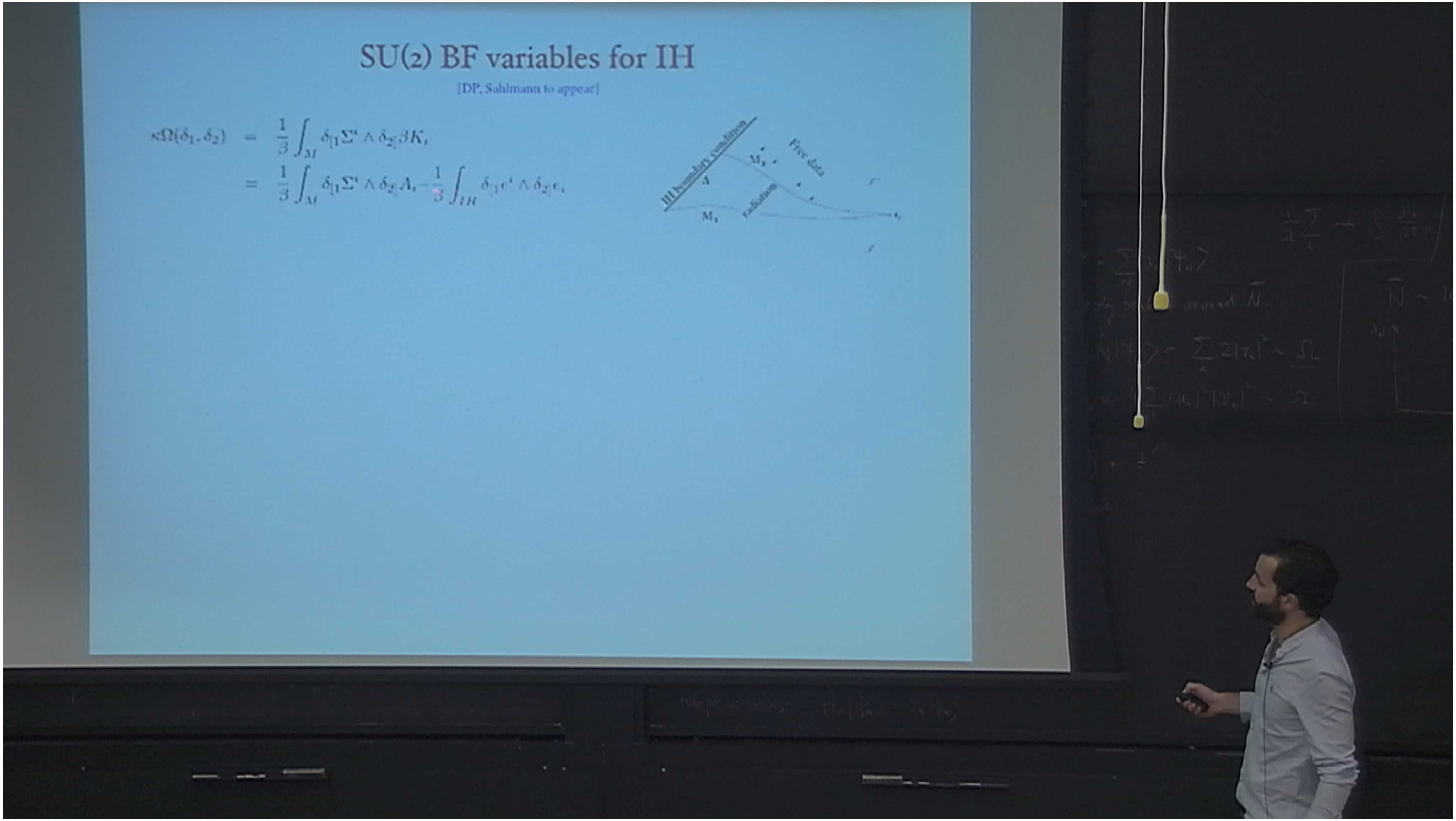
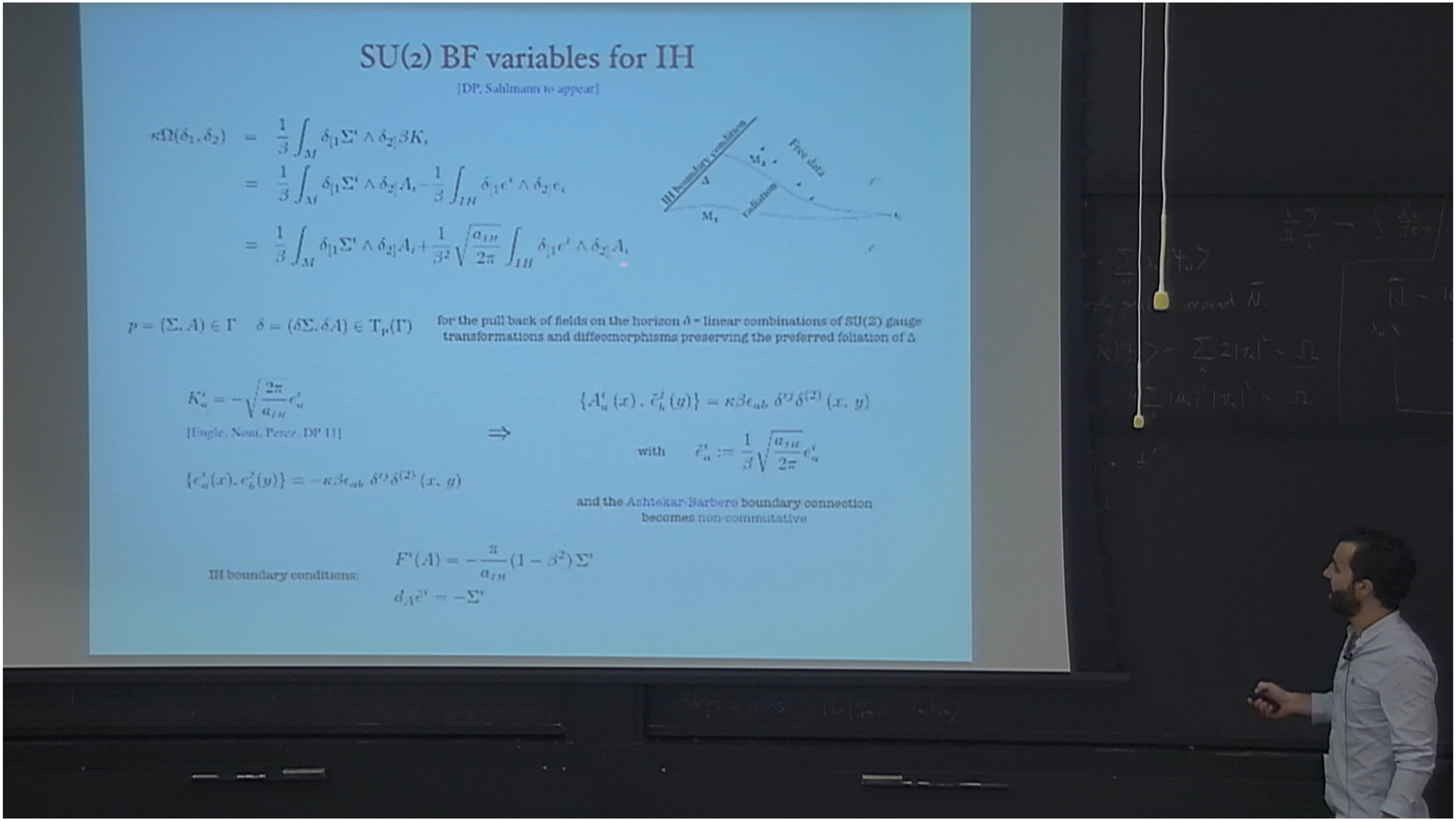


SU(2) BF variables for IH
[DP, Sahlmann to appear]
$\kappa\Omega(\delta_1, \delta_2) = \frac{1}{\beta}\int_M \delta_{[1}\Sigma^i \wedge \delta_{2]}\beta K_i$
$= \frac{1}{\beta}\int_M \delta_{[1}\Sigma^i \wedge \delta_{2]}A_i - \frac{1}{\beta}\int_{IH} \delta_{[1}e^i \wedge \delta_{2]}e_i$
IH boundary condition
Δ
M_2
Free data
radiation
M_1



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$p = (\Sigma, A) \in \Gamma \quad \delta = (\delta\Sigma, \delta A) \in T_p(\Gamma)$
for the pull back of fields on the horizon δ = linear combinations of SU(2) gauge transformations and diffeomorphisms preserving the preferred foliation of Δ
$K^i_a = -\sqrt{\frac{2\pi}{a_{IH}}}e^i_a$
[Engle, Noui, Perez, DP 11]
$\{e^i_a(x), e^j_b(y)\} = -\kappa\beta\epsilon_{ab}\,\delta^{ij}\delta^{(2)}(x,y)$
$\Rightarrow$
$\{A^i_a(x), \tilde{e}^j_b(y)\} = \kappa\beta\epsilon_{ab}\,\delta^{ij}\delta^{(2)}(x,y)$
with $\tilde{e}^i_a := \frac{1}{\beta}\sqrt{\frac{a_{IH}}{2\pi}}e^i_a$
and the Ashtekar-Barbero boundary connection becomes non-commutative
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$F^i(A) = -\frac{\pi}{a_{IH}}(1-\beta^2)\Sigma^i$
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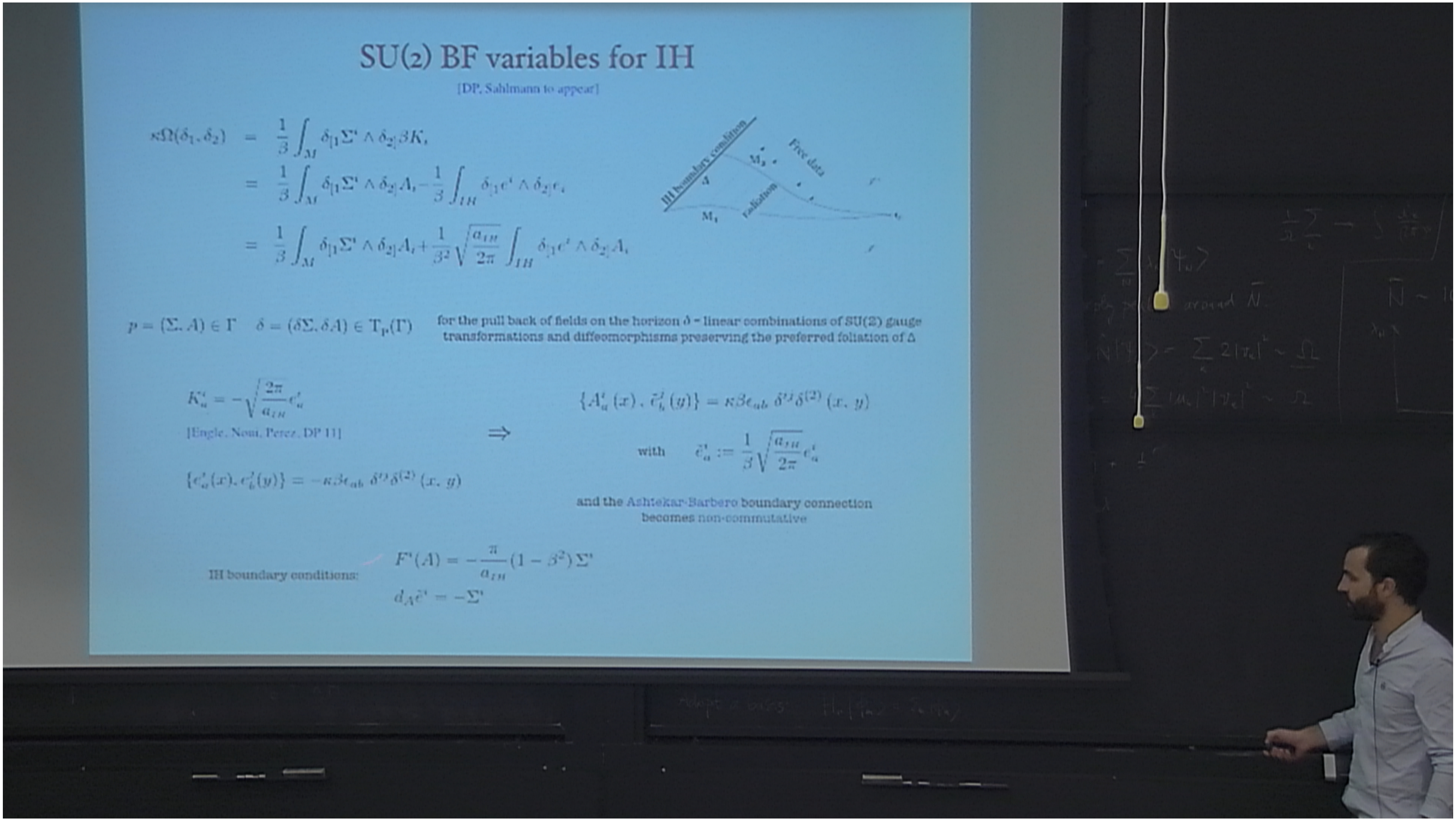

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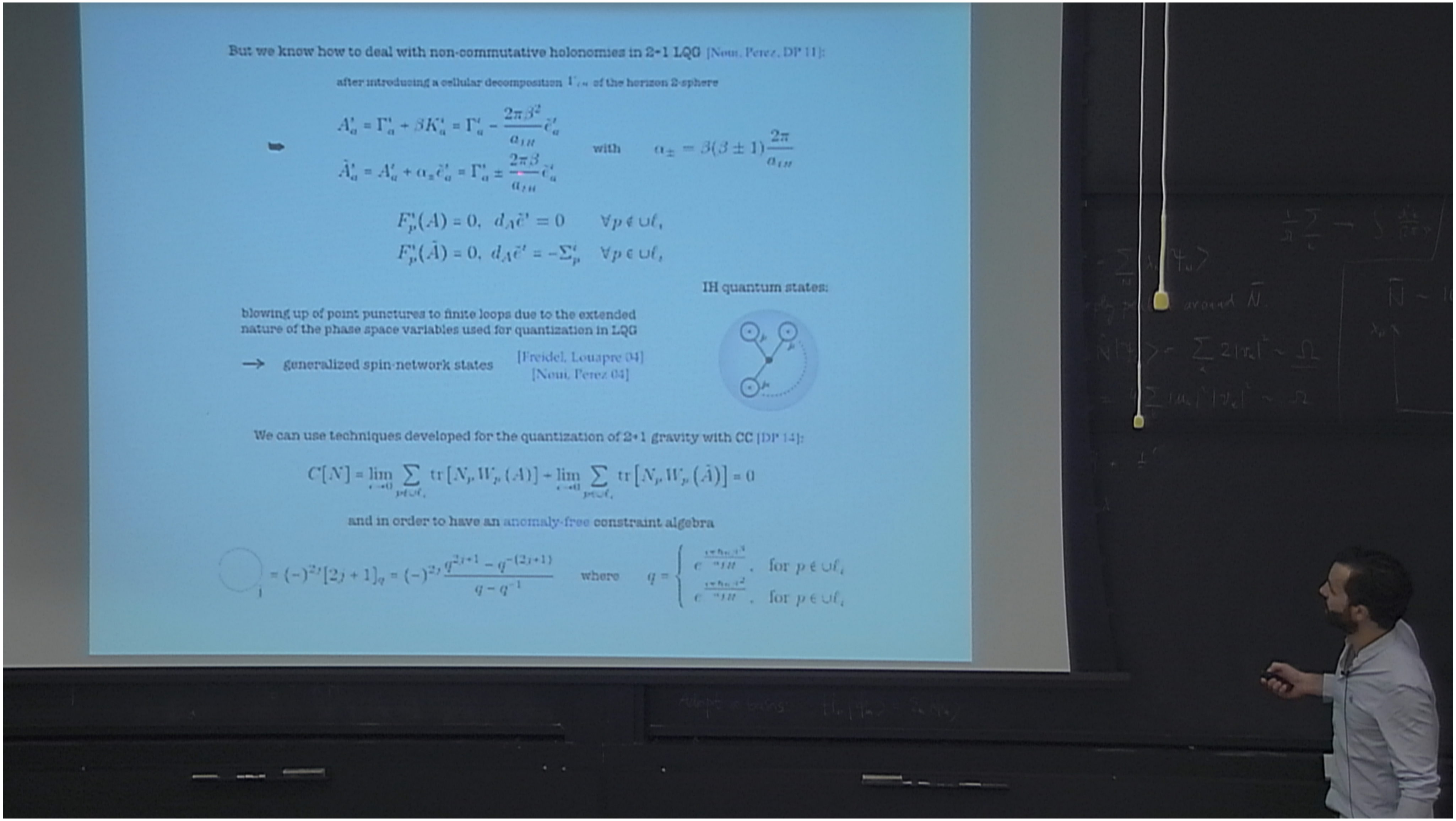
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blowing up of point punctures to finite loops due to the extended nature of the phase space variables used for quantization in LQG
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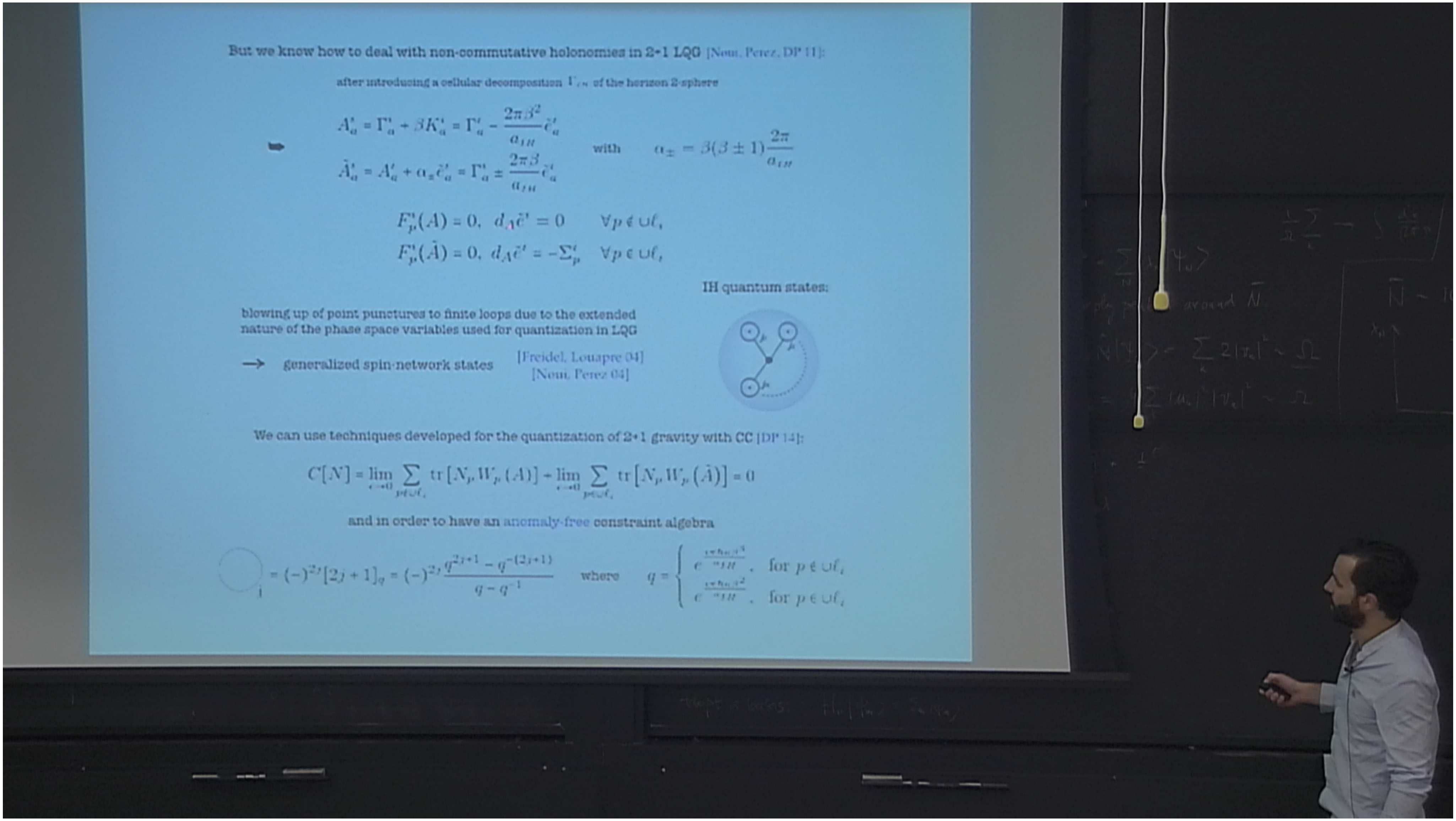

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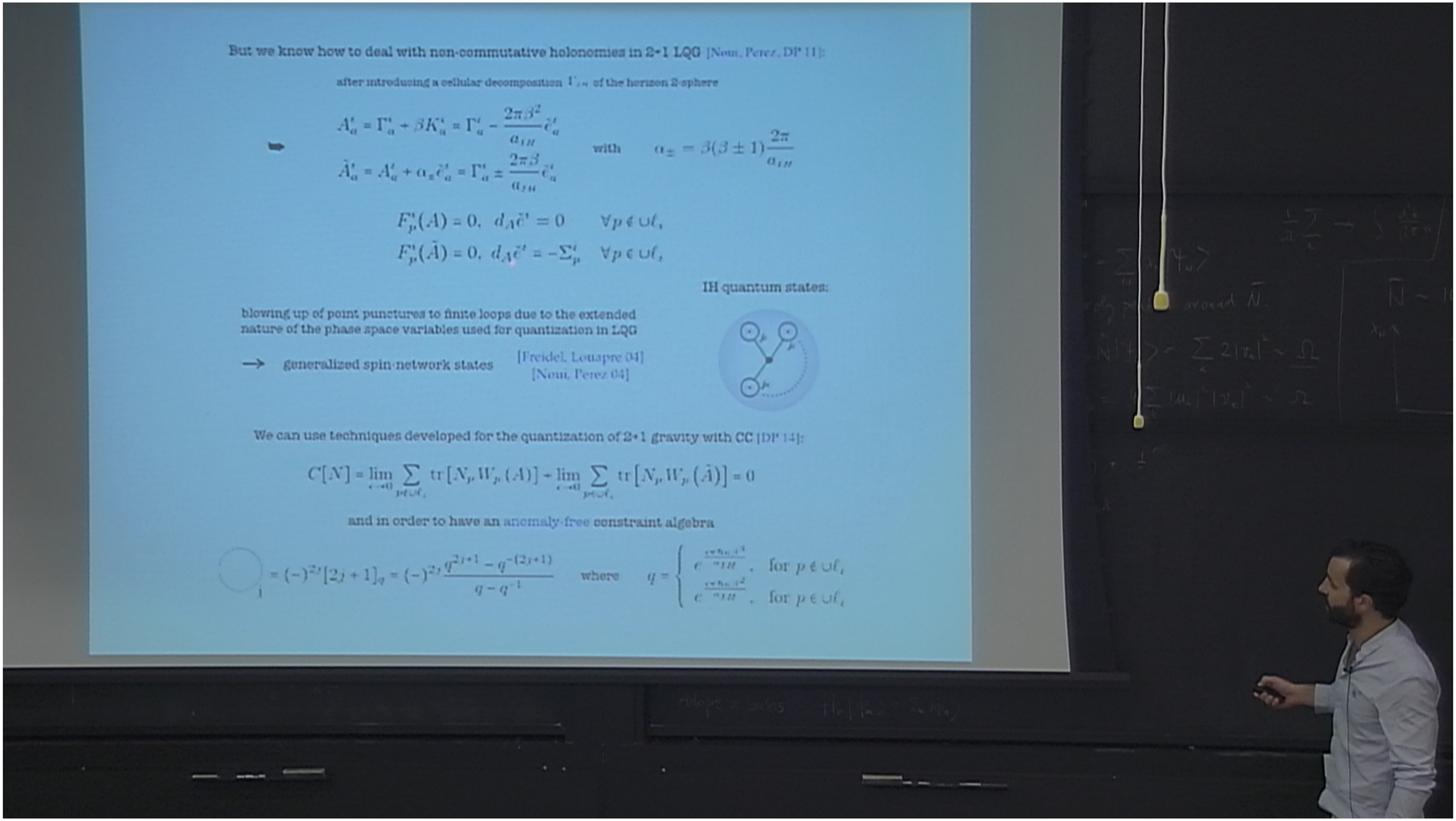
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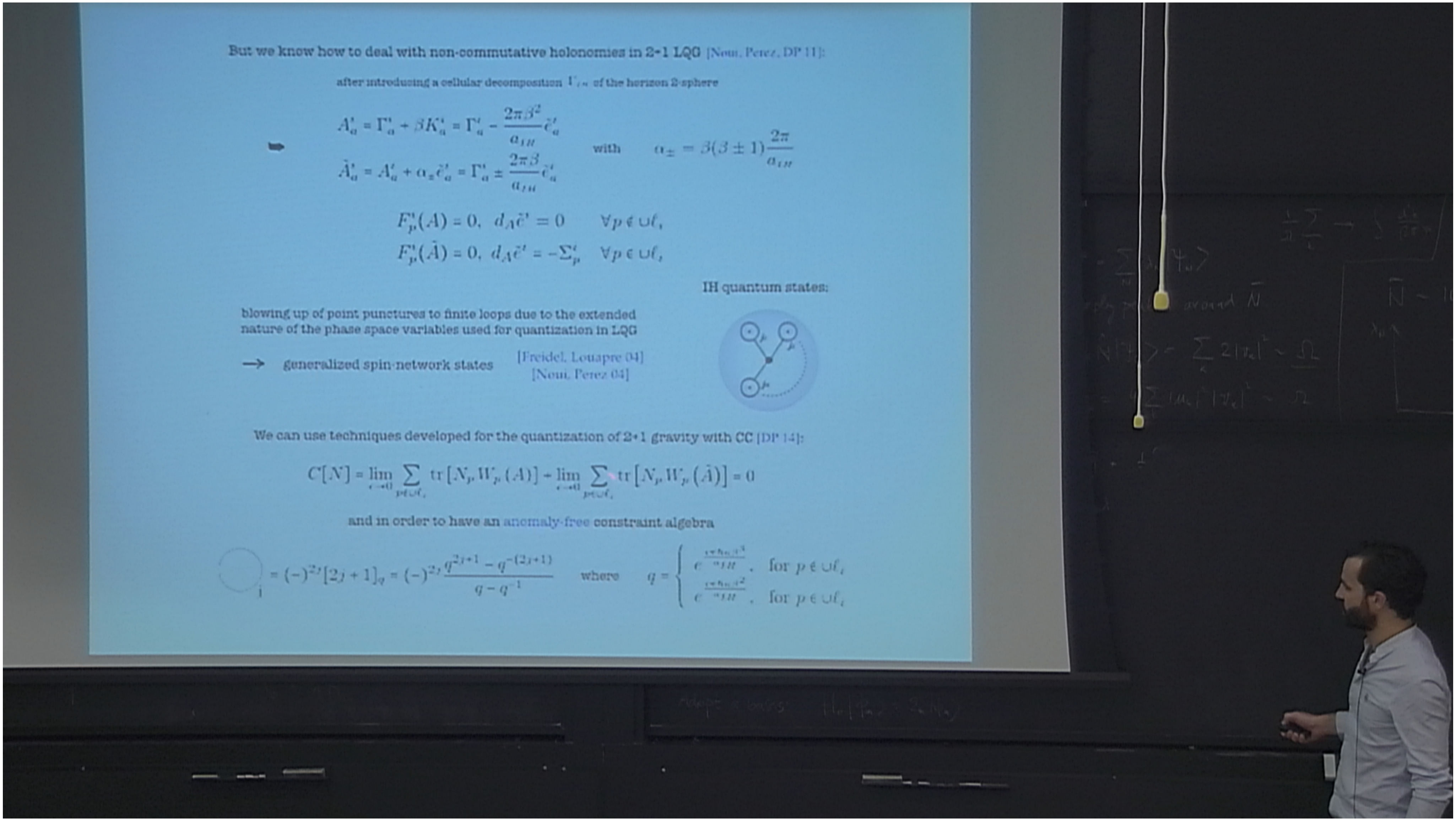

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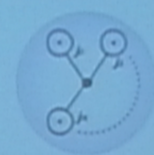

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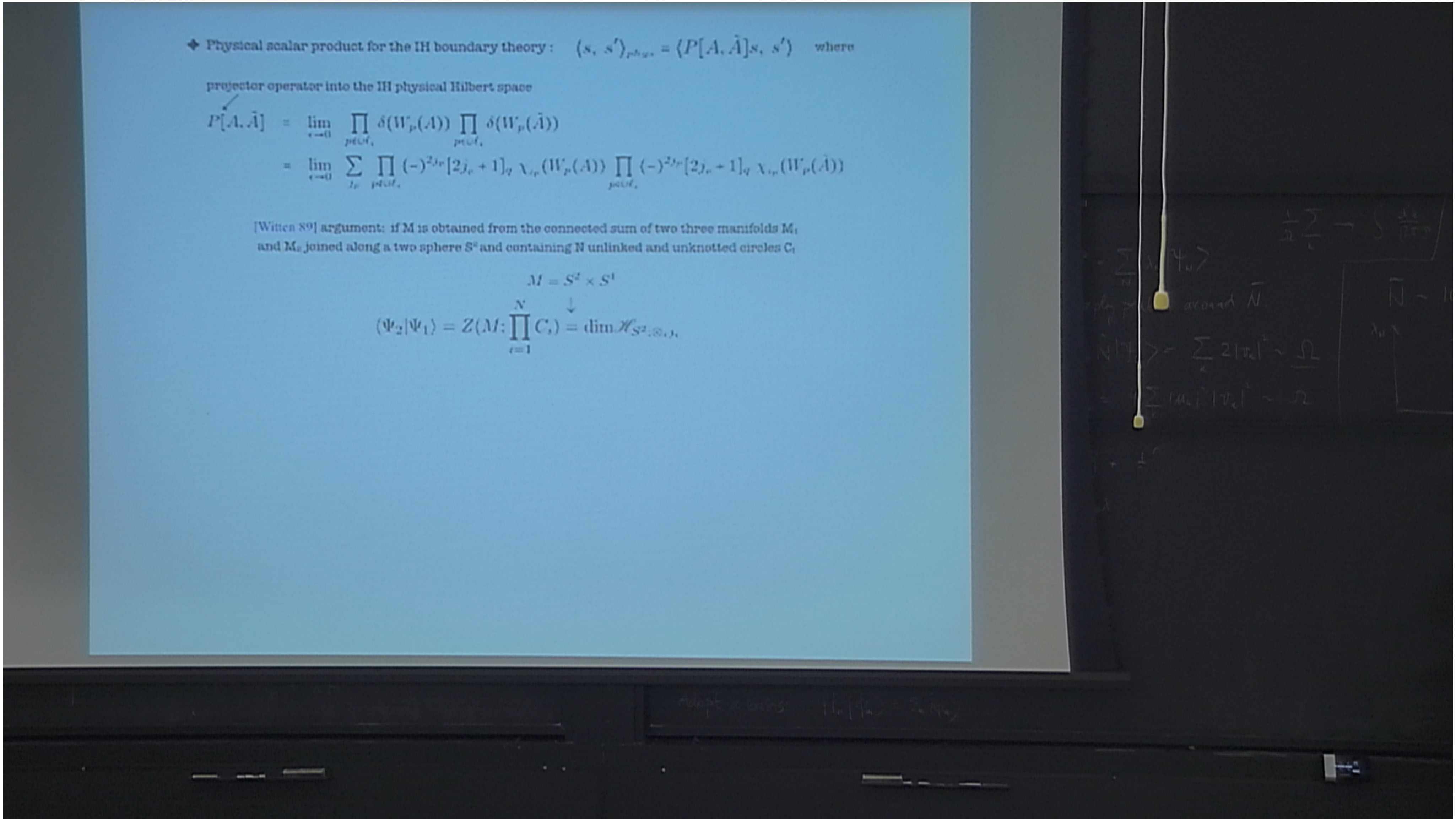

Physical scalar product for the IH boundary theory : $\langle s, s' \rangle_{phys} = \langle P[A,\tilde{A}]s, s' \rangle$ where
projector operator into the IH physical Hilbert space
$P[A,\tilde{A}] = \lim_{\epsilon\to 0} \prod_{p\in\cup\ell_i} \delta(W_p(A)) \prod_{p\in\cup\ell_i} \delta(W_p(\tilde{A}))$
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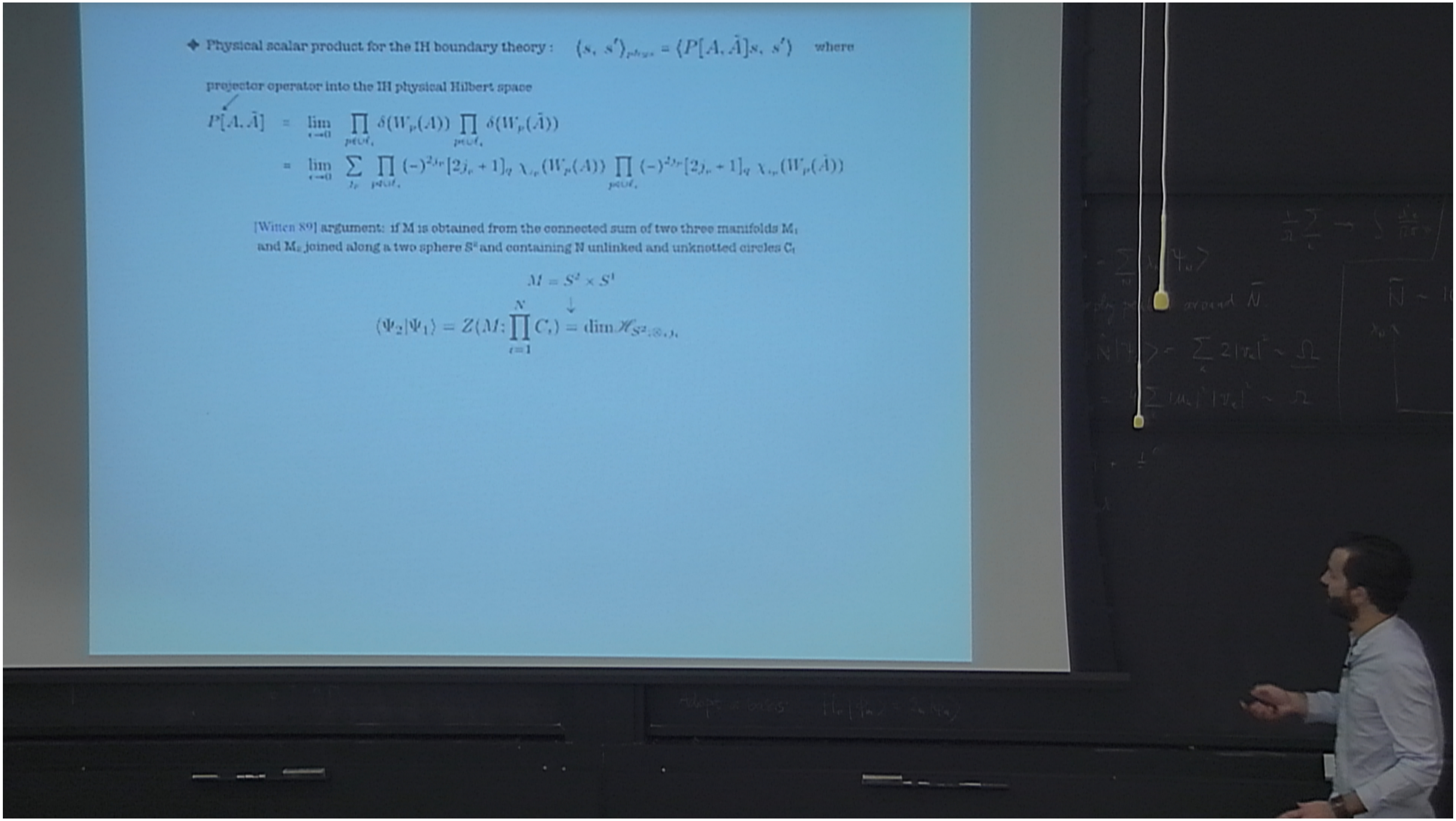
[Witten 89] argument: if M is obtained from the connected sum of two three manifolds M_1
and M_2 joined along a two sphere S^2 and containing N unlinked and unknotted circles C_i
$M = S^2 \times S^1$
$\langle \Psi_2 | \Psi_1 \rangle = Z(M; \prod_{i=1}^{N} C_i) = \dim \mathscr{H}_{S^2, \otimes_i j_i}$



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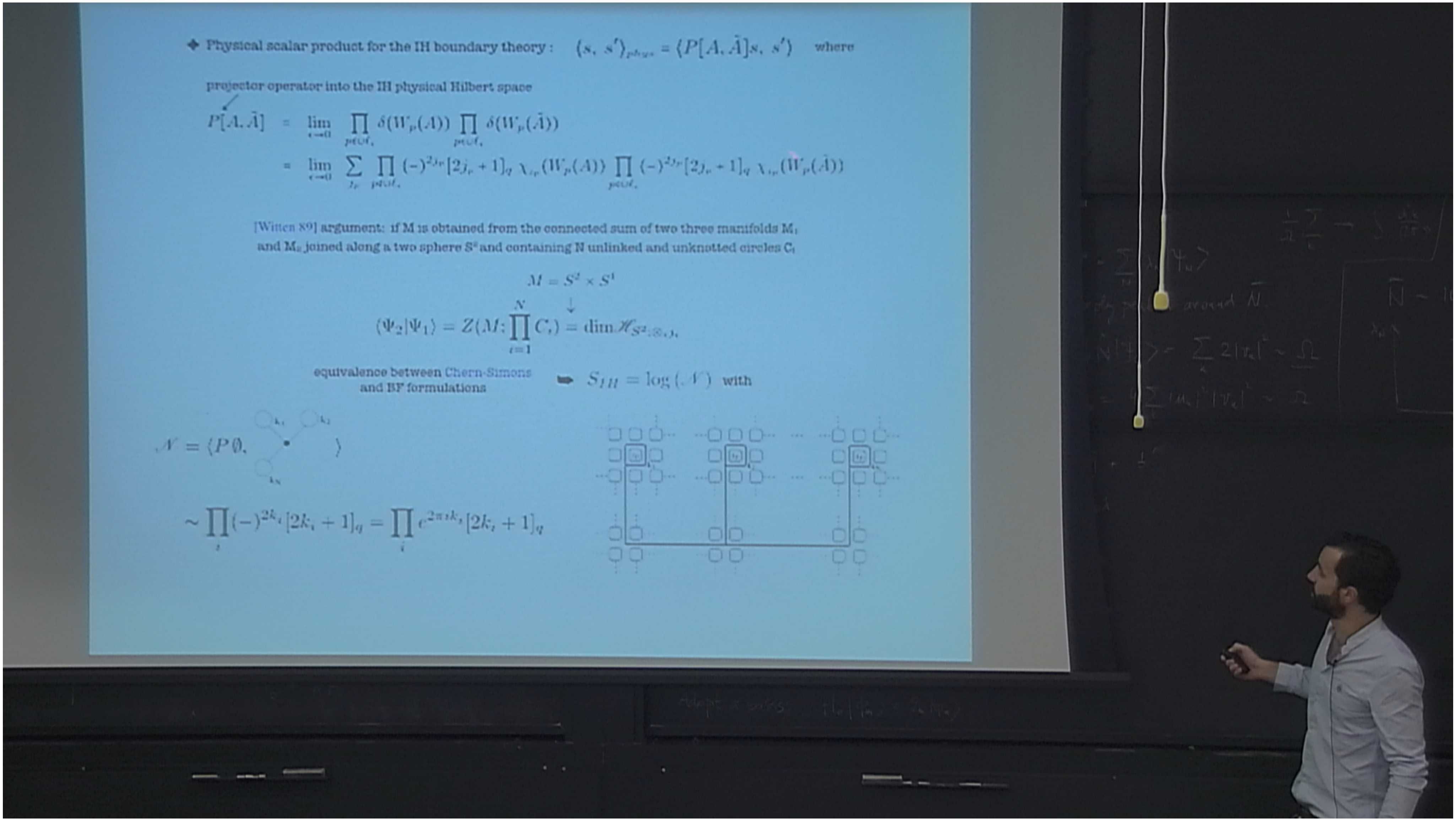
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$S_{IH} = \log(\mathcal{N})$ with
$\mathcal{N} = \langle P \emptyset, \rangle$
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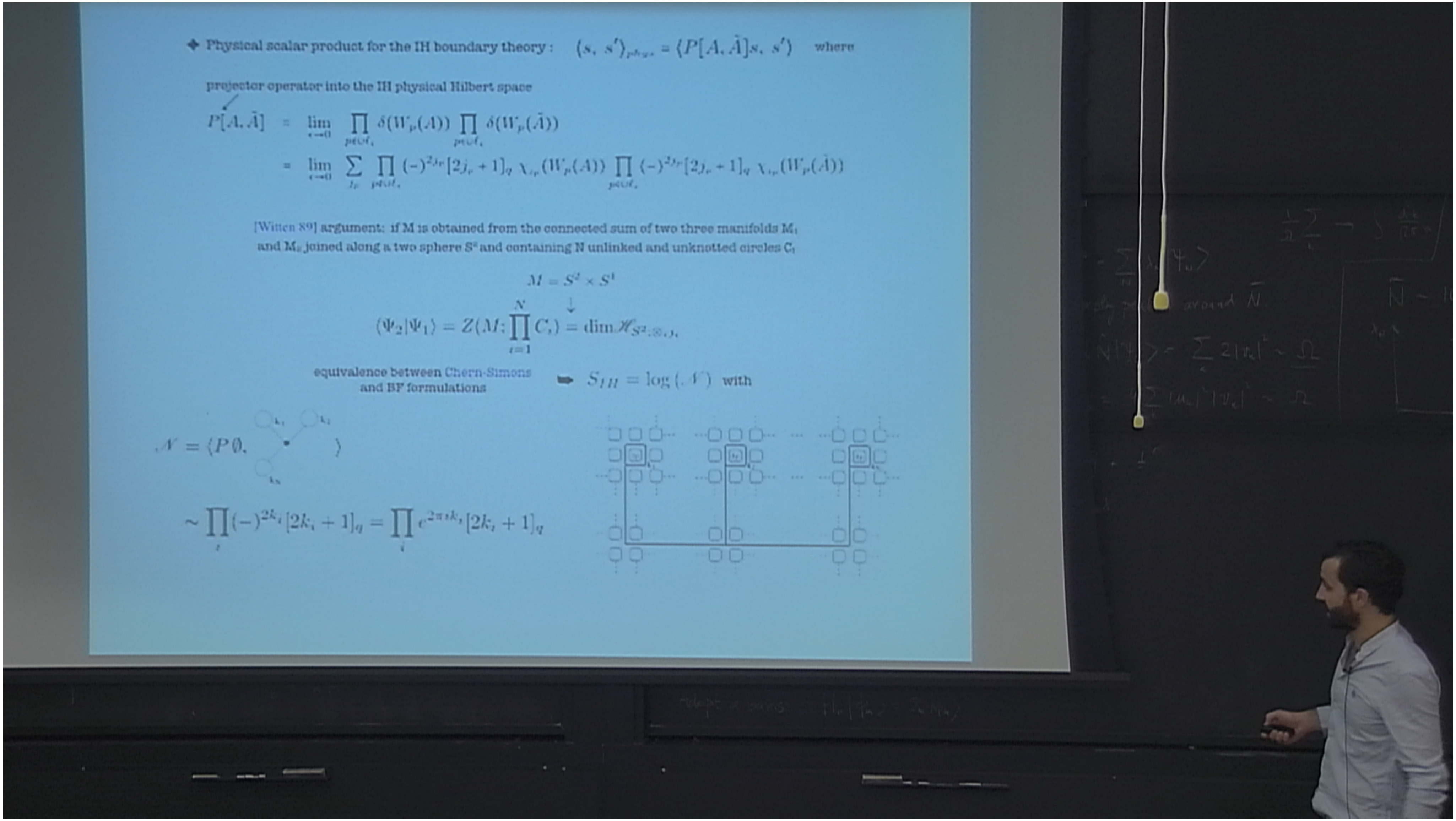
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𝒩 = ⟨P ∅, ⟩
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k₂
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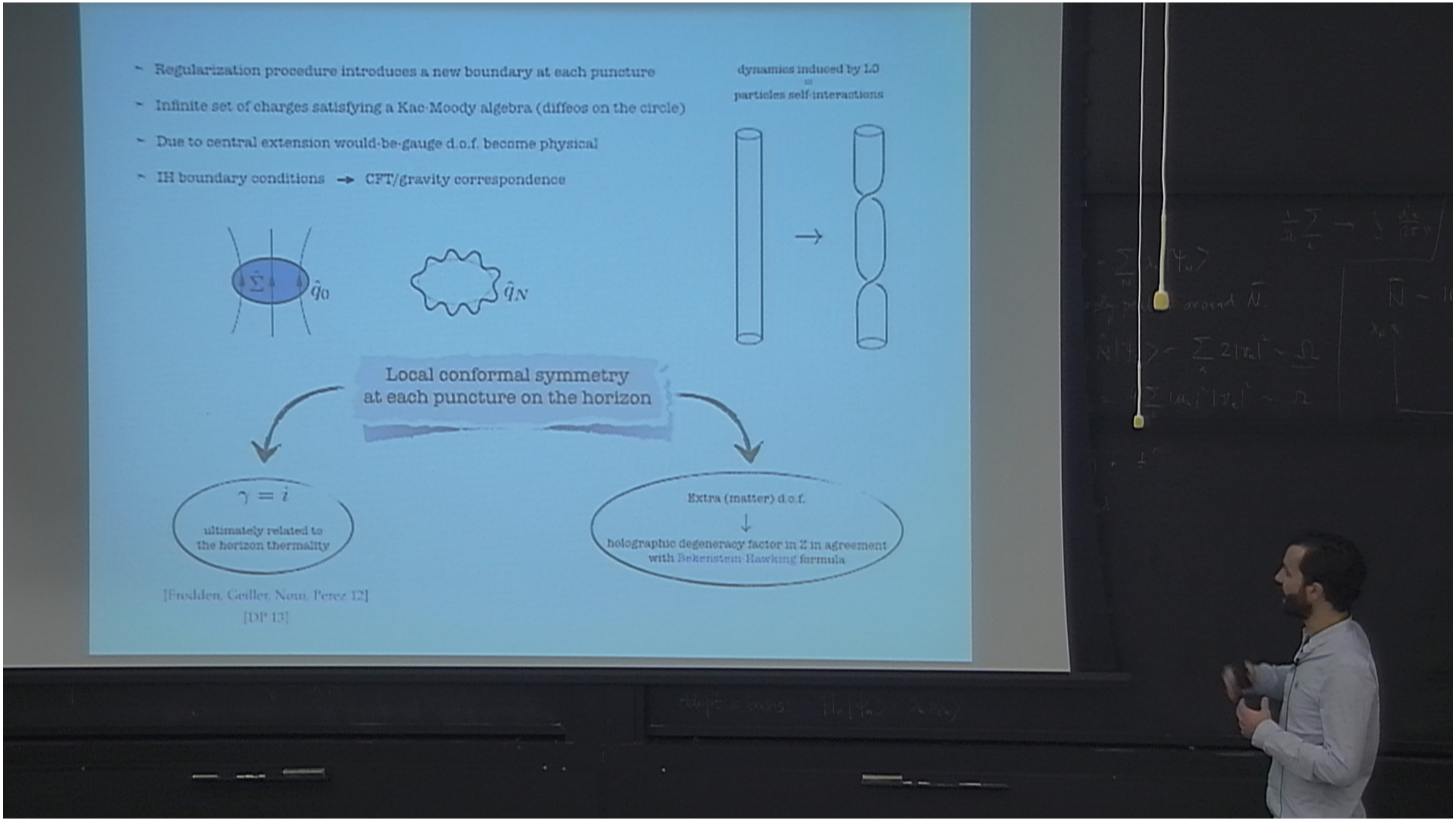
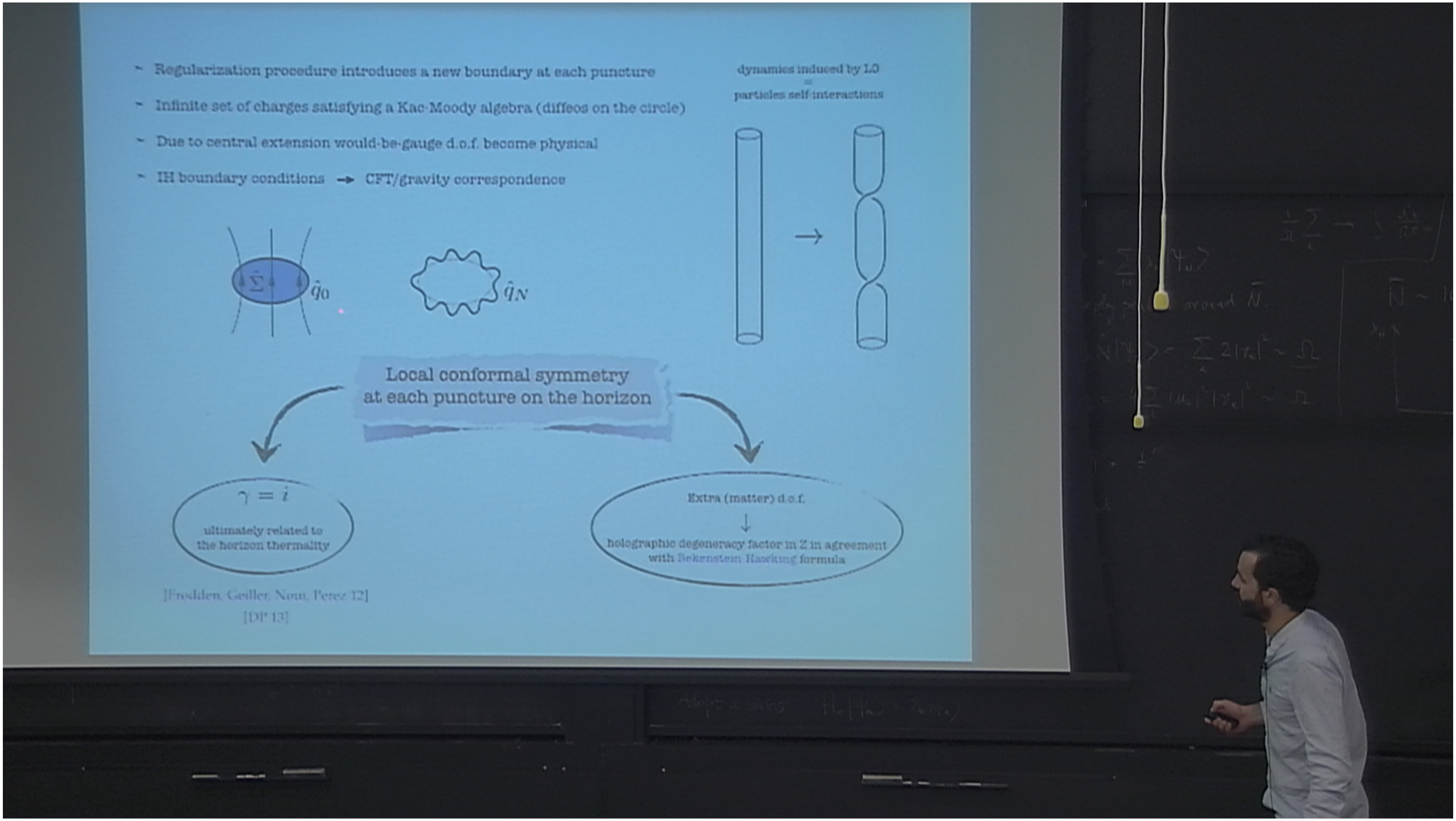


- Regularization procedure introduces a new boundary at each puncture
- Infinite set of charges satisfying a Kac-Moody algebra (diffeos on the circle)
- Due to central extension would-be-gauge d.o.f. become physical
- IH boundary conditions → CFT/gravity correspondence
dynamics induced by L0
=
particles self-interactions
$\hat{\Sigma}$
$\hat{q}_0$
$\hat{q}_N$
Local conformal symmetry
at each puncture on the horizon
$\gamma = i$
ultimately related to
the horizon thermality
[Frodden, Geiller, Noui, Perez 12]
[DP 13]
Extra (matter) d.o.f.
holographic degeneracy factor in Z in agreement
with Bekenstein-Hawking formula



Regularization procedure introduces a new boundary at each puncture
Infinite set of charges satisfying a Kac-Moody algebra (diffeos on the circle)
Due to central extension would-be-gauge d.o.f. become physical
IH boundary conditions → CFT/gravity correspondence
dynamics induced by 1.0
=
particles self-interactions
$\hat{\Sigma}$
$\hat{q}_0$
$\hat{q}_N$
Local conformal symmetry at each puncture on the horizon
$\gamma = i$
ultimately related to the horizon thermality
Extra (matter) d.o.f.
holographic degeneracy factor in Z in agreement with Bekenstein-Hawking formula
[Frodden, Geiller, Noui, Perez 12]
[DP 13]

About the full theory:

If we see each spin network intertwiner as a micro-BH, then this new regularization can provide an alternative way to couple matter dof in LQG

⇨ Unified CFT description of gravity and matter at the Planck scale

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SM valid up to the Planck scale [Froggatt, Nielsen 95]:
top quark and Higgs masses predicted from the "Multiple Point Principle" assumption,
i.e. the Standard Model effective Higgs potential should have two degenerate minima (vacua),
one of which should be at the Planck scale, where it vanishes!

Scenario supported by the recent NNLO calculation of
[Degrassi, Di Vita, Elias-Miró, Espinosa, Giudice, Isidori, Strumia 13].

(see also ['t Hooft 14])

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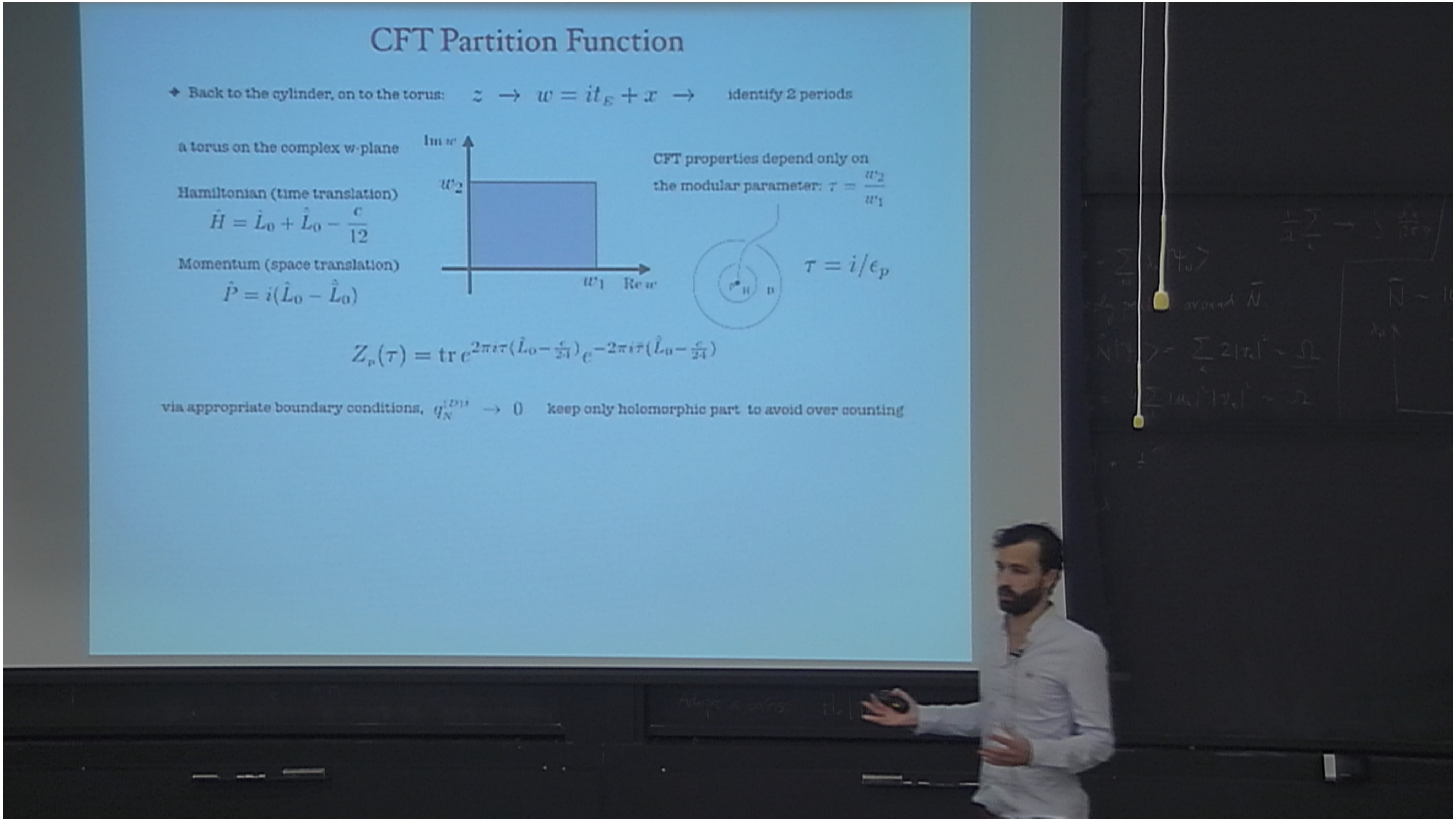
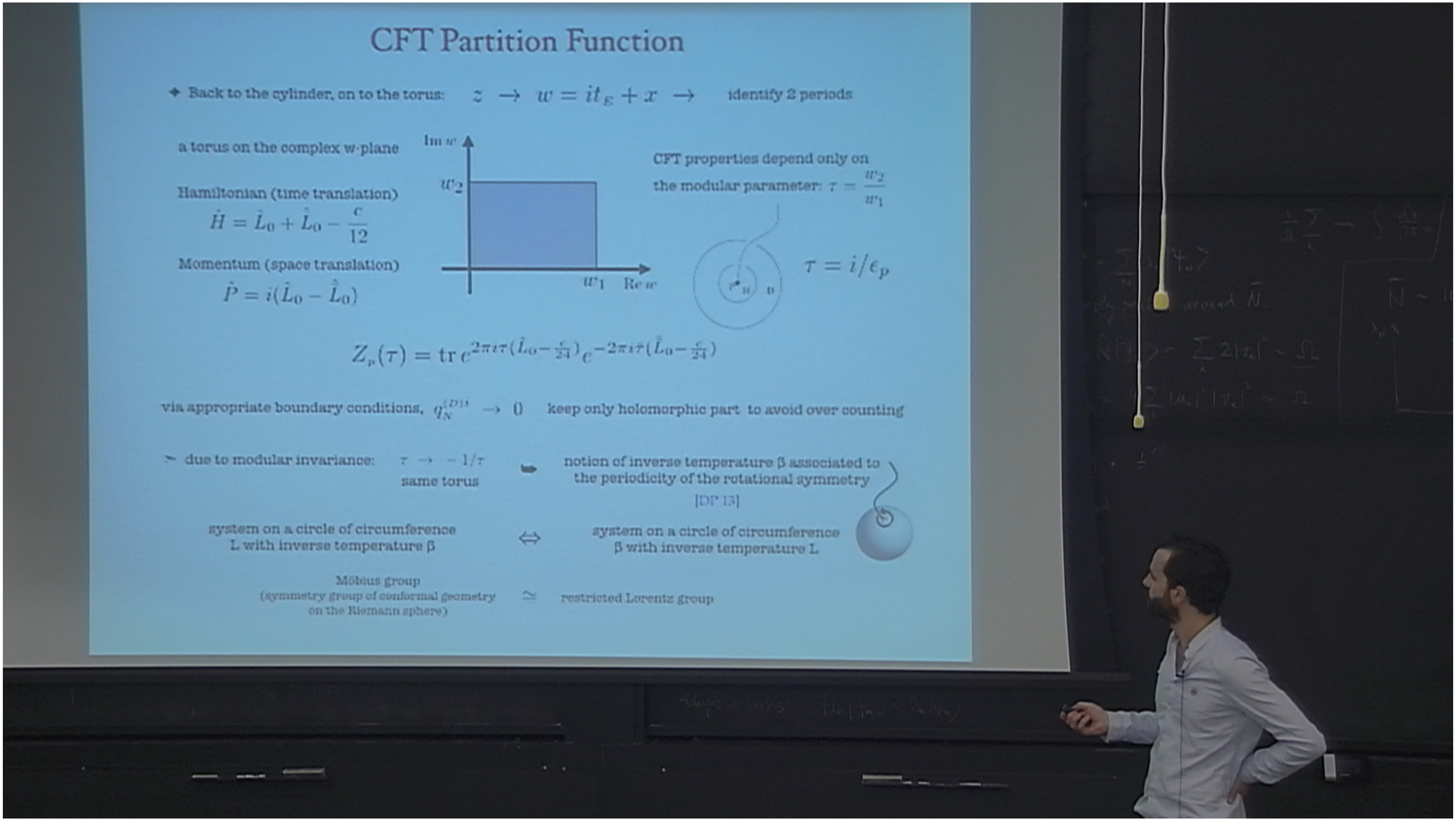

CFT Partition Function
Back to the cylinder, on to the torus: $z \to w = it_E + x \to$ identify 2 periods
a torus on the complex w-plane
Hamiltonian (time translation)
$\hat{H} = \hat{L}_0 + \hat{\bar{L}}_0 - \frac{c}{12}$
Momentum (space translation)
$\hat{P} = i(\hat{L}_0 - \hat{\bar{L}}_0)$
Im w
w_2
w_1
Re w
CFT properties depend only on the modular parameter: $\tau = \frac{w_2}{w_1}$
$\tau = i/\epsilon_p$
$Z_p(\tau) = \mathrm{tr}\, e^{2\pi i\tau(\hat{L}_0 - \frac{c}{24})} e^{-2\pi i\bar{\tau}(\hat{\bar{L}}_0 - \frac{c}{24})}$
via appropriate boundary conditions, $q_N^{(D)} \to 0$ keep only holomorphic part to avoid over counting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due to modular invariance: $\tau \to -1/\tau$
same torus
notion of inverse temperature β associated to the periodicity of the rotational symmetry
[DP 13]
system on a circle of circumference L with inverse temperature β
⇔
system on a circle of circumference β with inverse temperature L
Möbius group (symmetry group of conformal geometry on the Riemann sphere)
≃
restricted Lorentz group