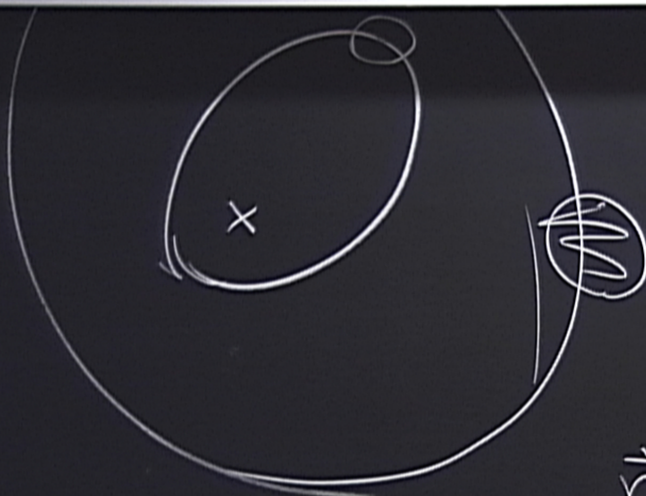
Title: PHYS 781 - Lecture 19

Date: Nov 14, 2014 12:00 PM

URL: http://pirsa.org/14110142

Abstract:



$$\frac{d\vec{L}_a}{dt} = \frac{G M_a M_J \; \vec{r}_a \times (\vec{r}_J - \vec{r}_a)}{|r_a - r_J|^3}$$

$$= \frac{G M_a M_J}{|r_a - r_J|^3} \, \vec{r}_a \times \vec{r}_J$$

$$\vec{L}_a = G M_a M_J \int dt \, \frac{(\vec{r}_a \times \vec{r}_J)}{|\vec{r}_a - \vec{r}_J|^3}$$

$$\vec{r}_a = \sum_n \vec{A}_n e^{i n \omega_a t}$$

$$\vec{r}_J = \sum_m \vec{B}_n e^{i m \omega_J t}$$

$$\sum_{n=-\infty} A_n e$$

$$\vec{r}_J = \sum_{m=-\infty}^{+\infty} \vec{B}_n e^{im\omega_J t}$$

$$\vec{L}_a = G M_a M_J \int_0^t dt \sum_{mn} C_{mn}\, e^{in\omega_a t}\, e^{im\omega_J t}$$

$$= G M_a M_J \sum_{mn} C_{mn} \frac{[e^{i(n\omega_a + m\omega_J)t} - 1]}{n\omega_a + m\omega_J}$$

$$\frac{1}{r_J^2} \text{ or } \frac{1}{r_a^2} \sim [C_{mn}] = \frac{1}{L^2}$$

$$\Delta L_{a,(mn)} \sim \frac{G M_a M_J}{r^2} \times \frac{1}{n\omega_a + m\omega_J} \sim M_a v_a r_a$$

$$\omega_{mn} = \left|\frac{m}{n}\right| \omega_J$$

$$n(\omega_a - \omega_{mn}) \sim \frac{G M_J}{r_a^3 v_a}$$

$$f(r,v) = \beta e^{-H(r,v)/kT} = \beta e^{\frac{-mv^2}{nT} + \frac{mM}{rkT}}$$

$$\frac{kT}{m} = \frac{\pi\sigma^2}{3\pi - 8}$$

$$\frac{f(r,v)}{f(\infty,v)} = e^{M_{BH}G/r(a^2)} = 2$$

$$\langle (v-\mu)^2 \rangle$$

$$e^{v^2/2a^2}$$

1.1 pc

$$r = \frac{M_{BH}G}{\sigma^2 \ln(2)(\pi/(3\pi-8))}$$

$$a^2 = \sigma^2 \frac{(3\pi-8)}{\pi}$$

For an isoth. sphere: $\nabla^2 \phi_{isoth} = 4\pi G \rho$ where $\rho = \frac{\sigma^2}{2\pi G r^2}$

$\hookrightarrow \nabla^2 \phi_{isoth} = \frac{2\sigma^2}{r^2} \Rightarrow$ the Grav. field $\nabla \phi_{isoth} = \frac{-2\sigma^2}{r}$

For the BH: $\phi_{BH} = \frac{G M_{BH}}{r} \Rightarrow$ the Grav field $\nabla \phi_{BH} = -\frac{G M_{BH}}{r^2}$

$$\nabla \phi_{isoth} = \nabla \phi_{BH} \Rightarrow \boxed{r = \frac{G M_{BH}}{2\sigma^2}}$$

$$t \gtrsim t_c$$

$$t \sim t_c$$

$$t_c \sim \frac{N}{\ln N}\, t_{dyn} \qquad t_{dyn} = \left(\frac{GM}{R^3}\right)^{-1/2}$$

$$N = \frac{M}{M_*}$$

$$M = \int \rho \, d^3x$$

$$M = 4\pi \int \rho r^2 dr$$

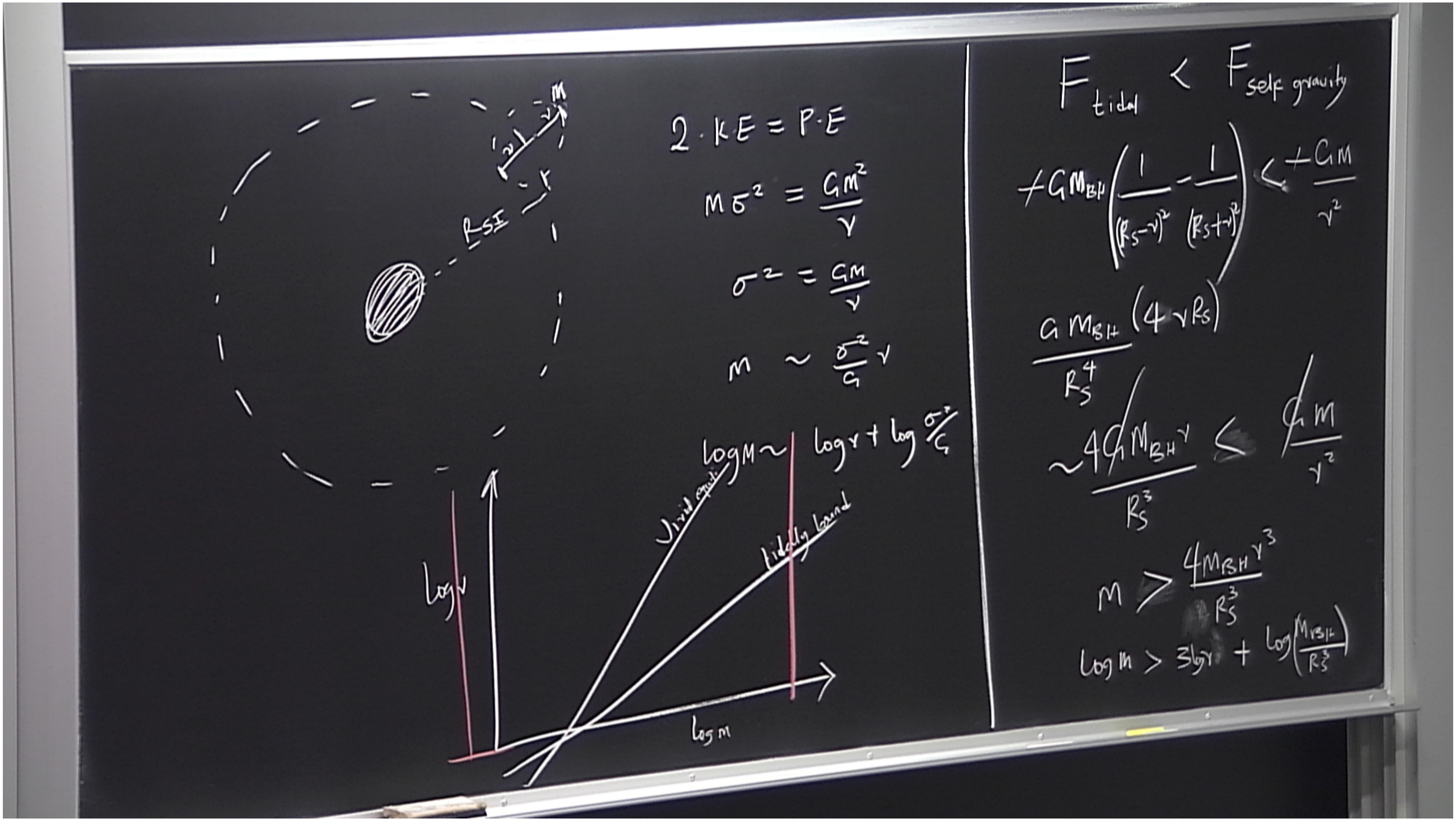
$$M = \frac{2\sigma^2 R}{G}$$

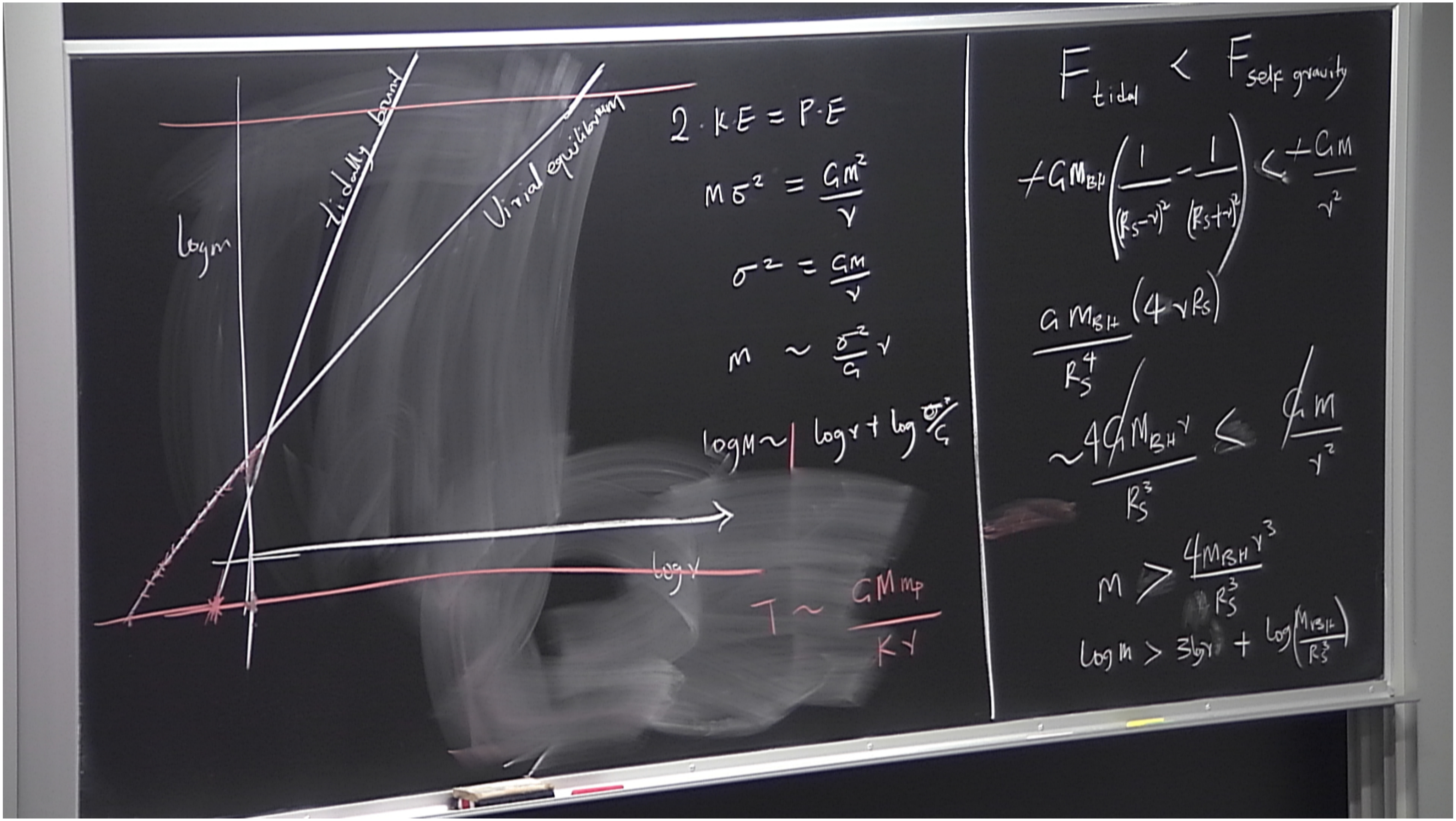
$$t_{uni} \sim \frac{M}{\ln N} \left(\frac{GM}{R^3}\right)^{-1/2}$$

$$t_{universe} \sim \frac{\sqrt{2}\, \sigma R^2}{G M_* \left(\ln\left(\frac{2\sigma^2}{G M_*}\right) + \ln R\right)}$$

$$t_{universe} \sim 13.7 \times 3.16 \times 10^{16}\, s$$

$$R \sim 8.2 \times 10^{18}\, cm$$


m
r
r
R_SI
2·KE = P·E
Mσ² = Gm²/r
σ² = Gm/r
m ~ σ²/G r
log M ~ log r + log σ²/G
Log r
Log m
Virial equil.
tidally bound
F_tidal < F_self gravity
+GM_BH (1/(R_S−r)² − 1/(R_S+r)²) < +Gm/r²
GM_BH/R_S⁴ (4 r R_S)
~4GM_BH r/R_S³ ≤ Gm/r²
m > 4M_BH r³/R_S³
Log m > 3 log r + log(M_BH/R_S³)



log m
tidally bound
Virial equilibrium
log r
$2 \cdot KE = P \cdot E$
$M\sigma^2 = \frac{GM^2}{r}$
$\sigma^2 = \frac{GM}{r}$
$M \sim \frac{\sigma^2}{G} r$
$\log M \sim \log r + \log \frac{\sigma^2}{G}$
$T \sim \frac{GMm_p}{kr}$
$F_{tidal} < F_{self\ gravity}$
$GM_{BH}\left(\frac{1}{(R_s-r)^2} - \frac{1}{(R_s+r)^2}\right) < \frac{GM}{r^2}$
$\frac{GM_{BH}}{R_s^4}(4 r R_s)$
$\sim \frac{4GM_{BH} r}{R_s^3} \leq \frac{GM}{r^2}$
$M > \frac{4M_{BH} r^3}{R_s^3}$
$\log m > 3\log r + \log\left(\frac{M_{BH}}{R_s^3}\right)$

$$r_a^2 V_a$$

$$\nabla\phi = \frac{\sigma_T F_{rad}}{m_p c}$$

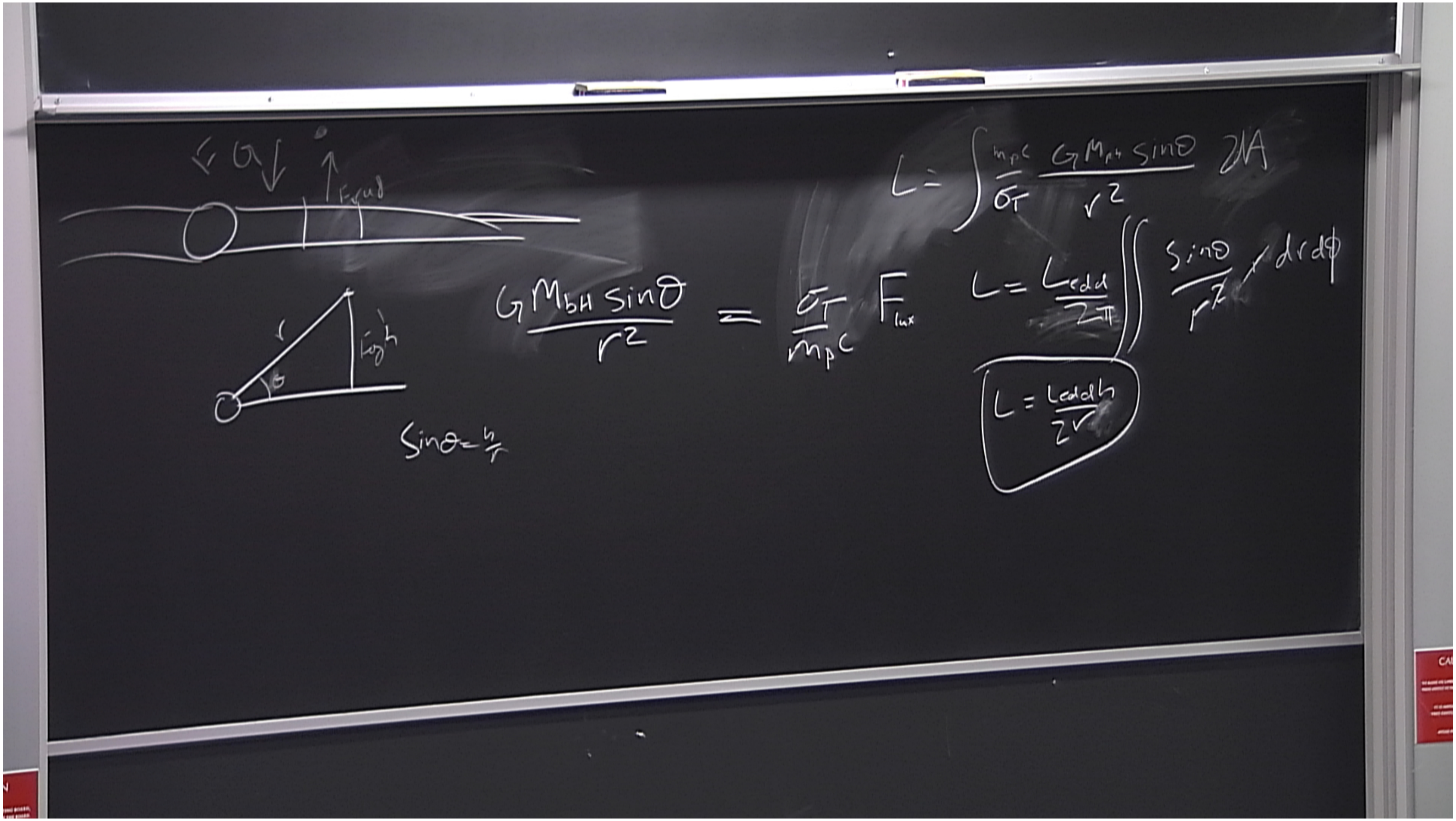
$$L = \int_S F_{rad} \cdot dS = \frac{m_p c}{\sigma_T} \int \nabla\phi \cdot dS$$

$$L = \frac{m_p c}{\sigma_T} \int \underline{\nabla^2\phi}\, dV$$

$$L = \frac{4\pi G c m_p}{\sigma_T} \int \rho\, dV$$

$$L_{edd} = \frac{4\pi G M m_p c}{\sigma_T}$$

$$\frac{L_{edd}}{L_\odot} \simeq 3.2\times10^4 \left(\frac{M}{M_\odot}\right)$$


F_rad
G M_BH sinθ / r^2 = σ_T / m_p c F_lux
sinθ = h/r
L = ∫ m_p c / σ_T G M_BH sinθ / r^2 2dA
L = L_edd / 2π ∫∫ sinθ / r^2 r dr dφ
L = L_edd h / 2r

$$L = \sigma_B T^4 \, 2\pi R^2$$

$$L = \frac{dE}{dt} = \frac{GM}{R}\dot{m} = \sigma_B T^4 \, 2\pi R^2$$

$$\boxed{T^4 = \frac{GM\dot{m}}{2\pi R^3 \sigma_B}}$$

$$F_{tidal} < F_{self\ gravity}$$

$$-GM_{BH}\left(\frac{1}{(R_S - r)^2} - \frac{1}{(R_S + r)^2}\right) < -\frac{Gm}{r^2}$$

$$\frac{GM_{BH}}{R_S^4}(4 r R_S)$$

$$\sim \frac{4\cancel{G}M_{BH} r}{R_S^3} \leq \frac{\cancel{G}m}{r^2}$$

$$m > \frac{4 M_{BH} r^3}{R_S^3}$$

$$\log m > 3\log r + \log\left(\frac{M_{BH}}{R_S^3}\right)$$

$r_a^2 V_a$

$$dL = 2\,dS\,\sigma_{SB} T^4$$

mass : $dM = \Sigma\, dS$

surface m. density

$$\hookrightarrow dL = \frac{2\,dM}{\Sigma}\sigma_{SB} T^4$$

$$dU = -\frac{GM\,dM}{r}$$

$$dL_{pot} = \frac{GM\,dM}{r^2}\frac{dr}{dt} = \frac{GM\,dM}{r^2} v$$

$$dL = dL_{pot}$$

$$\Rightarrow T = \left(\frac{GM\Sigma v}{2\sigma_{SB} r^2}\right)^{1/4}$$

$$\dot{M} = 2\pi r \Sigma v$$

$\boxed{\text{Jeff McClintock}}$

$$\Rightarrow T = \left(\frac{G}{4\pi\sigma_{SB}}\right)^{1/4} M^{1/4} \dot{M}^{1/4} r^{-3/4}$$