

Title: The Universe as a Cosmic String

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Abstract: <span>We are investigating modifications of general relativity that are operative at the largest observable scales. In this context, we are investigating the model of brane induced gravity in 6D, a higher dimensional generalization of the DGP model. As opposed to different claims in the literature, we have proven the quantum stability of the theory in a weakly coupling regime on a Minkowski background. In particular, we have shown that the Hamiltonian of the linear theory is bounded from below. This result opened a new window of opportunity for consistent modified Friedmann cosmologies. In our recent work it is shown that a brane with FRW symmetries necessarily acts as a source of cylindrically symmetric gravitational waves, so called Einstein-Rosen waves. Their existence essentially distinguishes this model from its codimension-one counterpart and necessitates to solve the non-linear system of bulk and brane-matching equations. A numerical analysis is performed and two qualitatively different and dynamically separated classes of cosmologies are derived: degravitating solutions for which the Hubble parameter settles to zero despite the presence of a non-vanishing energy density on the brane and super-accelerating solutions for which Hubble grows unbounded. The parameter space of both the stable and unstable regime is derived and observational consequences are discussed: It is argued that the degravitating regime does not allow for a phenomenologically viable cosmology. On the other hand, the super-accelerating solutions are potentially viable, however, their unstable behavior questions their physical relevance.</span>

# The Universe as a Cosmic String

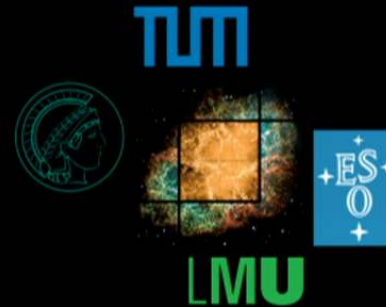
Brane Induced Cosmology in 6D

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Perimeter  
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Phys. Rev. D 88, 064047 (2013)

# Motivation



★ Consistent competitor theory to GR

★ Problem of naturalness

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = M_{\text{Pl}}^{-2} T_{\mu\nu}$$



Fits the observed accelerated expansion

Is  $\Lambda$  robust quantum mechanically?

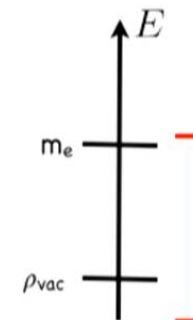
→ Consider EFT valid up to cutoff  $M$  just above  $m_e$

$$\rho_{\text{obs}} = M_{\text{Pl}}^2 \Lambda + \# m_e^4 + \dots$$

$\sim 10^{-12}(\text{eV})^4$                        $\sim 10^{24}(\text{eV})^4$

→ The two terms must cancel up to  $\sim 36$  decimal places!

→ Known particles (e.g. electron) are already problematic



# IR Modification of Gravity



- ★ Problem cannot be solved solely by UV physics
- ★  $\Lambda$  is inferred only gravitationally

Does the resolution reside in the IR of the gravitational sector?

e.g. massive gravity

- ★ Einstein's equations in 4D:  $\Lambda$  curves space-time inevitably

$$G_{\mu\nu}^{(4)} = -\Lambda g_{\mu\nu}^{(4)} \quad \longrightarrow \quad 3H^2 = \Lambda$$

Can we relax that equation by introducing extra dimensions?



# Degravitation (e.g. Dvali, Gabadadze, Shifman, 2003)



## ★ Toy model

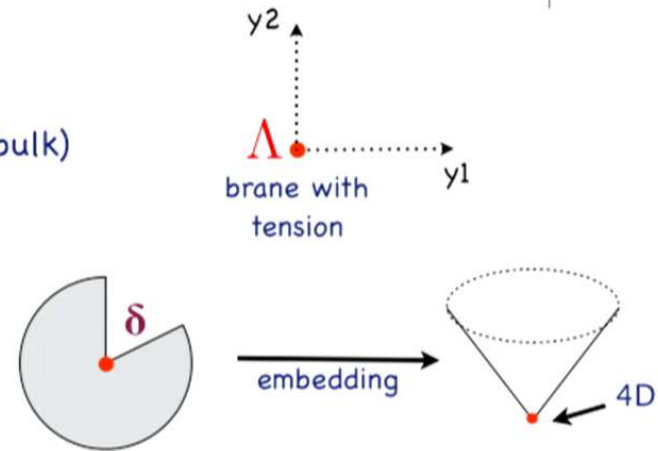
→ 4D brane with tension  $\Lambda$  in 6D space (bulk)

$$\mathcal{S} = \mathcal{S}_{\text{EH}}^{(6)} + \int d^4x \sqrt{-h} (-2\Lambda)$$

→ Explicit cosmic string solution:

$$ds_{(6)}^2 = \eta_{\mu\nu} dx^\mu dx^\nu + dr^2 + \left(1 - \frac{\delta}{2\pi}\right) r^2 d\phi^2$$

→ Deficit angle:  $\delta = \frac{\Lambda}{M_6^4}$



## ★ Mechanism

$$G_{\mu\nu}^{(6)} = G_{\mu\nu}^{(4)} + K_{\mu\nu} = -\frac{\Lambda}{M_6^4} \delta^{(2)}(y) g_{\mu\nu} \longrightarrow H^2 = 0$$

↑ intrinsic + extrinsic curvature terms

# Brane Induced Gravity (BIG)



★ The BIG model (Dvali, Gabadadze, Porrati, 2000)

$$\mathcal{S} = \mathcal{S}_{\text{EH}} + \mathcal{S}_{\text{BIG}} + \mathcal{S}_{\text{m}}[h]$$

(4+n)D gravity term  $\mathcal{S}_{\text{EH}} = M_D^{D-2} \int d^D X \sqrt{-g} \mathcal{R}^{(D)}$

Induced 4D gravity term  $\mathcal{S}_{\text{BIG}} = M_{\text{Pl}}^2 \int d^4 x \sqrt{-h} \mathcal{R}^{(4)}$

Localized matter fields on induced geometry h  $\mathcal{S}_{\text{m}}[h]$

→ D=5: DGP model

★ Weak field limit on Minkowski

(i) For distances  $r \gg r_c$ :  $V(r) \propto \frac{1}{r^{1+n}}$



Gravity leaks into the bulk

(ii) For distances  $r \ll r_c$ :  $V(r) \propto \frac{1}{r}$



Gravity confined

→ Crossover between 4D and (4+n)D  $r_c(M_{\text{Pl}}, M_D, R)$

*Ghost*

*or No Ghost*

# From a No-Go to a No-Ghost Theorem



## ★ Claims in the literature

(e.g. Dubovsky et al., 2002)

→ BIG with  $D > 5$  has a linear ghost on Minkowski background

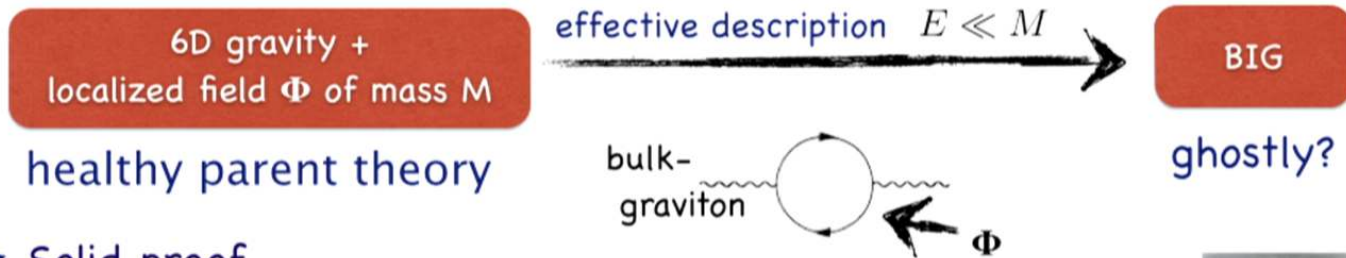
## ★ Degree of freedom with a wrong sign kinetic term

→ Classical Hamiltonian unbounded from below

→ Spoils quantum description

Existence of negative norm states

## ★ **But:** Simple counter-argument:



## ★ Solid proof

→ Express the Hamiltonian in terms of independent d.o.f.





*Cosmology*

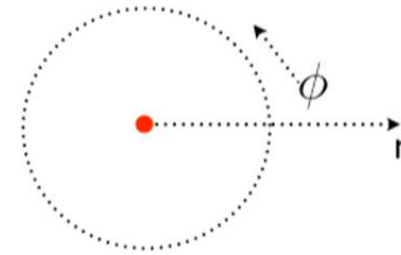
# Cosmology in 6D



★ Challenge: Derive modified Friedmann equations

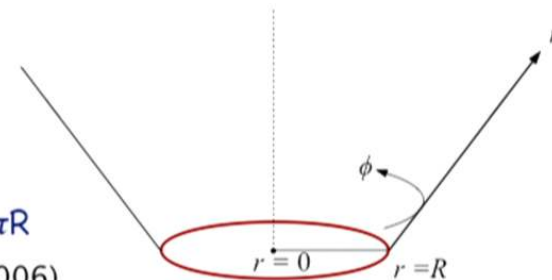
★ Assumptions

- Homogeneous and isotropic along spatial brane directions
- Source-free bulk (implies cylindrical symmetry)



★ Logarithmic divergencies

- Promote brane to a ring of circumference  $2\pi R$   
(Peloso, Sorbo, Tasinato, 2006) (Kaloper, Kiley, 2006)



- Brane localized energy momentum tensor:  $T^{(5)a}_b = \frac{1}{2\pi R} \text{diag}(-\rho, P, P, P, P_\phi)$
- Stabilization requires pressure component  $P_\phi$  in compact direction
- Results are insensitive to regularization as long as  $HR \ll 1$

# Einstein-Rosen Waves



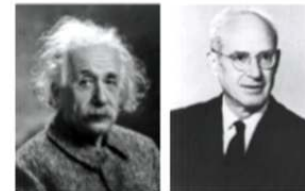
★ In 5D (DGP): Embedding in static bulk (Deffayet, 2001)

→ ensured by analog of Birkhoff's theorem in planar symmetry  
(e.g. Taub, 1951)

★ In 6D: Symmetries allow for cylindrical waves

→ Generalization of 4D Einstein-Rosen waves

→ Cylindrical symmetric ansatz (valid in vacuum)



$$ds_6^2 = e^{2(\eta-3\alpha)} (-dt^2 + dr^2) + e^{2\alpha} d\vec{x}^2 + e^{-6\alpha} r^2 d\phi^2$$

→  $\alpha(t,r)$  fulfills 2D linear wave equation in vacuum:  $\partial_t^2 \alpha = \partial_r^2 \alpha + \frac{1}{r} \partial_r \alpha$

→  $\eta(t,r)$  completely fixed, no dynamical field

→ Incoming  $\alpha$ -waves affect brane curvature evolution through  $e^{2\alpha} d\vec{x}^2$

# Brane Matching Equations



★ Brane position:  $r_0(t)$

★ Induced metric on the brane:  $ds_5^2 = -d\tau^2 + e^{2\alpha_0} d\vec{x}^2 + R^2 d\phi^2$

→ with proper time  $\tau$ :  $d\tau = \frac{e^{-3\alpha_0}}{\gamma} dt$  and  $\gamma \equiv \frac{e^{-\eta_0}}{\sqrt{1 - \left(\frac{dr_0}{dt}\right)^2}}$

→ Hubble parameter:  $H = \frac{d\alpha_0}{d\tau}$

→ demand fixed circumference:  $R \equiv r_0 e^{-3\alpha_0} = \text{const.}$

→ underlying stabilization mechanism effectively implemented by  $P_\phi$

★ Israel junction conditions at brane (Israel, 1966)

$\rho$ -matching:  $H^2 = \frac{\rho}{3M_{\text{Pl}}^2} + \frac{1}{r_c^2} (\gamma - 1)$  modified Friedmann equation

$P$ -matching:  $\dot{\rho} + 3H(\rho + P) = 0$

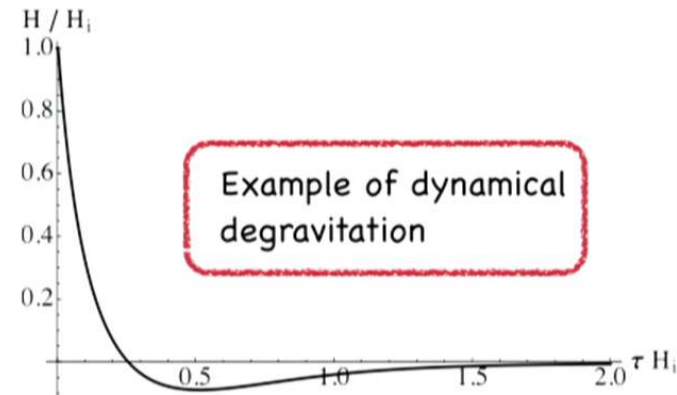
# Dynamical Degravitation



★ Consider a pure tension brane:  $\Lambda < \Lambda_{\text{crit}} = 2\pi M_6^4$

## I. Degravitating solution:

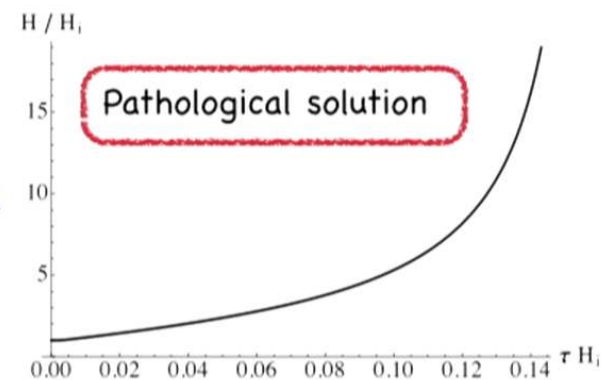
- Static deficit angle solution is approached
- Equation of state for  $P_\phi$  is  $> -1$
- Happens for dust and radiation, too



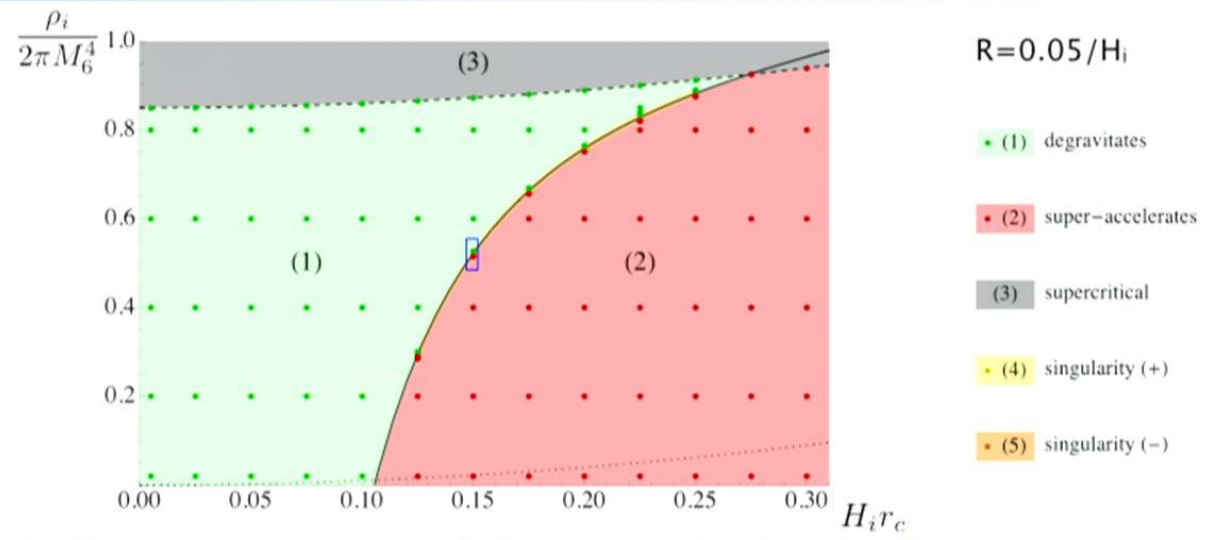
## II. Super-accelerating solution:

- Hubble grows unbounded
- Effective energy density as seen by 6D gravity becomes negative
- Equation of state for  $P_\phi$  falls rapidly below  $-1$
- Analog of self-accelerating branch in DGP

➔ Perturbative ghosts like in DGP?



# Island of Stability



★ Both regions separated by physical singularity

★ Analytic bounds for degravitating solutions

(i) stability bound:  $\frac{\rho}{2\pi M_6^4} > r_c^2 H^2 + 1 - \frac{9R^2}{2r_c^2}$

(ii) criticality bound:  $\frac{\rho}{2\pi M_6^4} \leq r_c^2 H^2 + 1 - 3|H|R$

}  $Hr_c < \sqrt{\frac{3}{2}} |H| R$

# Phenomenology



## ★ A viable solution requires

(i) 4D regime for early times:  $Hr_c \gg 1$

(ii) Insensitivity to unknown UV physics:  $HR \ll 1$

→ (i) and (ii) are mutually incompatible with  $Hr_c < \sqrt{\frac{3}{2}} |H| R$

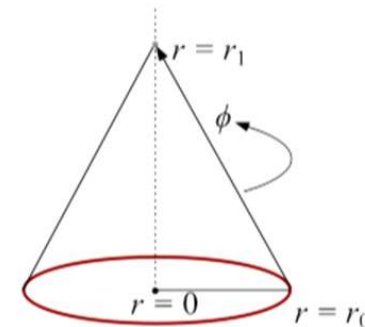
→ Degravitating solutions are ruled out!

→ Super-accelerating solutions are compatible with (i) and (ii) but are dismissed due to instability

## ★ Remaining window of opportunity

→ Supercritical regime

→ Static solution with naked singularity!

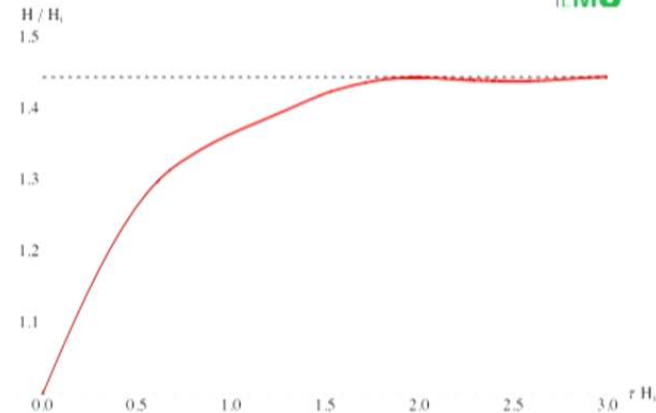


# Step back: Supercritical Cosmic String in 4D



## ★ First numerical results

- (i) Attractor: dS on-brane geometry
- (ii) Horizon separates brane from outer axis
- (iii) Growing 'cigar'-shaped universe



## ★ Analytical result

→ Scaling solution in interior:  $\tilde{\alpha}(\tilde{t}, \tilde{r}/\tilde{t})$  and  $\tilde{\eta}(\tilde{r}/\tilde{t})$

→ Analytic Relation between tension and Hubble:

$$\frac{\lambda}{2\pi M_{\text{Pl}}^2} - 1 \approx HR$$

in near-critical regime

# Summary



- ★ Linearized BIG on Minkowski is ghost-free in all dimensions
  
- ★ The theory allows for stable cosmological solutions
  - Dynamical degravitation
  - Cosmic string solution is approached
  - Phenomenologically not viable
  
- ★ Unstable solutions similar to self-accelerating branch in DGP model
  
- ★ Supercritical regime remains for investigation
  - First results without BIG:
    - Attractor solution corresponding to inflating brane
    - Formation of horizon

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