Title: From locality and operationalism to classical and quantum theory?

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Abstract: <p>We present a first principles approach to a probabilistic description of nature based on two guiding principles: spacetime locality and operationalism. No notion of time or metric is assumed, neither any specific physical model. Remarkably, the emerging framework converges with the recently proposed positive formalism of quantum theory, obtained constructively from known quantum physics. However, it also seems to embrace classical physics.</p>

From locality and operationalism to classical and quantum theory?

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Plan

In this talk I want to develop the structure of a physical description of nature from first principles.

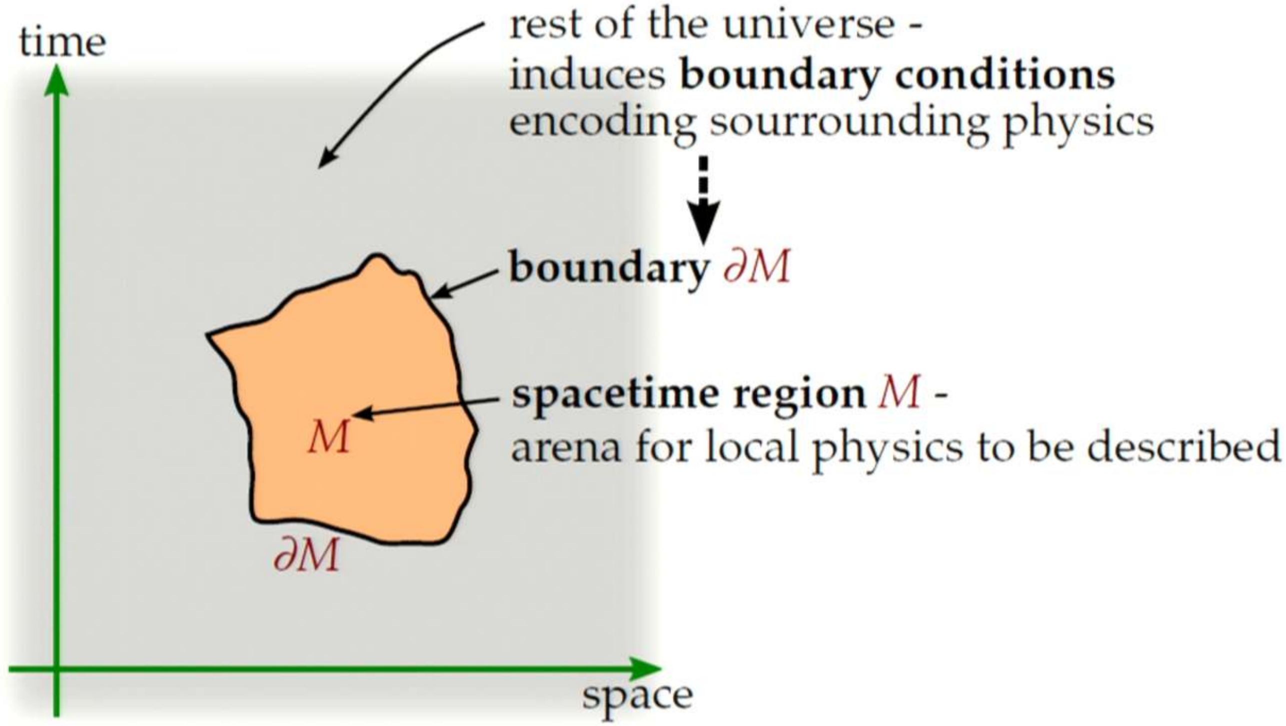
Classical physics will serve as an illustrative example.

In the end, an encounter with quantum theory awaits.

Guidelines

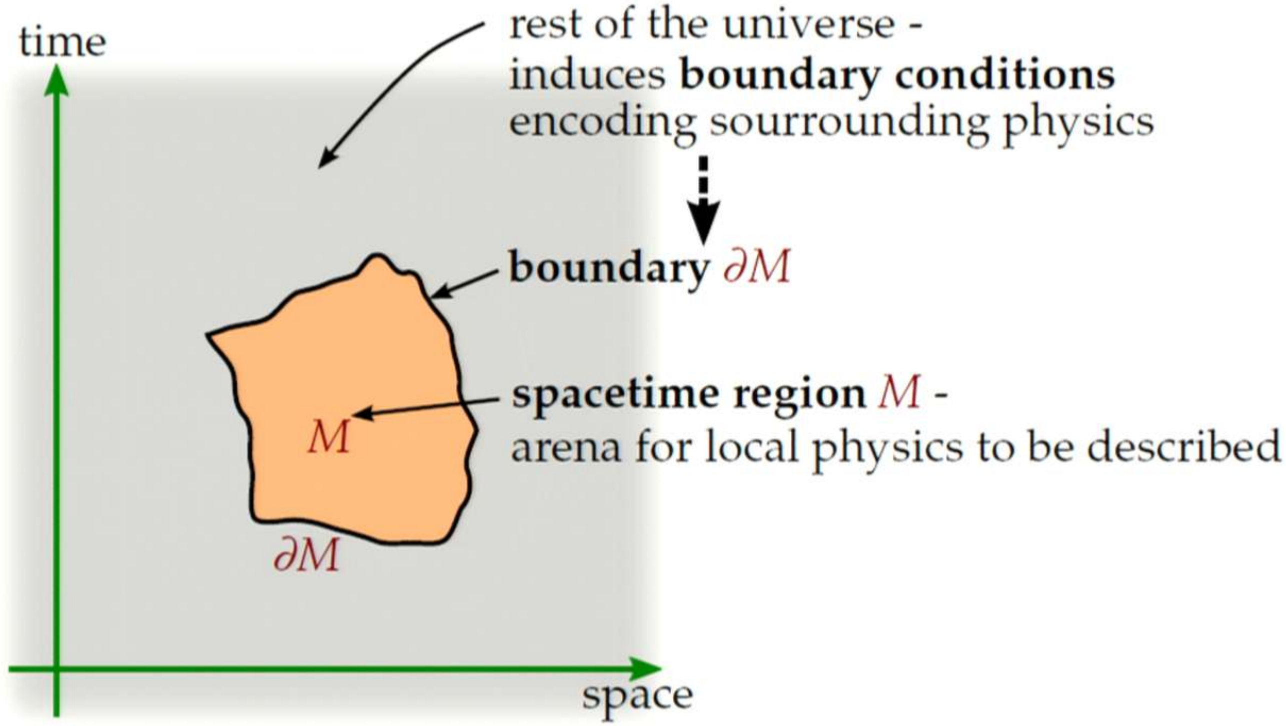
- **Locality**: We have learned that to understand and describe local physics, a knowledge or control of the immediate spatial and temporal surroundings is sufficient.
- **Operationalism**: In classical physics sweeping statements about physical reality independent of a possible observer are possible and even sensible. This is not so in quantum theory. Rather, we should be describing physics through the interaction with an observer or experimenter.

Locality and spacetime



Require a **notion of spacetime**:
spacetime regions and their **boundaries**.

Locality and spacetime



Require a **notion of spacetime**:
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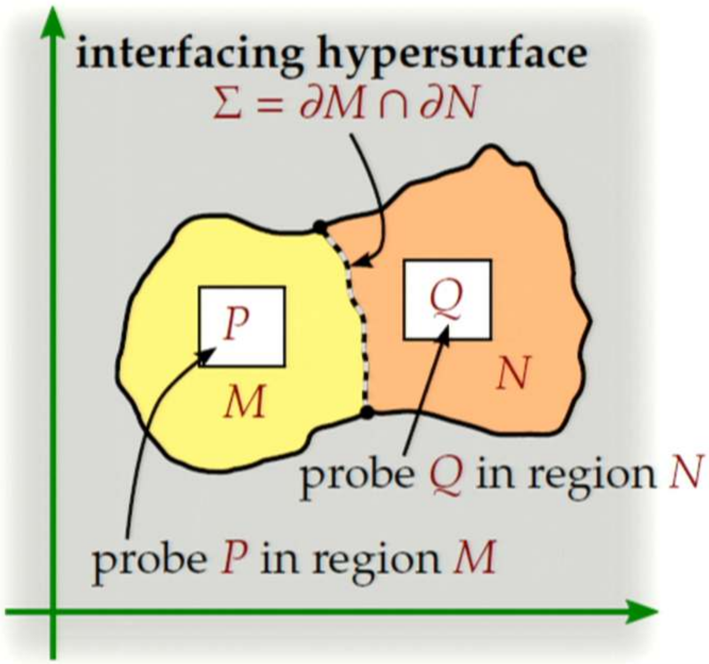
Composition

For a comprehensive description it is essential that we be able to relate the physics in adjacent spacetime regions.



Need an operation that allows to **combine probes** P, Q in adjacent spacetime regions M, N to a composite probe $P \diamond Q$ in the joint region $M \cup N$.

"Holography"

Information about local physics is communicated between adjacent regions through **boundary conditions** on **interfacing hypersurfaces**.

Towards a quantitative description of physics

Associate mathematical structures to the ingredients identified so far.

- For a **region** M we denote the space of **probes** in M by $\mathcal{P}_M$. We denote the null-probe by $\emptyset \in \mathcal{P}_M$. The composition of probes is a map $\diamond : \mathcal{P}_M \times \mathcal{P}_N \to \mathcal{P}_{M \cup N}$.

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- To a **hypersurface** Σ we associate a space $\mathcal{B}_\Sigma$ of **boundary conditions**. This encodes the possible physical information flows between the two regions adjacent to the hypersurface Σ.

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- To a **hypersurface** Σ we associate a space $\mathcal{B}_\Sigma$ of **boundary conditions**. This encodes the possible physical information flows between the two regions adjacent to the hypersurface Σ.
- To a **probe** P in a **spacetime region** M with **boundary condition** $b \in \mathcal{B}_{\partial M}$ we associate a **quantity**. We shall take this to be a **real number** and denote it by $(P, b)_M$. It encodes a property of the local physics in the interior as detected by the probe and subject to the boundary condition. Formally, $(\cdot, \cdot)_M : \mathcal{P}_M \times \mathcal{B}_{\partial M} \to \mathbb{R}$.

Quantities – Direct and Deterministic version

In the simplest case the **quantities** $(P, b)_M$

- correspond **directly** to measurable quantities and
- predict these with certainty, i.e., are **deterministic**.

An important special class of probes answers **YES/NO** questions. These are conveniently represented by quantities $(P, b)_M \in \{0, 1\}$.

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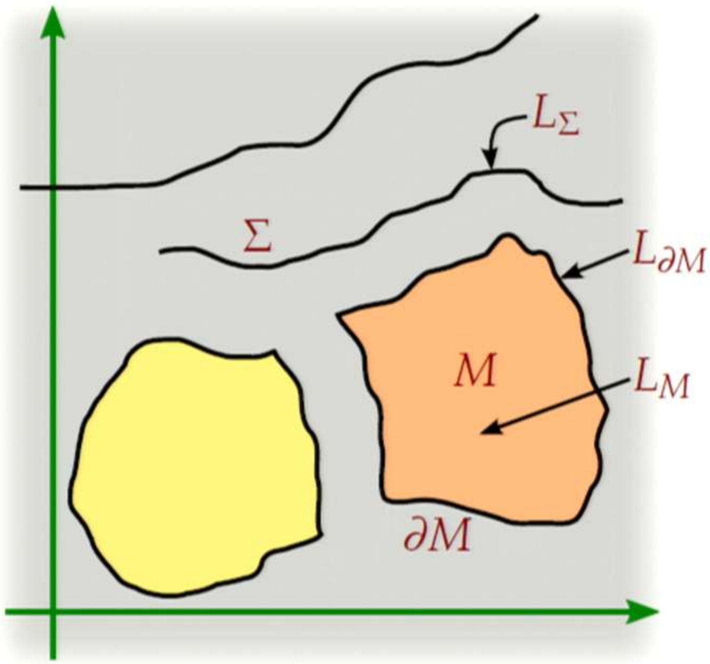
Classical physics

In classical physics the possible realities are **solutions** of the **equations of motions**, i.e. **field configurations** and **particle trajectories**. Locally:

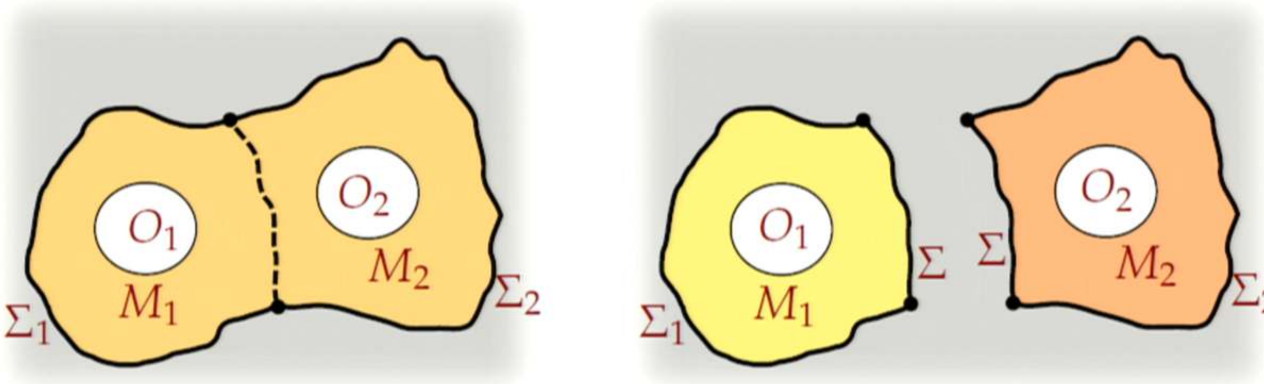


- Per **hypersurface** Σ : The **space** L_Σ **of solutions** near Σ. This is a space of **initial data**.
- Per **region** M : The **space** L_M **of solutions** in M. Forgetting the interior yields a map $L_M \to L_{\partial M}$.

Observables

In classical physics the role of **probes** may be taken by **observables**. An observable in a region M is a function $O : L_M \to \mathbb{R}$.

Consider regions M_1, M_2 with matching boundary components Σ and their **composition** to a joint region $M = M_1 \cup M_2$.



The joint observable $O = O_1 \diamond O_2$ is the product

$$O(\phi) = O(\phi|_{M_1}) \cdot O(\phi|_{M_2})$$

where $\phi \in L_M$.

Physical quantities

- A **boundary condition** on Σ is a boundary solution, i.e., $\mathcal{B}_\Sigma = L_\Sigma$.
- For a spacetime region M and boundary condition $\varphi \in L_{\partial M}$ the quantity for the **null-probe** is,

$$(\emptyset, \varphi)_M := \begin{cases} 1 & \text{if there is } \phi \in L_M \text{ with } \varphi = \phi|_{\partial M} \\ 0 & \text{otherwise} \end{cases}$$

This is the truth-value of whether a given boundary condition can be physically realized or not.

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Hierarchies of Probes

The probes that yield YES/NO answers may be organized into **hierarchies** yielding **partial orders**.

Consider an instrument with one light that shows either red or green. Consider **three** different probes associated with this instrument:

- Two probes for the two light states: $P(r)$ (red) and $P(g)$ (green)
- One probe for the unspecified state: $P(*)$

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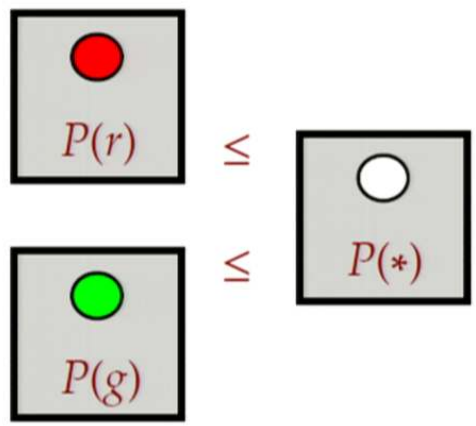
The unspecified state is more general than the others. This yields a hierarchy and a **partial order** on $\mathcal{P}_M$. Given any boundary condition b,

$$(P(r), b)_M \leq (P(*), b)_M$$
$$(P(g), b)_M \leq (P(*), b)_M$$



Hierarchies of Probes

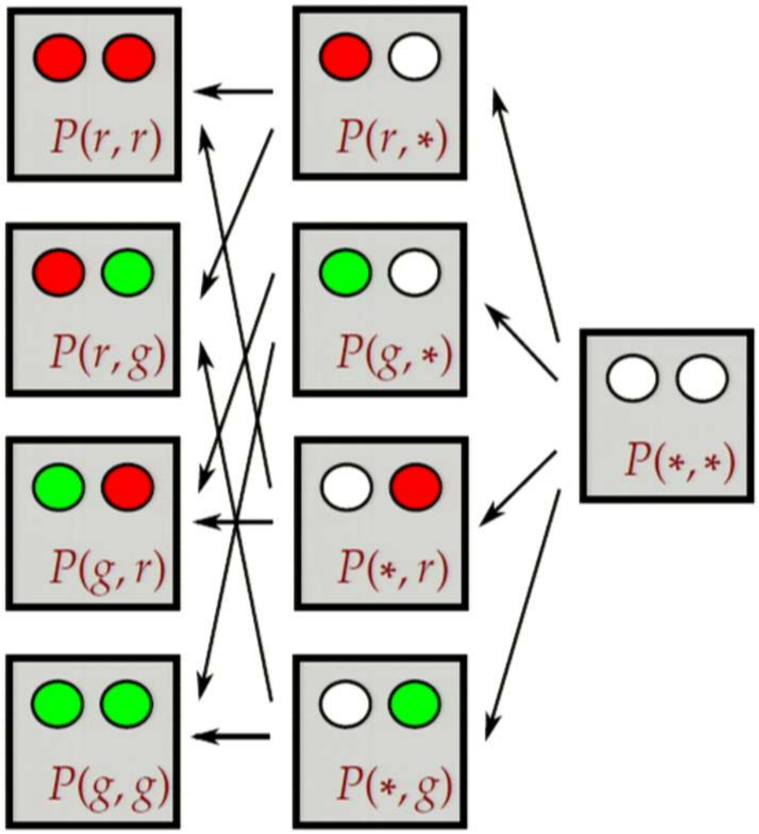
Hierarchies may become more complex when the instrument allows for more distinct readings.

We may also have **additive relations**. Here for example: For all b,

$$(P(r,r), b)_M + (P(r,g), b)_M = (P(r,*), b)_M.$$

In short:

$$P(r,r) + P(r,g) = P(r,*)$$



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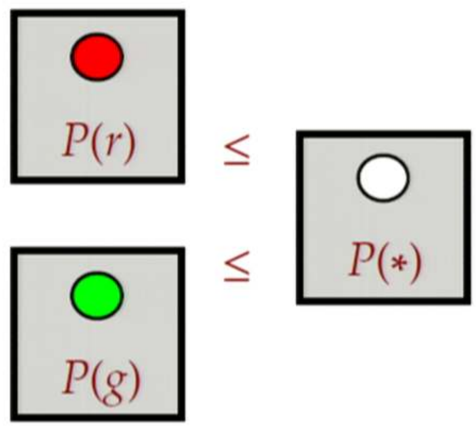


Quantities – Probabilistic version

The quantities $(P, b)_M$ might have a **probabilistic** rather than deterministic character. That is they might be

- **Probabilities** with $(P, b)_M \in [0, 1]$ or
- **Expectation values** with $(P, b)_M \in \mathbb{R}$.

Interesting quantities are then often **conditional** probabilities or expectation values.

Classical statistical physics

- We consider **boundary conditions** that are **probability densities** b on the space $L_{\partial M}$ of boundary solutions (which may be thought of as **statistical ensembles**).
- As before, **probes** are **observables**. Given an observable P in the spacetime region M with boundary condition b we define the associated quantity as,

$$(P, b)_M := \int_{L_M} P(\phi)\, b(\phi|_{\partial M})$$

Examples of physical quantities:

$$(\emptyset, b)_M$$

is the fraction of the boundary probability distribution b that is physically realizable.

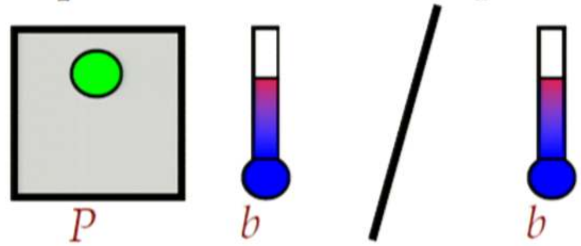
$$\frac{(P, b)_M}{(\emptyset, b)_M}$$

is the expectation value of P given the probability distribution induced by the boundary condition b.

Conditional quantities

Suppose we are interested in the **expectation value** of an observable encoded by probe P given a specific boundary condition b. (Assume not looking at the instrument is equivalent to the **null probe**.) This is:

$$\frac{(P, b)_M}{(\emptyset, b)_M}$$



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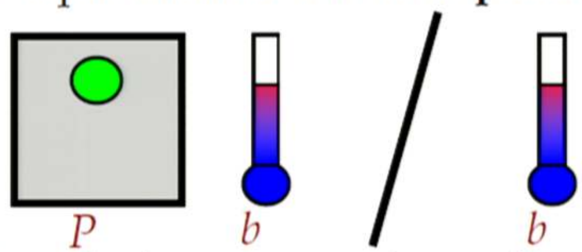
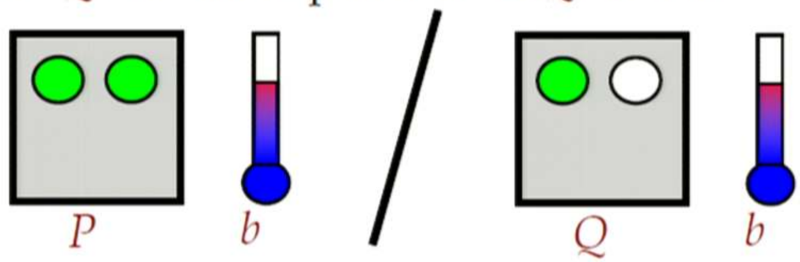


Suppose we are interested in the **relative expectation value** of a quantity P shown by an instrument given both a boundary condition b and a certain instrument state Q. We have probes $P \leq Q$. Then:

$$\frac{(P,b)_M}{(Q,b)_M}$$



Hierarchies of boundary conditions

Boundary conditions may also form **hierarchies**. This gives rise to a **partial order** on $\mathcal{B}_M$. Here, for all $P \in \mathcal{P}_M$:

$$(P, b_1)_M \leq (P, b)_M$$
$$(P, b_2)_M \leq (P, b)_M$$

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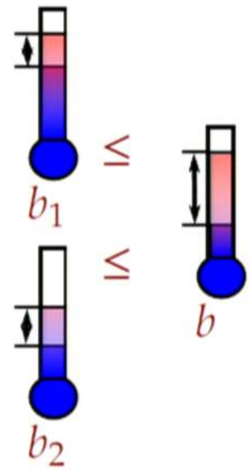

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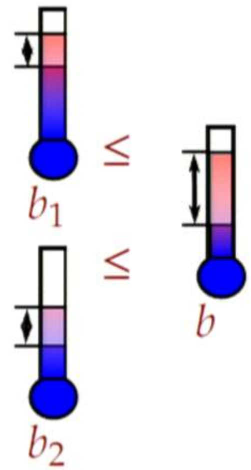
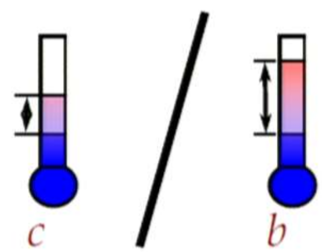


Since boundary conditions interact with the bulk it also makes sense to consider probabilities for boundary conditions conditioned on more general boundary conditions. Here for $c \leq b$,

$$\frac{(\emptyset, c)_M}{(\emptyset, b)_M}$$



Formalization of the probabilistic setting

Consider a setting where quantities are relative and give rise to **probabilities** and **real expectation values**. (As in the classical statistical setting, but with hierarchies of boundary conditions.)

- The spaces $\mathcal{B}_\Sigma$ of boundary conditions are **real vector spaces with a partial order**.
- A class $\mathcal{P}_M^+$ of **basic probes** (including the null-probe) on M give rise to **values** that are **positive linear functions** on $\mathcal{B}_{\partial M}$. (This is required for relative probabilities.)
- All **probes** on M give rise to **values** that are **real linear functions** on $\mathcal{B}_{\partial M}$. The space $\mathcal{P}_M$ of probes on M itself is a **real vector space with a partial order**.

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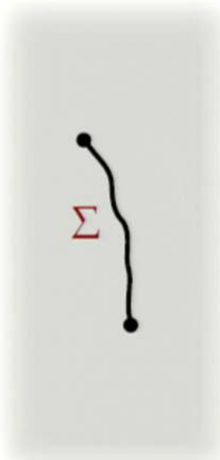
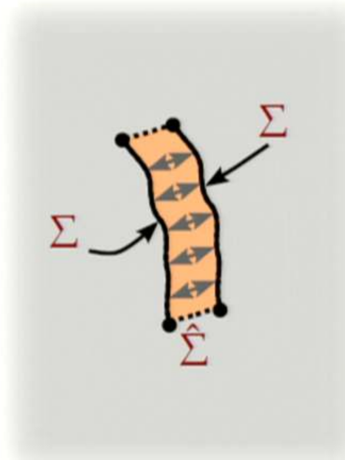
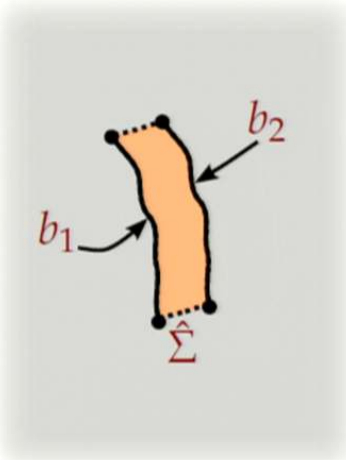
Slice regions



A **hypersurface** Σ gives rise to an infinitesimally thin **slice region** $\hat{\Sigma}$ by thickening. $\hat{\Sigma}$ has a boundary $\partial\hat{\Sigma}$ with two components, each a copy of Σ.



An inner product on boundary conditions



Putting **boundary conditions** on the two sides of a **slice region** allows evaluation with the **null probe**. This yields an **inner product** on the space of boundary conditions.

$$\mathcal{B}_\Sigma \times \mathcal{B}_\Sigma \to \mathbb{R} : \quad (b_1, b_2) \mapsto (\emptyset, (b_1, b_2))_{\hat{\Sigma}}$$

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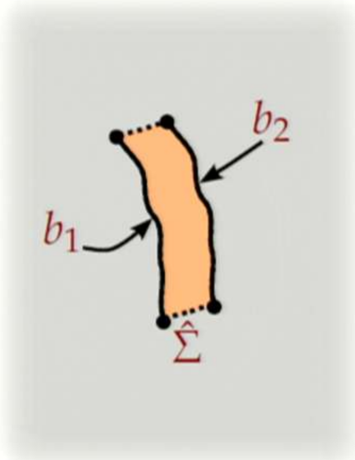

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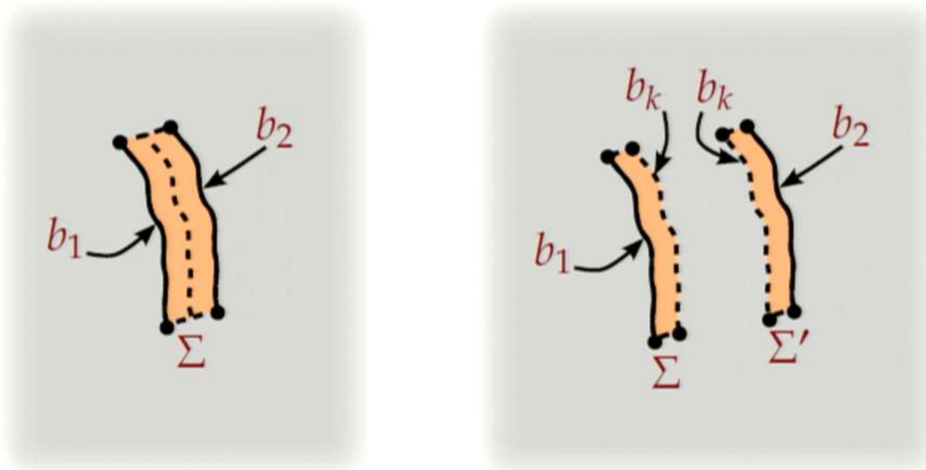
Different boundary conditions should encode different physics of adjacent regions. This means that the inner product must be **non-degenerate**. It may have **positive definite** and **negative definite** components. An **orthonormal basis** $\{b_k\}_{k \in I}$ has the property,

$$(\emptyset, (b_k, b_l))_{\hat{\Sigma}} = (-1)^{\sigma(k)} \delta_{k,l}.$$

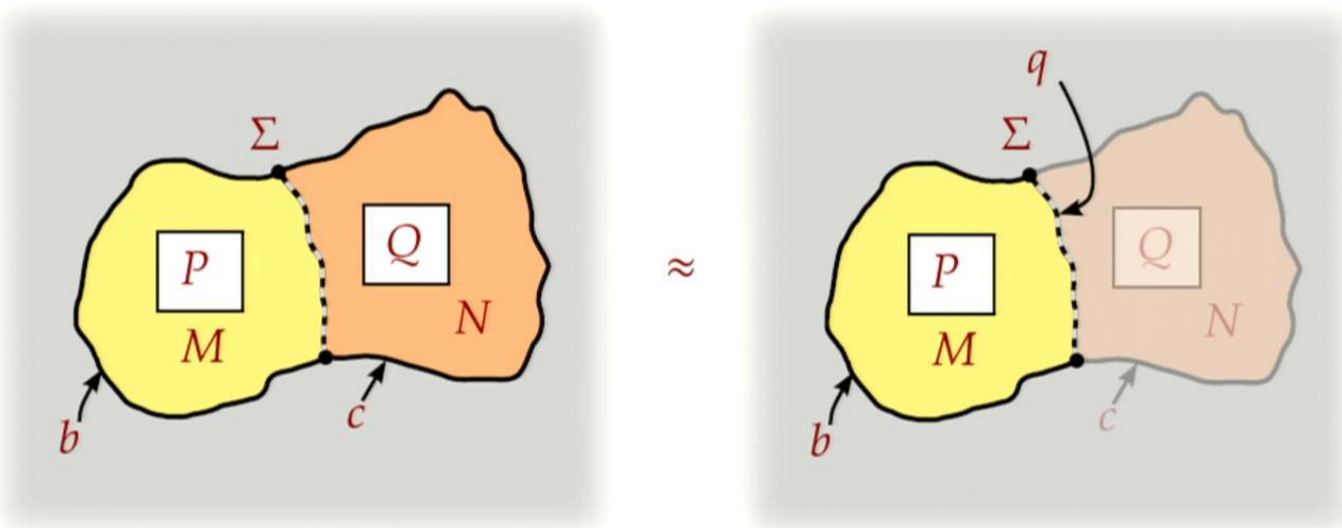
Composition of slice regions

The **completeness property** satisfied by the inner product translates into a geometric composition property of slice regions.



$$(\emptyset,(b_1,b_2))_{\hat{\Sigma}} \quad = \quad \sum_k (-1)^{\sigma(k)}\,(\emptyset,(b_1,b_k))_{\hat{\Sigma}}\,(\emptyset,(b_k,b_2))_{\hat{\Sigma}}$$

Composition through a boundary

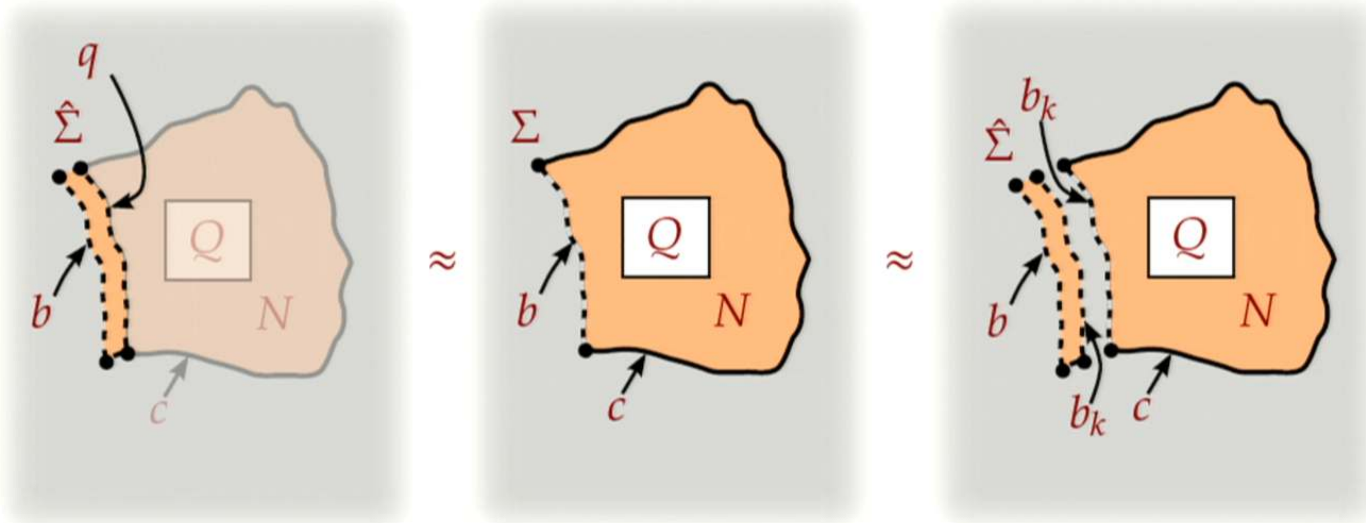


Consider the **composition of probes** P, Q in adjacent spacetime regions M, N. By locality, it must be possible to describe the effect of probe Q in N with c equivalently through a **boundary condition** q on the interfacing hypersurface $\Sigma = \partial M \cap \partial N$. Formally,

$$(P \diamond Q, (b, c))_{M \cup N} = (P, (b, q))_M.$$

Boundary condition from probe

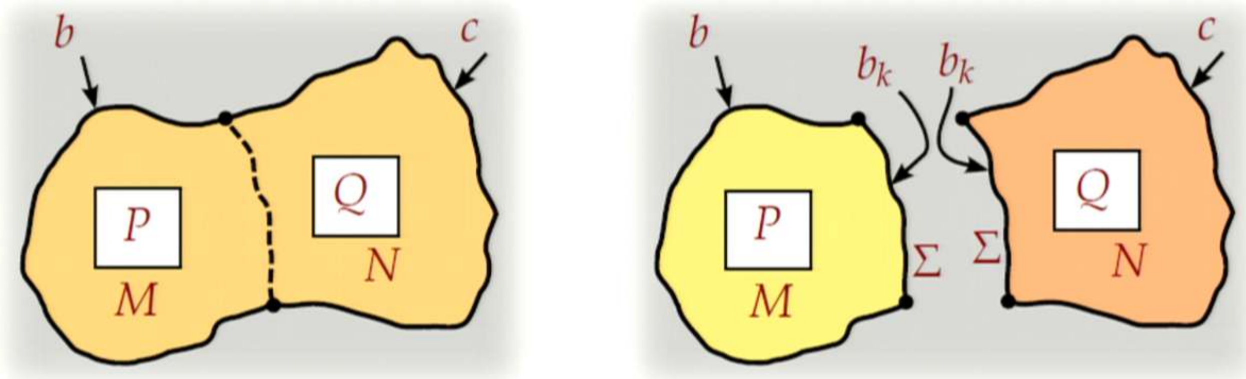
Specialize to the case that M is a **slice region** $\hat{\Sigma}$ with **null-probe** $\emptyset$. Use completeness to rewrite b in terms of a linear combination of b_k.



$$(\emptyset, (b, q))_{\hat{\Sigma}} \quad = \quad (Q, b)_N \quad = \sum_k (-1)^{\sigma(k)} (\emptyset, (b, b_k))_{\hat{\Sigma}} \, (Q, b_k)_N$$

Composition rule for probes

As a result we obtain the **composition rule for probes**.



$$(P \diamond Q, (b, c))_{M \cup N} \quad = \quad \sum_k (-1)^{\sigma(k)} (P, (b, b_k))_M \, (Q, (b_k, c))_N$$

Structures and rules

- **Boundary conditions**
- **Probes**
- Probe **composition rule** in spacetime

- Given boundary conditions $c \leq b \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{(\emptyset, c)_M}{(\emptyset, b)_M}$$

  is the **conditional probability** for c to be realized given b.

- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition b is given by,

$$\frac{(P, b)_M}{(\emptyset, b)_M}.$$

Structures and rules – of quantum theory*!

- **Boundary conditions** generalize **mixed states**
- **Probes** generalize **observables** and **quantum operations**
- Probe **composition rule** in spacetime arises from the **Feynman path integral***.
- Given boundary conditions $c \leq b \in \mathcal{B}_{\partial M}$ the quotient

$$\frac{(\emptyset, c)_M}{(\emptyset, b)_M}$$

  is the **conditional probability** for c to be realized given b.
  Transition amplitudes arise as a special cases of this.
- The **expected outcome** of a **probe** P in a spacetime region M given a boundary condition b is given by,

$$\frac{(P, b)_M}{(\emptyset, b)_M}.$$

  Conventional **expectation values** arise as special cases of this.

Quantum theory

A formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

Quantum theory – Positive Formalism

A formulation of quantum theory taking precisely this form emerges by following a constructive approach starting from **standard quantum theory**.

This is the **general boundary formulation of quantum theory** in the guise of the **Positive Formalism**. [RO 2012]

See my previous talk in this seminar in February 2013.

Classical vs. Quantum

The framework is able to describe both **classical** and **quantum** physics. How can we characterize the difference between classical and quantum?

Classical vs. Quantum

The framework is able to describe both **classical** and **quantum** physics. How can we characterize the difference between classical and quantum?

- In the **classical statistical case**, boundary conditions are essentially real valued functions on a set. Positivity is the positivity of functions. In particular, boundary conditions form a **lattice**.
- In the **quantum case**, boundary conditions are essentially self-adjoint operators on a Hilbert (or Krein) space. In particular, boundary conditions form an **anti-lattice**.

Is that the main difference? Are there others? What is its physical significance?

General remarks

The framework does not require fixed notions of **time** or of **causality**. This suggests its suitability

- in the **classical case**: as a basis for a **statistical theory of general relativity**
- in the **quantum case**: as a basis for a **quantum theory of gravity** (this is a main motivation for the GBF)

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The framework might be useful for constructing **hybrid theories**, combining classical and quantum parts.

References

Unfortunately, the main part of this talk has not been written down in any paper yet.

Positive formalism (Christmas paper):

R. O., *A positive formalism for quantum theory in the general boundary formulation*. arXiv:1212.5571.

Some applications of the GBF

- Conceptual basis for **spin foam approach** to quantum gravity (sometimes secretly so)
- Non-linear models:
  - **Three dimensional quantum gravity** is a TQFT and fits "automatically". [Witten 1988;. . .]
  - **Quantum Yang-Mills theory** in 2 dimensions for arbitrary regions and hypersurfaces with corners. [RO 2006]
  - **Yang-Mills theory in higher dimensions** is under investigation [Díaz 2014]
- New **S-matrix** type asymptotic amplitudes [Colosi, RO 2008; Colosi 2009; Dohse 2011; 2012]
- QFT in **curved spacetime**: dS, AdS and more [Colosi, Dohse 2009–]
- Rigorous and functorial quantization of linear and affine field theories without metric background. [RO 2010; 2011; 2012]
- **Unruh effect**. [Colosi, Rätzel 2012; Bianchi, Haggard, Rovelli 2013]
- Striking results for **fermions**: Hilbert spaces become **Krein spaces** and an **emergent notion of time**. [RO 2012]