Title: Quantum information geometric foundations: an overview

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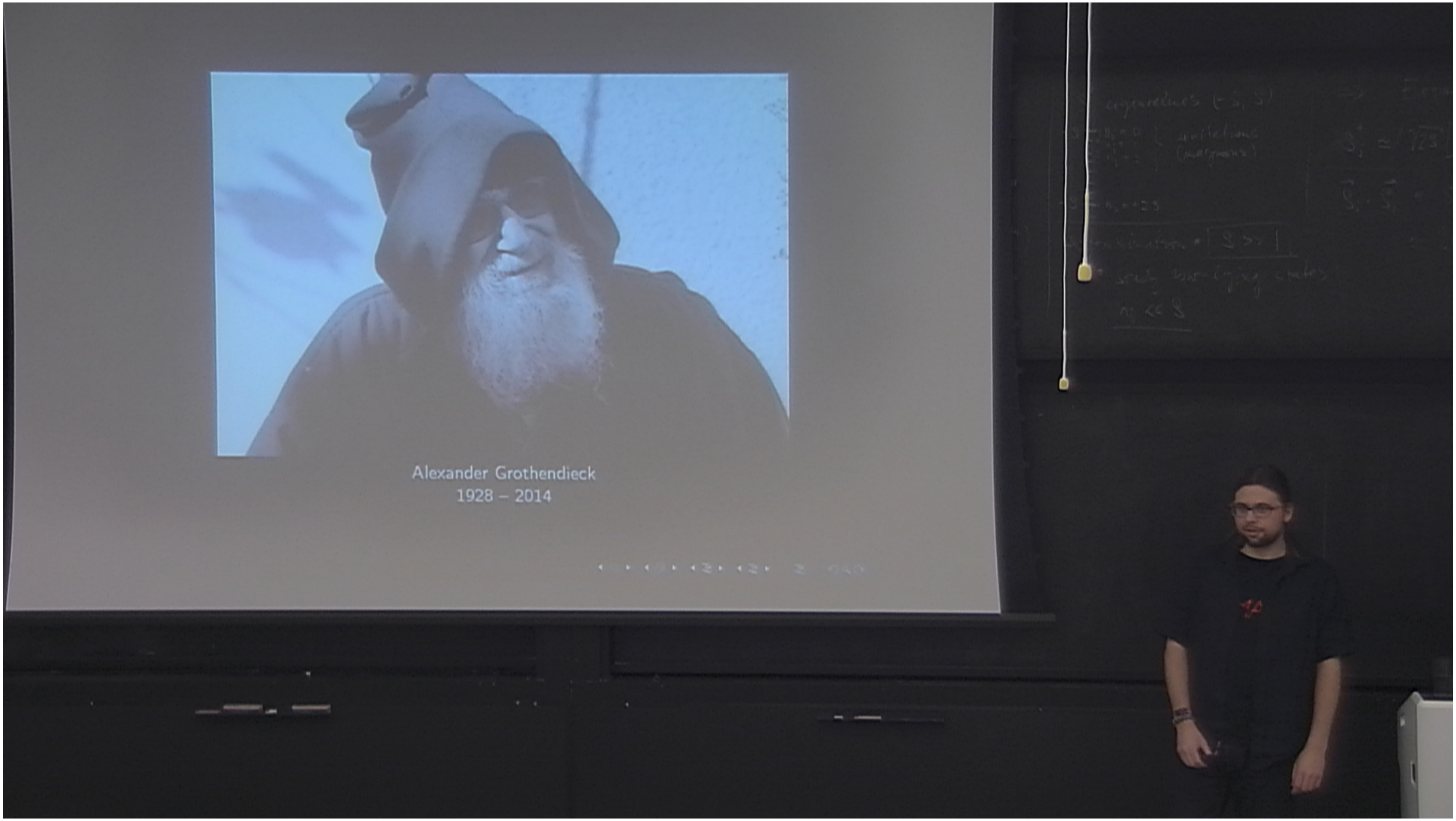
Abstract: I will present a new approach to information-theoretic foundations of quantum theory, that does not rely on probability theory, spectral theory, or Hilbert spaces. The direct nonlinear generalisations of quantum kinematics and dynamics are constructed using quantum information geometric structures over algebraic states of W*-algebras (quantum relative entropies and Poisson structure). In particular, unitary evolutions are generalised to nonlinear hamiltonian flows, while Luedersâ€™ rules are generalised to constrained relative entropy maximisations. Orthodox probability theory and quantum mechanics are special cases of this framework. I will also discuss the epistemic interpretation associated with this approach (rendering quantum theory as a framework for ontically noncommittal causal inference), as well as the possibility of deriving emergent space-times directly from quantum models.


Alexander Grothendieck
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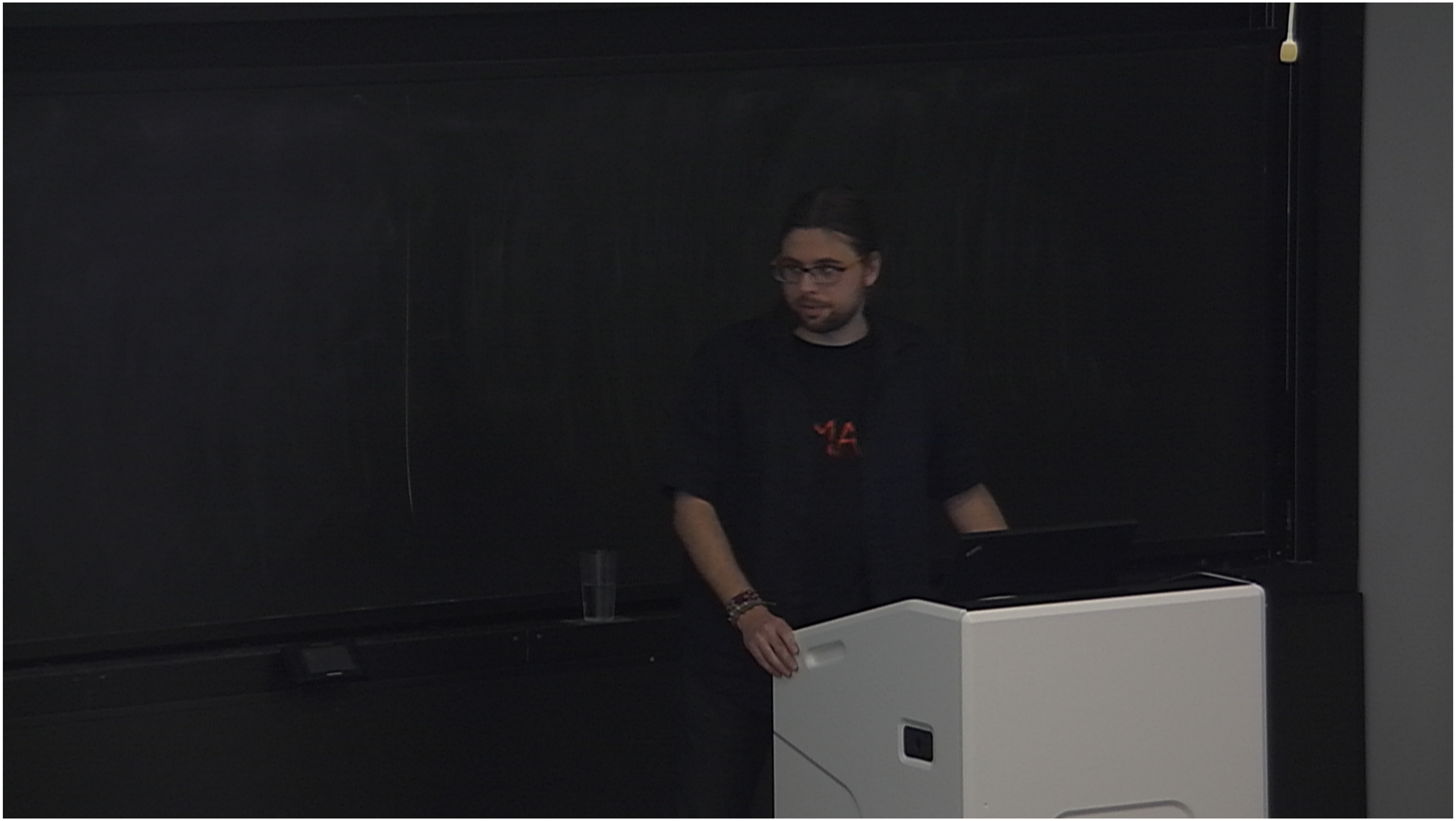
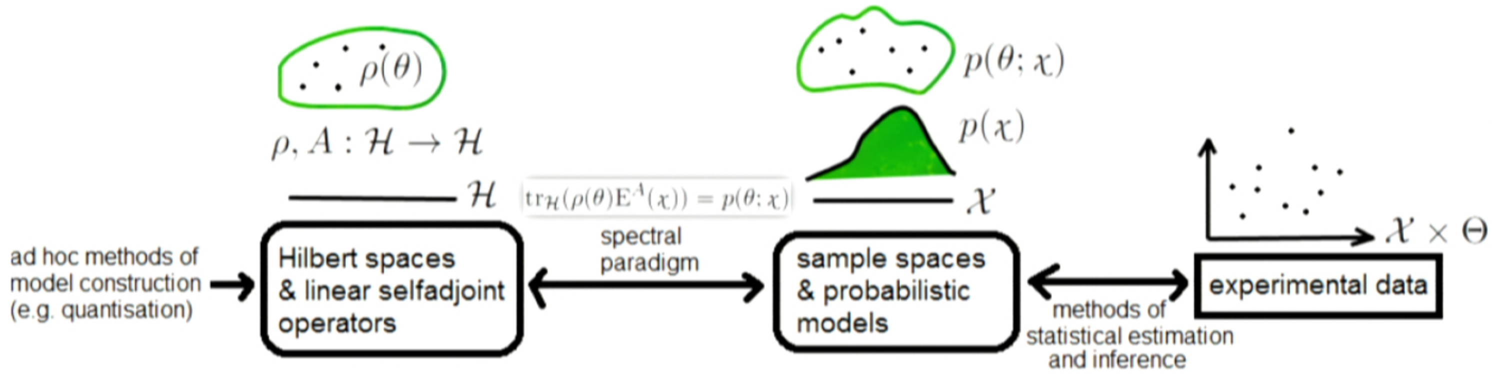
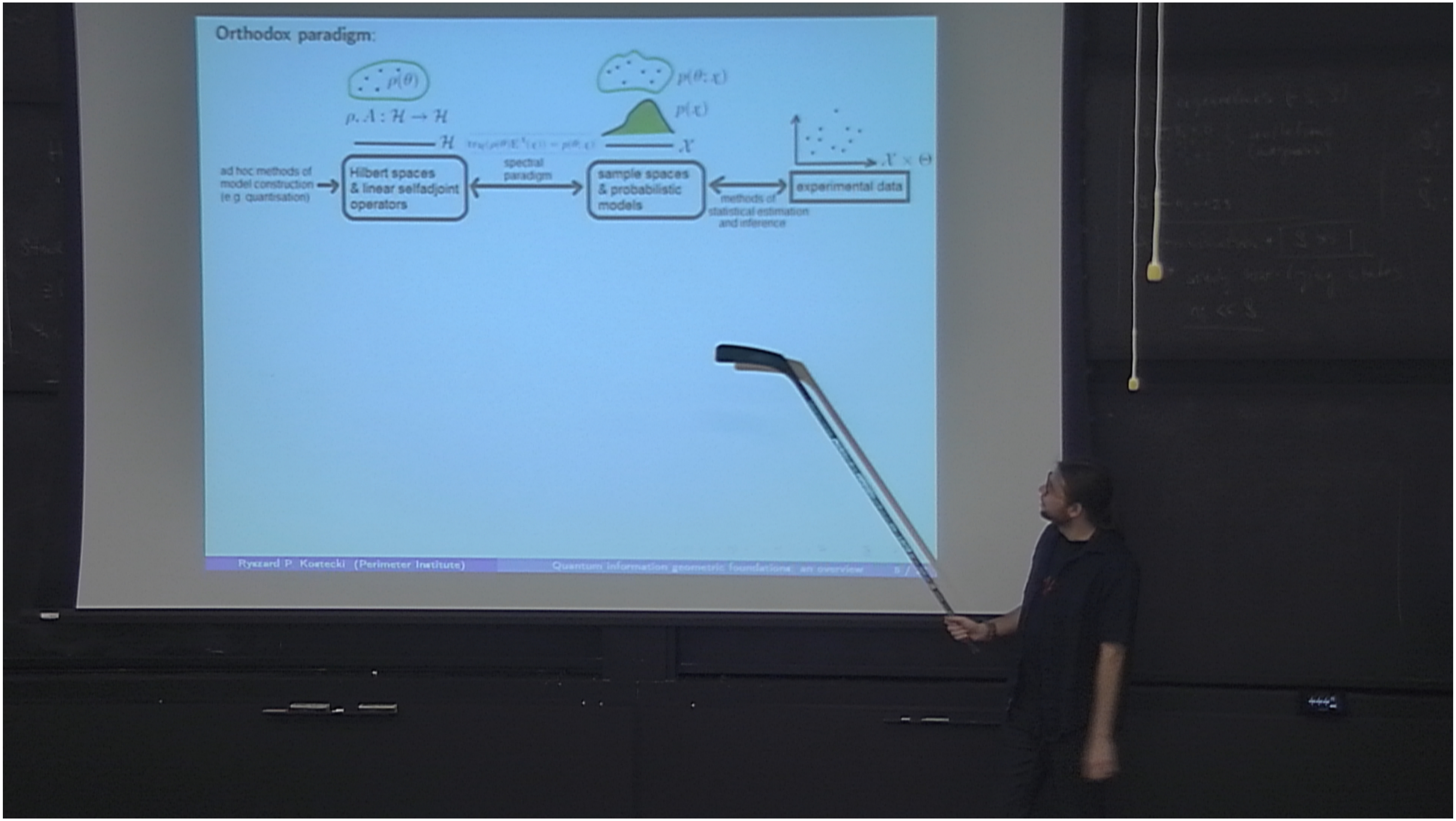


Orthodox paradigm:





Orthodox paradigm:
p(θ)
ρ, A : H → H
H
p(θ; x)
p(x)
X
X × Θ
ad hoc methods of model construction (e.g. quantisation)
Hilbert spaces & linear selfadjoint operators
spectral paradigm
sample spaces & probabilistic models
methods of statistical estimation and inference
experimental data
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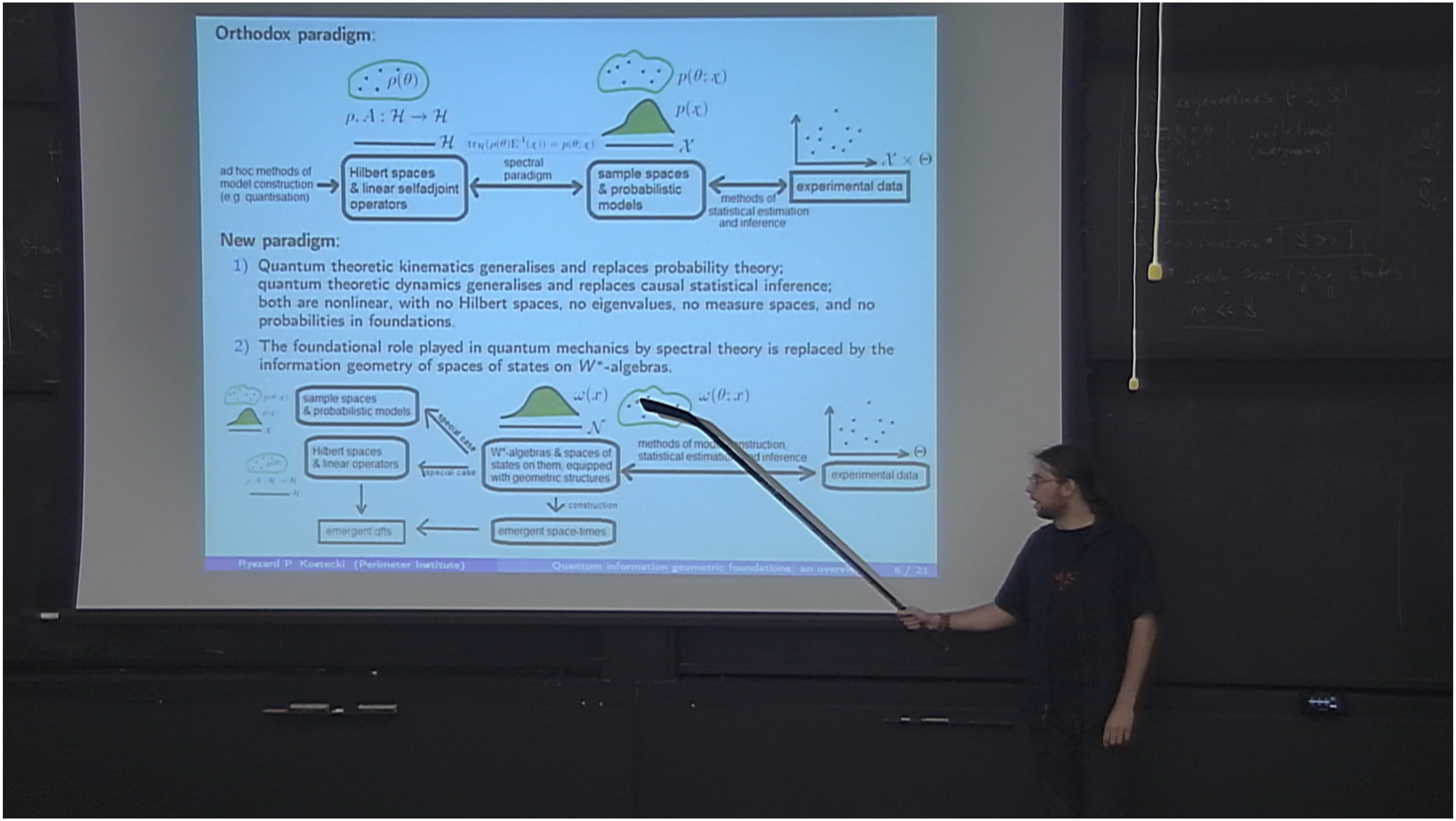
Orthodox paradigm:
$\rho(\theta)$
$\rho, A : \mathcal{H} \to \mathcal{H}$
$\mathcal{H}$
$p(\theta; x)$
$p(x)$
$\mathcal{X}$
$\mathcal{X} \times \Theta$
ad hoc methods of model construction (e.g. quantisation)
Hilbert spaces & linear selfadjoint operators
spectral paradigm
sample spaces & probabilistic models
methods of statistical estimation and inference
experimental data
New paradigm:
1) Quantum theoretic kinematics generalises and replaces probability theory; quantum theoretic dynamics generalises and replaces causal statistical inference; both are nonlinear, with no Hilbert spaces, no eigenvalues, no measure spaces, and no probabilities in foundations.
2) The foundational role played in quantum mechanics by spectral theory is replaced by the information geometry of spaces of states on W^*-algebras.
sample spaces & probabilistic models
Hilbert spaces & linear operators
special case
special case
$\omega(x)$
$\mathcal{N}$
$\omega(\theta; x)$
W^*-algebras & spaces of states on them, equipped with geometric structures
construction
emergent qfts
emergent space-times
experimental data
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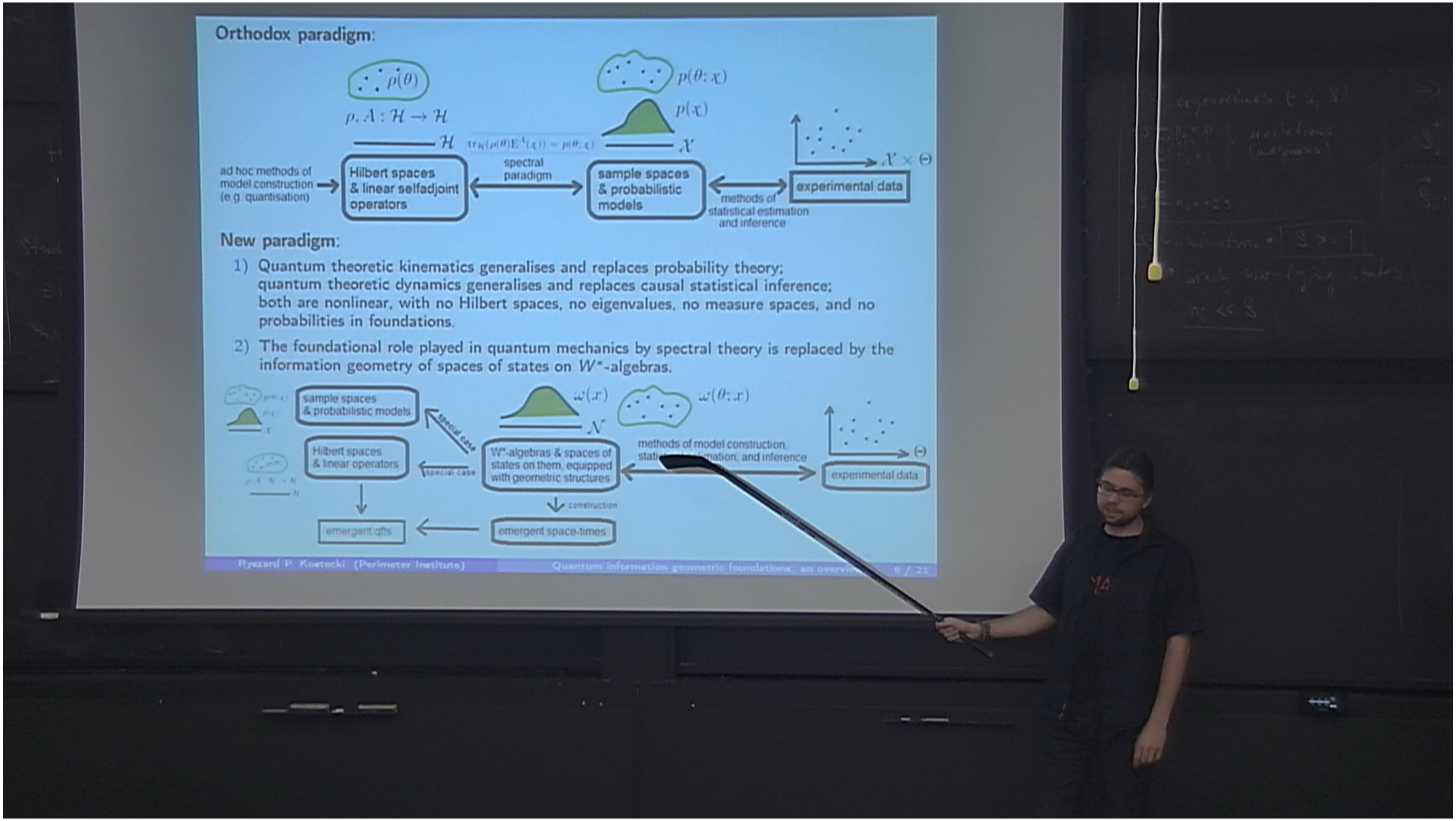
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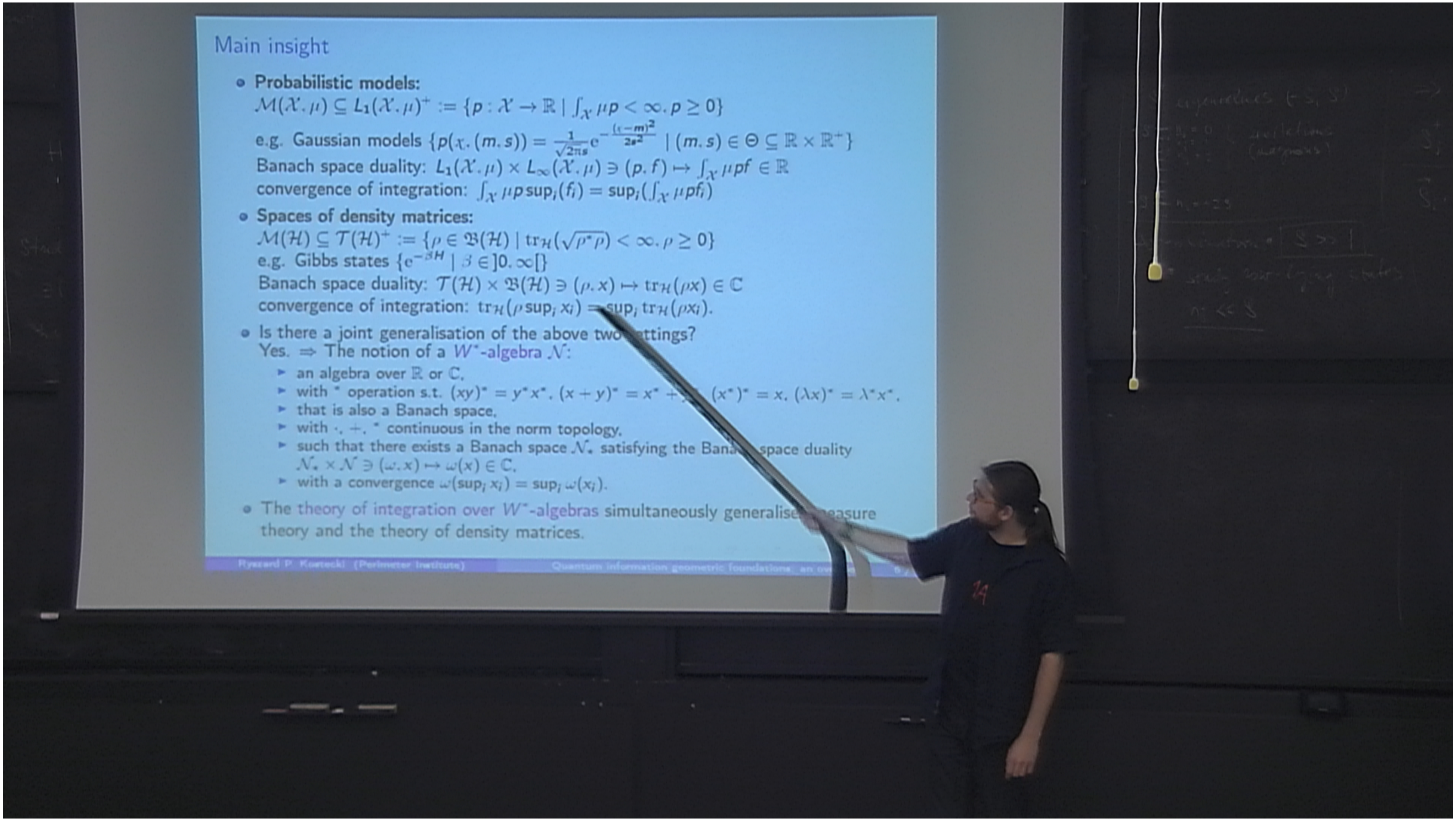
Main insight
Probabilistic models:
$\mathcal{M}(\mathcal{X},\mu) \subseteq L_1(\mathcal{X},\mu)^+ := \{p : \mathcal{X} \to \mathbb{R} \mid \int_{\mathcal{X}} \mu p < \infty, p \geq 0\}$
e.g. Gaussian models $\{p(x,(m,s)) = \frac{1}{\sqrt{2\pi s}} e^{-\frac{(x-m)^2}{2s^2}} \mid (m,s) \in \Theta \subseteq \mathbb{R} \times \mathbb{R}^+\}$
Banach space duality: $L_1(\mathcal{X},\mu) \times L_\infty(\mathcal{X},\mu) \ni (p,f) \mapsto \int_{\mathcal{X}} \mu p f \in \mathbb{R}$
convergence of integration: $\int_{\mathcal{X}} \mu p \sup_i(f_i) = \sup_i(\int_{\mathcal{X}} \mu p f_i)$
Spaces of density matrices:
$\mathcal{M}(\mathcal{H}) \subseteq \mathcal{T}(\mathcal{H})^+ := \{\rho \in \mathfrak{B}(\mathcal{H}) \mid \mathrm{tr}_{\mathcal{H}}(\sqrt{\rho^*\rho}) < \infty, \rho \geq 0\}$
e.g. Gibbs states $\{e^{-\beta H} \mid \beta \in]0,\infty[\}$
Banach space duality: $\mathcal{T}(\mathcal{H}) \times \mathfrak{B}(\mathcal{H}) \ni (\rho, x) \mapsto \mathrm{tr}_{\mathcal{H}}(\rho x) \in \mathbb{C}$
convergence of integration: $\mathrm{tr}_{\mathcal{H}}(\rho \sup_i x_i) = \sup_i \mathrm{tr}_{\mathcal{H}}(\rho x_i)$.
Is there a joint generalisation of the above two settings?
Yes. ⇒ The notion of a W^*-algebra $\mathcal{N}$:
an algebra over $\mathbb{R}$ or $\mathbb{C}$,
with * operation s.t. $(xy)^* = y^*x^*$, $(x+y)^* = x^* +$ ... $(x^*)^* = x$, $(\lambda x)^* = \lambda^* x^*$,
that is also a Banach space,
with $\cdot, +, ^*$ continuous in the norm topology,
such that there exists a Banach space $\mathcal{N}_*$ satisfying the Ban... space duality
$\mathcal{N}_* \times \mathcal{N} \ni (\omega, x) \mapsto \omega(x) \in \mathbb{C}$,
with a convergence $\omega(\sup_i x_i) = \sup_i \omega(x_i)$.
The theory of integration over W^*-algebras simultaneously generalise... ...easure theory and the theory of density matrices.
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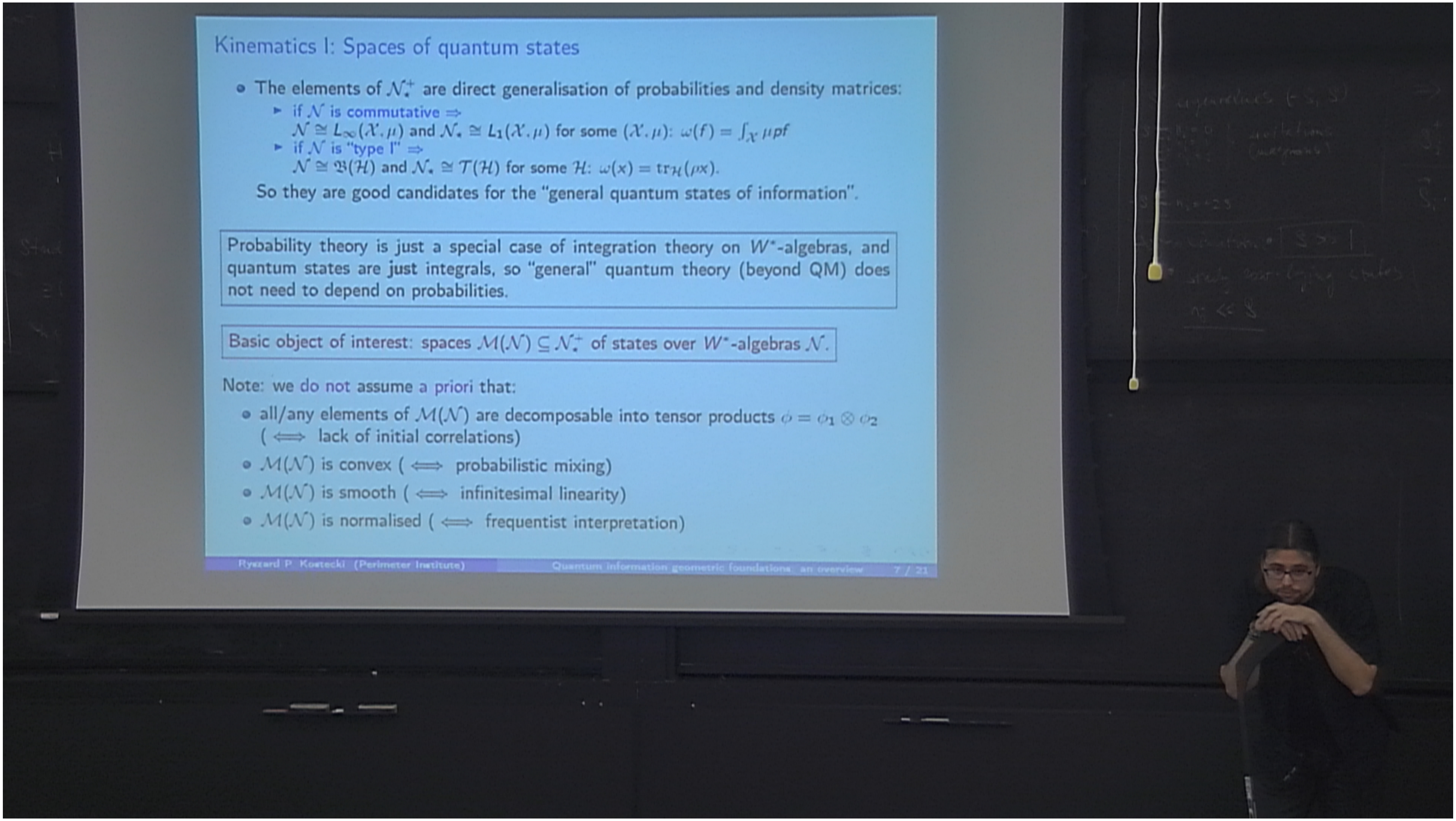
Kinematics I: Spaces of quantum states
The elements of $\mathcal{N}_\star^+$ are direct generalisation of probabilities and density matrices:
if $\mathcal{N}$ is commutative $\Rightarrow$
$\mathcal{N} \cong L_\infty(\mathcal{X}, \mu)$ and $\mathcal{N}_\star \cong L_1(\mathcal{X}, \mu)$ for some $(\mathcal{X}, \mu)$: $\omega(f) = \int_{\mathcal{X}} \mu p f$
if $\mathcal{N}$ is "type I" $\Rightarrow$
$\mathcal{N} \cong \mathfrak{B}(\mathcal{H})$ and $\mathcal{N}_\star \cong \mathcal{T}(\mathcal{H})$ for some $\mathcal{H}$: $\omega(x) = \mathrm{tr}_{\mathcal{H}}(\rho x)$.
So they are good candidates for the "general quantum states of information".
Probability theory is just a special case of integration theory on W^*-algebras, and quantum states are **just** integrals, so "general" quantum theory (beyond QM) does not need to depend on probabilities.
Basic object of interest: spaces $\mathcal{M}(\mathcal{N}) \subseteq \mathcal{N}_\star^+$ of states over W^*-algebras $\mathcal{N}$.
Note: we do not assume a priori that:
all/any elements of $\mathcal{M}(\mathcal{N})$ are decomposable into tensor products $\phi = \phi_1 \otimes \phi_2$ ($\iff$ lack of initial correlations)
$\mathcal{M}(\mathcal{N})$ is convex ($\iff$ probabilistic mixing)
$\mathcal{M}(\mathcal{N})$ is smooth ($\iff$ infinitesimal linearity)
$\mathcal{M}(\mathcal{N})$ is normalised ($\iff$ frequentist interpretation)
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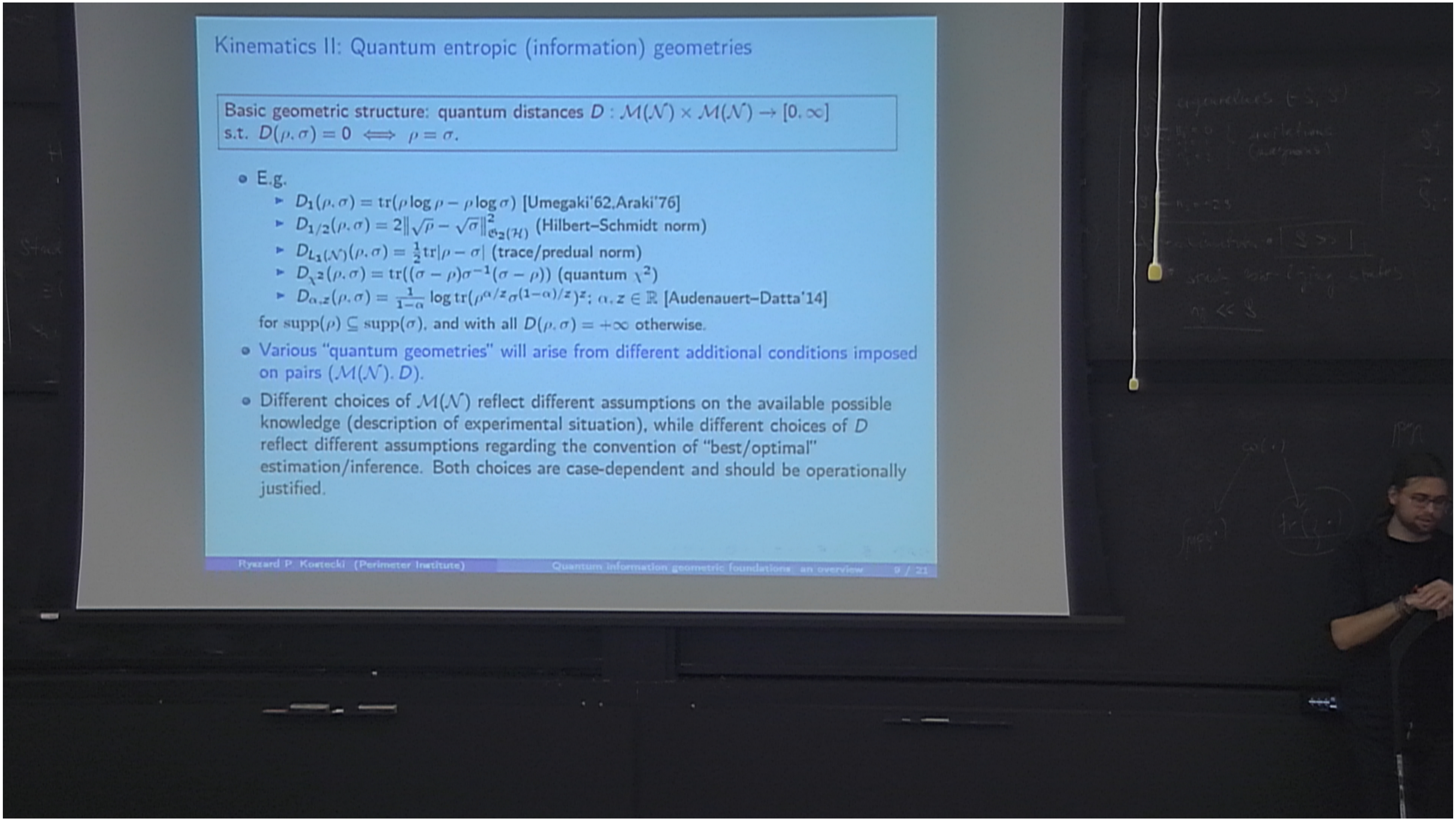
New kinematics: quantum information geometry

- Main principle: Consider expectations as more important than eigenvalues ($\Rightarrow$ spectral theory replaced by quantum information geometry)
- kinematic setting:
  (1) elementary objects of concern: subsets $\mathcal{M}(\mathcal{N})$ of states on W^*-algebras $\mathcal{N}$
  (2) nonlinear geometries of $\mathcal{M}(\mathcal{N})$ determined by quantum relative entropy functions $D(\cdot,\cdot)$ on $\mathcal{M}(\mathcal{N})$
  (3) observables defined as arbitrary functions $f : \mathcal{M}(\mathcal{N}) \to \mathbb{R}$
  (4) the commutator of $\mathcal{N}$ induces a Poisson structure $\{\cdot,\cdot\}$ that acts on smooth observables on $\mathcal{M}(\mathcal{N})$
- no Hilbert spaces, no probability theory in foundations (derived as special cases)
- emergent curved space-times


Kinematics II: Quantum entropic (information) geometries
Basic geometric structure: quantum distances $D : \mathcal{M}(\mathcal{N}) \times \mathcal{M}(\mathcal{N}) \to [0, \infty]$ s.t. $D(\rho, \sigma) = 0 \iff \rho = \sigma$.
E.g.
$D_1(\rho, \sigma) = \mathrm{tr}(\rho \log \rho - \rho \log \sigma)$ [Umegaki'62,Araki'76]
$D_{1/2}(\rho, \sigma) = 2\|\sqrt{\rho} - \sqrt{\sigma}\|^2_{\mathfrak{S}_2(\mathcal{H})}$ (Hilbert–Schmidt norm)
$D_{L_1(\mathcal{N})}(\rho, \sigma) = \frac{1}{2}\mathrm{tr}|\rho - \sigma|$ (trace/predual norm)
$D_{\chi^2}(\rho, \sigma) = \mathrm{tr}((\sigma - \rho)\sigma^{-1}(\sigma - \rho))$ (quantum χ^2)
$D_{\alpha,z}(\rho, \sigma) = \frac{1}{1-\alpha} \log \mathrm{tr}(\rho^{\alpha/z}\sigma^{(1-\alpha)/z})^z$; $\alpha, z \in \mathbb{R}$ [Audenauert–Datta'14]
for $\mathrm{supp}(\rho) \subseteq \mathrm{supp}(\sigma)$, and with all $D(\rho, \sigma) = +\infty$ otherwise.
Various "quantum geometries" will arise from different additional conditions imposed on pairs $(\mathcal{M}(\mathcal{N}), D)$.
Different choices of $\mathcal{M}(\mathcal{N})$ reflect different assumptions on the available possible knowledge (description of experimental situation), while different choices of D reflect different assumptions regarding the convention of "best/optimal" estimation/inference. Both choices are case-dependent and should be operationally justified.
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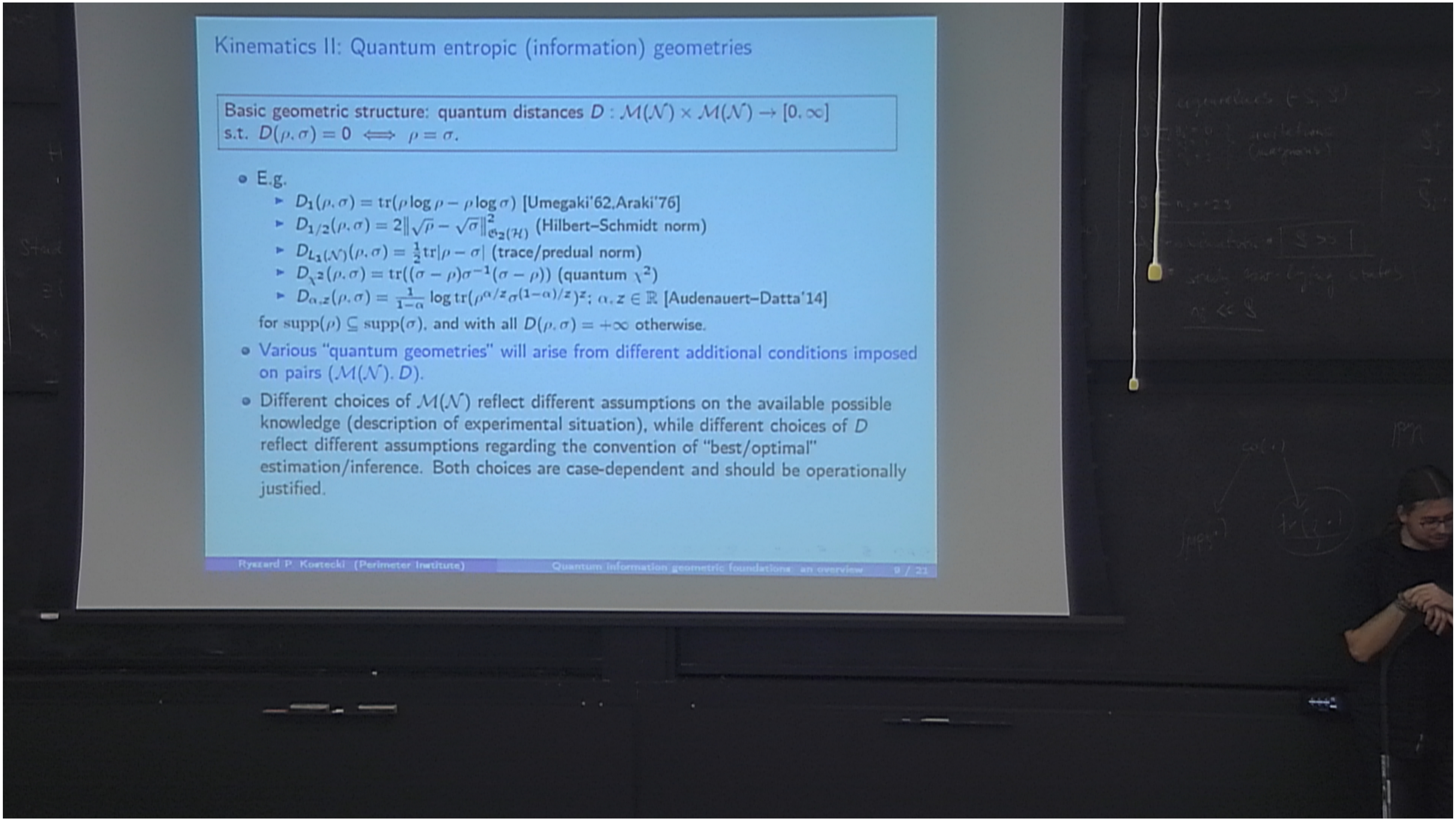
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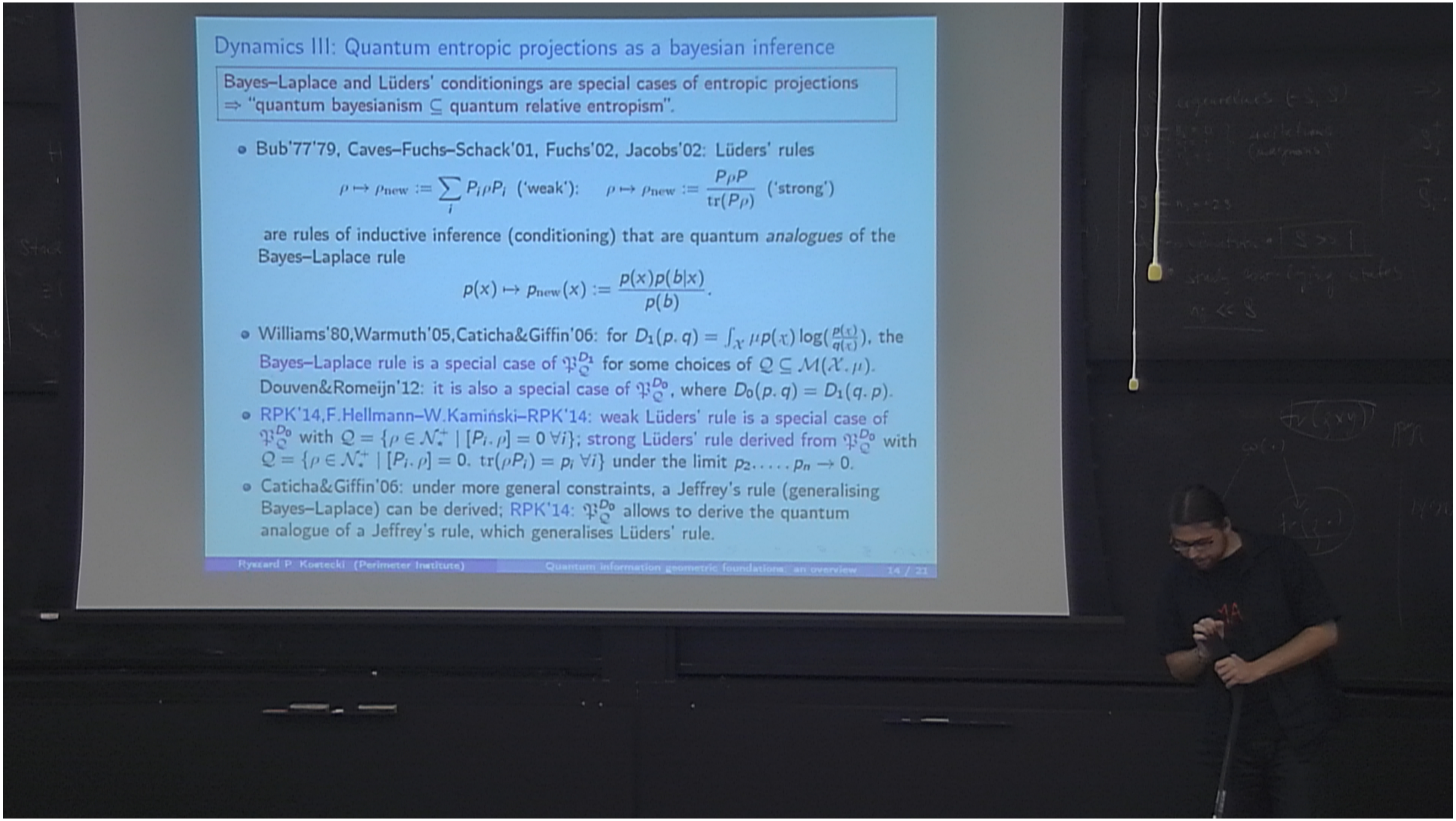
Dynamics III: Quantum entropic projections as a bayesian inference
Bayes–Laplace and Lüders' conditionings are special cases of entropic projections
⇒ "quantum bayesianism ⊆ quantum relative entropism".
Bub'77'79, Caves–Fuchs–Schack'01, Fuchs'02, Jacobs'02: Lüders' rules
$\rho \mapsto \rho_{\mathrm{new}} := \sum_i P_i \rho P_i$ ('weak'); $\rho \mapsto \rho_{\mathrm{new}} := \frac{P\rho P}{\mathrm{tr}(P\rho)}$ ('strong')
are rules of inductive inference (conditioning) that are quantum *analogues* of the Bayes–Laplace rule
$p(x) \mapsto p_{\mathrm{new}}(x) := \frac{p(x)p(b|x)}{p(b)}$.
Williams'80, Warmuth'05, Caticha&Giffin'06: for $D_1(p,q) = \int_{\mathcal{X}} \mu p(x) \log(\frac{p(x)}{q(x)})$, the Bayes–Laplace rule is a special case of $\mathfrak{P}^{D_1}_{\mathcal{Q}}$ for some choices of $\mathcal{Q} \subseteq \mathcal{M}(\mathcal{X},\mu)$.
Douven&Romeijn'12: it is also a special case of $\mathfrak{P}^{D_0}_{\mathcal{Q}}$, where $D_0(p,q) = D_1(q,p)$.
RPK'14, F.Hellmann–W.Kamiński–RPK'14: weak Lüders' rule is a special case of $\mathfrak{P}^{D_0}_{\mathcal{Q}}$ with $\mathcal{Q} = \{\rho \in \mathcal{N}^+_\star \mid [P_i, \rho] = 0 \; \forall i\}$; strong Lüders' rule derived from $\mathfrak{P}^{D_0}_{\mathcal{Q}}$ with $\mathcal{Q} = \{\rho \in \mathcal{N}^+_\star \mid [P_i, \rho] = 0, \; \mathrm{tr}(\rho P_i) = p_i \; \forall i\}$ under the limit $p_2, \ldots, p_n \to 0$.
Caticha&Giffin'06: under more general constraints, a Jeffrey's rule (generalising Bayes–Laplace) can be derived; RPK'14: $\mathfrak{P}^{D_0}_{\mathcal{Q}}$ allows to derive the quantum analogue of a Jeffrey's rule, which generalises Lüders' rule.
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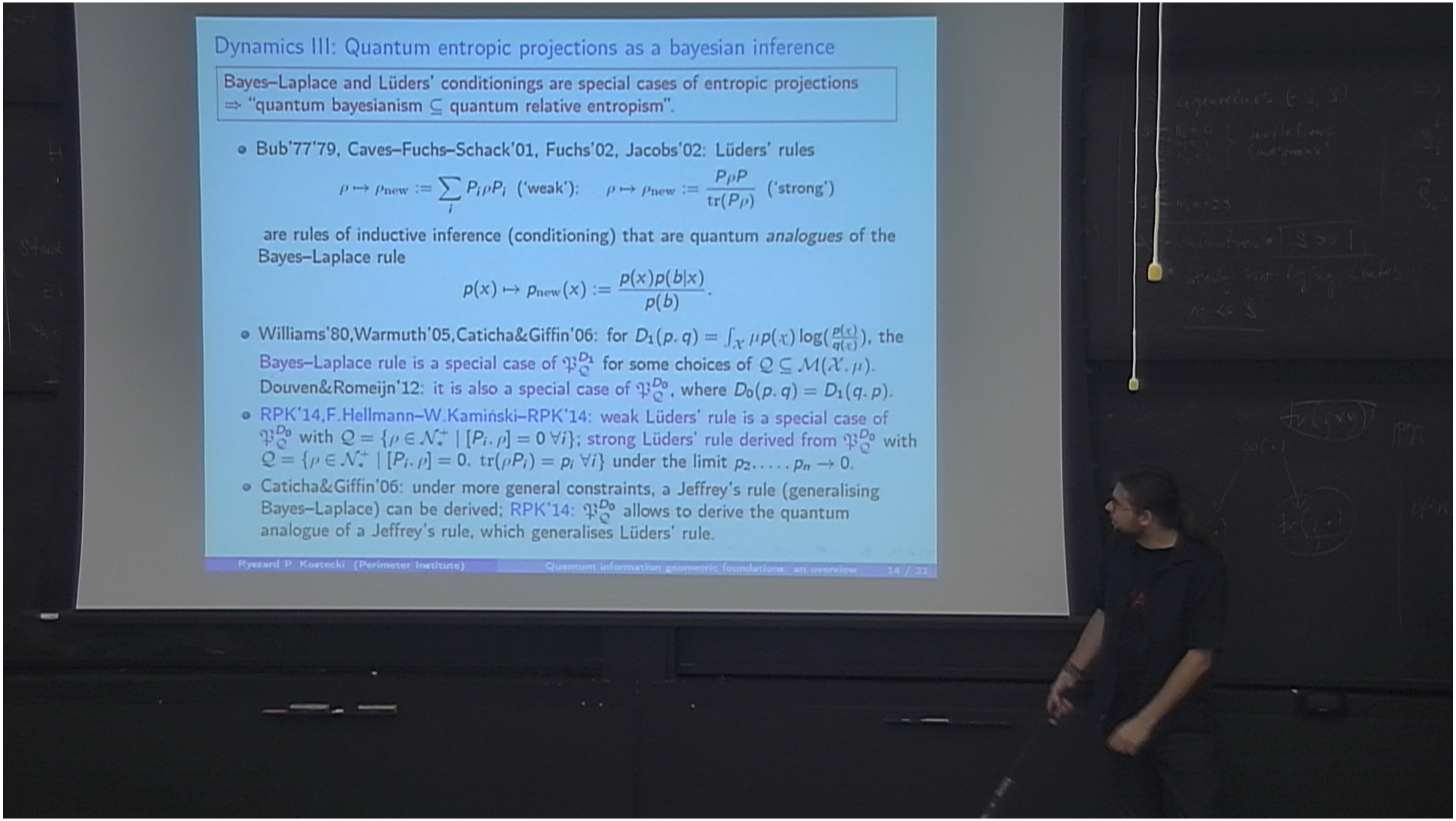
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$\rho \mapsto \rho_{\mathrm{new}} := \sum_i P_i \rho P_i$ ('weak'); $\rho \mapsto \rho_{\mathrm{new}} := \frac{P \rho P}{\mathrm{tr}(P\rho)}$ ('strong')
are rules of inductive inference (conditioning) that are quantum *analogues* of the Bayes–Laplace rule
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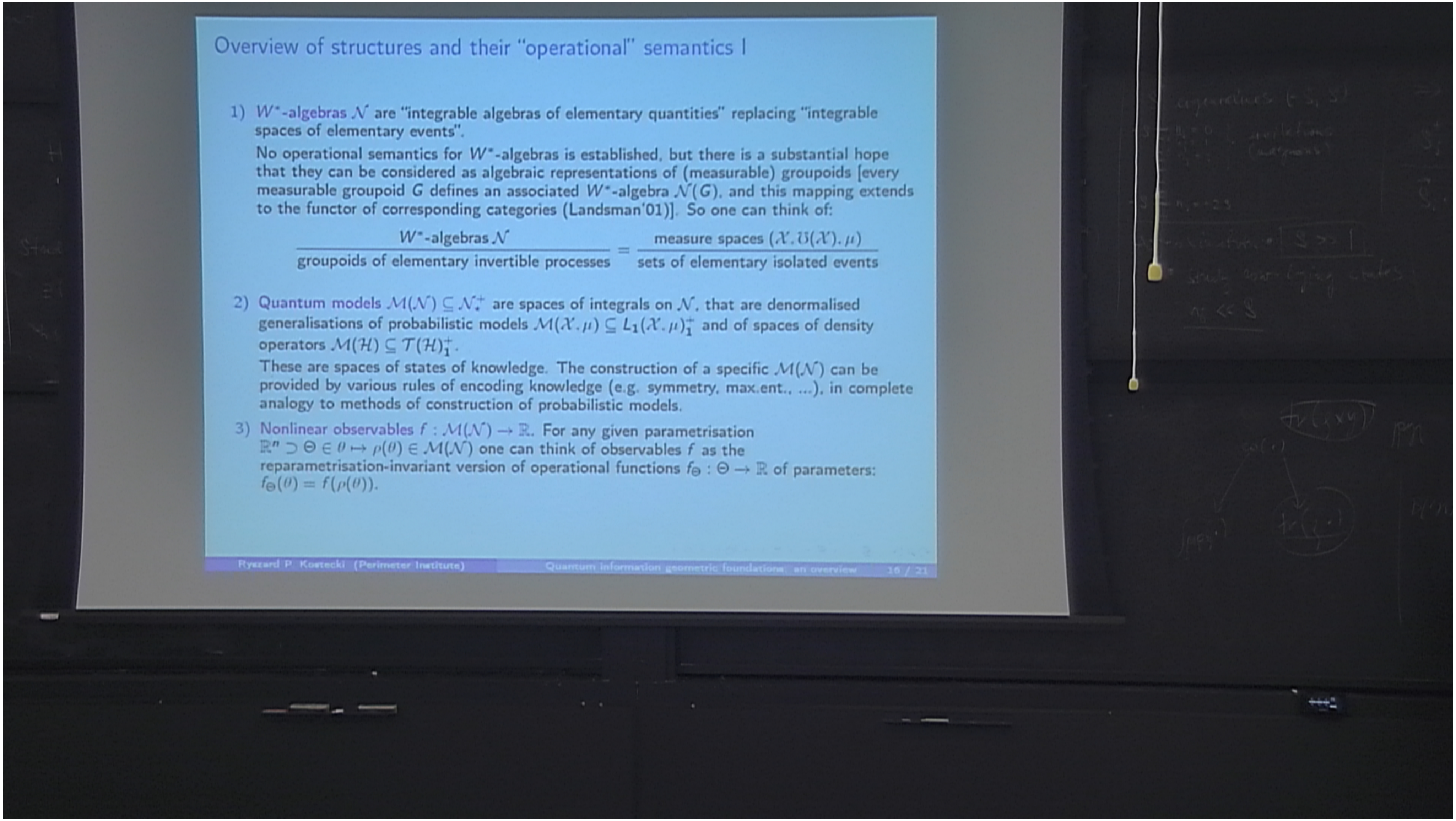
Dynamics IV: Causal inferences = entropic-hamiltonian dynamics

- Two elementary forms of quantum dynamics:
  - hamiltonian flows w_t^h generated by nonlinear hamiltonian vector fields $\{h, \cdot\}$
  - entropic projections $\mathfrak{P}_Q^D$ generated by quantum distances $D(\cdot, \cdot)$
- Interpretation:
  - $\{h, \cdot\}$ represents a convention of causality ("internal dynamics")
  - $D(\cdot, \cdot)$ represents the convention of "best estimation/inference"

A general form of quantum dynamics is defined as a causal inference $\mathfrak{P}_Q^D \circ w_t^h$.

- It generalises unitary evolution followed by a "projective measurement".
- Postulate: consider the setting of causal inferences $\mathfrak{P}_Q^D \circ w_t^h$ as an alternative to the paradigm of CP maps.
- Some virtues:
  - no requirement for lack of initial correlations
  - nonmarkovianity
  - consistent nonlinearity
  - direct relationship with geometric structures on quantum states, and with conventions of estimation
  - replacement for ad hoc techniques of perturbations of hamiltonians


Overview of structures and their "operational" semantics I
1) W^*-algebras $\mathcal{N}$ are "integrable algebras of elementary quantities" replacing "integrable spaces of elementary events".
No operational semantics for W^*-algebras is established, but there is a substantial hope that they can be considered as algebraic representations of (measurable) groupoids [every measurable groupoid G defines an associated W^*-algebra $\mathcal{N}(G)$, and this mapping extends to the functor of corresponding categories (Landsman'01)]. So one can think of:
$$\frac{W^*\text{-algebras } \mathcal{N}}{\text{groupoids of elementary invertible processes}} = \frac{\text{measure spaces } (\mathcal{X}, \mho(\mathcal{X}), \mu)}{\text{sets of elementary isolated events}}$$
2) Quantum models $\mathcal{M}(\mathcal{N}) \subseteq \mathcal{N}_*^+$ are spaces of integrals on $\mathcal{N}$, that are denormalised generalisations of probabilistic models $\mathcal{M}(\mathcal{X}, \mu) \subseteq L_1(\mathcal{X}, \mu)_1^+$ and of spaces of density operators $\mathcal{M}(\mathcal{H}) \subseteq \mathcal{T}(\mathcal{H})_1^+$.
These are spaces of states of knowledge. The construction of a specific $\mathcal{M}(\mathcal{N})$ can be provided by various rules of encoding knowledge (e.g. symmetry, max.ent., ...), in complete analogy to methods of construction of probabilistic models.
3) Nonlinear observables $f : \mathcal{M}(\mathcal{N}) \to \mathbb{R}$. For any given parametrisation $\mathbb{R}^n \supset \Theta \in \theta \mapsto \rho(\theta) \in \mathcal{M}(\mathcal{N})$ one can think of observables f as the reparametrisation-invariant version of operational functions $f_\Theta : \Theta \to \mathbb{R}$ of parameters: $f_\Theta(\theta) = f(\rho(\theta))$.
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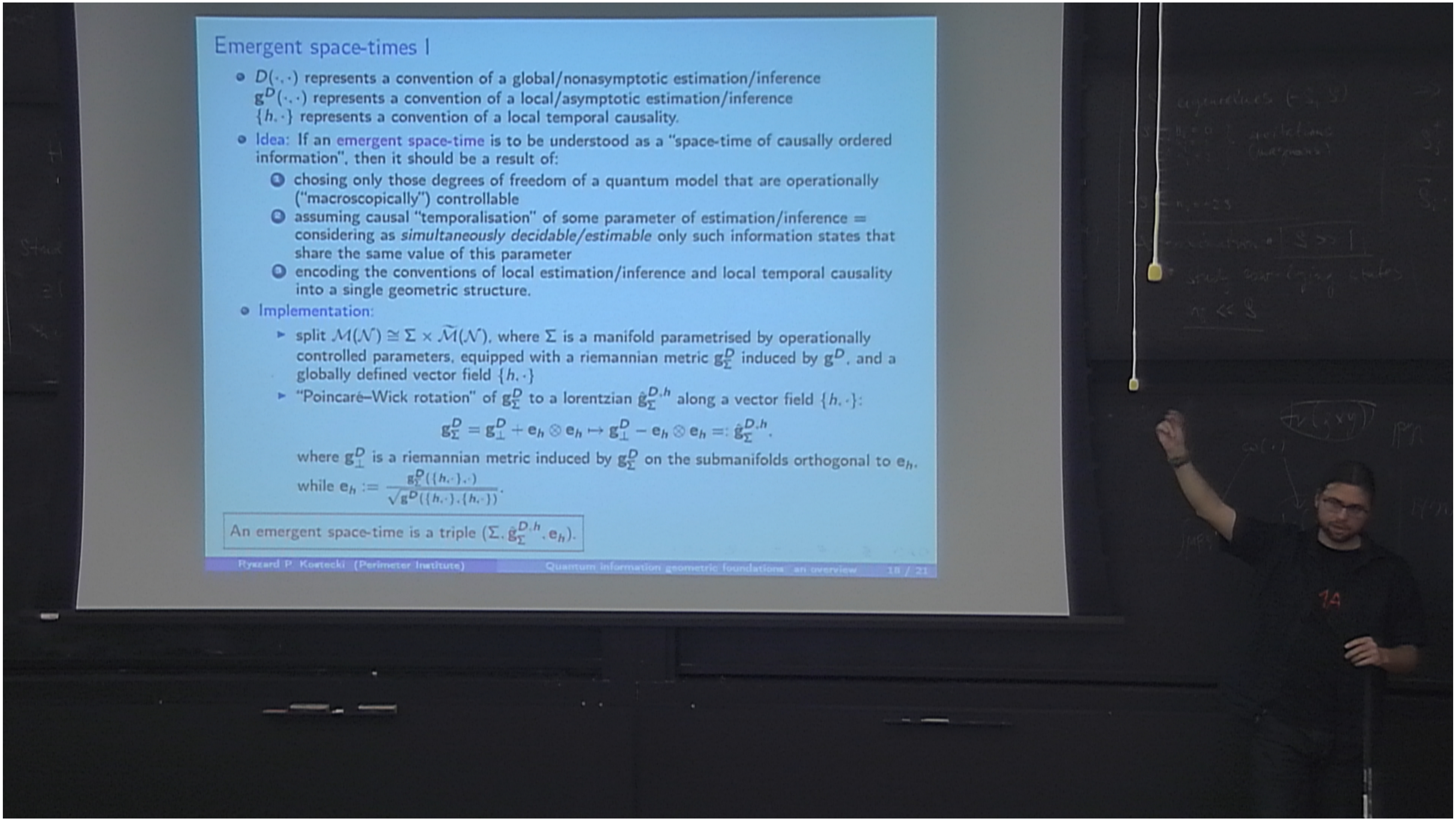
Emergent space-times I
$D(\cdot,\cdot)$ represents a convention of a global/nonasymptotic estimation/inference
$\mathbf{g}^D(\cdot,\cdot)$ represents a convention of a local/asymptotic estimation/inference
$\{h,\cdot\}$ represents a convention of a local temporal causality.
Idea: If an emergent space-time is to be understood as a "space-time of causally ordered information", then it should be a result of:
1 chosing only those degrees of freedom of a quantum model that are operationally ("macroscopically") controllable
2 assuming causal "temporalisation" of some parameter of estimation/inference = considering as *simultaneously decidable/estimable* only such information states that share the same value of this parameter
3 encoding the conventions of local estimation/inference and local temporal causality into a single geometric structure.
Implementation:
split $\mathcal{M}(\mathcal{N}) \cong \Sigma \times \widetilde{\mathcal{M}}(\mathcal{N})$, where Σ is a manifold parametrised by operationally controlled parameters, equipped with a riemannian metric $\mathbf{g}^D_\Sigma$ induced by $\mathbf{g}^D$, and a globally defined vector field $\{h,\cdot\}$
"Poincaré–Wick rotation" of $\mathbf{g}^D_\Sigma$ to a lorentzian $\hat{\mathbf{g}}^{D,h}_\Sigma$ along a vector field $\{h,\cdot\}$:
$$\mathbf{g}^D_\Sigma = \mathbf{g}^D_\perp + \mathbf{e}_h \otimes \mathbf{e}_h \mapsto \mathbf{g}^D_\perp - \mathbf{e}_h \otimes \mathbf{e}_h =: \hat{\mathbf{g}}^{D,h}_\Sigma,$$
where $\mathbf{g}^D_\perp$ is a riemannian metric induced by $\mathbf{g}^D_\Sigma$ on the submanifolds orthogonal to $\mathbf{e}_h$, while $\mathbf{e}_h := \frac{\mathbf{g}^D_\Sigma(\{h,\cdot\},\cdot)}{\sqrt{\mathbf{g}^D(\{h,\cdot\},\{h,\cdot\})}}$.
An emergent space-time is a triple $(\Sigma, \hat{\mathbf{g}}^{D,h}_\Sigma, \mathbf{e}_h)$.
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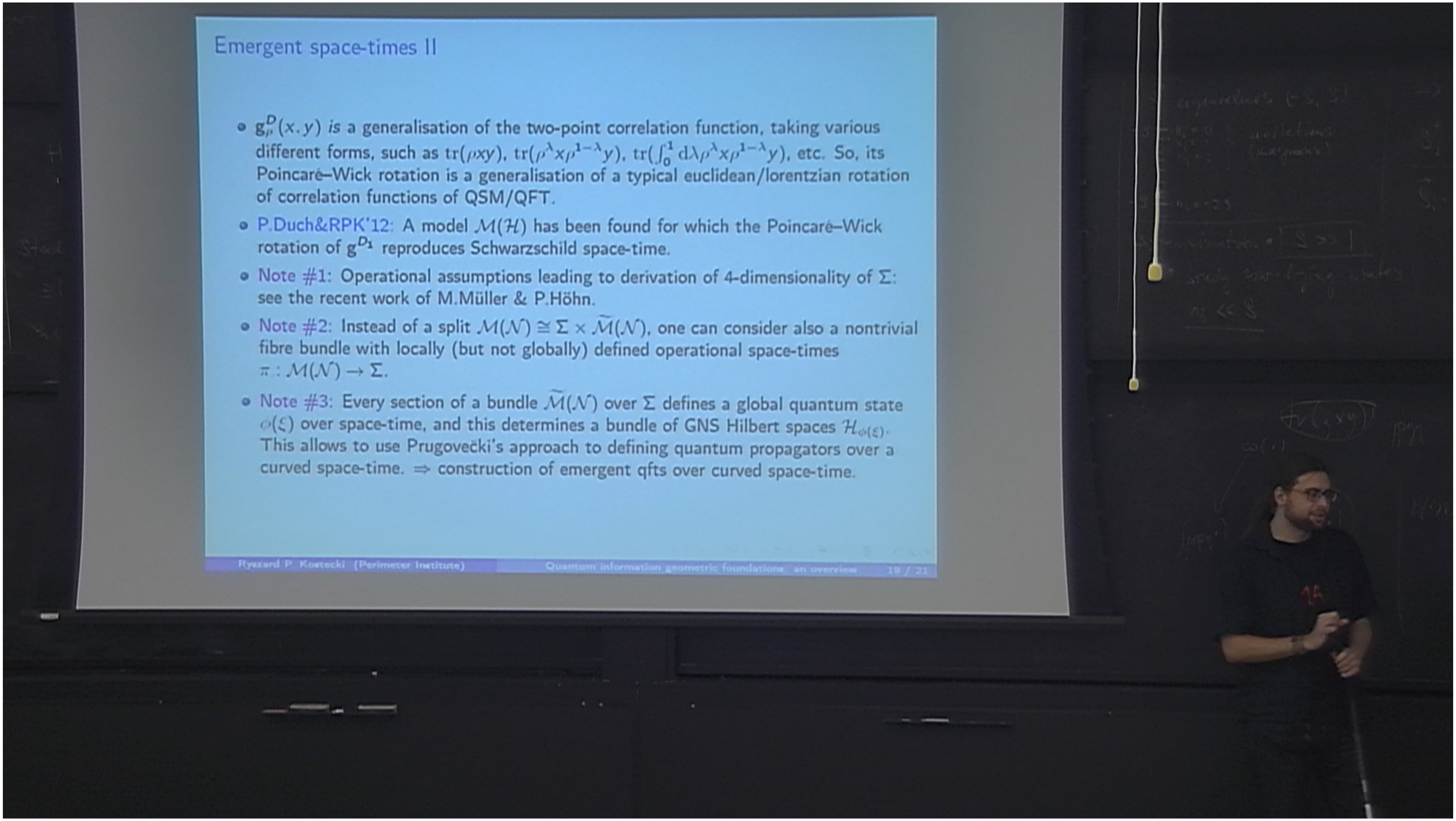
Emergent space-times II
$g^D_\rho(x,y)$ is a generalisation of the two-point correlation function, taking various different forms, such as $\mathrm{tr}(\rho xy)$, $\mathrm{tr}(\rho^\lambda x \rho^{1-\lambda} y)$, $\mathrm{tr}(\int_0^1 \mathrm{d}\lambda \rho^\lambda x \rho^{1-\lambda} y)$, etc. So, its Poincaré–Wick rotation is a generalisation of a typical euclidean/lorentzian rotation of correlation functions of QSM/QFT.
P.Duch&RPK'12: A model $\mathcal{M}(\mathcal{H})$ has been found for which the Poincaré–Wick rotation of g^{D_1} reproduces Schwarzschild space-time.
Note #1: Operational assumptions leading to derivation of 4-dimensionality of Σ: see the recent work of M.Müller & P.Höhn.
Note #2: Instead of a split $\mathcal{M}(\mathcal{N}) \cong \Sigma \times \widetilde{\mathcal{M}}(\mathcal{N})$, one can consider also a nontrivial fibre bundle with locally (but not globally) defined operational space-times $\pi : \mathcal{M}(\mathcal{N}) \to \Sigma$.
Note #3: Every section of a bundle $\widetilde{\mathcal{M}}(\mathcal{N})$ over Σ defines a global quantum state $\phi(\xi)$ over space-time, and this determines a bundle of GNS Hilbert spaces $\mathcal{H}_{\phi(\xi)}$. This allows to use Prugovečki's approach to defining quantum propagators over a curved space-time. $\Rightarrow$ construction of emergent qfts over curved space-time.
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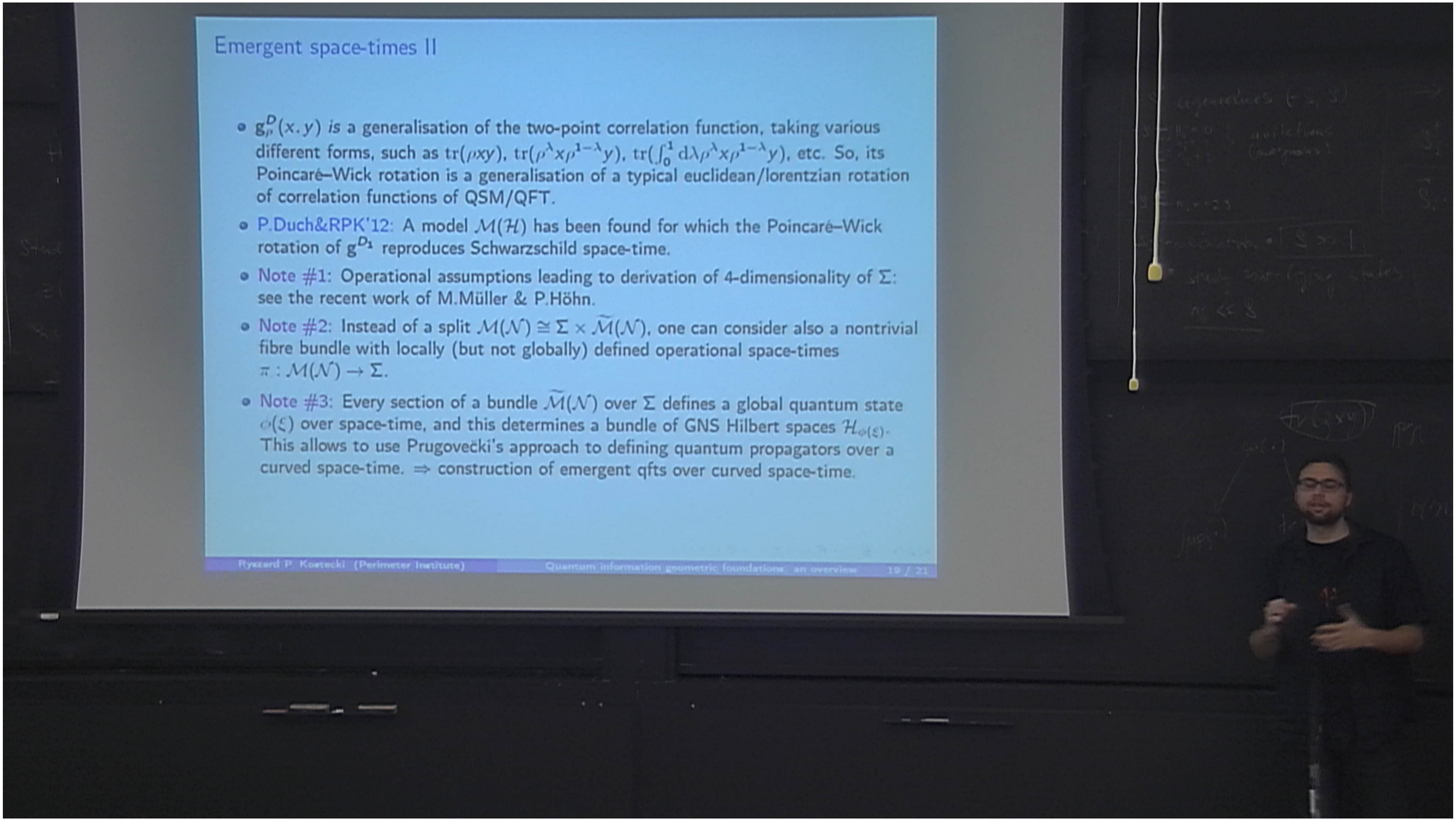
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P.Duch&RPK'12: A model $\mathcal{M}(\mathcal{H})$ has been found for which the Poincaré–Wick rotation of g^{D_1} reproduces Schwarzschild space-time.
Note #1: Operational assumptions leading to derivation of 4-dimensionality of Σ: see the recent work of M.Müller & P.Höhn.
Note #2: Instead of a split $\mathcal{M}(\mathcal{N}) \cong \Sigma \times \widetilde{\mathcal{M}}(\mathcal{N})$, one can consider also a nontrivial fibre bundle with locally (but not globally) defined operational space-times $\pi : \mathcal{M}(\mathcal{N}) \to \Sigma$.
Note #3: Every section of a bundle $\widetilde{\mathcal{M}}(\mathcal{N})$ over Σ defines a global quantum state $\phi(\xi)$ over space-time, and this determines a bundle of GNS Hilbert spaces $\mathcal{H}_{\phi(\xi)}$. This allows to use Prugovečki's approach to defining quantum propagators over a curved space-time. ⇒ construction of emergent qfts over curved space-time.
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