Title: Equivalence of wave-particle duality to entropic uncertainty

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Abstract: Interferometers capture a basic mystery of quantum mechanics: a single particle can exhibit wave behavior, yet that wave behavior disappears when one tries to determine the particle's path inside the interferometer. This idea has been formulated quantitatively as an inequality, e.g., by Englert and Jaeger, Shimony, and Vaidman, which upper bounds the sum of the interference visibility and the path distinguishability. Such wave-particle duality relations (WPDRs) are often thought to be conceptually inequivalent to Heisenberg's uncertainty principle, although this has been debated. Here we show that WPDRs correspond precisely to a modern formulation of the uncertainty principle in terms of entropies, namely the min- and max-entropies. This observation unifies two fundamental concepts in quantum mechanics. Furthermore, it leads to a robust framework for deriving novel WPDRs by applying entropic uncertainty relations to interferometric models (arXiv reference: 1403.4687).

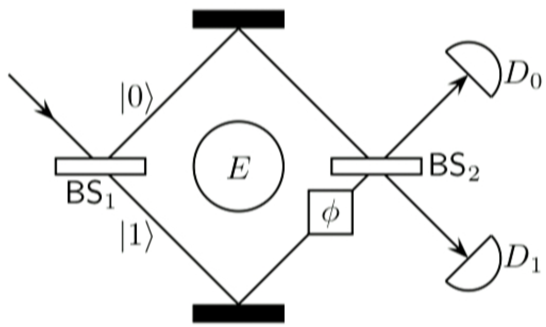
Centre for Quantum Technologies

National University of Singapore

Equivalence of wave-particle duality to entropic uncertainty

arXiv:1403.4687

Patrick Coles (IQC Waterloo)
Jed Kaniewski (TU Delft)
Stephanie Wehner (TU Delft)



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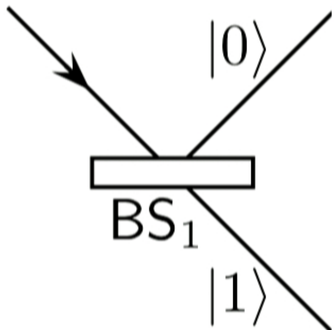
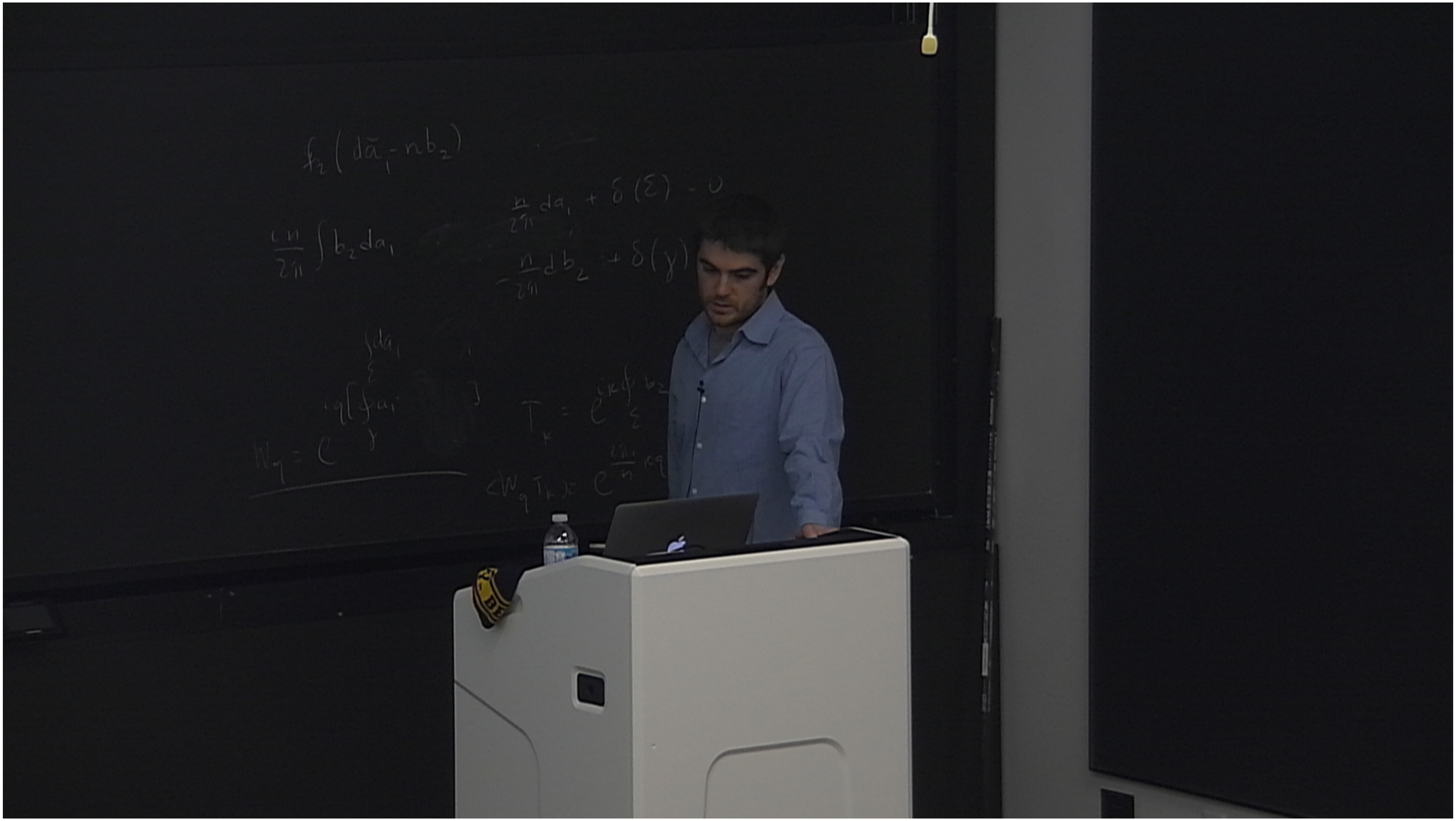
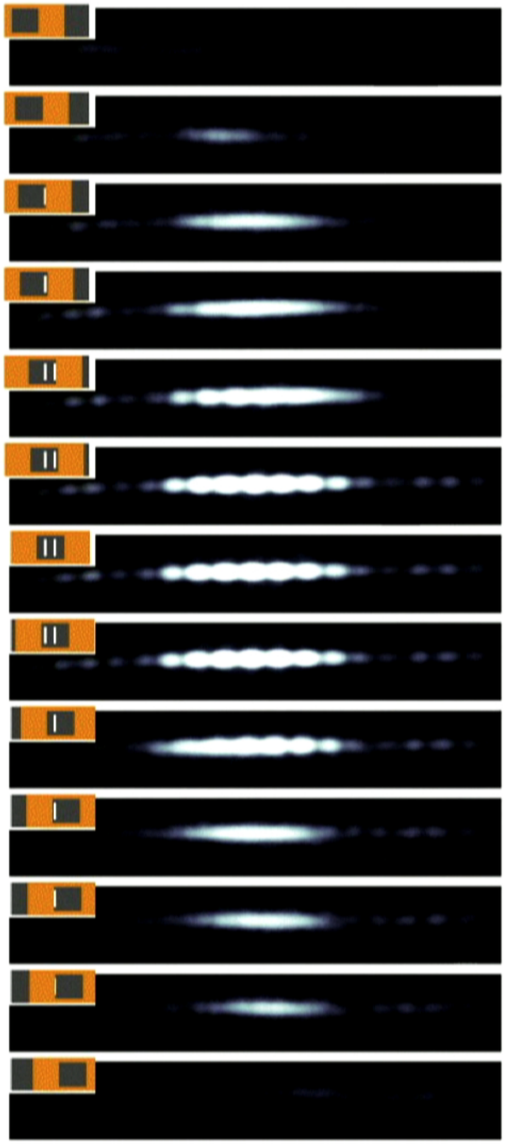




Wave-particle duality

The transition (from no interference to interference) can even be seen with single electrons.

Data from: "Controlled double-slit electron diffraction" Bach et al. NJP (2013)



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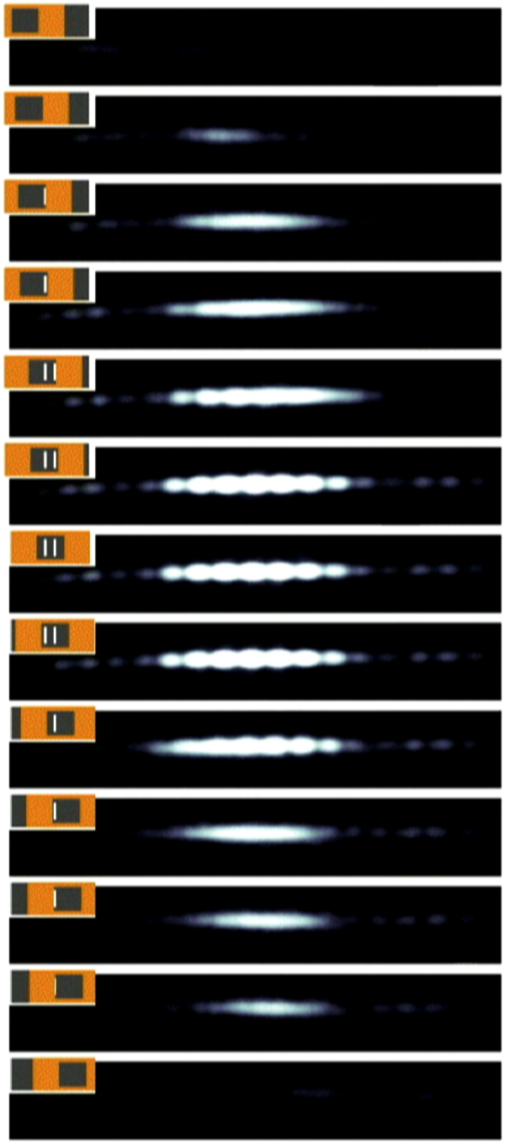
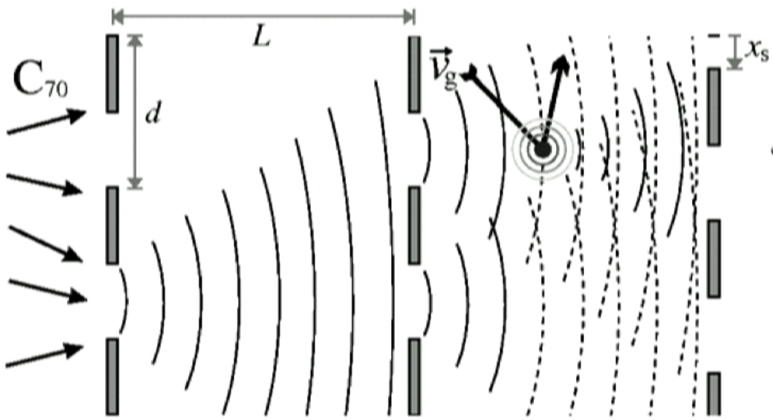
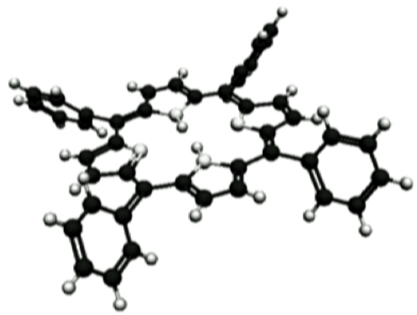


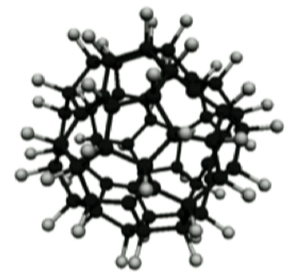
The great mystery:

Each kind of thing (bullet, electron, bacteria, ...) has the ability to exhibit wave behavior, i.e., produce interference. Likewise, each can exhibit particle behavior, i.e., have a well-defined path. But the two behaviors compete – you either get one or the other.

Wave-particle duality: big molecules

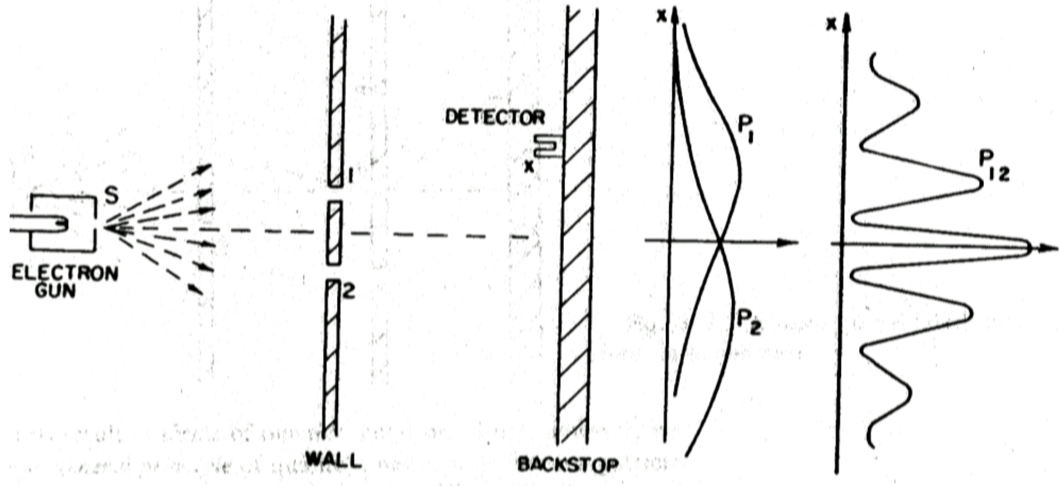






Wave-particle duality

While the behaviors are mysterious, we can get intuition for how they compete.

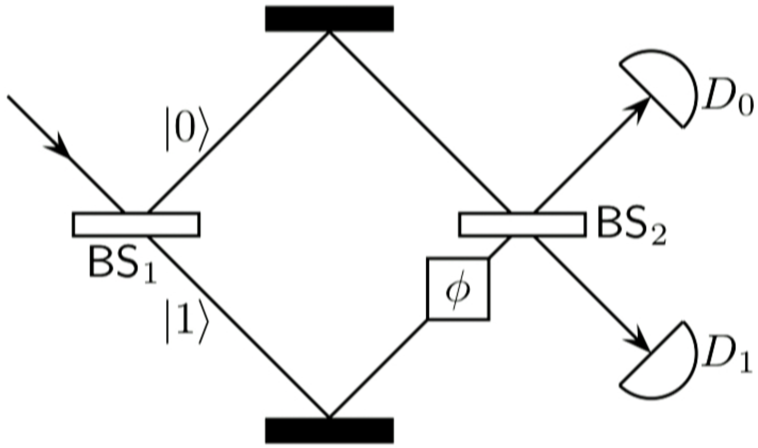


Wave-particle duality

getting quantitative

Simplification of double-slit:

Two-path interferometer for single photons (named after Mach and Zehnder).



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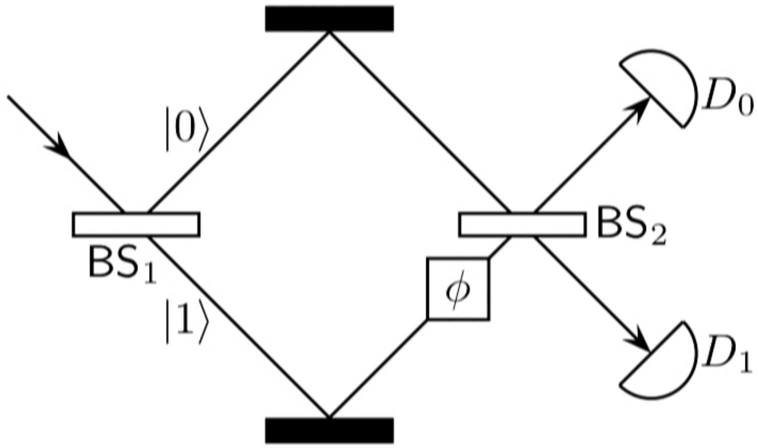

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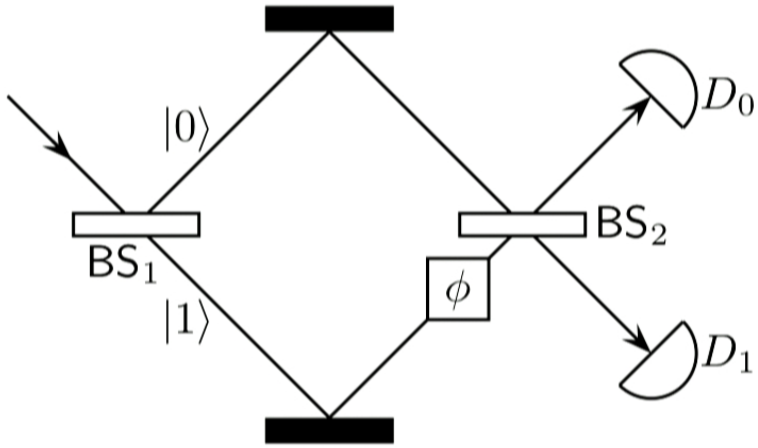
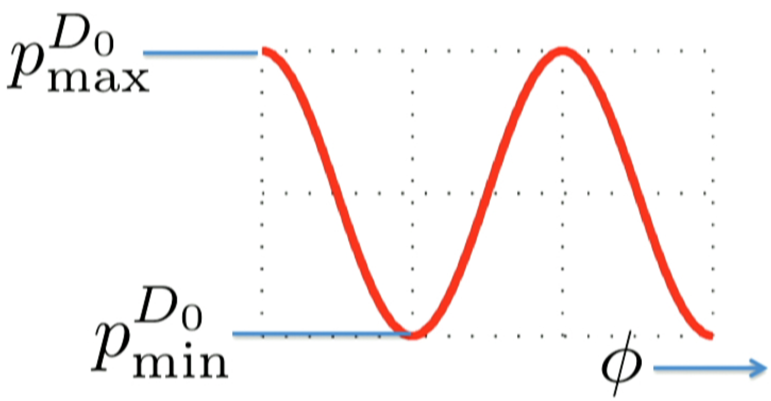


Fringe visibility

$$\mathcal{V} := \frac{p_{\max}^{D_0} - p_{\min}^{D_0}}{p_{\max}^{D_0} + p_{\min}^{D_0}}$$



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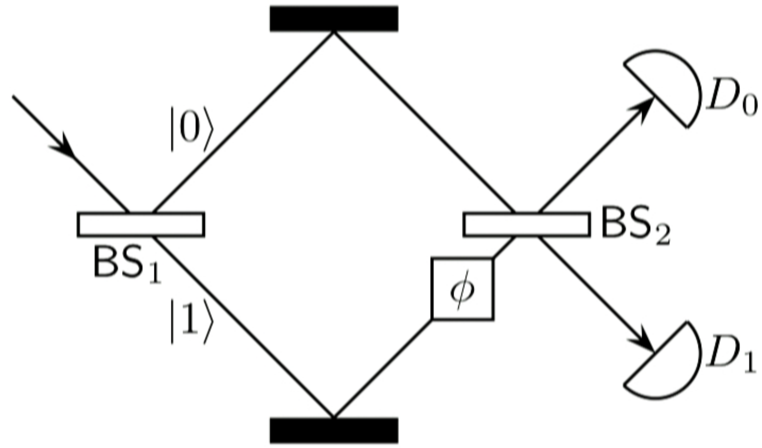

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Path predictability (e.g. asymmetric BS_1)

$$Z = \{|0\rangle, |1\rangle\}$$

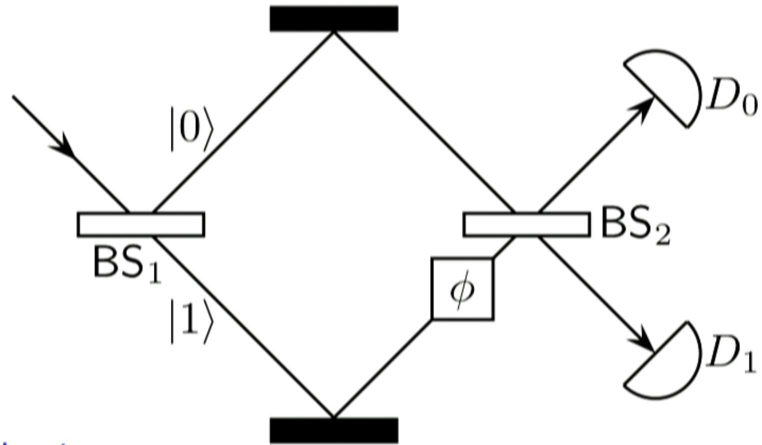
$$\mathcal{P} := 2p_{\text{guess}}(Z) - 1$$

probability of guessing Z correctly

Wave-particle duality

getting quantitative

Wooters, Zurek (1979)
Greenberger, Yasin (1988)
Englert (1996)



Wave-particle duality relation (WPDR):

$$\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1$$

Full particle behavior → No wave behavior
Full wave behavior → No particle behavior

Fringe visibility

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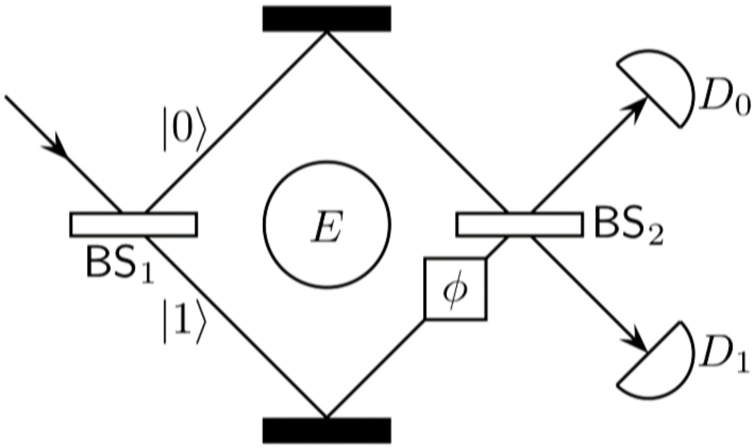
getting quantitative

Jaeger, Shimony, Vaidman (1995)
Englert (1996)

Let *E* be a (partial) which-path detector. *E* could be gas of atoms whose internal state is sensitive to presence of photon.



Stronger WPDR:

$$\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1$$

Path distinguishability

$$\mathcal{D} := 2p_{\text{guess}}(Z|E) - 1$$

probability of guessing Z correctly given E (i.e., given optimal measurement on E)

Fringe visibility

$$\mathcal{V} := \frac{p_{\text{max}}^{D_0} - p_{\text{min}}^{D_0}}{p_{\text{max}}^{D_0} + p_{\text{min}}^{D_0}}$$

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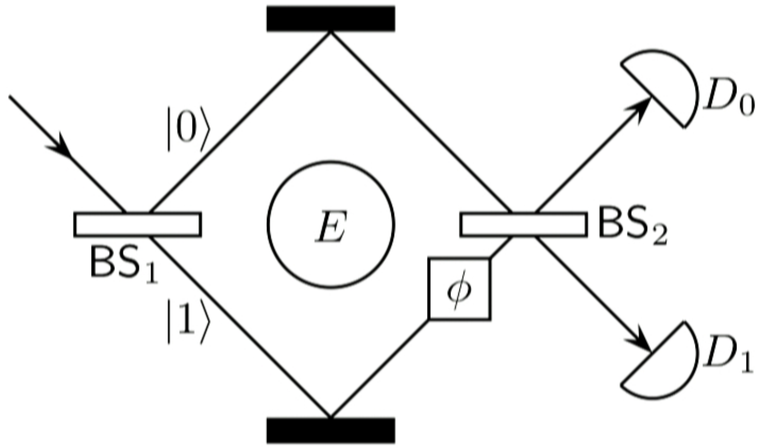

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WPDRs

Where do they come from?

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Is wave-particle duality a fundamental principle of quantum mechanics, or is it a corollary of some other principle?

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Is it a consequence of position/momentum uncertainty principle?

$$\Delta q \Delta p \geqslant \hbar/2 \quad ?$$

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This was intensely debated in 1990's:

"Path detection and the uncertainty principle" Storey et al. Nature (1994).
"Complementarity and uncertainty" Englert, Scully, Walther. Nature (1995), and Reply by Storey et al.
"Uncertainty over complementarity?" Wiseman, Harrison. Nature (1995).

Looks to be inconclusive / still open to debate

WPDRs

Where do they come from?

Consider: $\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1$

WPDRs

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Several authors showed that this WPDR is equivalent to Robertson's uncertainty relation for particular qubit observables

$$\Delta X \Delta Z \geqslant \frac{1}{2} |\langle \psi | [X, Z] | \psi \rangle|$$

Busch and Shilladay (2006)
Bjork et al. (1999)
Durr and Rempe (2000)
Bosyk et al. (2013)

Qubit observables:

$$\hat{P} = \sigma_z$$
$$\hat{V}_\phi = (\cos\phi)\ \sigma_x + (\sin\phi)\ \sigma_y$$

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Variances:

$$(\Delta\hat{P})^2 = 1 - P^2$$
$$(\Delta\hat{V}_\phi)^2 = 1 - V^2\cos^2(\theta - \phi)$$

Plugging into Robertson's relation gives:

$$(1 - P^2)[1 - V^2\cos^2(\theta - \phi)]$$
$$\geqslant P^2 V^2 \cos^2(\theta - \phi) + V^2 \sin^2(\theta - \phi)$$

WPDRs

Where do they come from?

<u>So we have</u>

$$\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1 \iff \Delta X \Delta Z \geqslant \frac{1}{2}|\langle\psi|[X,Z]|\psi\rangle|$$

$$\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1 \iff \text{??????}$$

WPDRs

Where do they come from?

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$$\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1 \quad \Longleftrightarrow \quad \Delta X \Delta Z \geqslant \frac{1}{2}|\langle\psi|[X,Z]|\psi\rangle|$$

$$\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1 \quad \Longleftrightarrow \quad ??????$$

Note that distinguishability involves conditioning on system *E*. This is not so natural for standard deviation, but is quite natural for *entropies*. Could the *D-V* relation be related to the *entropic* uncertainty principle?

$$\mathcal{D} := 2p_{\text{guess}}(Z|E) - 1$$

WPDRs

Consider again: $\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1$

Bosyk et al. [Phys. Scr. (2013)] considered entropic uncertainty relations (EURs), of the form:

$$H_q(P) + H_q(V) \geqslant \mathcal{B}_q$$

for Renyi entropies:

$$H_q(P) = \frac{1}{1-q} \ln\left[\left(\frac{1+P}{2}\right)^q + \left(\frac{1-P}{2}\right)^q\right]$$

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They argue that such EURs are <u>inequivalent</u> to the P-V relation!

But Maassen & Uffink (1988) proved an EUR that involves different q's, for example,

$$H_\infty(P) + H_{1/2}(V) \geqslant 1$$

Our first result: This EUR is equivalent to the P-V relation!!!!

WPDRs

$$H_\infty(P) + H_{1/2}(V) \geqslant 1$$

INVITATION: Plug these formulas in to obtain *P-V* relation

$$H_\infty(P) = 1 - \log(1 + \mathcal{P})$$
$$H_{1/2}(V) = \log\left(1 + \sqrt{1 - \mathcal{V}^2}\right)$$

WPDRs

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APOLOGY: In what follows, I will switch notation:

$$H_\infty(P) \to H_{\min}(Z) \qquad H_{1/2}(V) \to H_{\max}(W)$$

Goals of our work

1.) Unify a vast literature on WPDRs. Many complicated versions of WPDRs have been formulated, for exotic scenarios involving quantum beam splitters or quantum erasure or for alternative interferometers like the double slit. We show that all these WPDRs correspond to special cases of a single inequality.

2.) Show that WPDRs come from the uncertainty relation for the min- and max-entropies. Hence we unify the entropic uncertainty principle with the wave-particle duality principle.

3.) Provide a general, robust framework for discussing WPDRs and deriving novel WPDRs. We illustrate this by deriving a novel WPDR for a quantum beam splitter.

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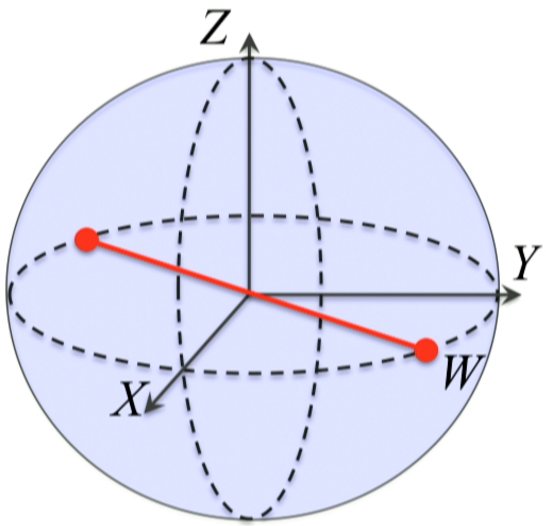
3.) Provide a general, robust framework for discussing WPDRs and deriving novel WPDRs. We illustrate this by deriving a novel WPDR for a quantum beam splitter.

4.) Uncertainty relations can be applied in two different ways. We emphasize the distinction between preparation and measurement WPDRs.

Main Result

For a binary interferometer (i.e., two interfering paths), we identify particle and wave behaviors with the knowledge of complementary qubit observables:

$$\text{which-path:} \quad Z = \{|0\rangle, |1\rangle\}$$
$$\text{which-phase:} \quad W = \{|w_\pm\rangle\}, \quad |w_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm e^{i\phi_0}|1\rangle)$$



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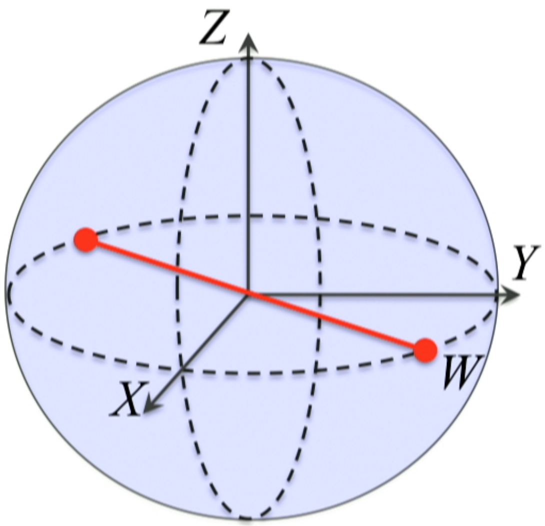
lack of particle behavior: $H_{\min}(Z|E_1)$

lack of wave behavior: $\min_{W \in XY} H_{\max}(W|E_2)$



E_1, E_2 : some other quantum systems that help to reveal the behavior

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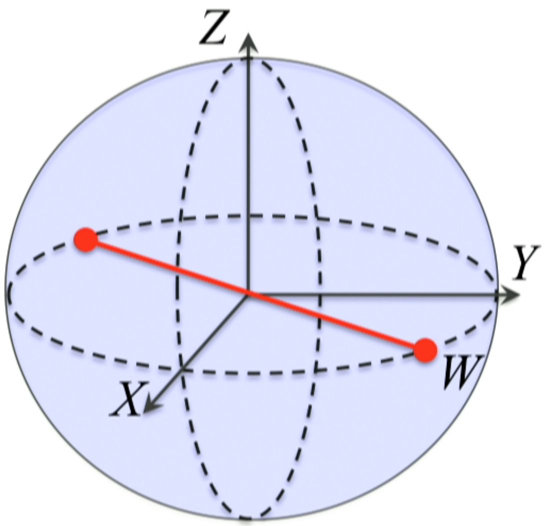


E_1, E_2 : some other quantum systems that help to reveal the behavior

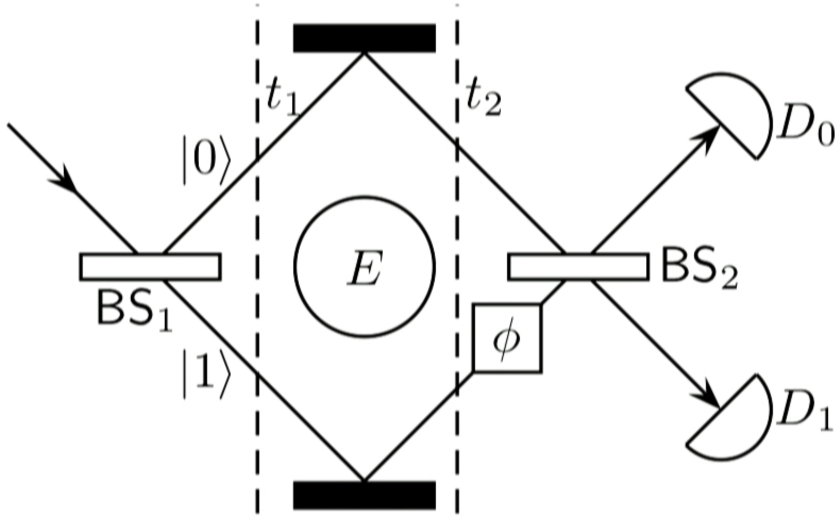
Our general WPDR:

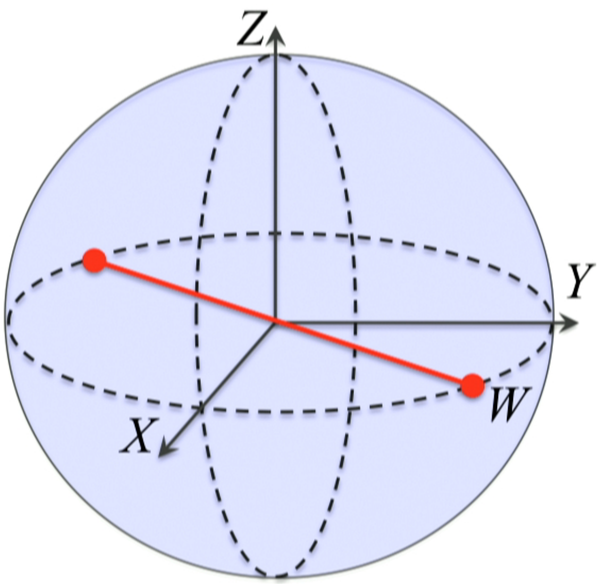
$$H_{\min}(Z|E_1) + \min_{W \in XY} H_{\max}(W|E_2) \geqslant 1$$

Majority of WPDRs in literature are special cases of this relation.

Revisiting Distinguishability-Visibility tradeoff

Recall scenario: photon interacts with E inside interferometer





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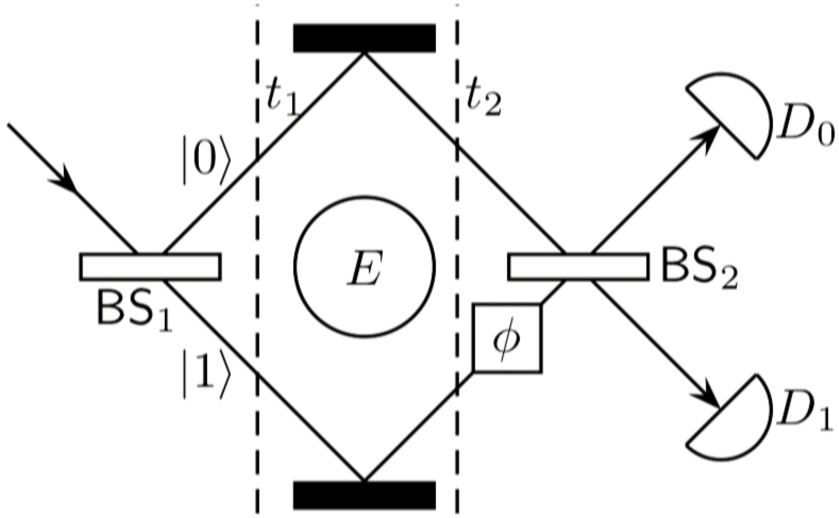
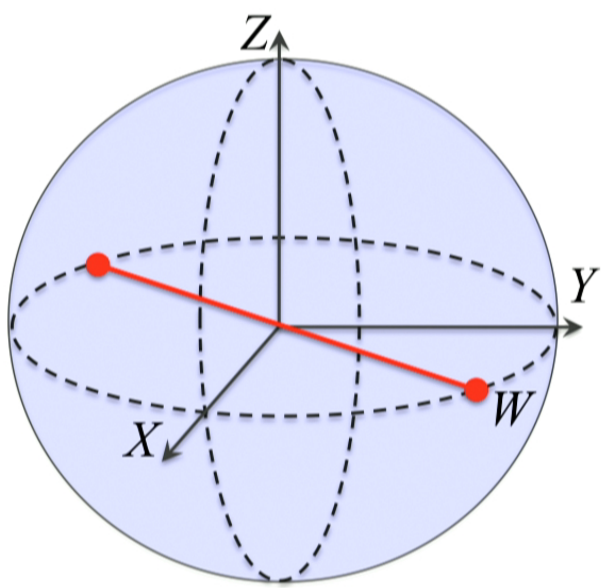


Recall scenario: photon interacts with *E* inside interferometer



Apply uncertainty relation at time t_2

$$H_{\min}(Z)_{t_2} + \min_{W \in XY} H_{\max}(W)_{t_2} \geqslant 1$$

$$\mathcal{P}^2 + \mathcal{V}^2 \leqslant 1$$

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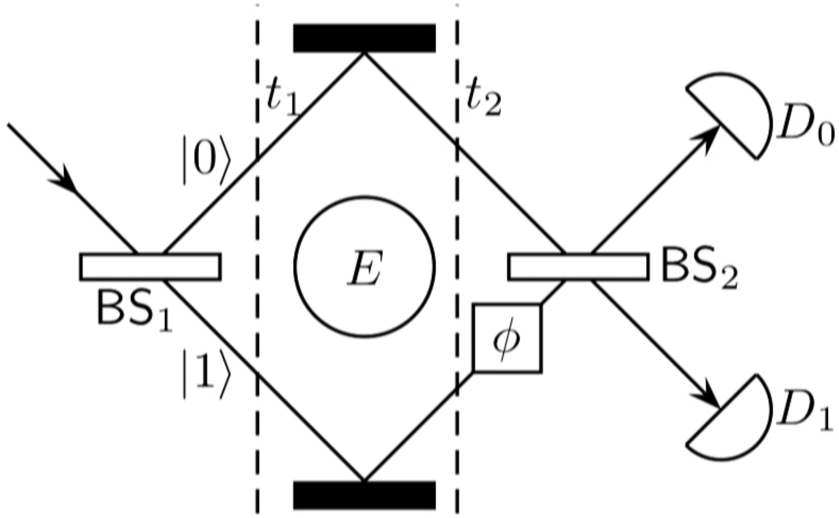

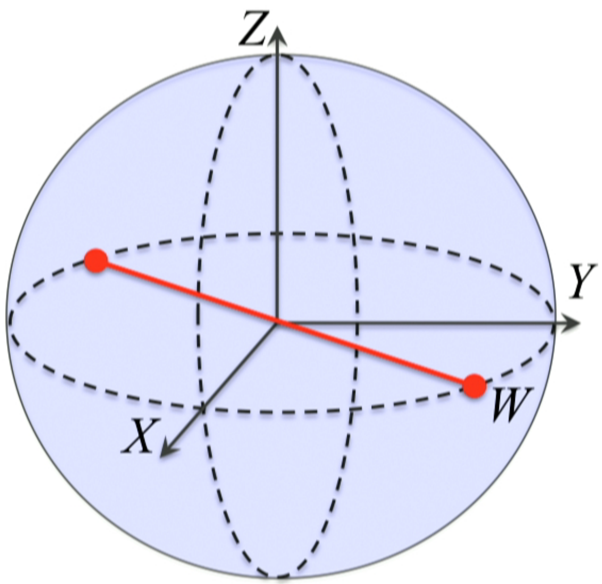

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Operational meaning of entropies

Quantum key distribution

$$H_{\min}(Z|E_1) + \min_{W \in XY} H_{\max}(W|E_2) \geqslant 1$$

Used to prove security of QKD

H_{min} : randomness extraction
H_{max} : data compression

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Guessing games

classical-quantum state ρ_{XB}

$$H_{\min}(X|B) = -\log p_{\text{guess}}(X|B)$$

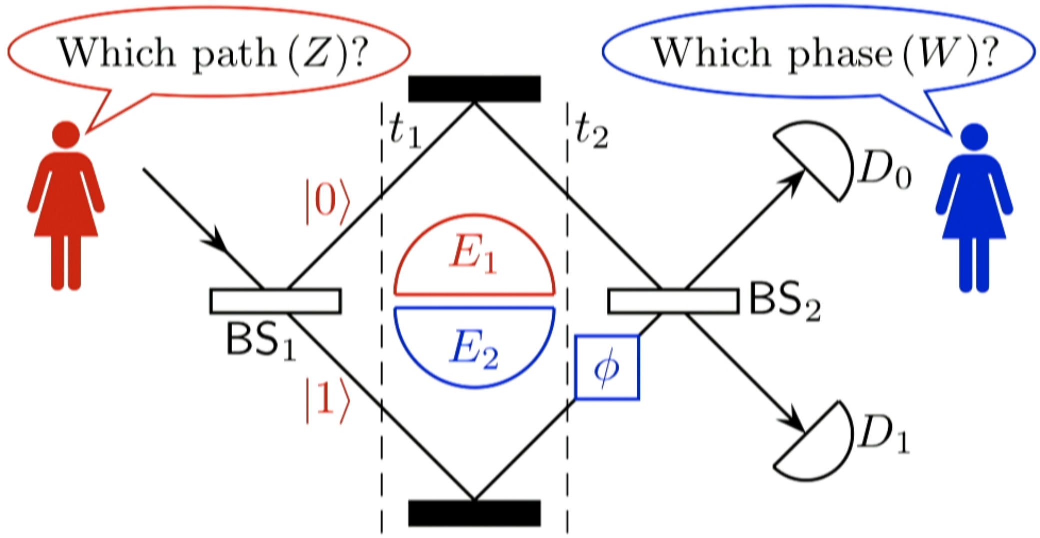
When X is binary, we show that:

$$H_{\max}(X|B) \leqslant \log\left(1 + \sqrt{1 - (2p_{\text{guess}}(X|B) - 1)^2}\right)$$

Complementary Guessing Games

Consider two games

Game #1: We ask Alice to guess which path the quanton takes, given that she has access to E_1.



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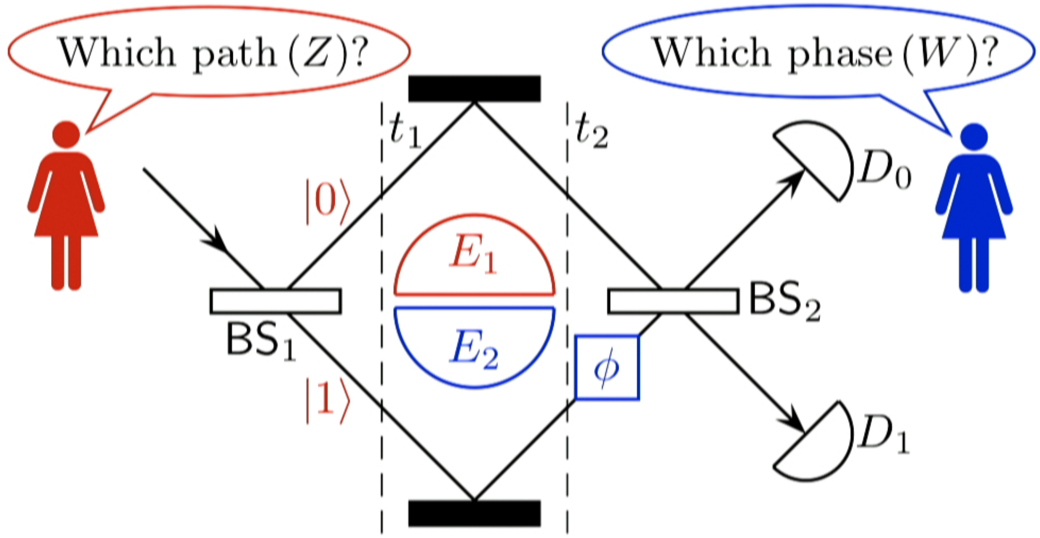

Complementary Guessing Games

Consider two games

Game #1: We ask Alice to guess which path the quanton takes, given that she has access to E_1.

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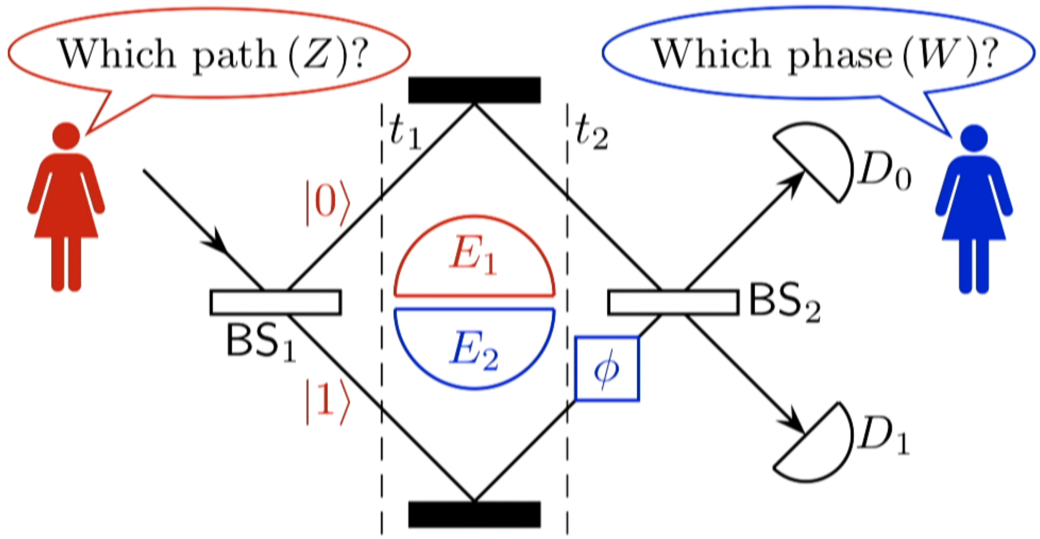

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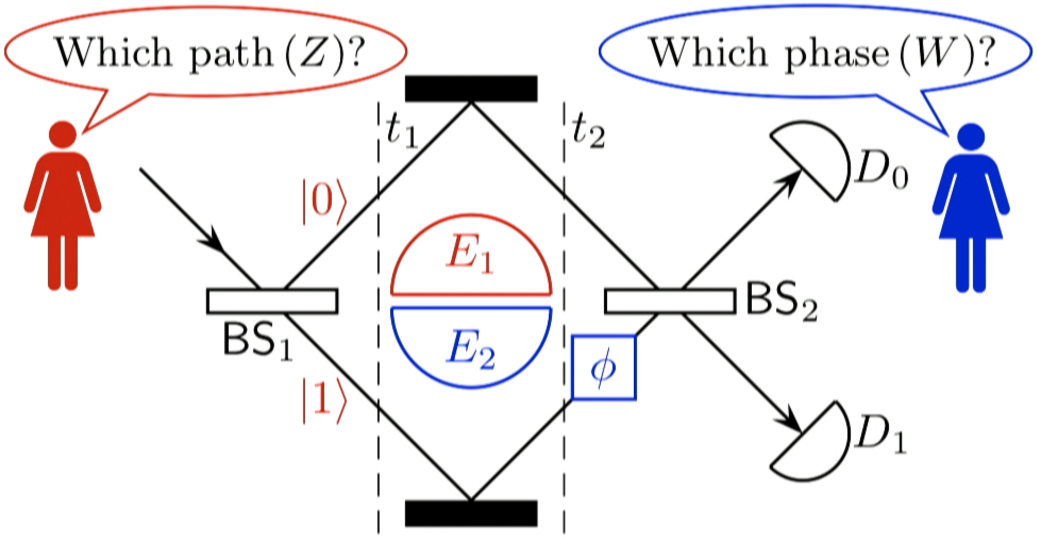

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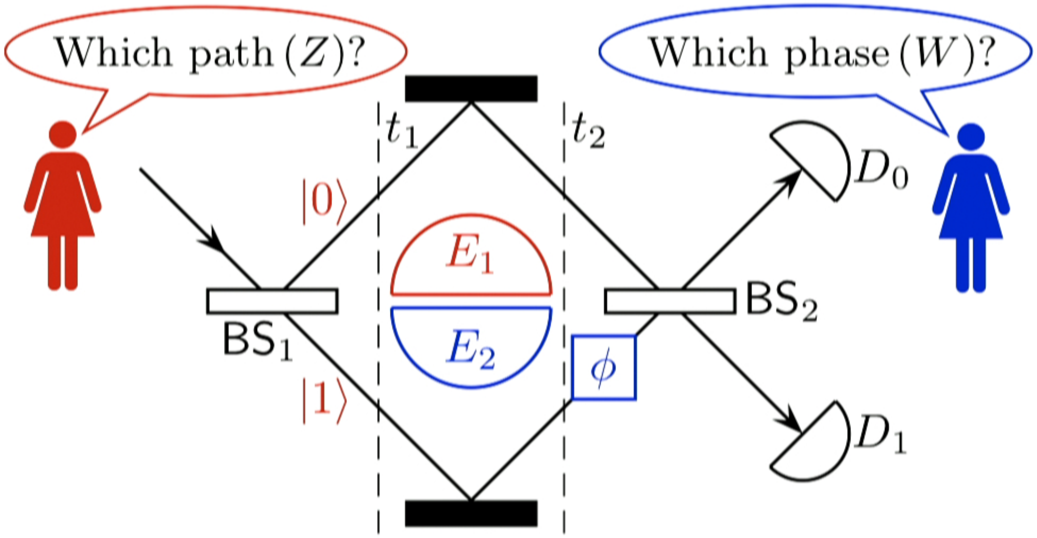

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Generic measures

$$\mathcal{D}_g := 2p_{\text{guess}}(Z|E_1) - 1,$$

$$\mathcal{V}_g := \max_{W \in XY} [2p_{\text{guess}}(W|E_2) - 1]$$

Rearrange into traditional WPDR form

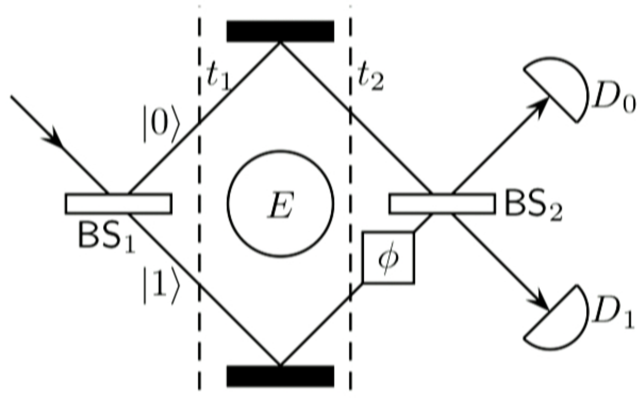
$$\mathcal{D}_g^2 + \mathcal{V}_g^2 \leqslant 1$$

Preparation Uncertainty

Remark

We applied the preparation uncertainty relation at time t_2 to derive the WPDR.

Preparation uncertainty restricts one's ability to predict *future* measurements.



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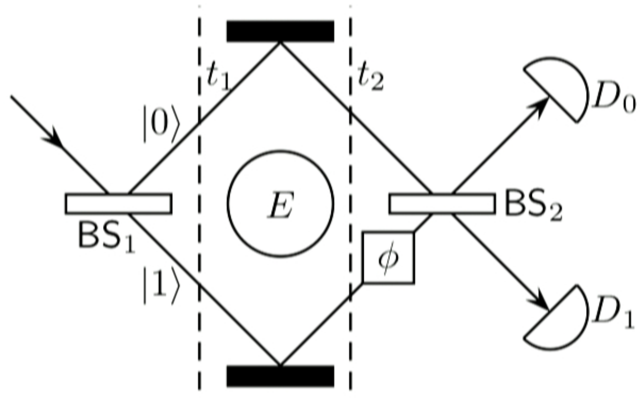
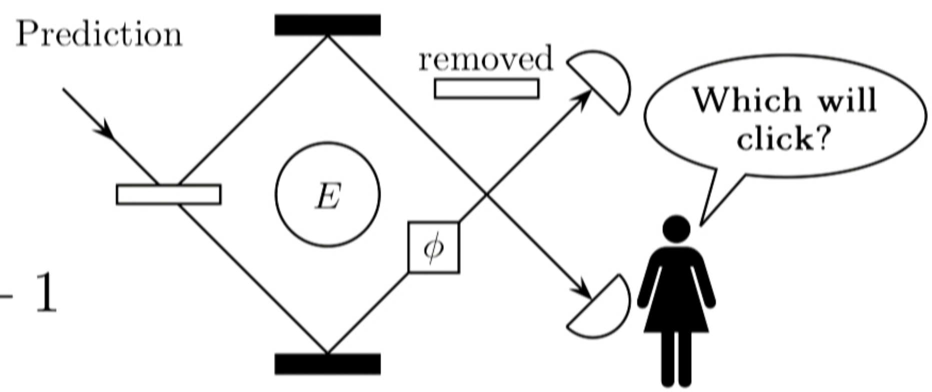


To measure *P* or *D*, one removes the second beam splitter (BS_2) and tries to *predict* which detector clicks.

$$\mathcal{D} := 2p_{\text{guess}}(Z|E)_{t_2} - 1$$



Measurement Uncertainty

Preparation uncertainty

- Fixed input state; complementary output measurements

Measurement uncertainty

- Fixed output measurement; complementary input ensembles:

$$Z_i = \{|0\rangle, |1\rangle\} \qquad W_i = \{|w_\pm\rangle\} \quad |w_\pm\rangle = (|0\rangle \pm e^{i\phi}|1\rangle)/\sqrt{2}$$

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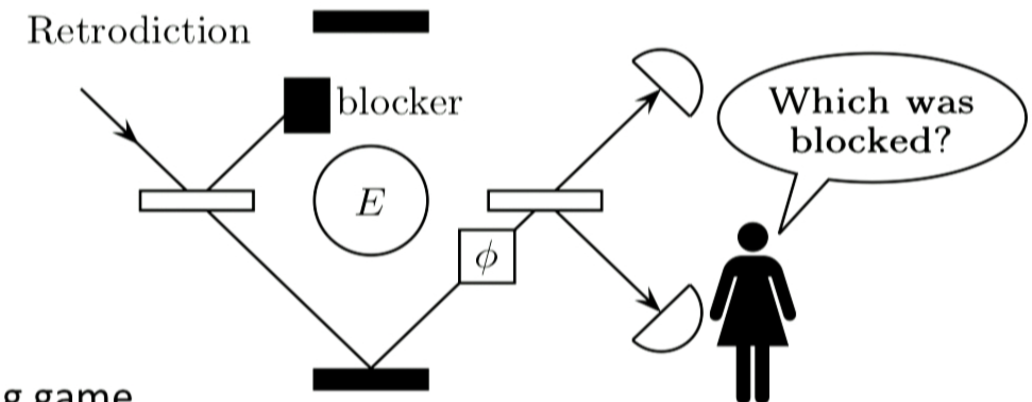

Guessing game

The Z_i states are generated by Bob flipping a coin and blocking either the top or bottom arm depending on flip outcome. Alice tries to guess Bob's coin flip, given E and given which detector clicks, denoted by C.

$$\mathcal{D}_i := 2p_{\text{guess}}(Z_i|EC) - 1$$

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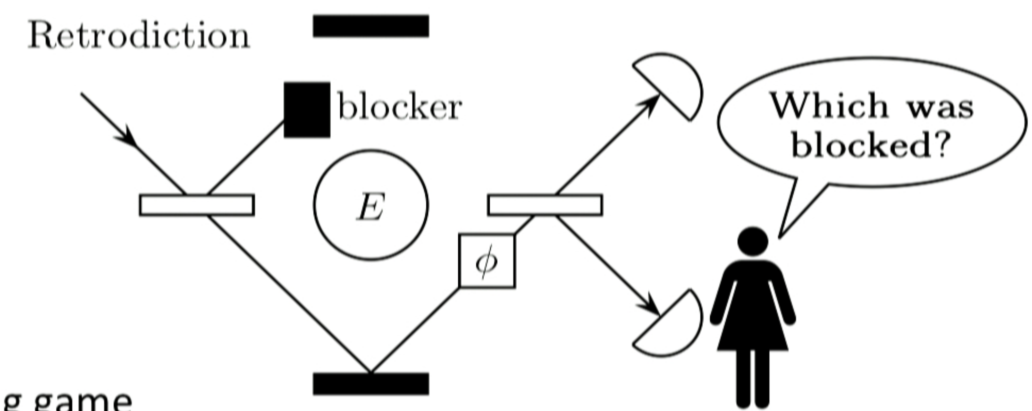

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Preparation vs. Measurement Uncertainty

Output distinguishability

$$\mathcal{D} := 2p_{\text{guess}}(Z|E)_{t_2} - 1$$

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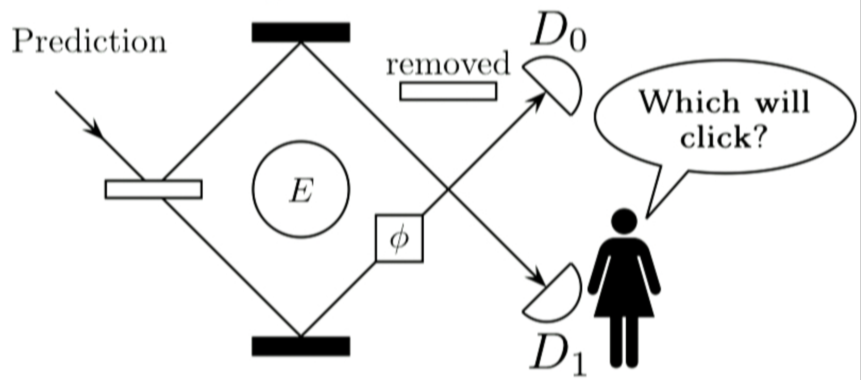

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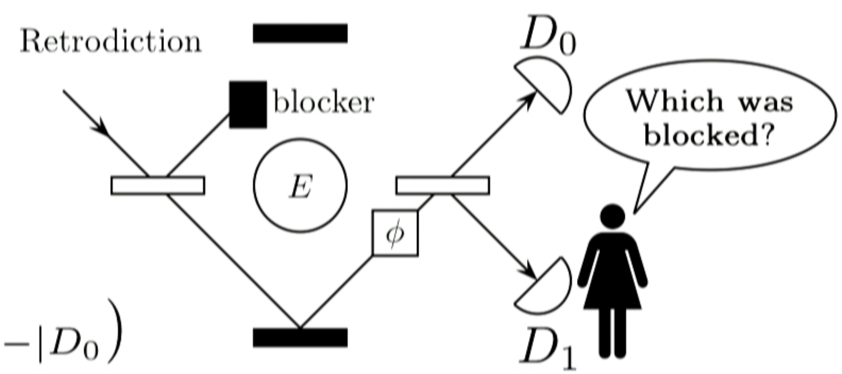

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$$\mathcal{D}^2 + \mathcal{V}^2 \leqslant 1$$

Addresses question of how well Alice can prepare a state with low uncertainty in *Z* and *W*.

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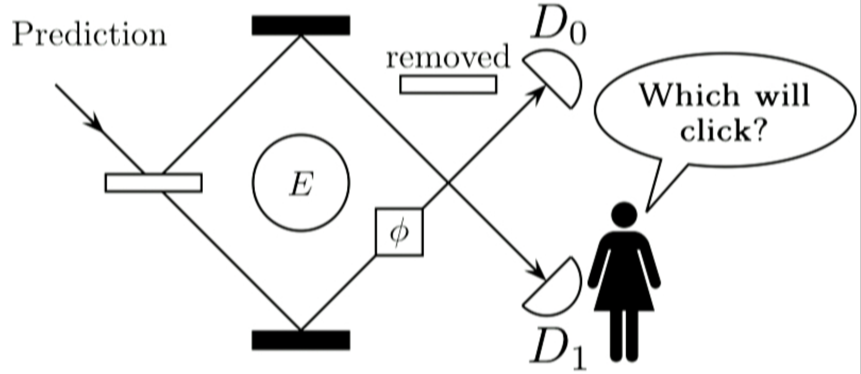

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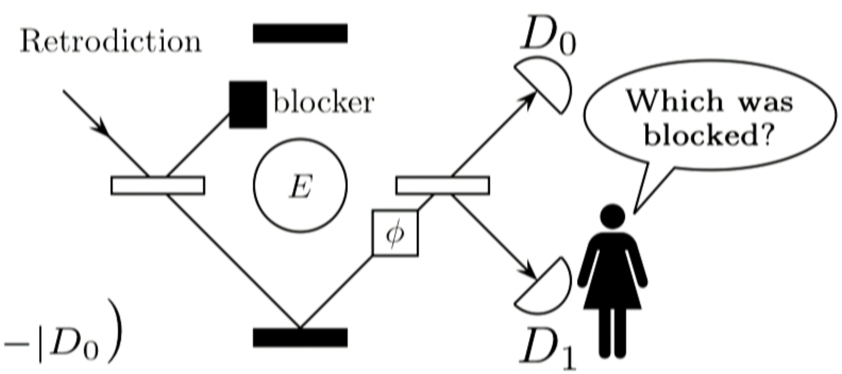

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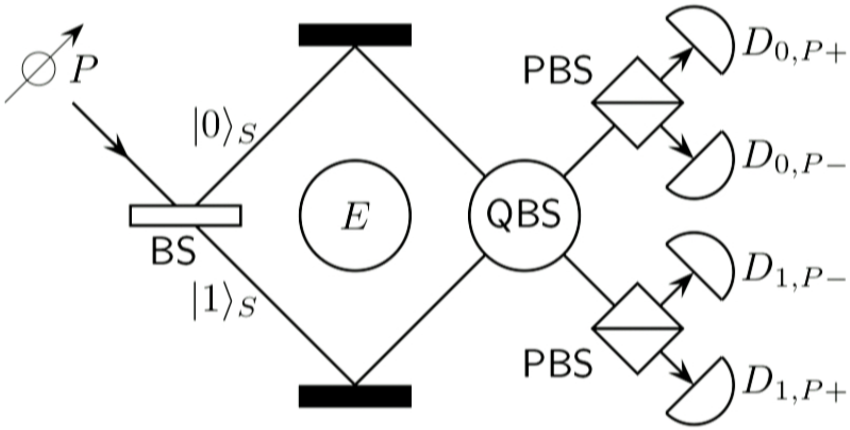

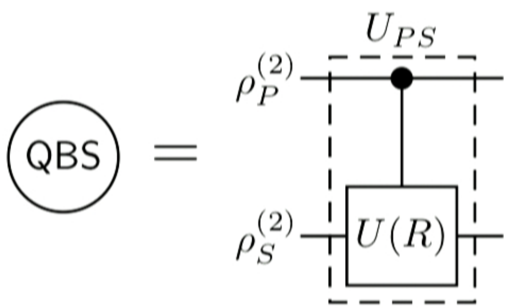

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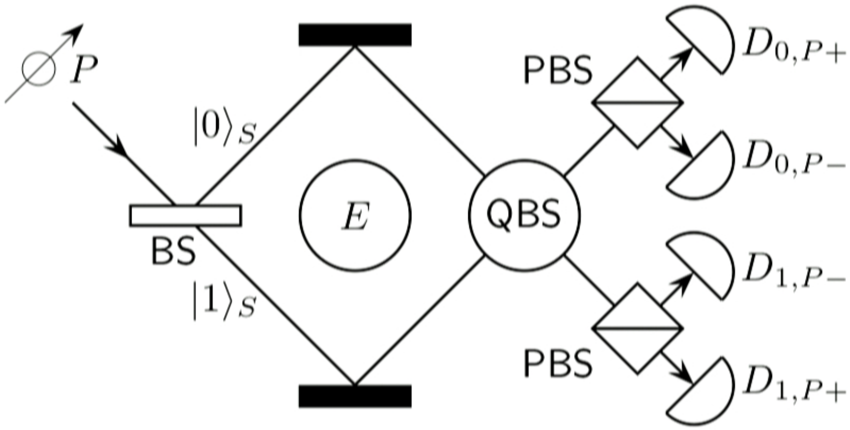
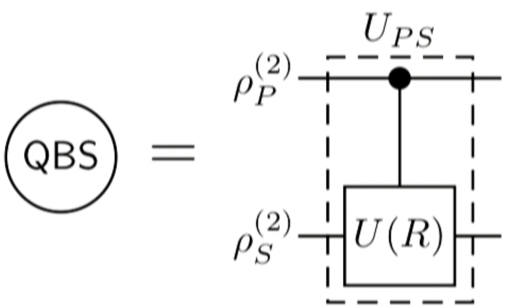


Feeding in a polarization superposition means that BS_2 is in a superposition of "absent" and "present".



$$\rho_P^{(2)} = |\psi_P^{(2)}\rangle\langle\psi_P^{(2)}|$$

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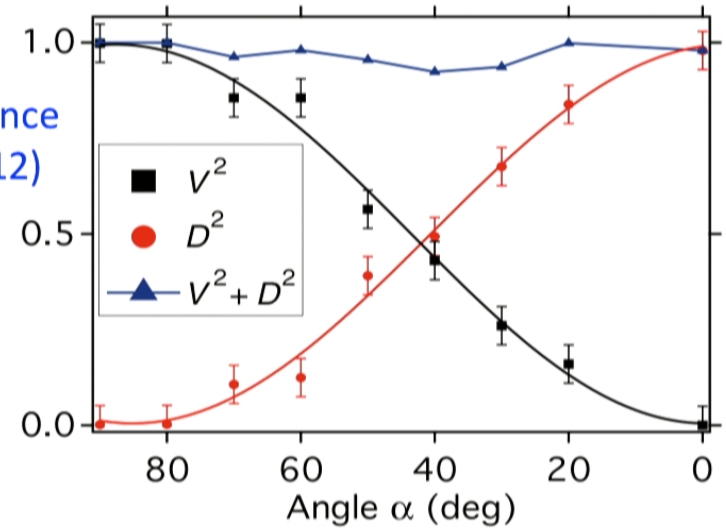
Science (2012)



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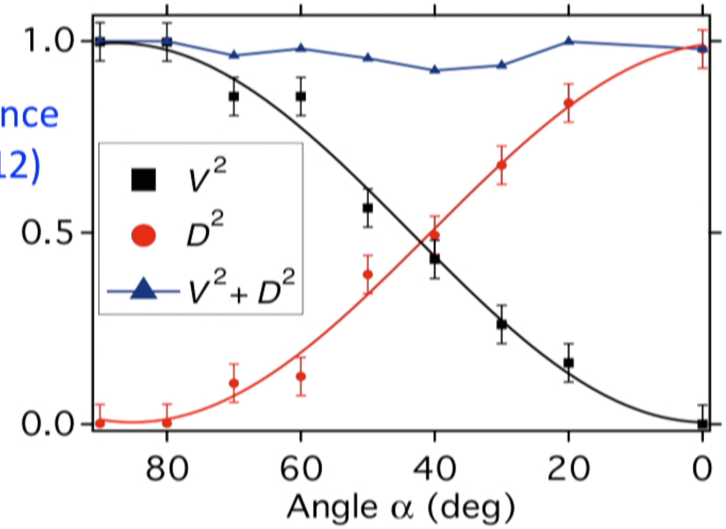

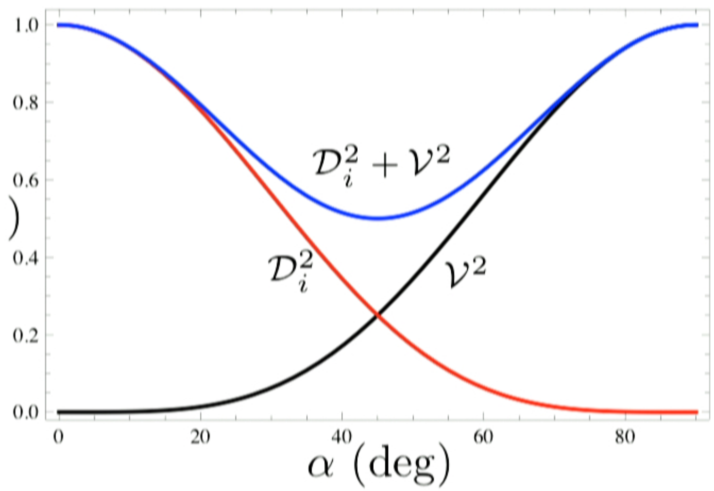

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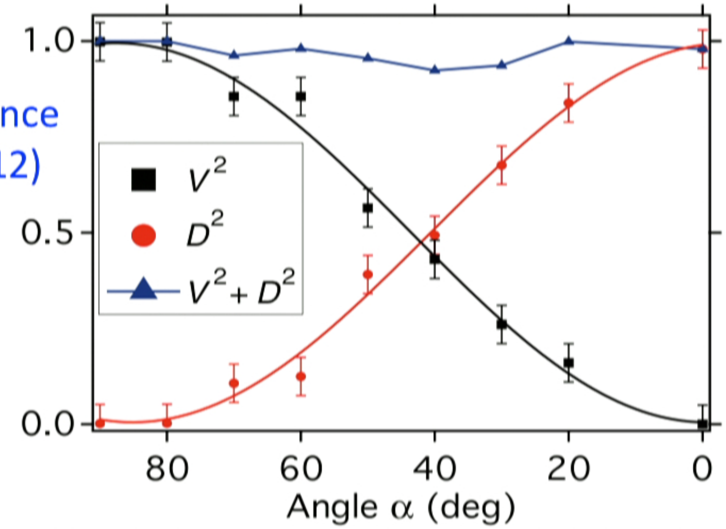

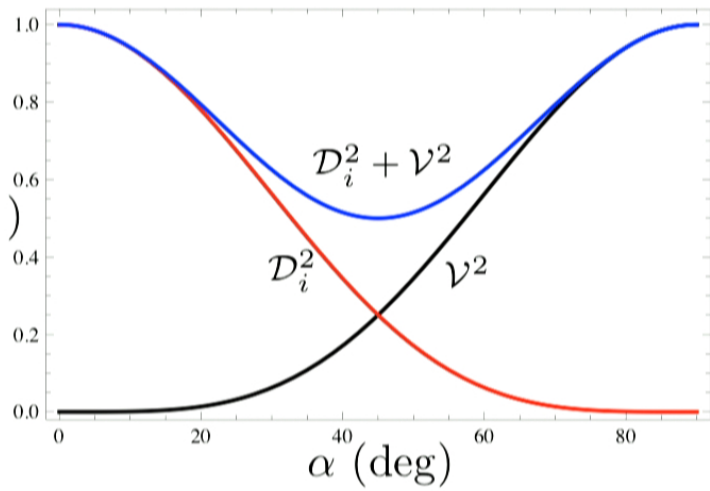

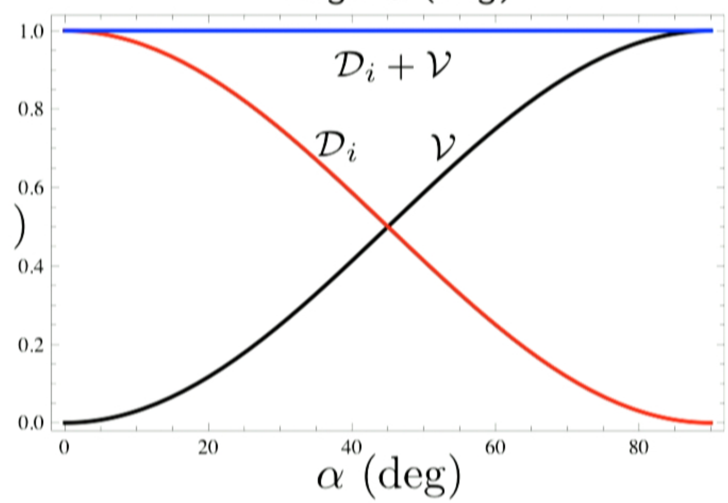

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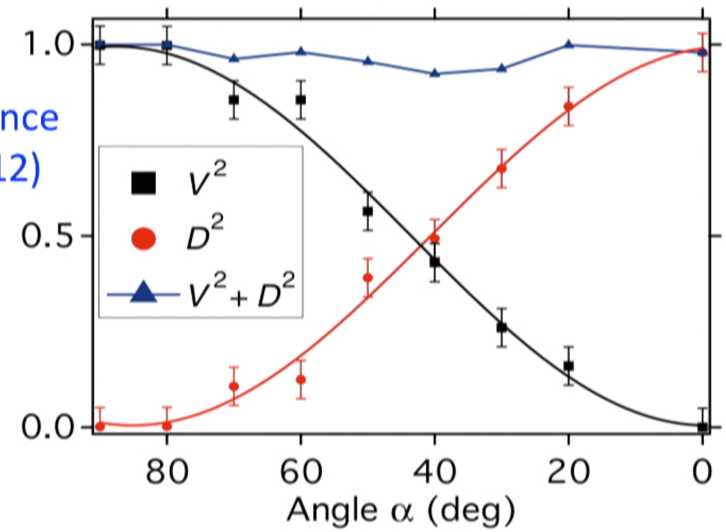

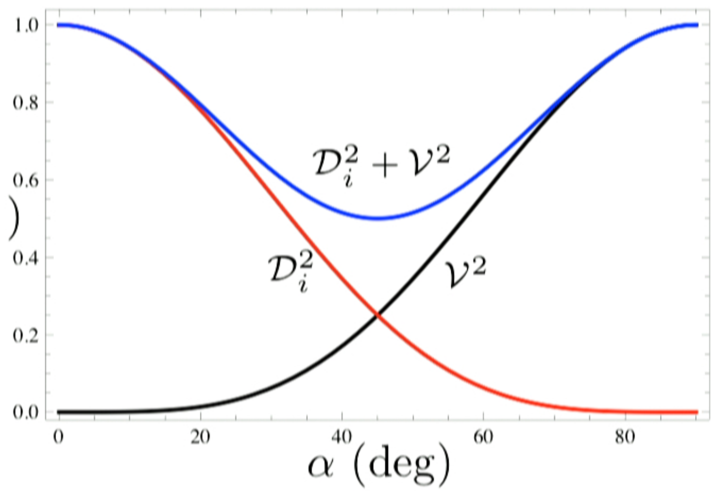

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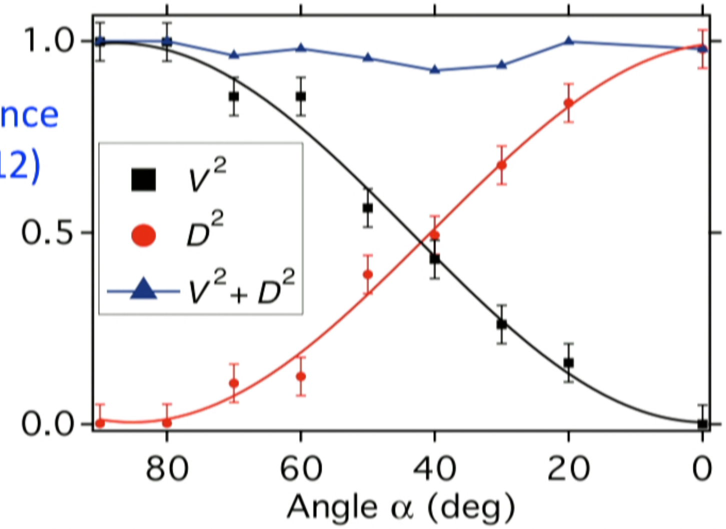

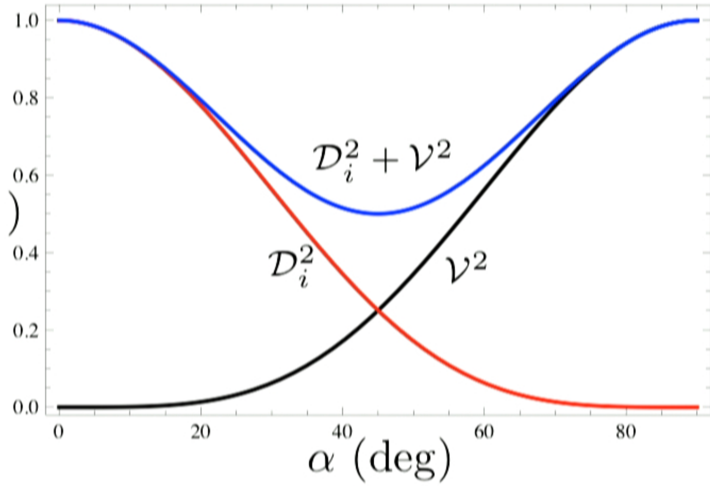

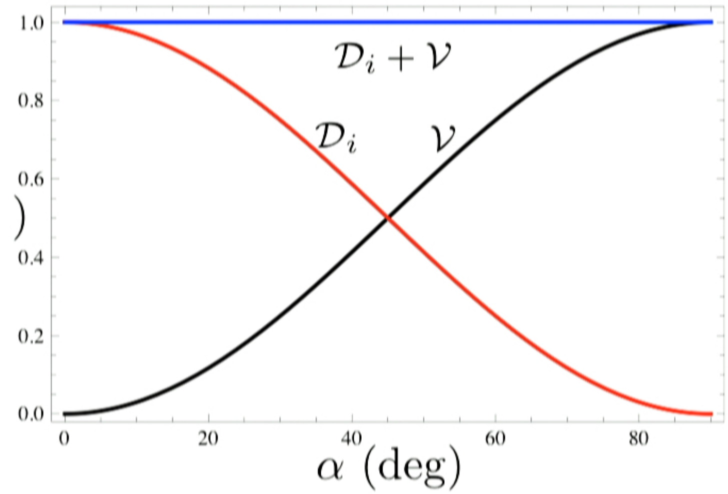

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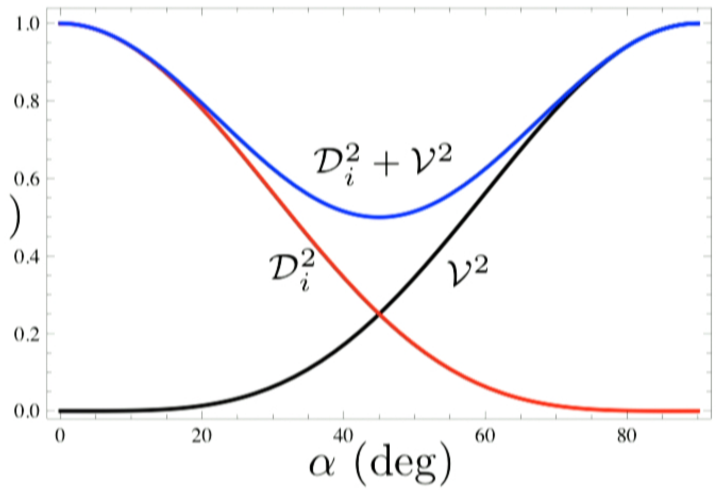

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Our framework easily provides a tight relation that captures beam splitter's coherence. We condition distinguishability on the final polarization.

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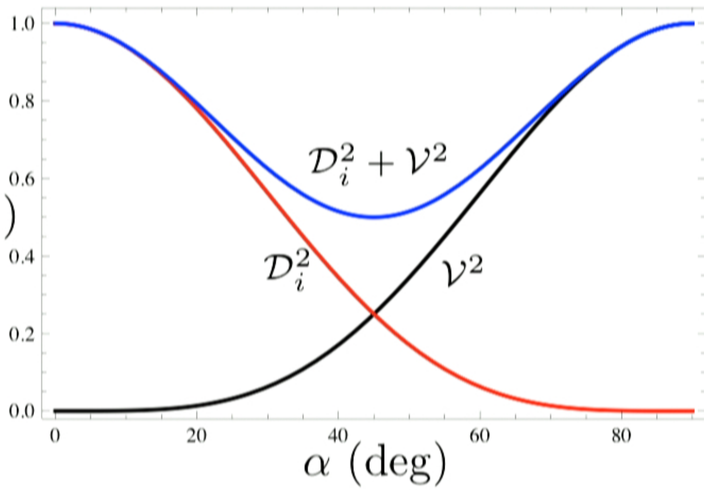
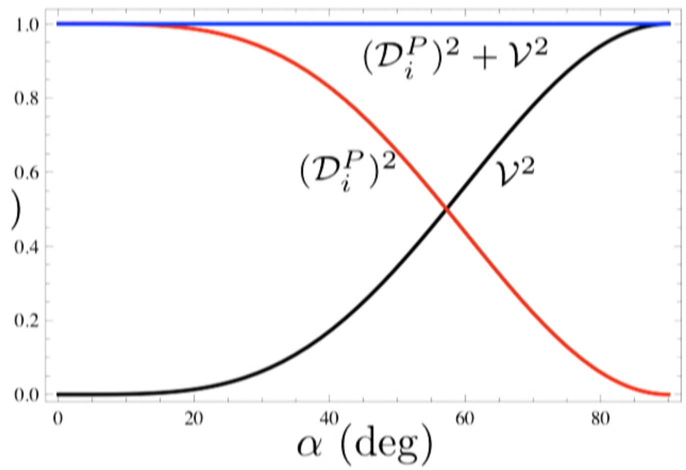


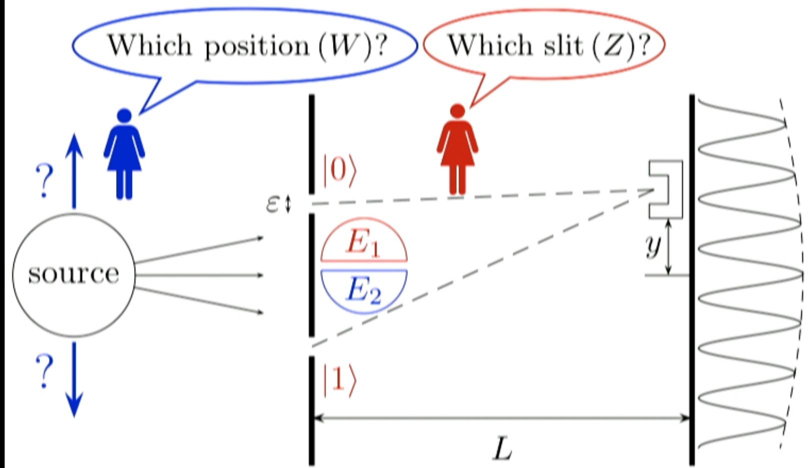
$$(\mathcal{D}_i^P)^2 + \mathcal{V}^2 \leqslant 1$$



Binary interferometers

Two interfering paths

Double Slit



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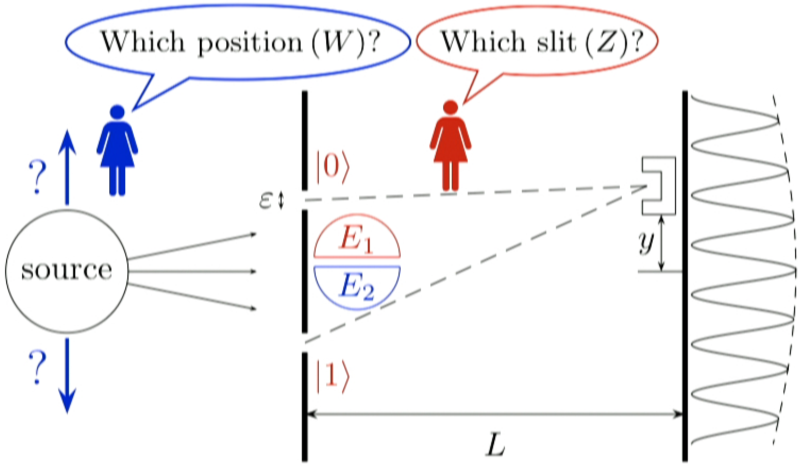

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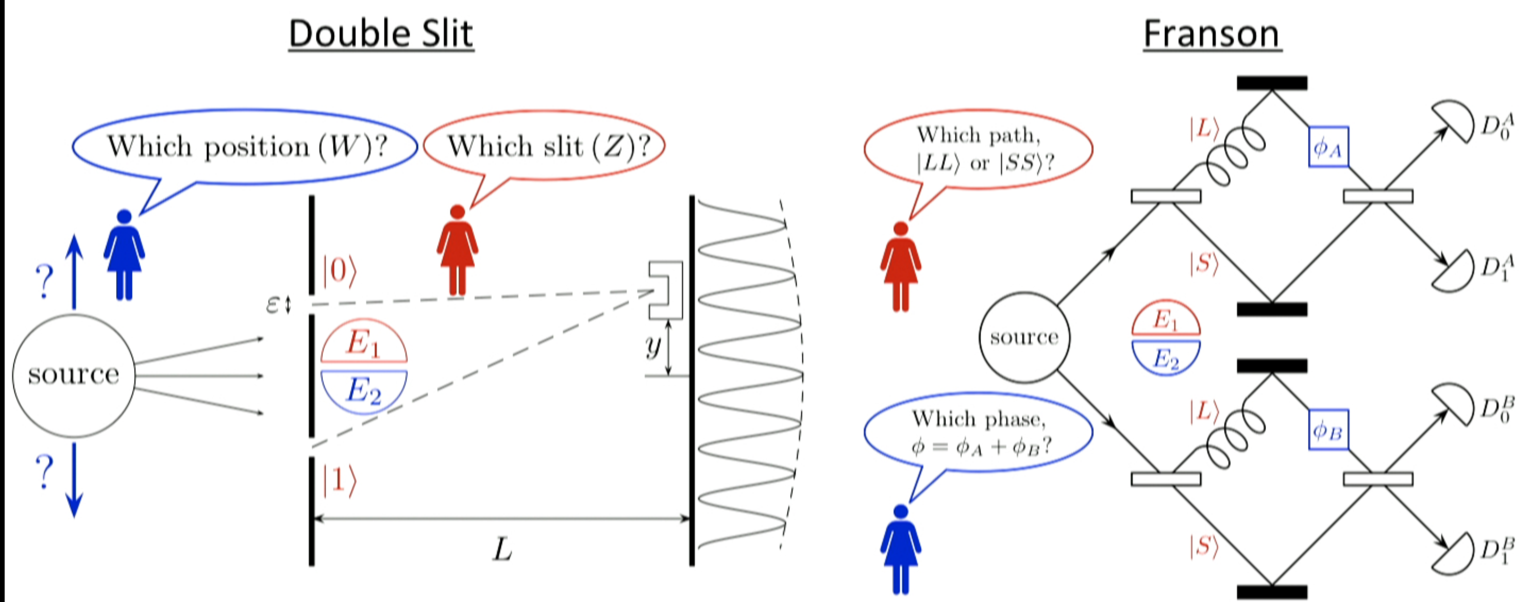

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