Title: Spin-charge scattering in Luttinger Liquids

Date: Nov 04, 2014 03:30 PM

URL: http://pirsa.org/14110096

Abstract: I will discuss the violation of spin-charge separation in generic Luttinger liquids and investigate its effect on the relaxation, thermal and electrical transport of genuine spin-1/2 electron liquids in ballistic quantum wires. We will identify basic scattering processes compatible with the symmetry of the problem and conservation laws that lead to the decay of plasmons into the spin modes and also discuss Brownian backscattering of spin excitations. I will present a closed set of coupled kinetic equations for the spin-charge excitations and solve the problem of electrical and thermal conductance of interacting electrons for an arbitrary relation between the quantum wire length and spin-charge thermalization length.

Spin-charge scattering in generic Luttinger liquids

Alex Levchenko

(Perimeter Institute, November 5, 2014)

PRB **82**, 115413 (2010) PRB **83**, 041303 (2011)

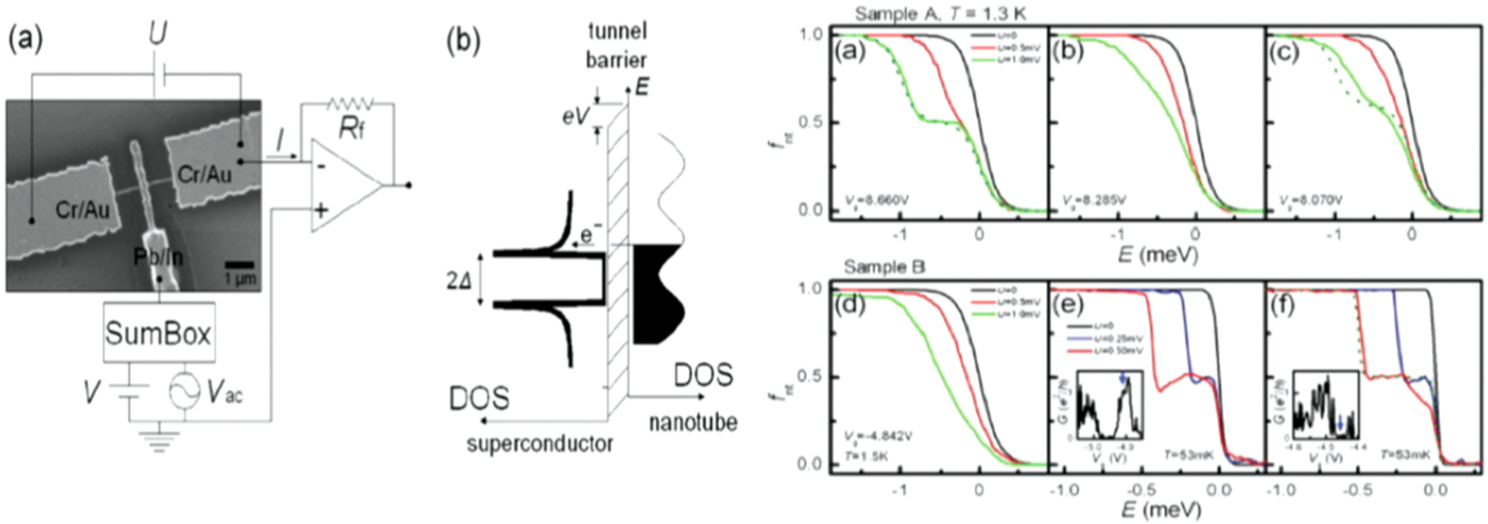
PRL **106**, 196402 (2011) PRL **109** 036405 (2012)

Department of Physics and Astronomy

MICHIGAN STATE UNIVERSITY

Nonequilibrium Tunneling Spectroscopy in Carbon Nanotubes

Yung-Fu Chen *et. al.*, PRL (2009)



$$I(V) \approx \frac{1}{eR_T}\int_{-\infty}^{\infty} dE\, n_s(E+eV)\, n_{\rm nt}(E)\,(f_{\rm nt}(E) - f_s(E+eV)).$$

Nonequilibrium distribution

1) Enhanced relaxation at *T>1.5K*   2) Relaxation length *L>1µm*

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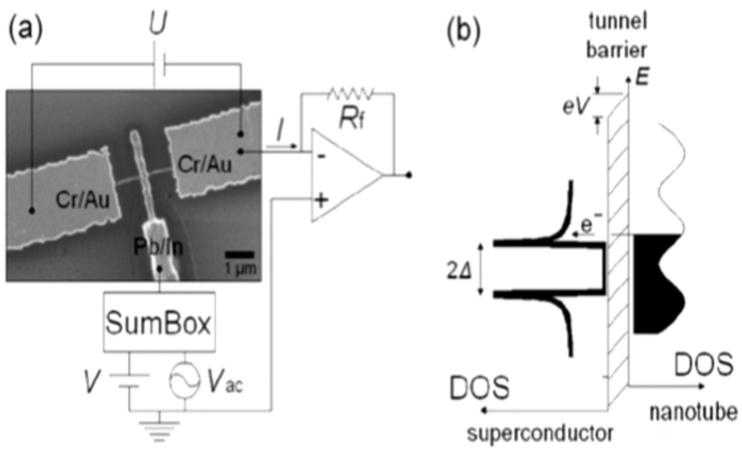

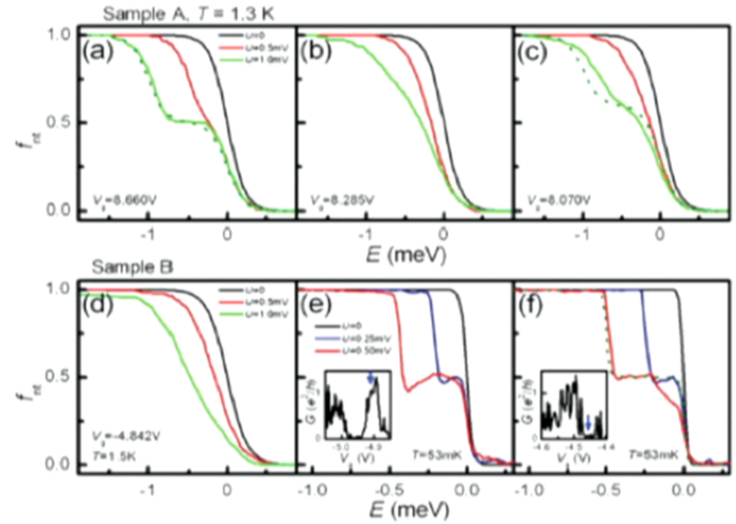

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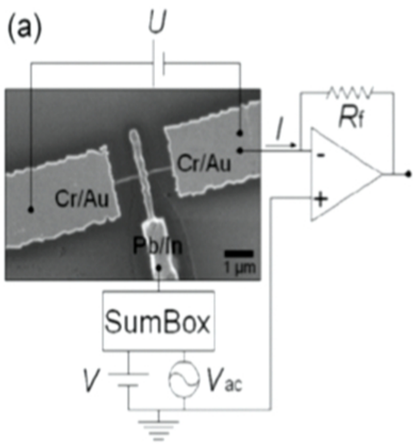

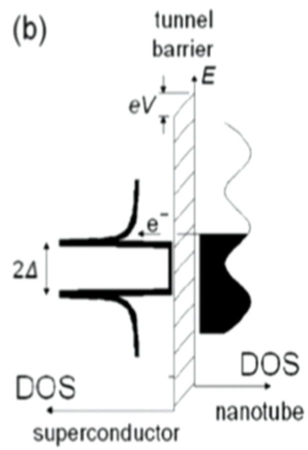

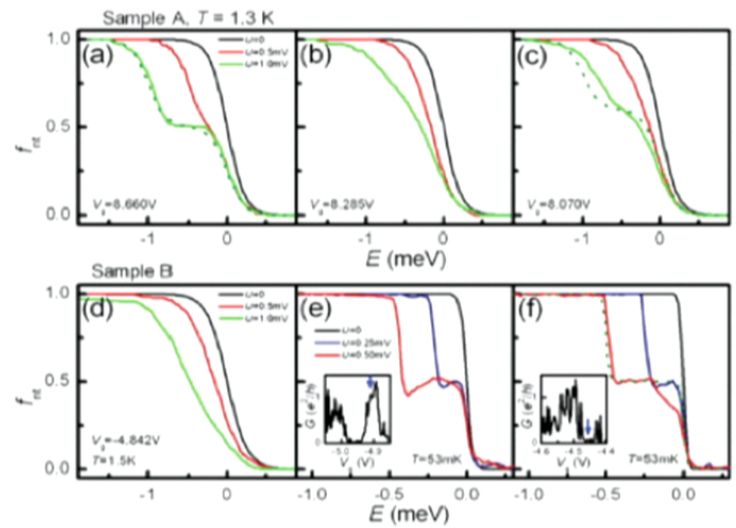

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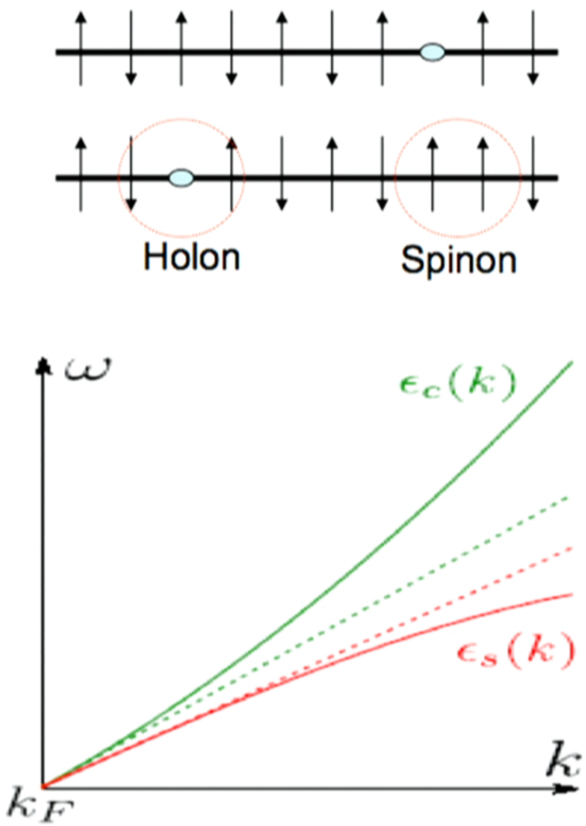
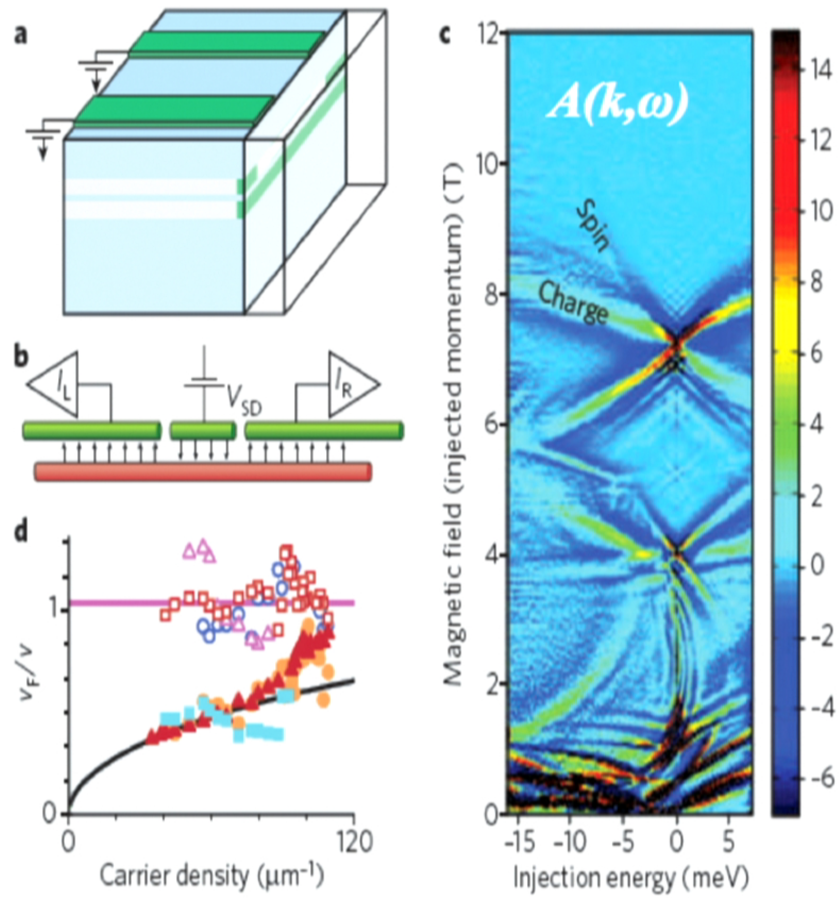
Spin-Charge Separation

Auslaender *et. al.* Science (2005)





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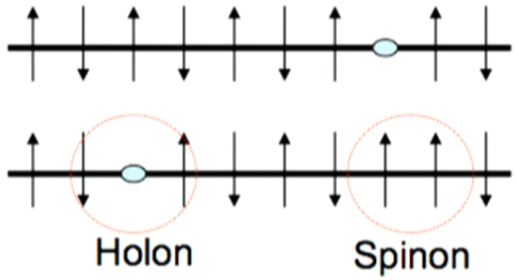
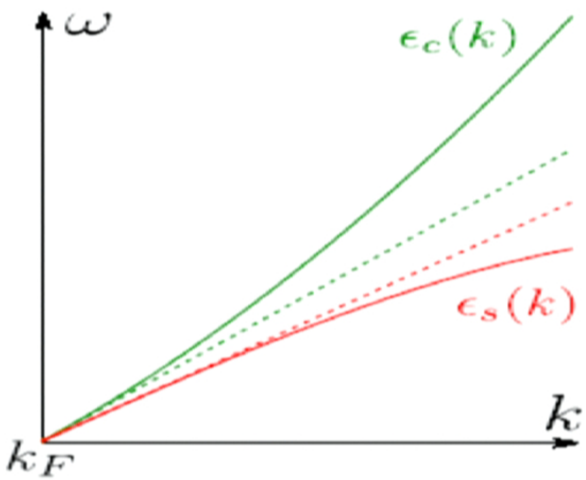
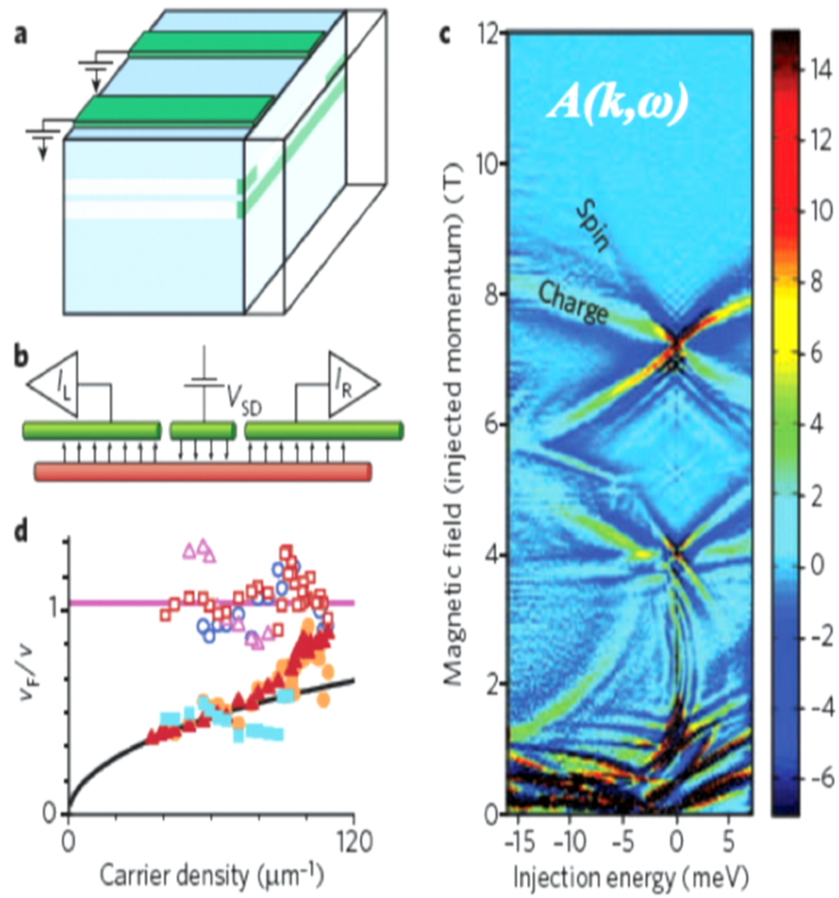
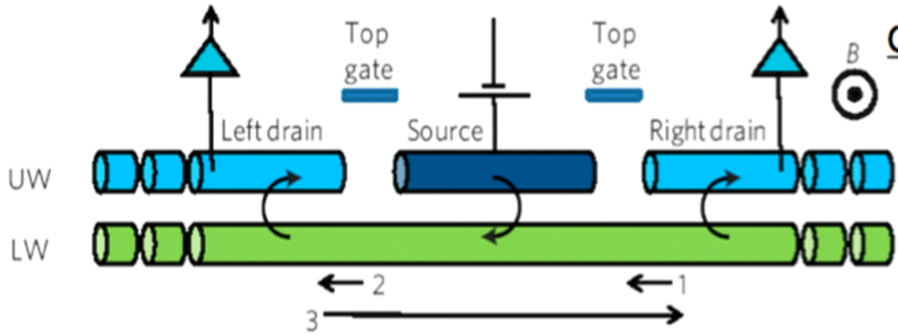






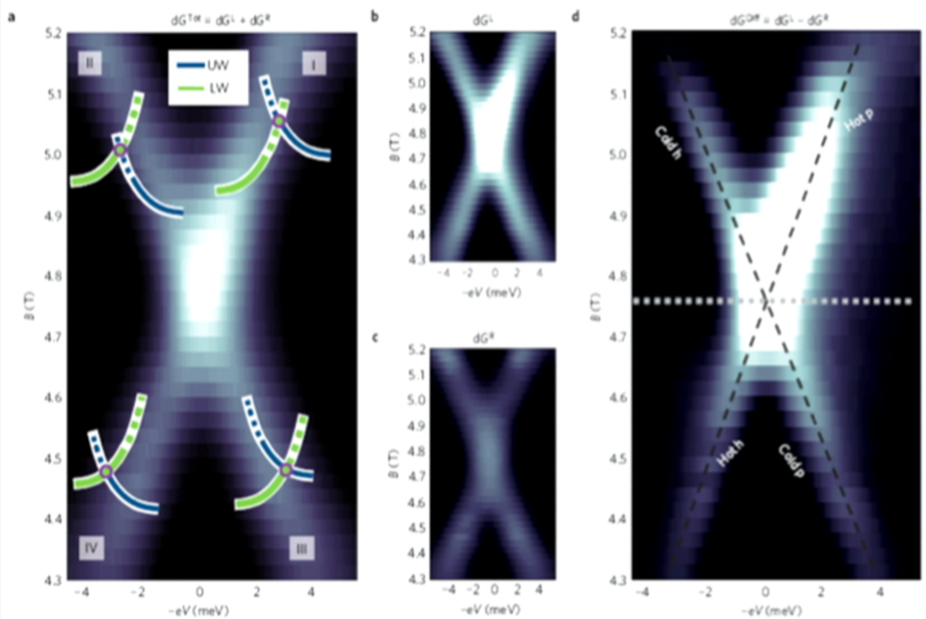
Energy Relaxation of Hot Electrons in QW



G. Barak *et al.*, Nature Phys. (2010)

Relaxation of hot electrons (holes) in a double-wire setup

$$I_T \propto \int d\varepsilon\, T(\varepsilon)[f_1(\varepsilon) - f_2(\varepsilon)]$$



Bounds for the energy relaxation rate

- For electrons $\tau_e < 10^{-11}$s
- For holes $\tau_h >> 10^{-11}$s

Distinct asymmetry in relaxation in contrast to Fermi Liquids where

$$\tau^{-1} \propto \frac{\varepsilon^2 + T^2}{E_F}$$

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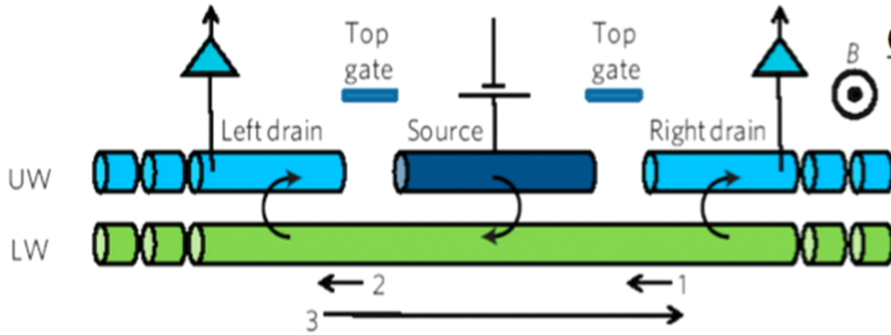

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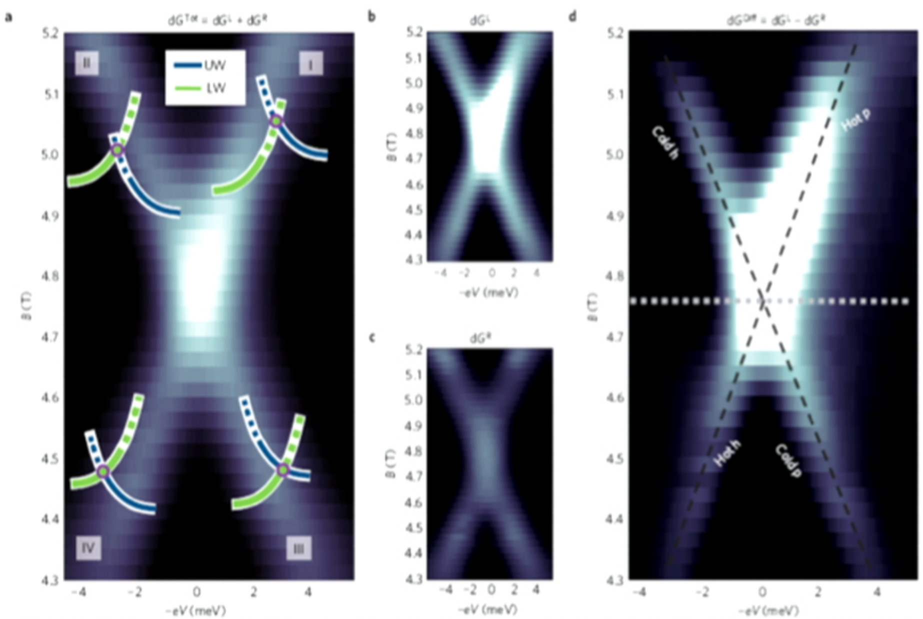

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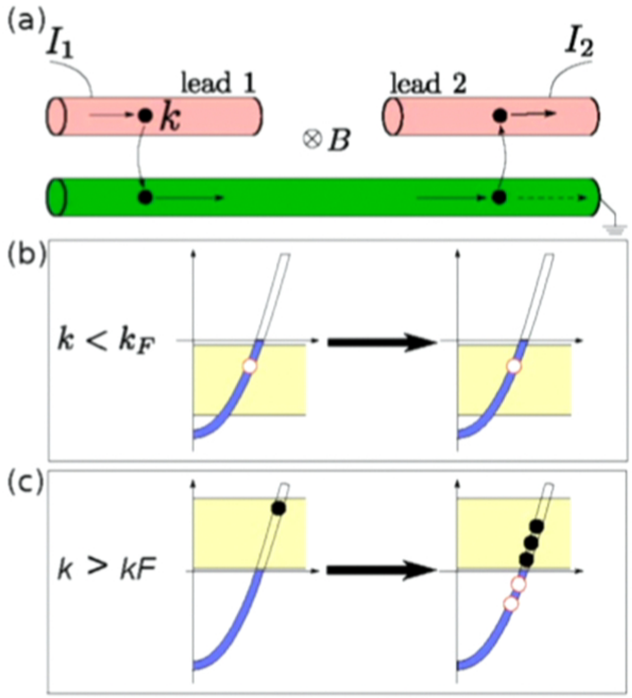

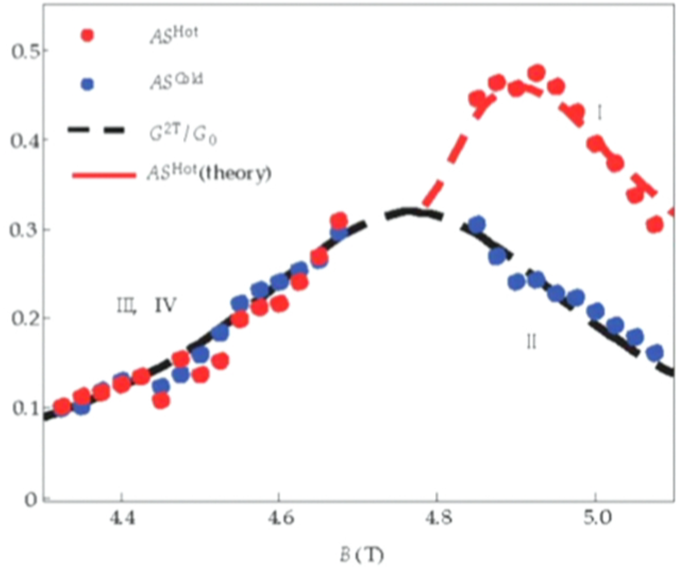

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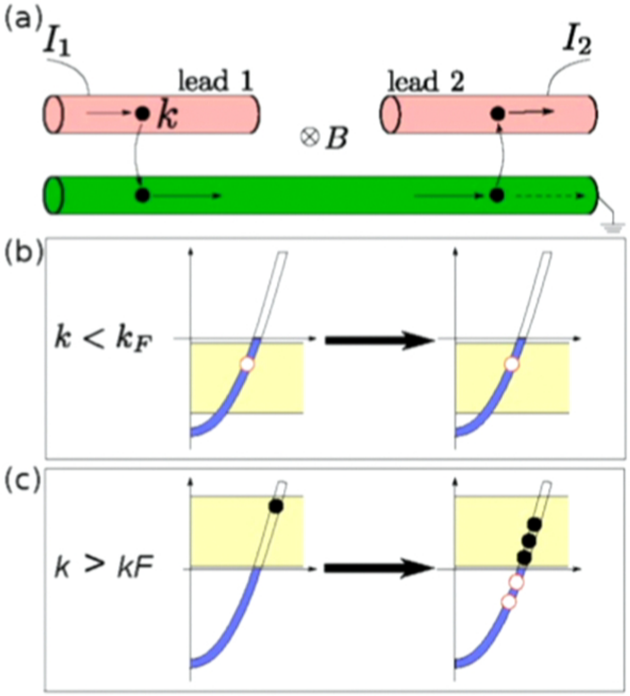
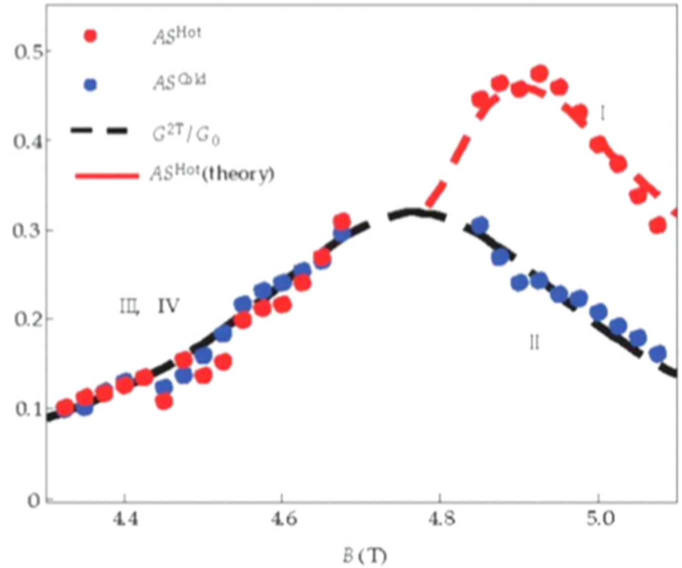
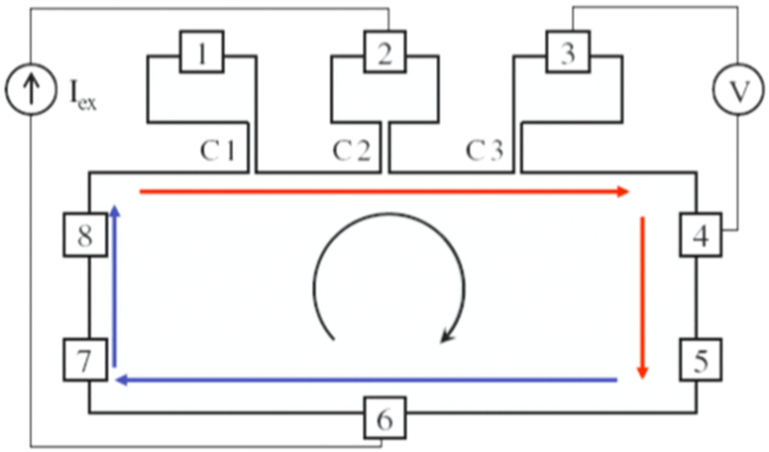




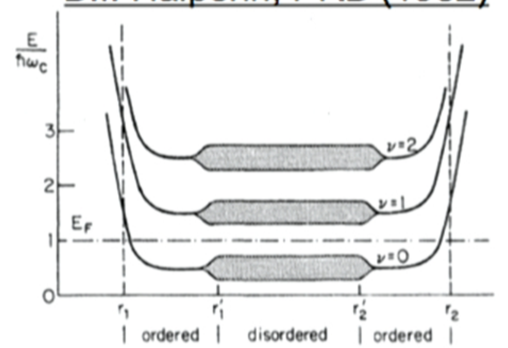
Chiral Heat Transport in the QHR

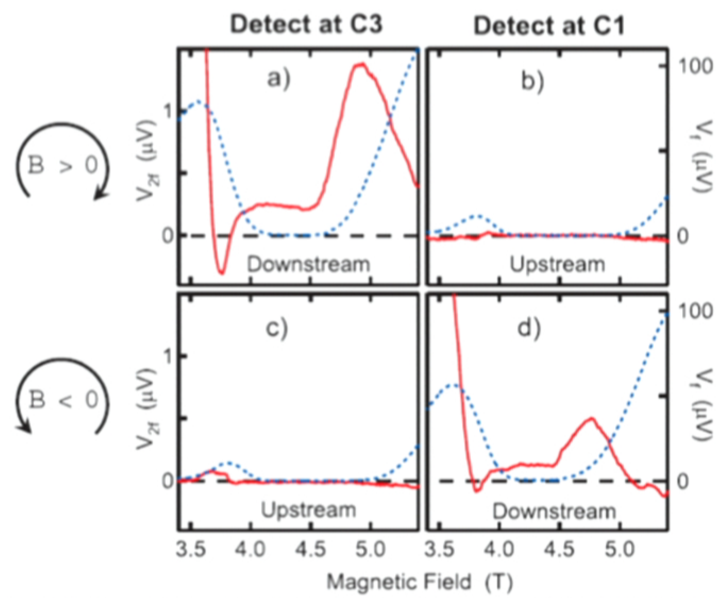
G. Granger *et al.*, PRL (2009)

Transport at the Quantum Hall Regime at ν=1:
thermopower and cooling rate



B.I. Halperin, PRB (1982)





Chiral heat transfer!

Thermal decay length L_Q~60μm

$$\frac{V_{2f}(40\mu m)}{V_{2f}(20\mu m)} \sim \frac{1}{5}$$

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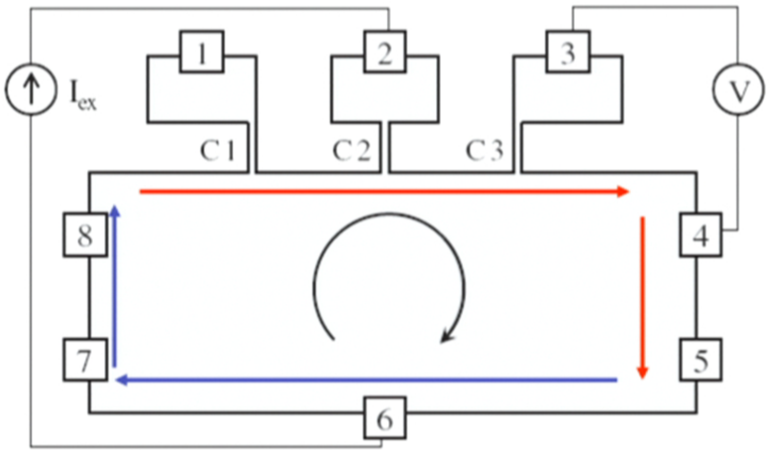

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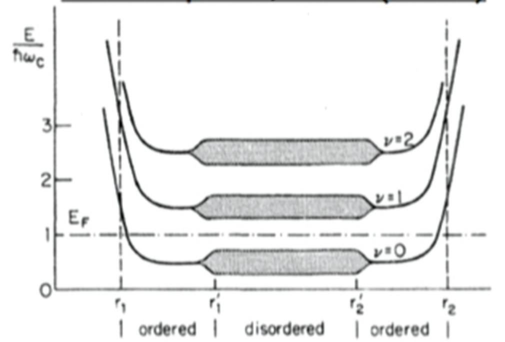
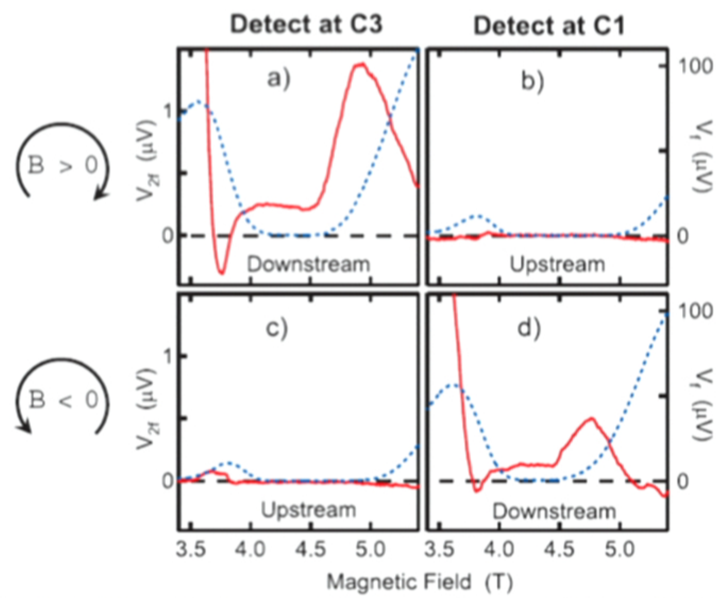




Chiral heat transfer!

Thermal decay length $L_Q \sim 60\mu m$

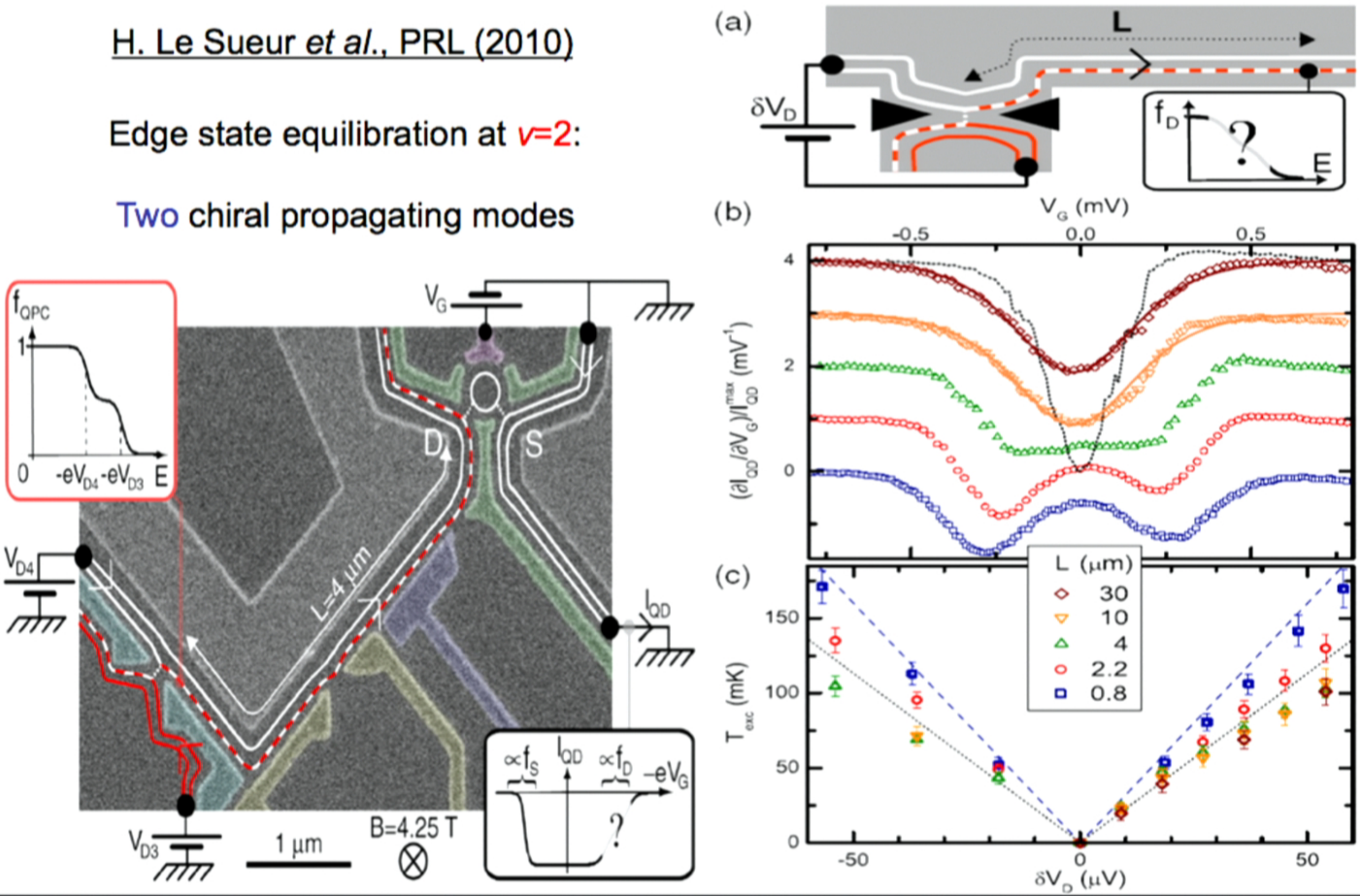
$$\frac{V_{2f}(40\mu m)}{V_{2f}(20\mu m)} \sim \frac{1}{5}$$

Out of equilibrium edge-channel spectroscopy

H. Le Sueur *et al.*, PRL (2010)

Edge state equilibration at ν=2:

Two chiral propagating modes



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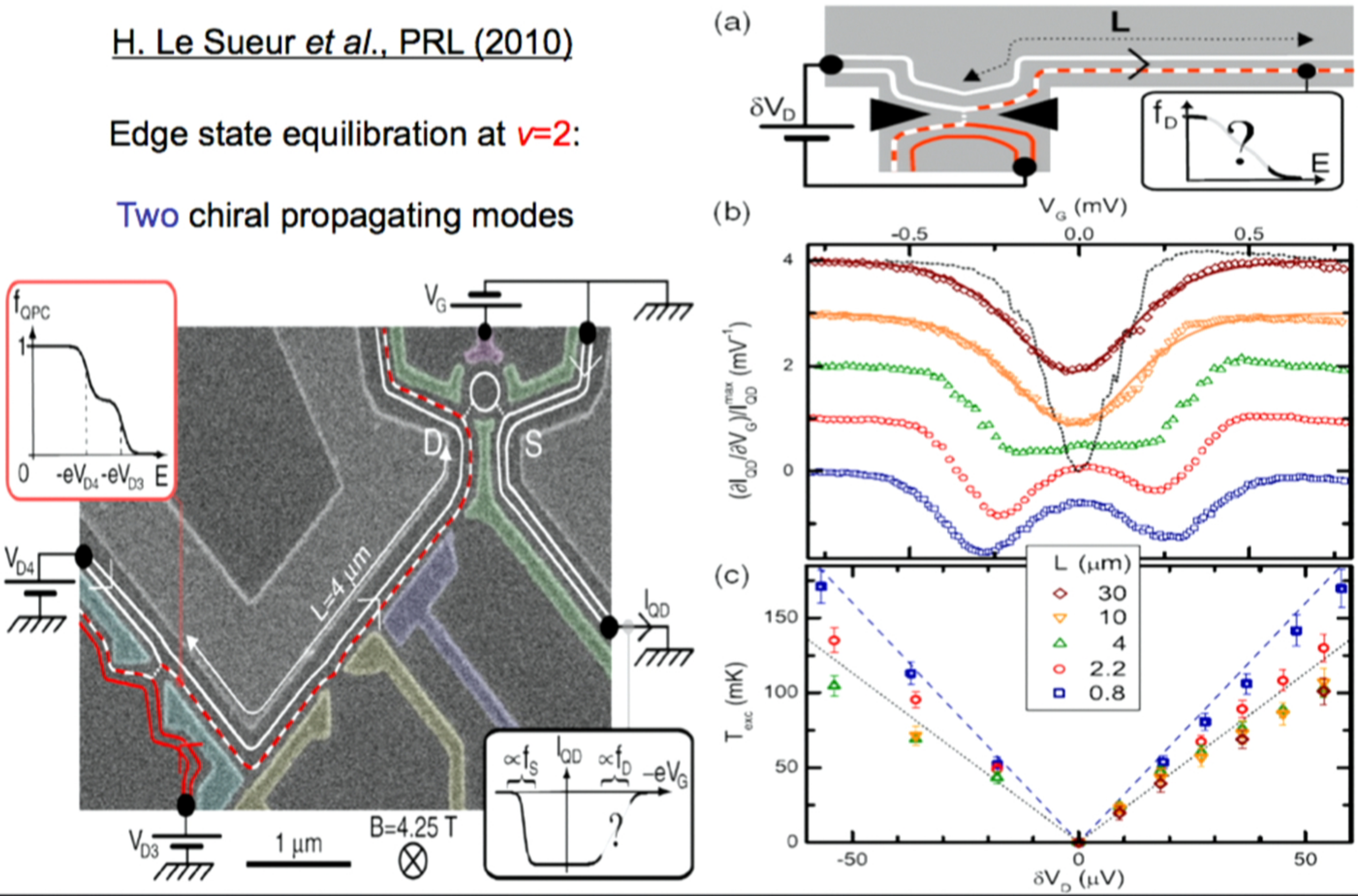

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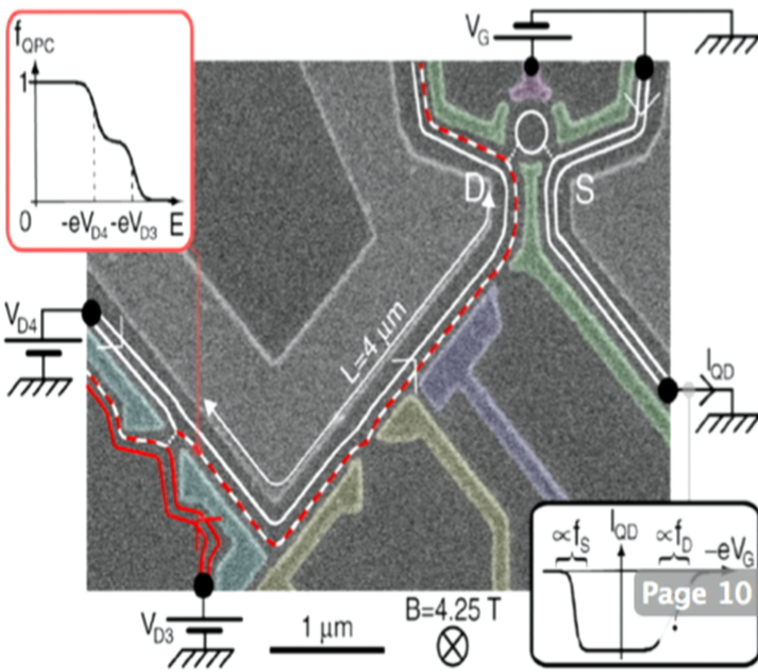

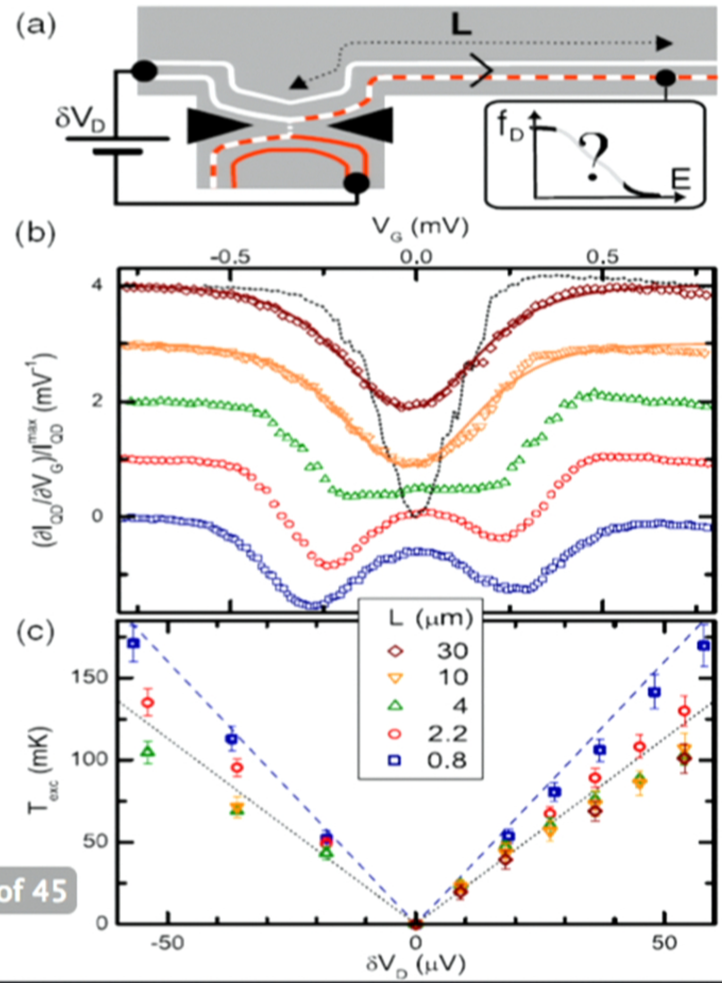

More Experiments

- Quantum Hall interferometers
- Scanning gate microscopy with quantum spin Hall edge states (so far equilibrium probes)
- 1D cold atom systems…

Target Questions

- What is the mechanism of relaxation?
- Which processes set relevant τ and L scales?
- Why there exists distinct asymmetry between relaxation of hot electrons and hot holes?
- Implications for transport coefficients if any?...

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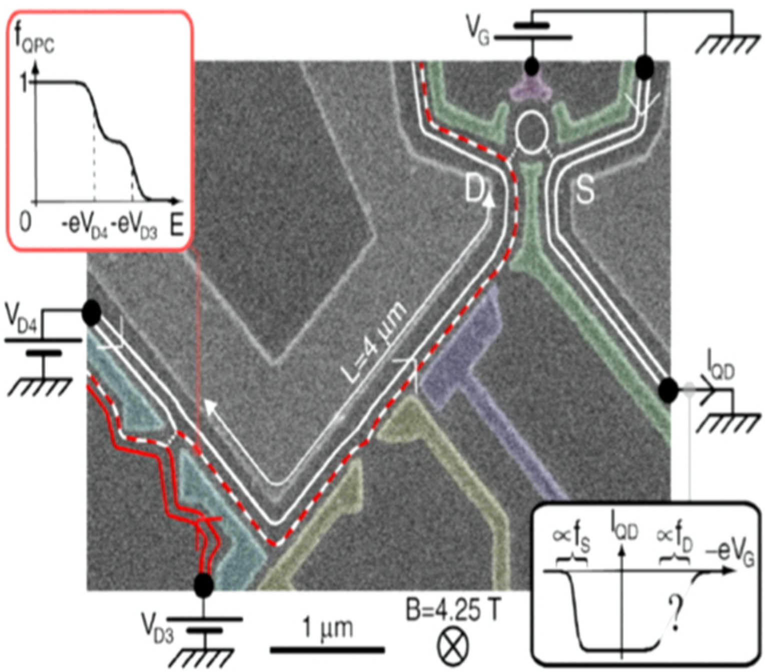

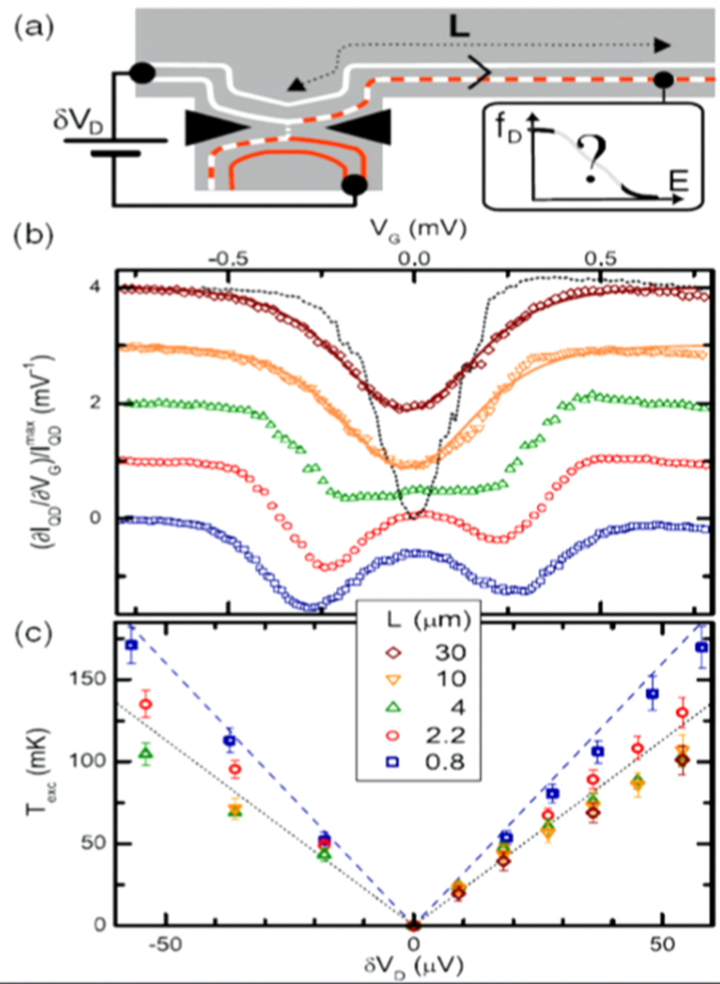

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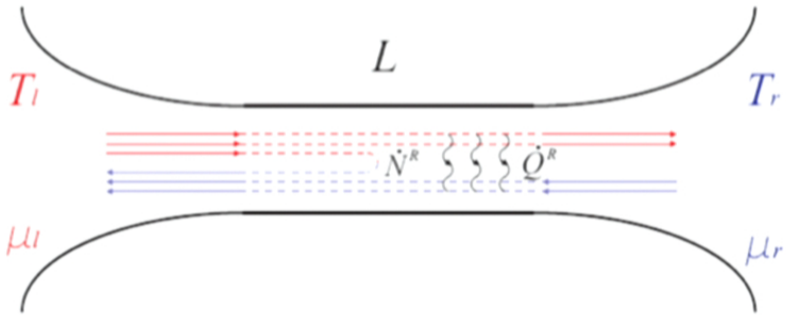
Transport coefficients in 1D quantum wire



Wire is biased by small voltage and/or temperature difference

$$\mu_l - \mu_r = eV \qquad T_l - T_r = \Delta T$$

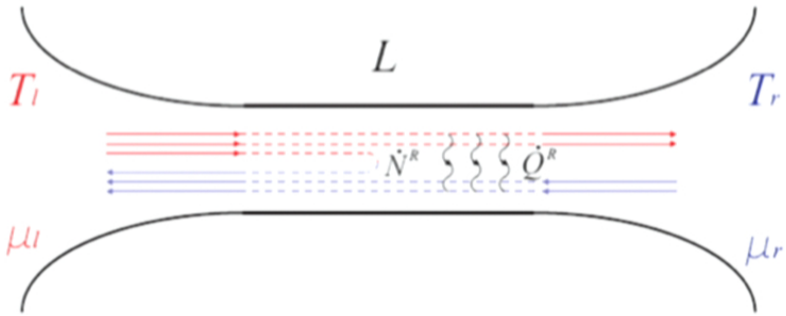
In the linear response

$$I = GV|_{\Delta T=0} \quad I_Q = K\Delta T|_{I=0} \qquad V = -S\Delta T|_{I=0} \quad I_Q = \Pi I|_{\Delta T=0}$$

Conductance — Thermal conductance — Thermopower — Peltier coefficient

In what follows we will concentrate on a limit of a single channel QW

Transport coefficients in 1D quantum wire



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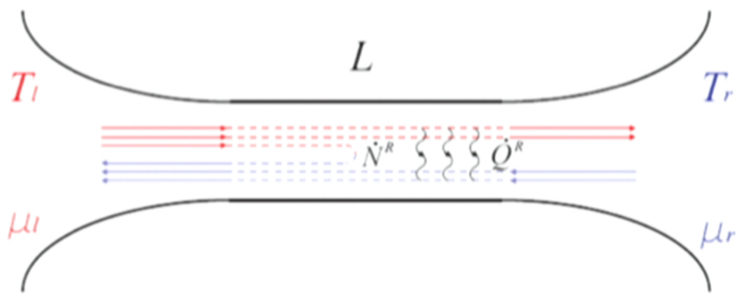
Non-interacting electrons



right-movers

right-movers

$$f_p^{(0)} = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T_l} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T_r} + 1}$$

$$j^{R/L}(x) = \int_{-\infty}^{\infty} \frac{dp}{h} \theta(\pm p) v_p f_{p,x},$$

$$G_0 = \frac{2e^2}{h} \frac{1}{e^{-\mu/T} + 1}$$

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$$\Pi_0 = ST = \frac{1}{e}[\mu e^{-\mu/T} + T(1 + e^{-\mu/T})\ln(1 + e^{-\mu/T})]$$

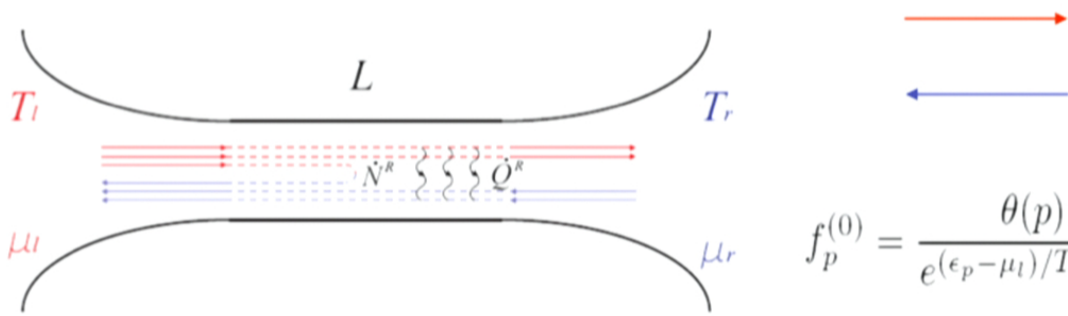
Wiedemann-Franz law

$$K = \frac{\pi^2}{3e^2} TG$$

Mott formula

$$S = \frac{\pi^2 T}{3e} \frac{d \ln G}{d\mu}$$

Non-interacting electrons



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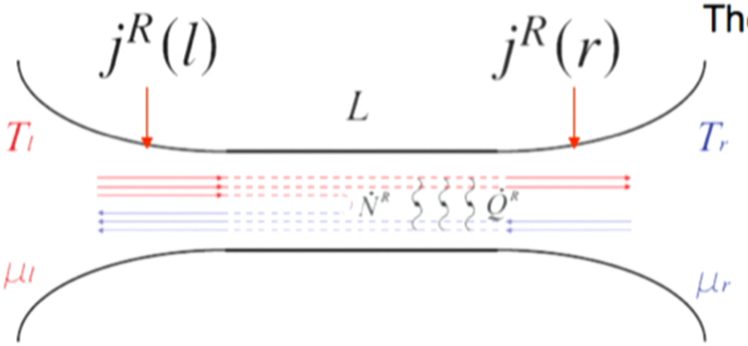
Interacting case - Conservation laws



The rate of change in a number of right-movers

$$\dot{N}^R = j^R(r) - j^R(l)$$

From the total current conservation

$$j = j^R(x) + j^L(x)$$

$$\underbrace{j^R(l) + j^L(r)}_{2eV/h} = \frac{I}{e} - \dot{N}^R$$

$$\boxed{\frac{2e^2}{h}V = I - e\dot{N}^R}$$

$$\underbrace{j_E^R(l) + j_E^L(r)}_{2e\mu V/h + 2\pi^2 T\Delta T/3h} = j_E - \dot{E}^R$$

$$I_Q = j_E - \mu j \quad \dot{Q}^R = \dot{E}^R - \mu\dot{N}^R$$

$$\boxed{\frac{2\pi^2 T\Delta T}{3h} = I_Q - \dot{Q}^R}$$

Brief Summary

Momentum and energy conservations enforce both rates to be zero

for two-particle scattering in 1D regardless of the strength of interaction!

$$\dot{N}^R = 0 \qquad \dot{Q}^R = 0$$

Both conductance (G) and thermopower (S) remain unaffected by interactions

$G=G_0$ and $S=S_0$

D. Maslov and M. Stone PRB (1995)

Thermal conductance (K) remains the same as for non-interacting electrons and there is no relaxation

$K=K_0$ and $\tau^{-1}=0$

R. Fazio, F. Hekking, D. Khmelnitskii PRL (1998)

Models are to be used, not believed
H. Theil (Principles of Econometrics)

Theory – Part - I

Kinetics beyond the LL limit

This is quite a three-pipe problem
Sherlock Holmes, by Sir A.C. Doyle
(The red headed league)

Three-particle collisions

BE ⟹ $\mathcal{I}\{f_{p_1}\} = -\sum_{\substack{p_2 p_3 \\ p_{1'} p_{2'} p_{3'}}} W [\underbrace{f_{p_1} f_{p_2} f_{p_3} (1-f_{p_{1'}})(1-f_{p_{2'}})(1-f_{p_{3'}})}_{\text{"out"-rate}}$

"in"-rate $\underbrace{- f_{p_{1'}} f_{p_{2'}} f_{p_{3'}} (1-f_{p_1})(1-f_{p_2})(1-f_{p_3})}]$.

Generalized Fermi-golden rule:

$$T \equiv V + VG_0T.$$

$$G_0 = \frac{1}{E_i - H_0 + i\eta}, \quad (\eta \to 0^+),$$

$$V = \frac{1}{2L}\sum_{k_1 k_2 q}\sum_{\sigma_1\sigma_2} V_q c^\dagger_{k_1+q\sigma_1} c^\dagger_{k_2-q\sigma_2} c_{k_2\sigma_2} c_{k_1\sigma_1}.$$

H_0 – single-particle Hamiltonian
V – generic two-body interaction

⟹ Iterate to the second order in V

Second term generates 3 particle process:

$$T=(1-VG_0)^{-1}V \rightarrow \delta T=VG_0V$$

⇓

Irreducible scattering rate:

$$W_{123;1'2'3'} = \frac{2\pi}{\hbar}\left|\langle 1'2'3'|VG_0V|123\rangle_c\right|^2 \delta(E_i - E_f)$$

Initial state: $|123\rangle = c^\dagger_{k_1\sigma_1} c^\dagger_{k_2\sigma_2} c^\dagger_{k_3\sigma_3}|0\rangle,$

Final state: $|1'2'3'\rangle = c^\dagger_{k_{1'}\sigma_{1'}} c^\dagger_{k_{2'}\sigma_{2'}} c^\dagger_{k_{3'}\sigma_{3'}}|0\rangle$

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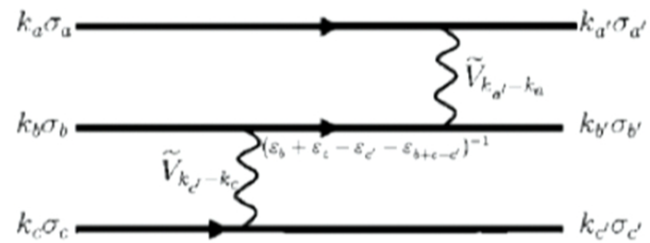
Scattering Amplitude

$\widetilde{V}_q = V_q + V_{-q}$

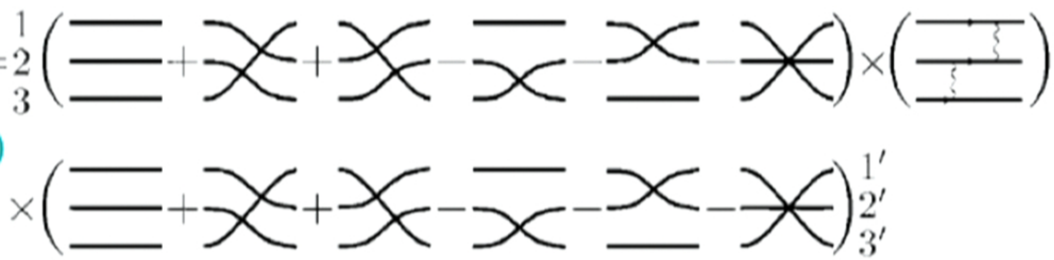
(a)

$$\frac{1}{4L^2}\frac{\widetilde{V}_{a'-a}\widetilde{V}_{c'-c}\;\delta_{a+b+c,a'+b'+c'}}{\varepsilon_b+\varepsilon_c-\varepsilon_{c'}-\varepsilon_{b+c-c'}+i\eta}\delta_{\sigma_{a'},\sigma_a}\delta_{\sigma_{b'},\sigma_b}\delta_{\sigma_{c'},\sigma_c} \equiv$$



(b)

$\langle 1'2'3'|VG_0V|123\rangle_c =$



Sirenko *et. al*. PRB (1994)
Lunde *et. al*. PRB (2007)
AL *et. al*. PRB (2011)

Amplitude consists of 36 terms and the scattering rate thus has 36^2=1296 terms!

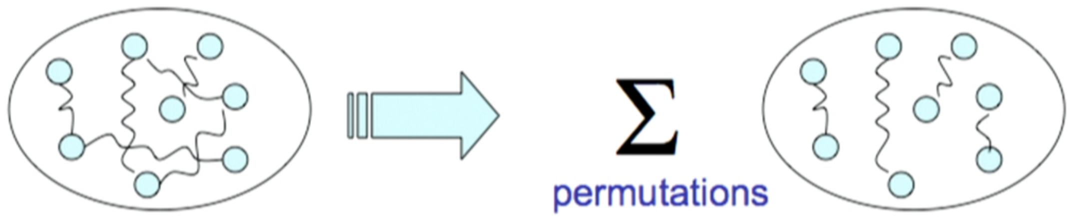
$$A(123\rightarrow 1'2'3') = \underbrace{A(11',22',33')}_{\text{direct}} + \underbrace{A(12',23',31')+A(13',21',32')-A(11',23',32')-A(13',22',31')-A(12',21',33')}_{\text{exchange contributions}}$$

$$\dot{N}^R = \sum_{p>0}\dot{f}_p = \sum_{p>0}\mathcal{I}\{f_p\} \qquad \dot{E}^R = \sum_{p>0}\varepsilon_p\dot{f}_p = \sum_{p>0}\varepsilon_p\mathcal{I}\{f_p\}$$

Exactly integrable quantum many-body models

B. Sutherland, *Beautiful Models* (Secs. 1.3– 1.5.)

For some two-body interaction potentials V_p, scattering of the particles of the N-body system factorizes into a sequence of two-body collisions.



It means that *irreducible* three-particle scattering amplitude for the *integrable* potentials must be exactly zero → ***A=0***!

- Contact interaction $V_p = const$
- Calogero-Sutherland model $V_p = |p|$
- Fermionic Lieb-Liniger model $V_p = p^2$
- Yang-Gaudin………

Generic Coulomb interaction

$$V(x) = \frac{e^2}{\kappa}\left[\frac{1}{\sqrt{x^2+4d^2}} - \frac{1}{\sqrt{x^2+4w^2}}\right]$$

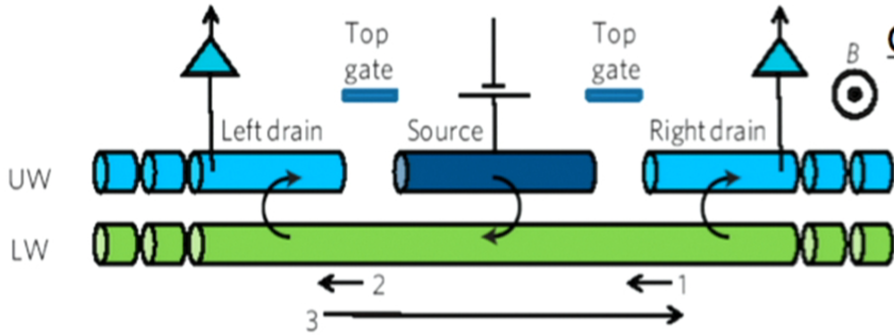
screened

$$V_q \propto 1-(qd)^2 \ln\left(\frac{1}{|q|d}\right)$$

unscreened

$$V_q \propto \ln\left(\frac{1}{|q|w}\right)\left[1+(qw)^2\right]$$

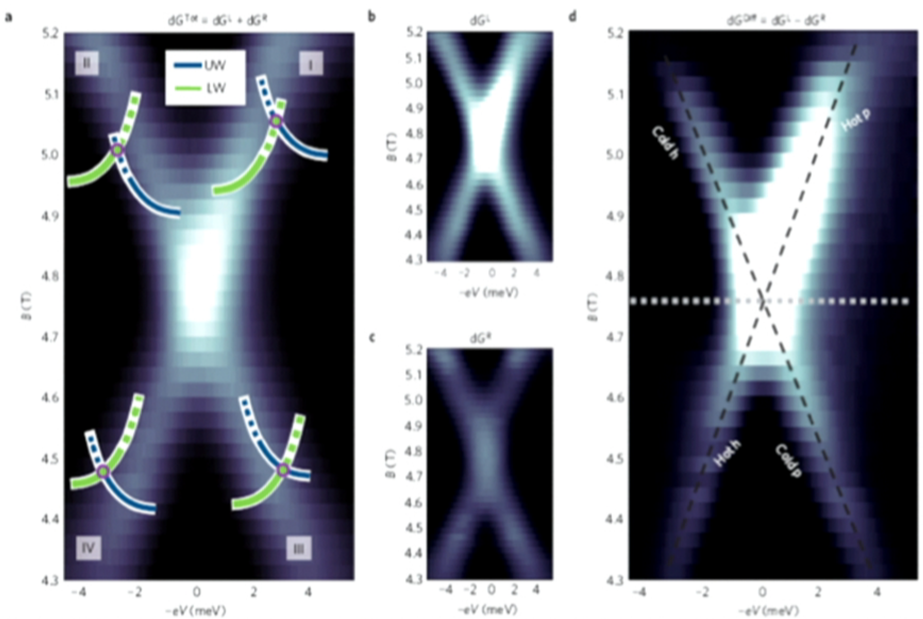
Energy Relaxation of Hot Electrons in QW



G. Barak *et al.*, Nature Phys. (2010)

Relaxation of hot electrons (holes) in a double-wire setup

$$I_T \propto \int d\varepsilon\, T(\varepsilon)[f_1(\varepsilon) - f_2(\varepsilon)]$$



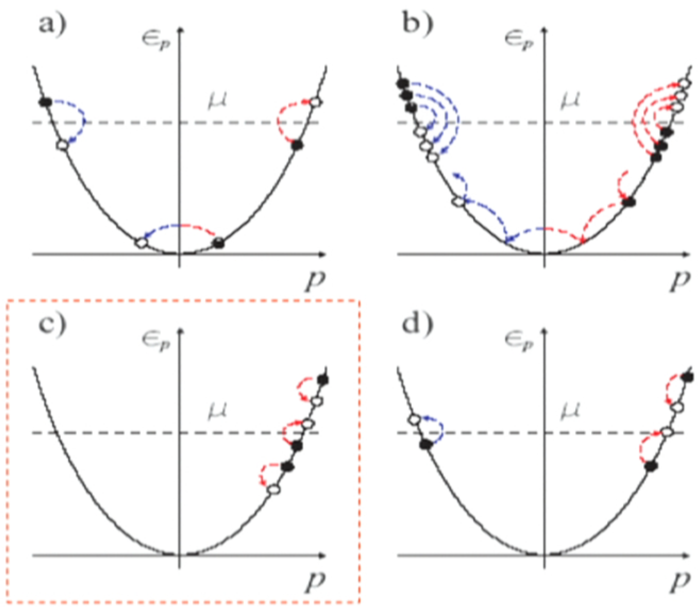
Bounds for the energy relaxation rate

- For electrons $\tau_e < 10^{-11}$s
- For holes $\tau_h >> 10^{-11}$s

Distinct asymmetry in relaxation in contrast to Fermi Liquids where

$$\tau^{-1} \propto \frac{\varepsilon^2 + T^2}{E_F}$$

Partially equilibrated wires



For non-interacting electrons

$$f_p = \frac{\theta(p)}{e^{(\epsilon_p - \mu_l)/T_l} + 1} + \frac{\theta(-p)}{e^{(\epsilon_p - \mu_r)/T_r} + 1}$$

Interacting case: Partial equilibration

$$\underbrace{f_p = \frac{\theta(p)}{e^{(\varepsilon_p - p u^R - \mu^R)/T^R} + 1} + \frac{\theta(-p)}{e^{(\varepsilon_p - p u^L - \mu^L)/T^L} + 1}}$$

in general contains 6-parameters

Intra-branch equilibration conserves independently:

Number of particles $\mu(x) \rightarrow N^{L(R)}$, energy $T(x) \rightarrow E^{L(R)}$ and momentum $u(x) \rightarrow P^{L(R)}$

Thermal transport - generalities

Neglect backscattering at the moment: as we already saw $N^R \propto \exp(-\varepsilon_F/T) \ll 1$

$$\dot{N}^R = 0$$

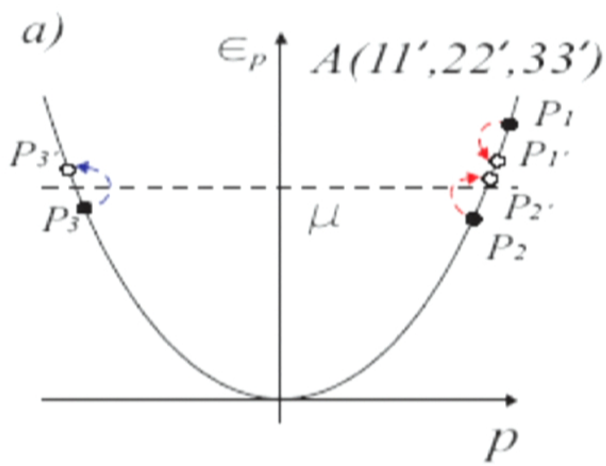
Currents of left- and right-movers are conserved independently then

$$\partial_x j^R(x) = 0 \quad \partial_x j^L(x) = 0$$

Total energy and momentum currents are also conserved

$$\partial_x j_P(x) = 0 \quad \partial_x j_E(x) = 0$$

Momentum and energy currents of right-movers alone are *not* conserved



$$j_P^R(x+\Delta x) - j_P^R(x) = \dot{P}^R \quad j_E^R(x+\Delta x) - j_E^R(x) = \dot{E}^R$$

All currents and relaxation rates we express in terms of $u^{R(L)}(x)$, $T^{R(L)}(x)$ and $\mu^{R(L)}(x)$

…In essence – hydrodynamic approach…

Thermal conductance (K)

$$\frac{j_Q}{\Delta T} = K_0 \mathcal{F}(L/\ell_Q) \qquad \mathcal{F}(z) = \frac{\frac{1}{\beta}\sinh(z/2) + \cosh(z/2)}{\alpha \sinh(z/2) + \cosh(z/2)}$$

Sommerfeld functions

$$\alpha = 1 + \frac{29\pi^2 T^2}{120\mu^2} \qquad \beta = 1 - \frac{7\pi^2 T^2}{40\mu^2}$$

Short wire limit $L \ll L_Q$

$$\mathcal{F}(L \ll \ell_Q) = 1 - \frac{\pi^2}{30}\frac{T^2}{\mu^2}\frac{L}{\ell_Q}$$

Scattering length (spinless case)

$$\ell_Q^{-1} = \frac{864 \ln^2 4}{7\pi} k_F \lambda^2_{k_F w} \left(\frac{e^2}{\hbar v_F \kappa}\right)^4 \frac{T^3}{\mu^3}$$

Long wire limit $L_Q \ll L$

$$\mathcal{F}(L \gg \ell_Q) = 1 - \frac{\pi^2}{30}\frac{T^2}{\mu^2}(1 - e^{-L/\ell_Q})$$

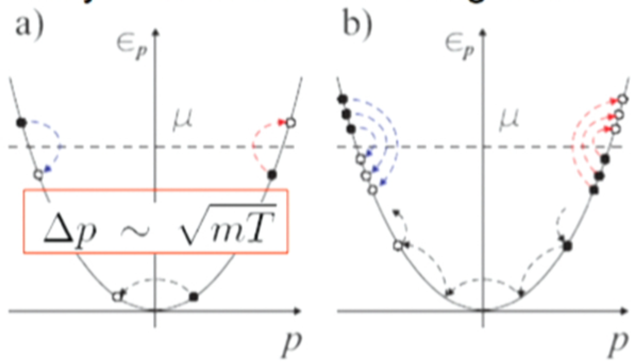

$\ell_Q^{-1} =$


$\ell_Q^{-1} =$
CAUTION

Electrical transport - generalities

Return to electron backscattering

Very short wires | Long wires



$$\Delta p \sim \sqrt{mT}$$

Over time *t* momentum change is

$$(\Delta p)^2 \sim Bt$$

Since $\pm T/v_F$ once during τ_{eee}

$$(\Delta p)^2 \sim (T/v_F)^2 t/\tau_{eee} \quad \text{for } t \gg \tau_{eee}$$

$$B \sim \frac{T^2}{v_F^2 \tau_{eee}}$$

Electrons experience multiple collisions

Typical momentum change $\delta p \sim T/v_F \ll p_F$

It is convenient to think of this collision as a process in which a deep hole, corresponding to the outgoing electron state, is backscattered by electron excitations close to the Fermi level.

Scattering rate $1/\tau_{eee} \longrightarrow \delta p \sim \pm T/v_F$!

$$\Delta \dot{N}^R \sim \frac{1}{t}\underbrace{\left(\frac{\Delta p \Delta x}{h}\right)}_{\text{phase space}} \overbrace{\left(e^{-\mu^R/T} - e^{-\mu^L/T}\right)}^{\text{probability}}$$

$$\approx -\frac{\Delta\mu \Delta x B}{h\sqrt{mT^3}} e^{-\mu/T}$$

$$\frac{G}{G_0} = 1 - \frac{\pi^2 T^2}{12 E_F^2}$$

Applications

G, K, S, τ

Spin-1/2 Mobile Impurity in LL

$H=H_0+H_d+H_{\text{int-1}}+H_{\text{int-2}}+H_{\text{int-3}}$

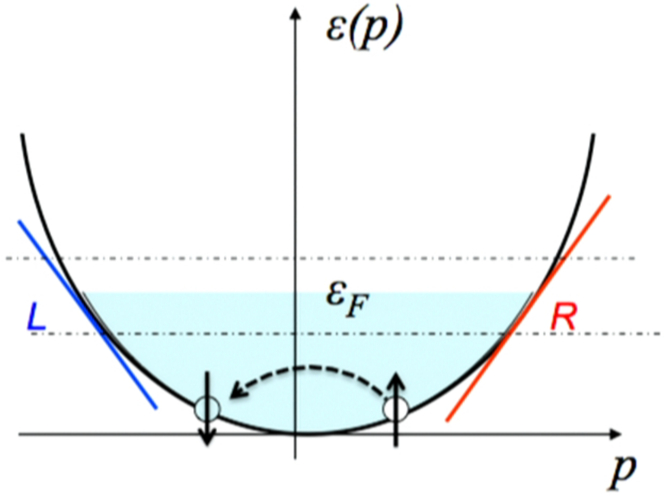
$H_0=(v/2\pi)\int dx\,[K(\partial\theta)^2+K^{-1}(\partial\varphi)^2]$

$H_d=\int dx\, d^\dagger(x)[\varepsilon(p)-iv_d\partial]d(x)$

$H_{\text{int-1}}=\int dx\,[V_R\rho_R+V_L\rho_L]d^\dagger(x)d(x)$

$H_{\text{int-2}}=\int dx\, V^\rho_{RL}\,\rho_R\rho_L d^\dagger(x)d(x)$

$H_{\text{int-3}}=\int dx\, V^\sigma_{RL}\,(\mathbf{S}\bullet\boldsymbol{\sigma}_{ss'})d_s^\dagger(x)d_{s'}(x)$



1. Luttinger limit plus band curvature
2. Edge singularities
3. Finite life-time (relaxation due to three-particle processes)

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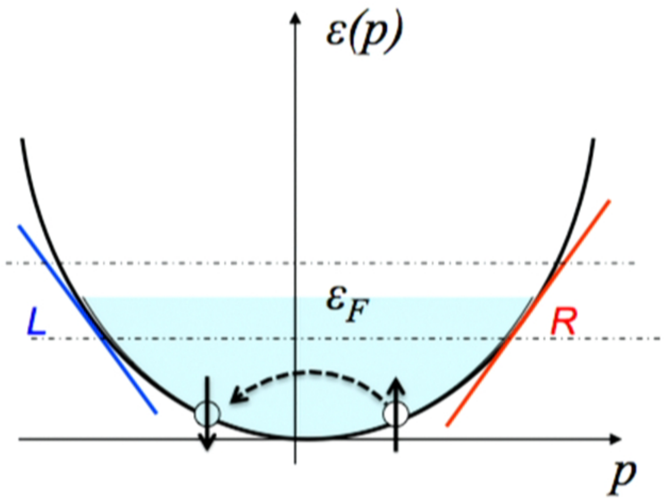

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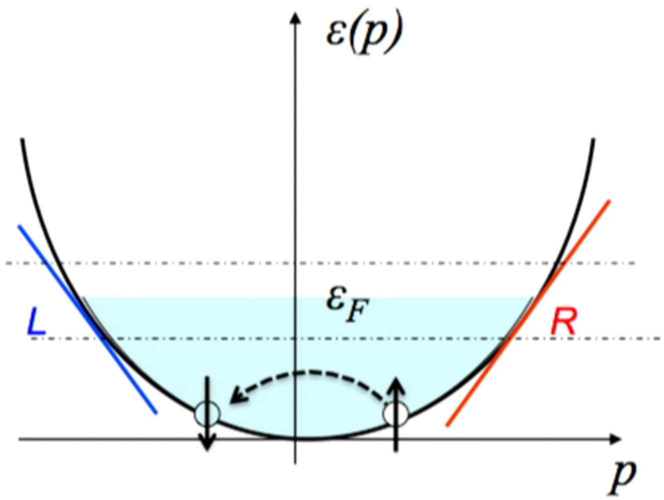

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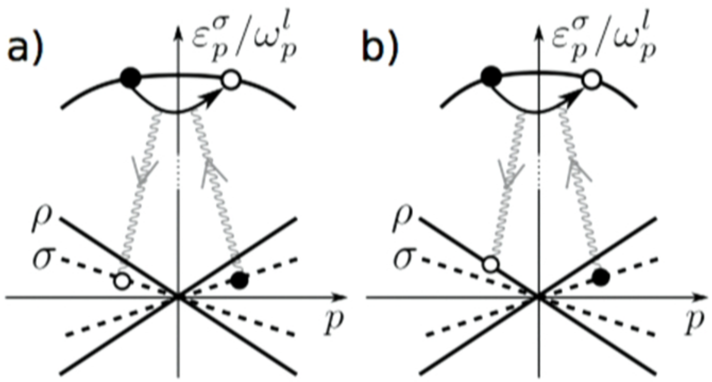
Backscattering of Spinons



Diffusion in momentum space

$$B = \sum_q q^2 W(q)$$

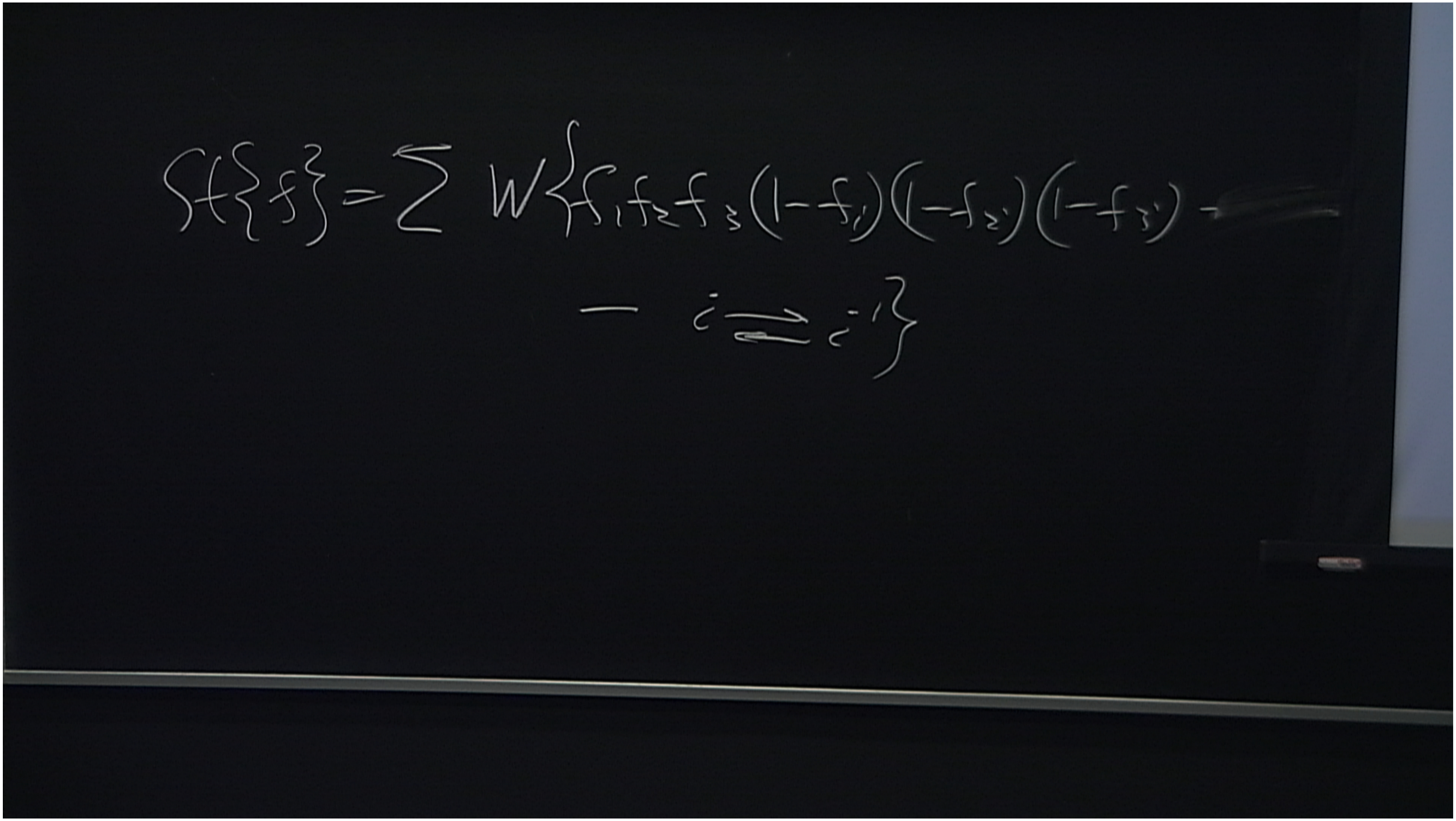
$$\mu = T/B \qquad B \propto \left(\frac{T}{E_F}\right)^3$$

From three particle collisions

$$W(q) = \sum_{p_2 p_3}^{\sigma_2\sigma_3} \sum_{p_2' p_3'}^{\sigma_2'\sigma_3'} \Lambda_{pp'}^{\sigma\sigma'} f(p_2')[1-f(p_2)] f(p_3')[1-f(p_3)]$$

From the effective model

$$W(q) = \sum_{ll'} \sum_{kk'}^{\sigma\sigma'} \Gamma_{ll'}^{\sigma\sigma'}(q;kk') n_l(k)[1+n_{l'}(k')]$$


$S\{f\} = \sum W \{ f_1 f_2 f_3 (1-f_{1'})(1-f_{2'})(1-f_{3'})$
$- \; i \rightleftharpoons i' \}$

Collision operator and zero modes

Linearized collision integral:

$$F_p = f_p + f_p(1-f_p)\psi_p$$

$$\dot{\psi}_1 = -L\{\psi_1\}$$

$$L\{\psi_1\} = \frac{1}{f_1(1-f_1)} \sum_{p_2, p_3} \sum_{p_{1'}, p_{2'}, p_{3'}} W_{123;1'2'3'} (\psi_1 + \psi_2 + \psi_3 - \psi_{1'} - \psi_{2'} - \psi_{3'}) ,$$

Let us view L as an operator and define eignevalue problem

$$L\{\psi_n\} = \omega_n \psi_n \Longrightarrow \psi_n \propto \exp(-\omega_n t)$$

Zero modes:

- $\psi_E(p) = \epsilon_p$ – energy conservation;
- $\psi_P(p) = p$ – momentum conservation;
- $\psi_N(p) = \text{const}$ – conservation of total number of particles;
- $\psi_{\Delta N}(p) = \text{sgn}(p)$ – conserved difference in number of left- and right-movers.

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Eigenvalues and energy relaxation rate

Introducing Even/Odd eigenmodes $\psi_p^{\pm}\sqrt{f_p(1-f_p)}\,[\psi_{p_F+p}\pm\psi_{p_F-p}]$

Eigenvalue problem reduces to an integral equation

$$\underbrace{\omega_n\psi^{ns}_{p_1} = \gamma_0\left[\frac{1}{2}[p_1^2+\varkappa^2]\psi^{ns}_{p_1} - \int dp_2\,\frac{3^{\delta_s}(p_2-p_1)\psi^{ns}_{p_2}}{2\sinh\left(\frac{v_F(p_2-p_1)}{2T}\right)}\right]}$$

Kernel depends only on a difference of arguments

$$\int \frac{k e^{ikx}\,dk}{\sinh(\pi k/2)} = \frac{2}{\cosh^2 x}$$

Quantum mechanics in the Poschl-Teller potential

$$\left[\frac{d^2}{dx^2} + \Omega_n + \frac{2\times 3^{\delta_s}}{\cosh^2(x)}\right]\psi^{ns}_x = 0$$

$$\tau_E^{-1} = \frac{\pi^3 r_s^4}{64\hbar}\,\frac{T^3}{\mu^2}$$

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$\dot{\psi}_1 = -L\{\psi_1\}$

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Thank you!


THAT CONCLUDES MY TWO-HOUR PRESENTATION. ANY QUESTIONS?
DID YOU INTEND THE PRESENTATION TO BE INCOMPREHENSIBLE, OR DO YOU HAVE SOME SORT OF RARE "POWER-POINT" DISABILITY?
ARE THERE ANY QUESTIONS ABOUT THE CONTENT?
THERE WAS CONTENT?
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8/9/03