

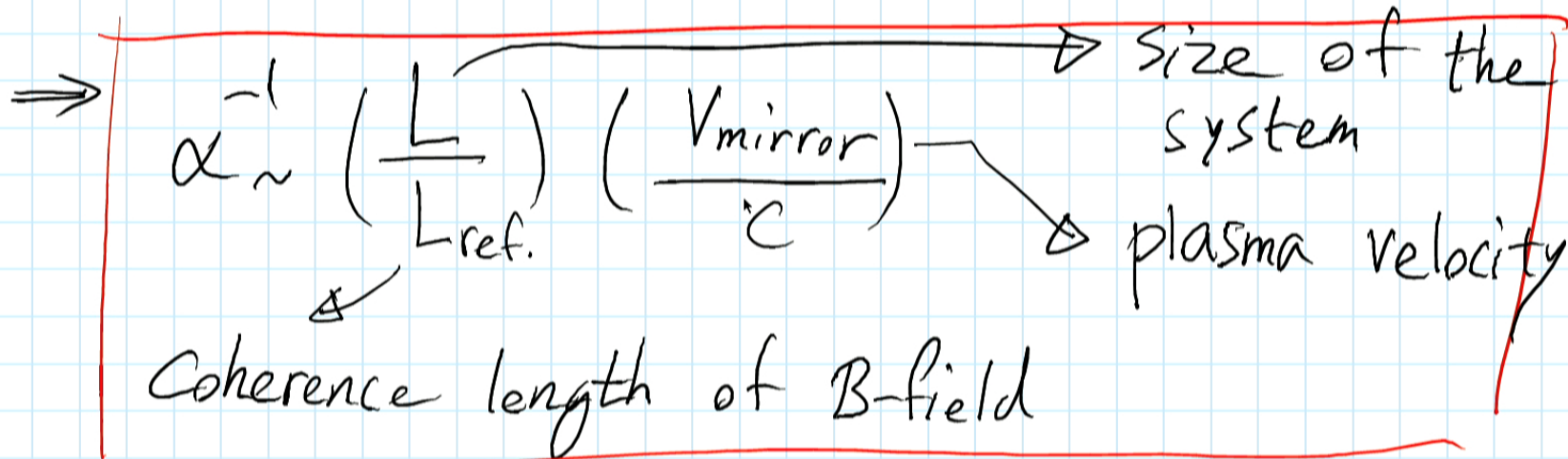
Title: PHYS 781 - Lecture 25

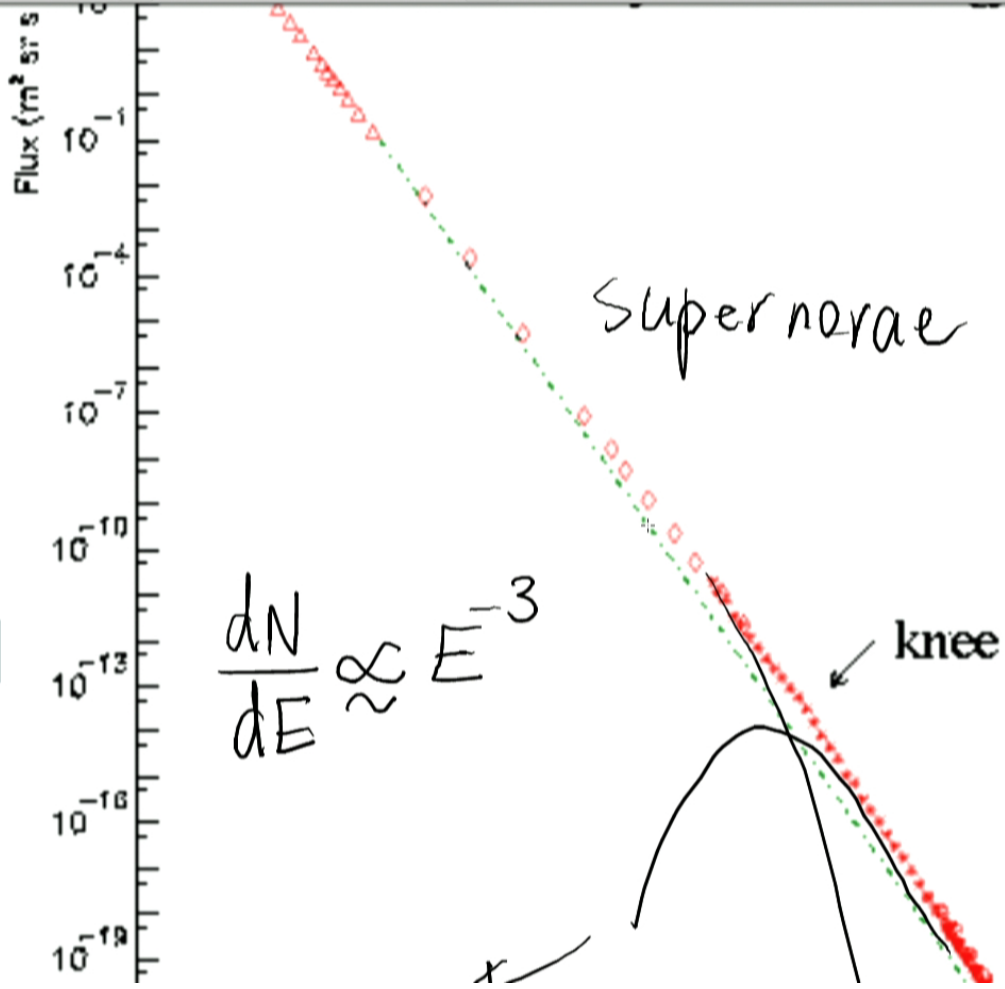
Date: Nov 28, 2014 12:00 PM

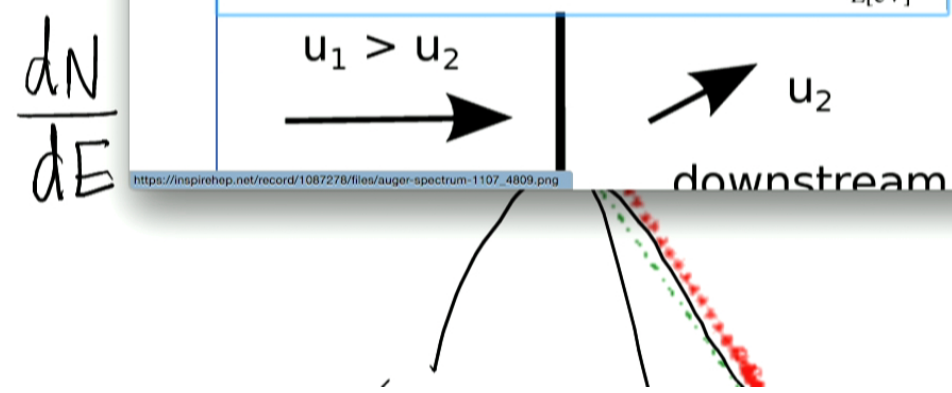
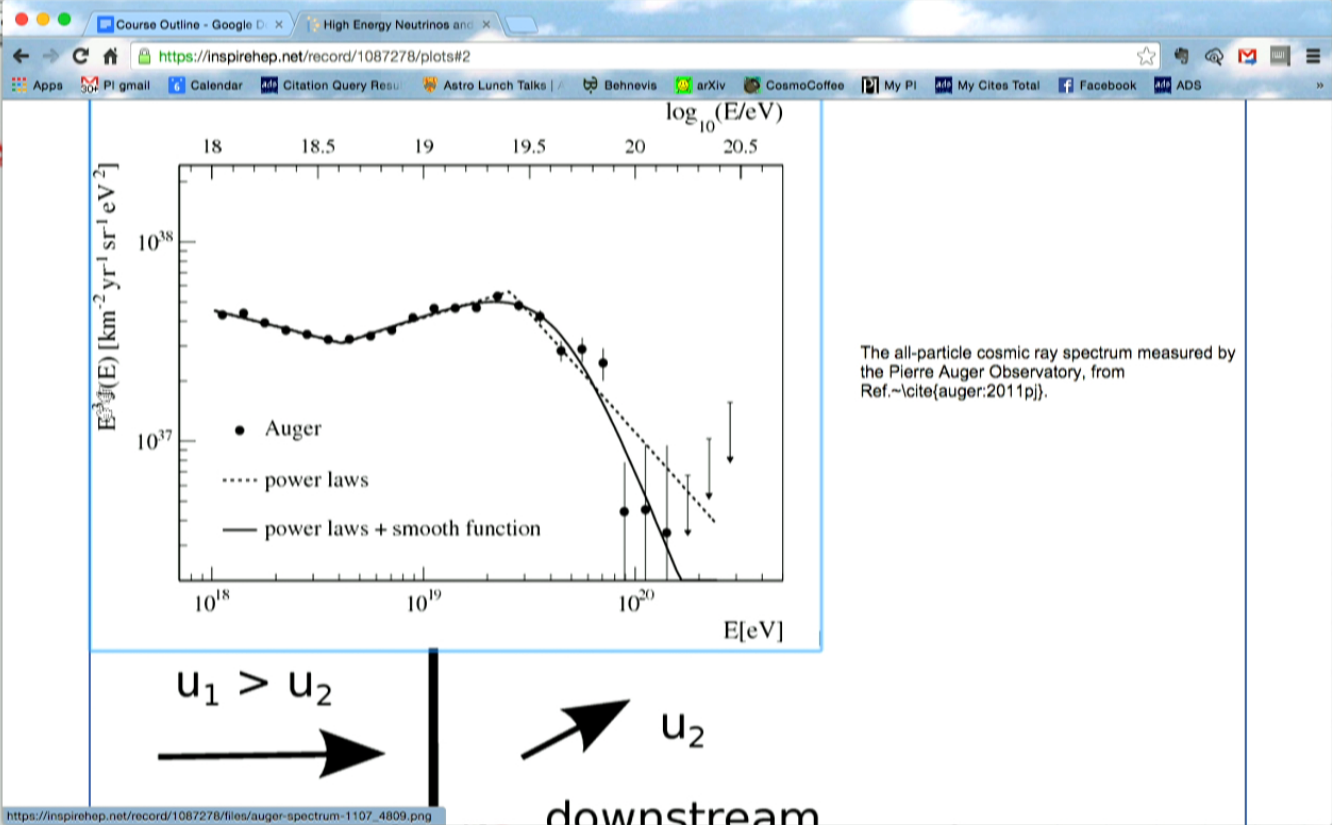
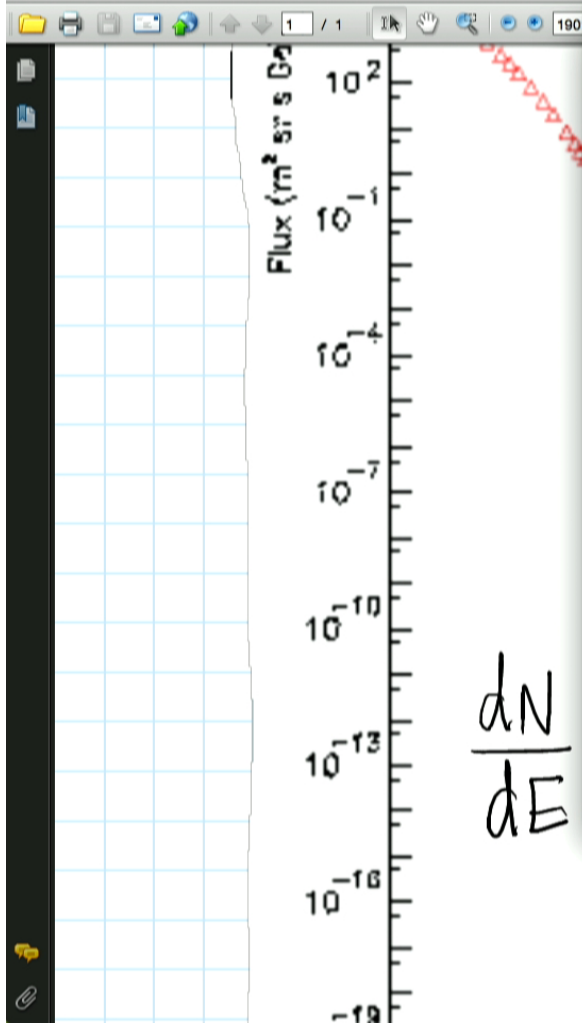
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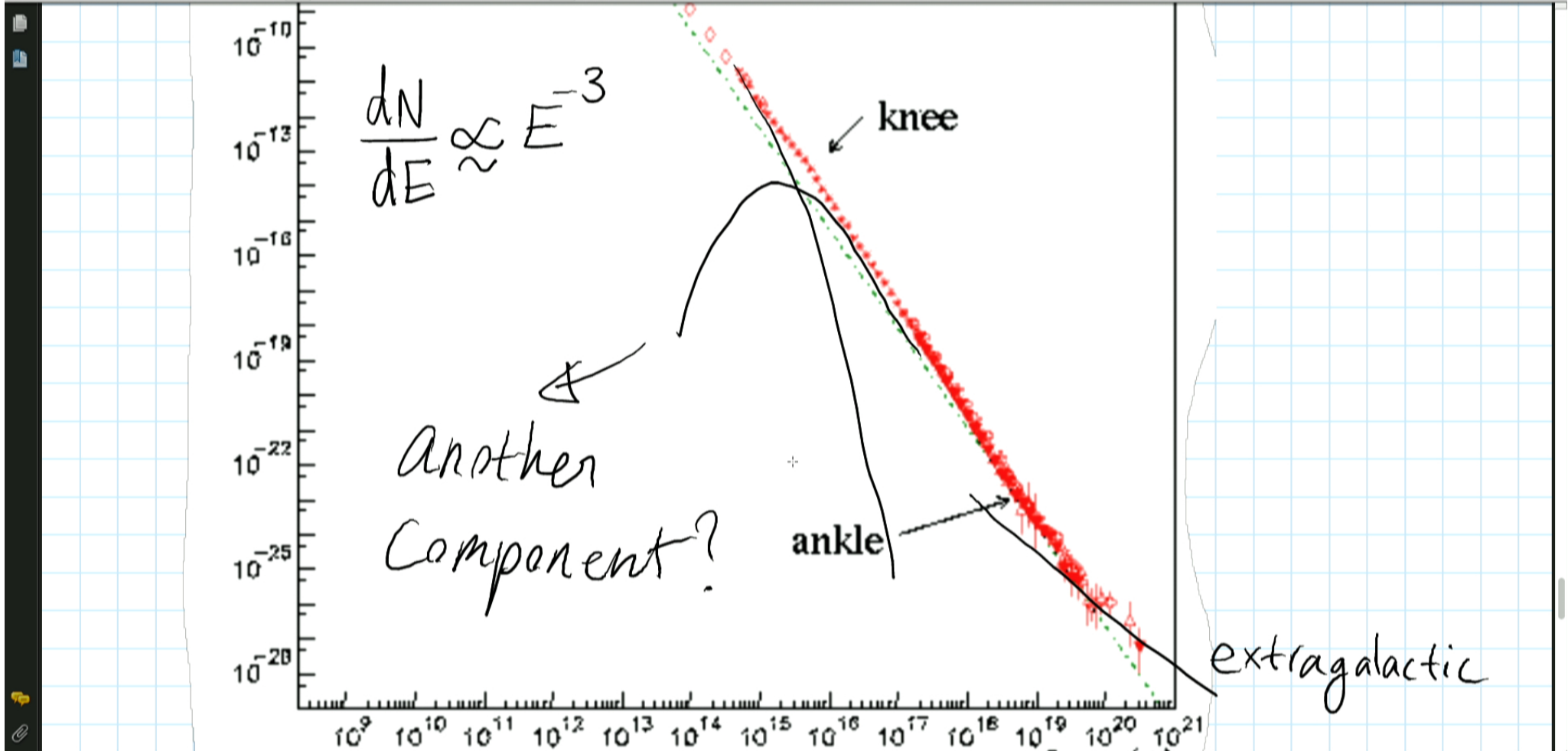
Abstract:

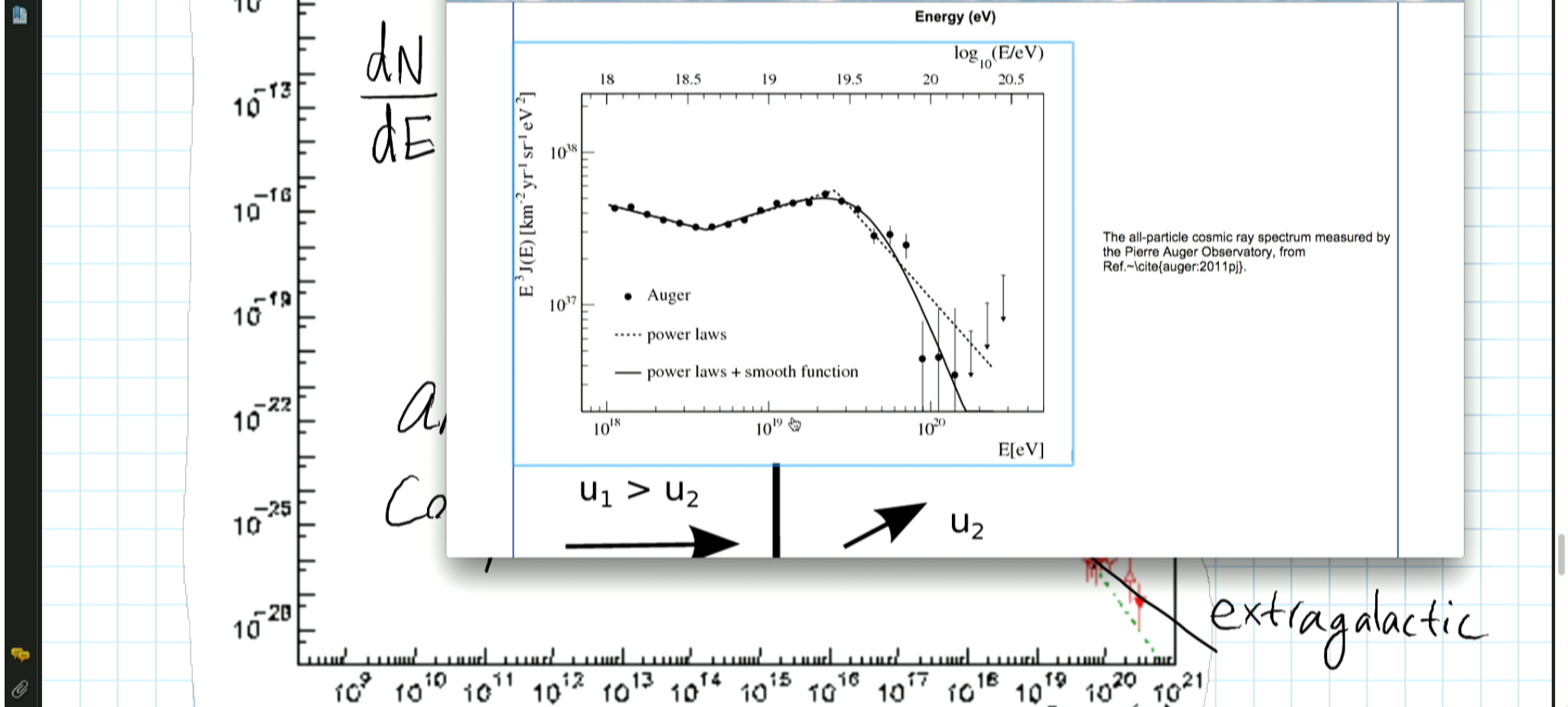
$$\alpha^2 = \frac{t_{\text{ref.}}}{t_{\text{esc.}} \langle \Delta h \epsilon^2 \rangle} \sim \frac{c^2 t_{\text{ref.}}^2}{L^2 (v_{\text{mirror}}^2 / c^2)}$$

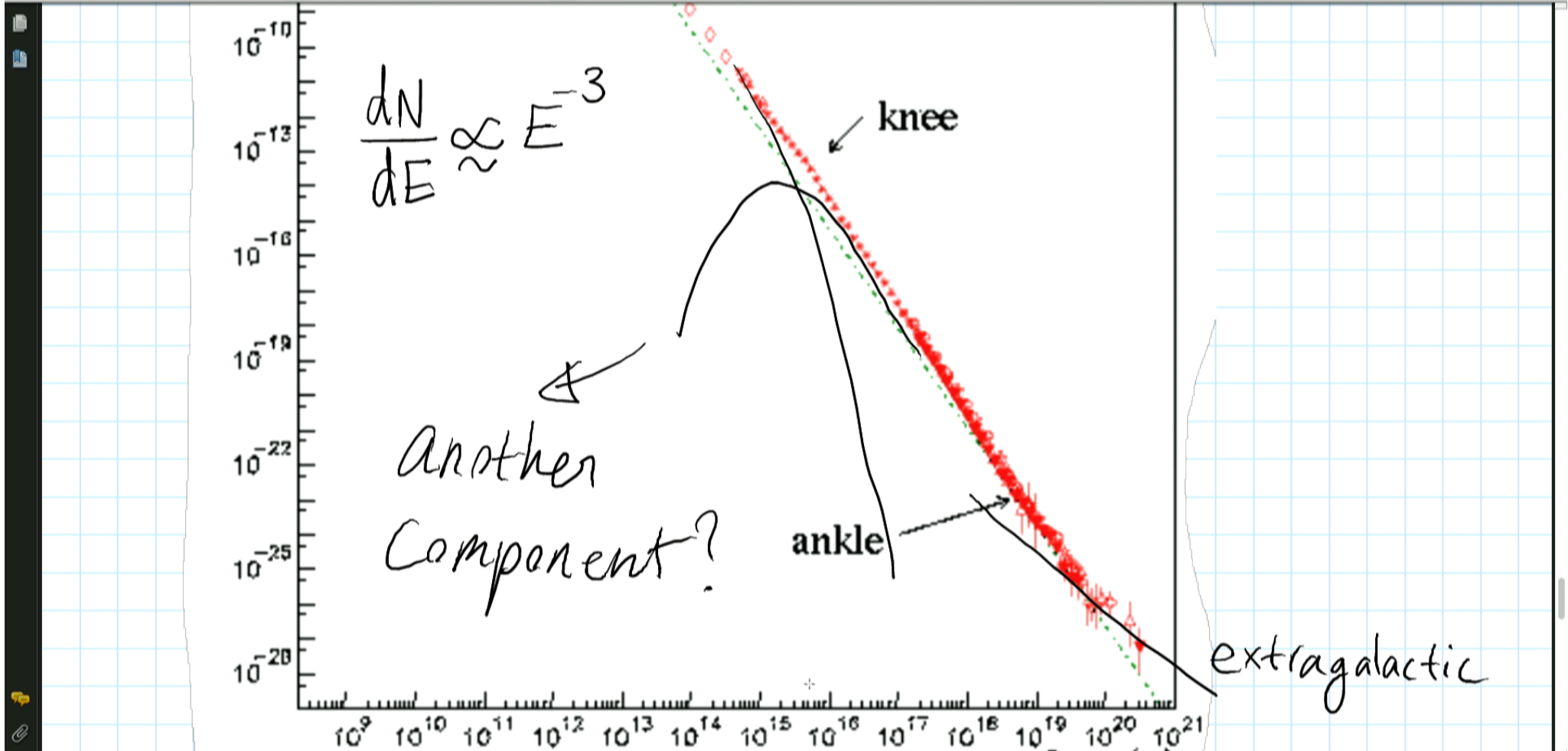


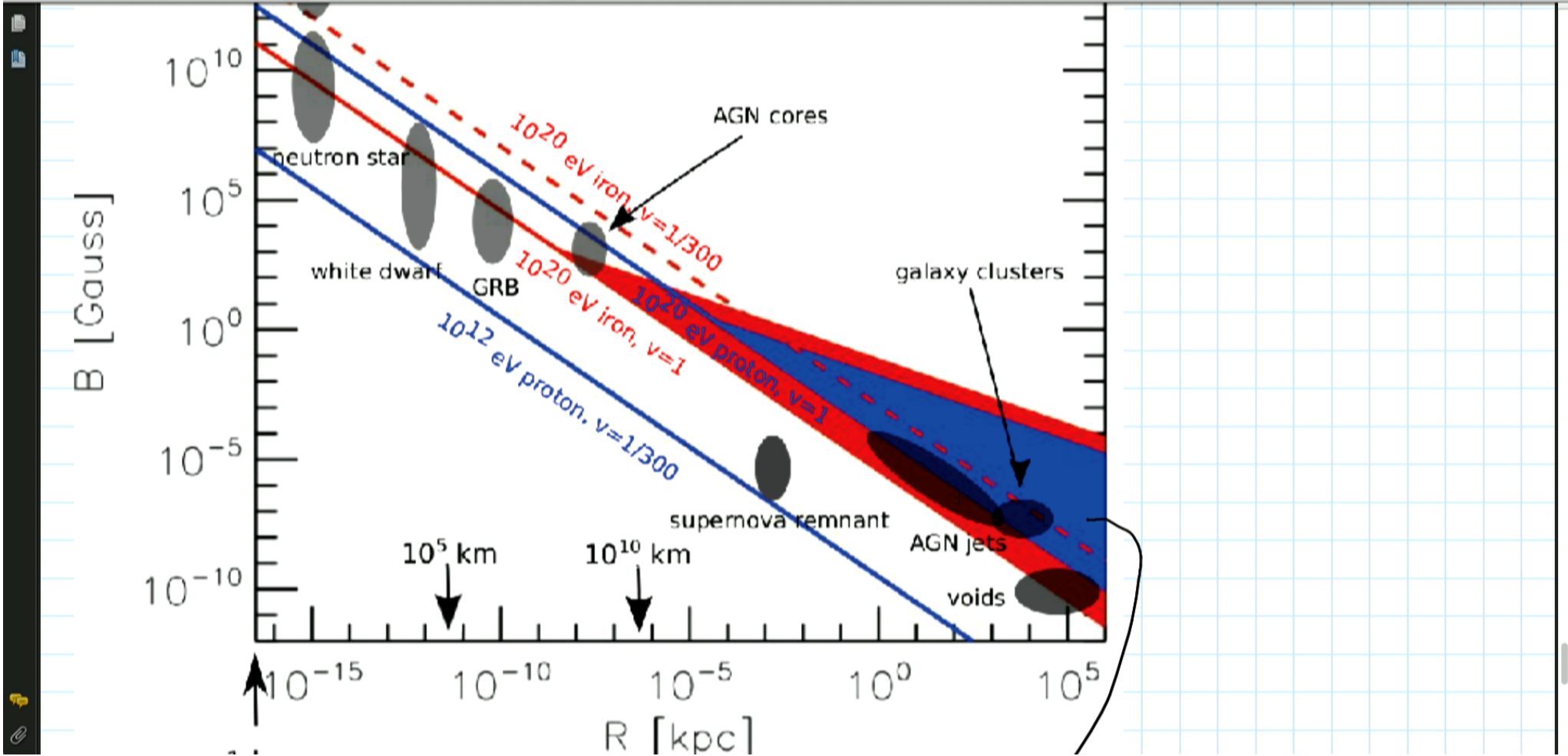












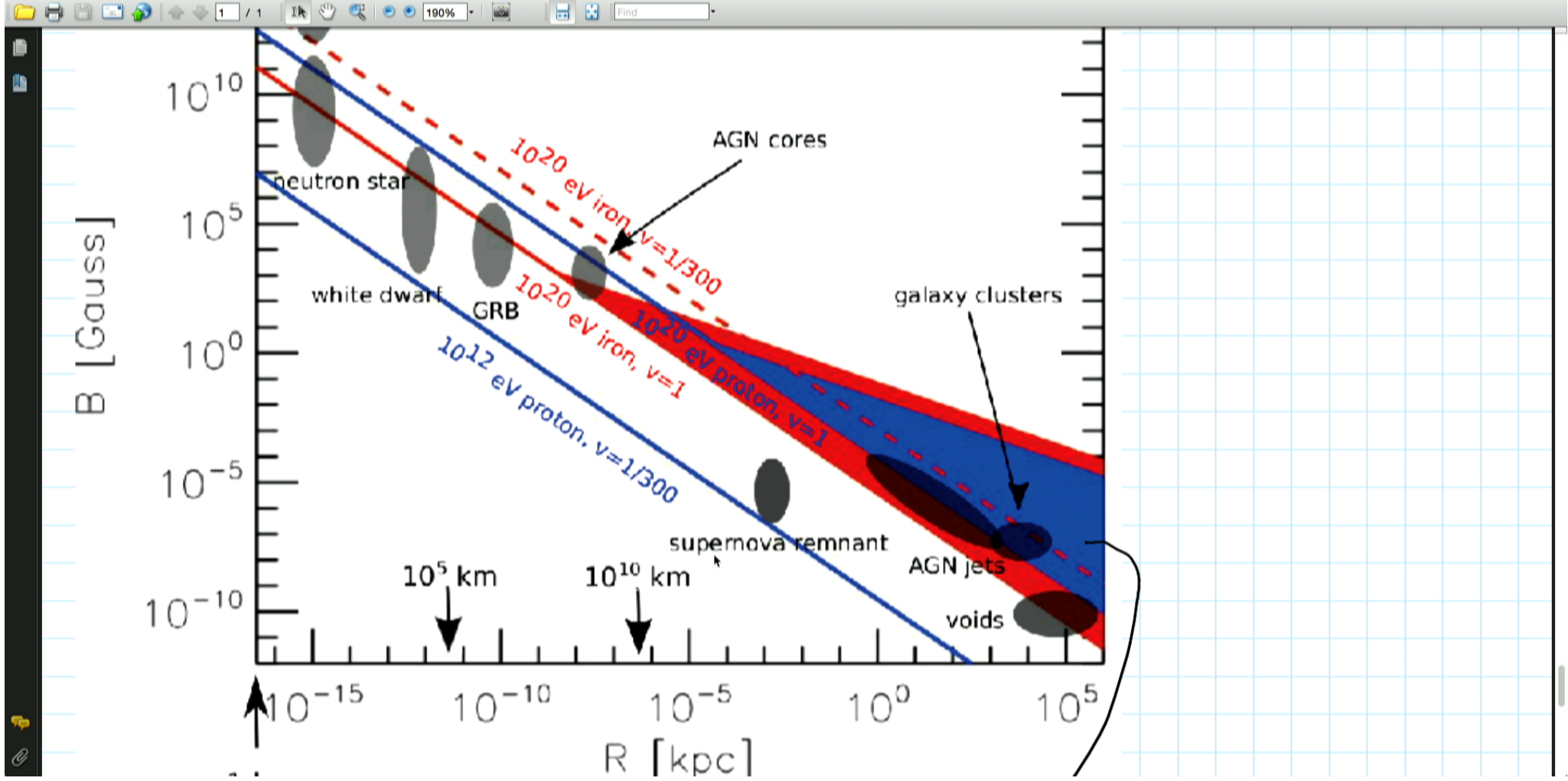


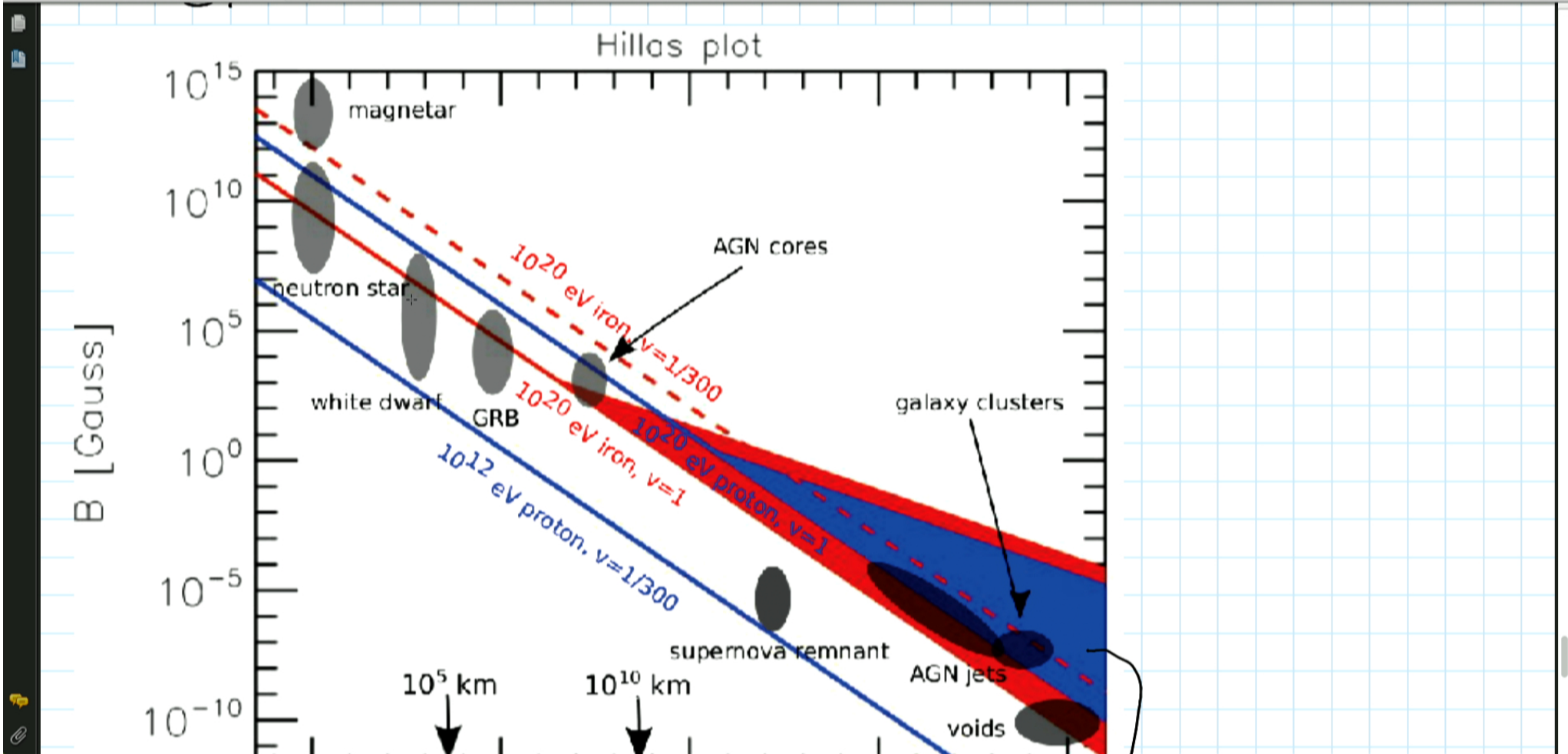
$$\omega p_{\perp} = eB$$

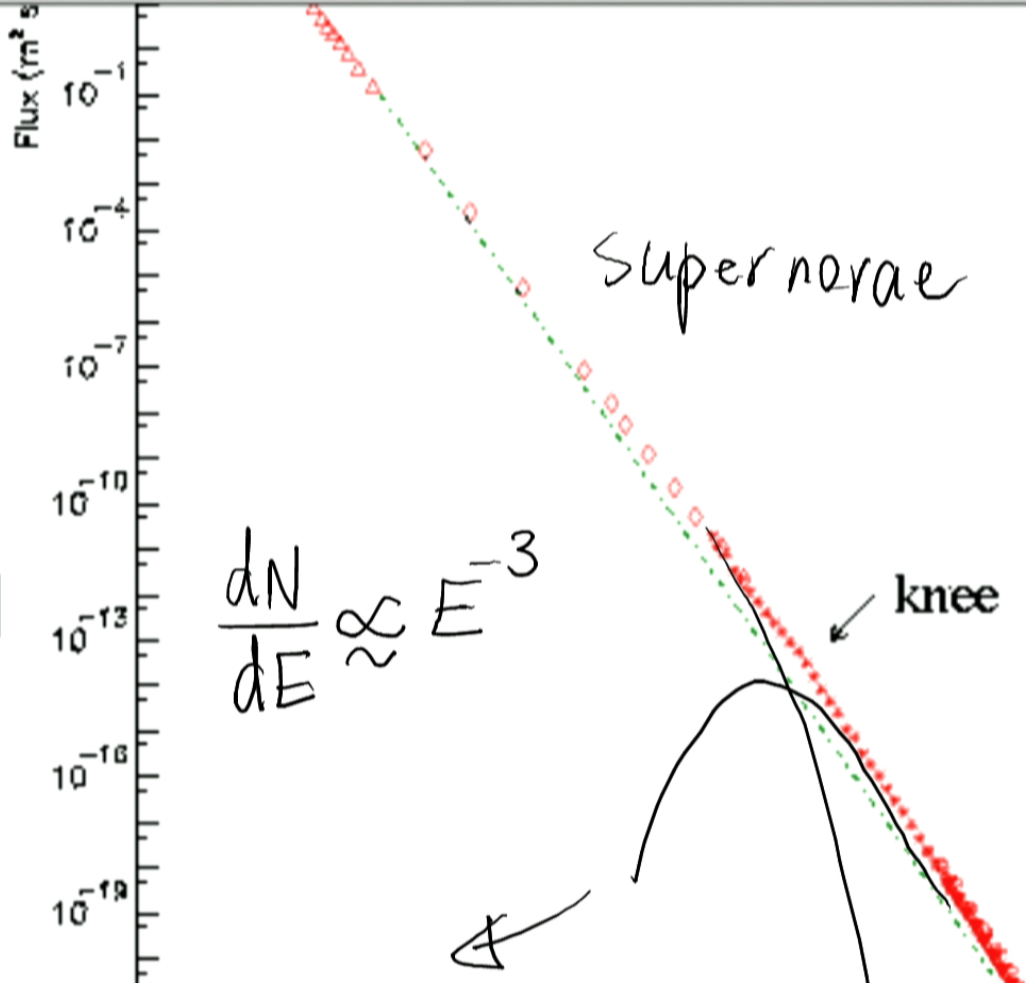
$$\omega = \frac{v_{\perp}}{r}$$

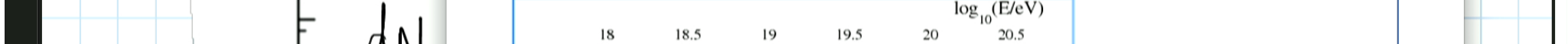
$$\frac{p_{\perp} v_{\perp}}{r} = eB$$

$$\sin^2 \theta \frac{E}{r} = eB$$

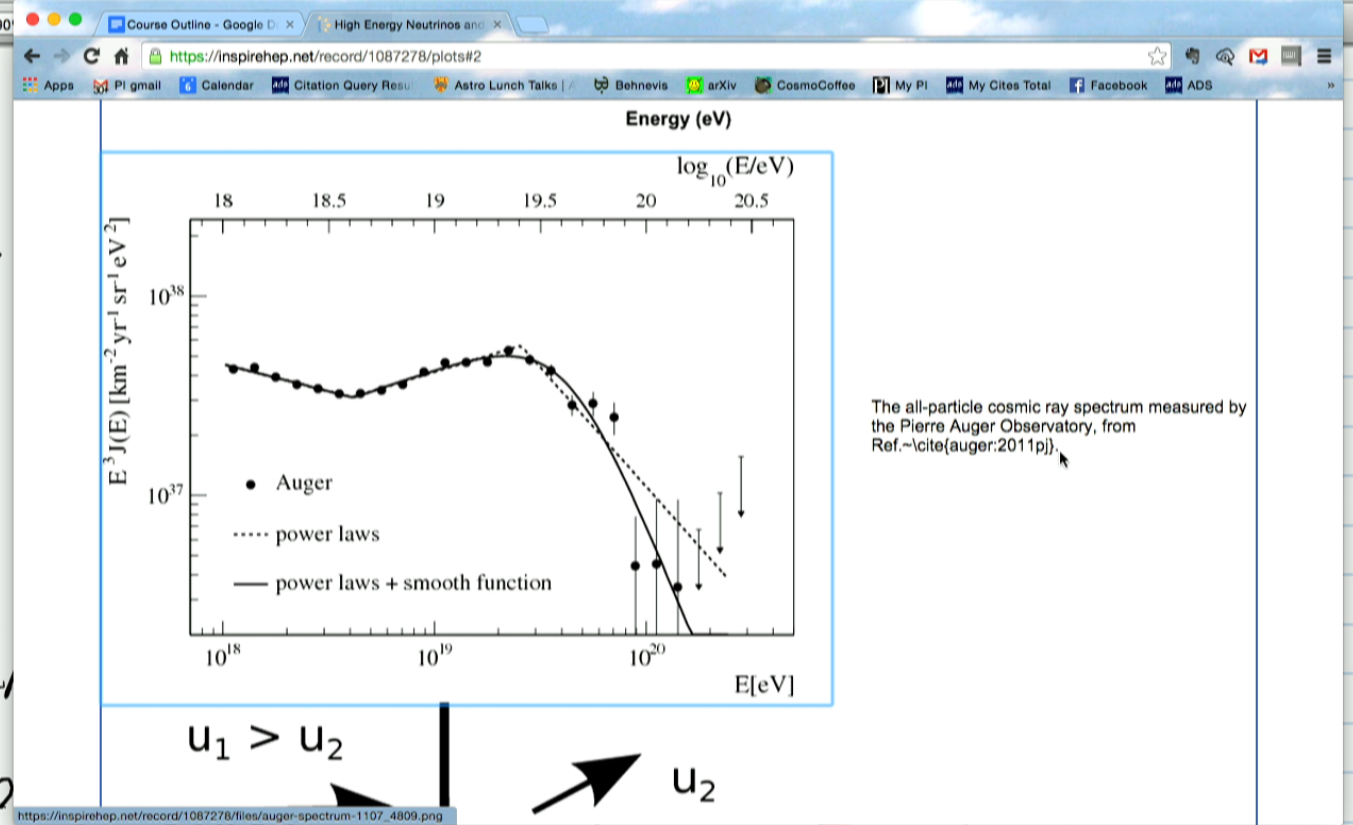




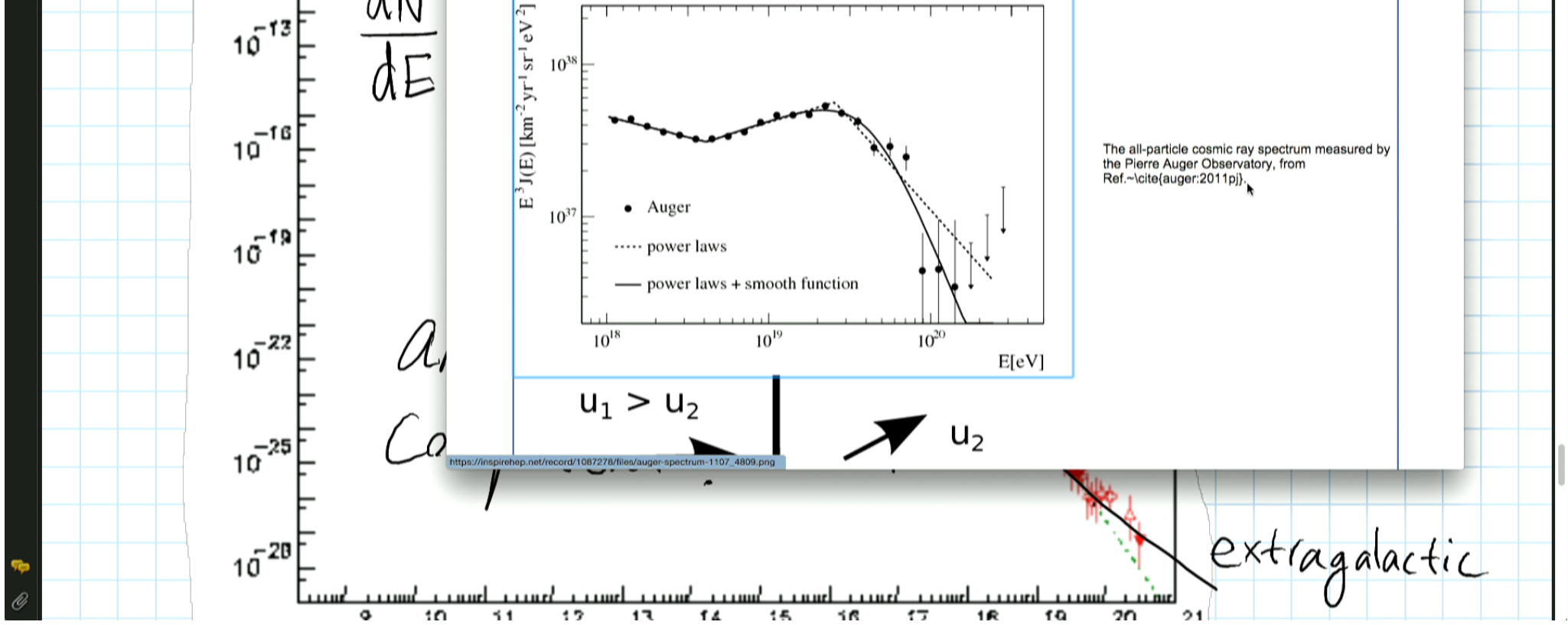




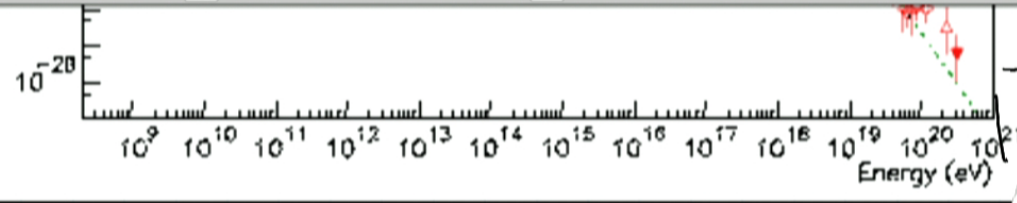
$\frac{dN}{dE}$



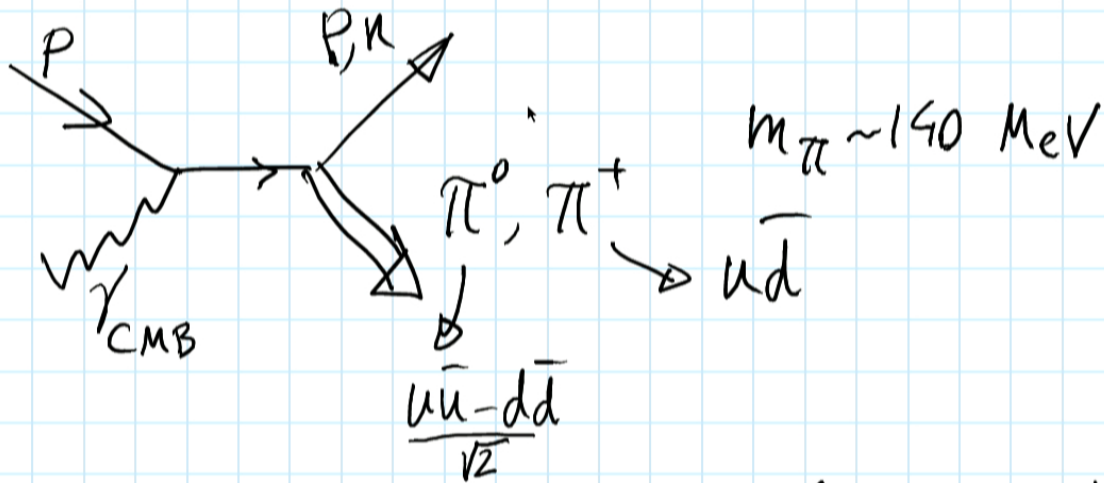
The all-particle cosmic ray spectrum measured by the Pierre Auger Observatory, from Ref. [cite{auger:2011p}]



extragalactic



→ GZK  
cut-off



$$E_{\text{CMB}} E_p \gtrsim m_{\pi}^2 \Rightarrow 10^{-4} \text{ eV} \times E_p \gtrsim 10^{16} \text{ eV} \Rightarrow E_p \gtrsim 10^{20} \text{ eV}$$

possibly seen by Pierre Auger observatory

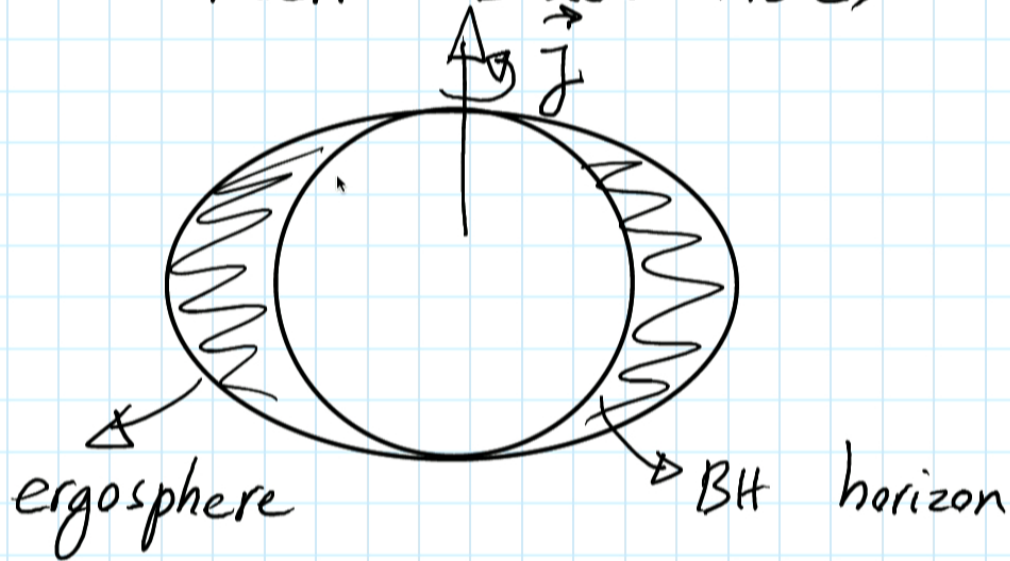
ergosphere

→ BH horizon

particles that don't rotate  
fast enough have negative energies

Penrose process: a particle can  
gain energy by emitting a negative- $E$   
particle (not very efficient)

# Kerr Black Holes



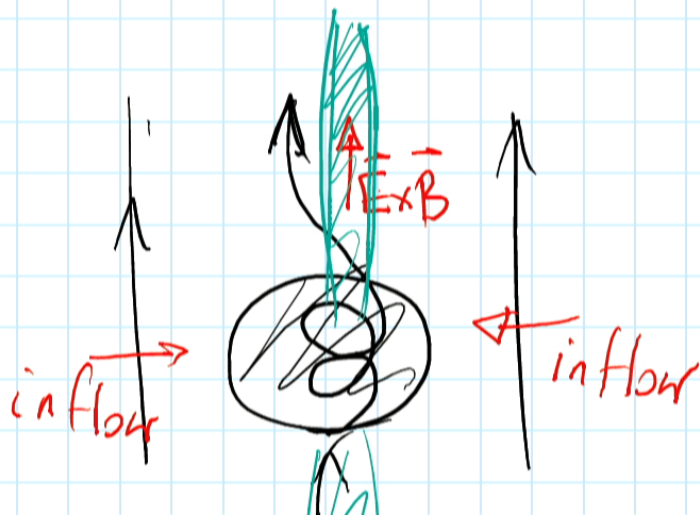
particles that don't rotate

fast enough have negative energies



# Blanford-Znajek Process:

Extraction of energy by twisting  
polar B-field lines



$$\begin{aligned} \text{Power} &\approx (\vec{E} \times \vec{B}_\phi) \times \pi r^2 \\ &\approx \left(\frac{\omega r}{c}\right)^2 B_z^2 \times \pi r^2 \end{aligned}$$

Additional annotations in the diagram include  $\frac{v \times B_z}{c}$  and  $\frac{v}{c} B_z$  with arrows pointing towards the main equation, likely representing the velocity of the field lines and the resulting electric field component.

$$\lambda L = \frac{G m \dot{m}}{R} \quad \dot{m} = \frac{R L M}{G M} = \frac{4 \pi c m_H R_s}{\sigma_T}$$

$$\dot{m} = \frac{4 \pi m_H 2 G M}{\sigma_T c}$$

$$L_{\text{Edd}} = \frac{4 \pi c G M m_H}{\sigma_T}$$

$$\int_m^{2m} \frac{dm'}{m'} = \int_0^t \frac{80 \pi G M_H}{\sigma_T c} dt \Rightarrow t = \frac{c \sigma_T \ln(2)}{8 \pi G M_H} \sim 1.56 \times 10^7 \text{ years}$$

$$4.9 \times 10^{14} \text{ s}$$

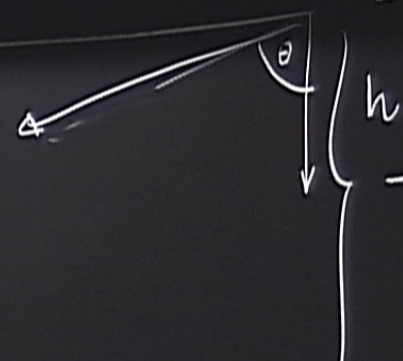
$$\dot{M} = 1.6 \cdot 10^{26} \text{ g/s}$$

$$\dot{V} = vA = \frac{c}{10} (2\pi r) h = \frac{2\pi r^2 c}{10^2} \approx 17 \times 10^{36} \text{ cm}^3/\text{s}$$

$$\rho = \frac{\dot{M}}{\dot{V}} = 8.2 \times 10^{-11} \text{ g/cm}^3$$

$$\frac{1}{8\pi} B^2 = \frac{1}{2} (v_T)^2 \rho \Rightarrow B = \left(\frac{c}{10}\right) (4\pi \rho)^{1/2} = 3 \cdot 10^{41} \text{ Gauss}$$

Sun



$$c_s^2 = \frac{dP}{d\rho}$$

$$\frac{dP}{dz} = - \int \frac{GM}{r^3} \frac{h}{2}$$

$$\frac{dP}{d\rho} = - \int \frac{GM}{r^3} \frac{h}{2} \frac{dz}{d\rho}$$

$$\frac{dP}{\rho} c_s^2 = - \frac{GM}{r^3} \frac{h}{2} dh$$

$$\ln\left(\frac{P}{P_c}\right) c_s^2 = - \frac{GM}{4r^3} \left(\frac{h}{2}\right)^2$$

$$t_s^2 = \left( \frac{h}{C_s} \right)^2 \sim -\ln \left( \frac{\rho_e}{\rho_c} \right) \left( \frac{4r^3}{GM} \right) \sim \frac{T^2}{4\pi^2}$$