

Title: PHYS 781 - Lecture 16

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URL: <http://pirsa.org/14110057>

Abstract:



Assignment #4

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Lagrangian: $L = T - U = \frac{1}{2} m v^2 + \frac{GmM_0}{r}$
 $= \frac{1}{2} m [\dot{r}^2 + r^2 \dot{\theta}^2] + \frac{GmM_0}{r}$

$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta}$$

$$P_r = \frac{\partial L}{\partial \dot{r}} = m \dot{r}$$

$$v = \sqrt{\dot{r}^2 + r^2 \dot{\theta}^2}$$

Hamiltonian:

$$H = \sum_i \dot{q}_i p_i - L$$

Assignment #4



Lagrangian: $L = T - U = \frac{1}{2} m \dot{r}^2 + \frac{GmM_0}{r}$

$\dot{r} = \sqrt{\dot{r}^2 + r^2 \dot{\theta}^2}$

$L = \frac{1}{2} m [\dot{r}^2 + r^2 \dot{\theta}^2] + \frac{GmM_0}{r}$

$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta}$

$P_r = \frac{\partial L}{\partial \dot{r}} = m \dot{r}$

Hamiltonian:

$H = \sum_i \dot{q}_i p_i - L$

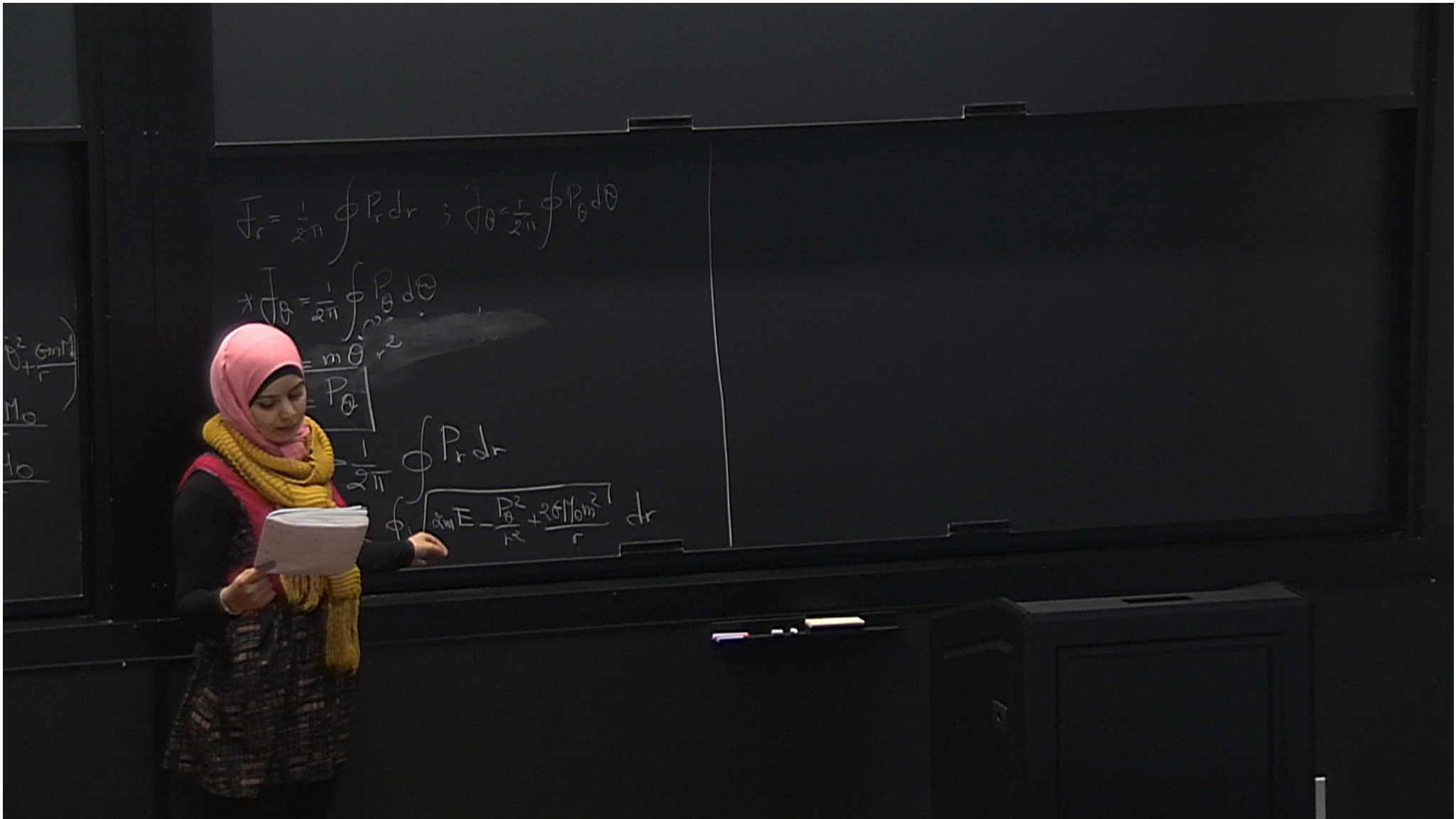
$= P_r \dot{r} + P_\theta \dot{\theta} - \left(\frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 + \frac{GmM_0}{r} \right)$

$= \frac{1}{2} m \dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 - \frac{GmM_0}{r}$

$= \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} - \frac{GmM_0}{r}$

Hamiltonian in Action variables:

$H = \frac{P_r^2}{2m} + \frac{P_\theta^2}{2mr^2} - \frac{GmM_0}{r}$



$$J_r = \frac{1}{2\pi} \oint P_r dr ; J_\theta = \frac{1}{2\pi} \oint P_\theta d\theta$$

$$* J_\theta = \frac{1}{2\pi} \oint P_\theta d\theta$$

$$= m \dot{\theta} r^2$$

$$\boxed{J_\theta = P_\theta}$$

$$* J_r = \frac{1}{2\pi} \oint P_r dr$$

$$= \frac{1}{2\pi} \oint \sqrt{\alpha_m E - \frac{P_\theta^2}{r^2} + \frac{2G\gamma m^2}{r}} dr$$

$$J_r = \frac{1}{2\pi} \left[\right]$$



$$J_r = \frac{1}{2\pi} \oint P_r dr ; J_\theta = \frac{1}{2\pi} \oint P_\theta d\theta$$

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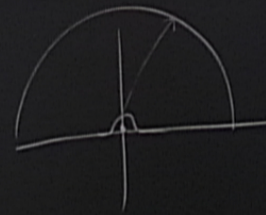
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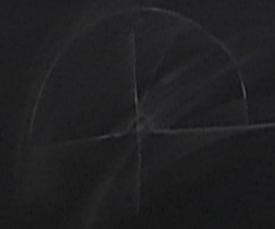
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$$* J_r = \frac{1}{2\pi} \oint P_r dr$$

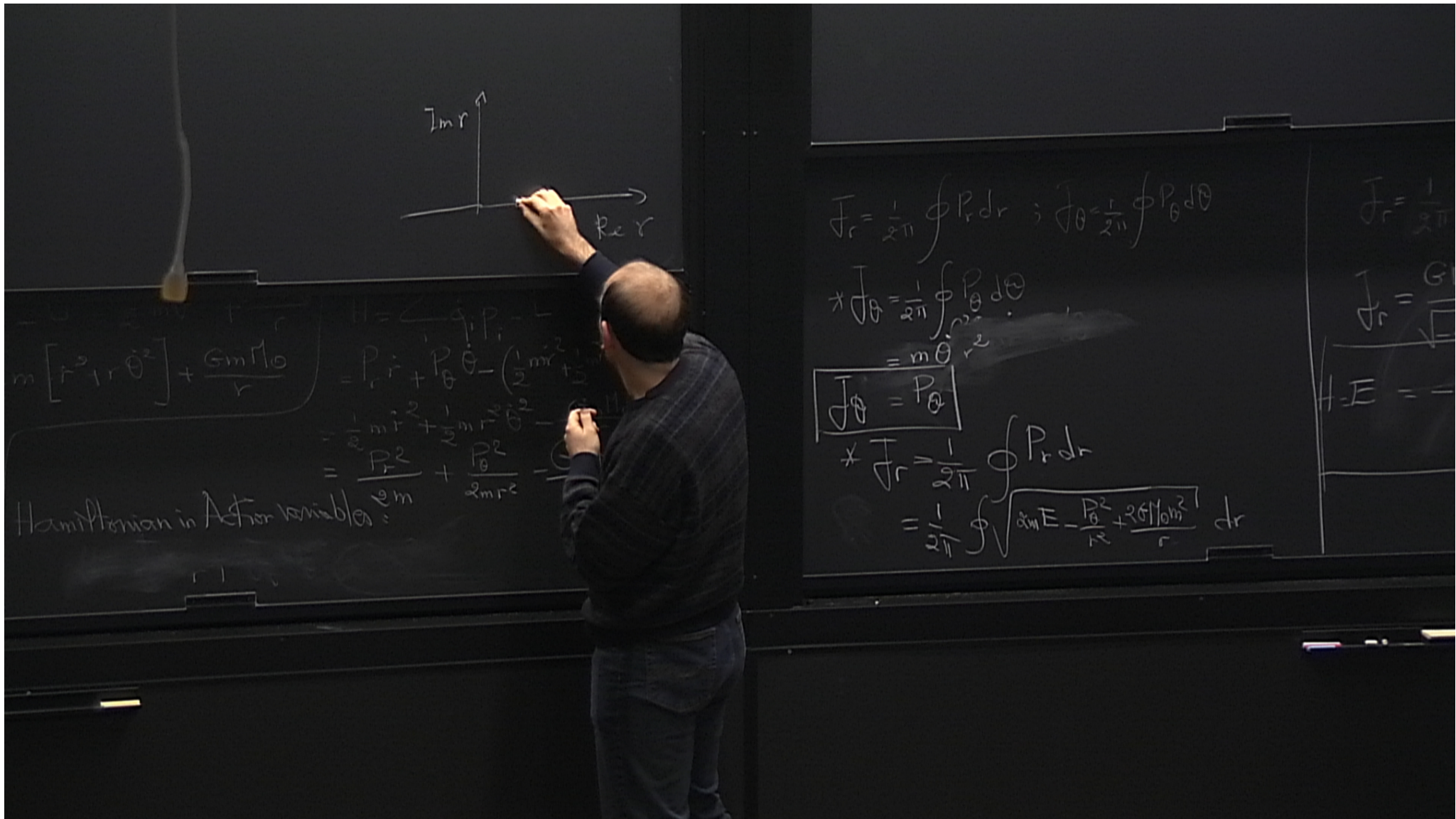
$$= \frac{1}{2\pi} \oint \sqrt{2mE - \frac{P_\theta^2}{r^2} + \frac{2GM_0 m^2}{r}} dr$$

$$J_r = \frac{1}{2\pi} \left[-2\pi P_\theta + 2\pi \frac{GM_0 m^2}{\sqrt{-2mE}} \right] \quad \omega_r = \frac{\partial H}{\partial J_r} = \frac{G^2 M_0^2 m^3}{(J_r + J_\theta)^3}$$

$$J_r = \frac{GM_0^2 m^2}{\sqrt{-2mE}} - J_\theta$$

$$H = E = \frac{2M_0^2 m^3}{(J_r + J_\theta)^2}$$

$$\omega_\theta = \frac{\partial H}{\partial J_\theta} = \frac{G^2 M_0^2 m^3}{(J_r + J_\theta)^3}$$



$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

Hamiltonian in Action variables: $\frac{e^2 m}{2mr^2} - \frac{1}{r}$

$$H = H_0 + \delta H_1$$

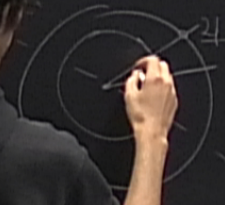
$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMM_0}{r} - \frac{m}{r^2 - 2r\gamma + \gamma^2}$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r}$$

Hamiltonian in Action variables: $\frac{e^2 m}{2mr^2} - \frac{1}{r}$

$$H = H_0 + \delta H_1$$

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GM M_0}{r} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos \gamma}}$$



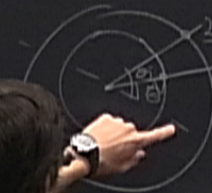
$$H = H_0 + \delta H_1$$

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm_0}{r} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos \gamma}}$$

$$\gamma =$$

$$\gamma = \theta - \theta_j$$

$$\theta = \omega t$$

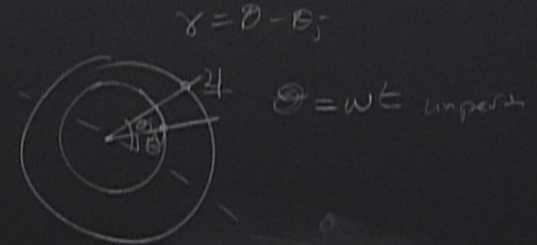


$$H = H_0 + \delta H_1$$

$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm_0}{r} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos \gamma}}$$

$$H = G - \frac{GM_j m}{\sqrt{r_j^2 + r^2}}$$

$\gamma =$



$$H = H_0 + \delta H_1$$

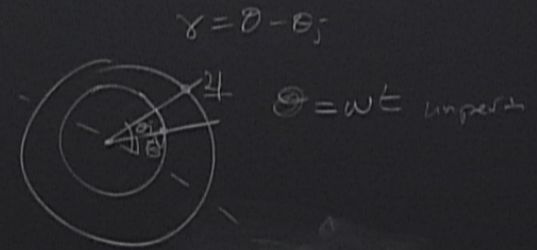
$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm_0}{r} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos \gamma}}$$

//
 $\gamma = 0$ circular

$$H = \frac{G^2 M_0^2 m^3}{J_0^2} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos(\theta - \omega t)}}$$

$$\frac{\partial H}{\partial \theta} = 0$$

$$\gamma =$$



$$H = H_0 + \delta H_1$$

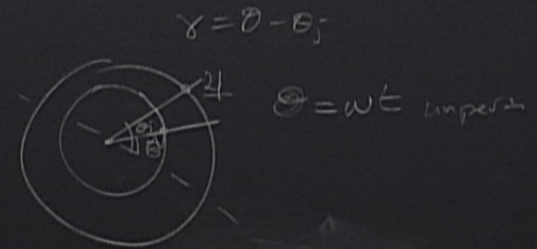
$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm_0}{r} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos \gamma}}$$

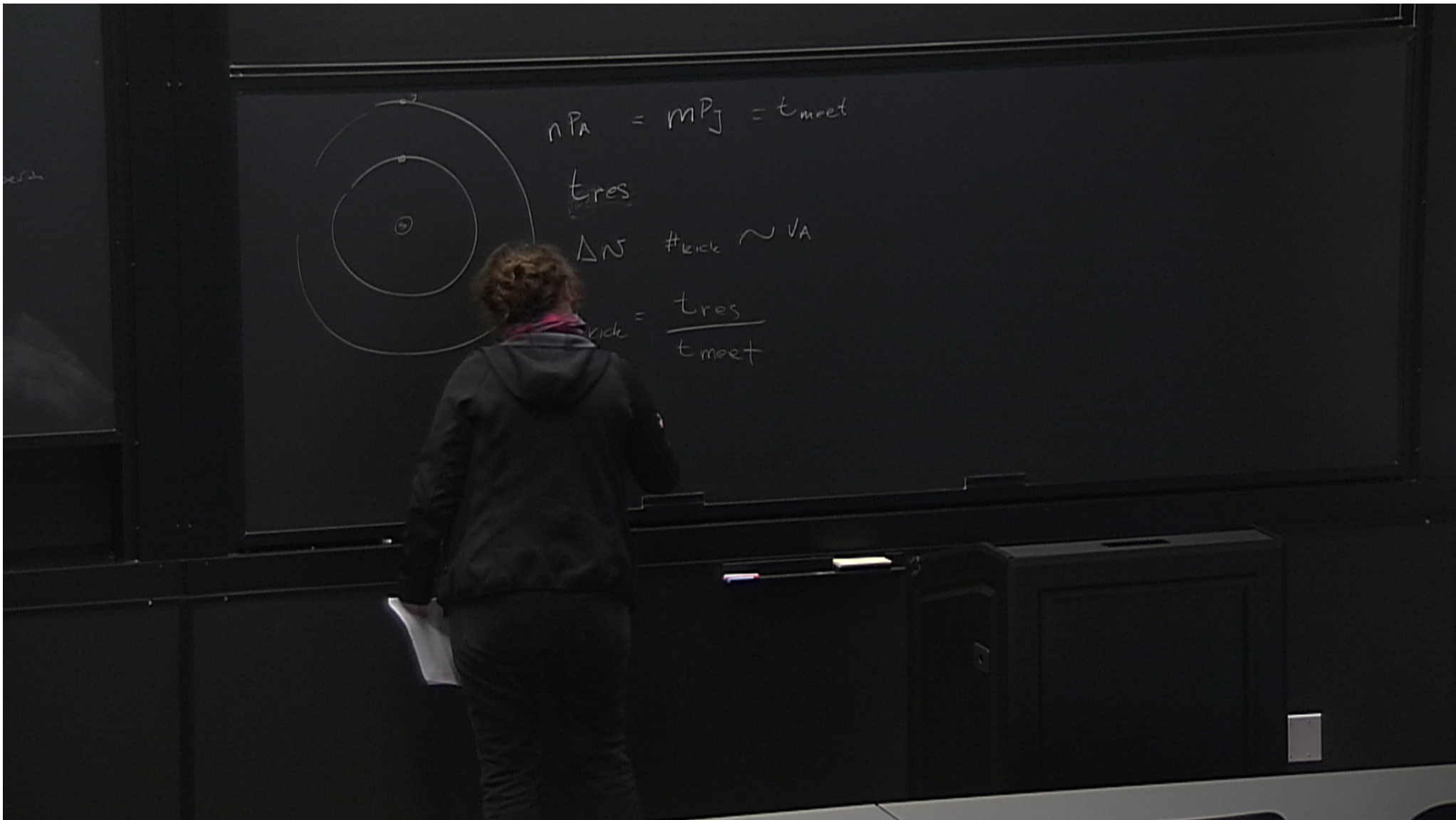
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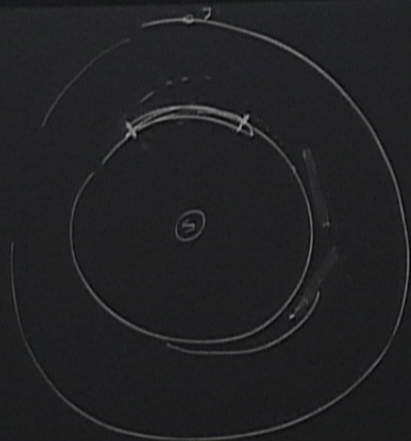
$$H = \frac{G^2 M_0^2 m^3}{2 J_0^2} - \frac{GM_j m}{\sqrt{r_j^2 + r^2 - 2r_j r \cos(\theta - \omega t)}}$$

$$\frac{\partial H}{\partial \theta} = 0$$

$$\gamma =$$







$$\frac{n P_A}{m P_J} = \frac{\tilde{t}}{t}$$

t_{res}

$$\Delta N \cdot \#_{kick} \sim v_A$$

$$\#_{kick} = \frac{t_{res}}{\tilde{t}}$$

$$\Delta N \frac{t_{res}}{t_{meet}} \sim v_A$$

$$\Delta N = \frac{G M_J}{r_A^2} \frac{P_A}{a}$$

$$\frac{G M_J}{r_A^2} \frac{P_A}{a} t_{res}$$

$$S = -Et + L\theta = \frac{M^2 \mu^2 G^2}{2L} t + L\theta$$

$$M = n$$

$$\mu = \frac{1}{2}$$

$$2L =$$

$$\frac{\partial S}{\partial L} = \beta = \theta - \underbrace{\frac{M^2 \mu^2 G^2}{L^3}}_{\omega_0} +$$

$$\theta = \omega_0 t + \beta$$

$$S = -Et + L\theta = \frac{M^2 \mu^3 G^2}{2L} t + L\theta$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\mathcal{H}_E = \frac{M^2 \mu^3 G^2}{2L^2} - E - \frac{\epsilon M M G}{\sqrt{r^2 + r_J^2 - 2\cos((\omega_b - \omega_a)t + \beta) r r_J}}$$

$$\frac{\partial S}{\partial L} = \beta = \theta - \underbrace{\frac{M^2 \mu^3 G^2}{L^3}}_{\omega_0} +$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = \frac{\partial \mathcal{H}_E}{\partial \beta}$$

$$= \epsilon(\mu M)$$

$$S = -Et + L\theta = \frac{M^2 \mu^3 G^2}{2L} t + L\theta$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = \frac{-M^2 \mu^3 G^2}{2L^2} - E - \frac{\epsilon M M G}{\sqrt{r^2 + r_J^2 - 2\cos((\omega_b - \omega_J)t + \beta) r r_J}}$$

$$\frac{\partial S}{\partial L} = \beta = \theta - \underbrace{\frac{M^2 \mu^3 G^2}{L^3}}_{\omega_0} +$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} =$$

$$-\sin((\omega_b - \omega_J)t + \beta)$$

$$S = -Et + L\theta = \frac{M^2 \mu G^2}{2L} t + L\theta$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = -\frac{M^2 \mu G^2}{2L^2} - E - \frac{\epsilon \mu}{\sqrt{r^2 + r_j^2 - 2rr_j \cos(\omega_0 - \omega_j + \beta)}}$$

$$\frac{\partial S}{\partial L} = \beta = \theta -$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = -\frac{\partial \mathcal{L}}{\partial \beta}$$

$$= \epsilon \left(\frac{\mu M G r r_j \sin(\omega_0 - \omega_j + \beta)}{\sqrt{r^2 + r_j^2 - 2rr_j \cos(\omega_0 - \omega_j + \beta)}} \right)$$

$$S = -Et + L\theta = \frac{M^2 \mu^2 G^2}{2L} t + L\theta$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = -$$

$$MMG$$

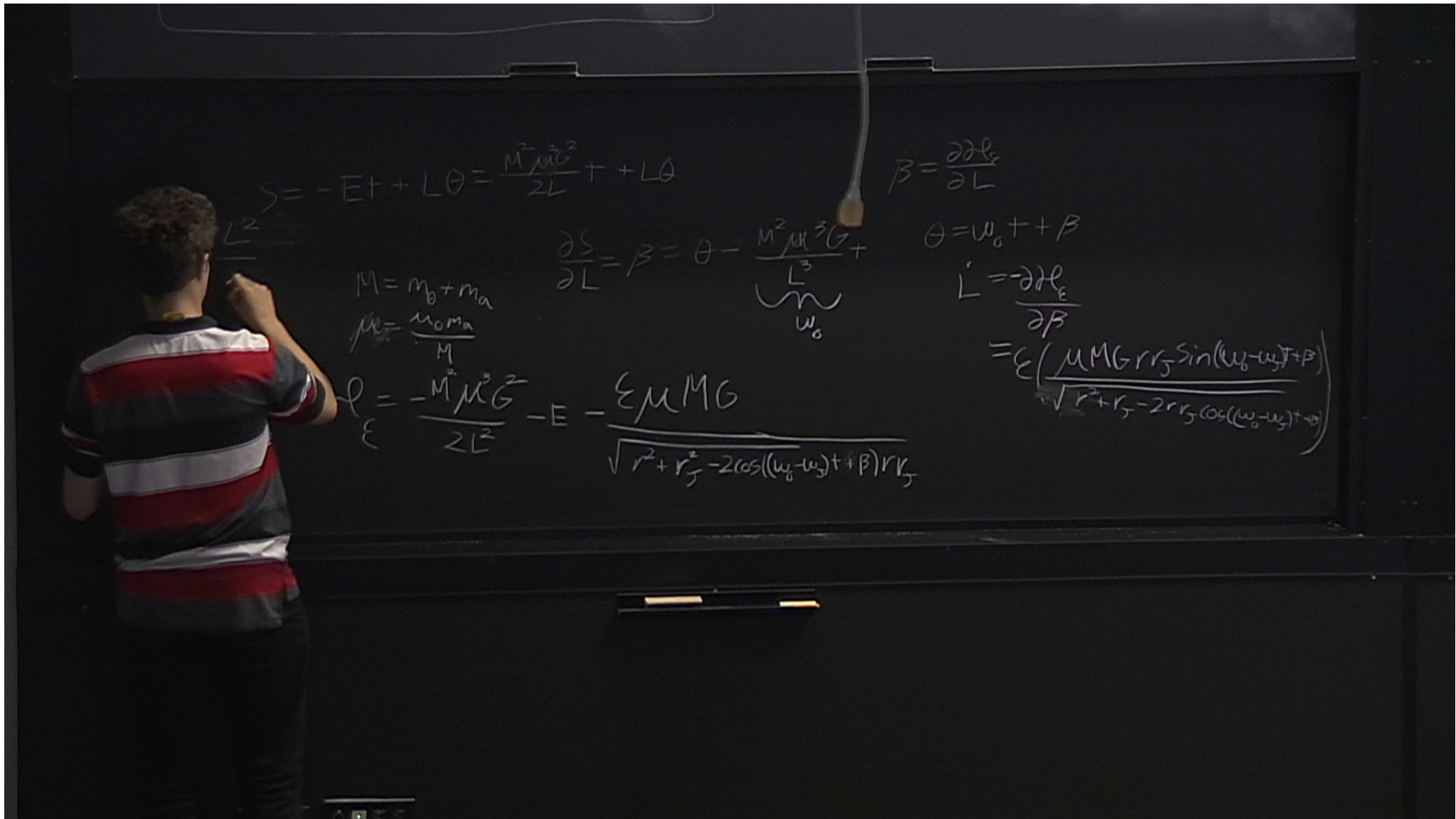
$$r^2 + r_J^2 - 2r \cos((\omega_b - \omega_J)t + \beta) r r_J$$

$$\beta = \frac{\partial \mathcal{L}}{\partial L}$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = -\frac{\partial \mathcal{L}}{\partial \beta}$$

$$= \epsilon \left(\frac{MMG r r_J \sin((\omega_b - \omega_J)t + \beta)}{\sqrt{r^2 + r_J^2 - 2r \cos((\omega_b - \omega_J)t + \beta) r r_J}} \right)$$



$$S = -Et + L\theta = \frac{M^2 \mu^3 G^2}{2L} t + L\theta$$

$$\beta = \frac{\partial \mathcal{L}}{\partial L}$$

$$\frac{L^2}{2}$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\frac{\partial S}{\partial L} = \beta = \theta - \underbrace{\frac{M^2 \mu^3 G^2}{L^3}}_{\omega_0} +$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = -\frac{\partial \mathcal{H}}{\partial \beta}$$

$$= \epsilon \left(\frac{\mu M G r r_J \sin((\omega_0 - \omega_J)t + \beta)}{\sqrt{r^2 + r_J^2 - 2r r_J \cos((\omega_0 - \omega_J)t + \beta)}} \right)$$

$$\mathcal{L} = -\frac{M^2 \mu^3 G^2}{2L^2} - E - \frac{\epsilon \mu M G}{\sqrt{r^2 + r_J^2 - 2r r_J \cos((\omega_0 - \omega_J)t + \beta)}}$$

$$S = -Et + L\theta = \frac{M^2 \mu^3 G^2}{2L} t + L\theta$$

$$r = \frac{L^2}{M\mu G}$$

$$M = m_b + m_a$$

$$\mu = \frac{m_b m_a}{M}$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = -\frac{M^2 \mu^3 G^2}{2L^2} - E - \frac{\epsilon M \mu G}{\sqrt{r^2 + r_J^2 - 2r r_J \cos((\omega_b - \omega_J)t + \beta)}}$$

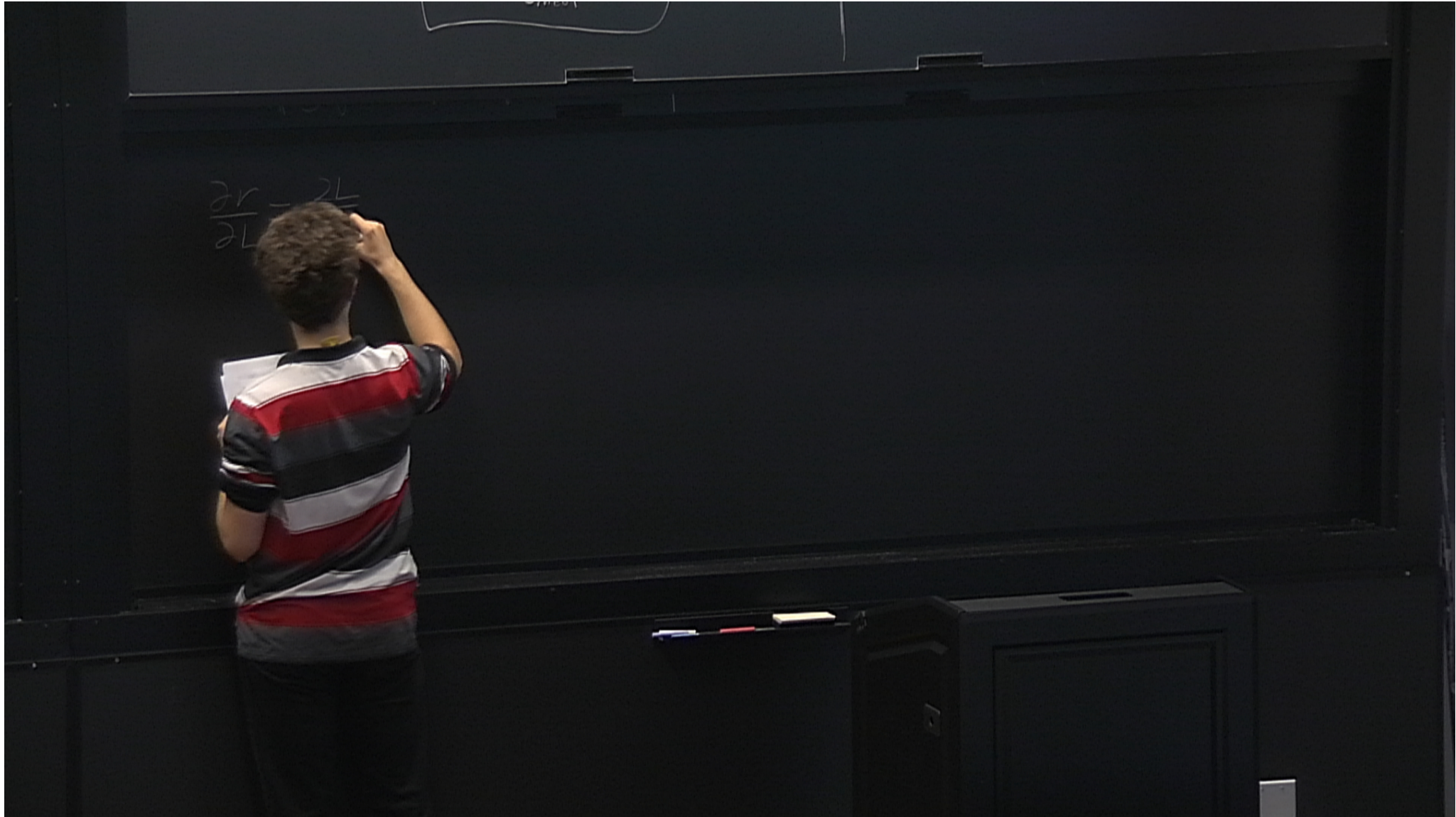
$$\frac{\partial S}{\partial L} = \beta = \theta - \underbrace{\frac{M^2 \mu^3 G^2}{L^3}}_{\omega_0} +$$

$$\beta = \frac{\partial \mathcal{L}}{\partial L}$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = -\frac{\partial \mathcal{L}}{\partial \beta}$$

$$= \epsilon \left(\frac{M \mu G r r_J \sin((\omega_b - \omega_J)t + \beta)}{\sqrt{r^2 + r_J^2 - 2r r_J \cos((\omega_b - \omega_J)t + \beta)}} \right)$$



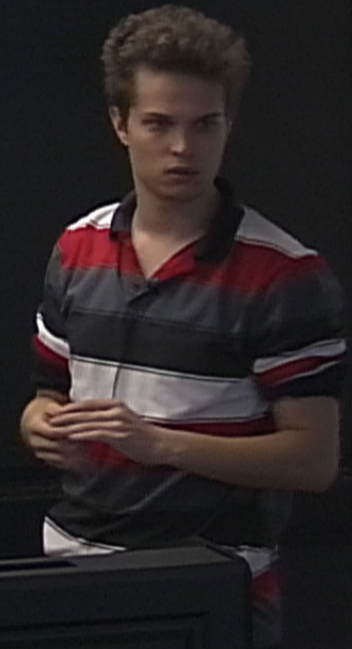
$$\frac{\partial r}{\partial L} = \frac{2L}{M\mu G}$$
$$\dot{\beta} = \frac{\epsilon \mu M G}{\sqrt{\quad}} \rightarrow (r-2)$$

$$\frac{\partial r}{\partial L} = \frac{2L}{M \mu G}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_j^2 - 2rr_j \cos(\omega_0 - \omega_j) + \beta})^3} \frac{\partial r}{\partial L}$$

$$\frac{\partial r}{\partial L} = \frac{2L}{M\mu G}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_j^2 - 2rr_j \cos(\omega_0 - \omega_j) + \beta})^3} \frac{\partial r}{\partial L}$$





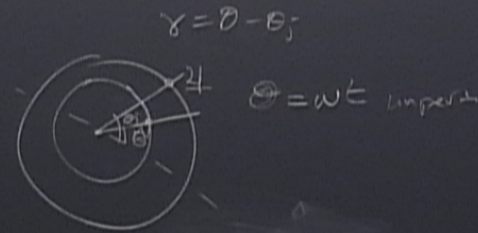
$$H = \frac{p_r^2}{2m} + \frac{p_\theta^2}{2mr^2} - \frac{GMm_0}{r} - \frac{GMjm}{\sqrt{r_j^2 + r^2 - 2rr_j \cos \gamma}}$$

//
 $\gamma = 0$ circular

$$H = \frac{G^2 M_0^2 m^3}{2 J_\theta^2} - \frac{GMjm}{\sqrt{r_j^2 + r^2 - 2rr_j \cos(\theta - \theta_j)}}$$

$$\frac{\partial H}{\partial \theta} = 0$$

$$\vec{L} = \vec{r} \times \vec{F}$$



$$r = \frac{L}{m\omega G}$$

$$M = m_0 + m_a$$

$$\mu = \frac{m_0 m_a}{M}$$

$$\frac{\partial \mathcal{L}}{\partial \epsilon} = -\frac{M^2 \mu^2 G^2}{2L^2} - E - \frac{\epsilon \mu M G}{\sqrt{r^2 + r_j^2 - 2r r_j \cos((\omega_0 - \omega_j)t + \beta)}}$$

$$\frac{\partial S}{\partial L} = \beta = \theta - \frac{m_0 m_a}{L} \omega$$

$$\theta = \omega_0 t + \beta$$

$$\dot{L} = -\frac{\partial \mathcal{L}}{\partial \beta}$$

$$= \epsilon \left(\frac{\mu M G r r_j \sin((\omega_0 - \omega_j)t + \beta)}{\sqrt{r^2 + r_j^2 - 2r r_j \cos((\omega_0 - \omega_j)t + \beta)}} \right)$$

$$\frac{\partial r}{\partial L} = \frac{2L}{4\mu MG}$$

$$\dot{\beta} = \frac{\epsilon \mu MG}{(\sqrt{r^2 + r_0^2 - 2rr_0 \cos(\omega_0 - \omega_1)t + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = \int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right]$$

$$= -\frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta}$$

$$\frac{\partial r}{\partial L} = \frac{2L}{M\mu G}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_0^2 - 2rr_0 \cos(\omega_0 - \omega_f)t + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = \int d\theta \left[\frac{1}{\omega_A} \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right] \right]$$

$$= -\frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta}$$

$$\frac{v}{t_{meet}} \sim v_A$$

$$\frac{\partial r}{\partial L} = \frac{2L}{MAG} \quad \dot{\beta} = \frac{\epsilon \mu M G (r - 2r_0 \cos(\omega_0 - \omega_1)t + \beta)}{(\sqrt{r^2 + r_0^2 - 2rr_0 \cos(\omega_0 - \omega_1)t + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = - \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = - \int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx - \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right]$$

$$= + \frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta)$$

$$= \frac{1}{\omega_A^2}$$

$$\frac{\partial r}{\partial L} = \frac{2L}{M\mu G}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_g^2 - 2rr_g \cos(\omega_0 - \omega_f)T + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = - \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = - \int d\theta \frac{\partial H}{\partial \theta}$$

$$\begin{aligned} \frac{\partial H}{\partial \theta} &\approx - \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right] \\ &= + \frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta) \\ &= \frac{1}{\omega_A^2} \int d\theta \end{aligned}$$

$$\frac{L}{t_{\text{meet}}} \sim v_A$$

$$\frac{\partial r}{\partial L} = \frac{2L}{MAG} \quad \dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_g^2 - 2rr_g \cos(\omega_0 - \omega_f)t + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = - \int dt \frac{\partial H}{\partial \theta} d\theta = - \int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx - \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right]$$

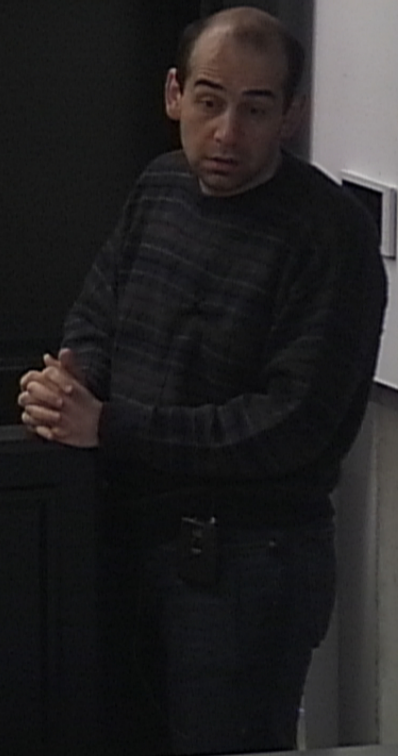
$$= + \frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta)$$

$$\frac{\partial r}{\partial L} = \frac{2L}{MMG}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_2^2 - 2rr_2 \cos(\omega_0 - \omega_2) + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = - \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = - \int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx - \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right]$$

$$= + \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta)$$



$$\frac{\partial r}{\partial L} = \frac{2L}{MMG}$$

$$\dot{\beta} = \frac{\epsilon \mu M G}{(\sqrt{r^2 + r_j^2 - 2rr_j \cos(\omega_0 - \omega_j) t + \beta})^3} \frac{\partial r}{\partial L}$$

$$\Delta J_\theta = - \int \frac{dt}{d\theta} \frac{\partial H}{\partial \theta} d\theta = - \int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx - \int d\theta \frac{\partial H}{\partial \theta} \left[1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right]$$

$$= \frac{1}{\omega_A} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta)$$



$$\frac{\partial r}{\partial L} = \frac{2L}{MMG}$$

$$\Delta J_\theta = -\int g \frac{dt}{d\theta}$$



$$\dot{\beta} = \frac{\epsilon \mu M G}{(r^2 + r_f^2 - 2rr_f \cos(\omega_0 - \omega_f)t + \beta)} \frac{\partial r}{\partial L}$$

$$= -\int d\theta \frac{\partial H}{\partial \theta} \frac{1}{\omega_A + \dot{\beta}(\theta)} \approx -\frac{1}{\omega_A} \int d\theta \left[\frac{\partial H}{\partial \theta} \left(1 - \frac{\dot{\beta}(\theta)}{\omega_A} \right) \right]$$

$$= +\frac{1}{\omega_A^2} \int d\theta \frac{\partial H}{\partial \theta} \dot{\beta}(\theta)$$