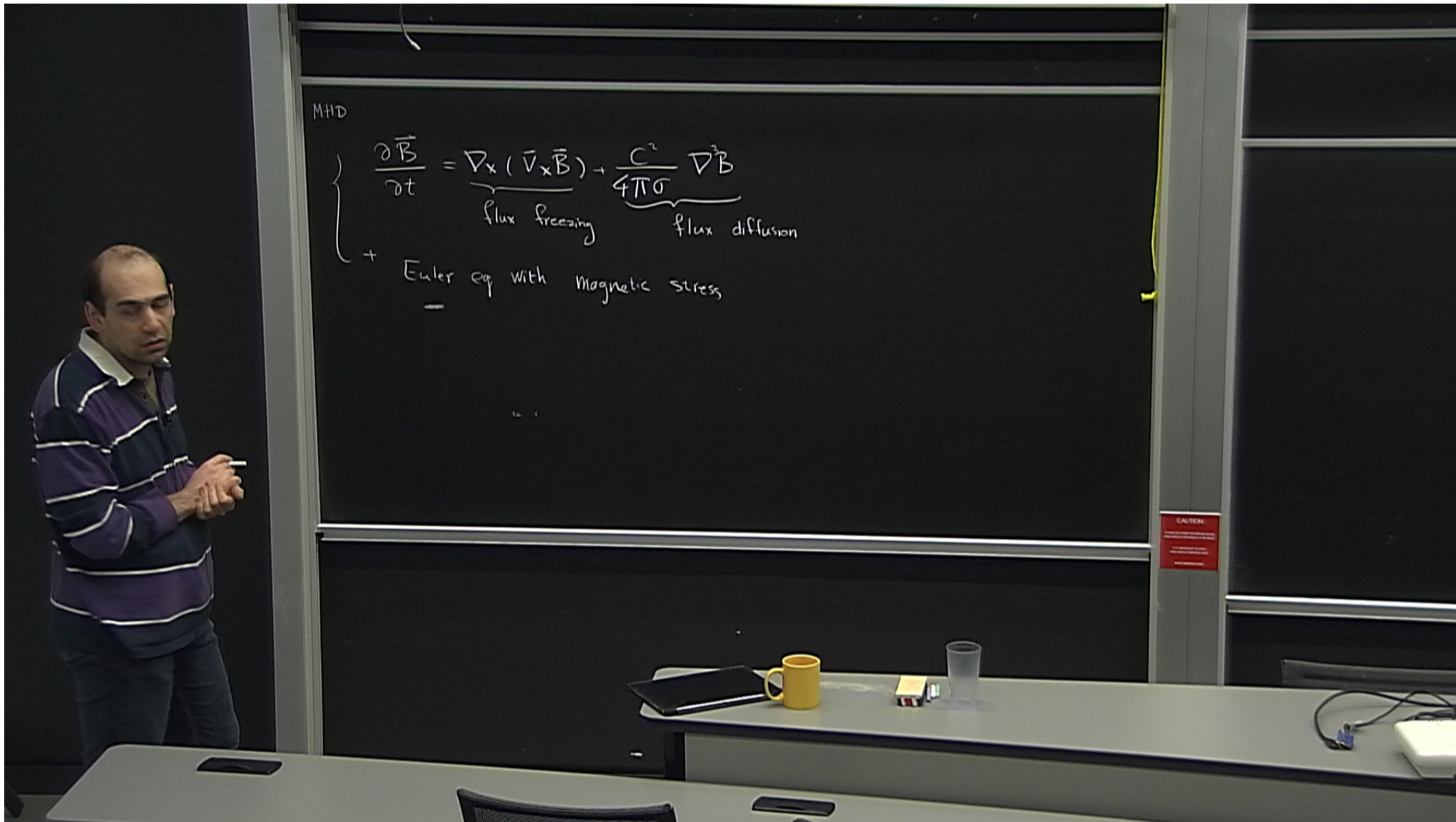


Title: PHYS 781 - Lecture 21

Date: Nov 20, 2014 02:00 PM

URL: <http://pirsa.org/14110055>

Abstract:



MHD

$$\left\{ \begin{array}{l} \frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{V} \times \vec{B})}_{\text{flux freezing}} + \underbrace{\frac{c^2}{4\pi\sigma} \nabla^2 \vec{B}}_{\text{flux diffusion}} \\ + \text{Euler eq with magnetic stress} \end{array} \right.$$

MHD

$$\left\{ \begin{array}{l} \frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{v} \times \vec{B})}_{\text{flux freezing}} + \underbrace{\frac{c^2}{4\pi\sigma} \nabla^2 \vec{B}}_{\text{flux diffusion}} \\ + \text{Euler eq with magnetic stress} \end{array} \right.$$

Biermann battery : How to produce mag. field

$$\text{erg} = \text{g} \cdot \text{cm}^2 \text{s}^{-2}$$

$$\vec{J} = \sigma (\vec{E} + \vec{V} \times \vec{B})$$

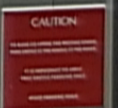
$$\vec{J} = -\bar{n}_e e (\vec{v}_e - \vec{v}_p)$$

$$\left[\frac{E^2}{8\pi} \right] = \frac{\text{erg}}{\text{cm}^3}$$

$$\left[\frac{e^2}{r} \right] \sim \text{erg}$$

$$[e] = \sqrt{\text{erg} \cdot \text{cm}}$$

$$[\sigma] = \frac{[e] \text{cm}^3 \cdot \text{cm} \text{s}^{-1}}{\text{erg}^{1/2} \text{cm}^{-3/2}} = \frac{\text{cm}^{1/2} \text{cm}^{-2} \text{s}^{-1}}{\text{cm}^{-3/2}}$$



Biermann battery : How to produce mag field

$$\text{erg} = \text{g} \cdot \text{cm}^2 \text{s}^{-2}$$

$$\vec{J} = \sigma (\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

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$$\frac{d(\vec{v}_e - \vec{v}_p)}{dt} \approx -\frac{\nabla P_e}{m_e n_e} + \frac{\nabla P_p}{m_p n_p} - \frac{e}{m_e} (\vec{E} + \frac{\vec{v}_e \times \vec{B}}{c}) - \frac{(v_e - v_p)}{\tau}$$

Biermann battery : How to produce mag field

$$\text{erg} = \text{g} \cdot \text{cm}^2 \text{s}^{-2}$$

$$\vec{J} = \sigma (\vec{E} + \frac{\vec{v}}{c} \times \vec{B})$$

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$$\frac{d(\vec{v}_e - \vec{v}_p)}{dt} \approx -\frac{\nabla P_e}{m_e n_e} + \frac{\nabla P_p}{m_p n_p} - \frac{e}{m_e} (\vec{E} + \frac{\vec{v}_e - \vec{v}_p}{c} \times \vec{B})$$

$$- \frac{(v_e - v_p)}{\tau}$$

$$\left[\frac{E^2}{8\pi} \right] = \frac{\text{erg}}{\text{cm}^3}$$

$$\left[\frac{e^2}{r} \right] \sim \text{erg}$$

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$$[\sigma] = \frac{[e] \text{cm}^3 \cdot \text{cm} \text{s}^{-1}}{\text{erg}^{1/2} \text{cm}^{-3/2}} = \frac{\text{cm}^{1/2}}{\text{s}}$$

steady state)

$$v_e - v_p = - \frac{\tau e}{m_e} (\bar{E} + \bar{v} \times \bar{B})$$

w/o pressure

$$\bar{J} = -n_e e (v_e - v_p) = \underbrace{\frac{n_e e^2 \tau}{m_e}}_{\sigma} (\bar{E} + \bar{v} \times \bar{B})$$

with pressure

$$\sigma \rightarrow \infty \Leftrightarrow \tau \rightarrow \infty$$

$$\bar{E} = - \frac{\bar{v} \times \bar{B}}{c} - \frac{\nabla p_e}{en_e}$$

$$= g \cdot \text{cm}^2 \text{ s}^{-2}$$

$$\frac{1}{\omega} \approx 5^{-1}$$

steady state) w/o pressure

$$v_e - v_p = - \frac{\tau e}{m_e} (\bar{E} + \frac{\vec{v}}{c} \times \vec{B})$$

$$\vec{j} = -n_e e (v_e - v_p) = \underbrace{\frac{n_e e^2 \tau}{m_e}}_{\sigma} (\bar{E} + \frac{\vec{v}}{c} \times \vec{B})$$

$$\nabla \times [\alpha \nabla f] = \nabla \alpha \times \nabla f + \alpha \nabla^2 f$$

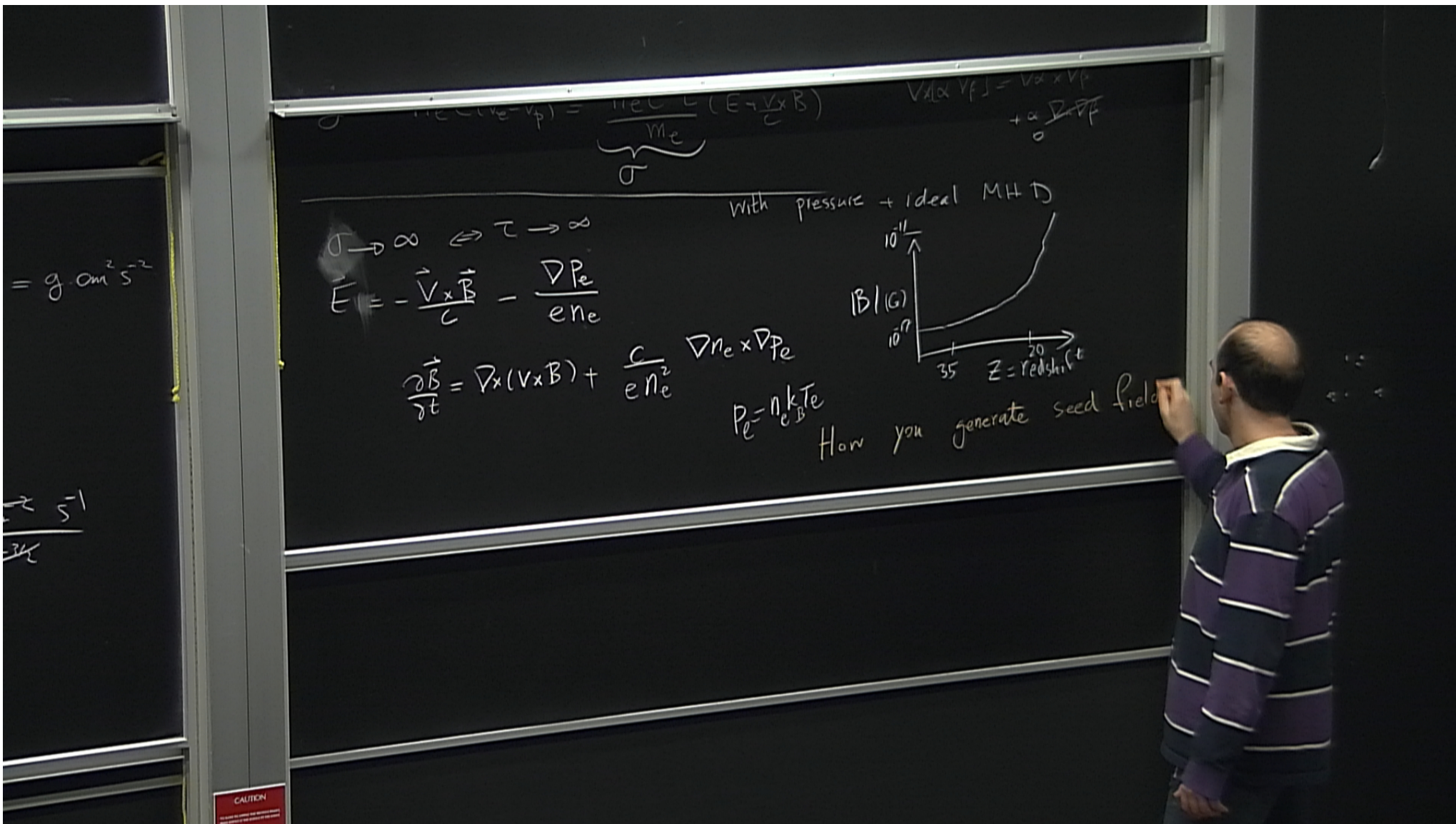
With pressure + ideal MHD

$$\sigma \rightarrow \infty \Leftrightarrow \tau \rightarrow \infty$$

$$\vec{E} = - \frac{\vec{v} \times \vec{B}}{c} - \frac{\nabla p_e}{en_e}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) - \frac{c}{en_e^2} \nabla n_e \times \nabla p_e$$





$\Omega m^2 s^{-2}$

Dynamo mechanism

$$\frac{B \cdot \text{Battery}}{v \times (v \times B)} \sim \frac{\frac{c}{e} L^{-2} (k_B T_e)}{v B}$$
$$\sim \left(\frac{c}{v}\right) \cdot \frac{1}{e} \cdot \frac{(k_B T_e)}{L B}$$

Dynamo mechanism

$$\frac{B \cdot \text{Battery}}{D \times (v \times B)} \sim \frac{\frac{c}{e} L^{-2} (k_B T_e)}{v B}$$

$$\sim \left(\frac{c}{v}\right) \cdot \frac{1}{e} \cdot \frac{(k_B T_e)}{L B}$$

Dynamo mechanism

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

$$\frac{B \cdot \text{Battery}}{v \times (v \times B)} \sim \frac{c}{e} \frac{L^{-2} (k_B T_e)}{v B}$$

$$\sim \left(\frac{c}{v} \right) \cdot \frac{1}{e} \cdot \frac{(k_B T_e)}{L B}$$

$$\sim \left(\frac{c}{v} \right) \times 10^3 \times \frac{10^{-16} \times 10^4}{10^{-6} \times 10^{18}}$$

$$\sim \frac{10^{13-12}}{10^{12}} \sim 10^{-11} \left(\frac{T_e}{10^4 \text{ K}} \right) \left(\frac{B}{\mu\text{G}} \right)^{-1}$$

$$\times \left(\frac{L}{\text{pc}} \right)^{-1} \left(\frac{v}{300 \text{ km/s}} \right)$$

$$\Omega m^2 s^{-2}$$

Dynamo mechanism

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

$$\dot{B}_i = M_{ij} B_j$$

$$\frac{B_{\text{Battery}}}{v \times (v \times B)} \sim \frac{c}{e} \frac{L^{-2} (k_B T_e)}{v B}$$

$\nearrow v \quad B \rightarrow 10^4 K$

$$\sim \left(\frac{c}{v}\right) \cdot \frac{1}{e} \cdot \frac{(k_B T_e)}{L B}$$

$$\sim \left(\frac{c}{v}\right) \times 10^3 \times \frac{10^{10}}{10^{-16} \times 10^4}$$

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$$\Omega m^2 s^{-2}$$

Dynamo mechanism

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$$

$$\lambda \propto \frac{\text{rotation freq.}}{\Omega}$$

$$\dot{B}_i = M_{ij} B_j$$

$$B \propto e^{\lambda t}$$

$$\frac{B \cdot \text{Battery}}{v \times (v \times B)} \sim \frac{c}{e} \frac{L^{-2} (k_B T_e)}{v B}$$

$$\begin{aligned} & \left(\frac{c}{v} \right) \cdot \frac{1}{e} \cdot \frac{(k_B T_e)}{L B} \\ & \sim \left(\frac{c}{v} \right) \times 10^3 \times \frac{10^{10} \times 10^{-16} \times 10^4}{10^{-6} \times 10^{18}} \\ & \sim \frac{10^{13-12}}{10^{12}} \sim 10^{-11} \left(\frac{T_e}{10^4 \text{ K}} \right) \left(\frac{B}{\mu\text{G}} \right)^{-1} \\ & \quad \times \left(\frac{L}{\text{pc}} \right)^{-1} \left(\frac{v}{300 \text{ km/s}} \right) \end{aligned}$$

$$\Omega \text{ s}^{-2}$$

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Magneto-Rotational instability

$\lambda \propto \text{rotation freq.}$
 Ω

$$\text{growth} \propto e^{-\Omega t} = e^{2\pi(t/\text{period})}$$

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Magneto-Rotational instability in disks

$$\lambda = \text{epicyclic freq.} = \sqrt{\frac{1}{r^3} \frac{d}{dr} [r^4 \Omega^2]}$$

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MHD

$$\left\{ \begin{array}{l} \frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{v} \times \vec{B})}_{\text{flux freezing}} + \underbrace{\frac{c^2}{4\pi\sigma} \nabla^2 \vec{B}}_{\text{flux diffusion}} \end{array} \right.$$

+ Euler eq with magnetic stress
after MRI develops

$$\alpha = 0.01 - 0.1$$

$$\frac{B^2}{8\pi} \sim \alpha (\text{Pressure})$$

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$$\left[\begin{array}{l} P_B \ll P \rightarrow \text{Dynamo} \\ \rightarrow P_B \gg P \rightarrow \end{array} \right. \text{equipartition}$$

MHD

$$\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{v} \times \vec{B})}_{\text{flux freezing}} + \underbrace{\frac{c^2}{4\pi\sigma} \nabla^2 \vec{B}}_{\text{flux diffusion}}$$

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$$\begin{array}{l} P_B \ll P \rightarrow \text{Dynamo} \\ \rightarrow P_B \gg P \rightarrow \text{reconnection} \end{array} \quad \begin{array}{l} \text{equipartition} \\ \nearrow \end{array}$$

after MRI develops, $\frac{B}{8\pi} \sim \alpha (\text{Pressure})$

$$\begin{aligned} & P_B \ll P_{th} \rightarrow \text{Dynamo} \\ & \rightarrow P_B \gg P_{th} \rightarrow \text{reconnection} \end{aligned} \quad \text{equipartition}$$

Ambipolar Diffusion

$$\begin{aligned} \frac{\partial \vec{B}}{\partial t} &= \nabla \times (\vec{v}_e \times \vec{B}) \\ &= \nabla \times (\vec{v}_{neutral} \times \vec{B}) + \frac{m_p c}{4\pi \rho_i \rho_n \sigma v_{th}} \vec{B} \cdot \nabla \vec{B} \end{aligned}$$

Biermann battery

Ohm's law

$$\vec{J} = \sigma (\vec{E} + \frac{\vec{v} \times \vec{B}}{c})$$

$$\vec{J} = -\bar{n}_e e (\vec{v}_e - \vec{v}_p)$$

$$\frac{d(\vec{v}_e - \vec{v}_p)}{dt} = -\frac{\nabla P_e}{m_e n_e} + \frac{\nabla P_p}{m_p n_p} - \frac{e}{m_e} (\vec{E} + \frac{\vec{v} \times \vec{B}}{c}) - \frac{(\vec{v}_e - \vec{v}_p)}{\tau}$$

after MRI develops, $\frac{B}{8\pi} \sim \alpha (\text{Pressure})$

$P_B \ll P_{th} \rightarrow \text{Dynamo}$
 $\rightarrow P_B \gg P_{th} \rightarrow \text{reconnection}$

equipartition

Ambipolar Diffusion

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v}_e \times \vec{B})$$

$$= \nabla \times (\vec{v}_{neutral} \times \vec{B}) + \frac{m_e c}{4\pi p_i p_n \sigma v_{th}} \vec{B}^2 \nabla^2 \vec{B}$$

$\rightarrow$ low ionization $\Rightarrow$ flux diffusion

In planet formation $\rightarrow$ what replaces MRI for accretion

Biermann battery

Ohm's law

$$\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B})$$

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MHD waves

$$\vec{B} = \vec{B}_0 + \delta\vec{B}$$

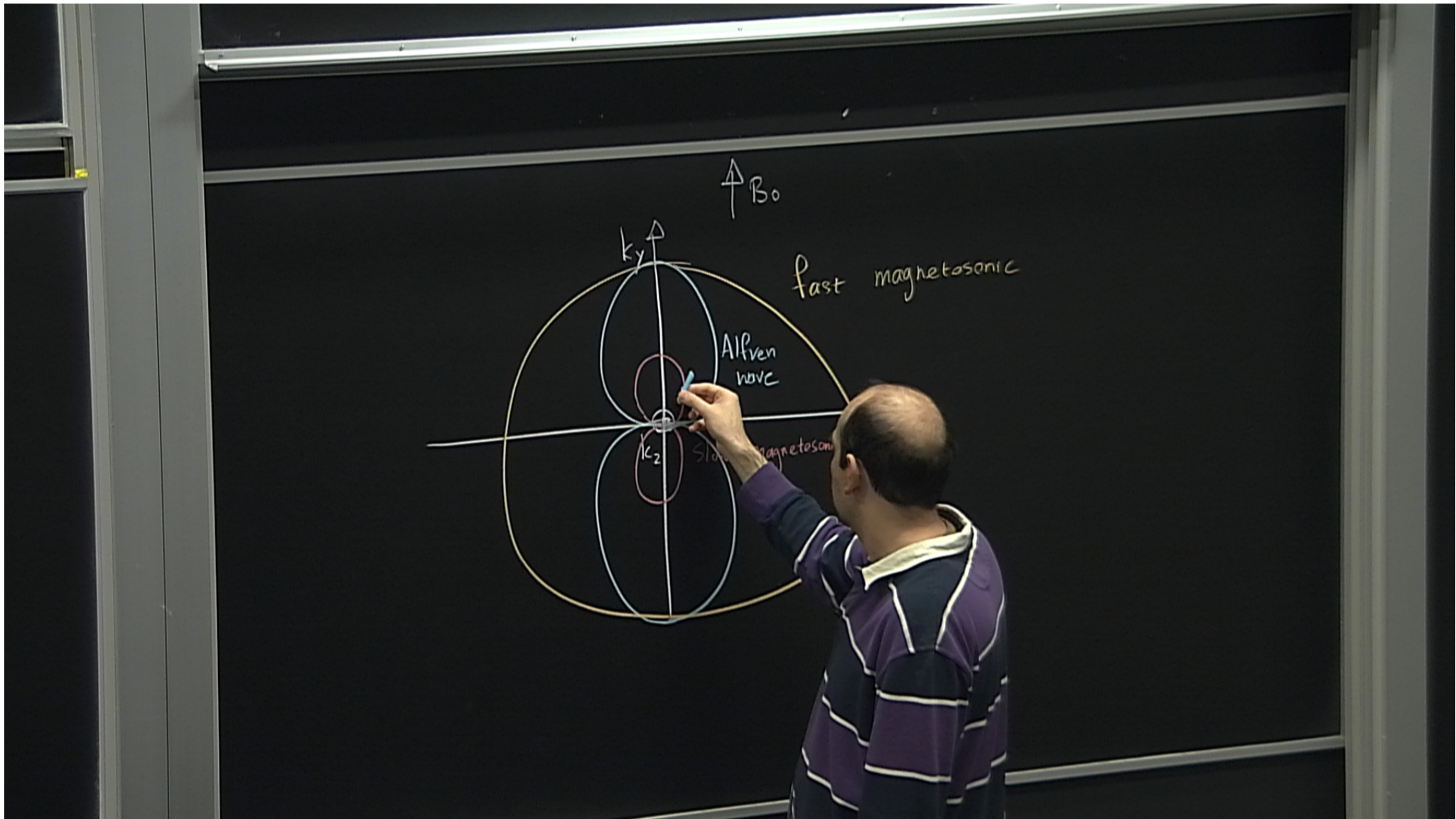
const. $\vec{B}$ in background

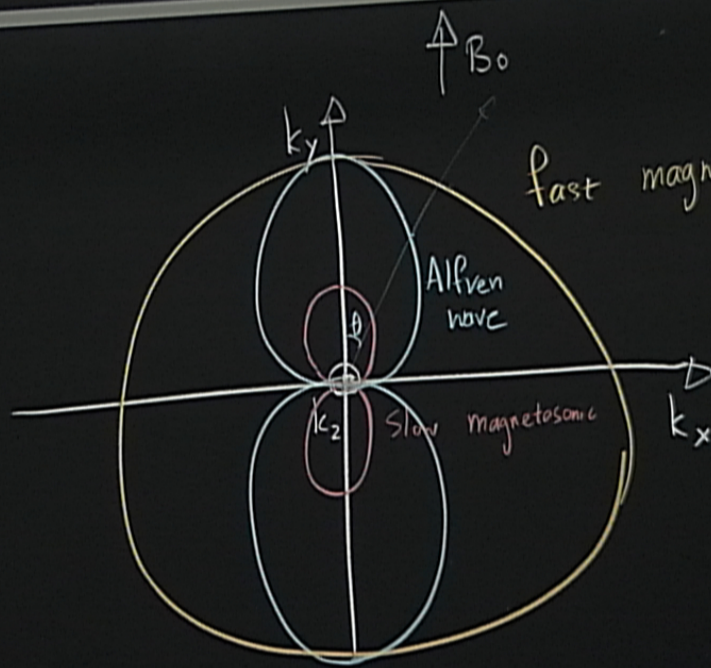
$$\vec{\nabla} \cdot \delta\vec{B} = 0, \quad \delta\dot{\vec{B}} = \vec{\nabla} \times (\delta\vec{v} \times \vec{B})$$

$$\delta\dot{\rho} + \rho_0 \vec{\nabla} \cdot \delta\vec{v} = 0, \quad \delta\dot{\vec{v}} = -c_s^2 \frac{\vec{\nabla} \delta\rho}{\rho_0} - \frac{1}{4\pi} \vec{B}_0 \times (\vec{\nabla} \times \delta\vec{B})$$

$$\begin{aligned} \vec{\nabla} &\rightarrow i\vec{k} \\ \partial_t &\rightarrow -i\omega \end{aligned}$$

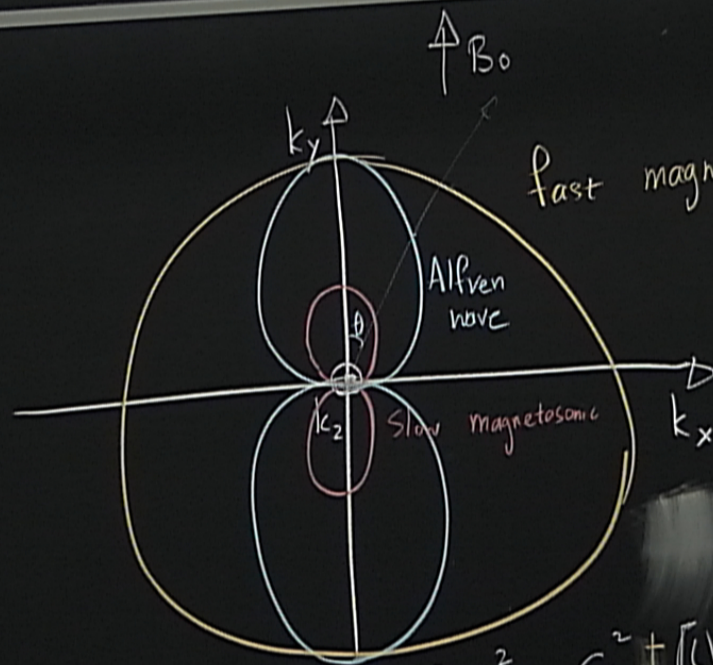
$$M(\omega, \vec{k}) \begin{pmatrix} \delta\vec{B} \\ \delta\vec{v} \\ \delta\rho \end{pmatrix} = 0 \Rightarrow \det M = 0$$





Fast magnetosonic
Alfven:

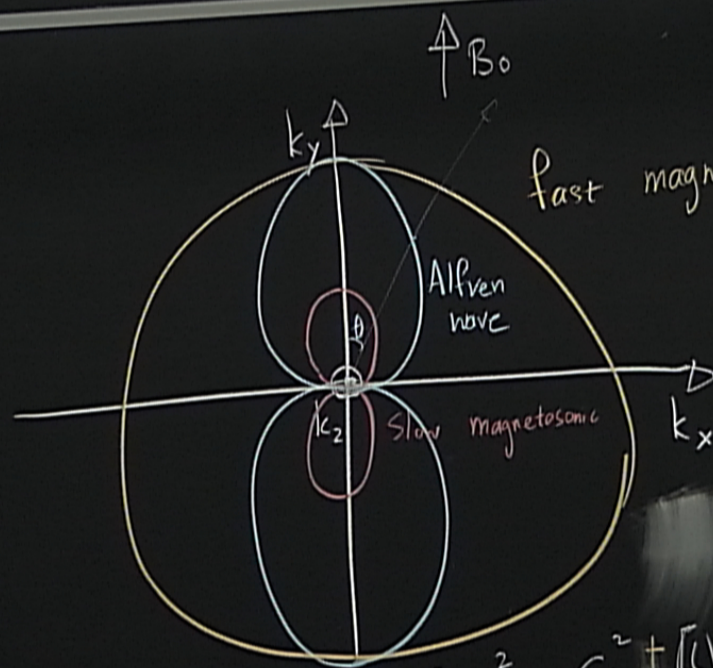
$$\frac{\omega^2}{k^2} = v_A^2 = \frac{B_0^2 c^2}{4\pi n}$$



Fast magnetosonic

Alfvén: $\frac{\omega^2}{k^2} = V_A^2 = \frac{B^2 C_s^2 \theta}{4\pi\rho}$

$$V_{ms}^2 = \frac{1}{2} \left\{ V_{A0}^2 + C_s^2 \pm \left[(V_{A0}^2 + C_s^2)^2 - 4V_{A0}^2 C_s^2 \cos^2 \theta \right]^{1/2} \right\}$$



$$V_{A0}^2 = \frac{B^2}{4\pi\rho}$$

Alfvén: $\frac{\omega^2}{k^2} = V_A^2 = \frac{B^2 \cos^2 \theta}{4\pi\rho}$

Fast: $\max[V_A, C_s] < V_F < \sqrt{V_A^2 + C_s^2}$

slow: $0 < V_S < V_A$

$$V_{MS}^2 = \frac{1}{2} \left\{ V_{A0}^2 + C_s^2 \pm \left[(V_{A0}^2 + C_s^2)^2 - 4V_{A0}^2 C_s^2 \cos^2 \theta \right]^{1/2} \right\}$$

$$C_s \rightarrow V_{A0}$$

$$V_{A0}^2 = \frac{B^2}{4\pi\rho}$$

fast magnetosonic

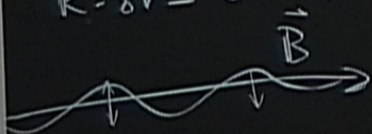
$$\text{Alfven: } \frac{\omega^2}{k^2} = V_A^2 = \frac{B^2 \cos^2 \theta}{4\pi\rho}$$

$$\text{fast: } \max[V_{A0} C_s] < V_f < \sqrt{V_{A0}^2 + C_s^2}$$

$$\text{slow: } 0 < V_s < V_A$$

for Alfven

$$\vec{k} \cdot \delta \vec{v} = 0 \Rightarrow \delta p = 0$$



$$V_{ms}^2 = \frac{1}{2} \left\{ V_{A0}^2 + C_s^2 \pm \left[(V_{A0}^2 + C_s^2)^2 - 4V_{A0}^2 C_s^2 \cos^2 \theta \right]^{1/2} \right\}$$