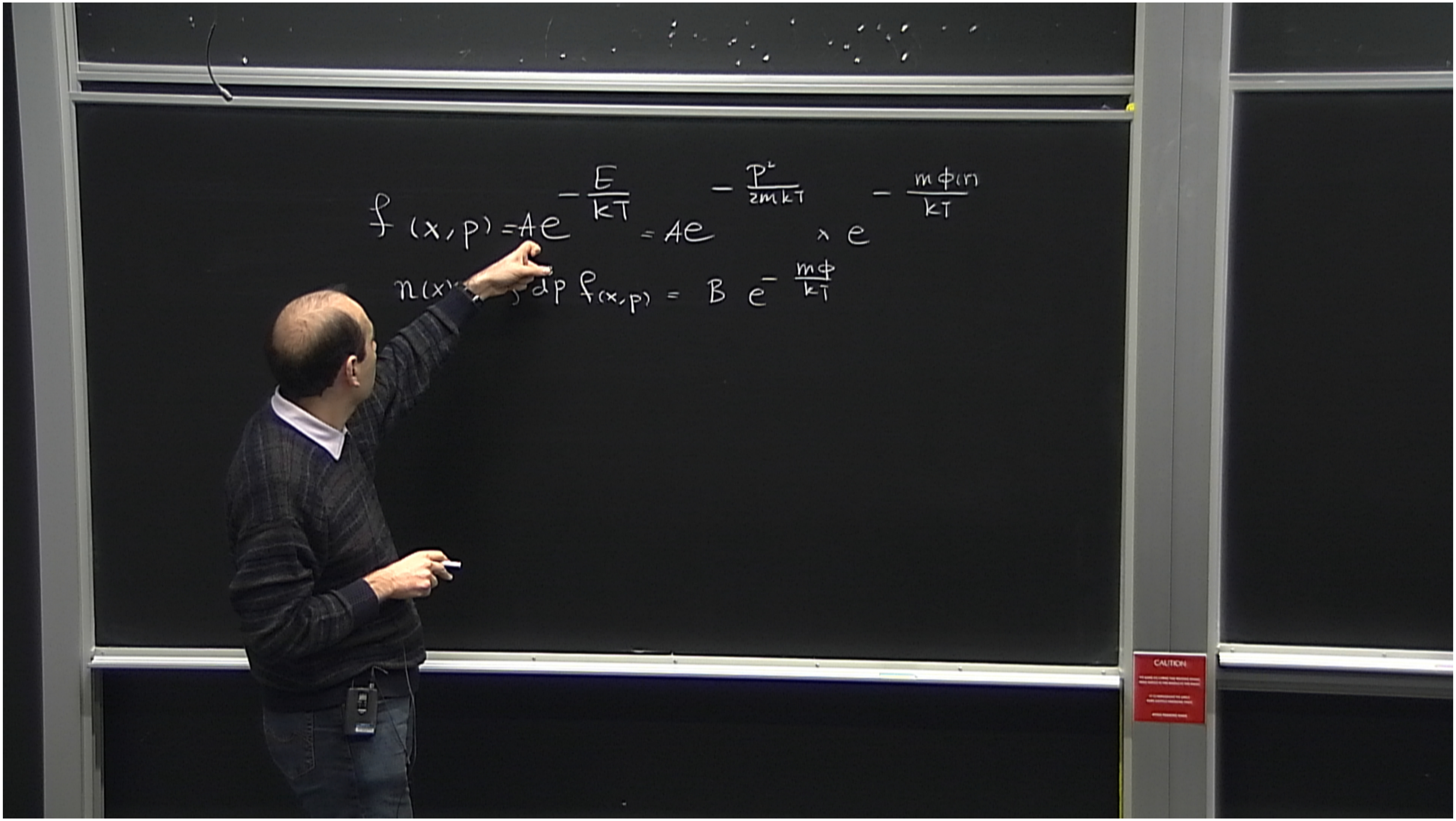
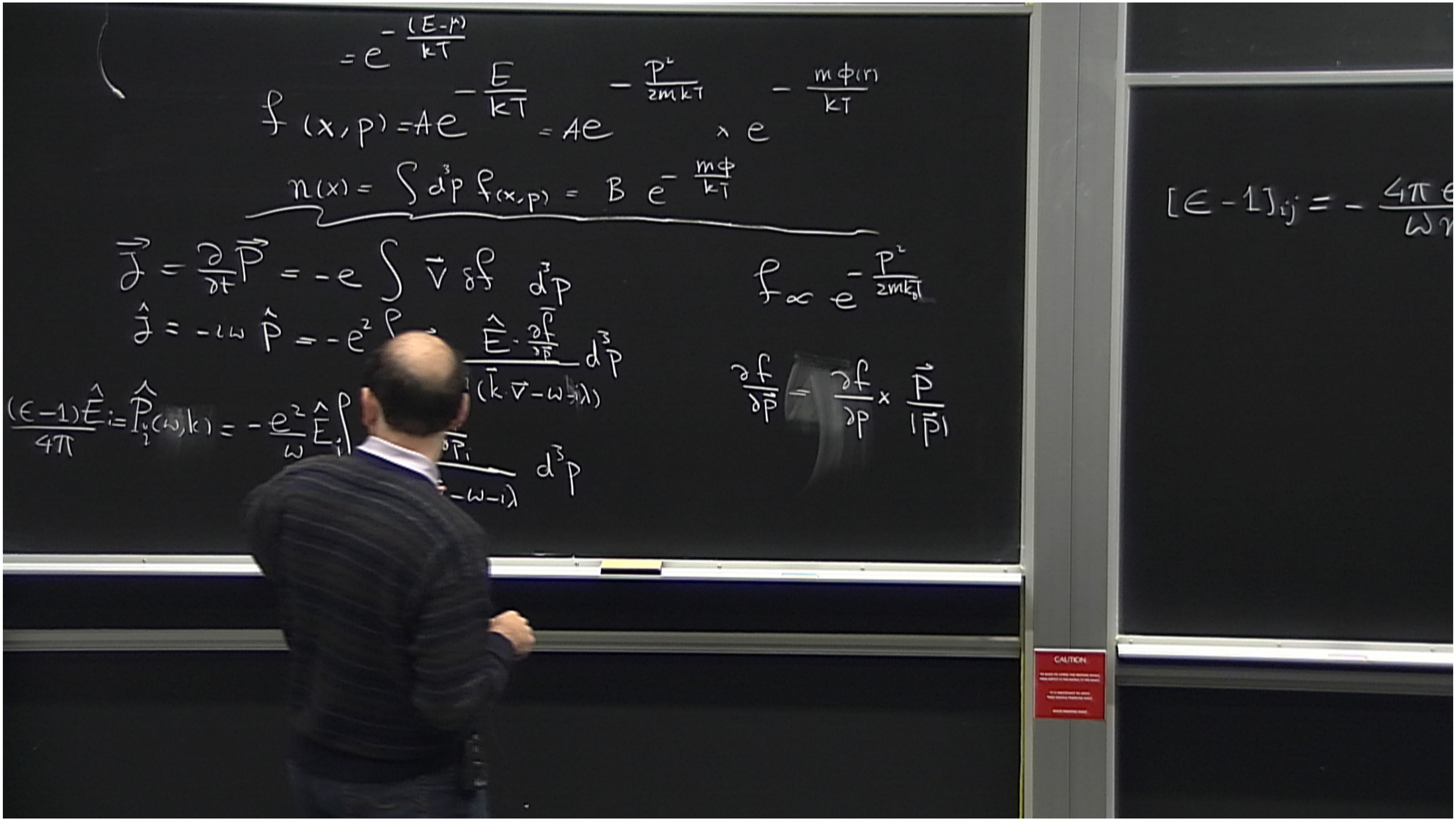
Title: PHYS 781 - Lecture 18

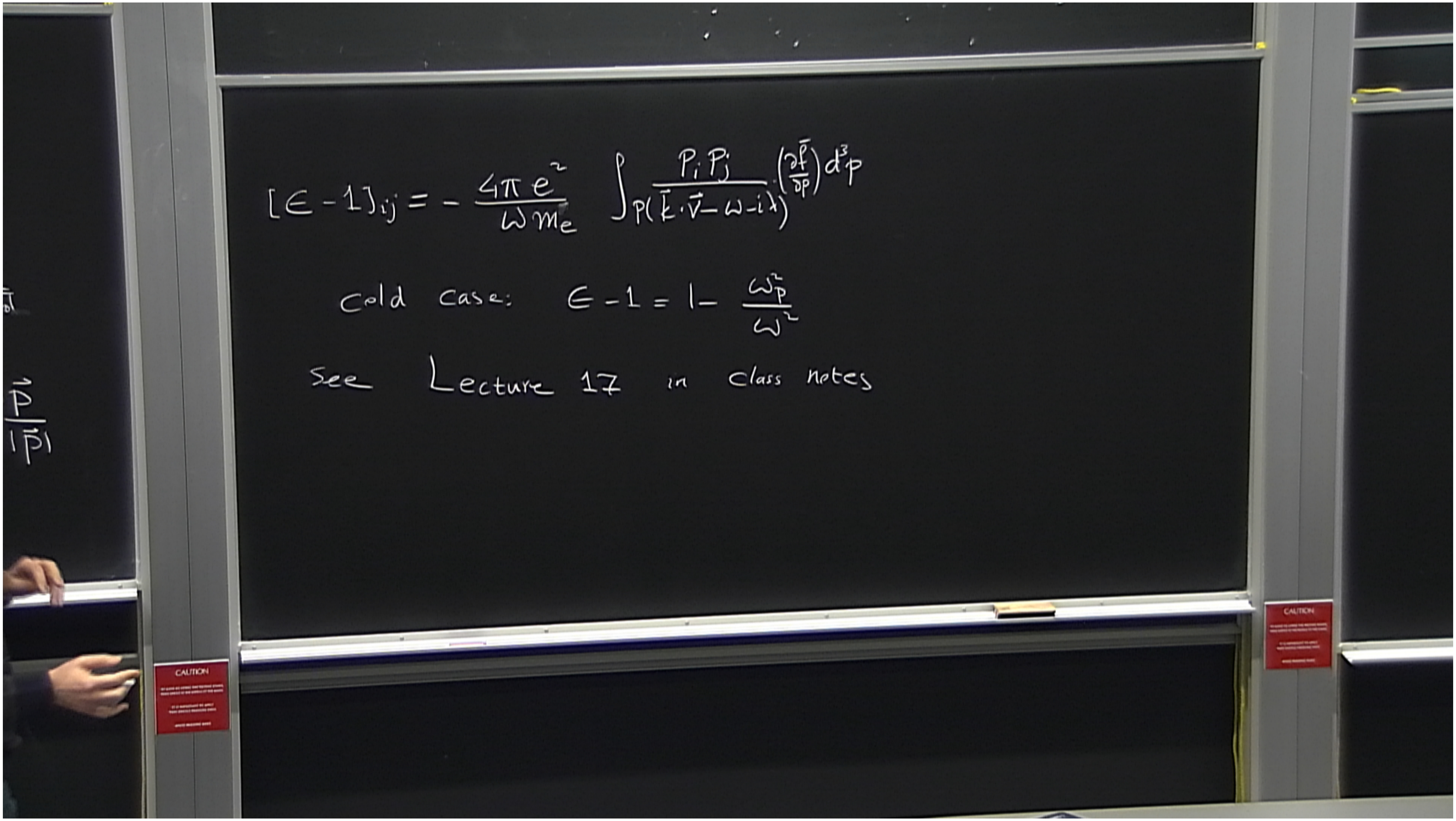
Date: Nov 13, 2014 02:00 PM

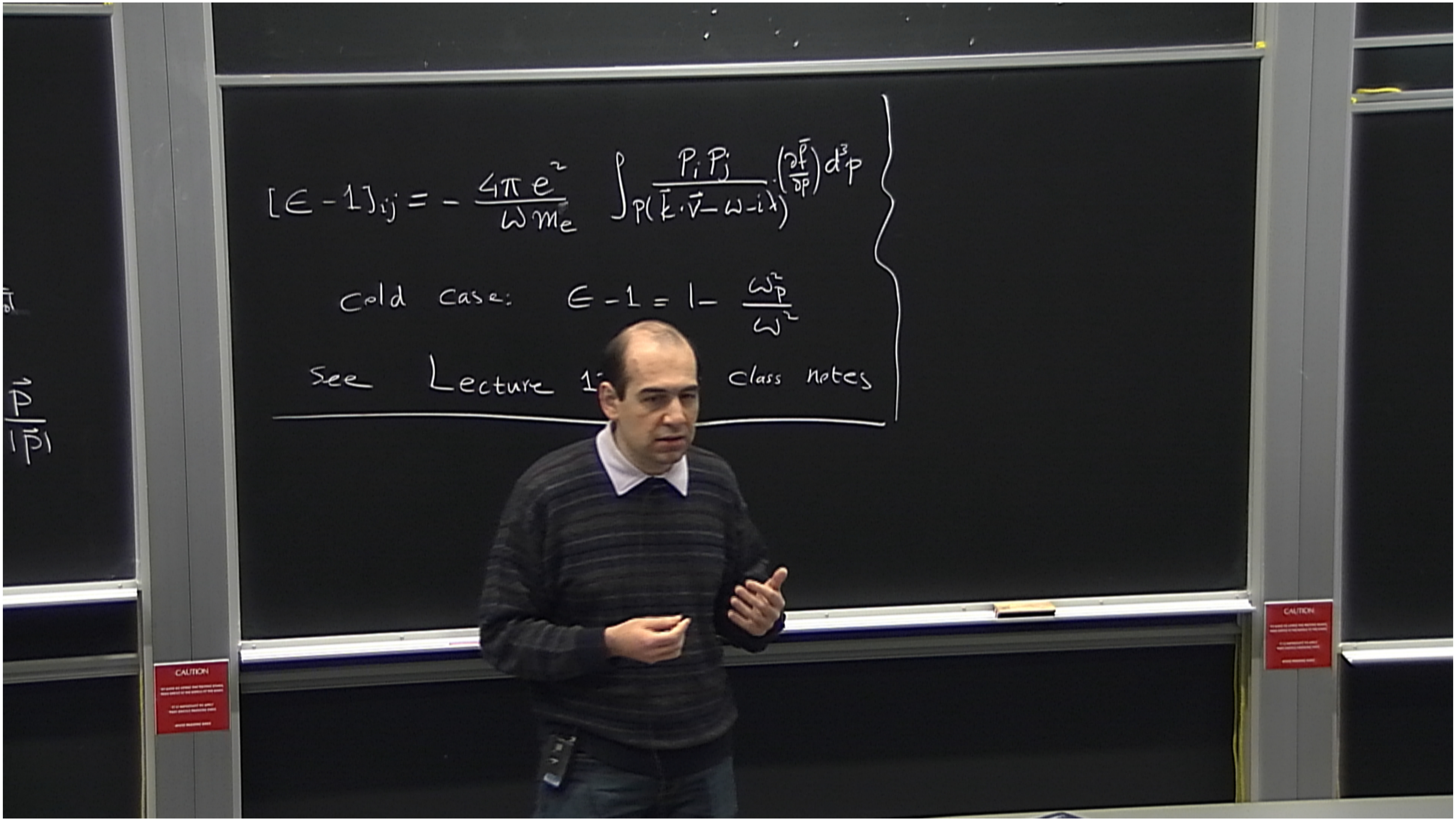
URL: http://pirsa.org/14110054

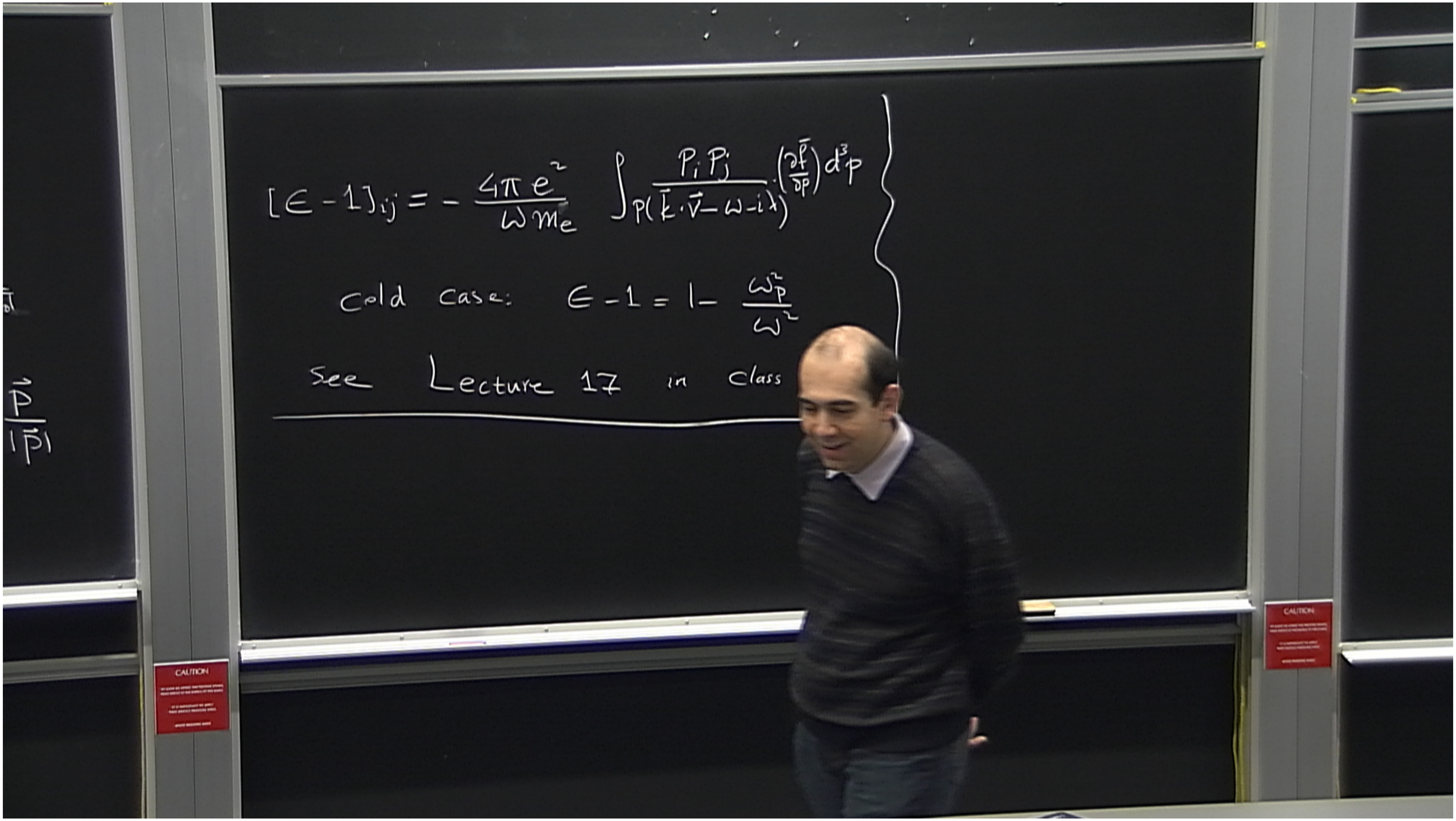
Abstract:

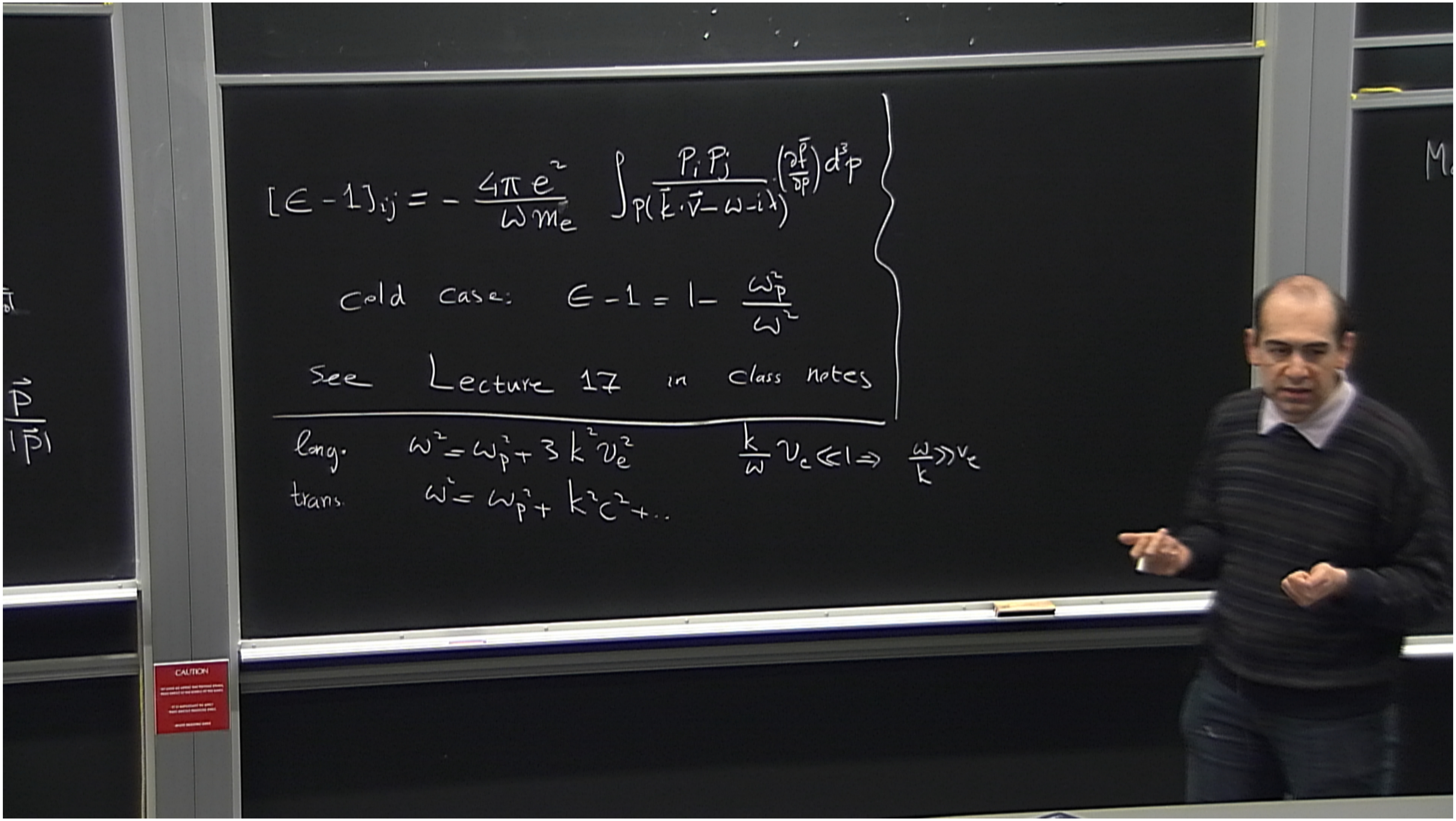

$f(x,p) = A e^{-\frac{E}{kT}} = A e^{-\frac{p^2}{2mkT}} \times e^{-\frac{m\phi(r)}{kT}}$
$n(x) \ldots \int dp\, f_{(x,p)} = B\, e^{-\frac{m\phi}{kT}}$

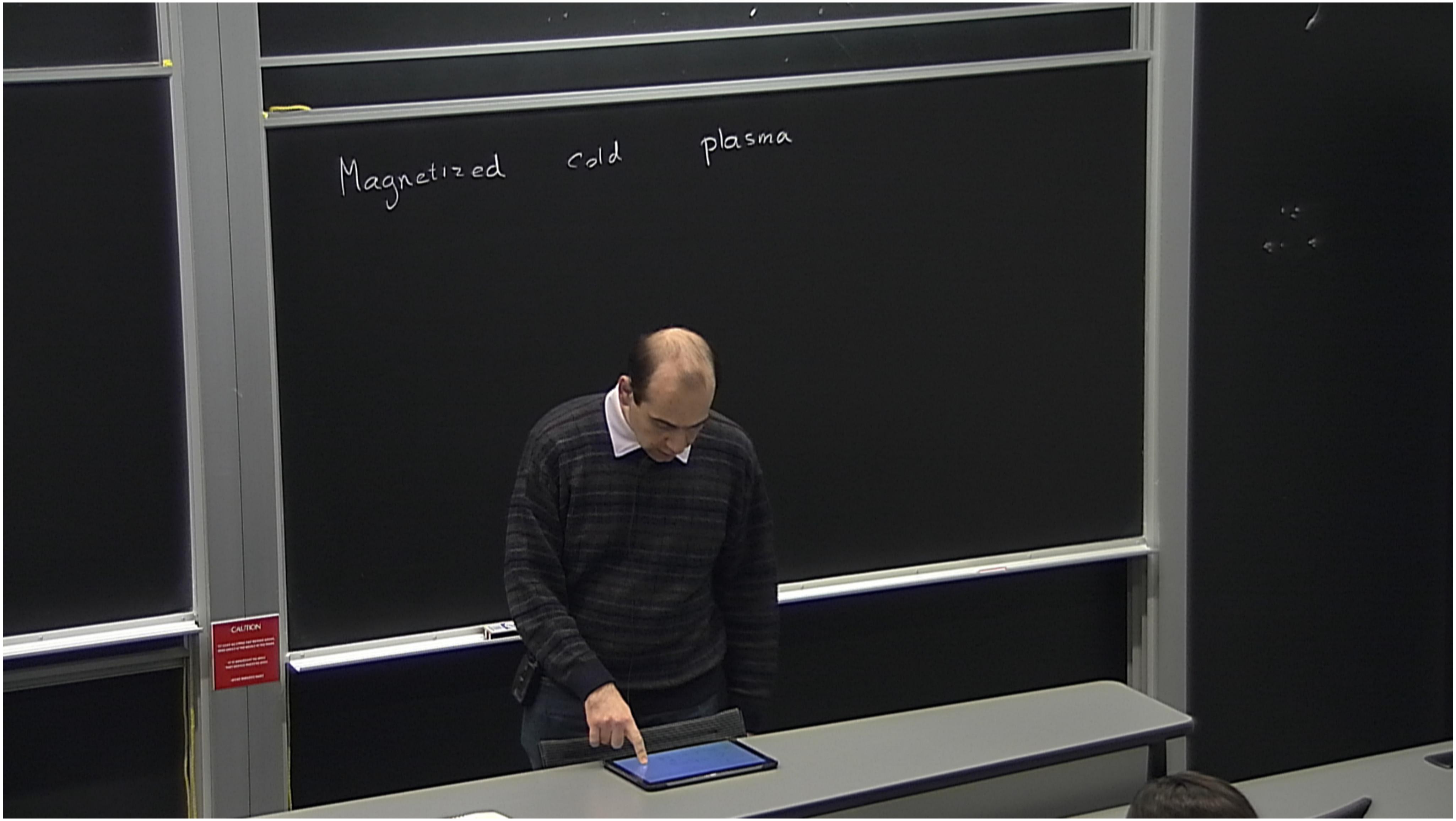

$= e^{-\frac{(E-\mu)}{kT}}$
$f(x,p) = Ae^{-\frac{E}{kT}} = Ae^{-\frac{p^2}{2mkT}} \times e^{-\frac{m\phi(r)}{kT}}$
$n(x) = \int d^3p\, f(x,p) = B\, e^{-\frac{m\phi}{kT}}$
$\vec{J} = \frac{\partial}{\partial t}\vec{P} = -e\int \vec{v}\,\delta f\, d^3p$
$f \propto e^{-\frac{p^2}{2mk_BT}}$
$\frac{\partial f}{\partial \vec{p}} = \frac{\partial f}{\partial p} \times \frac{\vec{p}}{|\vec{p}|}$
$[\epsilon - 1]_{ij} = -\frac{4\pi e}{\omega}$

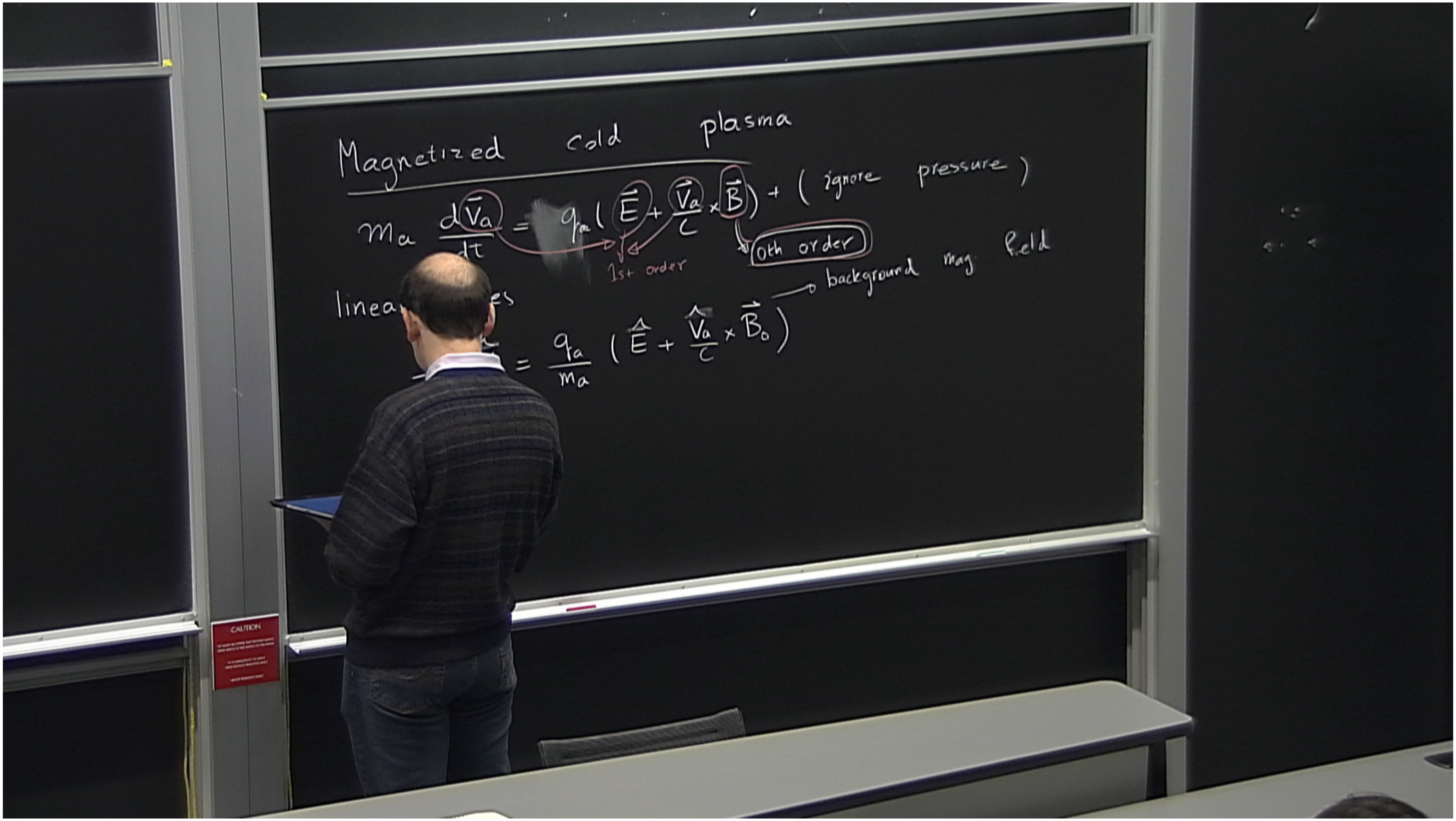

$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega m_e} \int \frac{p_i p_j}{p(\vec{k}\cdot\vec{v} - \omega - i\lambda)} \left(\frac{\partial \bar{f}}{\partial p}\right) d^3p$
Cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$
See Lecture 17 in class notes

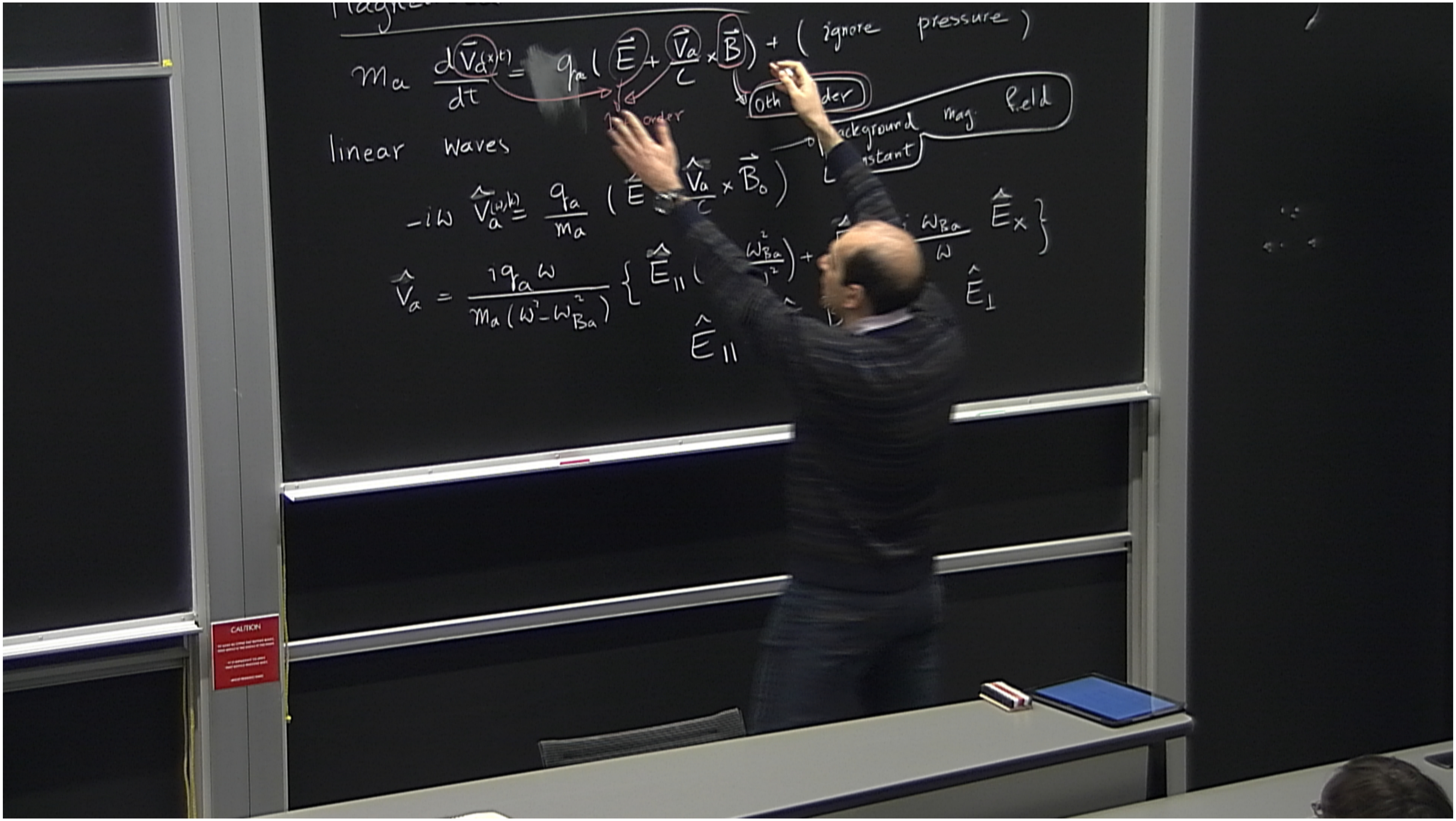

$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega m_e} \int \frac{p_i p_j}{p(\vec{k}\cdot\vec{v} - \omega - i\lambda)} \left(\frac{\partial \bar{f}}{\partial p}\right) d^3p$
Cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$
See Lecture 1
class notes

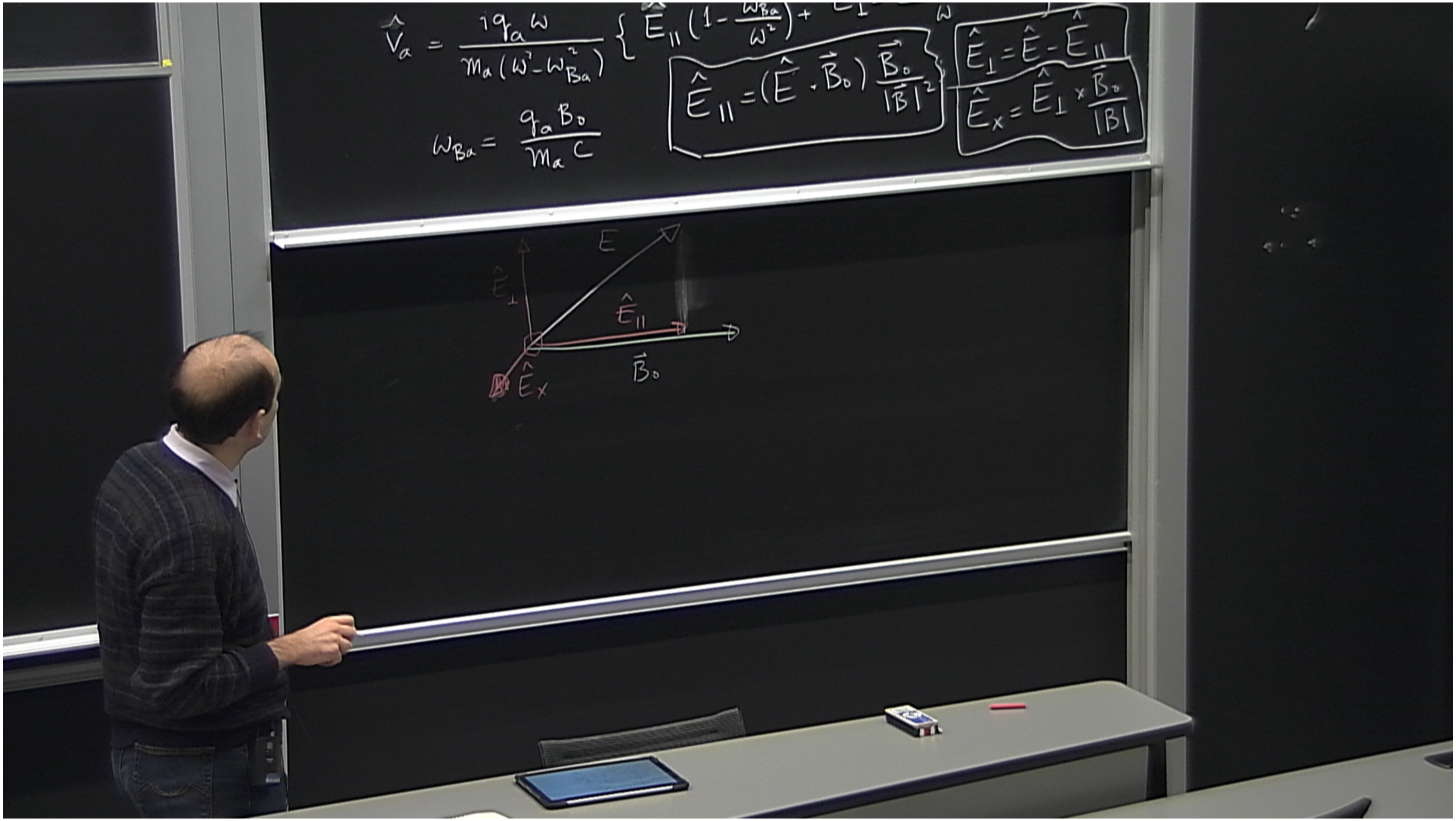

$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega m_e} \int \frac{p_i p_j}{p(\vec{k}\cdot\vec{v} - \omega - i\lambda)} \left(\frac{\partial \bar{f}}{\partial p}\right) d^3p$
Cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$
See Lecture 17 in class
$\frac{\vec{p}}{|\vec{p}|}$

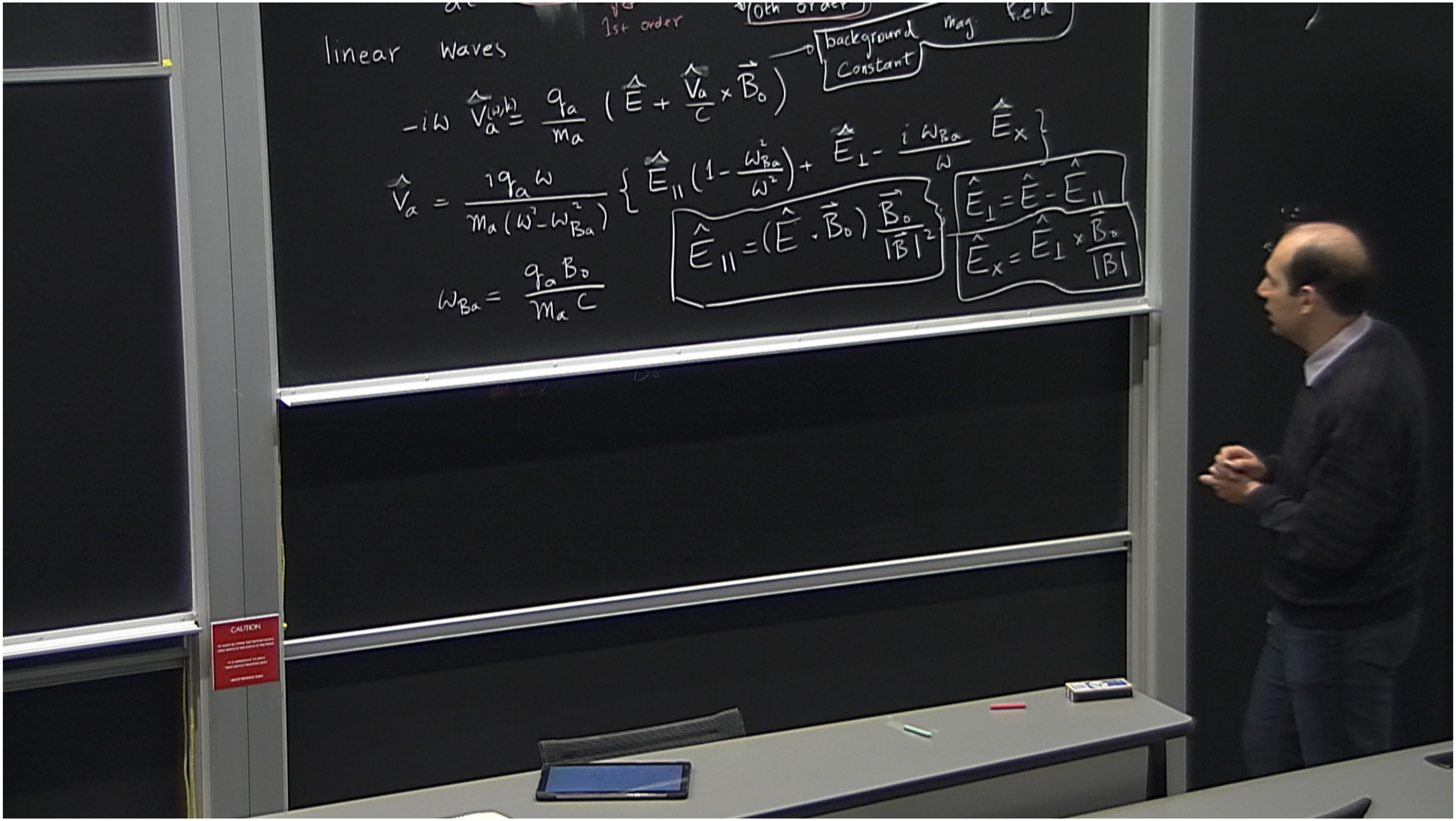

$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega m_e} \int \frac{p_i p_j}{p(\vec{k}\cdot\vec{v} - \omega - i\lambda)} \left(\frac{\partial \bar{f}}{\partial p}\right) d^3p$
cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$
See Lecture 17 in class notes
long. $\omega^2 = \omega_p^2 + 3k^2 v_e^2$
$\frac{k}{\omega} v_e \ll 1 \Rightarrow \frac{\omega}{k} \gg v_e$
trans. $\omega^2 = \omega_p^2 + k^2 c^2 + \ldots$


Magnetized cold plasma
CAUTION

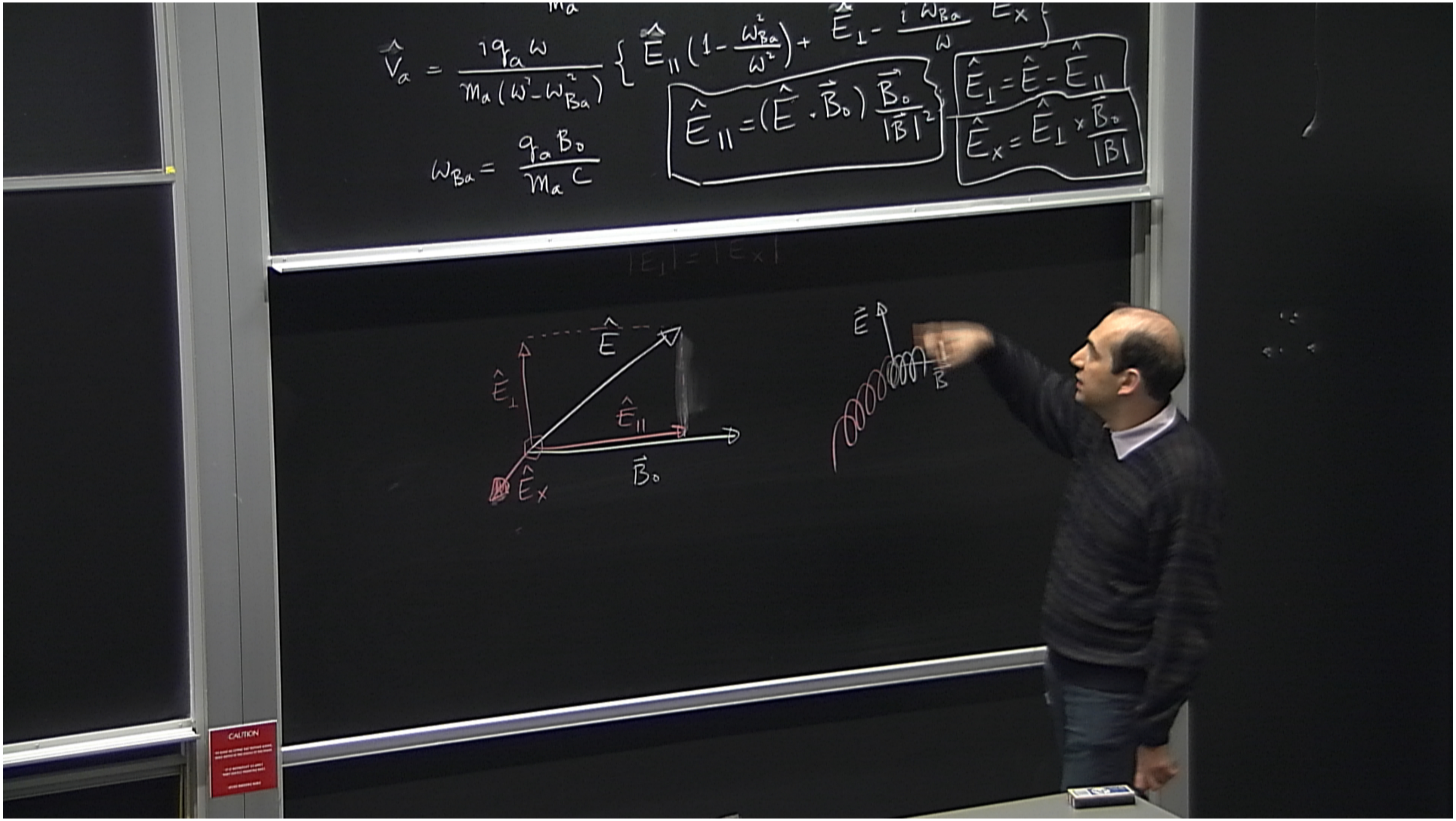

Magnetized cold plasma
(ignore pressure)
1st order
0th order
background mag field

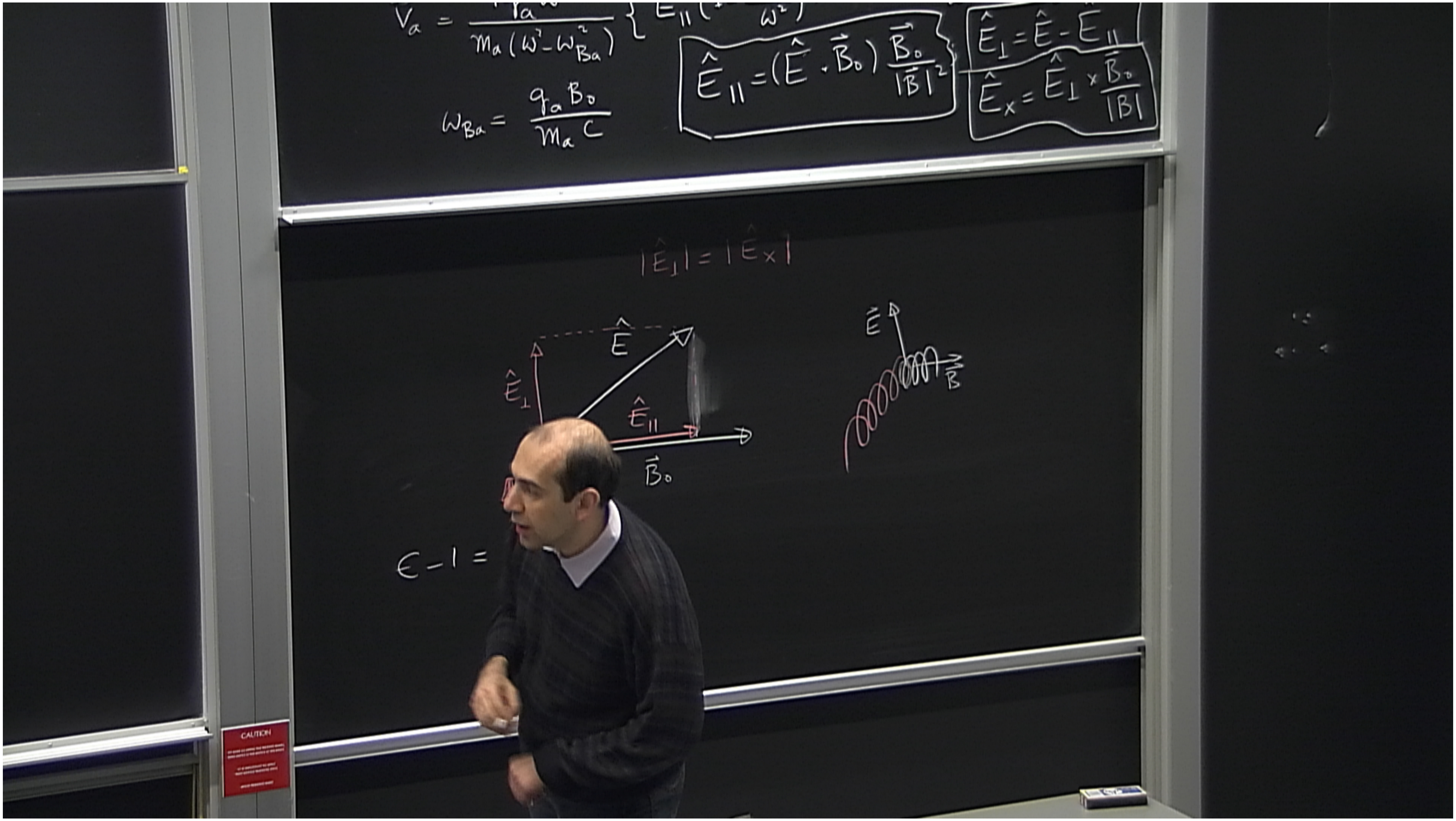

(ignore pressure)
0th
order
mag.
field
linear waves
CAUTION


$\hat{V}_a = \frac{i q_a \omega}{m_a(\omega^2 - \omega_{Ba}^2)}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$
$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$
$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
E
$\hat{E}_{\perp}$
$\hat{E}_{\parallel}$
$\vec{B}_0$
$\hat{E}_x$

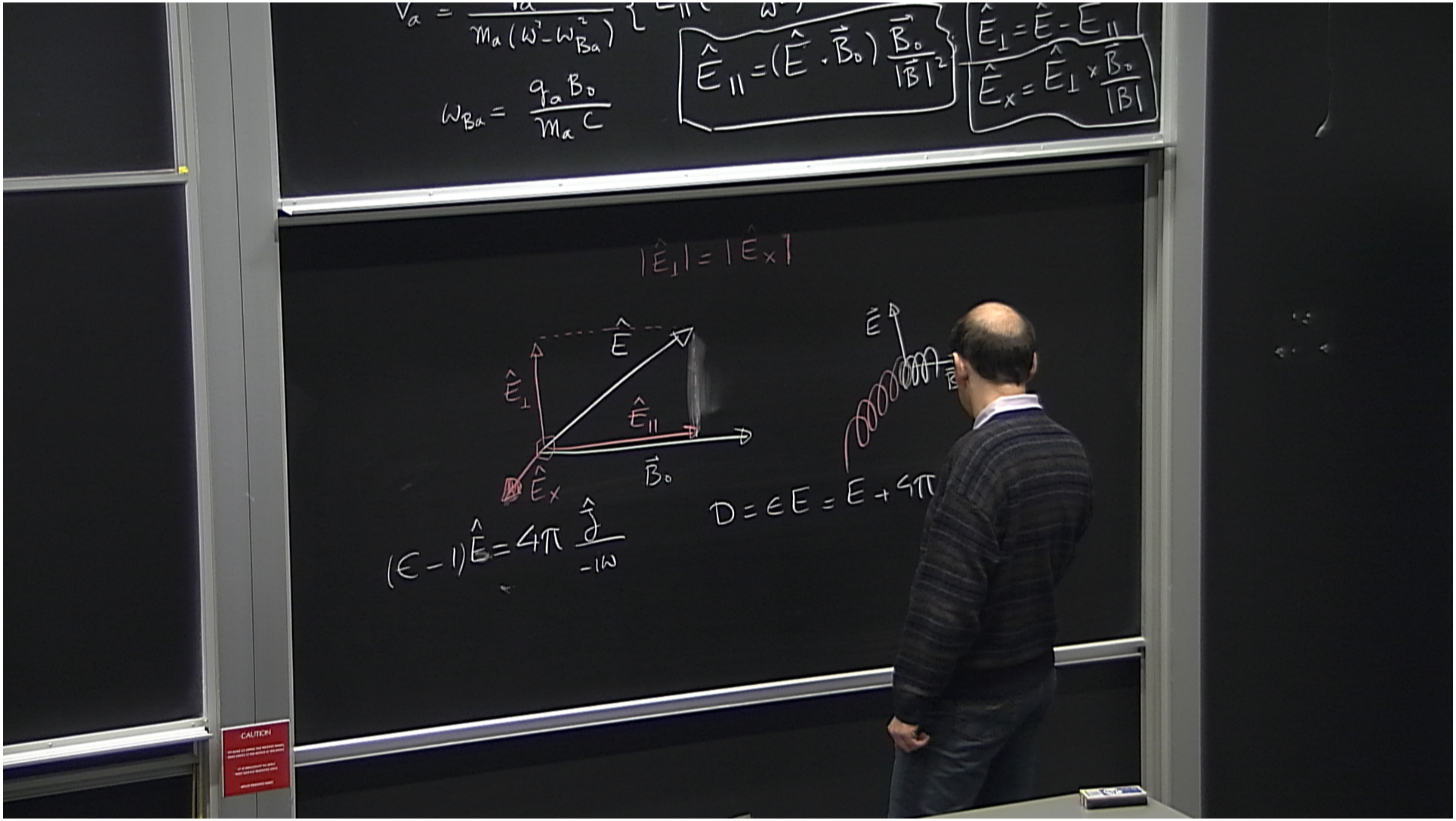


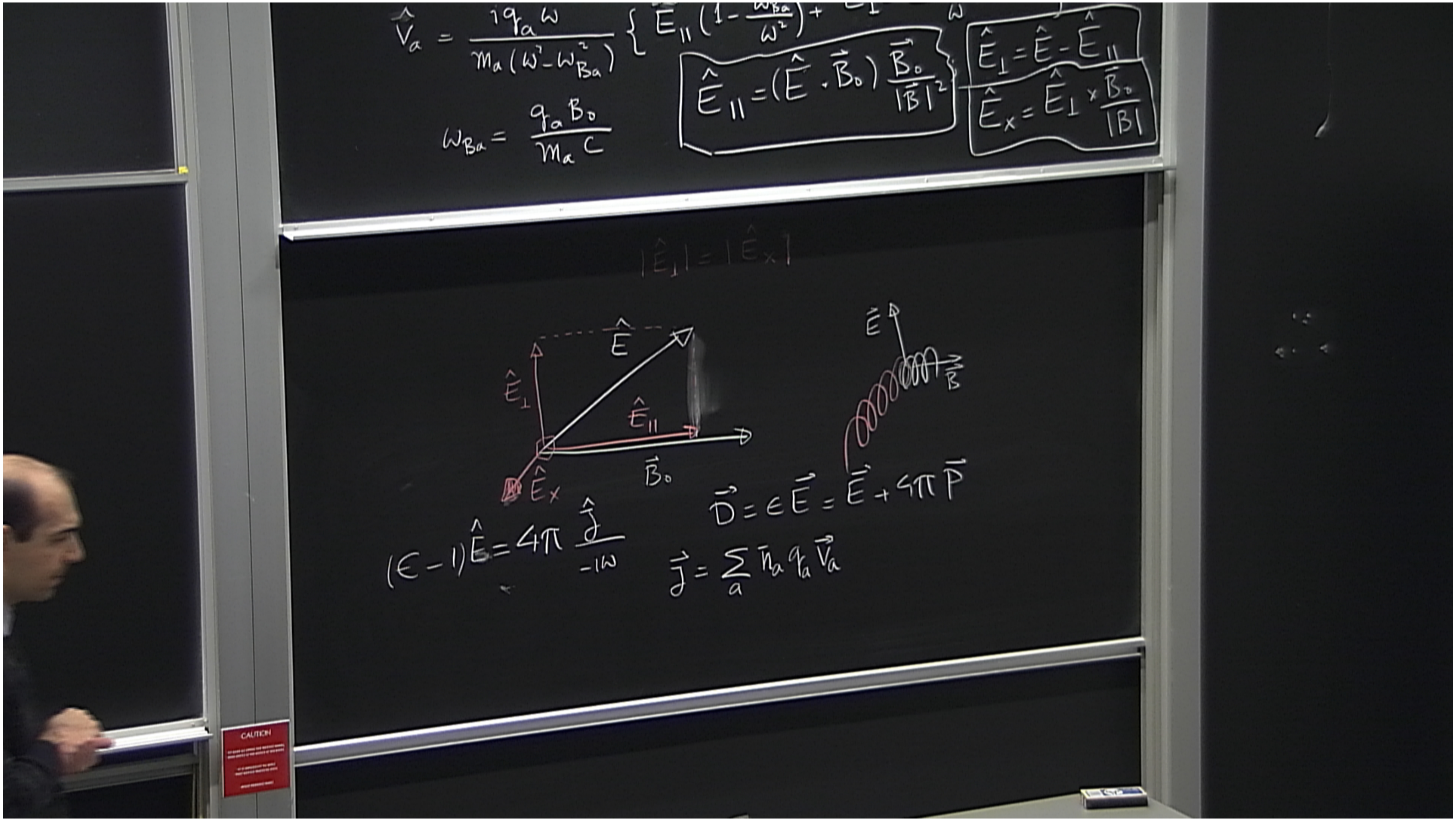
1st order
0th order
linear waves
background mag field
Constant
$-i\omega \hat{V}_a^{(\omega,k)} = \frac{q_a}{m_a}\left(\hat{E} + \frac{\hat{V}_a}{c} \times \vec{B}_0\right)$
$\hat{V}_a = \frac{i q_a \omega}{m_a(\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel}\left(1 - \frac{\omega_{Ba}^2}{\omega^2}\right) + \hat{E}_{\perp} - \frac{i\omega_{Ba}}{\omega} \hat{E}_x \right\}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|\vec{B}|^2}$
$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$
$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|\vec{B}|}$
$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
CAUTION

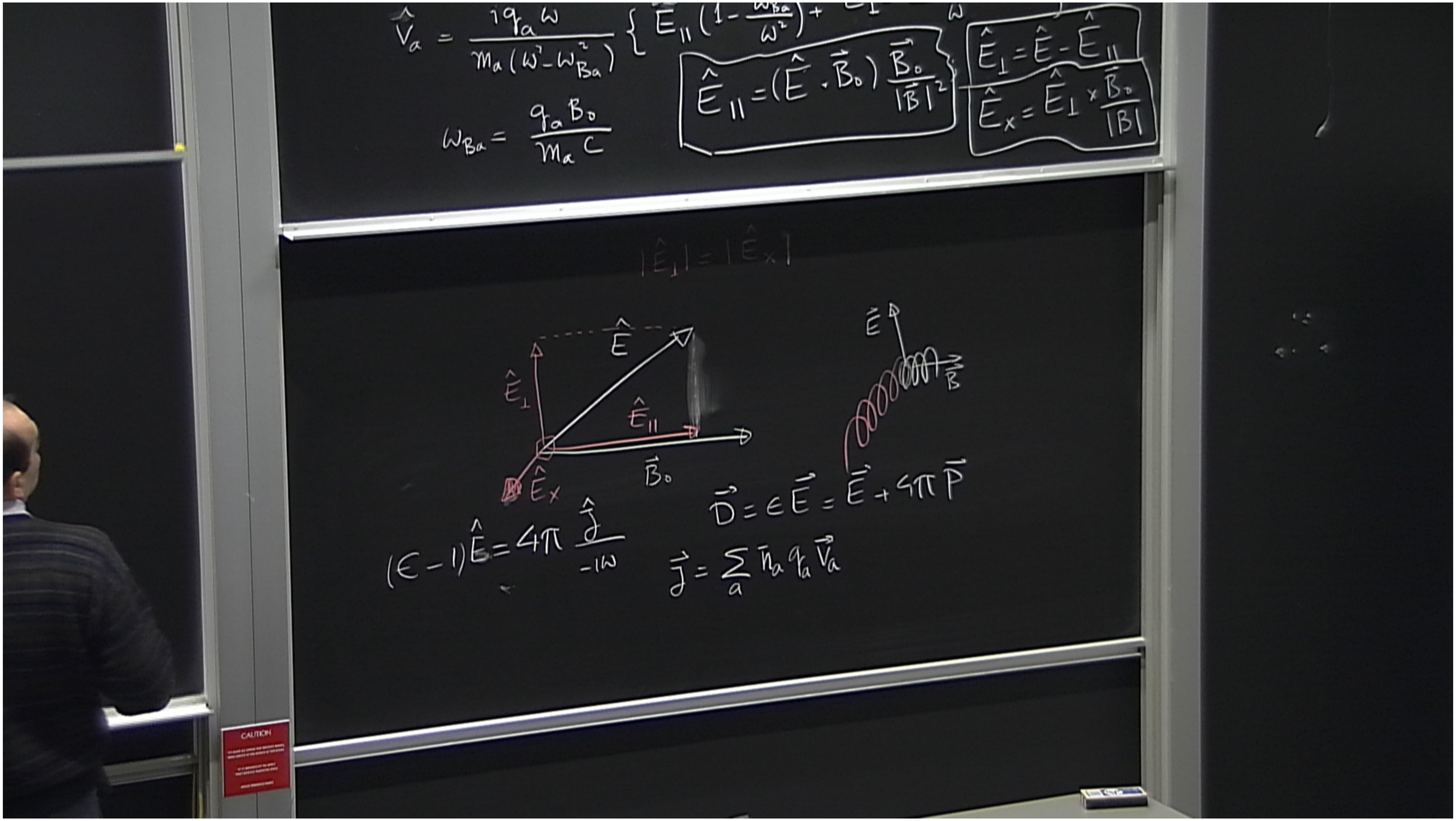

$\hat{V}_a = \frac{i q_a \omega}{m_a(\omega^2 - \omega_{Ba}^2)} \{ \hat{E}_{\parallel} (1 - \frac{\omega_{Ba}^2}{\omega^2}) + \hat{E}_{\perp} - \frac{i\omega_{Ba}}{\omega} \hat{E}_x \}$
$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$
$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$
$\hat{E}$
$\hat{E}_{\perp}$
$\hat{E}_{\parallel}$
$\vec{B}_0$
$\hat{E}_x$
$\vec{E}$
$\vec{B}$
CAUTION



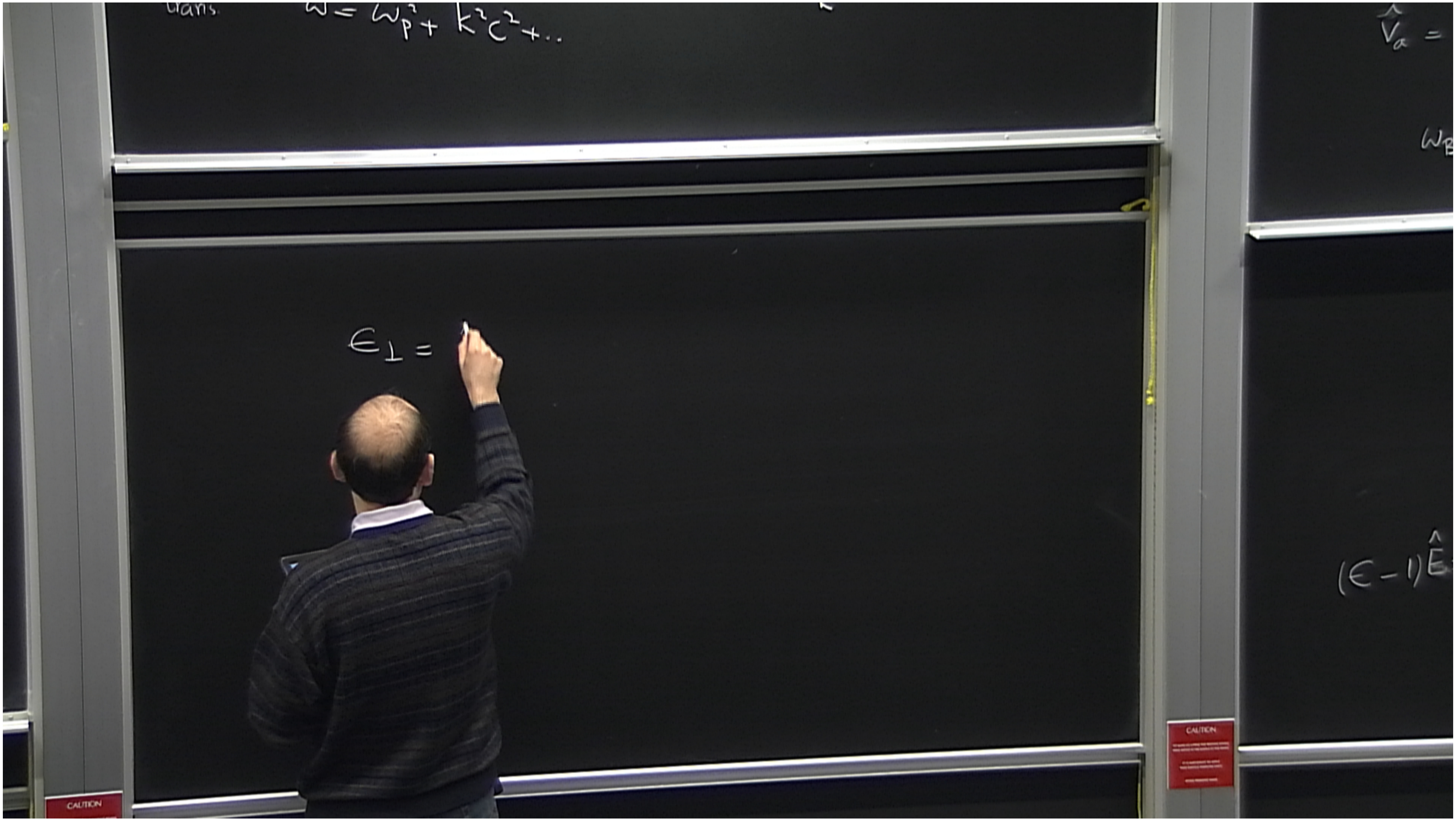
$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_{\perp} = \hat{E} - E_{\parallel}$
$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$
$|\hat{E}_{\perp}| = |\hat{E}_x|$
$\epsilon - 1 =$


$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$
$\hat{E}_{x} = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$
$|\hat{E}_{\perp}| = |\hat{E}_x|$
$\hat{E}$
$\hat{E}_{\perp}$
$\hat{E}_{\parallel}$
$\vec{B}_0$
$\hat{E}_x$
$\vec{E}$
$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{J}}{-i\omega}$
$D = \epsilon E = E + 4\pi$


$\hat{V}_a = \frac{i q_a \omega}{m_a(\omega^2 - \omega_{Ba}^2)}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_\perp = \hat{E} - \hat{E}_\parallel$
$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|B|}$
$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$|\hat{E}_\perp| = |\hat{E}_x|$
$\hat{E}$
$\hat{E}_\perp$
$\hat{E}_\parallel$
$\vec{B}_0$
$\hat{E}_x$
$\vec{E}$
$\vec{B}$
$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$
$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{J}}{-i\omega}$
$\vec{J} = \sum_a \bar{n}_a q_a \vec{V}_a$
CAUTION



$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$
$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$
$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$
$|\hat{E}_{\perp}| = |\hat{E}_x|$
$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$
$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{J}}{-i\omega}$
$\vec{J} = \sum_a n_a q_a \vec{V}_a$


$\omega = \omega_p^2 + k^2c^2 + ..$
$\epsilon_\perp =$
$(\epsilon - 1)\hat{E}$
CAUTION

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$$

$$\boxed{\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}}$$

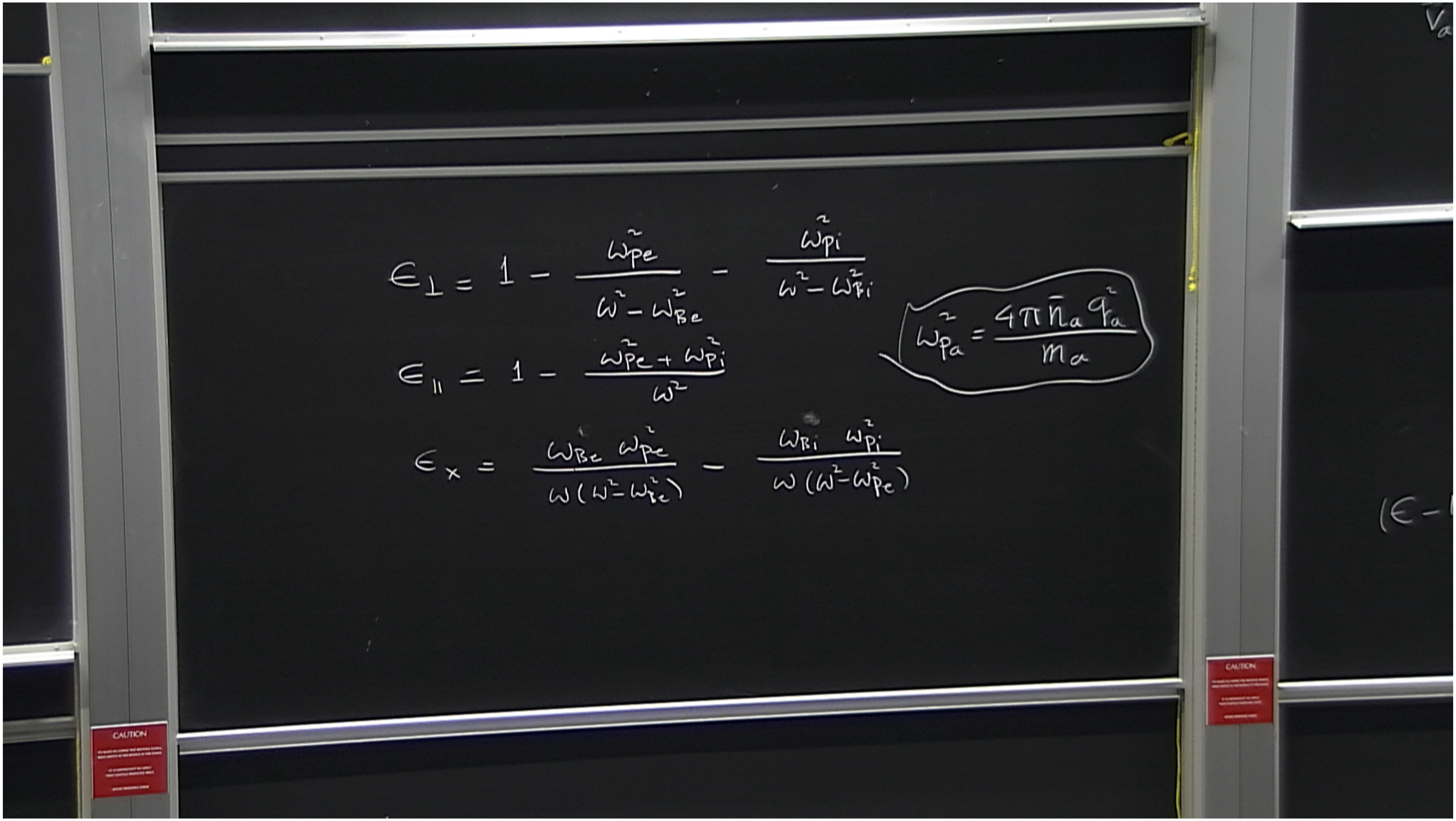
$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

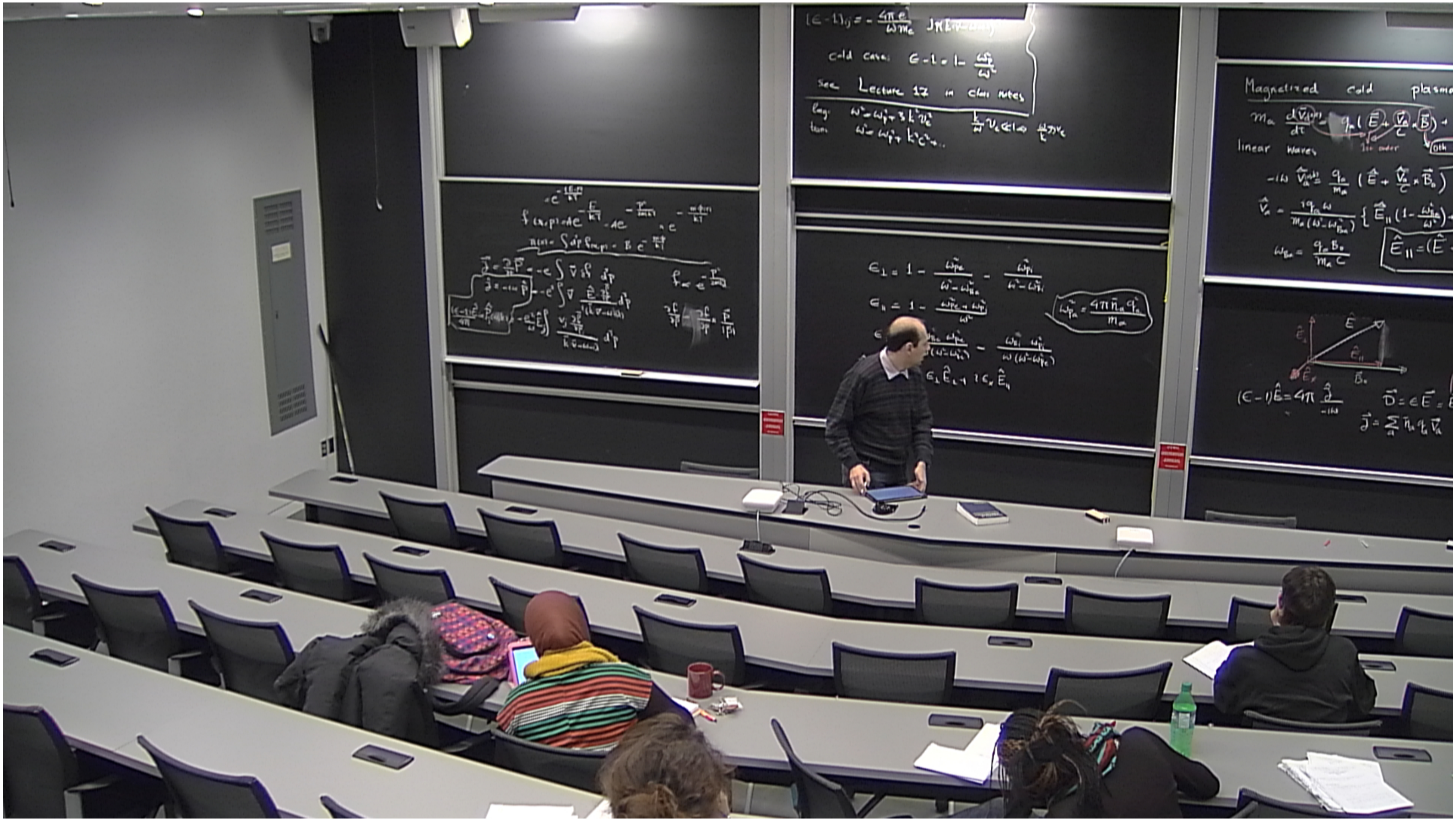
$$\epsilon_{x} = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

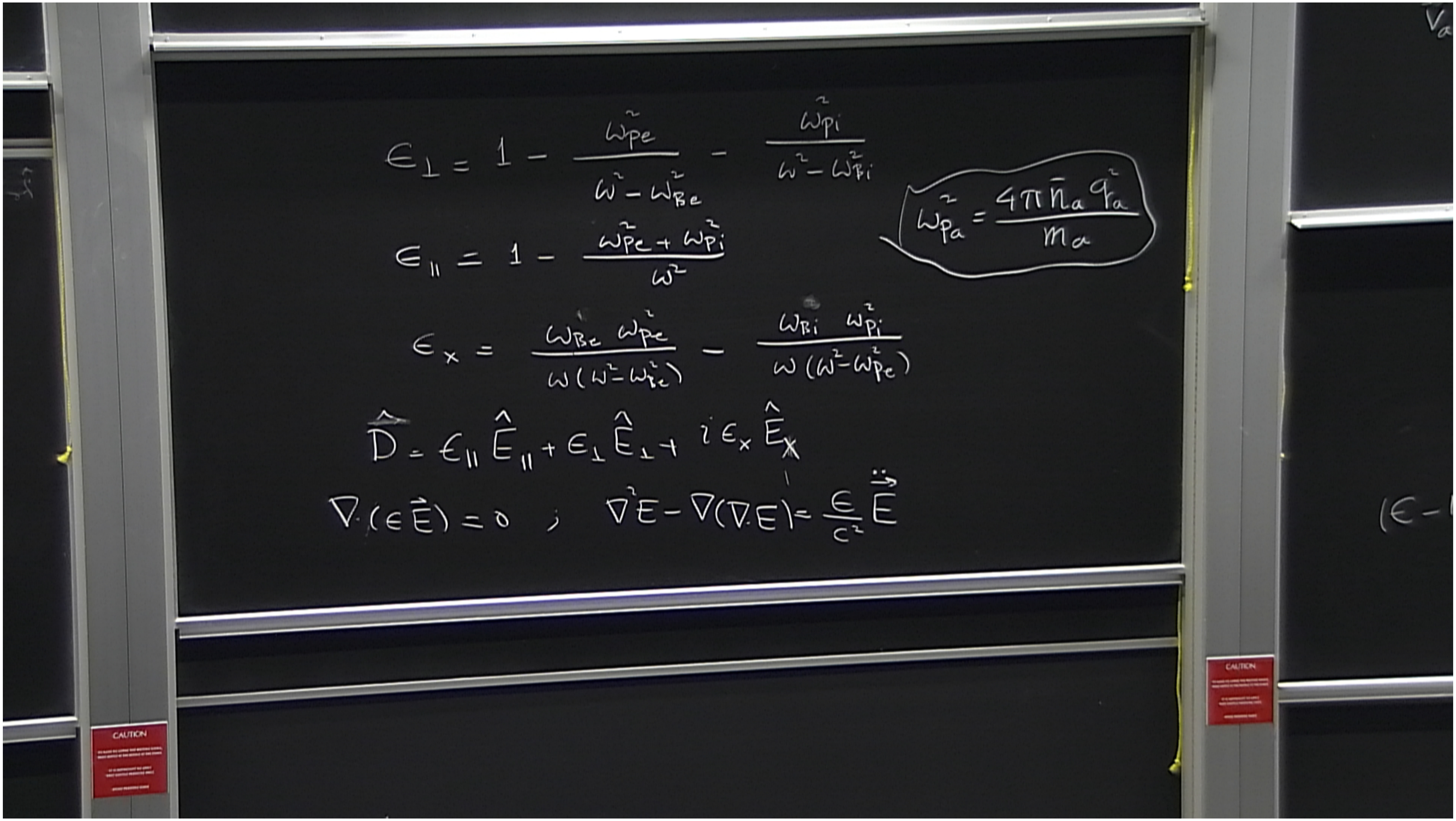
$\hat{V}_a =$

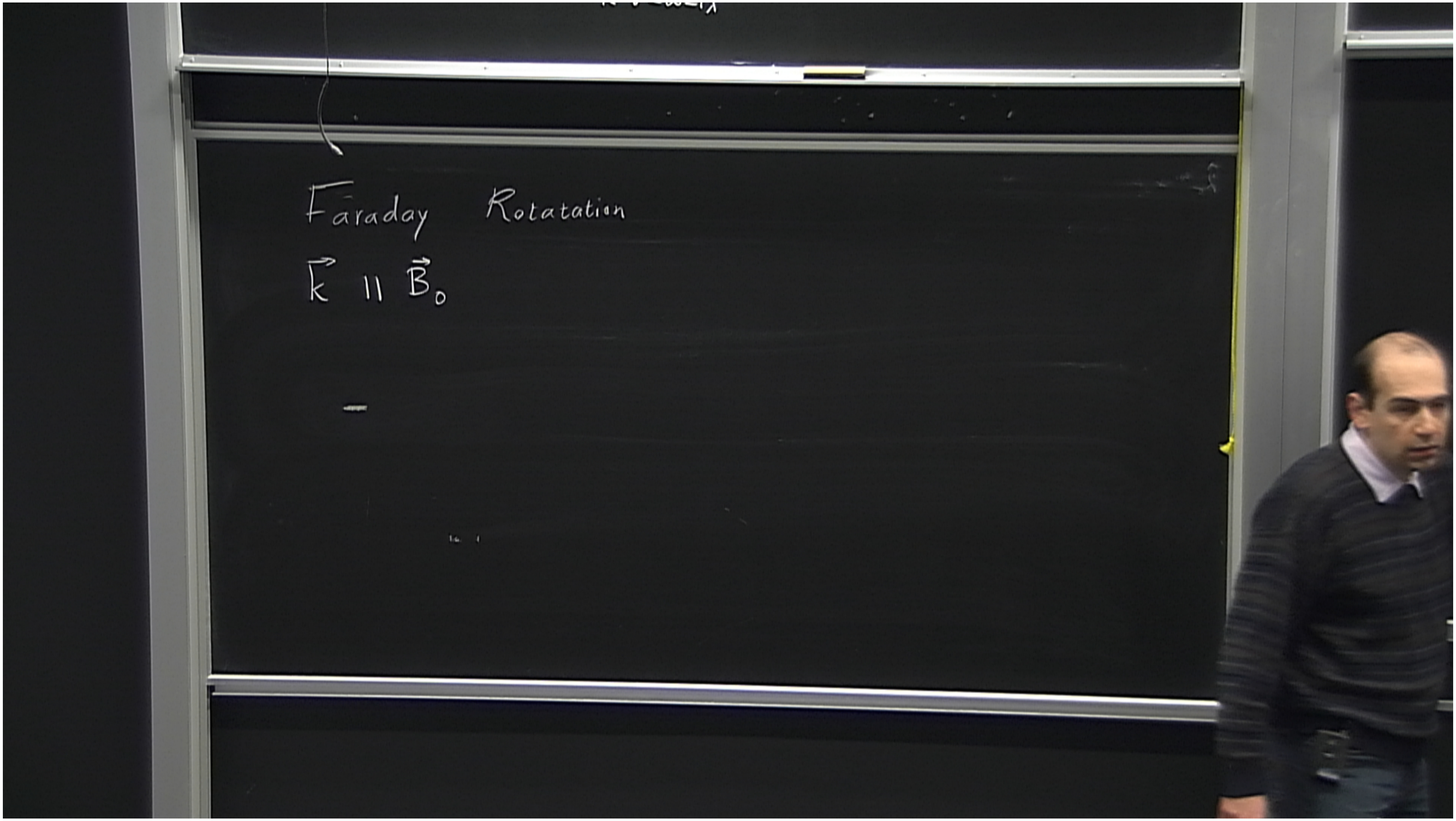
ω_B

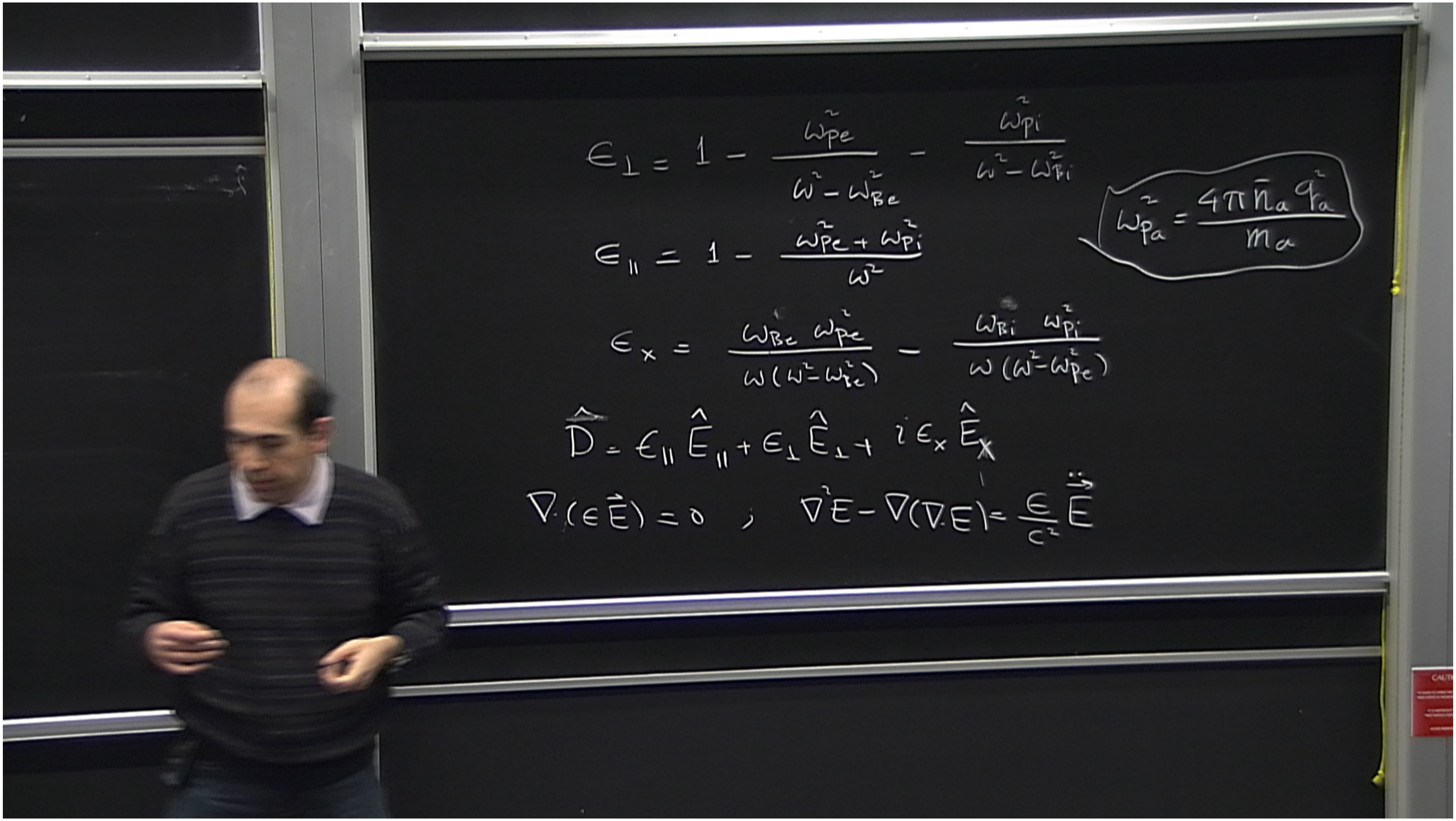
$(\epsilon - 1)\hat{E}$

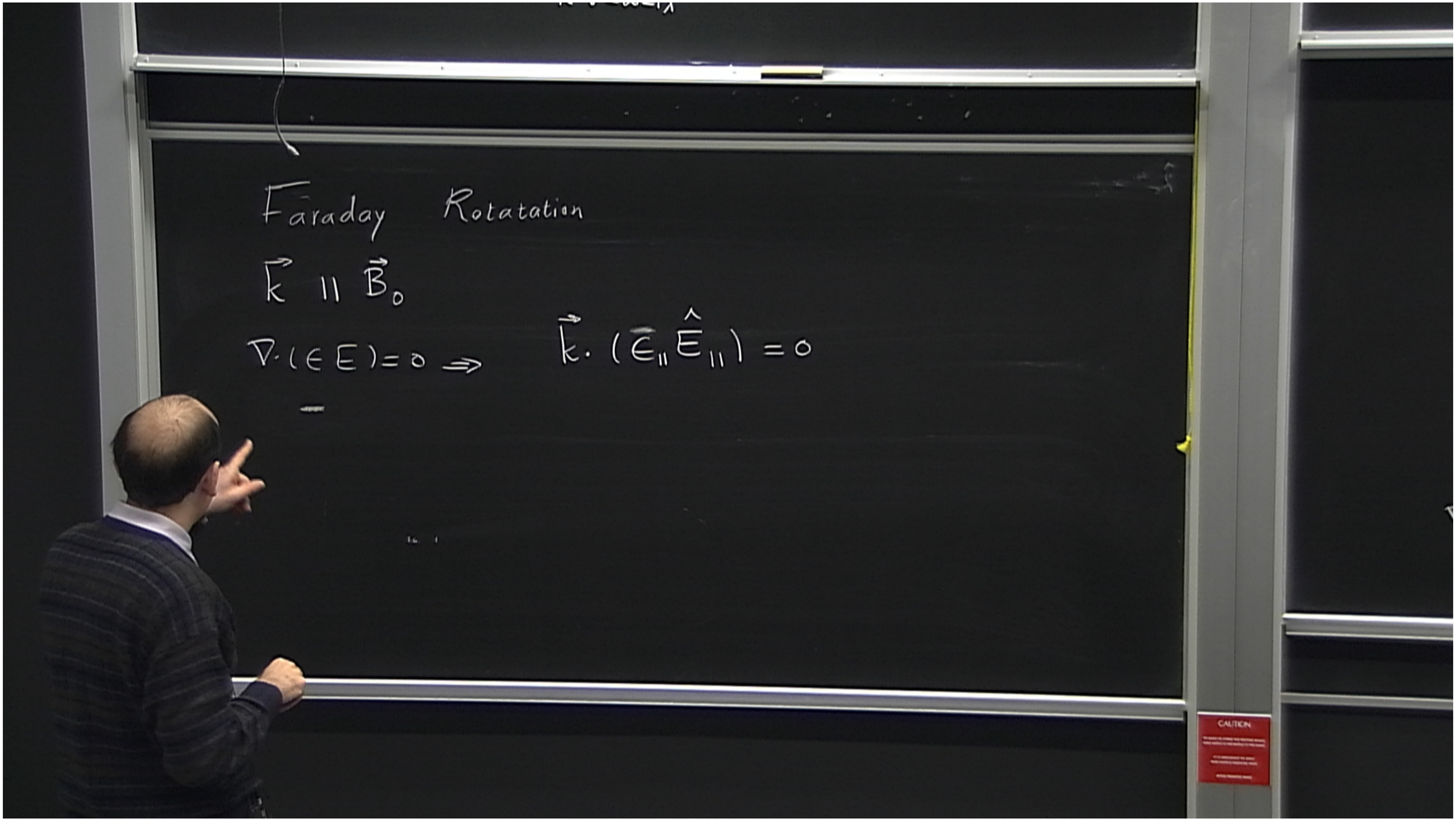

$\epsilon_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$
$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$
$\epsilon_x = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$
$\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}$

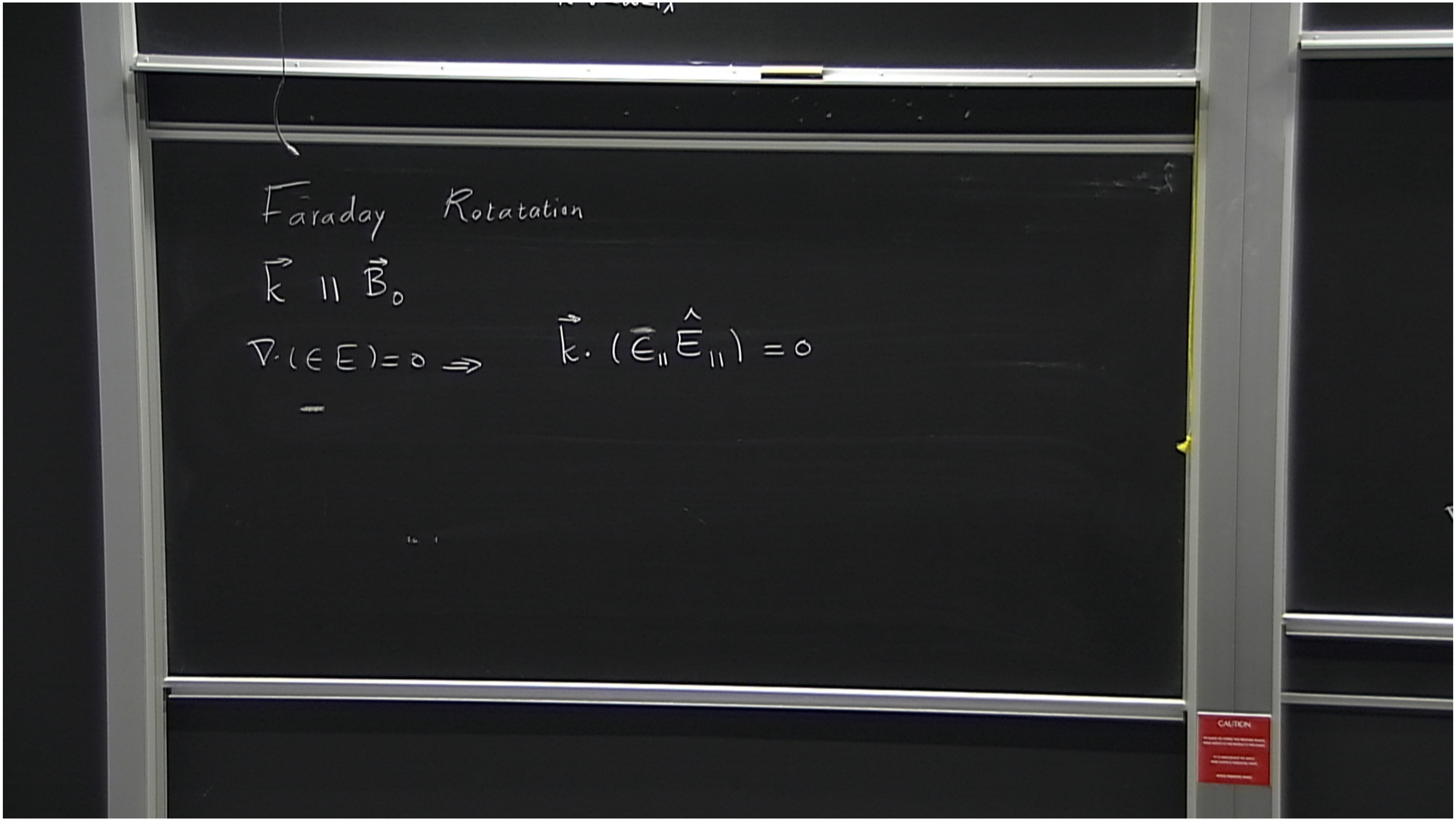


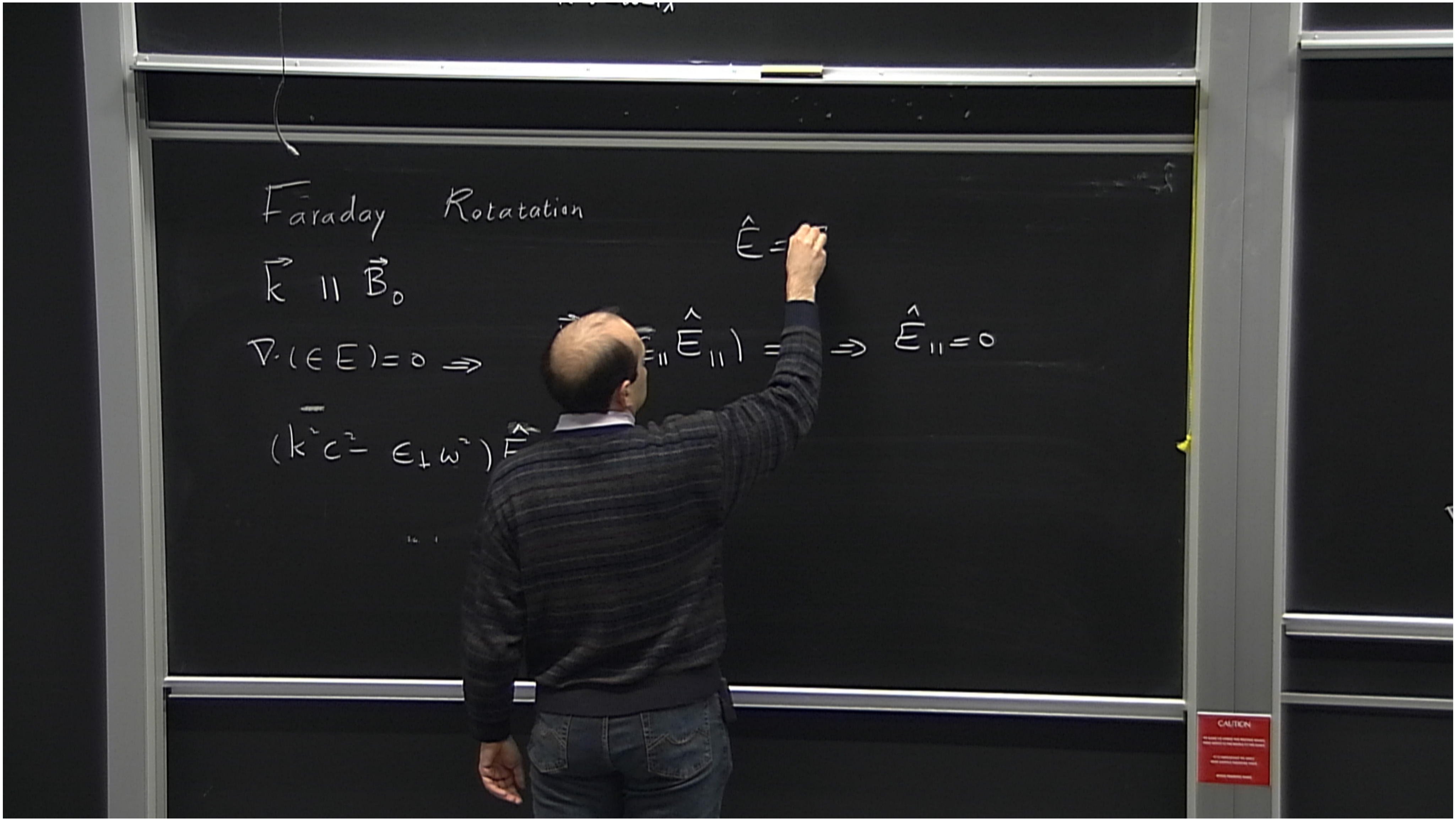

$\epsilon_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$
$\boxed{\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}}$
$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$
$\epsilon_x = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$
$\hat{D} = \epsilon_\parallel \hat{E}_\parallel + \epsilon_\perp \hat{E}_\perp + i\epsilon_x \hat{E}_x$
$\nabla\cdot(\epsilon\vec{E}) = 0 \;;\; \nabla^2 E - \nabla(\nabla\cdot E) = \frac{\epsilon}{c^2}\ddot{\vec{E}}$


Faraday Rotatation
$\vec{k} \parallel \vec{B}_0$

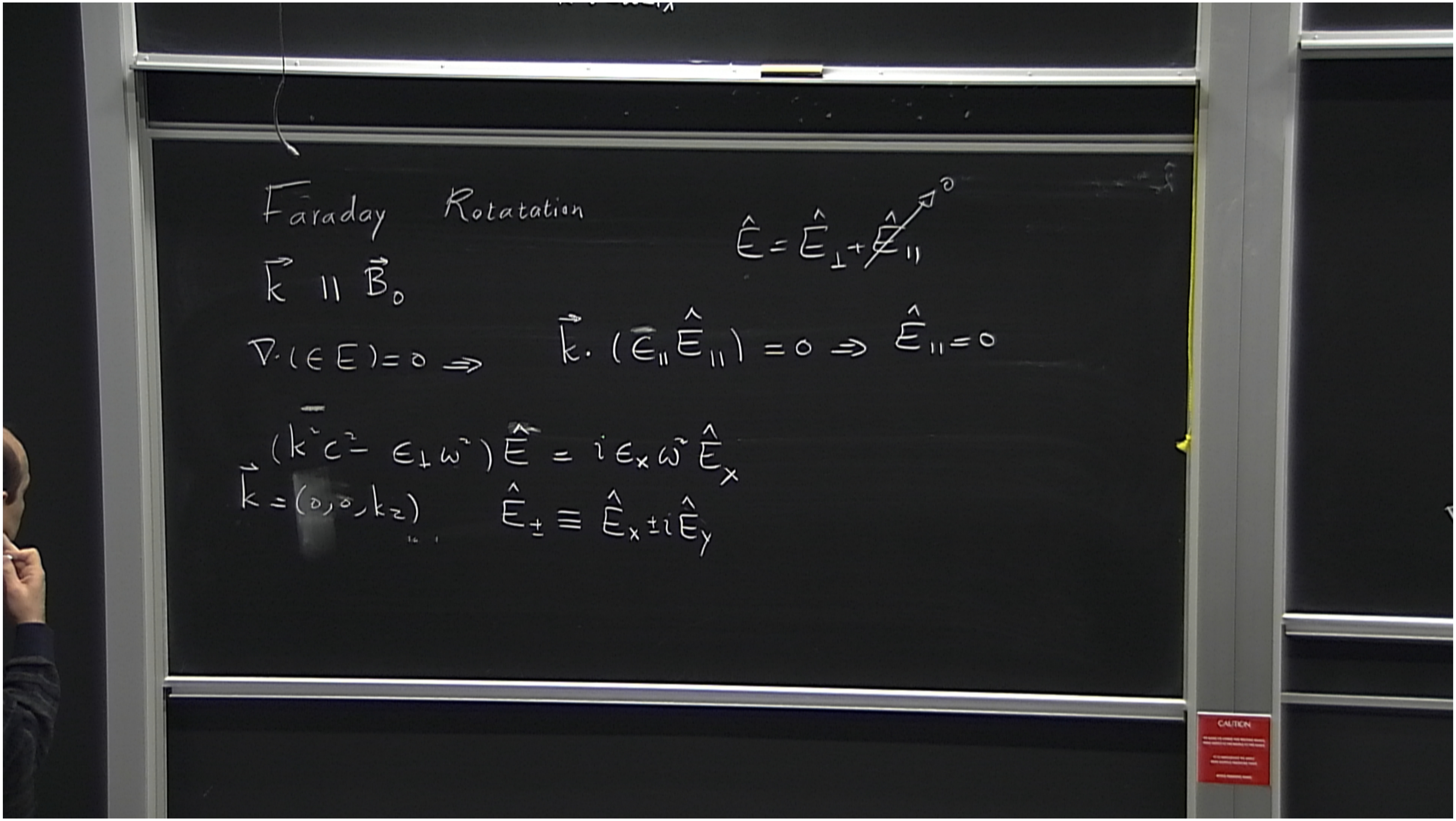

$\epsilon_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$
$\boxed{\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}}$
$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$
$\epsilon_x = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$
$\hat{D} = \epsilon_\parallel \hat{E}_\parallel + \epsilon_\perp \hat{E}_\perp + i\epsilon_x \hat{E}_x$
$\nabla\cdot(\epsilon \vec{E}) = 0 \;;\; \nabla^2 E - \nabla(\nabla\cdot E) = \frac{\epsilon}{c^2}\ddot{\vec{E}}$

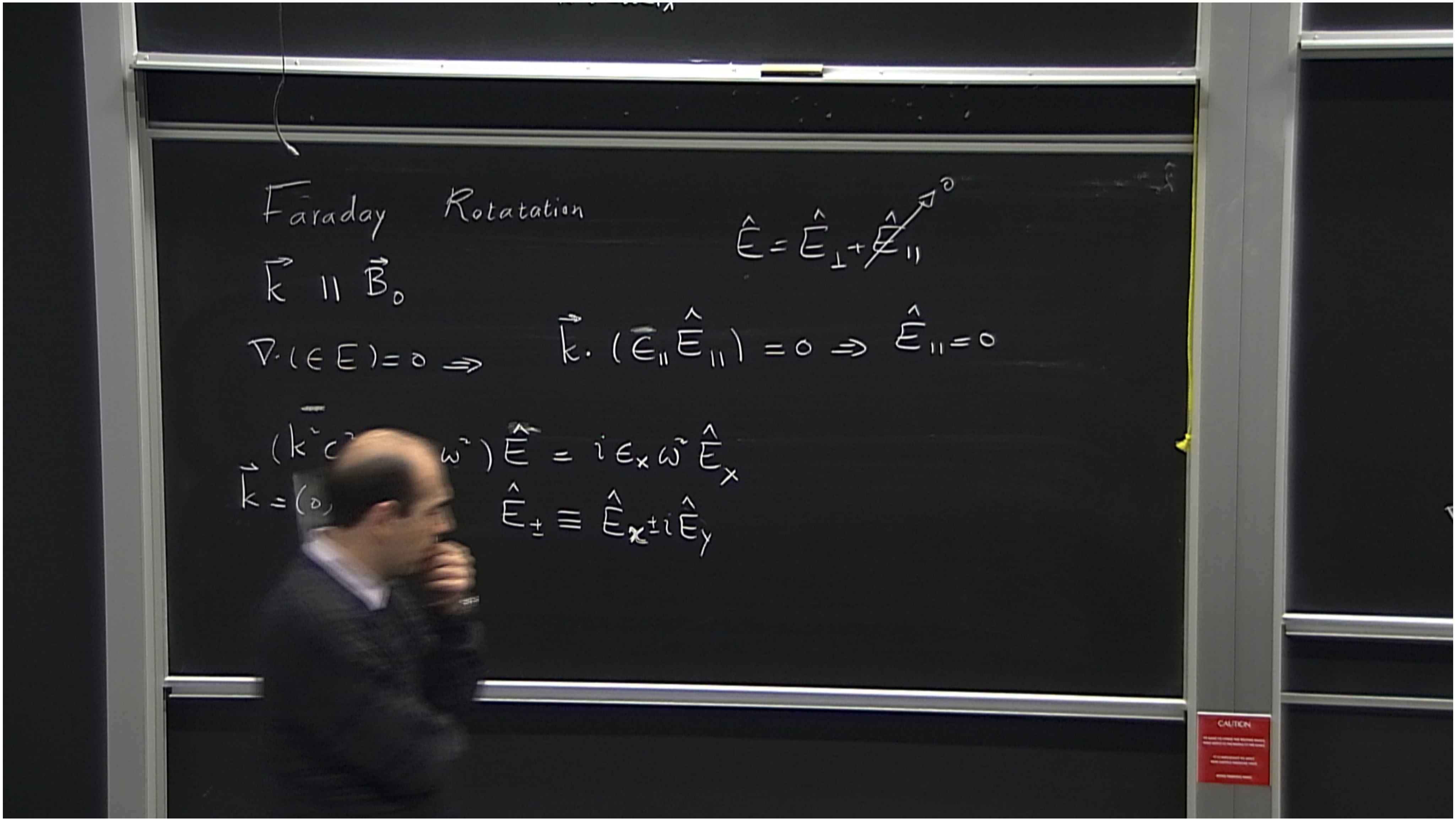

Faraday Rotatation
$\vec{k} \parallel \vec{B}_0$
$\nabla \cdot (\epsilon E) = 0 \Rightarrow \quad \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0$

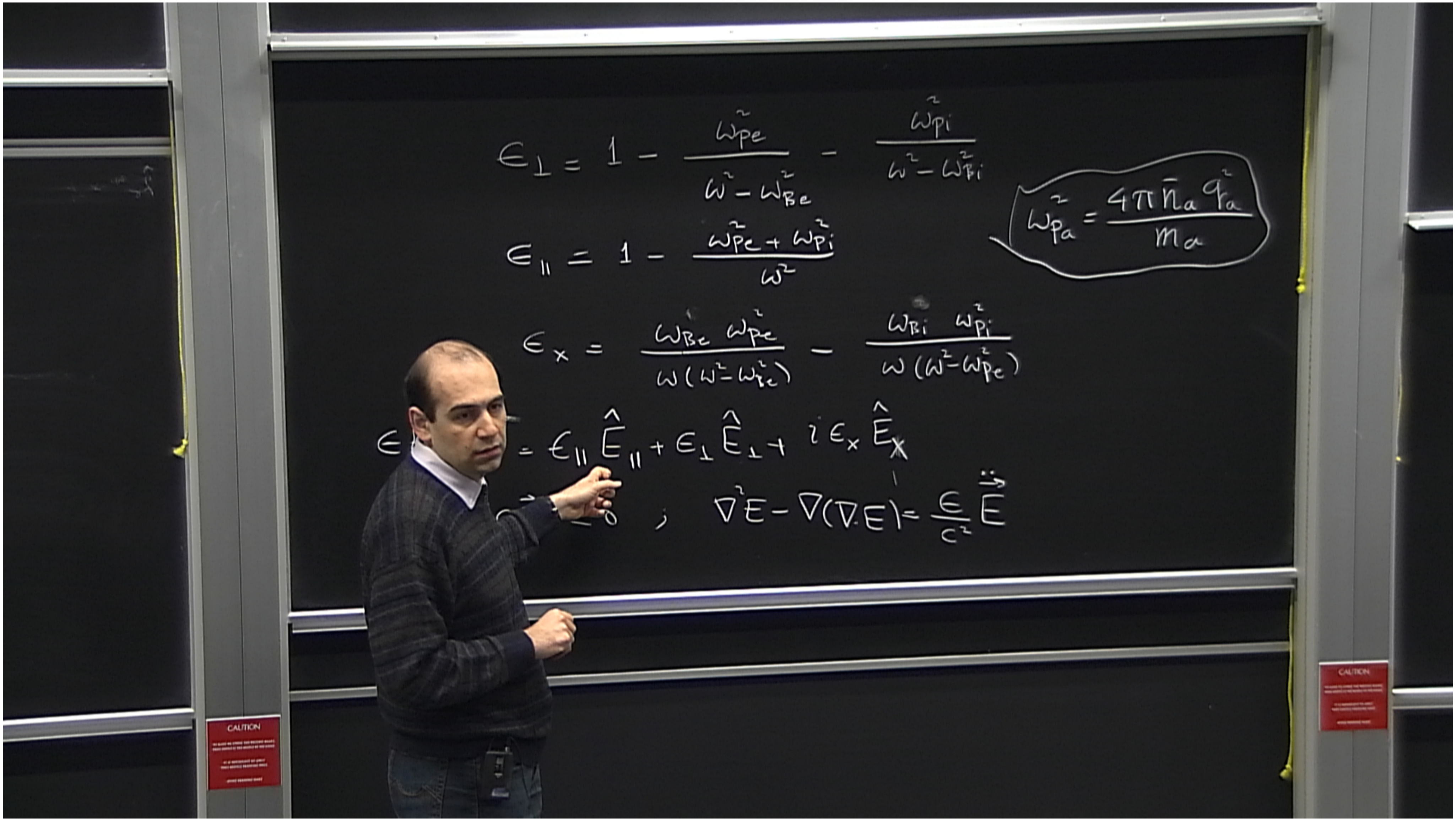

Faraday Rotatation
$\vec{k} \parallel \vec{B}_0$
$\nabla\cdot(\epsilon E)=0 \Rightarrow$
$\vec{k}\cdot(\epsilon_{\parallel}\hat{E}_{\parallel})=0$

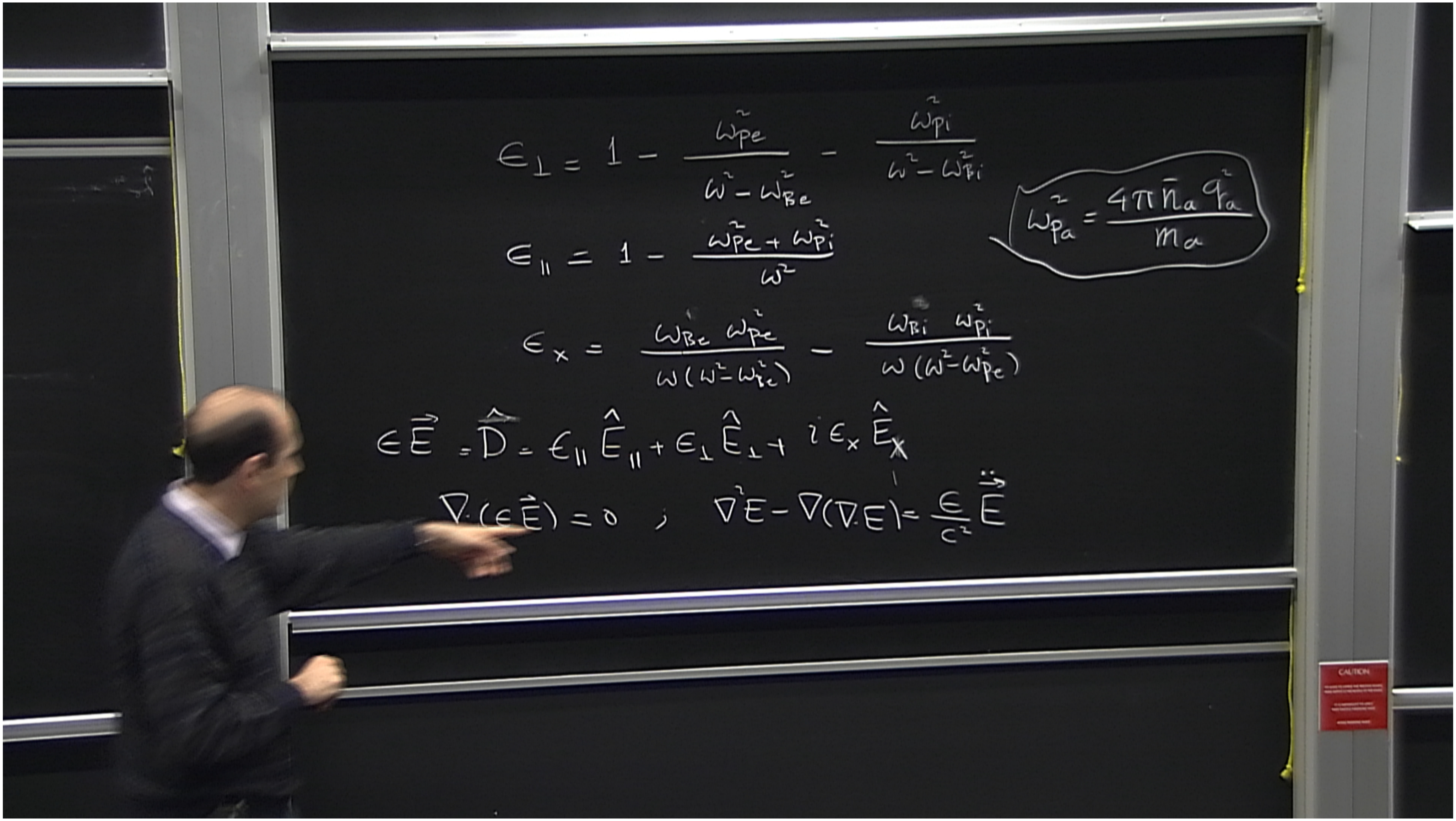


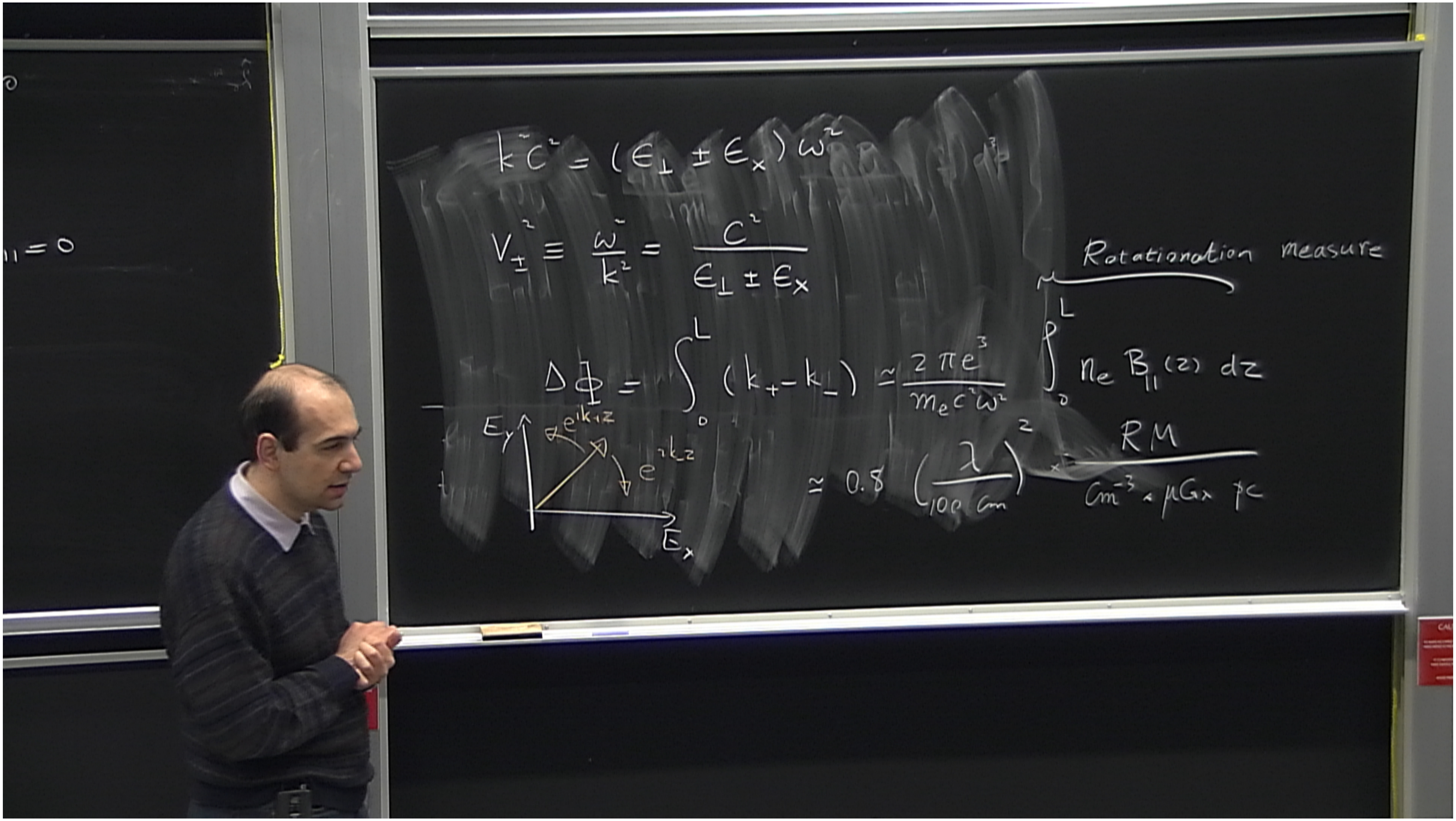
Faraday Rotatation
$\vec{k} \parallel \vec{B}_0$
$\nabla \cdot (\epsilon E) = 0 \Rightarrow$
$\Rightarrow \hat{E}_{\parallel} = 0$
$\hat{E} =$
$(k^2 c^2 - \epsilon_{\pm} \omega^2)$

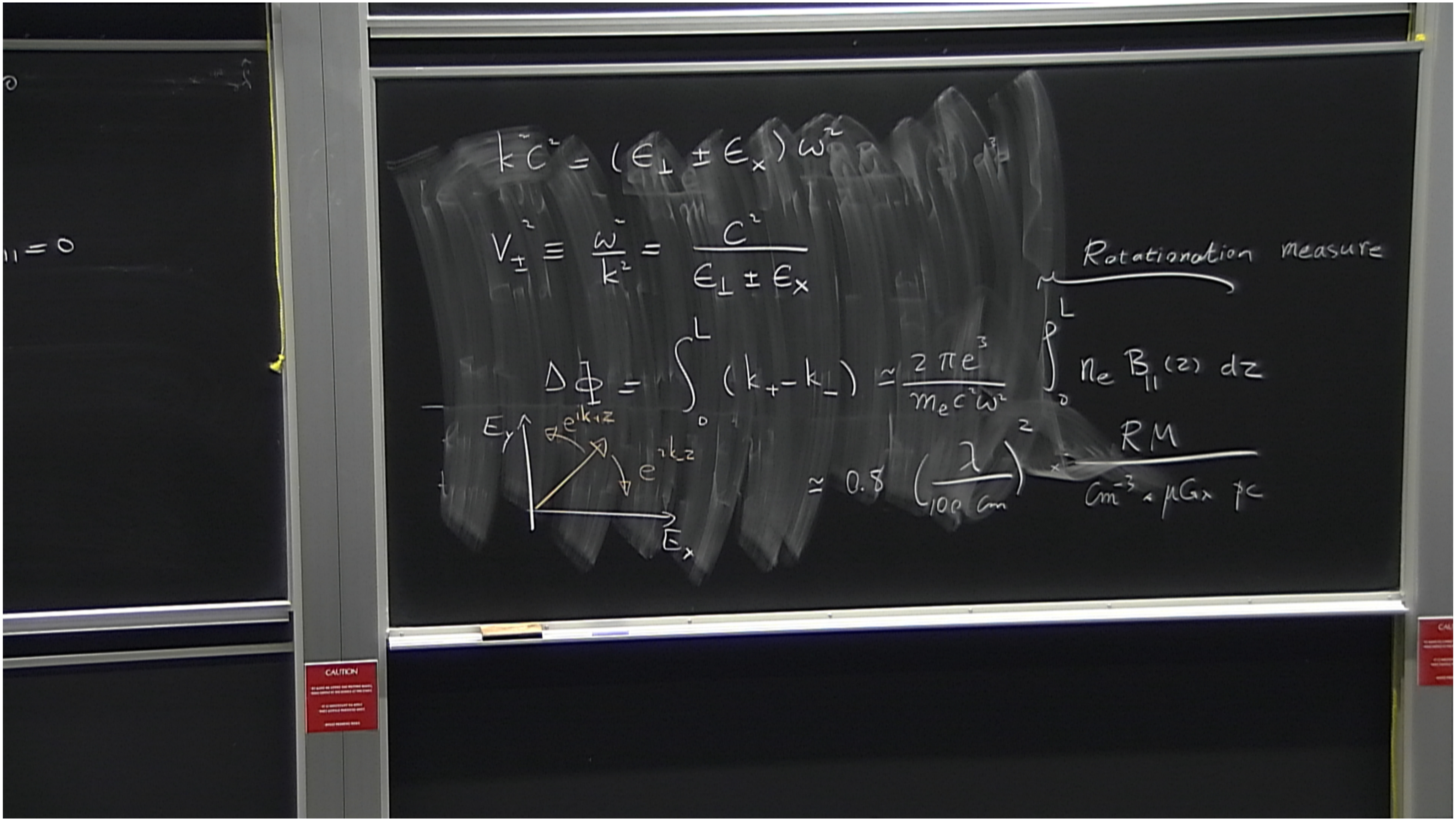

Faraday Rotatation
$\hat{E} = \hat{E}_\perp + \cancel{\hat{E}_\parallel}^{\,0}$
$\vec{k} \parallel \vec{B}_0$
$\nabla\cdot(\epsilon E)=0 \Rightarrow$
$\vec{k}\cdot(\epsilon_\parallel \hat{E}_\parallel)=0 \Rightarrow \hat{E}_\parallel=0$
$(k^2c^2-\epsilon_\perp\omega^2)\hat{E} = i\epsilon_x\omega^2\hat{E}_x$
$\vec{k}=(0,0,k_z)$
$\hat{E}_\pm \equiv \hat{E}_x \pm i\hat{E}_y$

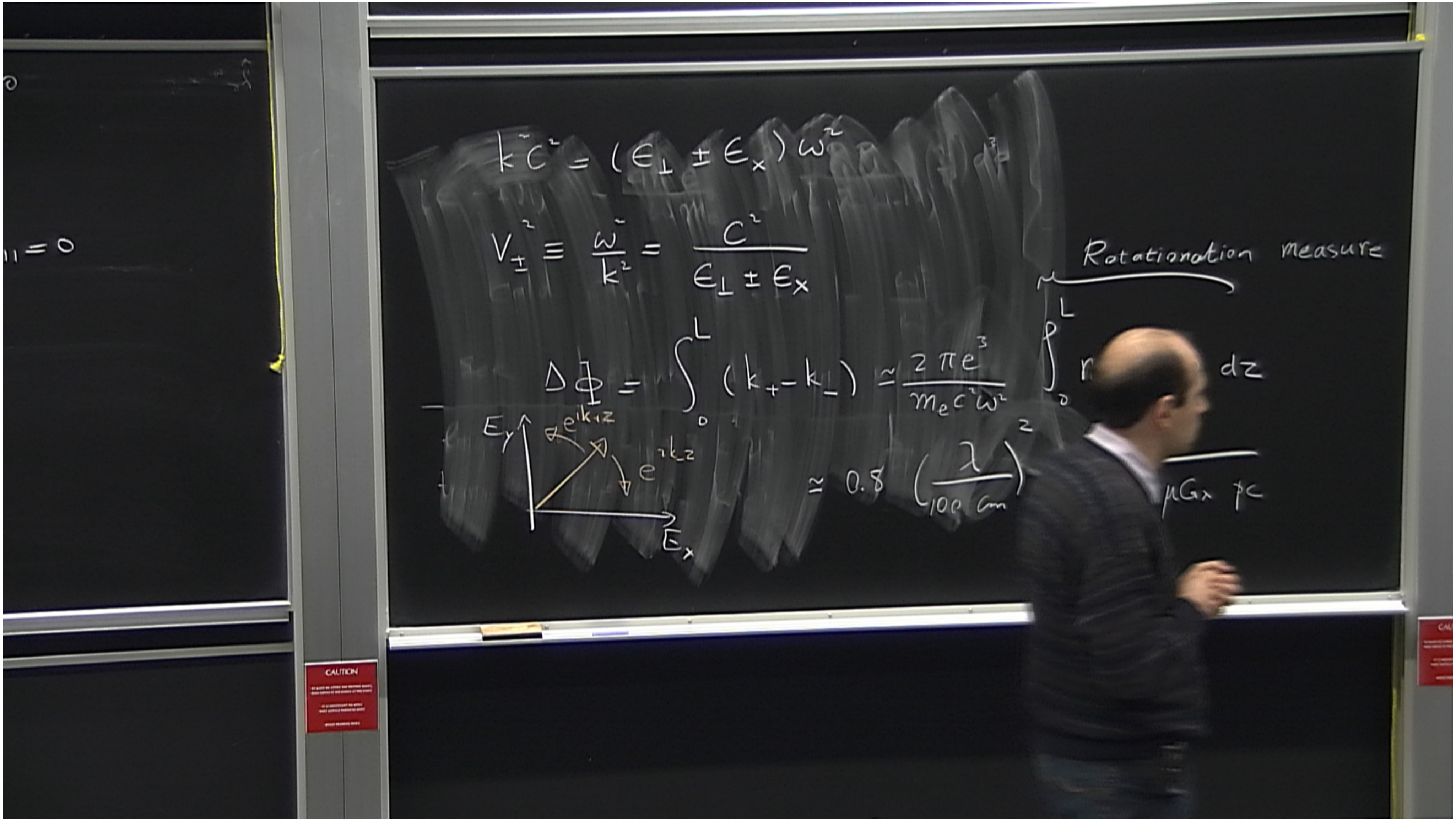

Faraday Rotatation
$\hat{E} = \hat{E}_\perp + \hat{E}_{\parallel}$
$\vec{k} \parallel \vec{B}_0$
$\nabla\cdot(\epsilon E) = 0 \Rightarrow$
$\vec{k}\cdot(\epsilon_{\parallel}\hat{E}_{\parallel}) = 0 \Rightarrow \hat{E}_{\parallel} = 0$
$\omega^2)\hat{E} = i\epsilon_x\omega^2\hat{E}_x$
$\hat{E}_\pm \equiv \hat{E}_x \pm i\hat{E}_y$

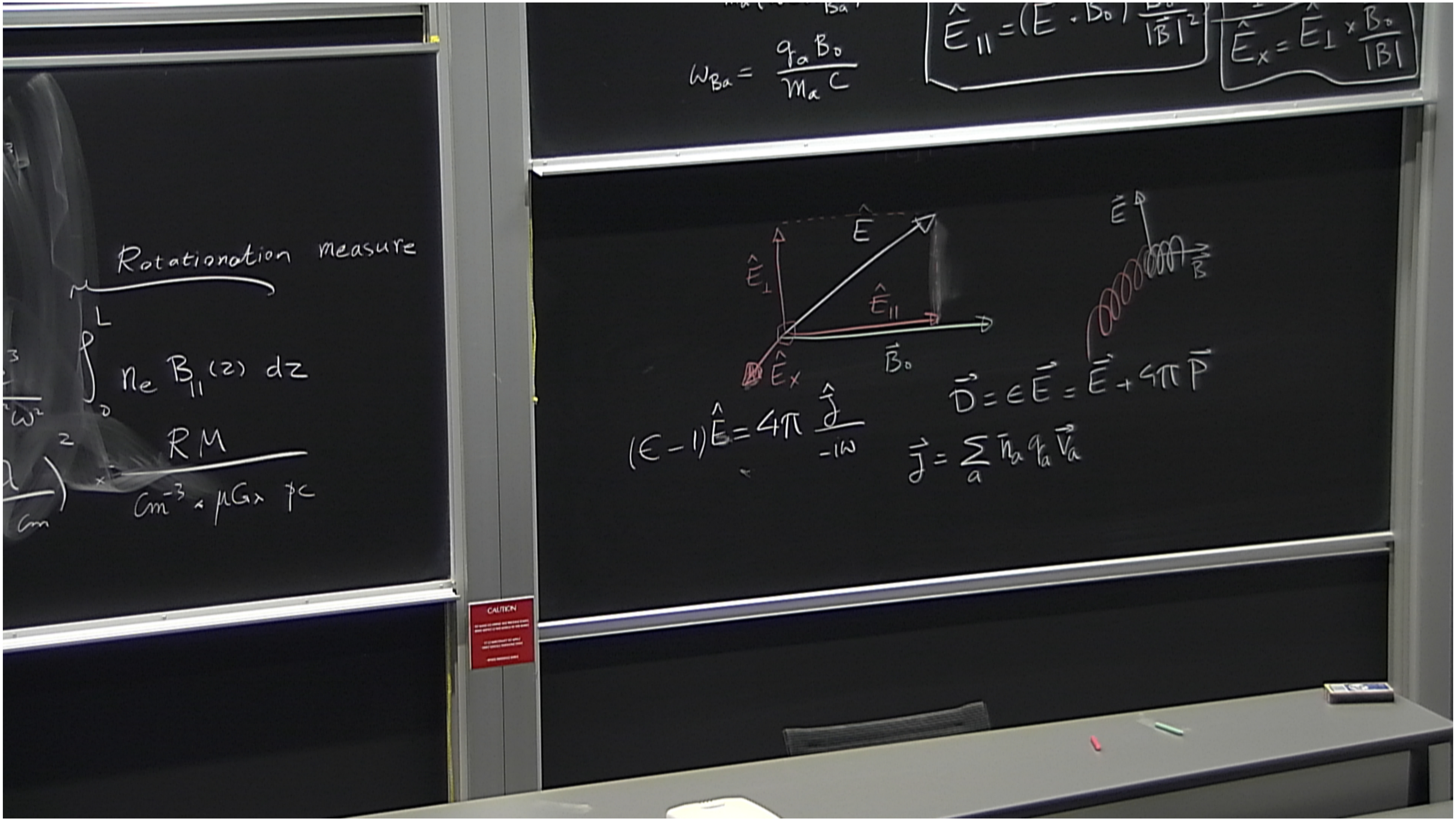

$\epsilon_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$
$\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}$
$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$
$\epsilon_x = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$
$= \epsilon_\parallel \hat{E}_\parallel + \epsilon_\perp \hat{E}_\perp + i\epsilon_x \hat{E}_x$
$\nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2}\ddot{\vec{E}}$


$\epsilon_\perp = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{Be}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{Bi}^2}$
$\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}$
$\epsilon_\parallel = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$
$\epsilon_x = \frac{\omega_{Be}\,\omega_{pe}^2}{\omega(\omega^2 - \omega_{Be}^2)} - \frac{\omega_{Bi}\,\omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$
$\epsilon \vec{E} = \hat{D} = \epsilon_\parallel \hat{E}_\parallel + \epsilon_\perp \hat{E}_\perp + i\epsilon_x \hat{E}_x$
$\nabla\cdot(\epsilon\vec{E}) = 0 \ ; \ \nabla^2 E - \nabla(\nabla\cdot E) = \frac{\epsilon}{c^2}\ddot{\vec{E}}$


Rotationation measure
RM


$k^2c^2 = (\epsilon_\perp \pm \epsilon_x)\,\omega^2$
$V_\pm^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_\perp \pm \epsilon_x}$
Rotationation measure
$\Delta\Phi = \int_0^L (k_+ - k_-) \simeq \frac{2\pi e^3}{m_e c^2 \omega^2} \int_0^L n_e B_\parallel(z)\, dz$
$\simeq 0.8 \left(\frac{\lambda}{100\ \mathrm{cm}}\right)^2 \times \frac{RM}{\mathrm{cm}^{-3} \times \mu G \times pc}$
E_y
e^{ik_+z}
e^{ik_-z}
E_x


Rotationation measure
dz
CAUTION


$\omega_{Ba} = \frac{q_a B_0}{m_a c}$
$\hat{E}_{\parallel} = (\hat{E}\cdot \vec{B}_0)\frac{\vec{B}_0}{|B|^2}$
$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|B|}$
Rotationotion measure
$\int_0^L n_e B_\parallel(z)\, dz$
RM
$cm^{-3} \times \mu G \times pc$
$\hat{E}$
$\hat{E}_\perp$
$\hat{E}_\parallel$
$\hat{E}_x$
$\vec{B}_0$
$\vec{E}$
$\vec{B}$
$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{J}}{-i\omega}$
$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$
$\vec{J} = \sum_a n_a q_a \vec{v}_a$