

Title: PHYS 781 - Lecture 18

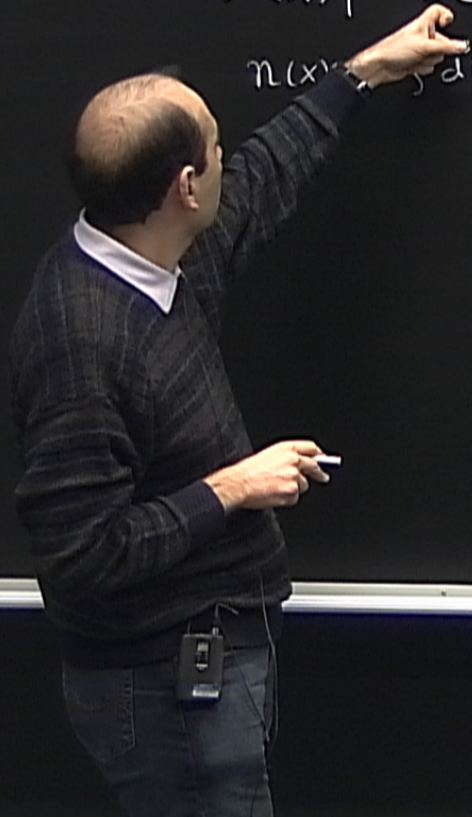
Date: Nov 13, 2014 02:00 PM

URL: <http://pirsa.org/14110054>

Abstract:

$$f(x, p) = A e^{-\frac{E}{kT}} = A e^{-\frac{p^2}{2mkT}} e^{-\frac{m\phi(x)}{kT}}$$

$$n(x) = \int dp f(x, p) = B e^{-\frac{m\phi}{kT}}$$



CAUTION  
 DO NOT TOUCH THE BOARD SURFACE  
 IT IS HOT TO THE TOUCH  
 AND MAY BE DAMAGED BY OILS  
 FROM YOUR HANDS

$$= e^{-\frac{(E-\mu)}{kT}}$$

$$f(x, p) = A e^{-\frac{E}{kT}} = A e^{-\frac{p^2}{2mkT}} e^{-\frac{m\phi(x)}{kT}}$$

$$n(x) = \int d^3p f(x, p) = B e^{-\frac{m\phi}{kT}}$$

$$\vec{j} = \frac{\partial \vec{P}}{\partial t} = -e \int \vec{v} \delta f d^3p$$

$$\hat{j} = -i\omega \hat{P} = -e^2 \hat{P} = \frac{\hat{E} \cdot \frac{\partial f}{\partial \vec{p}}}{(\vec{k} \cdot \nabla - \omega + i\lambda)} d^3p$$

$$\frac{(\epsilon - 1)\hat{E}_i}{4\pi} = \hat{P}_i(\omega, k) = -\frac{e^2}{\omega} \hat{E}_i \int \frac{p_i}{\omega - \omega_i} d^3p$$

$$f \propto e^{-\frac{p^2}{2mkT}}$$

$$\frac{\partial f}{\partial \vec{p}} = \frac{\partial f}{\partial p} \times \frac{\vec{p}}{|\vec{p}|}$$

$$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega^2}$$

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left(\frac{\partial \vec{f}}{\partial \vec{p}}\right) d^3 p$$

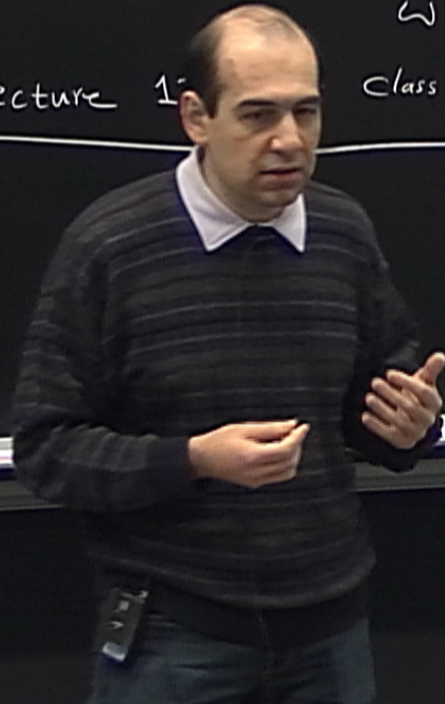
cold case:  $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 17 in class notes

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\lambda)} \left( \frac{\partial \bar{f}}{\partial p} \right) d^3 p$$

cold case:  $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 1 class notes



CAUTION  
DO NOT REACH FOR THE BOARD  
OR THE CHALK  
WHILE THE BOARD IS ON  
OR THE CHALK IS IN USE

CAUTION  
DO NOT REACH FOR THE BOARD  
OR THE CHALK  
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OR THE CHALK IS IN USE

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left( \frac{\partial \bar{f}}{\partial p} \right) d^3 p$$

cold case:  $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

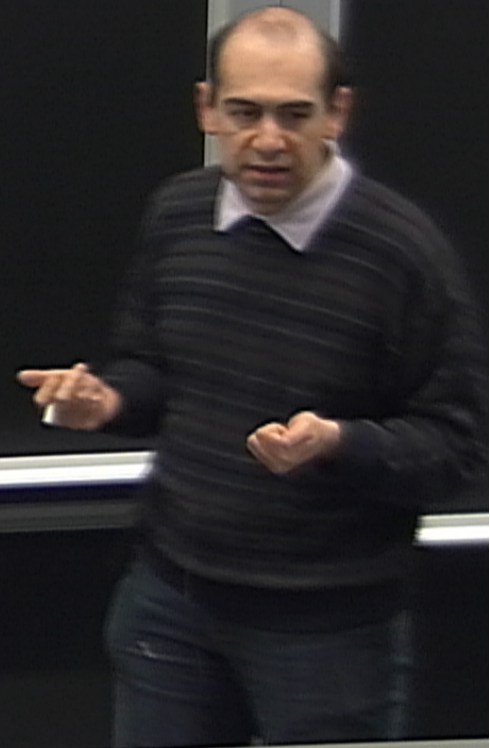
See Lecture 17 in class

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left( \frac{\partial \bar{f}}{\partial p} \right) d^3 p$$

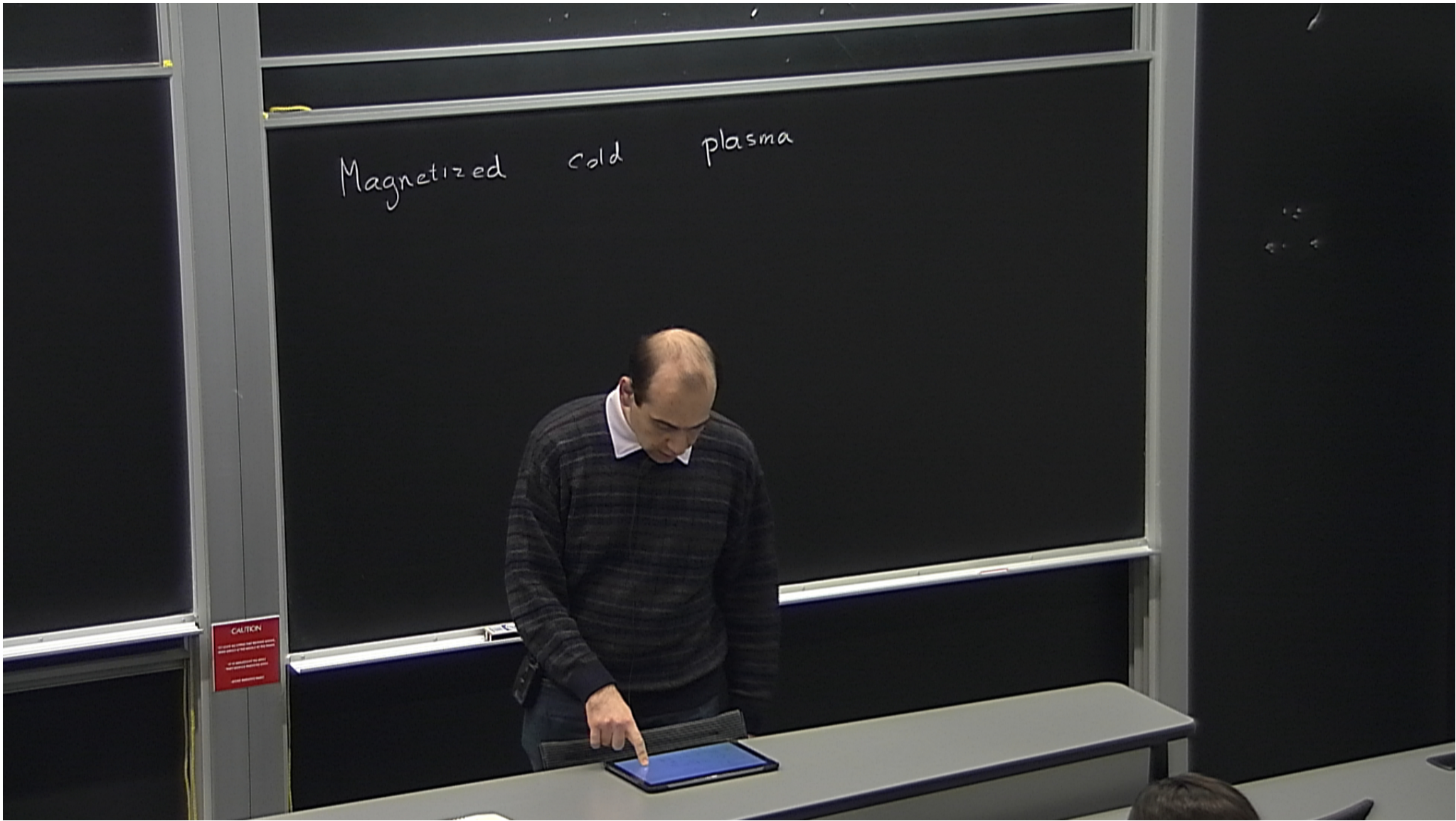
cold case:  $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 17 in class notes

long.  $\omega^2 = \omega_p^2 + 3k^2 v_e^2$        $\frac{k}{\omega} v_e \ll 1 \Rightarrow \frac{\omega}{k} \gg v_e$   
 trans.  $\omega^2 = \omega_p^2 + k^2 c^2 + \dots$



CAUTION  
 Do not touch the chalkboard when it is hot.  
 Do not touch the chalkboard when it is hot.  
 Do not touch the chalkboard when it is hot.





Magnetized cold plasma

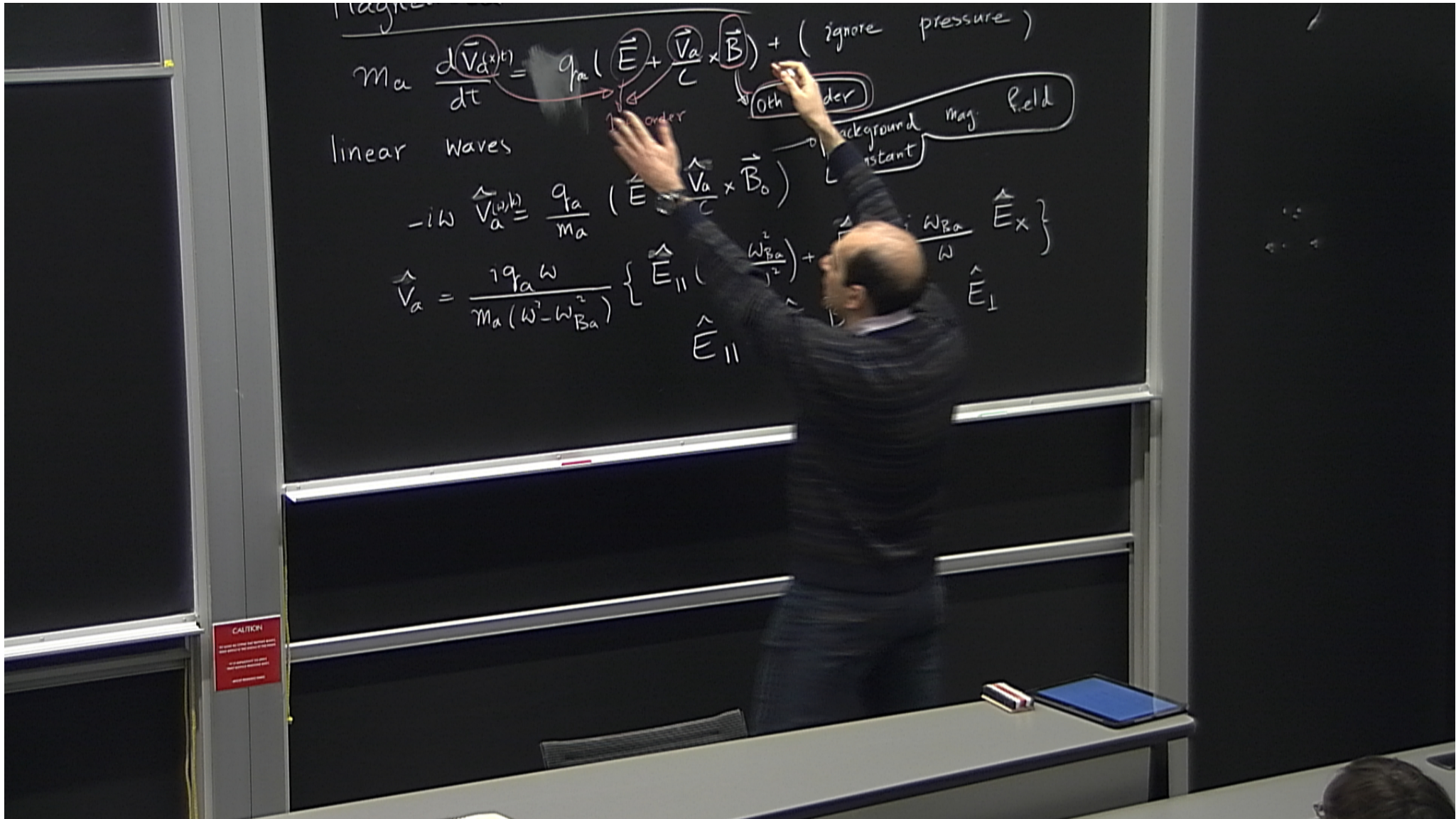
$$m_a \frac{d\vec{v}_a}{dt} = q_a \left( \vec{E} + \frac{\vec{v}_a \times \vec{B}}{c} \right) + (\text{ignore pressure})$$

linear approximation

$$= \frac{q_a}{m_a} \left( \vec{E} + \frac{\vec{v}_a \times \vec{B}_0}{c} \right)$$

1st order      0th order      background mag field

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
OR THE BOARD SURFACE  
OR THE BOARD SURFACE



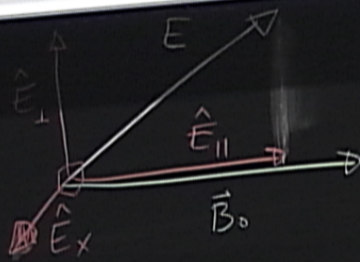
$$\hat{V}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{||} \left( 1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|\vec{B}_0|}$$



linear waves

$$-i\omega \hat{V}_a = \frac{q_a}{m_a} \left( \hat{E} + \hat{V}_a \times \vec{B}_0 \right)$$

$$\hat{V}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left( 1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \hat{E}_{\perp} - \frac{i\omega_{Ba}}{\omega} \hat{E}_x \right\}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\begin{cases} \hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel} \\ \hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|\vec{B}_0|} \end{cases}$$

background mag field  
Constant

1st order (0th order)

CAUTION  
DO NOT TOUCH THE BOARD SURFACE  
AS IT IS EXTREMELY HOT  
AND COULD CAUSE BURNS  
IF TOUCHED.



$$\hat{V}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left( 1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \hat{E}_{\perp} - \frac{i \omega_{Ba}}{\omega} E_x \right\}$$

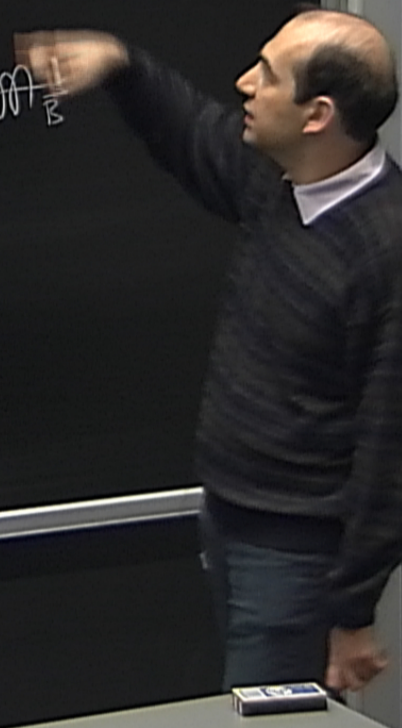
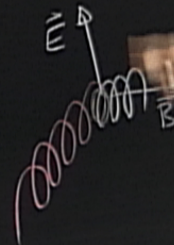
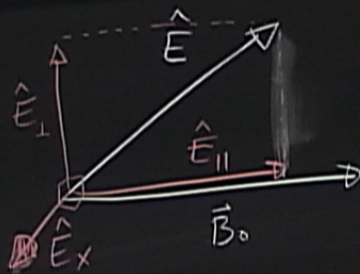
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_{\perp}| = |\hat{E}_x|$$



CAUTION

$$V_a = \frac{\hbar a^2}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ E_{||} (\omega^2) \right.$$

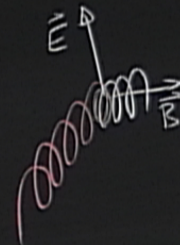
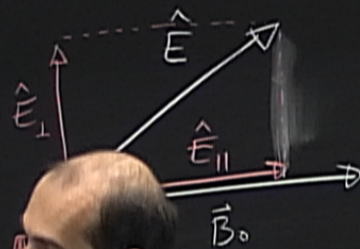
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_\perp| = |\hat{E}_x|$$



$$\epsilon - 1 =$$

CAUTION

$$V_a = \frac{v_a}{m_a(\omega^2 - \omega_{Ba}^2)}$$

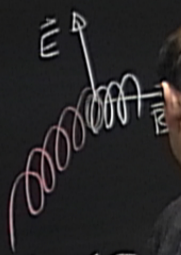
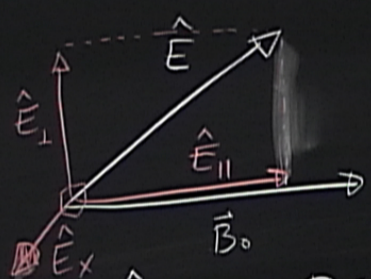
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|B|}$$

$$|\hat{E}_{\perp}| = |\hat{E}_{\times}|$$



$$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$D = \epsilon E = E + 4\pi$$

CAUTION

$$\hat{V}_a = \frac{q_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left( 1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

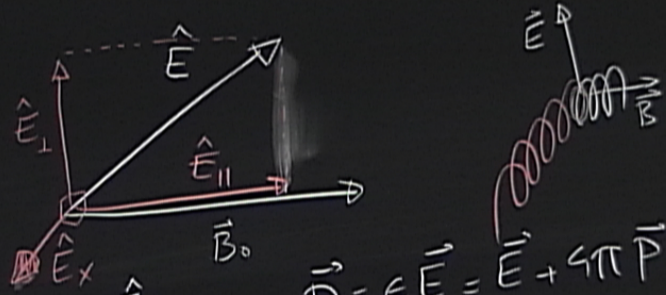
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \frac{\hat{E}_{\perp} \times \vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_{\perp}| = |\hat{E}_x|$$



$$(\epsilon - 1) \hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{j} = \sum_a n_a q_a \vec{V}_a$$

CAUTION



$$\hat{V}_a = \frac{q_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left( 1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

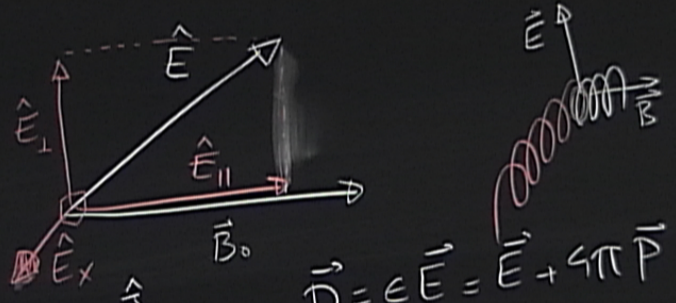
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \frac{\hat{E}_{\perp} \times \vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_{\perp}| = |\hat{E}_x|$$



$$(\epsilon - 1) \hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{j} = \sum_a n_a q_a \vec{v}_a$$

CAUTION

trans.

$$\omega = \omega_p^2 + k^2 c^2 + \dots$$

$$\hat{V}_a =$$

$$\omega_p$$

$$\epsilon_{\perp} =$$

$$(\epsilon - 1)\hat{E}$$

CAUTION

CAUTION  
Do not touch the screen when it is hot.  
Do not touch the screen when it is hot.  
Do not touch the screen when it is hot.

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\epsilon_x = \frac{\omega_{pe}^2 \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi}^2 \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}$$

$$(\epsilon - 1)\hat{E}$$

$$\epsilon_{\perp} = 1 - \frac{\tilde{\omega}_{pe}^2}{\omega^2 - \tilde{\omega}_{pe}^2} - \frac{\tilde{\omega}_{pi}^2}{\omega^2 - \tilde{\omega}_{pi}^2}$$

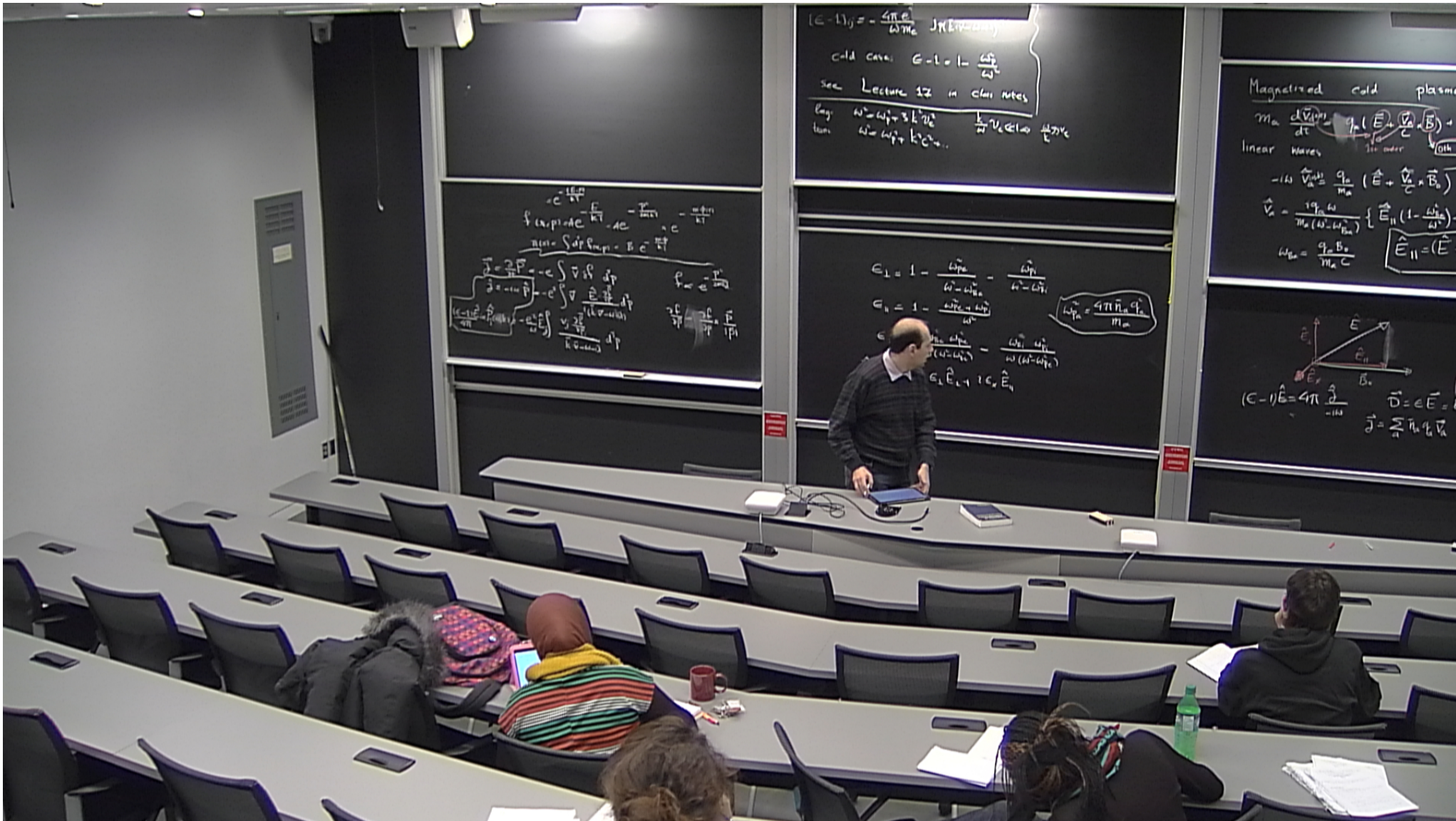
$$\epsilon_{\parallel} = 1 - \frac{\tilde{\omega}_{pe}^2 + \tilde{\omega}_{pi}^2}{\omega^2}$$

$$\epsilon_x = \frac{\tilde{\omega}_{pe} \tilde{\omega}_{pe}^2}{\omega(\omega^2 - \tilde{\omega}_{pe}^2)} - \frac{\tilde{\omega}_{pi} \tilde{\omega}_{pi}^2}{\omega(\omega^2 - \tilde{\omega}_{pe}^2)}$$

$$\tilde{\omega}_{pa}^2 = \frac{4\pi\tilde{n}_a q_a^2}{m_a}$$

CAUTION

CAUTION



$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi n_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe} \omega_{pi}}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi} \omega_{pe}}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla \cdot (\epsilon \vec{E}) = 0 ; \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi\bar{n}_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe} \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi} \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla \cdot (\epsilon \vec{E}) = 0, \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$



# Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon E) = 0 \Rightarrow \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon E) = 0 \Rightarrow \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0$$

Faraday Rotation

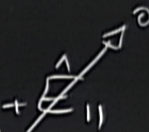
$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon \vec{E}) = 0 \Rightarrow \vec{\epsilon}_{\parallel} \hat{E}_{\parallel} = 0 \Rightarrow \hat{E}_{\parallel} = 0$$

$$(\vec{k}^2 c^2 - \epsilon_{\perp} \omega^2) \hat{E}_{\perp}$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\hat{E} = \hat{E}_\perp + \hat{E}_\parallel$$


$$\nabla \cdot (\epsilon E) = 0 \Rightarrow \vec{k} \cdot (\epsilon_\parallel \hat{E}_\parallel) = 0 \Rightarrow \hat{E}_\parallel = 0$$

$$(k^2 c^2 - \epsilon_\perp \omega^2) \hat{E} = i \epsilon_x \omega^2 \hat{E}_x$$

$$\vec{k} = (0, 0, k_z) \quad \hat{E}_\pm \equiv \hat{E}_x \pm i \hat{E}_y$$

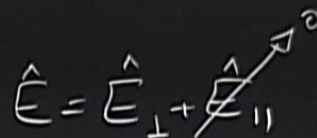
Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon \vec{E}) = 0 \Rightarrow \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0 \Rightarrow \hat{E}_{\parallel} = 0$$

$$(k^2 - \omega^2) \hat{E} = i \epsilon_x \omega^2 \hat{E}_x$$

$$\vec{k} = (0, 0, k_z)$$
$$\hat{E}_{\pm} \equiv \hat{E}_x \pm i \hat{E}_y$$

$$\hat{E} = \hat{E}_{\perp} + \hat{E}_{\parallel}$$


$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

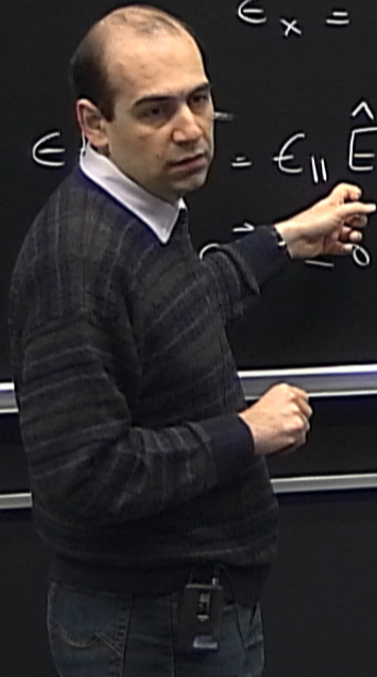
$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi n_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe}^2 \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\epsilon = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla^2 \mathbf{E} - \nabla(\nabla \cdot \mathbf{E}) = \frac{\epsilon}{c^2} \ddot{\mathbf{E}}$$



$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi\bar{n}_a q_a^2}{m_a}$$

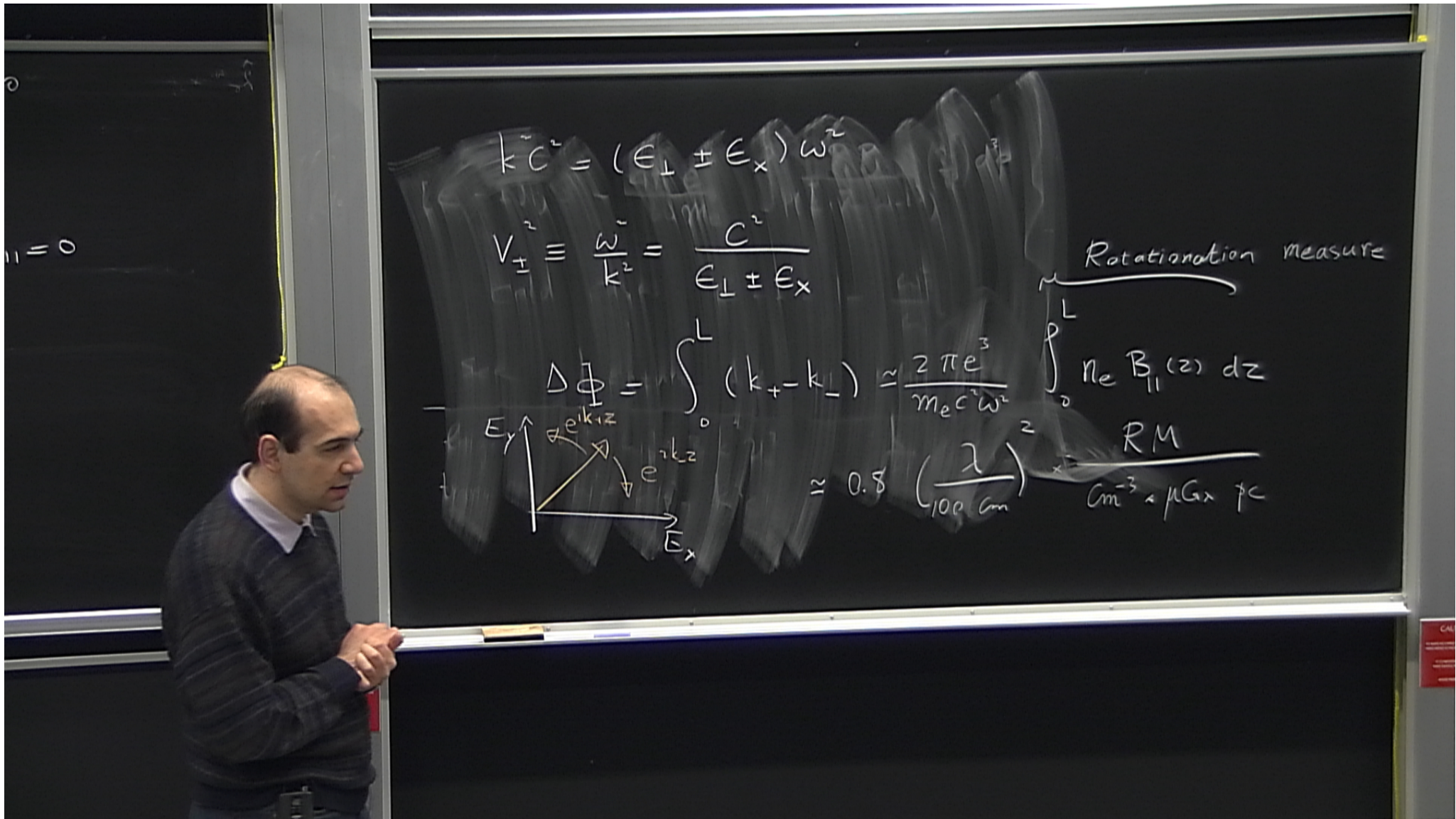
$$\epsilon_x = \frac{\omega_{pe}^2 \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\epsilon \vec{E} = \hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla \cdot (\epsilon \vec{E}) = 0, \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$





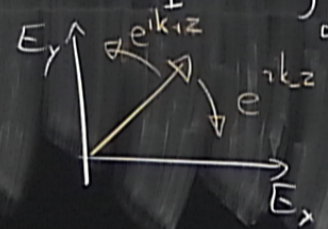


$$k^2 c^2 = (\epsilon_{\perp} \pm \epsilon_x) \omega^2$$

$$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_{\perp} \pm \epsilon_x}$$

Rotation measure

$$\Delta\phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} \int_0^L n_e B_{\parallel}(z) dz$$



$$\approx 0.8 \left( \frac{\lambda}{100 \text{ cm}} \right)^2$$

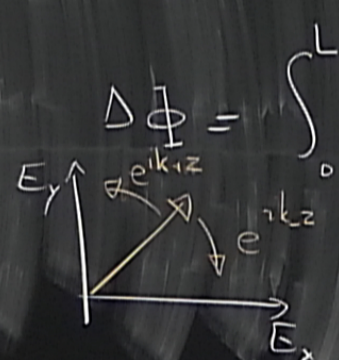
RM

$$\text{cm}^{-3} \mu\text{G} \times \text{pc}$$

$n_1 = 0$

$$k^2 c^2 = (\epsilon_L \pm \epsilon_x) \omega^2$$

$$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_L \pm \epsilon_x}$$



$$\Delta \Phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} n_e B_{||}(z) dz$$

$$\approx 0.8 \left( \frac{\lambda}{100 \text{ cm}} \right)^2$$

Rotation measure

$$n_e B_{||}(z) dz$$

RM

$$\text{cm}^{-3} \mu\text{G} \times \text{pc}$$

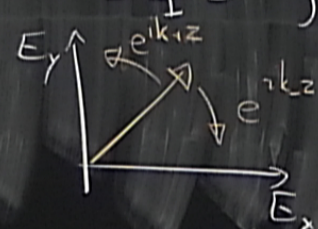
CAUTION  
DO NOT TOUCH THE BOARD  
OR THE SURROUNDING EQUIPMENT  
IF YOU HAVE ANY QUESTIONS  
PLEASE ASK THE STAFF

$\omega = 0$

~~$k^2 c^2 = (\epsilon_L \pm \epsilon_x) \omega^2$~~

~~$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_L \pm \epsilon_x}$~~

~~$\Delta \Phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} \approx 0.8 \left( \frac{\lambda}{100 \text{ nm}} \right)$~~



Rotation measure

$\int_0^L r dz$

$\mu G \times \tau c$

CAUTION  
 DO NOT TOUCH THE BOARD  
 OR THE BOARDER AT ANY TIME  
 WHILE THE BOARD IS IN USE

CAUTION

Rotation measure

$$\int n_e B_{||} dz$$

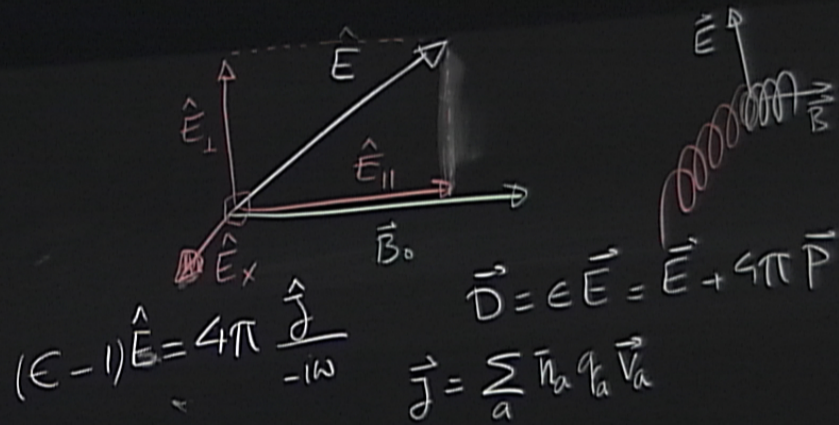
RM

$$\text{cm}^{-3} \mu\text{G} \times \text{pc}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\vec{E} \cdot \vec{B}_0) / |\vec{B}_0|^2$$

$$\hat{E}_\perp = \hat{E} \times \vec{B}_0 / |\vec{B}_0|$$



$$(\epsilon - 1)\hat{E} = 4\pi \frac{\vec{j}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{j} = \sum_a n_a q_a \vec{v}_a$$

CAUTION