

Title: PHYS 781 - Lecture 18

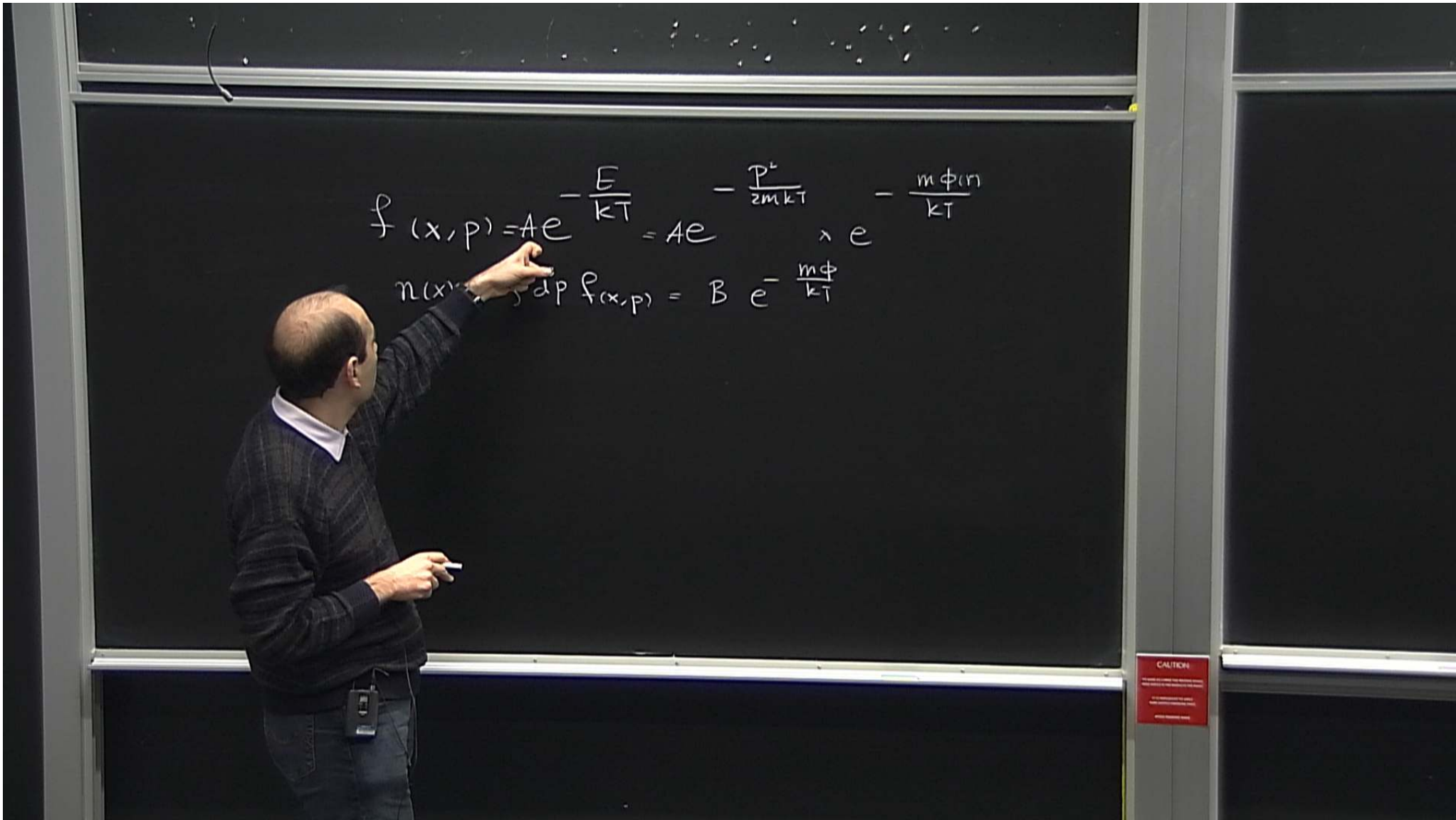
Date: Nov 13, 2014 02:00 PM

URL: <http://pirsa.org/14110054>

Abstract:

$$f(x, p) = A e^{-\frac{E}{kT}} = A e^{-\frac{p^2}{2mkT}} e^{-\frac{m\phi(x)}{kT}}$$

$$n(x) = \int dp f(x, p) = B e^{-\frac{m\phi}{kT}}$$



$$= e^{-\frac{(E-\mu)}{kT}}$$

$$f(x, p) = A e^{-\frac{E}{kT}} = A e^{-\frac{p^2}{2mkT}} e^{-\frac{m\phi(x)}{kT}}$$

$$n(x) = \int d^3p f(x, p) = B e^{-\frac{m\phi}{kT}}$$

$$\vec{j} = \frac{\partial \vec{P}}{\partial t} = -e \int \vec{v} \delta f d^3p$$

$$\hat{j} = -i\omega \hat{P} = -e^2 \hat{P} = \frac{\hat{E} \cdot \frac{\partial f}{\partial \vec{p}}}{(\vec{k} \cdot \nabla - \omega + i\lambda)} d^3p$$

$$\frac{(\epsilon - 1)\hat{E}_i}{4\pi} = \hat{P}_i(\omega, k) = -\frac{e^2}{\omega} \hat{E}_i \int \frac{p_i}{\omega - i\lambda} d^3p$$

$$f \propto e^{-\frac{p^2}{2mkT}}$$

$$\frac{\partial f}{\partial \vec{p}} = \frac{\partial f}{\partial p} \times \frac{\vec{p}}{|\vec{p}|}$$

$$[\epsilon - 1]_{ij} = -\frac{4\pi e^2}{\omega^2}$$

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\bar{k} \cdot \bar{v} - \omega - i\lambda)} \left(\frac{\partial \bar{f}}{\partial \bar{p}} \right) d^3 p$$

cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 17 in class notes

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left(\frac{\partial \bar{f}}{\partial \vec{p}} \right) d^3 p$$

cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 1 class notes

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left(\frac{\partial \bar{f}}{\partial p} \right) d^3 p$$

cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

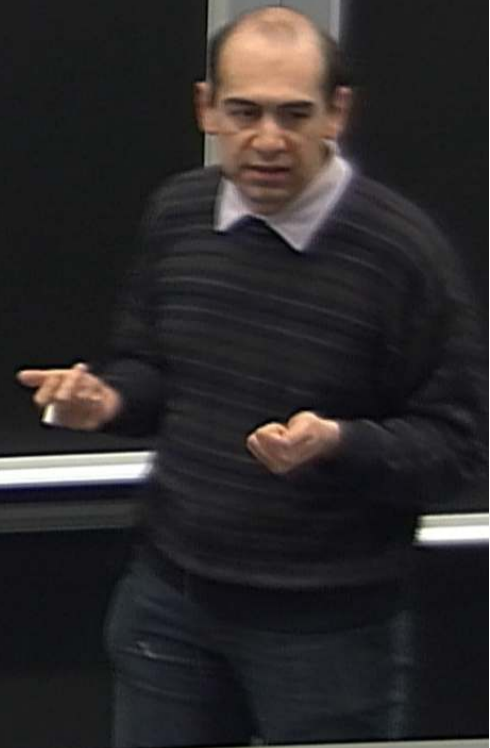
See Lecture 17 in class

$$[\epsilon - 1]_{ij} = - \frac{4\pi e^2}{\omega m_e} \int \frac{P_i P_j}{P(\vec{k} \cdot \vec{v} - \omega - i\eta)} \left(\frac{\partial \bar{f}}{\partial p} \right) d^3 p$$

cold case: $\epsilon - 1 = 1 - \frac{\omega_p^2}{\omega^2}$

See Lecture 17 in class notes

long. $\omega^2 = \omega_p^2 + 3k^2 v_e^2$ $\frac{k}{\omega} v_e \ll 1 \Rightarrow \frac{\omega}{k} \gg v_e$
 trans. $\omega^2 = \omega_p^2 + k^2 c^2 + \dots$





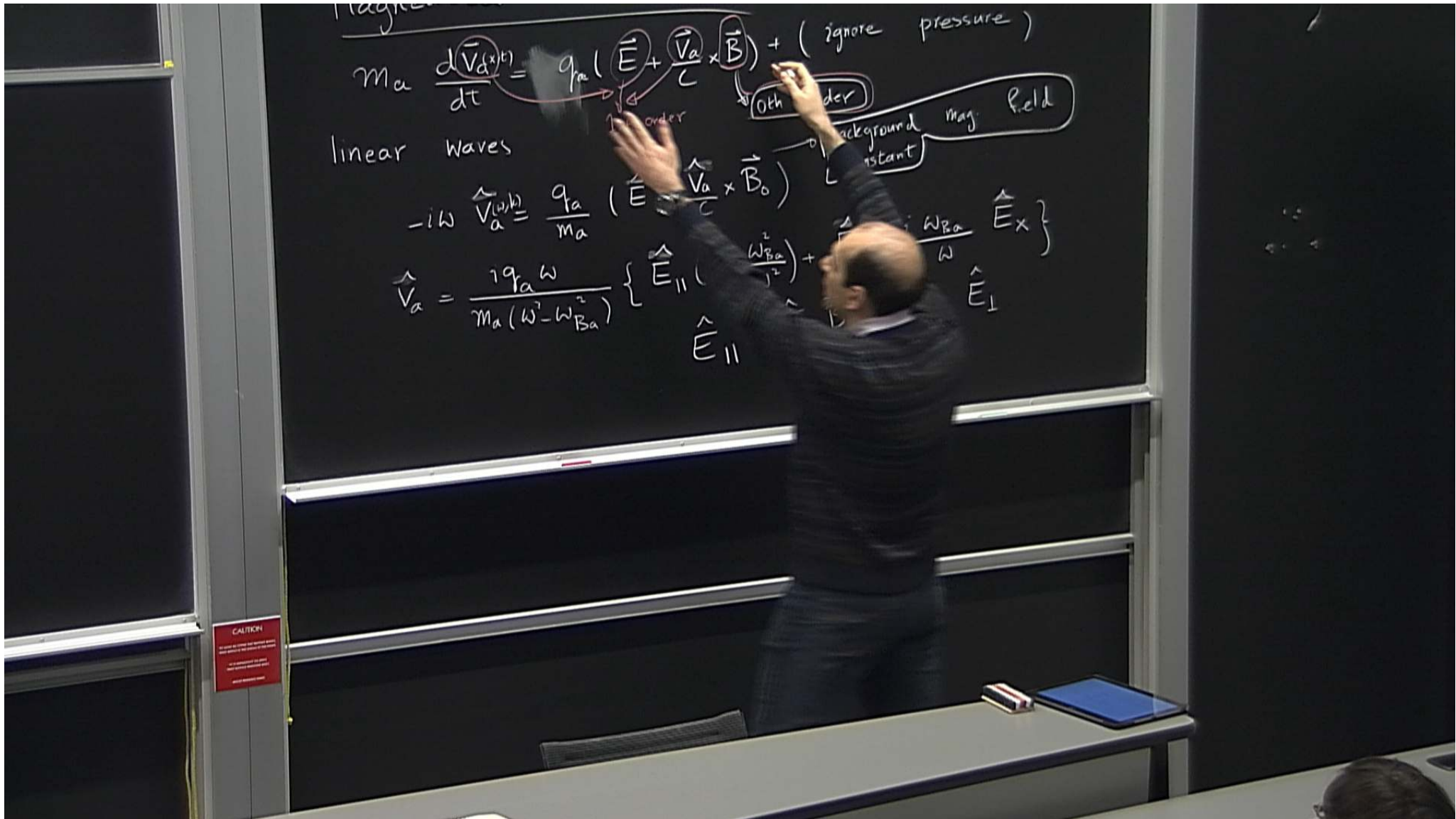
Magnetized cold plasma

$$m_a \frac{d\vec{v}_a}{dt} = q_a \left(\vec{E} + \frac{\vec{v}_a \times \vec{B}}{c} \right) + (\text{ignore pressure})$$

linear approximation

$$= \frac{q_a}{m_a} \left(\vec{E} + \frac{\vec{v}_a \times \vec{B}_0}{c} \right)$$

1st order 0th order background mag field



Magn

$$m_a \frac{d\langle \vec{v}_a^{(x,t)} \rangle}{dt} = q_a \left(\vec{E} + \frac{\vec{v}_a}{c} \times \vec{B} \right) + (\text{ignore pressure})$$

linear waves

$$-i\omega \hat{v}_a^{(\omega,k)} = \frac{q_a}{m_a} \left(\vec{E} - \frac{\hat{v}_a \times \vec{B}_0}{c} \right)$$

order

0th order

background mag field constant

$$\hat{v}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left(\frac{\omega_{Ba}^2}{\omega^2} \right) + \frac{\omega_{Ba}^2}{\omega} \hat{E}_{\perp} \right\}$$

\hat{E}_{\parallel}

\hat{E}_{\perp}

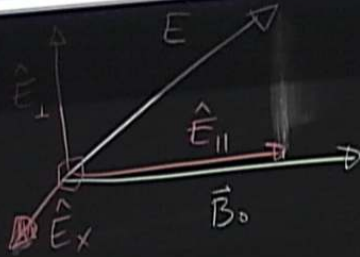
$$\hat{V}_a = \frac{1q_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{||} \left(1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|B|}$$



linear waves

$$-i\omega \hat{V}_a = \frac{q_a}{m_a} \left(\hat{E} + \hat{V}_a \times \vec{B}_0 \right)$$

1st order $\left\{ \begin{array}{l} \text{background} \\ \text{Constant} \end{array} \right.$ mag field

$$\hat{V}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left(1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \hat{E}_{\perp} - \frac{i\omega_{Ba}}{\omega} \hat{E}_x \right\}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\left\{ \begin{array}{l} \hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel} \\ \hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|\vec{B}_0|} \end{array} \right.$$



CAUTION
DO NOT TOUCH THE BOARD SURFACE
OR THE BOARD FRAME

$$\hat{V}_a = \frac{iq_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left(1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \hat{E}_{\perp} - \frac{i \omega_{Ba}}{\omega} E_x \right\}$$

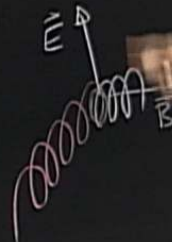
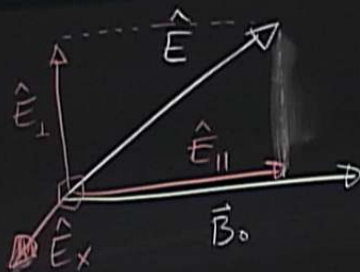
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = (\hat{E} \cdot \vec{B}_0) \frac{B_0}{|\vec{B}|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \hat{E}_{\perp} \times \frac{\vec{B}_0}{|\vec{B}|}$$

$$|\hat{E}_{\perp}| = |E_x|$$



CAUTION

$$V_a = \frac{\hbar \omega_a}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ E_{||} (\omega^2) \right.$$

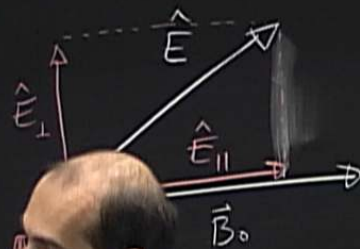
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_\perp| = |\hat{E}_x|$$



$$\epsilon - 1 =$$

CAUTION

$$V_a = \frac{v_a}{m_a(\omega^2 - \omega_{Ba}^2)}$$

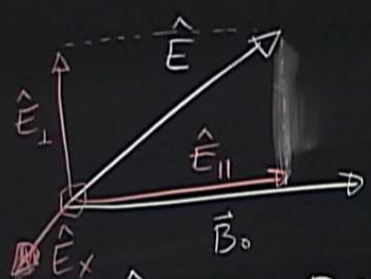
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|B|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|B|}$$

$$|\hat{E}_\perp| = |\hat{E}_x|$$



$$(\epsilon - 1)\hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$D = \epsilon E = E + 4\pi$$

CAUTION

$$\hat{V}_a = \frac{q_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{||} \left(1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

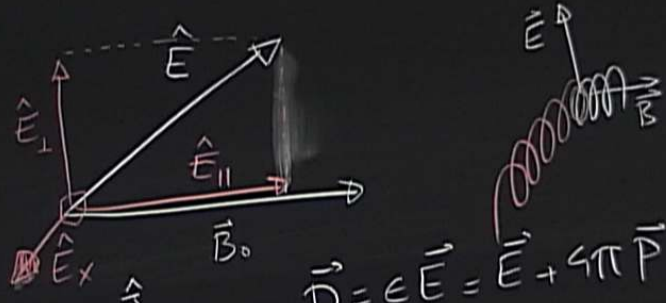
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\hat{E} \cdot \vec{B}_0) \frac{\vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_\perp = \hat{E} - \hat{E}_{||}$$

$$\hat{E}_x = \hat{E}_\perp \times \frac{\vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_\perp| = |\hat{E}_x|$$



$$(\epsilon - 1) \hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\hat{j} = \sum_a \bar{n}_a q_a \hat{V}_a$$

CAUTION

$$\hat{V}_a = \frac{q_a \omega}{m_a (\omega^2 - \omega_{Ba}^2)} \left\{ \hat{E}_{\parallel} \left(1 - \frac{\omega_{Ba}^2}{\omega^2} \right) + \dots \right\}$$

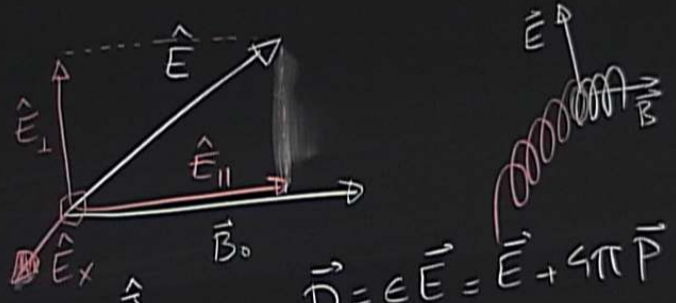
$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{\parallel} = \frac{(\hat{E} \cdot \vec{B}_0) \vec{B}_0}{|\vec{B}_0|^2}$$

$$\hat{E}_{\perp} = \hat{E} - \hat{E}_{\parallel}$$

$$\hat{E}_x = \frac{\hat{E}_{\perp} \times \vec{B}_0}{|\vec{B}_0|}$$

$$|\hat{E}_{\perp}| = |\hat{E}_x|$$



$$(\epsilon - 1) \hat{E} = 4\pi \frac{\hat{j}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{j} = \sum_a n_a q_a \vec{V}_a$$

CAUTION

trans.

$$\omega = \omega_p^2 + k^2 c^2 + \dots$$

$$\hat{V}_a =$$

$$\omega_p$$

$$\epsilon_{\perp} =$$

$$(\epsilon - 1)\hat{E}$$

CAUTION

CAUTION

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\epsilon_x = \frac{\omega_{pe}^2 \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi}^2 \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\omega_{pa}^2 = \frac{4\pi \bar{n}_a q_a^2}{m_a}$$

$$(\epsilon - 1)\hat{E}$$

$$\epsilon_{\perp} = 1 - \frac{\tilde{\omega}_{pe}^2}{\omega^2 - \tilde{\omega}_{pe}^2} - \frac{\tilde{\omega}_{pi}^2}{\omega^2 - \tilde{\omega}_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\tilde{\omega}_{pe}^2 + \tilde{\omega}_{pi}^2}{\omega^2}$$

$$\epsilon_x = \frac{\tilde{\omega}_{pe} \tilde{\omega}_{pi}^2}{\omega(\omega^2 - \tilde{\omega}_{pe}^2)} - \frac{\tilde{\omega}_{pi} \tilde{\omega}_{pe}^2}{\omega(\omega^2 - \tilde{\omega}_{pi}^2)}$$

$$\tilde{\omega}_{pa}^2 = \frac{4\pi\tilde{n}_a q_a^2}{m_a}$$

CAUTION

CAUTION

$$\epsilon_{\perp} = 1 - \frac{\tilde{\omega}_{pe}^2}{\omega^2 - \tilde{\omega}_{pe}^2} - \frac{\tilde{\omega}_{pi}^2}{\omega^2 - \tilde{\omega}_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\tilde{\omega}_{pe}^2 + \tilde{\omega}_{pi}^2}{\omega^2}$$

$$\tilde{\omega}_{pa}^2 = \frac{4\pi\tilde{n}_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\tilde{\omega}_{pe} \tilde{\omega}_{pe}^2}{\omega(\omega^2 - \tilde{\omega}_{pe}^2)} - \frac{\tilde{\omega}_{pi} \tilde{\omega}_{pi}^2}{\omega(\omega^2 - \tilde{\omega}_{pi}^2)}$$

$$\hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla \cdot (\epsilon \hat{E}) = 0 ; \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi\bar{n}_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe} \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi} \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\nabla \cdot (\epsilon \vec{E}) = 0, \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon E) = 0 \Rightarrow \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

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Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

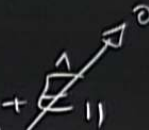
$$\nabla \cdot (\epsilon E) = 0 \Rightarrow$$

$$(k^2 c^2 - \epsilon_{\perp} \omega^2) \hat{E}_{\perp} =$$

$$\hat{E}_{\parallel} = \Rightarrow \hat{E}_{\parallel} = 0$$

Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\hat{E} = \hat{E}_\perp + \hat{E}_\parallel$$


$$\nabla \cdot (\epsilon E) = 0 \Rightarrow \vec{k} \cdot (\epsilon_\parallel \hat{E}_\parallel) = 0 \Rightarrow \hat{E}_\parallel = 0$$

$$(k^2 c^2 - \epsilon_\perp \omega^2) \hat{E} = i \epsilon_x \omega^2 \hat{E}_x$$

$$\vec{k} = (0, 0, k_z) \quad \hat{E}_\pm \equiv \hat{E}_x \pm i \hat{E}_y$$

CAUTION

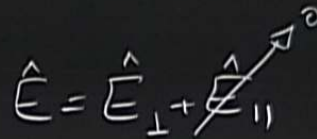
Faraday Rotation

$$\vec{k} \parallel \vec{B}_0$$

$$\nabla \cdot (\epsilon \vec{E}) = 0 \Rightarrow \vec{k} \cdot (\epsilon_{\parallel} \hat{E}_{\parallel}) = 0 \Rightarrow \hat{E}_{\parallel} = 0$$

$$(\vec{k} \cdot \vec{c} - \omega^2) \hat{E} = i \epsilon_x \omega^2 \hat{E}_x$$

$$\hat{E}_{\pm} \equiv \hat{E}_x \pm i \hat{E}_y$$

$$\hat{E} = \hat{E}_{\perp} + \hat{E}_{\parallel}$$


$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

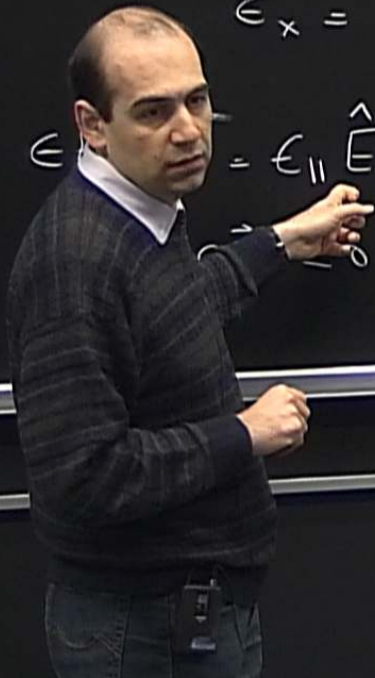
$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

$$\omega_{pa}^2 = \frac{4\pi n_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe}^2 \omega_{pi}^2}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi}^2 \omega_{pe}^2}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\vec{\epsilon} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

$$\vec{\nabla} \cdot \vec{E} - \nabla(\nabla \cdot \vec{E}) = \frac{\epsilon}{c^2} \ddot{\vec{E}}$$



$$\epsilon_{\perp} = 1 - \frac{\omega_{pe}^2}{\omega^2 - \omega_{pe}^2} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{pi}^2}$$

$$\epsilon_{\parallel} = 1 - \frac{\omega_{pe}^2 + \omega_{pi}^2}{\omega^2}$$

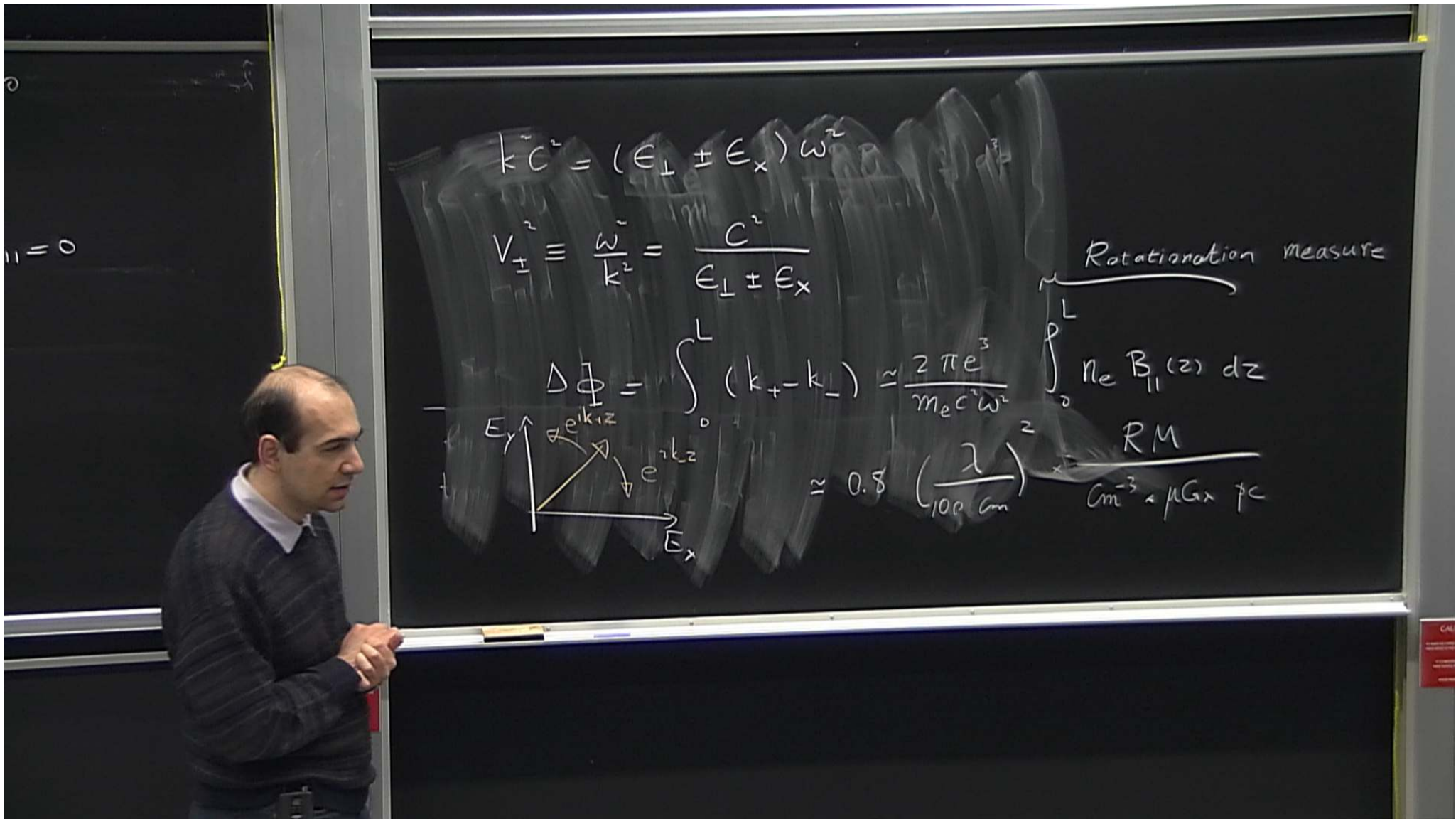
$$\omega_{pa}^2 = \frac{4\pi\bar{n}_a q_a^2}{m_a}$$

$$\epsilon_x = \frac{\omega_{pe} \omega_{pe}'}{\omega(\omega^2 - \omega_{pe}^2)} - \frac{\omega_{pi} \omega_{pi}'}{\omega(\omega^2 - \omega_{pe}^2)}$$

$$\epsilon \vec{E} = \hat{D} = \epsilon_{\parallel} \hat{E}_{\parallel} + \epsilon_{\perp} \hat{E}_{\perp} + i\epsilon_x \hat{E}_x$$

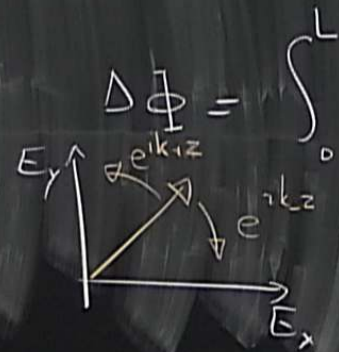
$$\nabla \cdot (\epsilon \vec{E}) = 0, \quad \nabla^2 E - \nabla(\nabla \cdot E) = \frac{\epsilon}{c^2} \ddot{E}$$





$$k^2 c^2 = (\epsilon_L \pm \epsilon_x) \omega^2$$

$$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_L \pm \epsilon_x}$$



$$\Delta \Phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} \int_0^L n_e B_{||}(z) dz$$

$$\approx 0.8 \left(\frac{\lambda}{100 \text{ cm}} \right)^2$$

Rotation measure

RM

$cm^{-3} \mu G \times pc$

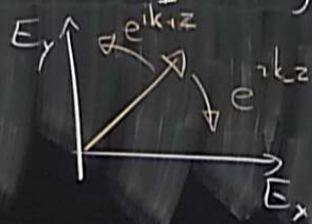
$n_1 = 0$

$$k^2 c^2 = (\epsilon_L \pm \epsilon_x) \omega^2$$

$$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_L \pm \epsilon_x}$$

Rotation measure

$$\Delta\phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} n_e B_{||}(z) dz$$



$$\approx 0.8 \left(\frac{\lambda}{100 \text{ cm}} \right)^2$$

RM

$$\text{cm}^{-3} \mu\text{G} \times \text{pc}$$

CAUTION
DO NOT TOUCH THE BOARD
OR THE SURROUNDING AREA
WHILE THE BOARD IS
BEING USED

$\psi = 0$

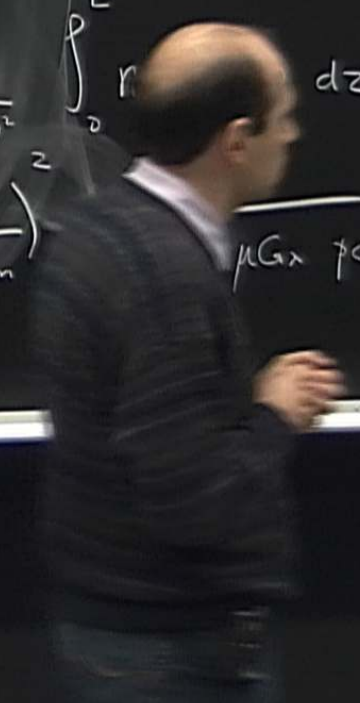
$$k^2 c^2 = (\epsilon_L \pm \epsilon_x) \omega^2$$

$$V_{\pm}^2 \equiv \frac{\omega^2}{k^2} = \frac{c^2}{\epsilon_L \pm \epsilon_x}$$

Rotation measure

$$\Delta \phi = \int_0^L (k_+ - k_-) dz \approx \frac{2\pi e^3}{m_e c^2 \omega^2} \int_0^L n dz$$

$$\approx 0.8 \left(\frac{\lambda}{100 \text{ cm}} \right) \mu G \times \text{pc}$$



CAUTION
 DO NOT TOUCH THE BOARD
 OR THE CHALKBOARD ERASER
 OR THE CHALKBOARD MARKERS

Rotation measure

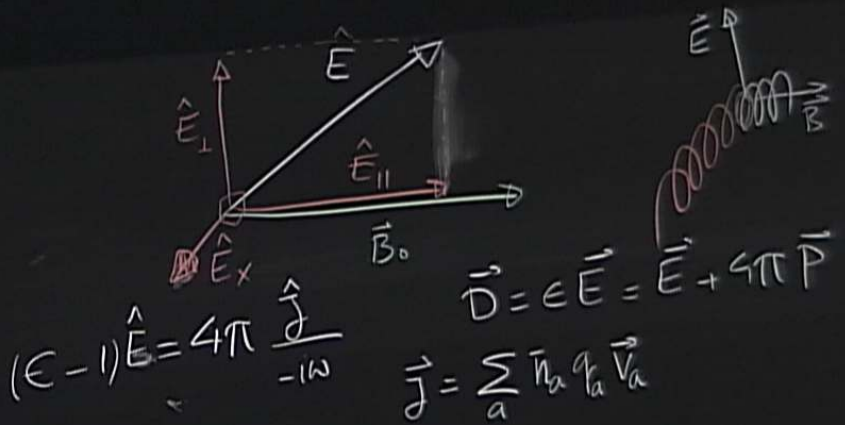
$$\int n_e B_{||} dz$$

RM

$$\text{cm}^{-3} \mu\text{G} \times \text{pc}$$

$$\omega_{Ba} = \frac{q_a B_0}{m_a c}$$

$$\hat{E}_{||} = (\mathbf{E} \cdot \mathbf{B}_0) / |\mathbf{B}_0|^2 \quad \hat{E}_\perp = \hat{E} \times \mathbf{B}_0 / |\mathbf{B}_0|$$



$$(\epsilon - 1) \hat{E}_{||} = 4\pi \frac{\hat{j}_{||}}{-i\omega}$$

$$\vec{D} = \epsilon \vec{E} = \vec{E} + 4\pi \vec{P}$$

$$\vec{j} = \sum_a n_a q_a \vec{v}_a$$

CAUTION