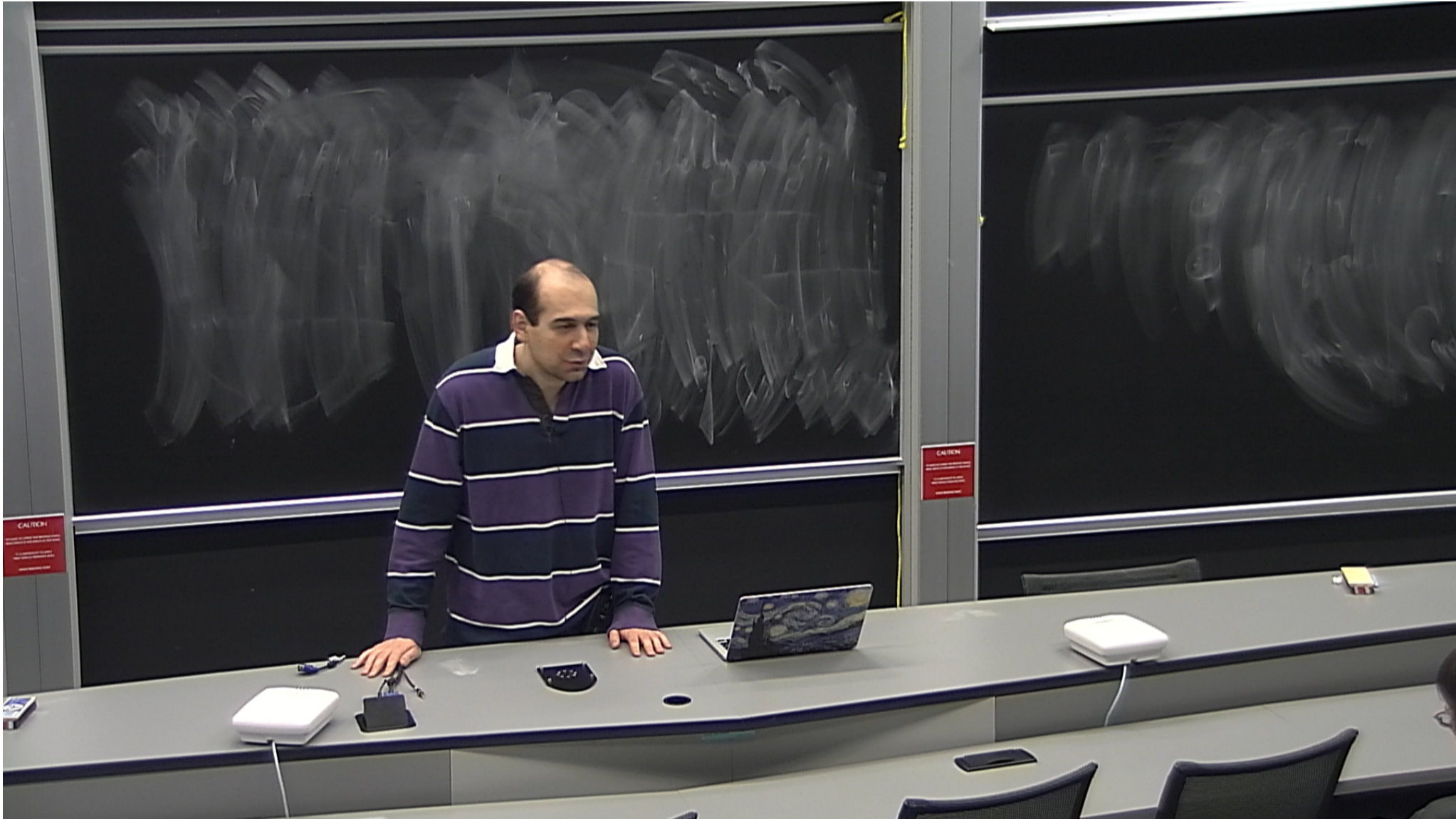


Title: PHYS 781 - Lecture 15

Date: Nov 06, 2014 02:00 PM

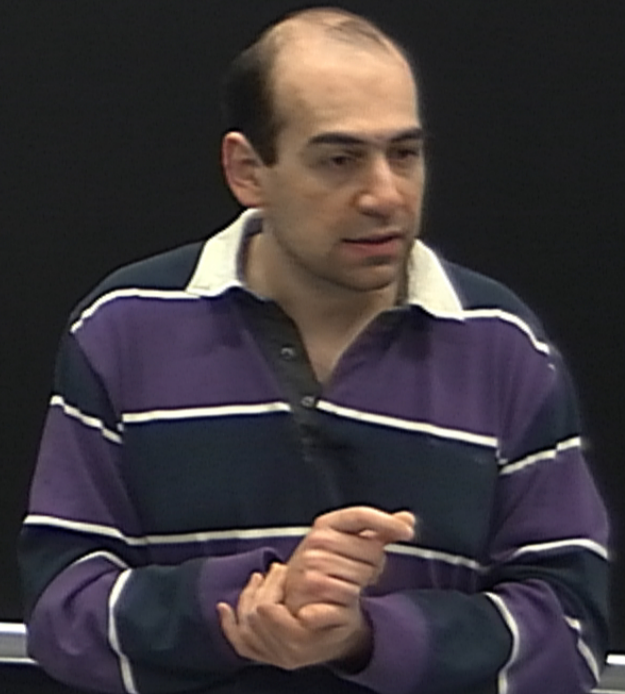
URL: <http://pirsa.org/14110053>

Abstract:



Plasma freq.: $\omega_p^2 = \frac{4\pi e^2 n}{m_e}$

Debye length: $\lambda_D = \frac{kT}{4\pi n e^2}$



Plasma freq.: $\omega_p^2 = \frac{4\pi e^2 n}{m_e} \Rightarrow \frac{\omega_p}{2\pi} = 9 \text{ kHz} \left(\frac{n}{1 \text{ cm}^{-3}} \right)^{1/2}$

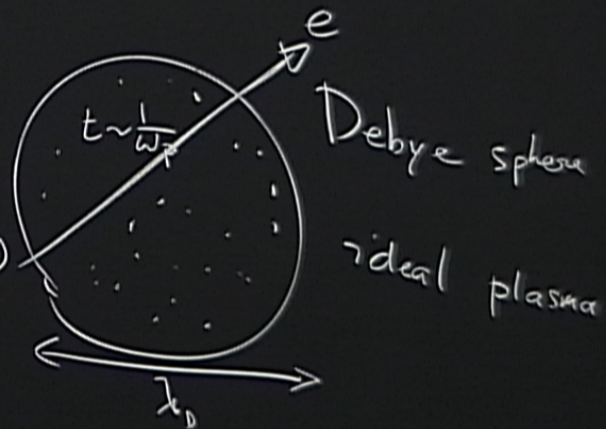
Debye length: $\lambda_D = \frac{kT}{4\pi n e^2} \rightarrow \text{neutral plasma}$

Plasma freq: $\omega_p^2 = \frac{4\pi e^2 n}{m_e} \Rightarrow \frac{\omega_p}{2\pi} = 9 \text{ kHz} \left(\frac{n}{1 \text{ cm}^{-3}} \right)^{1/2}$

Debye length: $\lambda_D^2 = \frac{kT}{4\pi n e^2} \rightarrow \text{neutral plasma}$

$$\omega_p \times \lambda_D \sim v_{th} \sim \sqrt{\frac{kT}{m_e}}$$

$$\lambda_D \sim 7 \text{ cm } T(K)^{1/2} n(\text{cm}^{-3})^{-1/2}$$



CMB

$$T = 3 \text{ K}$$

$$E = h\nu = kT$$

$$\lambda \nu = c \Rightarrow \lambda = \frac{c}{\nu} = \frac{ch}{kT} \sim \frac{3 \times 10^{10} \times 10^{-27}}{1}$$

CMB

$$T = 3 \text{ K}$$

$$E = h\nu = kT$$

$$\lambda \nu = c \Rightarrow \lambda = \frac{c}{\nu} = \frac{ch}{kT} \sim \frac{3 \times 10^{10} \times 6 \times 10^{-27}}{10^{-16} \times 3} \sim \text{cm}$$

CMB

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$$\nu \sim 100 \text{ GHz}$$

CAUTION

DO NOT TOUCH THE SURFACE OF THE MONITOR.

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$$\frac{\partial f_a}{\partial t} + \vec{v} \cdot \frac{\partial f_a}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial f_a}{\partial \vec{p}} = C_a[f]$$

$$\downarrow$$

$$q_a(\vec{E} + \vec{v} \times \vec{B}) - m_a \vec{\nabla} \phi_g$$

$$| \text{time} < t_{c,ab} \iff L < \lambda_{c,ab} \rightarrow \text{Collisi.} \\ (t_{c,ep} \gg t_{c,ee} \& t_{c,eb})$$

$$t_{c,ab} \gg t_{c,aa} \& t_{c,bb} \\ \text{if } m_a \gg m_b \text{ or } m_b \gg m_a$$

$$\frac{\partial f_a}{\partial t} + \vec{v} \cdot \frac{\partial f_a}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial f_a}{\partial \vec{p}} = C_a[f]$$

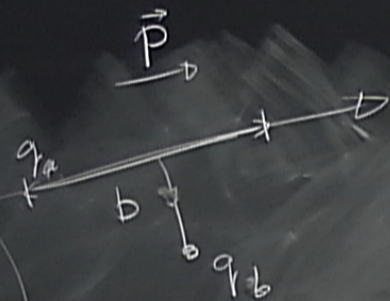
$$q_a (\vec{E} + \vec{v} \times \vec{B}) - m_a \vec{\nabla} \phi_g$$

$$(t_{\text{time}} < t_{c,ab} \leftrightarrow L < \lambda_{c,ab}) \rightarrow \text{Collisionless}$$

$$(t_{c,ep} \gg t_{c,ee} \& t_{c,pp})$$

$$t_{c,ab} \gg t_{c,aa} \& t_{c,bb}$$

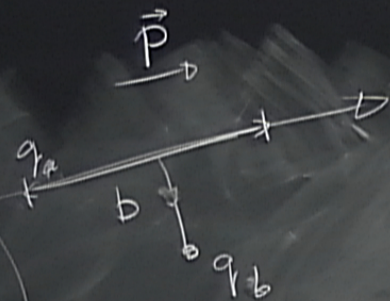
$$\text{if } m_a \gg m_b \text{ or } m_b \gg m_a$$



$$\Delta P_{\perp} \approx \frac{q_a q_b}{b^2} \frac{2b}{v_{rel}}$$

$$\begin{aligned} \Delta P_{tot}^2 &= \Delta P_{\perp}^2 \times \# = t_c \int (2\pi b db) n v_{rel} \cdot \frac{4}{b^2} \frac{q_a^2 q_b^2}{v_{rel}^2} \\ &= \frac{8\pi n q^4}{v_{rel}} t_c \left(\ln \left(\frac{b_{max}}{b_{min}} \right) \right) \end{aligned}$$

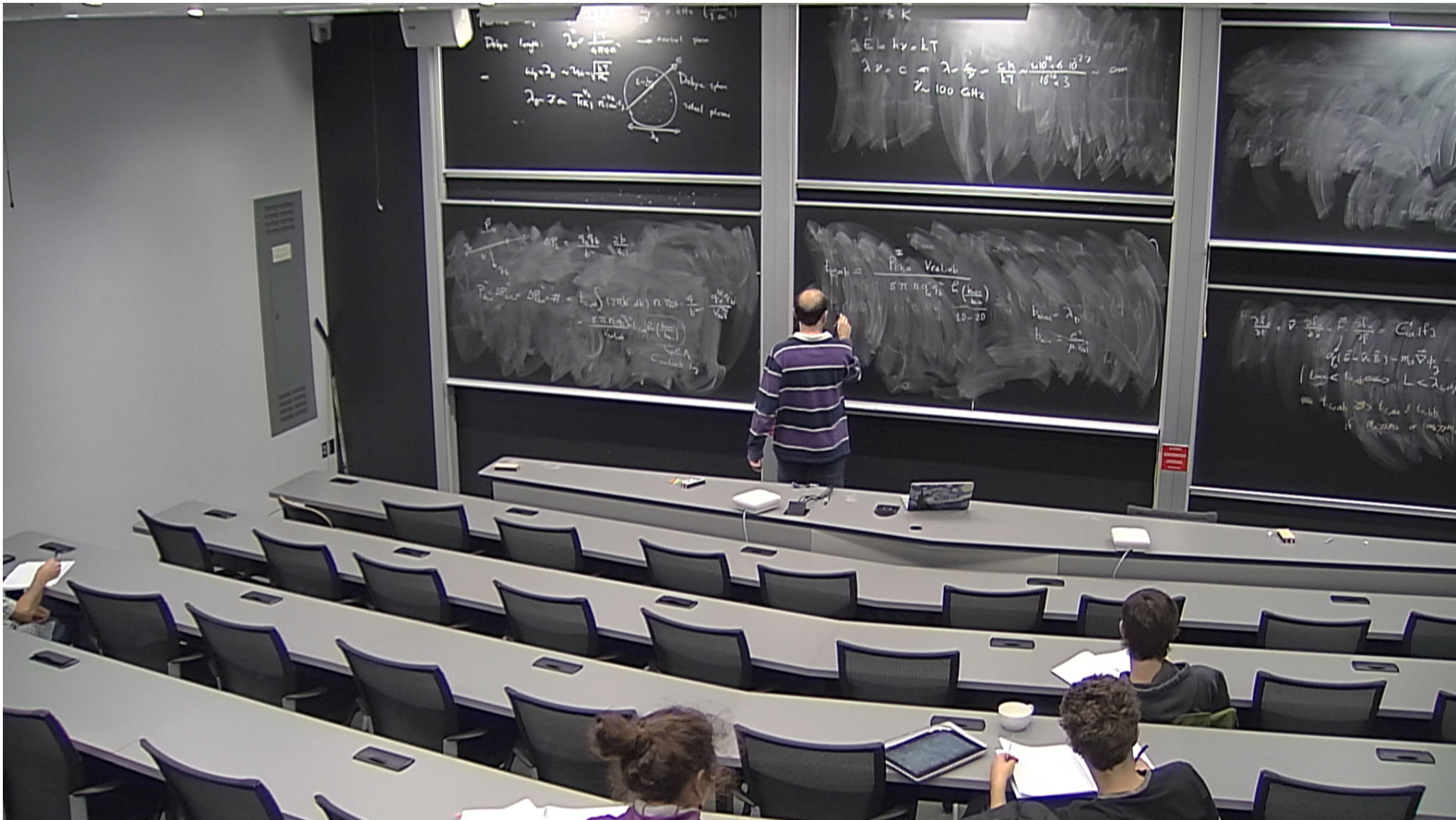
$\ln \Lambda$
 Coulomb log.

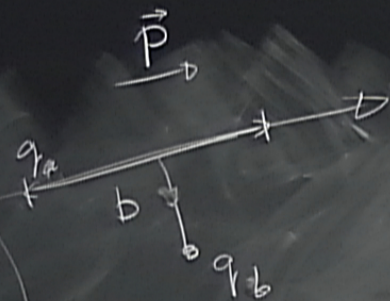


$$\Delta P_{\perp} \approx \frac{q_a q_b}{b^2} \frac{2b}{v_{rel}}$$

$$\begin{aligned} P_{tra}^2 &= \Delta P_{tra}^2 = \Delta P_{\perp,a}^2 \times \# = t_{c,ab} \int (2\pi b db) n v_{rel} \cdot \frac{4}{b^2} \frac{q_a^2 q_b^2}{v_{rel}^2} \\ &= \frac{8\pi n q_a^2 q_b^2 t_{c,ab}}{v_{rel,ab}} \ln\left(\frac{b_{max}}{b_{min}}\right) \\ &\quad \swarrow \ln \Lambda \\ &\quad \text{Coulomb log.} \end{aligned}$$







$$\Delta P_{\perp} \approx \frac{q_a q_b}{b^2} \frac{2b}{v_{rel}}$$

$$\begin{aligned} P_{\perp a}^2 &= \Delta P_{\perp a}^2 = \Delta P_{\perp a}^2 \times \# = t_{c,ab} \int (2\pi b db) n v_{rel} \frac{4}{b^2} \frac{q_a^2 q_b^2}{v_{rel}^2} \\ &= \frac{8\pi n q_a^2 q_b^2 t_{c,ab}}{v_{rel,ab}} \ln\left(\frac{b_{max}}{b_{min}}\right) \\ &\quad \swarrow \text{Coulomb log.} \end{aligned}$$

$$t_{c,ab} = \frac{z^2 P_{th,a} V_{rel,ab}}{8\pi n q_a^2 q_b^2 \underbrace{\ln\left(\frac{b_{max}}{b_{min}}\right)}_{10-20}} \propto$$

$$m_a \sqrt{\frac{1}{m_a} + \frac{1}{m_b}}$$

$$\mu = m_a^{-1} + m_b^{-1}$$

$$b_{max} = \lambda_D$$

$$b_{min} = \max\left[\frac{e^2}{\mu V_{rel}}, \frac{\hbar}{\mu V_{rel}}\right]$$

$$P_{th,a}^2 = kT m_a$$

$$V_{rel} = V_a - V_b \Rightarrow V_{rel}^2 = V_a^2 + V_b^2 = kT \left(\frac{1}{m_a} + \frac{1}{m_b}\right)$$

$$m_i \approx A \times 2 \times 10^3 m_e \gg m_e$$

$$t_{c,ii} \propto \sqrt{m_i}$$

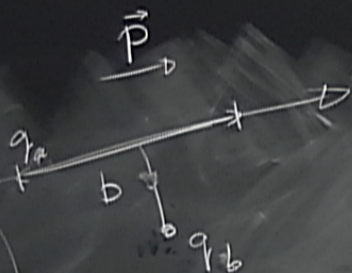
$$t_{c,ie} \propto \frac{m_i}{\sqrt{m_e}}$$

$$t_{c,ee} \propto \sqrt{m_e}$$

$$\cancel{t_{c,ei} \propto \sqrt{m_e}}$$

$$t_{c,ee} : t_{c,ie} : t_{c,ii}$$

$$1 : \frac{m_i}{m_e} : \sqrt{\frac{m_i}{m_e}}$$



$$\Delta P_{\perp} \approx \frac{q_a q_b}{b^2} \frac{2b}{v_{rel}}$$

$$P_{\perp}^2 = \Delta P_{\perp}^2 = \Delta P_{\perp}^2 \times \# = t_{c,ab} \int (2\pi b db) n v_{rel} \frac{4}{b^2} \frac{q_a^2 q_b^2}{v_{rel}^2}$$

$$L = m_a v_{rel} b > \hbar$$

$$= \frac{8\pi n q_a^2 q_b^2}{v_{rel,ab}} t_{c,ab} \ln\left(\frac{b_{max}}{b_{min}}\right)$$

$\ln \Lambda$
Coulomb log.

$$m_i \approx A \times 2 \times 10^3 m_e \gg m_e$$

$$t_{c,ii} \propto \sqrt{m_i}$$

$$t_{c,ie} \propto \frac{m_i}{\sqrt{m_e}}$$

$$t_{c,ee} \propto \sqrt{m_e}$$

Show $t_{c,ie} \sim t_{c,ei}$

$$t_{\text{Brem}} \propto \frac{T^{1/2}}{n}$$

$$t_{c,ee} : t_{c,ie} : t_{c,ii}$$

$$1 : \frac{m_i}{m_e} : \sqrt{\frac{m_i}{m_e}} \rightarrow 10^7 \text{ K}$$

$$t_{c,ee} \sim \left(\frac{30 \text{ yr}}{h \Lambda / 10} \right) \frac{T^{3/2} (\text{keV})}{n (\text{cm}^{-3})}$$

$$t_{c,ie} \sim 2 \times 10^3 \left(\frac{A}{Z^2} \right) t_{c,ee}$$

on cosmological scales and $T \sim 10 \text{ keV}$
 $t_{c,ie} \sim H^{-1}$

$$f(\vec{x}, \vec{p}, t) \quad \vec{E}(\vec{x}, t) \quad \vec{B}(\vec{x}, t)$$

$$\frac{\partial f_e}{\partial t} + \vec{v} \cdot \frac{\partial f_e}{\partial \vec{x}} - e \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_e}{\partial \vec{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + \vec{v} \cdot \frac{\partial f_i}{\partial \vec{x}} + Ze \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_i}{\partial \vec{p}} = 0$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j}$$

$$\nabla \cdot \vec{E} = 4\pi \rho$$

$$\rho \equiv e \int d^3p (Z f_i - f_e) \quad ; \quad \vec{j} \equiv e \int d^3p \vec{v} (Z f_i - f_e)$$

$$f(\vec{x}, \vec{p}; t) \quad \vec{E}(\vec{x}, t) \quad \vec{B}(\vec{x}, t)$$

$$\frac{\partial f_e}{\partial t} + v \cdot \frac{\partial f_e}{\partial \vec{x}} - e \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_e}{\partial \vec{p}} = 0$$

$$\frac{\partial f_i}{\partial t} + v \cdot \frac{\partial f_i}{\partial \vec{x}} + Ze \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_i}{\partial \vec{p}} = 0$$

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$$f(\vec{x}, \vec{p}; t) \quad \vec{E}(\vec{x}, t) \quad \vec{B}(\vec{x}, t)$$

$$\left\{ \begin{array}{l} \frac{\partial f_e}{\partial t} + \vec{v} \cdot \frac{\partial f_e}{\partial \vec{x}} - e \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_e}{\partial \vec{p}} = 0 \\ \frac{\partial f_i}{\partial t} + \vec{v} \cdot \frac{\partial f_i}{\partial \vec{x}} + Ze \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_i}{\partial \vec{p}} = 0 \end{array} \right.$$

$$\bar{f}(\vec{p}) = e^{-\vec{p}^2 / (2m kT)}$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{B} = 0 \\ \nabla \cdot \vec{E} = 4\pi \rho \end{array} \right.$$

constraints

$$\rho \equiv e \int d^3p (Z f_i - f_e) \quad ; \quad \vec{j} \equiv e \int d^3p \vec{v} (Z f_i - f_e)$$

$$f(\vec{x}, \vec{p}; t) \quad \vec{E}(\vec{x}, t) \quad \vec{B}(\vec{x}, t)$$

$$\left\{ \begin{array}{l} \frac{\partial f_e}{\partial t} + \vec{v} \cdot \frac{\partial f_e}{\partial \vec{x}} - e \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_e}{\partial \vec{p}} = 0 \\ \frac{\partial f_i}{\partial t} + \vec{v} \cdot \frac{\partial f_i}{\partial \vec{x}} + Ze \left(\vec{E} + \frac{\vec{v} \times \vec{B}}{c} \right) \cdot \frac{\partial f_i}{\partial \vec{p}} = 0 \end{array} \right.$$

$$\nabla \times \vec{E} = -\frac{1}{c} \frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c} \frac{\partial \vec{E}}{\partial t} + 4\pi \vec{j}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{B} = 0 \\ \nabla \cdot \vec{E} = 4\pi \rho \end{array} \right.$$

Constraints

$$\rho \equiv e \int d^3p (Z f_i - f_e) \quad ; \quad \vec{j} \equiv e \int d^3p \vec{v} (Z f_i - f_e)$$

$$\bar{f}(p) = e^{-\frac{\vec{p}^2}{2mKT}} = e^{-\frac{E}{KT}}$$

$$\vec{E}(\vec{x}, t) = \int \hat{E}_{\vec{k}, \omega} e^{i(\vec{k} \cdot \vec{x} - \omega t)} d^3k d\omega$$

$\vec{P}$: polarization vector

$$\vec{D} = \vec{E} + 4\pi \vec{P}$$

$$\frac{\partial \vec{P}}{\partial t} = \vec{j} ; \quad \vec{\nabla} \cdot \vec{P} = -\rho$$

$$\bar{P}(p) = e^{-\frac{p^2}{2m kT}}$$

$$= e^{-\frac{E}{kT}}$$

constraints

$$\sum f_i - f_e$$

$$\vec{E}(x,t) = \int \hat{E}_{\vec{k},\omega} e^{i(\vec{k} \cdot \vec{x} - \omega t)} d^3k d\omega$$

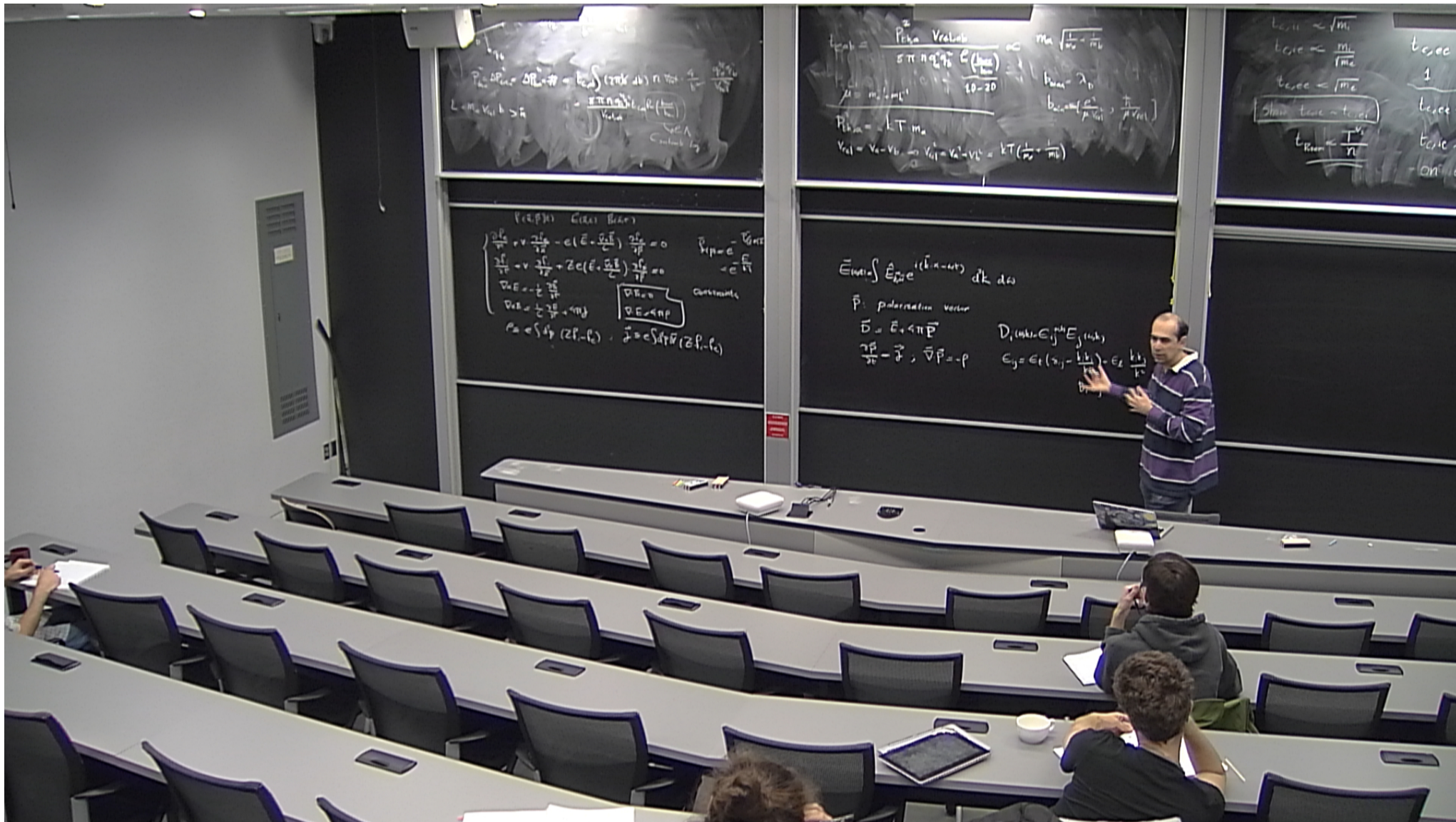
$\vec{P}$: polarization vector

$$\vec{D} = \vec{E} + 4\pi \vec{P}$$

$$\frac{\partial \vec{P}}{\partial t} = \vec{j} ; \vec{\nabla} \cdot \vec{P} = -\rho$$

$$D_i(\omega, k) = \epsilon_{ij}(\omega, k) E_j(\omega, k)$$

$$\epsilon_{ij} = \epsilon_t \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + \epsilon_l \frac{k_i k_j}{k^2}$$



$$\vec{E}(x,t) = \int \hat{E}_{\vec{k},\omega} e^{i(\vec{k} \cdot \vec{x} - \omega t)} d^3k d\omega$$

$\vec{P}$: polarization vector

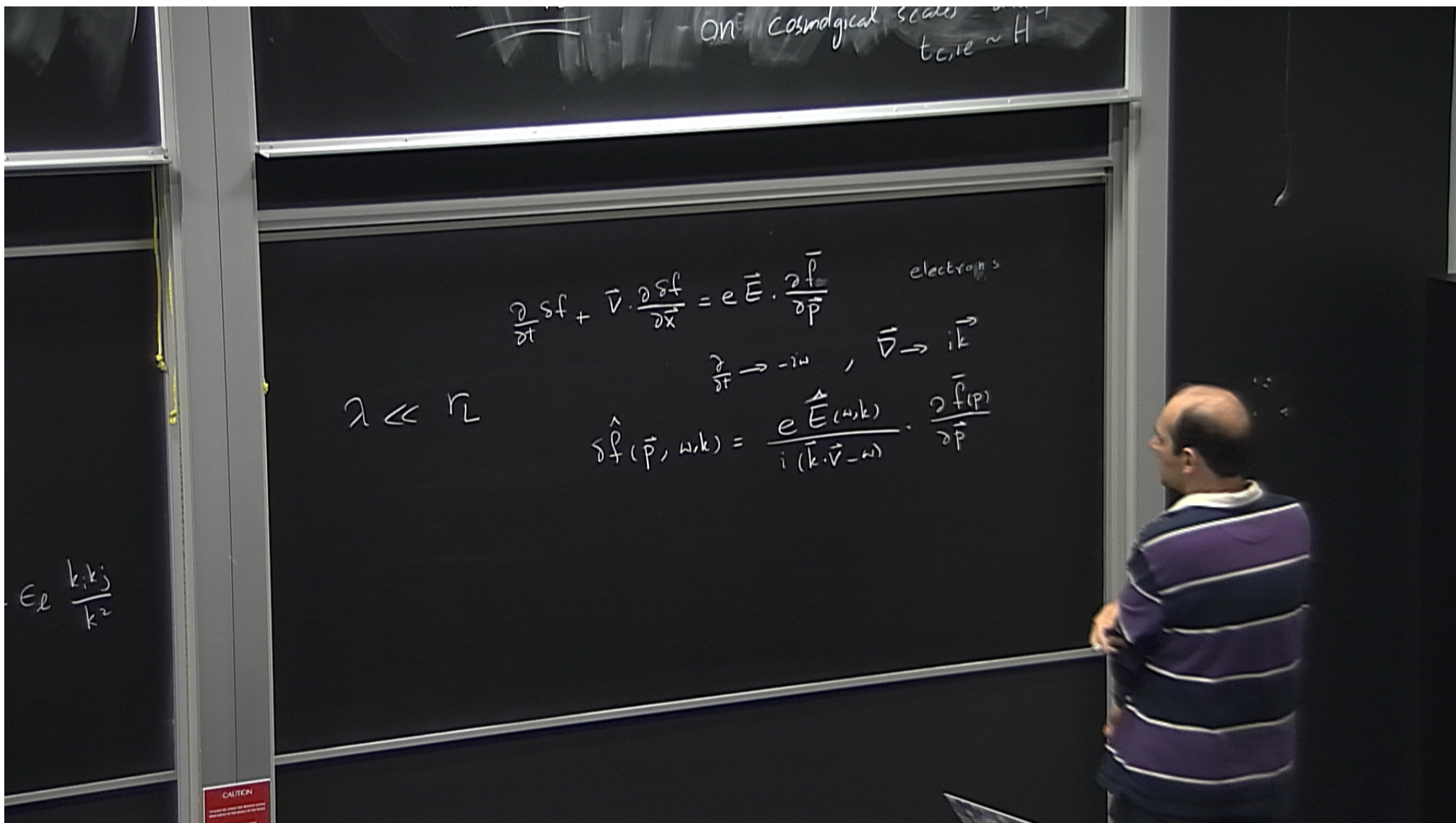
$$\vec{D} = \vec{E} + 4\pi\vec{P}$$

$$\frac{\partial \vec{P}}{\partial t} = \vec{J} ; \quad \vec{\nabla} \cdot \vec{P} = -\rho$$

$$D_i(\omega, k) = \epsilon_{ij}(\omega, k) E_j(\omega, k)$$

$$\epsilon_{ij} = \epsilon_t \left(\delta_{ij} - \frac{k_i k_j}{k^2} \right) + \epsilon_l \frac{k_i k_j}{k^2}$$

$B_i B_j$



$$\lambda \ll r_L$$

$$\frac{\partial}{\partial t} sf + \vec{v} \cdot \frac{\partial sf}{\partial \vec{x}} = e \vec{E} \cdot \frac{\partial \bar{f}}{\partial \vec{p}}$$

electrons

$$\frac{\partial}{\partial t} \rightarrow -i\omega, \quad \vec{v} \rightarrow i\vec{k}$$

$$\hat{\delta f}(\vec{p}, \omega, k) = \frac{e \hat{E}(\omega, k)}{i(\vec{k} \cdot \vec{v} - \omega)} \cdot \frac{\partial \bar{f}(\vec{p})}{\partial \vec{p}}$$

Landau
damping

$$\frac{ik \cdot \hat{E}}{4\pi} = -e \int sf d\vec{p}$$