Title: Quantum supergravity and exact holography

Date: Nov 11, 2014 02:00 PM

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Abstract: <span>I will present recent results on the computation of finite N corrections in supergravity in the context of AdS2/CFT1 and AdS4/ABJM holography. I will show how to use localisation in supergravity to compute all perturbative and nonperturbative charge corrections to the entropy of supersymmetric black holes including complicated number theoretic objects called Kloosterman sums. These are essential to recover an integer which can be identified as the number of black hole ground states. I will then explain how these techniques can be used on M-theory on AdS4xS^7 to compute the exact perturbative AdS4 partition function, the Airy function, as predicted from ABJM theory on a three sphere.</span>

## Black Hole entropy follows Area Formula

$$S_{BH} = \frac{A}{4}, \quad (l_P = 1)$$

[Bekenstein, Hawking]

**Universal!**

**Finite Area Corrections??**

$$S_{BH} = \frac{A}{4} + c_1 \ln A + \mathcal{O}(1/A) \qquad A \gg 1$$

**Model dependent!**

**What can they tell us about the microstates?**

In Quantum Gravity we aim to see the black hole as an ensemble of quantum states

$\mathcal{H}_Q =$ 

Ensemble of
quantum states

$$S_{stat} = \ln d(Q)$$ [Boltzmann]

**When is black hole entropy equal to statistical entropy?**

$$S_{BH} \overset{?}{=} S_{stat}$$

**Is there?**

$$F\left(\frac{A}{4}\right) = d(Q) \in \mathbb{N}$$

**Very constraining!**

**Take into account Quantum Effects!**

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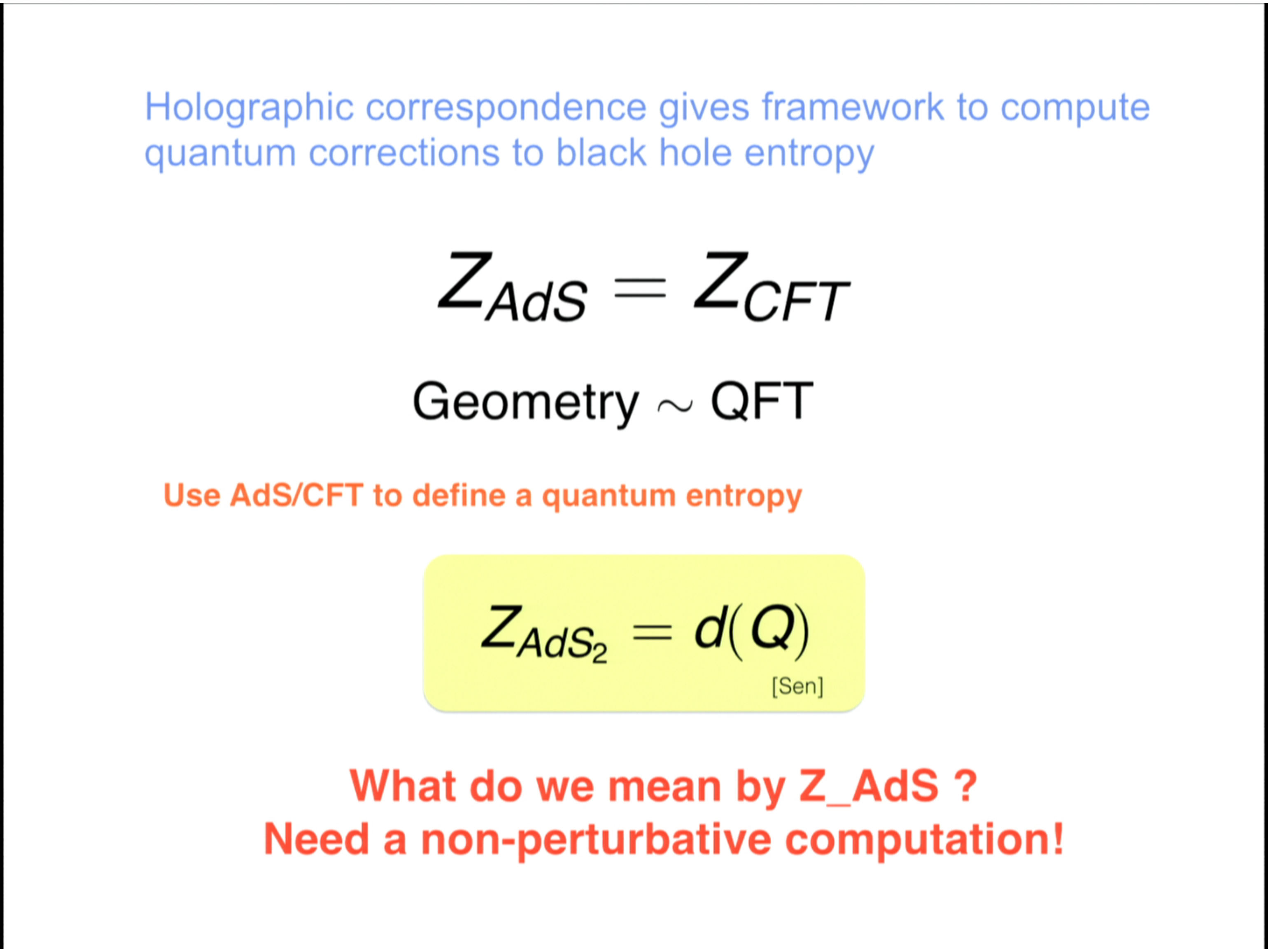
Any consistent theory of Quantum Gravity should pass the Integrality check!

**String Theory** = **Quantum Mechanics + Gravity**

**Then we should get integers!**

**In any phase (compactification) of the theory!**



Holographic correspondence gives framework to compute quantum corrections to black hole entropy

$$Z_{AdS} = Z_{CFT}$$

Geometry $\sim$ QFT

**Use AdS/CFT to define a quantum entropy**

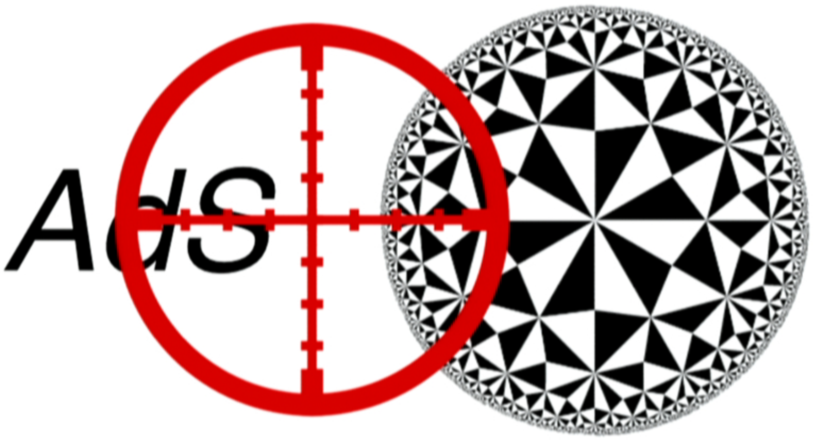
$$Z_{AdS_2} = d(Q)$$

[Sen]

**What do we mean by Z_AdS ?**
**Need a non-perturbative computation!**

# Localization in Supergravity allows to compute Z_AdS exactly

**Supersymmetric Localization**



**No known non-perturbative definition of String Theory!**
**How can we test our methods?!**

## Extract d(Q) from BPS state counting. This is our “experimental data”.

**Count BPS states from D-brane description (gN<<1)**

[Strominger & Vafa]

$$\ln \Omega_{micro} = \frac{A}{4} + \underset{\substack{\downarrow \\ \textbf{perturbative}}}{b \ln A} + \dots + \underset{\substack{\downarrow \\ \textbf{non-perturbative}}}{e^{-A}} + \dots$$

**Take result to Black Hole regime (gN>>1)**

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## Apply these ideas to other examples of Holography. Take for example AdS4/ABJM theory

**Partition function on S^3 well studied**

$$\ln Z_{S^3}^{ABJM} = -\frac{\sqrt{2}\pi}{3} N^{3/2} + c_1 \ln N + \ldots + e^{-N} + \ldots$$

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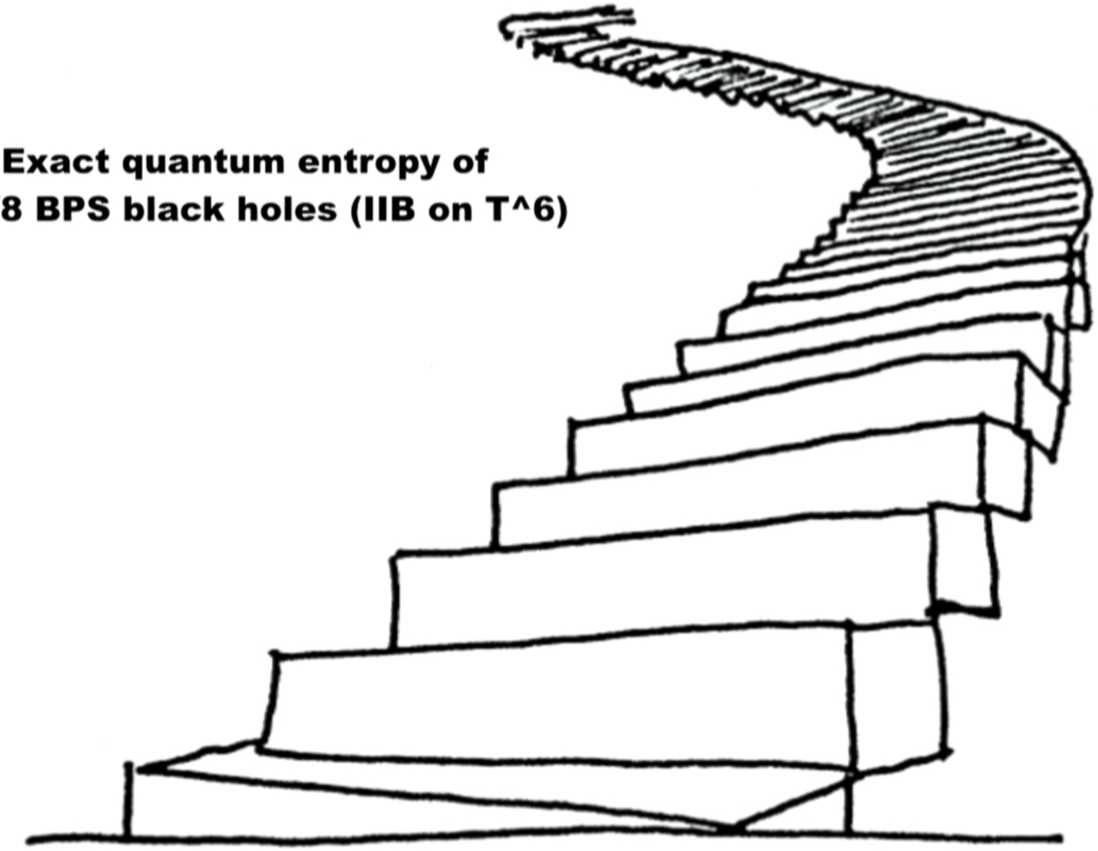
M-theory on $AdS_4 \times S^7$

**More generally…**

**Infinite N limit is well understood (Integrability, semiclassical methods)**
**Much less is known for finite N from the bulk!**

# PLAN:

- ✤ **Exact quantum entropy of 1/8 BPS black holes (IIB on T^6)**



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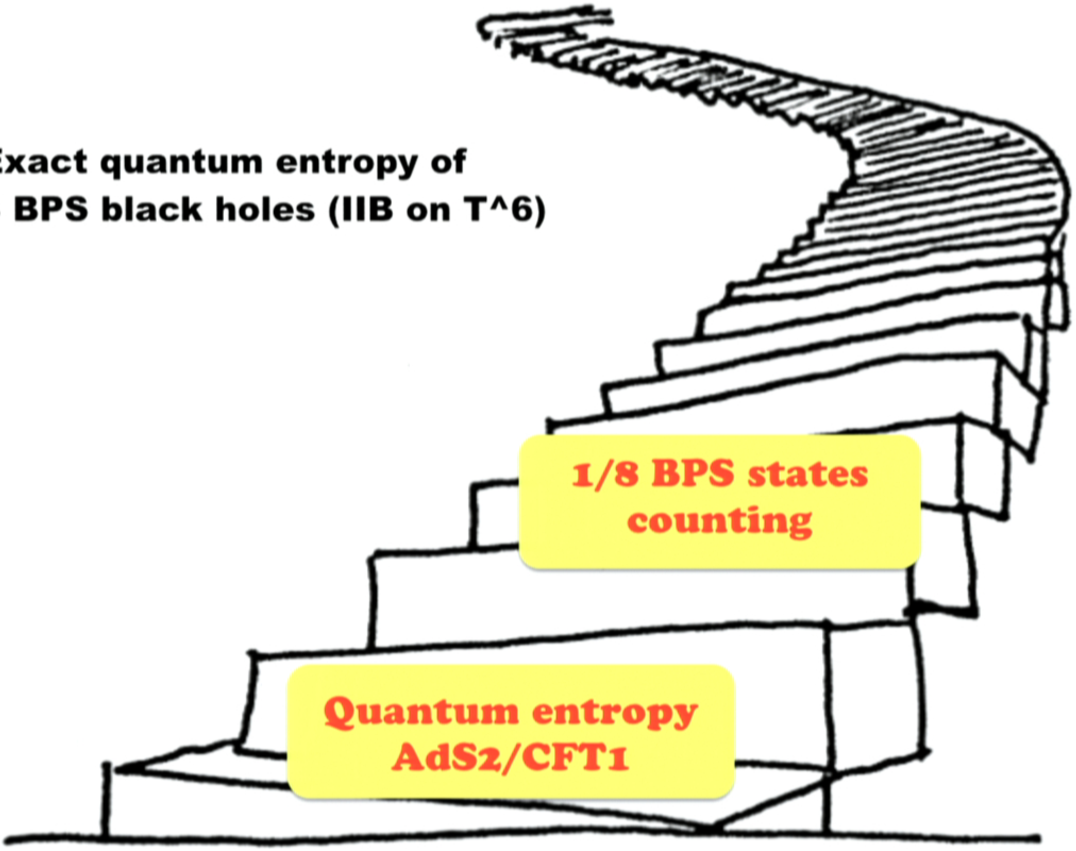
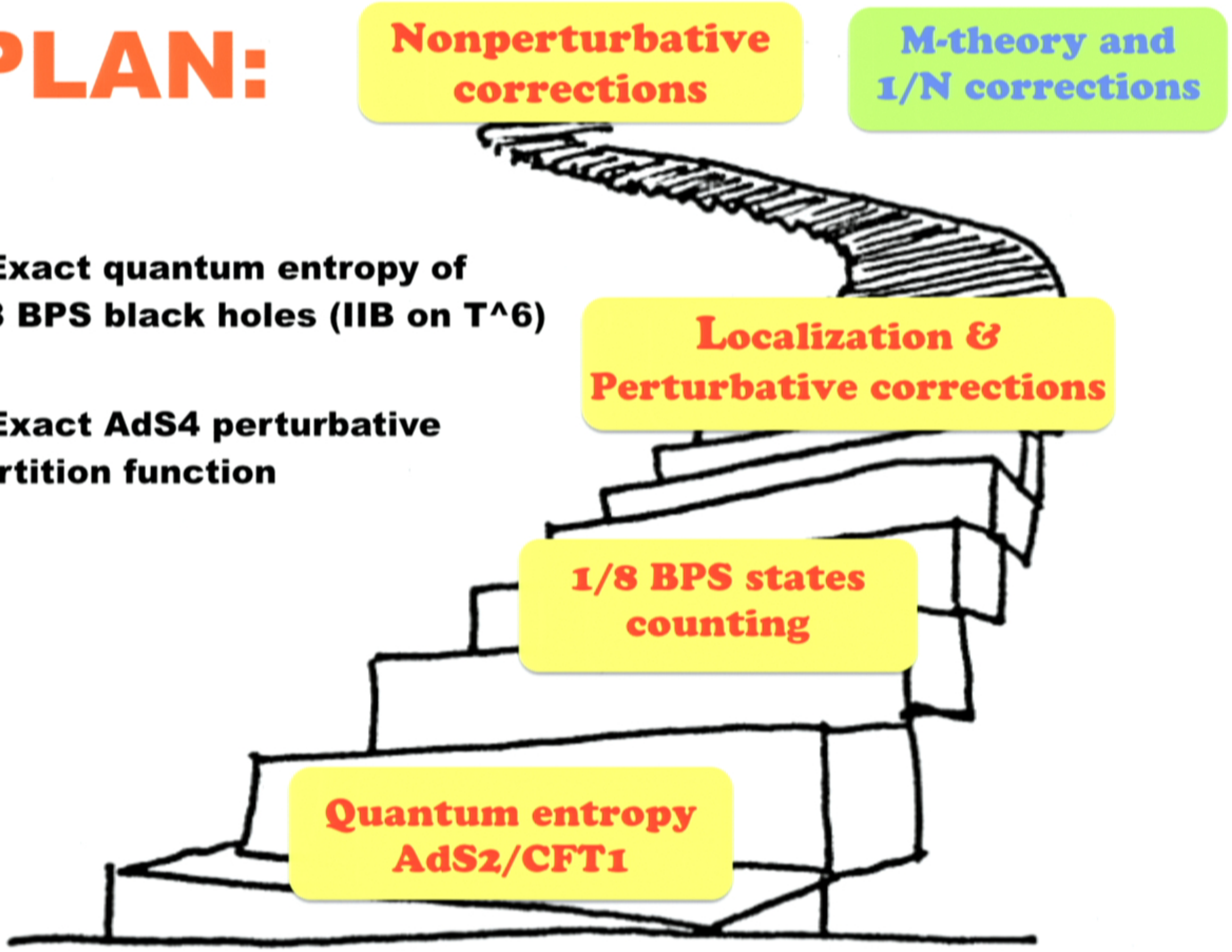


1/8 BPS states counting

Quantum entropy AdS2/CFT1

# PLAN:



Nonperturbative corrections

M-theory and 1/N corrections

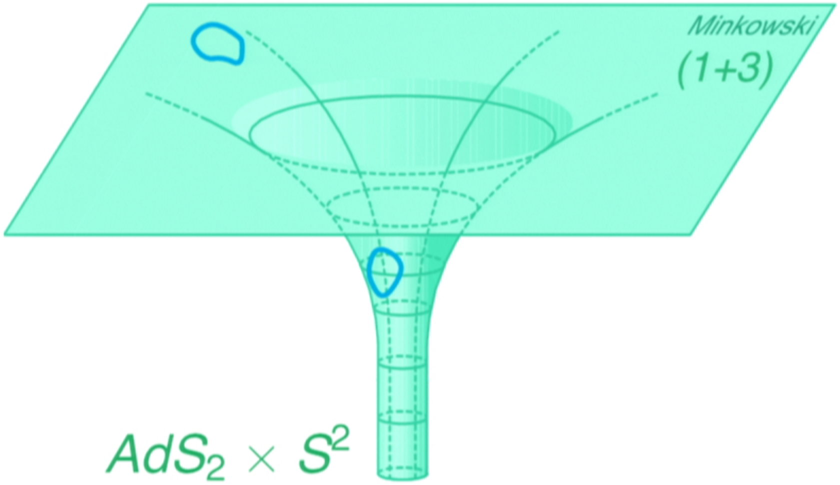
Localization & Perturbative corrections

1/8 BPS states counting

Quantum entropy AdS2/CFT1

- Exact quantum entropy of 1/8 BPS black holes (IIB on T^6)
- Exact AdS4 perturbative partition function

# Quantum entropy via AdS2/CFT1



# Quantum entropy is logarithm of AdS2 partition function

**Sen's proposal** $d(q) = Z^{ren}_{AdS_2}$

**Functional integral over AdS2 fluctuations**

$$Z_{AdS_2} = \int \exp\left[ i q_I \oint A^I + \int_{AdS_2} \mathcal{L}(R, A, \phi, \ldots) \right]$$

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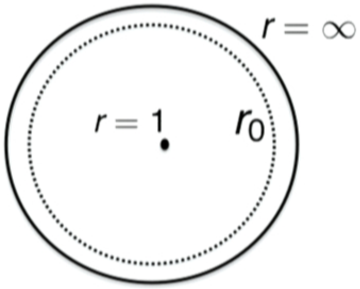
# AdS2 formalism implies Microcanonical Ensemble

Near horizon geometry is $AdS_2 \times S^2$

$$ds^2 = (r^2 - 1)d\theta^2 + \frac{dr^2}{r^2 - 1}$$



**Microcanonical Ensemble**

$$A_\theta = \underset{\text{fix}}{er} + \underset{\text{integrate}}{c}$$

**Charges are Fixed!**

**Introduce boundary Wilson Lines**

$$q_I \oint_{\partial AdS_2} A^I + \int_{AdS_2} \mathcal{L}$$

**EOM**

## Why AdS2 partition function? Two consistency checks.

**1. On-shell level**

$$Z_{AdS_2} \simeq e^{\frac{A}{4}}, \ A \gg 1$$

**2. AdS2/CFT1 dictionary**

$$Z_{CFT_1} = \lim_{r_0 \to \infty} \mathrm{Tr}_q \, e^{-2\pi r_0 H} = \mathrm{Tr}(1)|_{q,H=0}$$

$$Z_{AdS_2} = Z_{CFT_1}$$

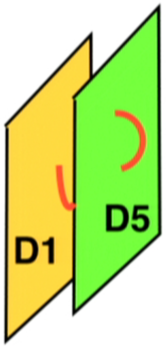
**IR limit of brane config.** $\left| \mathrm{BH} \right\rangle$ **= Ground States**

# 1/8 BPS Black Holes

## (IIB on T^6)

$Q\,|$  $\rangle = 0$

# 1/8 BPS state partition function is given by a Jacobi form



$$\mathbb{R}^{1,3} \times \tilde{S}^1 \times S^1 \times T^4$$

**IR**

$2d\ (4,4)\ SCFT$

$c_L = c_R = 6N_1 N_5$

**Put right movers in the ground state. Count left movers.**

$$F(\tau, z) = \sum_{n,l} \Omega(n,l) q^n y^l$$

**n:** *momenta* $S^1$

**l:** *momenta* $\tilde{S}^1$

$$F(\tau, z) = \frac{\vartheta(\tau, z)^2}{\eta(\tau)^6}$$

[Maldacena, Moore, Strominger]

$$\eta(\tau) = q^{1/24} \prod_{n=1}^{\infty} (1 - q^n) \qquad q = e^{2\pi i \tau} \ y = e^{2\pi i z}$$

$$v(\tau, z) = q^{1/8}(y^{1/2} - y^{-1/2}) \prod_{n=1}^{\infty} (1 - q^n)(1 - yq^n)(1 - y^{-1}q^n)$$

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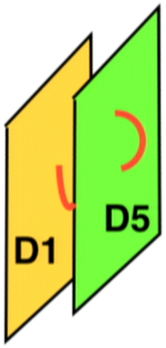

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## Multiplier matrix can be constructed explicitly from the generators of SL(2,Z)

**Explicit formula:**

$$T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \; S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$M^{-1}(T),\, M^{-1}(S) \to M^{-1}(\gamma = ST^{n_1} ST^{n_2} \ldots)$$

[continued fraction expansion]

$$M^{-1}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)_{\mu v} = C \sum_{\varepsilon=\pm} \sum_{n=0}^{c-1} \varepsilon e^{\frac{i\pi}{2rc}\left[d(v+1)^2 - 2(v+1)(2rn+\varepsilon(\mu+1)) + a(2rn+\varepsilon(\mu+1))^2\right]}$$

$\mu, v = 0, 1$

$$C := i\frac{\text{sign}(c) e^{-\frac{\pi i}{4}}}{\sqrt{2r|c|}} e^{-\frac{\pi i}{6}\Phi(\gamma)}$$

$\Phi(\gamma)$ : Rademacher phi function

**[Lisa Jeffrey; Dabholkar, Murthy, JG]**

**We have now a sum over exponentials! Good to compare with Bulk!**

## Index equals Degeneracy

**Compute spacetime index in the black hole background**

$$\Omega = \mathrm{Tr}_q(-1)^{J_{hor}}$$

Trace over J for fixed charges

**AdS2 naturally fixes the charges and not the chemical potentials**

$$S^2 \to J_{hor} = 0$$

**Microcanonical Ensemble**

$$\mathrm{Tr}_{q,J}(-1)^J = \mathrm{Tr}(1) = Z_{AdS_2}$$

[Sen 2009; Sen, Dabholkar, Murthy, JG 2010]

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## Bessel function encodes all perturbative corrections. Kloosterman sums are non-perturbative

$$\Omega(\Delta) = I_{7/2}(\pi\sqrt{\Delta}) + \sum_{c>1} Kl(\Delta)\, I_{7/2}\left(\frac{\pi}{c}\sqrt{\Delta}\right)$$

$$I_{7/2}(\pi\sqrt{\Delta}) \simeq e^{\pi\sqrt{\Delta}-4\ln(\sqrt{\Delta})+\ldots}, \quad \Delta \gg 1$$

$$\ln\Omega(\Delta) \sim \pi\sqrt{\Delta} - 4\ln(\sqrt{\Delta}) + \ldots + e^{\pi\sqrt{\Delta}(1/c-1)} + \ldots$$

**Bessel encodes all perturbative corrections to A/4**

**Kloosterman Sums are Non-perturbative**

# Localization in Supergravity

$$I_{7/2}(\pi\sqrt{\Delta}) = \frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty} \frac{dt}{t^{9/2}} e^{t+\frac{\pi^2\Delta}{4t}}$$

With Atish Dabholkar & Sameer Murthy

arXiv: 1012.0265; 1111.1161; 1305.2849

# Off-shell supergravity contains vacuum AdS2xS^2

## Use 4d N=2 off-shell superconformal formalism

Relevant Multiplets

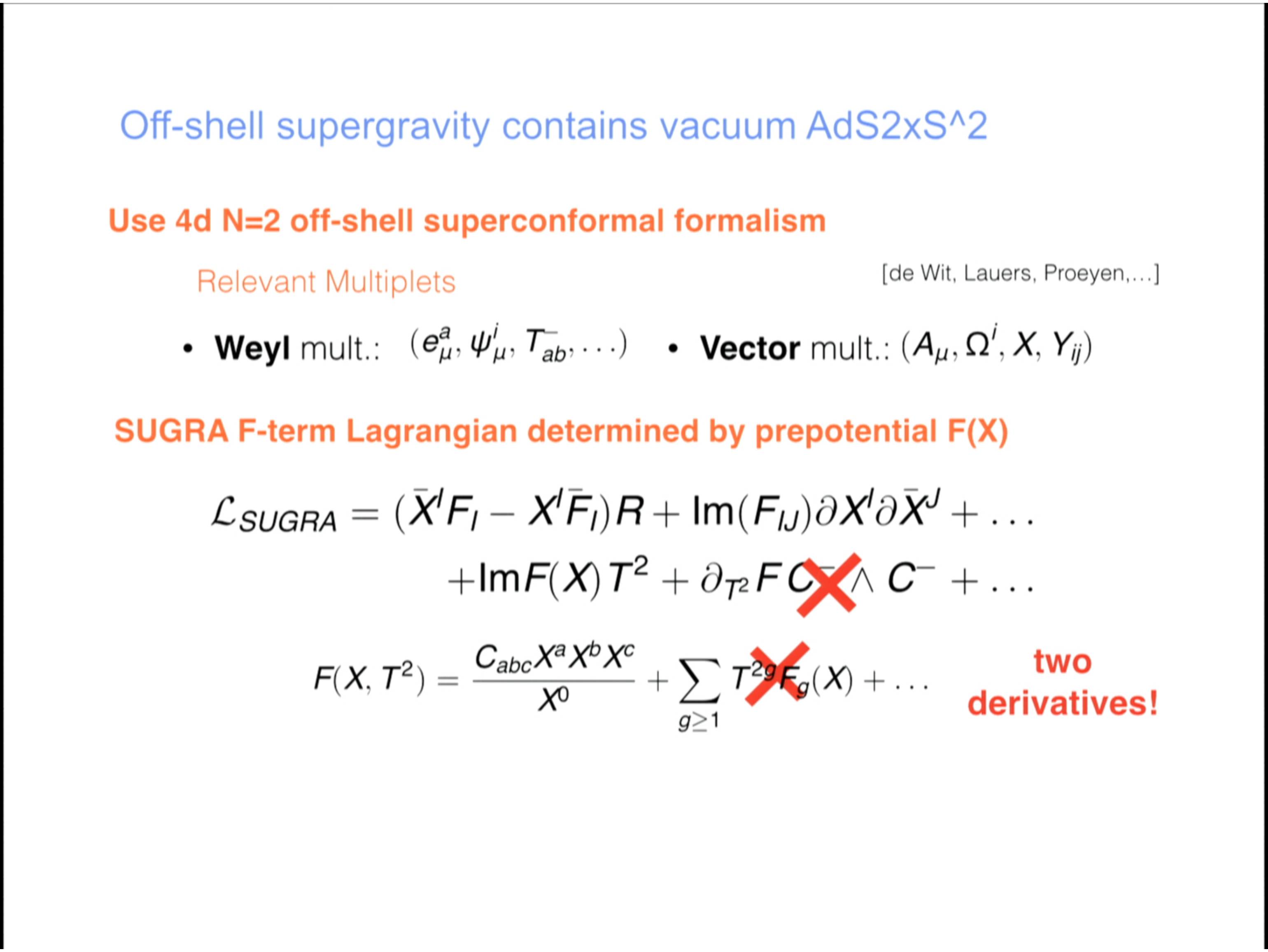
[de Wit, Lauers, Proeyen,...]

- **Weyl** mult.: $(e^a_\mu, \psi^i_\mu, T^-_{ab}, \ldots)$
- **Vector** mult.: $(A_\mu, \Omega^i, X, Y_{ij})$

## SUGRA F-term Lagrangian determined by prepotential F(X)

$$\mathcal{L}_{SUGRA} = (\bar{X}^I F_I - X^I \bar{F}_I) R + \mathrm{Im}(F_{IJ}) \partial X^I \partial \bar{X}^J + \ldots$$
$$+ \mathrm{Im} F(X)\, T^2 + \partial_{T^2} F\, C^- \wedge C^- + \ldots$$

$$F(X, T^2) = \frac{C_{abc} X^a X^b X^c}{X^0} + \sum_{g \geq 1} T^{2g} F_g(X) + \ldots$$



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**two derivatives!**

# Localization action has N_v parameter family of solutions

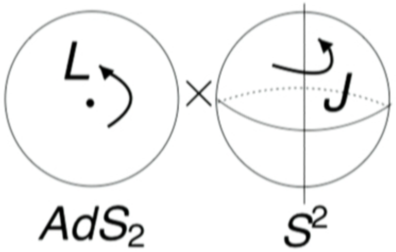
**Deform action**

$$S \to S - t \sum \delta\Big( (\delta\Psi)^\dagger \Psi \Big)$$

$$\delta V|_{bosonic} = (\delta\Psi)^\dagger \delta\Psi \geq 0$$

$$\delta^2 = L - J$$

[Sen, Mandal, Banerjee, Gupta 2009]



**Limit** $t \to \infty$ ,

**Localization BPS eqs:** $\delta\Psi = 0$

Weyl: $AdS_2 \times S^2$ ! [Murthy, Gupta 2012]

Vec: $X = X_{const} + \frac{C}{r}, \; Y = \frac{C}{r^2}, \; F_{\mu\nu} = F^{vac}_{\mu\nu}, \quad C = const$

[Dabholkar, JG, Murthy, 2010]

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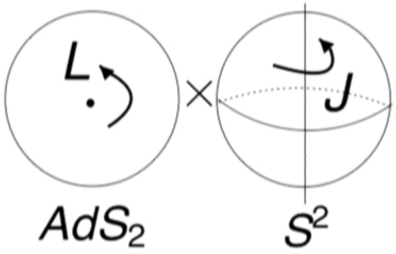

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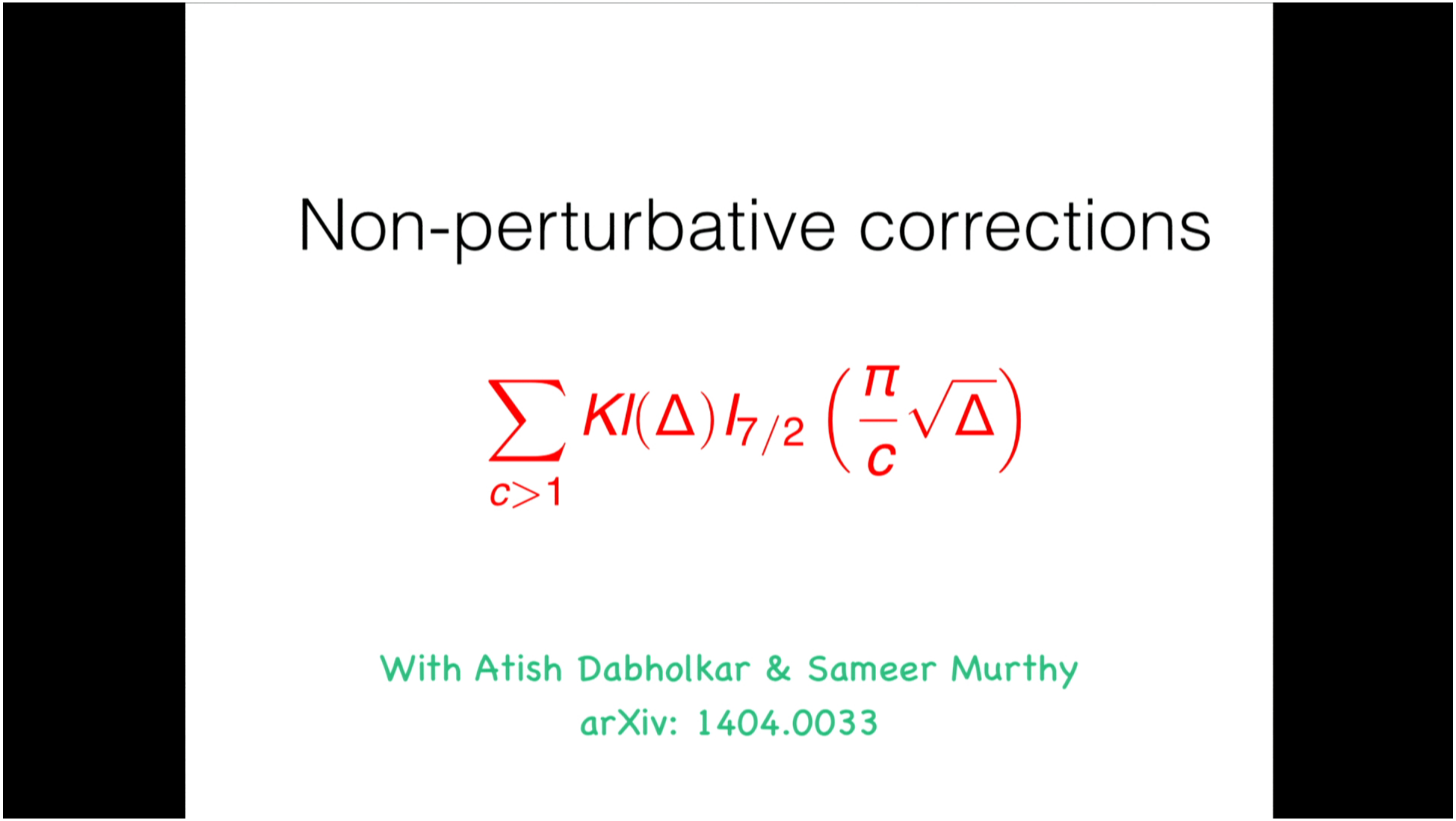


# Non-perturbative corrections

$$\sum_{c>1} Kl(\Delta)\, I_{7/2}\left(\frac{\pi}{c}\sqrt{\Delta}\right)$$

With Atish Dabholkar & Sameer Murthy
arXiv: 1404.0033

# Non-perturbative corrections suggest family of Z_c orbifolds

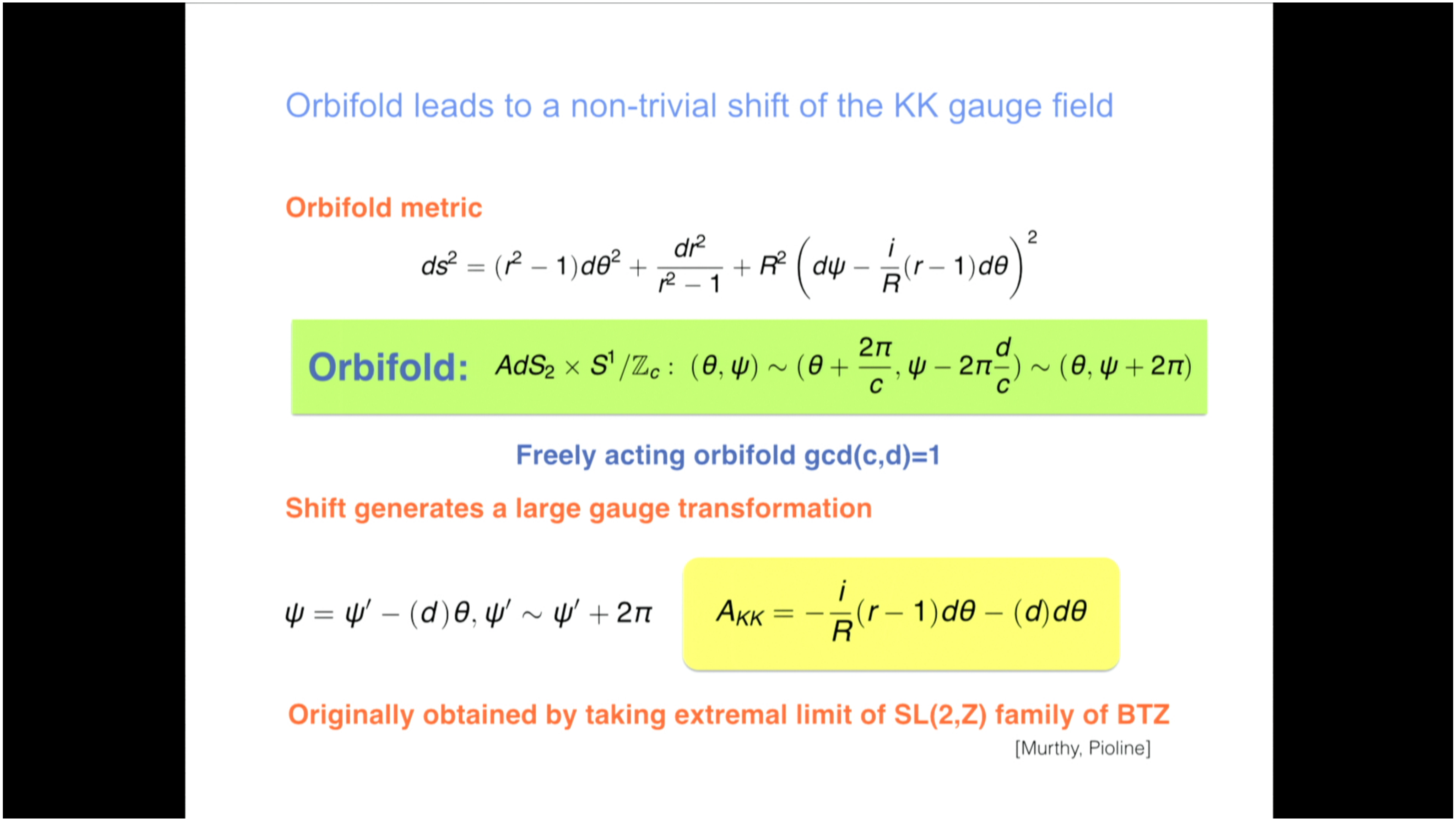
**Terms with c>1 are exponentially subleading**

$$\sum_{\substack{d\in\mathbb{Z}/c\mathbb{Z}\\ \gamma\in SL(2,Z)}} e^{2\pi i\frac{\Delta}{4}\frac{d}{c}} M^{-1}_{\mu 1}(\gamma) e^{-2\pi i\frac{1}{4}\frac{a}{c}} I_{7/2}\left(\frac{\pi\sqrt{\Delta}}{c}\right)$$

$$I_{7/2}\left(\frac{\pi\sqrt{\Delta}}{c}\right) \sim e^{\frac{\pi\sqrt{\Delta}}{c}},\ \Delta \gg 1$$

**Since** $\pi\sqrt{\Delta} \propto \mathrm{Ren}\left[vol(AdS_2)\right]$

**It suggests orbifold** $AdS_2/\mathbb{Z}_c \to vol(AdS_2)/c$



## Orbifold leads to a non-trivial shift of the KK gauge field

**Orbifold metric**

$$ds^2 = (r^2-1)d\theta^2 + \frac{dr^2}{r^2-1} + R^2\left(d\psi - \frac{i}{R}(r-1)d\theta\right)^2$$

**Orbifold:** $AdS_2 \times S^1/\mathbb{Z}_c: \ (\theta,\psi) \sim (\theta + \frac{2\pi}{c}, \psi - 2\pi\frac{d}{c}) \sim (\theta, \psi + 2\pi)$

**Freely acting orbifold gcd(c,d)=1**

**Shift generates a large gauge transformation**

$$\psi = \psi' - (d)\theta, \psi' \sim \psi' + 2\pi$$

$$A_{KK} = -\frac{i}{R}(r-1)d\theta - (d)d\theta$$

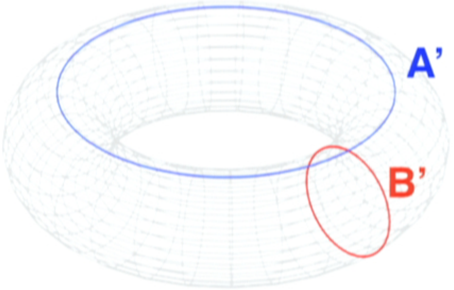
**Originally obtained by taking extremal limit of SL(2,Z) family of BTZ**

[Murthy, Pioline]

# Orbifold space has the topology of a solid torus.

**Topologically= Solid Torus**

$$\mathcal{M}(c,d) = AdS_2 \times S^1/\mathbb{Z}_c \simeq D^2 \times S^1$$



**A’: non-contractible**

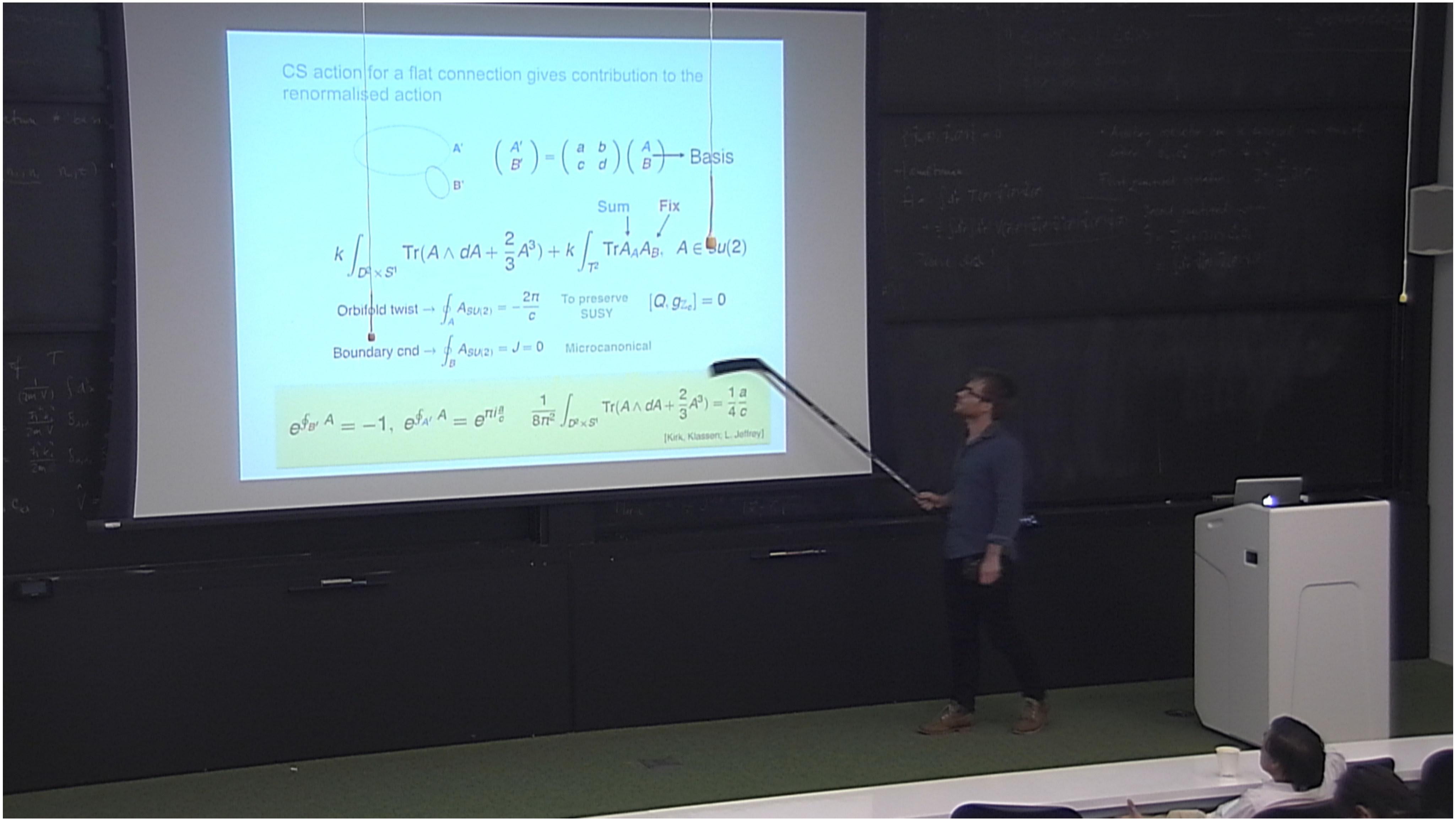
**B’: contractible**

**They differ on the homology of the cycle that becomes contractible**

$$B' \wedge A' = 1 \rightarrow \begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \quad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})$$

$\mathcal{M}(1,0)$ $\quad A' : \ \psi$ $\quad B' : \ \theta$

$\mathcal{M}(c,d)$ $\quad A' : \ c\psi + (d)\theta$ $\quad B' : \ a\psi - (b)\theta$



CS action for a flat connection gives contribution to the renormalised action

$$\begin{pmatrix} A' \\ B' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} \rightarrow \text{Basis}$$

Sum ↓ Fix ↙

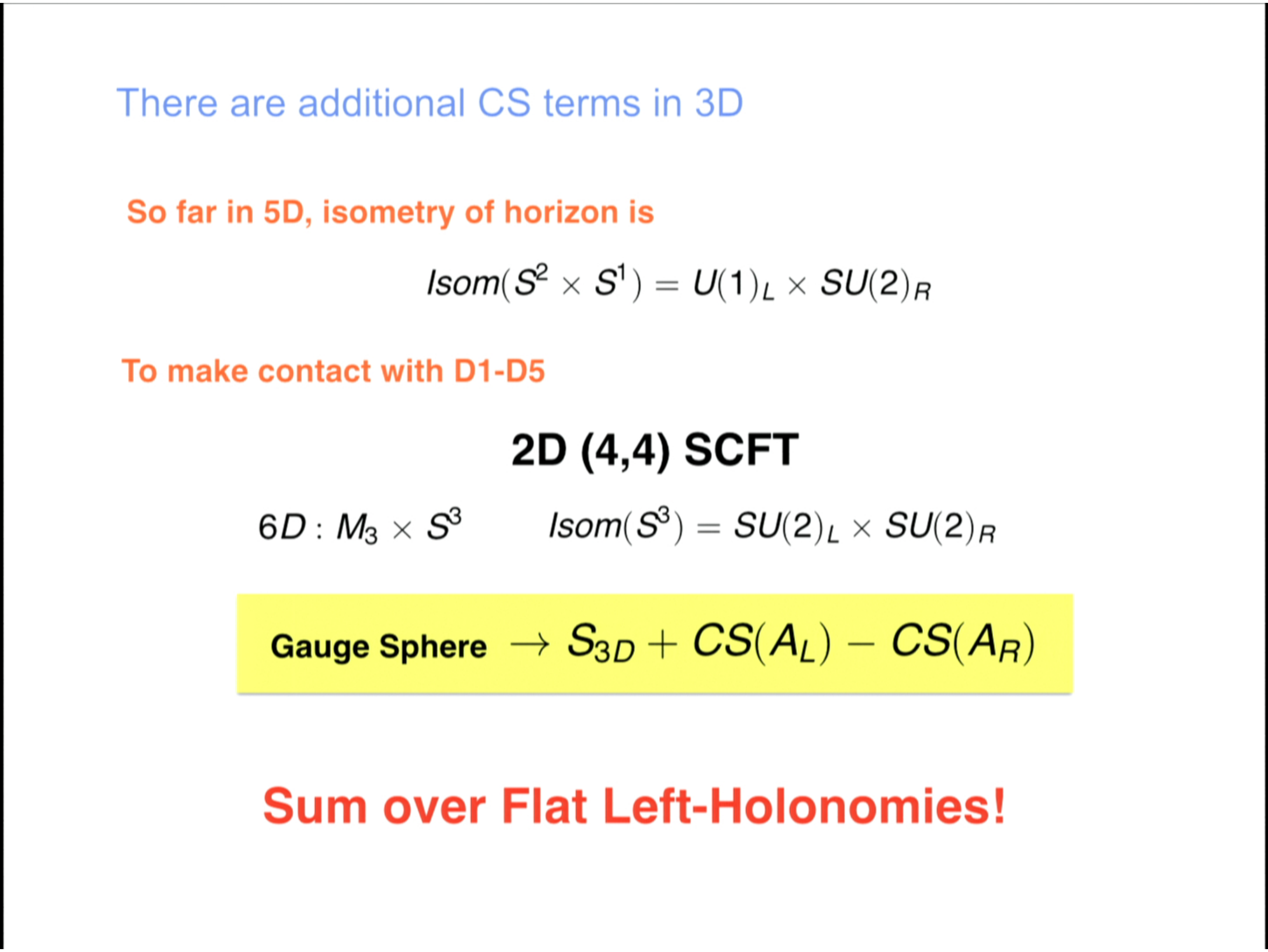
$$k \int_{D^2 \times S^1} \mathrm{Tr}(A \wedge dA + \frac{2}{3} A^3) + k \int_{T^2} \mathrm{Tr} A_A A_B, \quad A \in su(2)$$

Orbifold twist → $\oint_A A_{SU(2)} = -\frac{2\pi}{c}$ To preserve SUSY $[Q, g_{\mathbb{Z}_c}] = 0$

Boundary cnd → $\oint_B A_{SU(2)} = J = 0$ Microcanonical

$$e^{\oint_{B'} A} = -1, \quad e^{\oint_{A'} A} = e^{\pi i \frac{a}{c}} \qquad \frac{1}{8\pi^2} \int_{D^2 \times S^1} \mathrm{Tr}(A \wedge dA + \frac{2}{3} A^3) = \frac{1}{4} \frac{a}{c}$$

[Kirk, Klassen; L. Jeffrey]



## There are additional CS terms in 3D

**So far in 5D, isometry of horizon is**

$$Isom(S^2 \times S^1) = U(1)_L \times SU(2)_R$$

**To make contact with D1-D5**

**2D (4,4) SCFT**

$$6D : M_3 \times S^3 \qquad Isom(S^3) = SU(2)_L \times SU(2)_R$$

**Gauge Sphere** $\rightarrow S_{3D} + CS(A_L) - CS(A_R)$

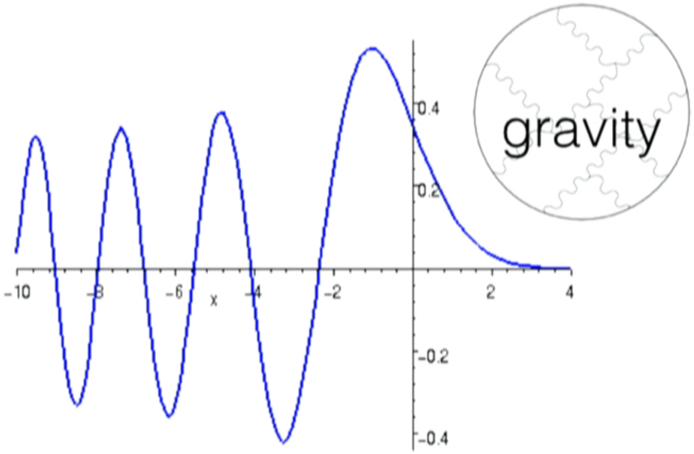
**Sum over Flat Left-Holonomies!**

# Summary

$$Z_{AdS_2}(A \propto \sqrt{\Delta}) = \quad \sum \quad e^{-S}$$

enough of Black holes…

# M-theory and the Ai(z)



With Atish Dabholkar & Nadav Drukker

arXiv: 1406.0505

# AdS4/ABJM

$U(N) \times U(N)$ Chern-Simons Matter theory

**2 CS with levels (k, -k) + 4 hypers in the fundamental**

$N$ M2-branes probing $\mathbb{C}^4/\mathbb{Z}_k$

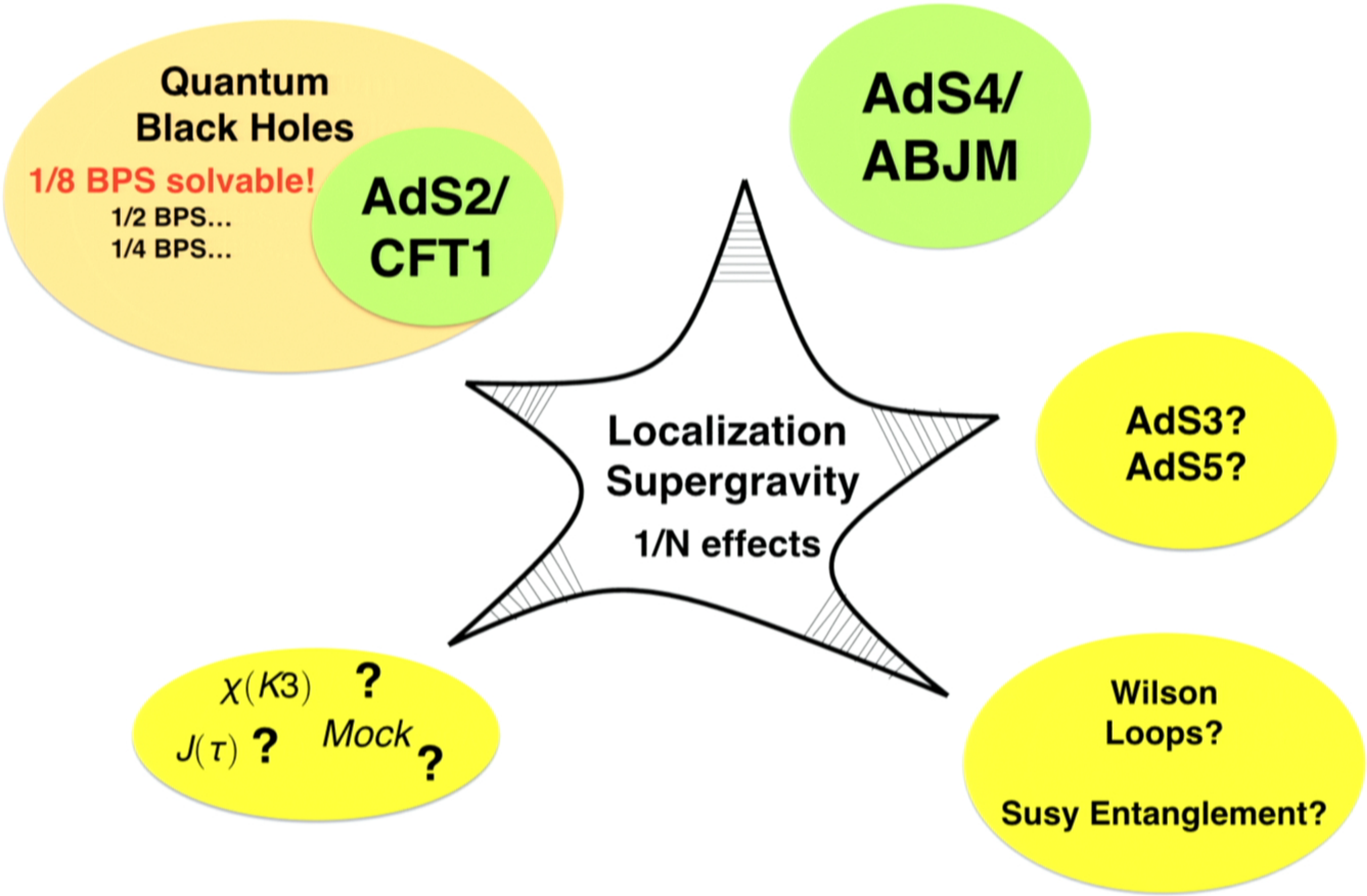
**Dual to**

**M-Theory on**

$AdS_4 \times S^7/\mathbb{Z}_k$ with electric flux $\int_{S^7} \star F_4 \propto N$

$S^7 \sim \mathbb{CP}^3 \times S^1$ **Large k limit ~ IIA string theory**

# Conclusions



**Quantum Black Holes**

**1/8 BPS solvable!**

**1/2 BPS...**

**1/4 BPS...**

**AdS2/ CFT1**

**AdS4/ ABJM**

**Localization**
**Supergravity**
**1/N effects**

**AdS3?**
**AdS5?**

$\chi(K3)$ **?**

$J(\tau)$ **?** *Mock* **?**

**Wilson Loops?**

**Susy Entanglement?**