Title: Topological Model for Domain Walls in (Super-)Yang-Mills Theories

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Abstract: We discuss a topological description of the confining phase of (Super-)Yang-Mills theories with gauge group SU(N) which encodes all the Aharonov-Bohm phases of configurations of non-local operators. This topological action shows an additional 1-form gauge symmetry. After the introduction of domain walls, this 1-form gauge symmetry demands the appearance of new fields on the worldvolume of the wall. These new fields have a topological Chern-Simons action at level N, also suggested by string theory constructions.

A Topological Model for Domain Walls in (Super-)Yang-Mills Theories

hep-th: 1405.4291 to appear in PRD

String Seminar - Perimeter Institute

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In collaboration with Alexander Pritzel

Based on:

- Acharya, Vafa (2001): hep-th/0103011
- Banks, Seiberg (2010): 1011.5120
- Aharony, Seiberg, Tachikawa (2013): 1305.0318
- Gaiotto (2013): 1306.5661
- Gukov, Kapustin (2013): 1307.4793
- Seiberg, Kapustin (2014): 1401.0740

Outline

1. Motivation
2. YM and vacuum structure
3. Topological action
4. Domain walls
5. Super-Yang-Mills
6. Dynamics of domain walls
7. Outlook

Motivation

Domain walls in SYM theories

- Tension exactly calculable [Dvali, Shifman '96]
- Topological phases in gauge theories
- Relation to D-branes in string theory

Topological approach

- Focus on topological subsector leads to significant simplifications
- Contains information about vacuum structure
- Very robust against deformations (even SUSY)

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M-/ string theory prediction [Acharya, Vafa '01]:

CS-term on domain walls (of level N for $SU(N)$ gauge group)

$\Rightarrow$ Use a **topological approach** to investigate domain walls in field theory

CS-term as a consequence of gauge invariance

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2. Yang-Mills theories and vacuum structure

Yang-Mills theory (basics)

All ingredients for the topological construction are already there in the non-supersymmetric case

Consider **pure Yang-Mills theory with gauge group $\mathbf{G} = \mathbf{SU}(\mathbf{N})/\mathbf{H}$** (with $H \subset \mathcal{C}$) $\Rightarrow$ algebra $g = su(N)$ (defines local dynamics)

Lagrangian density in terms of 't Hooft coupling $\lambda = g^2 N$

$$\boxed{\mathcal{L}_{YM} = -\frac{N}{2\lambda}\mathrm{Tr}(F \wedge *F) + \frac{i\theta}{8\pi^2}\mathrm{Tr}(F \wedge F)}$$

$\rightarrow$ develops a mass gap (confinement)

We assume that: Confining mechanism is condensation of monopoles

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Large N limit

Usual procedure for the large N limit:

$$\mathcal{L} = N^2 \mathcal{L}'(\Phi, \partial\Phi)$$

with $\mathcal{L}'$ independent of N

BUT: This leads to a θ/N instead of θ dependence, in conflict with

- 2π periodicity in θ
- θ-dependence of the energy

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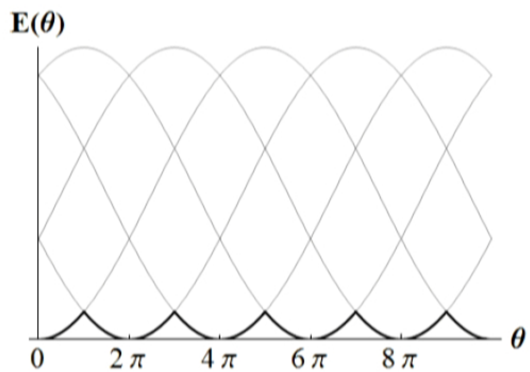
Solution by branch structure [Witten '79]

Not a single but **several branches** of the theory (quasi-stable in large N limit)



- single branch: $2\pi N$ periodicity
- collection: 2π periodicity
- labeled by additional parameter $k \in \{0, \ldots N-1\}$
- interchange roles for shift in θ

By the Witten effect [Witten '79]:

- Different charge of condensate for different branches
- Lowest energy for purely magnetically charged condensate
- Note: Topologically k and θ with similar effects

Classification of vacua

Not enough to fix local dynamics $\Rightarrow$ fix global gauge group (i.e. H) by classification via non-local operators

For $\mathbf{SU(N)/H}$: classification via charges in $\mathbb{Z}_N \times \mathbb{Z}_N$ (electric, magnetic) [Seiberg, Aharony, Tachikawa '13]

$\Rightarrow$ couple to discrete ($\mathbb{Z}_N$) gauge fields A (electric), $\tilde{A}$ (magnetic)

- **Line operators**

$$\exp\left[\oint_\gamma (iqA + im\tilde{A})\right]$$

- **Surface operators**

$$\exp\left[i\eta \int_\Sigma F\right]$$

Classification of vacua

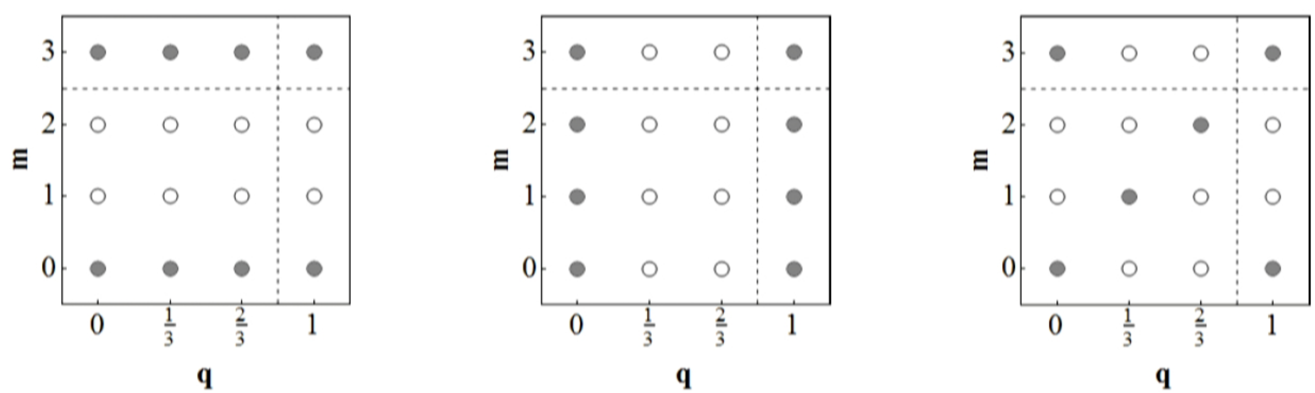
Fix global structure of the gauge group by choosing a maximal set of line operators satisfying the "**Dirac quantization**"

$$qm' - mq' \in \mathbb{Z}$$

Other line operators have to be supplemented by surface operator [Aharony, Seiberg, Tachikawa '13]

Example: $SU(3)$, $SU(3)/\mathbb{Z}_3$ (similar to one branch of $SU(N)$)

$SU(3)$ $\quad$ $(SU(3)/\mathbb{Z}_3)_{\theta=0}$ $\quad$ $(SU(3)/\mathbb{Z}_3)_{\theta=2\pi}$



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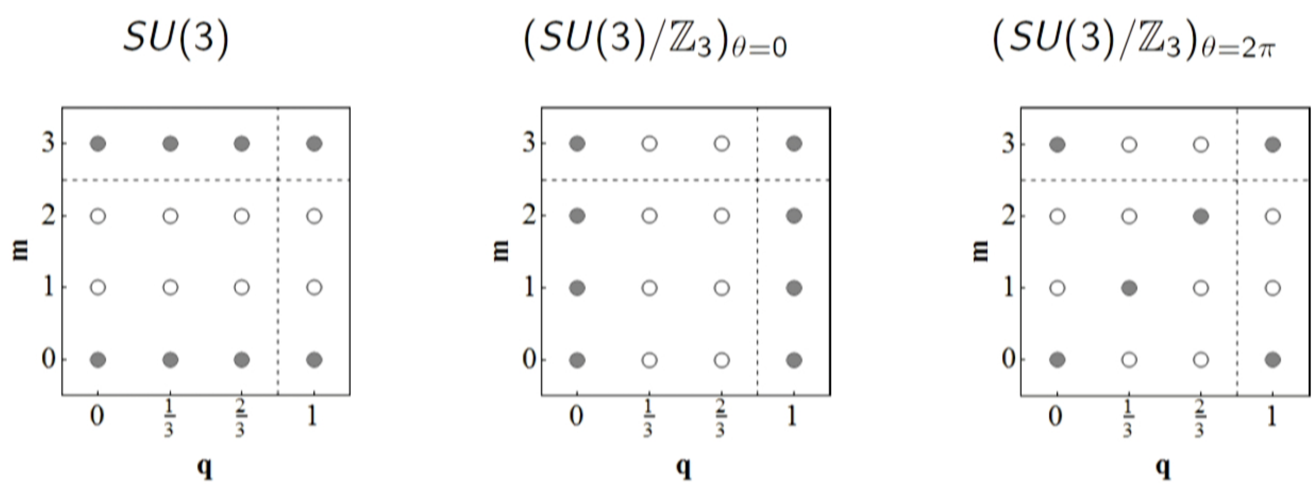

Construction of the topological action

- Consider a single branch of the $SU(N)$ theory
- $\mathbb{Z}_N$ theory with a magnetically charged condensate of charge N (c.f. dual superconductor)
- Embed the $\mathbb{Z}_N$ theory in a $U(1)$ symmetry

Starting action:

$$\boxed{S = \int h \wedge (d\varphi - N\tilde{A})}$$

gauge transformations:

$$\tilde{A} \to \tilde{A} + d\alpha, \quad \varphi \to \varphi + N\alpha$$

Derivation by Higgsing

Start with complex scalar field Φ of (magnetic) **charge N**:

$$\Phi \to e^{iN\alpha}\Phi$$

$\Rightarrow$ Covariant derivative: $D\Phi = (d - iN\tilde{A})\Phi$

Higgsing:

$$\langle|\Phi|\rangle = v \gg 1, \;\; m_{|\Phi|} \gg 1 \Rightarrow \Phi = ve^{i\varphi}$$

$$D\Phi = iv(d\varphi - N\tilde{A})$$

Euclidean action dominated by $d\varphi = N\tilde{A}$, fix by Lagrange multiplier

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Dualizing

[Gukov, Kapustin '13], [Kapustin, Seiberg '14]

- Dualize φ to 2-form B coupling to the vortices of φ, i.e. electric flux tubes
- Dualize $\tilde{A}$ to electric gauge field A

$$S = \int \left[h \wedge (d\varphi - N\tilde{A}) + \frac{i}{2\pi} d\varphi \wedge dB + \frac{i}{2\pi} d\tilde{A} \wedge dA \right]$$
$$= \frac{i}{2\pi} \int \tilde{F} \wedge (F - NB)$$

Form of BF action [Horowitz '89], dual description of $\mathbb{Z}_N$ theory

Describes **Aharonov-Bohm phases** associated to line and surface operators in a dual superconductor (with charge N condensate)

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Describes **Aharonov-Bohm phases** associated to line and surface operators in a dual superconductor (with charge N condensate)

Inclusion of the θ term

Consider the shift $\theta \to \theta + 2\pi$

- Condensate acquires electric charge by Witten effect
- Pure 't Hooft lines should **not** be gauge invariant anymore
- Combination of 't Hooft and Wilson lines should be gauge invariant

$\Rightarrow$ **Modify the 1-form transformations for $\tilde{\mathbf{A}}$**

$$\boxed{\tilde{A} \to \tilde{A} - \frac{\theta}{2\pi}\lambda}$$

Gauge invariant dyonic line operator (for $\theta = 2\pi$)

$$\delta \exp\left[i \oint_\gamma (qA + qN\tilde{A})\right] = 0$$

Inclusion of θ term

BUT: Action not gauge invariant with modified transformations

$\Rightarrow$ Add a new term that recovers gauge invariance

$$\boxed{S = \frac{i}{2\pi} \int \left[\tilde{F} \wedge (F - NB) - \frac{N\theta}{4\pi} B \wedge B \right]}$$

with 1-form transformations:

$$B \to B + d\lambda, \quad A \to A + N\lambda, \quad \tilde{A} \to \tilde{A} - \frac{\theta}{2\pi}\lambda$$

$\Rightarrow$ $\theta F \wedge F$ term with (as expected) $2\pi N$ periodicity

Complete $SU(N)$ action

Recover **all branches** for the **full SU(N)** theory

$\Rightarrow$ Introduce label $k \in \{0, \ldots, N-1\}$ with similar effect as θ term (topologically)

$$S = \frac{i}{2\pi} \int \left[\tilde{F} \wedge (F - NB) - \frac{N\theta}{4\pi} B \wedge B + \frac{Nk}{2} B \wedge B \right]$$

now: $\tilde{A} \to \tilde{A} - \frac{\theta}{2\pi}\lambda + k\lambda$

Similar action in [Kapustin, Seiberg '14] with different focus

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4. Domain walls

Domain walls

Fix $\theta = 0$: **Domain walls** are described by a **jump of k** on a codimension 1 surface $\mathcal{V}$, the worldvolume of the domain wall (we focus on $\Delta k = 1$)

$dk \neq 0$ leads to new contributions after gauge transformations

$$\Delta S = \frac{i}{2\pi} \int d \left[k\lambda \wedge F + \frac{Nk}{2} \lambda \wedge d\lambda \right] - \frac{iN}{4\pi} \int \left[dk \wedge (2\lambda \wedge B + \lambda \wedge d\lambda) \right]$$

$\Rightarrow$ For fundamental walls ($\Delta k = 1$):

$$\Delta S = -\frac{iN}{4\pi} \int_{\mathcal{V}} (2\lambda \wedge B + \lambda \wedge d\lambda)$$

Domain wall

We want: **Gauge invariance in the presence of domain walls**

⇒ Introduce **"boundary" field** $\mathcal{A}$ with 1-form transformation

$$\mathcal{A} \to \mathcal{A} - \lambda$$

And the worldvolume action

$$S_{\mathcal{V}} = -\frac{iN}{4\pi} \int (2\mathcal{A} \wedge B + \mathcal{A} \wedge d\mathcal{A})$$

⇒ **Chern-Simons term at level N** (necessary for gauge invariance)

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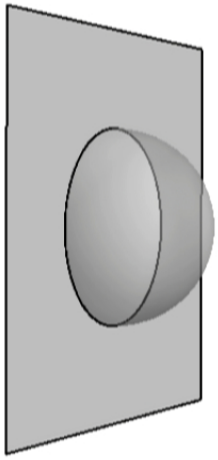
Flux tubes and domain walls

Another prediction by string theory: Electric flux tubes can end on domain walls [Witten '97]

- $\mathcal{A}$ as new field can cancel gauge non-invariant terms
- Invariant operator, for $\partial\Sigma \subset \mathcal{V}$

$$\exp\left[i \oint_{\partial\Sigma} \mathcal{A} + i \int_{\Sigma} B\right]$$

- Can end on wall

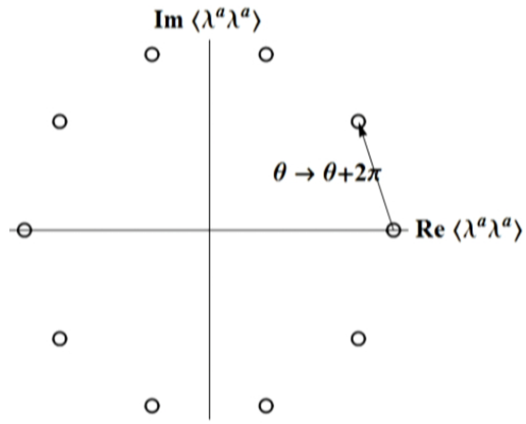


5. Super-Yang-Mills

Generalization to $\mathcal{N} = 1$ SYM

For fixed θ: **N degenerate vacua** described by **gaugino condensation**

$$\langle \mathrm{Tr}\lambda\lambda \rangle \propto N \exp\left[\frac{2\pi i}{N} k\right], \quad k \in \{0, \dots, N-1\}$$



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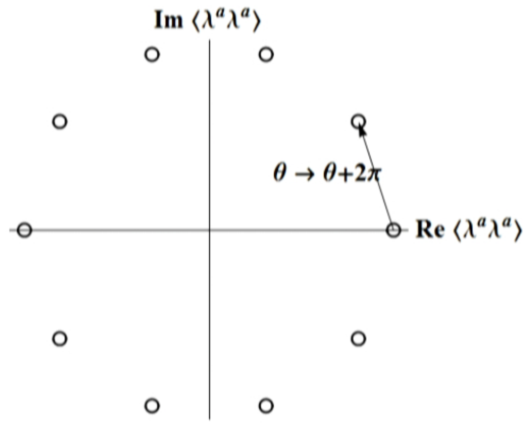

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Topological action for SYM

For the description of domain walls only keep one parameter, here θ

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6. Dynamics of domain walls

Dynamics of domain walls

In topological action we cannot consider **dynamics** and energetical issues

$\Rightarrow$ Need to add more input

- θ term is total divergence of a 3-form C (Chern-Simons 3-form)
- Introduce a kinetic term: $dC \wedge *dC \equiv F_4 \wedge *F_4$
- C couples to the worldvolume $\mathcal{V}$ of domain walls

Qualitatively:

$$\mathcal{L}_C = \frac{1}{2} F_4 \wedge *F_4 - \kappa k \, dC$$

κ: non-zero constant

Dynamics of domain walls

In domain wall background ($dk \neq 0$):

$$d(*F_4) - \kappa dk = 0$$

F_4 jumps to a constant value and contributes to the energy density $\propto F_4^2$

$\Rightarrow$ Reproduces the k^2 dependence of the energy [Witten '97]

BUT: For SYM phase of gaugino condensate is dynamical

- Acts as axionic particle
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Non-fundamental walls (Δk $\geq$ 2): [Acharya, Vafa '01], [Gaiotto, '13]

Abelian CS term at different level (suggested by reduction from $\mathcal{N} = 2$ SUSY)

vs.

Non-Abelian $U(\Delta k)$ CS term (suggested by string theory)

Possible help from:

- Fractional quantum Hall systems
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Outlook

- Relation to string theory
  - Identification of objects on both sides, D0-branes $\leftrightarrow$ baryons [Shifman, Gabadadze '99], interaction of multiple walls [Shifman, Armoni '03],...
  - Wall - antiwall annihilation processes
  - Strings with discrete charge
- Origin of the light wall tension (what are the degrees of freedom that build up the domain wall?)
- Dynamics $\Rightarrow$ beyond topological action
- Topological phases of matter

Thank you
for your attention!