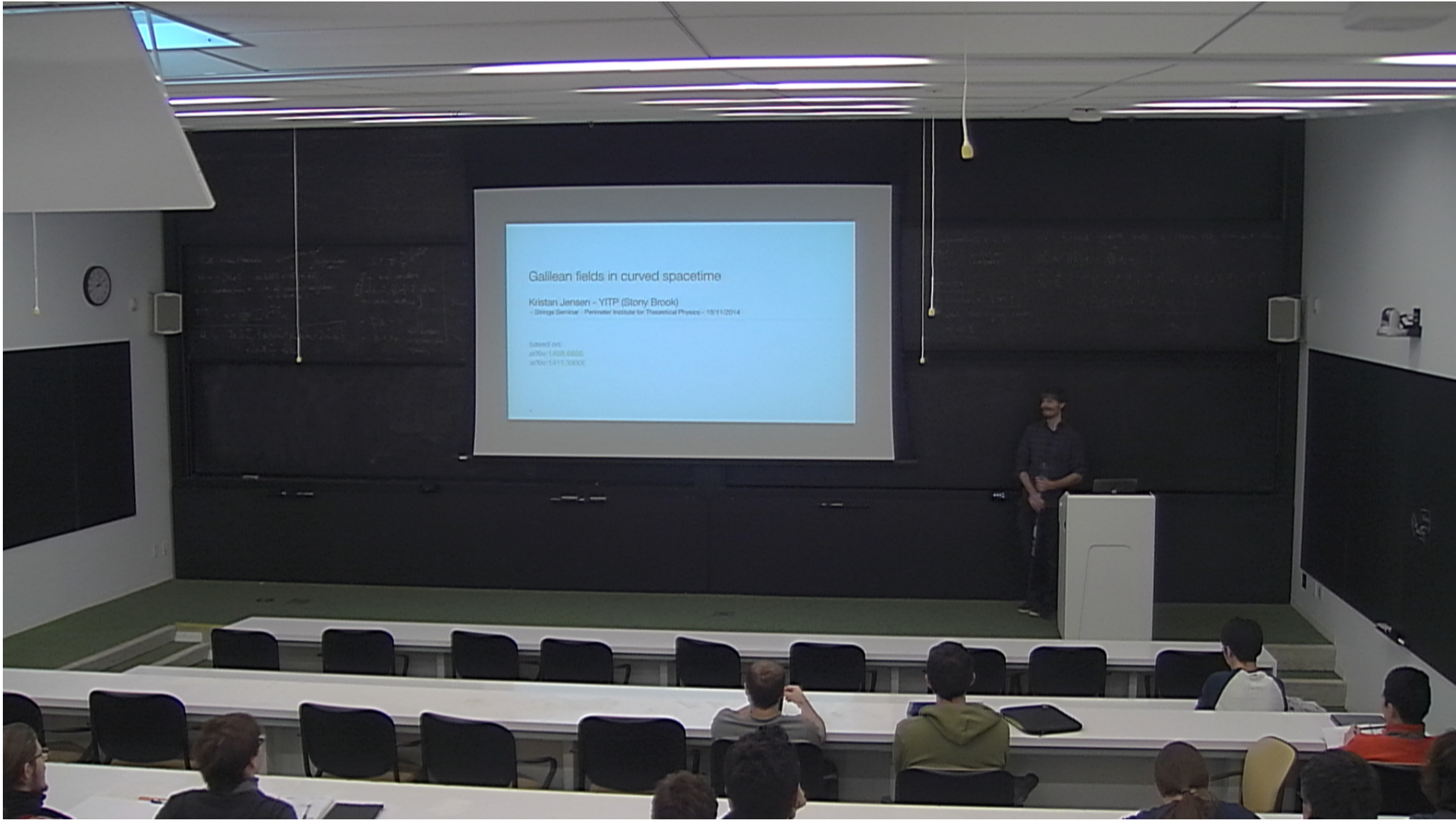


Title: Galilean fields in curved spacetime

Date: Nov 18, 2014 02:00 PM

URL: <http://pirsa.org/14110045>

Abstract: <p> I will discuss various aspects of non-relativistic field theories on a curved, background spacetime. First things first, we need to know what sort of geometry these theories couple to, as well as the symmetries we ought to impose. I will argue that Galilean-invariant theories should be coupled to a form of Newton-Cartan geometry in which one enforces a one-form shift symmetry, which amounts to a covariant version of invariance under Galilean boosts. I will focus on two main applications of this result, namely consequences of these symmetries at nonzero temperature and the relation to warped CFTs.</p>





## Some motivation

1. Folklore theorem in Galilean-invariant theory:

$$\mathcal{P}_i = \frac{m}{q} J_i$$

Is this a Ward identity?

2. What are non-relativistic anomalies?  
(think about the edge states of topologically non-trivial phases)
3. Are there observables (e.g. interrelations between transport) which are governed by symmetries alone? (FQHE)

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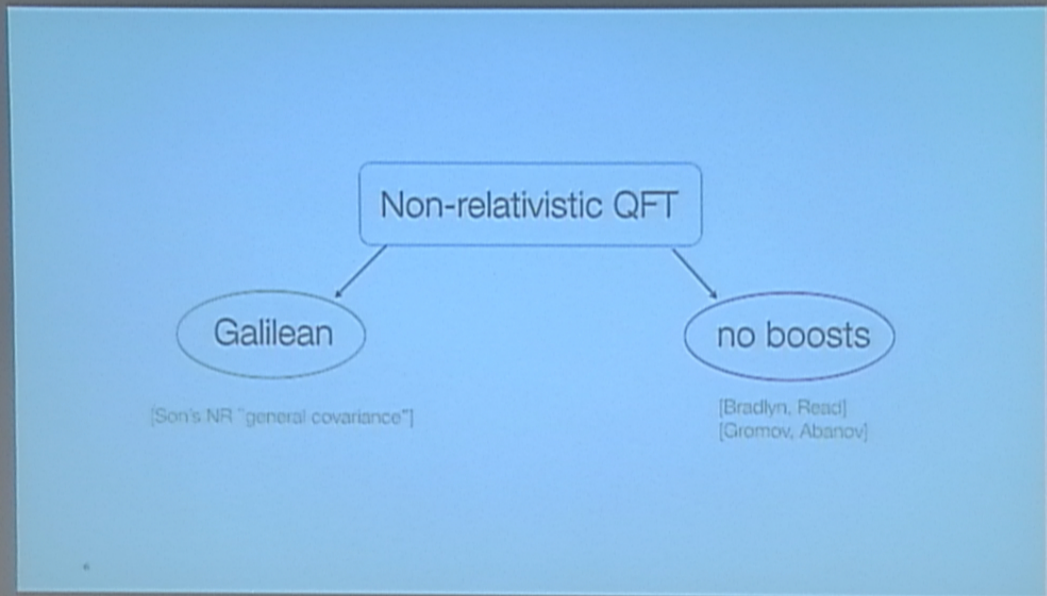
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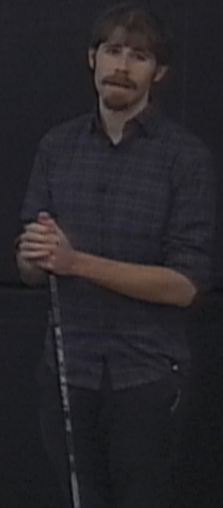
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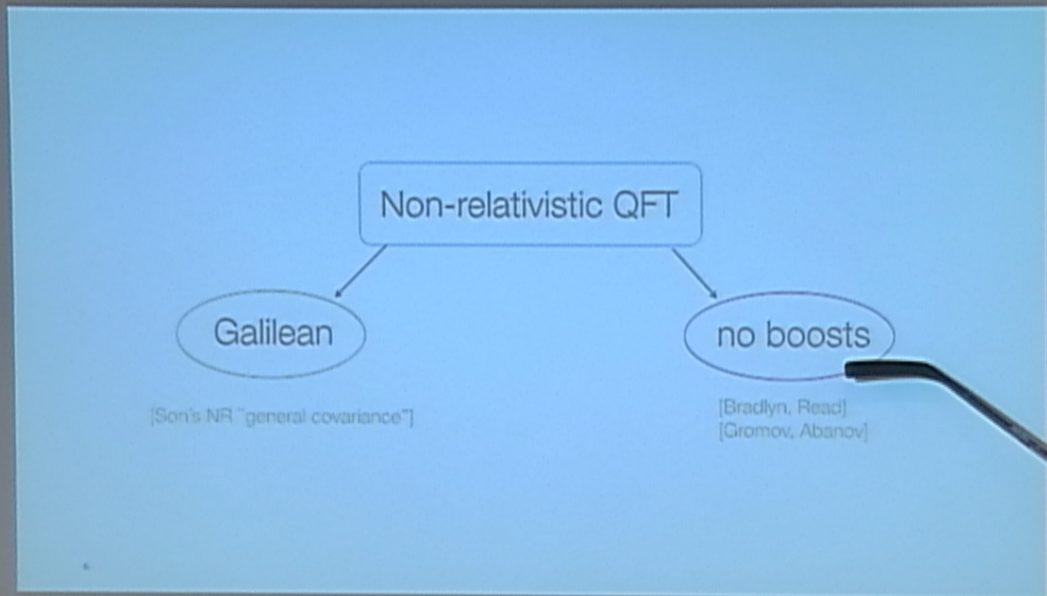
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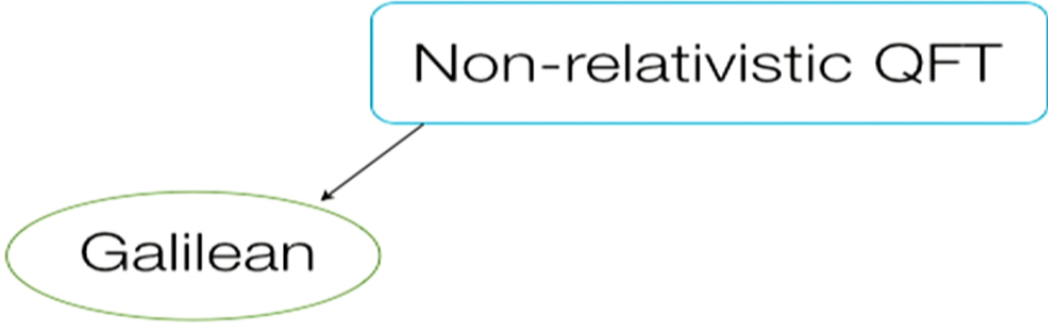




approx. rel.  $(-1, 1)$   $\rightarrow$  Fermi  
 $\rightarrow$   $\mu = 0$  (approx. rel. relativity)  
 $\mu = 0$  (approx. rel. relativity)  
 $\mu = 0$   
approx. rel. relativity  $\rightarrow$   $|S| > 1$   
approx. rel. relativity  $\rightarrow$   $v \ll c$







[Son's NR "general covariance"]



## The culprit

Galilean theories have a particle number symmetry  $[M, \cdot] = 0$

But:  $[P_i, K_j] = -i\delta_{ij} M$

So  $A_\mu$  which couples to  $J^\mu$  is not an ordinary  $U(1)$  connection

## Some personal motivation

We've learned a ton about relativistic QFT at  $T > 0$ :

- AdS/CFT-inspired revolution of relativistic fluid mechanics
- anomaly-induced transport
- constraints from symmetries on thermal partition functions

I'd like to see NR hydro (invented in 1840's [!]) brought up to the sophistication of its much younger sibling

(one practical consequence: hydrodynamic correlation functions)



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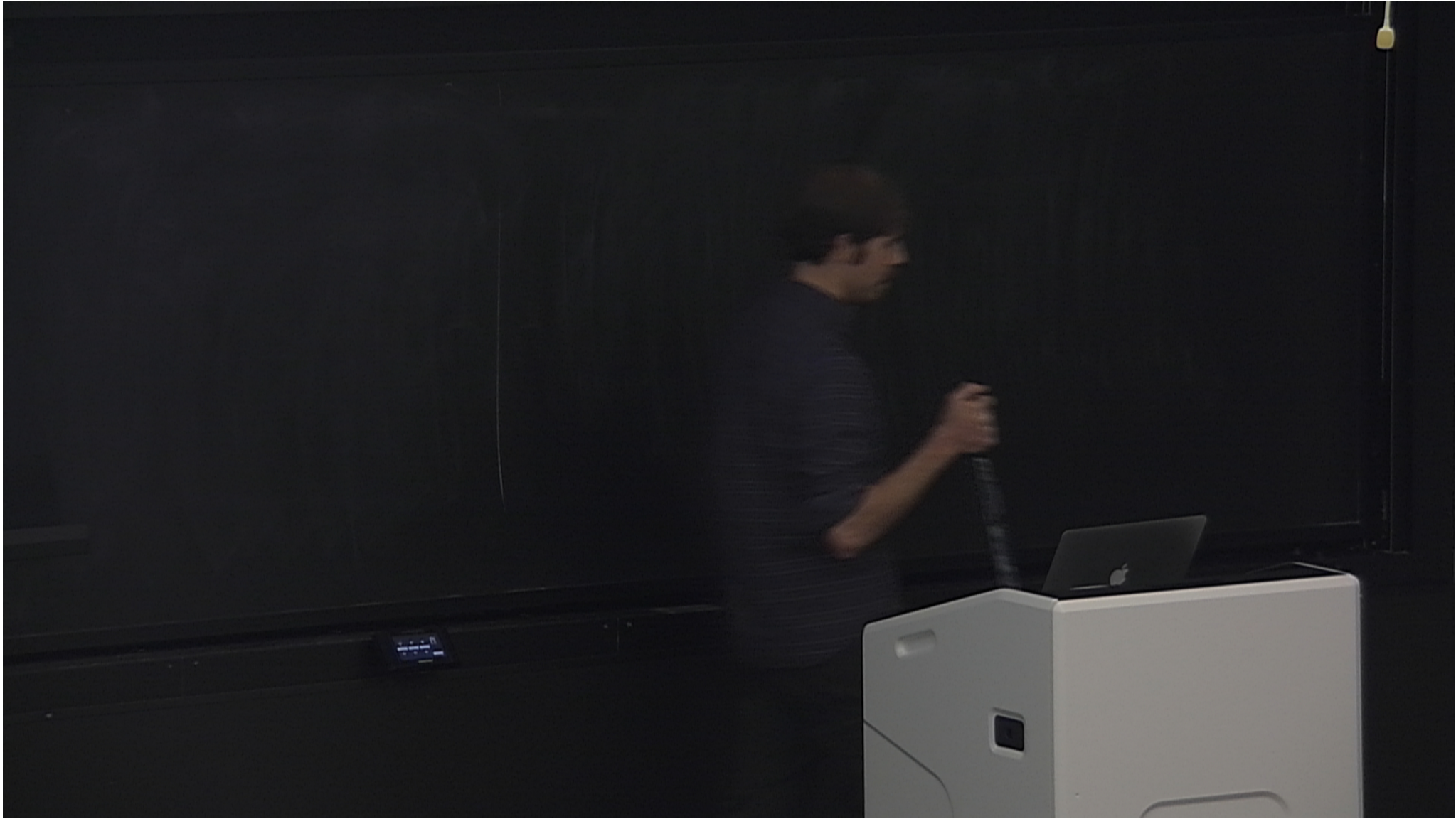
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hydrodynamics  $(S, \mathcal{E})$   $\rightarrow$   $E_{\text{hydro}}$   
 $\mathcal{E} = \rho + \mathcal{P}$  (hydrodynamics)  $S_{\text{hydro}} = \rho \mathcal{E}$   
 $\mathcal{P} = \rho \mathcal{E} - \rho \mathcal{E}$   
 $\rho \ll \rho$



## The plan

0. Motivation
- 1. Galilean free fields and Newton-Cartan geometry**
2. Some checks on the proposal
3. Warped CFTs in  $d=2$
4. Modernizing NR hydro

10

symmetries  $(\vec{v}, \vec{S})$   
 $N=0$  solutions  
(background)  
 $N=2,3$   
maximal  $\leq 1$   
spacetime symmetries  
 $n \ll 8$

## The plan

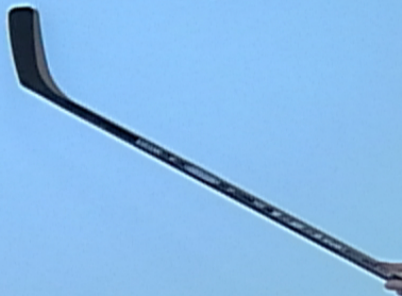
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## Galilean free field theory

$$S_{free} = \int d^d x \left\{ \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{ij}}{2m} \partial_i \Psi^\dagger \partial_j \Psi \right\}$$



Eigenvalues  $(-i, i)$   
 $\psi = 0$  solutions  
 $\psi = \psi_0 e^{i(kx - \omega t)}$   
 $\omega = v k$   
 $v = \frac{\omega}{k} = \frac{1}{m}$   
non-relativistic  $v \ll c$

## Galilean free field theory

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some features:

1. eom is Schrodinger equation
2. invariant under Schrodinger symmetries (Galilean CFT)

$$(H, P_i, R_{ij}, K_i, M, D, C)$$

12



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## Symmetry currents

energy current: 
$$\begin{cases} \mathcal{E}^0 = \frac{1}{2m} \delta^{ij} \partial_i \Psi^\dagger \partial_j \Psi \\ \mathcal{E}^i = \frac{1}{2m} (\dot{\Psi}^\dagger \partial^i \Psi + (\partial^i \Psi^\dagger) \dot{\Psi}) \end{cases} \quad \partial_\mu \mathcal{E}^\mu = 0$$

number current: 
$$\begin{cases} J^0 = m \Psi^\dagger \Psi \\ J^i = \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}^i \Psi \end{cases} \quad \partial_\mu J^\mu = 0$$

momentum & spatial stress: 
$$\begin{cases} \mathcal{P}^i = \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}^i \Psi \\ T^{ij} = \frac{1}{2m} \partial^i \Psi^\dagger \partial^j \Psi + \delta^{ij} \left( \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{kl}}{2m} \partial_k \Psi^\dagger \partial_l \Psi \right) \end{cases}$$

$$\dot{\mathcal{P}}^i + \partial_j T^{ij} = 0$$



## Symmetry currents

$$\text{energy current: } \begin{cases} \mathcal{E}^0 = \frac{1}{2m} \delta^{ij} \partial_i \Psi^\dagger \partial_j \Psi \\ \mathcal{E}^i = \frac{1}{2m} (\dot{\Psi}^\dagger \partial^i \Psi + (\partial^i \Psi^\dagger) \dot{\Psi}) \end{cases} \quad \partial_\mu \mathcal{E}^\mu = 0$$

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$$\dot{\mathcal{P}}^i + \partial_j T^{ij} = 0$$

## Coupling to spacetime

It's pretty clear what we got to do:

$$S_{free} = \int d^d x \left\{ \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{ij}}{2m} \partial_i \Psi^\dagger \partial_j \Psi \right\}$$

covariant  
measure

$v^\mu \overleftrightarrow{D}_\mu$

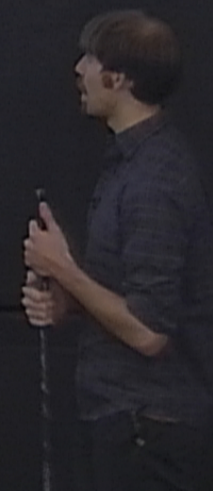
$h^{\mu\nu}$  (spatial co-metric)

$$D_\mu \Psi = (\partial_\mu - imA_\mu) \Psi$$

15

*Handwritten notes on a chalkboard:*

- $U(1)$  gauge theory
- $S = \int d^d x \left[ \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \right]$
- $S \rightarrow S + \int d^d x j^\mu A_\mu$
- $\nabla_\mu \psi = (\partial_\mu - imA_\mu) \psi$
- $\nabla_\mu \psi^\dagger = (\partial_\mu + imA_\mu) \psi^\dagger$
- $\mathcal{L} = \frac{i}{2} \psi^\dagger \overleftrightarrow{\partial}_\mu \psi - \frac{1}{2m} \nabla_i \psi^\dagger \nabla_i \psi - V(\psi)$





## Coupling to spacetime

$$S_{free} = \int d^d x \sqrt{\gamma} \left\{ \frac{i v^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{h^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$$

what is  $\sqrt{\gamma}$ ?

- from  $(v^\mu, h^{\mu\nu})$  we can get  $(n_\mu, h_{\mu\nu})$  satisfying:

$$n_\mu v^\mu = 1 \quad h^{\mu\nu} n_\nu = 0, \quad h_{\mu\nu} v^\nu = 0, \quad h_{\mu\rho} h^{\nu\rho} = P_\mu^\nu = \delta_\mu^\nu - v^\nu n_\mu$$

- then:

$$\gamma_{\mu\nu} \equiv n_\mu n_\nu + h_{\mu\nu}, \quad \gamma = \det(\gamma)$$

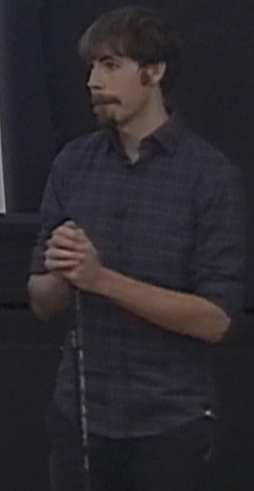
## Symmetries

$$S_{free} = \int d^d x \sqrt{\gamma} \left\{ \frac{iv^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{\hbar^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$$

Manifestly invariant under reparameterization/U(1)

18

Eigenvalues  $(-1, 1)$   
 $\Psi = 0$  (solutions  
(arguments))  
 $\Psi = 0$   
...  
 $\Psi = 0$   
...  
 $\Psi = 0$





## Symmetries

$$S_{free} = \int d^d x \sqrt{\gamma} \left\{ \frac{i v^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{h^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$$

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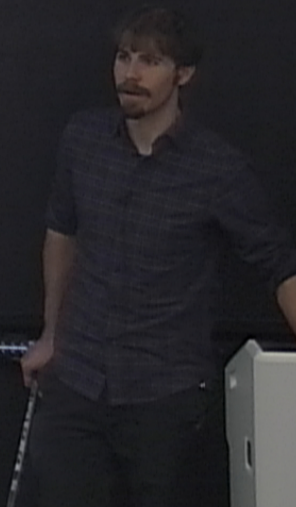
What is less obvious is that there is a shift symmetry:

$$\begin{cases} v^\mu \rightarrow v^\mu + h^{\mu\nu} \psi_\nu \\ A_\mu \rightarrow A_\mu + P_\mu^\nu \psi_\nu - \frac{1}{2} n_\mu h^{\nu\rho} \psi_\nu \psi_\rho \end{cases}$$

for any transverse  $\psi_\mu$

10

generators  $(S, \mathcal{L})$   
 $\mathcal{L} = 0$  (relations)  
 $\mathcal{L} = 0$  (constraints)  
 $\mathcal{L} = 0$   
maximization  $\mathcal{L} \rightarrow 1$   
sub- $\mathcal{L}$  (gauge states)  
 $\mathcal{L} \leq 0$



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## The proposal

Can now state a general proposal for coupling to spacetime:

1. the background fields are  $(v^\mu, h^{\mu\nu}, A_\mu)$ , or equivalently  $(n_\mu, h_{\mu\nu}, A_\mu)$
2. we ought to impose invariance under
  - reparameterizations
  - U(1)
  - the shift symmetry (Milne boosts)

## Symmetries

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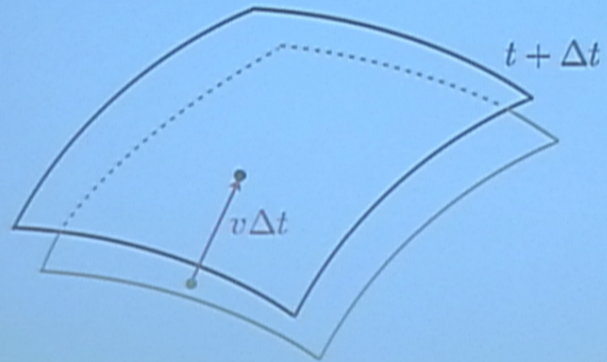
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generators  $(S, \vec{S})$   
 $\mathcal{H} = 0$  (constraints)  
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## Newton-Cartan geometry

We've unwittingly generated (one version of) Newton-Cartan geometry



23

coordinates  $(t, \vec{x})$   
 $\vec{x} = 0$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_0$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_1$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_2$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_3$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_4$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_5$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_6$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_7$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_8$  (spacelike hypersurface)  
 $\vec{x} = \vec{x}_9$  (spacelike hypersurface)

## Defining a derivative

The only thing that remains is to specify the analogue of the Levi-Civita

can fix it by demanding:

1.  $(n_\mu, h^{\mu\nu})$  constant
2. zero spatial torsion
3.  $-2h_{\rho[\mu}D_{\nu]}v^\rho = F_{\mu\nu}$

end result is:

$$\Gamma^\mu{}_{\nu\rho} = v^\mu \partial_\rho n_\nu + \frac{1}{2} h^{\mu\sigma} (\partial_\nu h_{\rho\sigma} + \partial_\rho h_{\nu\sigma} - \partial_\sigma h_{\nu\rho}) + h^{\mu\sigma} n_{(\nu} F_{\rho)\sigma}$$

24



## The relation to Son's "general covariance"

The Milne symmetry can be gauge-fixed:  $v^\mu \partial_\mu \propto \partial_0$

Resulting reparameterizations are constrained:  $h^{i\mu} \psi_\mu = e^\Phi \dot{\xi}^i$

$$n_\mu dx^\mu = e^\Phi (dx^0 - \beta_i dx^i), \quad h_{\mu\nu} dx^\mu \otimes dx^\nu = g_{ij} dx^i \otimes dx^j$$

this recovers Son's transformations on the nose, e.g.

$$\delta_\chi A_i = \mathcal{L}_\xi A_i + e^\Phi g_{ij} \dot{\xi}^j + \partial_i \Lambda$$

30



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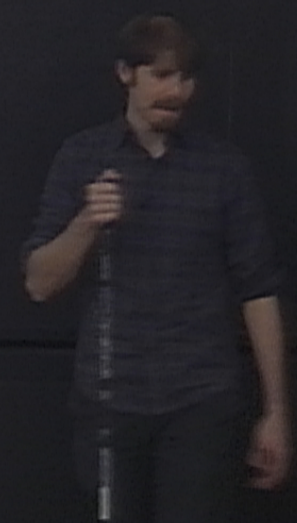
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30

generalized  $(S, S)$   
 $\mathcal{H} = 0$  (constraints)  
 $\mathcal{H}_i = 0$  (momentum)  
 $n = \dots$   
 $\mathcal{L}_\xi$   
 $\xi \ll \delta$





## Newton-Cartan from null reduction

We can get this structure from  $d+1$ -dim Lorentzian manifold with a null isometry:

$$G = n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$$

$$n^M \partial_M = \partial_-$$

see e.g.  
[Duval, et al] 1984  
[Julia, Nicolai] 1994  
[Christensen, Hartong, et al] 1311

Get this sort of geometry on “boundary” of Schrodinger spacetimes

$$g_{bulk} = \frac{c n_\mu n_\nu dx^\mu dx^\nu}{r^{2z}} + \frac{G + dr^2}{r^2} + \dots$$

[Son]  
[Balasubramanian, McGreevy]

31

$g_{\mu\nu} = \begin{pmatrix} -1 & & \\ & \delta_{ij} & \\ & & 0 \end{pmatrix}$   
 $\partial_- = \partial_{x^-}$   
 $\partial_+ = \partial_{x^+}$   
 $\partial_i = \partial_{x^i}$   
 $\partial_r = \partial_r$   
 $\partial_z = \partial_z$   
 $\partial_{x^i} = \partial_{x^i}$   
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## Newton-Cartan from null reduction

We can get this structure from  $d+1$ -dim Lorentzian manifold with a null isometry:

$$G = n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$$

$$n^M \partial_M = \partial_-$$

see e.g.  
[Duval, et al] 1984  
[Julia, Nicolai] 1994  
[Christensen, Hartong, et al] 1311

Get this sort of geometry on “boundary” of Schrodinger spacetimes

$$g_{bulk} = \frac{c n_\mu n_\nu dx^\mu dx^\nu}{r^{2z}} + \frac{G + dr^2}{r^2} + \dots$$

[Son]  
[Balasubramanian, McGreevy]

34



## Newton-Cartan from null reduction

$$G = n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$$

So reduction along  $x^-$  gives NC data  $(n_\mu, h_{\mu\nu}, A_\mu)$

Also:

- Milne boosts correspond to ambiguity in extraction of NC data from  $G$
- get the Milne-invariant, U(1)-non-invariant connection

37

generalises  $(S, S)$   
 $\rightarrow$  ...  
 $S = \dots$   
 $S \cdot S = \dots$   
 $n \ll 8$

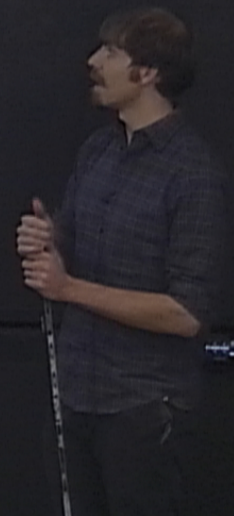
# Newton-Cartan from large c

[WIP w/ Andreas Karch]

It all works.

33

Newton-Cartan (1930s)  
Metric  $h_{\mu\nu}$  (signature  $(n-1, 1)$ )  
Connections  $\Gamma^\lambda_{\mu\nu}$  (torsionless)  
 $\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}$   
 $\Gamma^\lambda_{\mu\nu} = 0$  for  $\lambda = 0$   
Metric compatibility  $\nabla_\mu h_{\nu\rho} = 0$   
Compatibility with  $\partial_\mu$   
 $\partial_\mu h_{\nu\rho} = 0$   
 $\partial_\mu \Gamma^\lambda_{\nu\rho} = 0$   
 $\partial_\mu \Gamma^\lambda_{\nu\rho} = 0$   
 $\partial_\mu \Gamma^\lambda_{\nu\rho} = 0$





## What is a Warped CFT?

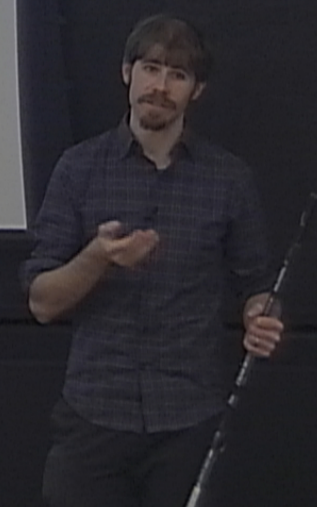
Suppose you have a 2d theory invariant under:

$$x^{\pm} \rightarrow x^{\pm} + c^{\pm}, \quad x^{-} \rightarrow \lambda x^{-} \quad [\text{Holman, Strominger}]$$

Then unitarity, locality enhance these symmetries to affine algebras

35

conformal  $(\mathfrak{h}, \mathfrak{g})$   $\rightarrow$   $\mathfrak{E}_{\mathfrak{h}}$   
solutions  
(warped)  
 $\mathfrak{S}_1 = \mathfrak{S}_2$   
 $\mathfrak{S}_1 \cdot \mathfrak{S}_2$   
maximization  $\mathfrak{S} > 1$   
scale  $\mathfrak{S} \ll 1$



## What is a Warped CFT?

Suppose you have a 2d theory invariant under:

$$x^{\pm} \rightarrow x^{\pm} + c^{\pm}, \quad x^{-} \rightarrow \lambda x^{-} \quad [\text{Hofman, Strominger}]$$

Then unitarity, locality enhance these symmetries to affine algebras

Two minimal cases:

1. Lorentz-invariance : get ordinary 2d CFT
2. R-moving translation enhances to Virasoro,  
L-moving translation to Kac-Moody

36





## Free scalar WCFT

If you have a complex scalar, you can also write

$$S' = \int d^2x \sqrt{\gamma} \left\{ c_1 w^\mu \varphi^* \overleftrightarrow{D}_\mu \varphi + c_2 \varphi^* \varphi \right\}$$

$$\varphi \rightarrow e^{-\Omega/2} \varphi$$