Title: Galilean fields in curved spacetime

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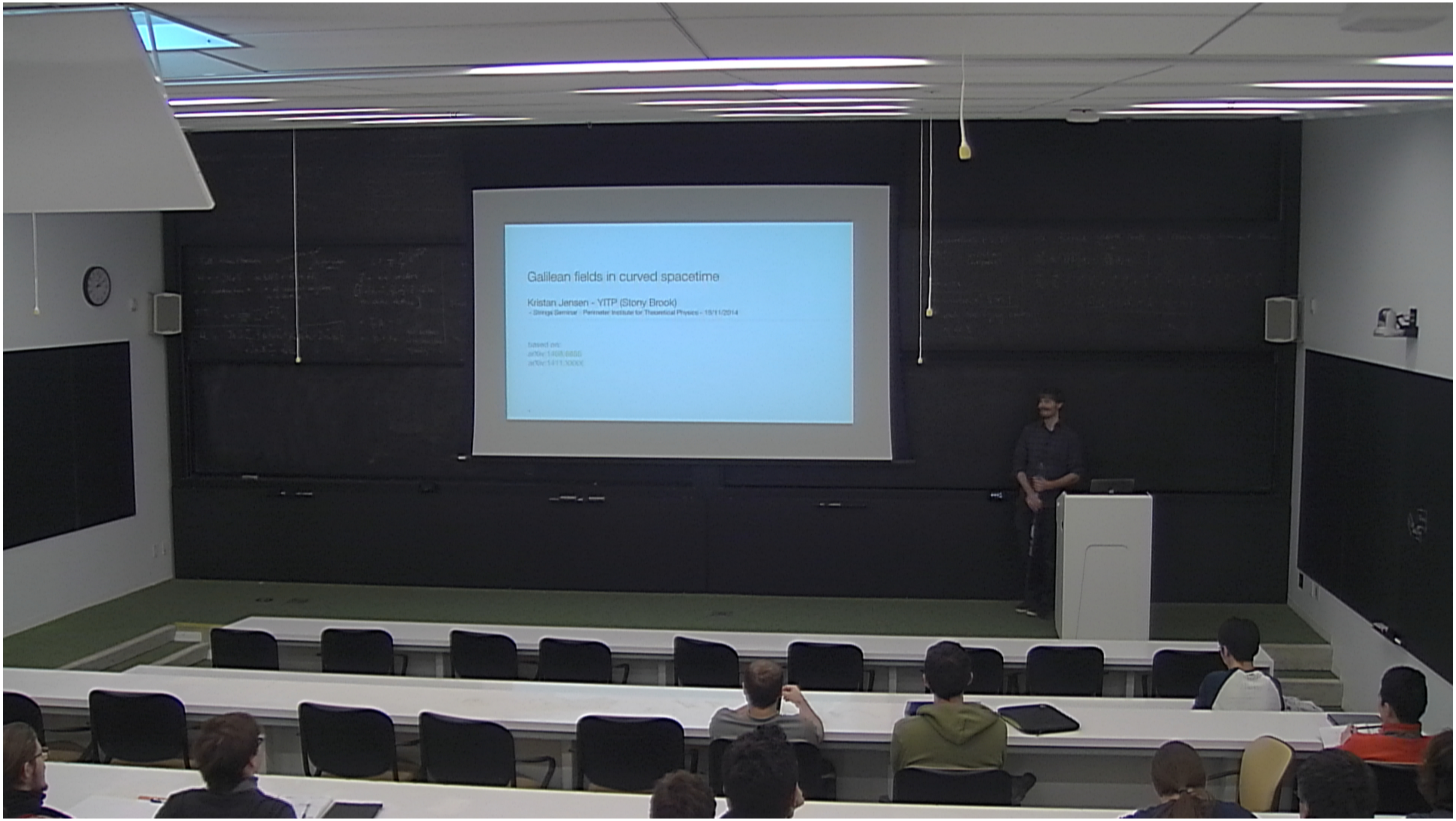
Abstract: <p> I will discuss various aspects of non-relativistic field theories on a curved, background spacetime. First things first, we need to know what sort of geometry these theories couple to, as well as the symmetries we ought to impose. I will argue that Galilean-invariant theories should be coupled to a form of Newton-Cartan geometry in which one enforces a one-form shift symmetry, which amounts to a covariant version of invariance under Galilean boosts. I will focus on two main applications of this result, namely consequences of these symmetries at nonzero temperature and the relation to warped CFTs.</p>


Galilean fields in curved spacetime
Kristan Jensen - YITP (Stony Brook)
- Strings Seminar - Perimeter Institute for Theoretical Physics - 19/11/2014
based on:

Some motivation

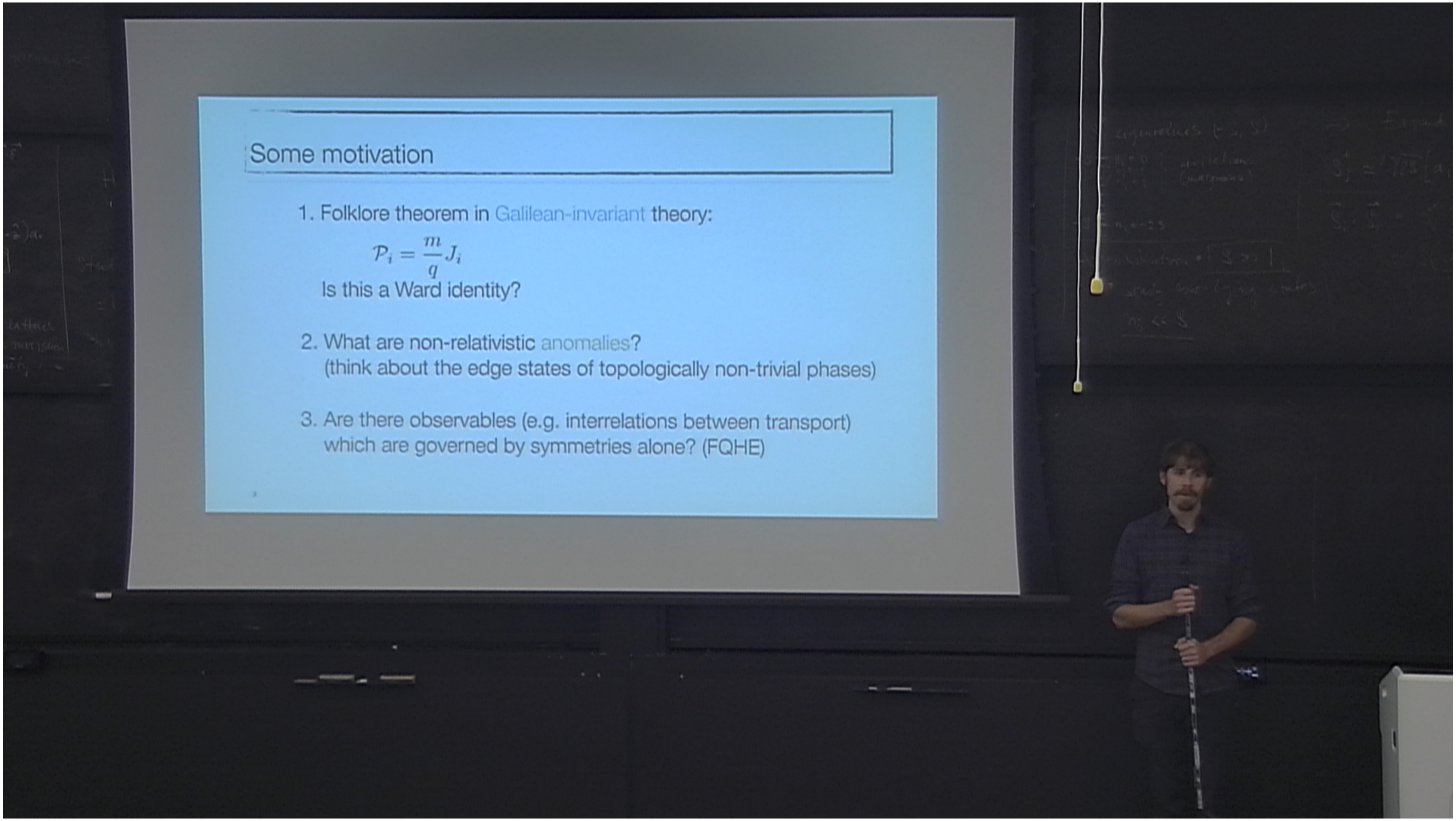
1. Folklore theorem in Galilean-invariant theory:

$$\mathcal{P}_i = \frac{m}{q} J_i$$

   Is this a Ward identity?

2. What are non-relativistic anomalies?
   (think about the edge states of topologically non-trivial phases)

3. Are there observables (e.g. interrelations between transport) which are governed by symmetries alone? (FQHE)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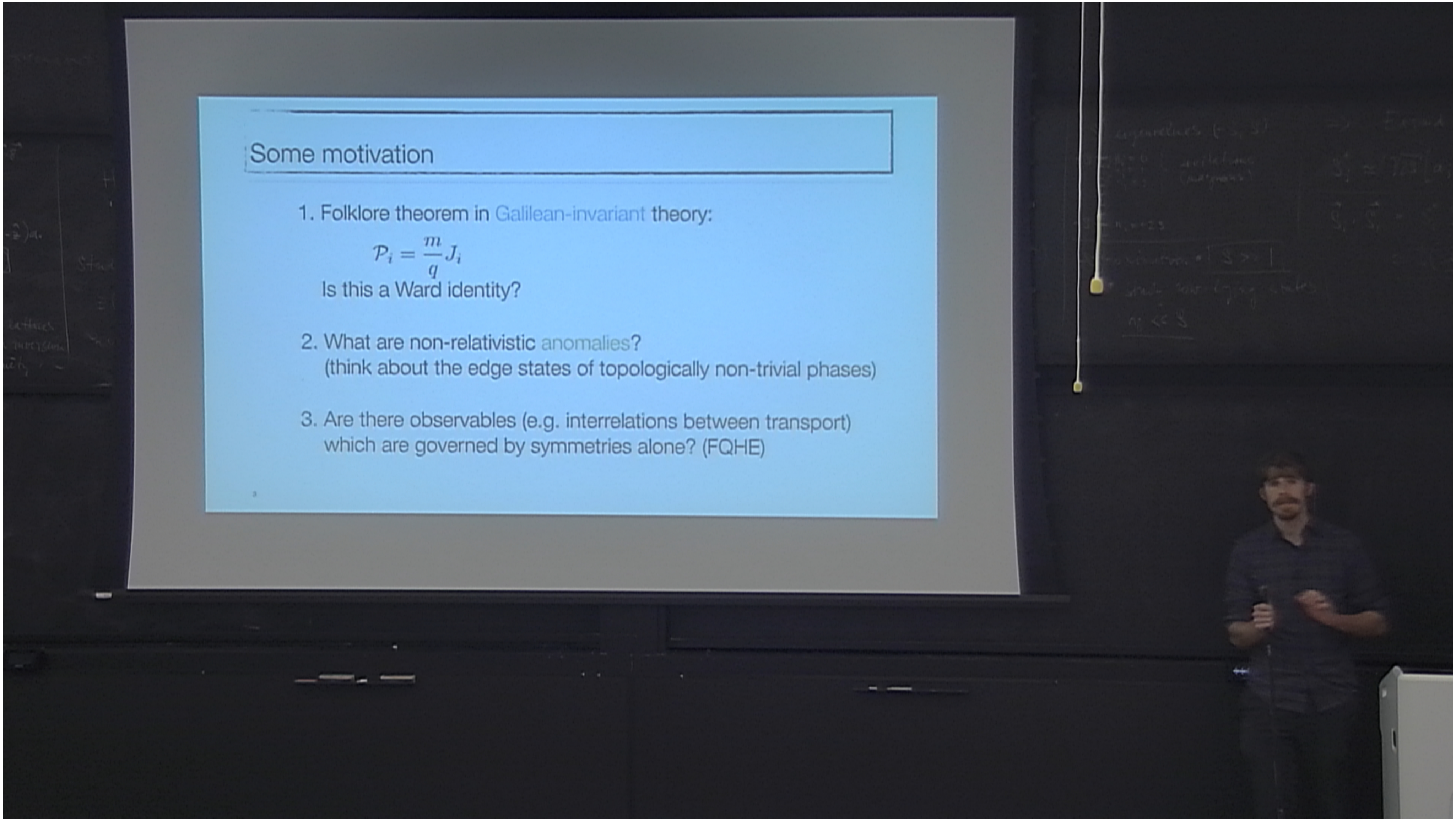
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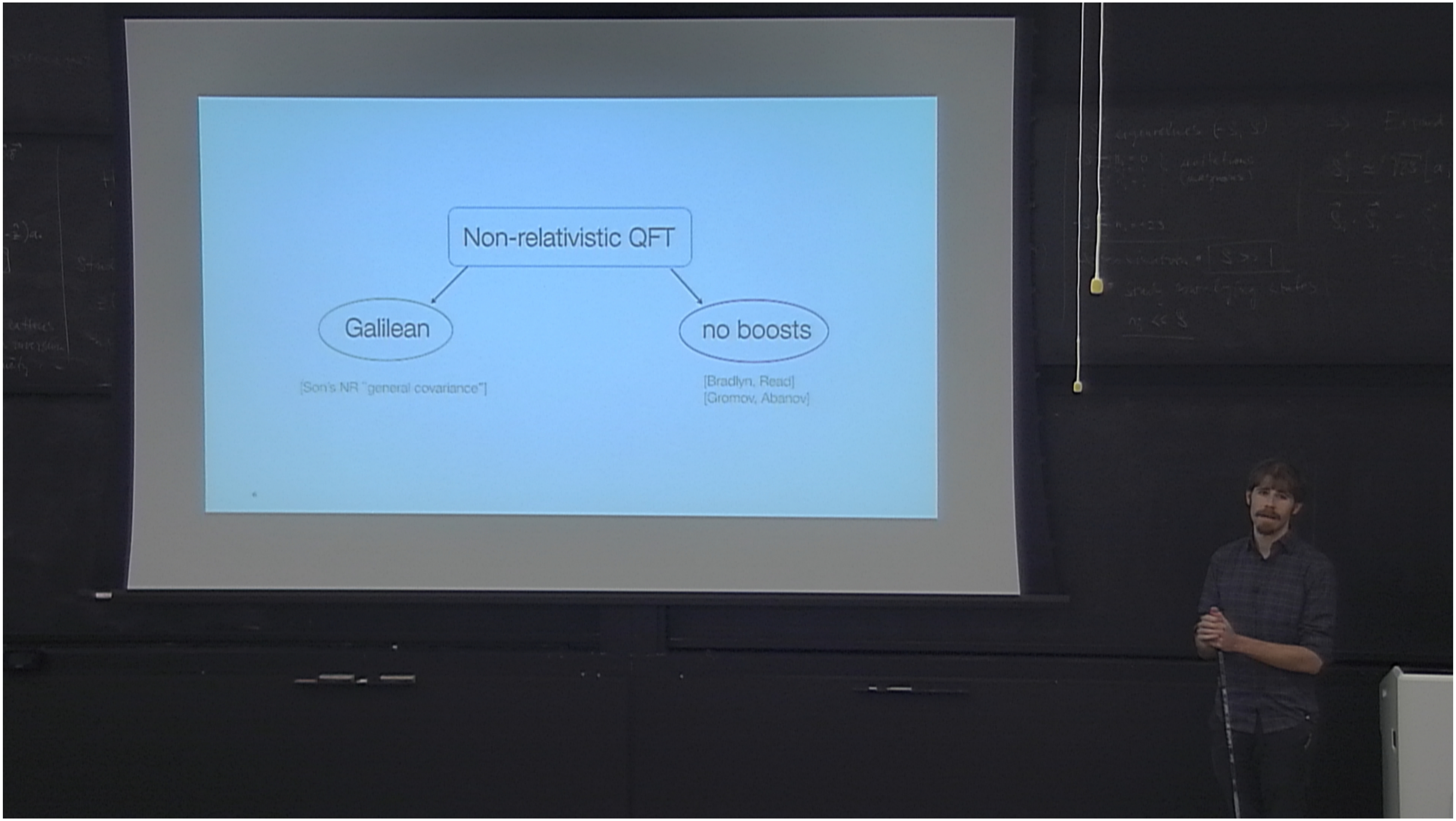
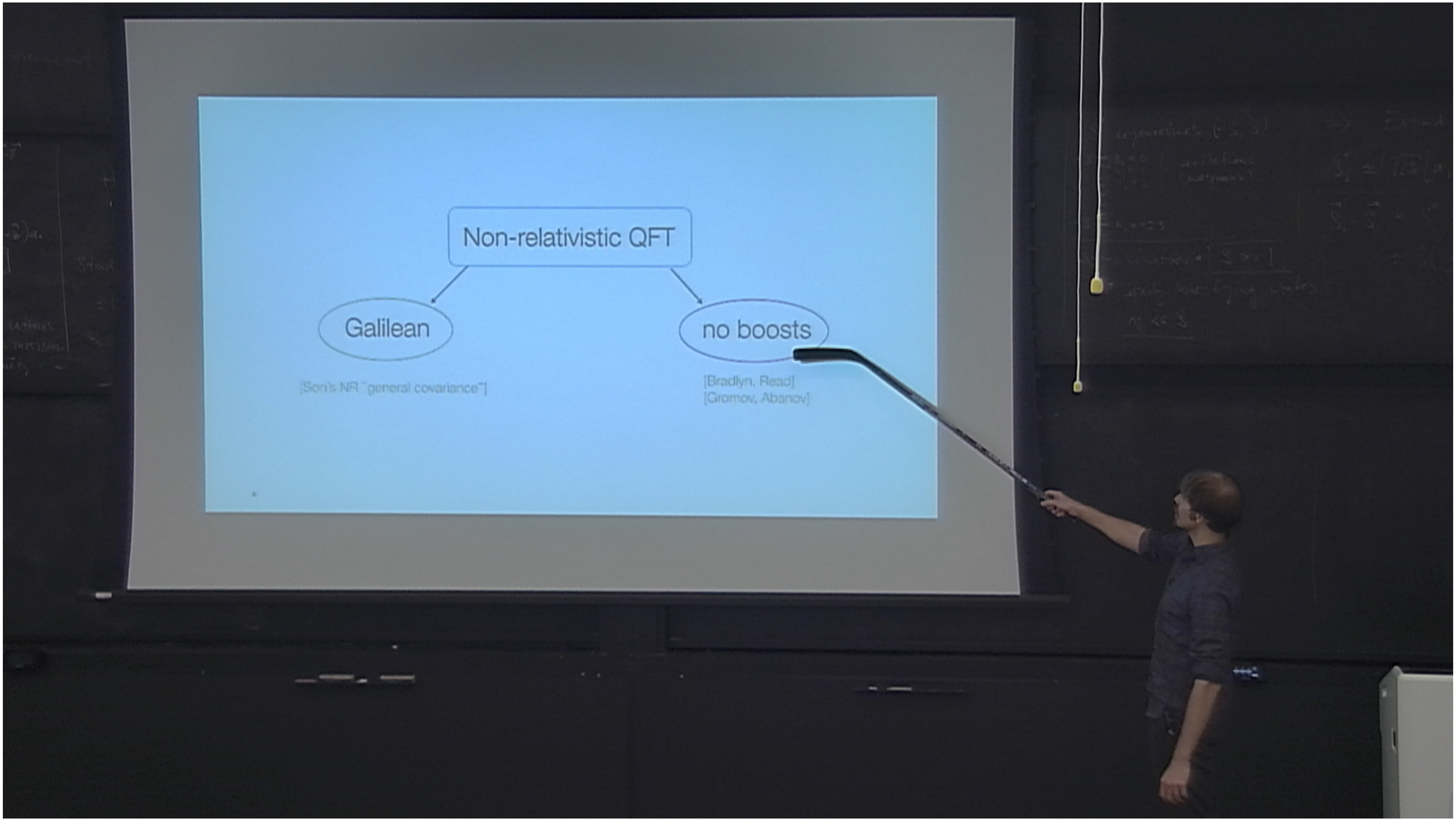

Non-relativistic QFT
Galilean
no boosts
[Son's NR "general covariance"]
[Bradlyn, Read]
[Gromov, Abanov]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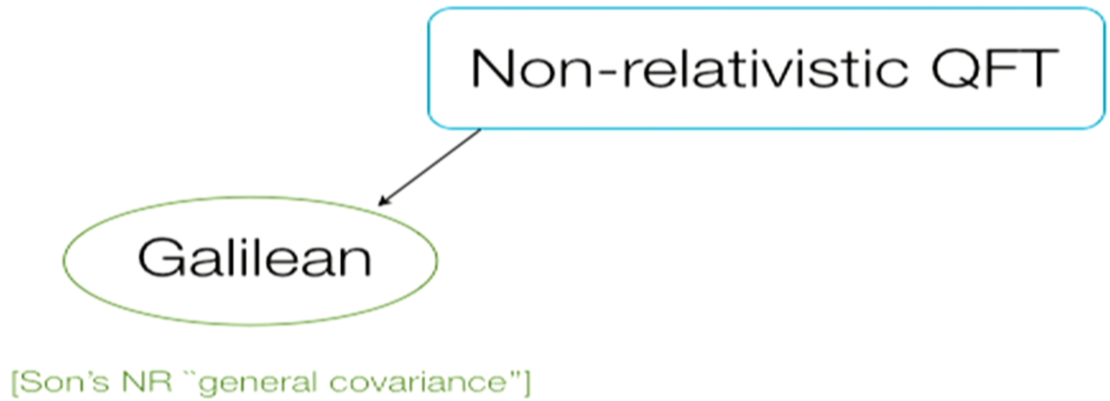

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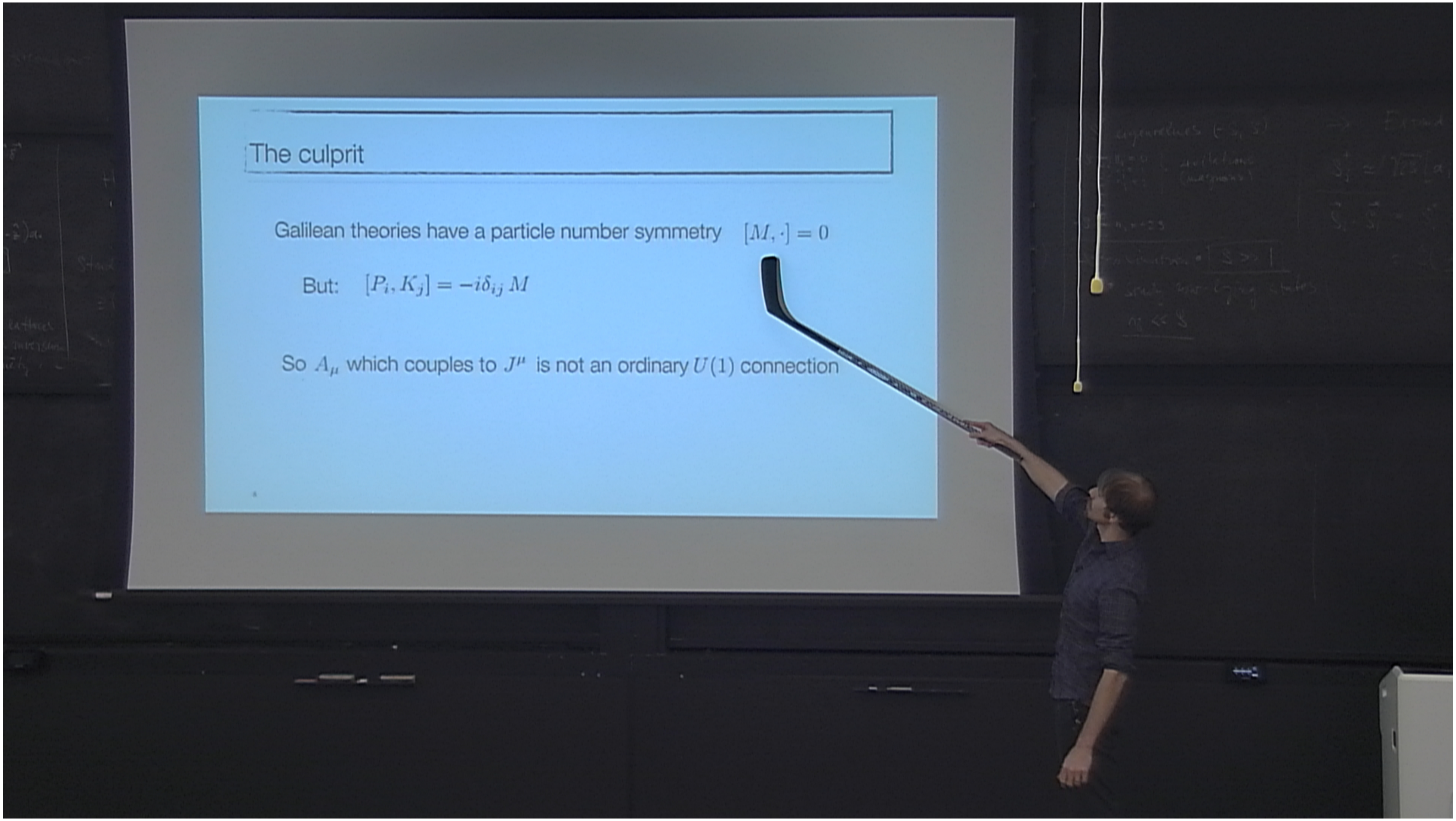
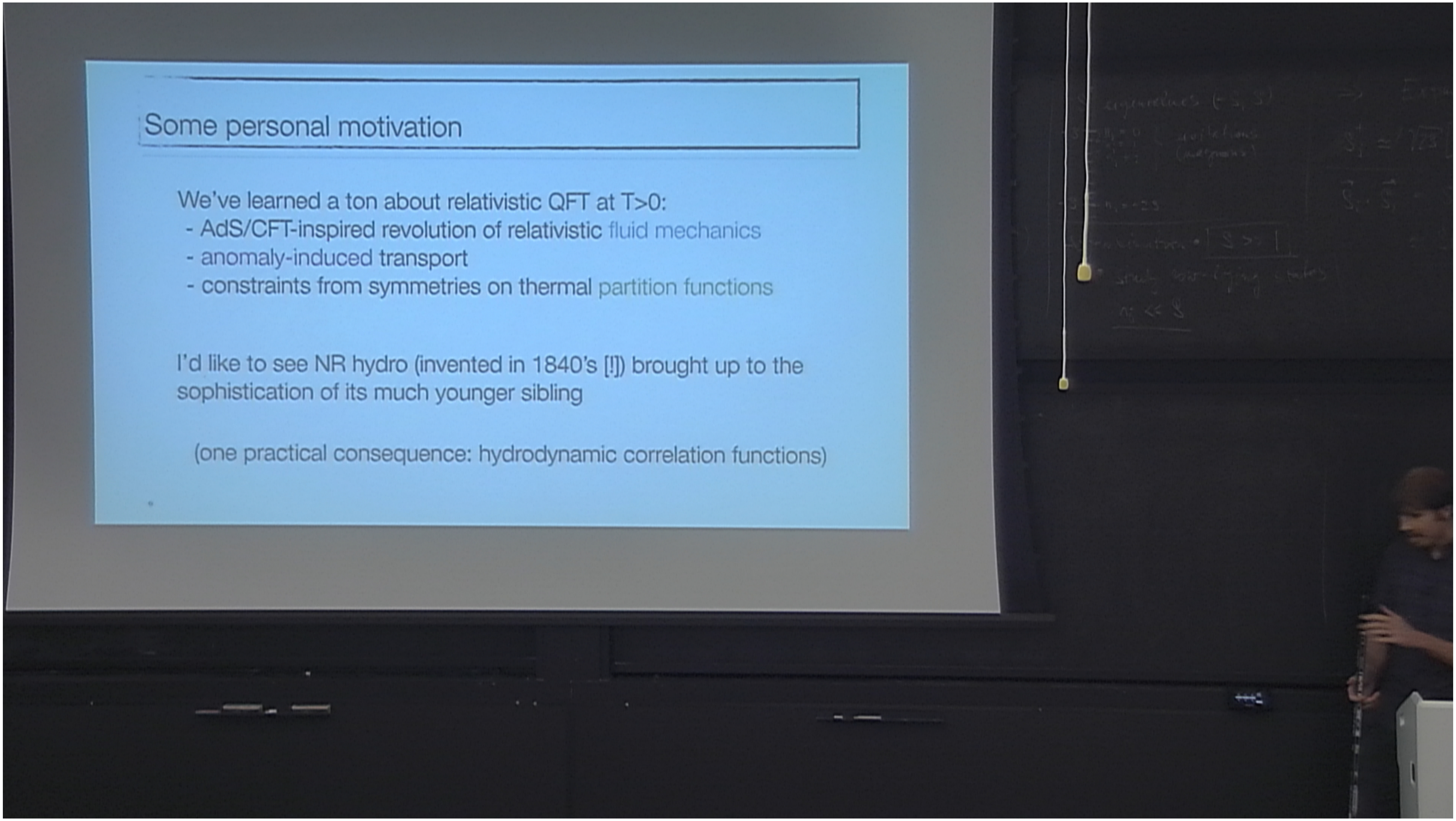
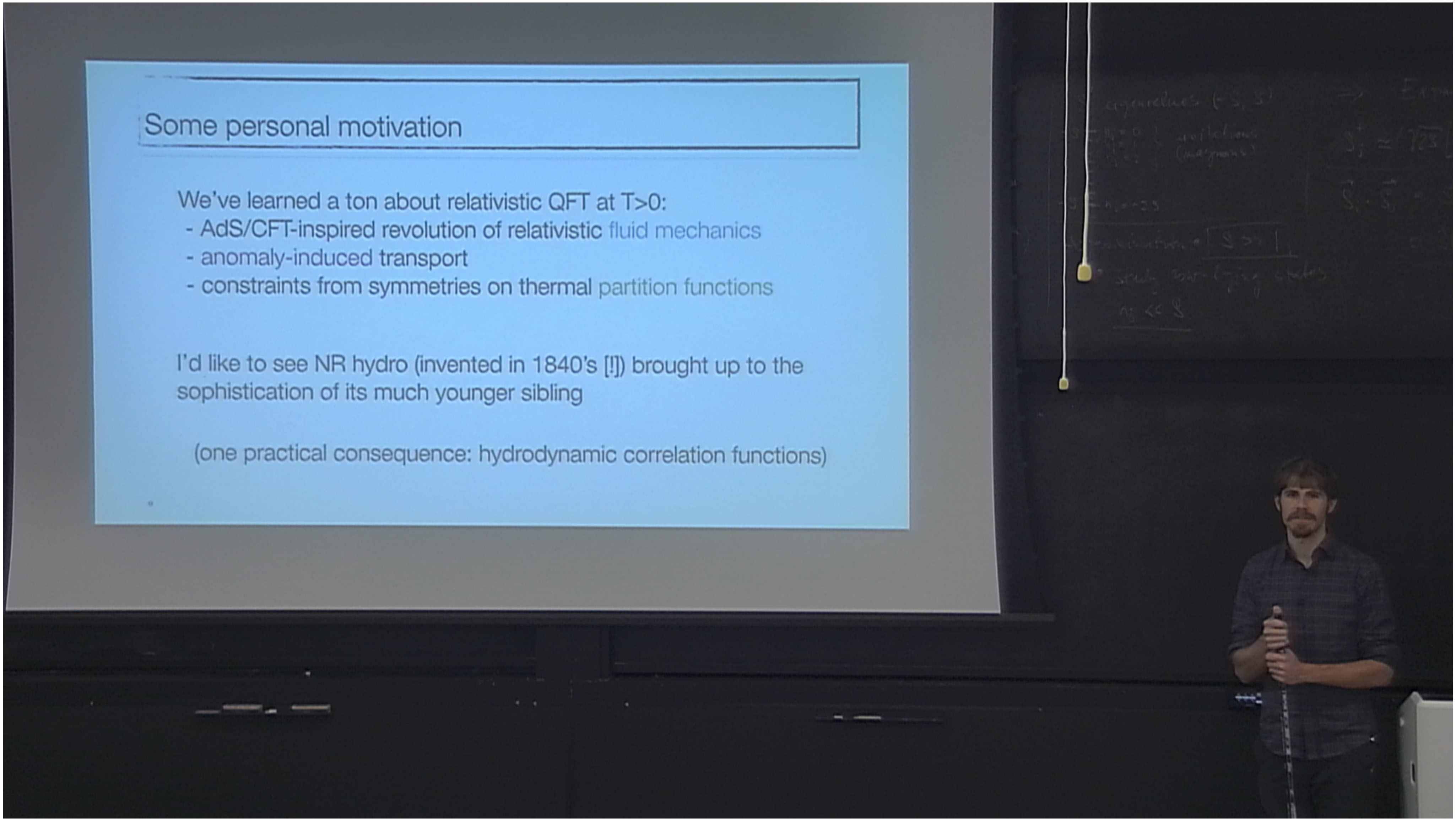

The culprit
Galilean theories have a particle number symmetry $[M, \cdot] = 0$
But: $[P_i, K_j] = -i\delta_{ij} M$
So A_μ which couples to J^μ is not an ordinary $U(1)$ connection


Some personal motivation
We've learned a ton about relativistic QFT at T>0:
- AdS/CFT-inspired revolution of relativistic fluid mechanics
- anomaly-induced transport
- constraints from symmetries on thermal partition functions
I'd like to see NR hydro (invented in 1840's [!]) brought up to the sophistication of its much younger sibling
(one practical consequence: hydrodynamic correlation functions)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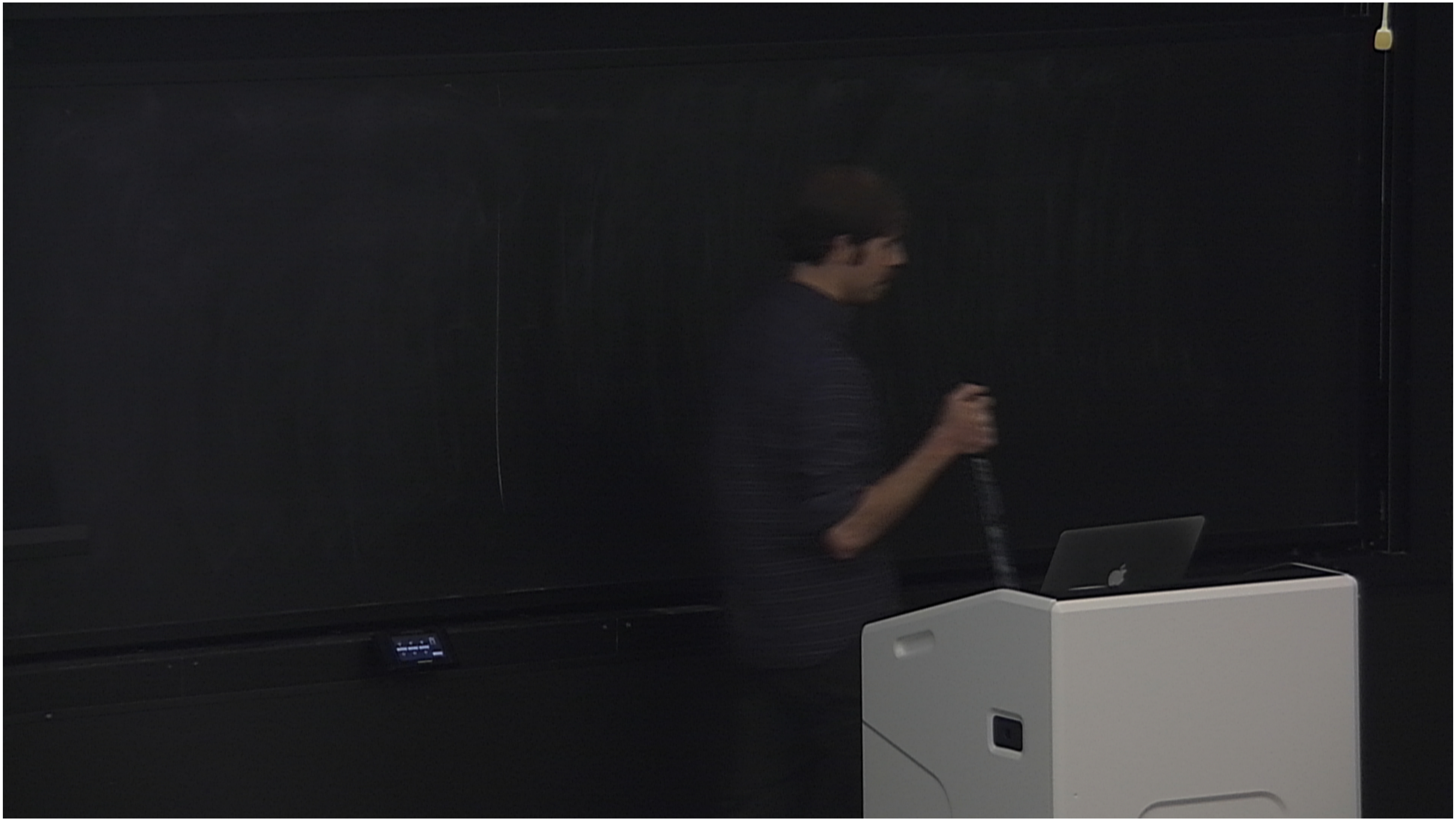

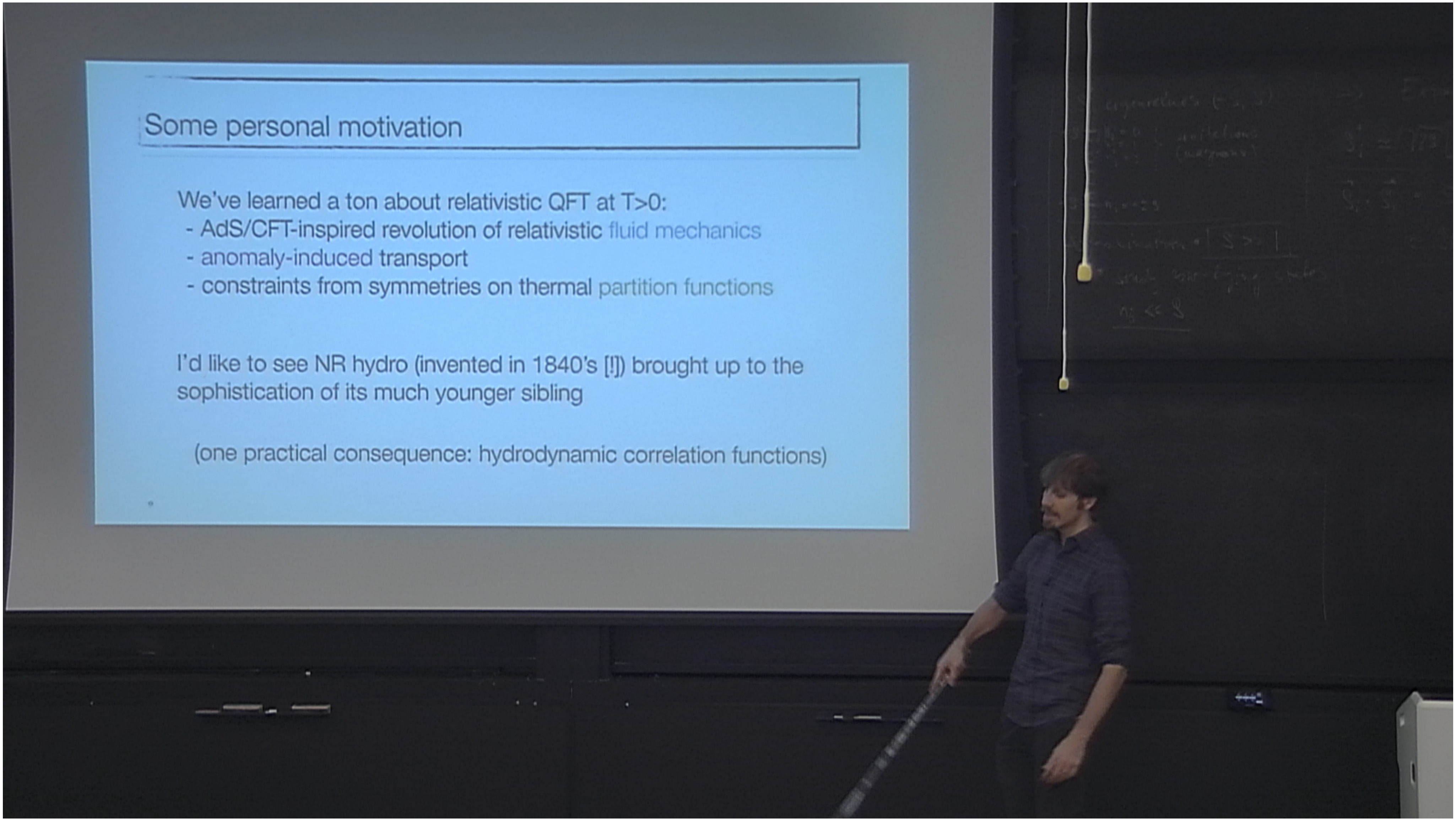
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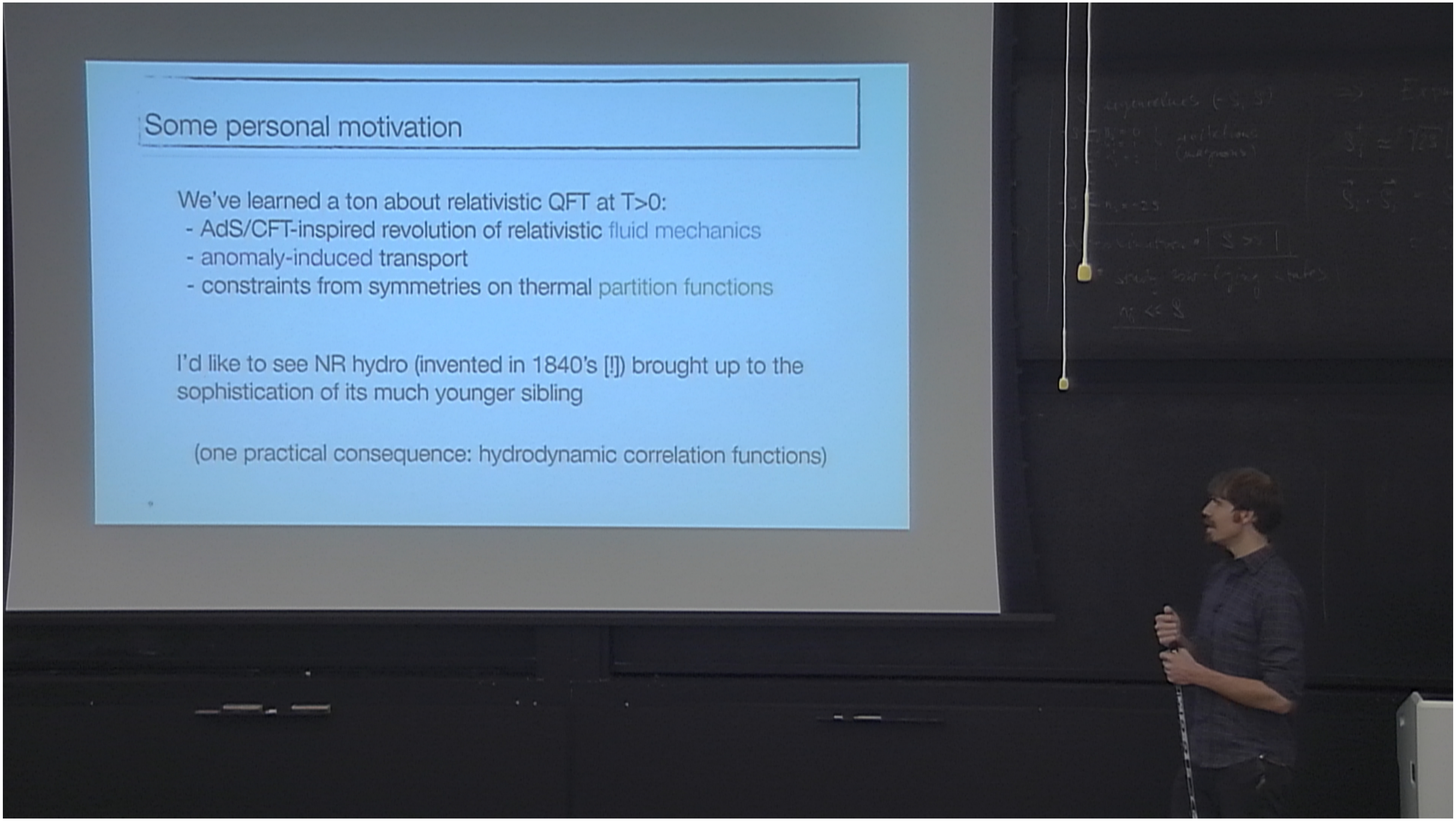
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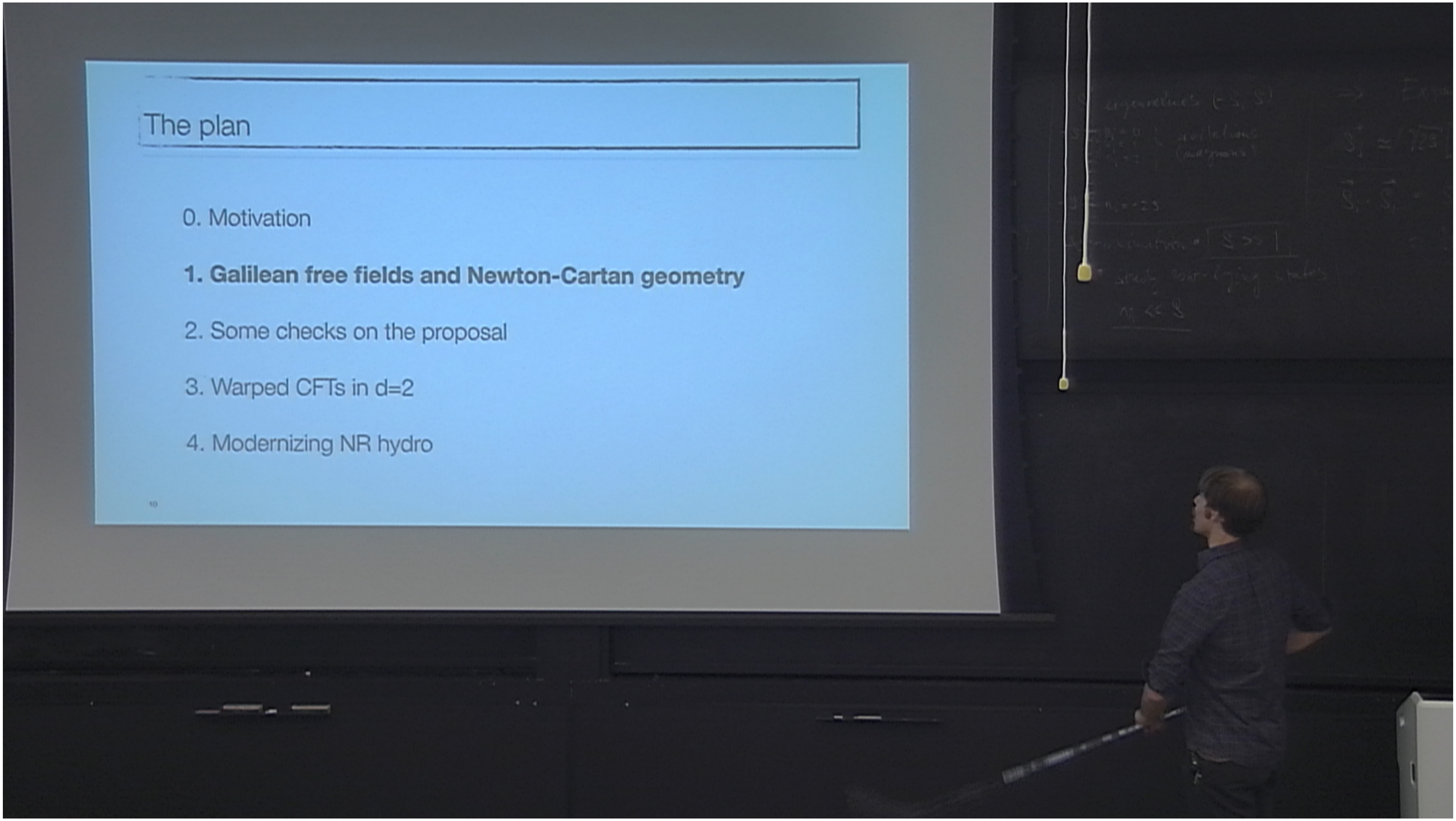
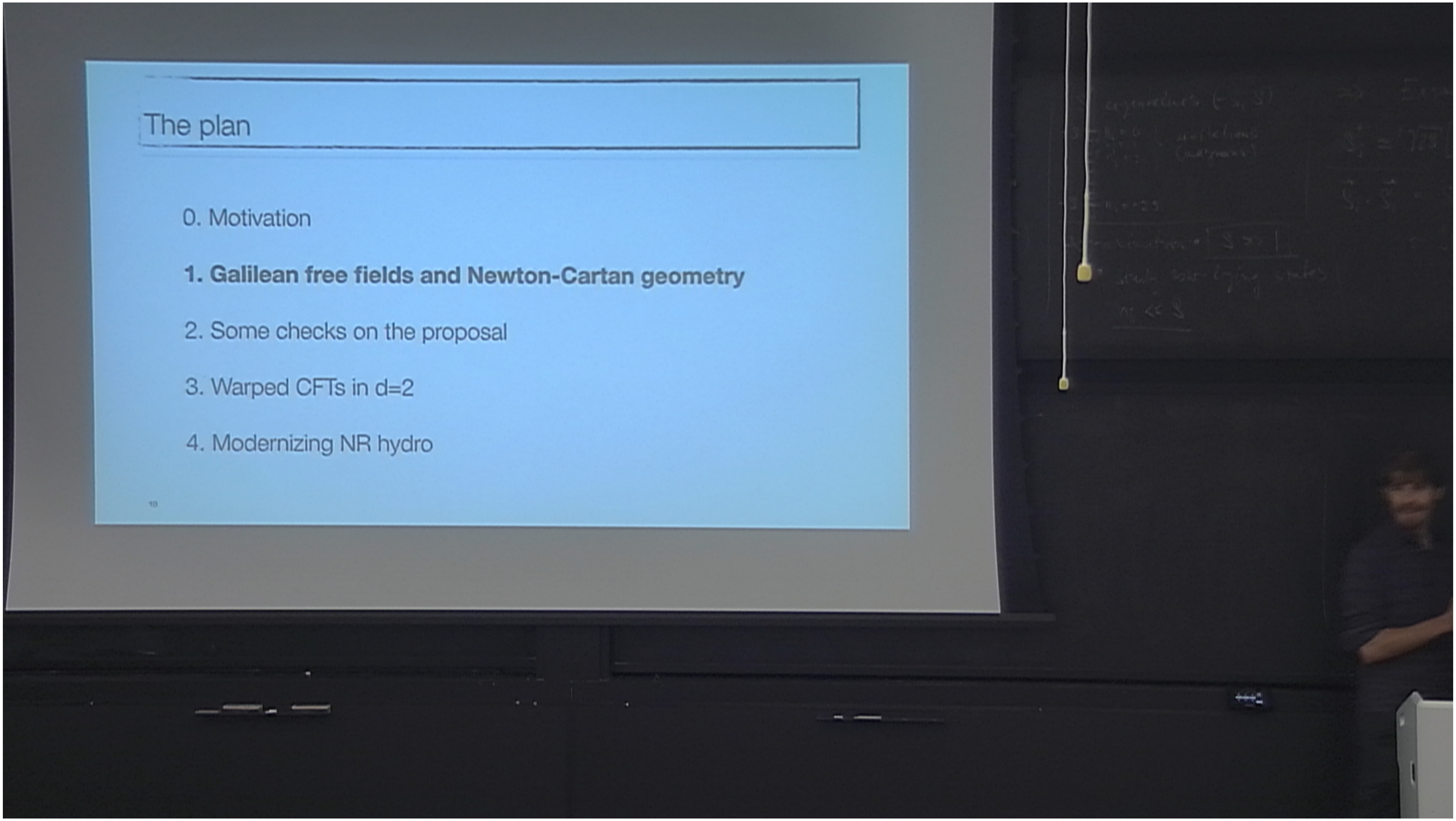

The plan
0. Motivation
1. Galilean free fields and Newton-Cartan geometry
2. Some checks on the proposal
3. Warped CFTs in d=2
4. Modernizing NR hydro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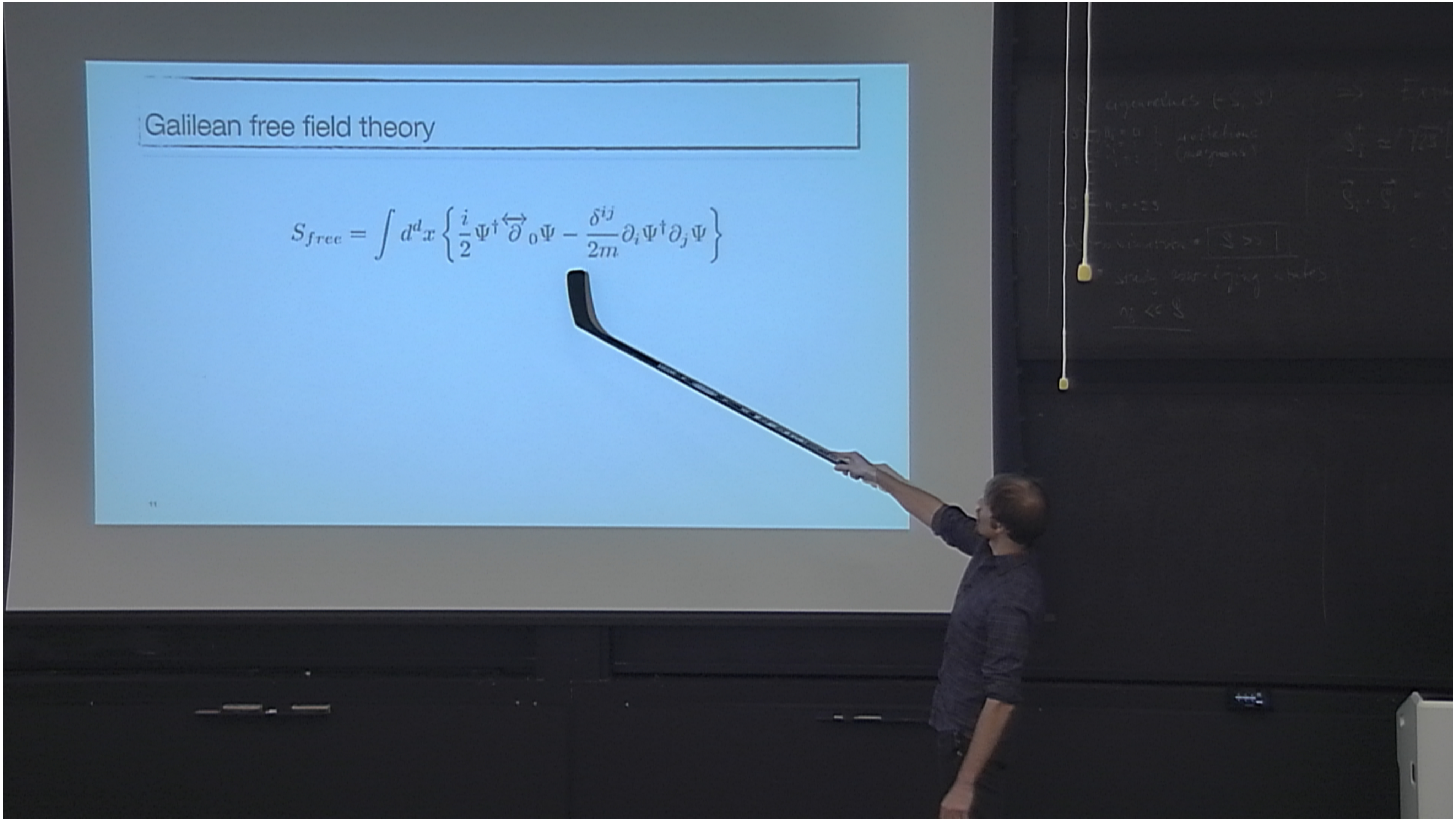
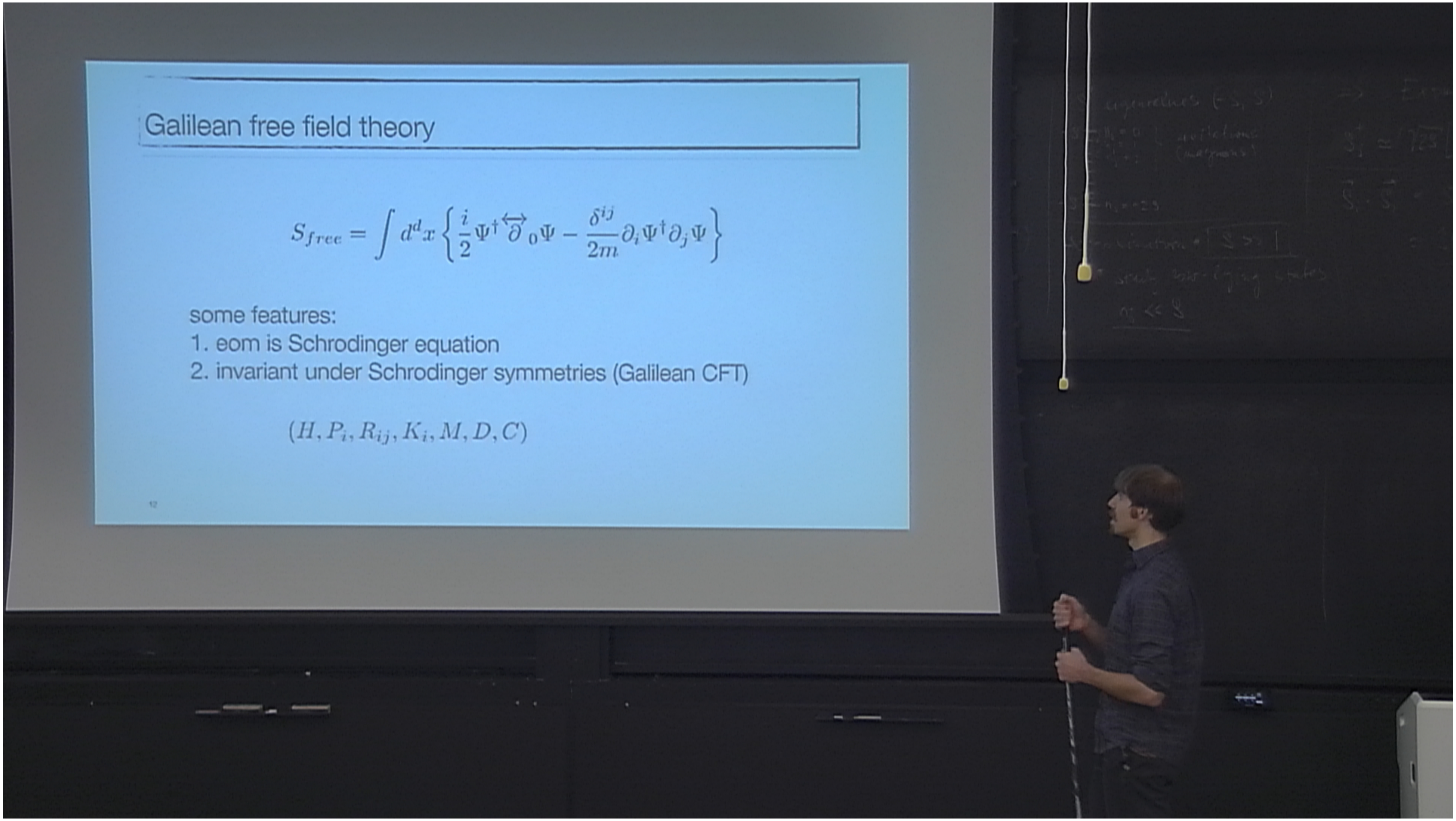

Galilean free field theory
$S_{free} = \int d^d x \left\{ \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{ij}}{2m} \partial_i \Psi^\dagger \partial_j \Psi \right\}$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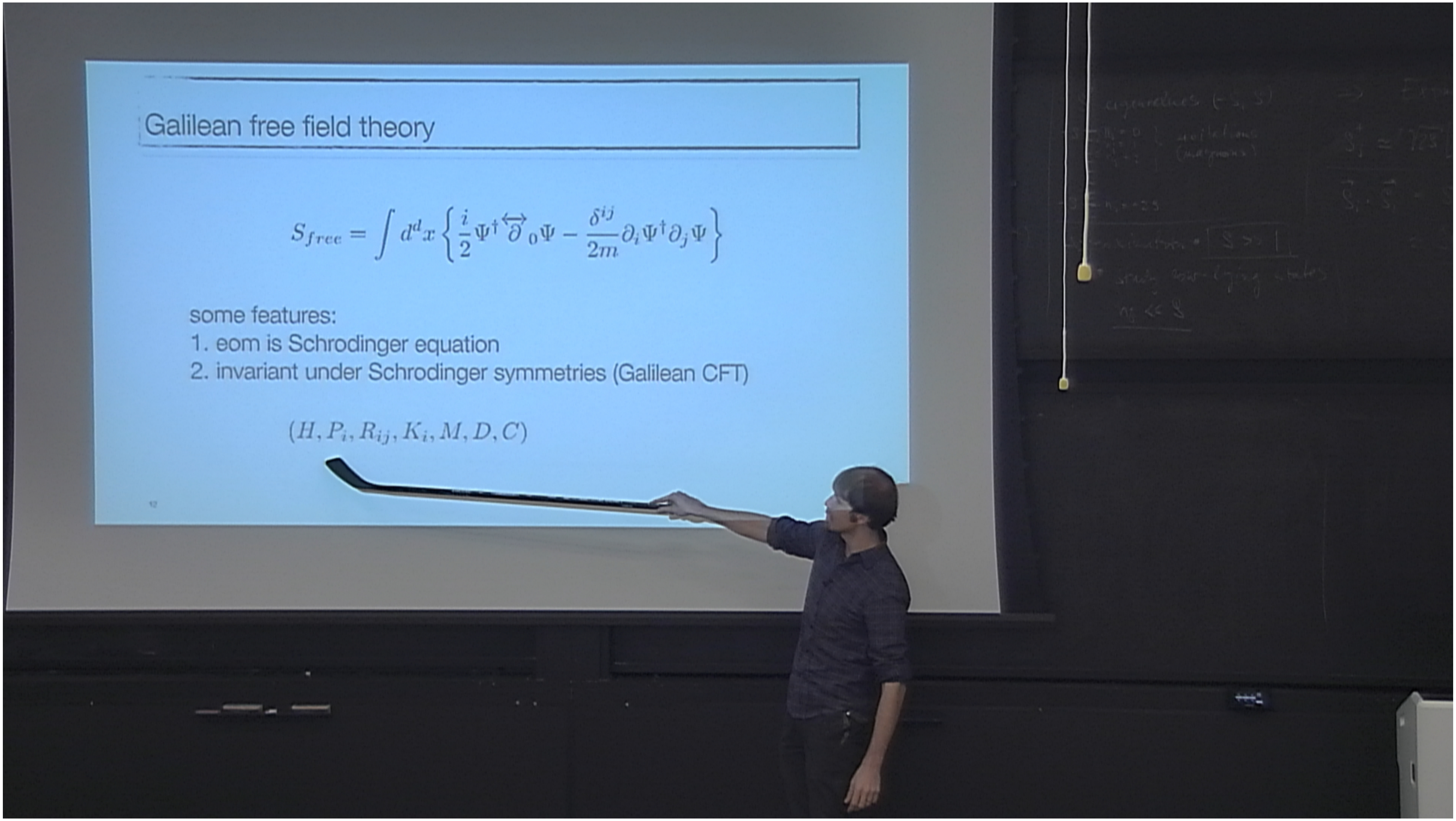
1. eom is Schrodinger equation
2. invariant under Schrodinger symmetries (Galilean CFT)
$(H, P_i, R_{ij}, K_i, M, D, C)$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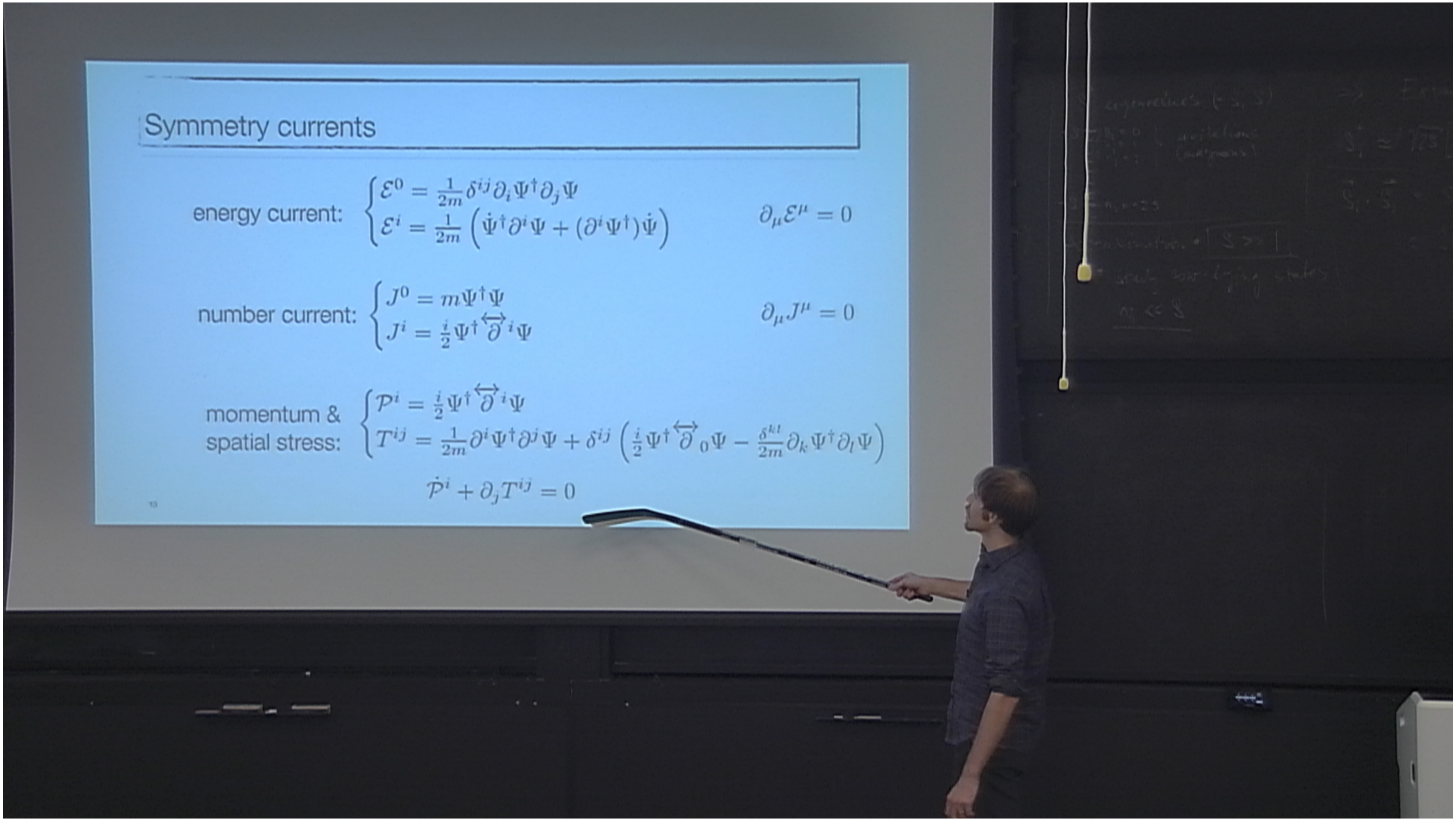
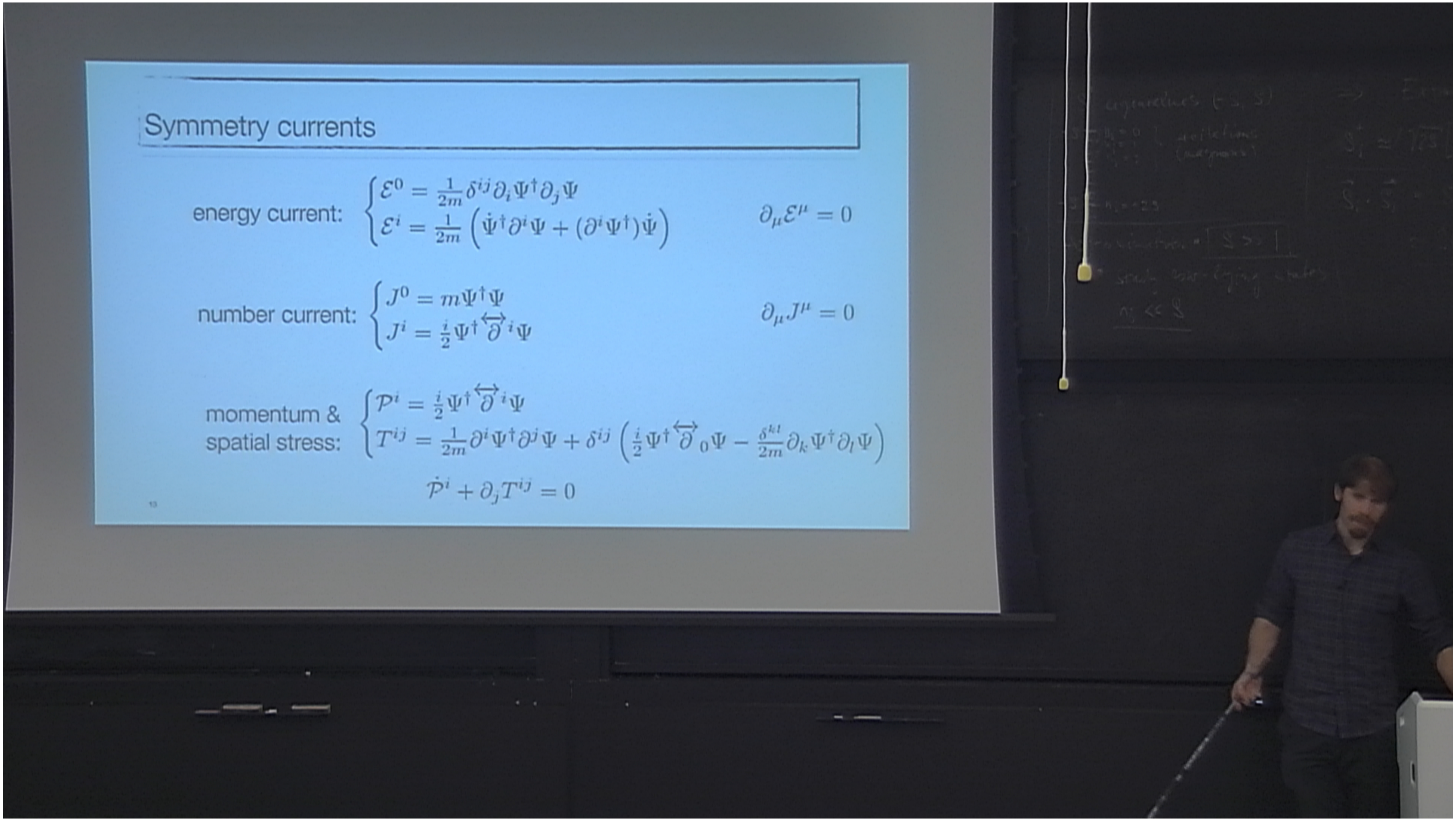

Symmetry currents
energy current:
$\mathcal{E}^0 = \frac{1}{2m}\delta^{ij}\partial_i\Psi^\dagger\partial_j\Psi$
$\mathcal{E}^i = \frac{1}{2m}\left(\dot{\Psi}^\dagger\partial^i\Psi + (\partial^i\Psi^\dagger)\dot{\Psi}\right)$
$\partial_\mu\mathcal{E}^\mu = 0$
number current:
$J^0 = m\Psi^\dagger\Psi$
$J^i = \frac{i}{2}\Psi^\dagger\overleftrightarrow{\partial}^i\Psi$
$\partial_\mu J^\mu = 0$
momentum &
spatial stress:
$\mathcal{P}^i = \frac{i}{2}\Psi^\dagger\overleftrightarrow{\partial}^i\Psi$
$T^{ij} = \frac{1}{2m}\partial^i\Psi^\dagger\partial^j\Psi + \delta^{ij}\left(\frac{i}{2}\Psi^\dagger\overleftrightarrow{\partial}_0\Psi - \frac{\delta^{kl}}{2m}\partial_k\Psi^\dagger\partial_l\Psi\right)$
$\dot{\mathcal{P}}^i + \partial_j T^{ij} = 0$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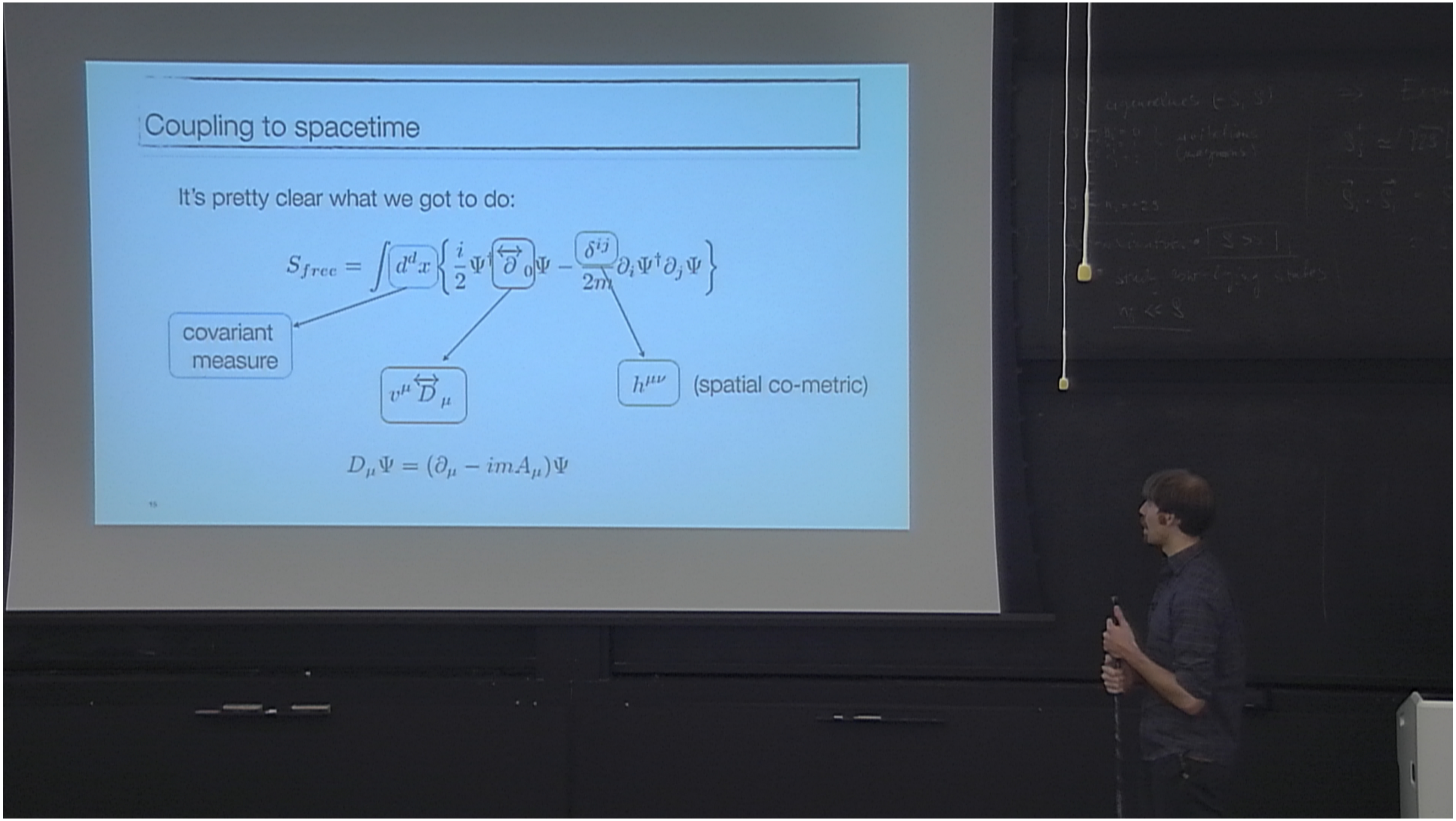
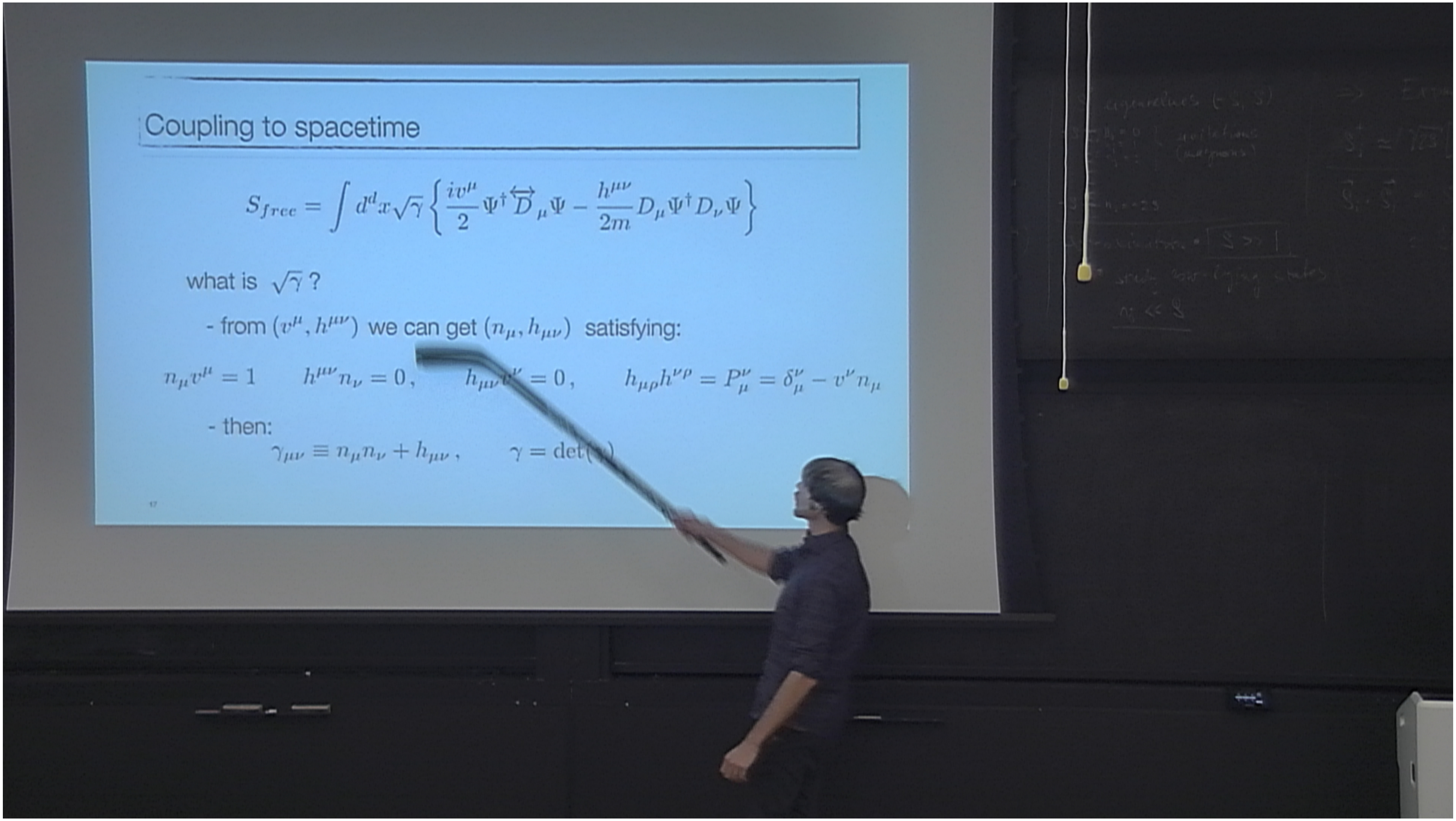
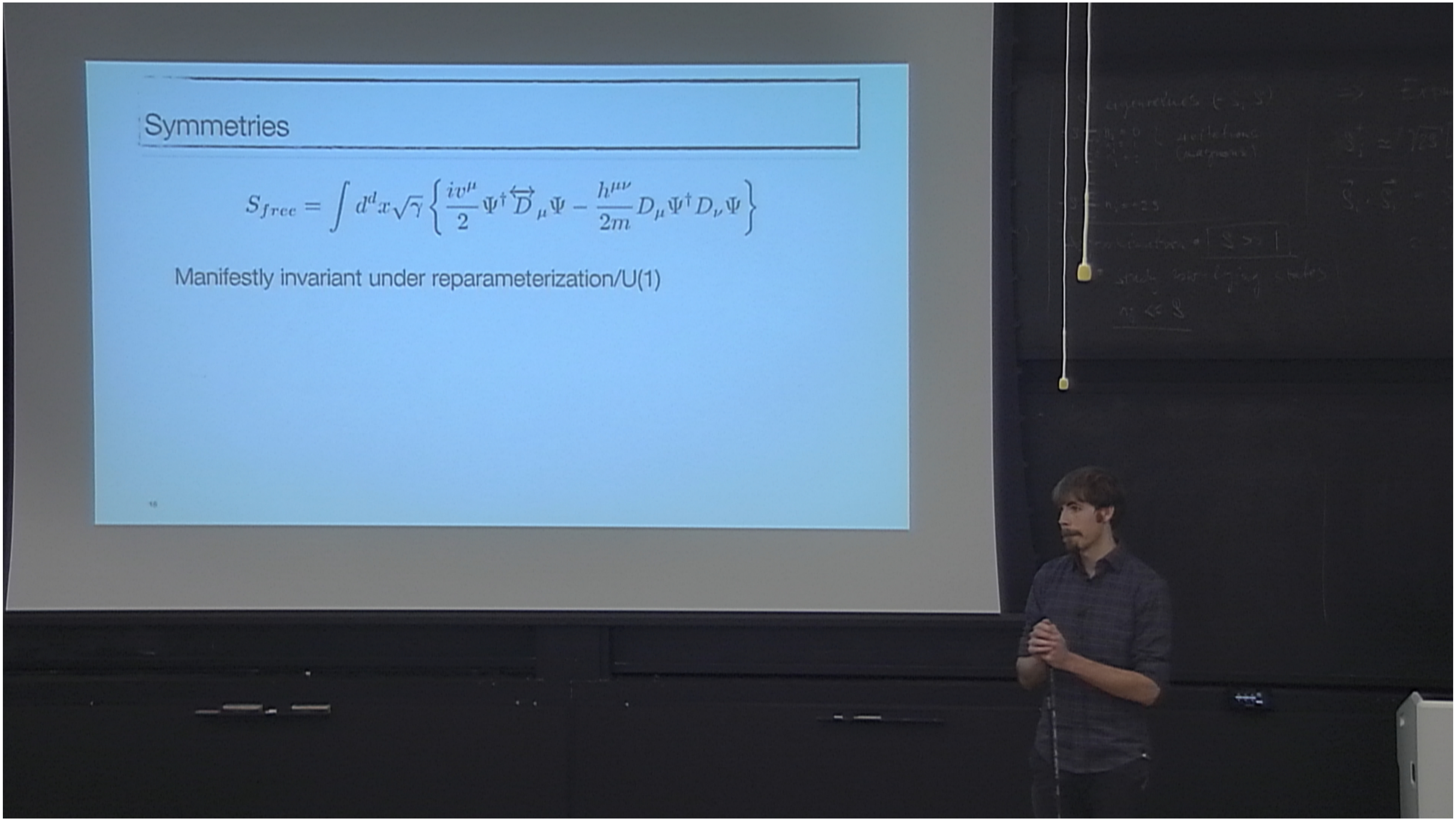

Coupling to spacetime
It's pretty clear what we got to do:
$S_{free} = \int d^d x \left\{ \frac{i}{2} \Psi^\dagger \overleftrightarrow{\partial}_0 \Psi - \frac{\delta^{ij}}{2m} \partial_i \Psi^\dagger \partial_j \Psi \right\}$
covariant measure
$v^\mu \overleftrightarrow{D}_\mu$
$h^{\mu\nu}$ (spatial co-metric)
$D_\mu \Psi = (\partial_\mu - imA_\mu)\Psi$


Coupling to spacetime
$S_{free} = \int d^dx \sqrt{\gamma} \left\{ \frac{iv^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{h^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$
what is $\sqrt{\gamma}$?
- from $(v^\mu, h^{\mu\nu})$ we can get $(n_\mu, h_{\mu\nu})$ satisfying:
$n_\mu v^\mu = 1 \qquad h^{\mu\nu} n_\nu = 0\,, \qquad h_{\mu\nu} v^\nu = 0\,, \qquad h_{\mu\rho} h^{\nu\rho} = P^\nu_\mu = \delta^\nu_\mu - v^\nu n_\mu$
- then:
$\gamma_{\mu\nu} \equiv n_\mu n_\nu + h_{\mu\nu}\,, \qquad \gamma = \det(\gamma)$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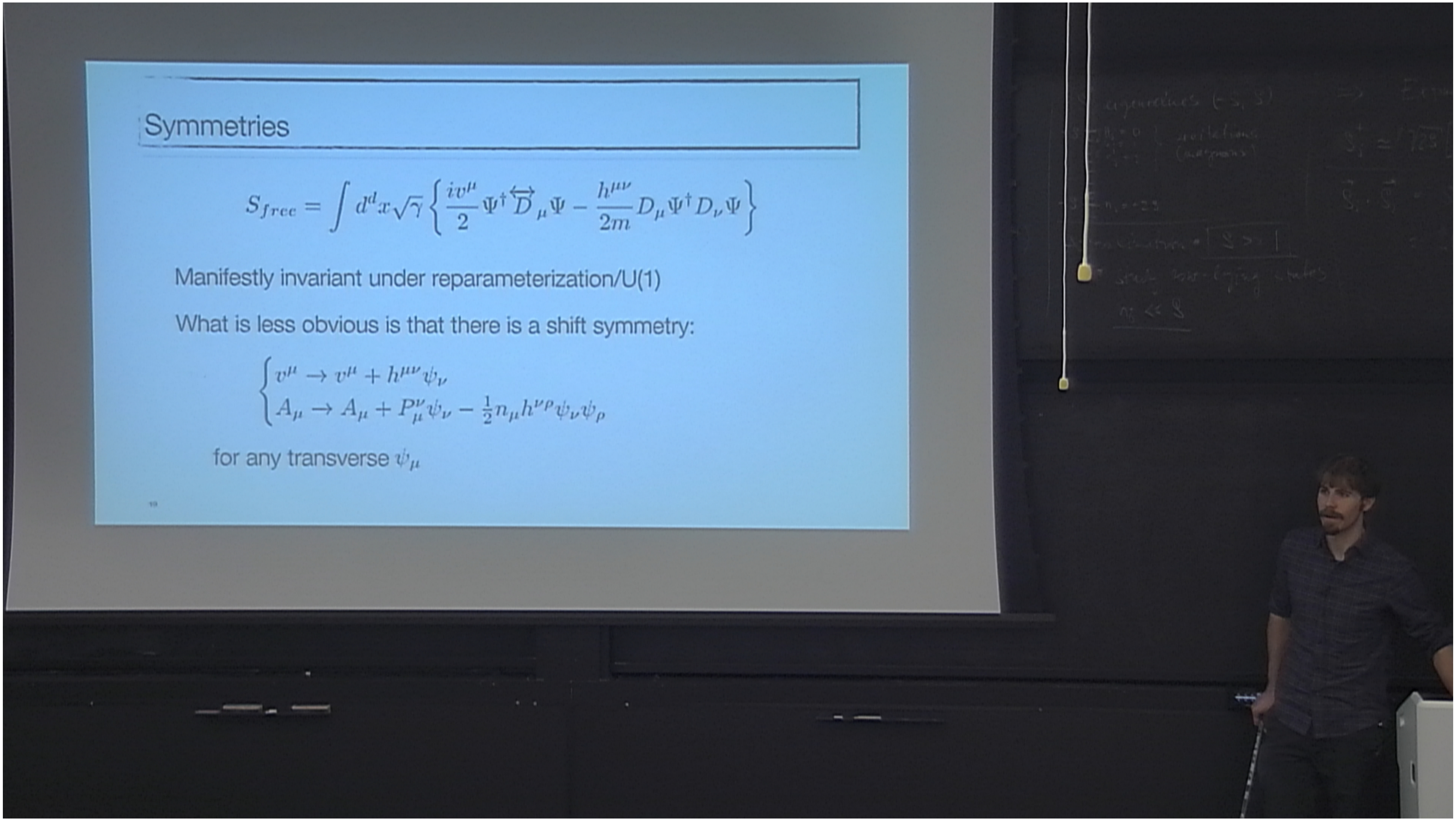
$$S_{free} = \int d^dx\sqrt{\gamma}\left\{\frac{iv^\mu}{2}\Psi^\dagger \overleftrightarrow{D}_\mu\Psi - \frac{h^{\mu\nu}}{2m}D_\mu\Psi^\dagger D_\nu\Psi\right\}$$
Manifestly invariant under reparameterization/U(1)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Manifestly invariant under reparameterization/U(1)
What is less obvious is that there is a shift symmetry:
$v^\mu \to v^\mu + h^{\mu\nu}\psi_\nu$
$A_\mu \to A_\mu + P^\nu_\mu \psi_\nu - \frac{1}{2} n_\mu h^{\nu\rho}\psi_\nu\psi_\rho$
for any transverse ψ_μ

Symmetries

$$S_{free} = \int d^d x \sqrt{\gamma} \left\{ \frac{i v^\mu}{2} \Psi^\dagger \overleftrightarrow{D}_\mu \Psi - \frac{h^{\mu\nu}}{2m} D_\mu \Psi^\dagger D_\nu \Psi \right\}$$

Manifestly invariant under reparameterization/U(1)

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$$\begin{cases} v^\mu \to v^\mu + h^{\mu\nu}\psi_\nu \\ A_\mu \to A_\mu + P^\nu_\mu \psi_\nu - \frac{1}{2} n_\mu h^{\nu\rho}\psi_\nu \psi_\rho \end{cases}$$

for any transverse ψ_μ

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The proposal

Can now state a general proposal for coupling to spacetime:

1. the background fields are $(v^\mu, h^{\mu\nu}, A_\mu)$, or equivalently $(n_\mu, h_{\mu\nu}, A_\mu)$

2. we ought to impose invariance under
   - reparameterizations
   - U(1)
   - the shift symmetry (Milne boosts)

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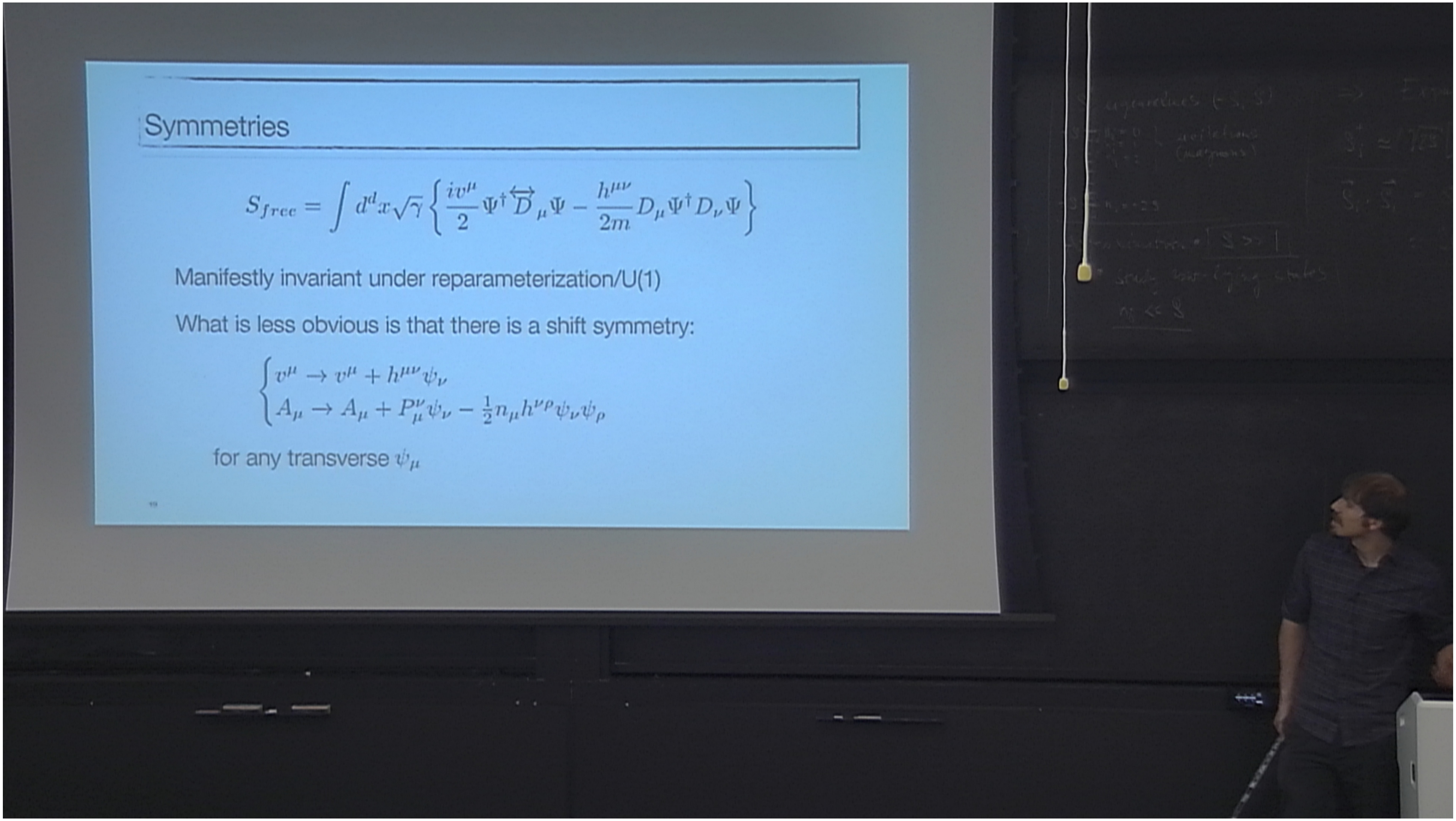
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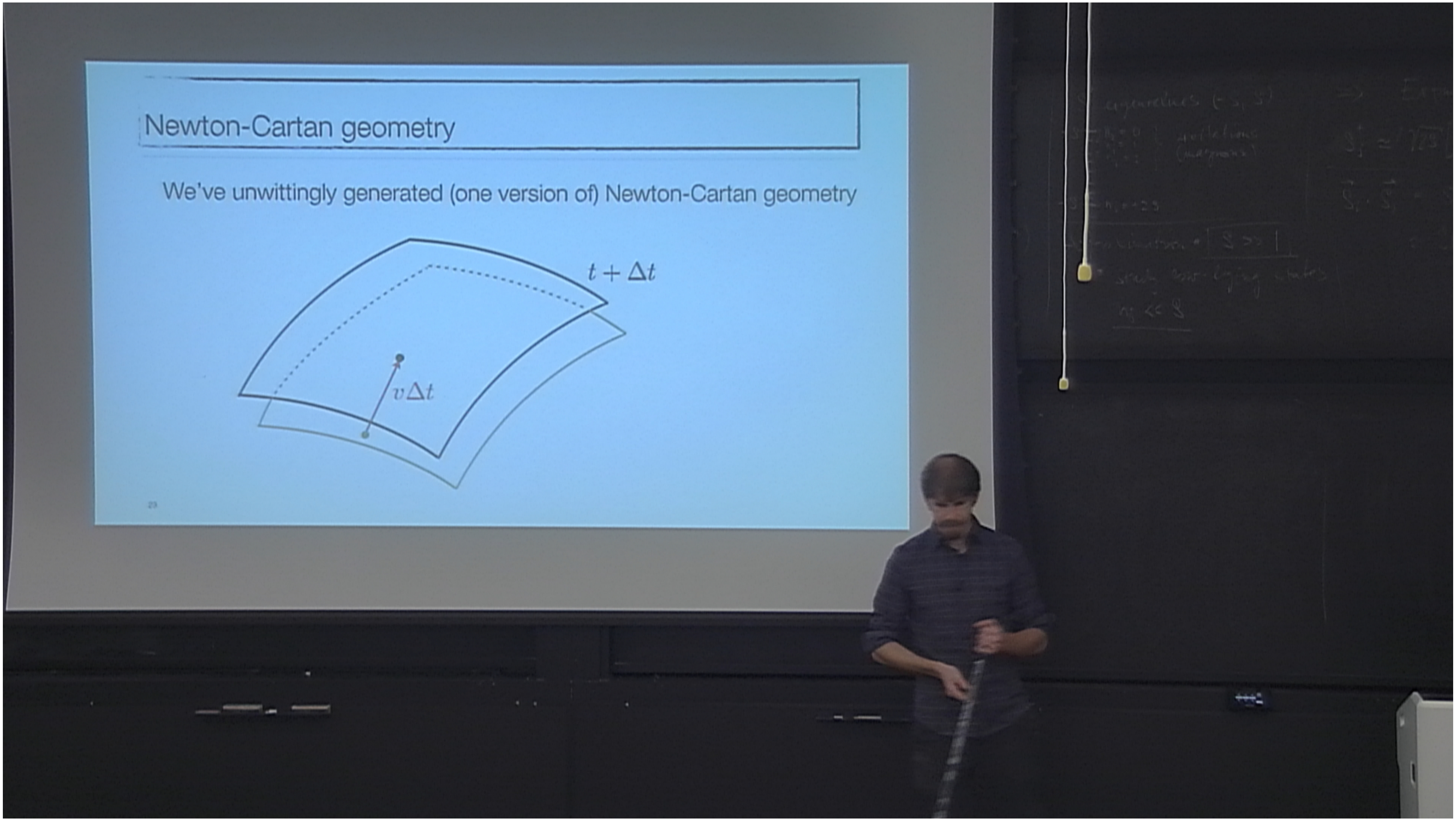
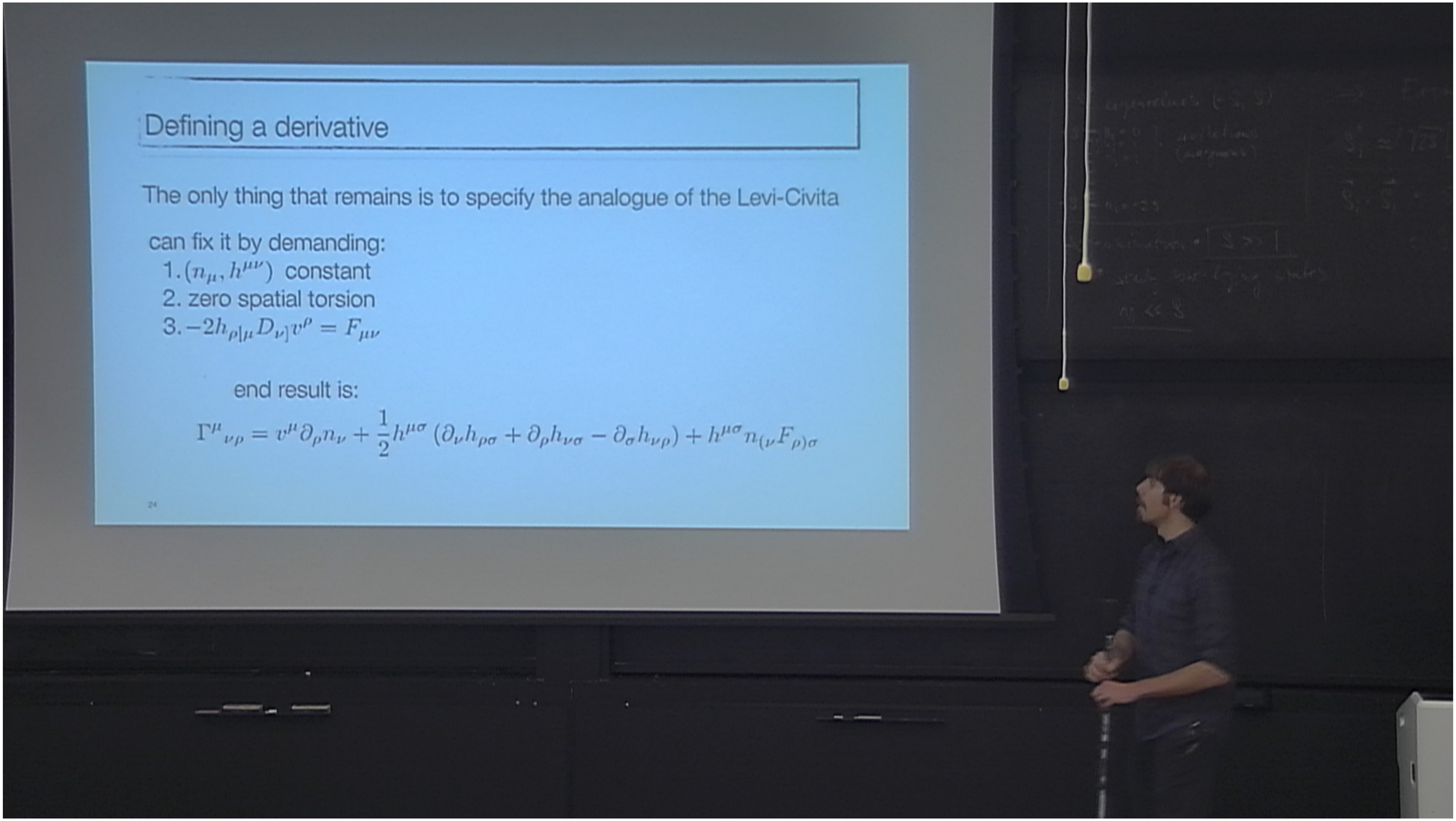
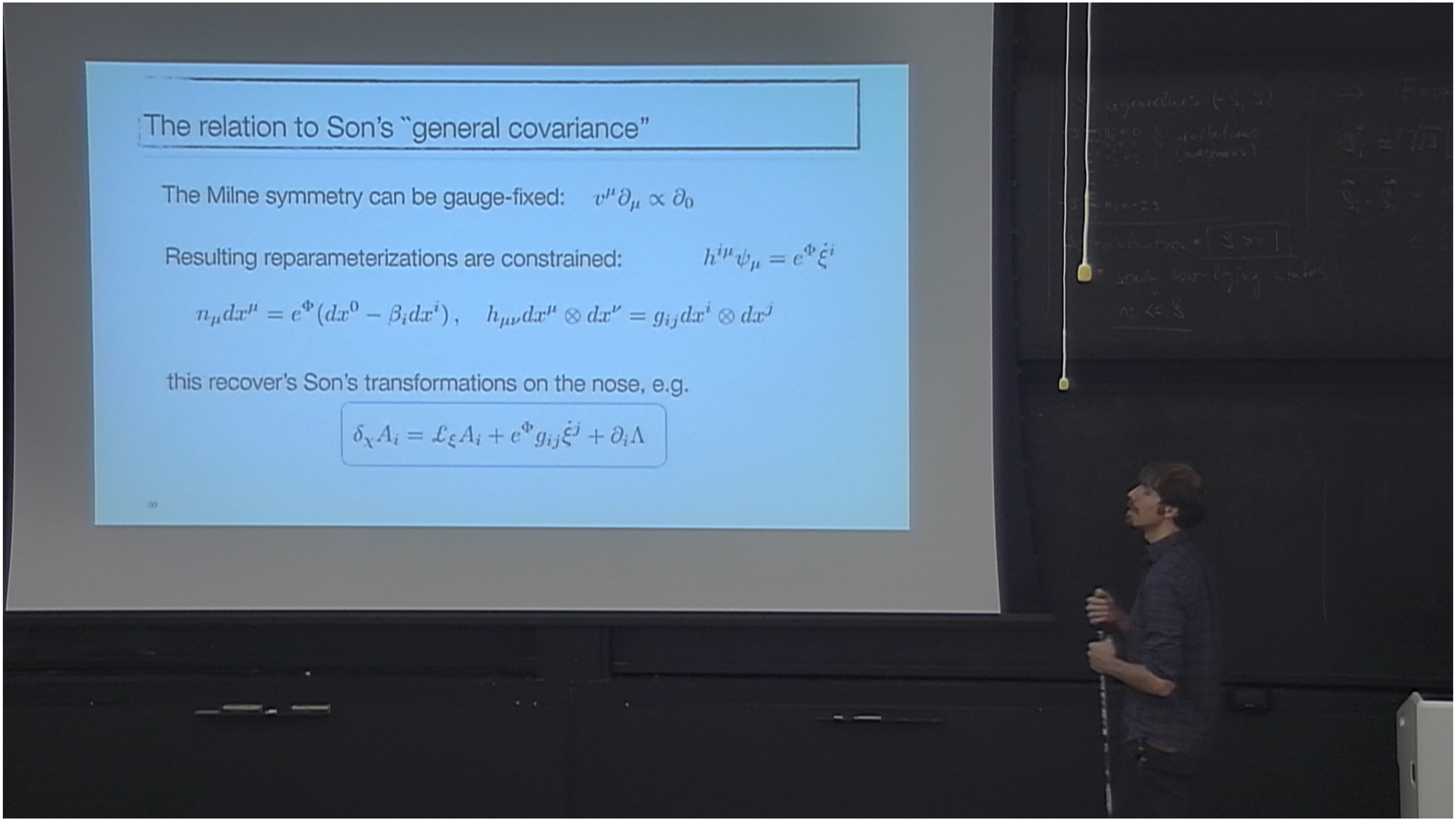
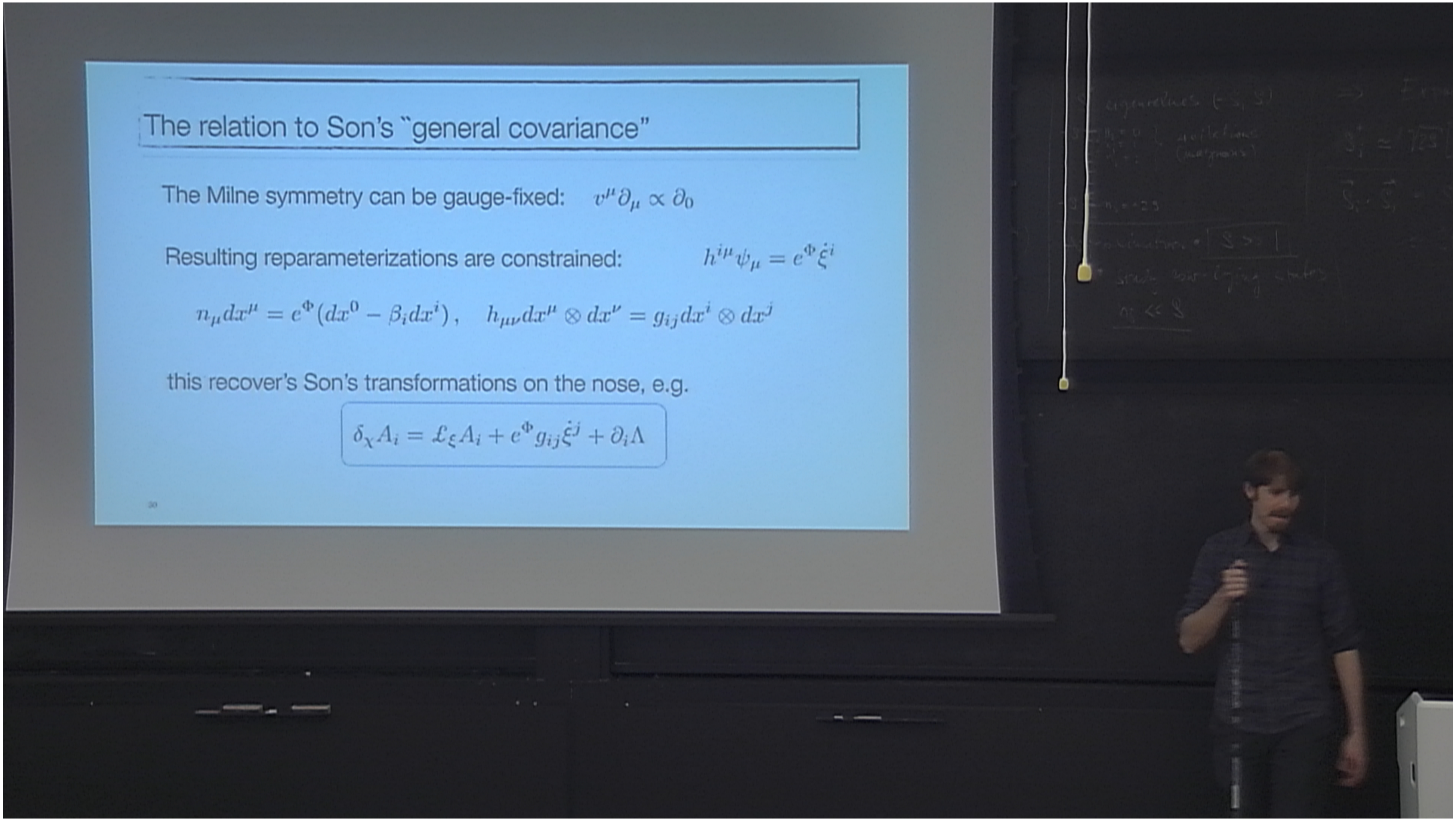

Newton-Cartan geometry
We've unwittingly generated (one version of) Newton-Cartan geometry
t + Δt
vΔt
29


Defining a derivative
The only thing that remains is to specify the analogue of the Levi-Civita
can fix it by demanding:
1. $(n_\mu, h^{\mu\nu})$ constant
2. zero spatial torsion
3. $-2h_{\rho[\mu}D_{\nu]}v^\rho = F_{\mu\nu}$
end result is:
$\Gamma^\mu{}_{\nu\rho} = v^\mu \partial_\rho n_\nu + \frac{1}{2}h^{\mu\sigma}\left(\partial_\nu h_{\rho\sigma} + \partial_\rho h_{\nu\sigma} - \partial_\sigma h_{\nu\rho}\right) + h^{\mu\sigma} n_{(\nu}F_{\rho)\sigma}$
24


The relation to Son's "general covariance"
The Milne symmetry can be gauge-fixed: $v^\mu \partial_\mu \propto \partial_0$
Resulting reparameterizations are constrained: $h^{i\mu}\psi_\mu = e^\Phi \dot{\xi}^i$
$n_\mu dx^\mu = e^\Phi (dx^0 - \beta_i dx^i)\,, \quad h_{\mu\nu} dx^\mu \otimes dx^\nu = g_{ij} dx^i \otimes dx^j$
this recover's Son's transformations on the nose, e.g.
$\delta_\chi A_i = \pounds_\xi A_i + e^\Phi g_{ij} \dot{\xi}^j + \partial_i \Lambda$
30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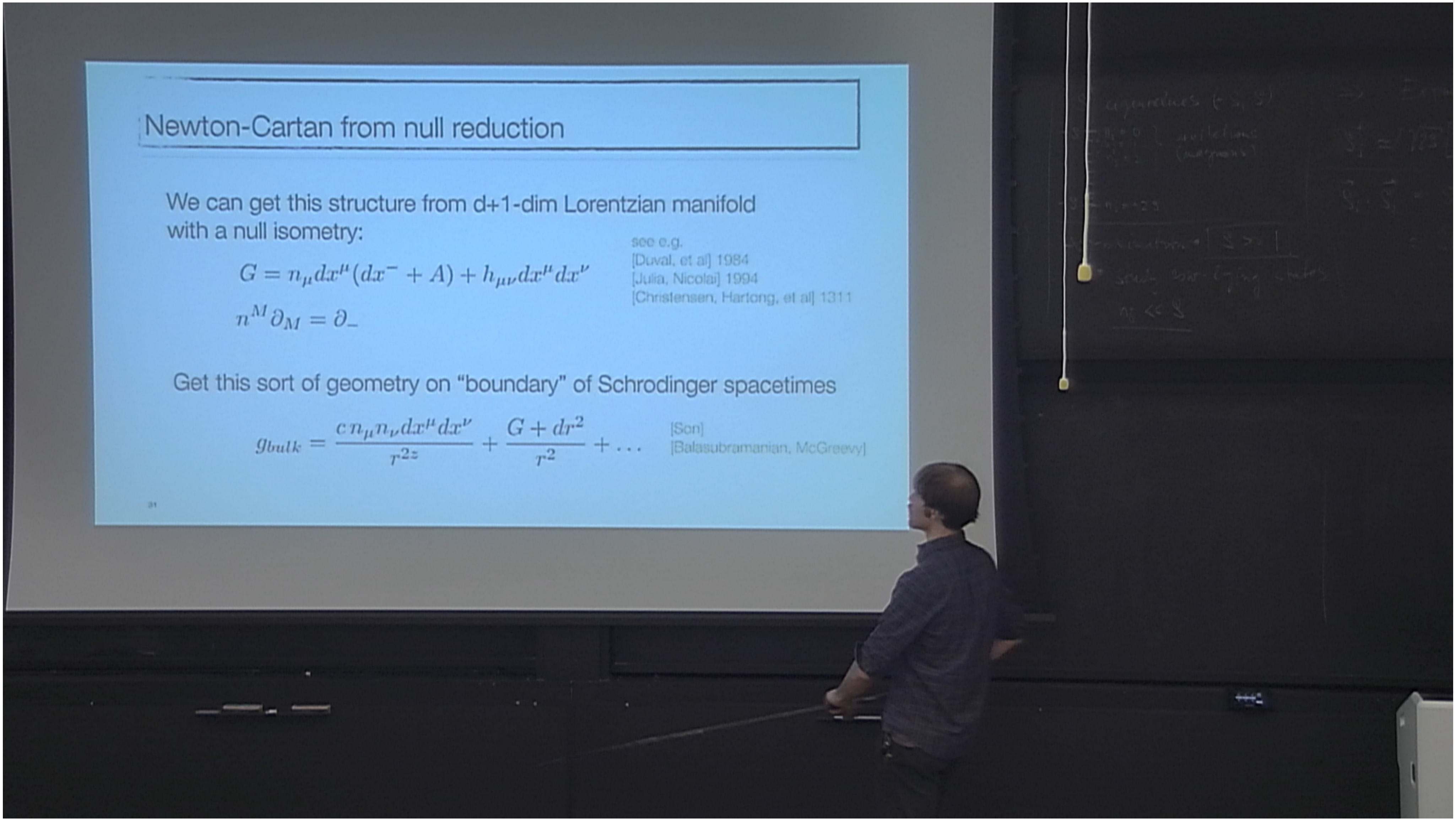
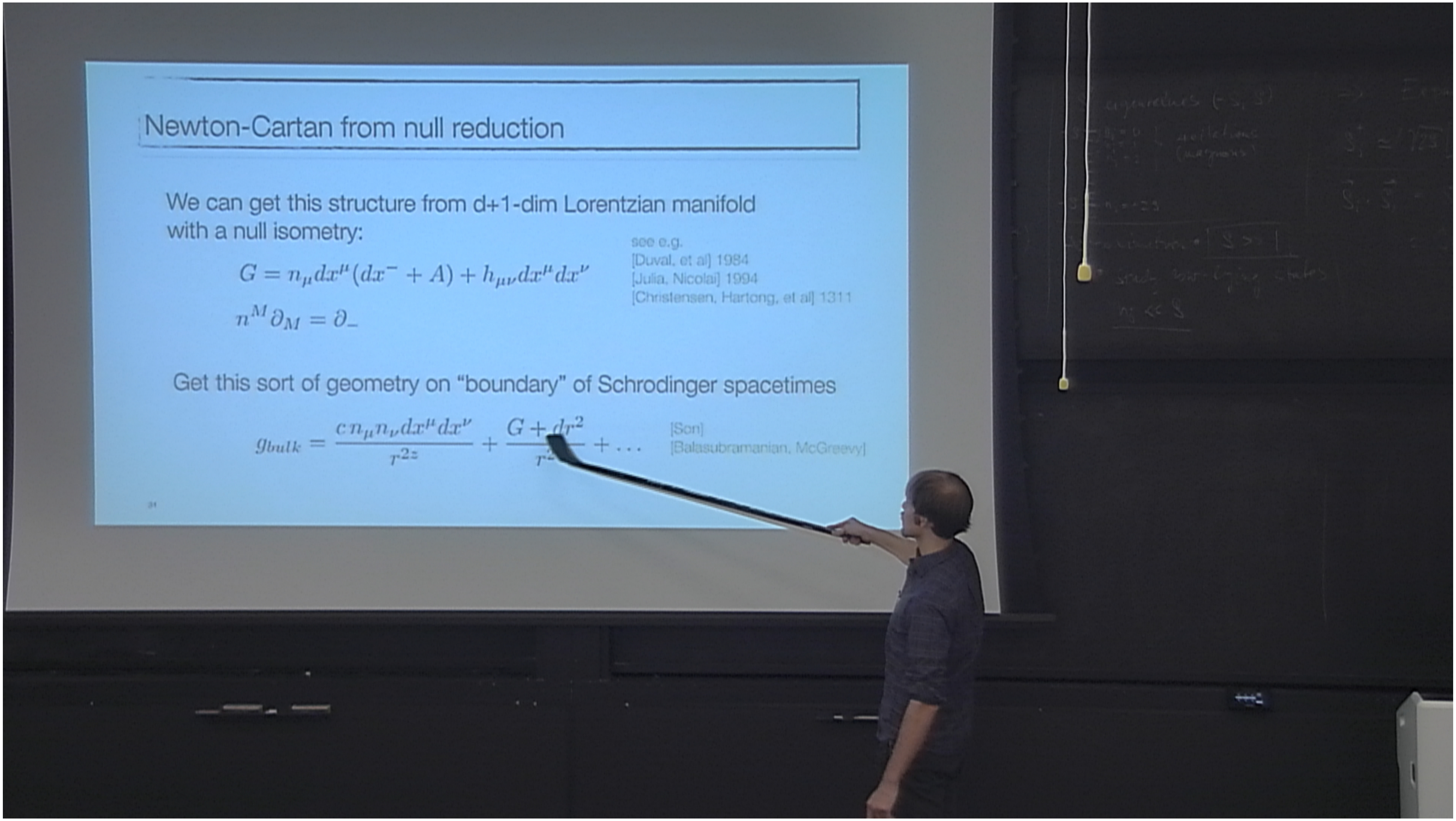

Newton-Cartan from null reduction
We can get this structure from d+1-dim Lorentzian manifold with a null isometry:
$G = n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$
$n^M \partial_M = \partial_-$
see e.g.
[Duval, et al] 1984
[Julia, Nicolai] 1994
[Christensen, Hartong, et al] 1311
Get this sort of geometry on "boundary" of Schrodinger spacetimes
$g_{bulk} = \frac{c\, n_\mu n_\nu dx^\mu dx^\nu}{r^{2z}} + \frac{G + dr^2}{r^2} + \dots$
[Son]
[Balasubramanian, McGreevy]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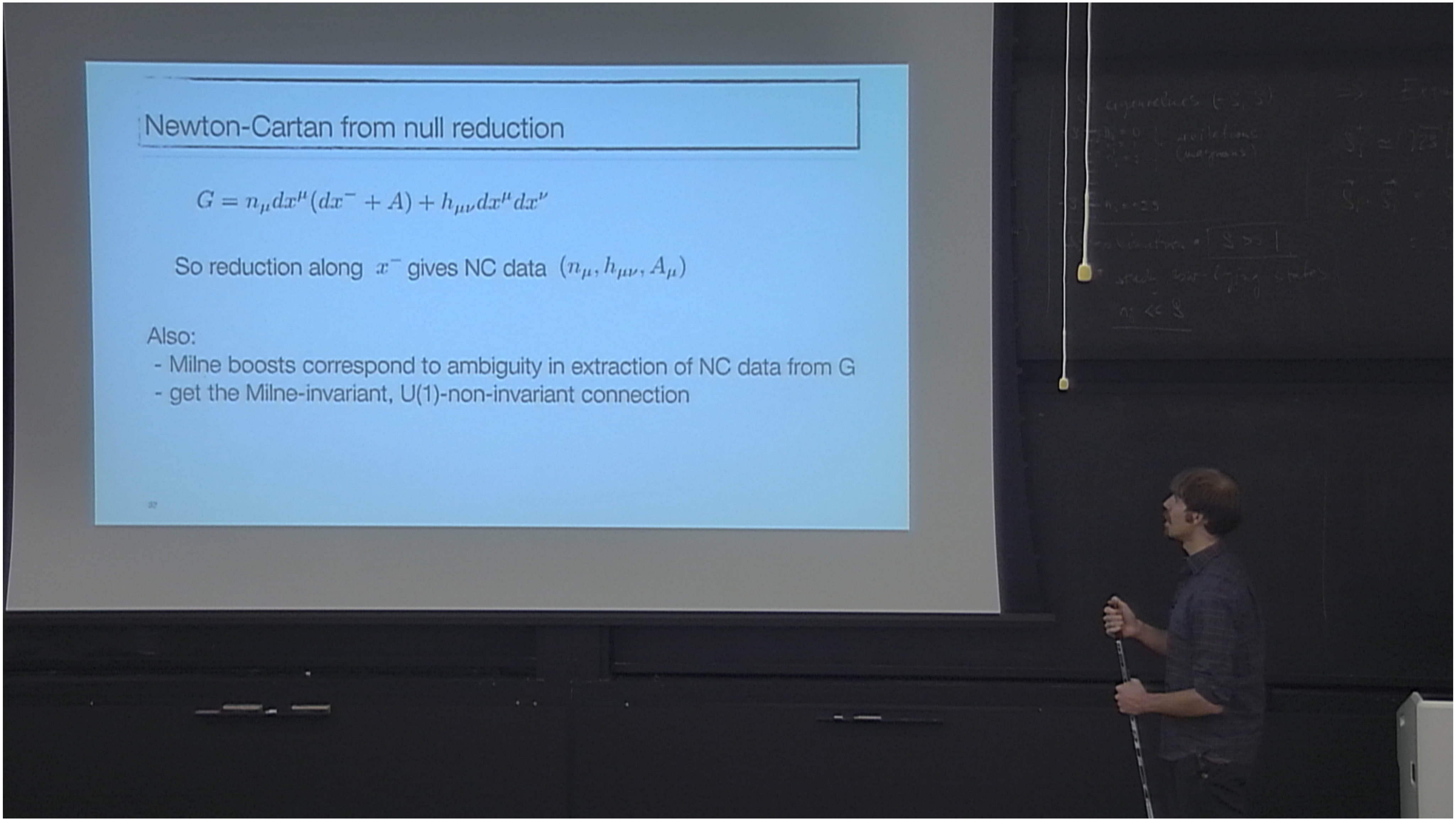
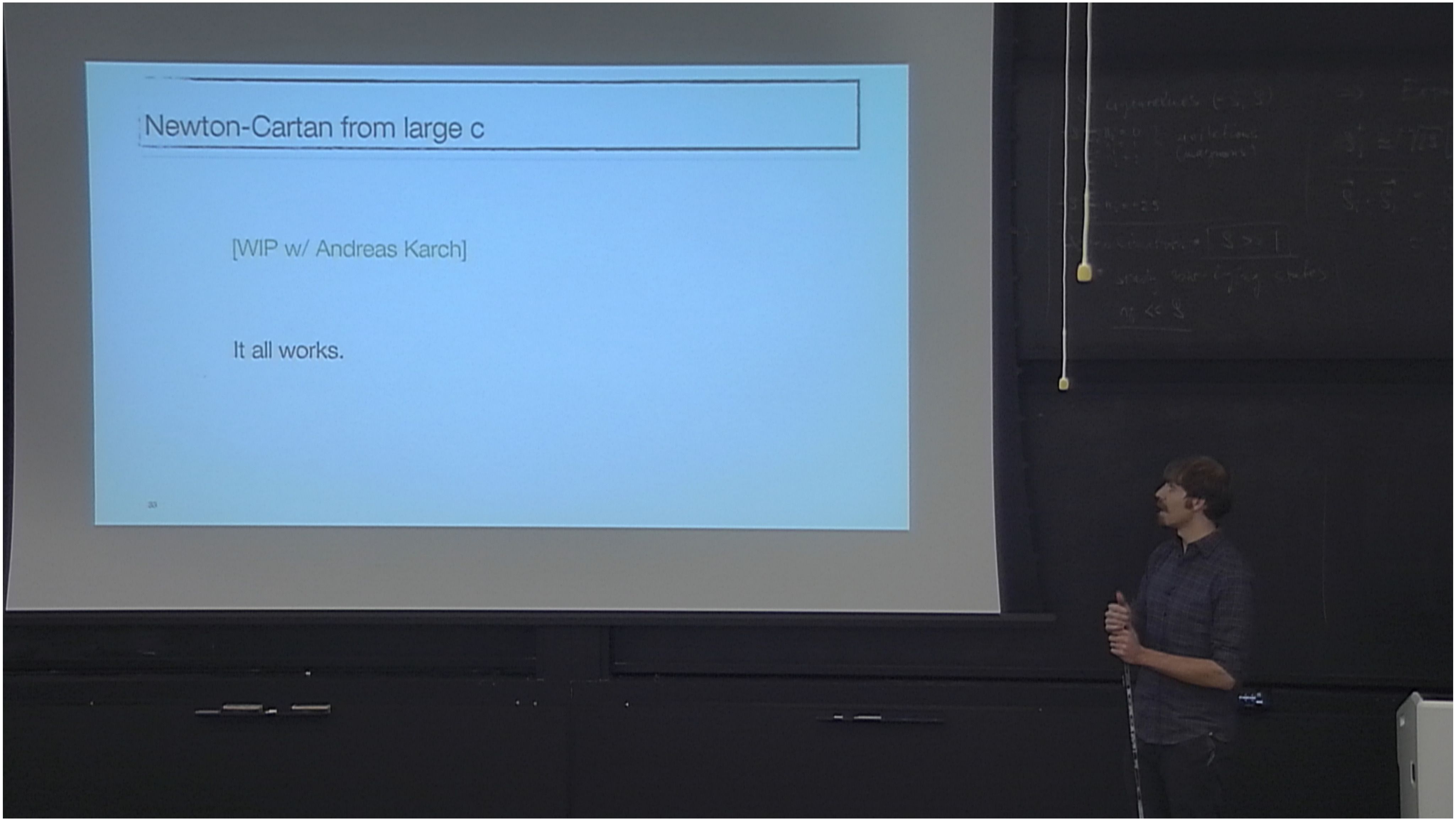
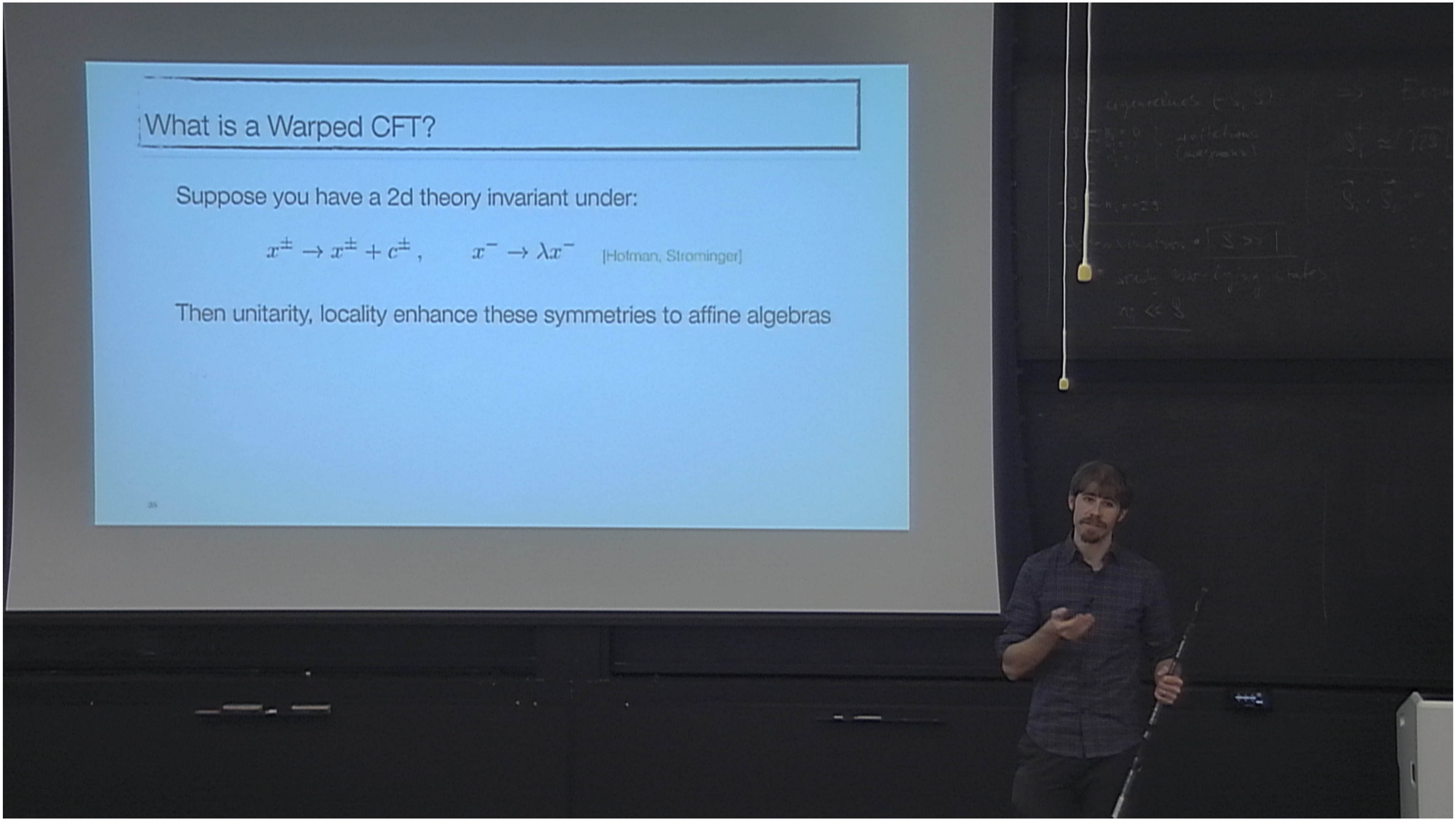
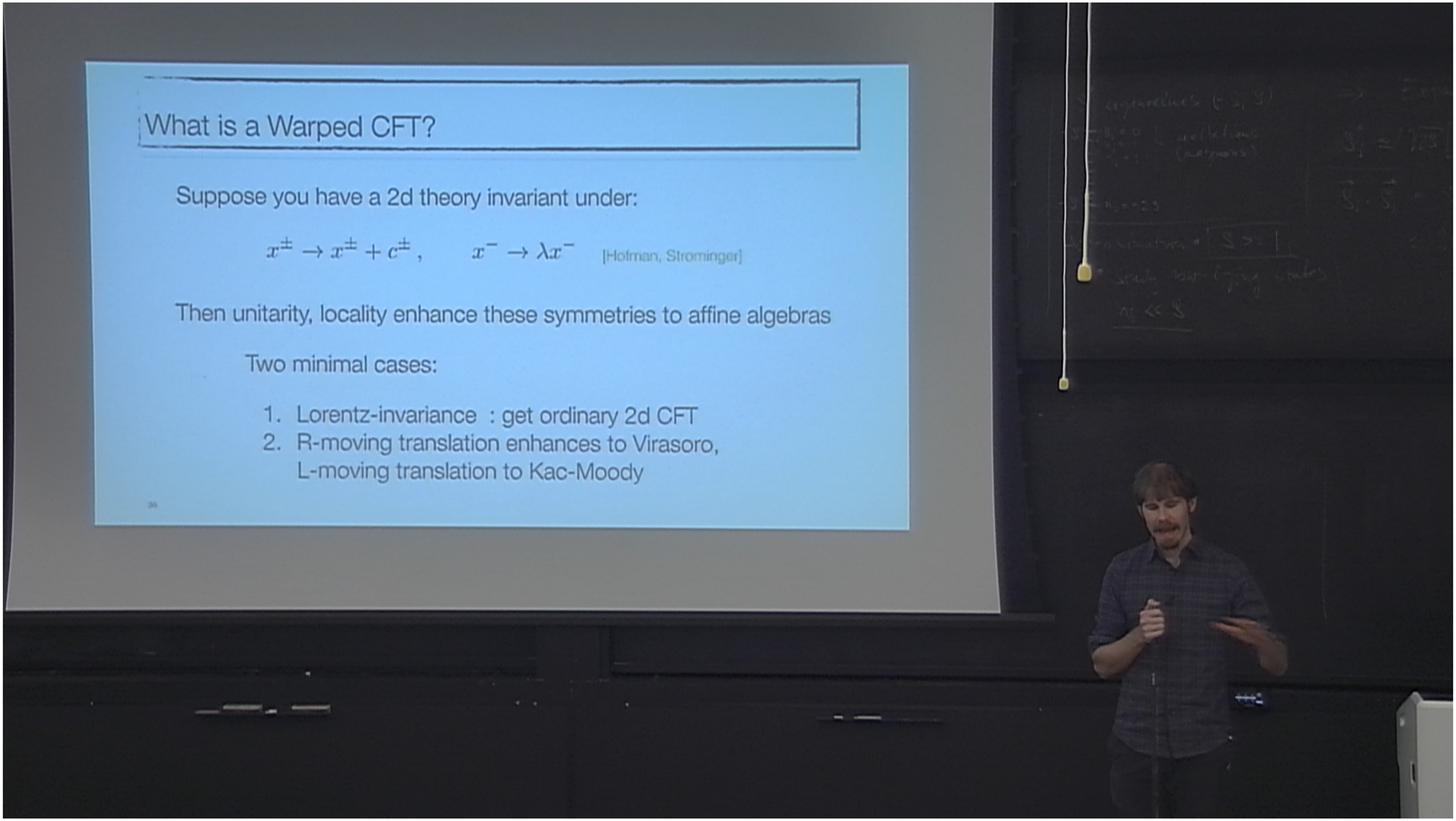

Newton-Cartan from null reduction
$G = n_\mu dx^\mu (dx^- + A) + h_{\mu\nu} dx^\mu dx^\nu$
So reduction along x^- gives NC data $(n_\mu, h_{\mu\nu}, A_\mu)$
Also:
- Milne boosts correspond to ambiguity in extraction of NC data from G
- get the Milne-invariant, U(1)-non-invariant connection


Newton-Cartan from large c
[WIP w/ Andreas Karch]
It all works.


What is a Warped CFT?
Suppose you have a 2d theory invariant under:
$x^\pm \to x^\pm + c^\pm\,, \qquad x^- \to \lambda x^-$
[Hofman, Strominger]
Then unitarity, locality enhance these symmetries to affine algebras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[Hofman, Strominger]
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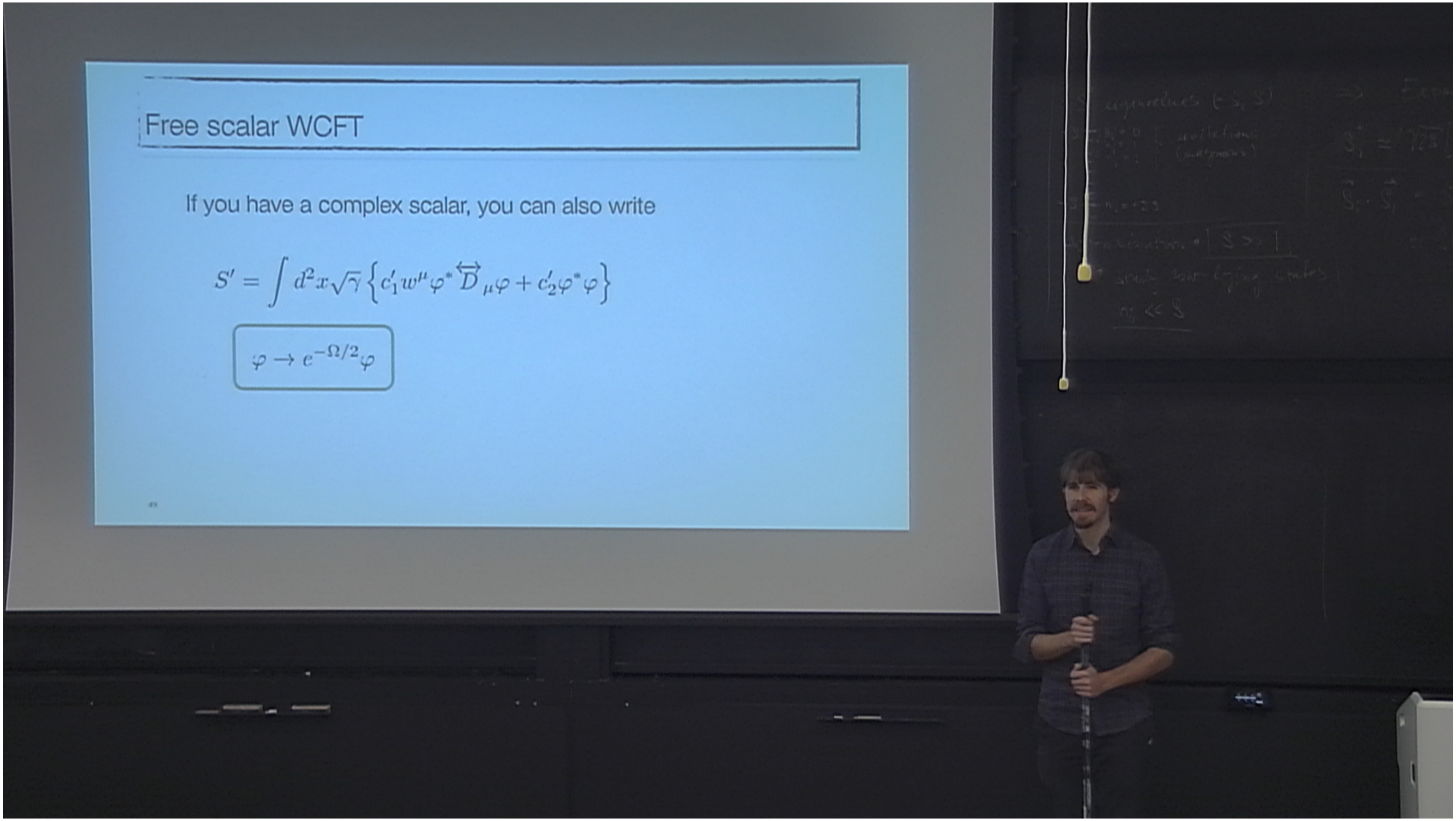
Two minimal cases:
1. Lorentz-invariance : get ordinary 2d CFT
2. R-moving translation enhances to Virasoro, L-moving translation to Kac-Moody


Free scalar WCFT
If you have a complex scalar, you can also write
$S' = \int d^2x \sqrt{\gamma} \left\{ c_1' w^\mu \varphi^* \overleftrightarrow{D}_\mu \varphi + c_2' \varphi^* \varphi \right\}$
$\varphi \to e^{-\Omega/2} \varphi$