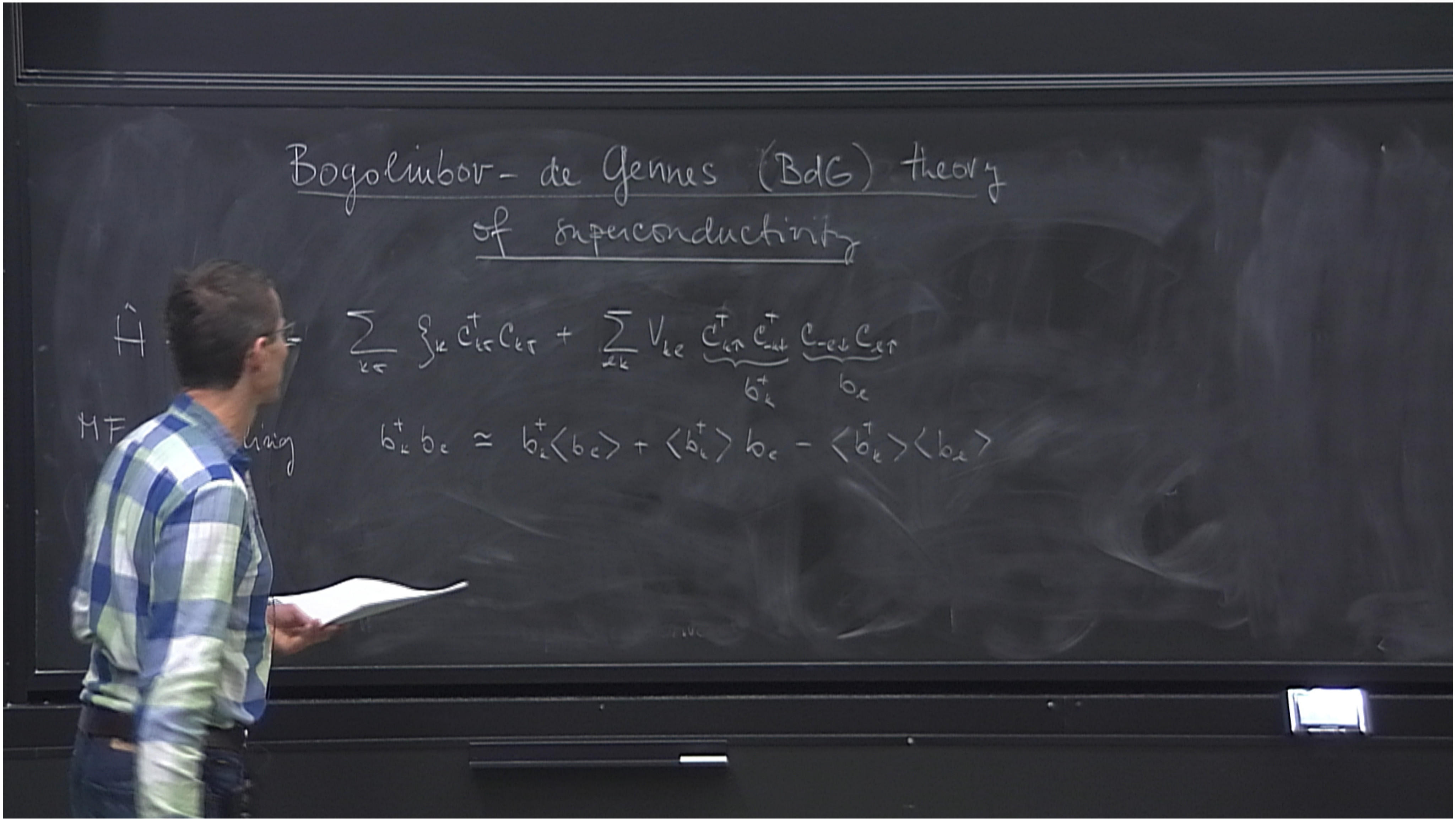
Title: Condensed Matter-15

Date: Nov 28, 2014 10:30 AM

URL: http://pirsa.org/14110043

Abstract:


Bogoliubov - de Gennes (BdG) theory
of superconductivity
$\hat{H} = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{kk'} V_{kk'} \underbrace{c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}}_{b^\dagger_k} \underbrace{c_{-k'\downarrow} c_{k'\uparrow}}_{b_{k'}}$
MF
$b^\dagger_k b_{k'} \simeq b^\dagger_k \langle b_{k'} \rangle + \langle b^\dagger_k \rangle b_{k'} - \langle b^\dagger_k \rangle \langle b_{k'} \rangle$

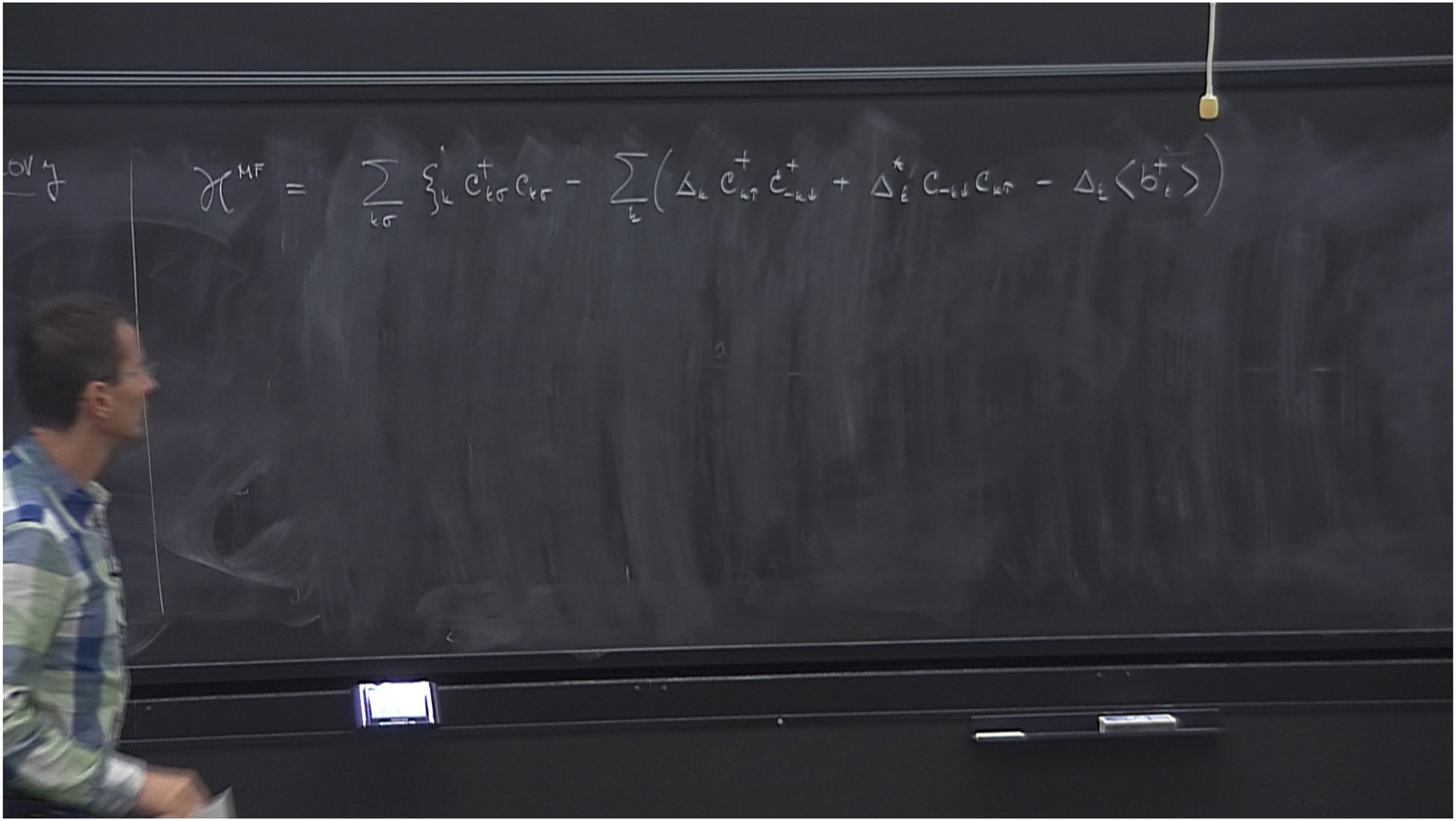
Bogoliubov - de Gennes (BdG) theory of superconductivity

$$\hat{H} - \mu \hat{N} = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} + \sum_{kl} V_{kl} \underbrace{c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow}}_{b^\dagger_k} \underbrace{c_{-l\downarrow} c_{l\uparrow}}_{b_l}$$

MF decoupling $\quad b^\dagger_k b_l \simeq b^\dagger_k \langle b_l \rangle + \langle b^\dagger_k \rangle b_l - \langle b^\dagger_k \rangle \langle b_l \rangle$

Define $\quad \Delta_k = -\sum_l V_{kl} \langle b_l \rangle = -\sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle$

$$\mathcal{H}^{MF} = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma}$$


$\mathcal{H}^{MF} = \sum_{k\sigma} \xi_k c^{+}_{k\sigma} c_{k\sigma} - \sum_{k} \left(\Delta_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow} + \Delta^{*}_{k} c_{-k\downarrow} c_{k\uparrow} - \Delta_k \langle b^{+}_{k} \rangle \right)$

$$\mathcal{H}^{MF} = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta_k^* c_{-k\downarrow} c_{k\uparrow} - \Delta_k \langle b_k^\dagger \rangle \right)$$

Diagonalize by Bogolubov-Valatin transformation

$$c_{k\uparrow} = u_k^* \gamma_{k\uparrow} + v_k \gamma^\dagger_{k\downarrow}$$

$$c^\dagger_{-k\downarrow} = - v^* \gamma_{k\downarrow} + u_k \gamma^\dagger_{k\uparrow}$$

$$\{ \gamma_{k\sigma}, \gamma^\dagger_{k'\sigma'} \} = \delta_{kk'} \delta_{\sigma\sigma'}$$

$$\Rightarrow |u_k|^2 + |v_k|^2 = 1$$

$$\mathcal{H}^{MF} = \sum_{k\sigma} \xi_k c^\dagger_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^\dagger_{k\uparrow} c^\dagger_{-k\downarrow} + \Delta^*_k c_{-k\downarrow} c_{k\uparrow} - \Delta_k \langle b^\dagger_k \rangle \right)$$

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$$\Rightarrow |u_k|^2 + |v_k|^2 = 1$$

$$H = \sum_{k\sigma} \xi_k \Big[(|u_k|^2 - |v_k|^2) \gamma^\dagger_{k\sigma} \gamma_{k\sigma} + 2|v_k|^2 + 2 (u_k v_k \gamma^\dagger_{k\uparrow} \gamma^\dagger_{k\downarrow} + h.c.) \Big]$$

$$+ \sum_{k\sigma} (\Delta_k u_k v^*_k + \Delta^*_k u^*_k v_k)(\gamma^\dagger_{k\sigma} \gamma_{k\sigma} + \tfrac{1}{2})$$

$$+ \sum_{k,\sigma} \Big[(\Delta^*_k v_k^2 - \Delta_k u_k^2) \gamma^\dagger_{k\uparrow} \gamma_{k\downarrow} + h.c. + \Delta_k \langle b^\dagger_k \rangle \Big]$$

$$\mathcal{H}^{MF} = \sum_{k\sigma} \xi_k c^{+}_{k\sigma} c_{k\sigma} - \sum_k \left(\Delta_k c^{+}_{k\uparrow} c^{+}_{-k\downarrow} + \Delta^{*}_k c_{-k\downarrow} c_{k\uparrow} - \Delta_k \langle b^{+}_k \rangle \right)$$

Diagonalize by <u>Bogolubov</u>-Valatin transformation

$$c_{k\uparrow} = u^{*}_k \gamma_{k\uparrow} + v_k \gamma^{+}_{k\downarrow}$$

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$$\Rightarrow |u_k|^2 + |v_k|^2 = 1$$

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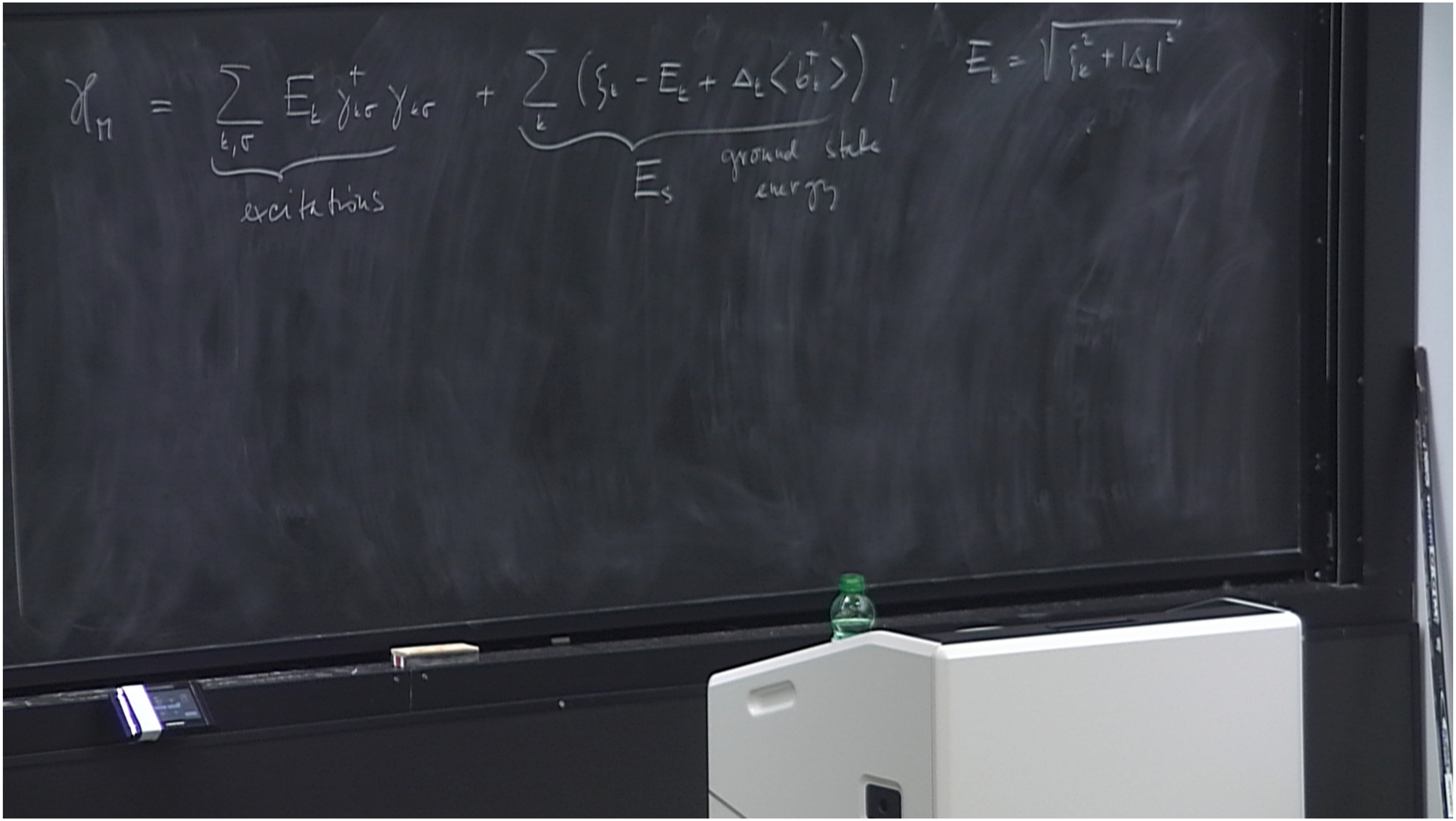
$$+ \sum_{k\sigma} (\Delta_k u_k v^{*}_k + \Delta^{*}_k u^{*}_k v_k)(\gamma^{+}_{k\sigma}\gamma_{k\sigma} + \tfrac{1}{2})$$

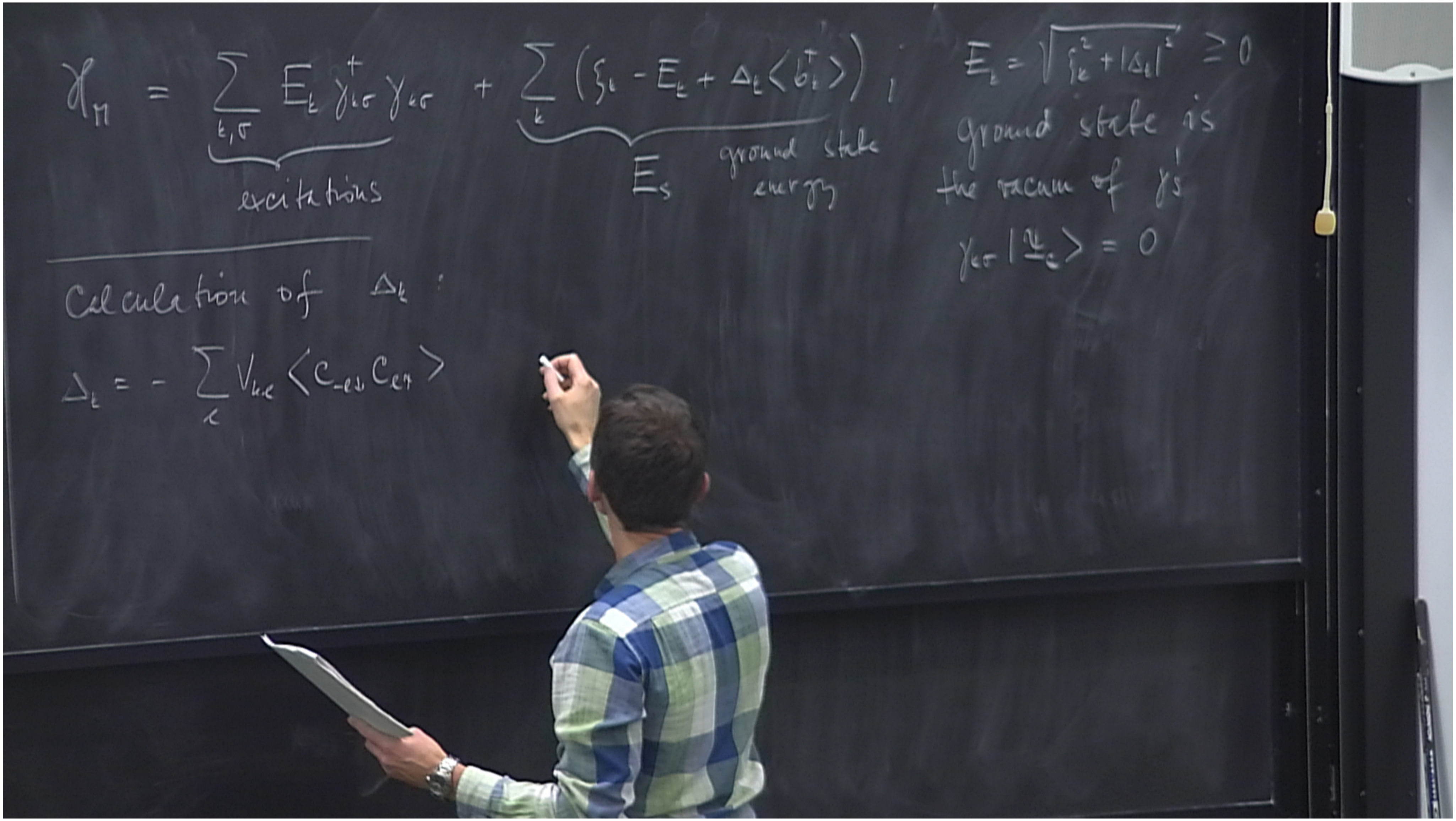
$$+ \sum_{k,\sigma} \Big[(\Delta^{*}_k v^{2}_k - \Delta_k u^{2}_k) \gamma^{+}_{k\uparrow}\gamma^{+}_{k\downarrow} + h.c. + \Delta_k \langle b^{+}_k \rangle \Big]$$

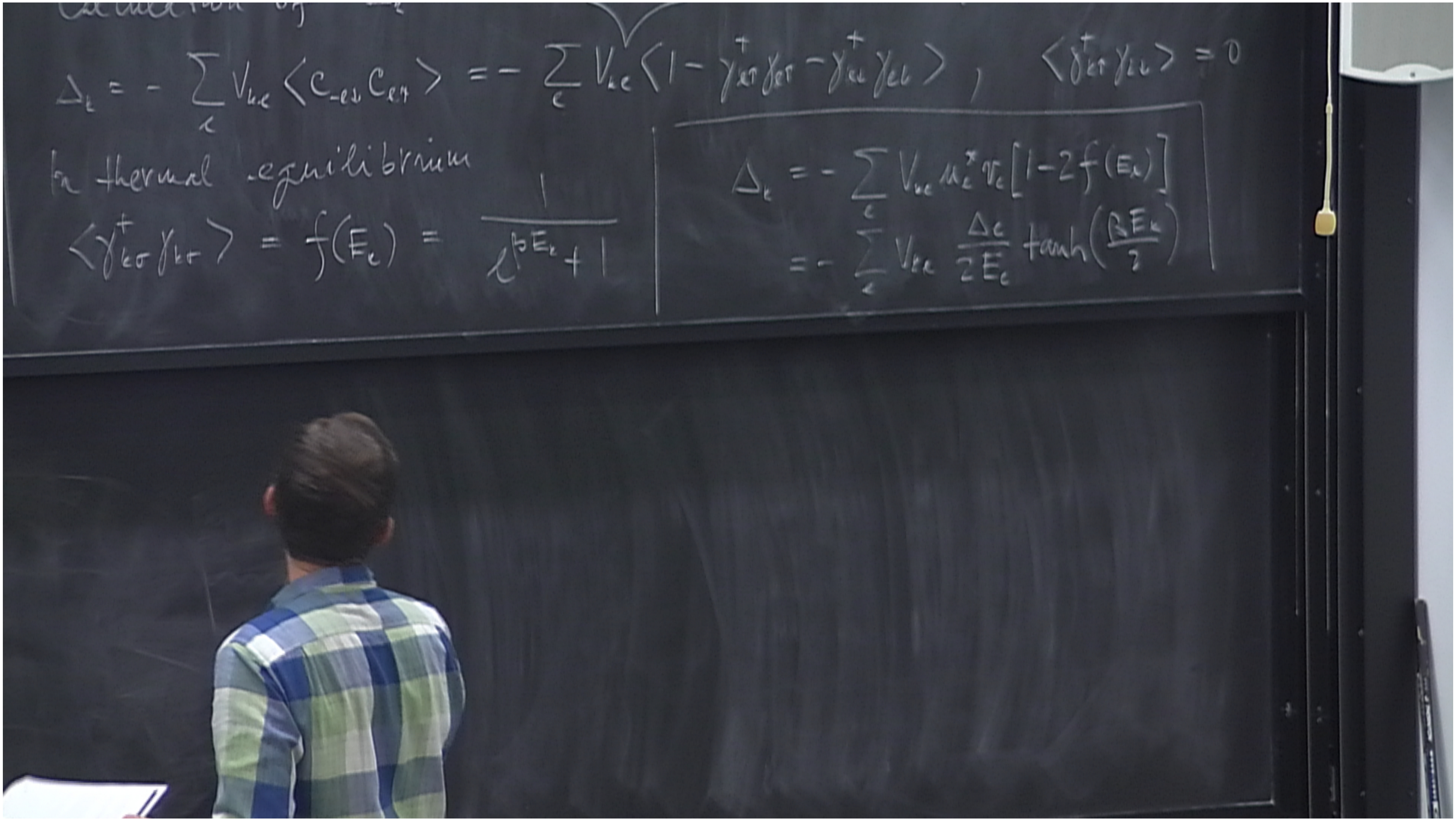
We require

$$2\xi_k u_k v_k + \Delta_k^* v_k^2 - \Delta_k u_k^2 = 0 \quad / \quad \frac{\Delta_k^*}{u_k^2}$$

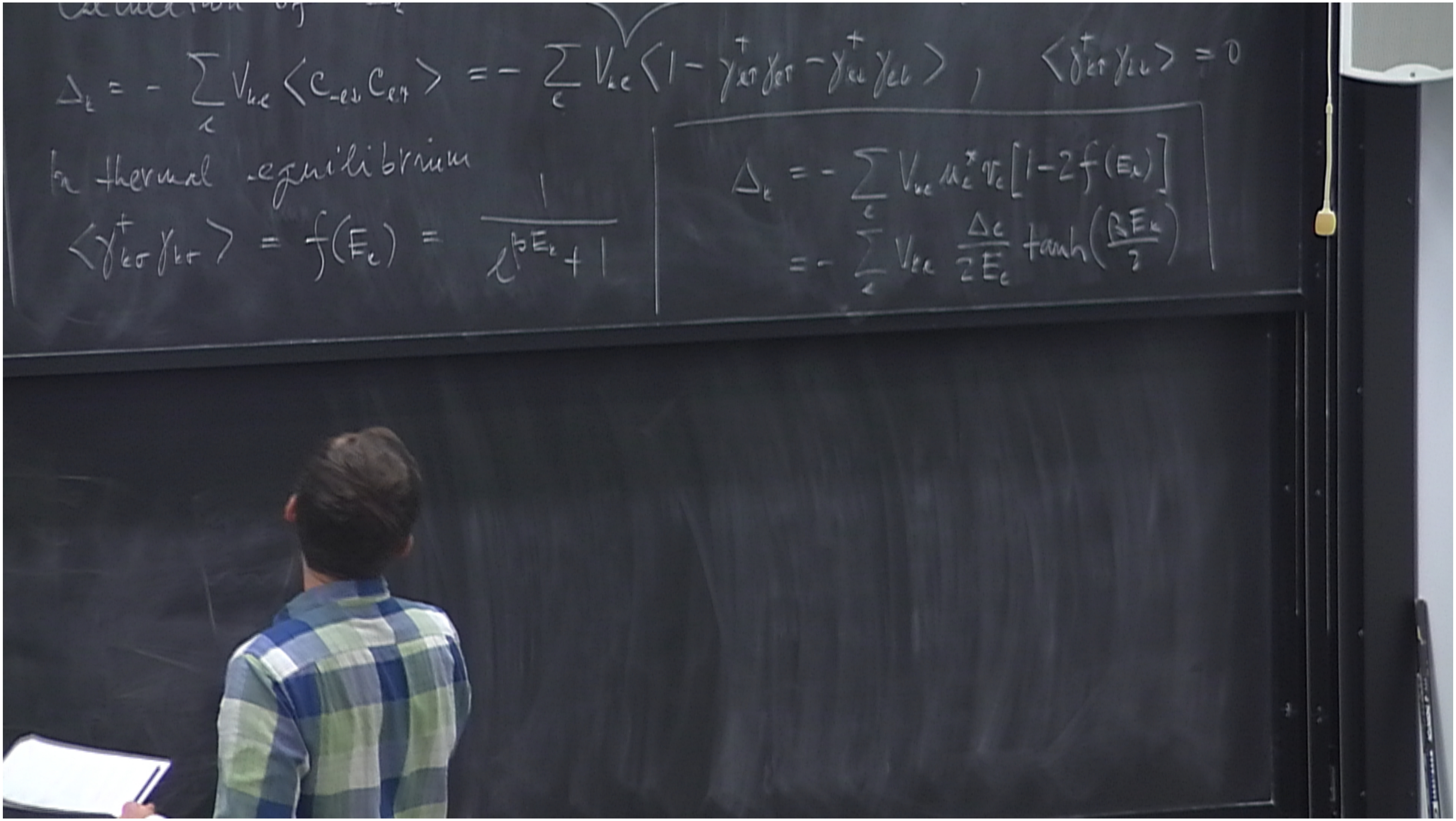
$$\frac{\Delta_k^* v_k}{u_k} = \underbrace{\sqrt{\xi_k^2 + |\Delta_k|^2}}_{E_k} - \xi_k \equiv E_k - \xi_k$$


$\mathcal{H}_M = \underbrace{\sum_{k,\sigma} E_k \gamma^{\dagger}_{k\sigma} \gamma_{k\sigma}}_{\text{excitations}} + \underbrace{\sum_k \left(\xi_k - E_k + \Delta_k \langle b^{\dagger}_k \rangle \right)}_{E_s \text{ ground state energy}} \; ; \quad E_k = \sqrt{\xi_k^2 + |\Delta_k|^2}$


$\mathcal{H}_M = \underbrace{\sum_{k,\sigma} E_k \gamma^\dagger_{k\sigma}\gamma_{k\sigma}}_{\text{excitations}} + \underbrace{\sum_k \left(\xi_k - E_k + \Delta_k \langle b^\dagger_k \rangle\right)}_{E_s \text{ ground state energy}}$;
$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2} \geq 0$
ground state is the vacum of γ's
$\gamma_{k\sigma} |\Psi_s\rangle = 0$
Calculation of Δ_k :
$\Delta_k = - \sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle$



$\Delta_k = -\sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle = -\sum_l V_{kl} \langle 1 - \gamma^\dagger_{l\uparrow}\gamma_{l\uparrow} - \gamma^\dagger_{l\downarrow}\gamma_{l\downarrow} \rangle$, $\langle \gamma^\dagger_{l\uparrow}\gamma_{l\downarrow} \rangle = 0$
In thermal equilibrium
$\langle \gamma^\dagger_{k\sigma}\gamma_{k\sigma} \rangle = f(E_k) = \frac{1}{e^{\beta E_k}+1}$
$\Delta_k = -\sum_l V_{kl} u_l^* v_l [1-2f(E_l)]$
$= -\sum_l V_{kl} \frac{\Delta_l}{2E_l} \tanh\left(\frac{\beta E_l}{2}\right)$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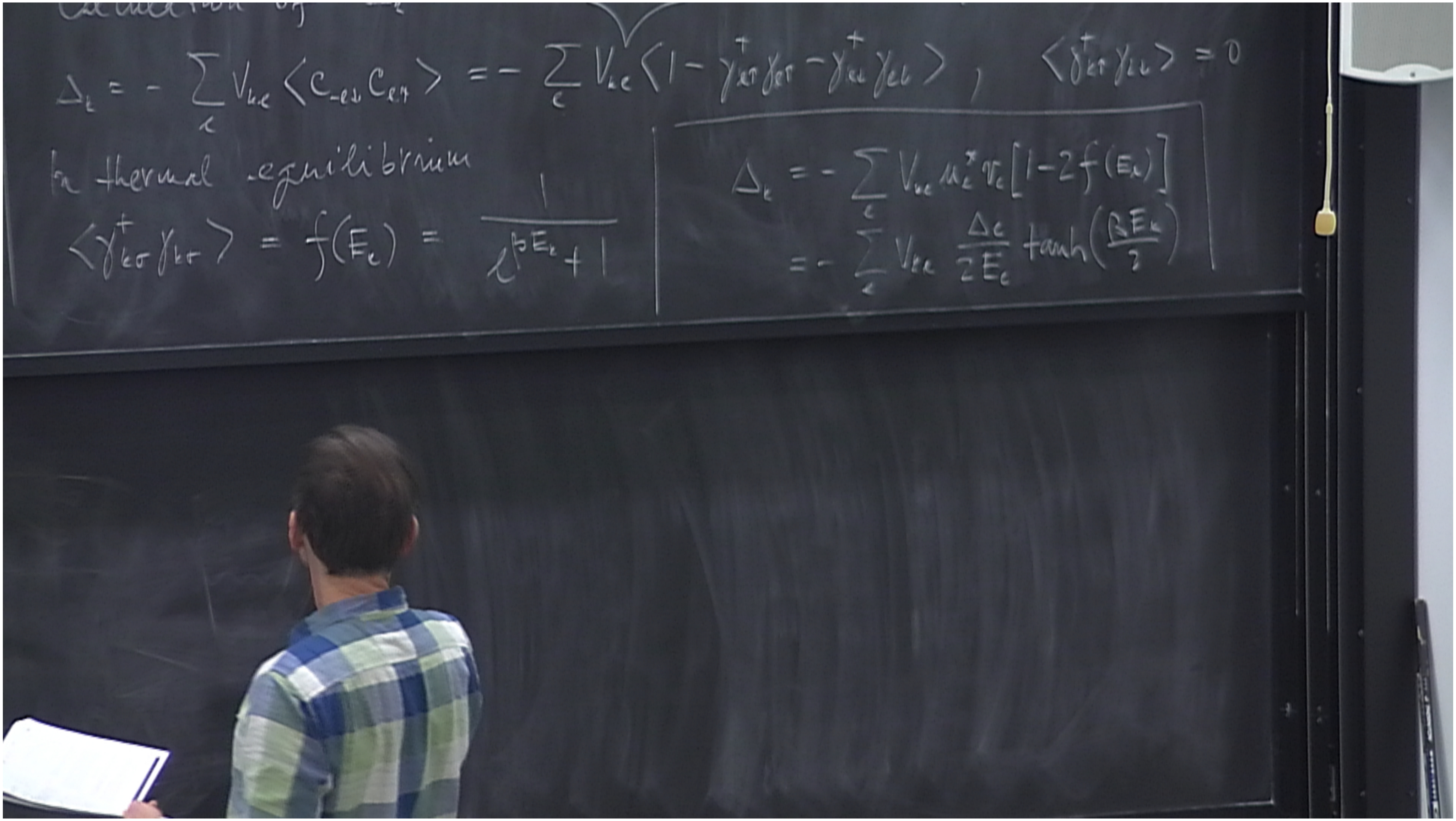

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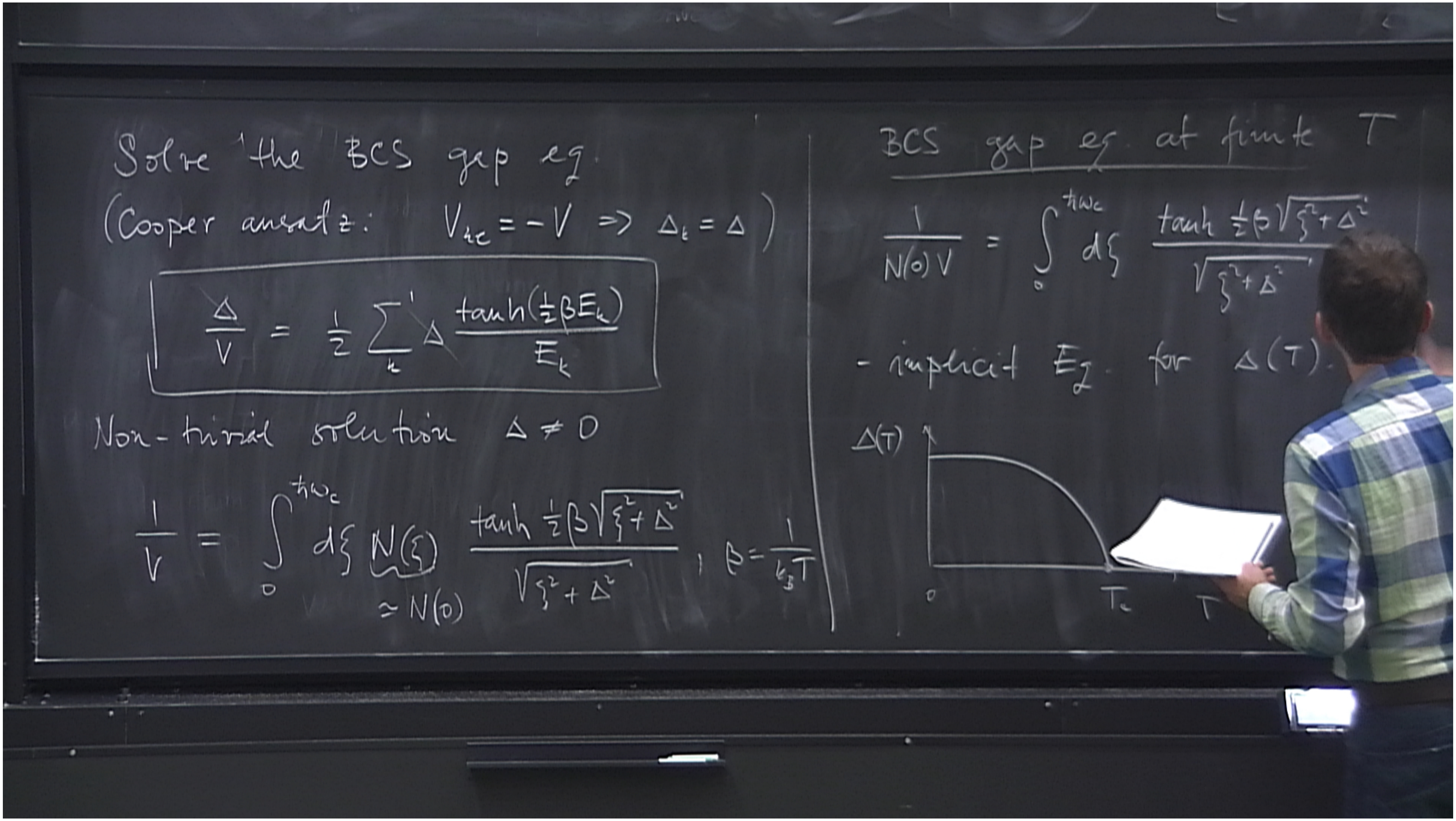
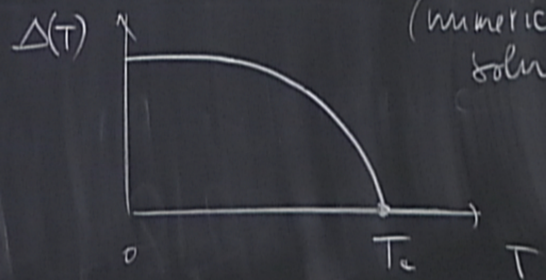

Solve the BCS gap eq.
(Cooper ansatz: $V_{kk'} = -V \Rightarrow \Delta_k = \Delta$)
$\boxed{\frac{\Delta}{V} = \frac{1}{2}\sum_k{}' \Delta \frac{\tanh(\frac{1}{2}\beta E_k)}{E_k}}$
Non-trivial solution $\Delta \neq 0$
$\frac{1}{V} = \int_0^{\hbar\omega_c} d\xi \underbrace{N(\xi)}_{\simeq N(0)} \frac{\tanh \frac{1}{2}\beta\sqrt{\xi^2+\Delta^2}}{\sqrt{\xi^2+\Delta^2}}, \quad \beta = \frac{1}{k_B T}$
BCS gap eq. at finite T
$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c} d\xi \frac{\tanh \frac{1}{2}\beta\sqrt{\xi^2+\Delta^2}}{\sqrt{\xi^2+\Delta^2}}$
- implicit Eq. for $\Delta(T)$.
$\Delta(T)$
0
T_c
T

BCS gap eq. at finite T

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c} d\xi \, \frac{\tanh \frac{1}{2}\beta\sqrt{\xi^2+\Delta^2}}{\sqrt{\xi^2+\Delta^2}}$$

$$\beta = \frac{1}{k_B T}$$

- implicit Eq. for $\Delta(T)$.



(numerical solution)

At $T=0$

$$\Delta(0) = 2\hbar\omega_c \, e^{-1/N(0)V}$$

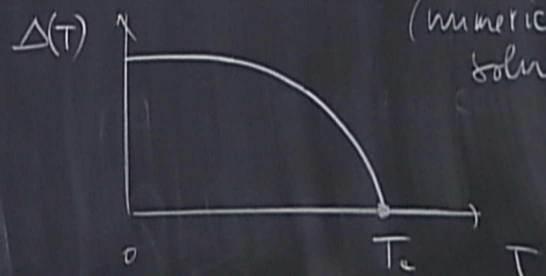
Δ)

$\beta = \frac{1}{k_B T}$

BCS gap eq. at finite T

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c} d\xi \, \frac{\tanh \frac{1}{2}\beta\sqrt{\xi^2+\Delta^2}}{\sqrt{\xi^2+\Delta^2}}$$

- implicit Eq. for $\Delta(T)$.



(numerical solution)

At T = 0

$$\Delta(0) = 2\hbar\omega_c \, e^{-1/N(0)V}$$

Determination of T_c

$\Delta(T_c) \to 0, \quad E_k \to \xi_k$

$x = \frac{1}{2}\frac{\xi}{k_B T}$

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c/k_B T} dx \, \frac{\tanh x}{x}$$

$$= \ln(A\beta\hbar\omega_c), \quad A = \frac{2\gamma}{\pi} \simeq 1.13$$

At $T = 0$

$$\boxed{\Delta(0) = 2\hbar\omega_c \, e^{-1/N(0)V}}$$

Determination of T_c

$\Delta(T_c) \to 0$, $\quad E_k \to \xi_k$

$$\frac{1}{N(0)V} = \int_0^{\hbar\omega_c/k_BT} dx \, \frac{\tanh x}{x}$$

$$= \ln(A\beta\hbar\omega_c), \quad A = \frac{2\gamma}{\pi} \simeq 1.13$$

$x = \frac{1}{2}\frac{\xi}{k_BT}$

Solve for β:

$$\boxed{k_BT_c = \frac{1}{\beta_c} = 1.13\,\hbar\omega_c \, e^{-1/N(0)V}}$$

BCS critical temperature

At $T=0$

$$\boxed{\Delta(0) = 2\hbar\omega_c e^{-1/N(0)V}}$$

Determination of T_c

$\Delta(T_c) \to 0, \quad E_k \to \xi_k$

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$$x = \frac{1}{2}\frac{\xi}{k_B T}$$

Solve for β:

$$\boxed{k_B T_c = \frac{1}{\beta_c} = 1.13\,\hbar\omega_c\, e^{-1/N(0)V}}$$

BCS critical temperature

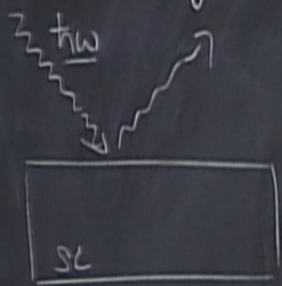
BCS ratio:

$$\boxed{\frac{\Delta(0)}{k_B T_c} = \frac{2}{1.13} \simeq 1.764}$$

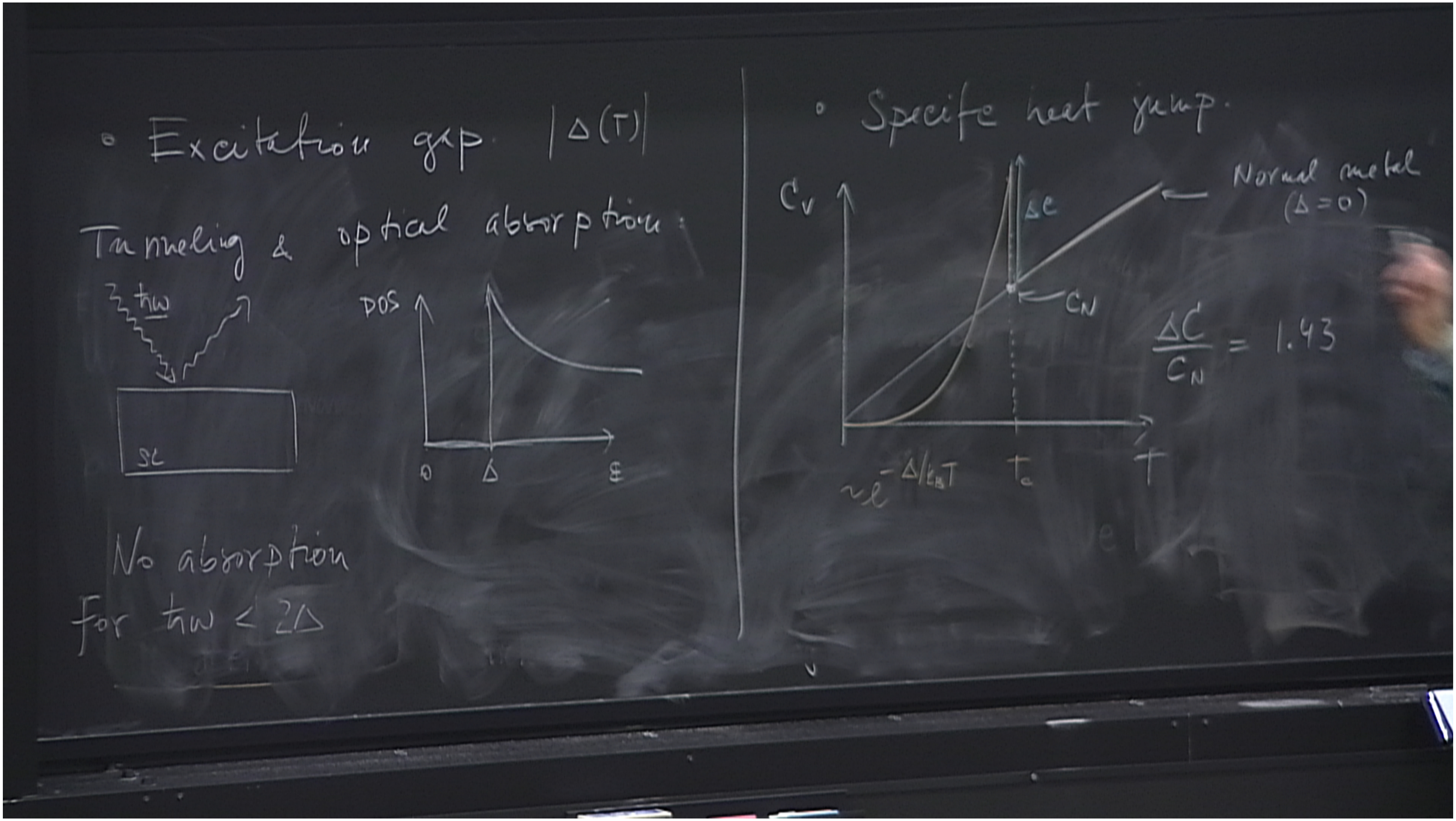
Experiment: 1.5 - 2.3

• Excitation gap $|\Delta(T)|$

Tunneling & optical absorption:

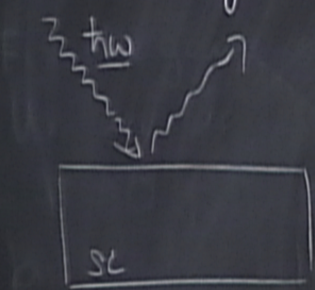


No absorption for $\hbar\omega < 2\Delta$


• Excitation gap $|\Delta(T)|$
Tunneling & optical absorption:
$\hbar\omega$
DOS
SC
0
Δ
E
No absorption
for $\hbar\omega < 2\Delta$
• Specific heat jump.
C_V
δC
Normal metal
$(\Delta = 0)$
C_N
$\frac{\Delta C}{C_N} = 1.43$
$\sim e^{-\Delta/k_B T}$
T_c
T

- Excitation gap $|\Delta(T)|$

Tunneling & optical absorption:



No absorption for $\hbar\omega < 2\Delta$

- Specific heat jump.

$$\frac{\Delta C}{C_N} = 1.43 \checkmark$$

$\simeq N(0)$ $\quad$ T_c $\quad$ T

EM properties, Ginzburg - Landau theory (1950)

GL

Describe Cooper pair condensate by a complex wavefunction

$$\Psi(\vec{r}) = |\Psi(\vec{r})|\, e^{i\varphi(\vec{r})}$$

$|\Psi|^2 = n_s$ "condensate density"

φ - condensate phase

T | | Experiment: 1.5 - 2.5

950)

ction

GL Free energy functional

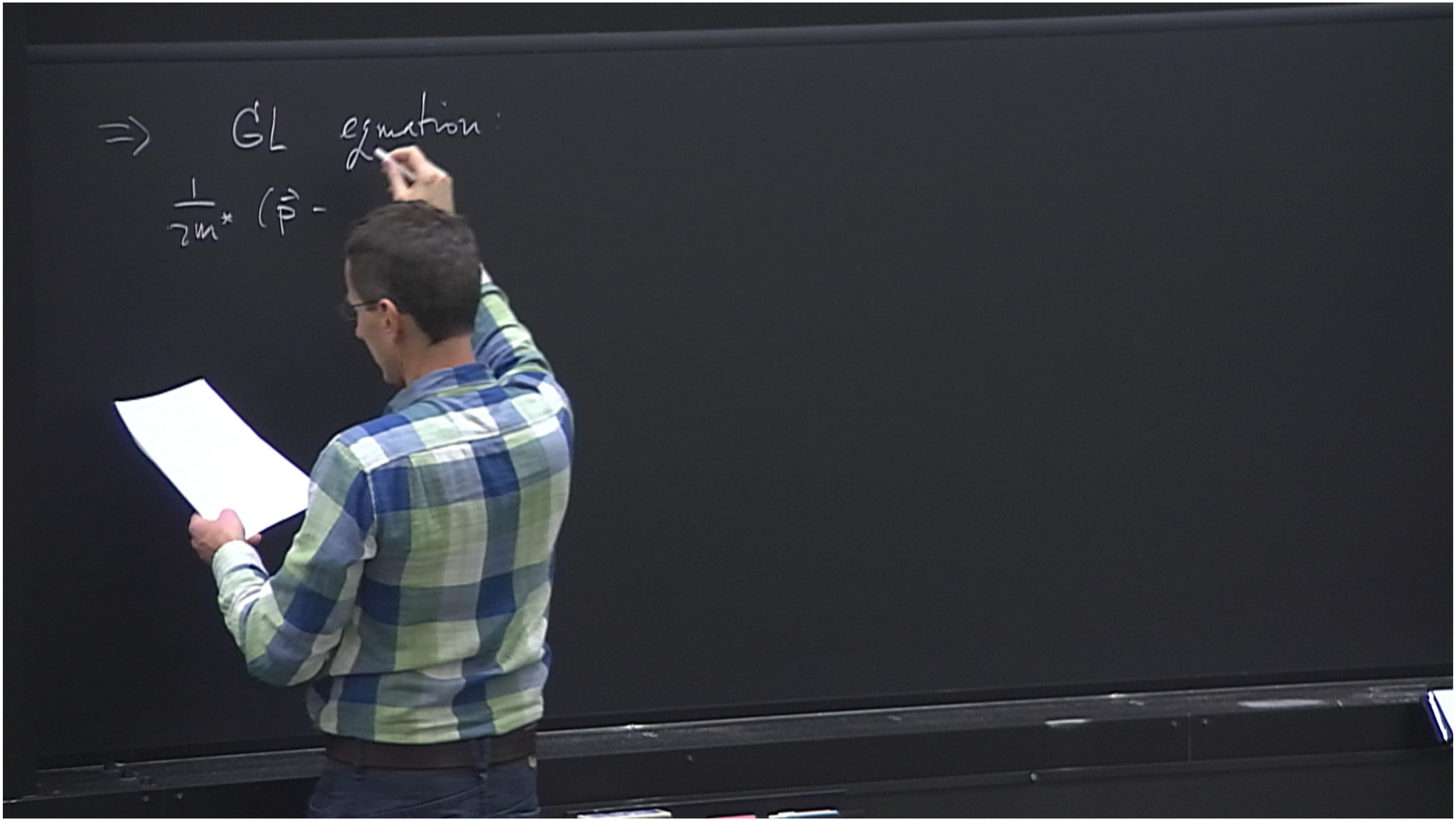
$$F[\Psi] = \int d^3r \left\{ \frac{1}{2m^*} \left| \left(\vec{p} - \frac{e^*}{c}\vec{A}\right)\Psi \right|^2 + \frac{1}{2}\alpha|\Psi|^2 + \beta|\Psi|^4 \right\}$$

$m^* \simeq 2m_e$, $e^* = 2e$ mass & charge of Cooper pairs

α, β - GL phenomenological parameters (material dependent)

$\alpha \simeq \alpha_0 (T - T_c)$, $\beta \sim$ temp indep

Find Ψ that minimizes $F[\Psi]$; $\frac{\delta F[\Psi]}{\delta \Psi^*} = 0$


⇒ GL equation:
$\frac{1}{2m^*}(\vec{p} -$

$\Rightarrow$ GL equation:

$$\frac{1}{2m^*}\left(\vec{p} - \frac{e^*}{c}\vec{A}\right)\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0$$

$\Rightarrow$ GL equation:

$$\frac{1}{2m^*}\left(\vec{p} - \frac{e^*}{c}\vec{A}\right)^2 \Psi + \alpha \Psi + \beta |\Psi|^2 \Psi = 0$$

uniform case, $\vec{A} = 0$, $\Psi = \text{const.}$

$$\alpha \Psi + \beta |\Psi|^2 \Psi = 0$$

$$|\Psi| = \sqrt{-\frac{\alpha}{\beta}}$$

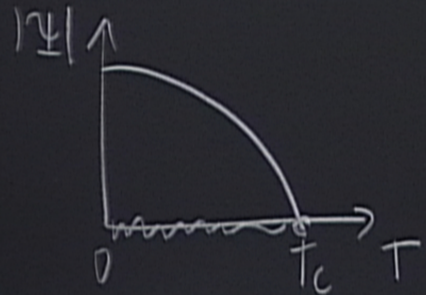
$\Rightarrow$ GL equation:

$$\frac{1}{2m^*}\left(\vec{p} - \frac{e^*}{c}\vec{A}\right)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0$$

uniform case, $\vec{A} = 0$, $\Psi = \text{const.}$

$$\alpha\cancel{\Psi} + \beta|\Psi|^2\cancel{\Psi} = 0$$

$$|\Psi| = \sqrt{-\frac{\alpha}{\beta}} = \sqrt{\frac{\alpha_0}{\beta}(T_c - T)}$$



$\Rightarrow$ GL equation:

$$\frac{1}{2m^*}\left(\vec{p} - \frac{e^*}{c}\vec{A}\right)^2\Psi + \alpha\Psi + \beta|\Psi|^2\Psi = 0$$

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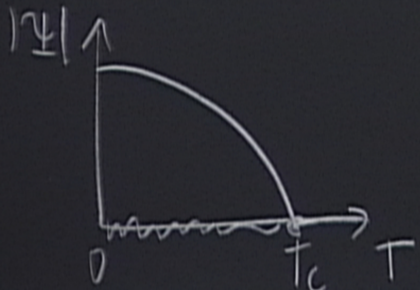
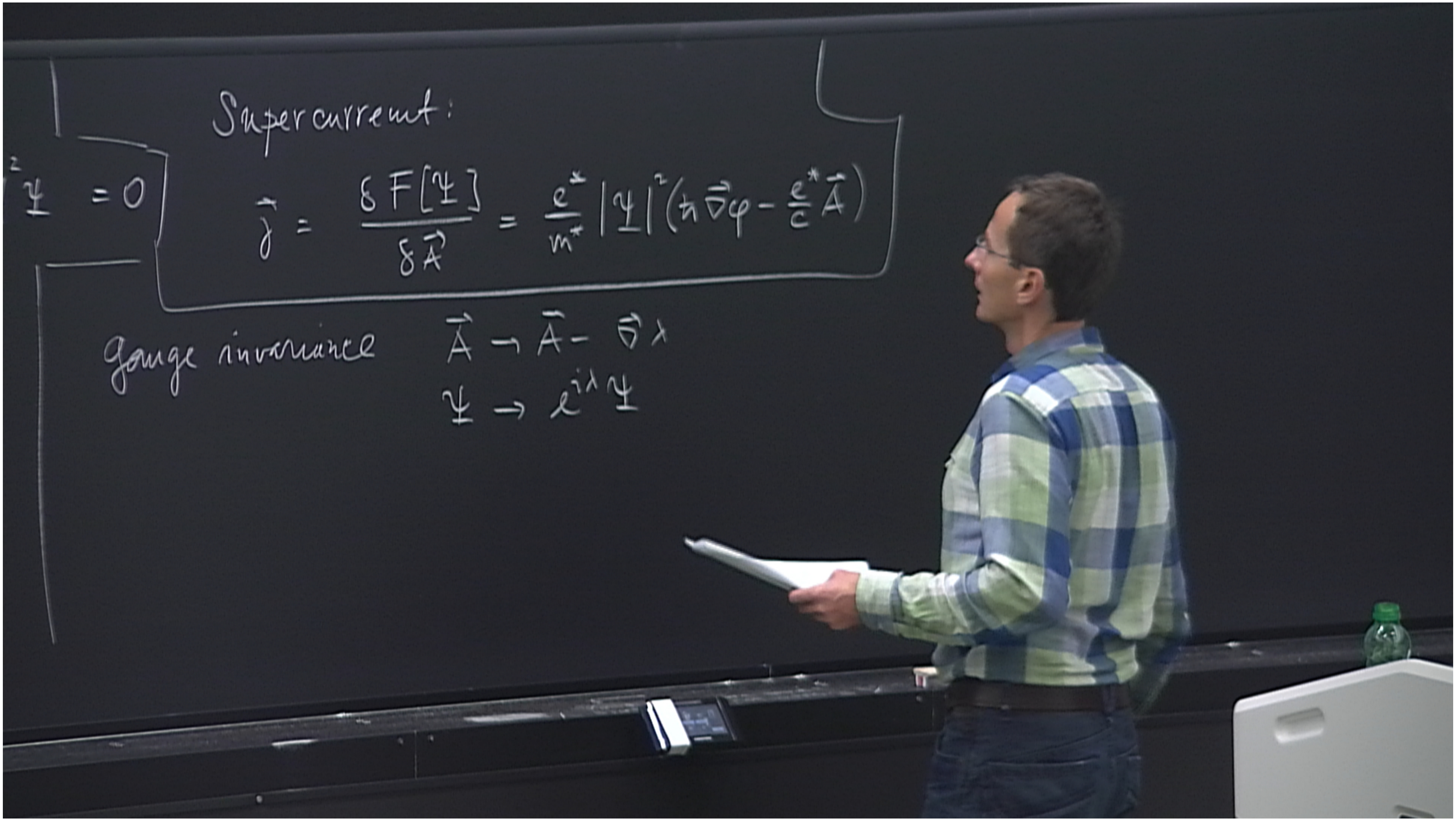


Supercurrent:

$$\vec{j} = \frac{\delta F[\Psi]}{\delta \vec{A}} =$$


Supercurrent:
$\vec{j} = \frac{\delta F[\Psi]}{\delta \vec{A}} = \frac{e^*}{m^*}|\Psi|^2\left(\hbar\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A}\right)$
gauge invariance
$\vec{A} \rightarrow \vec{A} - \vec{\nabla}\lambda$
$\Psi \rightarrow e^{i\lambda}\Psi$

$$\ldots^2 \Psi = 0$$

Supercurrent:

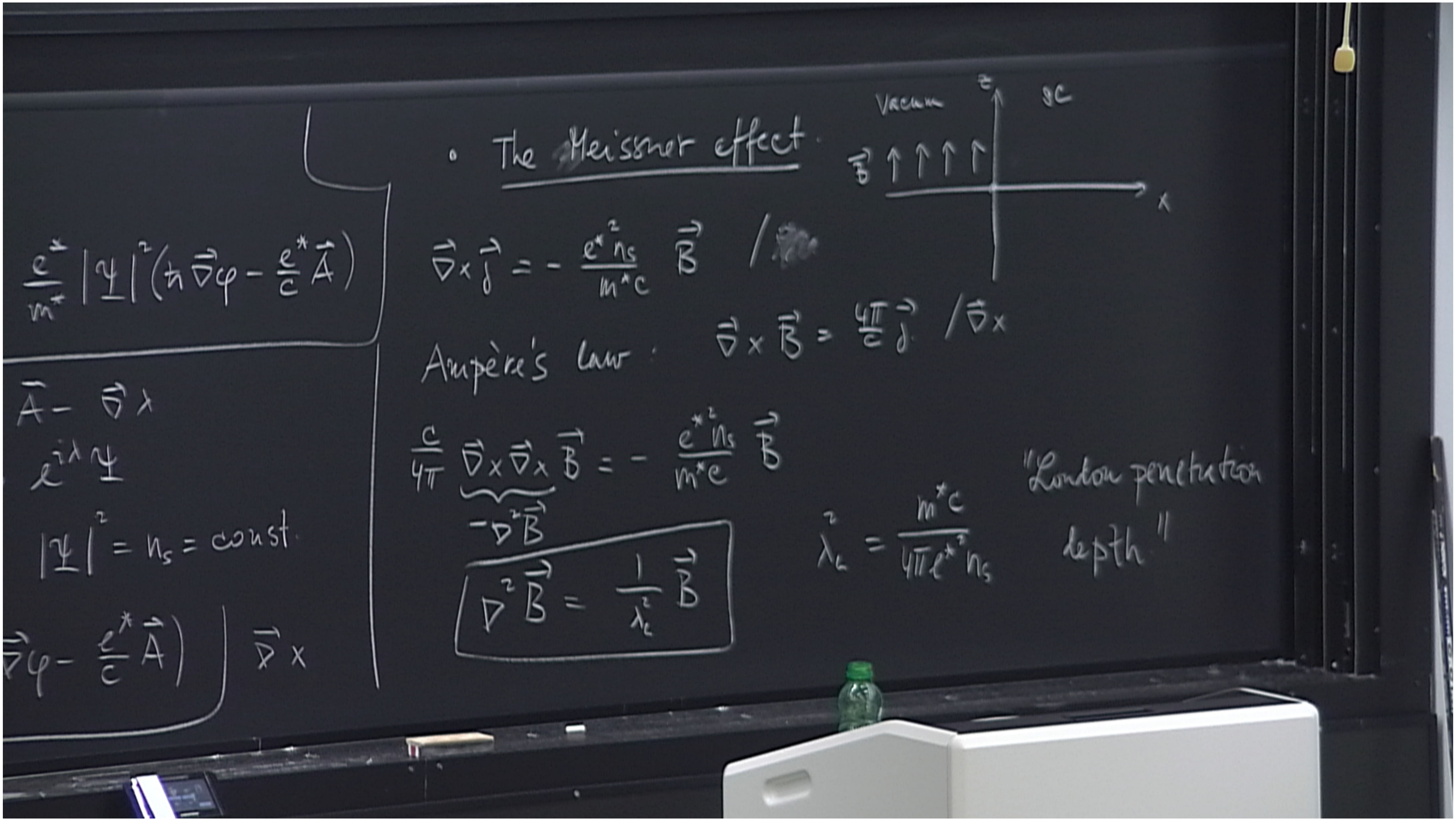
$$\boxed{\vec{j} = \frac{\delta F[\Psi]}{\delta \vec{A}} = \frac{e^*}{m^*}|\Psi|^2\left(\hbar\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A}\right)}$$

gauge invariance $\vec{A} \to \vec{A} - \vec{\nabla}\lambda$

$\Psi \to e^{i\lambda}\Psi$

London approximation: $|\Psi|^2 = n_s = \text{const.}$

$$\boxed{\vec{j} = \frac{e^* n_s}{m^*}\left(\hbar\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A}\right)}$$



$\frac{e^*}{m^*}|\Psi|^2(\hbar\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A})$
$\vec{A} - \vec{\nabla}\lambda$
$e^{i\lambda}\Psi$
$|\Psi|^2 = n_s = \text{const.}$
$\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A})$
$\vec{\nabla}\times$
• The Meissner effect.
Vacuum
z
SC
$\vec{B}$
x
$\vec{\nabla}\times\vec{j} = -\frac{e^{*2} n_s}{m^* c}\vec{B}$ /
Ampère's law: $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j}$ $/\vec{\nabla}\times$
$\frac{c}{4\pi}\underbrace{\vec{\nabla}\times\vec{\nabla}\times\vec{B}}_{-\nabla^2\vec{B}} = -\frac{e^{*2} n_s}{m^* c}\vec{B}$
$\boxed{\nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}}$
$\lambda_L^2 = \frac{m^* c}{4\pi e^{*2} n_s}$
"London penetration depth"

$$\frac{e^*}{m^*}|\Psi|^2\left(\hbar\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A}\right)$$

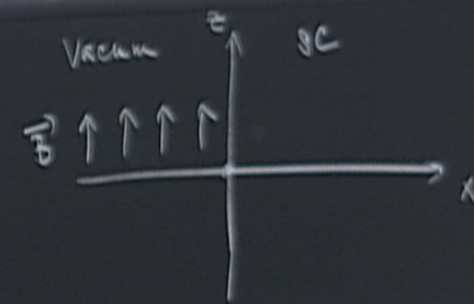
$$\vec{A} - \vec{\nabla}\lambda$$

$$e^{i\lambda}\Psi$$

$$|\Psi|^2 = n_s = \text{const.}$$

$$\vec{\nabla}\varphi - \frac{e^*}{c}\vec{A}\Big) \qquad \vec{\nabla}\times$$

• The Meissner effect.



$$\vec{\nabla}\times\vec{j} = -\frac{e^{*2} n_s}{m^* c}\vec{B} \quad /$$

Ampère's law: $\vec{\nabla}\times\vec{B} = \frac{4\pi}{c}\vec{j} \quad /\vec{\nabla}\times$

$$\frac{c}{4\pi}\underbrace{\vec{\nabla}\times\vec{\nabla}\times}_{-\nabla^2}\vec{B} = -\frac{e^{*2} n_s}{m^* c}\vec{B}$$

$$\boxed{\nabla^2\vec{B} = \frac{1}{\lambda_L^2}\vec{B}}$$

$$\lambda_L^2 = \frac{m^* c}{4\pi e^{*2} n_s}$$

"London penetration depth"

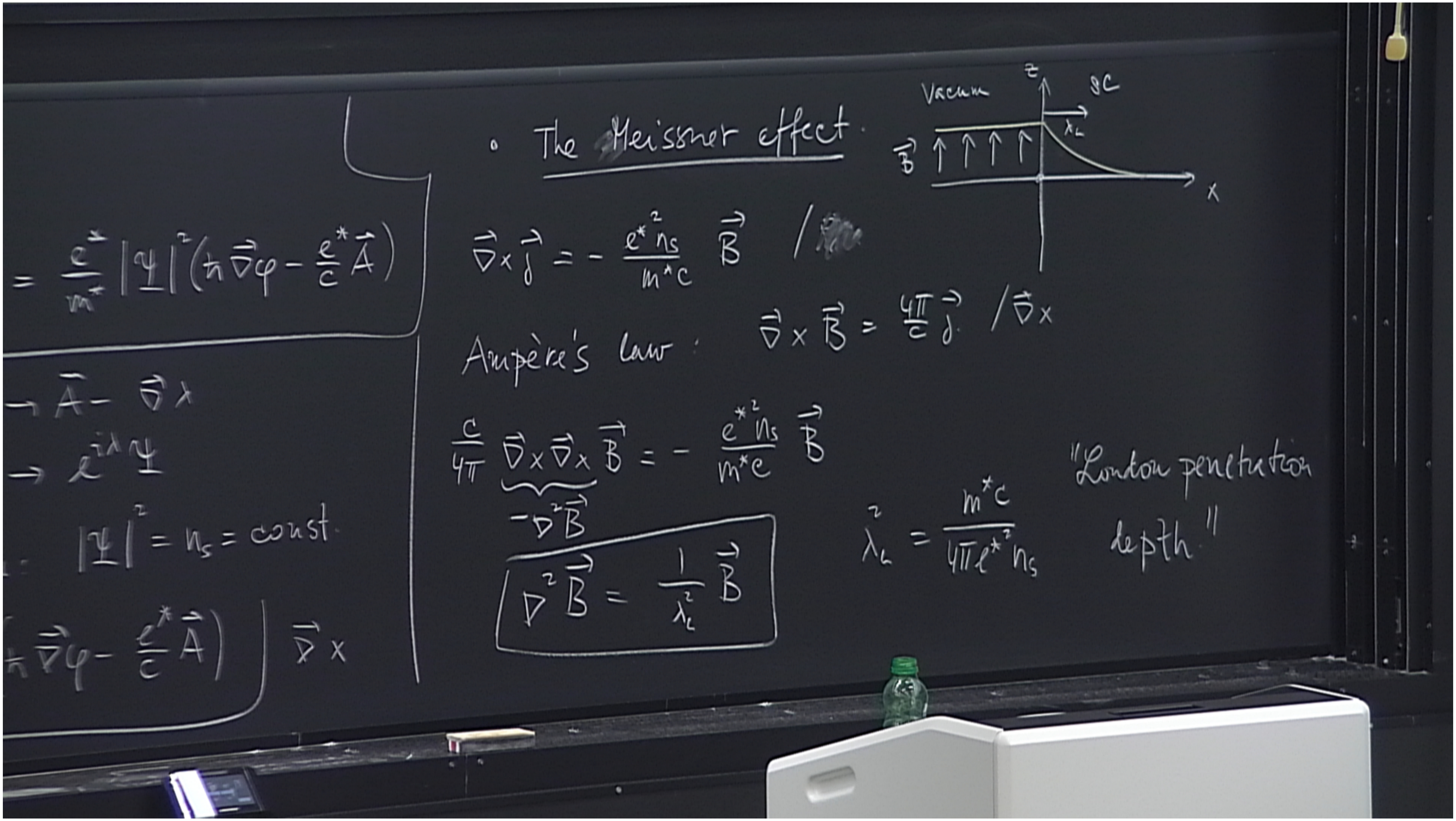
$$\vec{B} = B(x)\,\hat{z}$$

$$\frac{\partial^2}{\partial x^2} B = \frac{1}{\lambda_L^2} B$$

$$\Rightarrow \boxed{B(x) = B(0)\, e^{-x/\lambda_L}}$$

- magnetic field decays exponentially inside the SC.

$$\lambda_L \simeq 10^2 - 10^4\ \text{Å}$$



= $\frac{e^*}{m^*}|\Psi|^2(\hbar\vec\nabla\varphi - \frac{e^*}{c}\vec A)$
→ $\vec A - \vec\nabla\chi$
→ $e^{i\lambda}\Psi$
$|\Psi|^2 = n_s = \text{const.}$
$(\hbar\vec\nabla\varphi - \frac{e^*}{c}\vec A)$ $\vec\nabla\times$
• The Meissner effect.
Vacuum
SC
z
x
λ_L
$\vec B$
$\vec\nabla\times\vec j = -\frac{e^{*2}n_s}{m^*c}\vec B$
Ampère's law : $\vec\nabla\times\vec B = \frac{4\pi}{c}\vec j$ $/\vec\nabla\times$
$\frac{c}{4\pi}\underbrace{\vec\nabla\times\vec\nabla\times\vec B}_{-\nabla^2\vec B} = -\frac{e^{*2}n_s}{m^*c}\vec B$
$\boxed{\nabla^2\vec B = \frac{1}{\lambda_L^2}\vec B}$
$\lambda_L^2 = \frac{m^*c}{4\pi e^{*2}n_s}$
"London penetration depth"

$$\vec{B} = B(x)\,\hat{z}$$

$$\frac{\partial^2}{\partial x^2} B = \frac{1}{\lambda_L^2} B$$

$$\Rightarrow \boxed{B(x) = B(0)\, e^{-x/\lambda_L}}$$

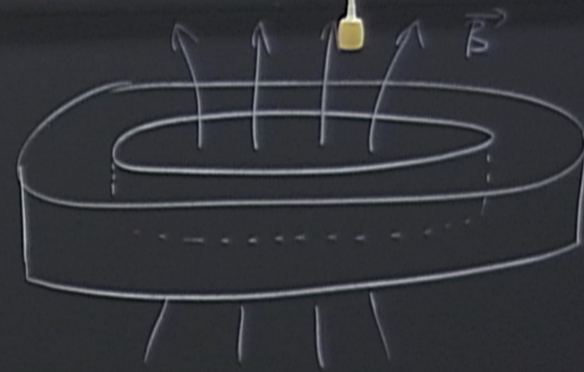
- magnetic field decays exponentially inside the SC.

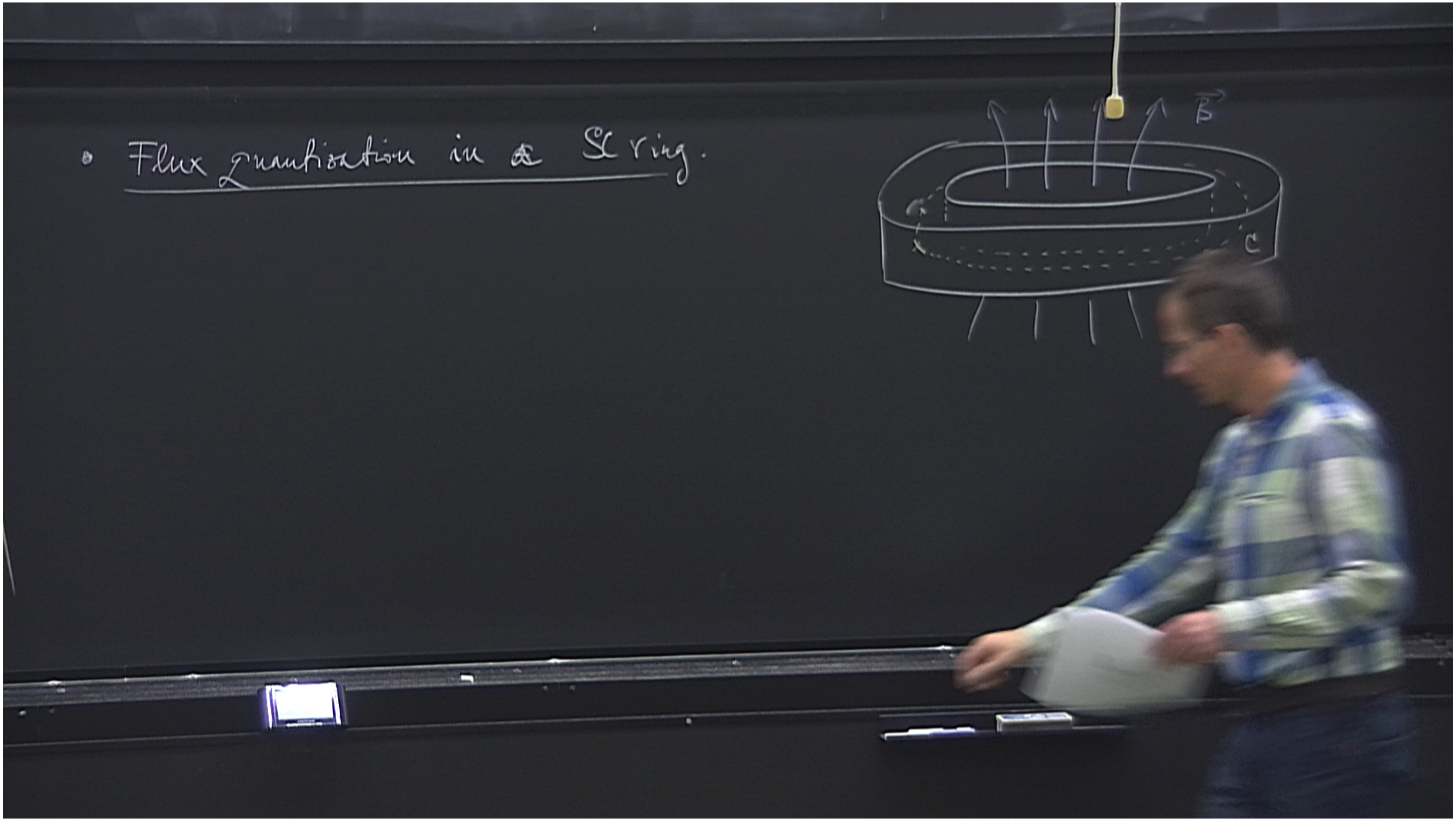
$$\lambda_L \simeq 10^2 - 10^4 \text{ Å}$$

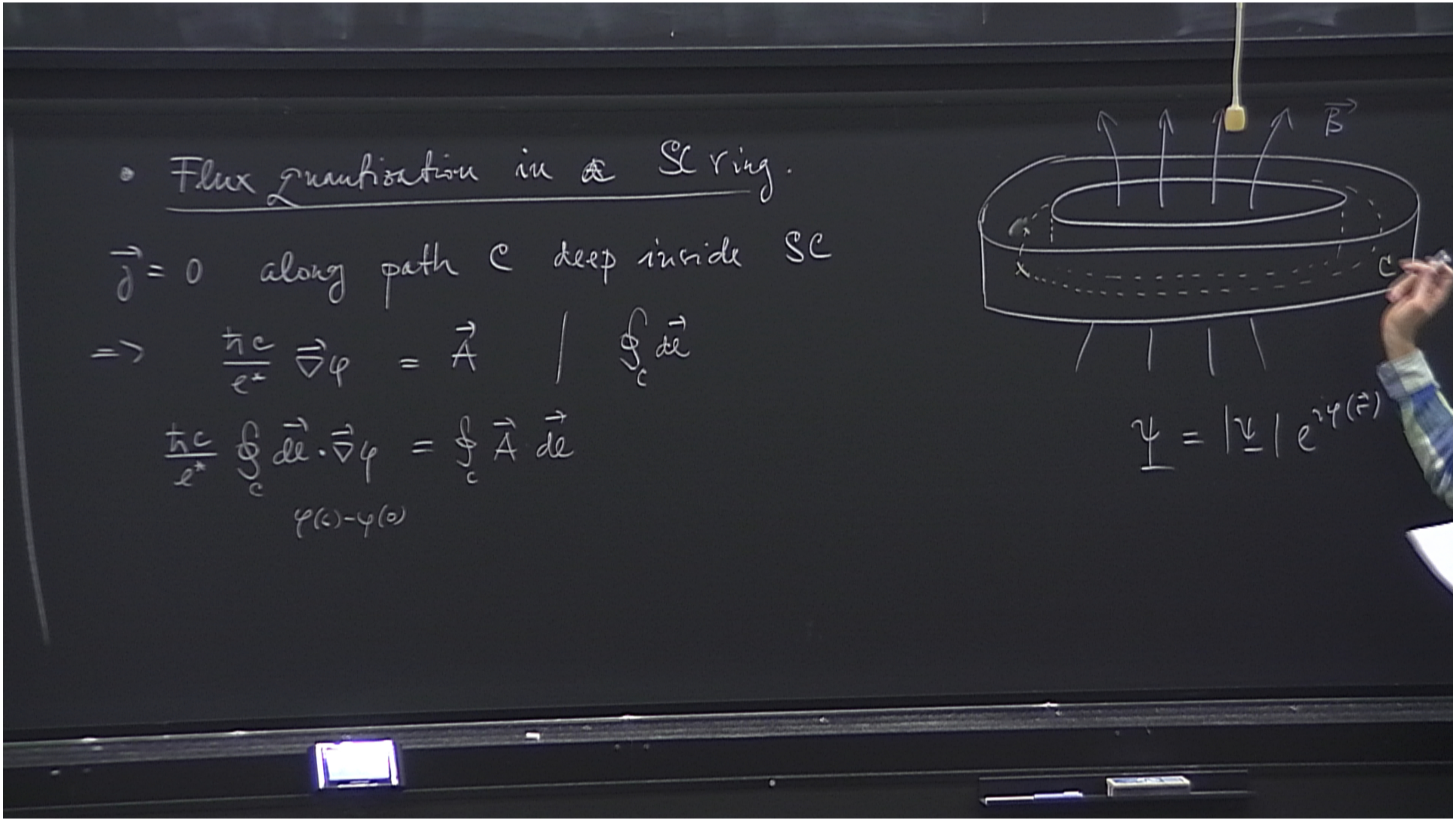
Shielding currents

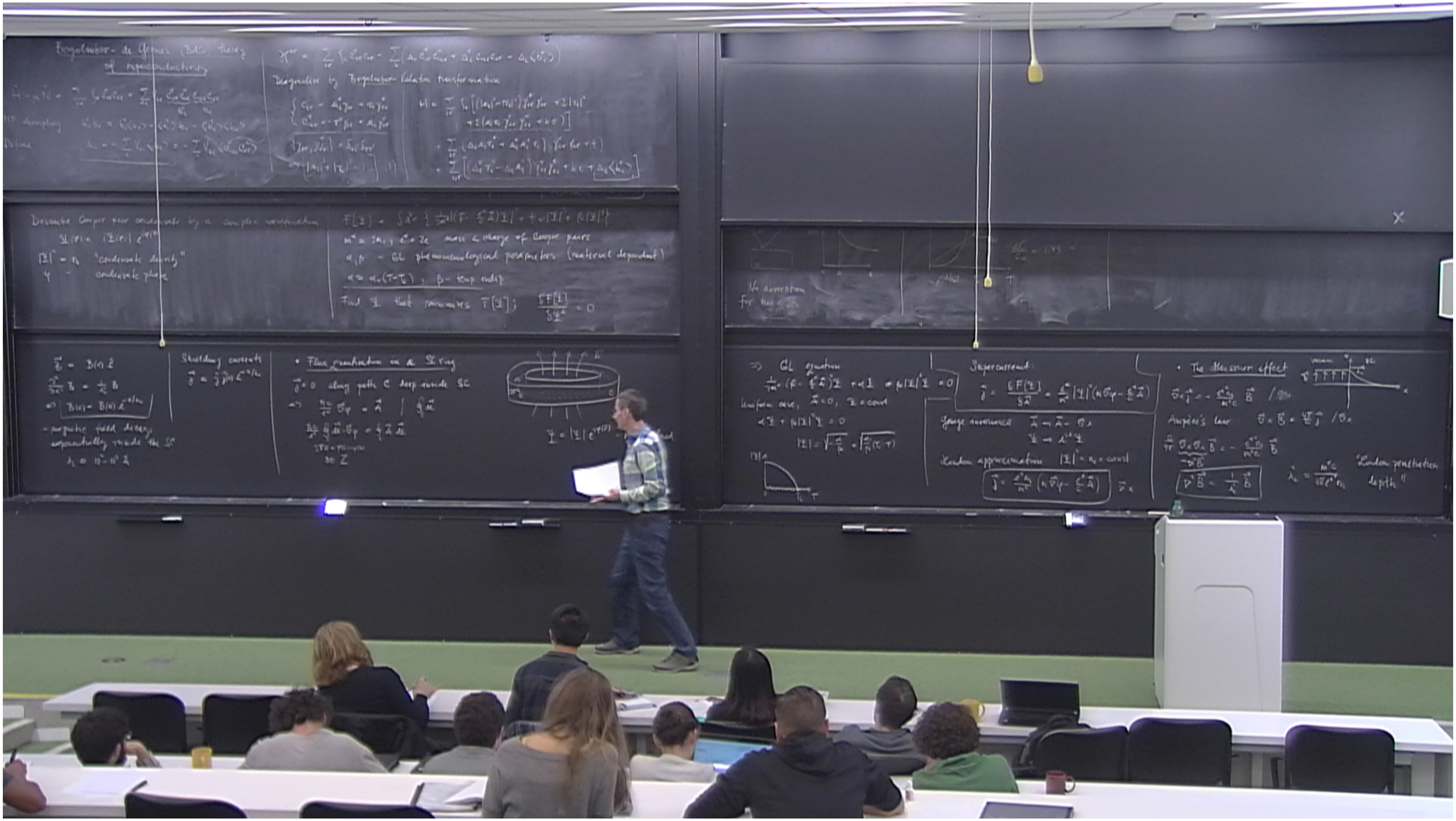
$$\vec{j} = \hat{y}\, j(0)\, e^{-x/\lambda_L}$$

- Flux quantization in a SC ring.




Flux quantisation in a SC ring.
$\vec{B}$
C


• Flux quantization in a SC ring.
$\vec{j} = 0$ along path C deep inside SC
$\Rightarrow \frac{\hbar c}{e^*} \vec{\nabla}\varphi = \vec{A}$ | $\oint_C d\vec{\ell}$
$\frac{\hbar c}{e^*} \oint_C d\vec{\ell} \cdot \vec{\nabla}\varphi = \oint_C \vec{A} \cdot d\vec{\ell}$
$\varphi(c) - \varphi(0)$
$\vec{B}$
C
$\Psi = |\Psi| e^{i\varphi(\vec{r})}$


Bogoliubov-de Gennes (BdG) theory
of superconductivity
Diagonalize by Bogoliubov-Valatin transformation
GL equation
Supercurrent
Gauge invariance
London approximation
The Meissner effect
Ampère's law
"London penetration depth"
Flux quantization in a SC ring
Shielding currents

...antization in a SC ring.

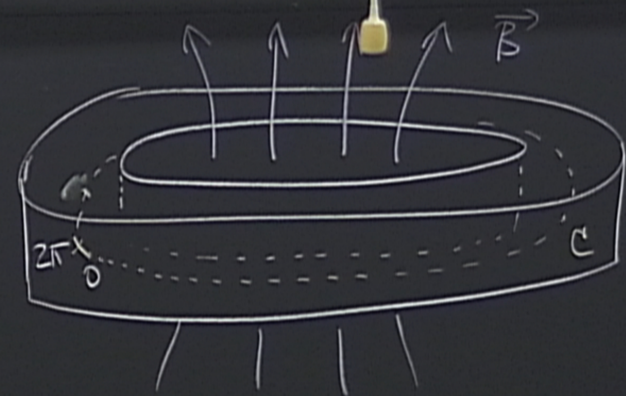
...g path C deep inside SC

$\vec{\nabla}\varphi = \vec{A}$ | $\oint_C d\vec{\ell}$

$\oint_C d\vec{\ell}\cdot\vec{\nabla}\varphi = \oint_C \vec{A}\, d\vec{\ell} = \int d^2S\, \underbrace{(\vec{\nabla}\times\vec{A})}_{\vec{B}} = \Phi$ ← mag flux

$\ldots(c)-\varphi(0)$

$$\boxed{\Phi = 2\pi s\,\frac{\hbar c}{e^*} = \Phi_0 s\,,\quad s\in\mathbb{Z}}$$



$\Psi = |\Psi|\, e^{i\varphi(\vec{r})}$ - single valued

$\Phi_0 = \frac{hc}{2e}$ "SC flux quantum"

"Little-Parks experiment"

...antization in a SC ring.

...g path C deep inside SC

$\ldots\varphi = \vec{A}$ | $\oint_c d\vec{\ell}$

$\ldots\cdot\vec{\nabla}\varphi = \oint_c \vec{A}\, d\vec{\ell} = \int d^2S\, \underbrace{(\vec{\nabla}\times\vec{A})}_{\vec{B}} = \Phi$ ← mag flux

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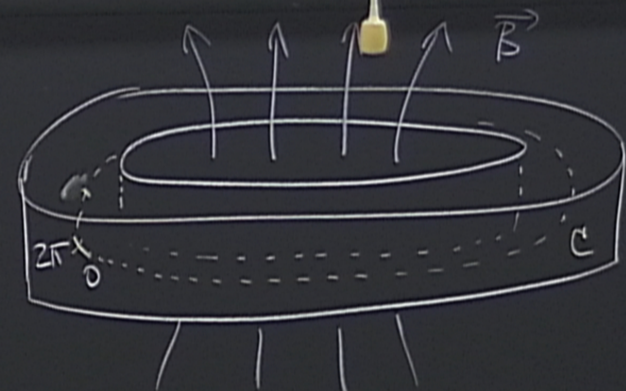

$\Psi = |\Psi|\, e^{i\varphi(\vec{r})}$ - single valued

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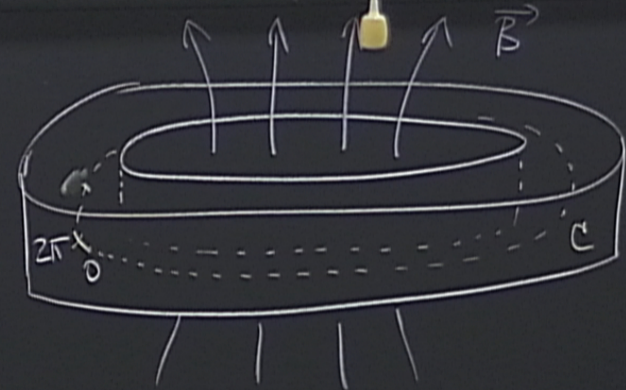


$\vec{\nabla}\varphi = \vec{A}$ | $\oint_C d\vec{\ell}$

$\Psi = |\Psi| e^{i\varphi(\vec{r})}$ - single valued

$\oint \vec{\nabla}\varphi = \oint_C \vec{A}\cdot d\vec{\ell} = \int d^2S\,\underbrace{(\vec{\nabla}\times\vec{A})}_{\vec{B}} = \Phi$ ↖ mag flux

...$(2\pi)-\varphi(0)$

$\Phi_0 = \frac{hc}{2e}$ "SC flux quantum"

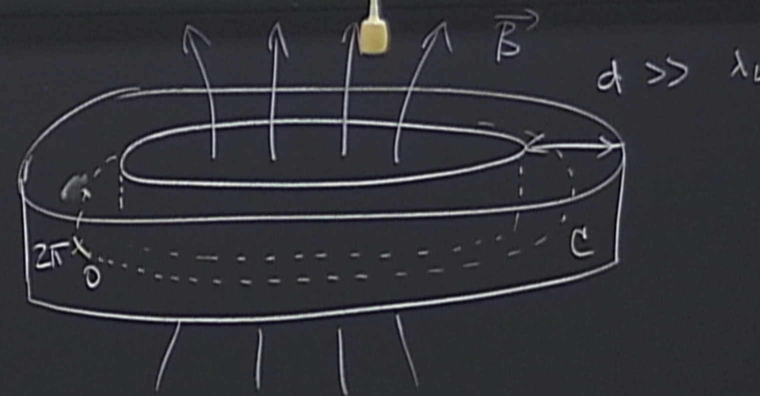
...$\mathbb{Z}$

$$\boxed{\Phi = 2\pi s\,\frac{\hbar c}{e^*} = \Phi_0\, s\,,\quad s\in\mathbb{Z}}$$

"Little-Parks experiment"

...antization in a SC ring.

...g path C deep inside SC



$\vec{\nabla}\varphi = \vec{A}$ | $\oint_C d\vec{l}$

$\oint_C d\vec{l}\cdot\vec{\nabla}\varphi = \oint_C \vec{A}\, d\vec{l} = \int d^2S\,\underbrace{(\vec{\nabla}\times\vec{A})}_{\vec{B}} = \Phi$ ← mag flux

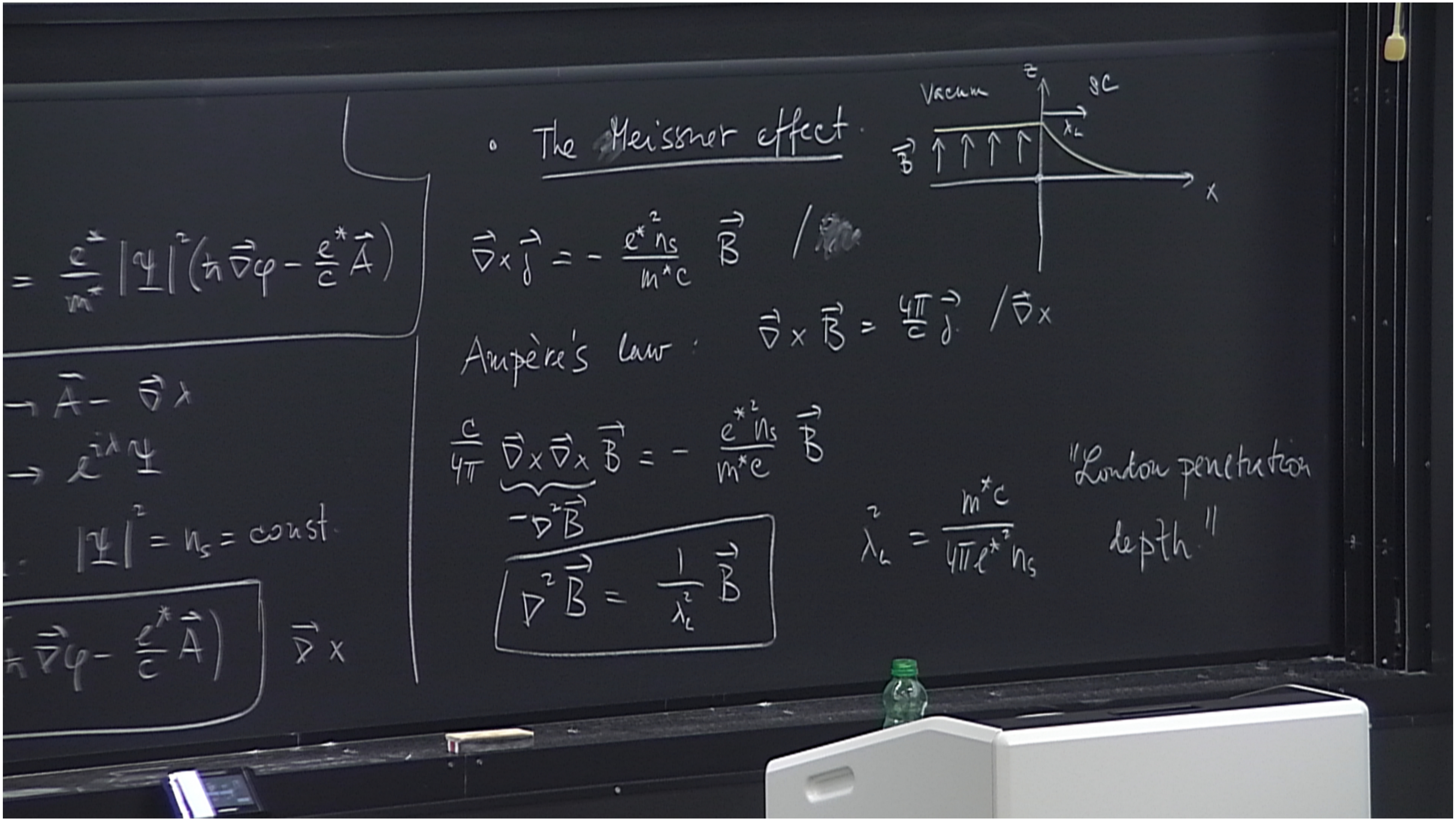
$\varphi(2\pi)-\varphi(0)$

$$\boxed{\Phi = 2\pi s\,\frac{\hbar c}{e^*} = \Phi_0\, s\,,\quad s\in\mathbb{Z}}$$

$\Psi = |\Psi|\, e^{i\varphi(\vec{r})}$ - single valued

$\Phi_0 = \frac{hc}{2e}$ "SC flux quantum"

"Little-Parks experiment"



$= \frac{e^*}{m^*}|\Psi|^2(\hbar\vec\nabla\varphi - \frac{e^*}{c}\vec A)$
$\to \vec A - \vec\nabla\lambda$
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"London penetration depth"