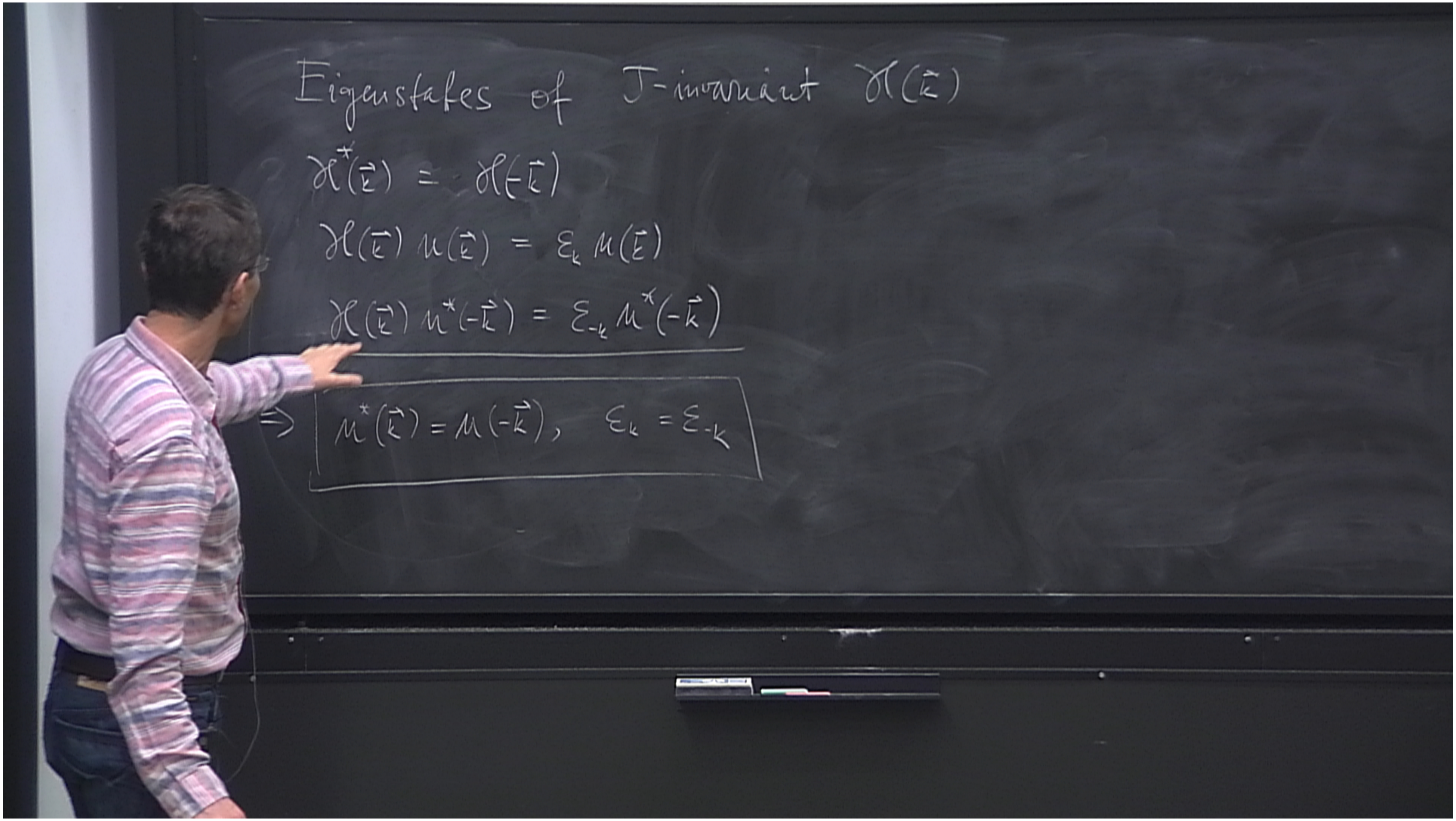
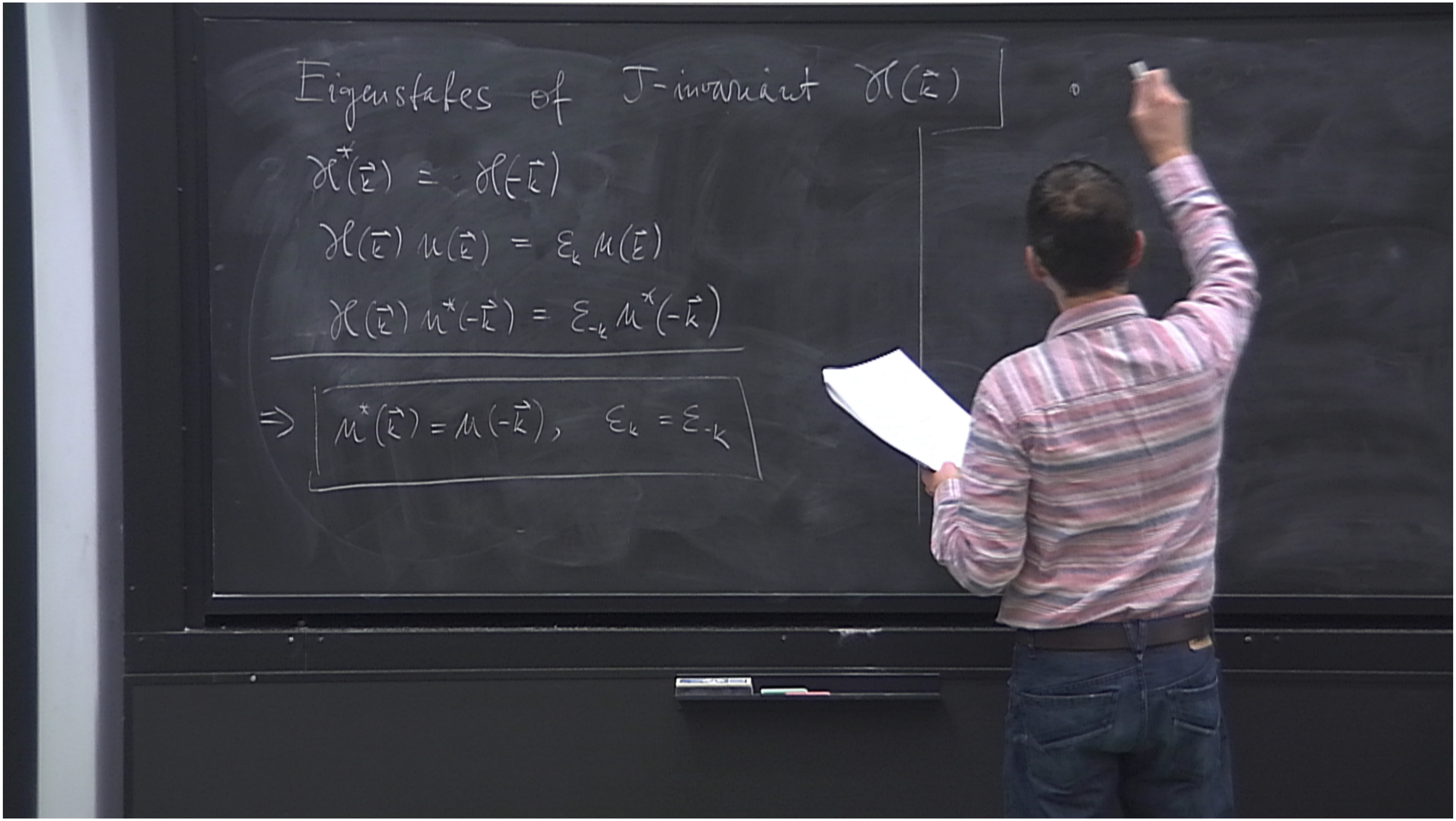
Title: 14/15 PSI - Condensed Matter-Lecture 13

Date: Nov 26, 2014 10:45 AM

URL: http://pirsa.org/14110041

Abstract:


Eigenstates of T-invariant $\mathcal{H}(\vec{k})$
$\mathcal{H}^*(\vec{k}) = \mathcal{H}(-\vec{k})$
$\mathcal{H}(\vec{k})\, u(\vec{k}) = \varepsilon_k\, u(\vec{k})$
$\mathcal{H}(\vec{k})\, u^*(-\vec{k}) = \varepsilon_{-k}\, u^*(-\vec{k})$
$\Rightarrow \boxed{u^*(\vec{k}) = u(-\vec{k}), \quad \varepsilon_k = \varepsilon_{-k}}$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$\Rightarrow \boxed{u^*(\vec{k}) = u(-\vec{k}), \quad \varepsilon_k = \varepsilon_{-k}}$

$\gamma(\vec{k})$

- Vanishing σ_{xy} and the Chern #
for $\mathcal{T}$-invariant systems.

Berry curvature:

$$\mathcal{F}_{ij}(-\vec{k}) = -i\left\{\langle \partial_i u(-\vec{k}) | \partial_j u(-\vec{k})\rangle - (i \leftrightarrow j)\right\}$$
$$= -i\left\{\langle \partial_i u^*(\vec{k}) | \partial_j u^*(\vec{k})\rangle - (i \leftrightarrow j)\right\}$$

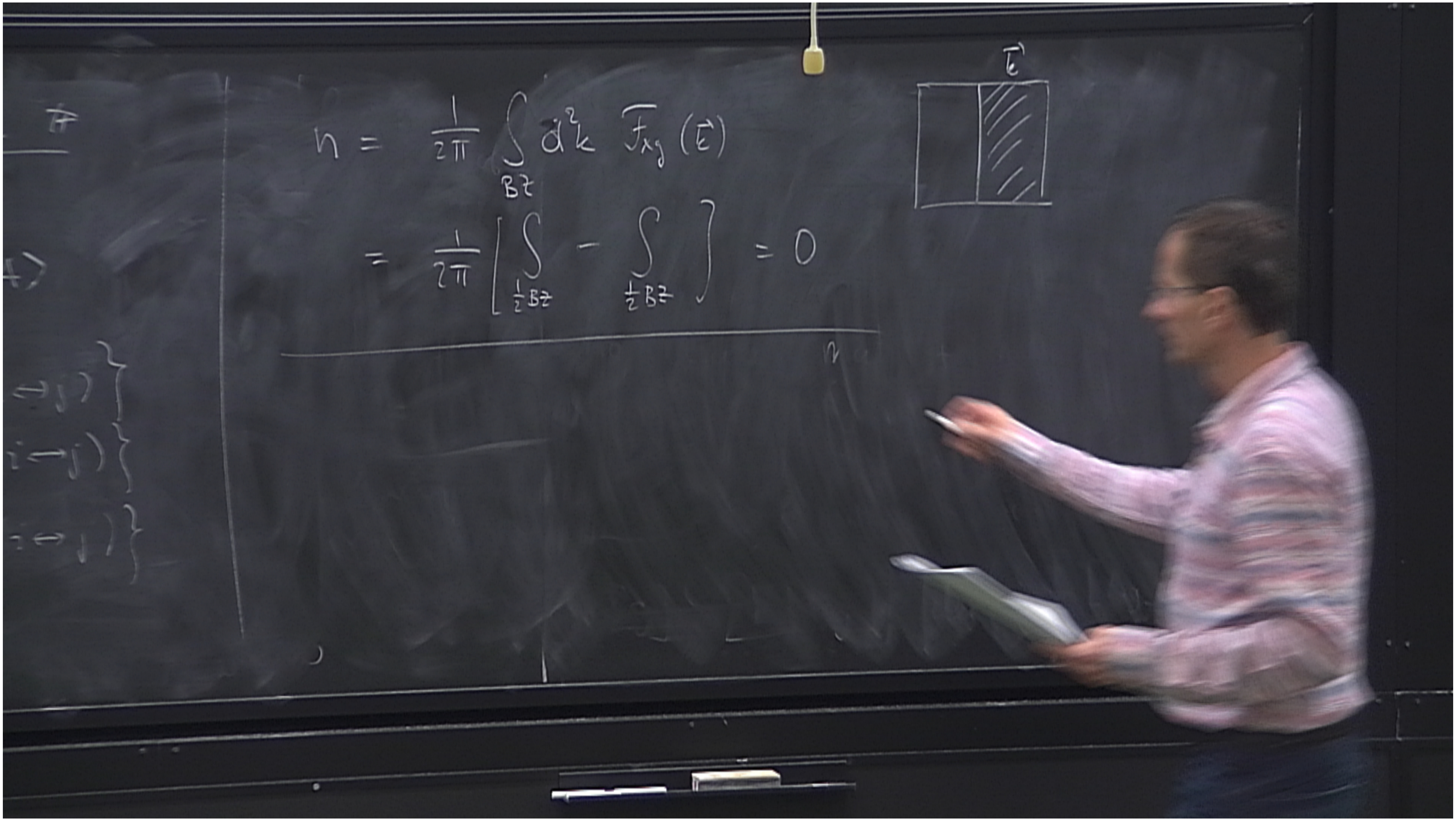
$(\vec{k})$

- Vanishing σ_{xy} and the Chern #
for T-invariant systems.

$$\langle A|B\rangle^* = \langle B|A\rangle$$

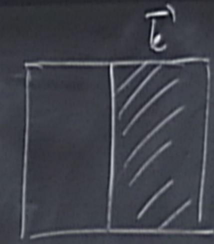
Berry curvature:

$$\begin{aligned}
\mathcal{F}_{ij}(-\vec{k}) &= -i\left\{\langle \partial_i u(-\vec{k}) | \partial_j u(-\vec{k})\rangle - (i\leftrightarrow j)\right\} \\
&= -i\left\{\langle \partial_i u^*(\vec{k}) | \partial_j u^*(\vec{k})\rangle - (i\leftrightarrow j)\right\} \\
&= -i\left\{\langle \partial_j u(\vec{k}) | \partial_i u(\vec{k})\rangle - (i\leftrightarrow j)\right\} \\
&= -\mathcal{F}_{ij}(\vec{k})
\end{aligned}$$


$n = \frac{1}{2\pi} \int_{BZ} d^2k \; F_{xy}(\vec{k})$
$= \frac{1}{2\pi} \left[\int_{\frac{1}{2}BZ} - \int_{\frac{1}{2}BZ} \right] = 0$
$\vec{k}$

$$n = \frac{1}{2\pi}\int_{BZ} d^2k \; \mathcal{F}_{xy}(\vec{k})$$

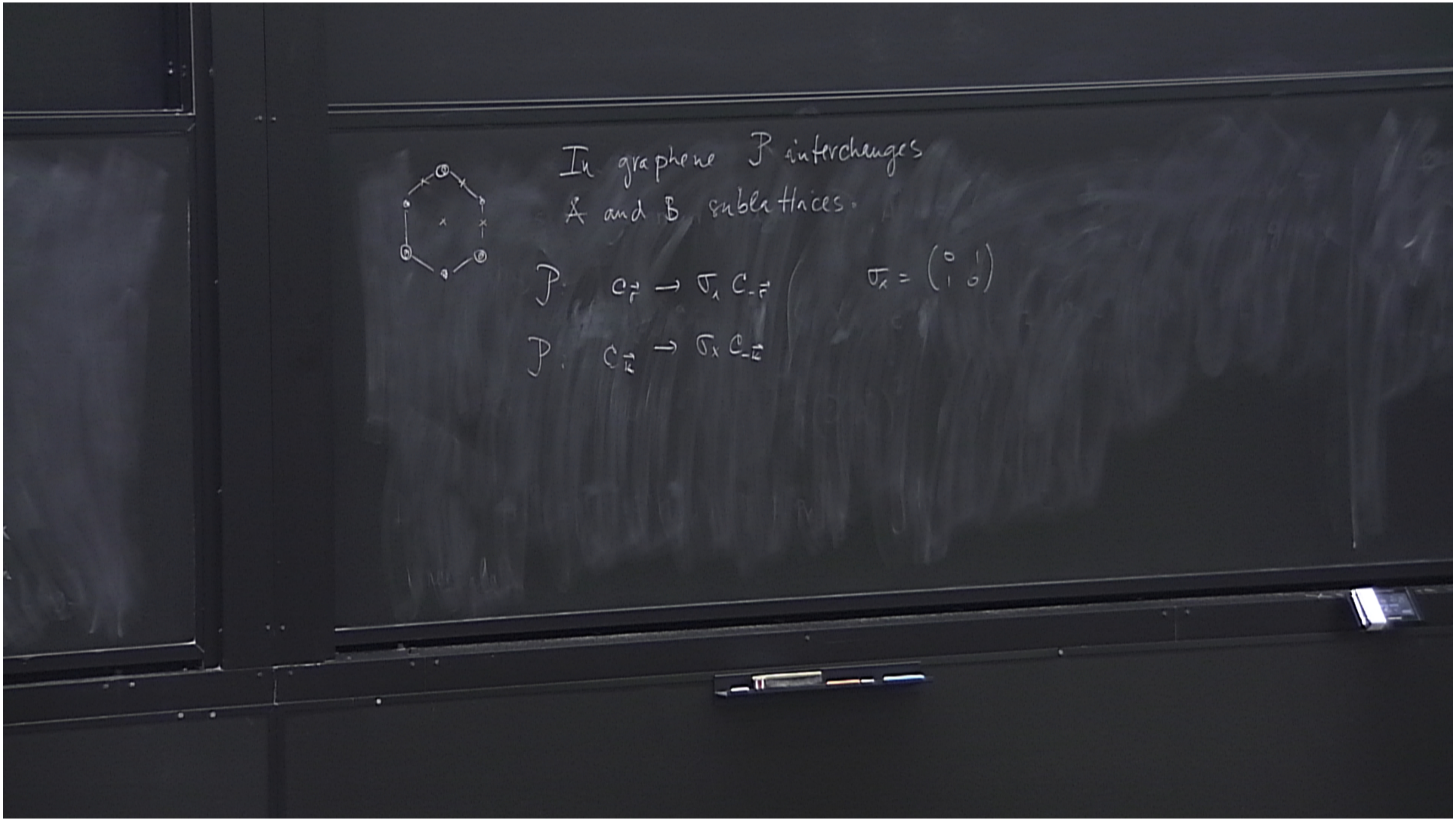
$$= \frac{1}{2\pi}\left[\int_{\frac{1}{2}BZ} - \int_{\frac{1}{2}BZ}\right] = 0$$

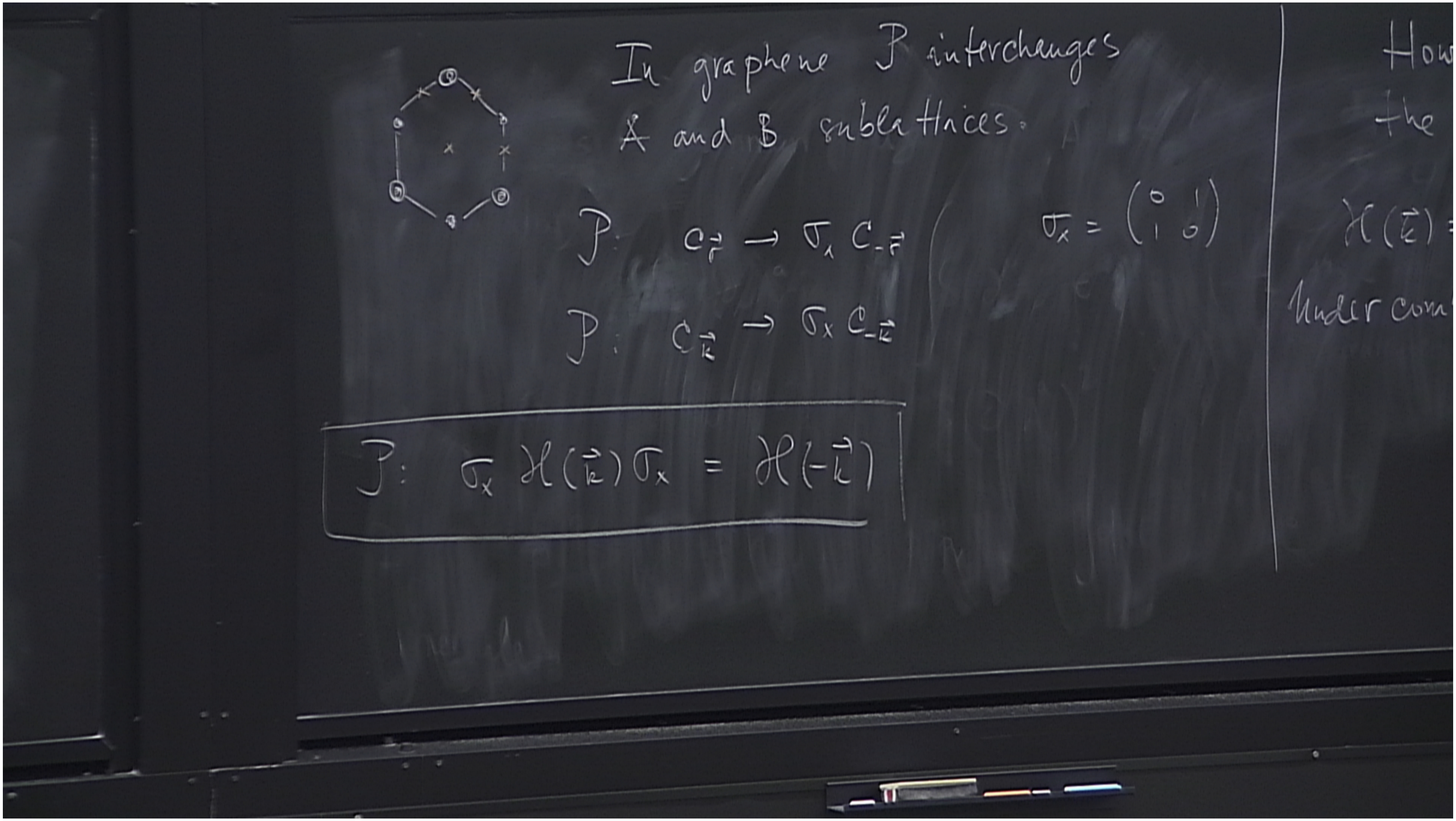


② Inversion symmetry

$P: \vec{r} \to -\vec{r}, \quad \vec{p} \to -\vec{p}$ $\Rightarrow$ no complex conjugation

Unitary: $P[\vec{r}, \vec{p}]P = [\vec{r}, \vec{p}]$


In graphene P interchanges
A and B sublattices.
$P: c_{\vec r} \to \sigma_x c_{-\vec r}$
$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$P: c_{\vec k} \to \sigma_x c_{-\vec k}$


In graphene $\mathcal{P}$ interchanges
A and B sublattices.
$\mathcal{P}: \; c_{\vec{r}} \rightarrow \sigma_x c_{-\vec{r}}$
$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
$\mathcal{P}: \; c_{\vec{k}} \rightarrow \sigma_x c_{-\vec{k}}$
$\mathcal{P}: \; \sigma_x \mathcal{H}(\vec{k}) \sigma_x = \mathcal{H}(-\vec{k})$
How
the
$\mathcal{H}(\vec{k})$
under com

How do T and P constrain the form of graphene H?

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\mathcal{H}(\vec{k}) = \vec{d}(\vec{k}) \cdot \vec{\sigma}$$

Under combination of T and P

...ges

$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

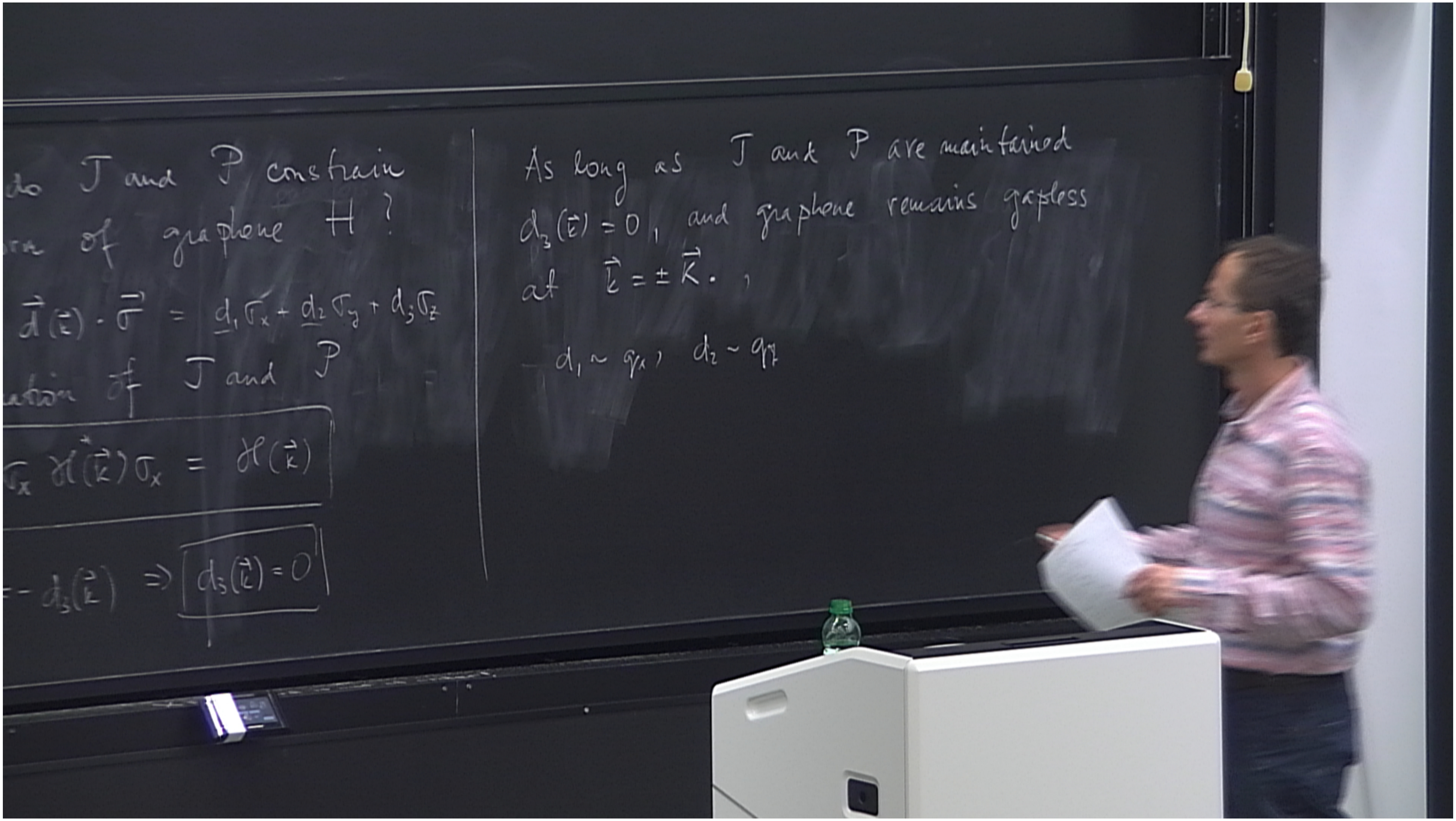
How do $\mathcal{T}$ and $\mathcal{P}$ constrain the form of graphene H?

$$\mathcal{H}(\vec{k}) = \vec{d}(\vec{k}) \cdot \vec{\sigma} = \underline{d_1}\,\sigma_x + \underline{d_2}\,\sigma_y + d_3\,\sigma_z$$

Under combination of $\mathcal{T}$ and $\mathcal{P}$

$$\boxed{\mathcal{T}\mathcal{P}: \quad \sigma_x \mathcal{H}^*(\vec{k})\,\sigma_x = \mathcal{H}(\vec{k})}$$

$$d_3(\vec{k}) = -\,d_3(\vec{k}) \Rightarrow \boxed{d_3(\vec{k}) = 0}$$


do T and P constrain
rm of graphene H ?
d(k) · σ = d1 σx + d2 σy + d3 σz
ation of T and P
σx H*(k) σx = H(k)
- d3(k) => d3(k) = 0
As long as T and P are maintained
d3(k) = 0, and graphene remains gapless
at k = ± K.
d1 ~ qx, d2 ~ qy

$- d_z \sigma_z$

$d_3(\vec{k}) = 0$, and graphene ...

at $\vec{k} = \pm \vec{\underline{\underline{K}}}$.

$d_1 \sim q_x$, $d_2 \sim q_y$

C_3 rotation symmetry further restricts Dirac points to be exactly at $\pm \vec{K}$

$$\Rightarrow \boxed{u^*(\vec{k}) = u(-\vec{k}), \quad \varepsilon_k = \varepsilon_{-k}}$$

$$= -i\{\langle \partial_j u(\vec{k}) | \partial_i u(\vec{k})\rangle - (\ldots$$

$$= -\mathcal{F}_{ij}(\vec{k})$$

Action of $\mathcal{T}$ & $\mathcal{P}$ in low-E theory

$$\left.\begin{aligned} \vec{k} &= \vec{K} + \vec{q} \quad (\text{valley } 1) \\ \vec{k} &= -\vec{K} + \vec{q} \quad (\text{valley } 2) \end{aligned}\right\} \text{Valleys are exchanged under both } \mathcal{T} \text{ \& } \mathcal{P}.$$

$$\boxed{\begin{aligned} \mathcal{T}: &\quad \tau_x \mathcal{H}^*_{eff}(\vec{q})\, \tau_x = \mathcal{H}_{eff}(-\vec{q}) \\ \mathcal{P}: &\quad \tau_x \sigma_x \mathcal{H}_{eff}(\vec{q})\, \sigma_x \tau_x = \mathcal{H}_{eff}(-\vec{q}) \end{aligned}}$$

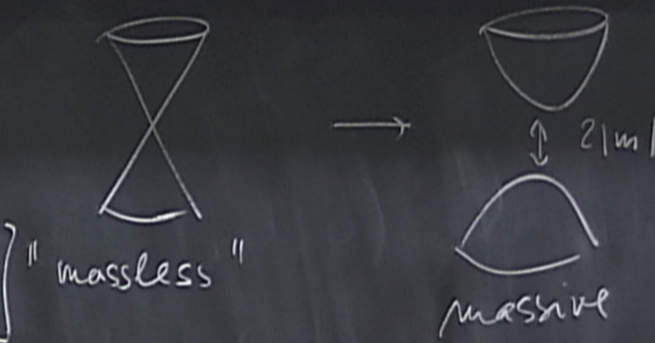
Action of $\mathcal{T}$ & $\mathcal{P}$ in low-$\underline{E}$ theory

$$\left.\begin{array}{ll} \vec{k} = \vec{K} + \vec{q} & \text{(valley 1)} \\ \vec{k} = -\vec{K} + \vec{q} & \text{(valley 2)} \end{array}\right\}$$ Valleys are exchanged undr both $\mathcal{T}$ & $\mathcal{P}$.

$$\boxed{\begin{array}{ll} \mathcal{T}: & \tau_x \, \mathcal{H}^*_{eff}(\vec{q}) \, \tau_x = \mathcal{H}_{eff}(-\vec{q}) \\ \mathcal{P}: & \tau_x \sigma_x \, \mathcal{H}_{eff}(\vec{q}) \, \sigma_x \tau_x = \mathcal{H}_{eff}(-\vec{q}) \end{array}}$$

• Mass terms in graphene

① Semenoff mass [PRL 53, 2449 (1984)]



$$\delta \mathcal{H}_s = \sigma_z m_s$$

$$T: \quad \tau_x \, \delta \mathcal{H}_s^* \, \tau_x = \delta \mathcal{H}_s \quad ✓$$

$$P: \quad \tau_x \sigma_x \, \delta \mathcal{H}_s \, \sigma_x \tau_x = -\delta \mathcal{H}_s \quad \times$$

Spectrum:

$$\mathcal{H}_{eff} = v_F(\tau_z \sigma_x q_x + \sigma_y q_y) + \sigma_z m_s$$

$$\mathcal{H}_{eff}^2 = v_F(q_x^2 + q_y^2) + m_s^2$$

$$E_q = \pm\sqrt{v_F^2 q^2 + m_s^2}$$

• Mass terms in graphene

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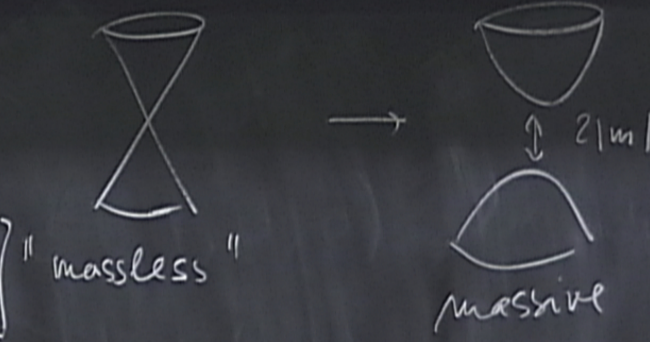

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$$\mathcal{T}: \quad \tau_x \, \delta\mathcal{H}_s^* \, \tau_x = \delta\mathcal{H}_s \quad ✓$$

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Spectrum:

$$\mathcal{H}_{eff} = v_F(\tau_z \sigma_x q_x + \sigma_y q_y) + \sigma_z m_s$$

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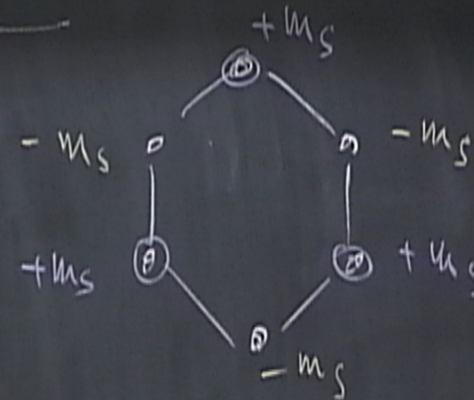
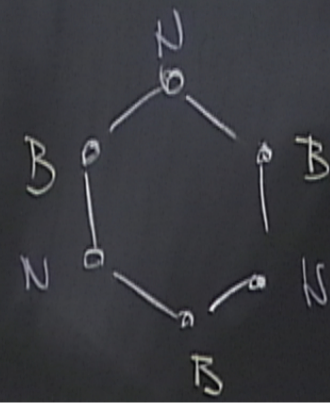
$$E_q = \pm\sqrt{v_F^2 q^2 + m_s^2}$$

Physical realization

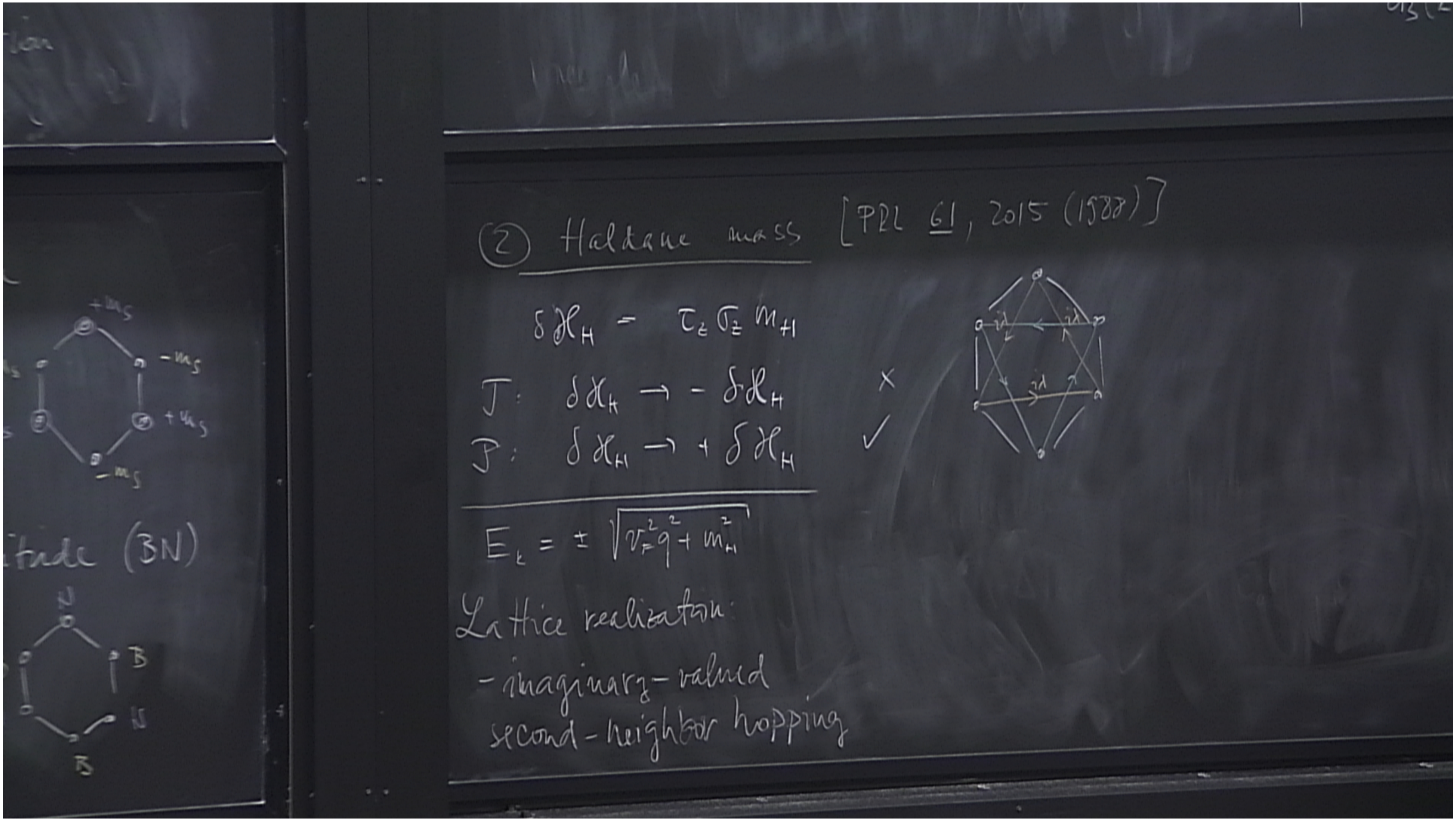
- staggered on-site potential



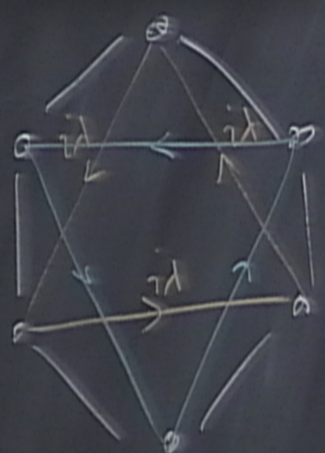
- realized by growing graphene on boron-nitride (BN) substrate



$+ \sigma_z m_s$


② Haldane mass [PRL 61, 2015 (1988)]
$\delta\mathcal{H}_H = \tau_z \sigma_z m_H$
$T: \delta\mathcal{H}_H \to -\delta\mathcal{H}_H$ ✗
$P: \delta\mathcal{H}_H \to +\delta\mathcal{H}_H$ ✓
$E_k = \pm\sqrt{v_F^2 q^2 + m_H^2}$
Lattice realization:
- imaginary-valued
second-neighbor hopping
$i\lambda$
$+m_s$
$-m_s$
(BN)
B
N

PRL 61, 2015 (1988)]



$M_H = 3\sqrt{3}\,\lambda_H$

The Chern

$$n = \frac{1}{2\pi}\int_{BZ} d^2k \; \mathcal{F}_{xy}(\vec{k})$$

$$\mathcal{F}_{ij} = \frac{1}{2}\,\hat{d}\cdot\left(\partial_i \hat{d} \times \partial_j \hat{d}\right)$$

$$= \frac{1}{2d^3}\,\vec{d}\cdot\left(\partial_i \vec{d} \times \partial_i \vec{d}\right)$$

The Chern #

$$n = \frac{1}{2\pi}\int_{BZ} d^2k \; \mathcal{F}_{xy}(\vec{k})$$

$$\mathcal{F}_{ij} = \frac{1}{2}\hat{d}\cdot\left(\partial_i \hat{d} \times \partial_j \hat{d}\right)$$

$$= \frac{1}{2d^3}\,\underline{\vec{d}\cdot\left(\partial_i \vec{d} \times \partial_i \vec{d}\right)}$$

Calculation for a single Dirac point.

$$\mathcal{H}_{eff}(\vec{q}) = \vec{d}(\vec{q})\cdot\vec{\sigma}$$

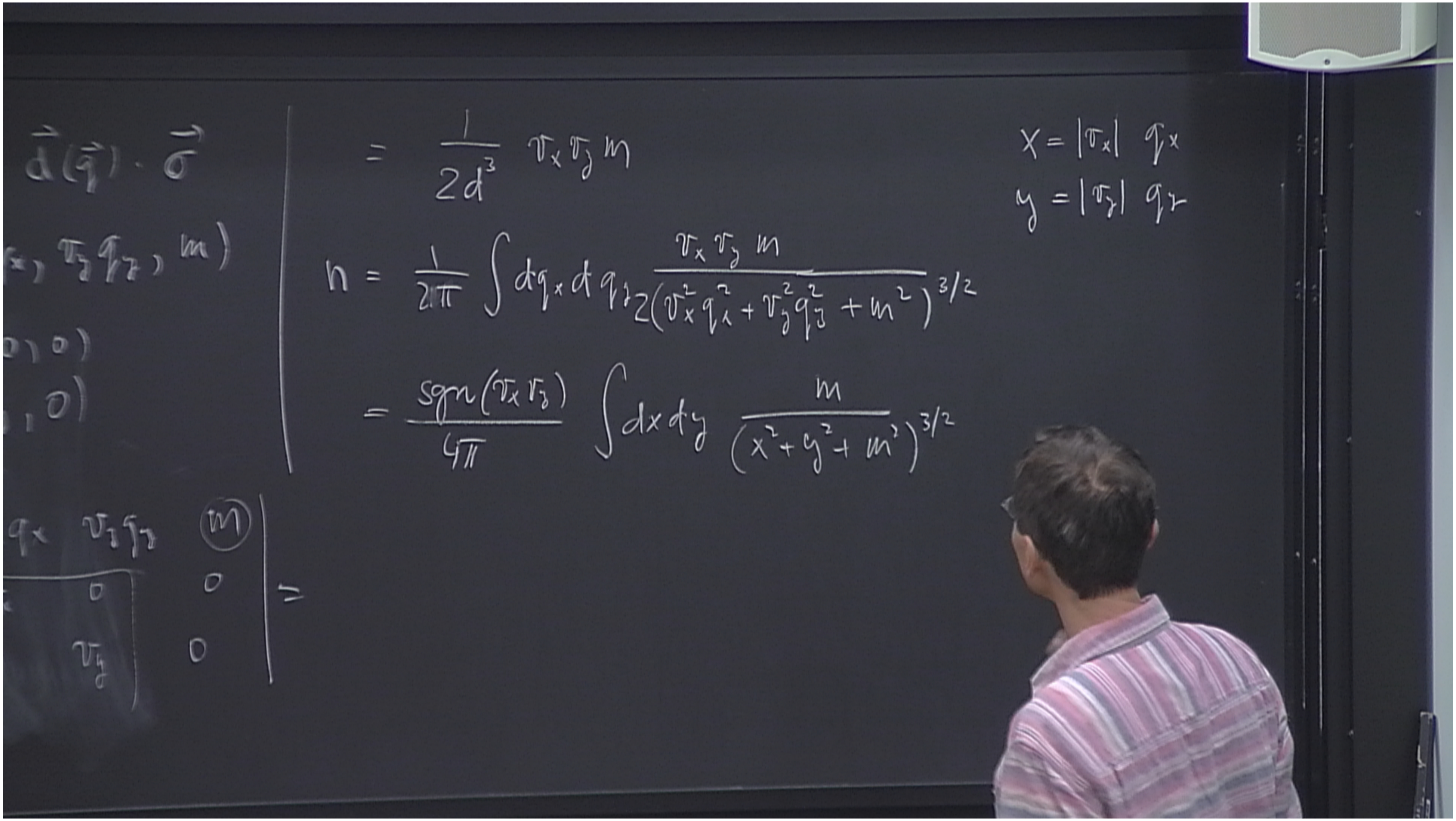
$$\vec{d}(\vec{q}) = (v_x q_x,\ v_y q_y,\ m)$$

$$\partial_x \vec{d} = (v_x, 0, 0)$$

$$\partial_y \vec{d} = (0, v_y, 0)$$

$$\partial_t \vec{d} = 0$$

$$\mathcal{F}_{xy} = \frac{1}{2d^3}\begin{vmatrix} v_x q_x & v_y q_y & m \\ v_x & 0 & 0 \\ 0 & v_y & 0 \end{vmatrix}$$


$\vec{d}(\vec{q})\cdot\vec{\sigma}$
$= \frac{1}{2d^3} v_x v_z m$
$n = \frac{1}{2\pi}\int dq_x\, dq_z \frac{v_x v_z m}{2(v_x^2 q_x^2 + v_z^2 q_z^2 + m^2)^{3/2}}$
$= \frac{\mathrm{sgn}(v_x v_z)}{4\pi}\int dx\, dy \frac{m}{(x^2+y^2+m^2)^{3/2}}$
$x = |v_x|\, q_x$
$y = |v_z|\, q_z$

- Semenoff:

Valley 1: $v_x = v_y = v_F$, $m = +m_S$

Valley 2: $-v_x = v_y = v_F$, $m = +m_S$

$$\left.\begin{array}{l} \text{Valley 1} \\ \text{Valley 2} \end{array}\right\} \quad \frac{1}{2}\,\mathrm{sgn}(v_F^2 m_S) + \frac{1}{2}\,\mathrm{sgn}(-v_F^2 m_S) = 0 \ \checkmark$$

- Haldane mass

1: $v_x = v_y = v_F$, $m = m_H$

2: $-v_x = v_y = v_F$, $m = -m_H$

- Semenoff:

Valley 1: $v_x = v_y = v_F\,, \quad m = +m_S$

Valley 2: $-v_x = v_y = v_F\,, \quad m = +m_S$

$$\tfrac{1}{2}\,\mathrm{sgn}(v_F^2 m_S) + \tfrac{1}{2}\,\mathrm{sgn}(-v_F^2 m_S) = 0 \ \checkmark$$

- Haldane mass

1: $v_x = v_y = v_F\,, \quad m = m_H$

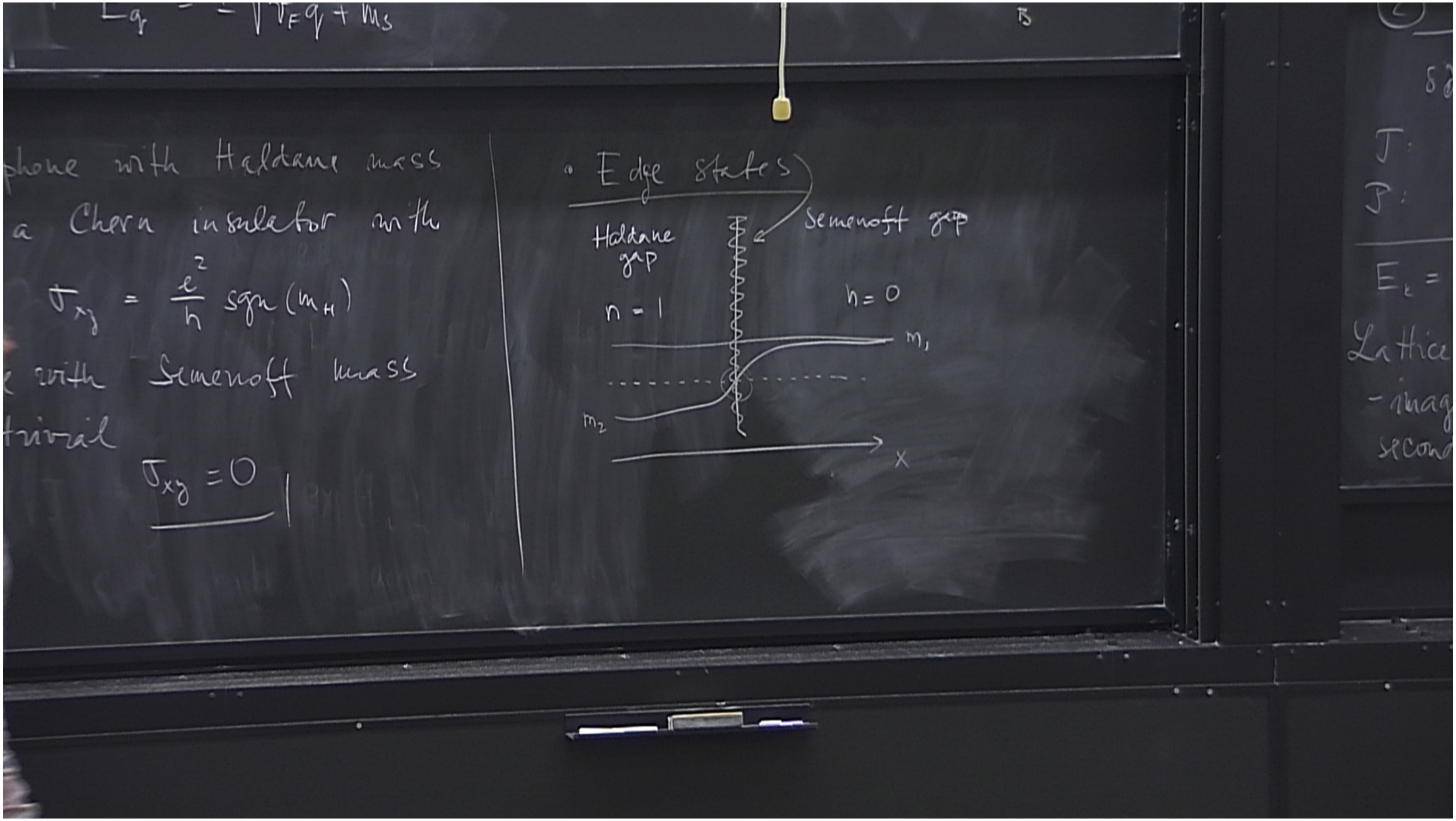
2: $-v_x = v_y = v_F\,, \quad m = -m_H$

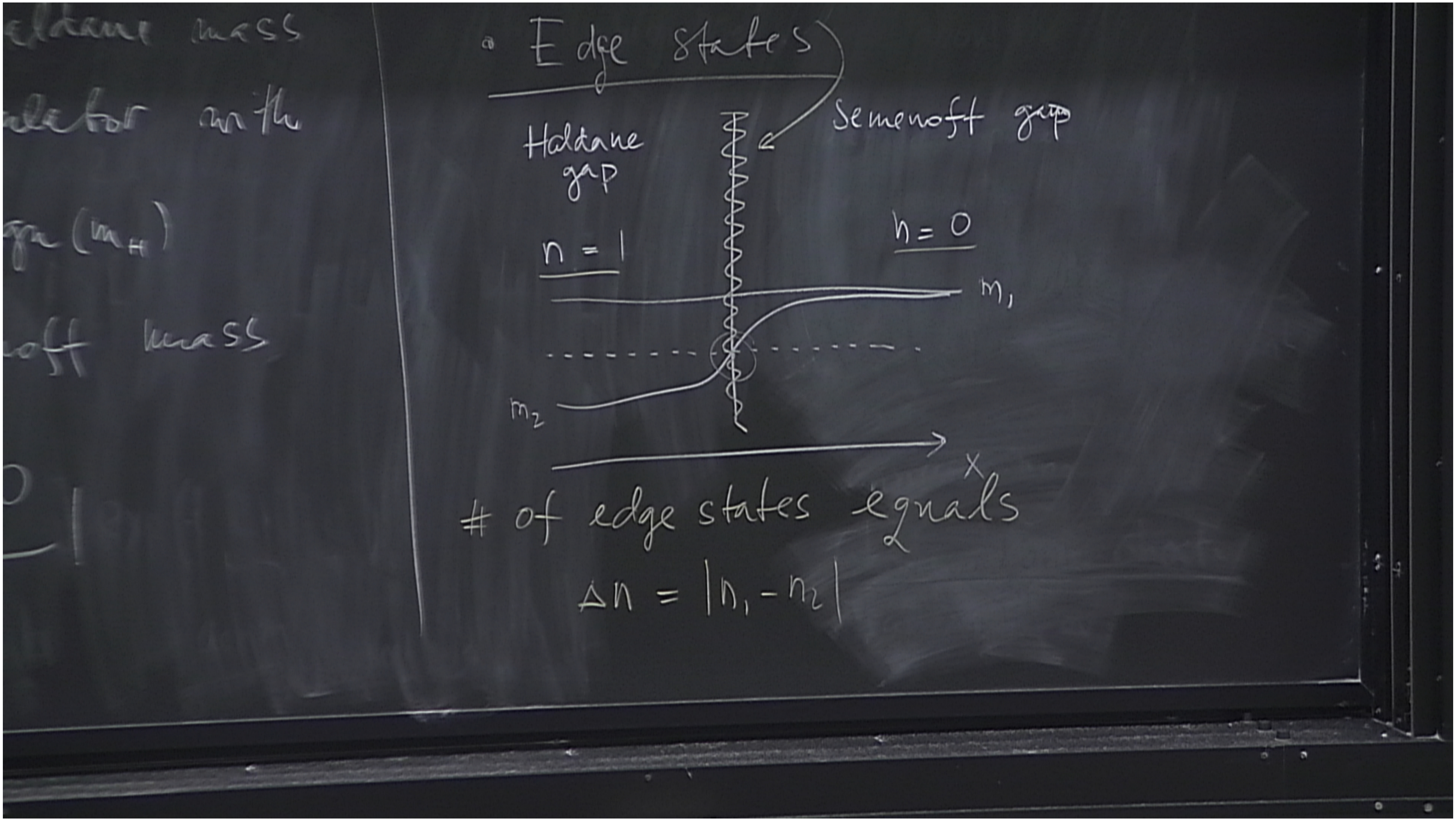
$$\underbrace{\tfrac{1}{2}\,\mathrm{sgn}(v_F^2 m_H) + \tfrac{1}{2}\,\mathrm{sgn}(v_F^2 m_H) = \mathrm{sgn}(m_H) = \pm 1}$$

Graphene
is a

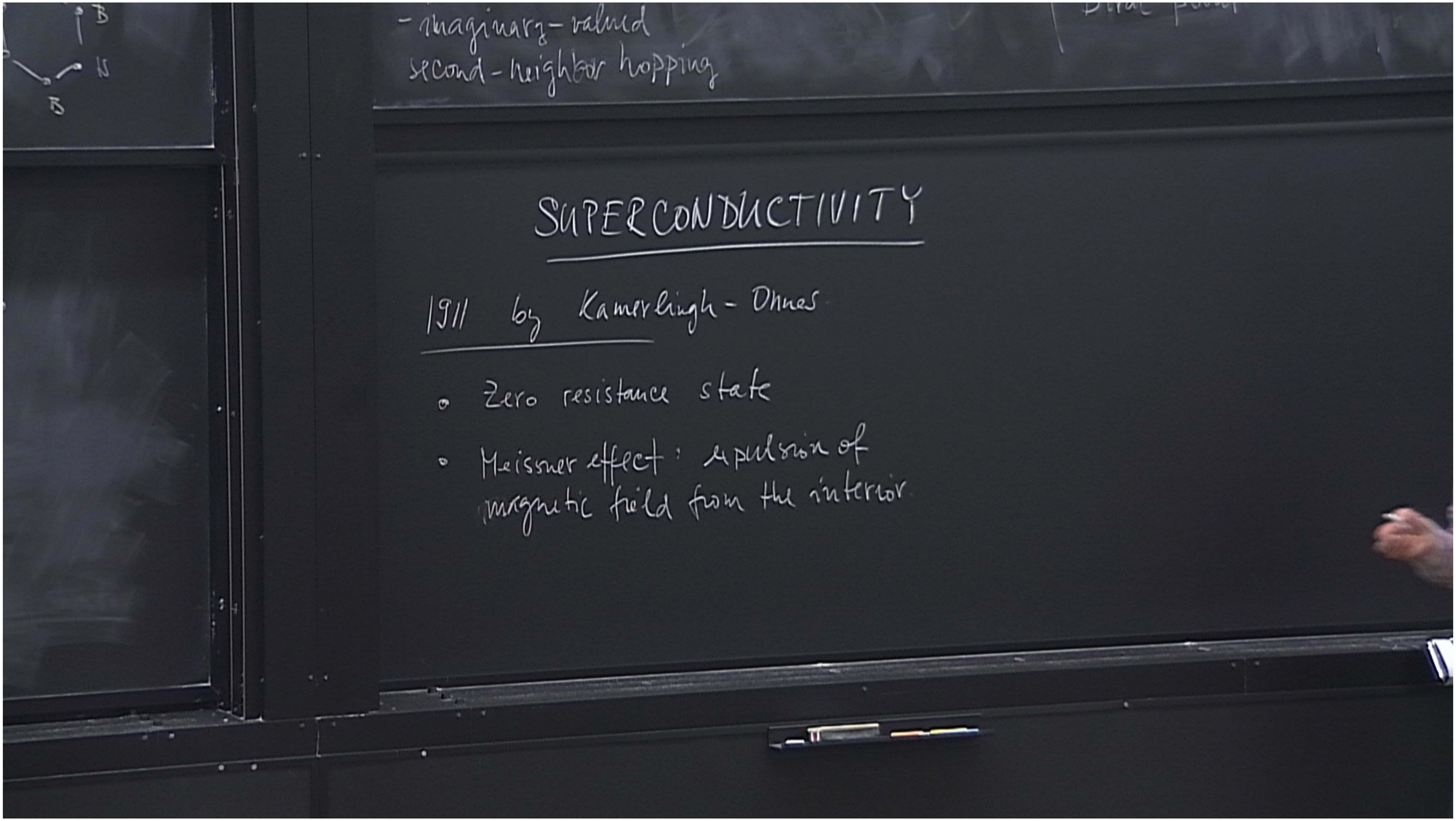
v_x

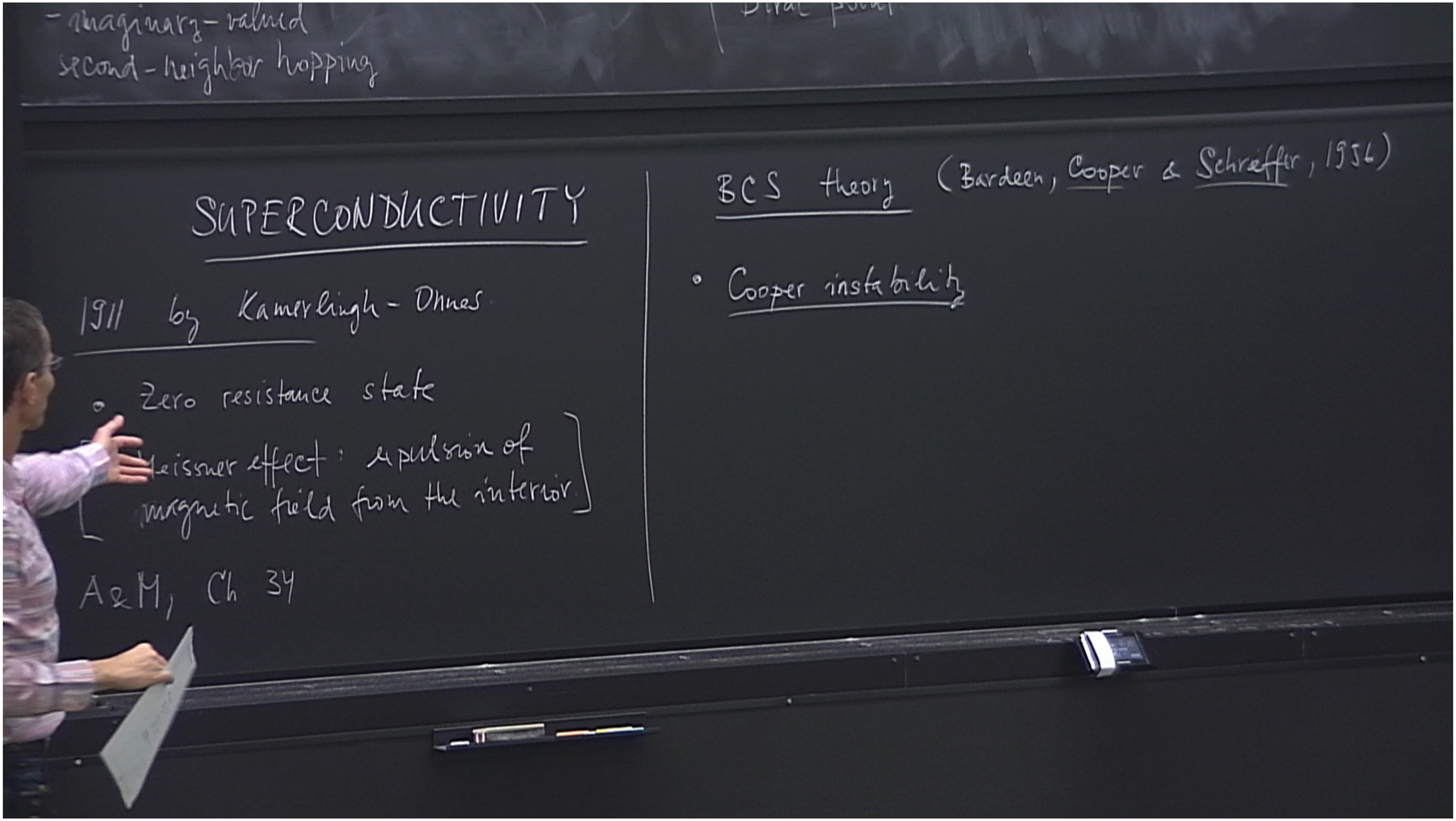
while with
is trivia


phone with Haldane mass
a Chern insulator with
$\sigma_{xy} = \frac{e^2}{h} \, \mathrm{sgn}(m_H)$
with Semenoff mass
trivial
$\sigma_{xy} = 0$
Edge states
Haldane gap
Semenoff gap
n = 1
n = 0
m_1
m_2
x
J:
P:
Lattice
- imag
second



Edge states
Semenoft gap
Haldane gap
n = 1
n = 0
m1
m2
x
# of edge states equals
Δn = |n1 - n2|
mass
mass


- imaginary-valued
second-neighbor hopping
B
N
B
SUPERCONDUCTIVITY
1911 by Kamerlingh-Onnes
• Zero resistance state
• Meissner effect: expulsion of magnetic field from the interior



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SUPERCONDUCTIVITY
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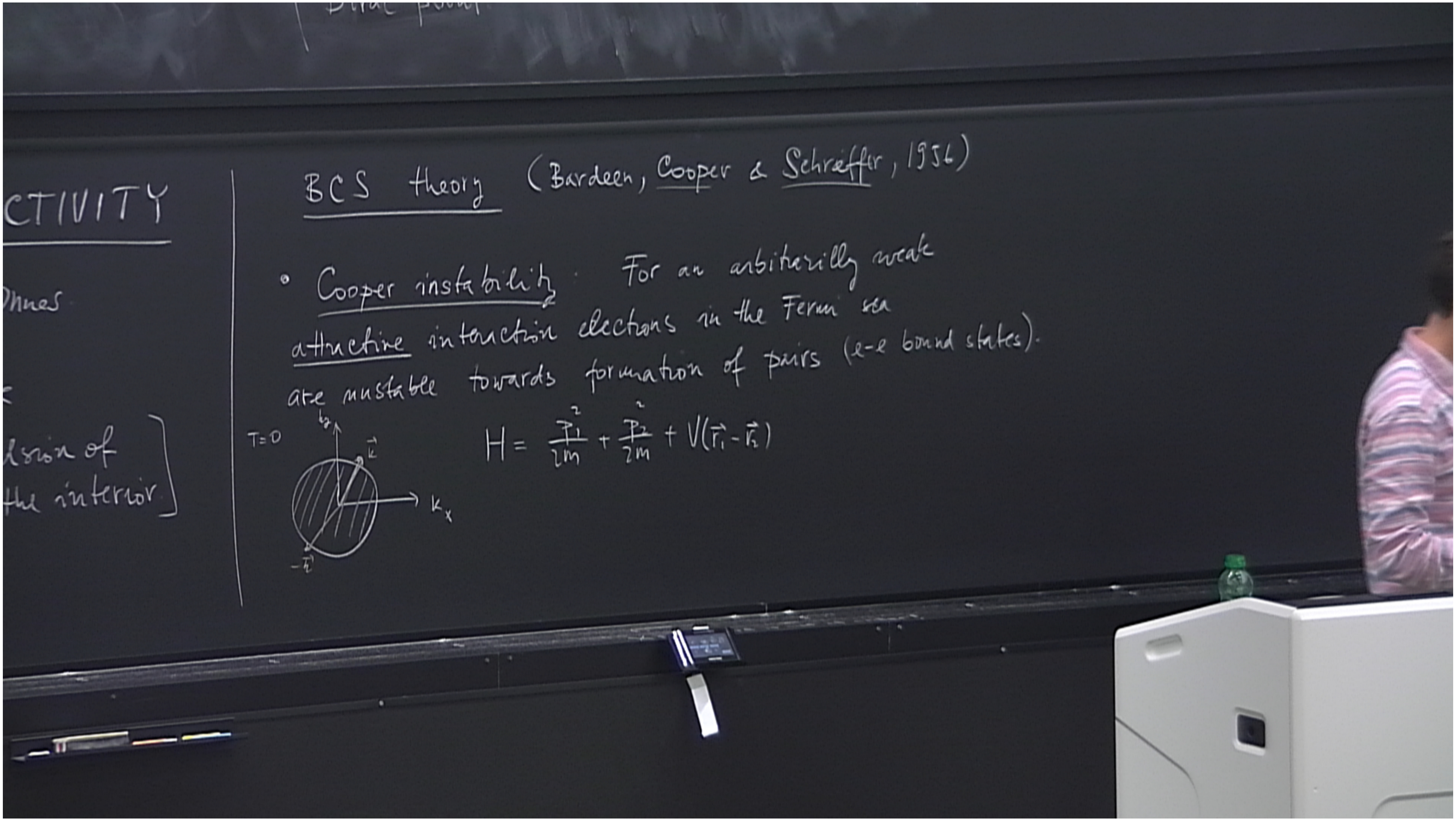
A&M, Ch 34
BCS theory (Bardeen, Cooper & Schrieffer, 1956)
• Cooper instability


CTIVITY
Onnes
sion of
the interior
BCS theory (Bardeen, Cooper & Schreffer, 1956)
• Cooper instability: For an arbitrarily weak attractive interaction electrons in the Fermi sea are unstable towards formation of pairs (e-e bound states).
T=0
k_y
$\vec{k}$
k_x
$-\vec{k}$
$H = \frac{p_1^2}{2m} + \frac{p_2^2}{2m} + V(\vec{r}_1 - \vec{r}_2)$

TY

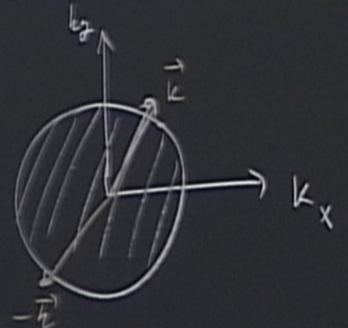
rtor.

BCS theory (Bardeen, Cooper & Schrieffer, 1956)

Assume:

- Cooper instability: For an arbitrarily weak attractive interaction electrons in the Fermi sea are unstable towards formation of pairs (e-e bound states).

T=0



$$H = \frac{\vec{p}_1^{\,2}}{2m} + \frac{\vec{p}_2^{\,2}}{2m} + V(\vec{r}_1 - \vec{r}_2)$$

$$\psi_0(\vec r_1, \vec r_2) = \sum_{|k|>k_F} g_k \, e^{i\vec k\cdot\vec r_1} e^{-i\vec k\cdot\vec r_2} \, (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$$

Fermi statistics implies $g_k = g_{-k}$

$$H\psi_0 = E\psi_0 \Rightarrow \boxed{(E - 2\varepsilon_k) g_k = \sum_{|k'|>k_F} V_{kk'} g_{k'}}$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \quad V_{kk'} = \frac{1}{\Omega}\int d^3r \, V(\vec r)\, e^{i(\vec k' - \vec k)\cdot\vec r}$$

Following Cooper take: $V_{kk'} = \begin{cases} -V, & \text{when } |\varepsilon_k - E_F|, |\varepsilon_{k'} - E_F| < \hbar\omega_c \\ 0, & \text{otherwise} \end{cases}$

$$= \mathrm{sgn}(m_H) = \pm 1$$

$$\sigma_{xy} = 0$$

$$\psi_0(\vec{r}_1 \vec{r}_2) = \sum_{|k|>k_F} g_k \, e^{i\vec{k}\cdot\vec{r}_1} e^{-i\vec{k}\cdot\vec{r}_2} \, (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$$

Fermi statistics implies $g_k = g_{-k}$

$$H\psi_0 = E\psi_0 \Rightarrow \boxed{(E - 2\varepsilon_k) g_k = \sum_{|k'|>k_F} V_{kk'} g_{k'}}$$

$$\varepsilon_k = \frac{\hbar^2 k^2}{2m}, \quad V_{kk'} = \frac{1}{\Omega} \int d^3r \, V(\vec{r}) \, e^{i(\vec{k}'-\vec{k})\cdot\vec{r}}$$

Following Cooper take: $V_{kk'} = \begin{cases} -V, & \text{when } |\varepsilon_k - E_F|, |\varepsilon_{k'} - E_F| < \hbar\omega_c \\ 0, & \text{otherwise} \end{cases}$

$$(E - 2\varepsilon_k) g_k = -V \sum_{k'}{}' g_{k'}$$

$'$: $|k'| > k_F$, $|\varepsilon_{k'} - E_F| < \hbar\omega_c$

$$g_k = V \frac{\sum' g_{k'}}{2\varepsilon_k - E} \quad / \sum_k{}'$$

$$\Rightarrow \boxed{\frac{1}{V} = \sum_k{}' \frac{1}{2\varepsilon_k - E}}$$

Solve for E

$$(E - 2\varepsilon_k) g_k = -V \sum_{k'}^{①} g_{k'} \qquad |k'| > k_F \qquad |\varepsilon_{k'} - E_F| < \hbar\omega_c$$

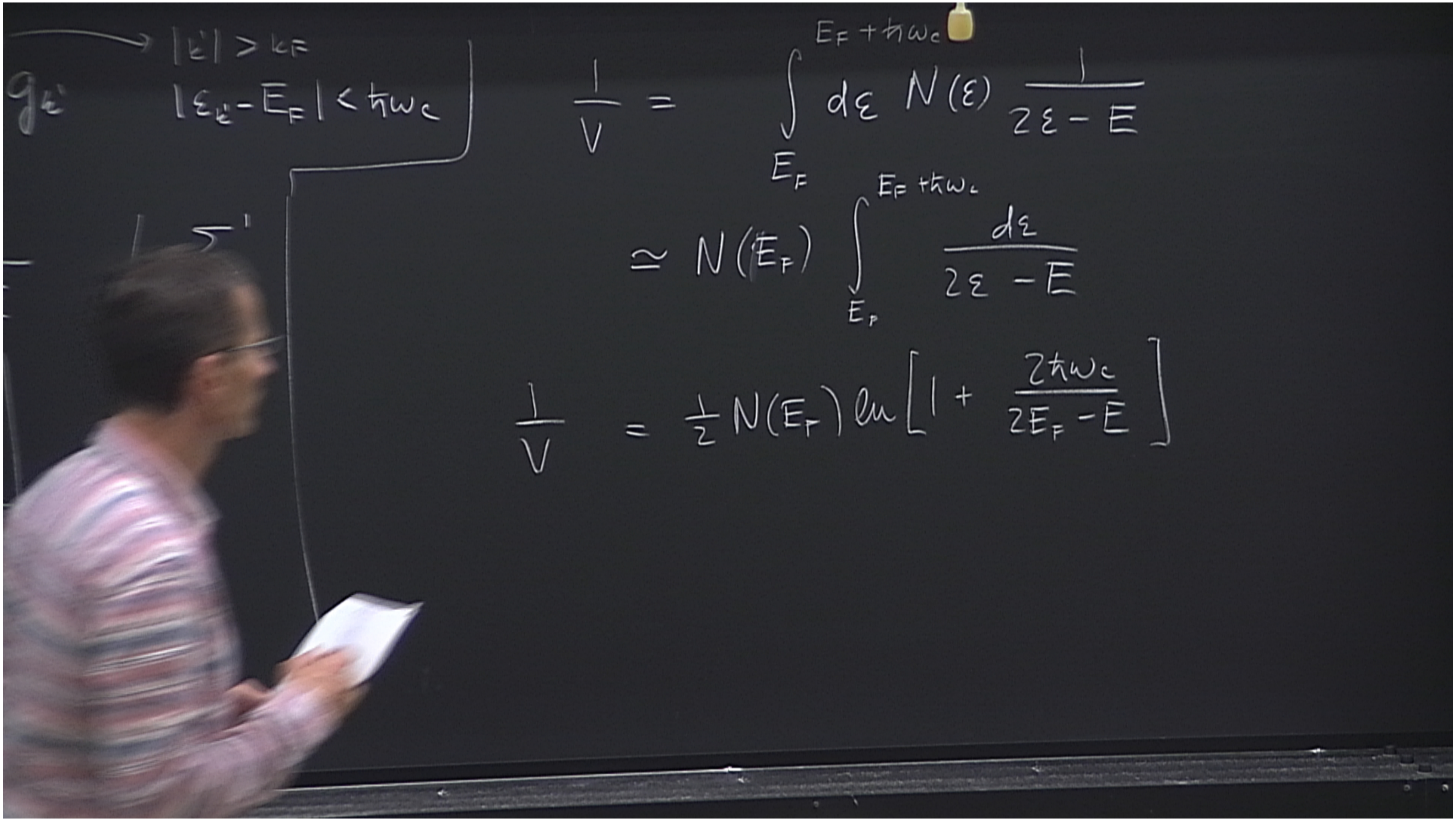
$$g_k = V \frac{\sum_{k'}' g_{k'}}{2\varepsilon_k - E} \quad / \sum_k'$$

$$\Rightarrow \boxed{\frac{1}{V} = \sum_k' \frac{1}{2\varepsilon_k - E}}$$

Solve for E.

$g_{k'}$

$-E_F|, |\varepsilon_{k'} - E_F| < \hbar\omega_c$



$|k'| > k_F$
$g_{k'} \quad |\varepsilon_{k'} - E_F| < \hbar\omega_c$
$\frac{1}{V} = \int_{E_F}^{E_F + \hbar\omega_c} d\varepsilon \, N(\varepsilon) \frac{1}{2\varepsilon - E}$
$\simeq N(E_F) \int_{E_F}^{E_F + \hbar\omega_c} \frac{d\varepsilon}{2\varepsilon - E}$
$\frac{1}{V} = \frac{1}{2} N(E_F) \ln\left[1 + \frac{2\hbar\omega_c}{2E_F - E}\right]$

$\longrightarrow |k'| > k_F$

$g_{k'} \qquad |\varepsilon_{k'} - E_F| < \hbar\omega_c$

$\Big| \quad {\sum_k}'$

$$\frac{1}{V} = \int_{E_F}^{E_F+\hbar\omega_c} d\varepsilon \, N(\varepsilon) \frac{1}{2\varepsilon - E}$$

$$\simeq N(E_F) \int_{E_F}^{E_F+\hbar\omega_c} \frac{d\varepsilon}{2\varepsilon - E}$$

$$\frac{1}{V} = \frac{1}{2} N(E_F) \ln\left[1 + \frac{2\hbar\omega_c}{2E_F - E}\right]$$

$$\cancel{1+} \frac{2\hbar\omega_c}{2E_F - E} = e^{2/VN(E_F)}$$

Weak coupling assumption

$$V N(E) \ll 1$$

$$\boxed{E \simeq 2E_F - 2\hbar\omega_c e^{-\frac{2}{VN(E_F)}}}$$

- bound state $E < 2E_F$
- non-perturbative in V