Title: 14/15 PSI - Condensed Matter-Lecture 11

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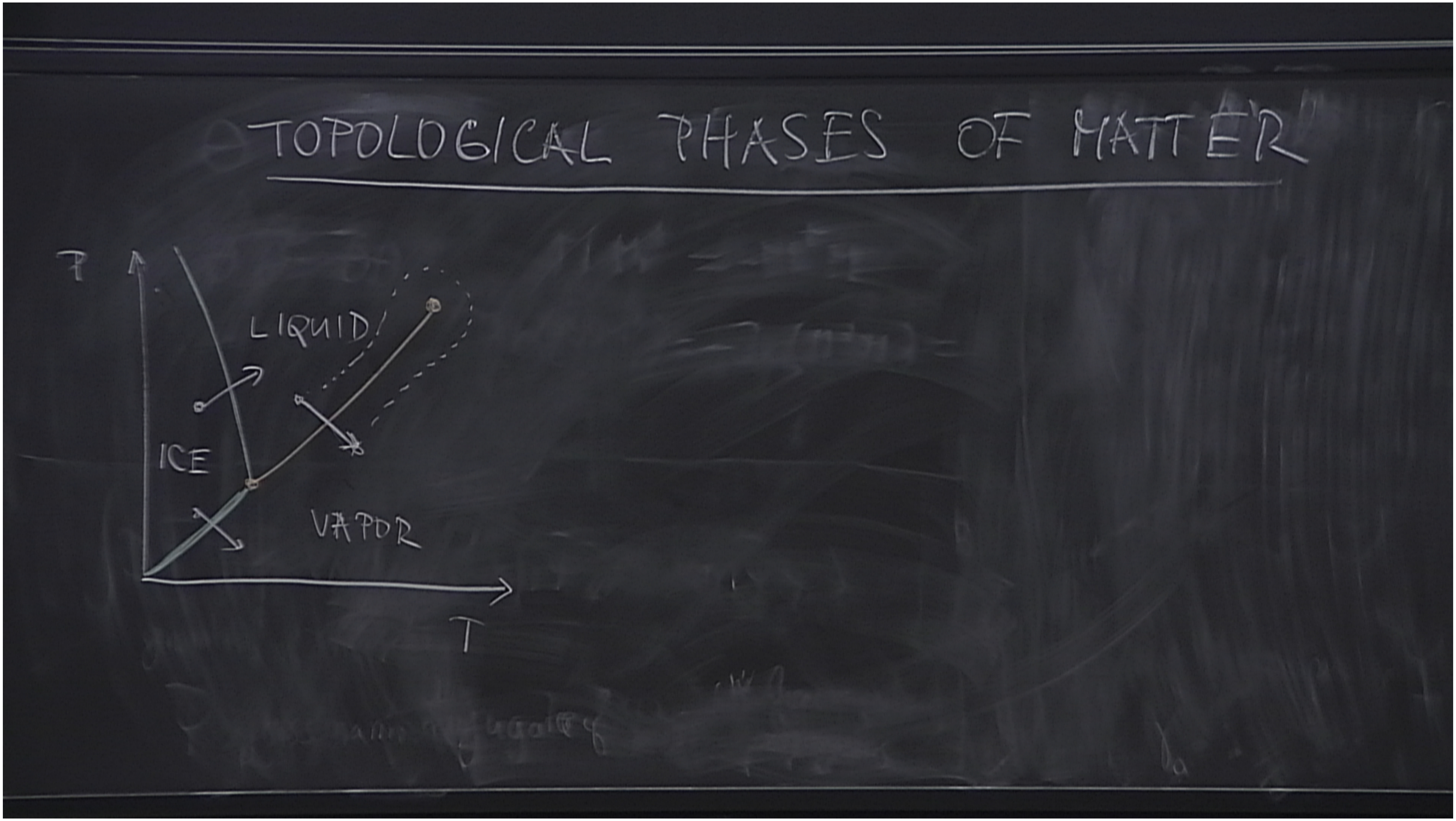
Abstract:


TOPOLOGICAL PHASES OF MATTER
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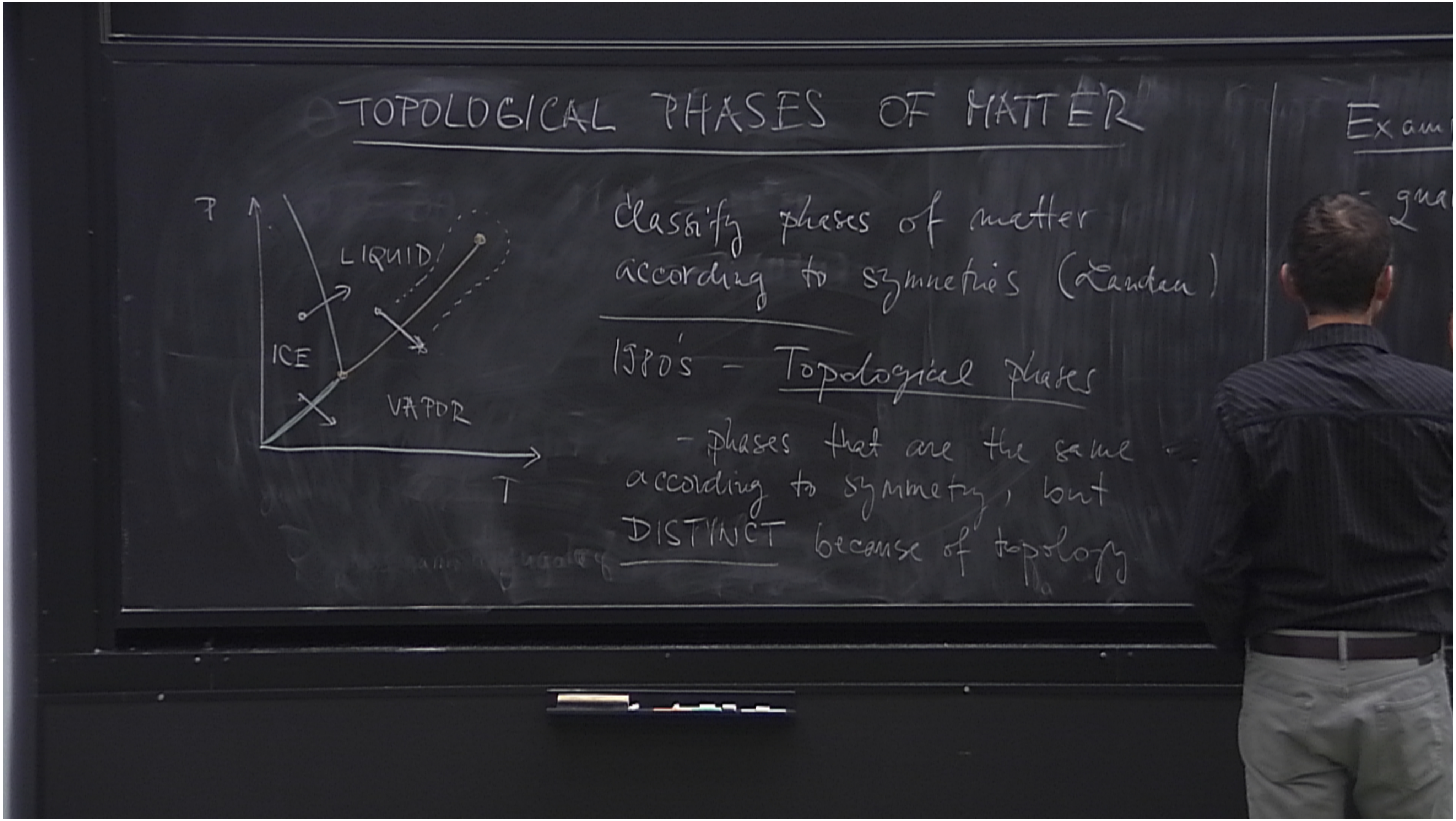
TOPOLOGICAL PHASES OF MATTER
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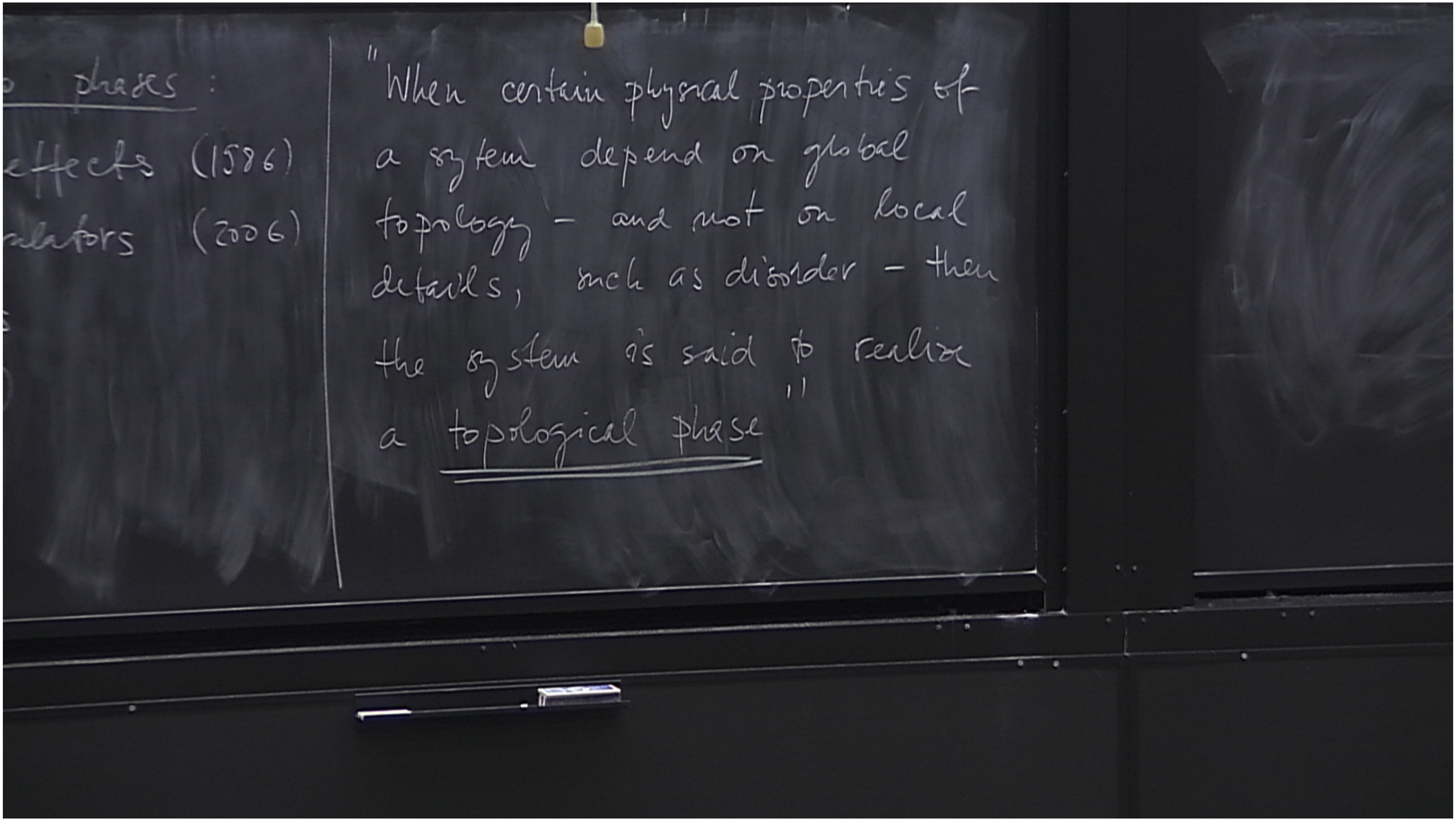
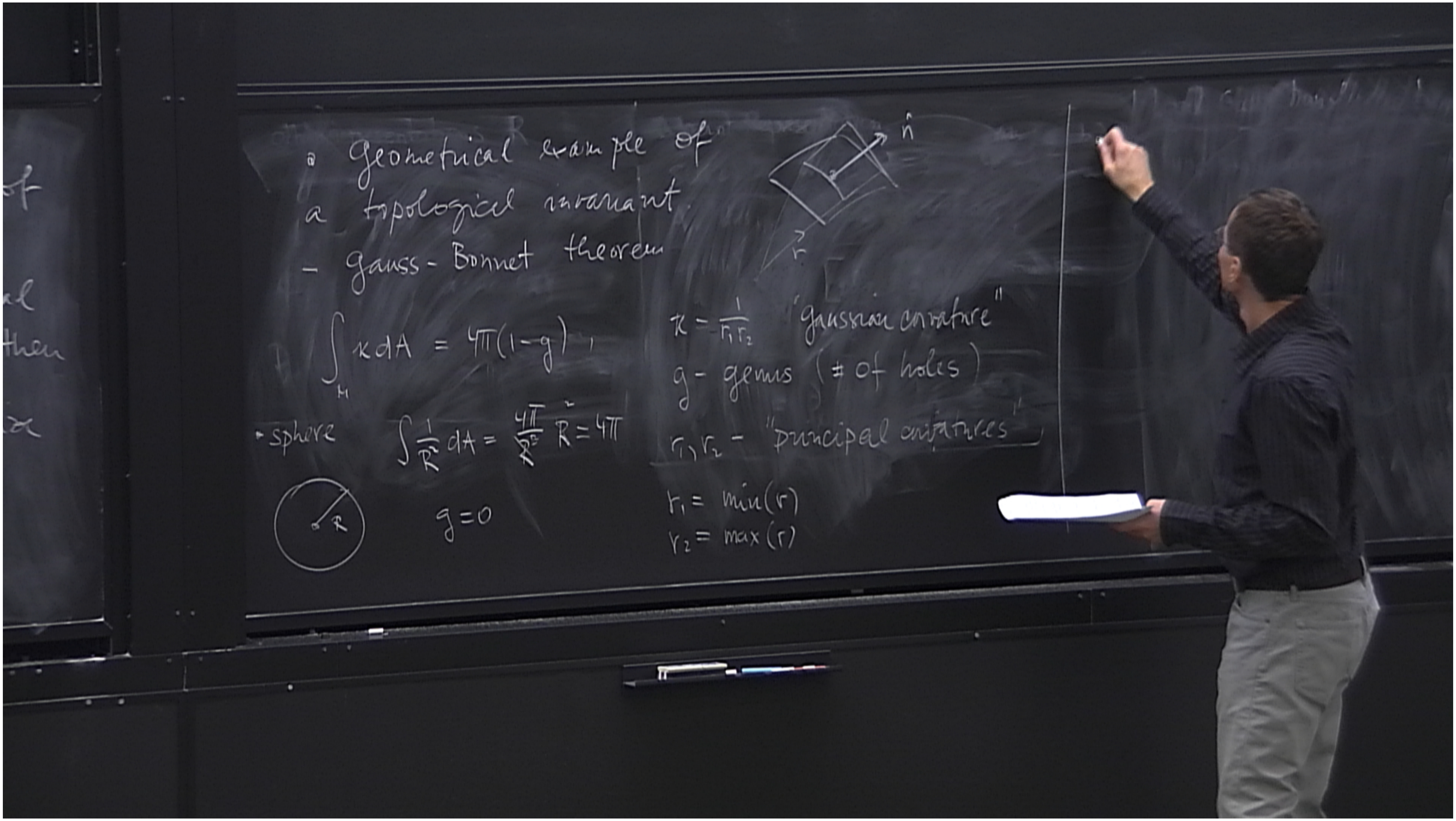
Classify phases of matter according to symmetries (Landau)
1980's – Topological phases
– phases that are the same according to symmetry, but DISTINCT because of topology

Examples of topo phases :

- quantum Hall effects (1986)
- topological insulators (2006)
- superconductors


phases :
"When certain physical properties of
a system depend on global
topology - and not on local
details, such as disorder - then
the system is said to realize
a topological phase"


Geometrical example of
a topological invariant
- Gauss-Bonnet theorem
∫_M κ dA = 4π(1-g),
sphere
∫ 1/R² dA = 4π/R² R² = 4π
g=0
R
n̂
r⃗
κ = 1/(r₁r₂) "gaussian curvature"
g - genus (# of holes)
r₁, r₂ - "principal curvatures"
r₁ = min(r)
r₂ = max(r)

Torus $(g=1)$

$$\int \kappa \, dA = 0$$

ture"

s)

Topological band theory

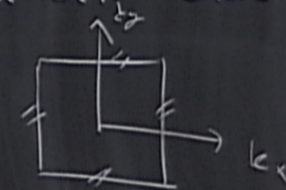
Bloch eq: $$H(\vec{k})\,|u_n(\vec{k})\rangle = E_n(\vec{k})\,|u_n(\vec{k})\rangle$$

$$|\psi_n(\vec{k})\rangle = e^{i\vec{k}\cdot\vec{r}}\,|u_n(\vec{k})\rangle$$

$$H(\vec{k}+\vec{G}) = H(\vec{k}), \quad \vec{G} \in \text{rec latt vectors}$$

$$|u_n(\vec{k}+\vec{G})\rangle = |u_n(\vec{k})\rangle$$

$$\Longrightarrow \quad \vec{k}+\vec{G} \simeq \vec{k}$$



Brillouin zone is a d-dimensional torus T^d.

use of topology

(1)

rms T^d.

Bloch eq is invariant under global transf

$$|u(\vec{k})\rangle = e^{i\phi}|u(\vec{k})\rangle$$

Local symmetry

$$|u(\vec{k})\rangle = e^{i\phi(\vec{k})}|u(\vec{k})\rangle \quad (2)$$

reminiscent of EM gauge transformation

This invites definition of of the "vector potential" or a "Berry connection"

$$\vec{A}(\vec{k}) = -i\langle u(\vec{k})|\vec{\nabla}_k u(\vec{k})\rangle$$

Under transf. (2) it transforms as

$$\vec{A}(\vec{k}) \to \vec{A}(\vec{k}) + \vec{\nabla}_k \phi_k(\vec{k})$$

$g = 0$

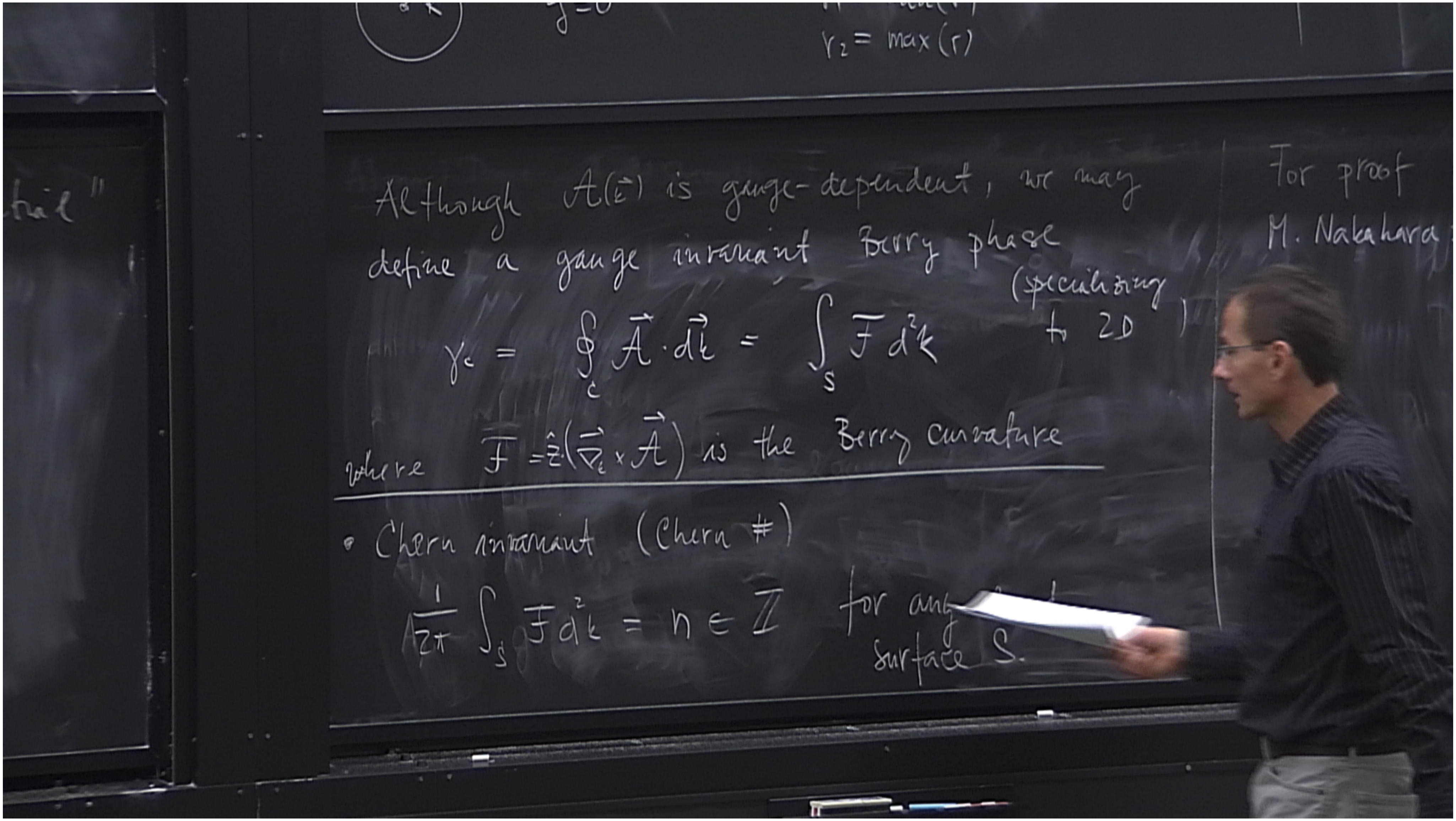
$r_1 = \min(r)$

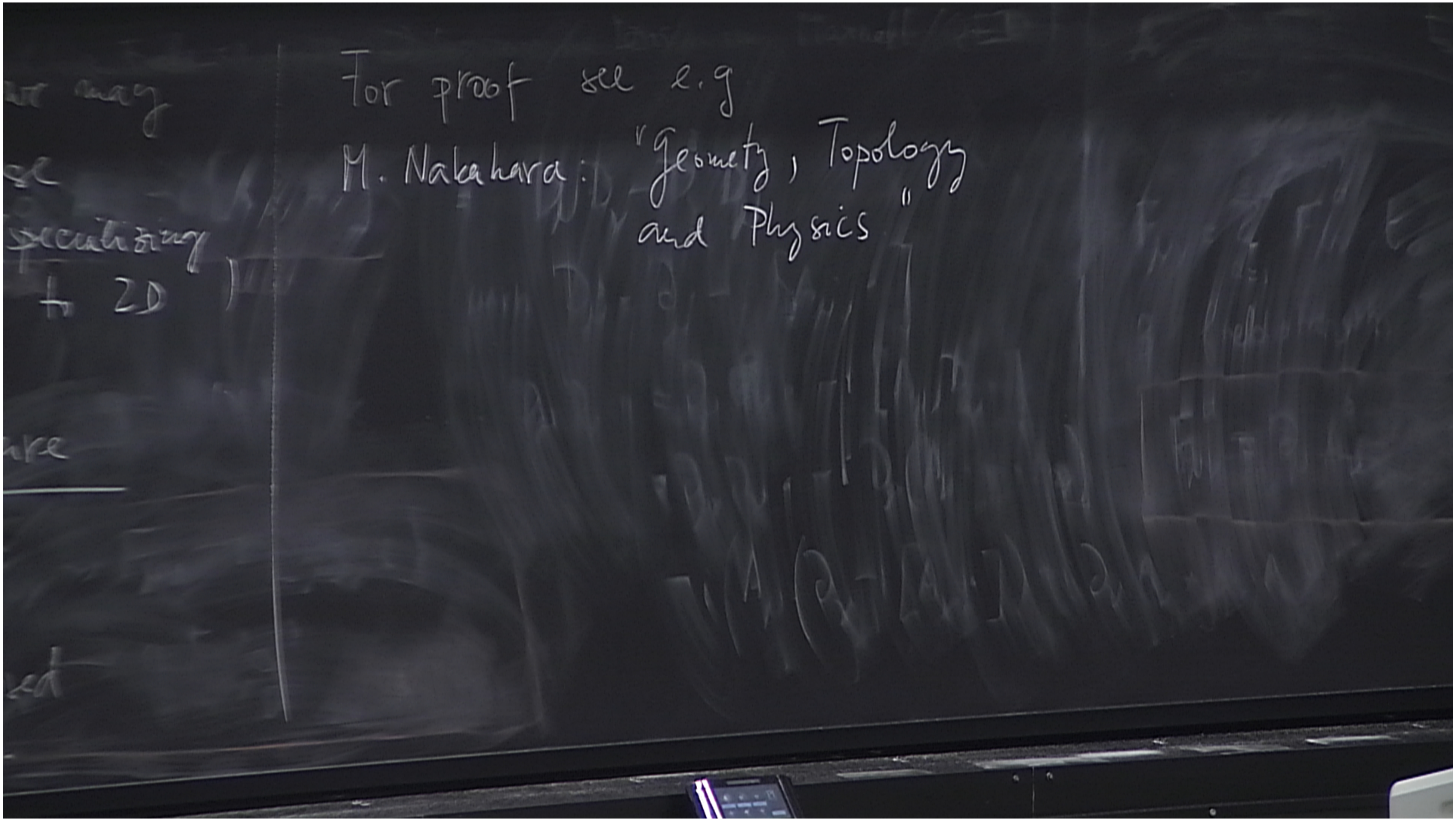
$r_2 = \max(r)$

Although $\mathcal{A}(\vec{k})$ is gauge-dependent, we may define a gauge invariant Berry phase (specializing to 2D)

$$\gamma_c = \oint_c \vec{\mathcal{A}} \cdot d\vec{k} = \int_S \mathcal{F}\, d^2k$$

where $\mathcal{F} = \hat{z}\cdot(\vec{\nabla}_k \times \vec{\mathcal{A}})$ is the Berry curvature


$r_2 = \max(r)$
Although $\mathcal{A}(\vec{k})$ is gauge-dependent, we may
define a gauge invariant Berry phase
(specializing to 2D)
$\gamma_c = \oint_c \vec{A} \cdot d\vec{k} = \int_S \mathcal{F} d^2k$
where $\mathcal{F} = \hat{z} \cdot (\vec{\nabla}_k \times \vec{A})$ is the Berry curvature
• Chern invariant (Chern #)
$\frac{1}{2\pi} \int_S \mathcal{F} d^2k = n \in \mathbb{Z}$ for any
surface S.
For proof
M. Nakahara


For proof see e.g
M. Nakahara: "Geometry, Topology
and Physics"

• **Example:** 2-level Hamiltonian

$$\boxed{H(\vec{k}) = \vec{d}(\vec{k})\cdot\vec{\sigma} + \cancel{\varepsilon(\vec{k})\sigma_0}}\,, \qquad \sigma_0 = \mathbb{1}_{2\times 2}$$

$\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ vector of Pauli matrices

$$\{\sigma_i, \sigma_j\} = 2\delta_{ij}\,, \qquad \sigma_i\sigma_j = i\varepsilon_{ijk}\sigma_k + \delta_{ij}$$

$$H(\vec{k}) = \begin{pmatrix} d_z & d_x - i d_y \\ d_x + i d_y & -d_z \end{pmatrix}$$

Energ

$H^2 =$

$= d$

$E^2(\vec{k})$

$E(\vec{k}$

Energy spectrum

$$H^2 = (d_i\sigma_i)(d_j\sigma_j) = d_i d_j(\sigma_i\sigma_j)$$

$$= d_i d_j(\delta_{ij} + \cancel{i\varepsilon_{ijk}\sigma_k}) = \vec{d}\cdot\vec{d}$$

$$E^2(\vec{k}) = |\vec{d}(\vec{k})|^2$$

$$\underline{\underline{E(\vec{k}) = \pm|\vec{d}(\vec{k})|}}$$

• Berry's

reminiscent of EM gauge transformation

$$\vec{\mathcal{A}}(\vec{k}) \to \vec{\mathcal{A}}(\vec{k}) + \vec{\nabla}_k \phi_k(\vec{k})$$

gy spectrum

$$(d_i \sigma_i)(d_j \sigma_j) = d_i d_j (\sigma_i \sigma_j)$$

$$d_i d_j (\delta_{ij} + i \cancel{\epsilon_{ijk}} \sigma_k) = \vec{d} \cdot \vec{d}$$

$$) = |\vec{d}(\vec{k})|^2$$

$$(\vec{k}) = \pm |\vec{d}(\vec{k})|$$

• Berry's phase

$$\boxed{\gamma_c = \frac{1}{2}\Omega}$$ ← the solid angle swept by $\hat{d}(\vec{k})$

$$\hat{d}(\vec{k}) = \frac{\vec{d}(\vec{k})}{|\vec{d}(\vec{k})|}$$

reminiscent of EM gauge transformation

$$\mathcal{A}(\vec{k}) \to \mathcal{A}(\vec{k}) + \vec{\nabla}_k \phi_k(\vec{k})$$

gy spectrum

$$(d_i\sigma_i)(d_j\sigma_j) = d_i d_j(\sigma_i\sigma_j)$$

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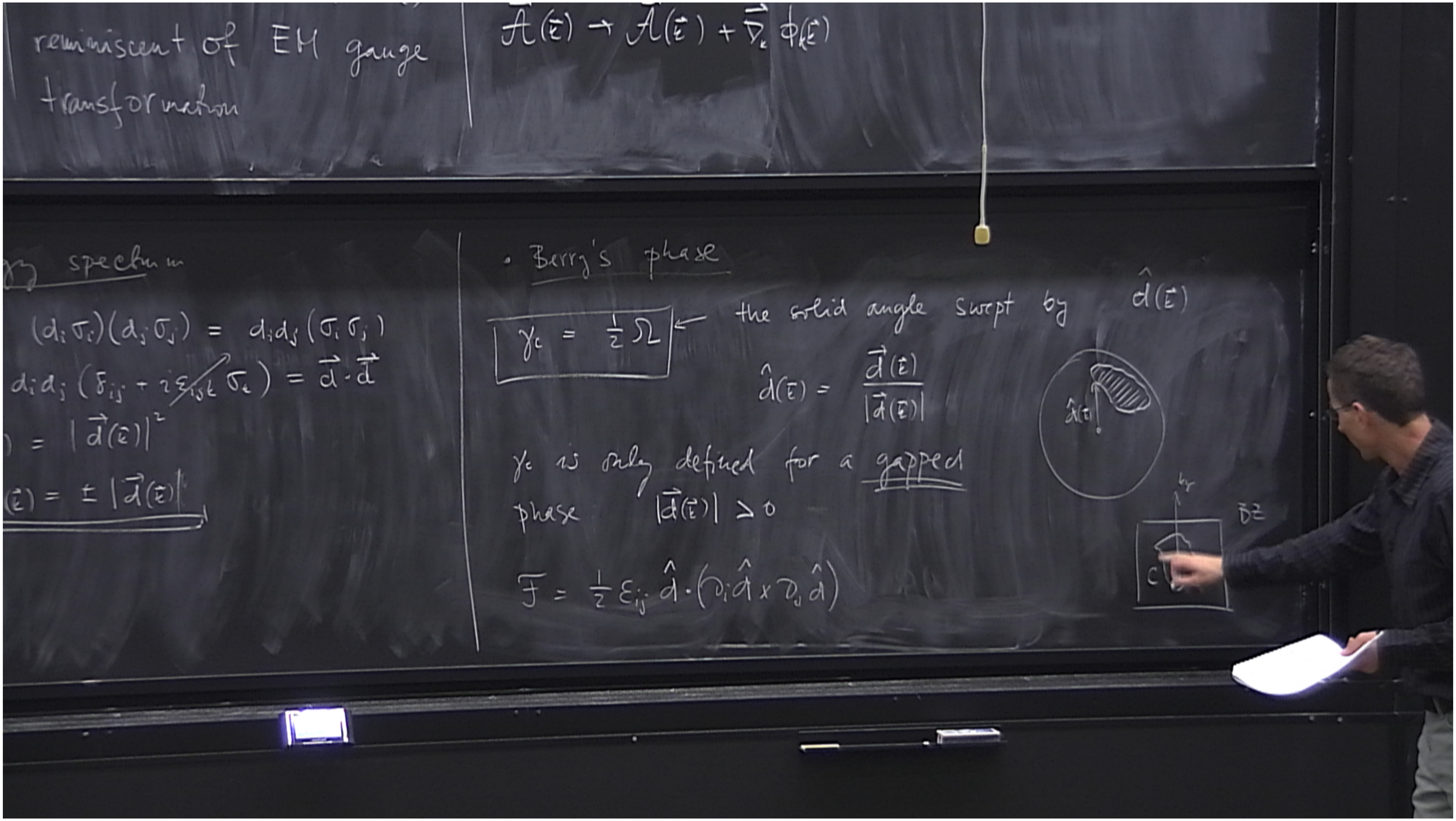
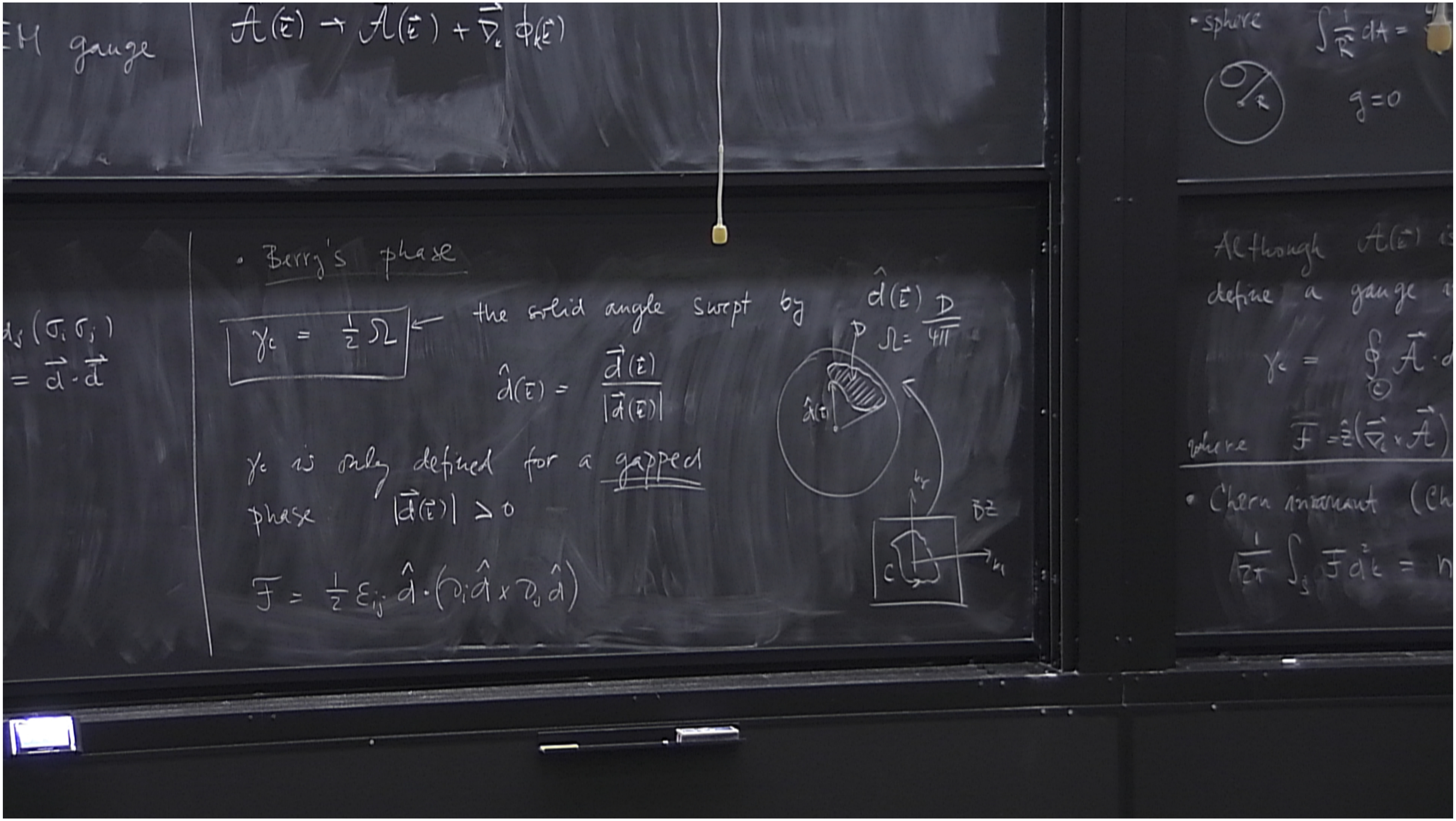
γ_c is only defined for a gapped phase $|\vec{d}(\vec{k})| > 0$

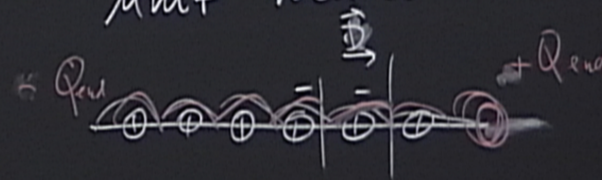

reminiscent of EM gauge transformation
$\mathcal{A}(\vec{k}) \to \mathcal{A}(\vec{k}) + \vec{\nabla}_k \phi_k(\vec{k})$
spectrum
$(d_i \sigma_i)(d_j \sigma_j) = d_i d_j (\sigma_i \sigma_j)$
$d_i d_j (\delta_{ij} + i \cancel{\epsilon_{ijk}} \sigma_k) = \vec{d} \cdot \vec{d}$
$= |\vec{d}(k)|^2$
$(k) = \pm |\vec{d}(k)|$
• Berry's phase
$\boxed{\gamma_c = \frac{1}{2} \Omega}$ ← the solid angle swept by $\hat{d}(\vec{k})$
$\hat{d}(k) = \frac{\vec{d}(k)}{|\vec{d}(k)|}$
γ_c is only defined for a gapped phase $|\vec{d}(\vec{k})| > 0$
$\mathcal{F} = \frac{1}{2} \epsilon_{ij} \hat{d} \cdot (\partial_i \hat{d} \times \partial_j \hat{d})$
$\hat{d}(k)$
k_y
BZ
C


EM gauge
$\mathcal{A}(\vec{k}) \to \mathcal{A}(\vec{k}) + \vec{\nabla}_k \phi_k(\vec{k})$
• sphere
$\int \frac{1}{R^2} dA =$
$g=0$
• Berry's phase
$\boxed{\gamma_c = \frac{1}{2}\Omega}$ ← the solid angle swept by $\hat{d}(\vec{k})$
$\Omega = \frac{D}{4\pi}$
$\hat{d}(\vec{k}) = \frac{\vec{d}(\vec{k})}{|\vec{d}(\vec{k})|}$
γ_c is only defined for a gapped phase $|\vec{d}(\vec{k})| > 0$
$\mathcal{F} = \frac{1}{2}\varepsilon_{ij}\hat{d}\cdot(\partial_i\hat{d}\times\partial_j\hat{d})$
BZ
$= \vec{d}\cdot\vec{d}$
Although $\mathcal{A}(\vec{k})$ is
define a gauge
$\gamma_c = \oint_C \vec{\mathcal{A}}\cdot$
where $\vec{\mathcal{F}} = \hat{z}(\vec{\nabla}_k \times \vec{\mathcal{A}})$
• Chern invariant (Ch
$\frac{1}{2\pi}\int_S \mathcal{F} d^2k = n$

Polarization & topology in 1D.

Electric polarization $\vec{P}$: dipole moment per unit volume.

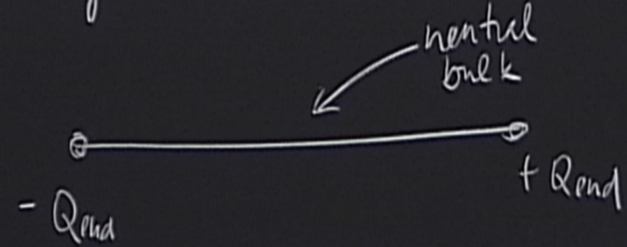
- bulk "bound" charges



$$\rho_b = -\vec{\nabla}\cdot\vec{P}$$

- surface charge $\sigma_b = \vec{P}\cdot\vec{n}$

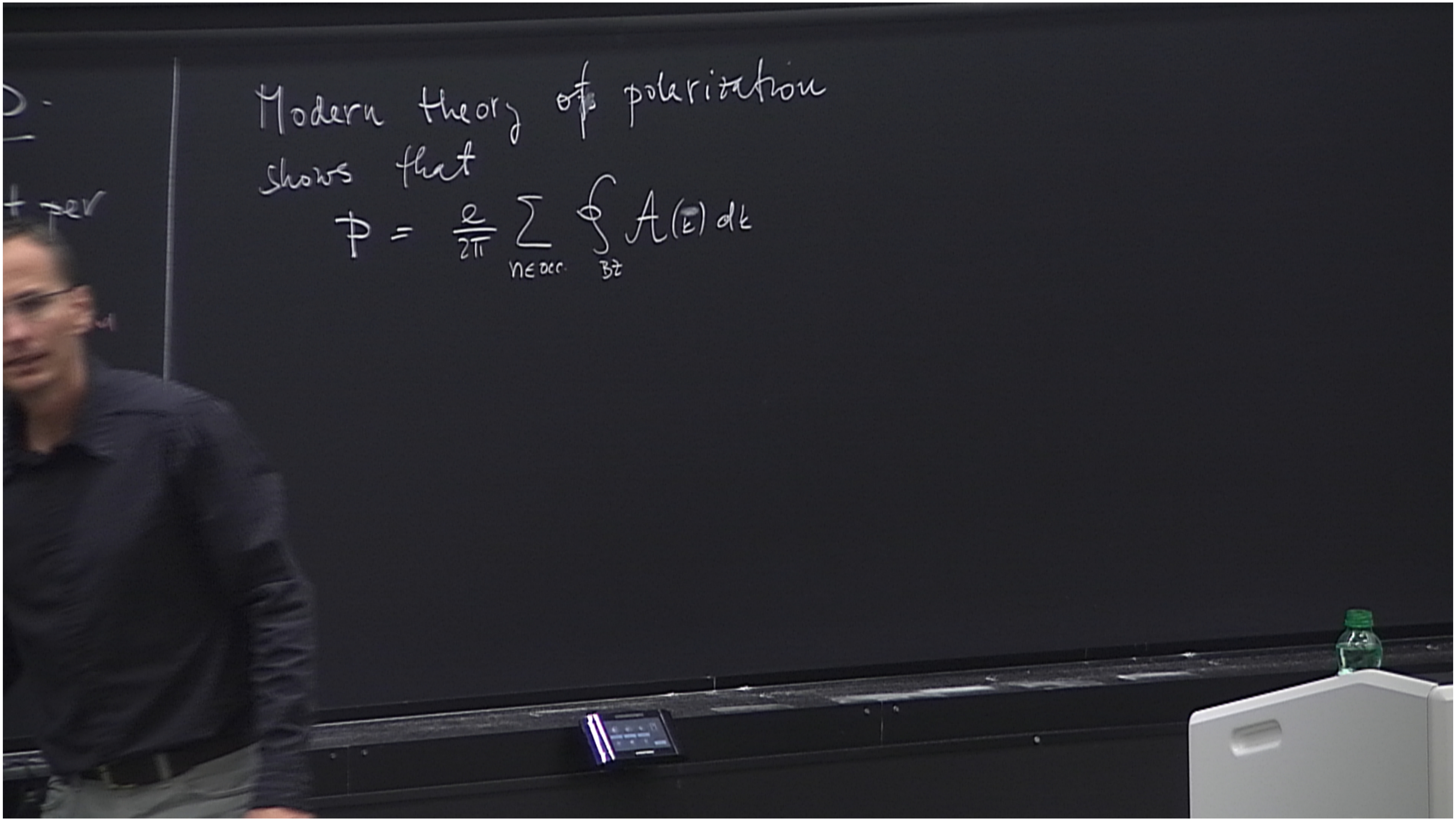
In 1D



Modern

shows

P


Modern theory of polarization
shows that
$P = \frac{e}{2\pi} \sum_{n \in occ.} \oint_{BZ} A(k)\, dk$

Modern theory of polarization shows that

$$P = \frac{e}{2\pi} \sum_{n \in occ.} \oint_{BZ} A(k)\, dk$$

[Bulk-boundary correspondence]

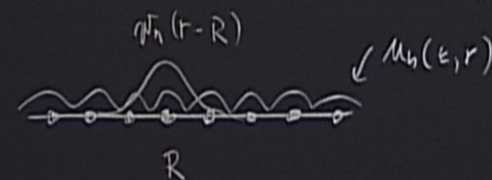
Surface charge Q_{end} is determined by bulk topology.

$\left(d_x + i d_y \quad -d_z\right)$

Proof (outline):

Introduce Wannier functions

$$w_n(r-R) = \frac{\sqrt{N}\,L}{2\pi} \int_{BZ} dk\, e^{ik(r-R)}\, u_n(k,r)$$



Inverse: $$u_n(k,r) = \frac{1}{\sqrt{N}} \sum_R e^{-ik(r-R)}\, w_n(r-R)$$

$$F = \frac{1}{2} \varepsilon_{ij}\, \hat{d} \cdot (\partial_i \hat{d} \times \partial_j \hat{d})$$

(k, r)

$r - R)$

$$A(k) = -i \frac{1}{N} \sum_{R, R'} \int_{cell} dr\, e^{ik(r-R)} (-i)(r-R') e^{-ik(r-R')} w_n^*(r-R) w_n(r-R')$$

$$\int dk \to \frac{1}{a} \delta_{RR'}$$

$$P = \frac{e}{Na} \sum_{n \in occ} \sum_{R} \int_{cell} dr\, (r-R) |w_n(r-R)|^2$$

$$= \frac{e}{L} \sum_{n \in occ} \int dr\, r |w_n(r)|^2$$

$\leftarrow$ dipole moment per unit length = polarization in 1D.