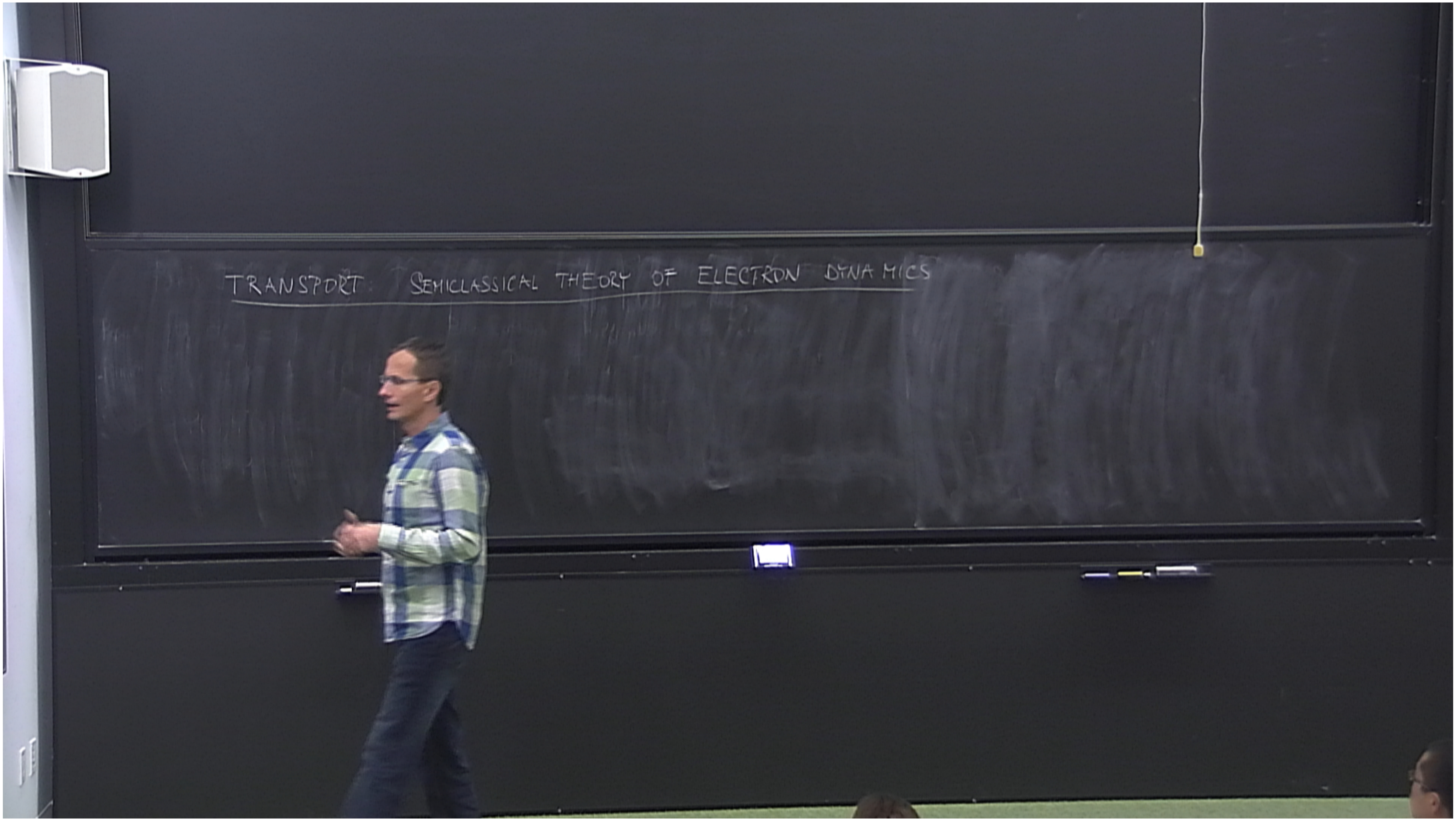
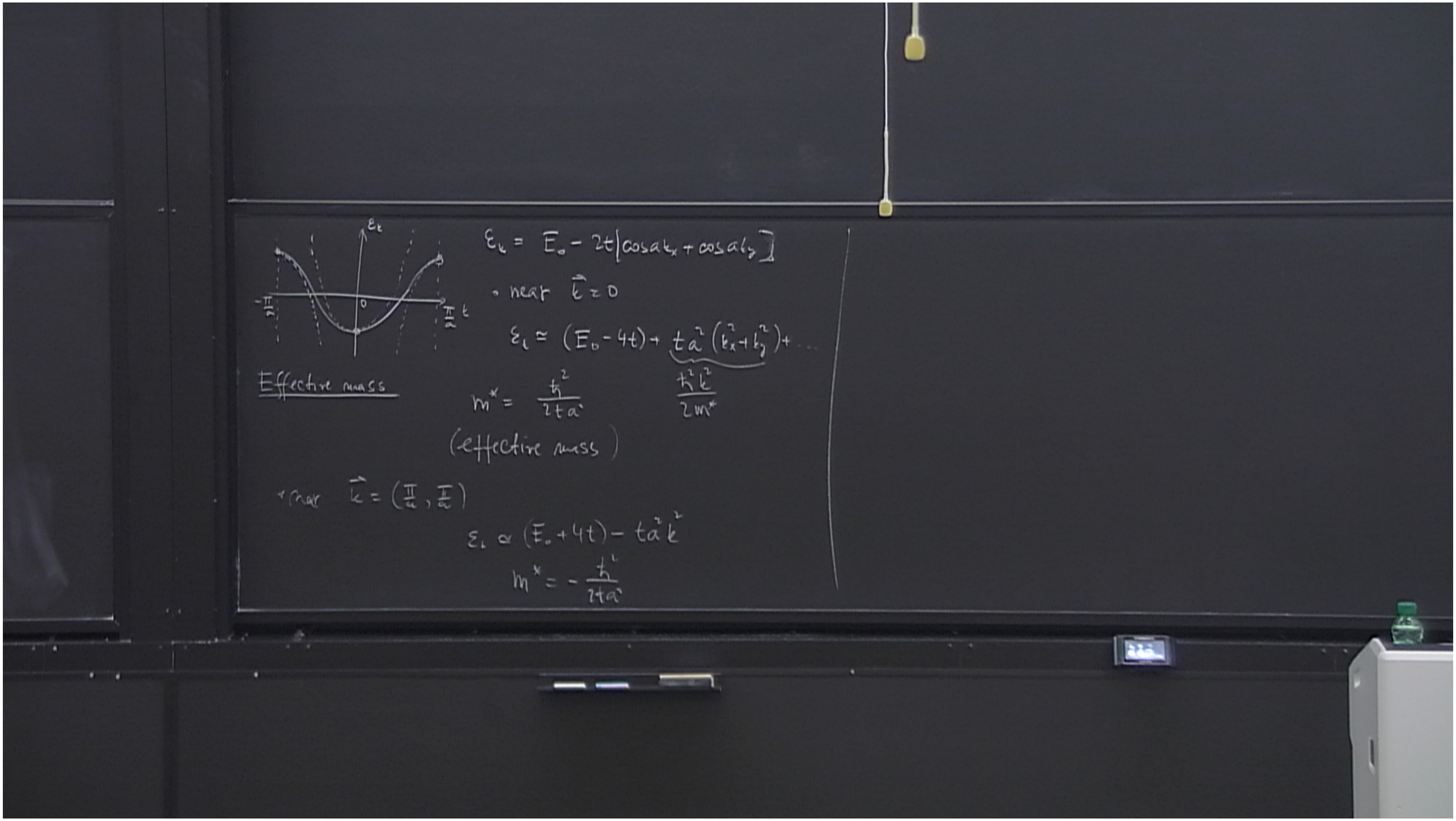
Title: 14/15 PSI - Condensed Matter-Lecture 10

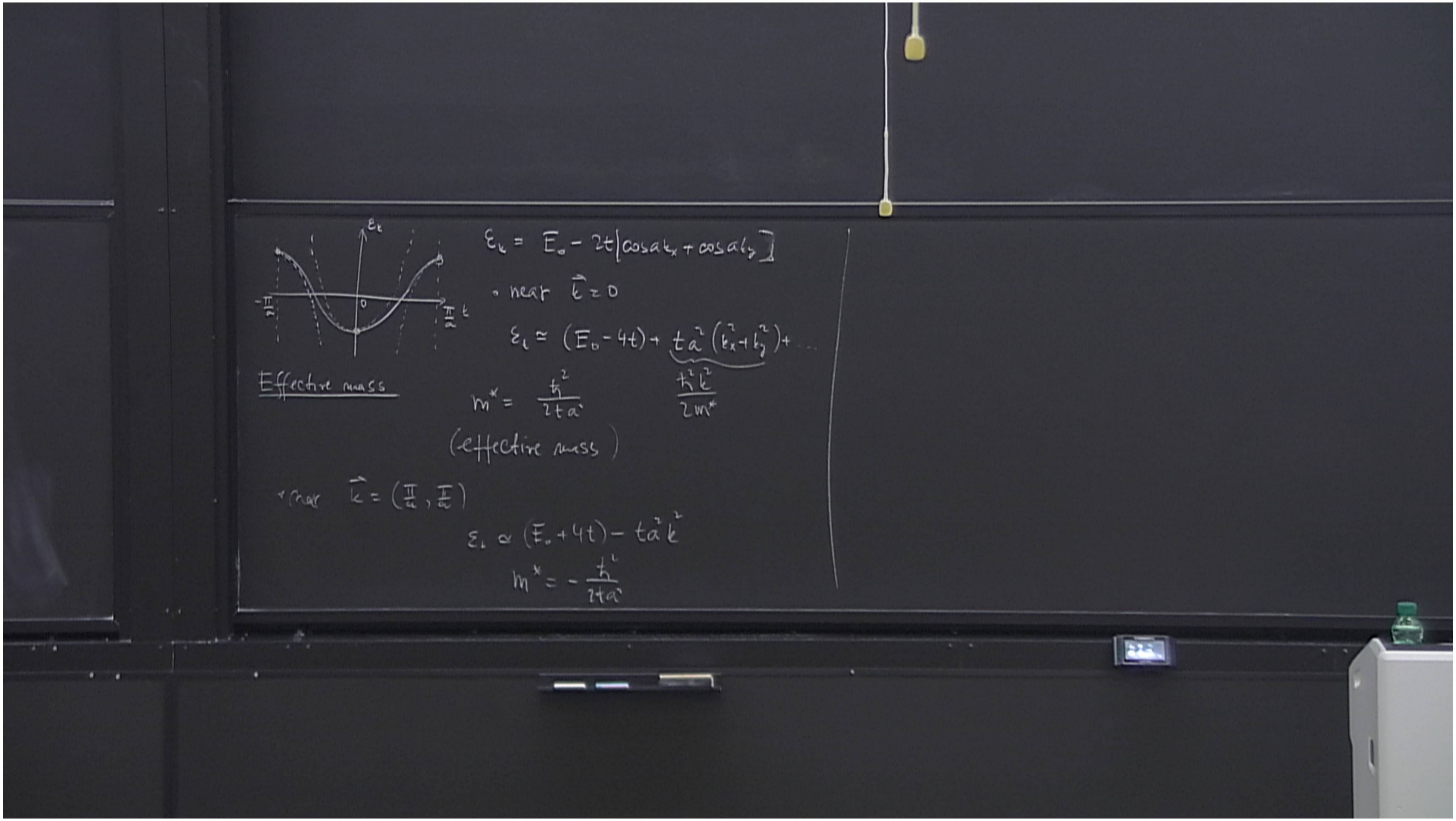
Date: Nov 21, 2014 10:45 AM

URL: http://pirsa.org/14110036

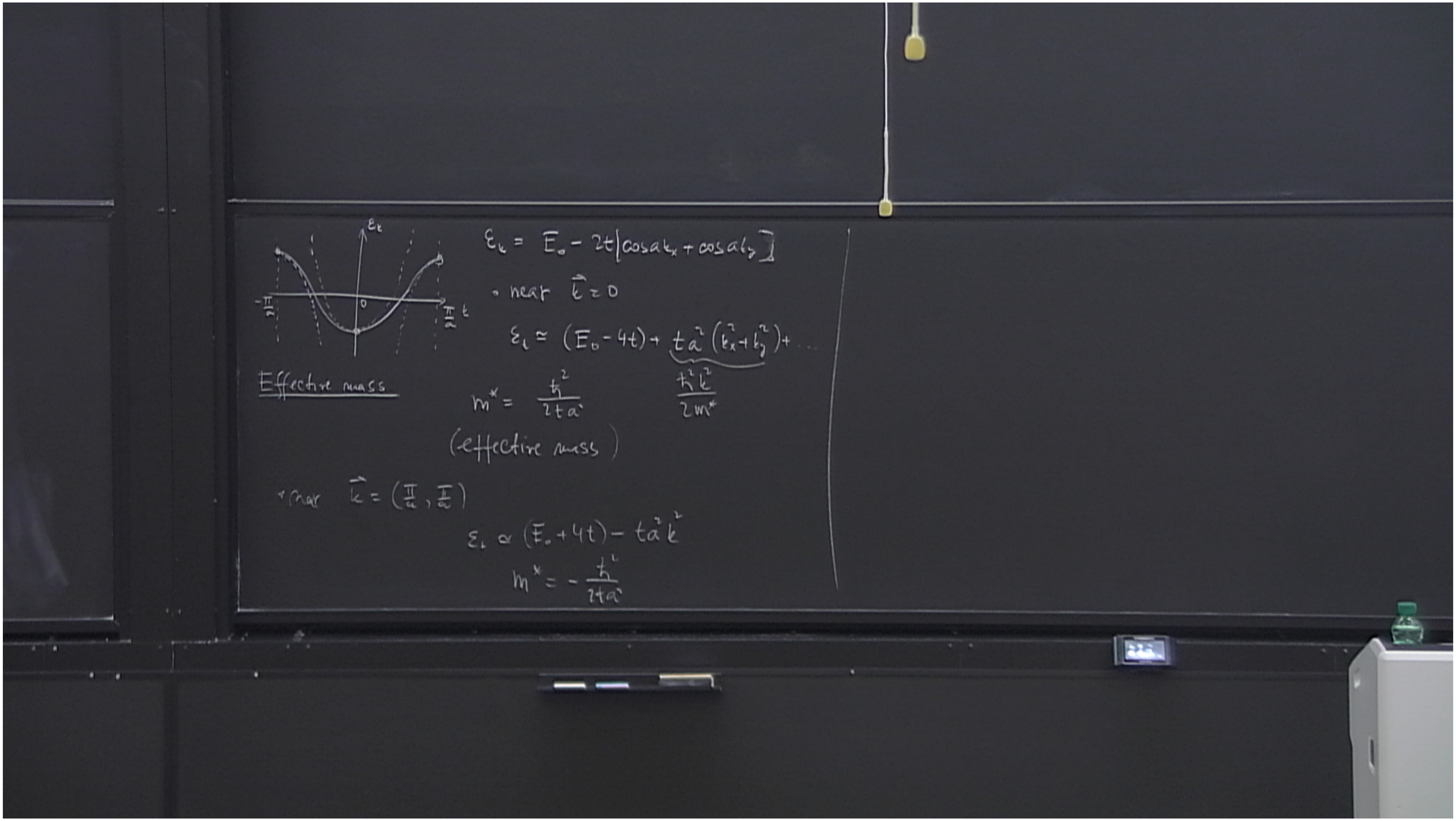
Abstract:

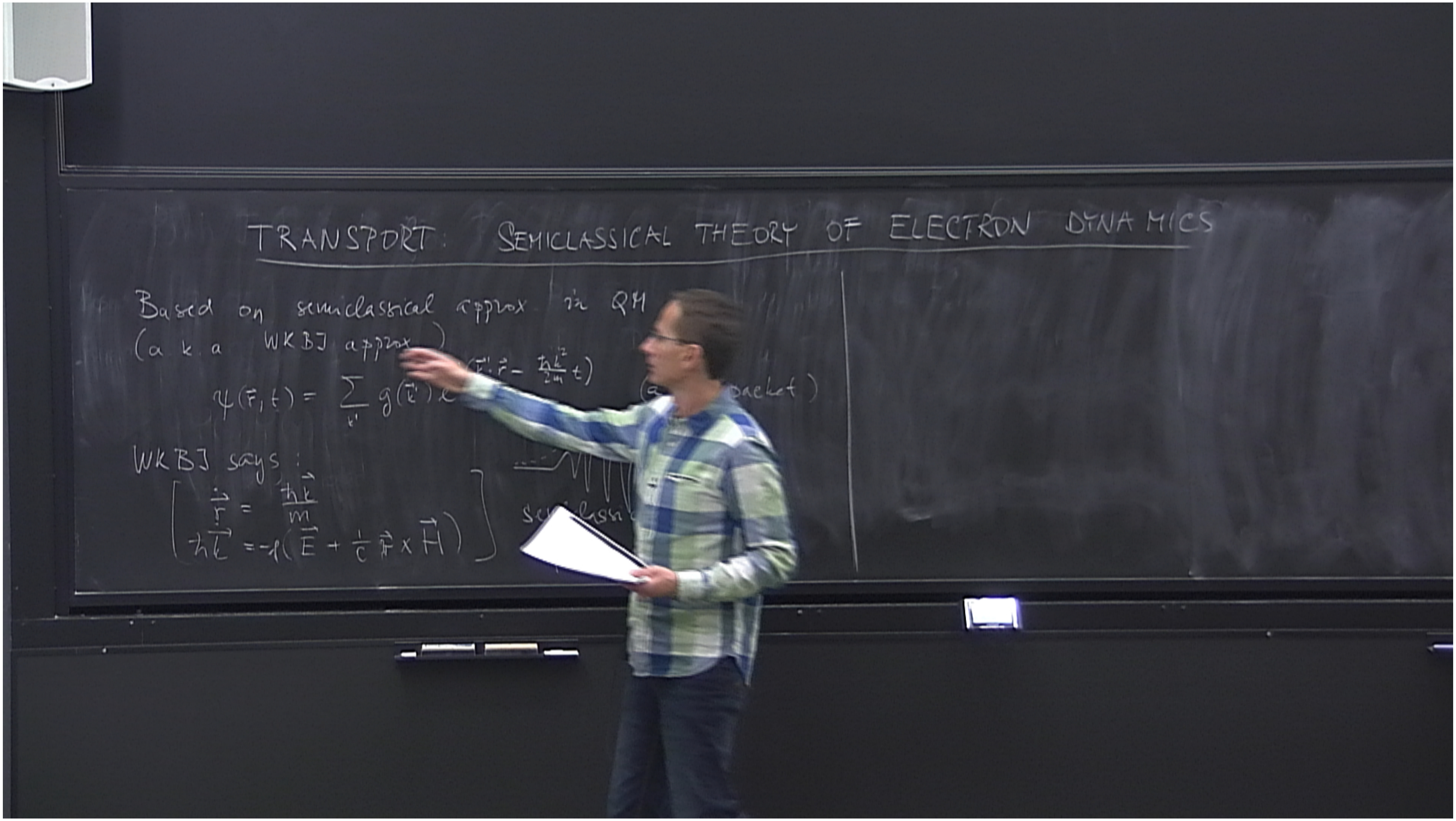

TRANSPORT: SEMICLASSICAL THEORY OF ELECTRON DYNAMICS

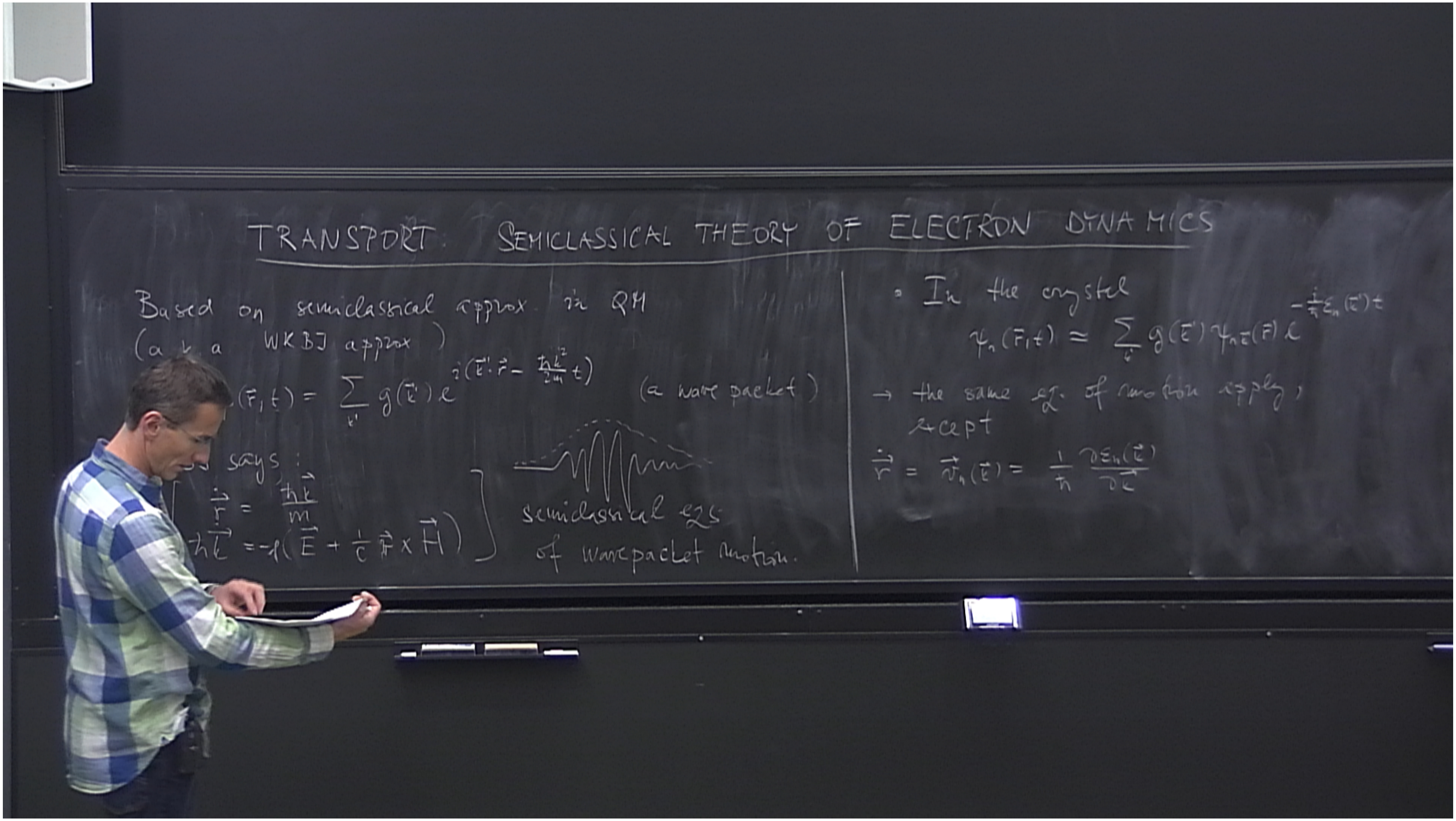

ε_k
$-\frac{\pi}{a}$
0
$\frac{\pi}{a}$ k
$\varepsilon_k = E_0 - 2t[\cos ak_x + \cos ak_y]$
• near $\vec{k} = 0$
$\varepsilon_k \simeq (E_0 - 4t) + \underbrace{ta^2(k_x^2 + k_y^2)}_{\frac{\hbar^2 k^2}{2m^*}} + \dots$
Effective mass
$m^* = \frac{\hbar^2}{2ta^2}$
(effective mass)
• near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \approx (E_0 + 4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$



ε_k
$-\frac{\pi}{a}$
0
$\frac{\pi}{a}$ k
$\varepsilon_k = E_0 - 2t[\cos ak_x + \cos ak_y]$
• near $\vec{k} = 0$
$\varepsilon_k \simeq (E_0 - 4t) + \underbrace{ta^2(k_x^2 + k_y^2)}_{\frac{\hbar^2 k^2}{2m^*}} + \dots$
Effective mass
$m^* = \frac{\hbar^2}{2ta^2}$
(effective mass)
• near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \simeq (E_0 + 4t) - ta^2 k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$


ε_k
$-\frac{\pi}{a}$
0
$\frac{\pi}{a}$ k
$\varepsilon_k = E_0 - 2t[\cos ak_x + \cos ak_y]$
• near $\vec{k} = 0$
$\varepsilon_k \simeq (E_0 - 4t) + \underbrace{ta^2(k_x^2 + k_y^2)}_{\frac{\hbar^2 k^2}{2m^*}} + \dots$
Effective mass
$m^* = \frac{\hbar^2}{2ta^2}$
(effective mass)
• near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \simeq (E_0 + 4t) - ta^2 k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$


TRANSPORT: SEMICLASSICAL THEORY OF ELECTRON DYNAMICS
Based on semiclassical approx in QM
(a.k.a WKBJ approx)
WKBJ says:

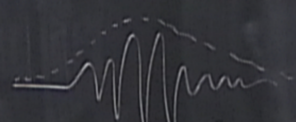

TRANSPORT: SEMICLASSICAL THEORY OF ELECTRON DYNAMICS
Based on semiclassical approx in QM
(a.k.a WKBJ approx)
$\psi(\vec r, t) = \sum_{k'} g(\vec k') e^{i(\vec k' \cdot \vec r - \frac{\hbar k'^2}{2m} t)}$
(a wave packet)
says:
$\dot{\vec r} = \frac{\hbar \vec k}{m}$
$\hbar \dot{\vec k} = -e(\vec E + \frac{1}{c} \dot{\vec r} \times \vec H)$
semiclassical eqs of wavepacket motion.
In the crystal
$\psi_n(\vec r, t) \simeq \sum_{k'} g(\vec k') \psi_{n\vec k'}(\vec r) e^{-\frac{i}{\hbar}\varepsilon_n(\vec k')t}$
→ the same eq. of motion apply, except
$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar} \frac{\partial \varepsilon_n(\vec k)}{\partial \vec k}$

TRANSPORT: SEMICLASSICAL THEORY OF ELECTRON DYNAMICS

Based on semiclassical approx in QM
(a.k.a. WKBJ approx)

$$\psi(\vec r, t) = \sum_{k'} g(\vec k')\, e^{i(\vec k' \cdot \vec r - \frac{\hbar k'^2}{2m} t)}$$

(a wave packet)



WKBJ says:

$$\left[\begin{array}{l} \dot{\vec r} = \dfrac{\hbar \vec k}{m} \\ \hbar \dot{\vec k} = -e\left(\vec E + \frac{1}{c} \dot{\vec r} \times \vec H\right) \end{array}\right]$$

semiclassical eqs of wavepacket motion.

- In the crystal

$$\psi_n(\vec r, t) \simeq \sum_{k'} g(\vec k')\, \psi_{n\vec k'}(\vec r)\, e^{-\frac{i}{\hbar}\varepsilon_n(\vec k') t}$$

→ the same eq. of motion apply, except

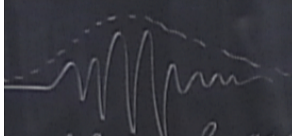
$$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar} \frac{\partial \varepsilon_n(\vec k)}{\partial \vec k}$$

$$\hbar \dot{\vec k} = -e\left[\vec E + \frac{1}{c} \vec v_n(\vec k) \times \vec H\right]$$

MICLASSICAL THEORY OF ELECTRON DYNAMICS

in QM

$\frac{\hbar k^2}{2m} t)$ (a wave packet)



semiclassical eqs of wavepacket motion.

- In the crystal

$$\psi_n(\vec{r},t) \simeq \sum_{k'} g(\vec{k}')\, \psi_{n\vec{k}'}(\vec{r})\, e^{-\frac{i}{\hbar}\varepsilon_n(\vec{k}')t}$$

→ the same eq. of motion apply, except

$$\dot{\vec{r}} = \vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}}$$

$$\hbar\dot{\vec{k}} = -e\left[\vec{E} + \frac{1}{c}\vec{v}_n(\vec{k})\times\vec{H}\right]$$

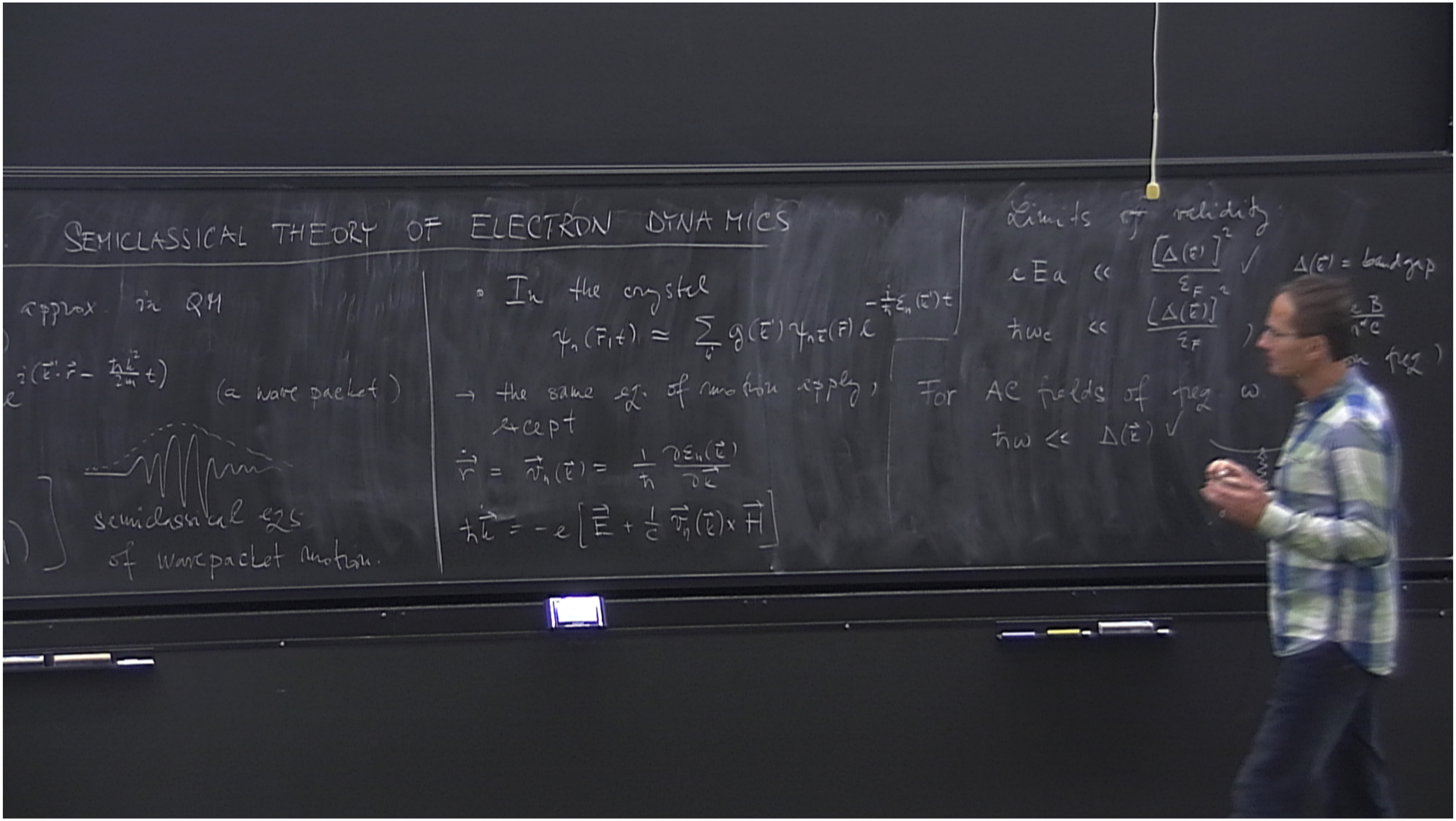
Limits of validity:

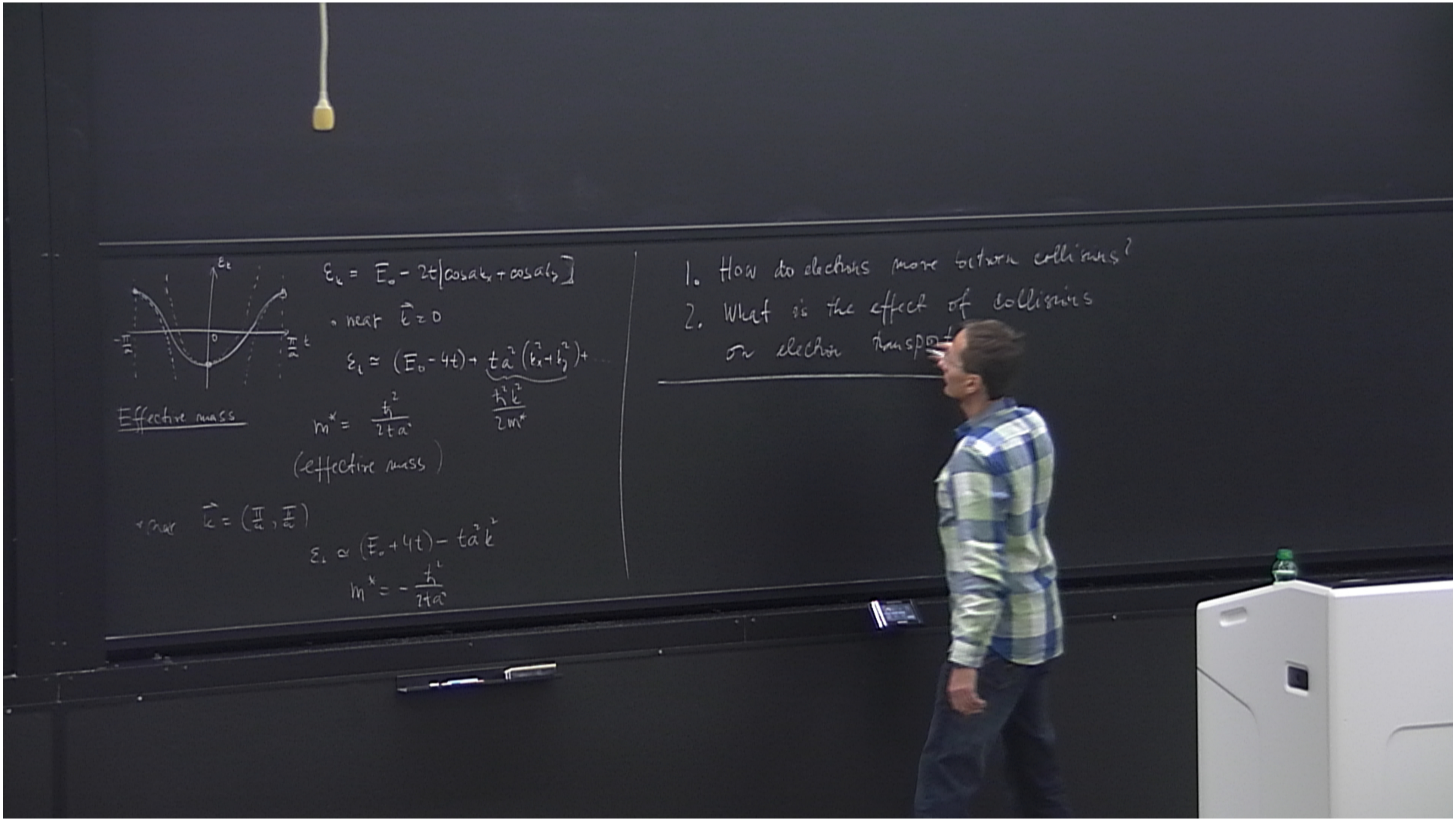
$eEa \ll \frac{[\Delta(\vec{k})]^2}{\varepsilon_F}$ ✓ $\Delta(\vec{k})$ = bandgap

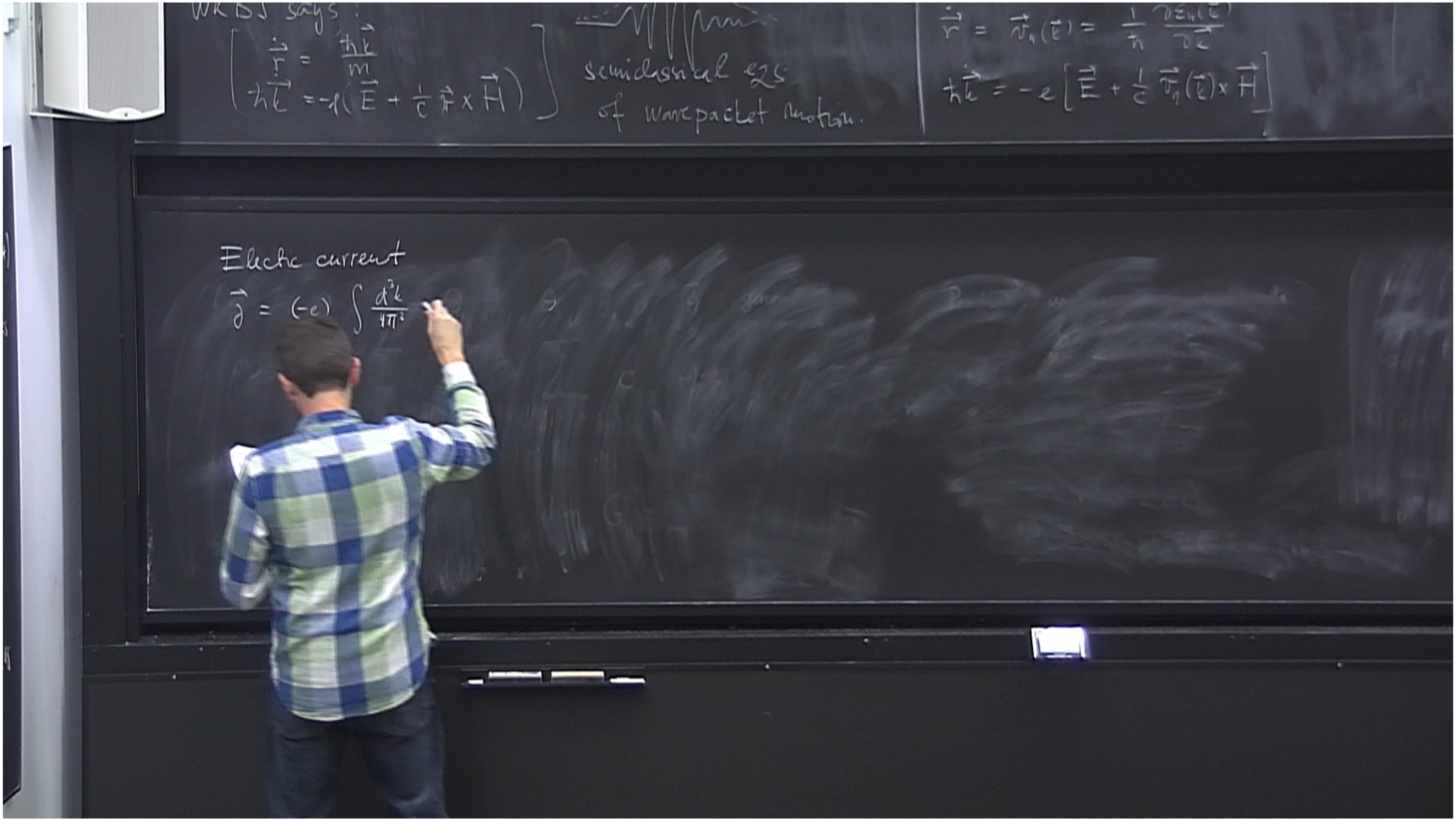
$\hbar\omega_c \ll \frac{[\Delta(\vec{k})]^2}{\varepsilon_F}$, $\omega_c = \frac{eB}{m^* c}$ (cyclotron freq)

For AC fields of freq ω

$\hbar\omega \ll \Delta(\vec{k})$ ✓


SEMICLASSICAL THEORY OF ELECTRON DYNAMICS
approx. in QM
(a wave packet)
semiclassical eqs of wavepacket motion.
In the crystal
$\psi_n(\vec r,t) \simeq \sum_{\vec k} g(\vec k')\,\psi_{n\vec k'}(\vec r)\,e^{-\frac{i}{\hbar}\varepsilon_n(\vec k')t}$
→ the same eq. of motion apply, except
$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec k)}{\partial \vec k}$
$\hbar\dot{\vec k} = -e\left[\vec E + \frac{1}{c}\vec v_n(\vec k)\times\vec H\right]$
Limits of validity
$eEa \ll \frac{[\Delta(\vec k)]^2}{\varepsilon_F}$ ✓
$\hbar\omega_c \ll \frac{[\Delta(\vec k)]^2}{\varepsilon_F}$
$\Delta(\vec k)$ = band gap
For AC fields of freq. ω
$\hbar\omega \ll \Delta(\vec k)$ ✓


1. How do electrons move between collisions?
2. What is the effect of collisions on electron transport
Effective mass
$\varepsilon_k = E_0 - 2t[\cos ak_x + \cos ak_y]$
near $\vec{k} = 0$
$\varepsilon_k \simeq (E_0 - 4t) + ta^2(k_x^2 + k_y^2) + \dots$
$m^* = \frac{\hbar^2}{2ta^2}$
$\frac{\hbar^2 k^2}{2m^*}$
(effective mass)
near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \simeq (E_0 + 4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$


semiclassical eqs
of wavepacket motion.
Electric current

WKBJ says:

$$\begin{cases} \dot{\vec{r}} = \dfrac{\hbar \vec{k}}{m} \\ \hbar\dot{\vec{k}} = -e\left(\vec{E} + \dfrac{1}{c}\dot{\vec{r}} \times \vec{H}\right) \end{cases}$$

semiclassical eqs of wavepacket motion.

$$\dot{\vec{r}} = \vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}}$$

$$\hbar\dot{\vec{k}} = -e\left[\vec{E} + \frac{1}{c}\vec{v}_n(\vec{k}) \times \vec{H}\right]$$

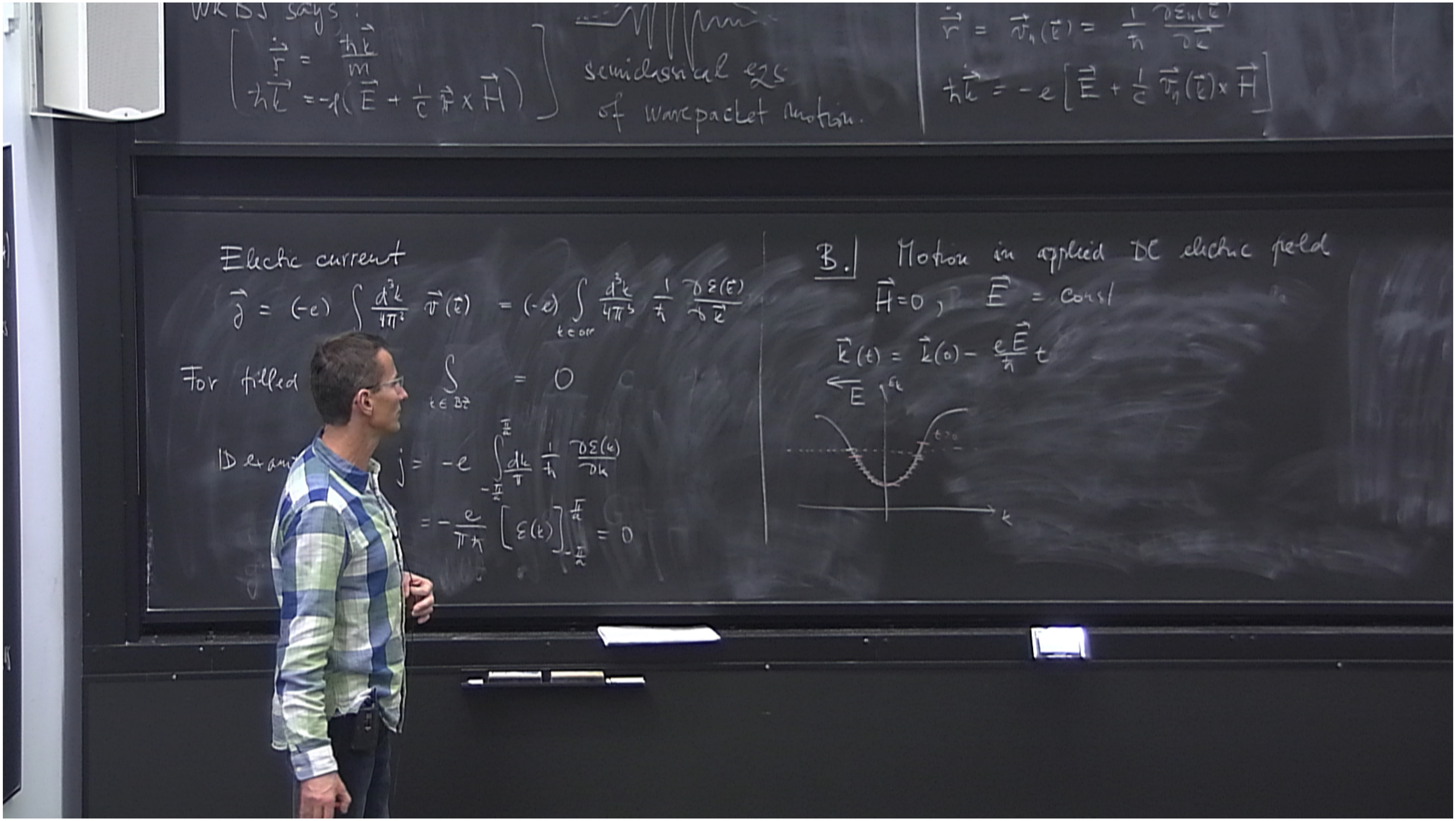
Electric current

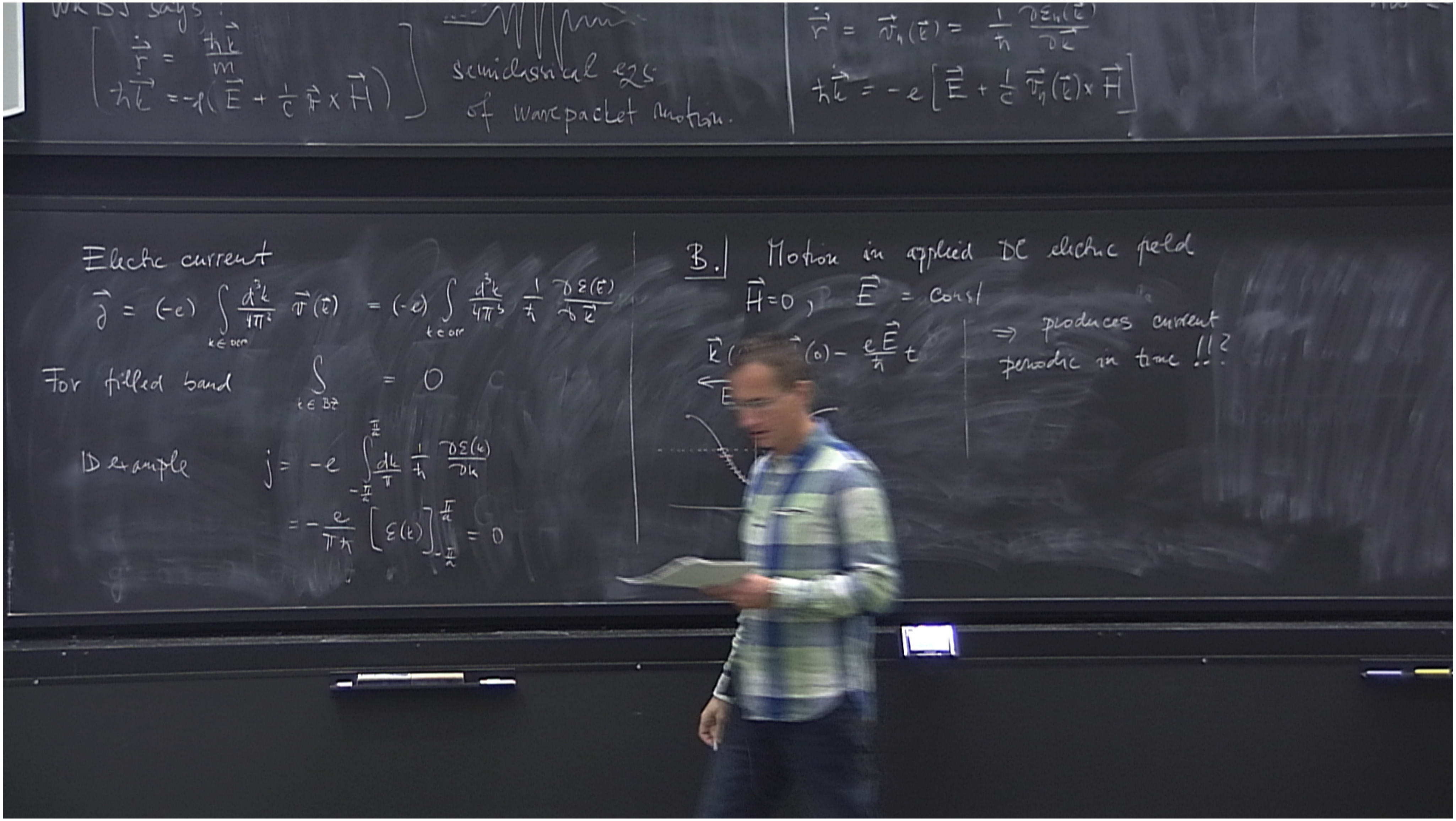
$$\vec{j} = (-e)\int_{k \in occ} \frac{d^3k}{4\pi^3}\,\vec{v}(\vec{k}) = (-e)\int_{k \in occ} \frac{d^3k}{4\pi^3}\,\frac{1}{\hbar}\frac{\partial \varepsilon(\vec{k})}{\partial \vec{k}}$$

For filled band $\displaystyle\int_{k \in BZ} = 0$

1D example $\displaystyle j = -e\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} \frac{dk}{\pi}\,\frac{1}{\hbar}\frac{\partial \varepsilon(k)}{\partial k}$

$$= -\frac{e}{\pi\hbar}\Big[\varepsilon(k)\Big]_{-\frac{\pi}{a}}^{\frac{\pi}{a}} = 0$$


semiclassical eqs
of wavepacket motion.
Electric current
For filled
B. Motion in applied DC electric field
$\vec{H}=0$, $\vec{E}$ = const
$\vec{k}(t) = \vec{k}(0) - \frac{e\vec{E}}{\hbar} t$


semiclassical eqs
of wavepacket motion.
Electric current
For filled band
1D example
B.| Motion in applied DC electric field
⇒ produces current
periodic in time !!?

WKBJ says:

$$\left[\begin{array}{l}\dot{\vec r} = \dfrac{\hbar \vec k}{m} \\ \hbar\dot{\vec k} = -e\left(\vec E + \dfrac{1}{c}\dot{\vec r}\times\vec H\right)\end{array}\right]$$

semiclassical eqs of wavepacket motion.

$$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar}\frac{\partial\varepsilon_n(\vec k)}{\partial \vec k}$$

$$\hbar\dot{\vec k} = -e\left[\vec E + \frac{1}{c}\vec v_n(\vec k)\times\vec H\right]$$

Electric current

$$\vec j = (-e)\int_{k\in occ}\frac{d^3k}{4\pi^3}\,\vec v(\vec k) = (-e)\int_{k\in occ}\frac{d^3k}{4\pi^3}\,\frac{1}{\hbar}\frac{\partial\varepsilon(\vec k)}{\partial\vec k}$$

For filled band $\int_{k\in BZ} = 0$

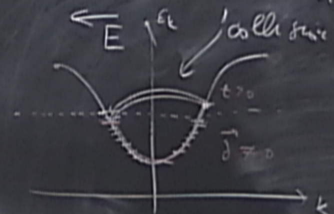
1D example

$$j = -e\int_{-\pi/a}^{\pi/a}\frac{dk}{\pi}\,\frac{1}{\hbar}\frac{\partial\varepsilon(k)}{\partial k} = -\frac{e}{\pi\hbar}\Big[\varepsilon(k)\Big]_{-\pi/a}^{\pi/a} = 0$$

B.| Motion in applied DC electric field

$\vec H = 0$, $\vec E$ = const

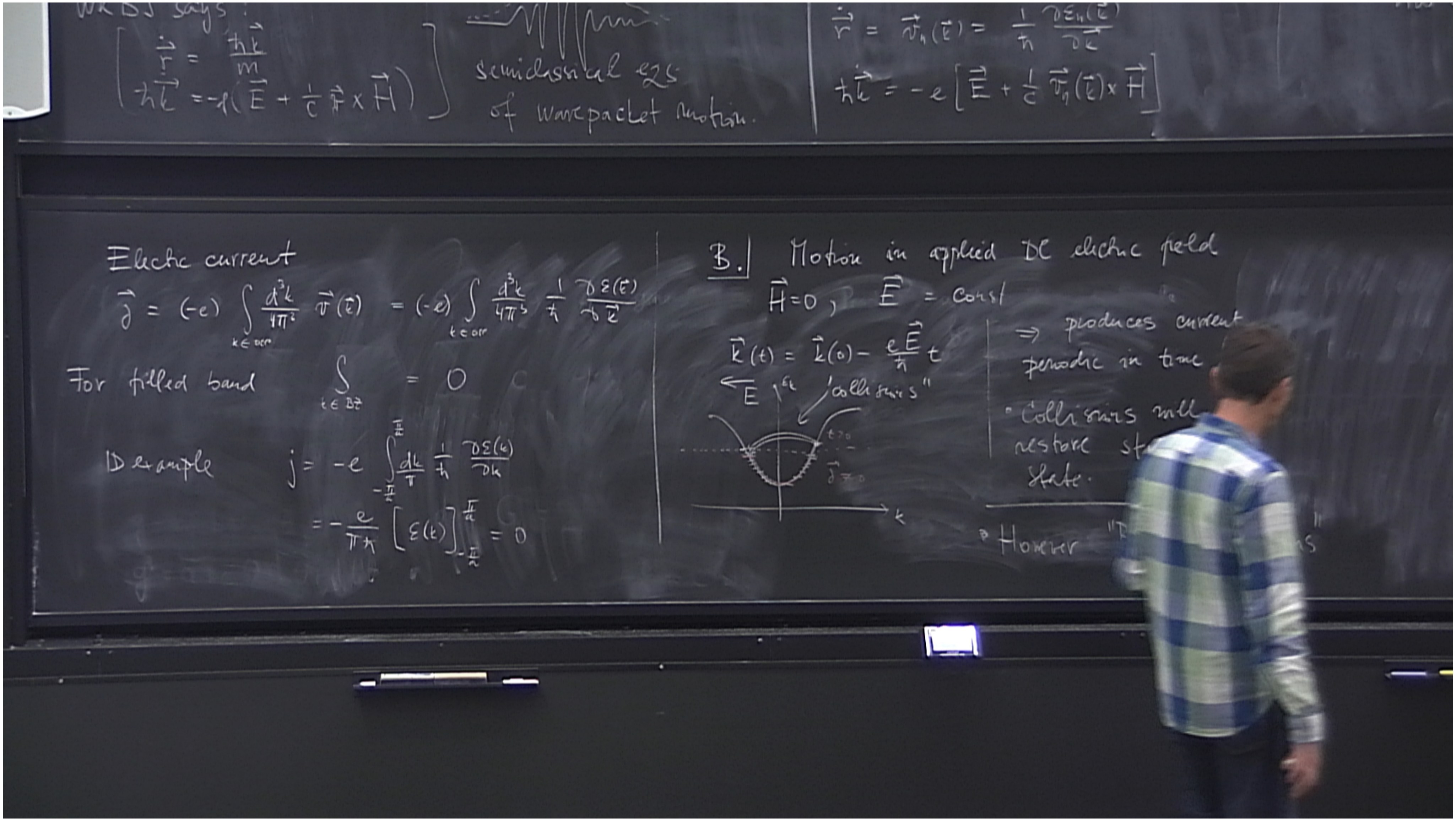
$$\vec k(t) = \vec k(0) - \frac{e\vec E}{\hbar}t$$

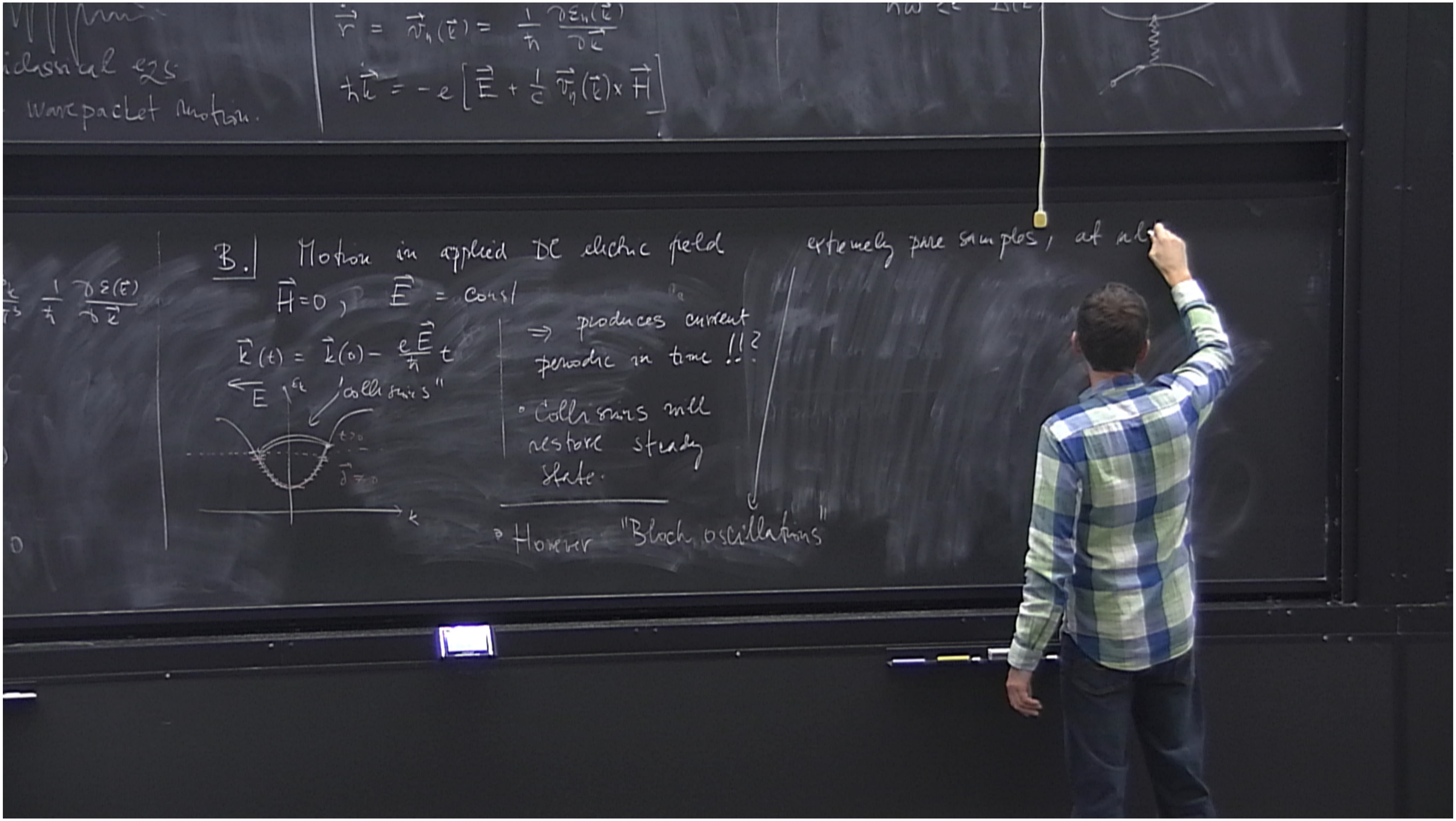


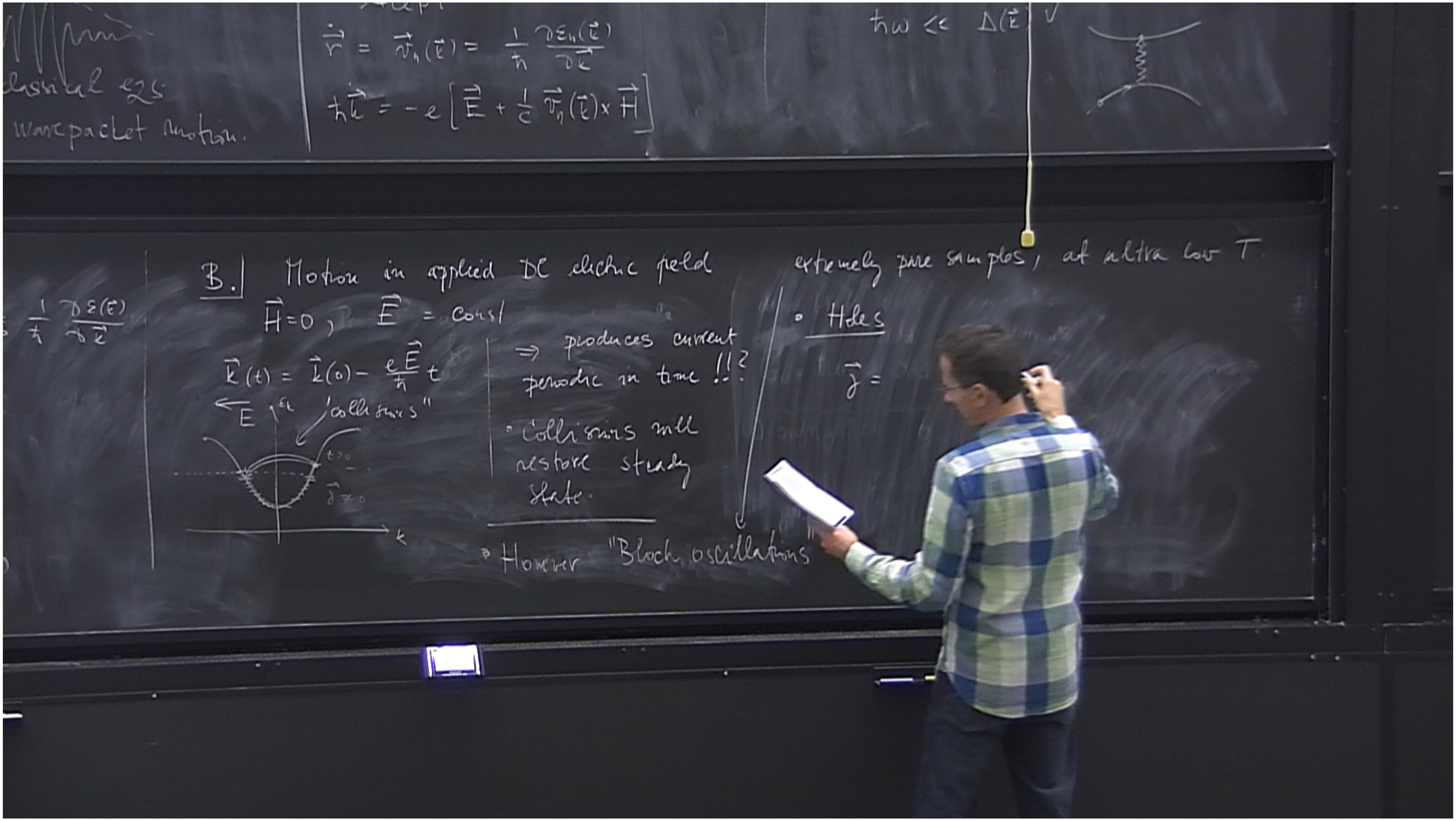
⇒ produces current periodic in time !!?

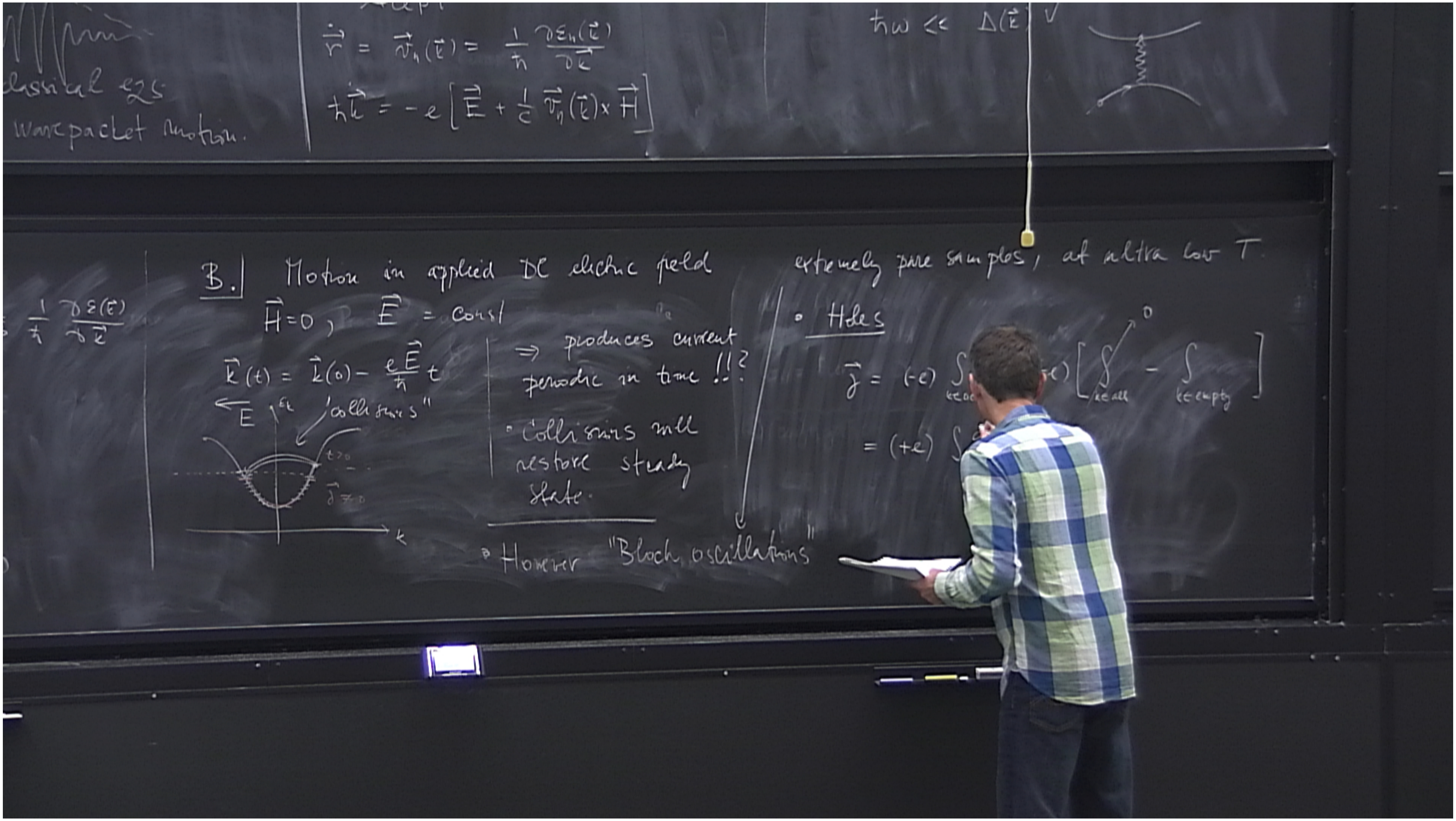
• Collisions will restore steady state.

• However "Bloch oscillations"

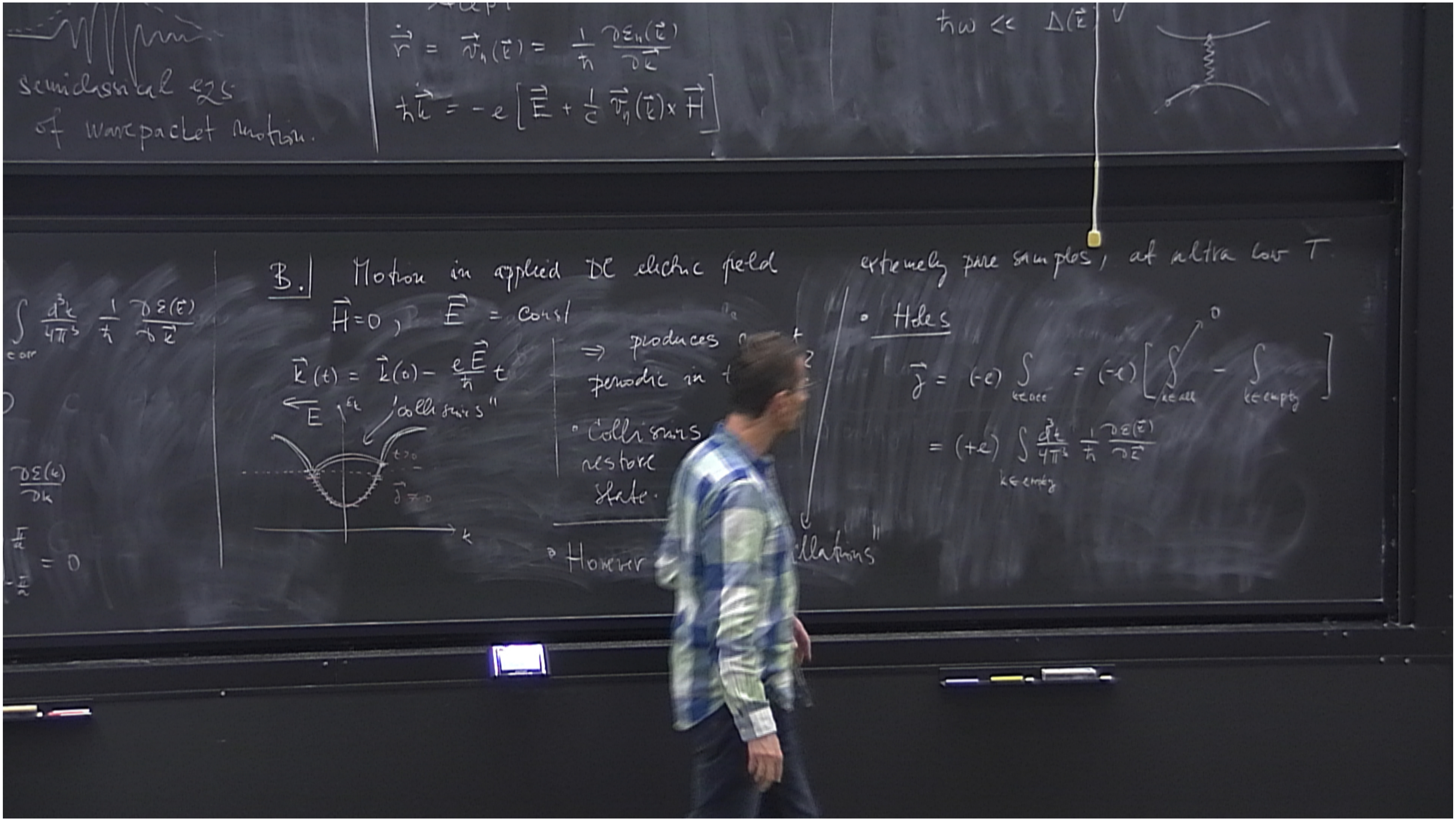

WKBJ says:
$\dot{\vec r} = \frac{\hbar \vec k}{m}$
$\hbar\dot{\vec k} = -e(\vec E + \frac{1}{c}\dot{\vec r}\times\vec H)$
semiclassical eqs of wavepacket motion.
$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec k)}{\partial \vec k}$
$\hbar\dot{\vec k} = -e\left[\vec E + \frac{1}{c}\vec v_n(\vec k)\times\vec H\right]$
Electric current
$\vec j = (-e)\int_{k\in occ}\frac{d^3k}{4\pi^3}\vec v(\vec k) = (-e)\int_{k\in occ}\frac{d^3k}{4\pi^3}\frac{1}{\hbar}\frac{\partial\varepsilon(\vec k)}{\partial\vec k}$
For filled band $\int_{k\in BZ} = 0$
1D example $j = -e\int_{-\frac{\pi}{a}}^{\frac{\pi}{a}}\frac{dk}{\pi}\frac{1}{\hbar}\frac{\partial\varepsilon(k)}{\partial k}$
$= -\frac{e}{\pi\hbar}\left[\varepsilon(k)\right]_{-\frac{\pi}{a}}^{\frac{\pi}{a}} = 0$
B. Motion in applied DC electric field
$\vec H = 0$, $\vec E$ = const
$\vec k(t) = \vec k(0) - \frac{e\vec E}{\hbar}t$
E
"collisions"
$t>0$
$\vec j = 0$
k
⇒ produces current periodic in time
• Collisions will restore st... state.
• However "R... ...ns"

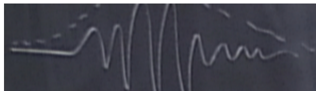

classical eqs
wavepacket motion.
B. Motion in applied DC electric field
H = 0, E = const
k(t) = k(0) − eE/ħ t
"collisions"
⇒ produces current periodic in time !!?
• Collisions will restore steady state.
• However "Bloch oscillations"
extremely pure samples, at al


wavepacket motion.
ħω << Δ(k)
B. Motion in applied DC electric field
H = 0 , E = const
k(t) = k(0) − eE/ħ t
"collisions"
⇒ produces current periodic in time !!?
∘ Collisions will restore steady state.
∘ However "Bloch oscillations"
extremely pure samples, at ultra low T
∘ Holes
j =



wavepacket motion.
B. Motion in applied DC electric field
⇒ produces current periodic in time !!?
Collisions will restore steady state.
However "Bloch oscillations"
extremely pure samples, at ultra low T
Holes


semiclassical eqs of wavepacket motion.
B. Motion in applied DC electric field
H = 0, E = const
produces
periodic in
'collisions'
Collisions restore state.
However
extremely pure samples, at ultra low T
Holes



semiclassical eqs of wavepacket motion.

$$\dot{\vec r} = \vec v_n(\vec k) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec k)}{\partial \vec k}$$

$$\hbar\dot{\vec k} = -e\left[\vec E + \frac{1}{c}\vec v_n(\vec k)\times\vec H\right]$$

$$\hbar\omega \ll \Delta(\vec k)$$

$$\int_{k\in occ}\frac{d^3k}{4\pi^3}\frac{1}{\hbar}\frac{\partial\varepsilon(\vec k)}{\partial\vec k}$$

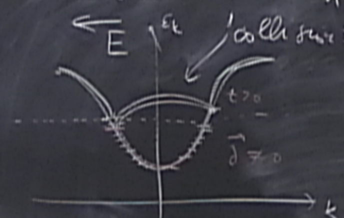
$$\frac{\partial\varepsilon(k)}{\partial k}$$

$$\int_{-\pi/a}^{\pi/a} = 0$$

B. Motion in applied DC electric field

$\vec H = 0$, $\vec E$ = const

$$\vec k(t) = \vec k(0) - \frac{e\vec E}{\hbar}t$$



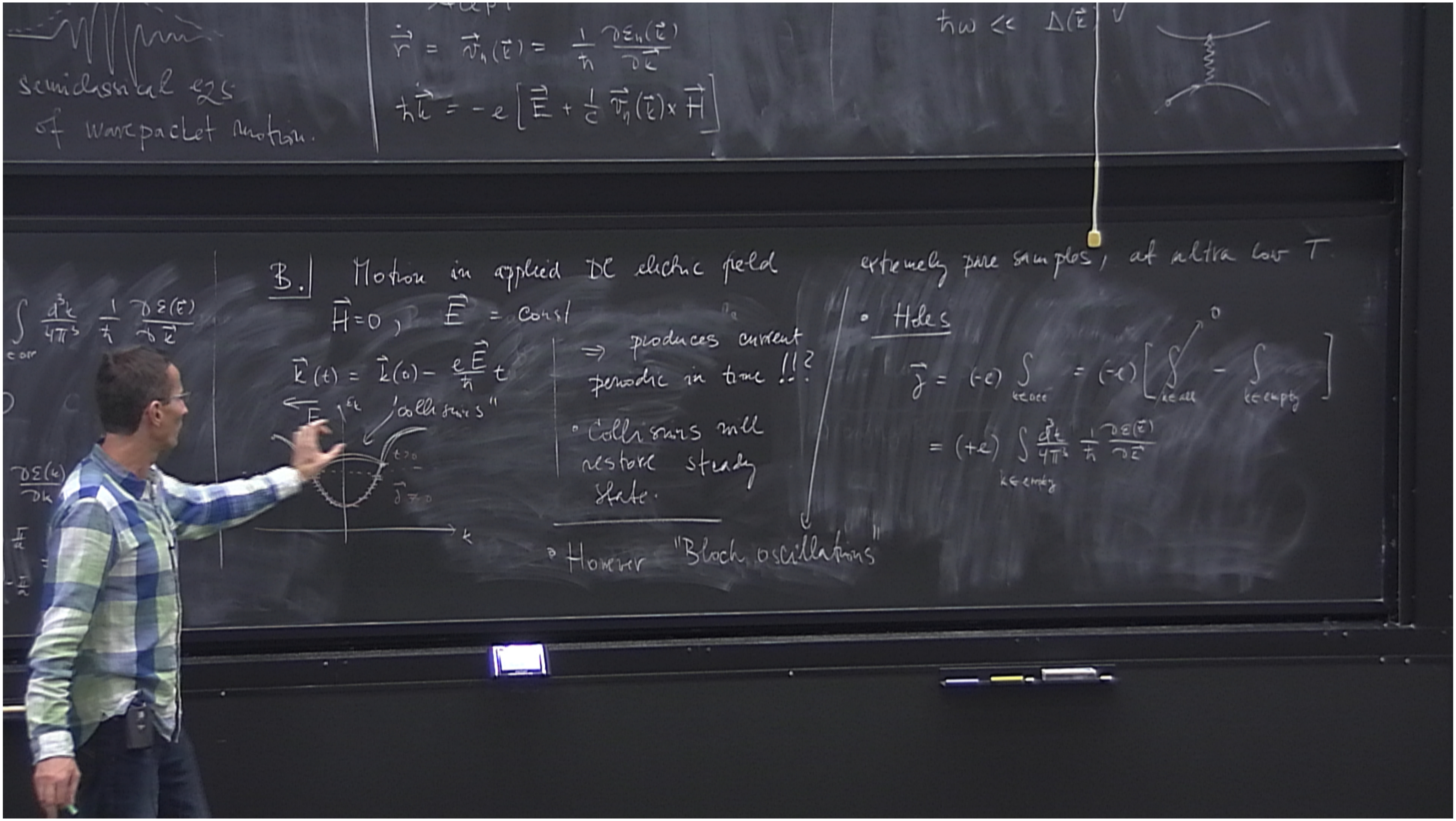
⇒ produces current periodic in time !!?

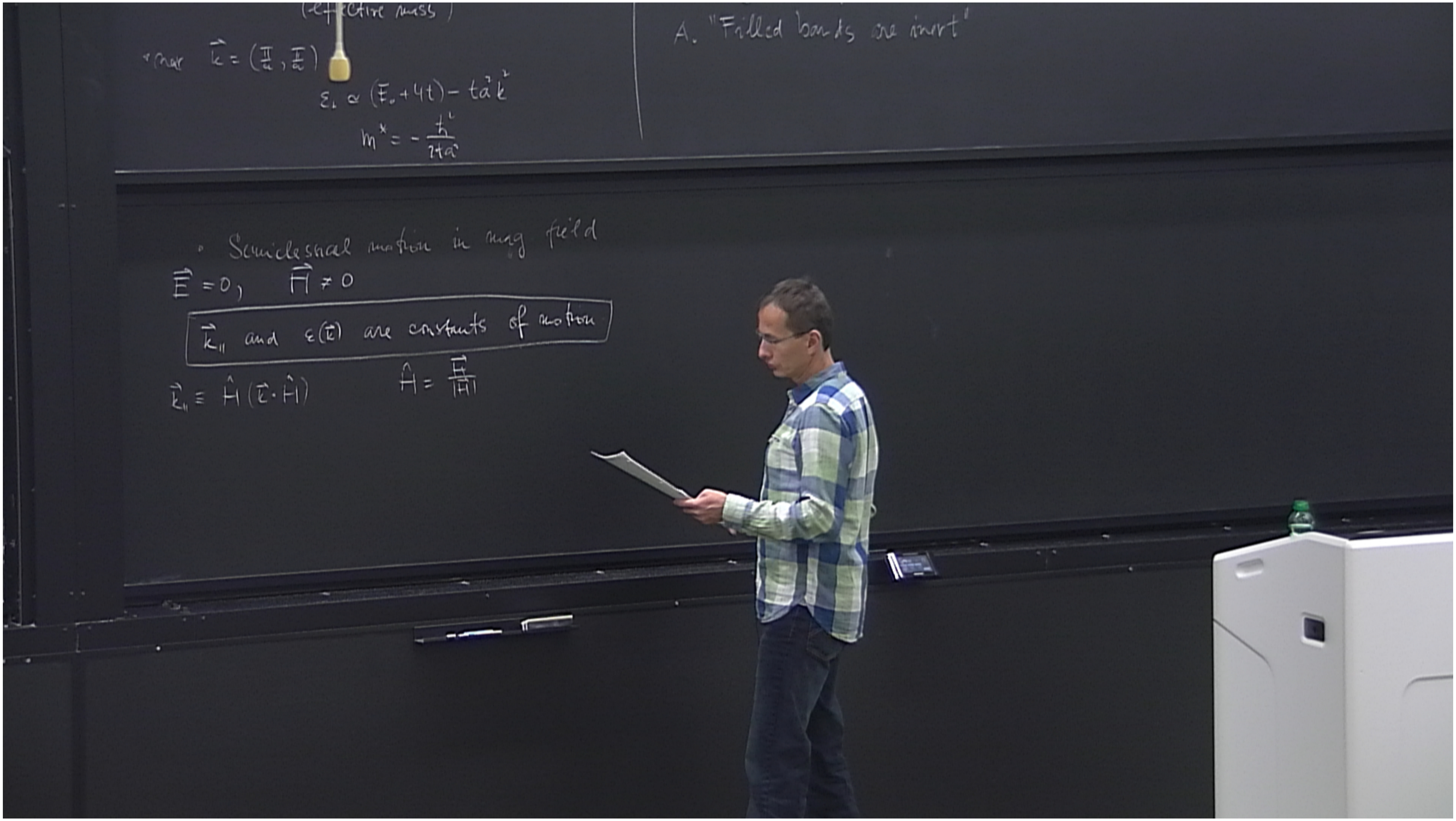
- Collisions will restore steady state.
- However "Bloch oscillations" extremely pure samples, at ultra low T

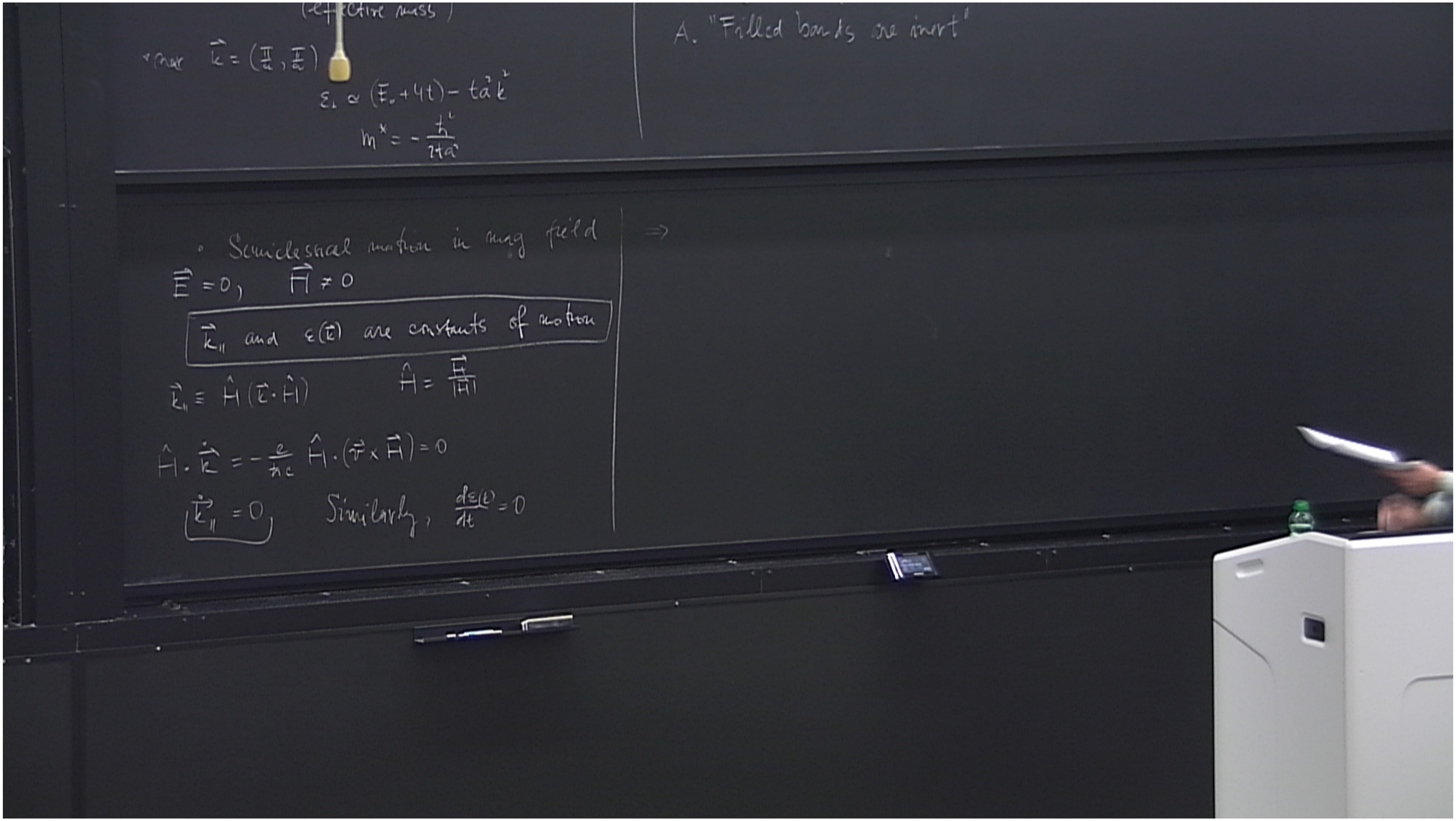
- Holes

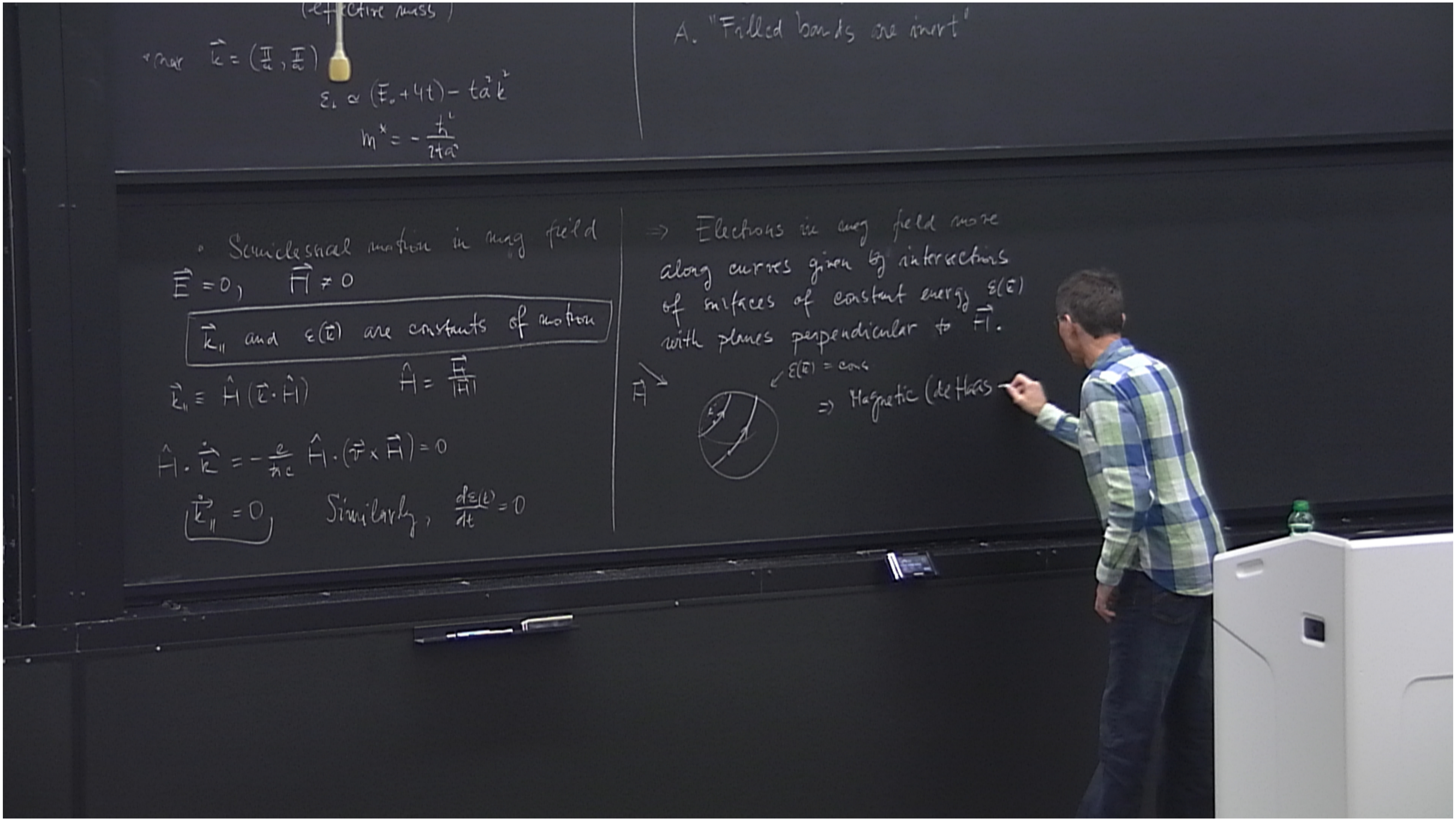
$$\vec j = (-e)\int_{k\in occ} = (-e)\left[\overbrace{\int_{k\in all}}^{0} - \int_{k\in empty}\right]$$

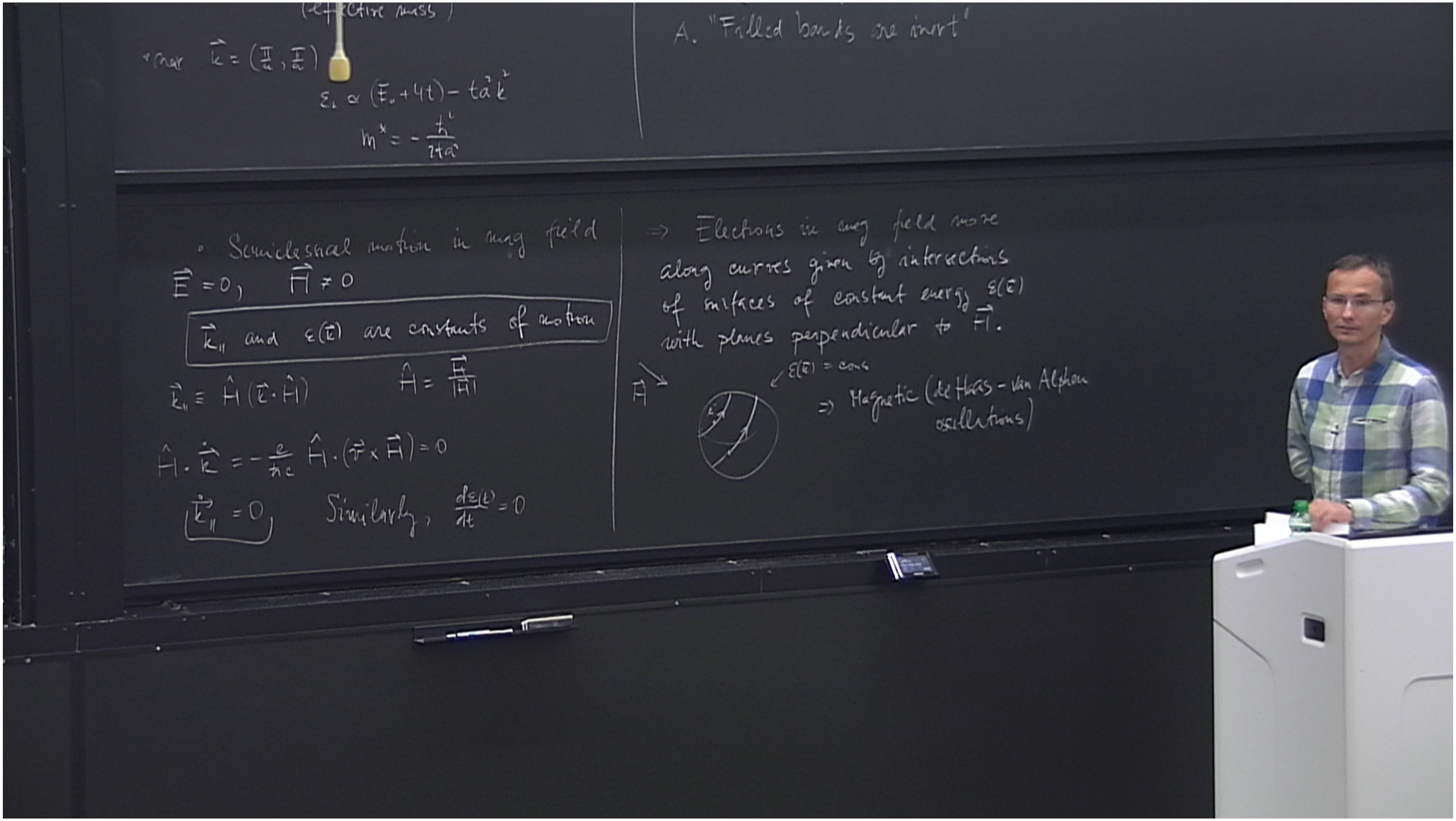
$$= (+e)\int_{k\in empty}\frac{d^3k}{4\pi^3}\frac{1}{\hbar}\frac{\partial\varepsilon(\vec k)}{\partial\vec k}$$


semiclassical eqs of wavepacket motion.
$\dot{\vec{r}} = \vec{v}_n(\vec{k}) = \frac{1}{\hbar}\frac{\partial \varepsilon_n(\vec{k})}{\partial \vec{k}}$
$\hbar\dot{\vec{k}} = -e\left[\vec{E} + \frac{1}{c}\vec{v}_n(\vec{k})\times\vec{H}\right]$
$\hbar\omega \ll \Delta(\vec{k})$ ✓
B. Motion in applied DC electric field
$\vec{H}=0, \quad \vec{E} = \text{const}$
$\vec{k}(t) = \vec{k}(0) - \frac{e\vec{E}}{\hbar}t$
⇒ produces current periodic in time !!?
"collisions"
◦ Collisions will restore steady state.
◦ However "Bloch oscillations"
extremely pure samples, at ultra low T
◦ Holes
$\vec{j} = (-e)\int_{k\in occ} = (-e)\left[\int_{k\in all} - \int_{k\in empty}\right]$
$= (+e)\int_{k\in empty}\frac{d^3k}{4\pi^3}\frac{1}{\hbar}\frac{\partial\varepsilon(\vec{k})}{\partial\vec{k}}$


(effective mass)
A. "Filled bands are inert"
$\varepsilon_1 \propto (E_0+4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$
Semiclassical motion in mag field
$\vec{E}=0, \quad \vec{H}\neq 0$
$\vec{k}_{\parallel}$ and $\varepsilon(\vec{k})$ are constants of motion
$\vec{k}_{\parallel} \equiv \hat{H}(\vec{k}\cdot\hat{H})$
$\hat{H} = \frac{\vec{H}}{|H|}$


(effective mass)
near $\vec{k} = \left(\frac{\pi}{a}, \frac{\pi}{a}\right)$
$\varepsilon_1 \simeq (E_0 + 4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$
A. "Filled bands are inert"
• Semiclassical motion in mag field
$\vec{E} = 0, \quad \vec{H} \neq 0$
$\vec{k}_\parallel$ and $\varepsilon(\vec{k})$ are constants of motion
$\vec{k}_\parallel \equiv \hat{H}(\vec{k}\cdot\hat{H}) \qquad \hat{H} = \frac{\vec{H}}{|\vec{H}|}$
$\hat{H}\cdot\dot{\vec{k}} = -\frac{e}{\hbar c}\hat{H}\cdot(\vec{v}\times\vec{H}) = 0$
$\dot{\vec{k}}_\parallel = 0$
Similarly, $\frac{d\varepsilon(t)}{dt} = 0$
$\Rightarrow$


(effective mass)
max $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \approx (E_0 + 4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$
A. "Filled bands are inert"
• Semiclassical motion in mag field
$\vec{E} = 0, \quad \vec{H} \neq 0$
$\vec{k}_\parallel$ and $\varepsilon(\vec{k})$ are constants of motion
$\vec{k}_\parallel \equiv \hat{H}(\vec{k}\cdot\hat{H}) \qquad \hat{H} = \frac{\vec{H}}{|\vec{H}|}$
$\hat{H}\cdot\dot{\vec{k}} = -\frac{e}{\hbar c}\hat{H}\cdot(\vec{v}\times\vec{H}) = 0$
$\dot{\vec{k}}_\parallel = 0$ Similarly, $\frac{d\varepsilon(\vec{k})}{dt} = 0$
⇒ Electrons in mag field move along curves given by intersections of surfaces of constant energy $\varepsilon(\vec{k})$ with planes perpendicular to $\vec{H}$.
$\varepsilon(\vec{k}) = $ const
$\vec{H}$
⇒ Magnetic (de Haas –


(effective mass)
near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$
$\varepsilon_k \approx (E_0 + 4t) - ta^2k^2$
$m^* = -\frac{\hbar^2}{2ta^2}$
A. "Filled bands are inert"
• Semiclassical motion in mag field
$\vec{E} = 0, \quad \vec{H} \neq 0$
$\boxed{\vec{k}_\parallel \text{ and } \varepsilon(\vec{k}) \text{ are constants of motion}}$
$\vec{k}_\parallel \equiv \hat{H}(\vec{k}\cdot\hat{H}) \qquad \hat{H} = \frac{\vec{H}}{|\vec{H}|}$
$\hat{H}\cdot\dot{\vec{k}} = -\frac{e}{\hbar c}\hat{H}\cdot(\vec{v}\times\vec{H}) = 0$
$\boxed{\dot{\vec{k}}_\parallel = 0}$ Similarly, $\frac{d\varepsilon(\vec{k})}{dt} = 0$
⇒ Electrons in mag field move along curves given by intersections of surfaces of constant energy $\varepsilon(\vec{k})$ with planes perpendicular to $\vec{H}$.
$\varepsilon(\vec{k}) = \text{const}$
$\vec{H}$
⇒ Magnetic (de Haas - van Alphen oscillations)

(effective mass)

near $\vec{k} = (\frac{\pi}{a}, \frac{\pi}{a})$

$\varepsilon_k \simeq (E_0 + 4t) - ta^2 k^2$

$m^* = -\frac{\hbar^2}{2ta^2}$

A. "Filled bands are inert"

- Semiclassical motion in mag field

$\vec{E} = 0, \quad \vec{H} \neq 0$

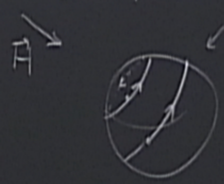
$\boxed{\vec{k}_\parallel \text{ and } \varepsilon(\vec{k}) \text{ are constants of motion}}$

$\vec{k}_\parallel \equiv \hat{H}(\vec{k} \cdot \hat{H}) \qquad \hat{H} = \frac{\vec{H}}{|H|}$

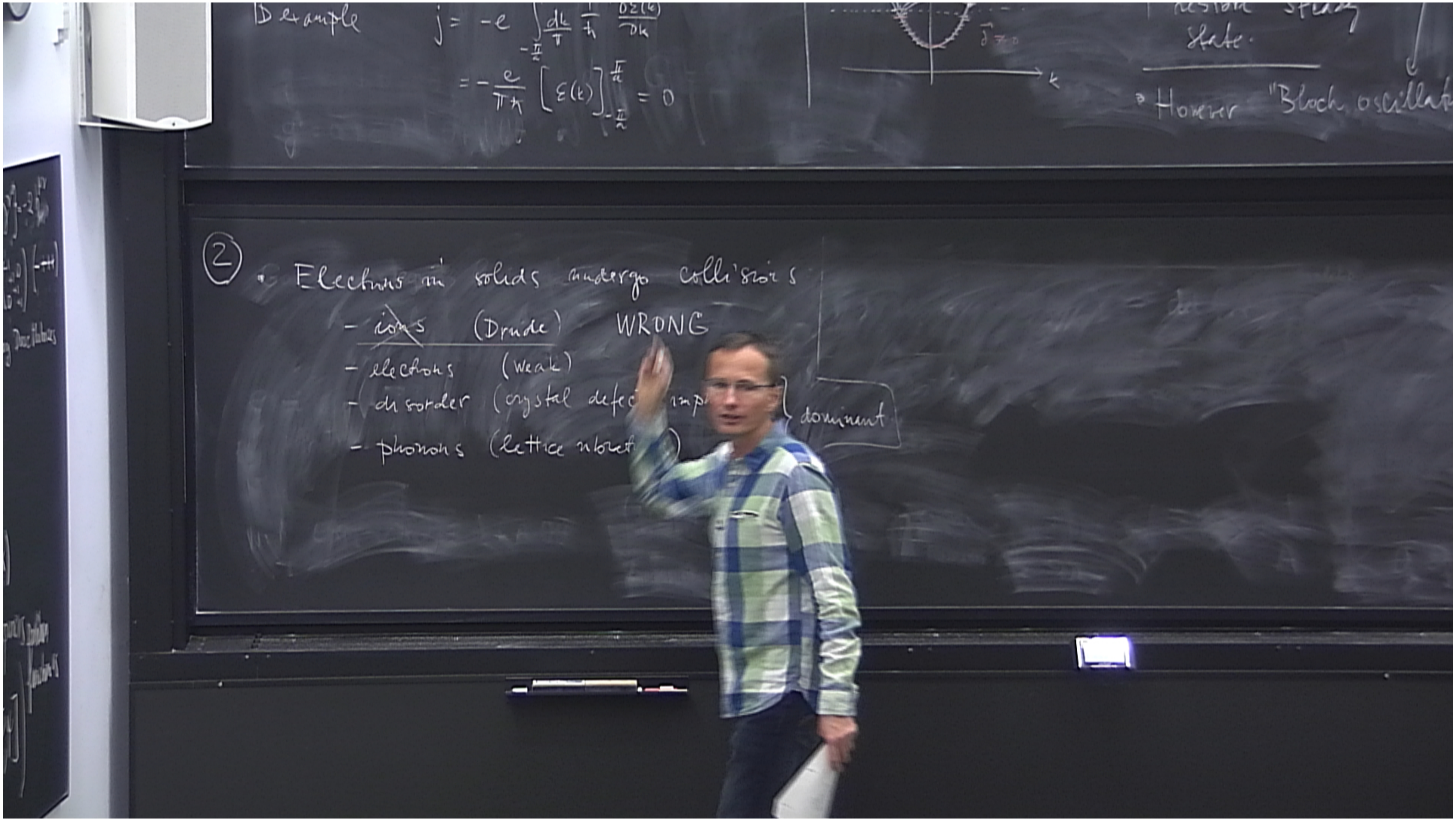
$\hat{H} \cdot \dot{\vec{k}} = -\frac{e}{\hbar c} \hat{H} \cdot (\vec{v} \times \vec{H}) = 0$

$\boxed{\dot{\vec{k}}_\parallel = 0}$ Similarly, $\frac{d\varepsilon(t)}{dt} = 0$

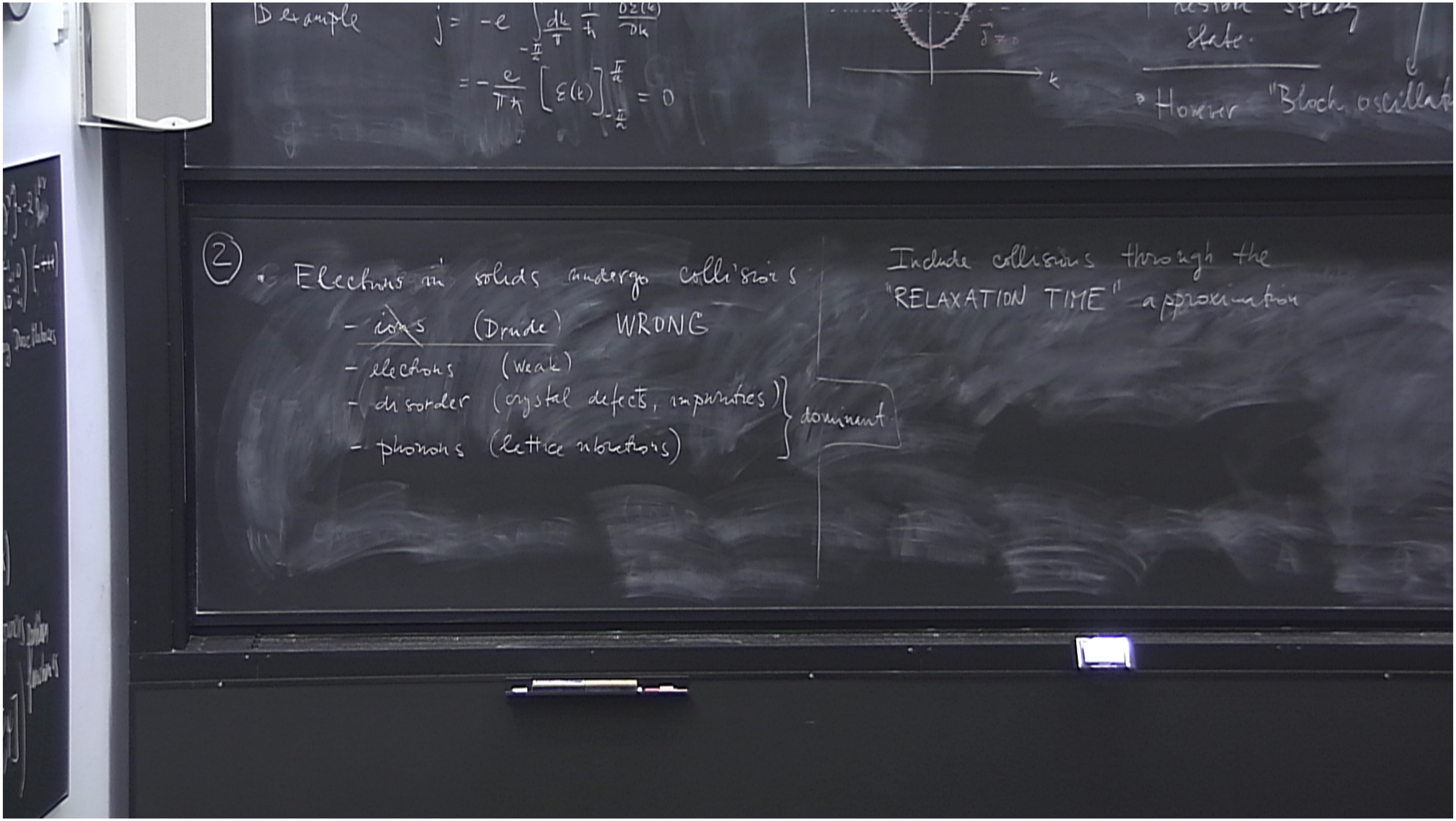
$\Rightarrow$ Electrons in mag field move along curves given by intersections of surfaces of constant energy $\varepsilon(\vec{k})$ with planes perpendicular to $\vec{H}$.



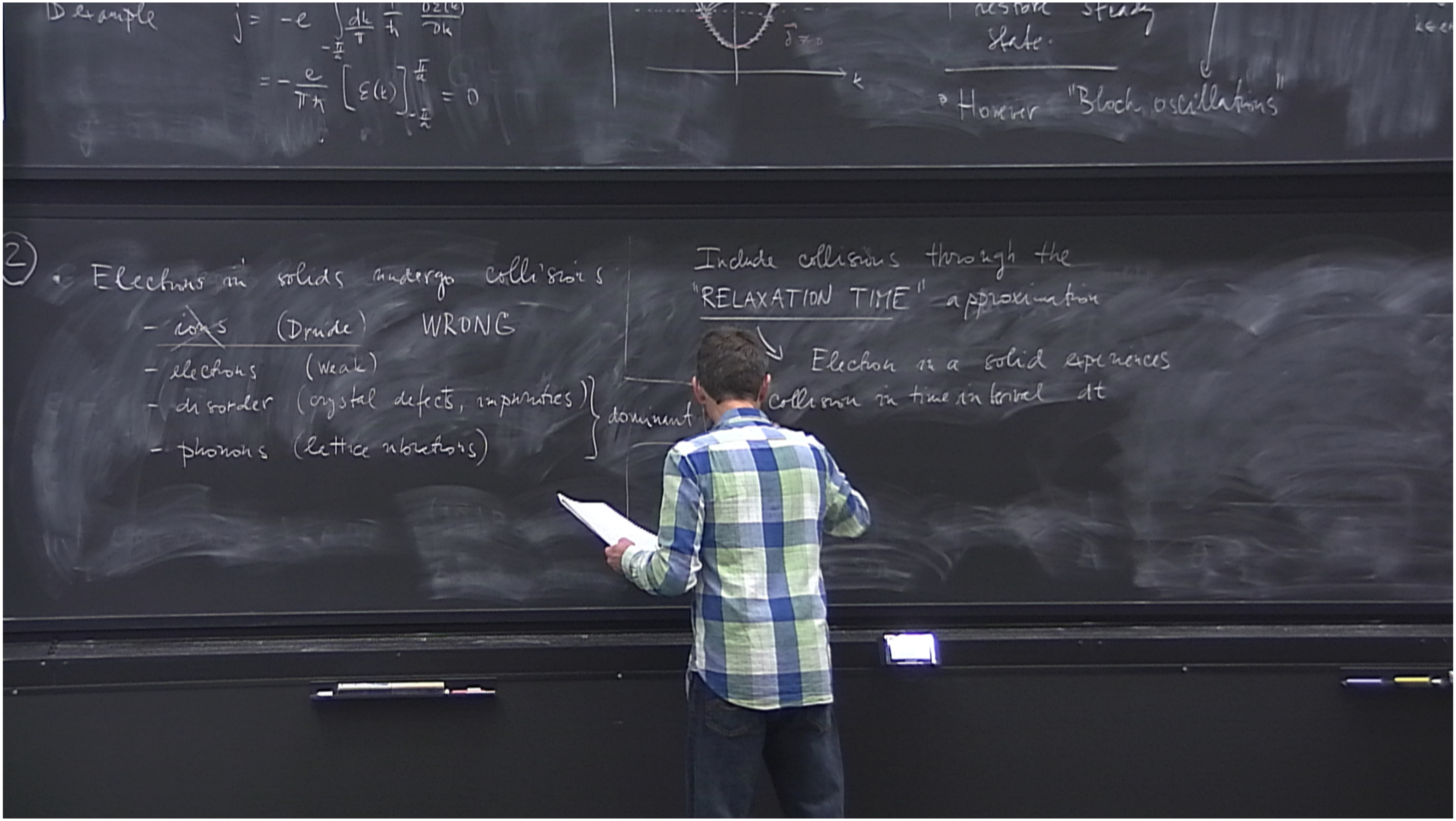
$\Rightarrow$ Magnetic (de Haas - van Alphen oscillations)



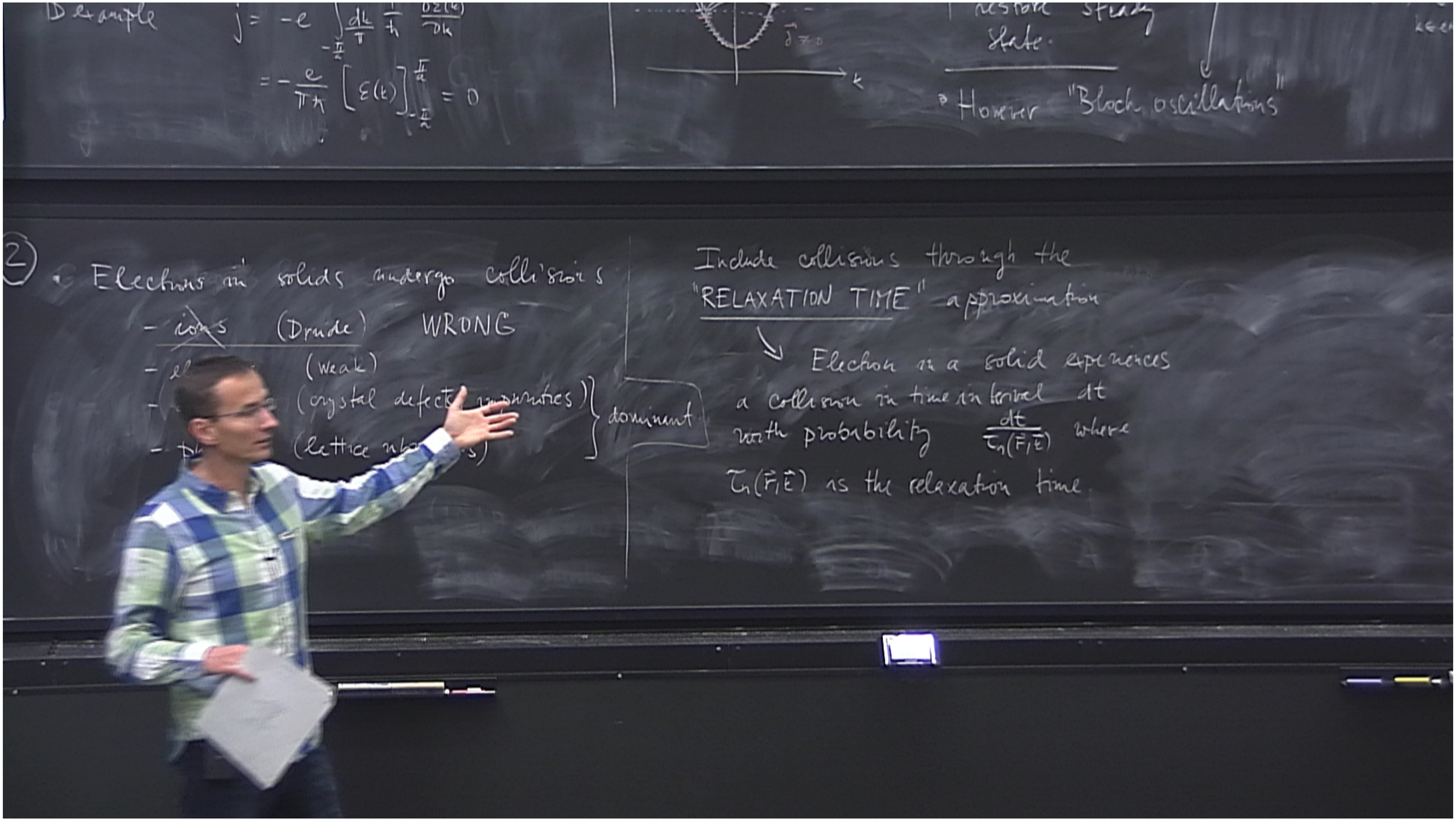
D example
"Bloch oscillat
2
• Electrons in solids undergo collisions
- ions (Drude) WRONG
- electrons (weak)
- disorder (crystal defect
- phonons (lattice vibrat
dominant

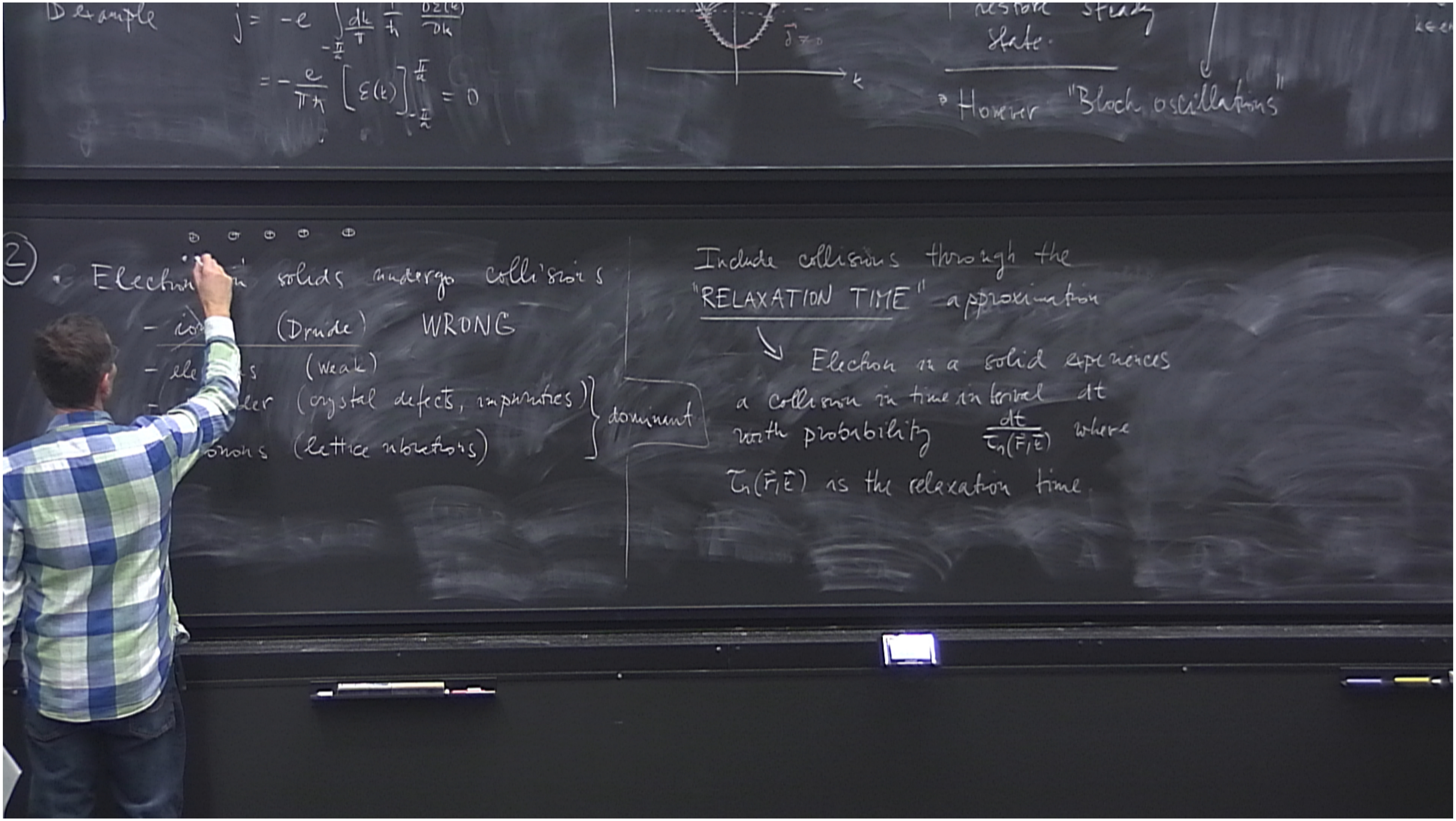


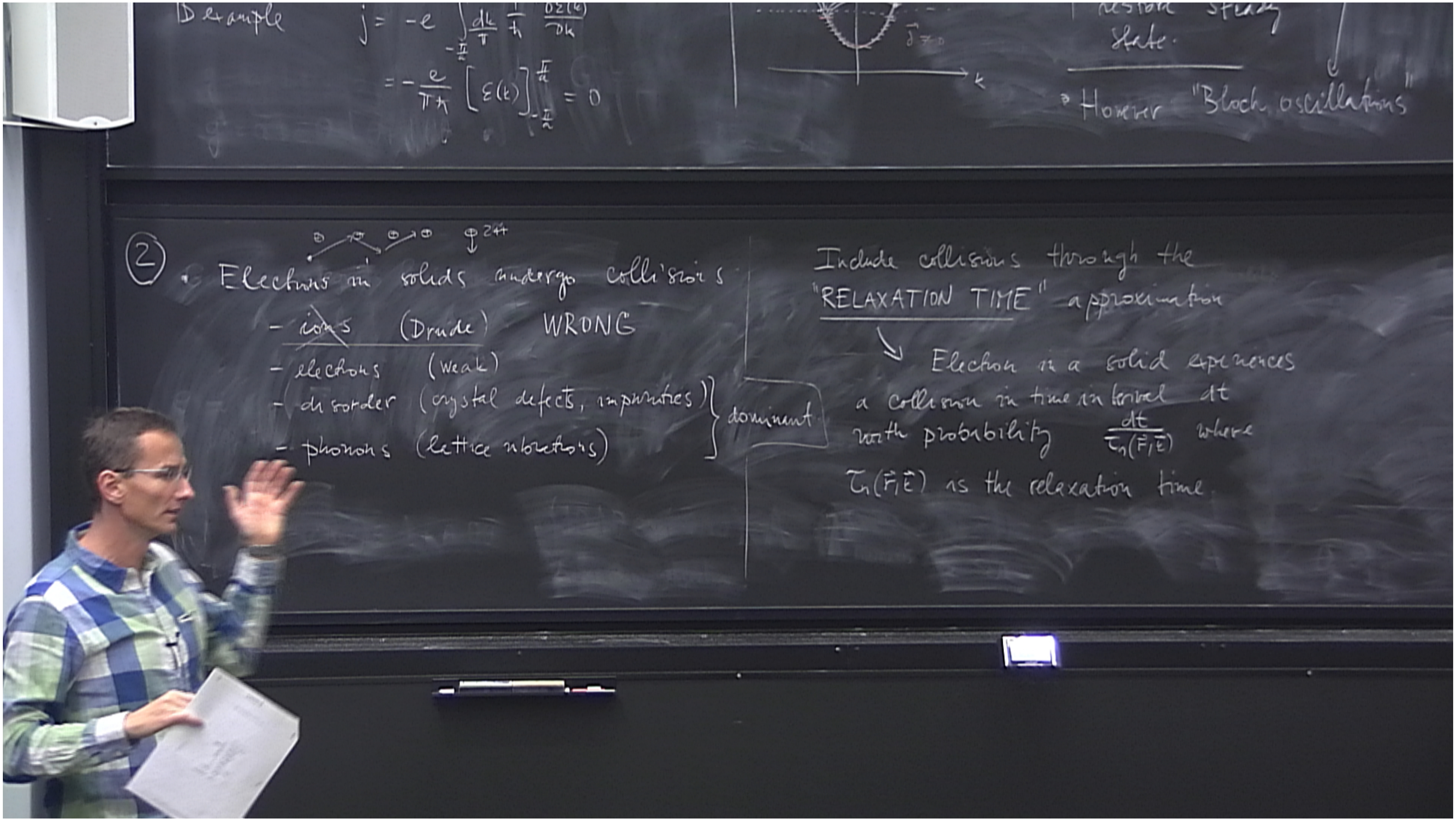
Example
$j = -e \int_{-\frac{\pi}{a}} \frac{dk}{\pi} \frac{1}{\hbar} \frac{\partial \varepsilon(k)}{\partial k}$
$= -\frac{e}{\pi\hbar} \left[\varepsilon(k) \right]_{-\frac{\pi}{a}}^{\frac{\pi}{a}} = 0$
state.
• However "Bloch oscillat
② • Electrons in solids undergo collisions
- ~~ions~~ (Drude) WRONG
- electrons (weak)
- disorder (crystal defects, impurities)
- phonons (lattice vibrations)
dominant
Include collisions through the "RELAXATION TIME" approximation

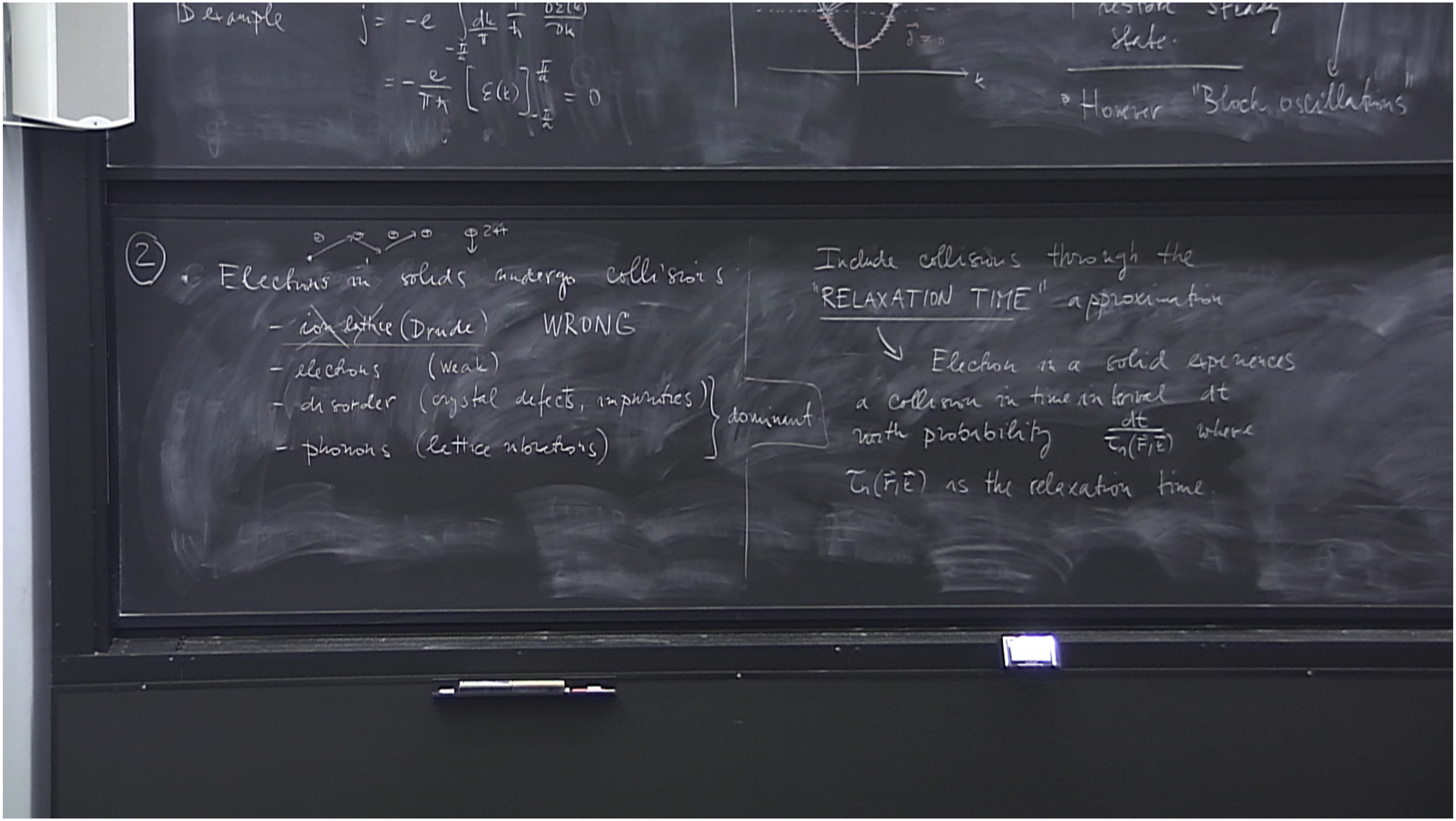


Electrons in solids undergo collisions
- ions (Drude) WRONG
- electrons (weak)
- disorder (crystal defects, impurities)
- phonons (lattice vibrations)
dominant
Include collisions through the
"RELAXATION TIME" approximation
Electron in a solid experiences
collision in time interval dt
However "Bloch oscillations"

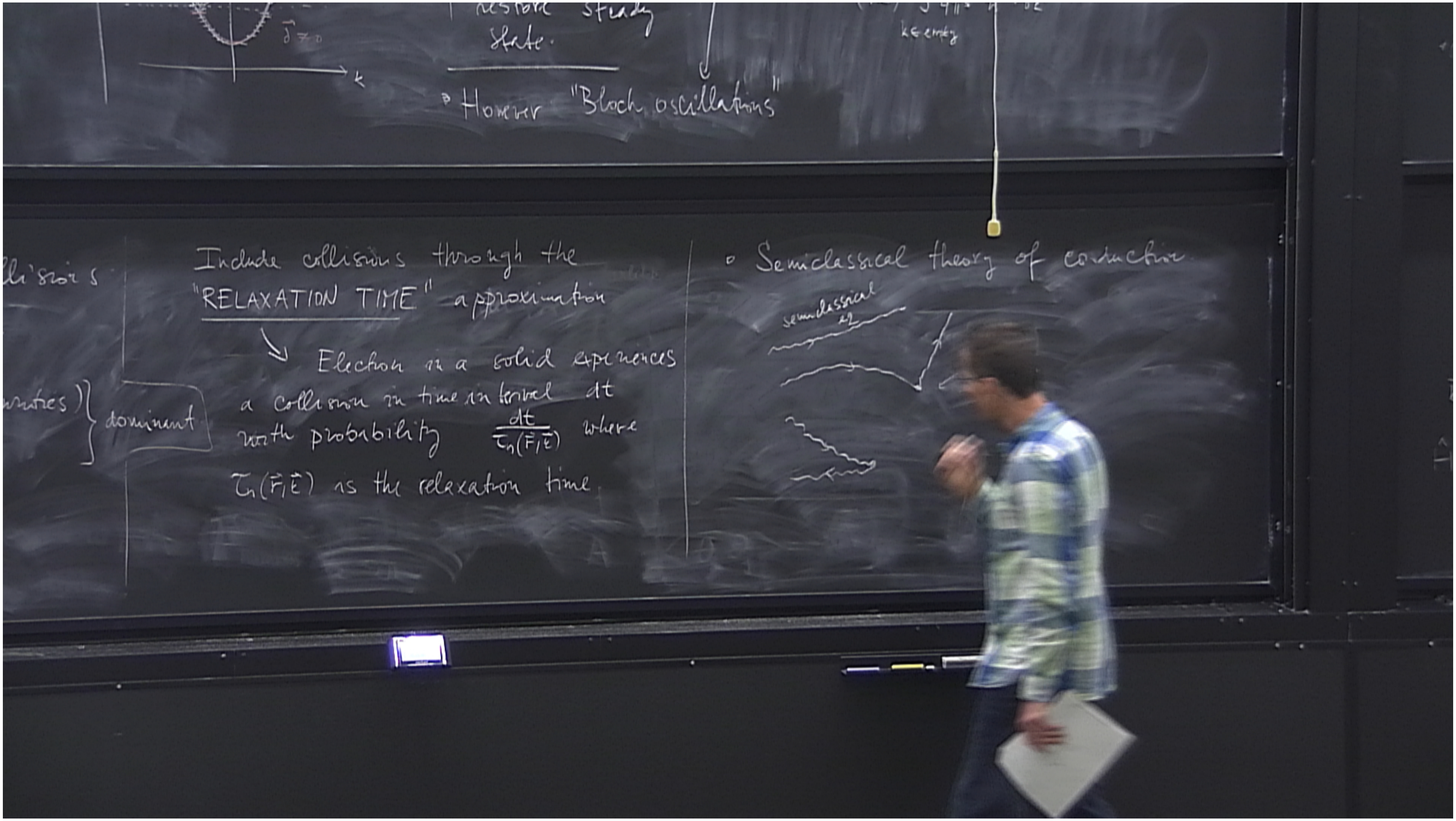

Electrons in solids undergo collisions
- ions (Drude) WRONG
(weak)
(crystal defects impurities)
dominant
Include collisions through the
"RELAXATION TIME" approximation
Electron in a solid experiences
a collision in time interval dt
with probability dt/τn(r,k) where
τn(r,k) is the relaxation time
state.
However "Bloch oscillations"


Example
$j = -e \int_{-\pi/a}^{\pi/a} \frac{dk}{\pi} \frac{1}{\hbar} \frac{\partial \varepsilon(k)}{\partial k}$
$= -\frac{e}{\pi\hbar} \left[\varepsilon(k) \right]_{-\pi/a}^{\pi/a} = 0$
restore steady state.
However "Bloch oscillations"
2
Electr solids undergo collisions
- ions (Drude) WRONG
- ele s (weak)
der (crystal defects, impurities)
onons (lattice vibrations)
dominant
Include collisions through the "RELAXATION TIME" approximation
Electron in a solid experiences a collision in time interval dt with probability $\frac{dt}{\tau_n(\vec{r}, \vec{k})}$ where
$\tau_n(\vec{r}, \vec{k})$ is the relaxation time

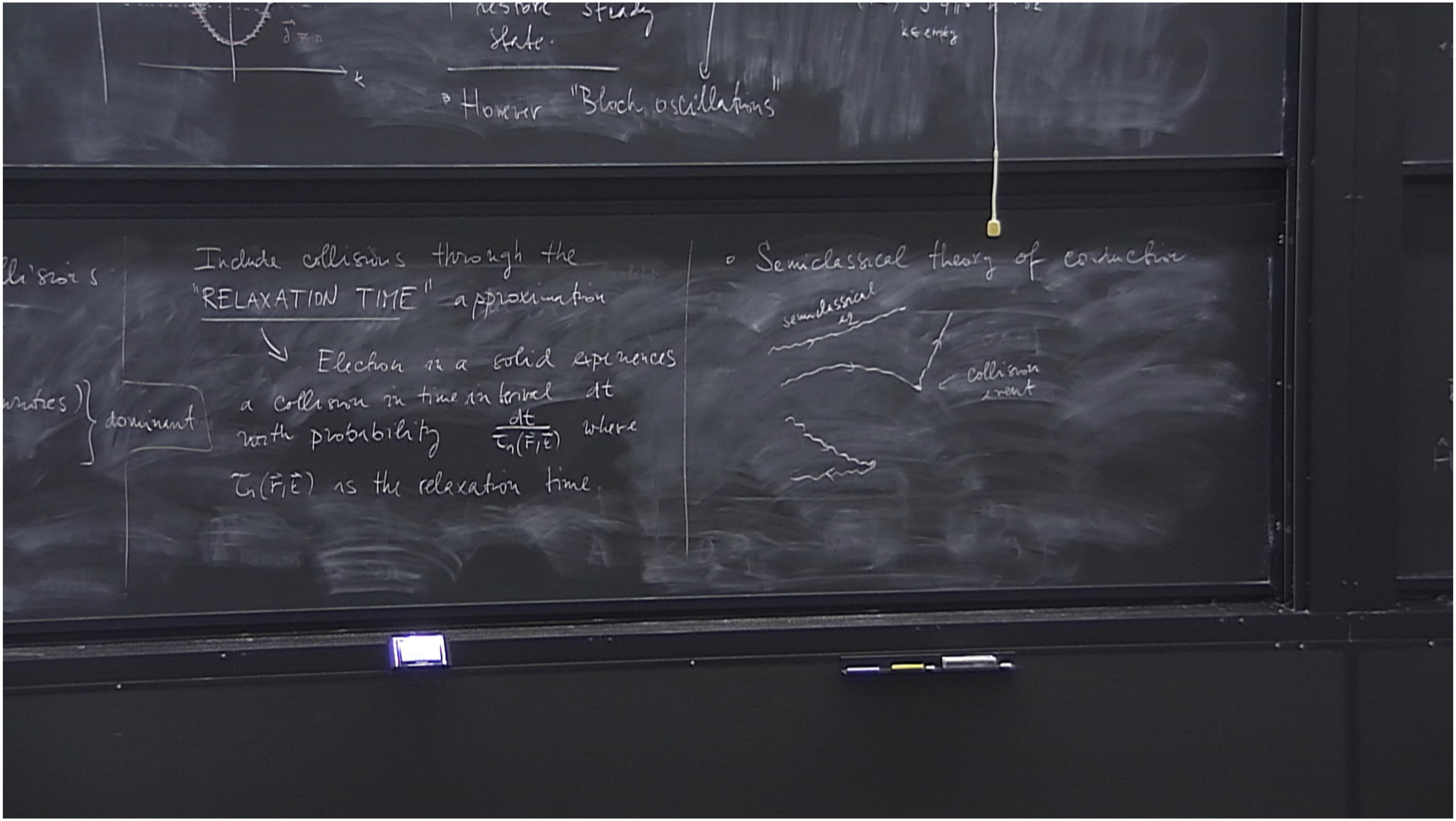

Example
$j = -e \int_{-\frac{\pi}{a}} \frac{dk}{\pi} \frac{1}{\hbar} \frac{\partial \varepsilon(k)}{\partial k}$
$= -\frac{e}{\pi\hbar} \left[\varepsilon(k)\right]_{-\frac{\pi}{a}}^{\frac{\pi}{a}} = 0$
state.
• However "Bloch oscillations"
② • Electrons in solids undergo collisions
– ~~ions~~ (Drude) WRONG
– electrons (weak)
– disorder (crystal defects, impurities)
– phonons (lattice vibrations)
dominant
Include collisions through the "RELAXATION TIME" approximation
Electron in a solid experiences a collision in time interval dt with probability $\frac{dt}{\tau_n(\vec{r},\vec{k})}$ where
$\tau_n(\vec{r},\vec{k})$ is the relaxation time



Example
$j = -e \int_{-\frac{\pi}{a}} \frac{dk}{\pi} \frac{1}{\hbar} \frac{\partial \varepsilon(k)}{\partial k}$
$= -\frac{e}{\pi\hbar} \left[\varepsilon(k) \right]_{-\frac{\pi}{a}}^{\frac{\pi}{a}} = 0$
state.
• However "Bloch oscillations"
②
• Electrons in solids undergo collisions
- ion lattice (Drude) WRONG
- electrons (weak)
- disorder (crystal defects, impurities)
- phonons (lattice vibrations)
dominant
Include collisions through the "RELAXATION TIME" approximation
Electron in a solid experiences a collision in time interval dt with probability $\frac{dt}{\tau_n(\vec{r},\vec{k})}$ where
$\tau_n(\vec{r},\vec{k})$ is the relaxation time.

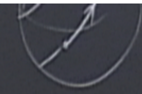


restore steady state.
However "Bloch oscillations"
Include collisions through the
"RELAXATION TIME" approximation
Electron in a solid experiences
a collision in time interval dt
with probability $\frac{dt}{\tau_n(\vec{r},\vec{k})}$ where
$\tau_n(\vec{r},\vec{k})$ is the relaxation time
dominant
Semiclassical theory of conduction
semiclassical


restore steady state.
• However "Bloch oscillations"
Include collisions through the "RELAXATION TIME" approximation
Electron in a solid experiences a collision in time interval dt with probability $\frac{dt}{\tau_n(\vec{r},\vec{k})}$ where
$\tau_n(\vec{r},\vec{k})$ is the relaxation time.
dominant
• Semiclassical theory of conduction
semiclassical eq
collision event

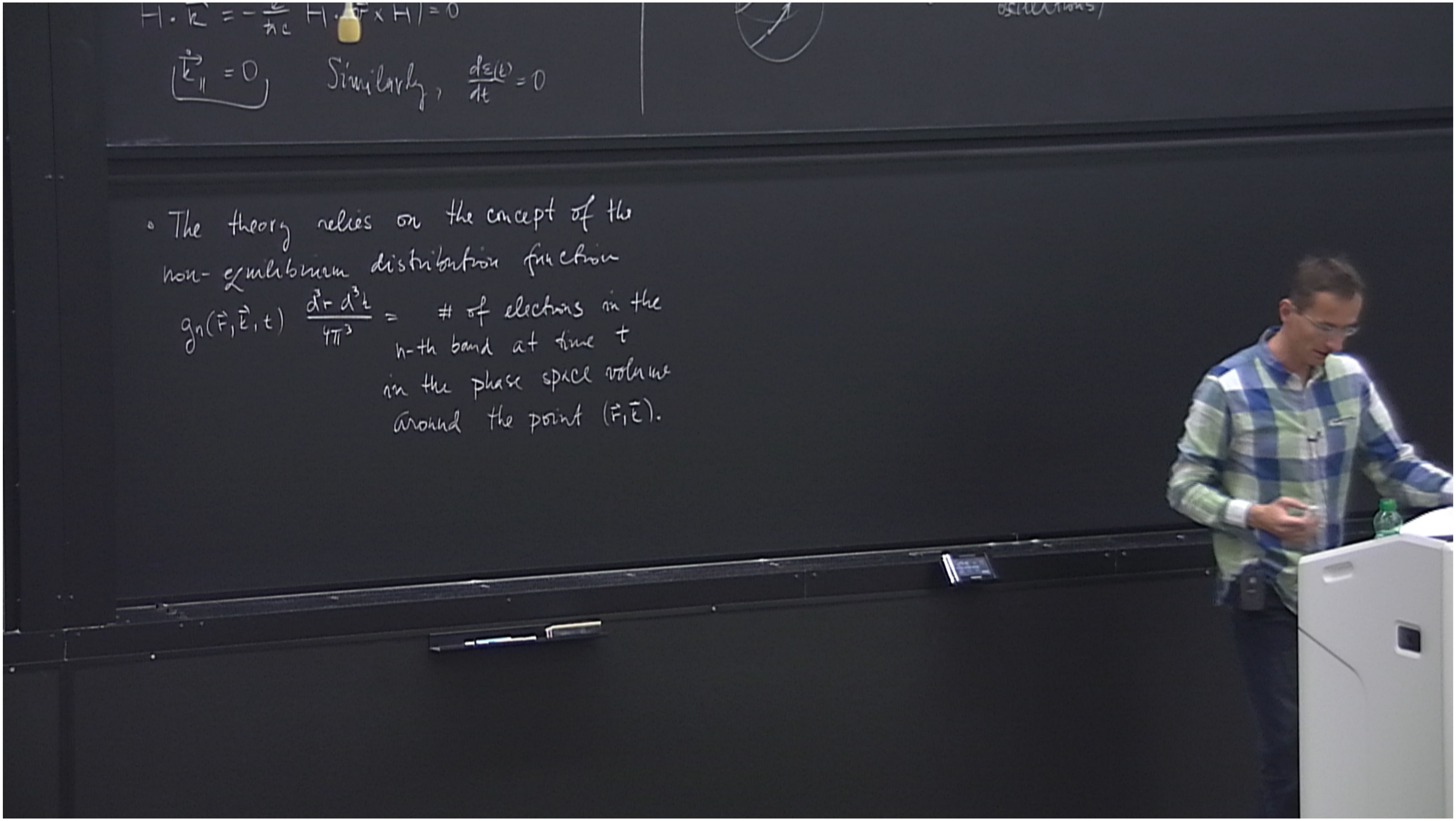
$$H \cdot \dot{\vec{k}} = -\frac{e}{\hbar c} H \cdot (\dot{\vec{r}} \times H) = 0$$

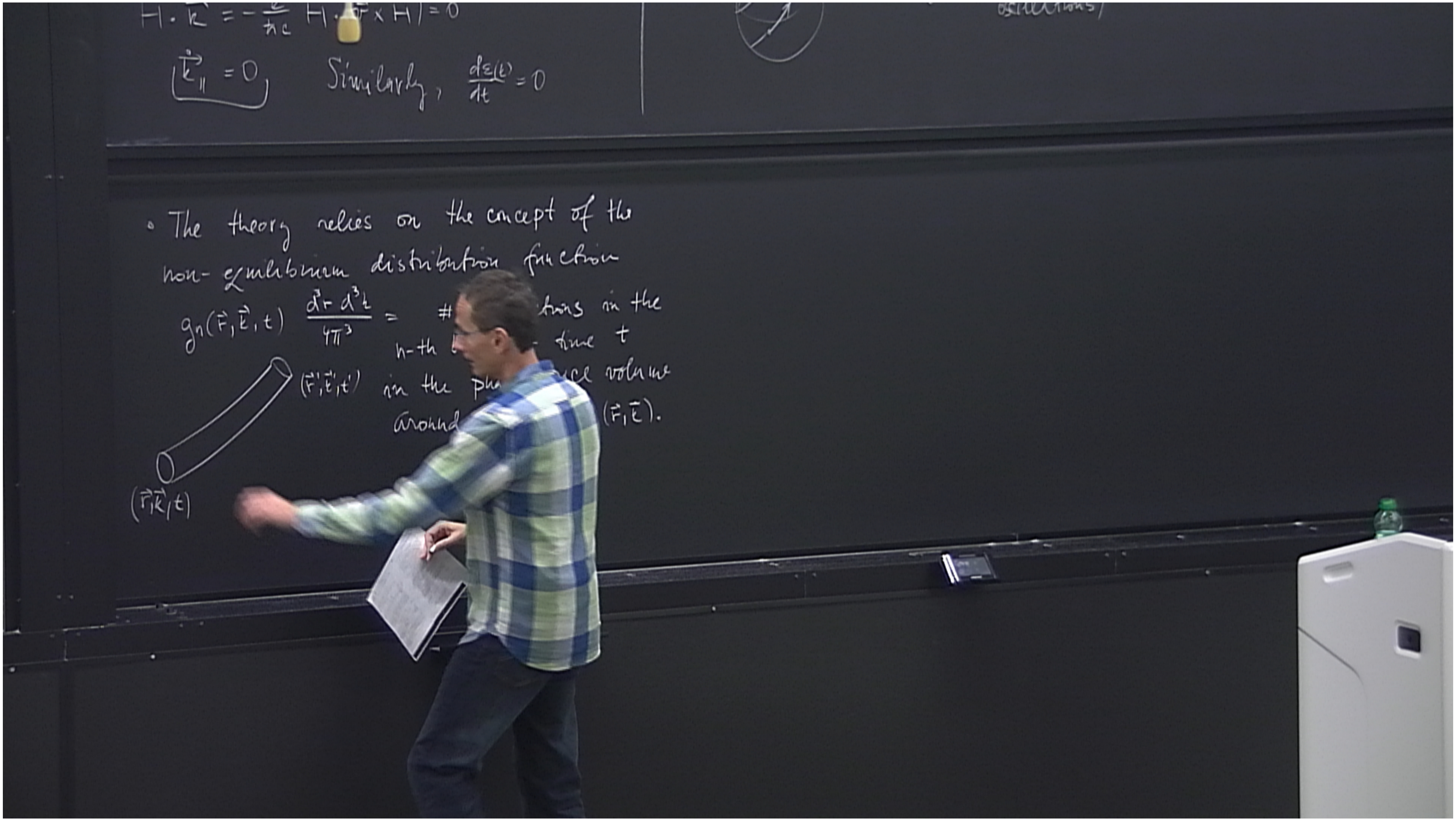
$$\boxed{\dot{\vec{k}}_\parallel = 0}, \quad \text{Similarly}, \quad \frac{d\varepsilon(t)}{dt} = 0$$

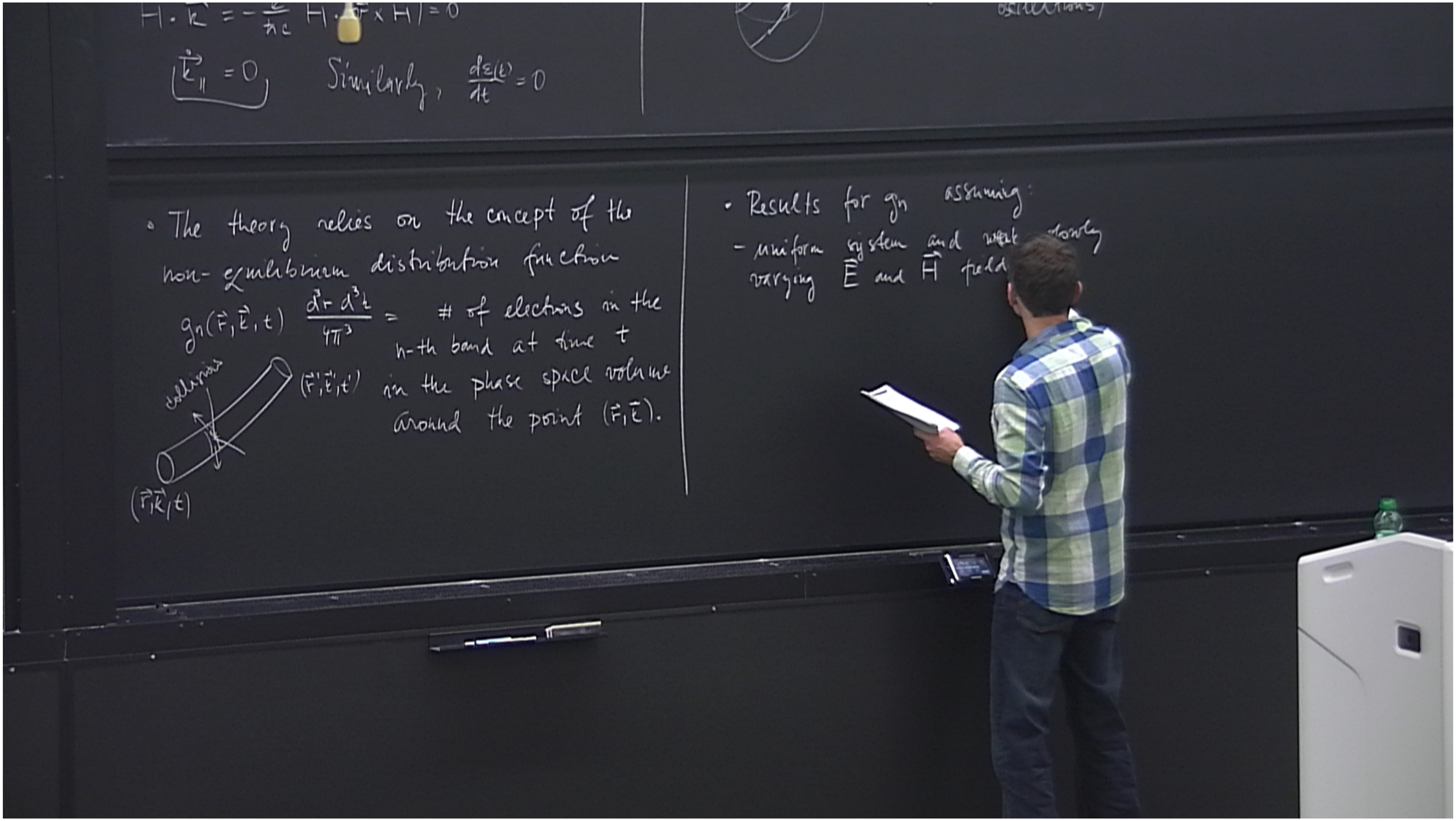


- The theory relies on the concept of the non-equilibrium distribution function

$$g_n(\vec{r}, \vec{k}, t) \, \frac{d^3r \, d^3k}{4\pi^3} = \text{\# of electrons in the } n\text{-th band at time } t \text{ in the phase space volume around the point } (\vec{r}, \vec{k}).$$


$\dot{\vec{k}}_{\parallel} = 0$
Similarly, $\frac{d\varepsilon(t)}{dt} = 0$
• The theory relies on the concept of the
non-equilibrium distribution function
$g_n(\vec{r},\vec{k},t)\,\frac{d^3r\,d^3k}{4\pi^3}$ = # of electrons in the
n-th band at time t
in the phase space volume
around the point $(\vec{r},\vec{k})$.


k⃗∥ = 0, Similarly, dε(t)/dt = 0
• The theory relies on the concept of the
non-equilibrium distribution function
gn(r⃗, k⃗, t) d³r d³k / 4π³ = #
n-th
in the
around
(r⃗, k⃗).
(r⃗, k⃗, t)
(r⃗′, k⃗′, t′)

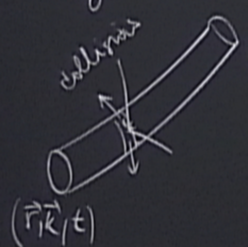

Similarly, dε(t)/dt = 0
• The theory relies on the concept of the non-equilibrium distribution function
gn(r, k, t) d³r d³k / 4π³ = # of electrons in the n-th band at time t in the phase space volume around the point (r, k).
collisions
(r', k', t')
(r, k, t)
• Results for gn assuming:
- uniform system and weak slowly varying E and H fields

$H \cdot \dot{\vec{k}} = -\frac{e}{\hbar c} H \cdot (\ldots \times H) = 0$

$\dot{\vec{k}}_{\parallel} = 0$ Similarly, $\frac{d\varepsilon(t)}{dt} = 0$

- The theory relies on the concept of the non-equilibrium distribution function

$g_n(\vec{r},\vec{k},t)\,\frac{d^3r\,d^3k}{4\pi^3}$ = # of electrons in the n-th band at time t in the phase space volume around the point $(\vec{r},\vec{k})$.



- Results for g_n assuming:
  - uniform system and weak, slowly varying $\vec{E}$ and $\vec{H}$ fields, and weak $\vec{\nabla} T$
  - $\tau_n(\vec{r},\vec{k}) = \tau_n(\varepsilon_n(\vec{k}))$

$$g_n(\vec{k}) = f(\vec{k}) + \tau(\varepsilon(\vec{k}))\left(-\frac{\partial f}{\partial \varepsilon}\right)\vec{v}(\vec{k})\cdot\left[-e\vec{E} + \frac{\varepsilon(\vec{k})-\mu}{T}(-\vec{\nabla} T)\right]$$

linear response theory

$\vec H) = 0$

...ly, $\frac{d\varepsilon(\vec k)}{dt} = 0$

$\vec H$

$\varepsilon(\vec k)$ = const

$\Rightarrow$ Magnetic (de Haas - van Alphen oscillations)

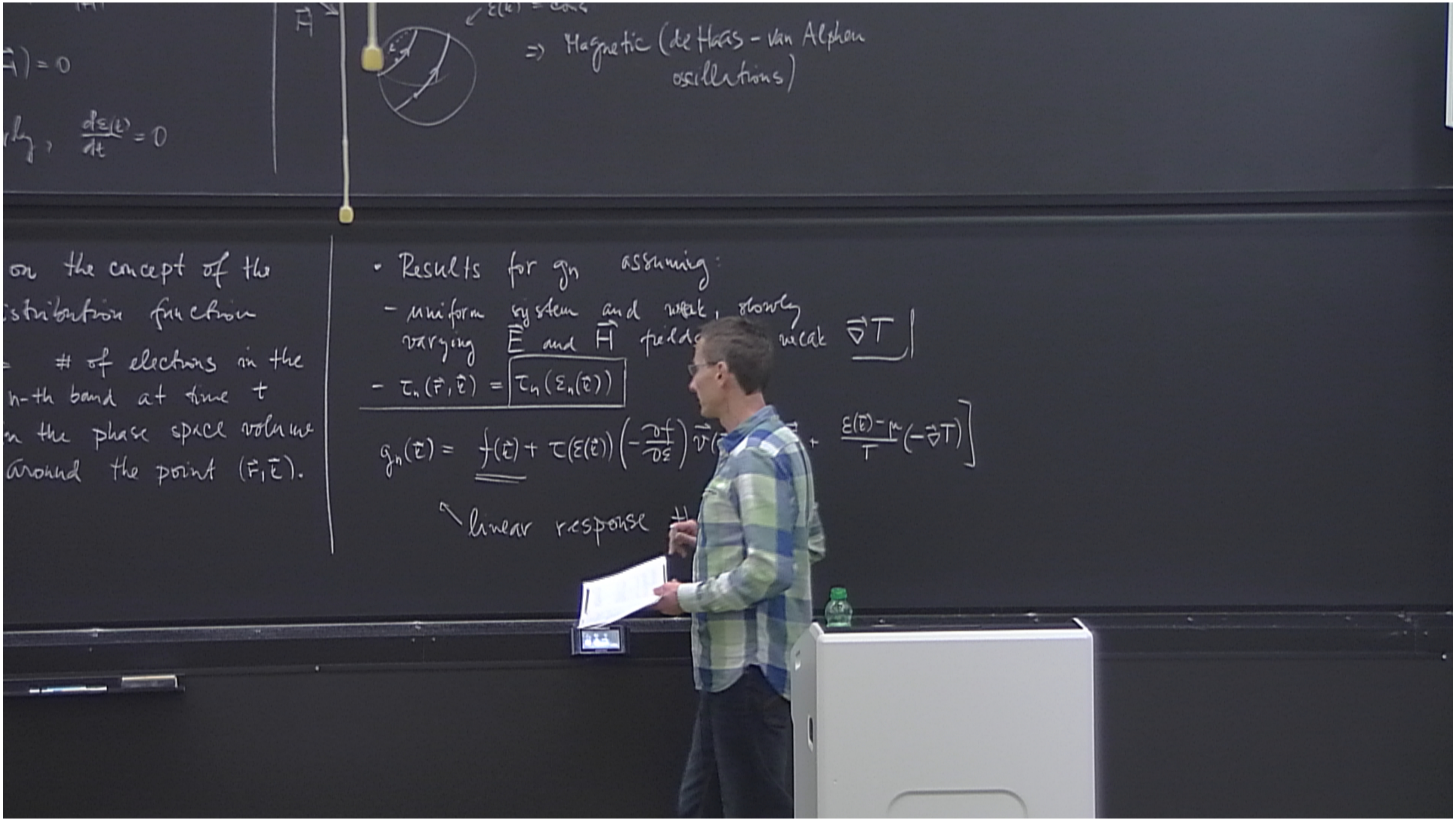
...on the concept of the
...istribution function
... = # of electrons in the
n-th band at time t
...n the phase space volume
around the point $(\vec r, \vec k)$.

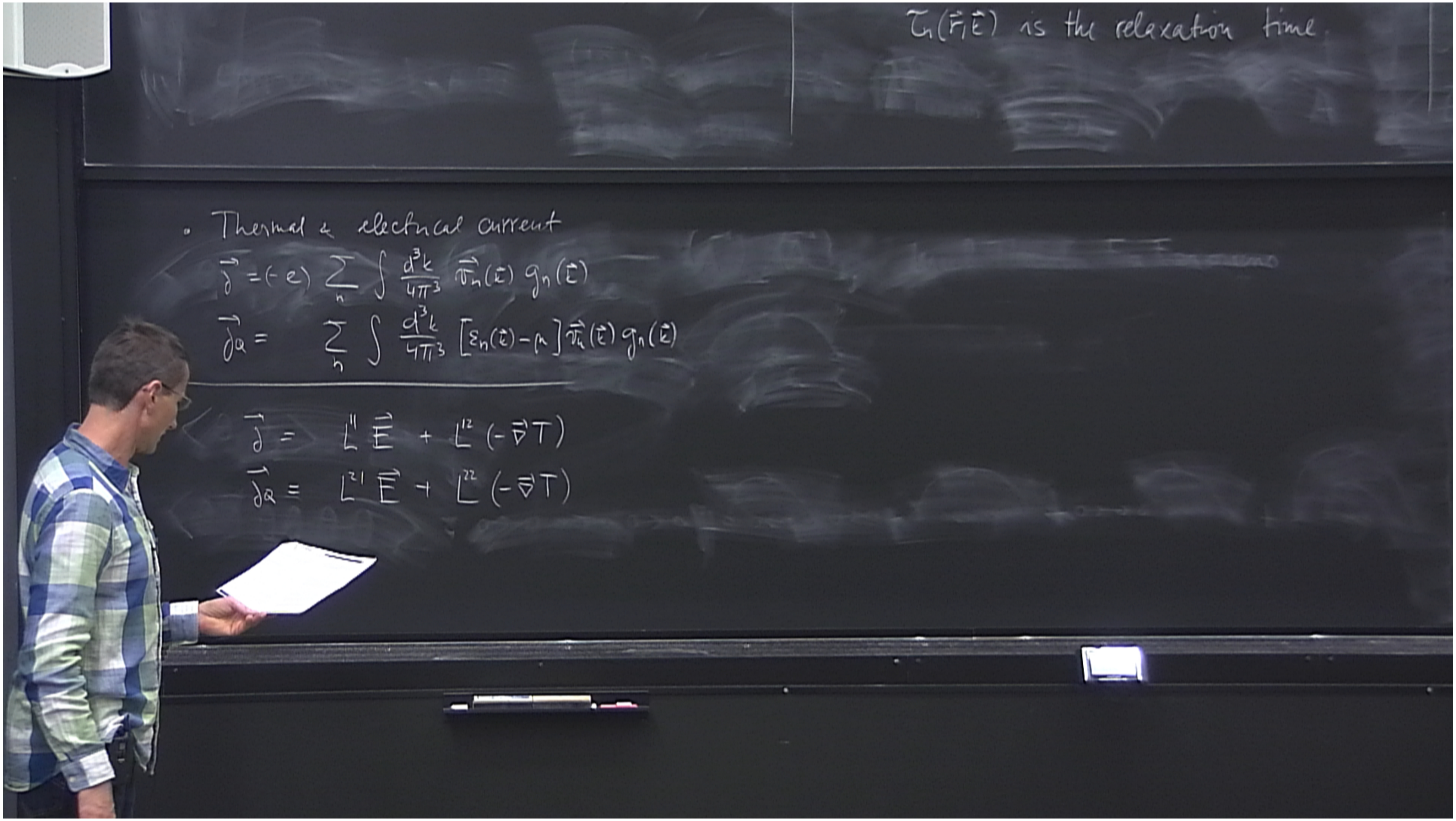
• Results for g_n assuming:

- uniform system and weak, slowly varying $\vec E$ and $\vec H$ fields, and weak $\vec\nabla T$
- $\tau_n(\vec r, \vec k) = \boxed{\tau_n(\varepsilon_n(\vec k))}$

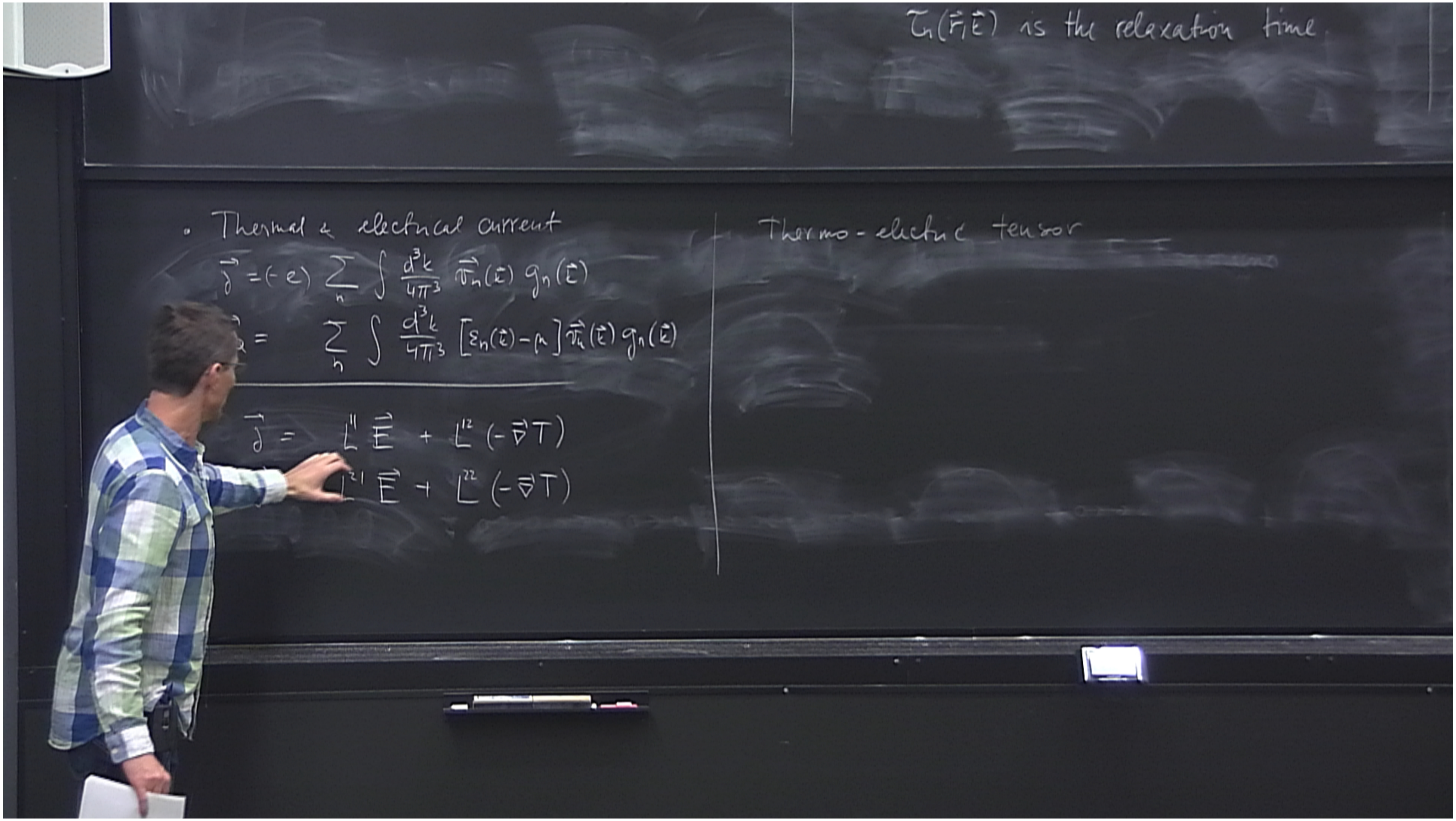
$$g_n(\vec k) = f(\vec k) + \tau(\varepsilon(\vec k))\left(-\frac{\partial f}{\partial \varepsilon}\right)\vec v(\vec k)\cdot\left[-e\vec E + \frac{\varepsilon(\vec k)-\mu}{T}(-\vec\nabla T)\right]$$

linear response theory


=> Magnetic (de Haas - van Alphen oscillations)
on the concept of the
istribution function
= # of electrons in the
n-th band at time t
n the phase space volume
around the point (r, k).
• Results for gn assuming:
- uniform system and weak, slowly
varying E and H fields
weak ∇T
- τn(r, k) = τn(εn(k))
linear response



$\tau_n(\vec{r},\vec{k})$ is the relaxation time
• Thermal & electrical current
$\vec{j} = (-e) \sum_n \int \frac{d^3k}{4\pi^3} \vec{v}_n(\vec{k})\, g_n(\vec{k})$
$\vec{j}_Q = \sum_n \int \frac{d^3k}{4\pi^3} \left[\varepsilon_n(\vec{k}) - \mu\right] \vec{v}_n(\vec{k})\, g_n(\vec{k})$
$\vec{j} = L^{11} \vec{E} + L^{12} (-\vec{\nabla} T)$
$\vec{j}_Q = L^{21} \vec{E} + L^{22} (-\vec{\nabla} T)$


$\tau_n(\vec{r},\vec{k})$ is the relaxation time
• Thermal & electrical current
$\vec{j} = (-e) \sum_n \int \frac{d^3k}{4\pi^3} \vec{v}_n(\vec{k})\, g_n(\vec{k})$
$\vec{Q} = \sum_n \int \frac{d^3k}{4\pi^3} [\varepsilon_n(\vec{k}) - \mu]\, \vec{v}_n(\vec{k})\, g_n(\vec{k})$
$\vec{j} = L^{11} \vec{E} + L^{12} (-\vec{\nabla} T)$
$L^{21} \vec{E} + L^{22} (-\vec{\nabla} T)$
Thermo-electric tensor

$\tau_n(\vec{r},\vec{k})$ is the relaxation time

Thermo-electric tensor

$L^{11} = \mathcal{L}^{(0)}, \quad L^{21} = T L^{12} = -\frac{1}{e}\mathcal{L}^{(1)}$

$L^{(22)} = \frac{1}{e^2 T}\mathcal{L}^{(2)}$

re $\mathcal{L}^{(\alpha)}_{ij} = e^2 \int \frac{d^3k}{4\pi^3}\left(-\frac{\partial f}{\partial \varepsilon}\right)\tau(\varepsilon(k))\,\underbrace{\vec{v}_i(k)\,\vec{v}_j(k)}_{\text{outer product}}\,[\varepsilon(k)-\mu]^\alpha$

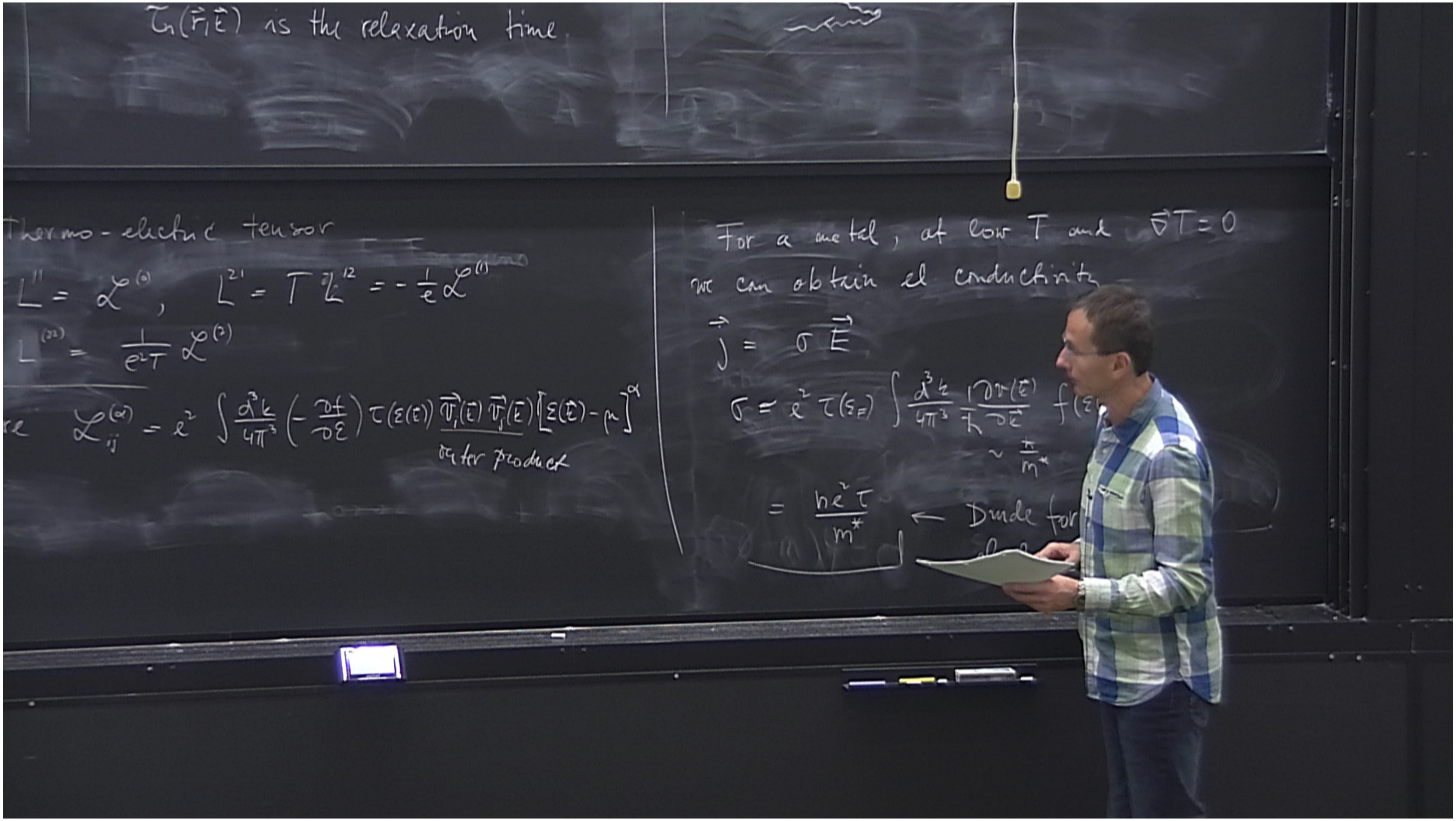
For a metal, at low T and $\vec{\nabla}T = 0$

we can obtain el conductivity

$\vec{j} = \sigma \vec{E}$

$\sigma = e^2\tau(\varepsilon_F)\int \frac{d^3k}{4\pi^3}\underbrace{\frac{1}{\hbar}\frac{\partial v(k)}{\partial k}}_{\sim \frac{\hbar}{m^*}} f(\varepsilon(k))$

$= \frac{ne^2\tau}{m^*}$ ← Drude formula for electric conductivity.


τ(r, k) is the relaxation time
Thermo-electric tensor
L11 = 𝓛(0), L21 = T L12 = −1/e 𝓛(1)
L(22) = 1/(e²T) 𝓛(2)
𝓛ij(α) = e² ∫ d³k/4π³ (−∂f/∂ε) τ(ε(k)) vi(k) vj(k) [ε(k) − μ]^α
outer product
For a metal, at low T and ∇T = 0
we can obtain el conductivity
j = σ E
σ = e² τ(εF) ∫ d³k/4π³ (1/ħ) ∂v(k)/∂k f(ε
~ n/m*
= ne²τ/m*
← Drude for