Title: 14/15 PSI - Condensed Matter-Lecture 7

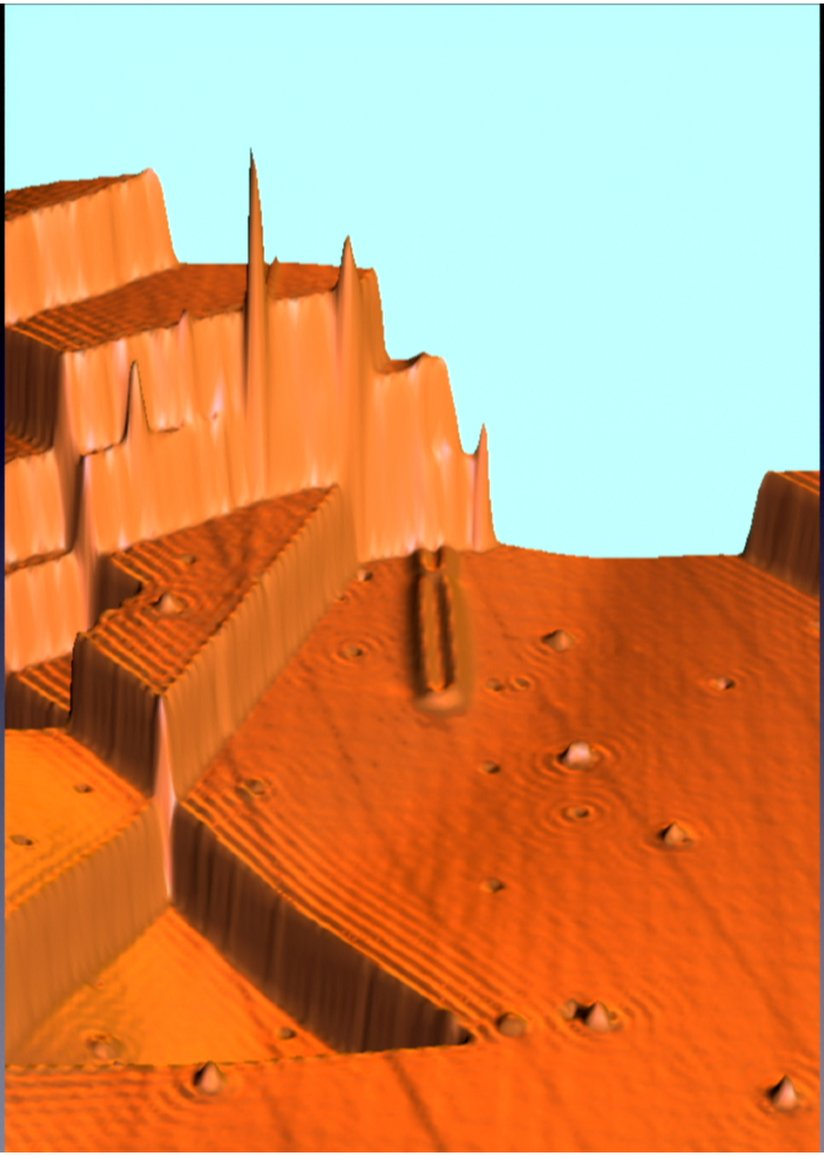
Date: Nov 18, 2014 10:45 AM

URL: http://pirsa.org/14110033

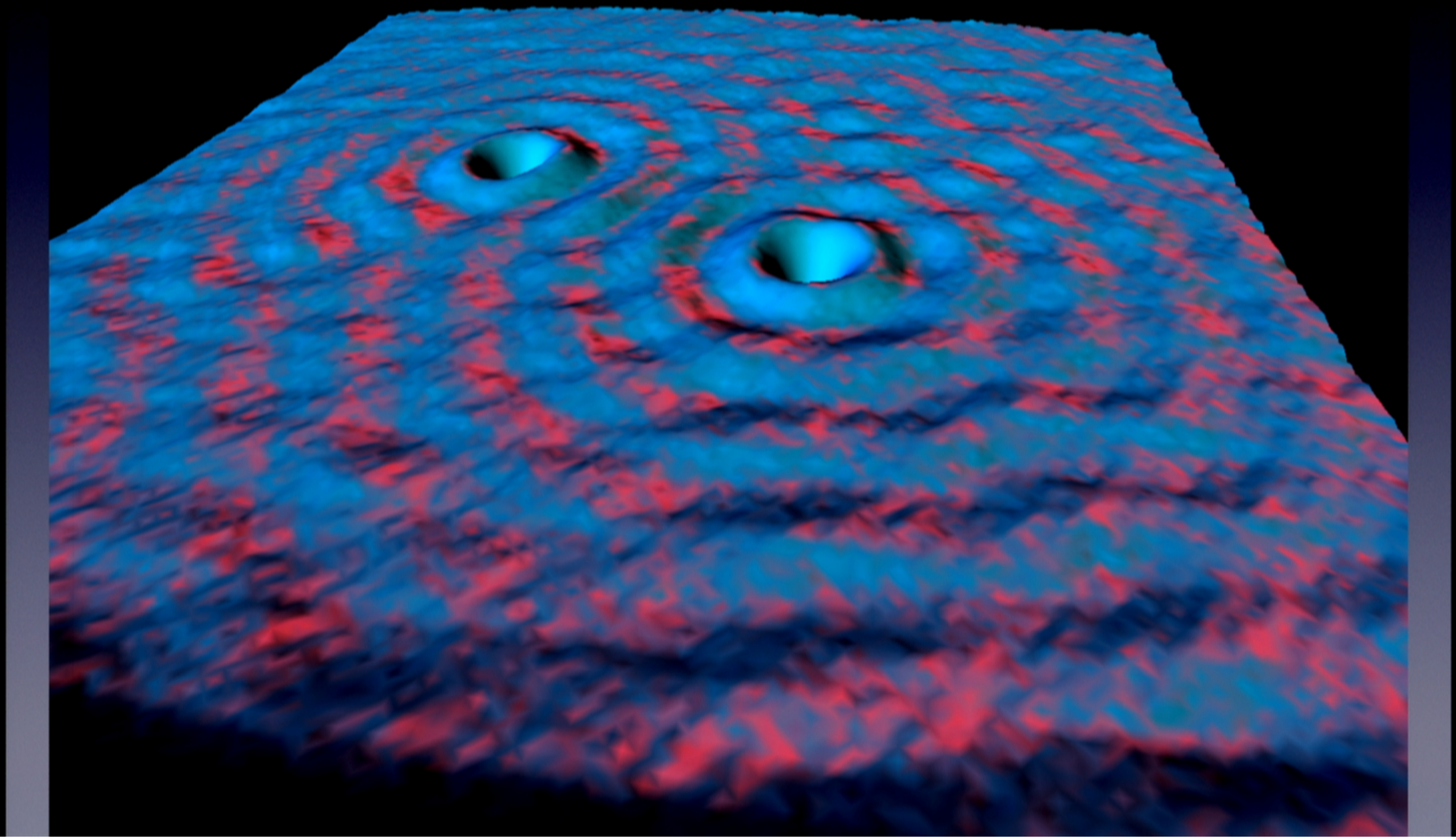
Abstract:

Friedel oscillations

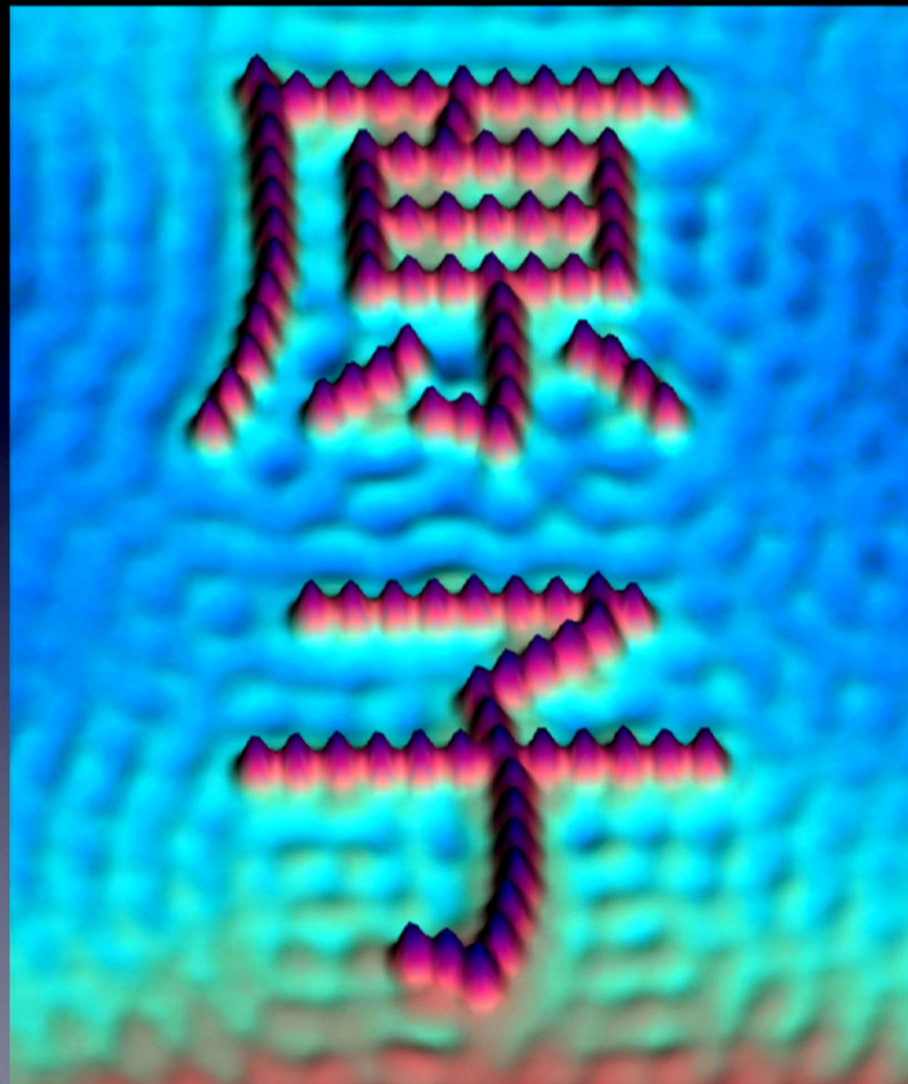
STM images of Cu surface wavefunction. [D. Eigler, IBM]

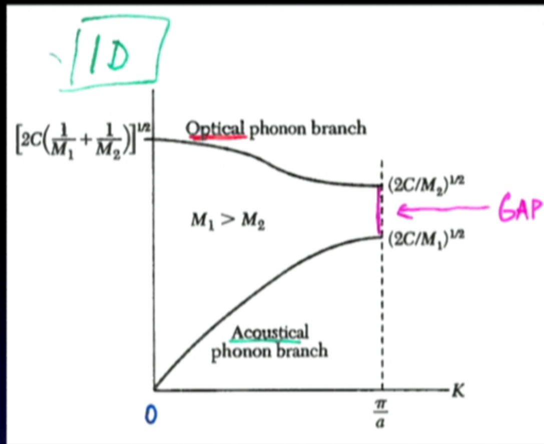


Unknown defects on Cu 111 surface



Iron on Cu





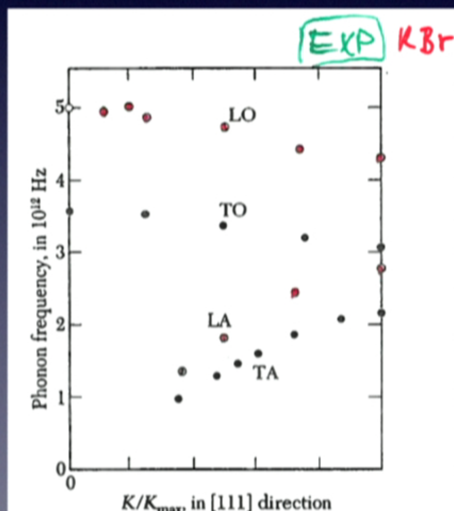


Figure 8b Dispersion curves in the [111] direction in KBr at 90 K, after A. D. B. Woods, B. N. Brockhouse, R. A. Cowley, and W. Cochran. The extrapolation to $K = 0$ of the TO, LO branches are called ω_T, ω_L.

the limiting frequencies at $K = 0$ and $K = K_{max} = \pi/a$.
The lattice constant is a.

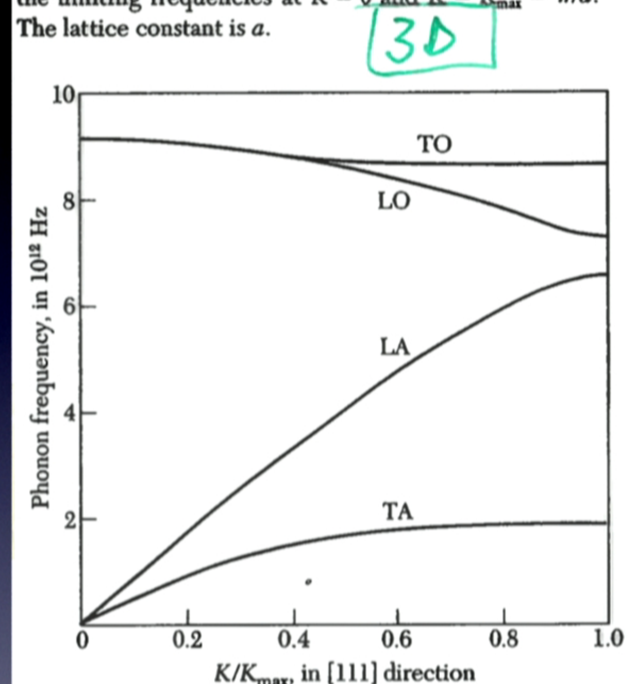
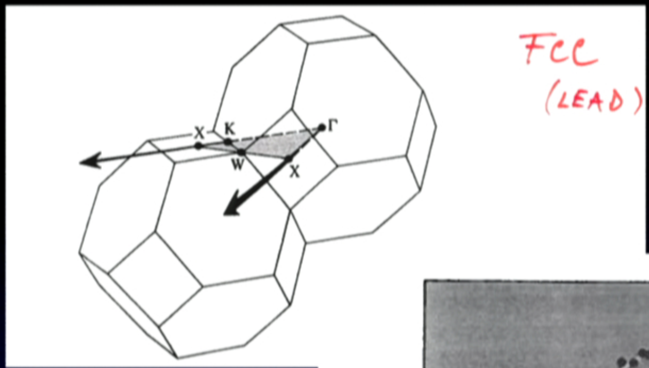


Figure 8a Phonon dispersion relations in the [111] direction in germanium at 80 K. The two TA phonon branches are horizontal at the zone boundary position, $K_{max} = (2\pi/a)(\frac{1}{2}\frac{1}{2}\frac{1}{2})$. The LO and TO branches coincide at $K = 0$; this also is a consequence of the crystal symmetry of Ge. The results were obtained with neutron inelastic scattering by G. Nilsson and G. Nelin.



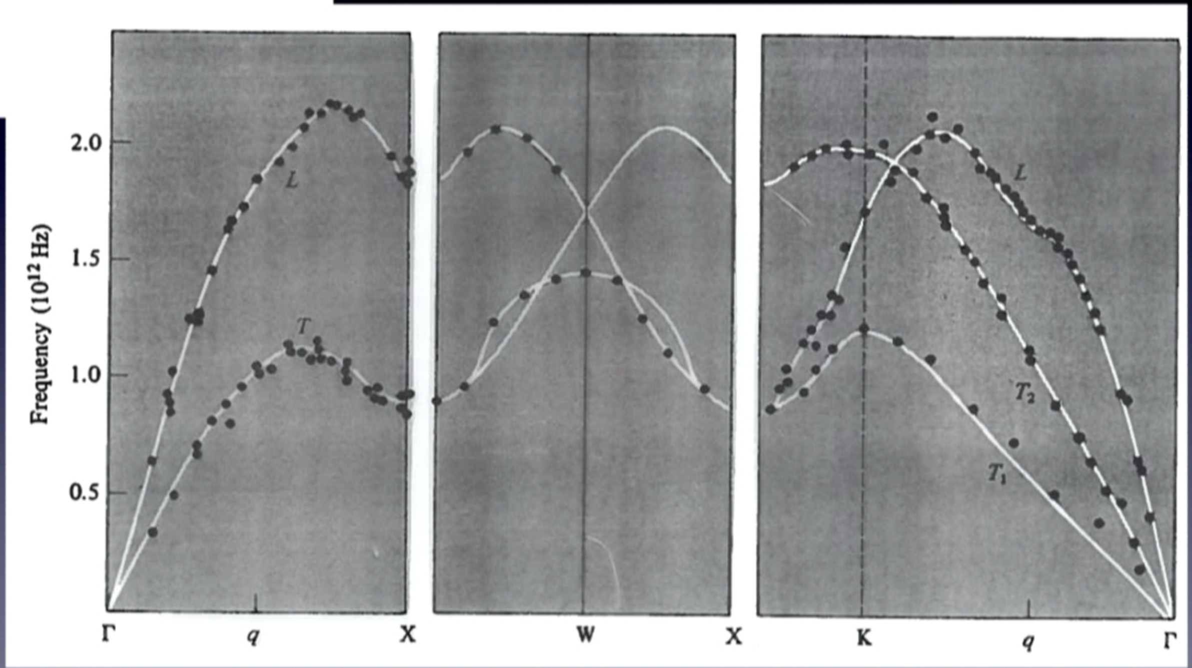
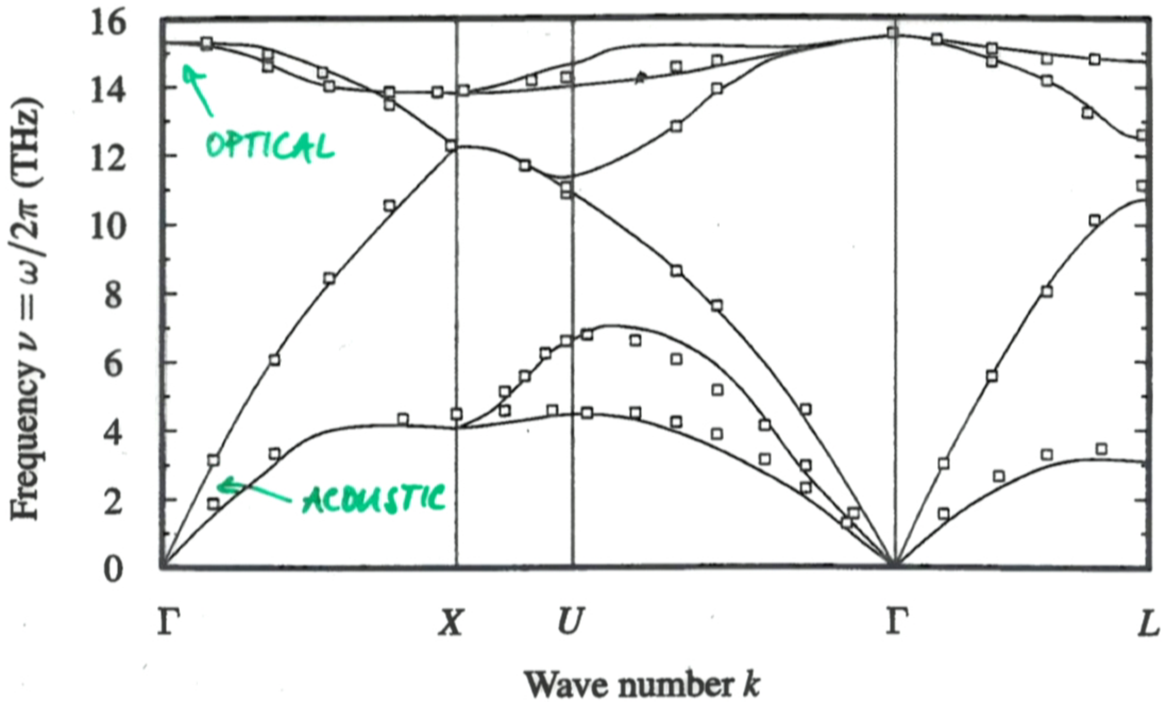


Figure 22.13
(a) Typical dispersion curves for the normal-mode frequencies in a monatomic Bravais lattice. The curves are for lead (face-centered cubic) and are plotted in a repeated-zone scheme along the edges of the shaded triangle shown in (b). Note that the two transverse branches are degenerate in the [100] direction. (After Brockhouse et al., *Phys. Rev.* **128**, 1099 (1962).)

PHONON SPECTRUM FOR SILICON

(Diamond lattice = FCC with 2-point basis)



Specific heat

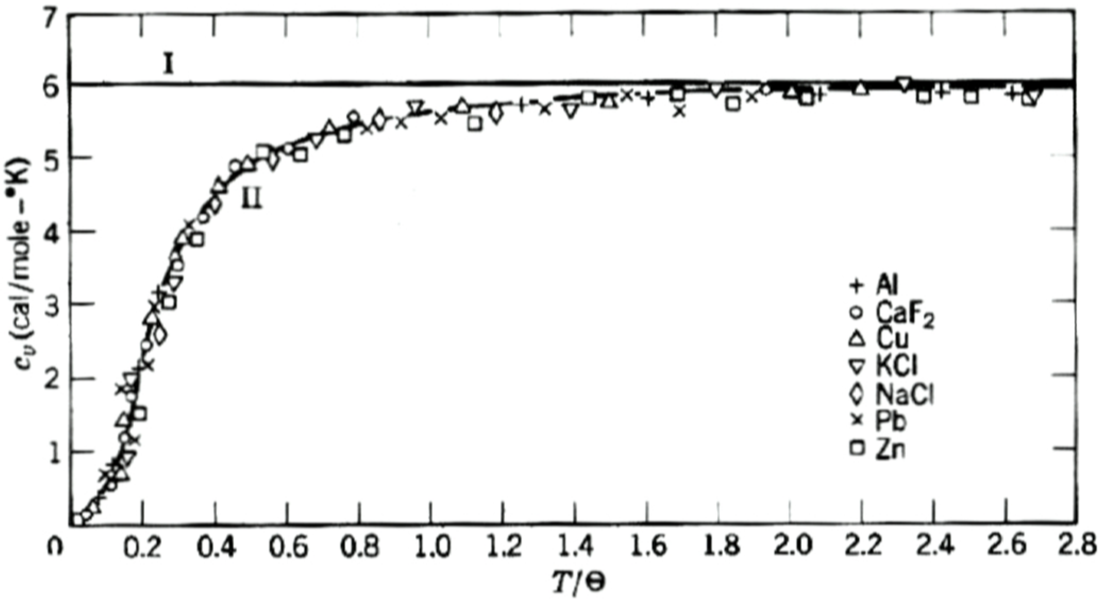
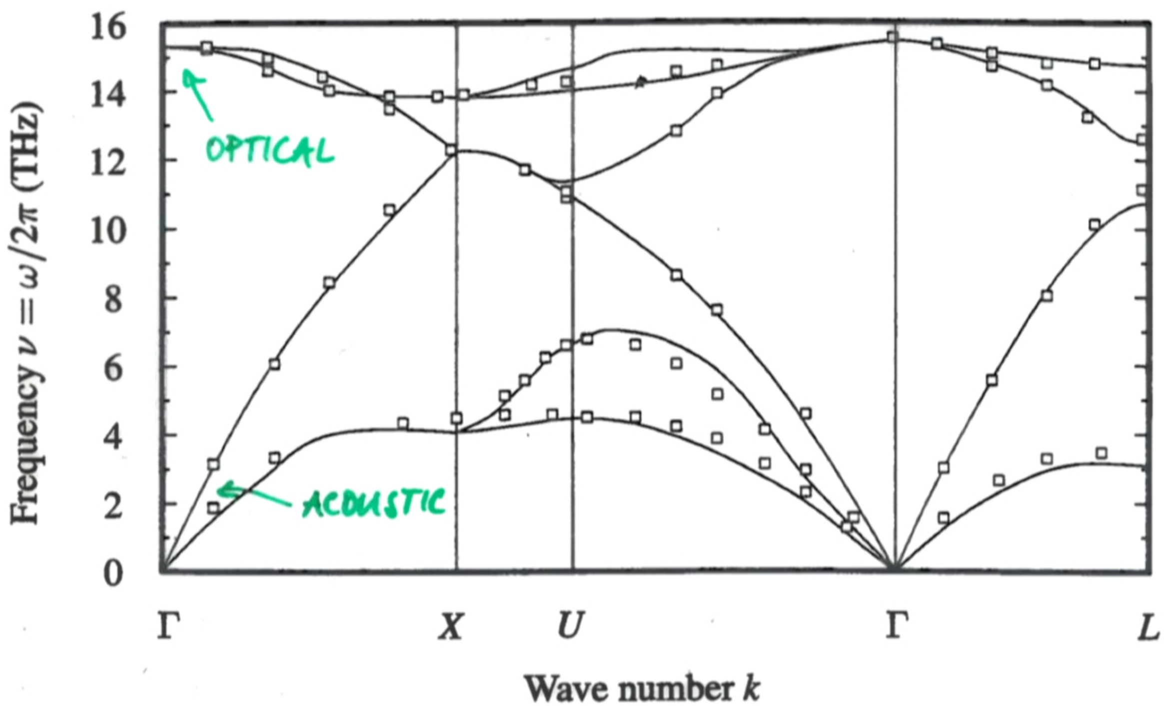


Figure 11-5 The measured specific heat at constant volume, as a function of temperature, for several materials. Horizontal line I represents the Dulong-Petit law, and curve II represents the predictions of the Debye theory.

PHONON SPECTRUM FOR SILICON

(Diamond lattice = FCC with 2-point basis)



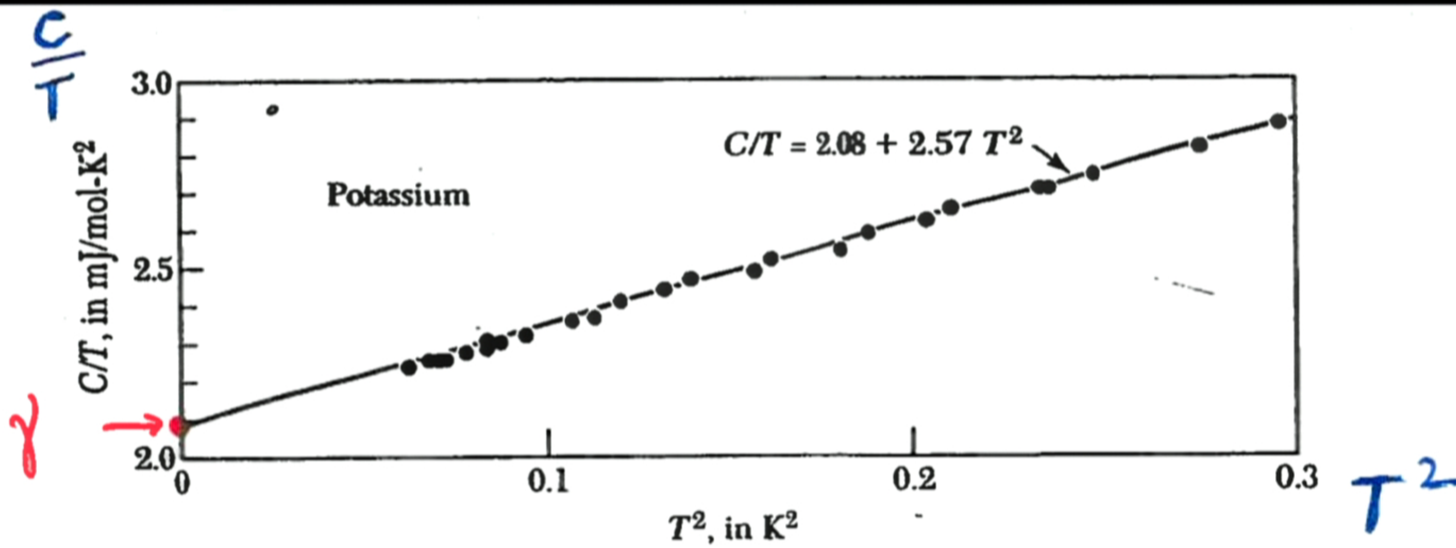


Figure 9 Experimental heat capacity values for potassium, plotted as C/T versus T^2. (After W. H. Lien and N. E. Phillips.)

Specific heat

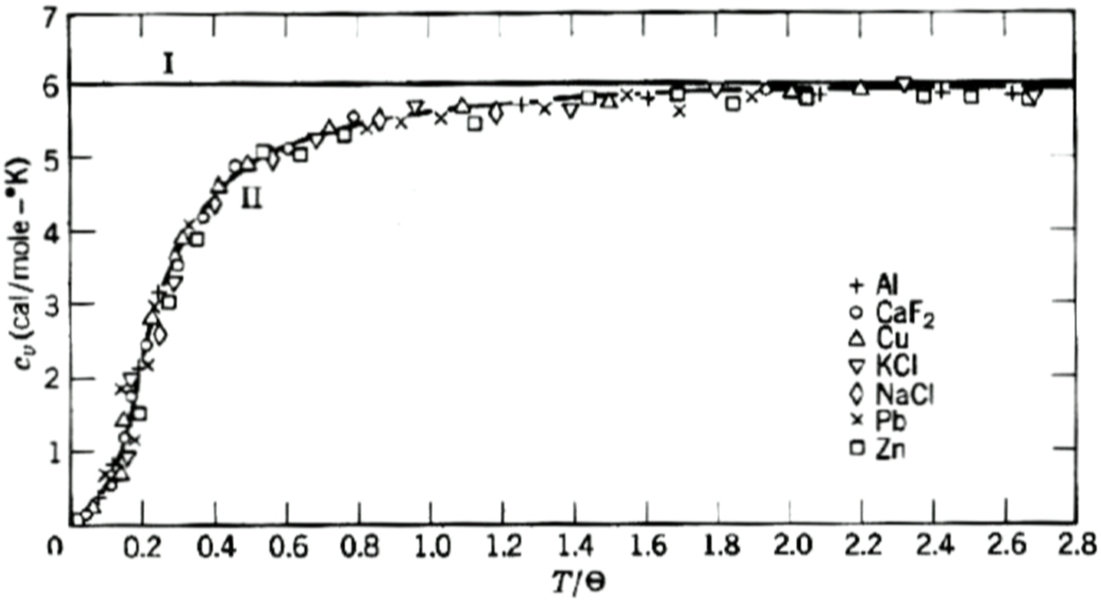


Figure 11-5 The measured specific heat at constant volume, as a function of temperature, for several materials. Horizontal line I represents the Dulong-Petit law, and curve II represents the predictions of the Debye theory.

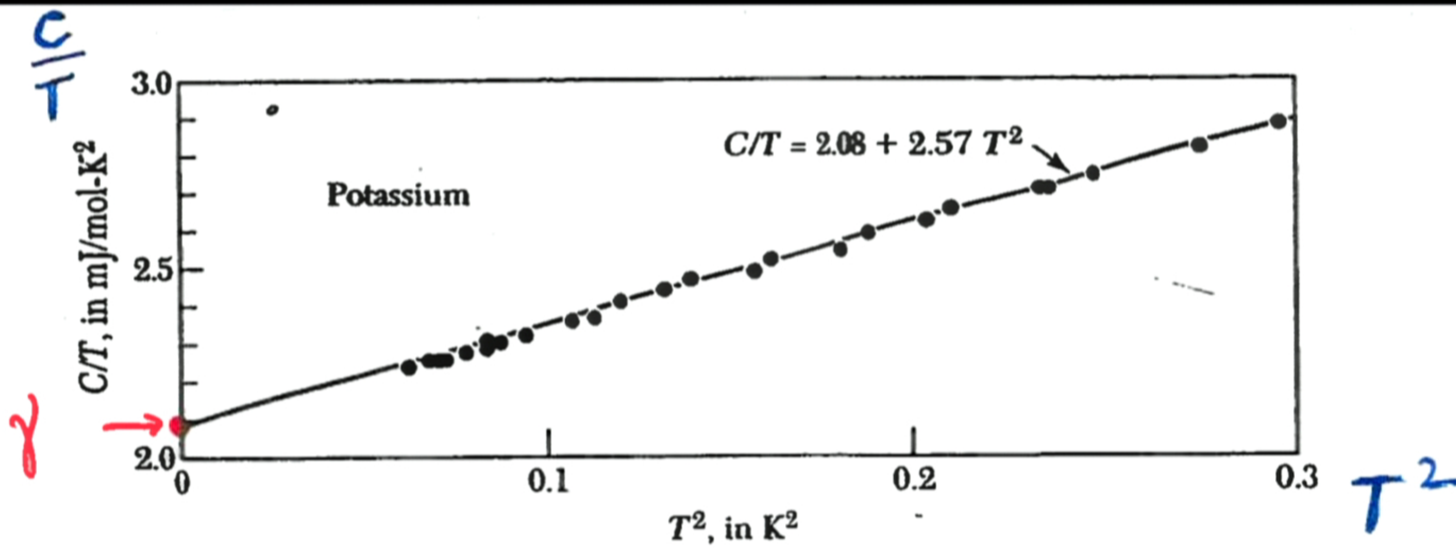
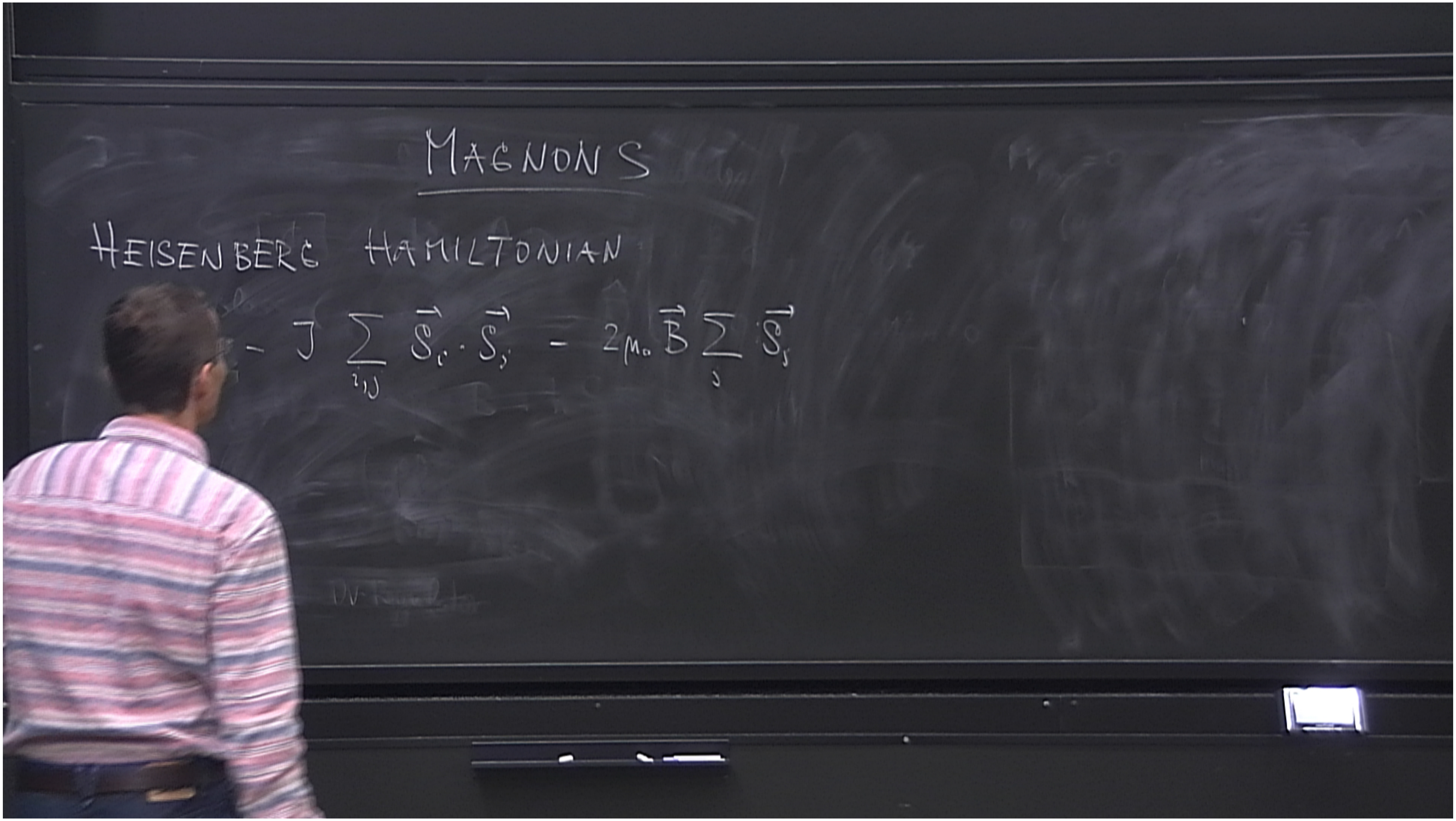
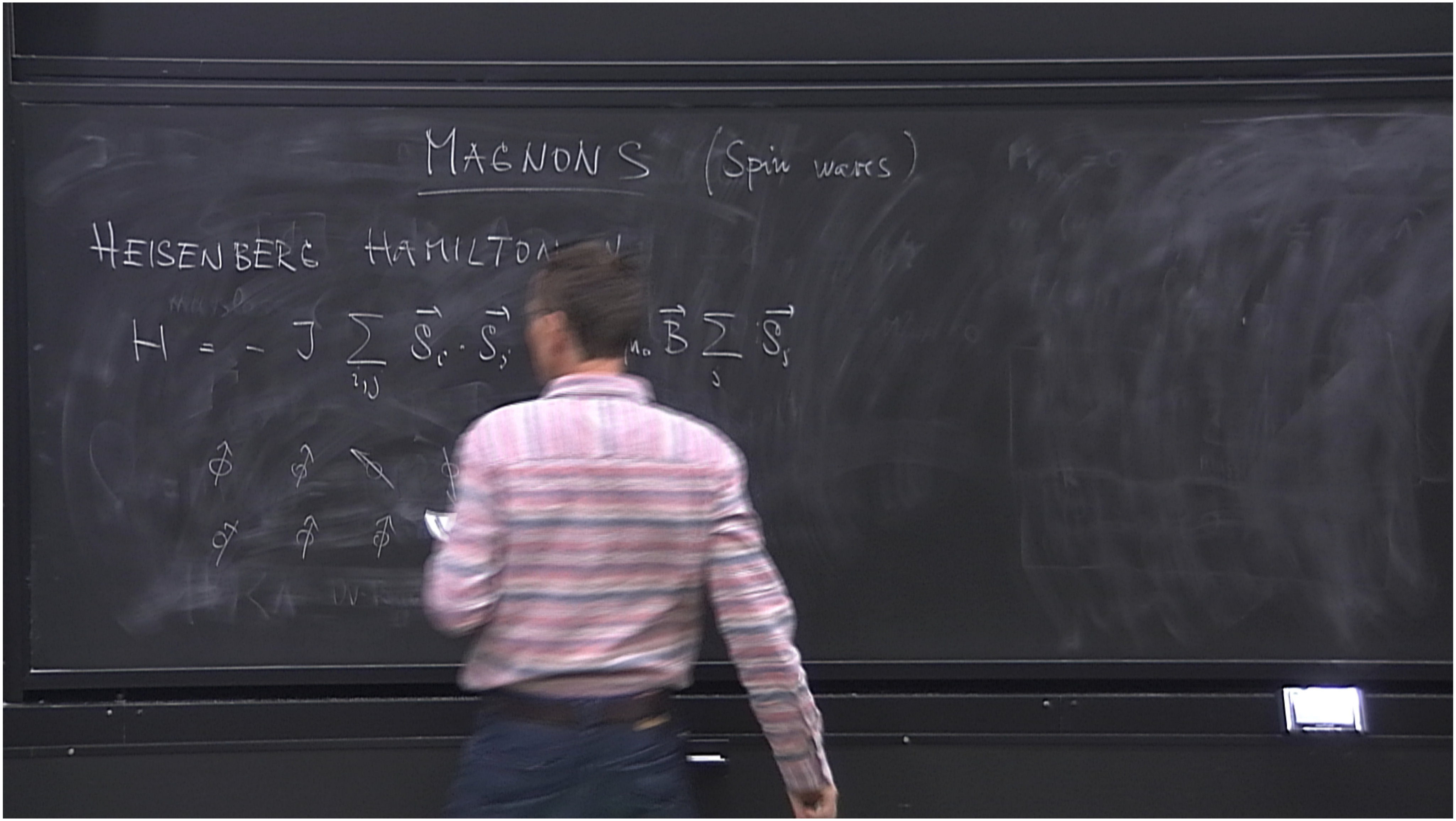


Figure 9 Experimental heat capacity values for potassium, plotted as C/T versus T^2. (After W. H. Lien and N. E. Phillips.)

MAGNONS

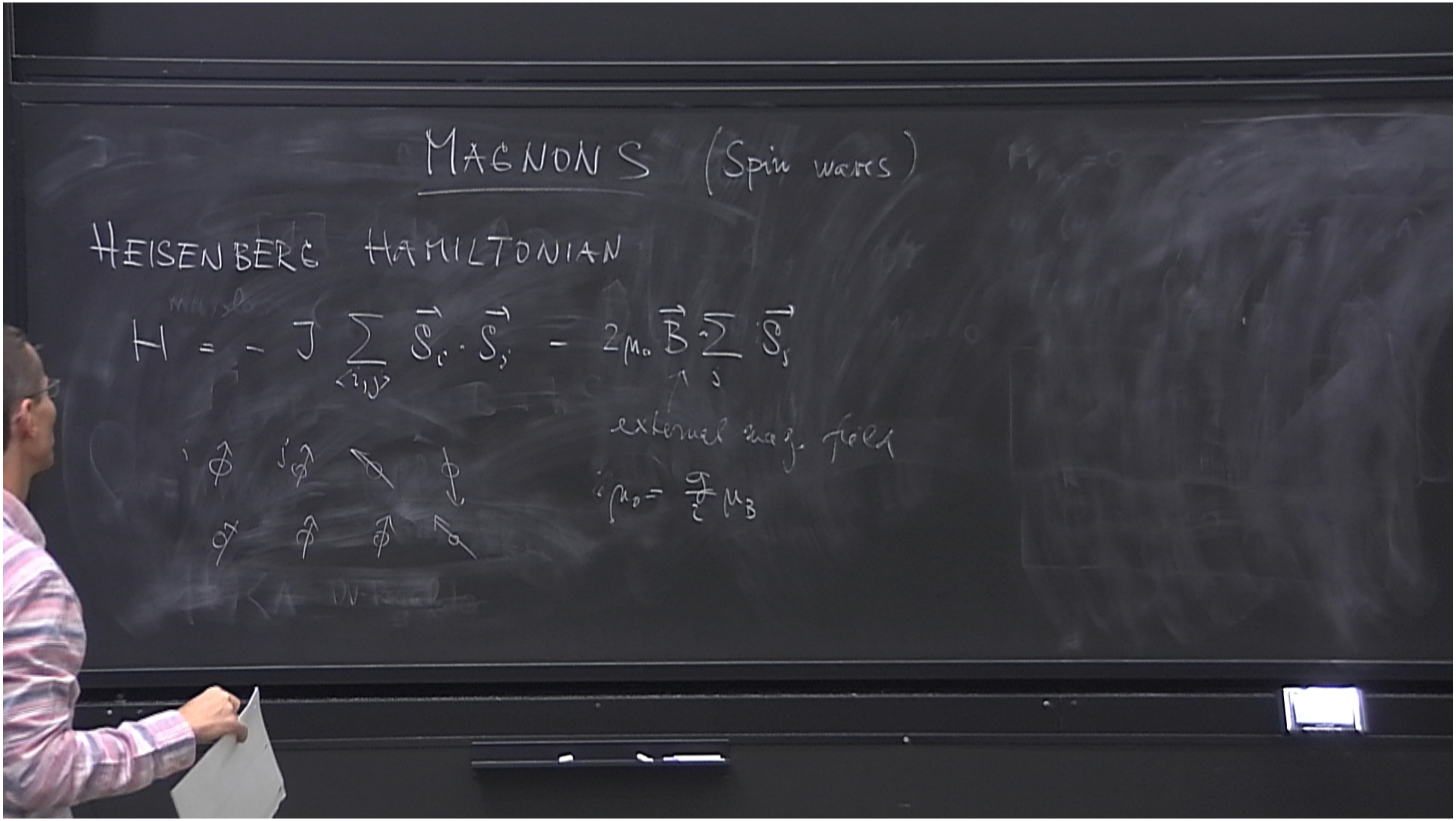

MAGNONS
HEISENBERG HAMILTONIAN
$- J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - 2\mu_0 \vec{B} \sum_i \vec{S}_i$

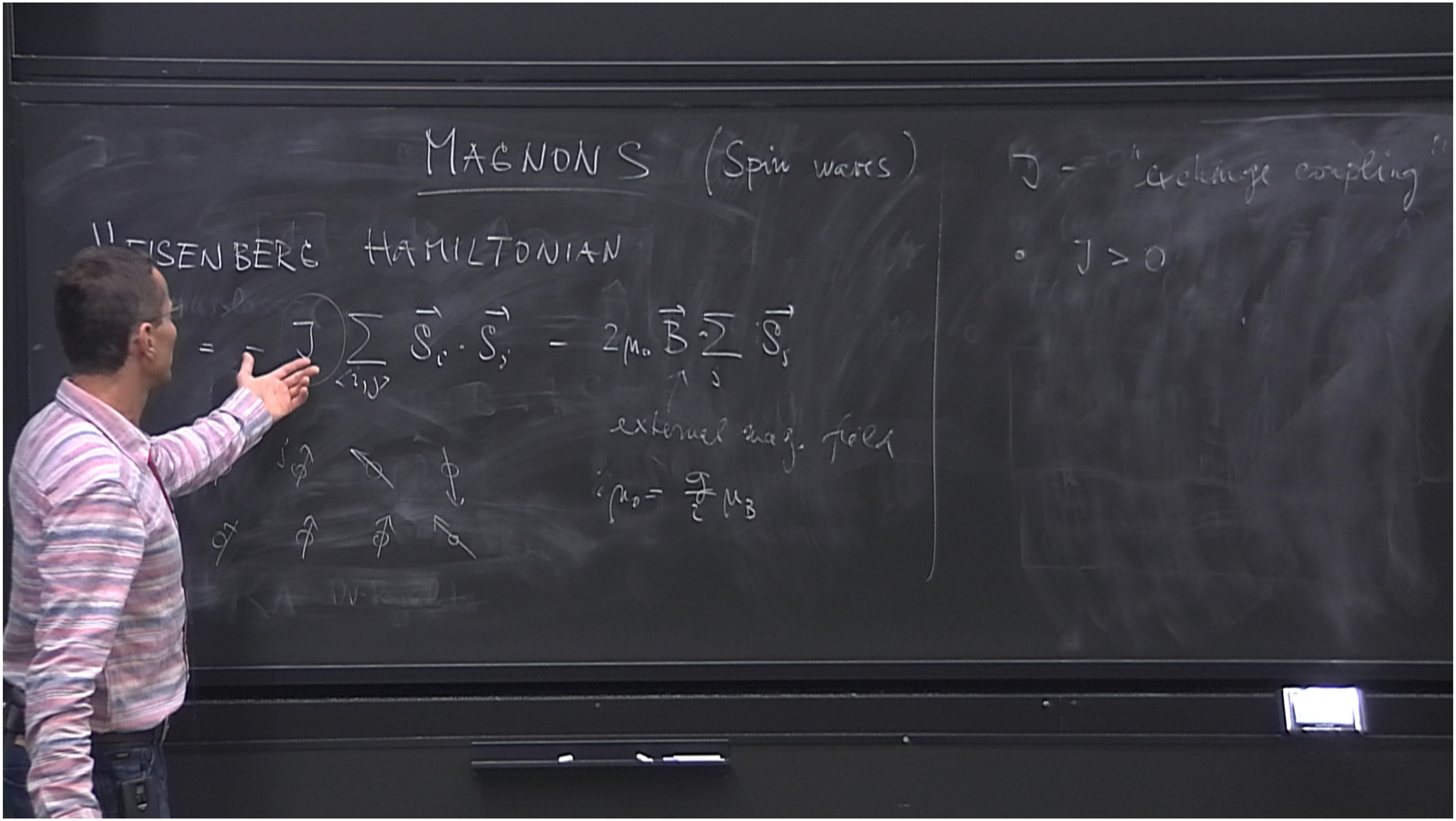

MAGNONS (Spin waves)
HEISENBERG

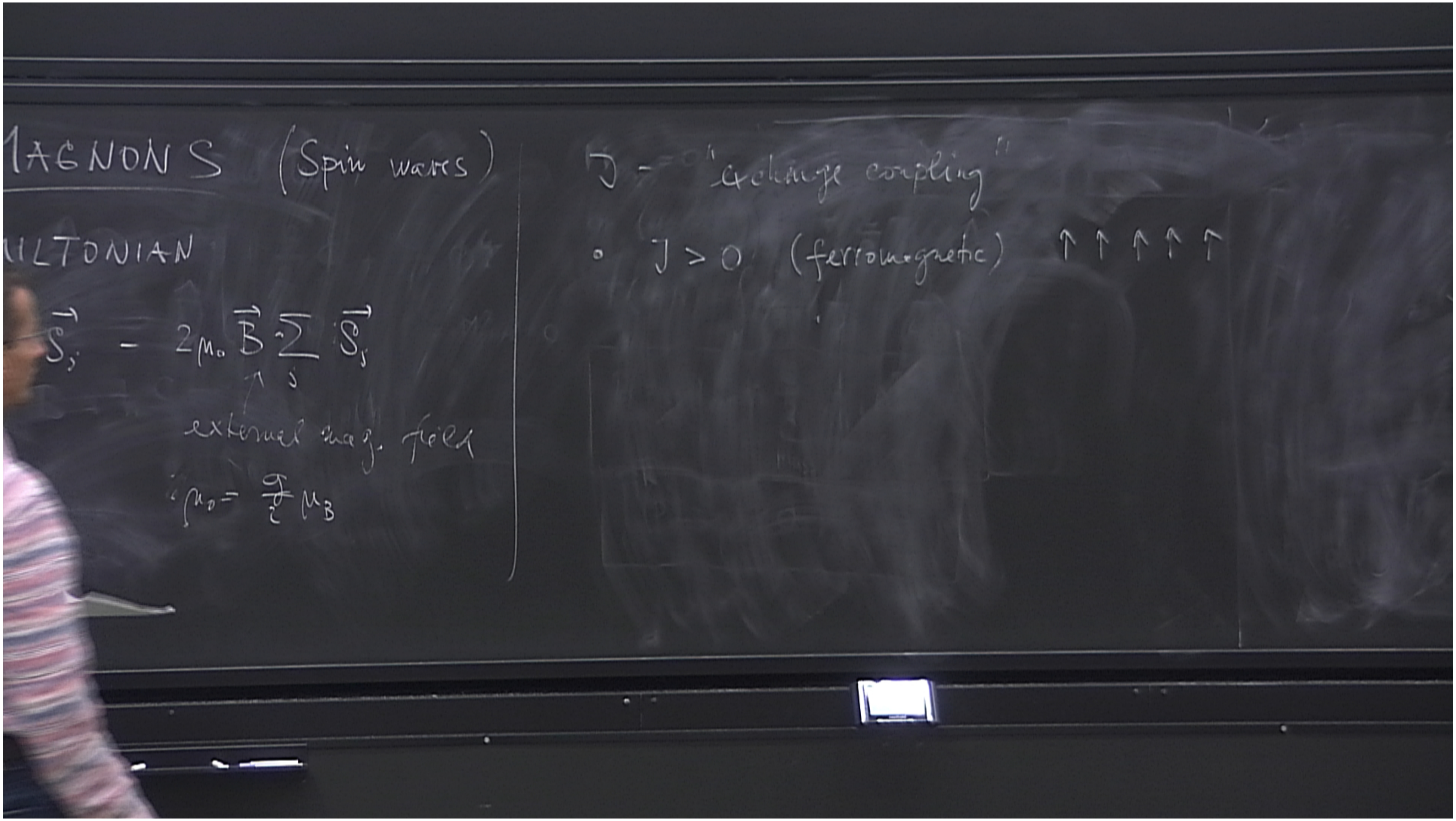
MAGNONS (Spin waves)

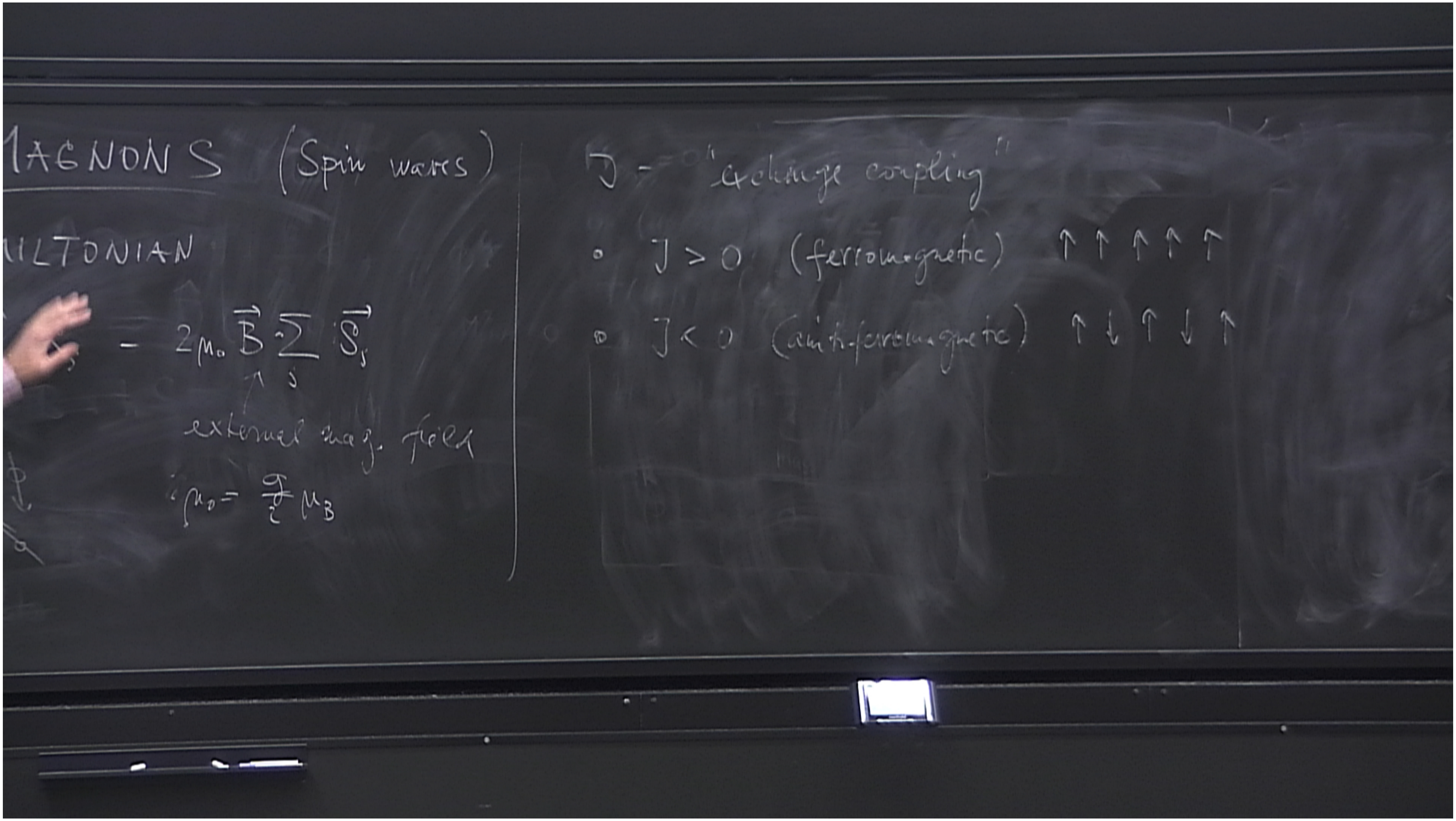
HEISENBERG HAMILTONIAN

$$H = -J \sum_{i,j} \vec{S}_i \cdot \vec{S}_j - 2\mu_0 \vec{B} \cdot \sum_i \vec{S}_i$$


MAGNONS (Spin waves)
HEISENBERG HAMILTONIAN
$H = -J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j - 2\mu_0 \vec{B} \cdot \sum_i \vec{S}_i$
external mag. field
$\mu_0 = \frac{g}{2} \mu_B$


MAGNONS (Spin waves)
EISENBERG HAMILTONIAN
= − J ∑⟨i,j⟩ S_i · S_j − 2μ₀ B · ∑_i S_i
external mag. field
μ₀ = g/2 μ_B
J − "exchange coupling"
J > 0


IAGNONS (Spin waves)
ILTONIAN
$\vec{S}_j - 2\mu_0 \vec{B} \cdot \sum_j \vec{S}_j$
external mag. field
$\mu_0 = \frac{g}{2}\mu_B$
J – "exchange coupling"
• J > 0 (ferromagnetic) ↑ ↑ ↑ ↑ ↑


AGNONS (Spin waves)
LTONIAN
$- 2\mu_0 \vec{B} \cdot \sum_j \vec{S}_j$
external mag. field
$\mu_0 = \frac{g}{2}\mu_B$
J = "exchange coupling"
J > 0 (ferromagnetic) ↑ ↑ ↑ ↑ ↑
J < 0 (anti-ferromagnetic) ↑ ↓ ↑ ↓ ↑

MAGNONS (Spin waves)

ILTONIAN

$\vec{S}_j - 2\mu_0 \vec{B} \cdot \sum_j \vec{S}_j$

external mag. field

$\mu_0 = \frac{g}{2}\mu_B$

J - "exchange coupling"

- J > 0 (ferromagnetic) ↑ ↑ ↑ ↑ ↑
- J < 0 (anti-ferromagnetic) ↑ ↓ ↑ ↓ ↑

J > 0 case

MAGNONS (Spin waves)

...ILTONIAN

$$\ldots \cdot \vec{S}_j \;-\; 2\mu_0 \vec{B} \cdot \sum_j \vec{S}_j$$

external mag. field

$\mu_0 = \frac{g}{2}\mu_B$

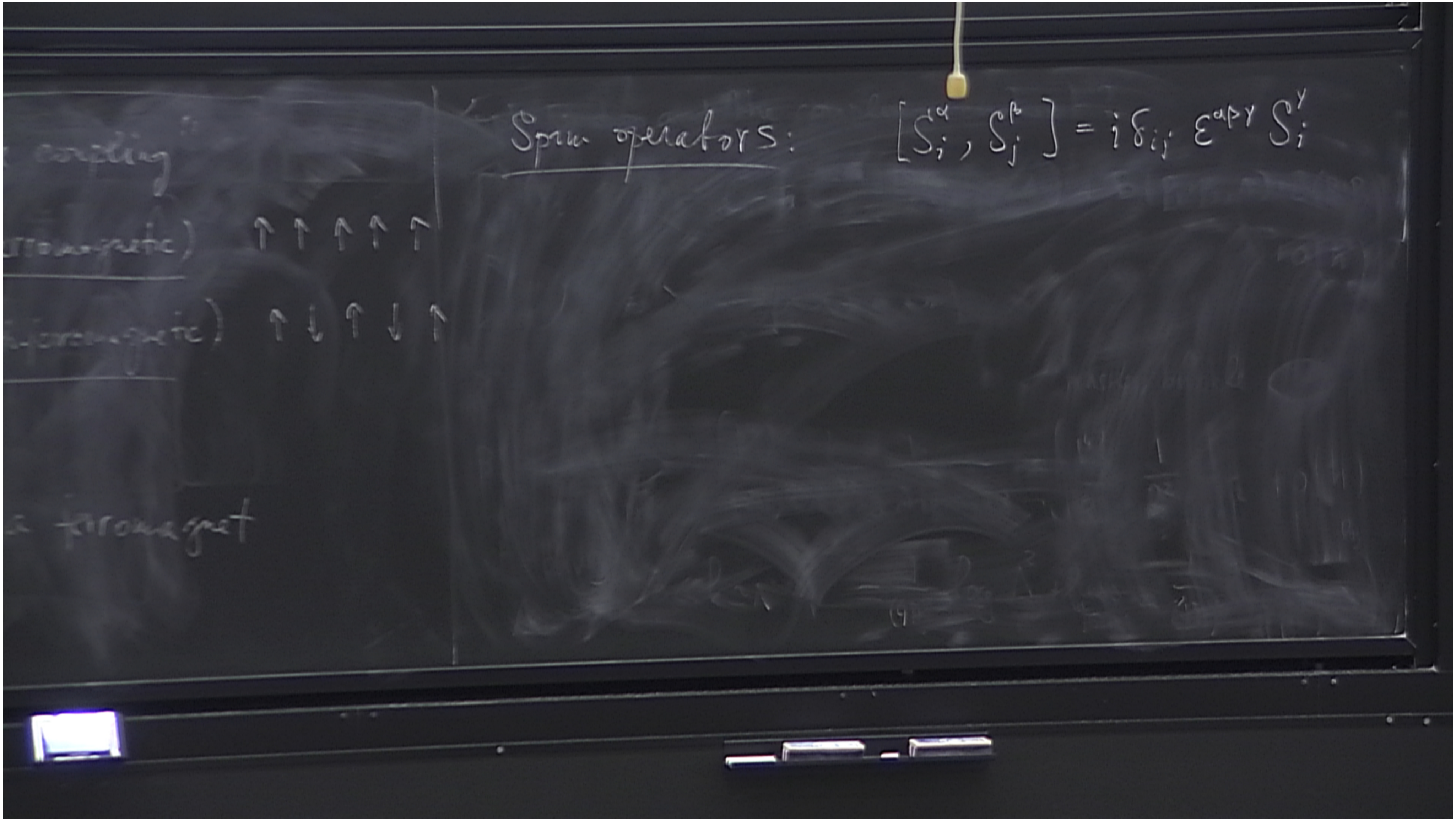
J — "exchange coupling"

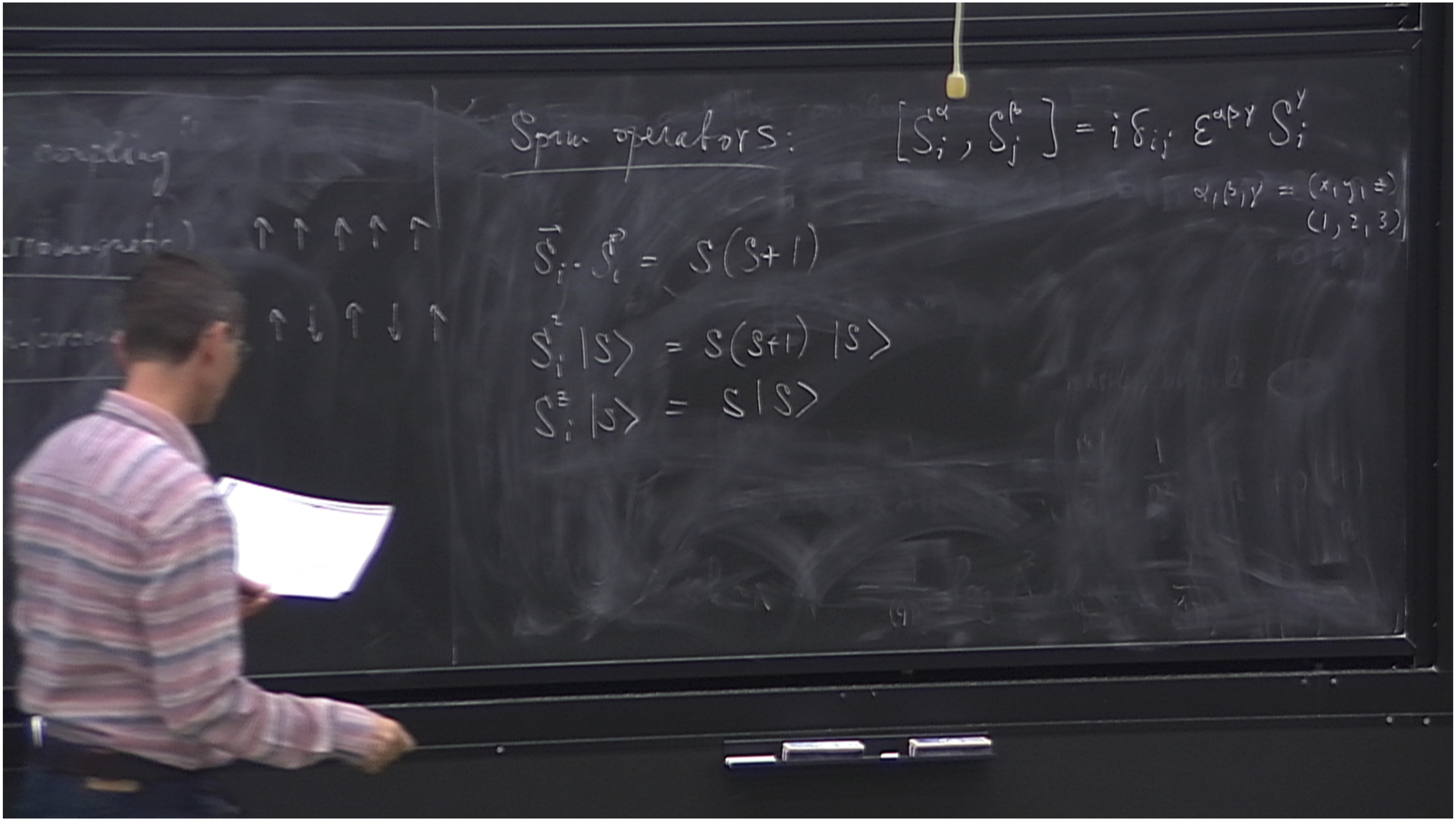
- J > 0 (ferromagnetic) ↑ ↑ ↑ ↑ ↑
- J < 0 (anti-ferromagnetic) ↑ ↓ ↑ ↓ ↑

J > 0 case

Ground state is a ferromagnet

↑ ↑ ↑ ↑ ↑ ↑


Spin operators: $[S_i^\alpha, S_j^\beta] = i\delta_{ij}\,\varepsilon^{\alpha\beta\gamma} S_i^\gamma$
coupling
↑ ↑ ↑ ↑ ↑
↑ ↓ ↑ ↓ ↑
ferromagnet


Spin operators:
$[S_i^\alpha, S_j^\beta] = i\delta_{ij}\,\varepsilon^{\alpha\beta\gamma} S_i^\gamma$
$\alpha,\beta,\gamma = (x,y,z)$
$(1,2,3)$
$\vec{S}_i \cdot \vec{S}_i = S(S+1)$
$S_i^2 |S\rangle = S(S+1)|S\rangle$
$S_i^z |S\rangle = S|S\rangle$
$\uparrow\uparrow\uparrow\uparrow\uparrow$
$\uparrow\downarrow\uparrow\downarrow\uparrow$

coupling

romagnetic) $\uparrow \uparrow \uparrow \uparrow \uparrow$

-ferromagnetic) $\uparrow \downarrow \uparrow \downarrow \uparrow$

a ferromagnet

Spin operators: $[S_i^\alpha, S_j^\beta] = i\delta_{ij}\,\varepsilon^{\alpha\beta\gamma} S_i^\gamma$

$\alpha, \beta, \gamma = (x, y, z)$
$(1, 2, 3)$

$\vec{S}_i \cdot \vec{S}_i = S(S+1)$

$S_i^2 |S\rangle = S(S+1)|S\rangle$

$S_i^z |S\rangle = S_z |S\rangle$

coupling

(ferromagnetic) ↑ ↑ ↑ ↑ ↑

(antiferromagnetic) ↑ ↓ ↑ ↓ ↑

a ferromagnet

Spin operators: $[S_i^{\alpha}, S_j^{\beta}] = i\delta_{ij}\,\varepsilon^{\alpha\beta\gamma} S_i^{\gamma}$

$\alpha, \beta, \gamma = (x, y, z)$
$(1, 2, 3)$

$\vec{S}_i \cdot \vec{S}_i = S(S+1)$

$S_i^2 |S\rangle = S(S+1)\,|S\rangle$

$S_i^z |S\rangle = S_z |S\rangle$

- Excited states: Holstein & Primakoff (1940) coupling"

(…romagnetic) $\uparrow\uparrow\uparrow\uparrow\uparrow$

(…ferromagnetic) $\uparrow\downarrow\uparrow\downarrow\uparrow$

… ferromagnet

Spin operators: $[S_i^\alpha, S_j^\beta] = i\delta_{ij}\,\varepsilon^{\alpha\beta\gamma} S_i^\gamma$

$\alpha,\beta,\gamma = (x,y,z)$
$(1,2,3)$

$$\vec{S}_i \cdot \vec{S}_i = S(S+1)$$

$$S_i^2 |S\rangle = S(S+1)|S\rangle$$

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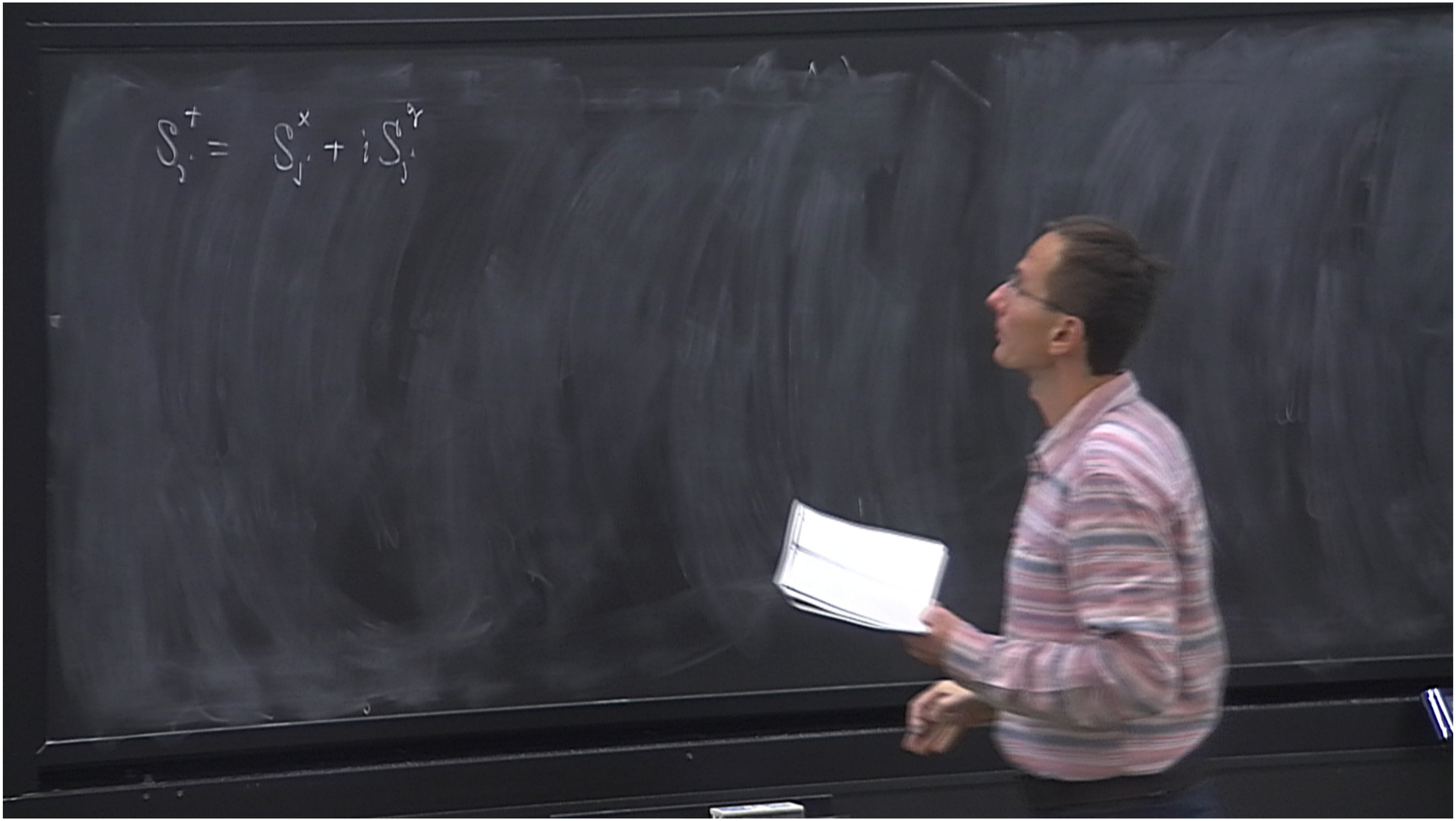
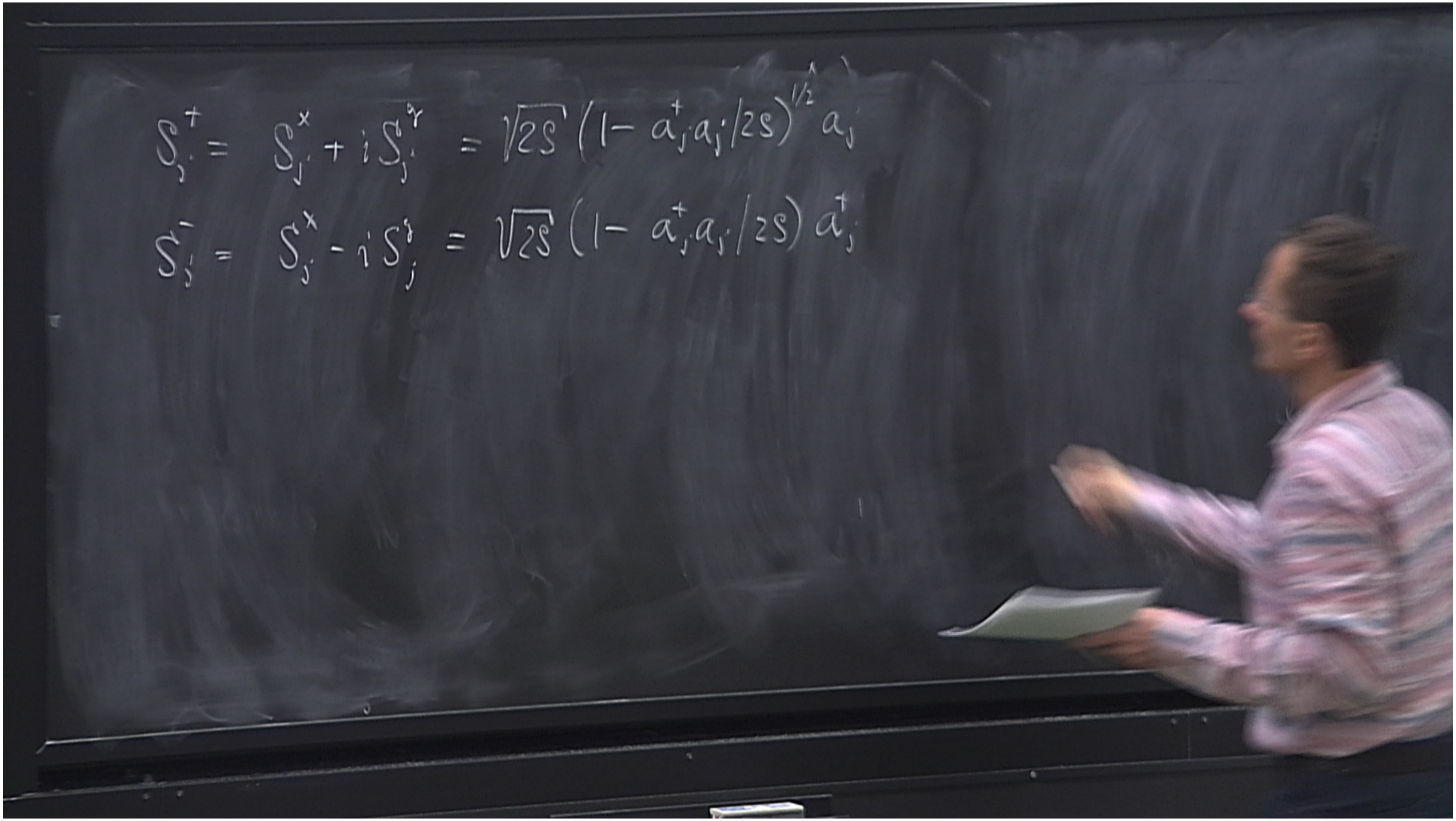
• Excited states: Holstein & Primakoff (1940)

Spins $\longrightarrow$ bosons


$S_j^+ = S_j^x + i S_j^y$



$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,(1 - a_j^+ a_j/2S)^{1/2}\, a_j$
$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\,(1 - a_j^+ a_j/2S)\, a_j^+$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\left(1 - a_j^\dagger a_j / 2S\right)^{1/2} a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\left(1 - a_j^\dagger a_j / 2S\right)^{1/2} a_j^\dagger$$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,\Big(1 - \overbrace{a_j^+ a_j}^{n_j}/2S\Big)^{1/2} a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\,\Big(1 - a_j^+ a_j/2S\Big)^{1/2} a_j^+$$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,\Big(1 - \overbrace{a_j^+ a_j}^{n_j}/2S\Big)^{1/2} a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\,\Big(1 - a_j^+ a_j/2S\Big)^{1/2} a_j^+$$

$$[a_j, a_k^+] = \delta_{jk}$$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,(1 - \overbrace{a_j^+ a_j}^{n_j}/2S)^{1/2}\, a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\,(1 - a_j^+ a_j/2S)^{1/2}\, a_j^+$$

$$[a_j, a_k^+] = \delta_{jk}$$

$$S_z^2 = S(S+1) - S_x^2 - S_y^2 = S(S+1) - \frac{1}{2}(S^+S^- + S^-S^+)$$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,(1 - \overbrace{a_j^+ a_j}^{n_j}/2S)^{1/2}\, a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\, a_j^+ (1 - a_j^+ a_j/2S)^{1/2}$$

$$[a_j, a_k^+] = \delta_{jk}$$

$$S_z^2 = S(S+1) - S_x^2 - S_y^2 = S(S+1) - \frac{1}{2}(S^+S^- + S^-S^+)$$

$$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,\Big(1 - \overbrace{a_j^+ a_j}^{n_j}/2S\Big)^{1/2} a_j$$

$$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\, a_j^+ \Big(1 - a_j^+ a_j/2S\Big)^{1/2}$$

$$[a_j, a_k^+] = \delta_{jk}$$

$$S_z^2 = S(S+1) - S_x^2 - S_y^2 = S(S+1) - \tfrac{1}{2}(S^+S^- + S^-S^+)$$

$$= (S - a^+ a)^2$$

$$\boxed{S_j^z = S - n_j}$$

coupling

(...omagnetic) ↑ ↑ ↑ ↑ ↑

(...ferromagnetic) ↑ ↓ ↑ ↓ ↑

ferromagnet

Spin operators: $[S_i^\alpha, S_j^\beta] = i\delta_{ij}\,\epsilon^{\alpha\beta\gamma} S_i^\gamma$

$\alpha,\beta,\gamma = (x,y,z)$
$(1,2,3)$

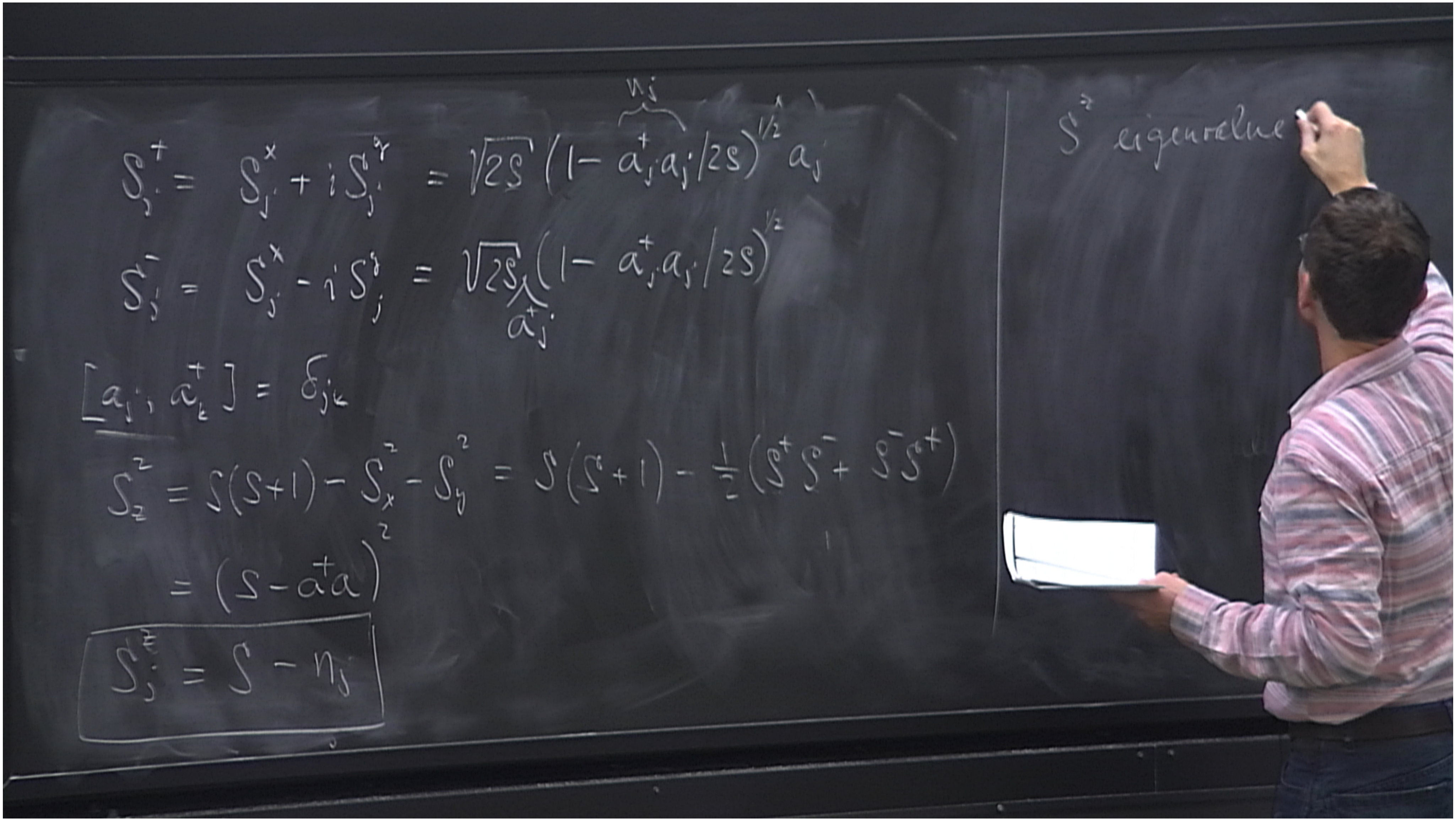
$$\vec{S}_i \cdot \vec{S}_i = S(S+1)$$

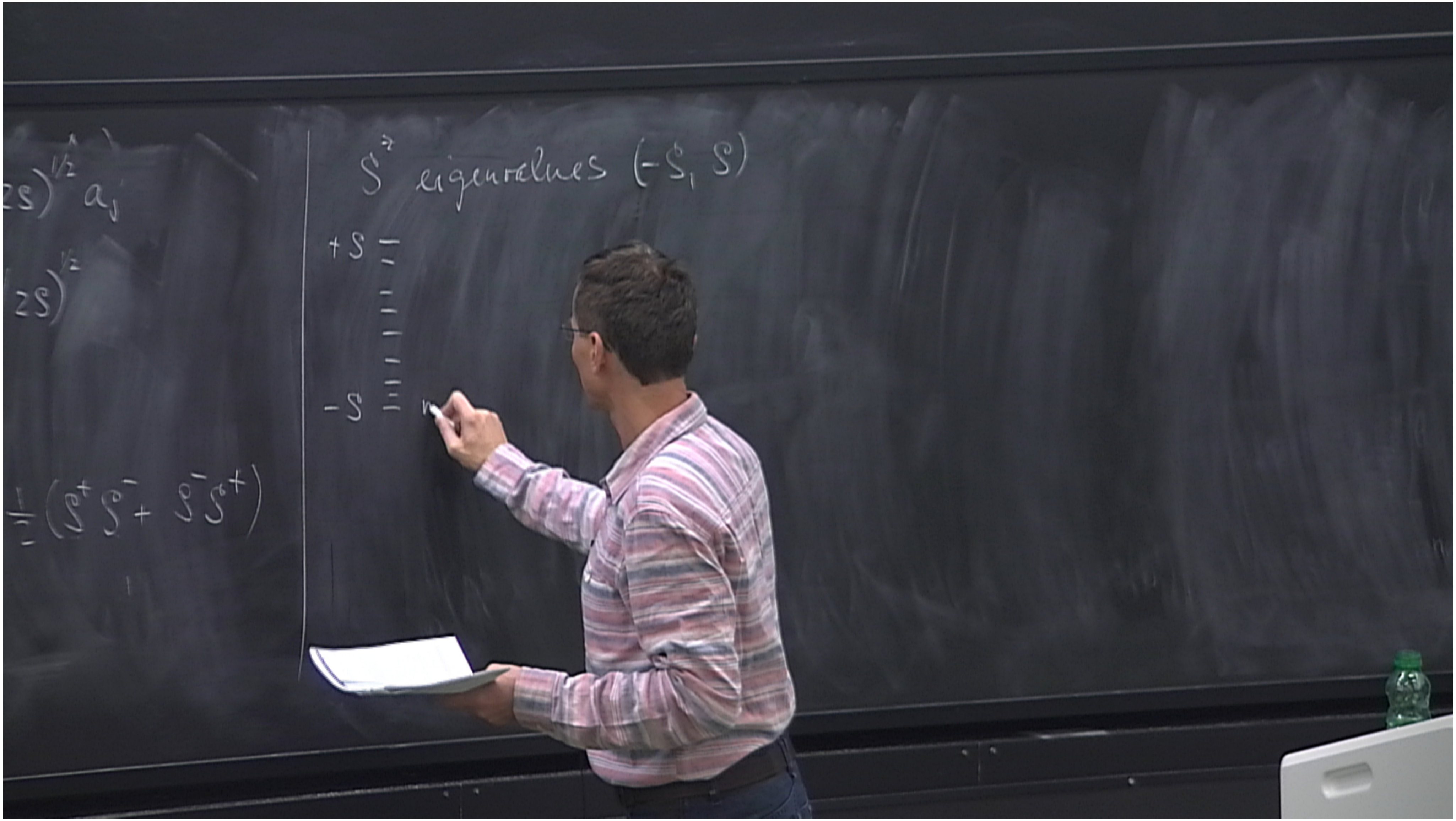
$$S_i^2 |S\rangle = S(S+1)|S\rangle$$

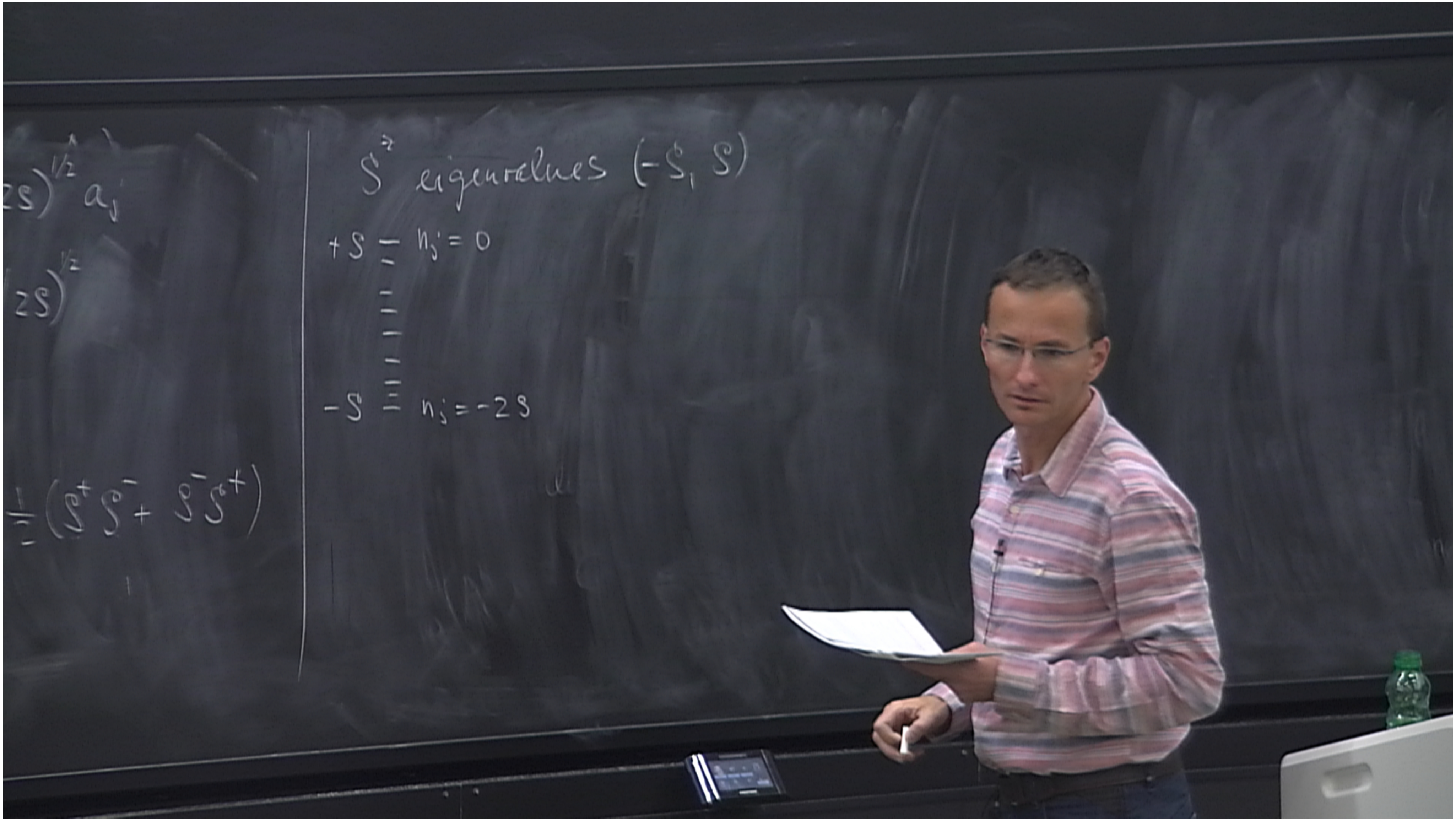
$$S_i^z |S\rangle = S_z |S\rangle$$

- Excited states: Holstein & Primakoff (1940)

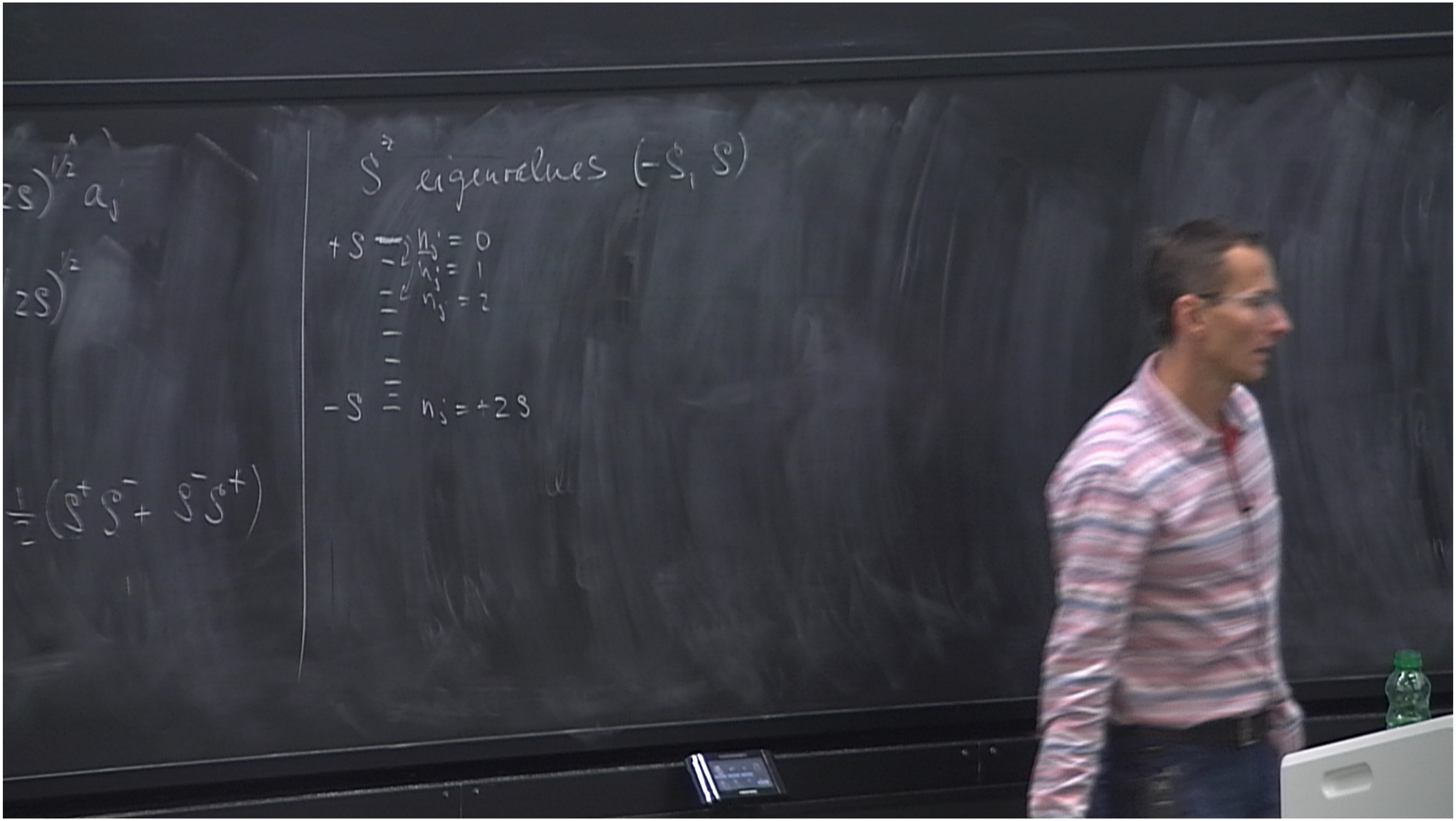
Spins $\longrightarrow$ bosons

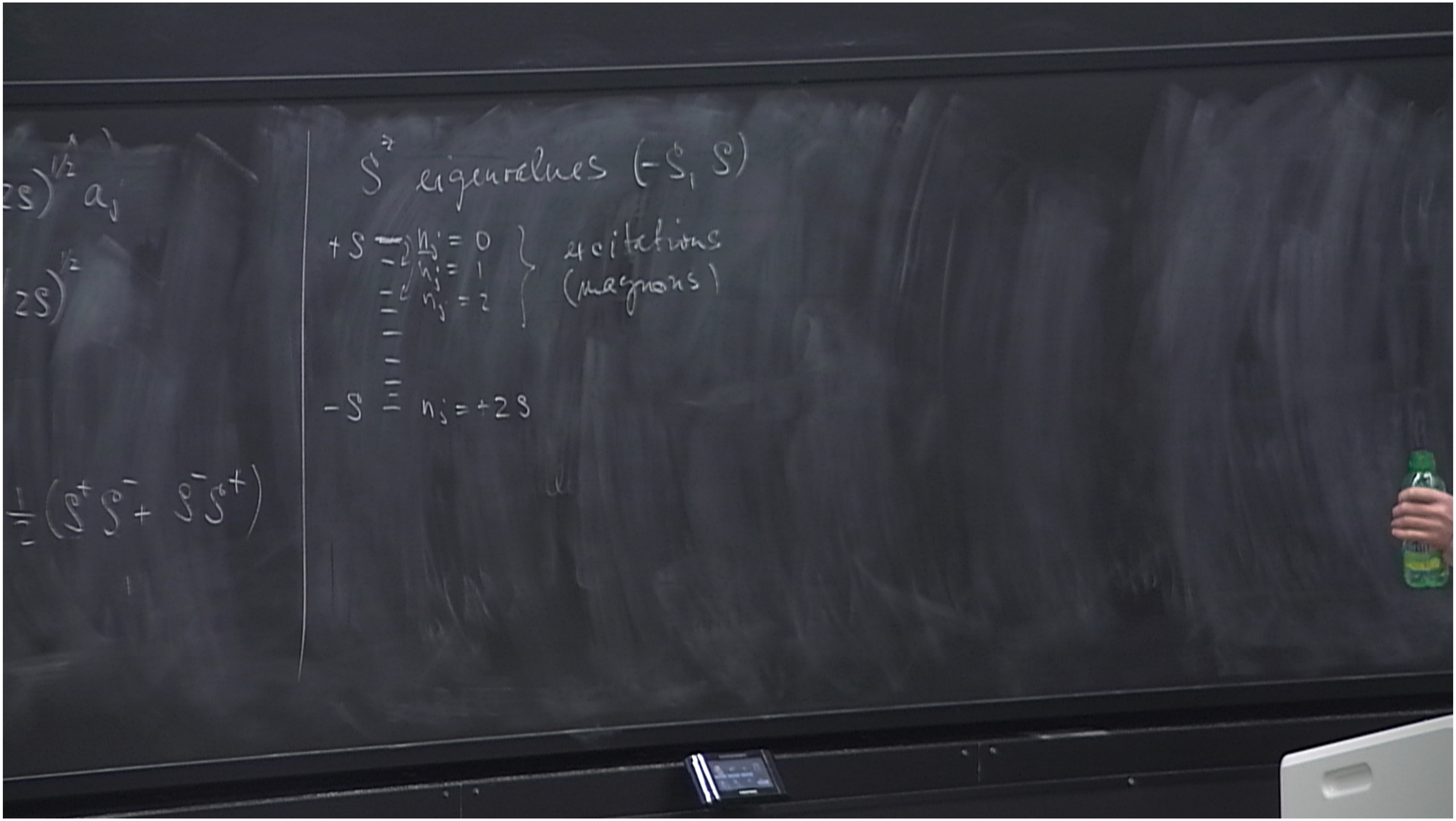

$S_j^+ = S_j^x + i S_j^y = \sqrt{2S}\,(1 - \underbrace{a_j^+ a_j}_{n_j}/2S)^{1/2}\, a_j$
$S_j^- = S_j^x - i S_j^y = \sqrt{2S}\, a_j^+ (1 - a_j^+ a_j/2S)^{1/2}$
$[a_j, a_k^+] = \delta_{jk}$
$S_z^2 = S(S+1) - S_x^2 - S_y^2 = S(S+1) - \frac{1}{2}(S^+S^- + S^-S^+)$
$= (S - a^+ a)^2$
$\boxed{S_j^z = S - n_j}$
S^z eigenvalue


S^z eigenvalues (-S, S)
+S
-S

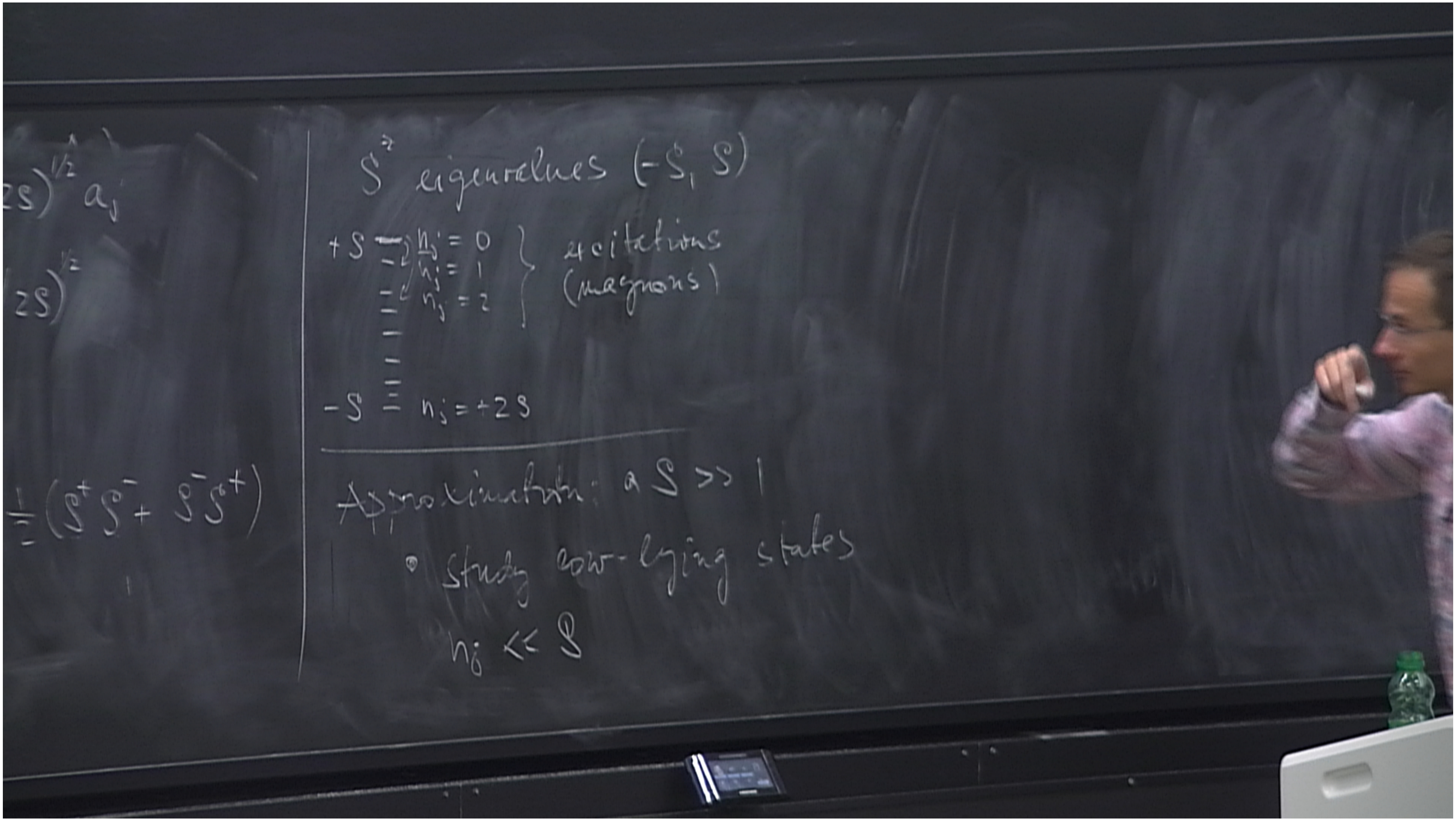


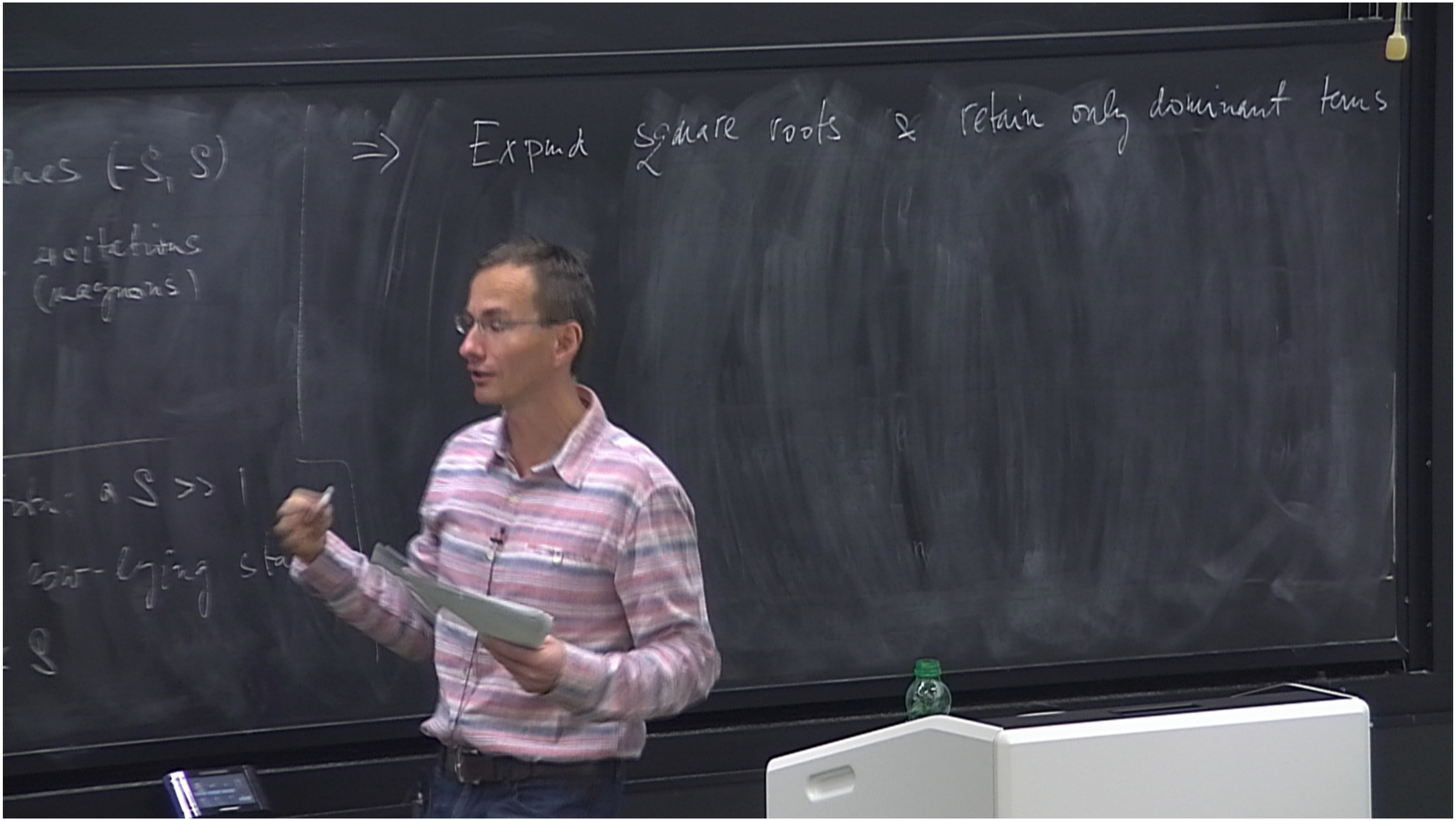
S^z eigenvalues (-S, S)
+S n_j = 0
-S n_j = -2S

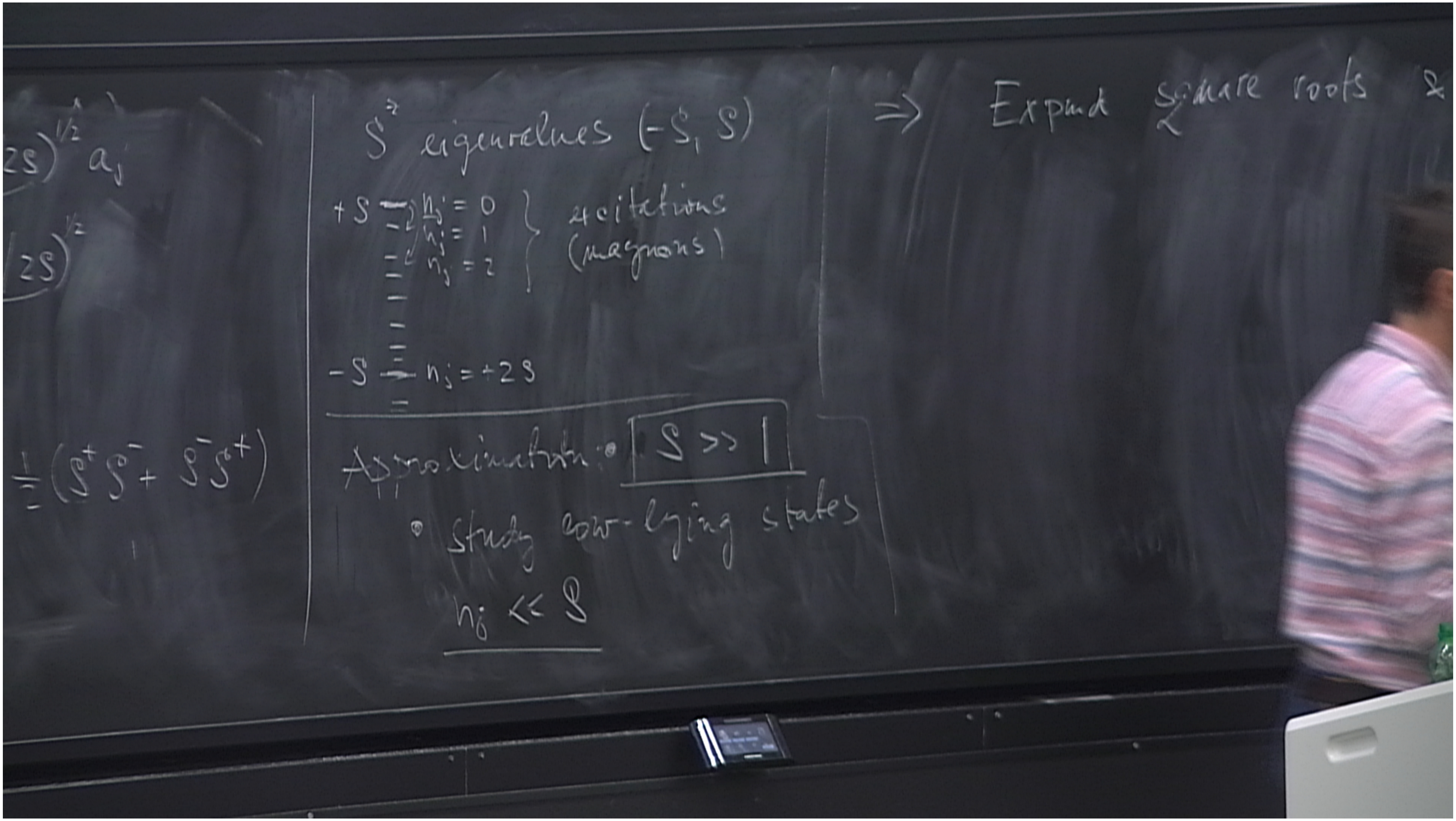

S^z eigenvalues (-S, S)
+S n_j = 0
n_j = 1
n_j = 2
-S n_j = +2S

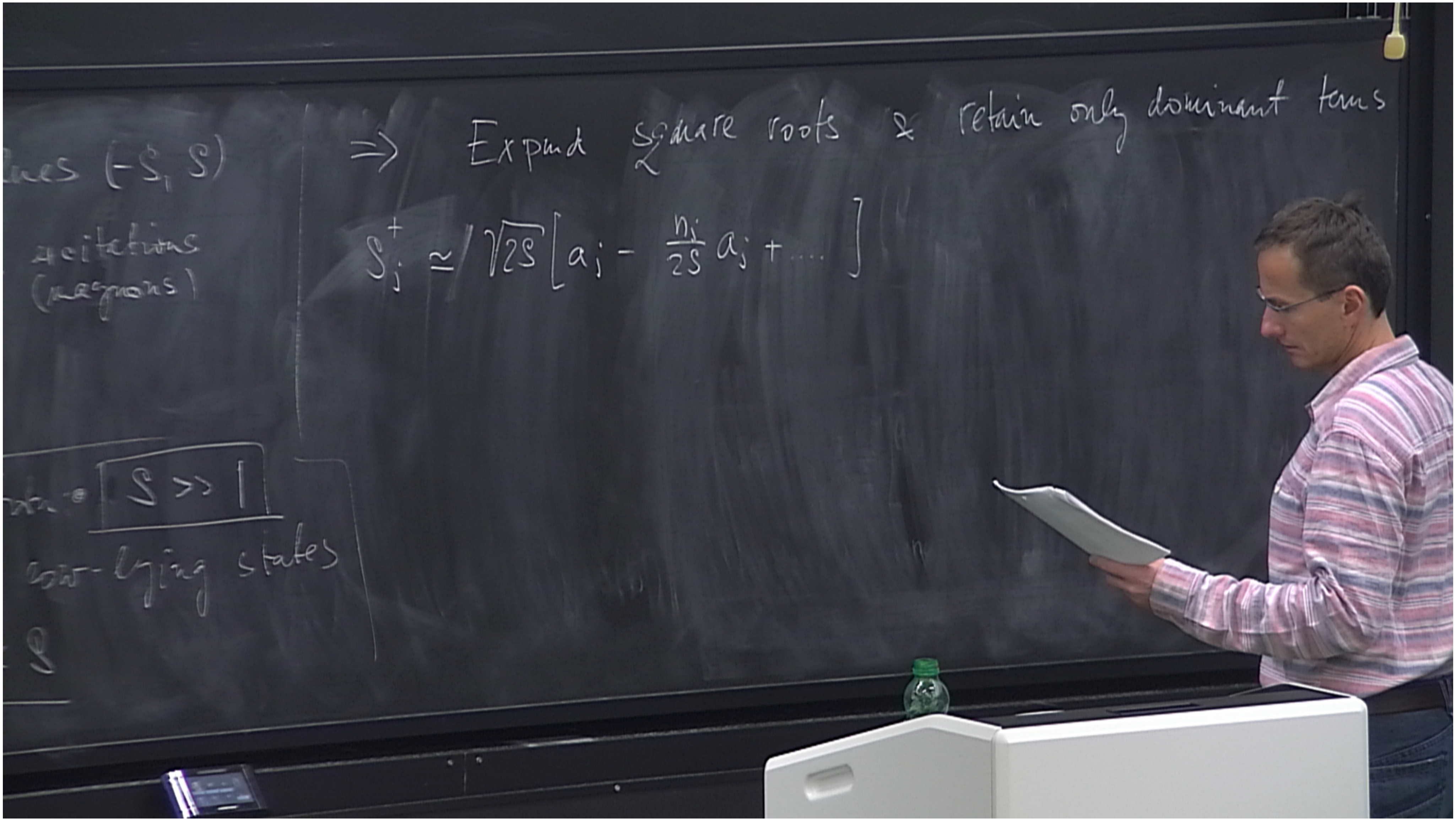


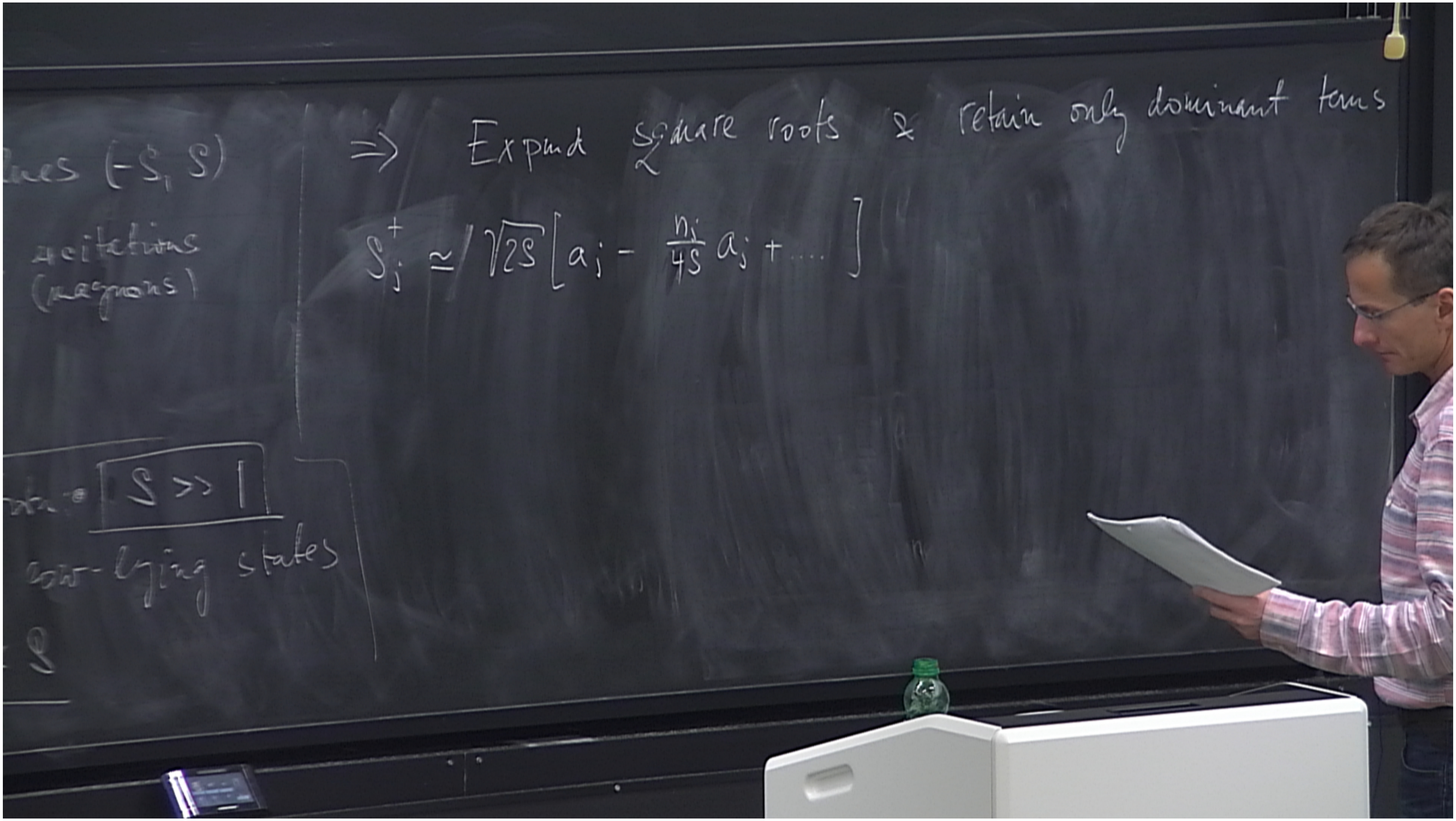
$2S)^{1/2} a_j$
$2S)^{1/2}$
$\frac{1}{2}(S^+S^- + S^-S^+)$
S^z eigenvalues $(-S, S)$
$+S$ $n_j = 0$
$n_j = 1$
$n_j = 2$
excitations
(magnons)
$-S$ $n_j = +2S$

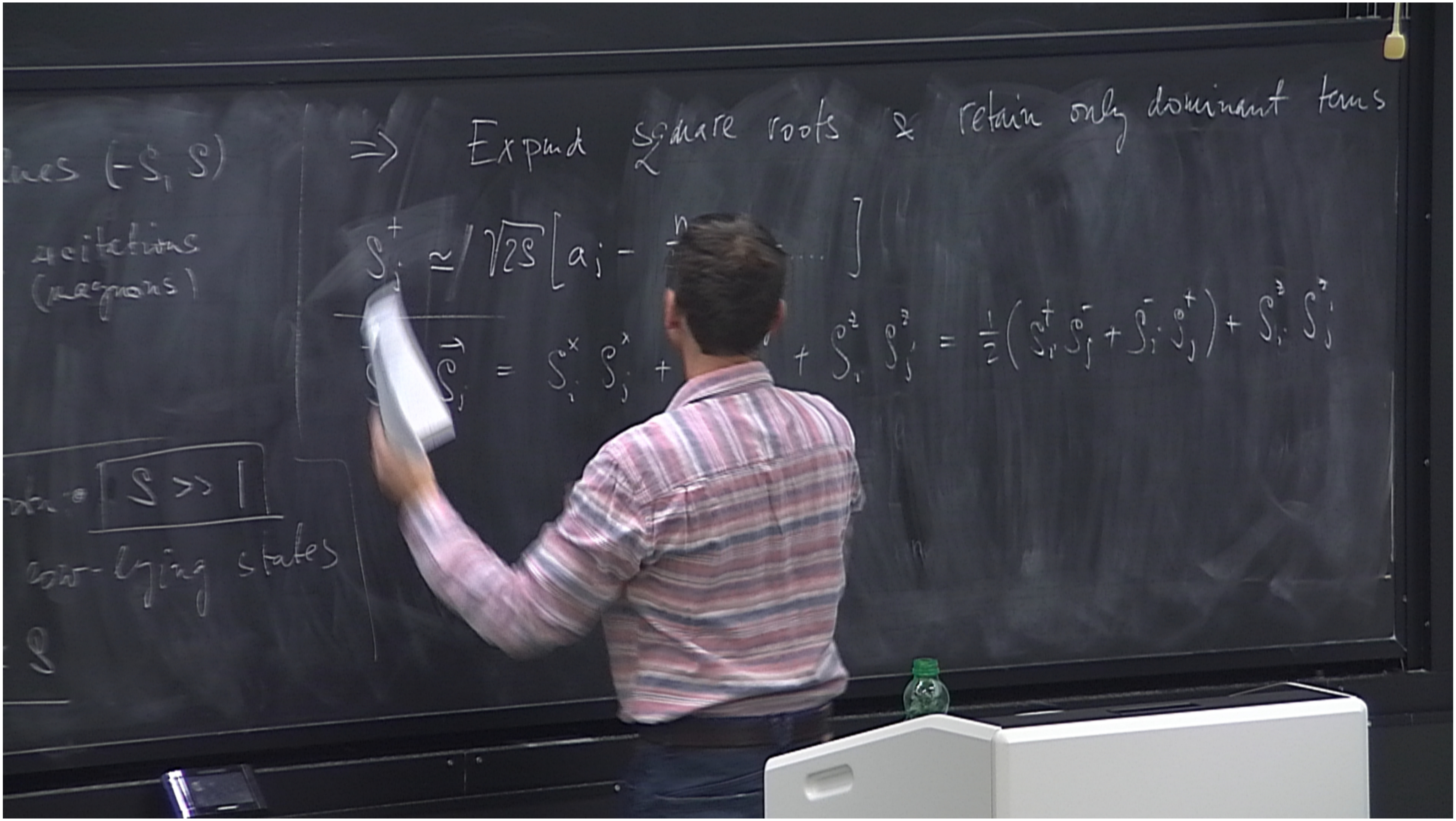

S^z eigenvalues (-S, S)
+S n_j = 0
n_j = 1
n_j = 2
excitations
(magnons)
-S n_j = 2S
Approximation: S >> 1
study low-lying states
n_j << S

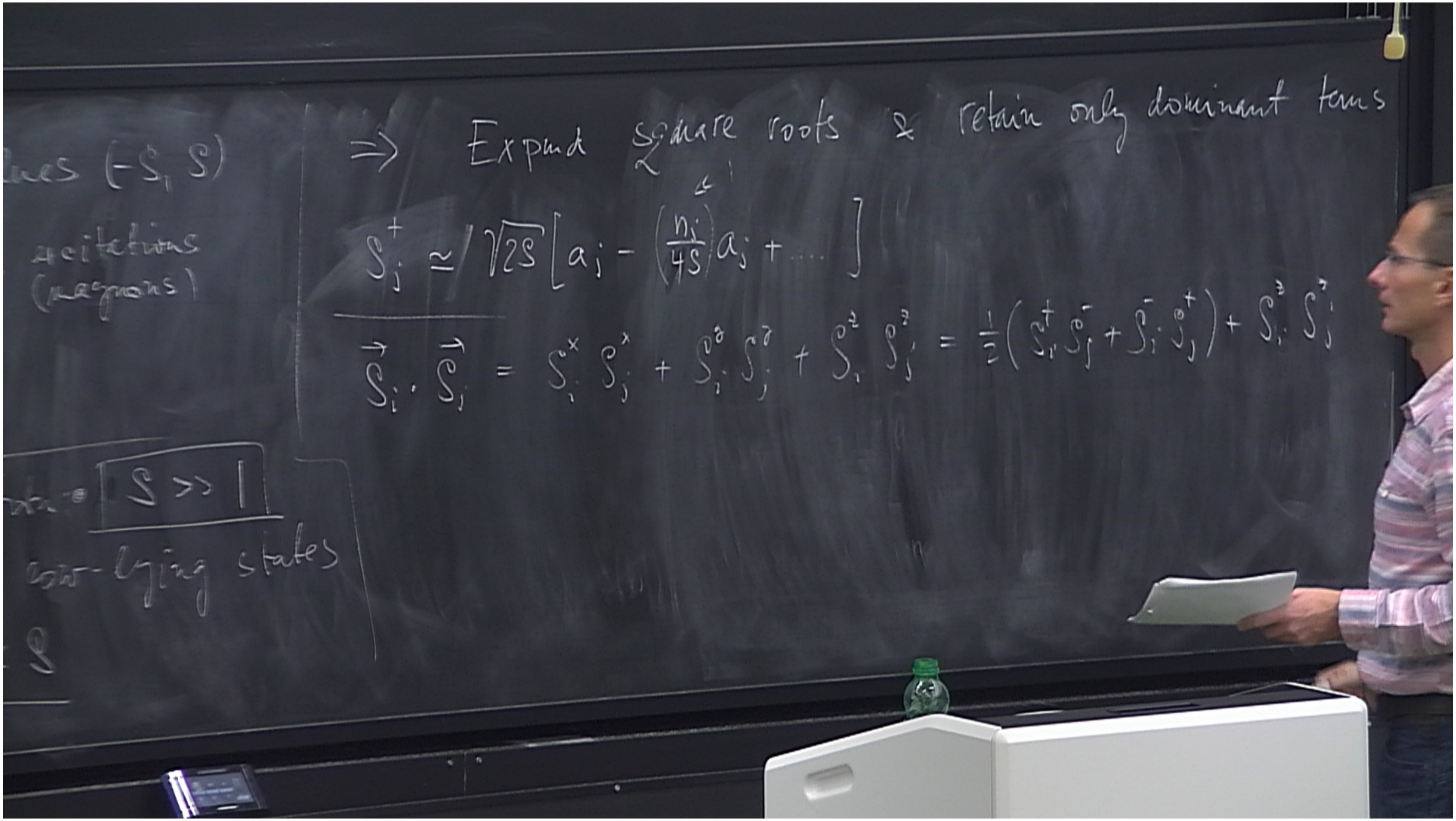

⇒ Expand square roots & retain only dominant terms

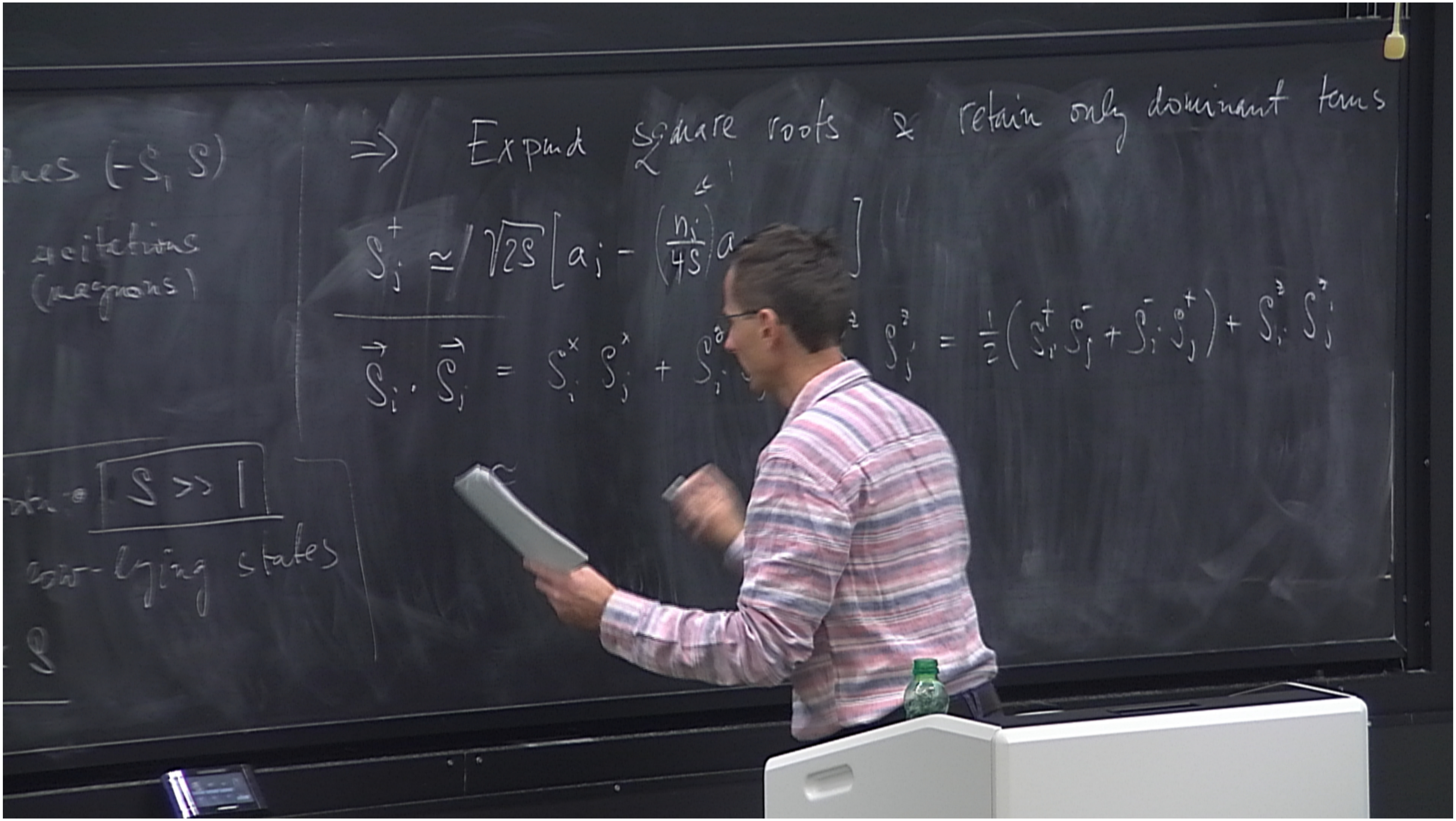

(2S)^1/2 a_j
(2S)^1/2
1/2 (S+S- + S-S+)
S^z eigenvalues (-S, S)
+S n_j = 0
n_j = 1
n_j = 2
excitations
(magnons)
-S n_j = +2S
Approximation: • S >> 1
• study low-lying states
n_j << S
⇒ Expand square roots &

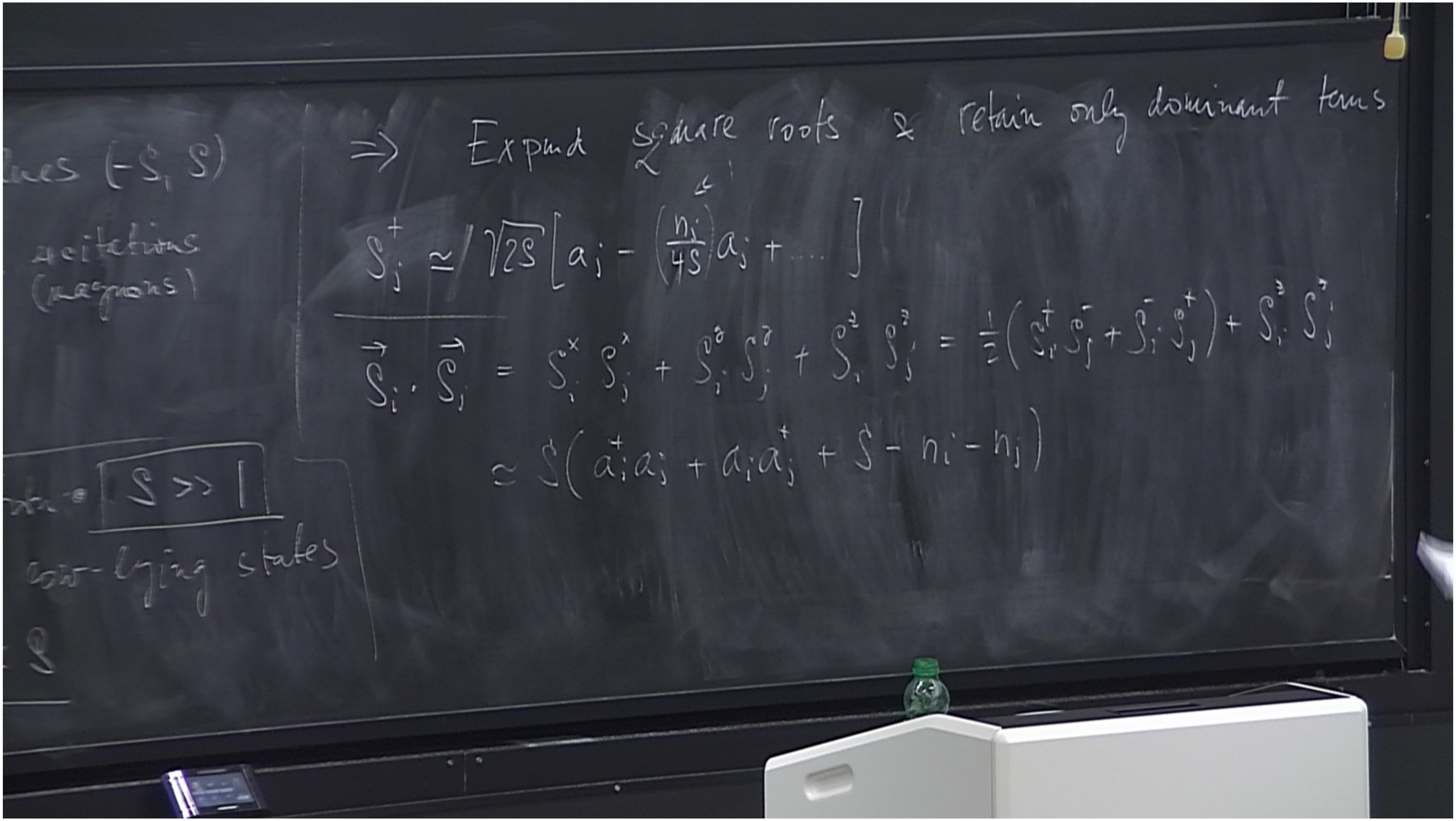

⇒ Expand square roots & retain only dominant terms
$S_j^+ \simeq \sqrt{2S}\left[a_j - \frac{n_j}{2S} a_j + \dots\right]$
(-S, S)
excitations (magnons)
$S >> 1$
low-lying states


⇒ Expand square roots & retain only dominant terms
$S_j^+ \simeq \sqrt{2S}\left[a_j - \frac{n_j}{4S} a_j + \ldots\right]$
(-S, S)
excitations
(magnons)
$S >> 1$
low-lying states

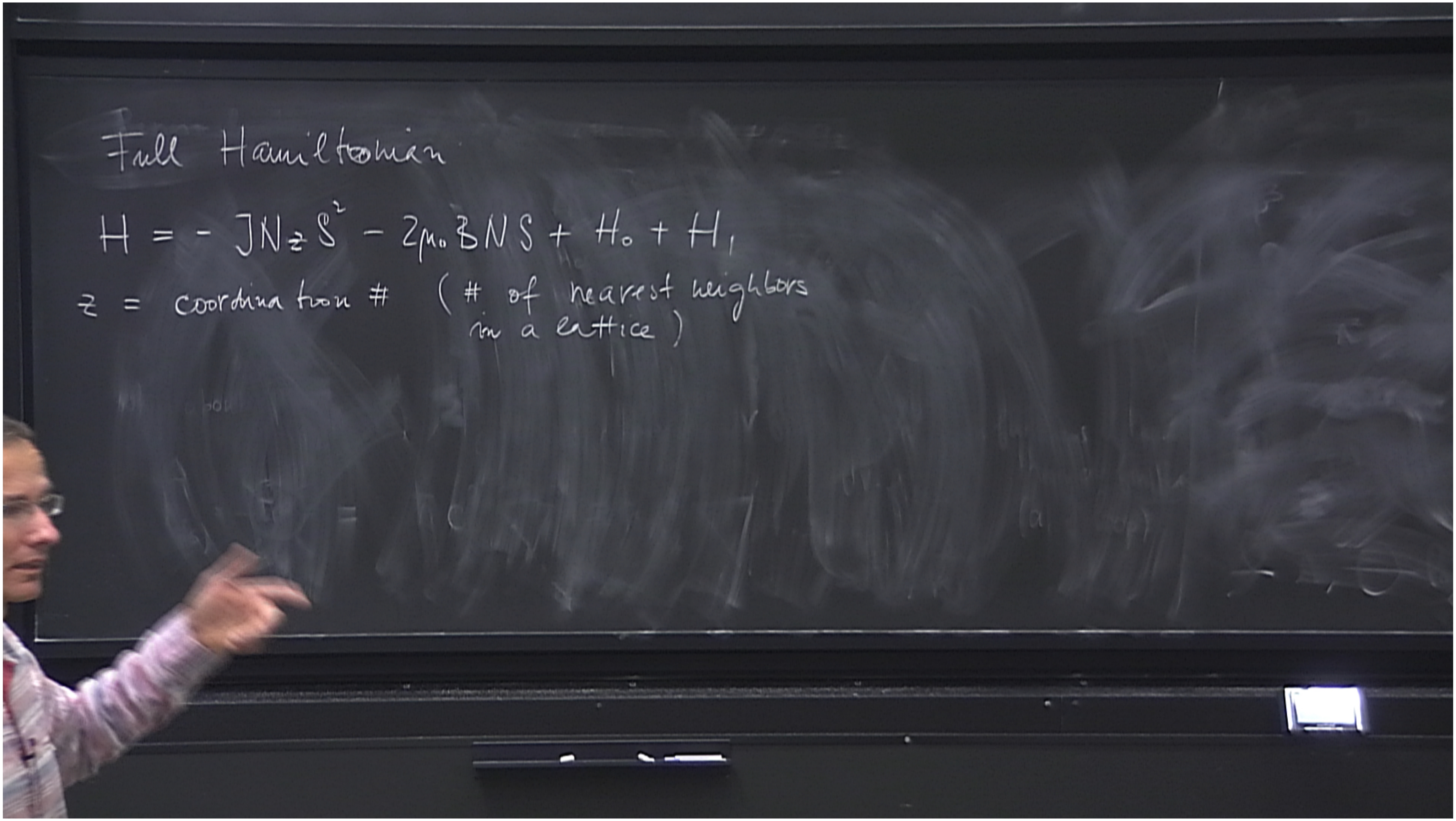

⇒ Expand square roots & retain only dominant terms
excitations
(magnons)
S >> 1
low-lying states

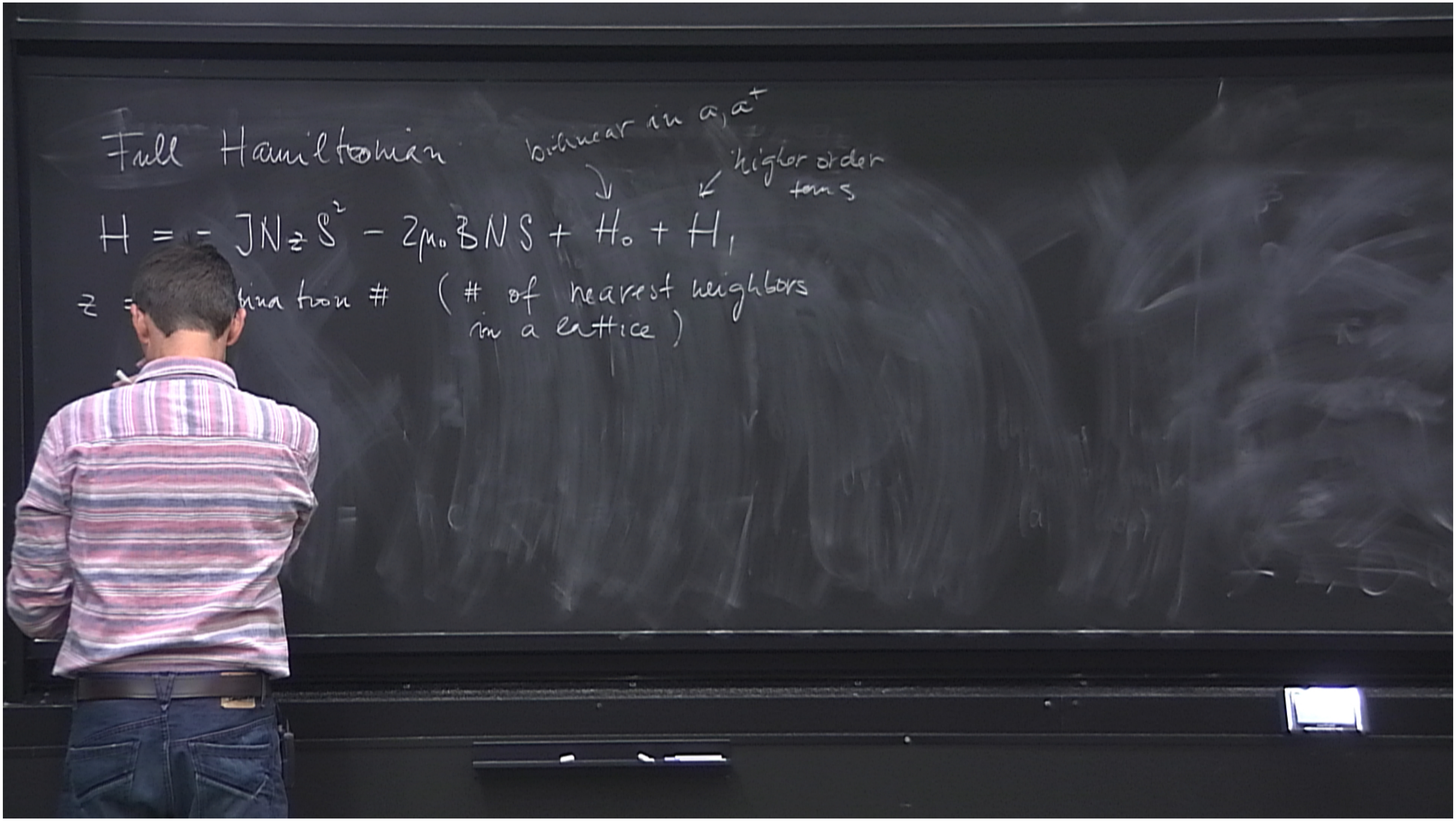

$(-S, S)$
(magnons)
$\Rightarrow$ Expand square roots & retain only dominant terms
$S_j^+ \simeq \sqrt{2S}\left[a_j - \left(\frac{n_j}{4S}\right)a_j + \dots\right]$
$\vec{S}_i \cdot \vec{S}_j = S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z = \frac{1}{2}\left(S_i^+ S_j^- + S_i^- S_j^+\right) + S_i^z S_j^z$
$S >> 1$
states

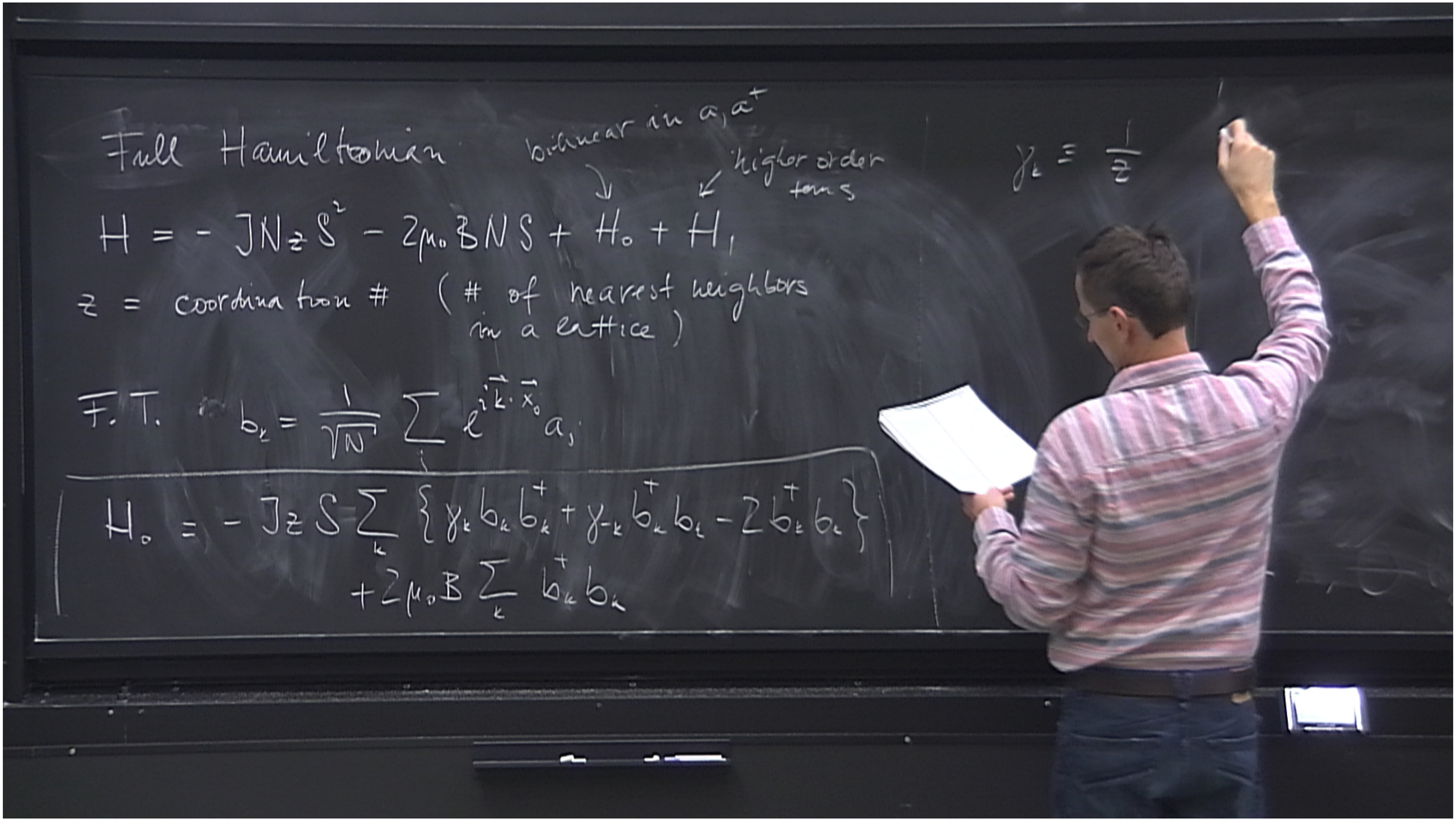

Expand square roots & retain only dominant terms
excitations
(magnons)
S >> 1
low-lying states



$\Rightarrow$ Expand square roots & retain only dominant terms
lues $(-S, S)$
excitations (magnons)
$\boxed{S \gg 1}$
low-lying states
$S_i^+ \simeq \sqrt{2S}\left[a_i - \left(\frac{n_i}{4S}\right)a_i + \ldots\right]$
$\vec{S}_i \cdot \vec{S}_j = S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z = \frac{1}{2}\left(S_i^+ S_j^- + S_i^- S_j^+\right) + S_i^z S_j^z$
$\simeq S\left(a_i^\dagger a_j + a_i a_j^\dagger + S - n_i - n_j\right)$


Full Hamiltonian
$H = - J N z S^2 - 2\mu_0 B N S + H_0 + H_1$
z = coordination # (# of nearest neighbors in a lattice)


Full Hamiltonian
bilinear in a, a+
higher order terms
H = - JNz S² - 2μ₀ BNS + H₀ + H₁
z =
ination # (# of nearest neighbors in a lattice)


Full Hamiltonian
bilinear in a, a^+
higher order terms
$H = -JNzS^2 - 2\mu_0 BNS + H_0 + H_1$
z = coordination # (# of nearest neighbors in a lattice)
F.T. $b_k = \frac{1}{\sqrt{N}} \sum e^{i\vec{k}\cdot\vec{x}_0} a_i$
$H_0 = -JzS\sum_k \{\gamma_k b_k b_k^+ + \gamma_{-k} b_k^+ b_k - 2 b_k^+ b_k\} + 2\mu_0 B \sum_k b_k^+ b_k$
$\gamma_k = \frac{1}{z}$

Full Hamiltonian

bilinear in $a, a^\dagger$; higher order terms

$$H = -JNzS^2 - 2\mu_0 B N S + H_0 + H_1$$

z = coordination # (# of nearest neighbors in a lattice)

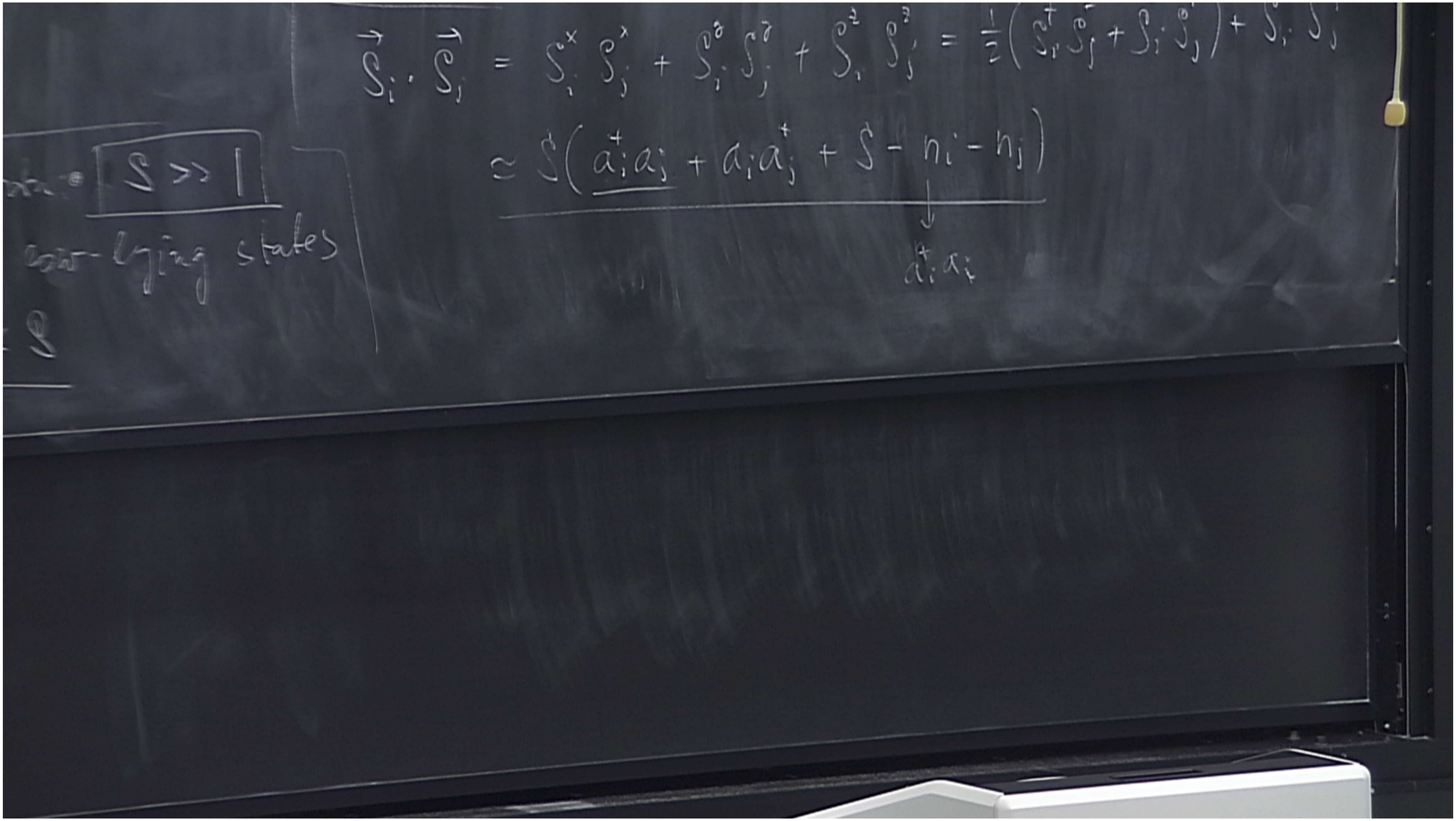
F.T. $b_k = \frac{1}{\sqrt{N}} \sum_i e^{i\vec{k}\cdot\vec{x}_i} a_i$

$$\boxed{H_0 = -JzS \sum_k \left\{ \gamma_k b_k b_k^\dagger + \gamma_{-k} b_k^\dagger b_k - 2 b_k^\dagger b_k \right\} + 2\mu_0 B \sum_k b_k^\dagger b_k}$$

$$\gamma_k \equiv \frac{1}{z} \sum_{\vec{\delta}} e^{i\vec{k}\cdot\vec{\delta}}$$

$\vec{\delta}$ = n.n. vectors

[$\vec{\delta} = (\hat{x}, -\hat{x}, \hat{y}, -\hat{y}, \hat{z}, -\hat{z})\, a_0$ for cubic lattice]


$\vec{S}_i \cdot \vec{S}_j = S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z = \frac{1}{2}(S_i^+ S_j^- + S_i^- S_j^+) + S_i^z S_j^z$
$\simeq S(a_i^\dagger a_j + a_i a_j^\dagger + S - n_i - n_j)$
$a_i^\dagger a_i$
$S \gg 1$
low-lying states

Full Hamiltonian

bilinear in a, a^{+} — higher order terms

$$H = -JNzS^2 - 2\mu_0 BNS + H_0 + H_1$$

z = coordination # (# of nearest neighbors in a lattice)

F.T. $b_k = \frac{1}{\sqrt{N}} \sum_i e^{i\vec{k}\cdot\vec{x}_i} a_i$

$$\boxed{H_0 = -JzS\sum_k \left\{ \gamma_k b_k b_k^{+} + \gamma_{-k} b_k^{+} b_k - 2 b_k^{+} b_k \right\} + 2\mu_0 B \sum_k b_k^{+} b_k}$$

$$\gamma_k = \frac{1}{z} \sum_{\vec{\delta}} e^{i\vec{k}\cdot\vec{\delta}}$$

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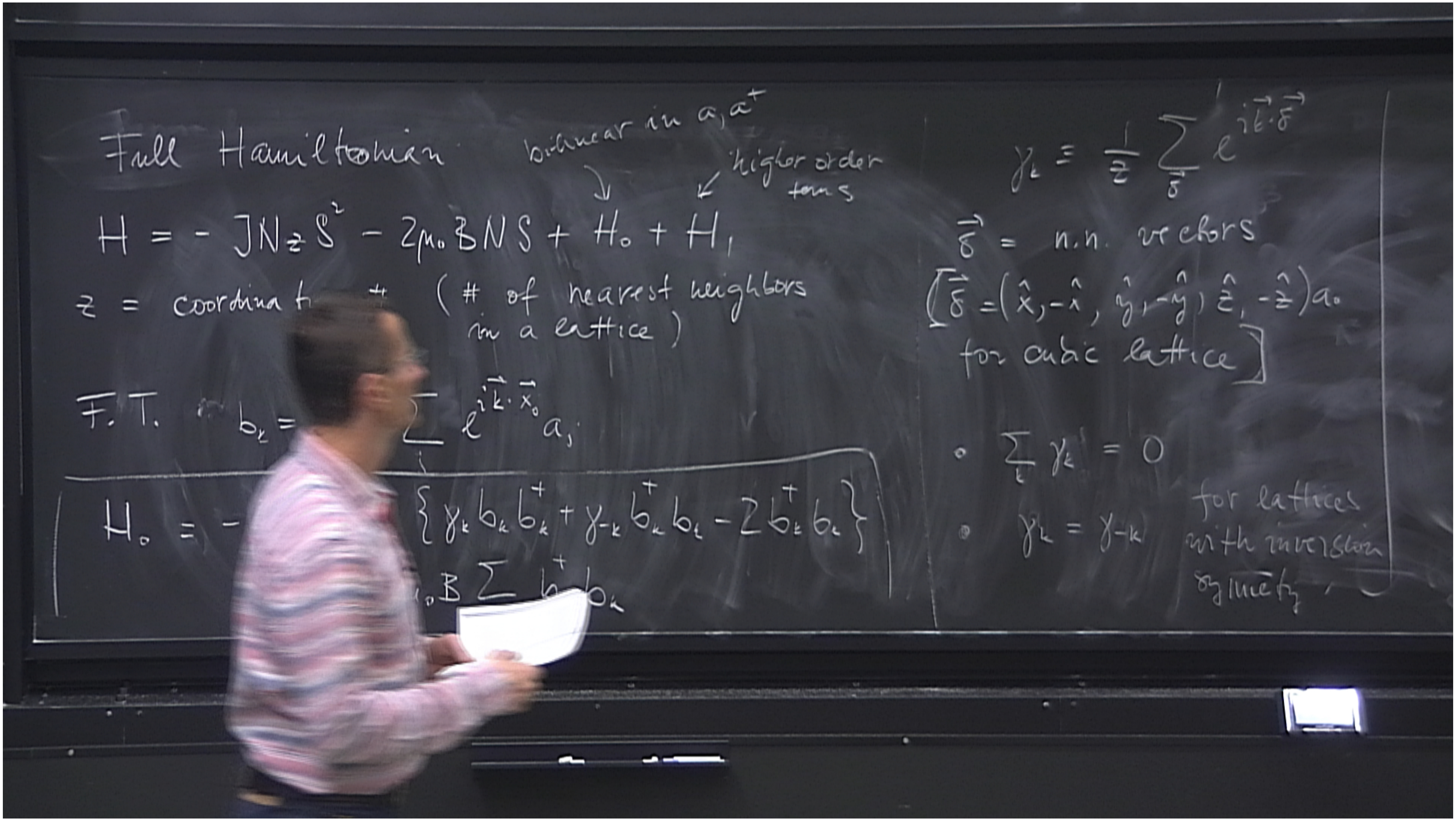
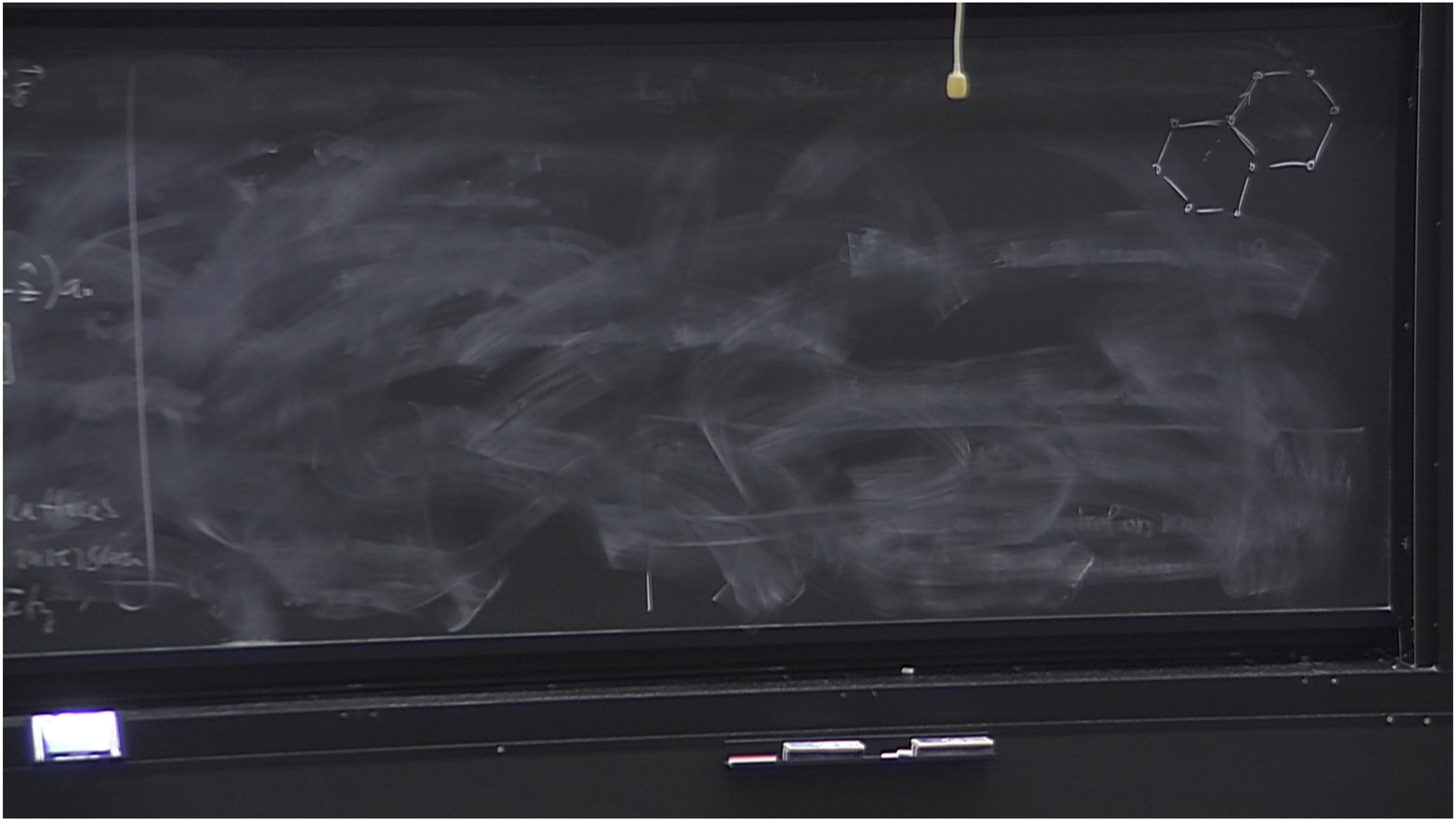
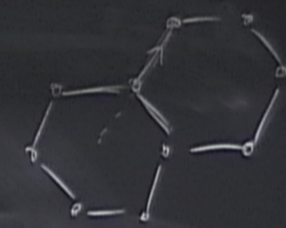
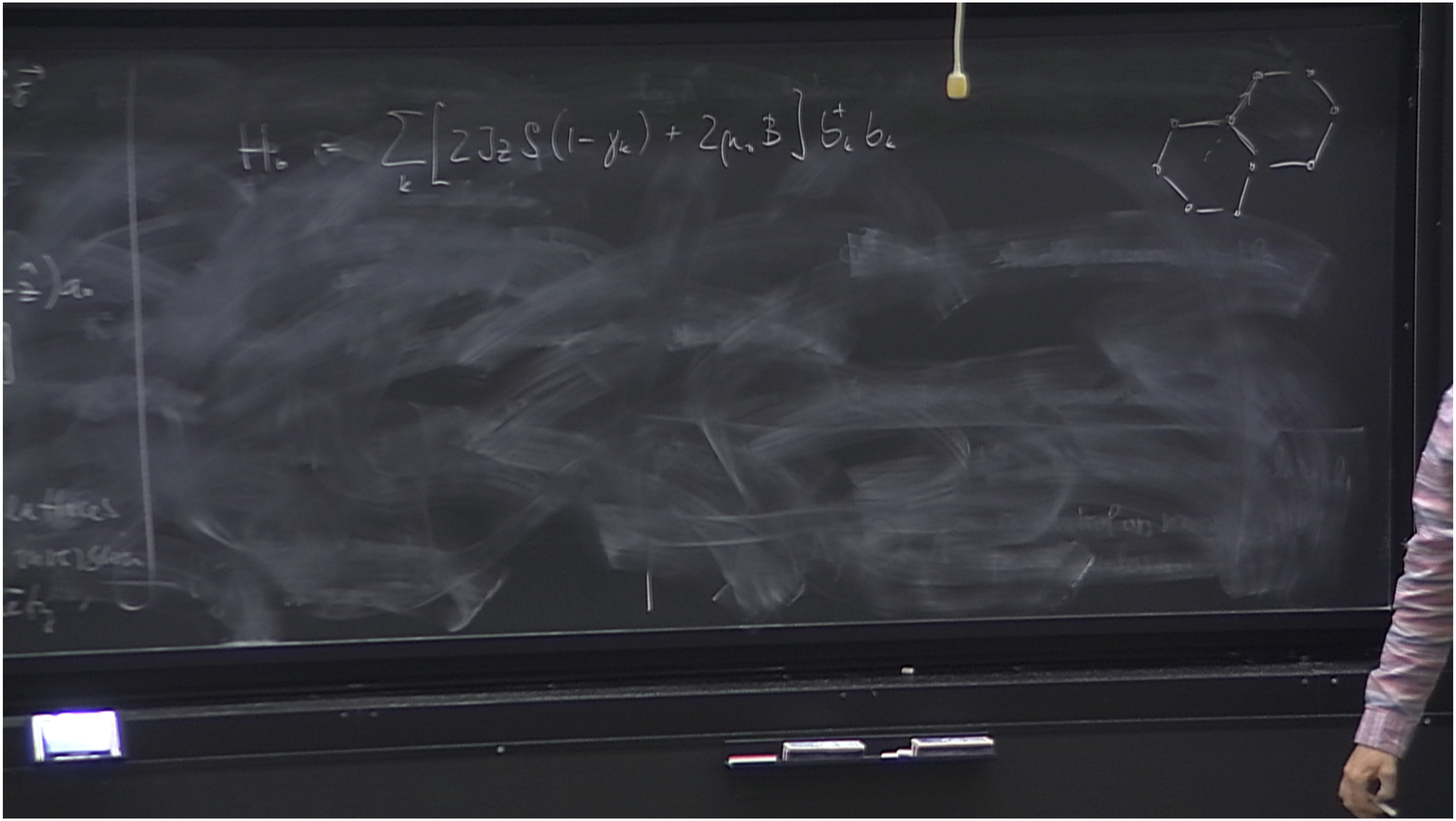

Full Hamiltonian
bilinear in a, a^+
higher order terms
$H = - J N z S^2 - 2\mu_0 B N S + H_0 + H_1$
z = coordina
(# of nearest neighbors in a lattice)
F. T. $b_k =$
$\sum e^{i\vec{k}\cdot\vec{x}_i} a_i$
$H_0 = -$
$\{\gamma_k b_k b_k^+ + \gamma_{-k} b_k^+ b_k - 2 b_k^+ b_k\}$
$B \sum b_k^+ b_k$
$\gamma_k = \frac{1}{z} \sum_{\vec{\delta}} e^{i\vec{k}\cdot\vec{\delta}}$
$\vec{\delta}$ = n.n. vectors
[$\vec{\delta} = (\hat{x}, -\hat{x}, \hat{y}, -\hat{y}, \hat{z}, -\hat{z}) a_0$
for cubic lattice]
• $\sum_k \gamma_k = 0$
• $\gamma_k = \gamma_{-k}$
for lattices with inversion symmetry

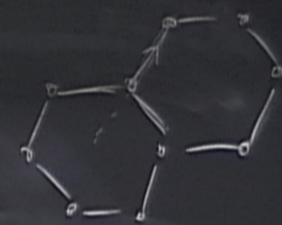





$H_0 = \sum_k \left[2 J_z S (1-\gamma_k) + 2\mu_B B \right] b_k^{+} b_k$

$$H_0 = \sum_k \underbrace{\left[2 J_z S (1-\gamma_k) + 2\mu_B B \right]}_{\hbar\omega_k} b_k^\dagger b_k$$

$$= \sum_k \hbar\omega_k \, b_k^\dagger b_k$$



$$H_0 = \sum_k \left[2 J_z S (1-\gamma_k) + 2\mu_B B \right] b_k^\dagger b_k$$

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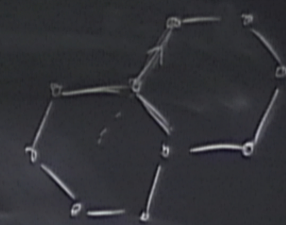
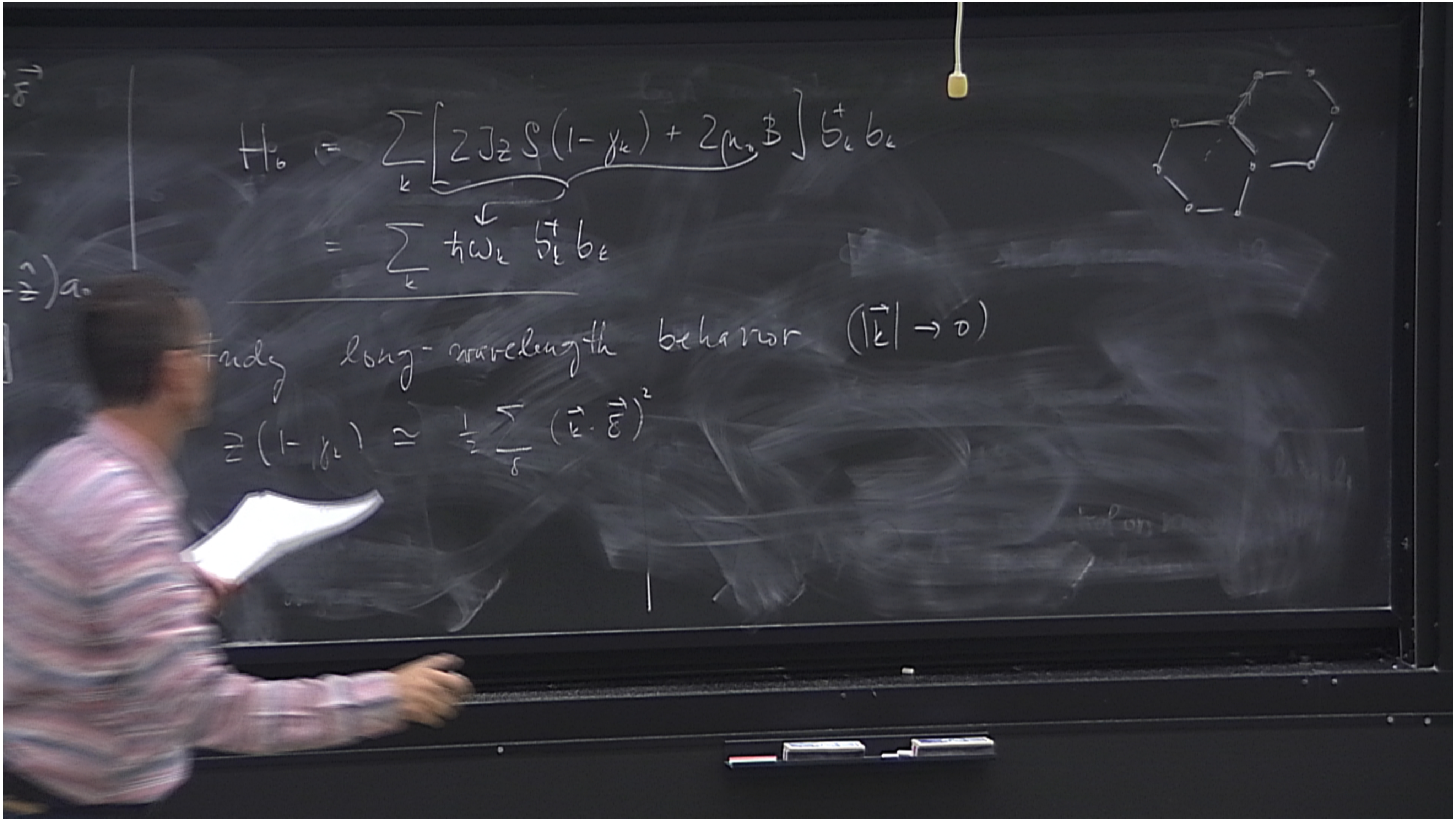
Study long-wavelength behavior $(|\vec{k}| \to 0)$

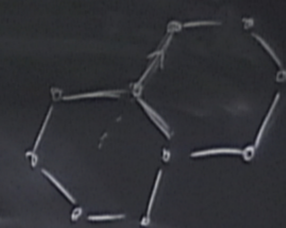




$H_0 = \sum_k \left[2 J_z S (1 - \gamma_k) + 2\mu_B B \right] b_k^\dagger b_k$
$= \sum_k \hbar\omega_k \, b_k^\dagger b_k$
tudy long-wavelength behavior $(|\vec{k}| \to 0)$
$\sum (1 - \gamma_k) \simeq \frac{1}{2} \sum_\delta (\vec{k} \cdot \vec{\delta})^2$

$$H_0 = \sum_k \Big[\underbrace{2 J_z S (1-\gamma_k) + 2\mu_B B}_{\hbar\omega_k} \Big] b_k^\dagger b_k$$

$$= \sum_k \hbar\omega_k \, b_k^\dagger b_k$$



Study long-wavelength behavior $(|\vec{k}| \to 0)$

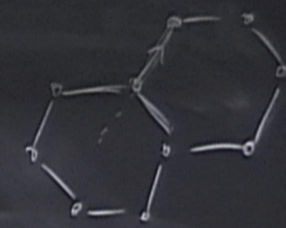
$$z(1-\gamma_k) \simeq \frac{1}{2}\sum_\delta (\vec{k}\cdot\vec{\delta})^2 + \ldots$$

lattices

inversion

$$H_0 = \sum_k \underbrace{\left[2JzS(1-\gamma_k) + 2\mu_0 B \right]}_{\hbar\omega_k} b_k^\dagger b_k$$

$$= \sum_k \hbar\omega_k \, b_k^\dagger b_k$$



Study long-wavelength behavior $(|\vec{k}| \to 0)$

$$z(1-\gamma_k) \simeq \frac{1}{2}\sum_\delta (\vec{k}\cdot\vec{\delta})^2 + \dots$$

SC, BCC, FCC

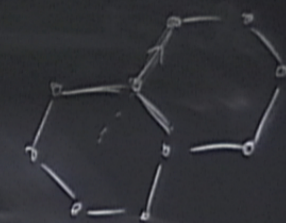
$$\hbar\omega_k \simeq 2\mu_0 B + JS\sum_\delta (\vec{k}\cdot\vec{\delta})^2$$

lattices

inversion

$$H_0 = \sum_k \underbrace{\left[2JzS(1-\gamma_k) + 2\mu_0 B\right]}_{} b_k^\dagger b_k$$

$$= \sum_k \hbar\omega_k \, b_k^\dagger b_k$$



Study long-wavelength behavior $(|\vec{k}| \to 0)$

$$z(1-\gamma_k) \simeq \frac{1}{2}\sum_\delta (\vec{k}\cdot\vec{\delta})^2 + \dots$$

$$\hbar\omega_k \simeq 2\mu_0 B + JS\sum_\delta (\vec{k}\cdot\vec{\delta})^2 \xrightarrow[\text{FCC}]{\text{SC, BCC}} 2\mu_0 B + 2JS(ka)^2$$

lattices

inversion

- Excitations are Magnons: non-interacting bosons with quadratic dispersion.

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- Magnon spec. heat

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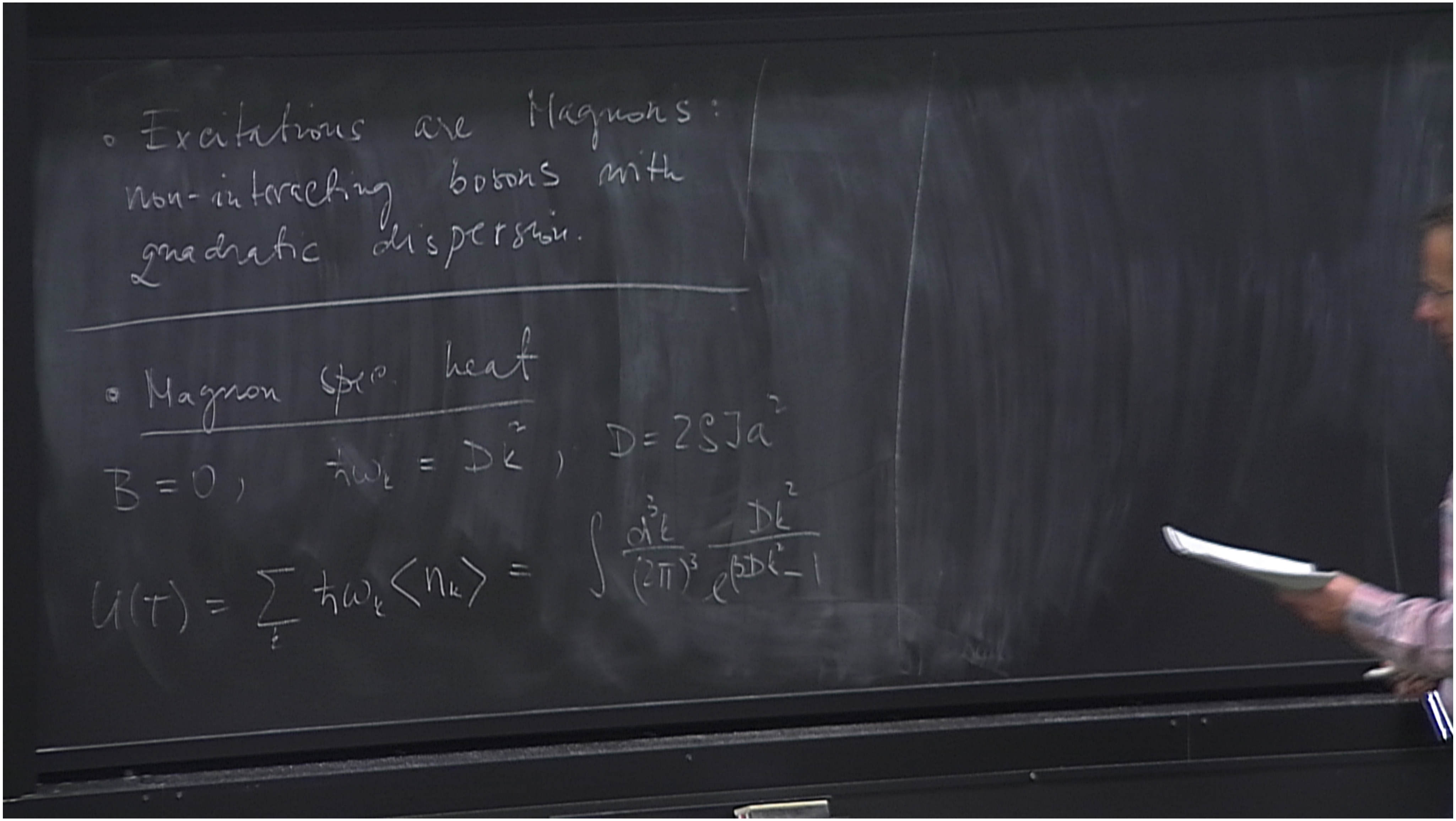
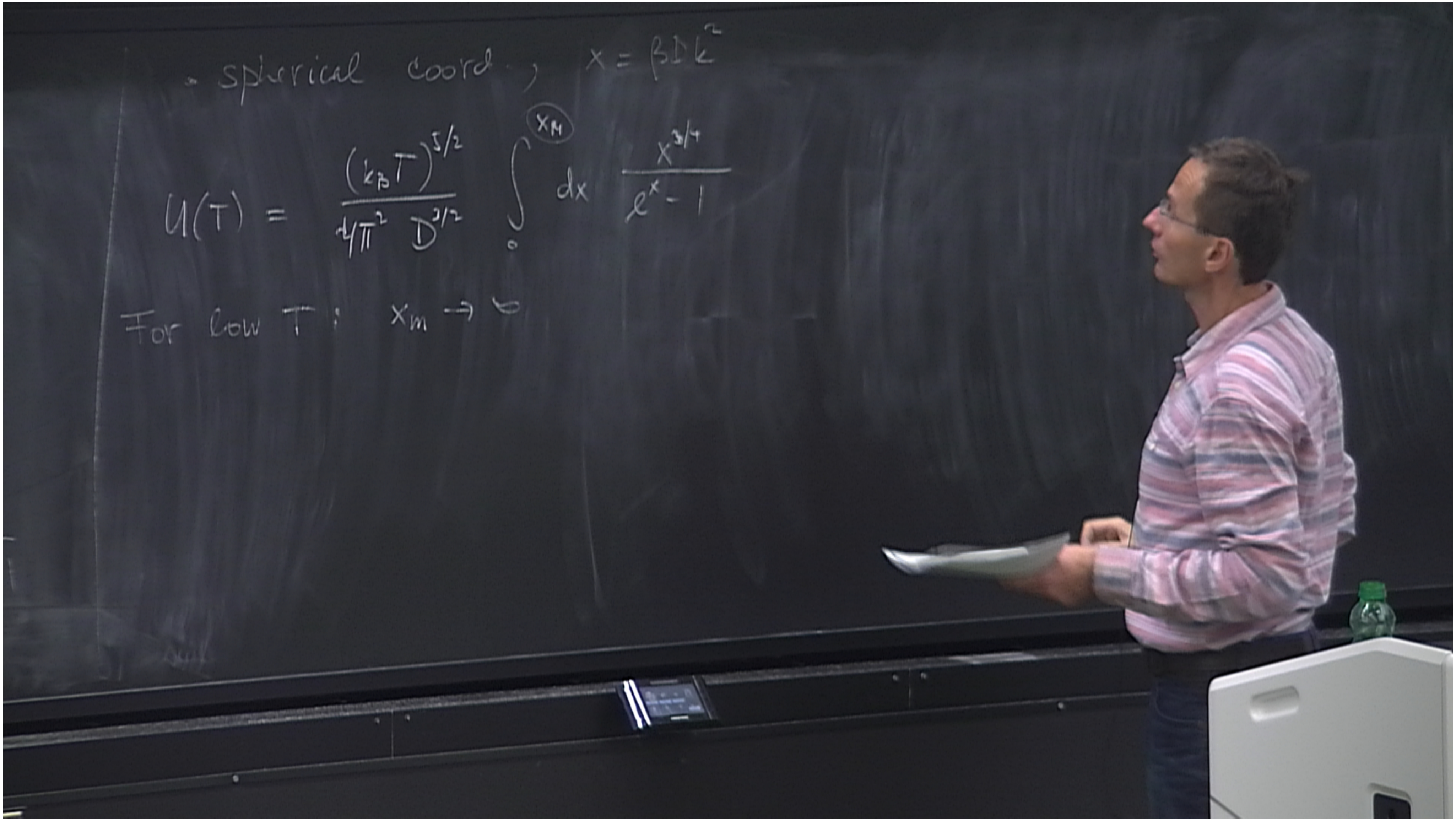
$B=0$, $\hbar\omega_k = Dk^2$, $D=2SJa^2$



∘ Excitations are Magnons:
non-interacting bosons with
quadratic dispersion.
∘ Magnon spec. heat
$B = 0, \quad \hbar\omega_k = Dk^2, \quad D = 2SJa^2$
$U(T) = \sum_k \hbar\omega_k \langle n_k \rangle = \int \frac{d^3k}{(2\pi)^3} \frac{Dk^2}{e^{\beta Dk^2} - 1}$

- spherical coord., $x = \beta D k^2$

$$U(T) = \frac{(k_B T)^{5/2}}{4\pi^2 D^{3/2}} \int_0^{x_M} dx \, \frac{x^{3/4}}{e^x - 1}$$


· spherical coord. , $x = \beta D k^2$
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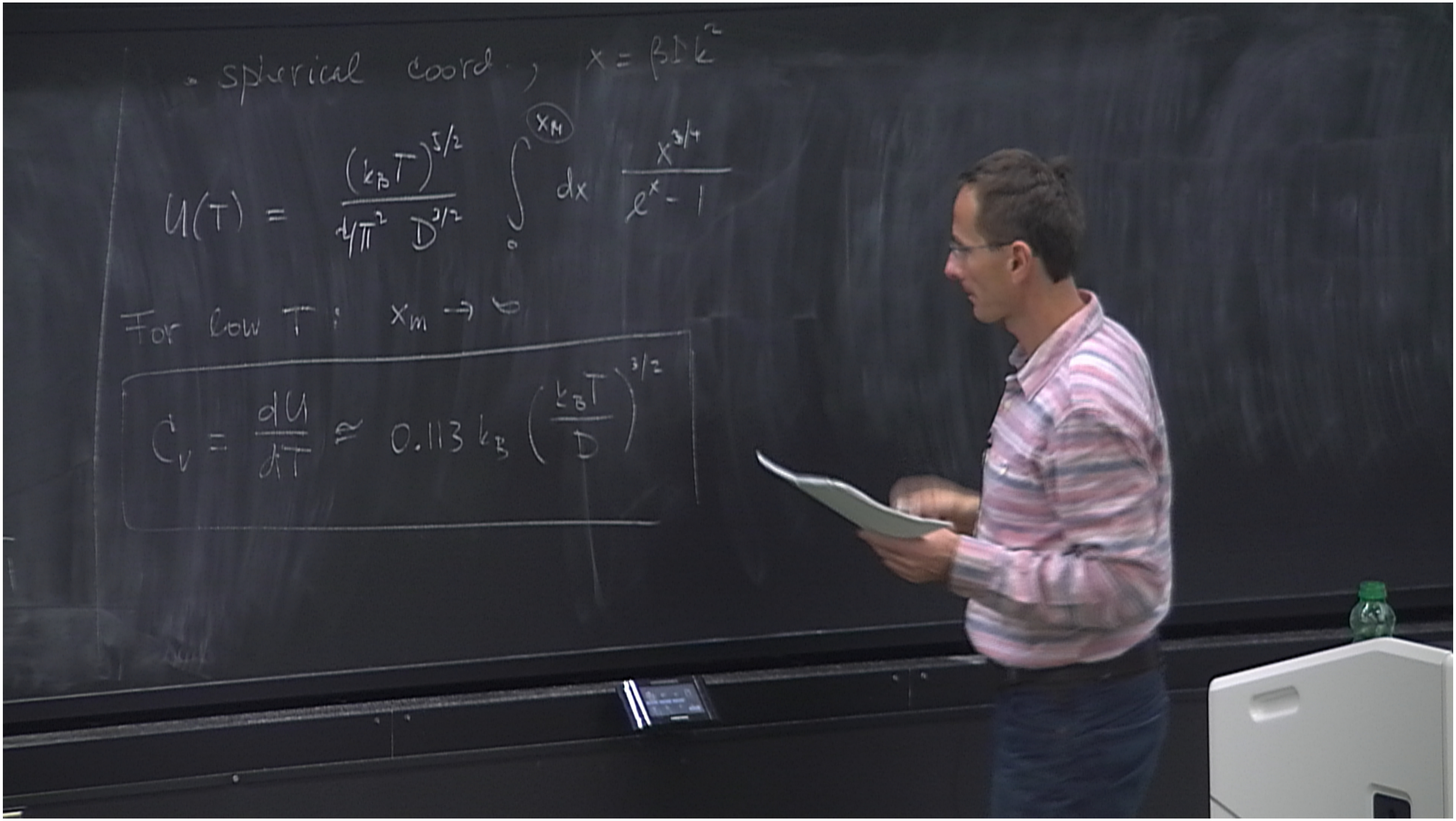
For low T : $x_m \to \infty$



• spherical coord., $x = \beta \Gamma k^2$
$U(T) = \frac{(k_B T)^{5/2}}{4\pi^2 D^{3/2}} \int_0^{x_M} dx \frac{x^{3/4}}{e^x - 1}$
For low T : $x_m \to \infty$
$C_V = \frac{dU}{dT} \simeq 0.113\, k_B \left(\frac{k_B T}{D}\right)^{3/2}$

$$x_m \to \infty$$

$$0.113\, k_B \left(\frac{k_B T}{D} \right)^{3/2}$$

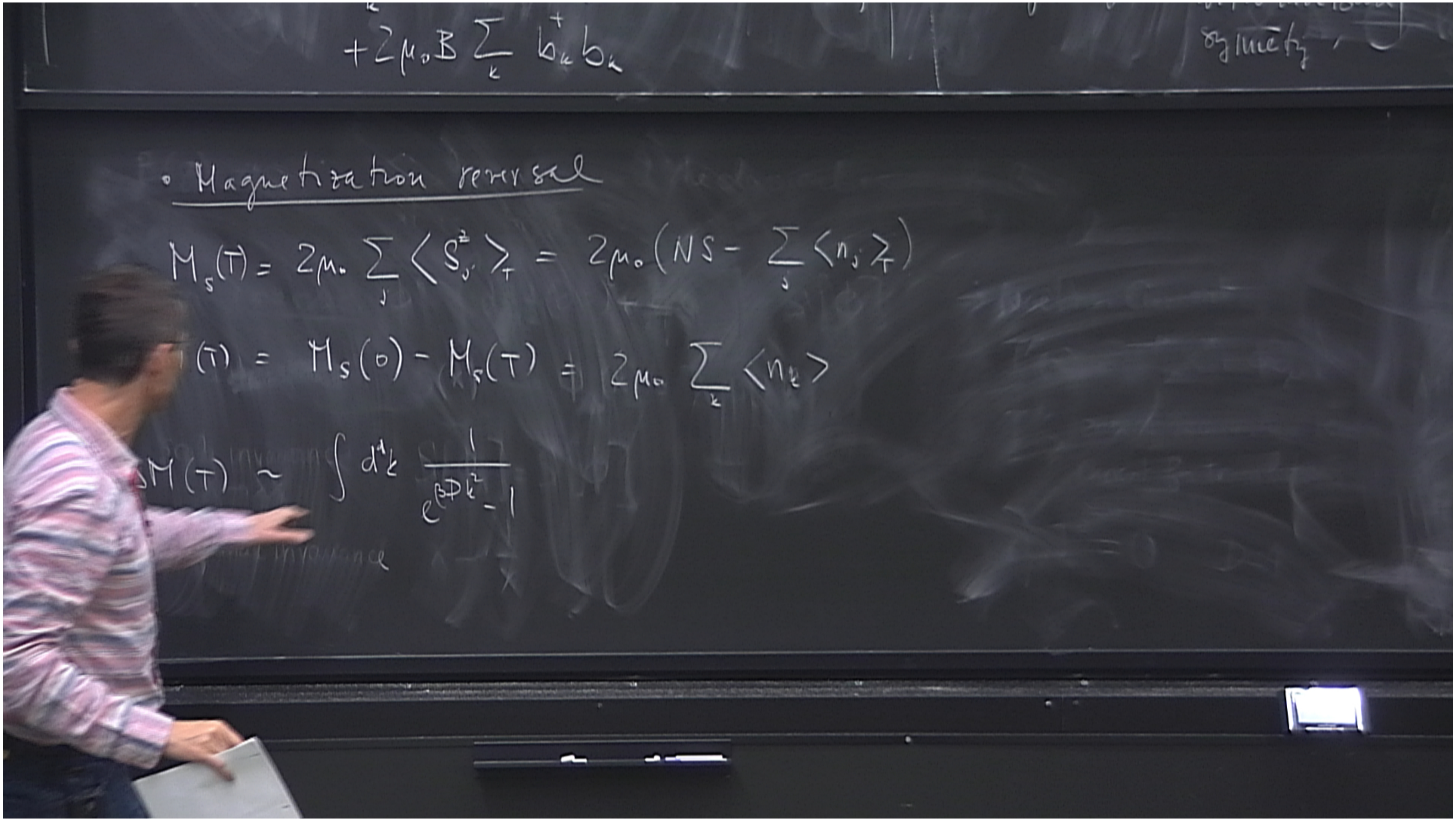
$$+2\mu_0 B \sum_k b_k^\dagger b_k$$

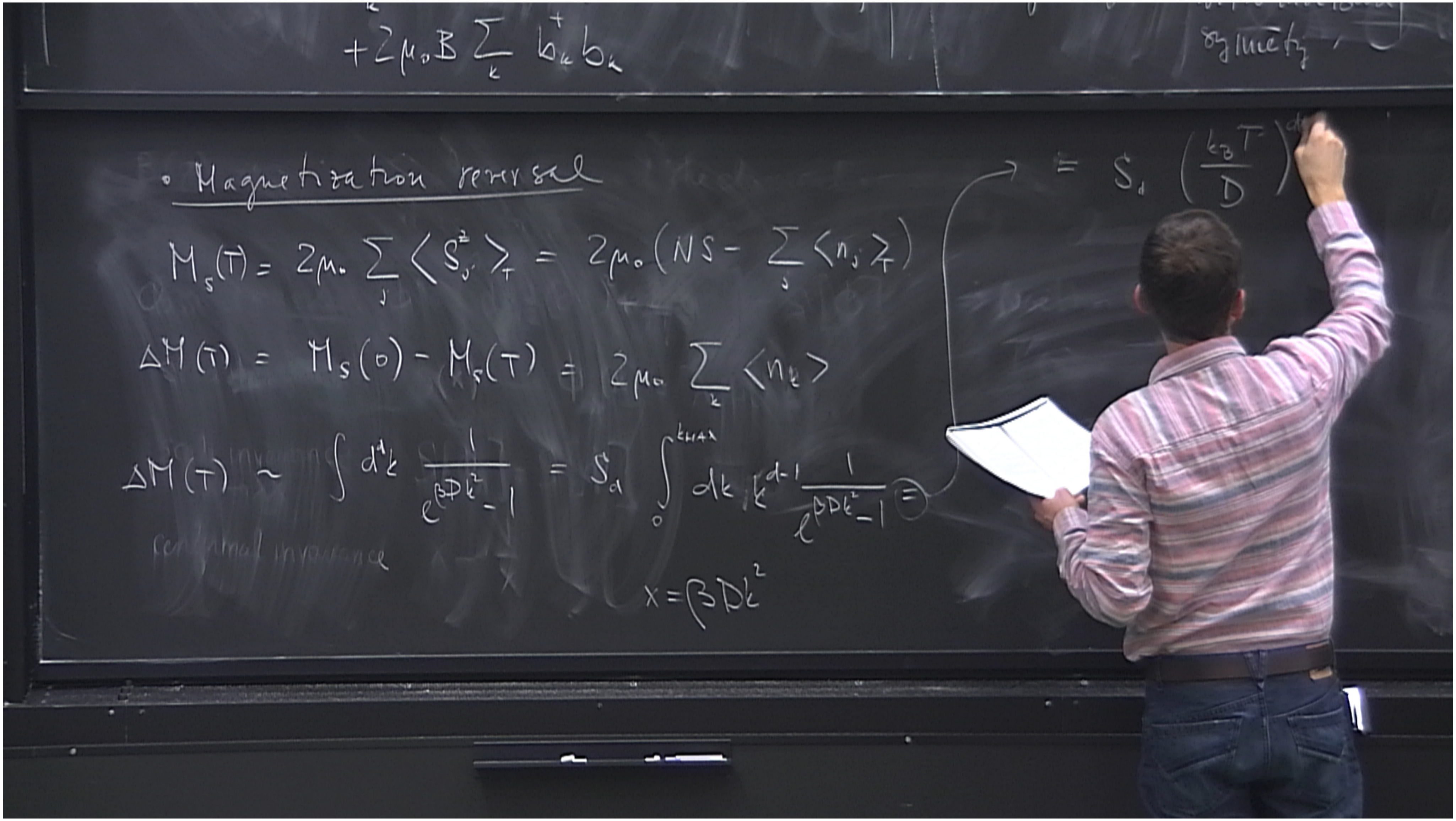
symmetry

- <u>Magnetization reversal</u>

$$M_s(T) = 2\mu_0 \sum_j \langle S_j^z \rangle_T = 2\mu_0 \left(NS - \sum_j \langle n_j \rangle_T \right)$$

$$\Delta M(T) = M_s(0) - M_s(T) = 2\mu_0 \sum_k \langle n_k \rangle$$


$+2\mu_0 B \sum_k b_k^\dagger b_k$
symmetry
• Magnetization reversal
$M_s(T) = 2\mu_0 \sum_j \langle S_j^z \rangle_T = 2\mu_0 \left(NS - \sum_j \langle n_j \rangle_T\right)$
$(T) = M_s(0) - M_s(T) = 2\mu_0 \sum_k \langle n_k \rangle$
$M(T) \sim \int d^d k \frac{1}{e^{\beta D k^2} - 1}$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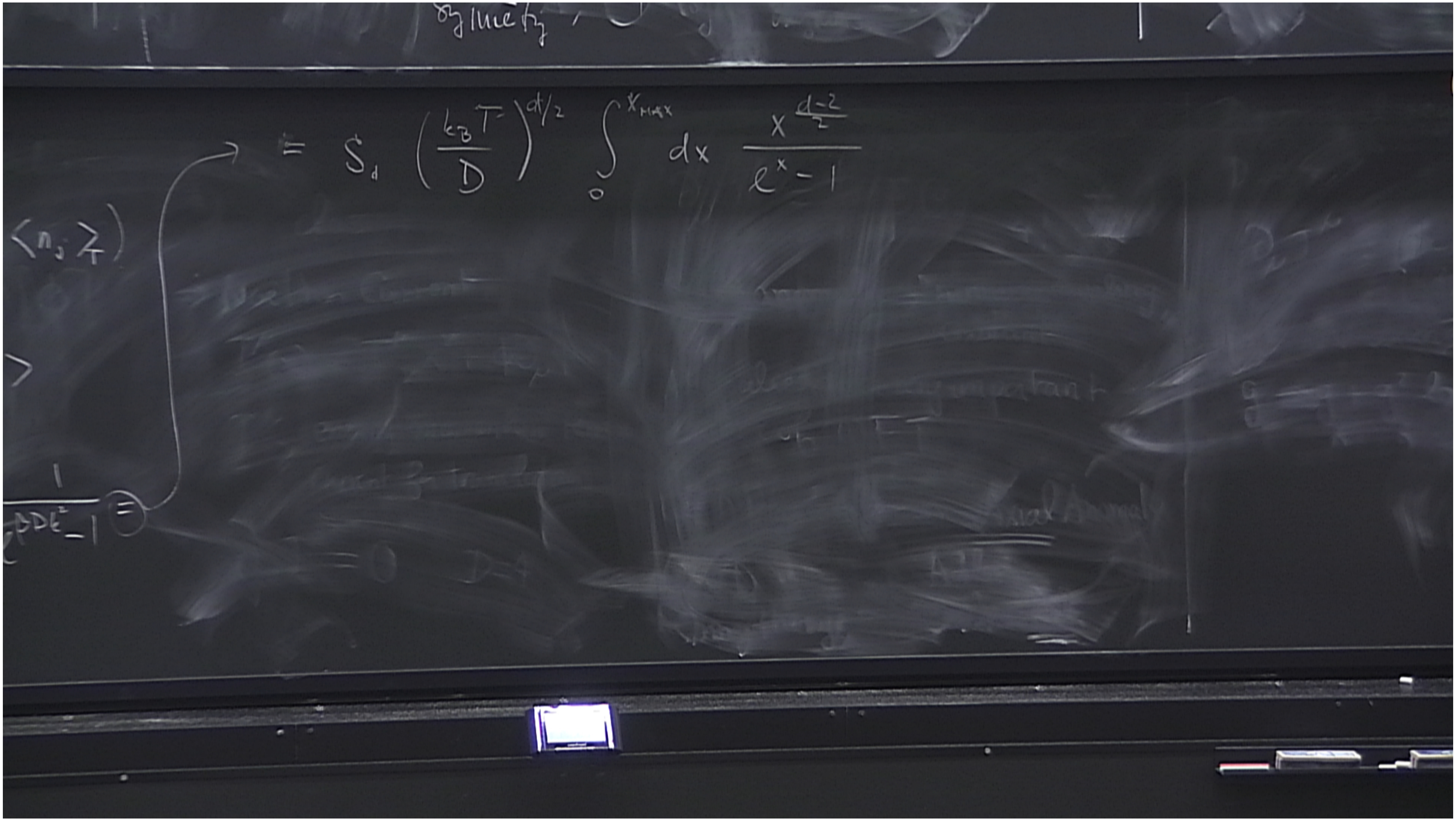
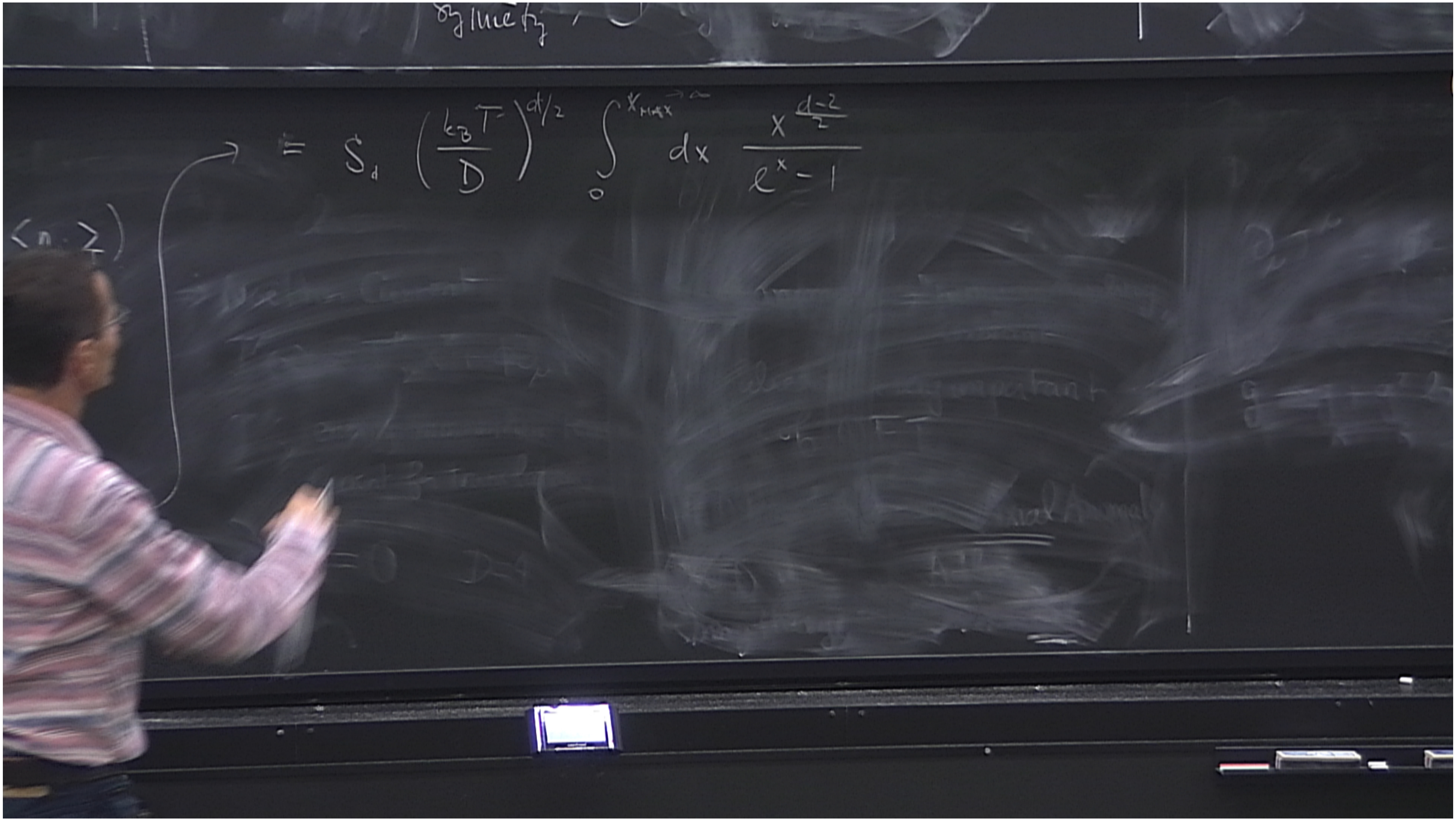
$\Delta M(T) \sim \int d^d k \frac{1}{e^{\beta D k^2} - 1} = S_d \int_0^{k_{max}} dk\, k^{d-1} \frac{1}{e^{\beta D k^2} - 1} =$
$x = \beta D k^2$
$= S_d \left(\frac{k_B T}{D} \right)^{d}$


symmetry
$= S_d \left(\frac{k_B T}{D}\right)^{d/2} \int_0^{x_{max}} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$
$\langle n_j \rangle_T)$
$\frac{1}{e^{\beta D k^2} - 1} =$


symmetry
$= S_d \left(\frac{k_B T}{D}\right)^{d/2} \int_0^{x_{max} \to \infty} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$

symmetry

$$\left(\frac{k_B T}{D}\right)^{d/2} \int_0^{x_{MAX} \to \infty} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$$

symmetry

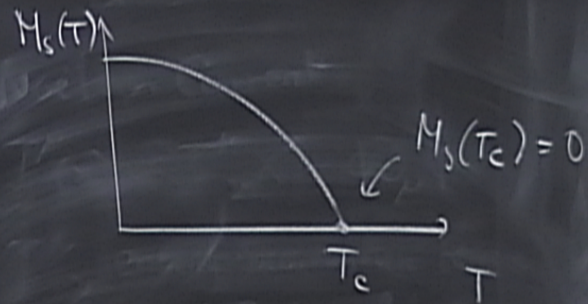
$$= S_d \left(\frac{k_B T}{D}\right)^{d/2} \int_0^{x_{max}\to\infty} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$$

$s - \sum_j \langle n_j \rangle_T)$

$\sum_k \langle n_k \rangle$

$dk \, k^{d-1} \frac{1}{e^{\beta D k^2} - 1} =$

$\beta D k^2$



symmetry ,

$$= S_d \left(\frac{k_B T}{D}\right)^{d/2} \int_0^{x_{max}\to\infty} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$$

$s - \sum_j \langle n_j \rangle_T)$

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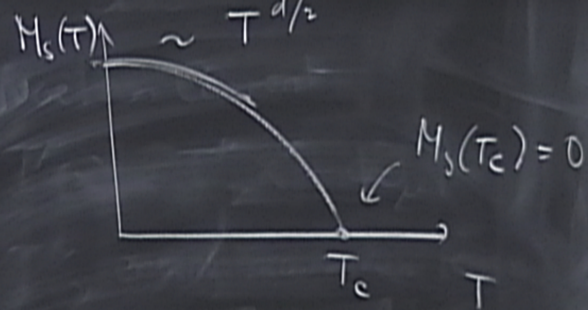
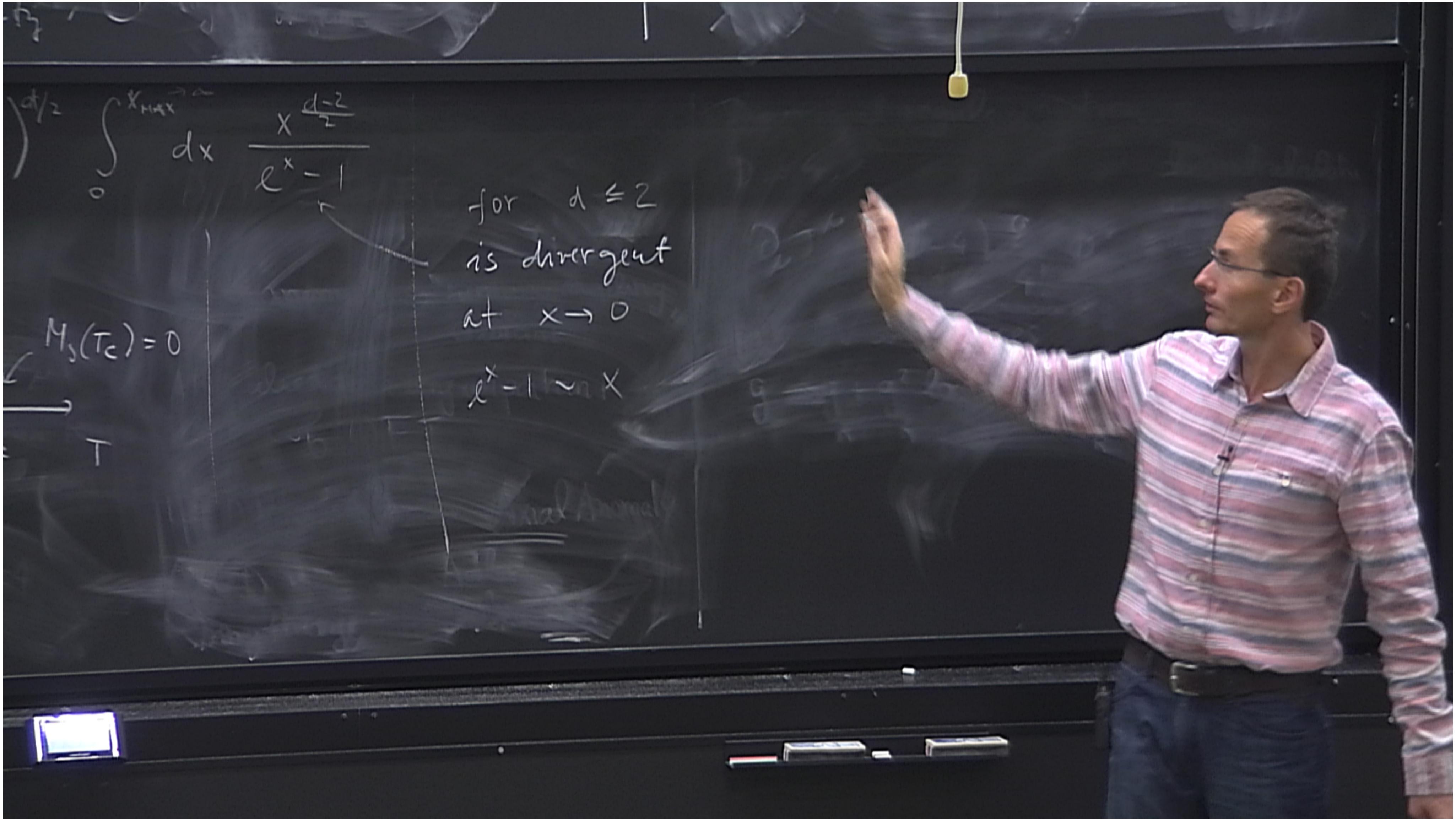
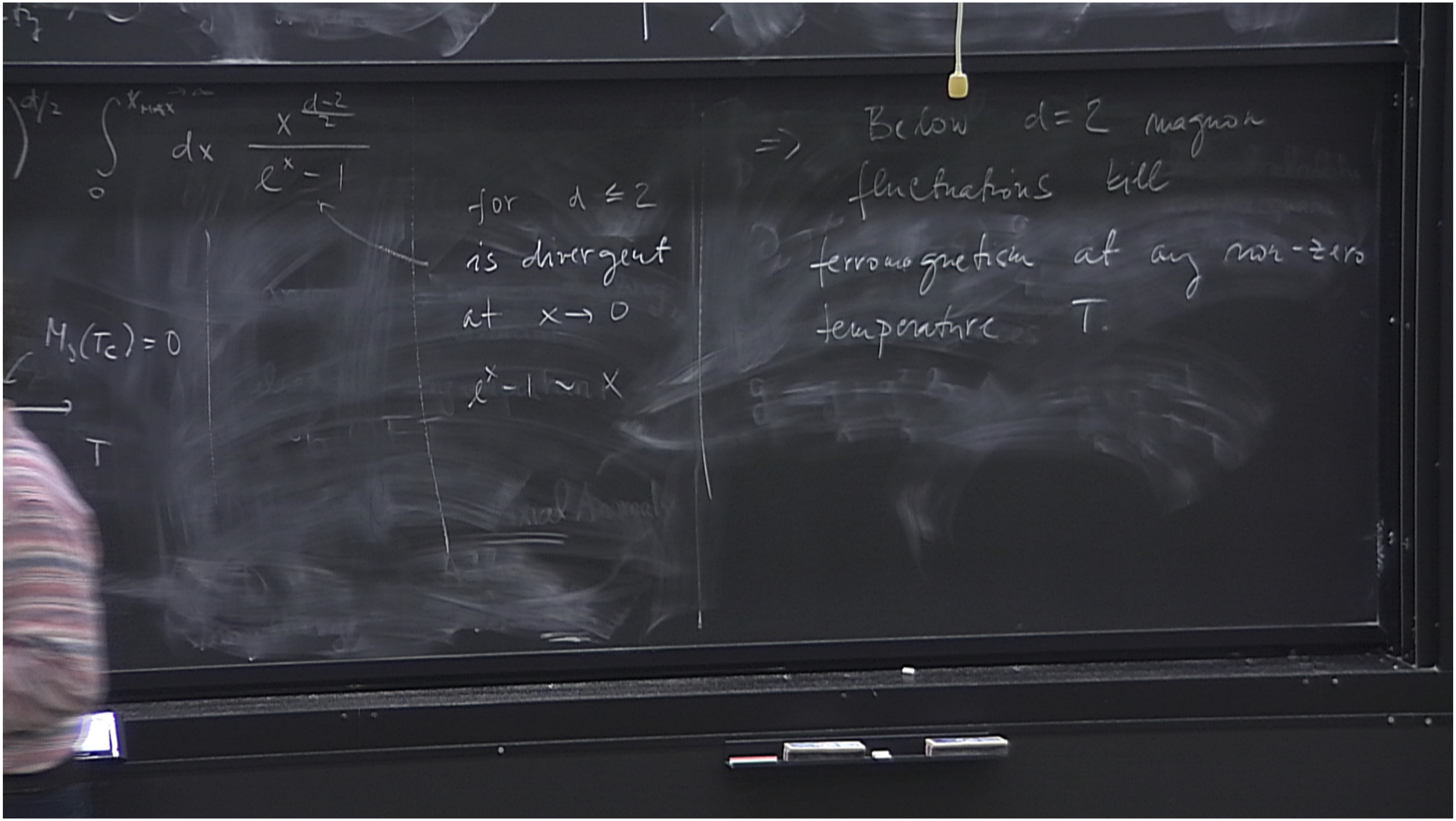
$dk \, k^{d-1} \frac{1}{e^{\beta D k^2} - 1} =$

$\beta D k^2$

$\sim T^{d/2}$




$\int_0^{x_{max}} dx \frac{x^{\frac{d-2}{2}}}{e^x - 1}$
for $d \leq 2$
is divergent
at $x \to 0$
$e^x - 1 \sim x$
$M_s(T_c) = 0$
T


$)^{d/2} \int_0^{x_{max}\to\infty} dx \, \frac{x^{\frac{d-2}{2}}}{e^x - 1}$
for $d \le 2$
is divergent
at $x \to 0$
$e^x - 1 \sim x$
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T
$\Rightarrow$ Below $d=2$ magnon
fluctuations kill
ferromagnetism at any non-zero
temperature T.

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for $d \le 2$
is divergent
at $x \to 0$

$e^x - 1 \sim x$

$M_s(T_c) = 0$

T

$\Rightarrow$ Below $d = 2$ magnon fluctuations kill ferromagnetism at any non-zero temperature T.

Agrees with Mermin-Wagner theorem.