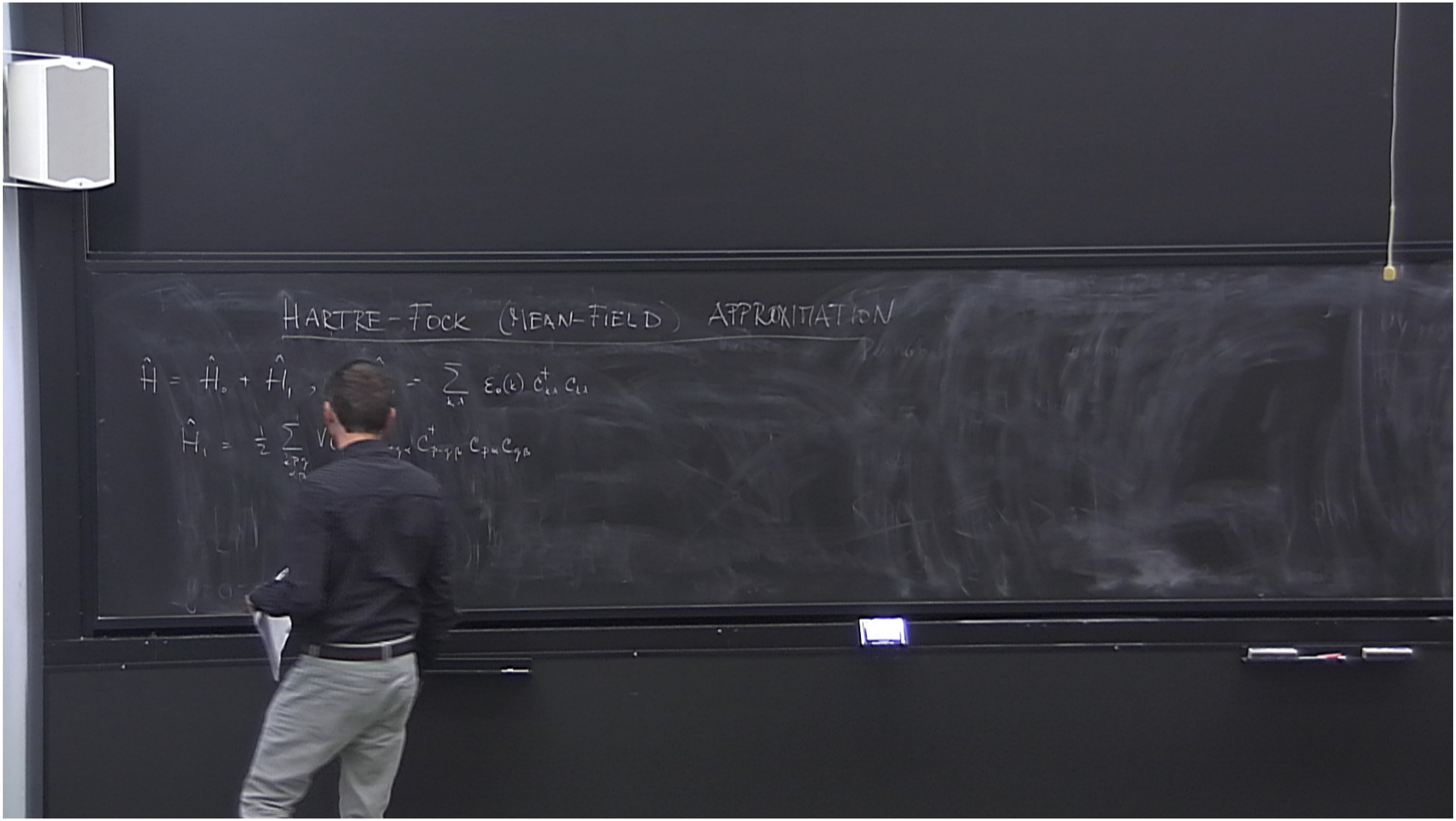
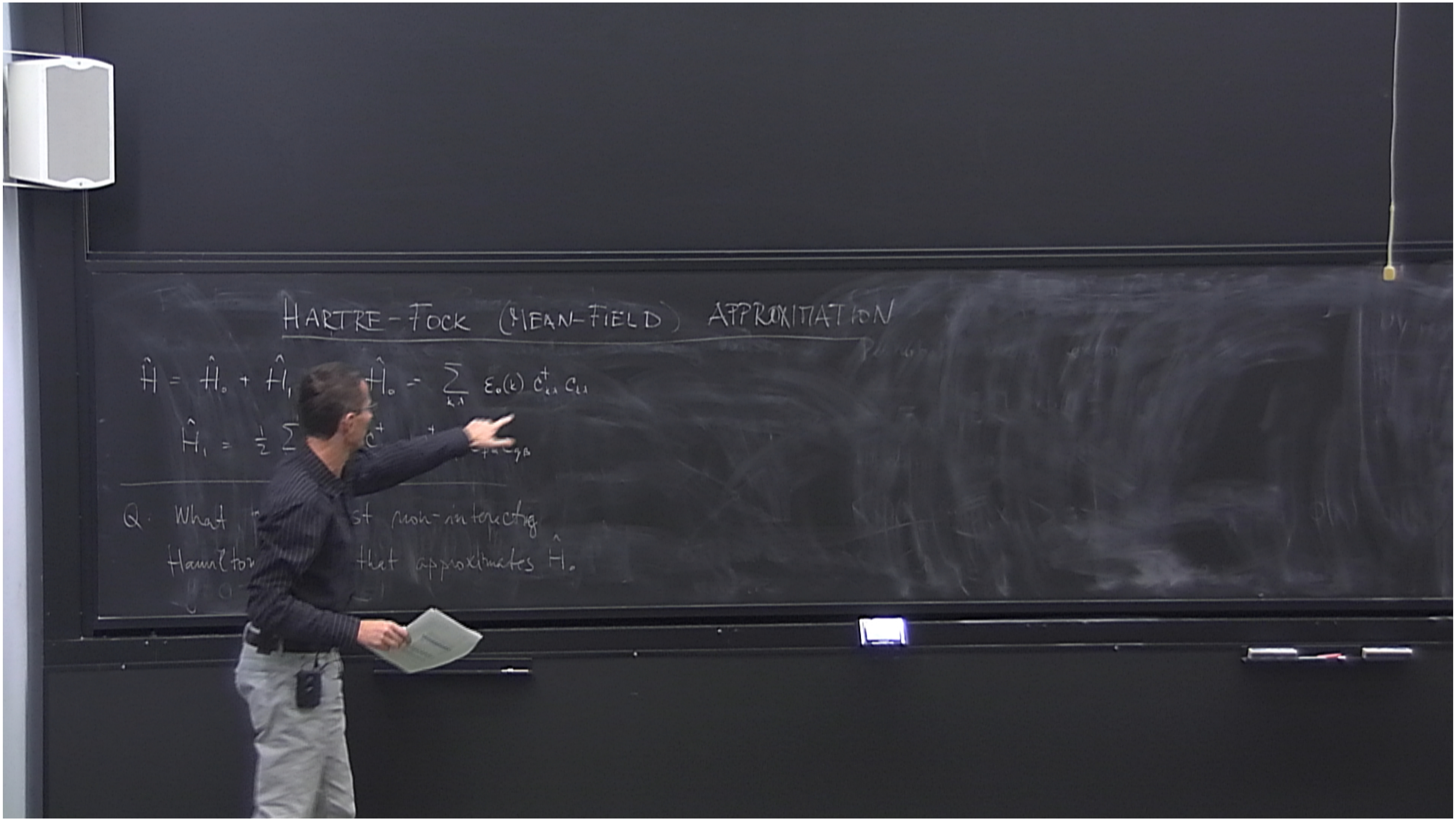
Title: 14/15 PSI - Condensed Matter-Lecture 4

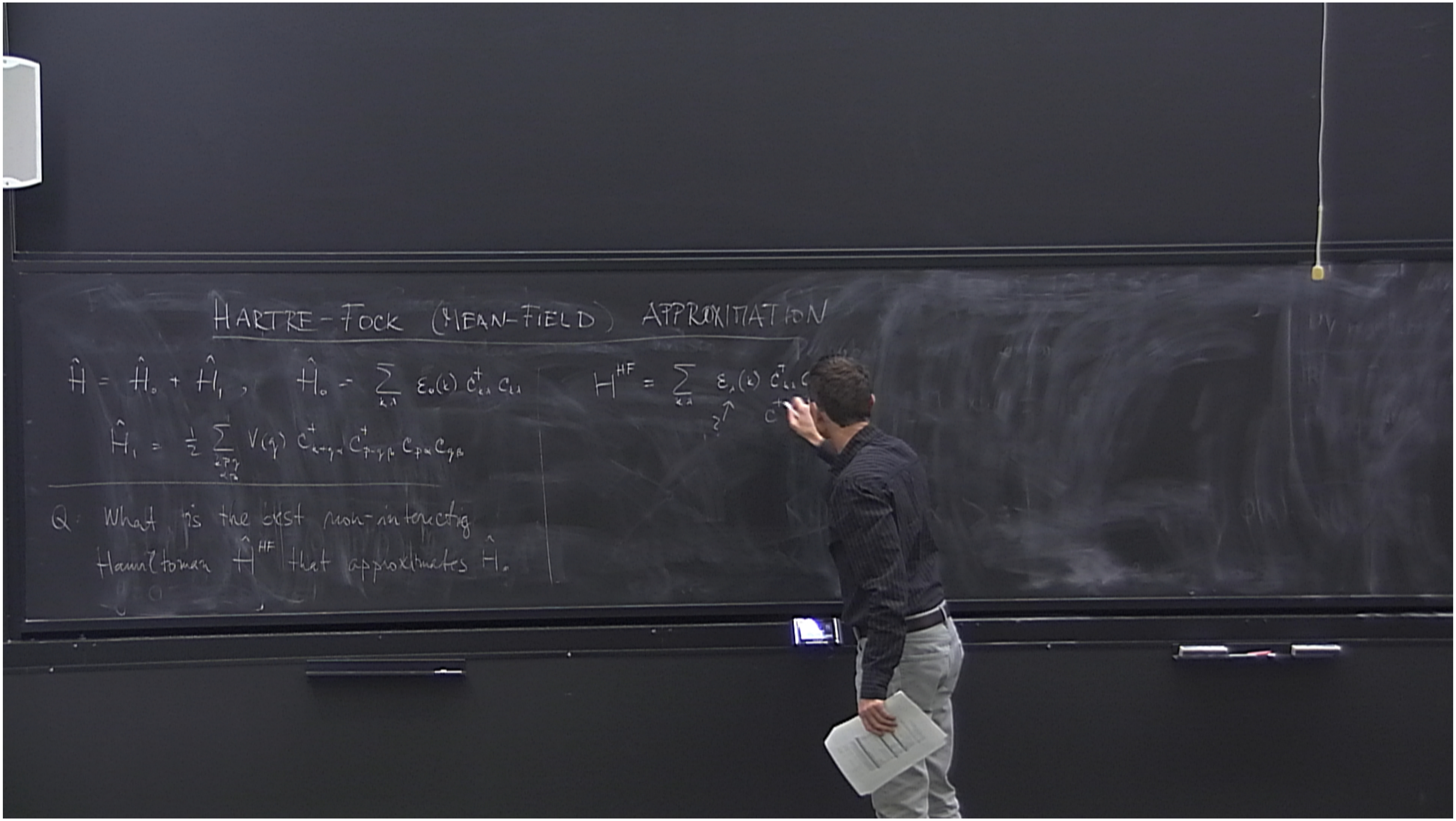
Date: Nov 13, 2014 10:45 AM

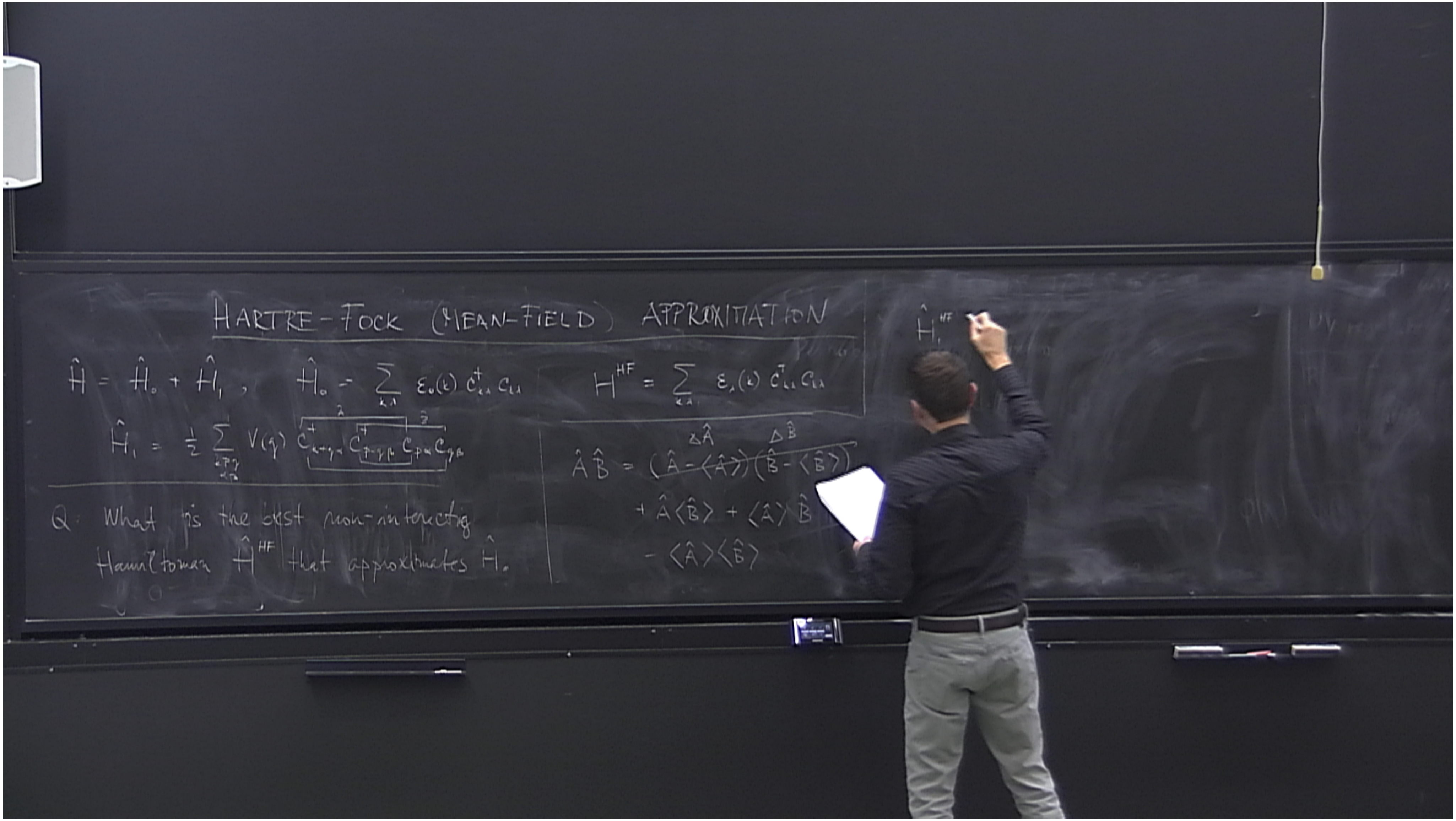
URL: http://pirsa.org/14110028

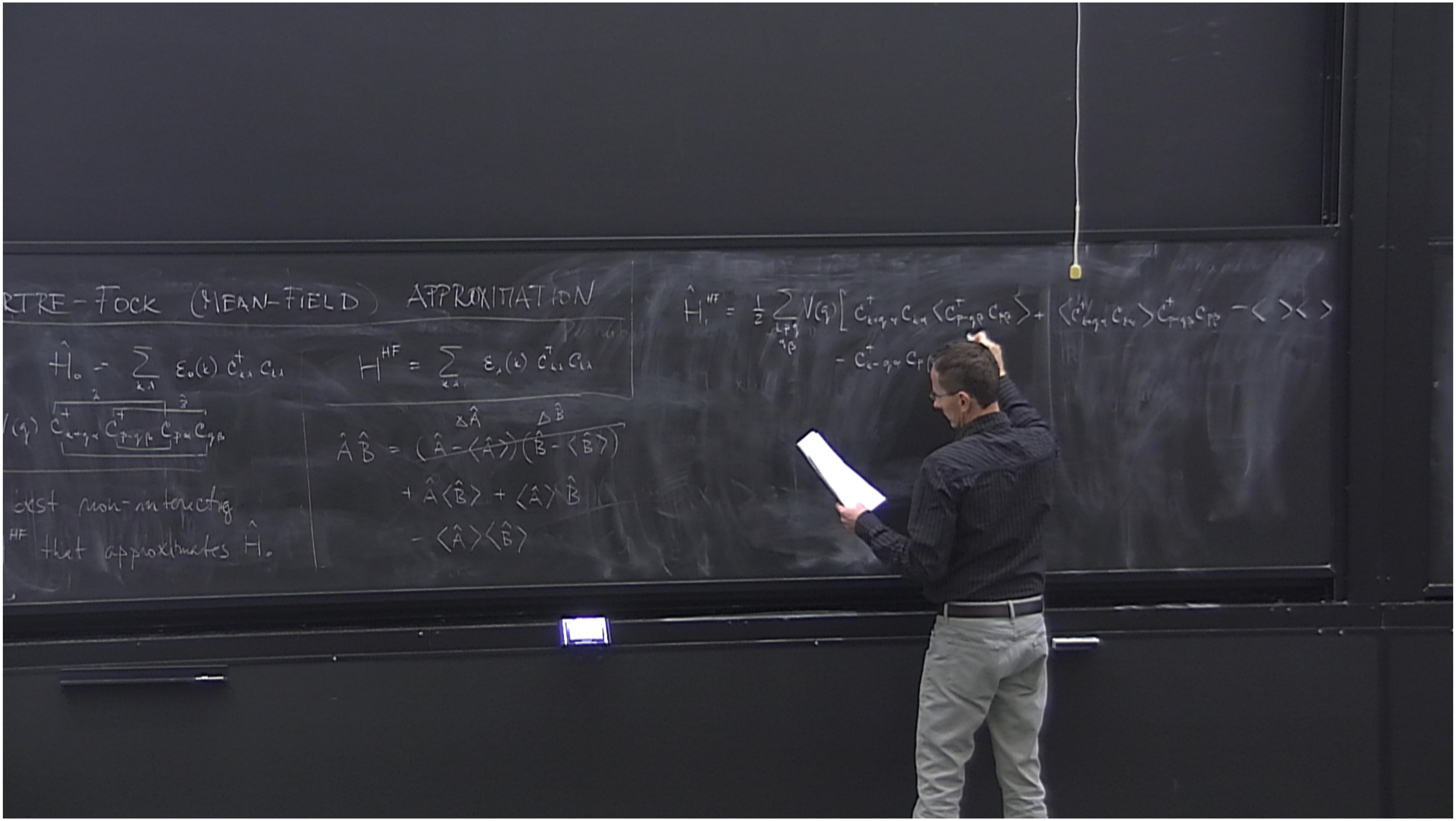
Abstract:

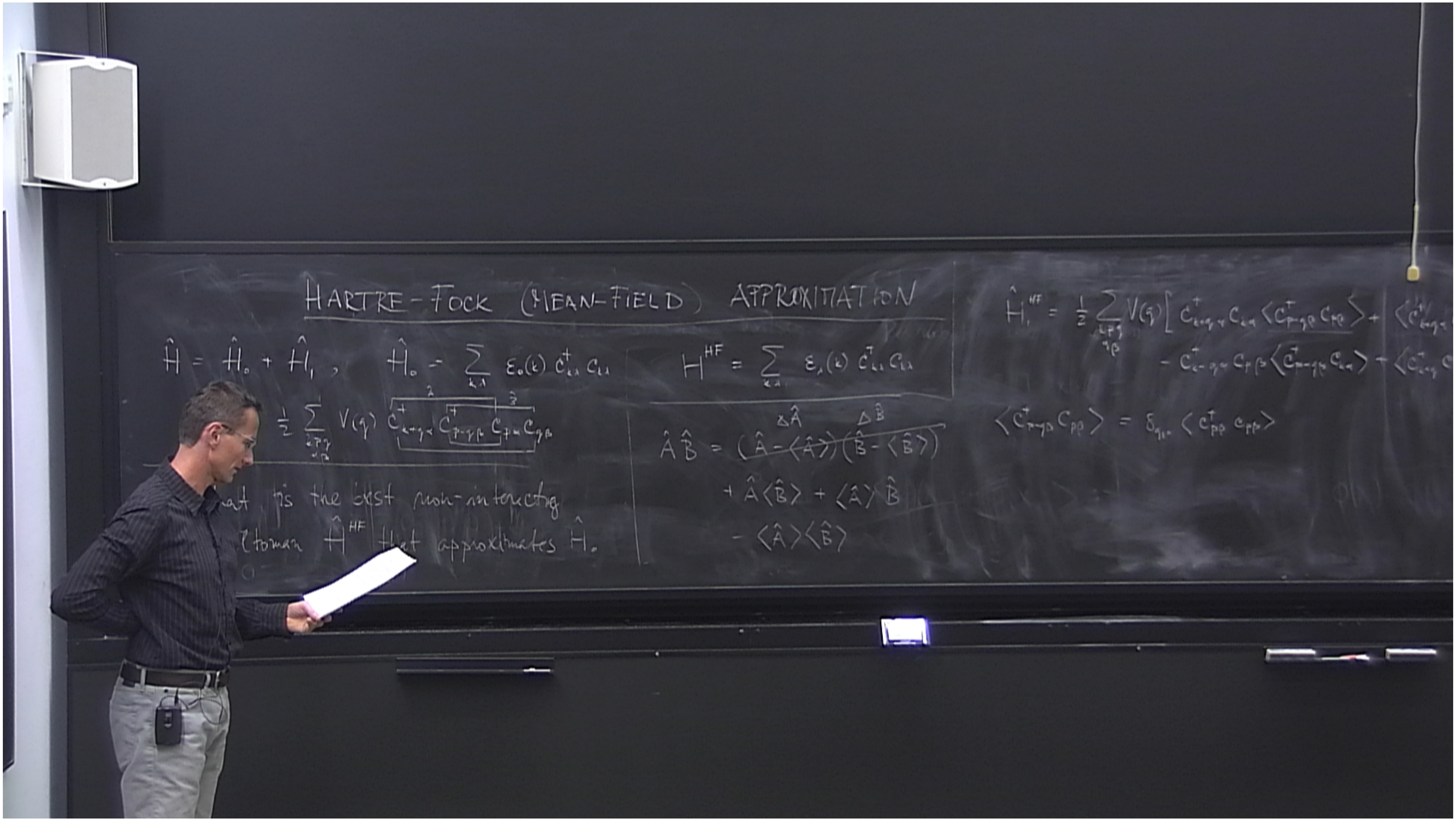

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION
$\hat{H} = \hat{H}_0 + \hat{H}_1$

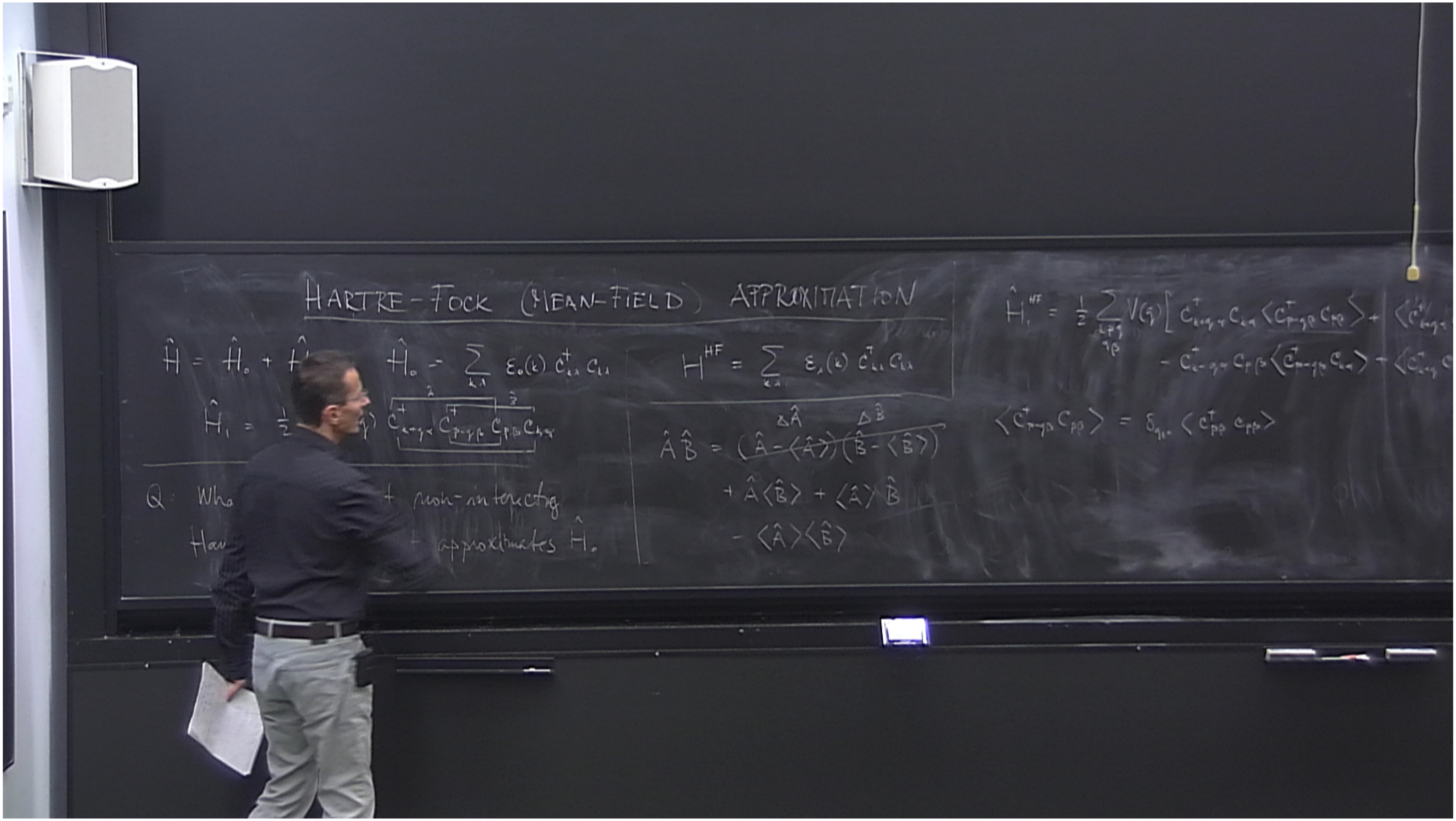

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION

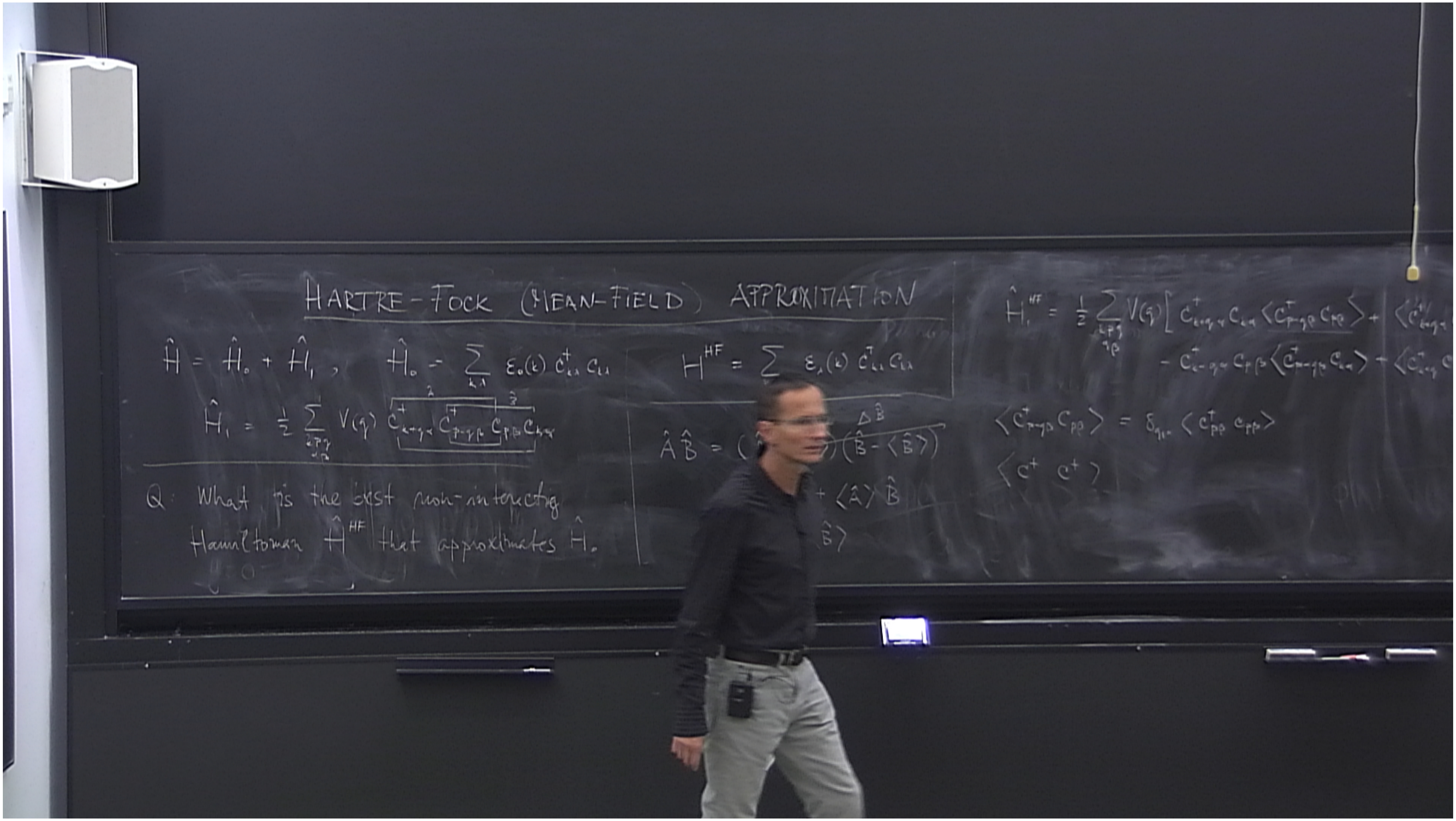

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION
$\hat{H} = \hat{H}_0 + \hat{H}_1$, $\hat{H}_0 = \sum_{k,\lambda} \varepsilon_0(k)\, c^\dagger_{k\lambda} c_{k\lambda}$
$H^{HF} = \sum_{k,\lambda} \varepsilon_\lambda(k)\, c^\dagger_{k\lambda}$
$\hat{H}_1 = \frac{1}{2} \sum V(q)\, c^\dagger_{k+q\,\alpha} c^\dagger_{p-q\,\beta} c_{p\,\alpha} c_{q\,\beta}$
Q: What is the best non-interacting
Hamiltonian $\hat{H}^{HF}$ that approximates $\hat{H}$.

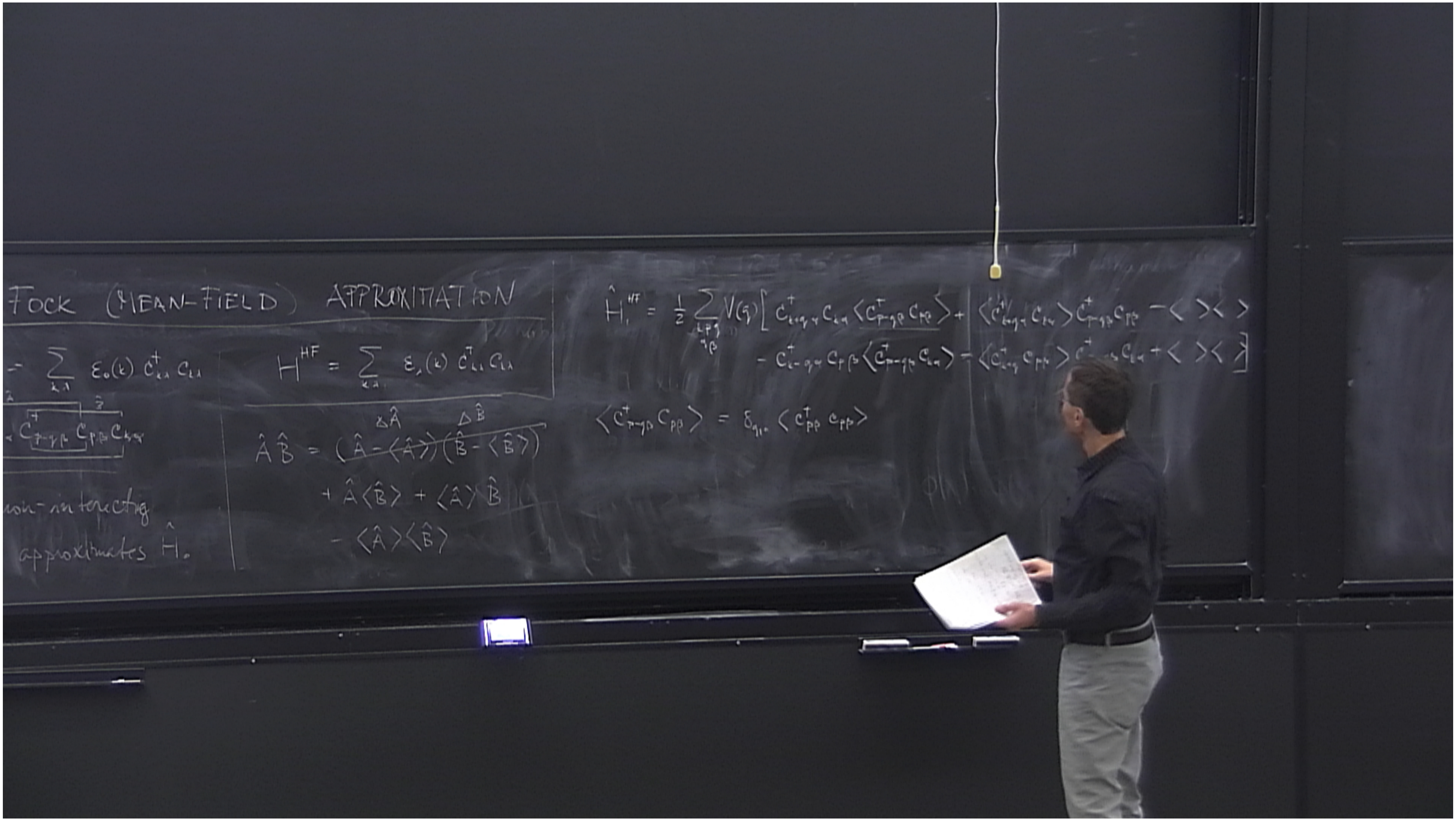

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION
Ĥ = Ĥ₀ + Ĥ₁ , Ĥ₀ = Σ ε₀(k) c†kλ ckλ
Ĥ₁ = ½ Σ V(q) c†k+q,α c†p-q,β cpα cqβ
Q: What is the best non-interacting Hamiltonian Ĥ^HF that approximates Ĥ.
H^HF = Σ ελ(k) c†kλ ckλ
ÂB̂ = (Â − ⟨Â⟩)(B̂ − ⟨B̂⟩)
+ Â⟨B̂⟩ + ⟨Â⟩B̂
− ⟨Â⟩⟨B̂⟩
Ĥ₁^HF

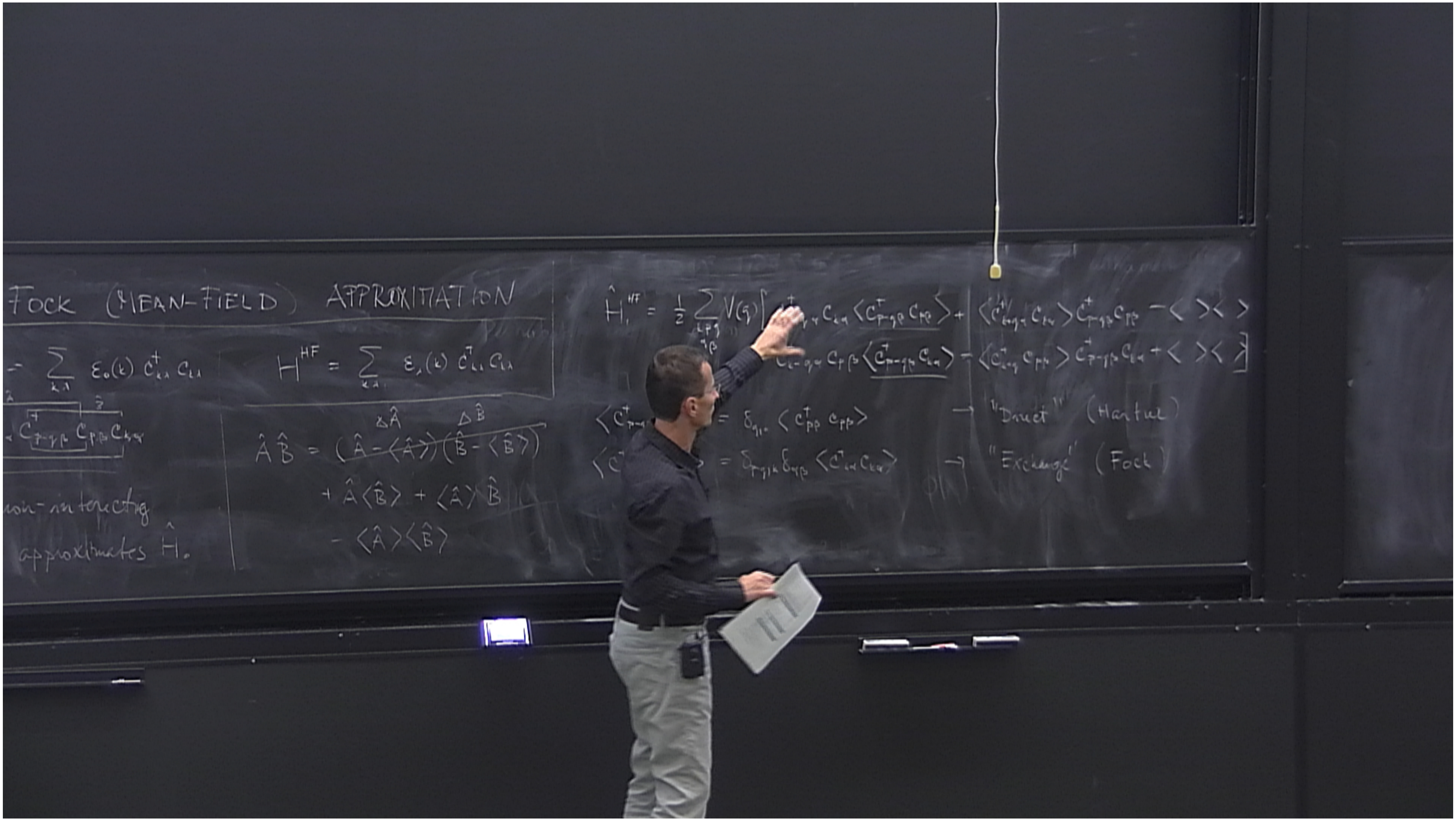

RTRE-FOCK (MEAN-FIELD) APPROXIMATION
best non-interacting
that approximates

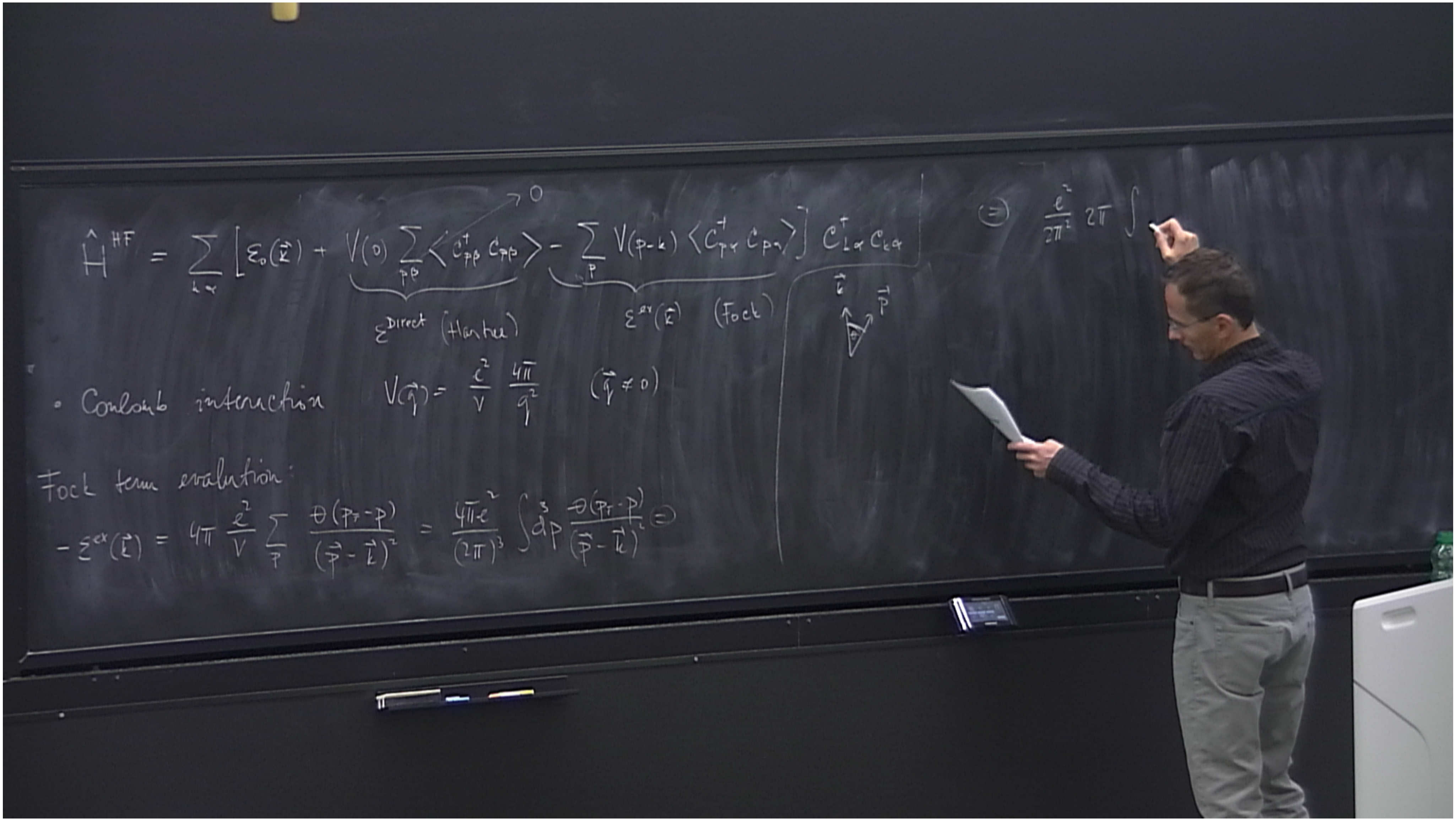

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION

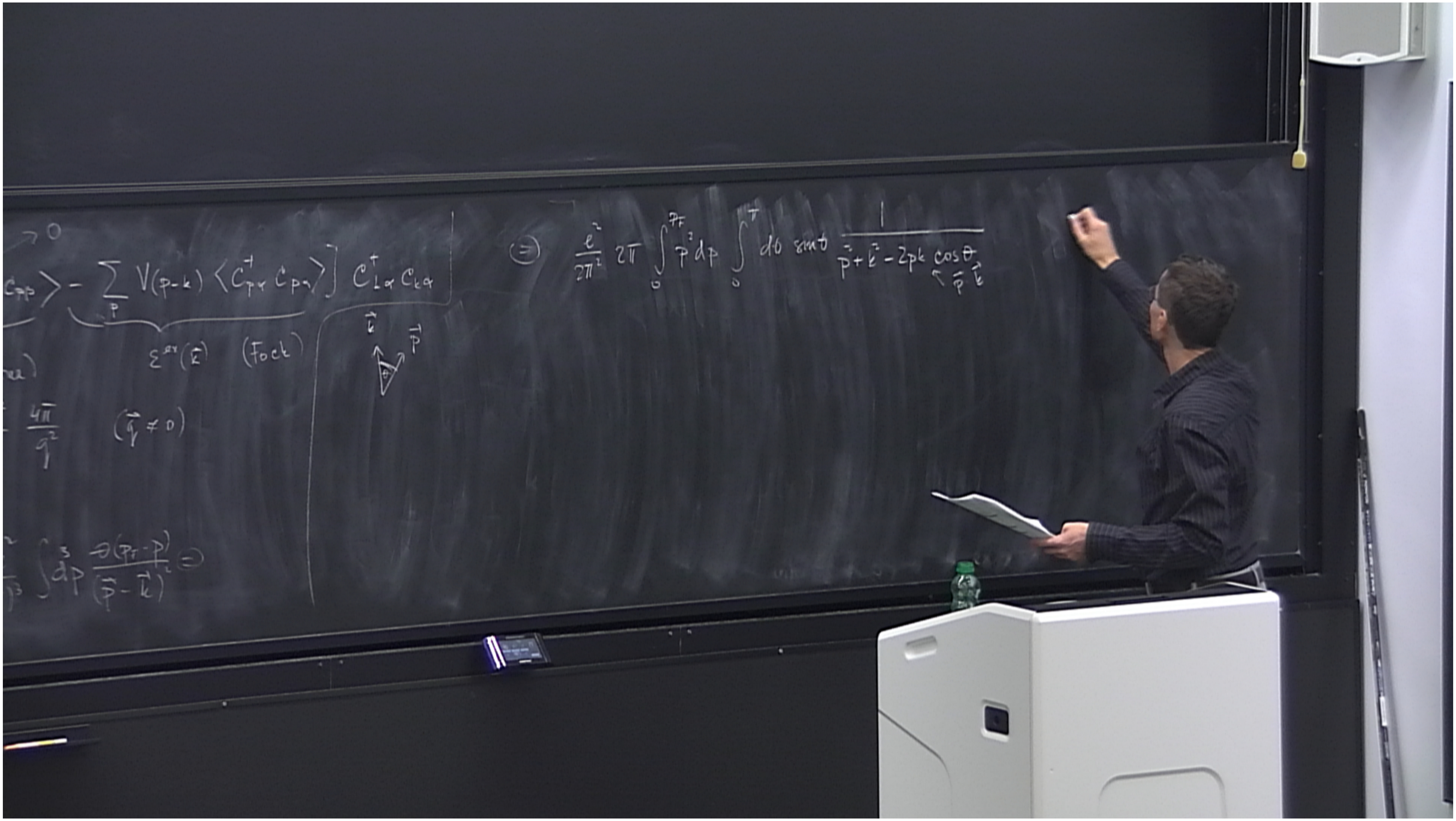

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION

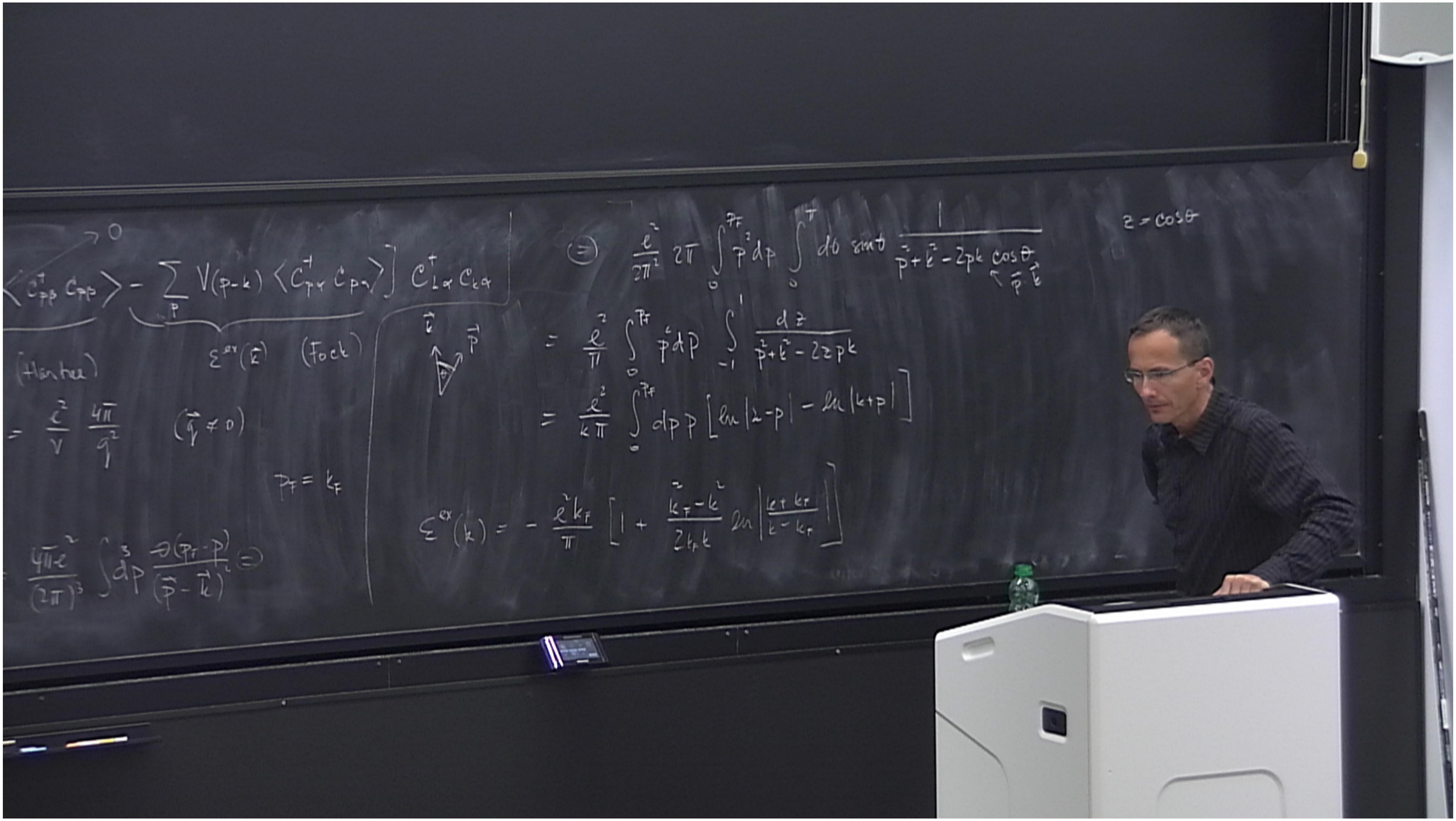

HARTRE-FOCK (MEAN-FIELD) APPROXIMATION
Q: What is the best non-interacting
Hamiltonian Ĥ^HF that approximates Ĥ.

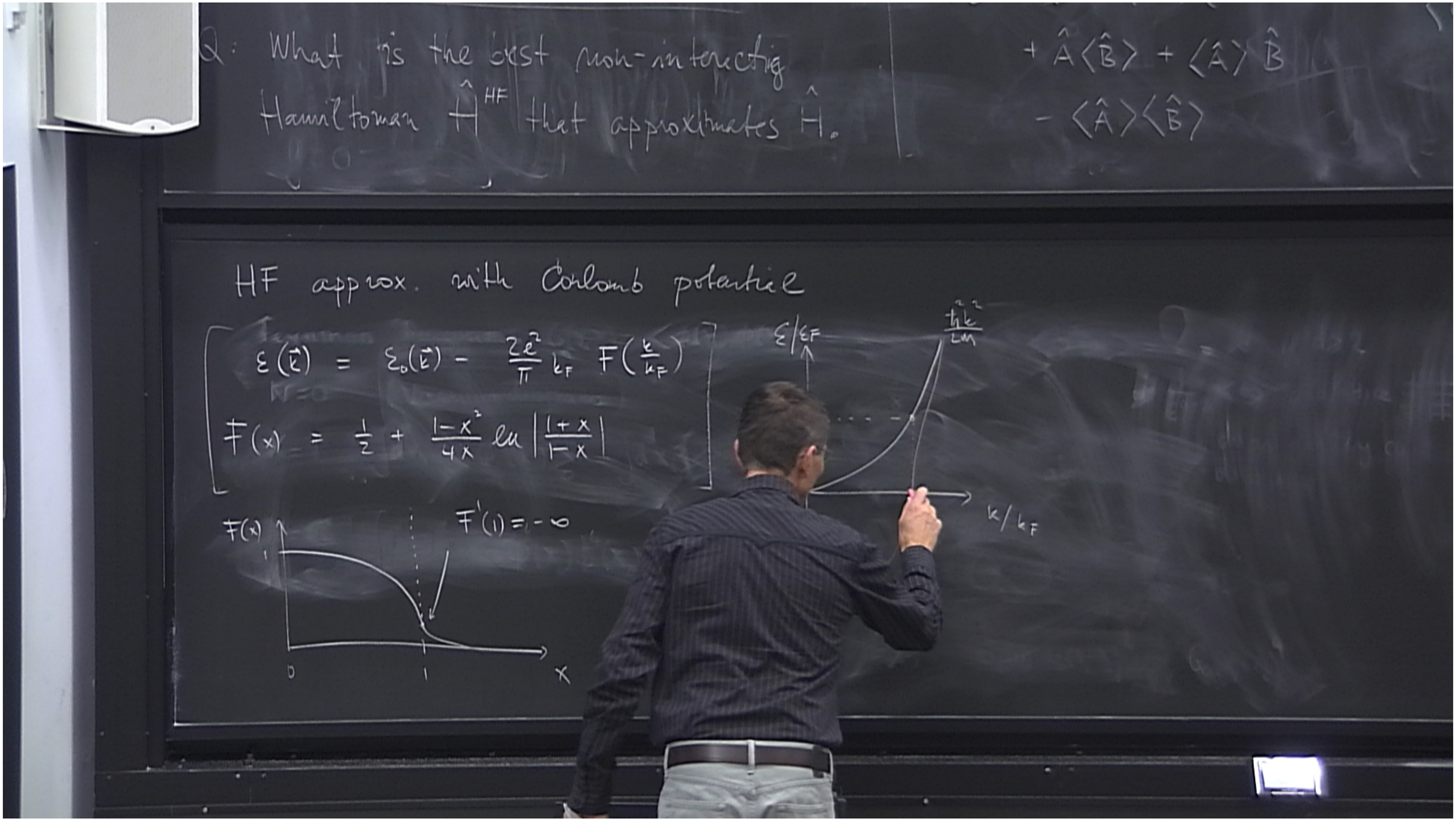

FOCK (MEAN-FIELD) APPROXIMATION
non-interacting
approximates Ĥ.

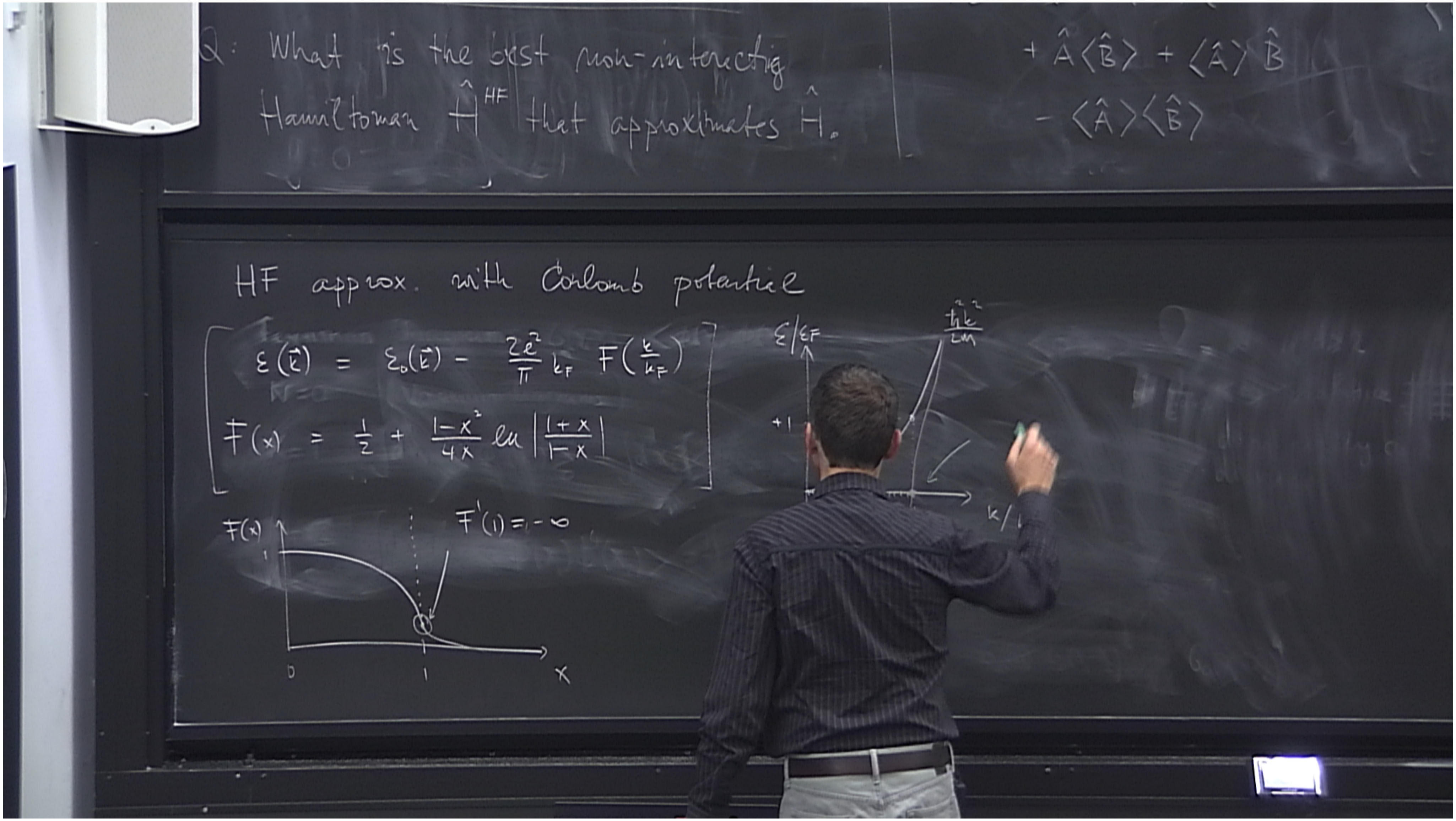

FOCK (MEAN-FIELD) APPROXIMATION
"Direct" (Hartree)
"Exchange" (Fock)
non-interacting
approximates Ĥ.

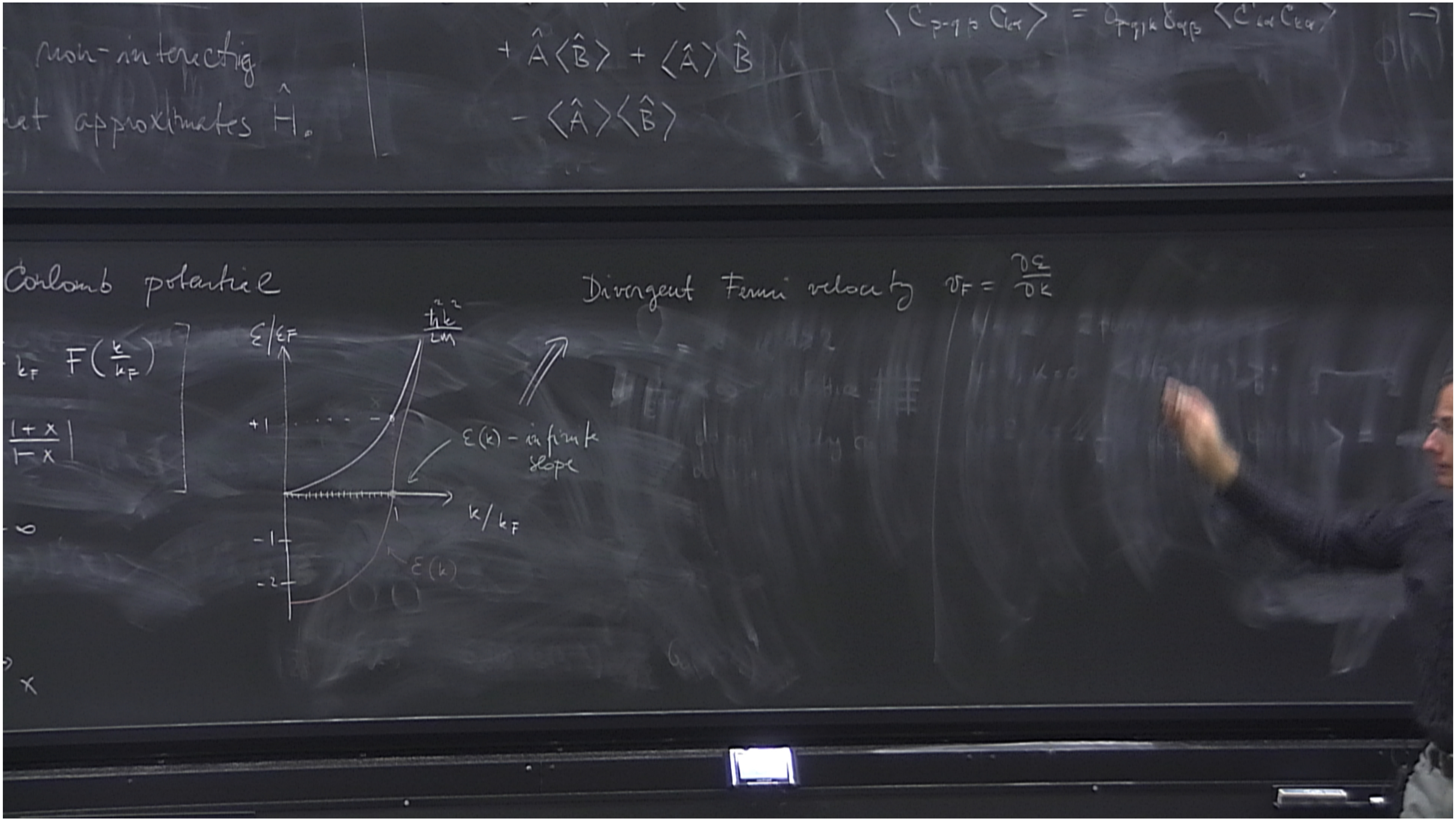

$\hat{H}^{HF} = \sum_{k\alpha} \left[\varepsilon_0(\vec{k}) + V(0) \sum_{p\beta} \langle c^{\dagger}_{p\beta} c_{p\beta} \rangle - \sum_{p} V(p-k) \langle c^{\dagger}_{p\alpha} c_{p\alpha} \rangle \right] c^{\dagger}_{k\alpha} c_{k\alpha}$
ε^{Direct} (Hartree)
$\varepsilon^{ex}(\vec{k})$ (Fock)
• Coulomb interaction $V(\vec{q}) = \frac{e^2}{V} \frac{4\pi}{q^2}$ $(\vec{q} \neq 0)$
Fock term evaluation:
$-\varepsilon^{ex}(\vec{k}) = 4\pi \frac{e^2}{V} \sum_{p} \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2} = \frac{4\pi e^2}{(2\pi)^3} \int d^3p \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2}$
$= \frac{e^2}{2\pi^2} 2\pi \int$

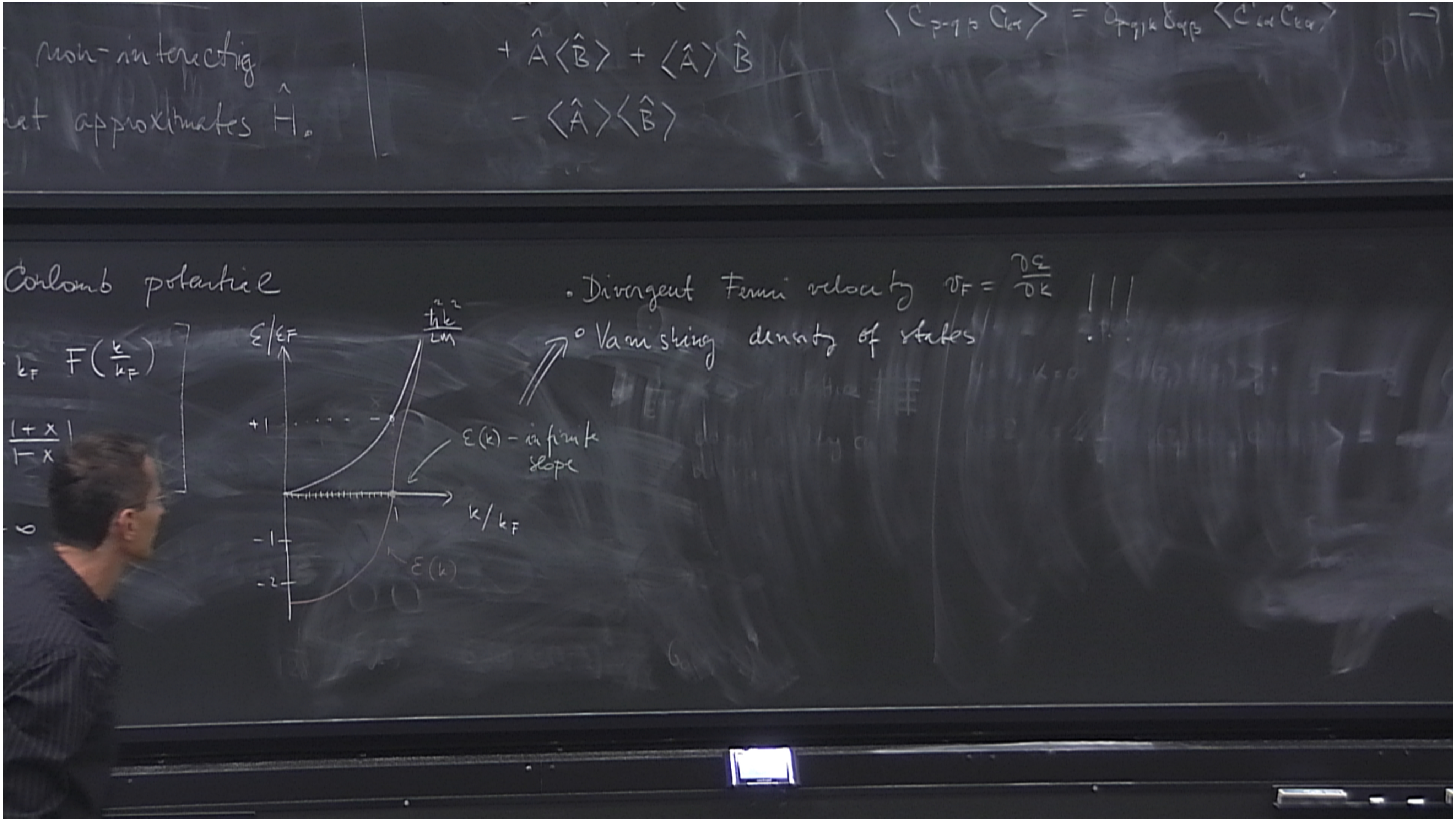

(Fock)

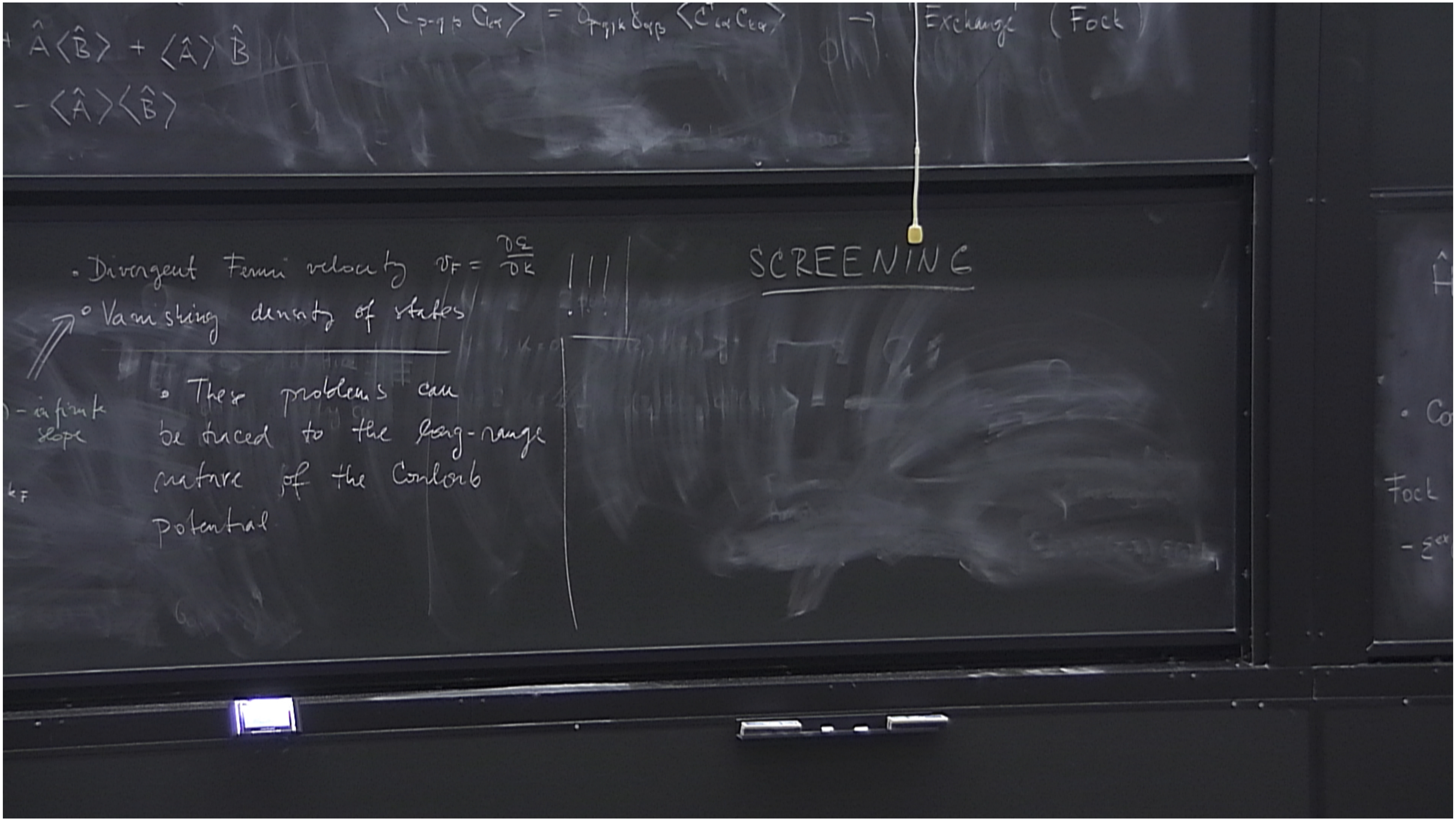

(Hartree)
(Fock)
$(\vec{q} \neq 0)$
$p_F = k_F$
$z = \cos\theta$

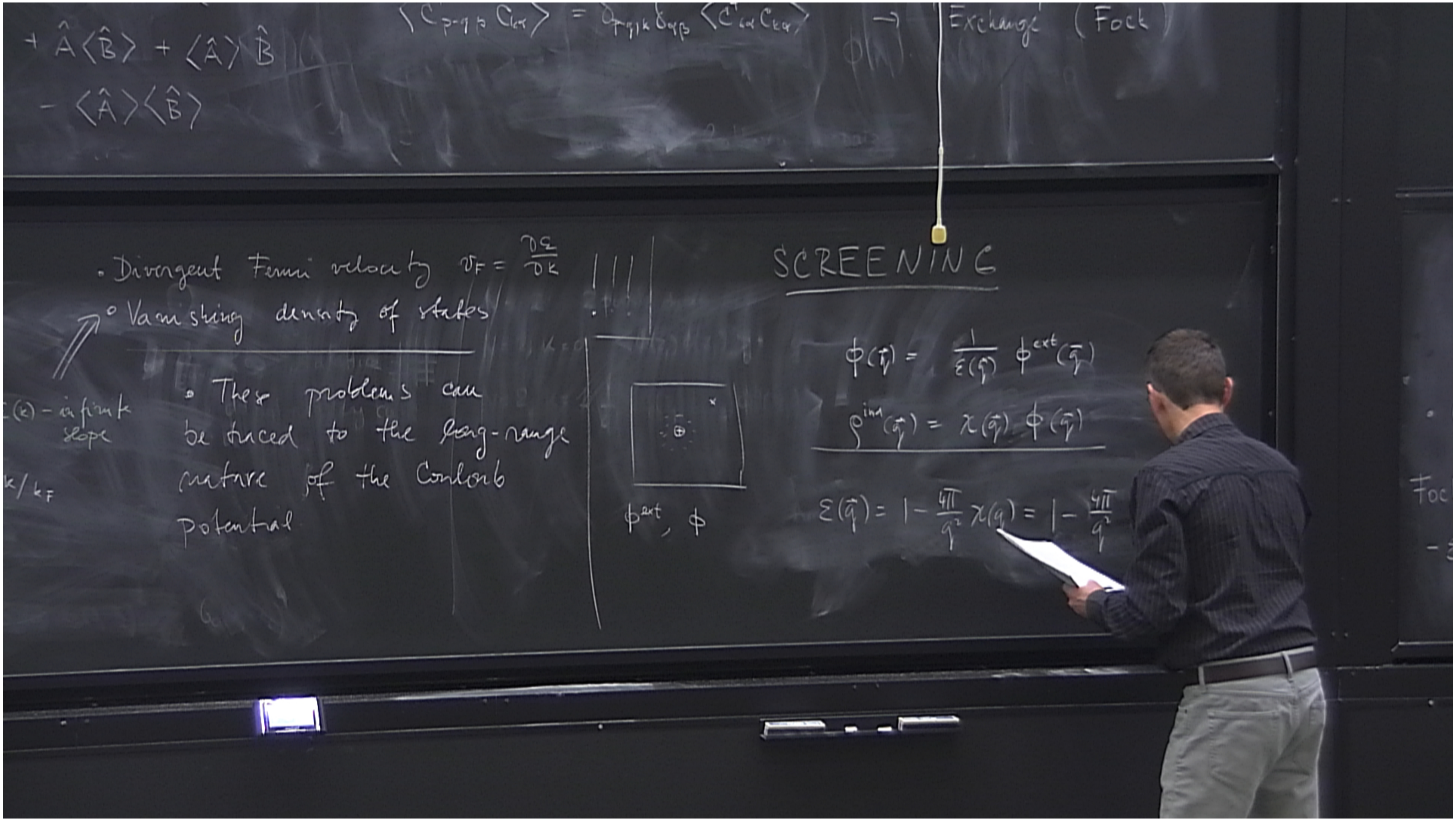

Q: What is the best non-interacting Hamiltonian $\hat{H}^{HF}$ that approximates $\hat{H}$.
$+ \hat{A}\langle\hat{B}\rangle + \langle\hat{A}\rangle\hat{B}$
$- \langle\hat{A}\rangle\langle\hat{B}\rangle$
HF approx. with Coulomb potential
$\varepsilon(\vec{k}) = \varepsilon_0(\vec{k}) - \frac{2e^2}{\pi} k_F F\left(\frac{k}{k_F}\right)$
$F(x) = \frac{1}{2} + \frac{1-x^2}{4x} \ln\left|\frac{1+x}{1-x}\right|$
$F(x)$
$F'(1) = -\infty$
x
$\varepsilon/\varepsilon_F$
$\frac{\hbar^2 k^2}{2m}$
k/k_F

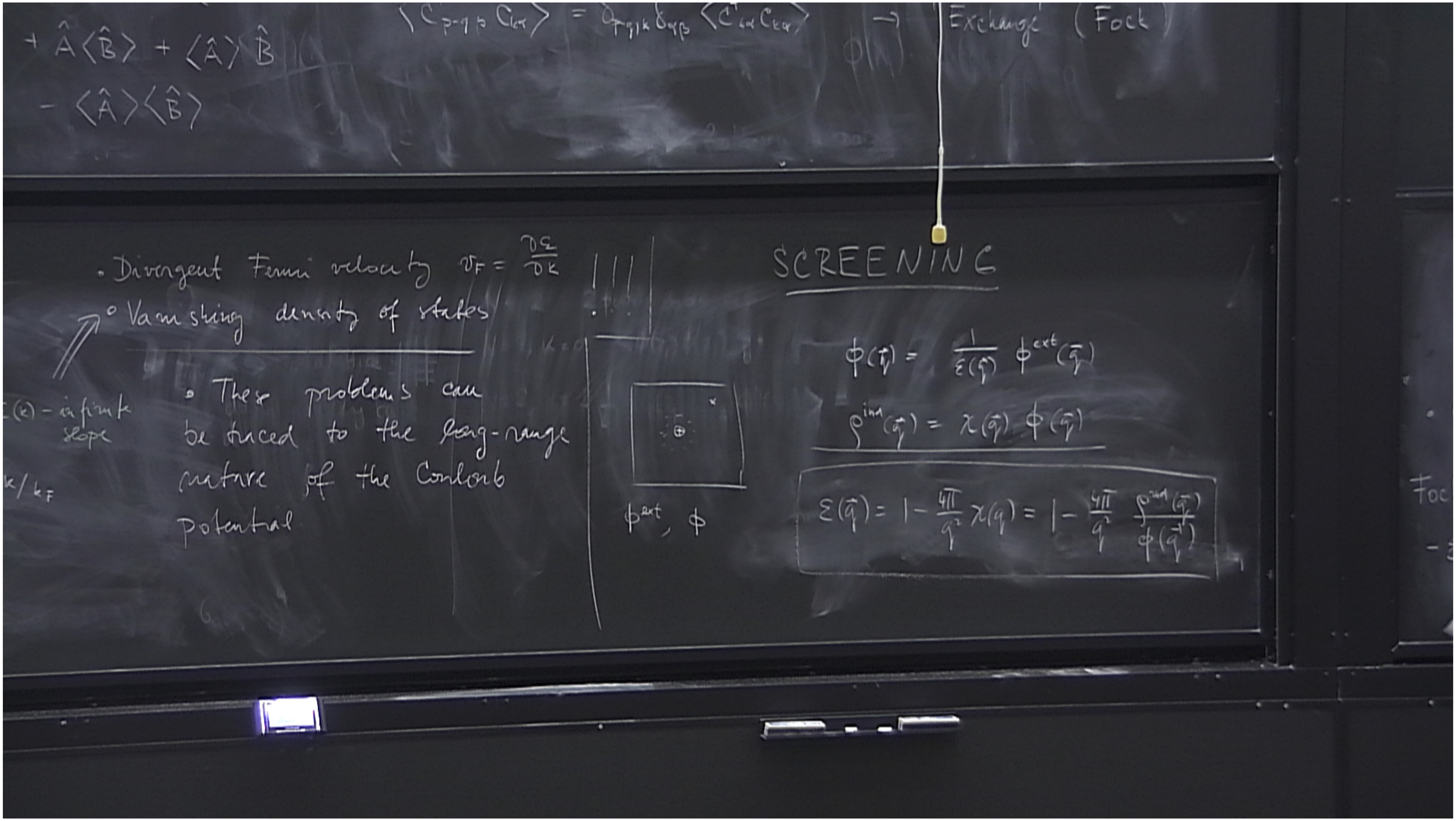

Q: What is the best non-interacting Hamiltonian $\hat{H}^{HF}$ that approximates $\hat{H}$.
$+ \hat{A}\langle\hat{B}\rangle + \langle\hat{A}\rangle\hat{B}$
$- \langle\hat{A}\rangle\langle\hat{B}\rangle$
HF approx. with Coulomb potential
$\varepsilon(\vec{k}) = \varepsilon_0(\vec{k}) - \frac{2e^2}{\pi} k_F \, F\left(\frac{k}{k_F}\right)$
$F(x) = \frac{1}{2} + \frac{1-x^2}{4x} \ln\left|\frac{1+x}{1-x}\right|$
$F(x)$
$F'(1) = -\infty$
x
$\varepsilon/\varepsilon_F$
$\frac{\hbar^2 k^2}{2m}$
+1

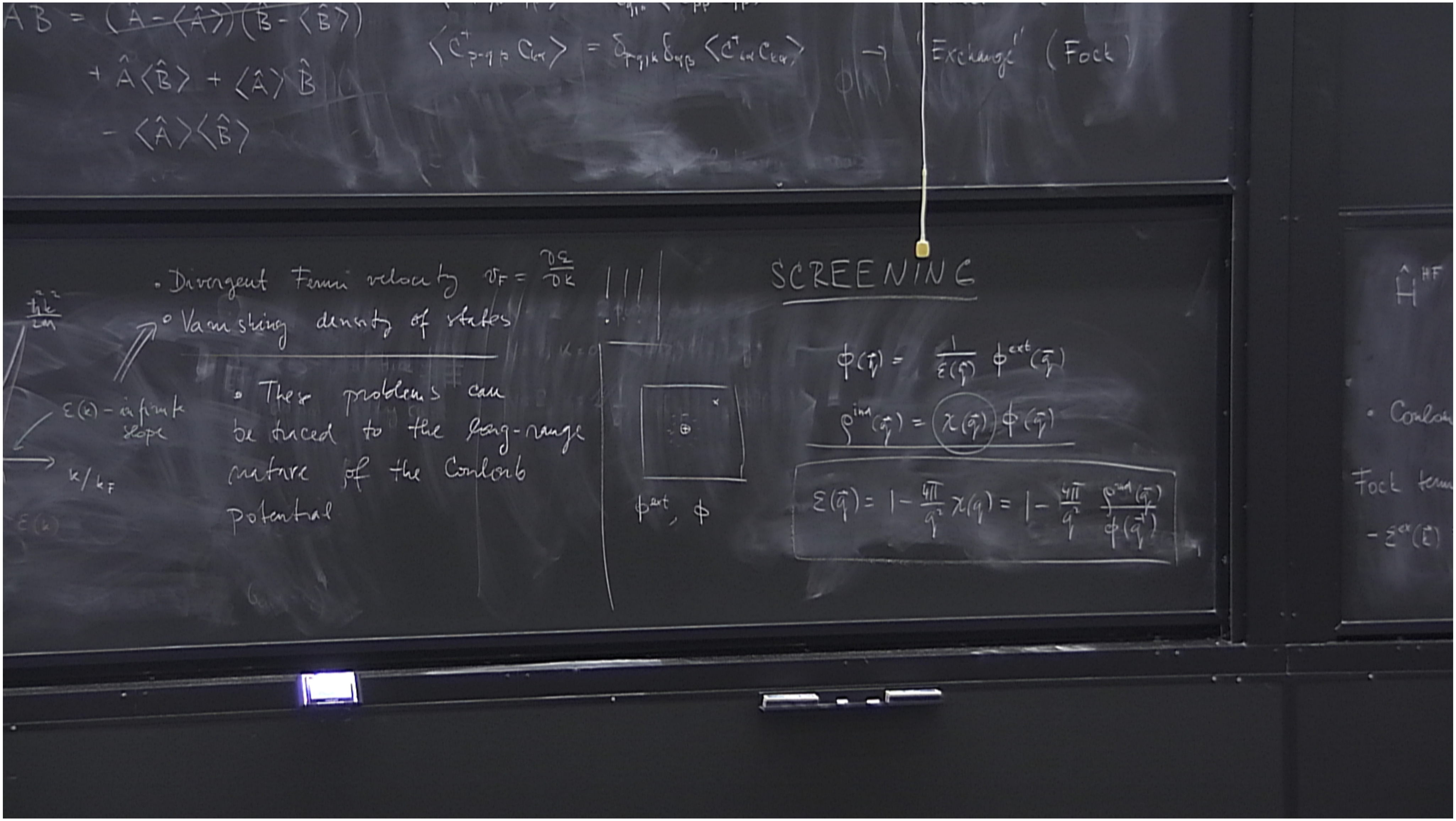

non-interacting
approximates Ĥ.
+ Â⟨B̂⟩ + ⟨Â⟩ B̂
− ⟨Â⟩⟨B̂⟩
Coulomb potential
Divergent Fermi velocity v_F = ∂ε/∂k
ε/ε_F
ħ²k²/2m
ε(k) − infinite slope
k/k_F
ε(k)

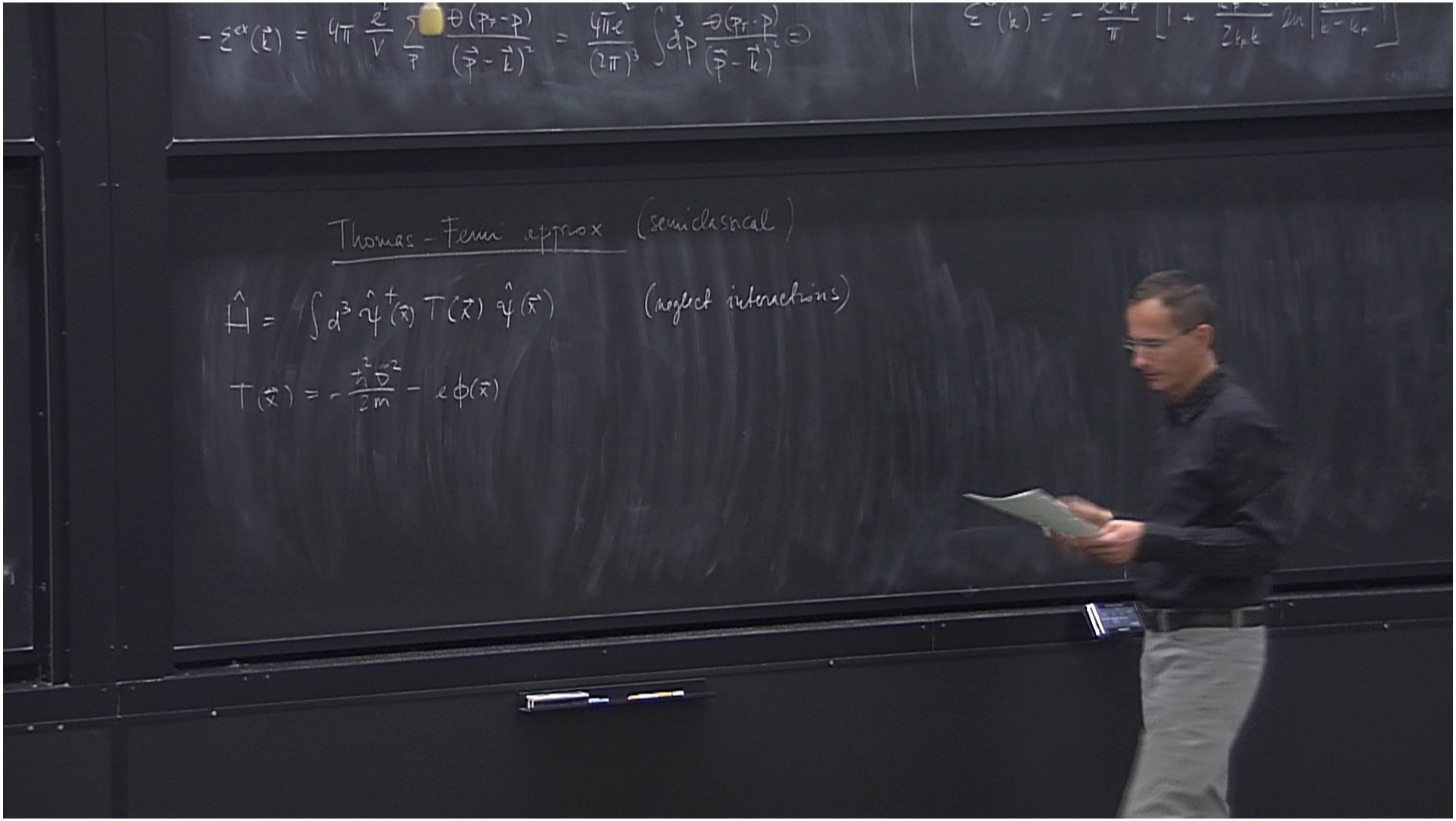

non-interacting
hat approximates $\hat{H}$.
$+ \hat{A}\langle\hat{B}\rangle + \langle\hat{A}\rangle\hat{B}$
$- \langle\hat{A}\rangle\langle\hat{B}\rangle$
Coulomb potential
$k_F \; F\left(\frac{k}{k_F}\right)$
$\left|\frac{1+x}{1-x}\right|$
$\varepsilon/\varepsilon_F$
$\frac{\hbar^2 k^2}{2m}$
+1
-1
-2
k/k_F
$\varepsilon(k)$ - infinite slope
$\varepsilon(k)$
• Divergent Fermi velocity $v_F = \frac{\partial\varepsilon}{\partial k}$!!!
• Vanishing density of states

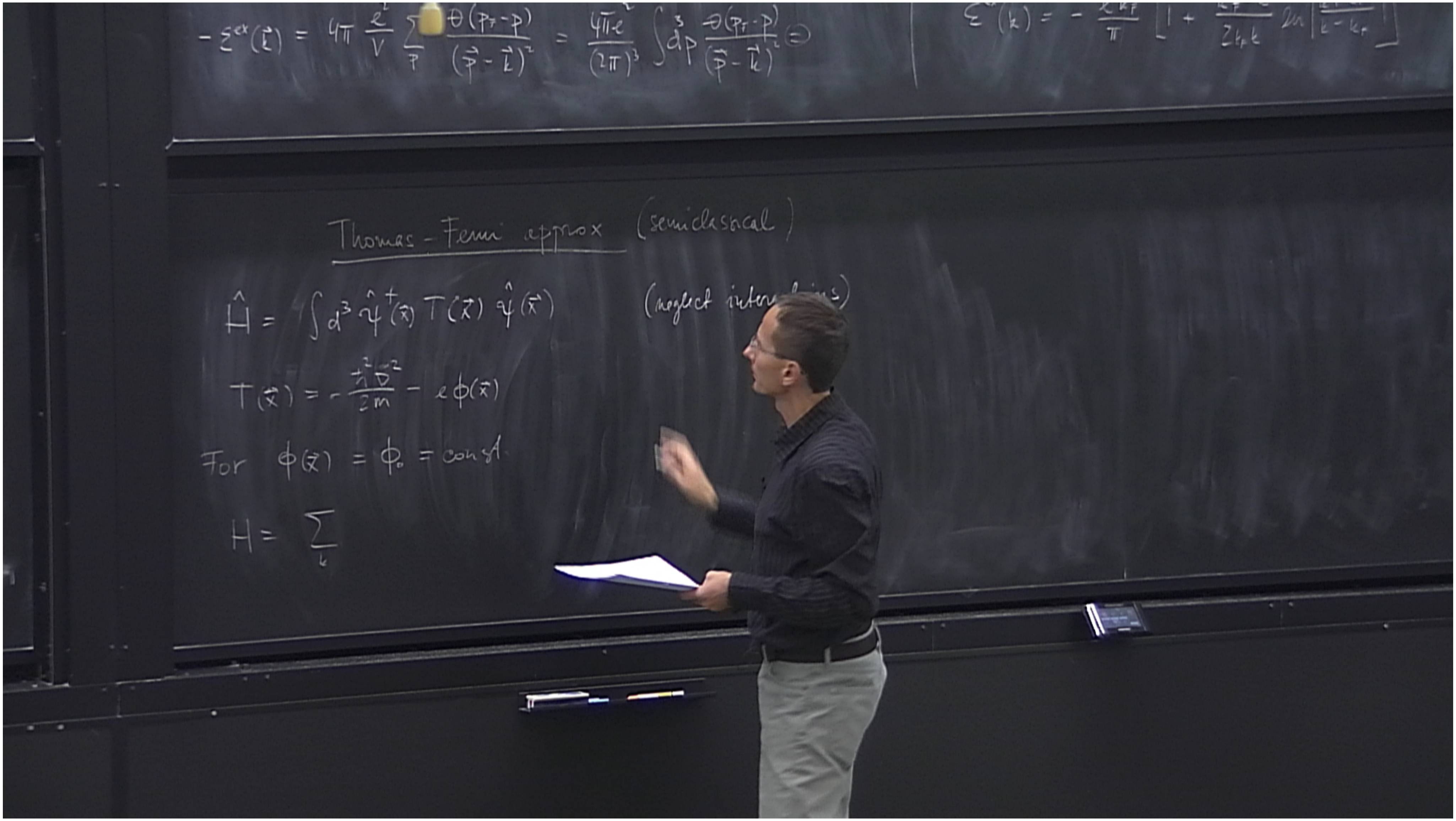

Â⟨B̂⟩ + ⟨Â⟩B̂
- ⟨Â⟩⟨B̂⟩
Exchange (Fock
• Divergent Fermi velocity
!!!
SCREENING
• Vanishing density of states
- infinite slope
• These problems can be traced to the long-range nature of the Coulomb potential
Fock

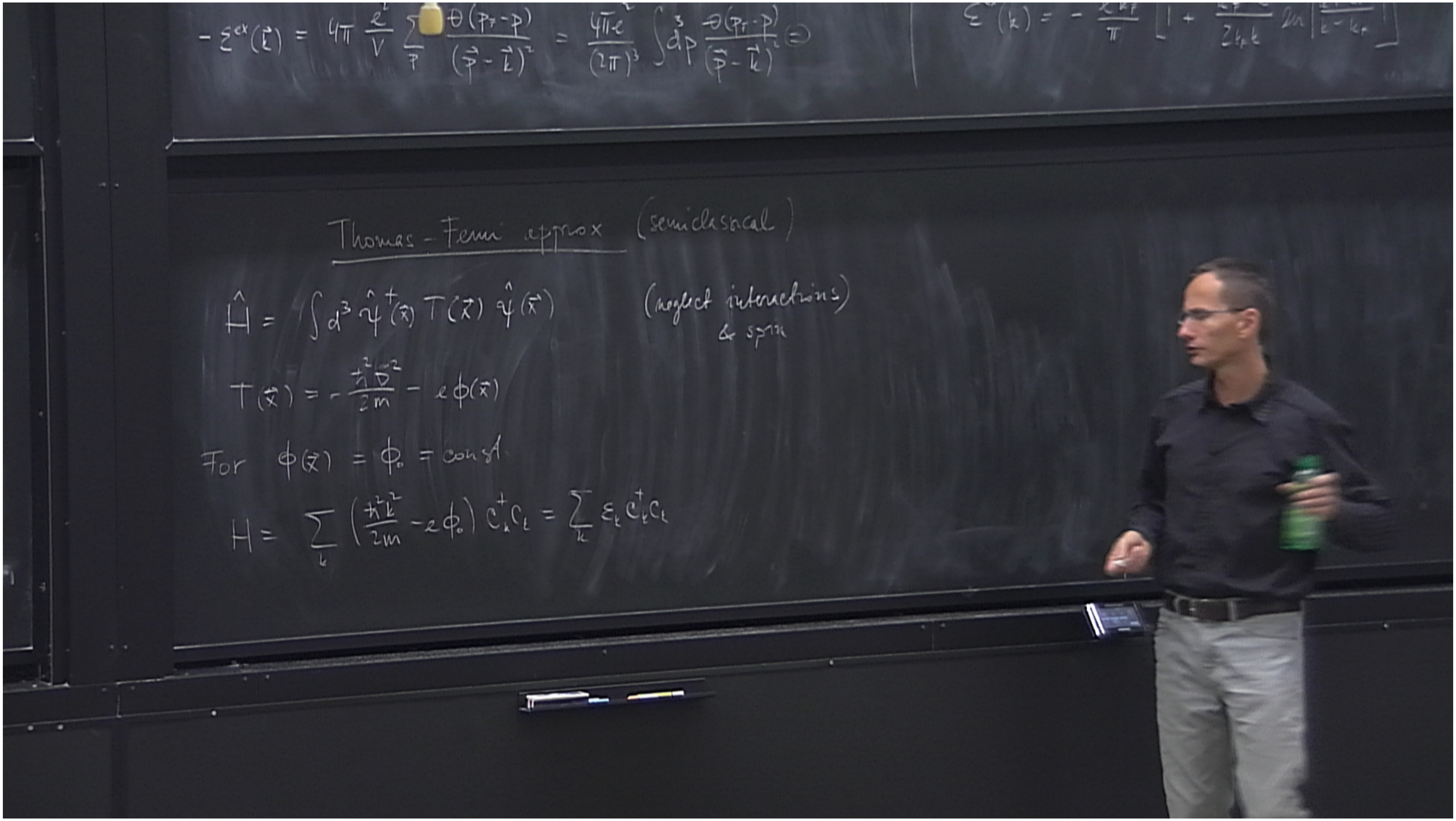

SCREENING
Divergent Fermi velocity
Vanishing density of states
These problems can be traced to the long-range nature of the Coulomb potential
Exchange (Fock)


SCREENING
• Divergent Fermi velocity
• Vanishing density of states
• These problems can be traced to the long-range nature of the Coulomb potential
Exchange (Fock)
slope

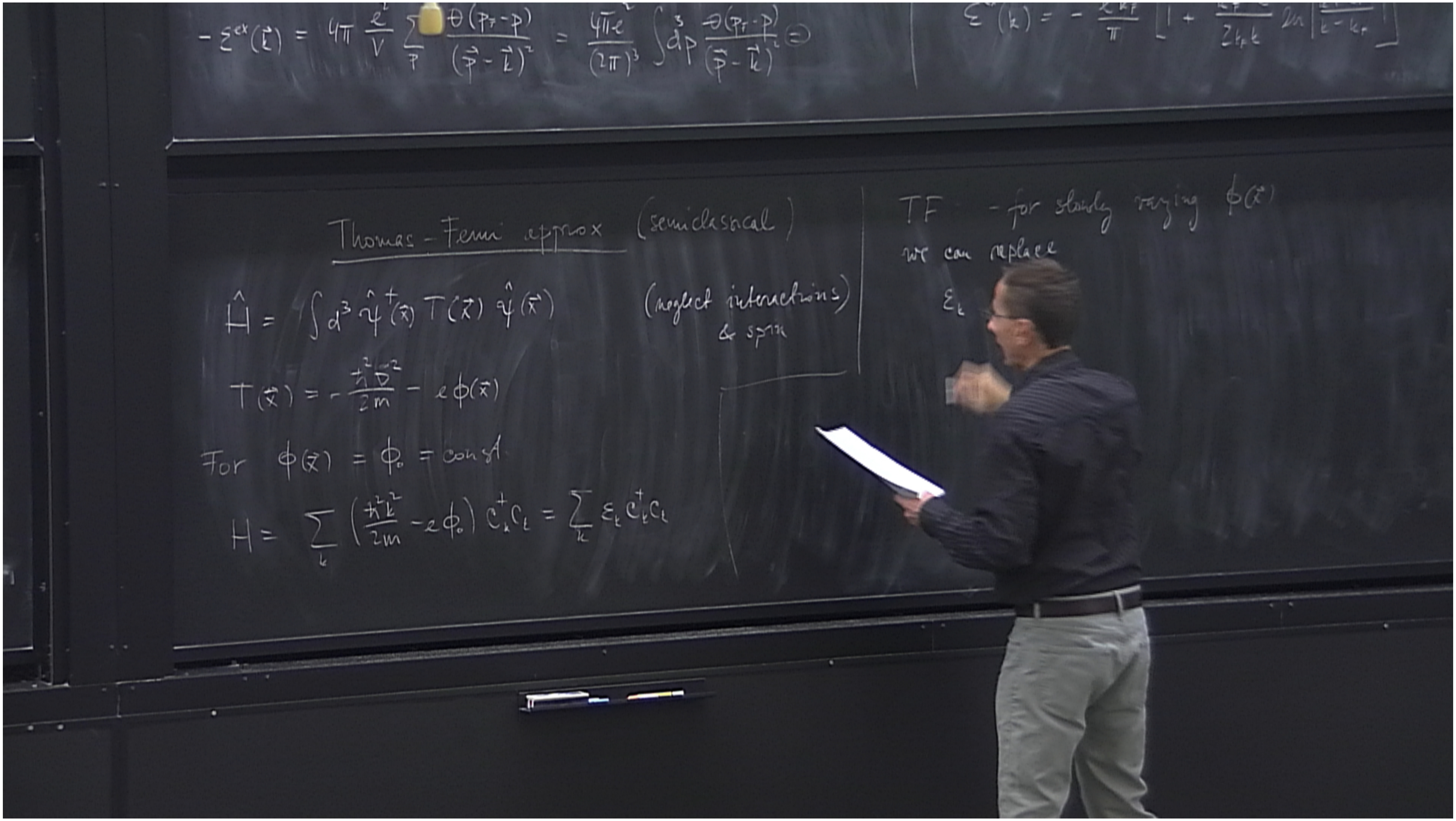

AB = (Â − ⟨Â⟩)(B̂ − ⟨B̂⟩)
+ Â⟨B̂⟩ + ⟨Â⟩B̂
− ⟨Â⟩⟨B̂⟩
'Exchange' (Fock)
· Divergent Fermi velocity v_F = ∂ε/∂k
· Vanishing density of states
· These problems can be traced to the long-range nature of the Coulomb potential
ε(k) − infinite slope
k/k_F
SCREENING
φ(q) = 1/ε(q) φ^ext(q)
ρ^ind(q) = χ(q) φ(q)
ε(q) = 1 − 4π/q² χ(q) = 1 − 4π/q² ρ^ind(q)/φ(q)
φ^ext, φ
Fock term

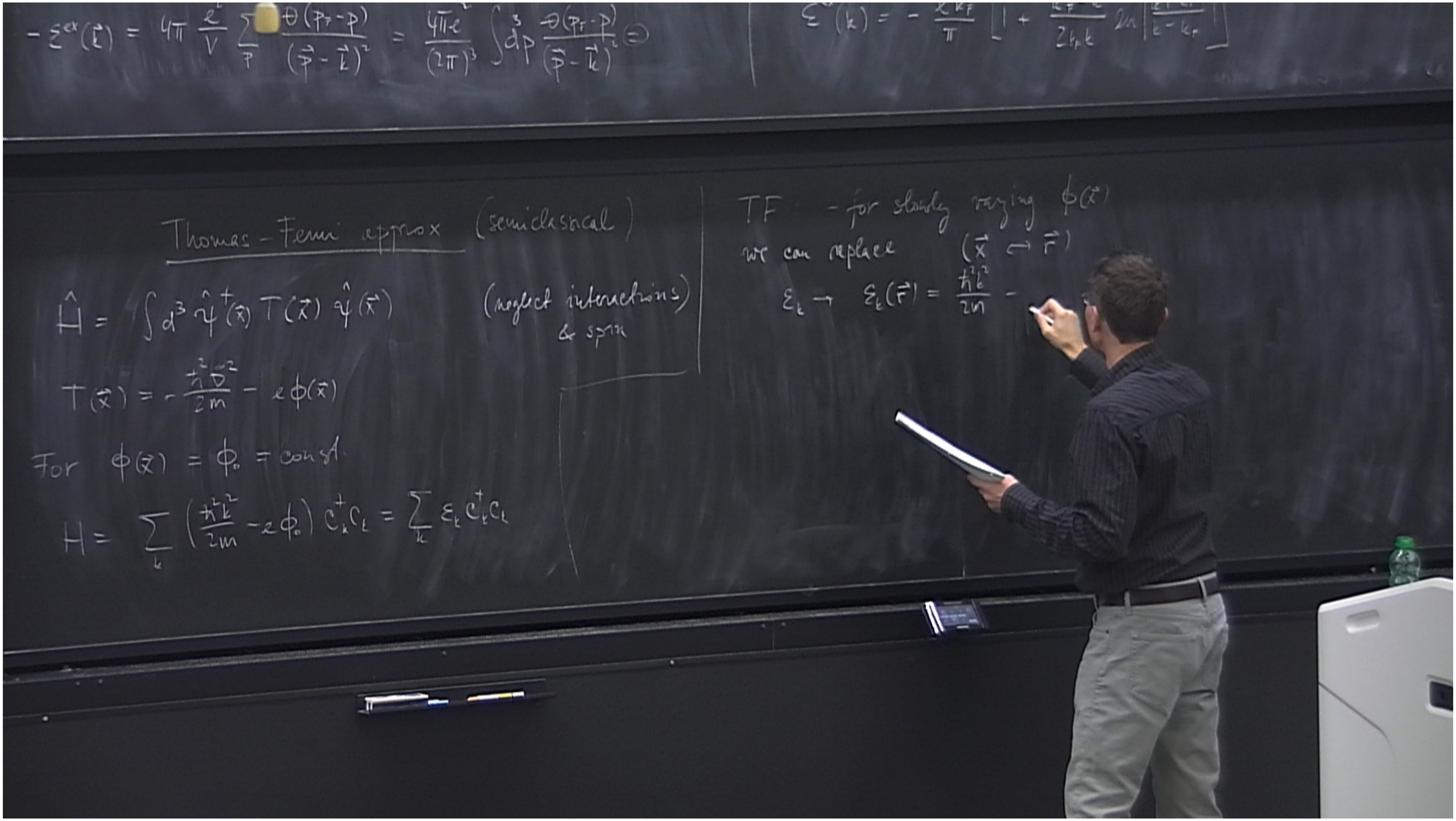

$-\Sigma^{ex}(\vec{k}) = 4\pi \frac{e^2}{V} \sum_{\vec{p}} \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2} = \frac{4\pi e^2}{(2\pi)^3} \int d^3p \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2}$
Thomas - Fermi approx (semiclassical)
$\hat{H} = \int d^3x \, \hat{\psi}^\dagger(\vec{x}) \, T(\vec{x}) \, \hat{\psi}(\vec{x})$
(neglect interactions)
$T(\vec{x}) = -\frac{\hbar^2 \vec{\nabla}^2}{2m} - e\phi(\vec{x})$


Thomas - Fermi approx (semiclassical)
(neglect interactions)
For φ(x) = φ₀ = const

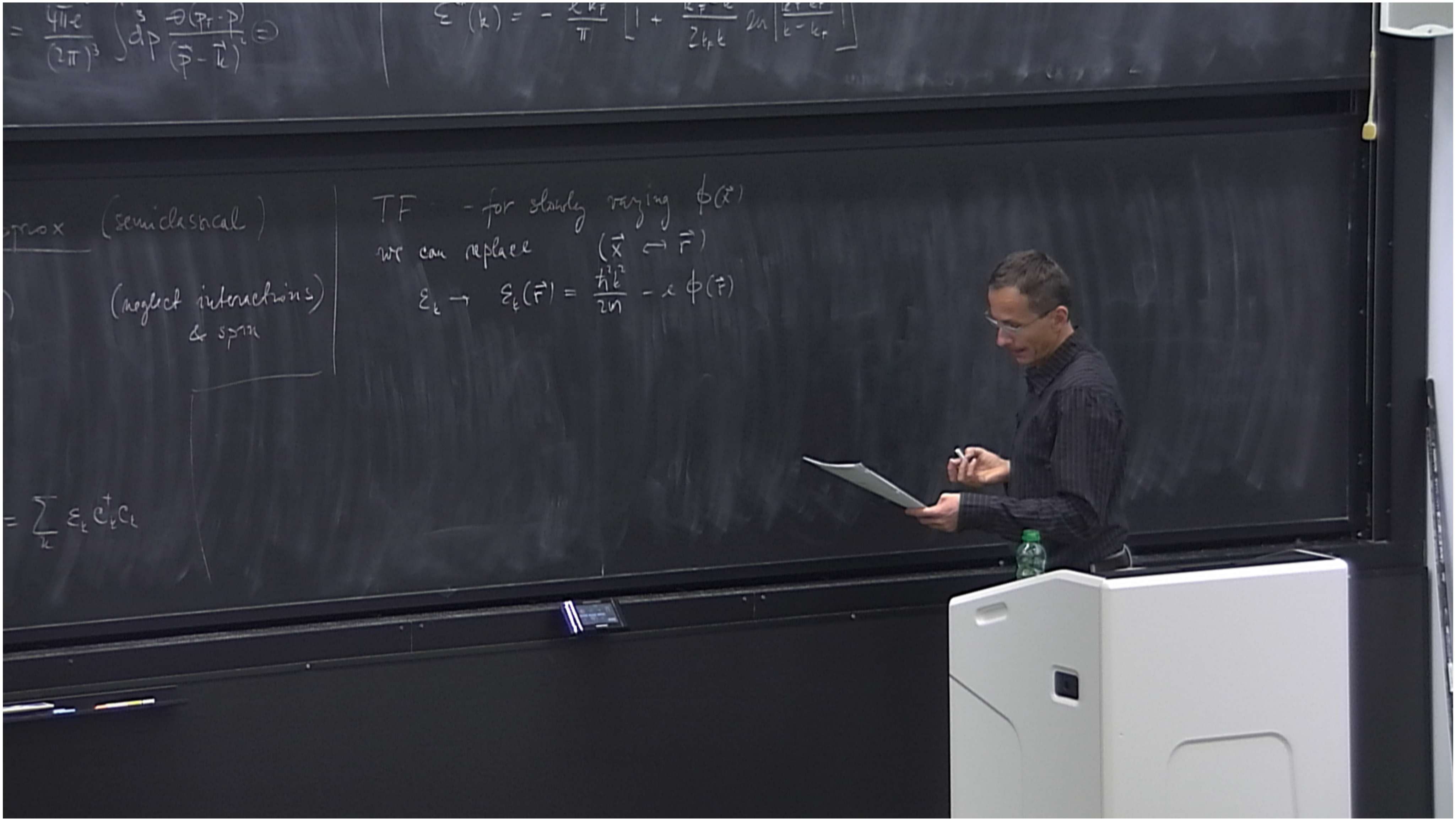


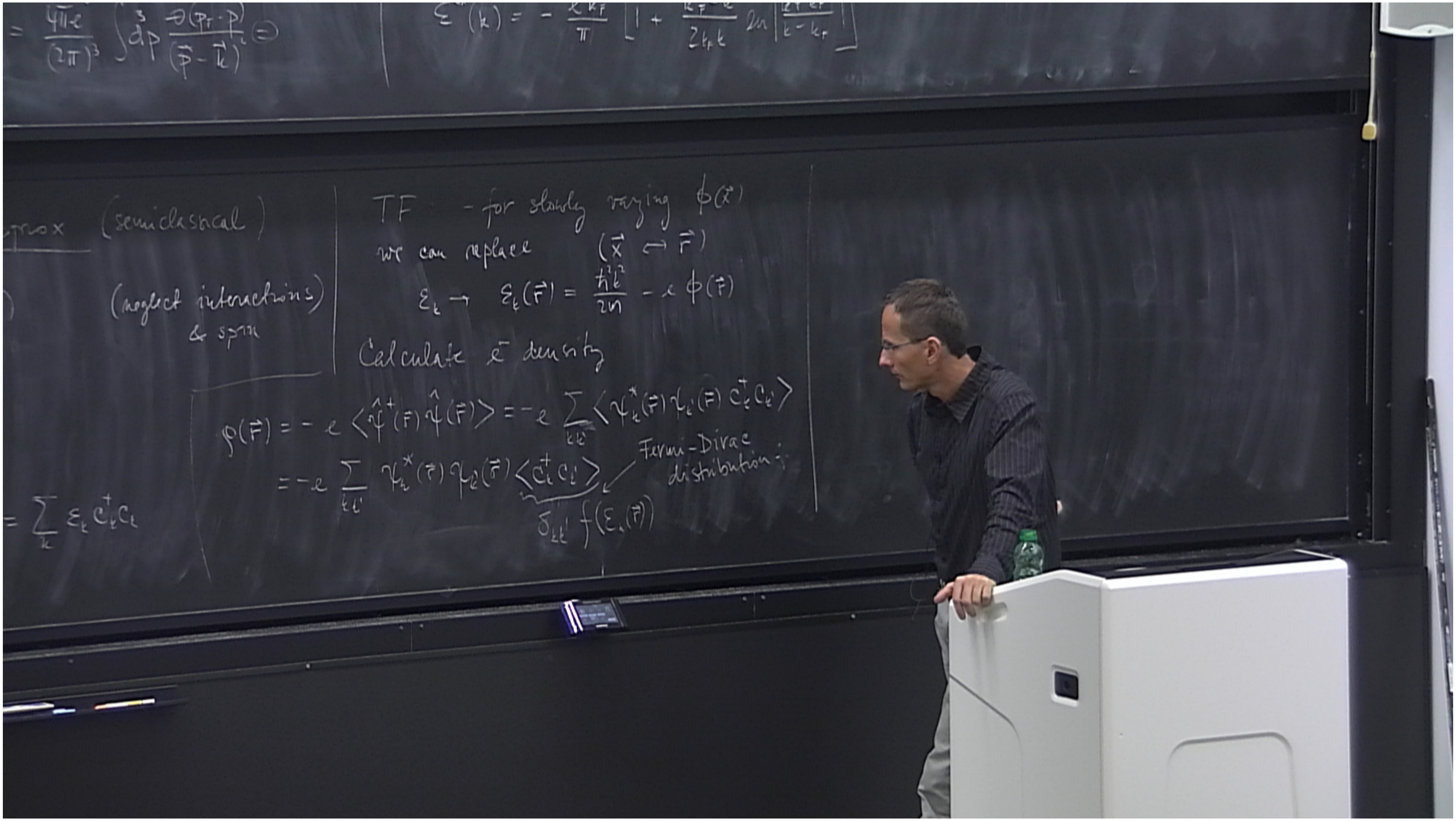
$-\Sigma^{ex}(\vec{k}) = 4\pi \frac{e^2}{V} \sum_{\vec{p}} \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2} = \frac{4\pi e^2}{(2\pi)^3} \int d^3p \frac{\theta(p_F - p)}{(\vec{p}-\vec{k})^2}$
Thomas - Fermi approx (semiclassical)
$\hat{H} = \int d^3 \hat{\psi}^\dagger(\vec{x})\, T(\vec{x})\, \hat{\psi}(\vec{x})$
(neglect interactions & spin)
$T(\vec{x}) = -\frac{\hbar^2 \nabla^2}{2m} - e\phi(\vec{x})$
For $\phi(\vec{x}) = \phi_0 = $ const
$H = \sum_k \left(\frac{\hbar^2 k^2}{2m} - e\phi_0\right) c_k^\dagger c_k = \sum_k \varepsilon_k c_k^\dagger c_k$

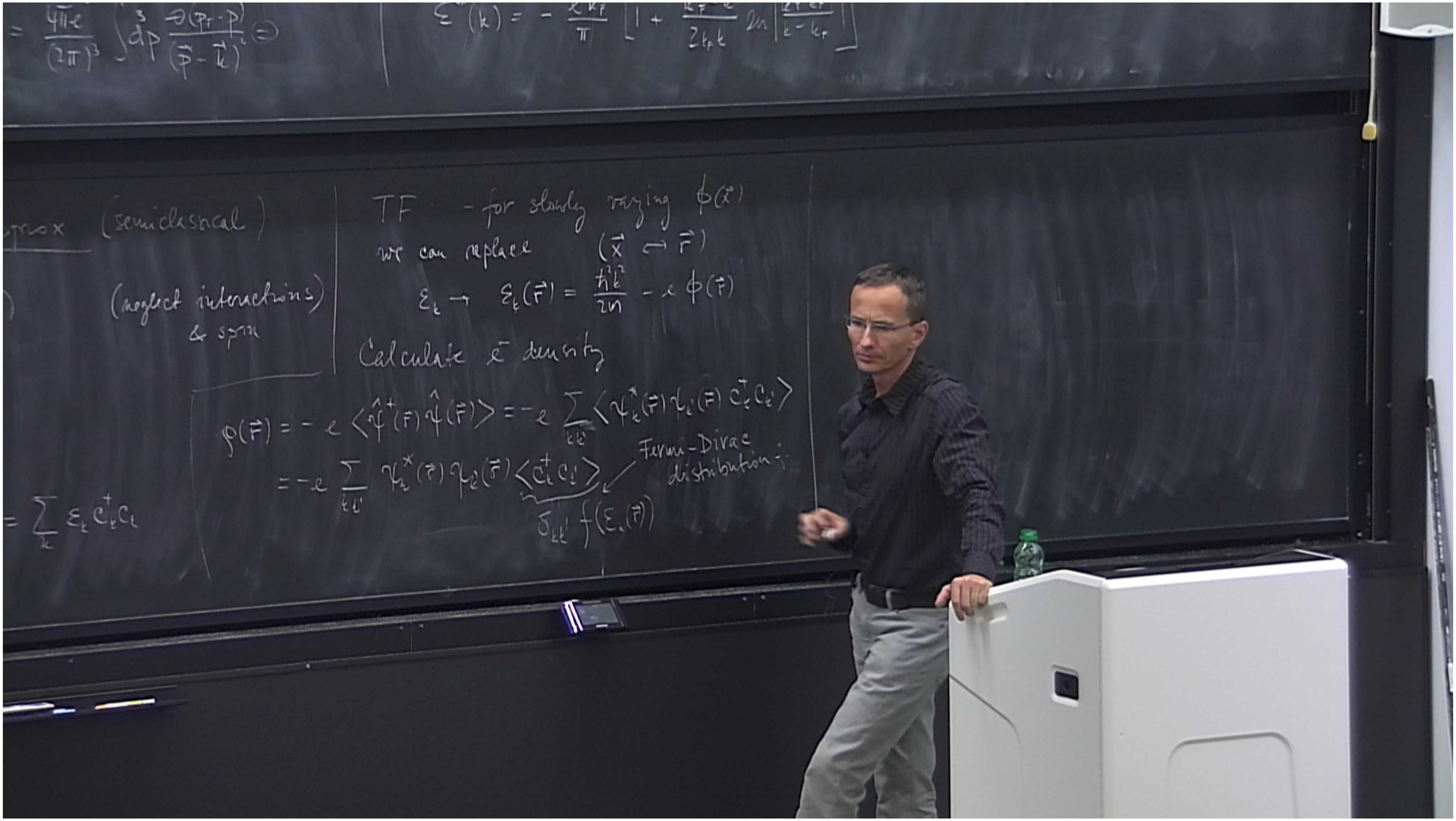

Thomas - Fermi approx (semiclassical)
(neglect interactions) & spin
For φ(x) = φ₀ = const
TF - for slowly varying φ(x)
we can replace

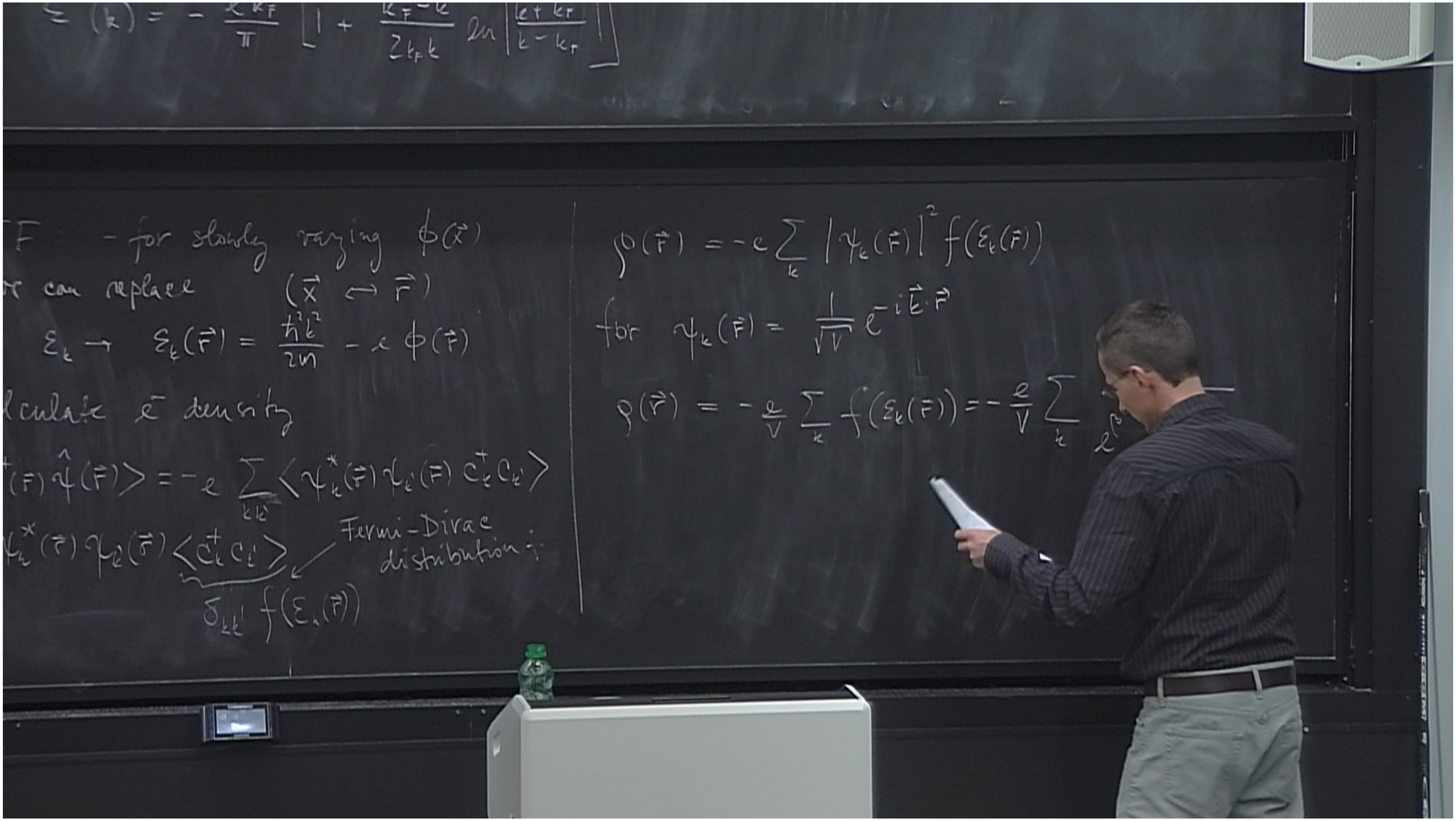


Thomas - Fermi approx (semiclassical)
Ĥ = ∫d³ ψ̂†(x⃗) T(x⃗) ψ̂(x⃗)
(neglect interactions & spin)
T(x⃗) = - ħ²∇²/2m - eφ(x⃗)
For φ(x⃗) = φ₀ = const
H = Σ_k (ħ²k²/2m - eφ₀) c†_k c_k = Σ_k ε_k c†_k c_k
TF - for slowly varying φ(x⃗)
we can replace (x⃗ → r⃗)
ε_k → ε_k(r⃗) = ħ²k²/2m -

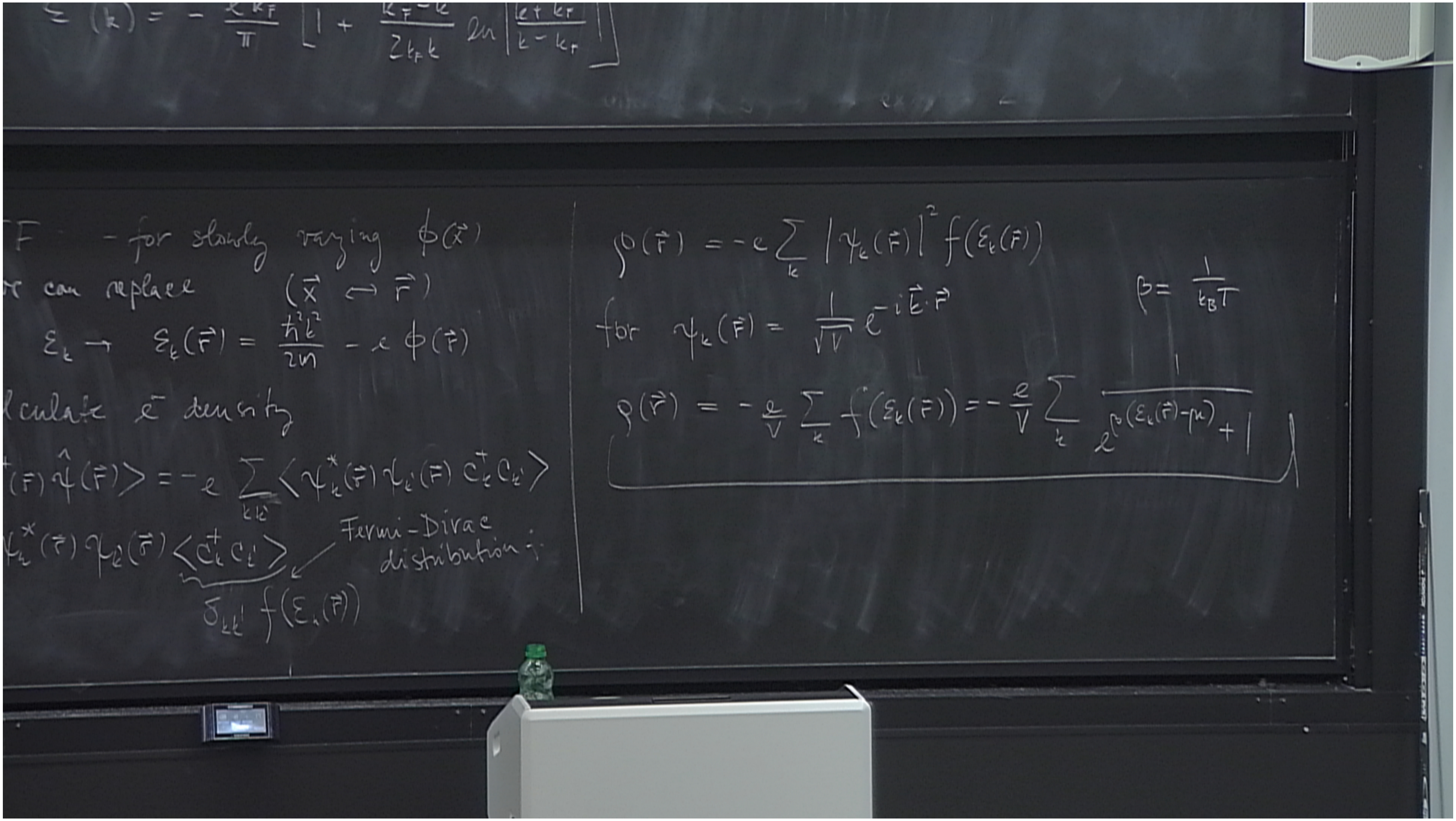

(semiclassical)
(neglect interactions)
& spin
TF - for slowly varying
we can replace

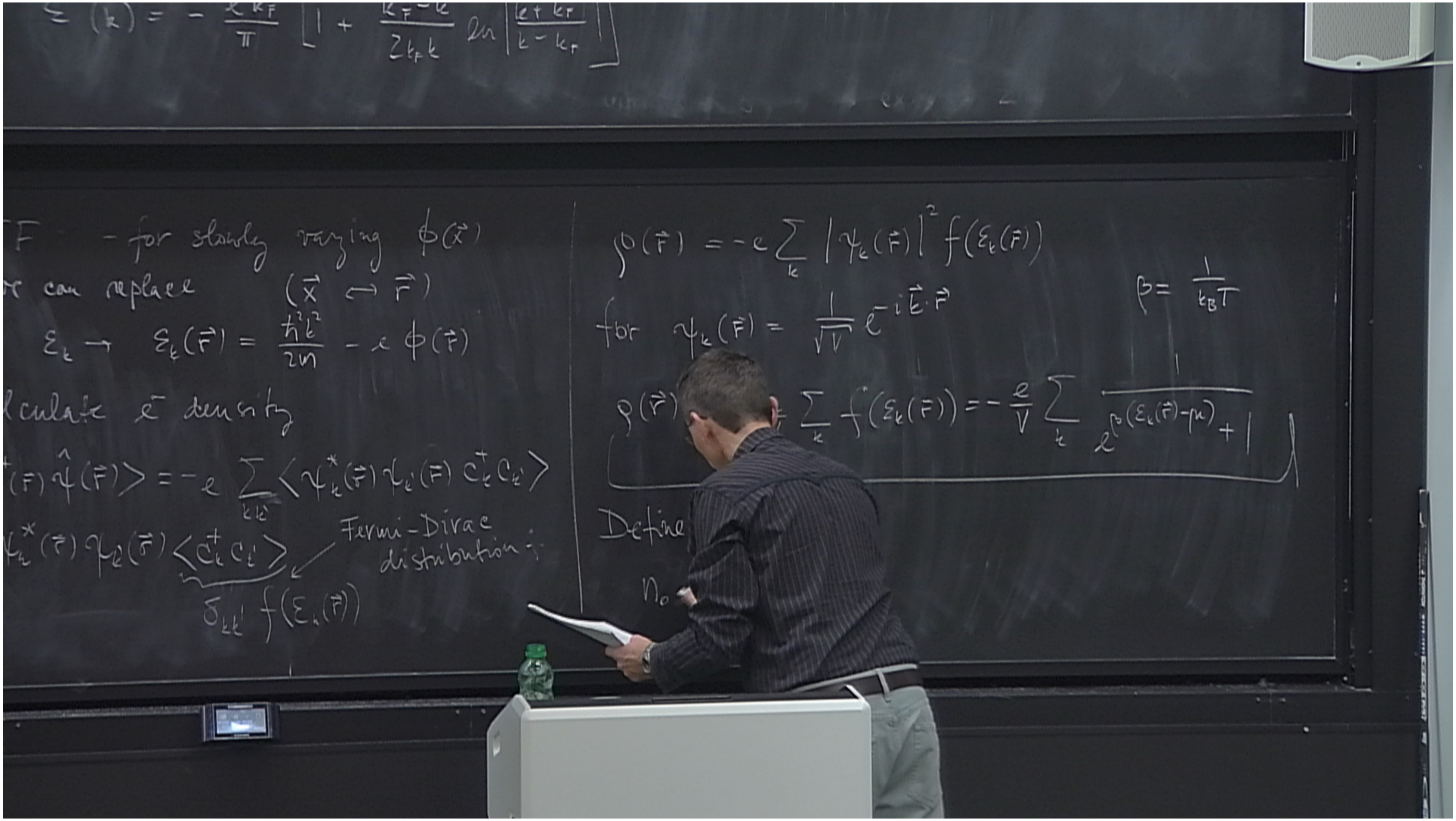

(semiclassical)
(neglect interactions)
& spin
TF - for slowly varying
we can replace
Calculate density
Fermi-Dirac
distribution

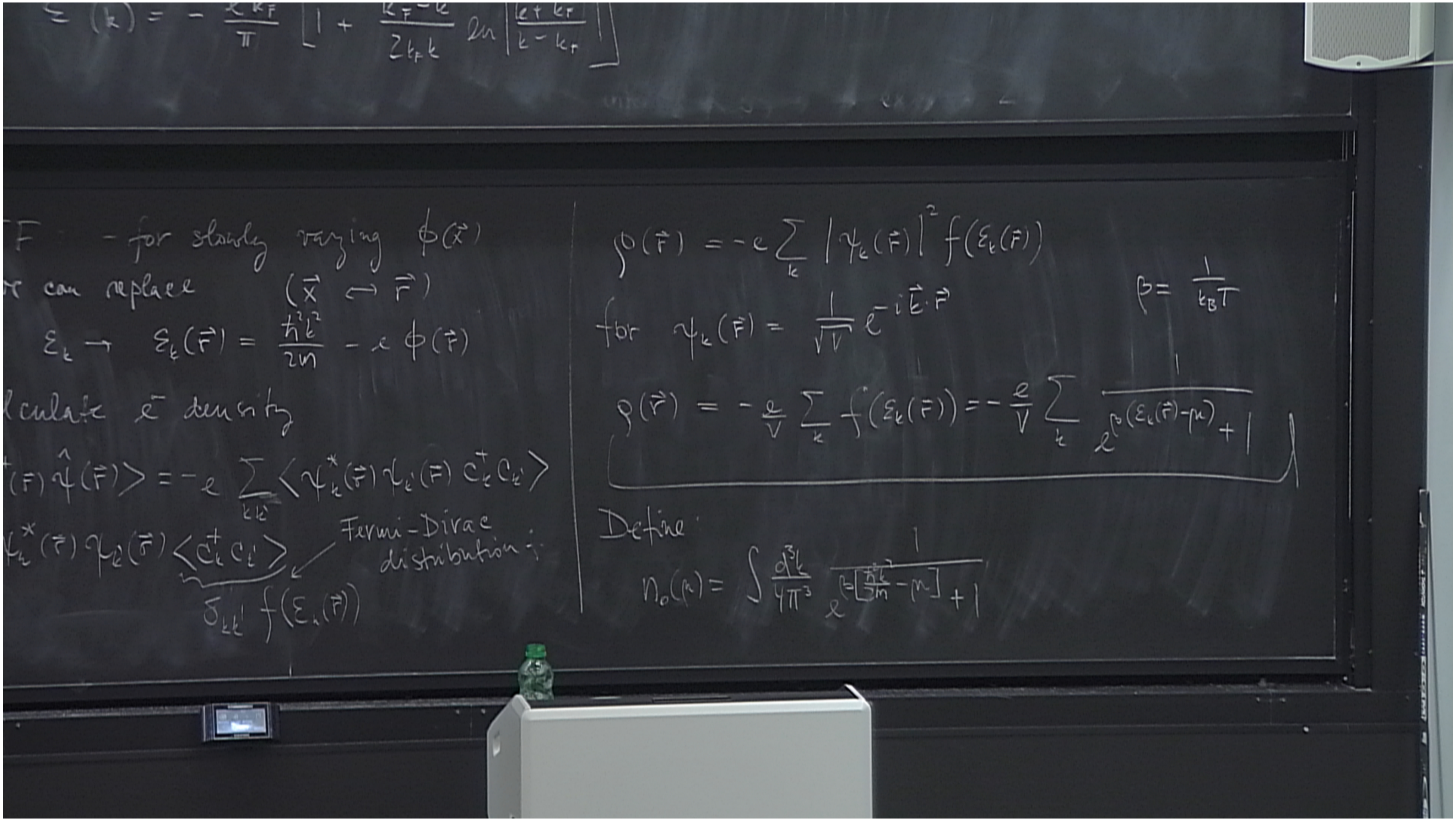

(semiclassical)
(neglect interactions)
& spin
TF - for slowly varying
we can replace
Calculate e density
Fermi-Dirac
distribution

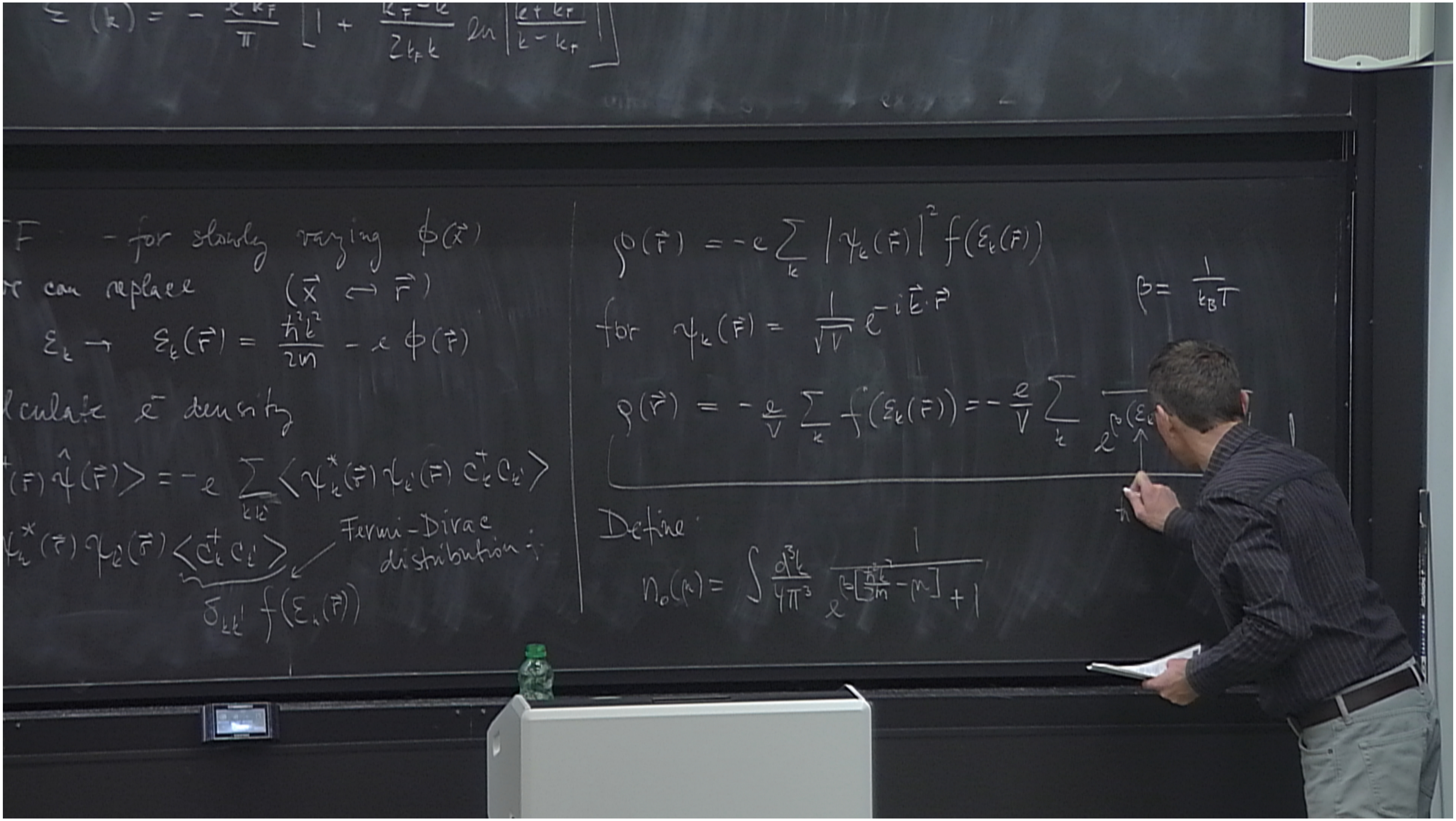


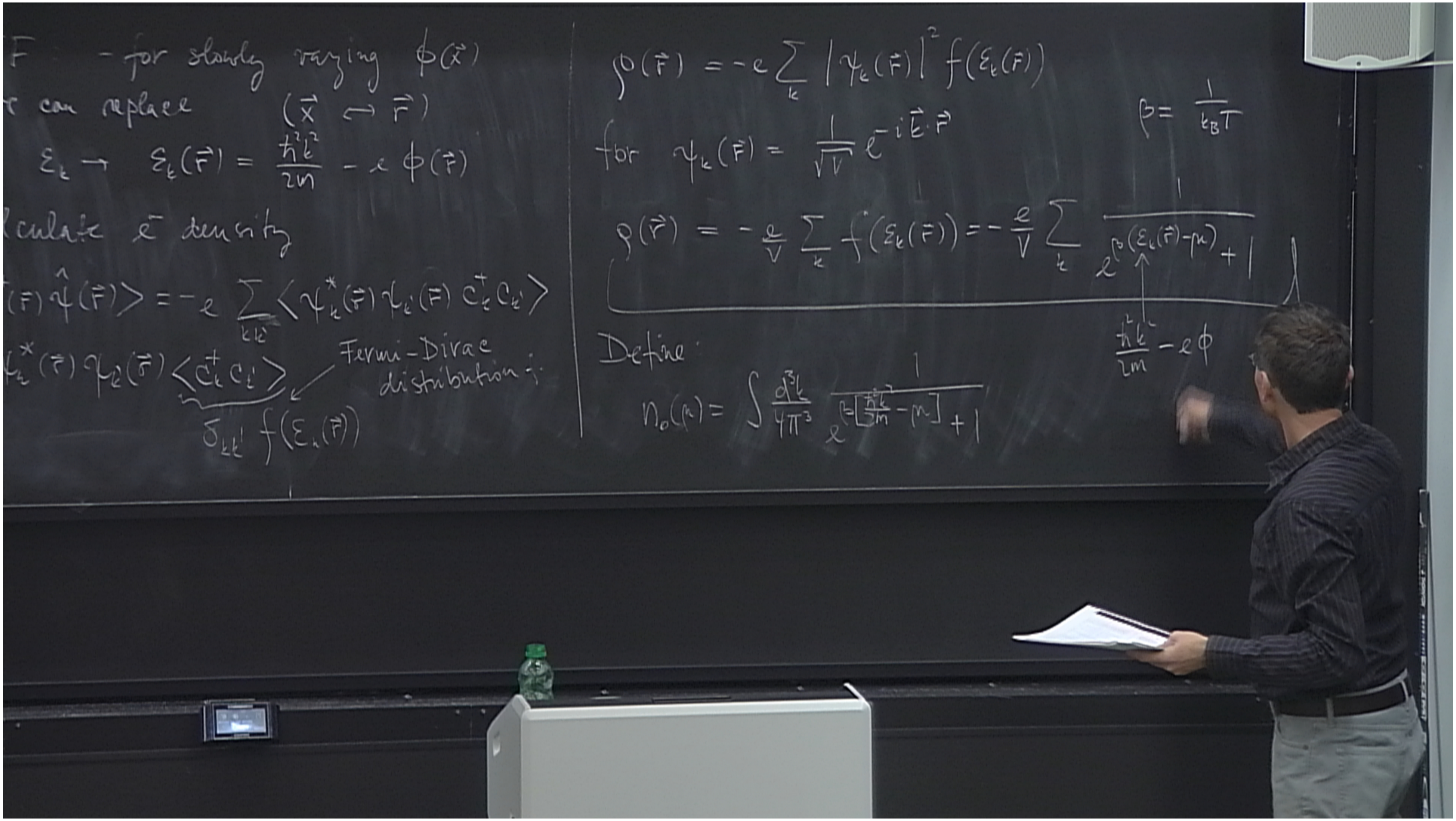
- for slowly varying
can replace
lculate e density
Fermi-Dirac
distribution
for

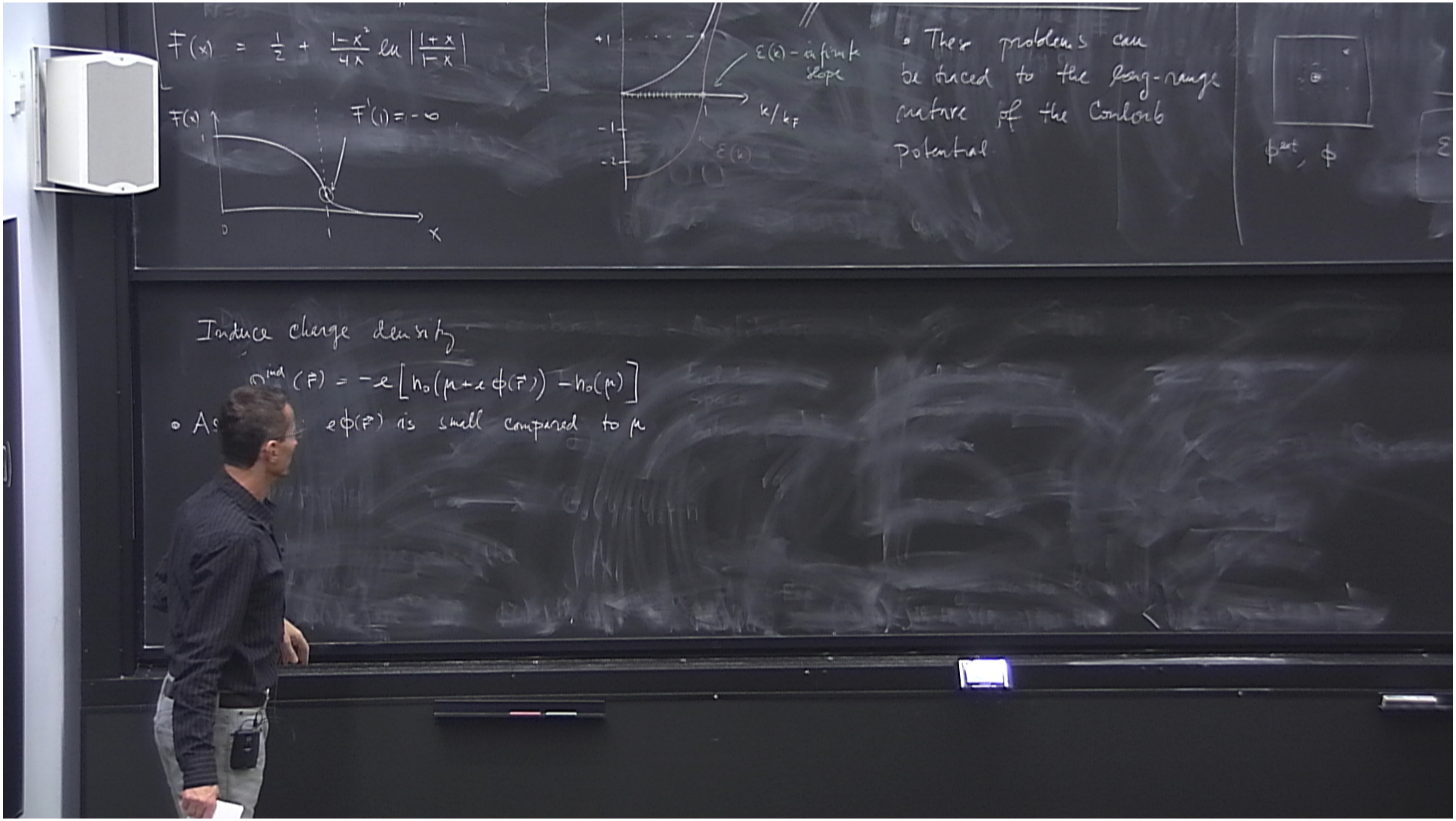

TF - for slowly varying ϕ(x)
can replace (x ↔ r)
ε_k → ε_k(r) = ħ²k²/2m − e ϕ(r)
calculate e⁻ density
Fermi-Dirac distribution
δ_kk' f(ε_k(r))
ρ(r) = −e Σ_k |ψ_k(r)|² f(ε_k(r))
for ψ_k(r) = (1/√V) e^{−ik·r}
β = 1/(k_B T)
ρ(r) = −(e/V) Σ_k f(ε_k(r)) = −(e/V) Σ_k 1/(e^{β(ε_k(r)−μ)} + 1)

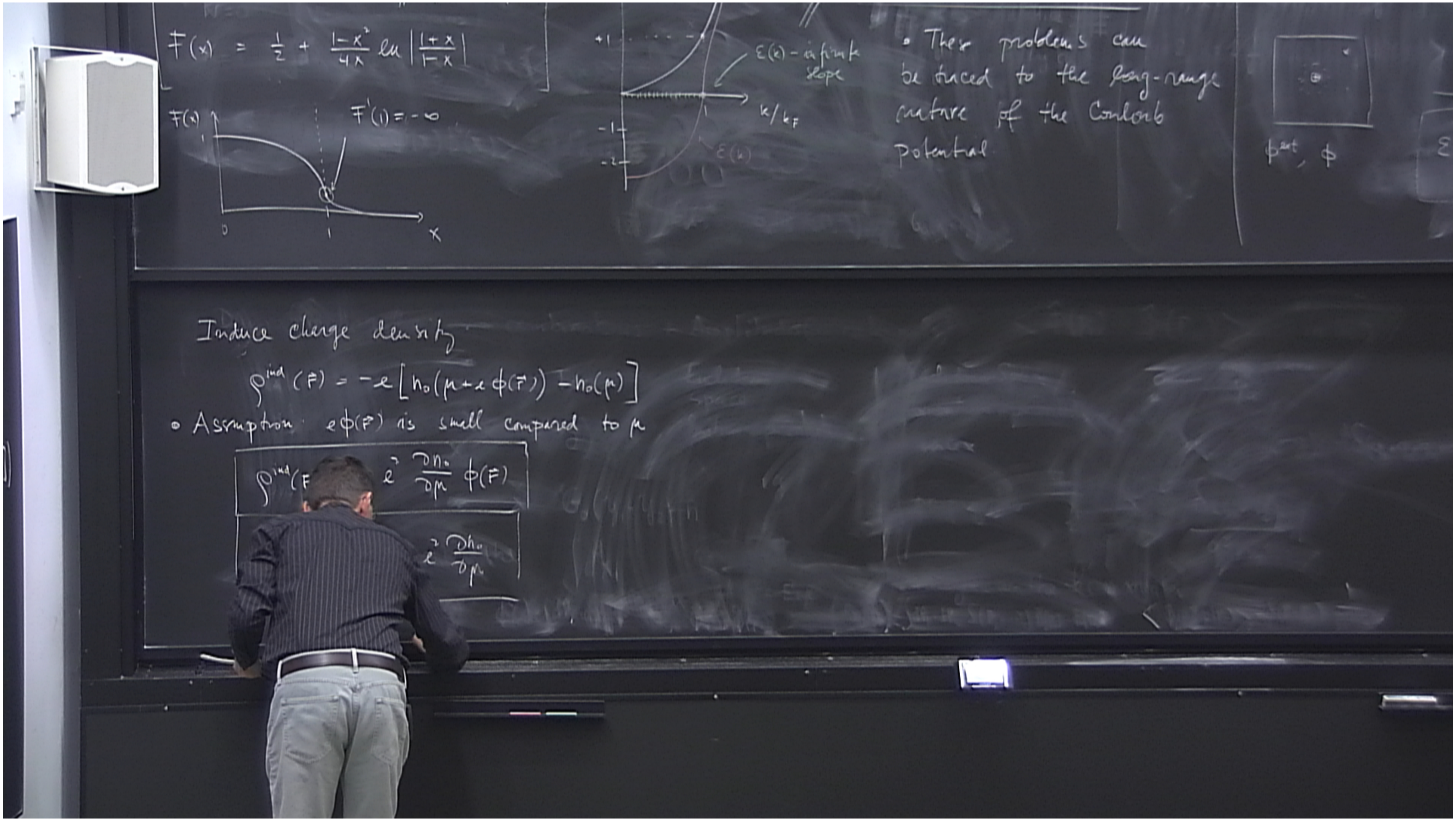

- for slowly varying
can replace
Fermi-Dirac distribution
Define


F - for slowly varying φ(x)
can replace (x ↔ r)
ε_k → ε_k(r) = ħ²k²/2m − e φ(r)
lculate e⁻ density
Fermi-Dirac distribution
ρ(r) = −e Σ_k |ψ_k(r)|² f(ε_k(r))
β = 1/k_B T
for ψ_k(r) = (1/√V) e^{−i k·r}
ρ(r) = −(e/V) Σ_k f(ε_k(r)) = −(e/V) Σ_k 1/(e^{β(ε_k(r)−μ)} + 1)
Define
n_0(μ) = ∫ d³k/4π³ 1/(e^{β[ħ²k²/2m − μ]} + 1)

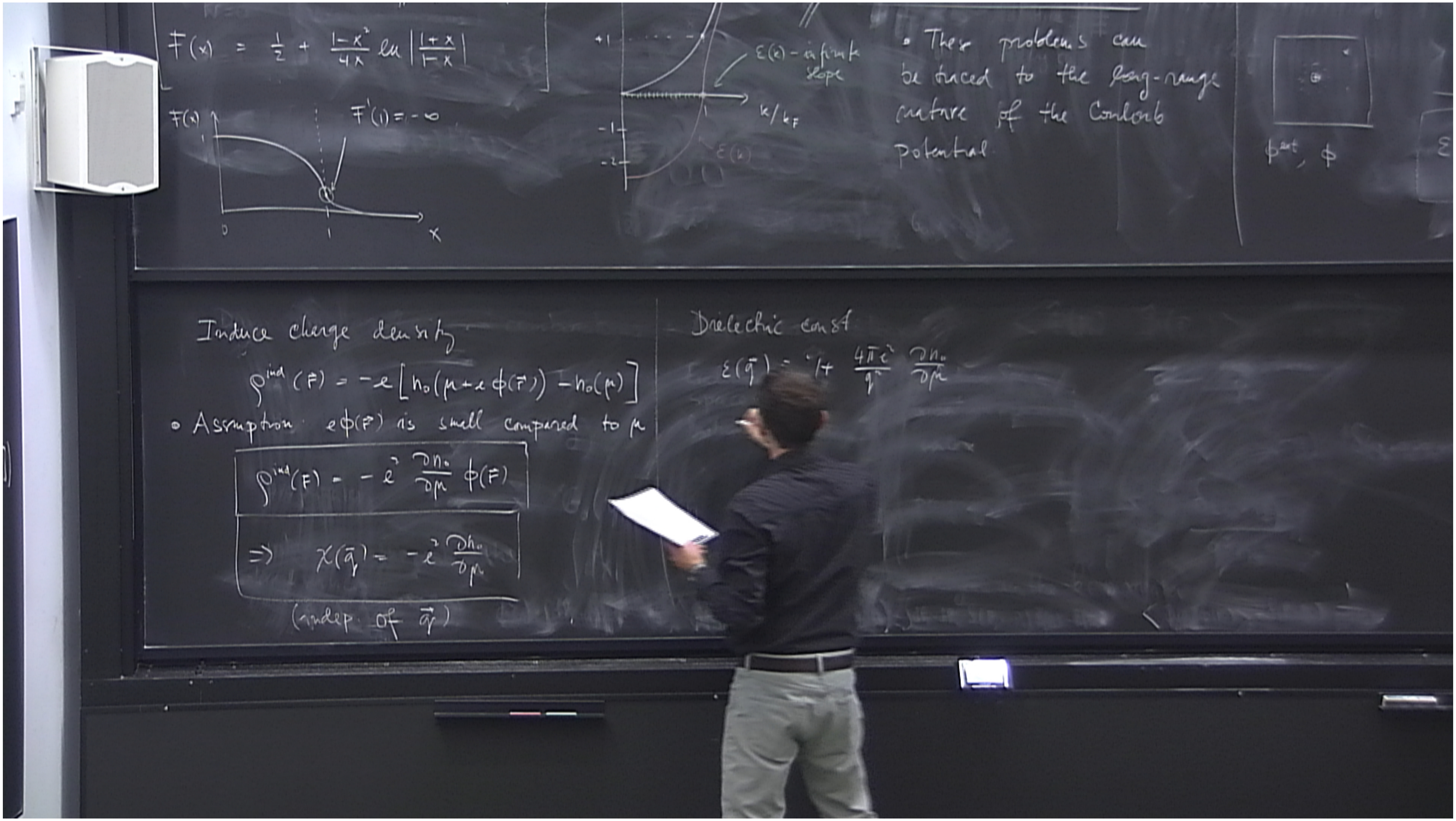

- for slowly varying φ(x⃗)
can replace (x⃗ ↔ r⃗)
ε_k → ε_k(r⃗) = ħ²k²/2m − e φ(r⃗)
lculate e⁻ density
Fermi-Dirac distribution
δ_kk' f(ε_k(r⃗))
ρ(r⃗) = −e Σ_k |ψ_k(r⃗)|² f(ε_k(r⃗))
β = 1/k_B T
for ψ_k(r⃗) = 1/√V e^{−i k⃗·r⃗}
ρ(r⃗) = −e/V Σ_k f(ε_k(r⃗)) = −e/V Σ_k
Define
n₀(μ) = ∫ d³k/4π³ 1/(e^{β[ħ²k²/2m − μ]} + 1)

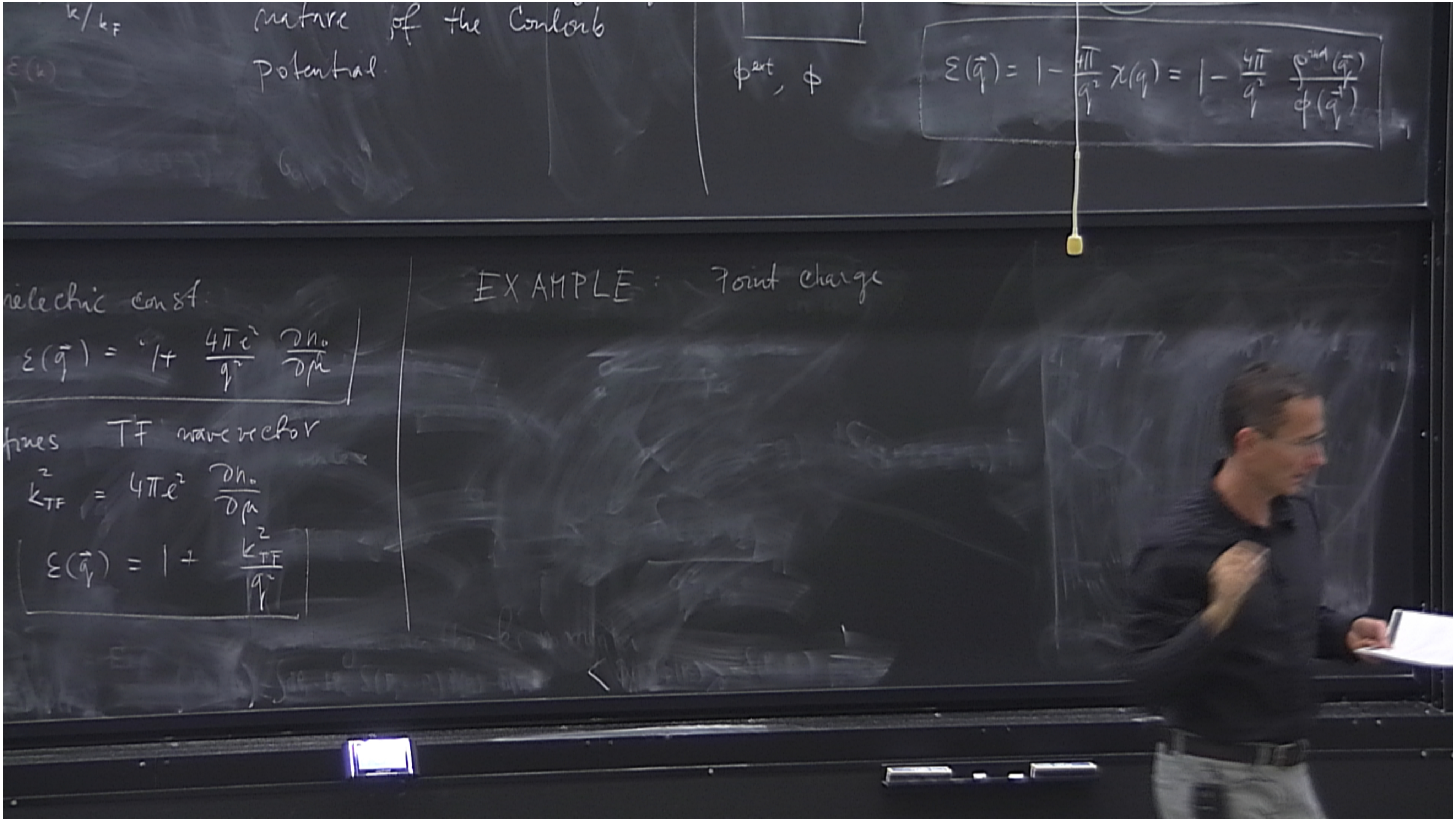

- for slowly varying
can replace
culate e density
Fermi-Dirac
distribution
for
Define

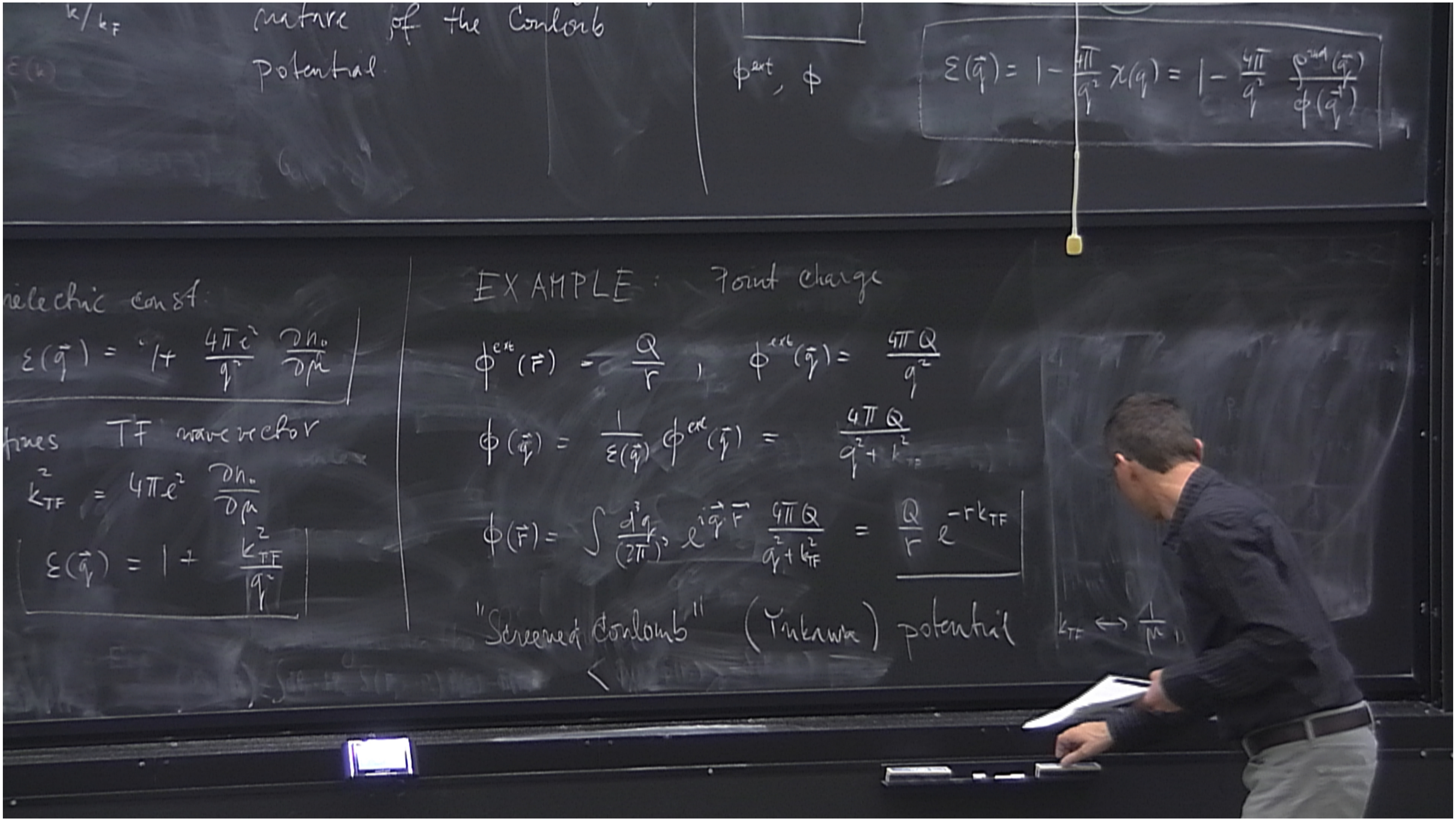

F(x) = 1/2 + (1 − x²)/(4x) ln |(1 + x)/(1 − x)|
F(x)
F′(1) = −∞
x
ε(k) − infinite slope
k/k_F
ε(k)
• These problems can be traced to the long-range nature of the Coulomb potential
φ^ext, φ
Induce charge density
ρ^ind(r) = −e[n₀(μ + eφ(r)) − n₀(μ)]
• As eφ(r) is small compared to μ

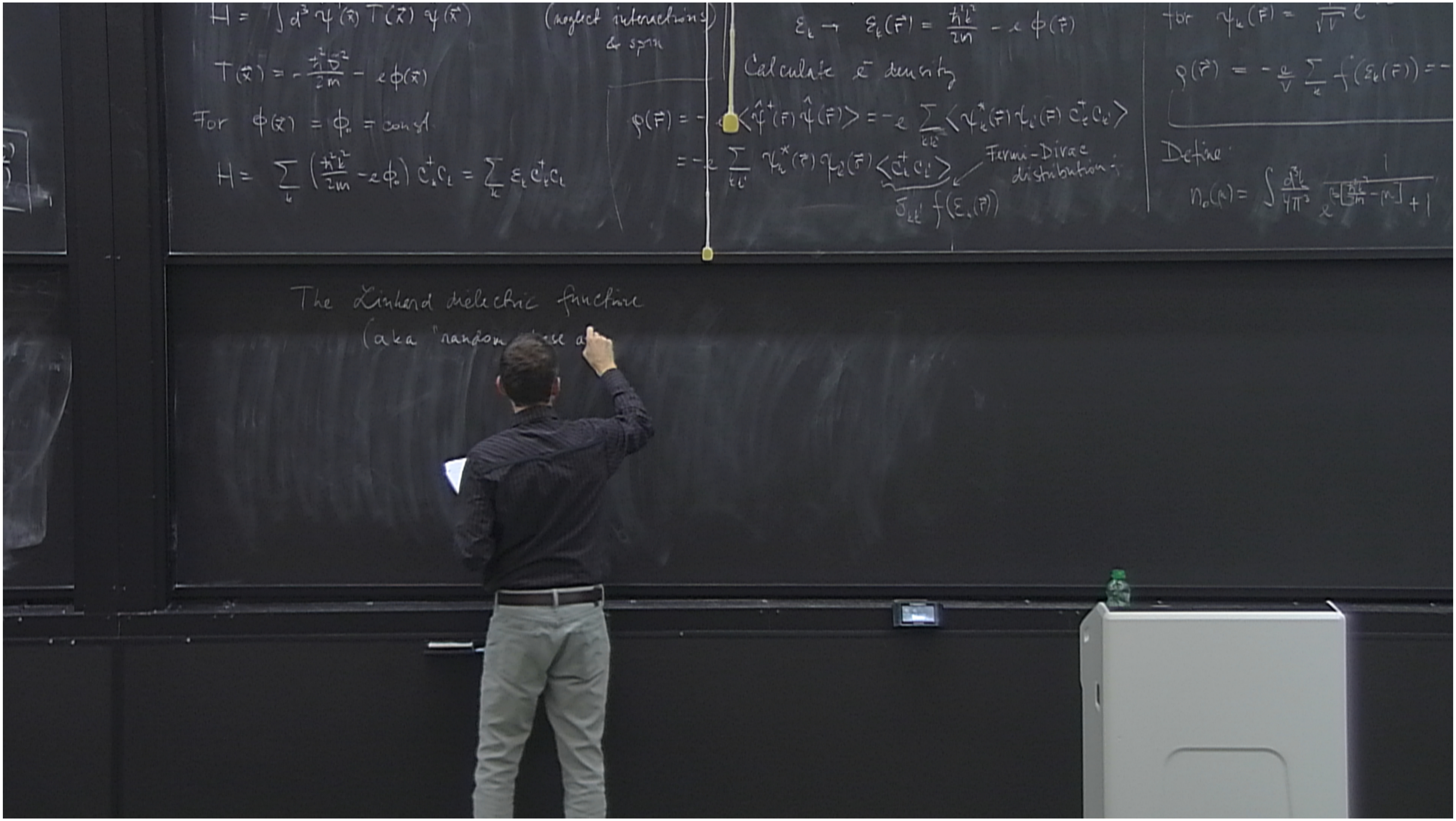


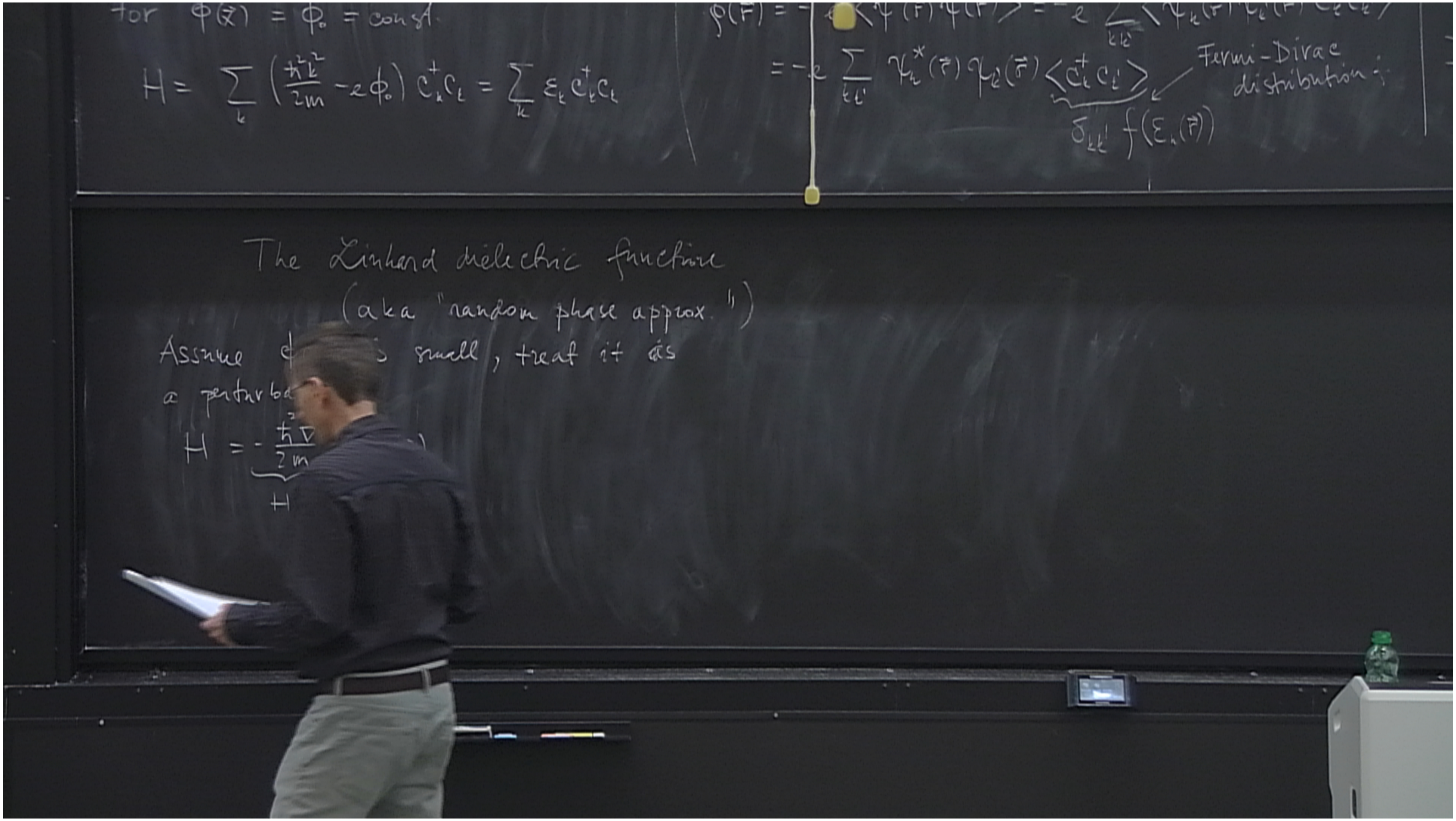
F(x) = 1/2 + (1−x²)/4x ln |(1+x)/(1−x)|
F'(1) = −∞
F(x)
x
ε(k) − infinite slope
k/k_F
ε(k)
• These problems can be traced to the long-range nature of the Coulomb potential
φ^ext, φ
Induce charge density
ρ^ind(r) = −e [n₀(μ + eφ(r)) − n₀(μ)]
• Assumption eφ(r) is small compared to μ

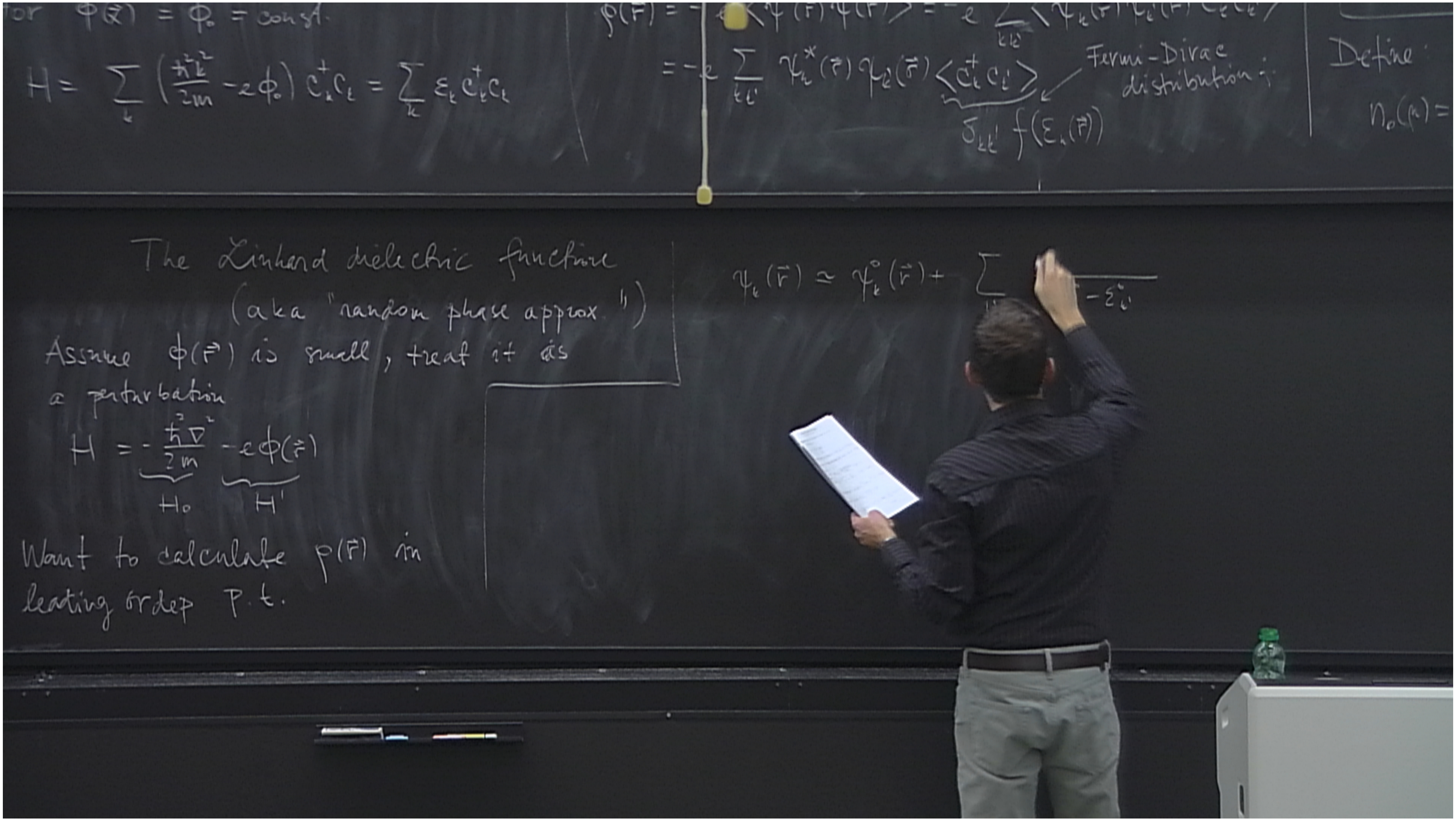

F(x) = 1/2 + (1 − x²)/(4x) ln |(1 + x)/(1 − x)|
F(x)
F'(1) = −∞
0
1
x
+1
−1
−2
ε(k) − infinite slope
k/k_F
• These problems can be traced to the long-range nature of the Coulomb potential
φ^ext, φ
Induce charge density
ρ^ind(r) = −e [n₀(μ + eφ(r)) − n₀(μ)]
• Assumption eφ(r) is small compared to μ
ρ^ind(r) = −e² ∂n₀/∂μ φ(r)
⇒ χ(q) = −e² ∂n₀/∂μ
(indep. of q)
Dielectric const
ε(q) = 1 + 4πe²/q² ∂n₀/∂μ

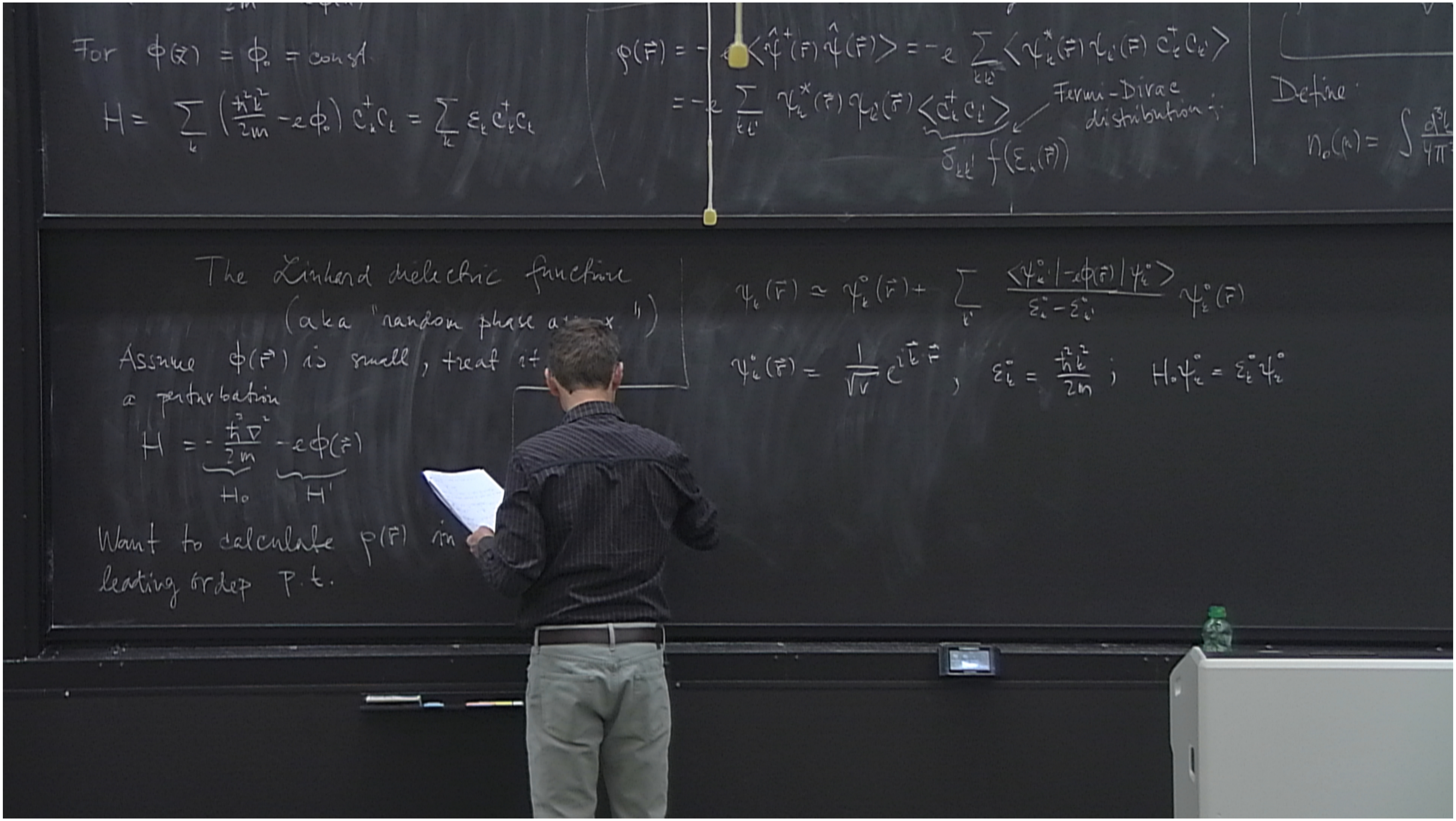

k/k_F
nature of the Coulomb
potential
φ^ext, φ
ε(q) = 1 − 4π/q² χ(q) = 1 − 4π/q² ρ^ind(q)/φ(q)
electric const
ε(q) = 1 + 4πe²/q² ∂n₀/∂μ
TF wavevector
k_TF² = 4πe² ∂n₀/∂μ
ε(q) = 1 + k_TF²/q²
EXAMPLE: Point charge

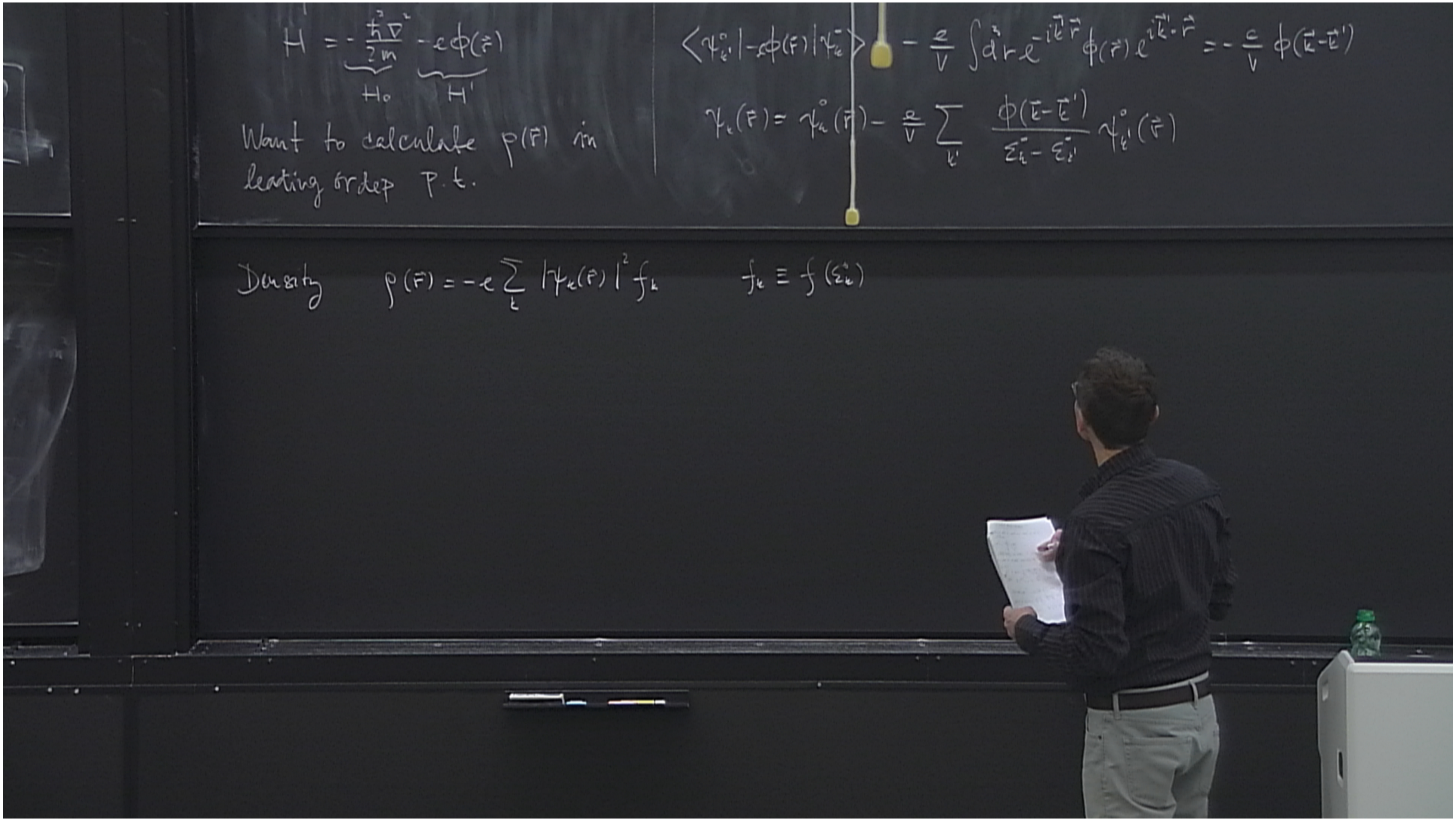

nature of the Coulomb
potential
EXAMPLE: Point charge
"Screened Coulomb" (Yukawa) potential
TF wavevector

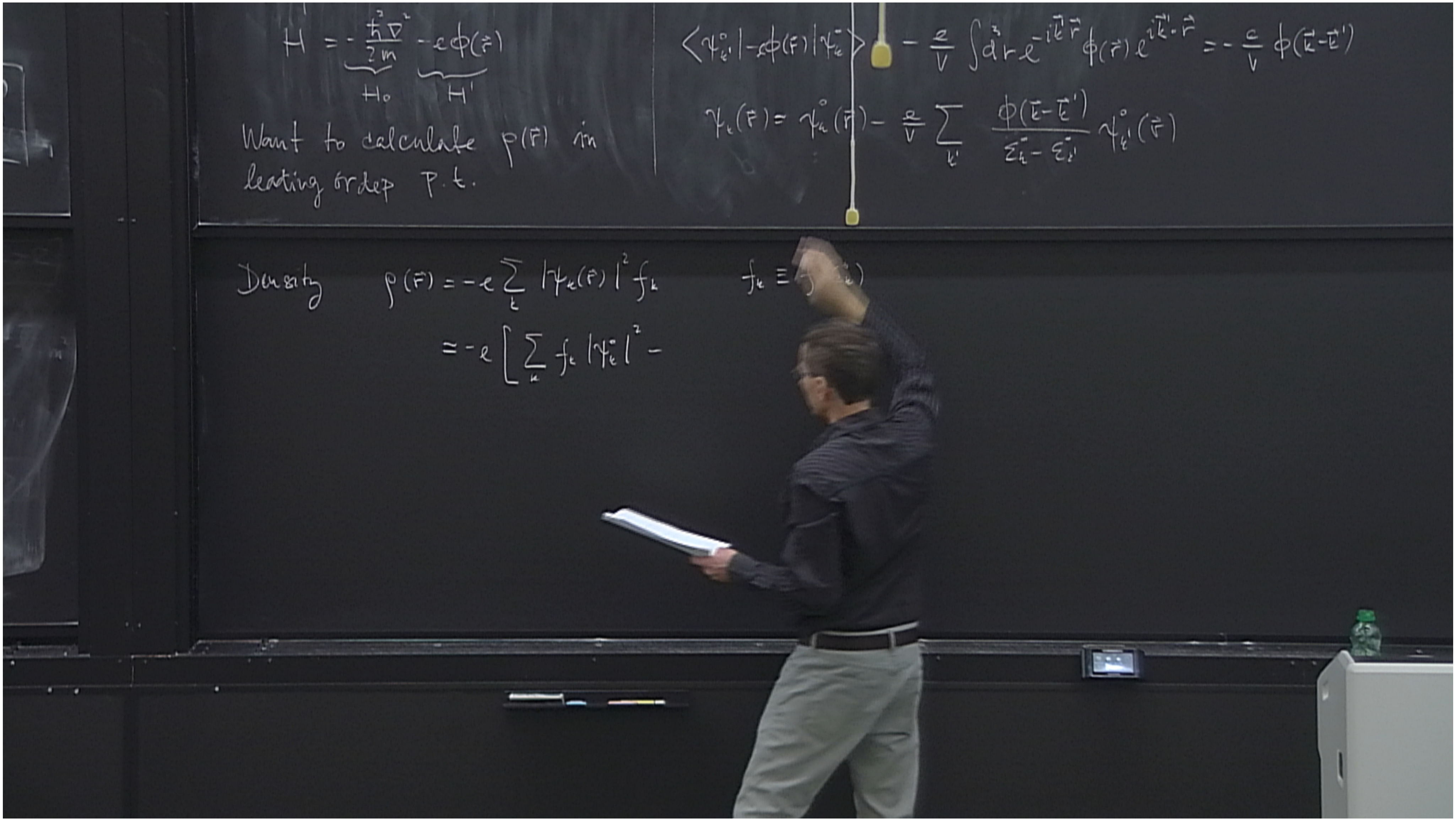

(neglect interactions
& spin)
For
Calculate e density
Fermi-Dirac
distribution
Define
The Lindhard dielectric function
(aka "random

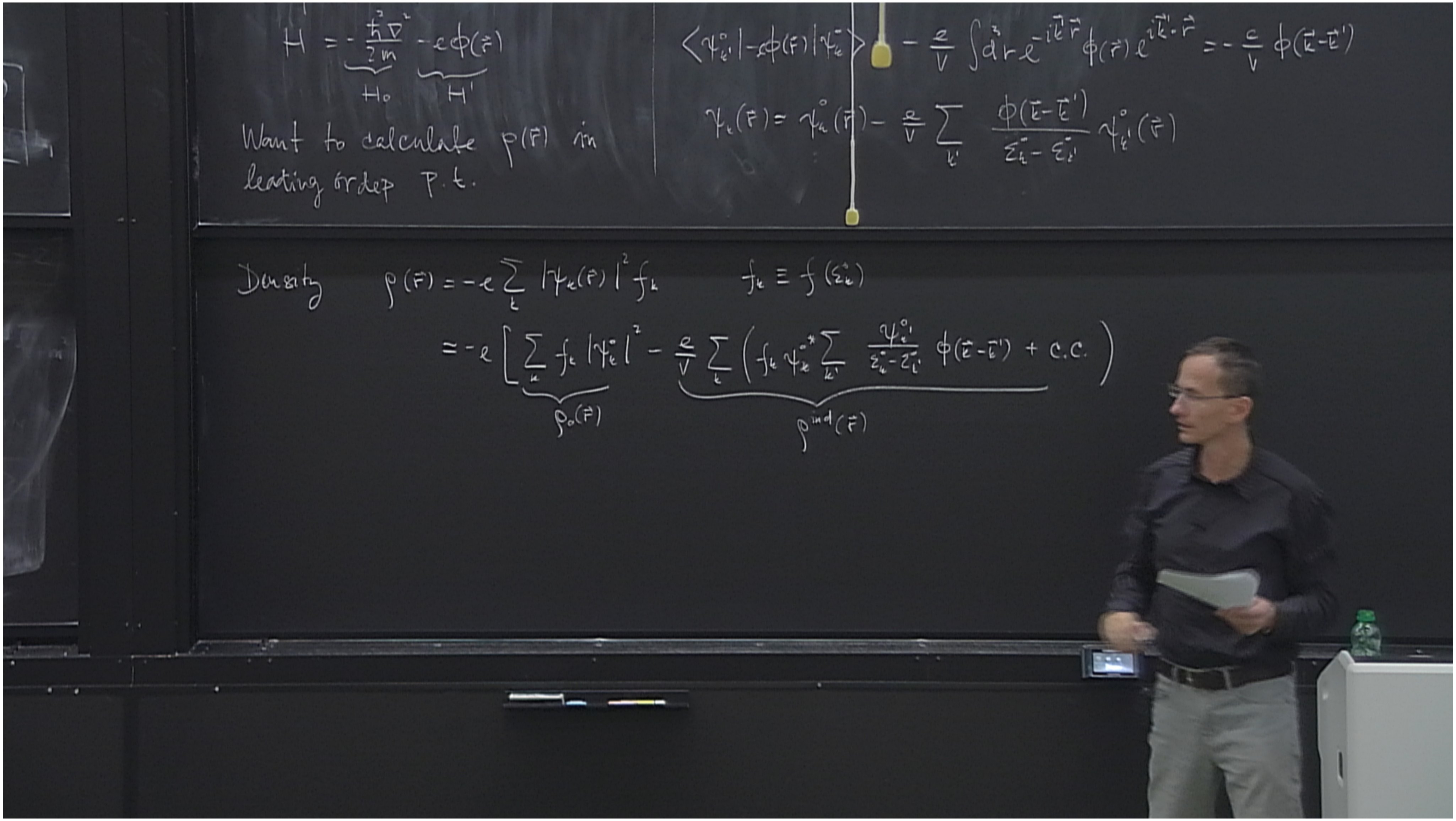

Fermi-Dirac distribution
The Linhard dielectric function
(aka "random phase approx.")
Assume
small, treat it as
a perturba

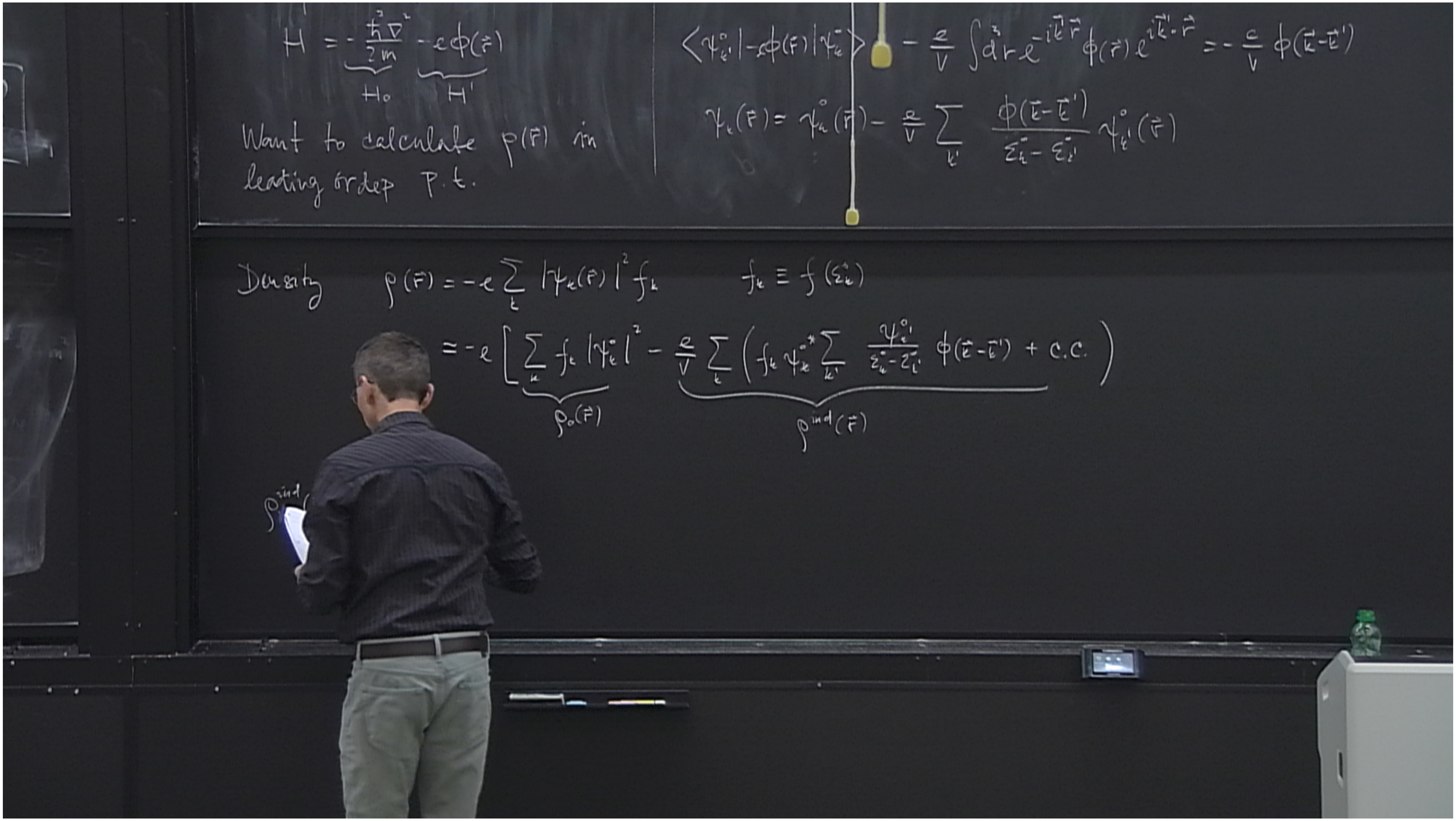

The Linhard dielectric function
(aka "random phase approx")
Assume φ(r) is small, treat it as
a perturbation
Want to calculate ρ(r) in
leading order p.t.
Fermi-Dirac
distribution
Define


For ϕ(x) = ϕ₀ = const
H = Σ_k (ħ²k²/2m − eϕ₀) c_k⁺ c_k = Σ_k ε_k c_k⁺ c_k
ρ(r) = −e ⟨ψ̂⁺(r) ψ̂(r)⟩ = −e Σ_kk' ⟨ψ_k*(r) ψ_k'(r) c_k⁺ c_k'⟩
= −e Σ_kk' ψ_k*(r) ψ_k'(r) ⟨c_k⁺ c_k'⟩
δ_kk' f(ε_k(r))
Fermi-Dirac distribution
Define
The Lindhard dielectric function
(aka "random phase approx")
Assume ϕ(r) is small, treat it as a perturbation
H = −ħ²∇²/2m − eϕ(r)
H₀
H'
Want to calculate ρ(r) in leading order p.t.
ψ_k(r) ≃ ψ_k°(r) + Σ_k' ⟨ψ_k'°| −eϕ(r) |ψ_k°⟩ / (ε_k° − ε_k'°) ψ_k'°(r)
ψ_k°(r) = (1/√V) e^(ik·r) , ε_k° = ħ²k²/2m ; H₀ψ_k° = ε_k°ψ_k°

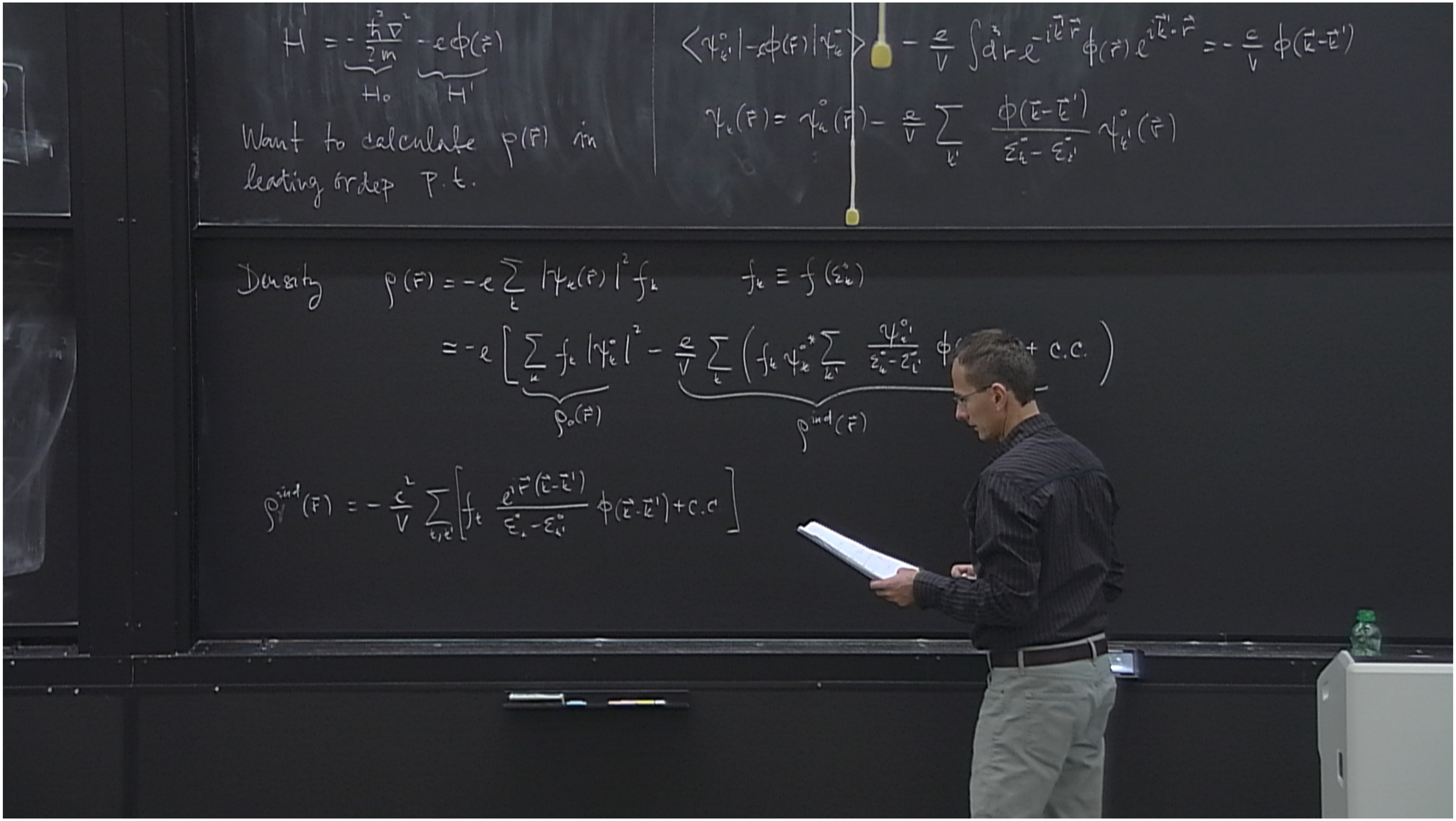

H = - ħ²∇²/2m - eφ(r)
H₀ H'
Want to calculate ρ(r) in
leading order p.t.
⟨ψ°k'| -eφ(r) |ψ°k⟩ = - e/V ∫d³r e^{-ik'·r} φ(r) e^{ik·r} = - e/V φ(k-k')
ψk(r) = ψ°k(r) - e/V Σk' φ(k-k')/(ε°k - ε°k') ψ°k'(r)
Density ρ(r) = -e Σk |ψk(r)|² fk
fk ≡ f(ε°k)

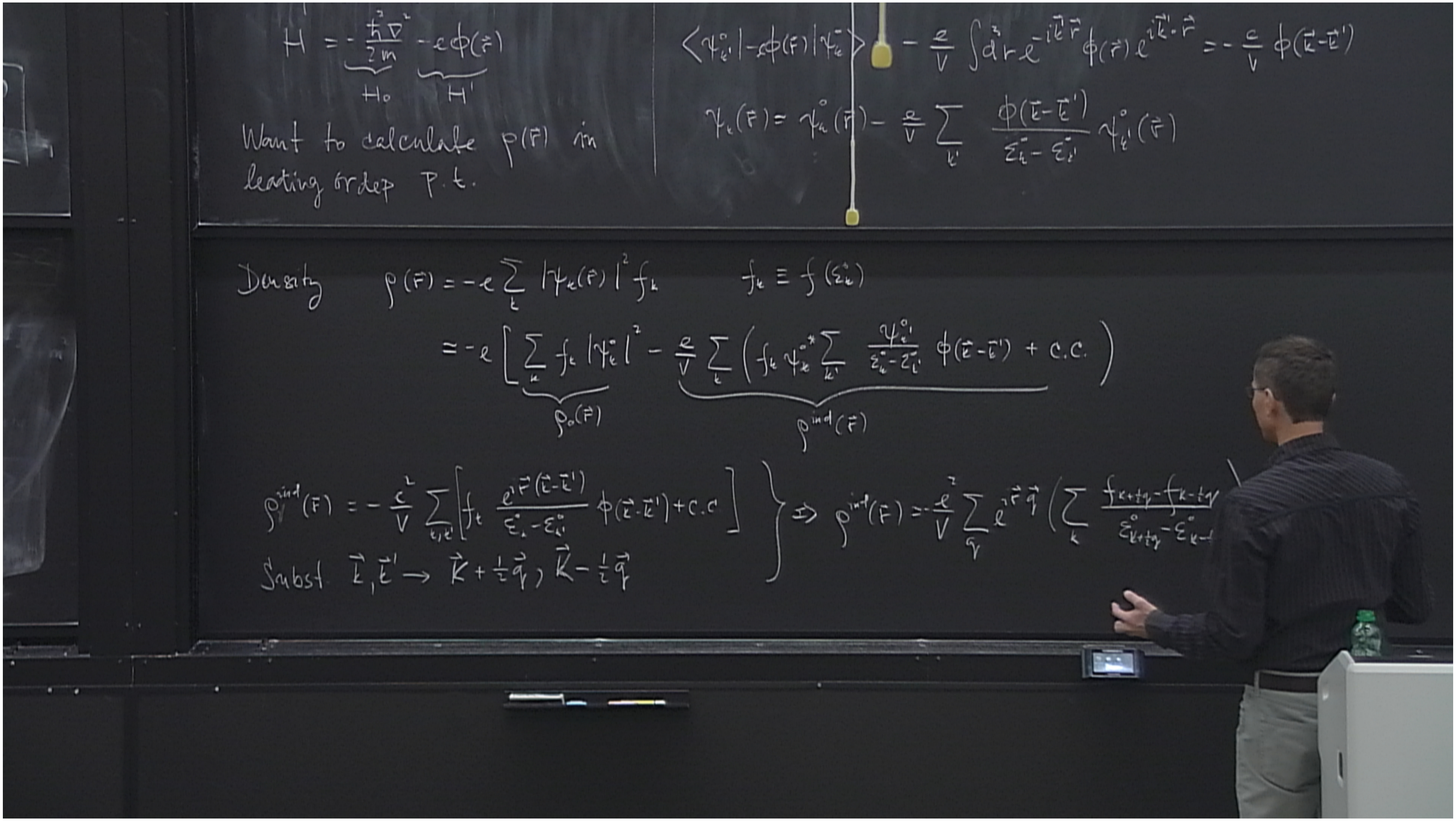

$H = -\frac{\hbar^2\nabla^2}{2m} - e\phi(\vec{r})$
H_0 H'
Want to calculate $\rho(\vec{r})$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec{r}) | \psi^0_k \rangle = -\frac{e}{V}\int d^3r\, e^{-i\vec{k}'\cdot\vec{r}}\phi(\vec{r})e^{i\vec{k}\cdot\vec{r}} = -\frac{e}{V}\phi(\vec{k}-\vec{k}')$
$\psi_k(\vec{r}) = \psi^0_k(\vec{r}) - \frac{e}{V}\sum_{k'} \frac{\phi(\vec{k}-\vec{k}')}{\varepsilon_k - \varepsilon_{k'}}\psi^0_{k'}(\vec{r})$
Density $\rho(\vec{r}) = -e\sum_k |\psi_k(\vec{r})|^2 f_k$
$f_k \equiv$
$= -e\left[\sum_k f_k |\psi^0_k|^2 - \right.$

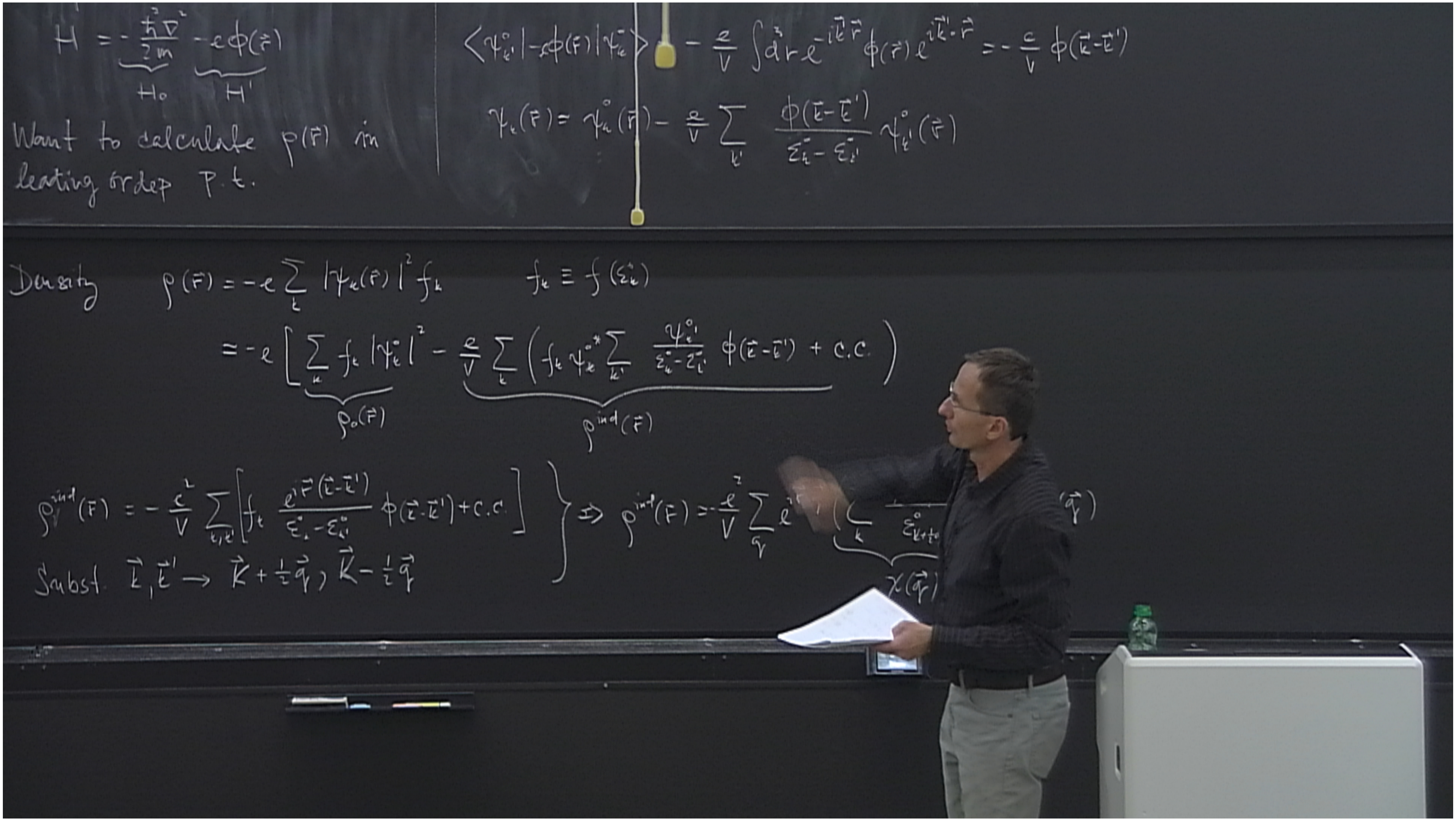

$H = -\frac{\hbar^2\nabla^2}{2m} - e\phi(\vec{r})$ (H_0, H')
Want to calculate $\rho(\vec{r})$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec{r}) | \psi^0_k \rangle = -\frac{e}{V}\int d^3r\, e^{-i\vec{k}'\cdot\vec{r}}\phi(\vec{r})e^{i\vec{k}\cdot\vec{r}} = -\frac{e}{V}\phi(\vec{k}-\vec{k}')$
$\psi_k(\vec{r}) = \psi^0_k(\vec{r}) - \frac{e}{V}\sum_{k'}\frac{\phi(\vec{k}-\vec{k}')}{\varepsilon^0_k - \varepsilon^0_{k'}}\psi^0_{k'}(\vec{r})$
Density $\rho(\vec{r}) = -e\sum_k |\psi_k(\vec{r})|^2 f_k$ $f_k \equiv f(\varepsilon^0_k)$
$= -e\left[\sum_k f_k|\psi^0_k|^2 - \frac{e}{V}\sum_k\left(f_k\psi^{0*}_k\sum_{k'}\frac{\psi^0_{k'}}{\varepsilon^0_k-\varepsilon^0_{k'}}\phi(\vec{k}-\vec{k}') + c.c.\right)\right.$
$\rho_0(\vec{r})$
$\rho^{ind}(\vec{r})$

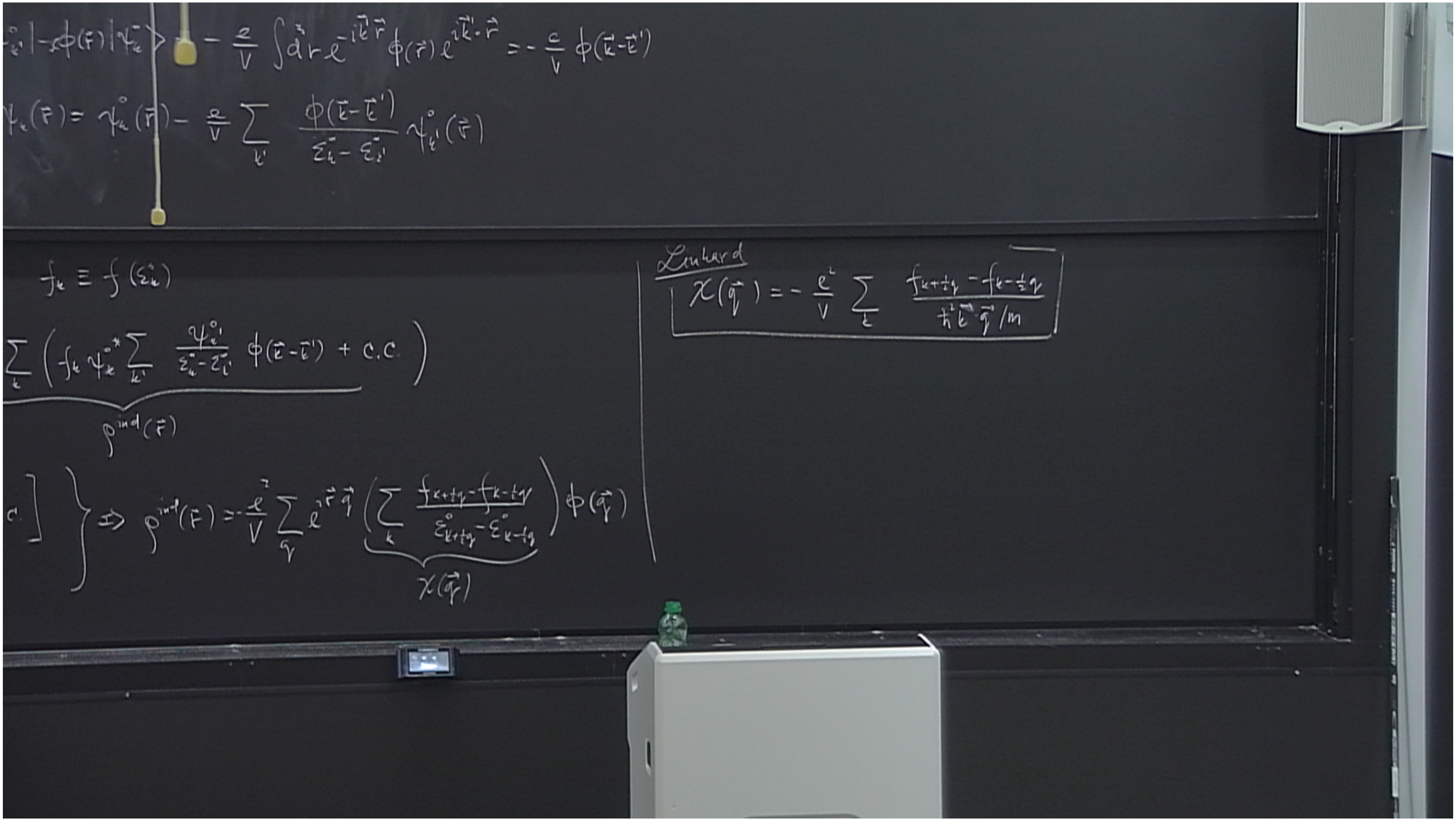


$H = -\frac{\hbar^2 \nabla^2}{2m} - e\phi(\vec{r})$
H_0 H'
Want to calculate $\rho(\vec{r})$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec{r}) | \psi^0_k \rangle = -\frac{e}{V} \int d^3r \, e^{-i\vec{k}'\cdot\vec{r}} \phi(\vec{r}) e^{i\vec{k}\cdot\vec{r}} = -\frac{e}{V} \phi(\vec{k}-\vec{k}')$
$\psi_k(\vec{r}) = \psi^0_k(\vec{r}) - \frac{e}{V} \sum_{k'} \frac{\phi(\vec{k}-\vec{k}')}{\varepsilon^0_k - \varepsilon^0_{k'}} \psi^0_{k'}(\vec{r})$
Density $\rho(\vec{r}) = -e \sum_k |\psi_k(\vec{r})|^2 f_k$
$f_k \equiv f(\varepsilon^0_k)$
$= -e \left[\sum_k f_k |\psi^0_k|^2 - \frac{e}{V} \sum_k \left(f_k \psi^{0*}_k \sum_{k'} \frac{\psi^0_{k'}}{\varepsilon^0_k - \varepsilon^0_{k'}} \phi(\vec{k}-\vec{k}') + c.c. \right) \right.$
$\rho_0(\vec{r})$
$\rho^{ind}(\vec{r})$


$H = -\frac{\hbar^2\nabla^2}{2m} - e\phi(\vec r)$, H_0, H'
Want to calculate $\rho(\vec r)$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec r) | \psi^0_k \rangle = -\frac{e}{V}\int d^3r\, e^{-i\vec k'\cdot\vec r}\phi(\vec r)e^{i\vec k\cdot\vec r} = -\frac{e}{V}\phi(\vec k-\vec k')$
$\psi_k(\vec r) = \psi^0_k(\vec r) - \frac{e}{V}\sum_{k'} \frac{\phi(\vec k-\vec k')}{\varepsilon^0_k - \varepsilon^0_{k'}}\psi^0_{k'}(\vec r)$
Density $\rho(\vec r) = -e\sum_k |\psi_k(\vec r)|^2 f_k$ $f_k \equiv f(\varepsilon^0_k)$
$= -e\left[\sum_k f_k|\psi^0_k|^2 - \frac{e}{V}\sum_k\left(f_k\psi^{0*}_k\sum_{k'}\frac{\psi^0_{k'}}{\varepsilon^0_k-\varepsilon^0_{k'}}\phi(\ldots) + c.c.\right)\right]$
$\rho_0(\vec r)$ $\rho^{ind}(\vec r)$
$\rho^{ind}(\vec r) = -\frac{e^2}{V}\sum_{k,k'}\left[f_k\frac{e^{i\vec r(\vec k-\vec k')}}{\varepsilon^0_k-\varepsilon^0_{k'}}\phi(\vec k-\vec k') + c.c.\right]$


$H = -\frac{\hbar^2\nabla^2}{2m} - e\phi(\vec r)$
H_0 H'
Want to calculate $\rho(\vec r)$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec r) | \psi^0_k \rangle = -\frac{e}{V}\int d^3r\, e^{-i\vec k'\cdot\vec r}\phi(\vec r)e^{i\vec k\cdot\vec r} = -\frac{e}{V}\phi(\vec k-\vec k')$
$\psi_k(\vec r) = \psi^0_k(\vec r) - \frac{e}{V}\sum_{k'}\frac{\phi(\vec k-\vec k')}{\varepsilon_k-\varepsilon_{k'}}\psi^0_{k'}(\vec r)$
Density $\rho(\vec r) = -e\sum_k |\psi_k(\vec r)|^2 f_k$ $f_k \equiv f(\varepsilon^0_k)$
$= -e\left[\sum_k f_k|\psi^0_k|^2 - \frac{e}{V}\sum_k\left(f_k\psi^{0*}_k\sum_{k'}\frac{\psi^0_{k'}}{\varepsilon^0_k-\varepsilon^0_{k'}}\phi(\vec k-\vec k') + c.c.\right)\right]$
$\rho_0(\vec r)$ $\rho^{ind}(\vec r)$
$\rho^{ind}(\vec r) = -\frac{e^2}{V}\sum_{k,k'}\left[f_k\frac{e^{i\vec r\cdot(\vec k-\vec k')}}{\varepsilon^0_k-\varepsilon^0_{k'}}\phi(\vec k-\vec k')+c.c.\right]$
Subst. $\vec k,\vec k' \to \vec K+\frac{1}{2}\vec q,\ \vec K-\frac{1}{2}\vec q$
$\Rightarrow \rho^{ind}(\vec r) = -\frac{e^2}{V}\sum_q e^{i\vec r\cdot\vec q}\left(\sum_k \frac{f_{k+\frac12 q}-f_{k-\frac12 q}}{\varepsilon^0_{k+\frac12 q}-\varepsilon^0_{k-\frac12 q}}\right.$


$H = -\frac{\hbar^2\nabla^2}{2m} - e\phi(\vec r)$
H_0 H'
Want to calculate $\rho(\vec r)$ in leading order p.t.
$\langle \psi^0_{k'} | -e\phi(\vec r) | \psi^0_k \rangle = -\frac{e}{V}\int d^3r\, e^{-i\vec k'\cdot\vec r}\phi(\vec r) e^{i\vec k\cdot\vec r} = -\frac{e}{V}\phi(\vec k - \vec k')$
$\psi_k(\vec r) = \psi^0_k(\vec r) - \frac{e}{V}\sum_{k'} \frac{\phi(\vec k - \vec k')}{\varepsilon^0_k - \varepsilon^0_{k'}} \psi^0_{k'}(\vec r)$
Density $\rho(\vec r) = -e\sum_k |\psi_k(\vec r)|^2 f_k$ $f_k \equiv f(\varepsilon^0_k)$
$= -e\left[\sum_k f_k |\psi^0_k|^2 - \frac{e}{V}\sum_k \left(f_k \psi^{0*}_k \sum_{k'} \frac{\psi^0_{k'}}{\varepsilon^0_k - \varepsilon^0_{k'}} \phi(\vec k - \vec k') + c.c. \right)\right]$
$\rho_0(\vec r)$ $\rho^{ind}(\vec r)$
$\rho^{ind}(\vec r) = -\frac{e^2}{V}\sum_{k,k'}\left[f_k \frac{e^{i\vec r(\vec k - \vec k')}}{\varepsilon^0_k - \varepsilon^0_{k'}} \phi(\vec k - \vec k') + c.c. \right]$
Subst. $\vec k, \vec k' \to \vec K + \frac{1}{2}\vec q, \vec K - \frac{1}{2}\vec q$
$\Rightarrow \rho^{ind}(\vec r) = -\frac{e^2}{V}\sum_q e^{i\vec q \cdot \vec r}$
$\chi(\vec q)$


Lindhard