Title: Quantum Field Theory II -15

Date: Nov 28, 2014 09:00 AM

URL: http://pirsa.org/14110024

Abstract:

1 | Spontaneous Continuous Symmetry Breaking & Nambu-Goldston

Abelian $U(1)$ sym

Scalar Complex $\phi = \phi_1 + i\phi_2$

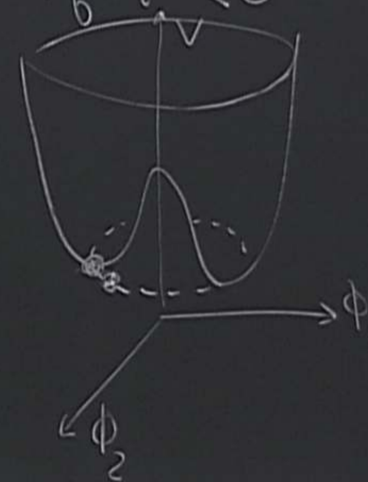
$\bar{\phi} = \phi_1 - i\phi_2$

Euclidean Signature (Lorentz)

$$S = \int d^D x \, \frac{1}{2} \partial_\nu \phi \, \partial^\nu \bar{\phi} + V(\phi)$$

$$V(\phi) = \frac{t}{2} \phi\bar{\phi} + \frac{g}{8}(\phi\bar{\phi})^2$$

if $t<0$



$\langle \phi \rangle = \phi_0 e^{i\alpha}$ α a p

$\phi_0 = \sqrt{\frac{-2t}{g}}$ degenerate ground s

$V = (|\phi| - \phi_0)^2 \cdot \frac{g}{2}\phi_0^2 +$

$\alpha_0 = 0$ $\phi = \begin{pmatrix} \phi_0 + H \\ \pi \end{pmatrix}$ $V=$

Goldstone Boson

$e^{i\alpha}$ α a phase $\quad$ $\text{Action} = \int d^D y \; \frac{1}{2}(\partial H)^2 + \frac{1}{2}(\partial \pi)^2 + V$

degenerate ground state $\quad$ H mass $g\phi_0^2$

π massless

$-\phi_0)^2 - \frac{g}{2}\phi_0^2 + cst$

$\phi_0 + H$

π

$$V = \left(H^2 + 2\pi\phi_0 + \pi^2\right)^2 \frac{g}{8} = \frac{g}{8}\Big(\underbrace{4\phi_0^2 H^2}_{\text{mass}} + 4H^3\phi_0 + 4\phi_0 H\pi^2 + 2H^2\pi^2 + H^4\Big)$$

Goldstone Boson

$e^{i\alpha}$ α a phase

$$\text{Action} = \int d^D x \left[\frac{1}{2}(\partial_\mu H)^2 + \frac{1}{2}(\partial_\mu \pi)^2 + V \right]$$

degenerate ground state

2 new fields Real-Scalar

H mass $g\,\phi_0^2 = M^2$

π massless : Nambu-Goldstone Boson

(general theorem)

$(-\phi_0)^2 \ \ \frac{g}{2}\phi_0^2 + cst$

$\phi_0 + H$

π

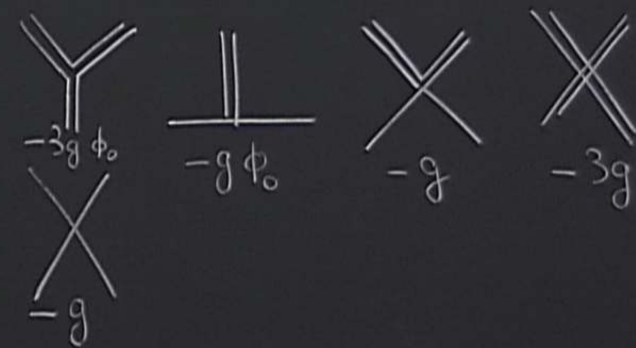
$H \text{ === } H = \frac{1}{p^2+M^2}$

$\pi \text{ — } \pi \quad \frac{1}{p^2}$

U(1) Sym.

$\Downarrow$

relations



$$V = \left(H^2 + 2\pi\phi_0 + \pi^2\right)^2 \frac{g}{8} = \frac{g}{8}\left(\overset{\text{mass}}{\underbrace{4\phi_0^2 H^2}} + 4H^3\phi_0 + 4\phi_0 H\pi^2 + 2H^2\pi^2 + H^4 + (\pi^2)^2\right)$$

$U(1)$ 1 Goldstone Boson. $U(1) \to \mathbb{1}d$.

$SU(2)$ several Goldstone Bosons

$SU(2) \to U(1)$ different patterns of S.S.B.

$G \to H$ subgroup

$\dim\left[G/H\right] = \#$ Goldstone Bosons

Linear - Sigma - Model

Nambu - Jona Lasinio model

Quantum Theory: the charge operator $Q = \int d^3\vec{x} \cdot J^0(x)$

SSB several vacua $|\Omega_\alpha\rangle$ $Q|\Omega_\alpha\rangle \neq 0$

no SSB: one vaccum $Q|\Omega\rangle = 0$

Φ charged scalar field

1 particle $(+e) \neq$ antiparticle $(-e)$

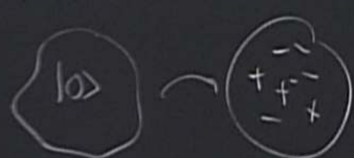
coupling to $U(1)$ gauge field

A_μ gauge potential

$t<0$ gain energy by creating pair.

$p, \bar{p}$ particles

classical vacuum is unstable

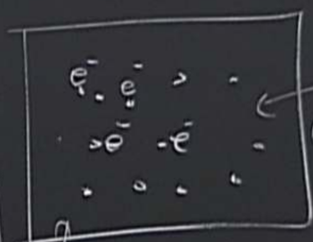


finite density of $p, \bar{p}$

Plasma physics (elementary)

Plasma waves or Langmuir waves

Langmuir, Tonks 28'



free electrons with density ρ

fixed ion background $+e$ density ρ

electrons charge e

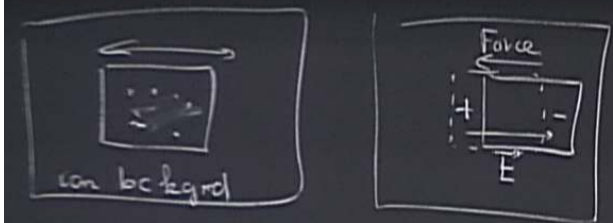
mass m

density ρ

neglect temperature

waves oscillating with frequency $\omega_p^2 = \dfrac{\rho e^2}{m \varepsilon_0}$

SSB several vacua $|\Omega_\alpha\rangle$ $\quad Q|\Omega_\alpha\rangle \neq 0 \quad | \quad Q|\Omega\rangle = 0$



oscillation of the electrons

current j

Resonance

$\bar{v}$ velocity of the electron

Maxwell equations

$$\cancel{\nabla \times B} - \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} = \mu_0 j$$

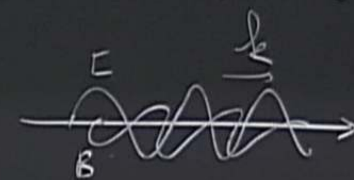
$B = 0$

current $\vec{j} = e\rho \cdot \vec{v}$

$$m\frac{dv}{dt} = e\left(\vec{E} + v \times \cancel{B}\right)$$

$$\frac{d^2 j}{dt^2} = -\frac{e^2 \rho}{\epsilon_0 m} j \qquad E \sim \frac{dj}{dt}$$

E M wave in the vacuum



2 polarizations transverse

$\vec{E} \cdot \vec{k} = 0$

speed $= c$

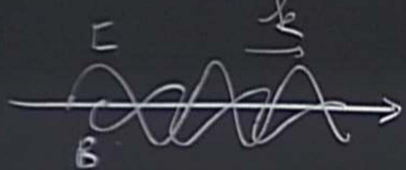
$K = [E, \vec{k}]$

$K^2 = 0$

In a plasma:

finite density of $p, \bar{p}$

EM wave in the vacuum



2 polarizations transverse

$\vec{E}\cdot\vec{k}=0$

speed = c $K=[E,\vec{k}]$

$K^2=0$

In a plasma:

you can have EM waves at rest !

3 directions (polarizations)

"photon" massive   quantum of plasma oscill.

3 polarizations   Plasmon $E_{pl} \simeq \hbar\omega_p$ $M_{pl}=\frac{\hbar\omega_p}{c^2}$

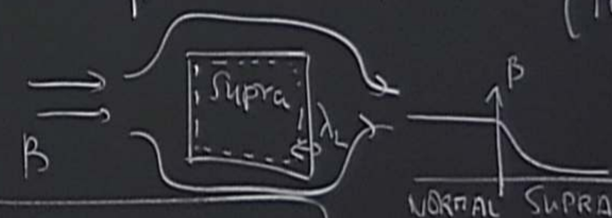
Plasmons in Supracond

Cooper Pairs $\pm 2e$, e

condensate $\approx$ plasma . d

$\frac{c}{\omega_{pl}} = \lambda_L$ London penetrati

(Meiss



finite density of $p, \bar{p}$

anizations transverse
$k = 0$
$d = c \qquad K = [E, \bar{h}]$
$K^2 = 0$
est !
(polarizations)
massive
...ons

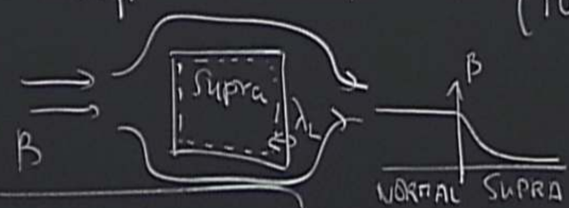
Plasmons in Supraconductors

Cooper Pairs $\pm 2e$, effective mass m^*

condensate $\approx$ plasma : density $\rho \qquad \omega_p^2 = \frac{\rho (2e)^2}{m^* \varepsilon_0}$

$\frac{c}{\omega_{pl}} = \lambda_L$ London penetration length

(Meissner Effect)



Landau-Ginzburg phenomeno-mological theory

charged condensate (BE type)

wave function $\Psi(x)$

quantum of plasma oscill.

Plasmon $E_{pl} \approx \hbar \omega_p \quad M_{pl} = \frac{\hbar \omega_p}{c^2}$

$$F_{LG} = \frac{1}{2m^*}\left|\left(-i\hbar\nabla - 2eA\right)\psi\right|^2 + \underbrace{\alpha|\psi|^2 + \frac{\beta}{2}|\psi^4|}_{\text{phenomenomenological potential}} + \frac{|B^2|}{2\mu_0}$$

covariant derivative

$F^2_{\mu\nu}$

$E = 0;\ B \neq 0$

no time dependence

finite density potential $\rho = |\psi|^2 \neq 0$

finite density of $p, \bar{p}$

$m \, \varepsilon_0$

Plasmons in Supraconductors

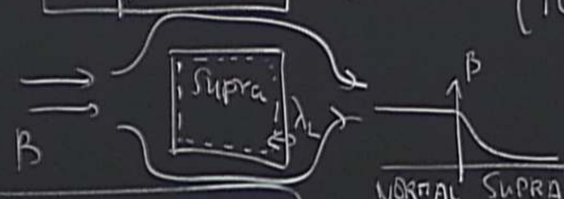
$\boxed{\frac{1}{c^2} = \varepsilon_0 \mu_0}$

Cooper Pairs $\pm 2e$, effective mass m^*

condensate $\approx$ plasma. density ρ

$\boxed{\omega_p^2 = \frac{\rho (2e)^2}{m^* \varepsilon_0}}$

$\boxed{\frac{c}{\omega_{pl}} = \lambda_L}$ London penetration length (Meissner Effect)



Landau-Ginzburg phenomeno-mological theory

charged condensate (BE type)

Wave function $\Psi(x)$

...zations transverse

$k = 0$

$= c$ $\quad K = [E, \frac{E}{\hbar}]$

$K^2 = 0$

!

(...larizations)

...ssive quantum of plasma oscill.

Plasmon $E_{pl} \simeq \hbar \omega_p$ $\quad M_{pl} = \frac{\hbar \omega_p}{c^2}$

$$F_{LG} = \frac{1}{2m^*}\left|\left(-i\hbar\nabla - 2eA\right)\psi\right|^2 + \underbrace{\alpha|\psi|^2 + \frac{\beta}{2}|\psi^4|}_{\text{phenomenological potential}} + \frac{|B^2|}{2\mu_0}$$

covariant derivative

$F_{\mu\nu}^2$

'63 P. Anderson.

$E = 0;\ B \neq 0$

no time dependence

finite density potential $\quad \rho = |\psi|^2 \neq 0$

$$\boxed{\text{Penetration length} \quad \lambda_L = \sqrt{\frac{m^*}{4e^2\mu_0|\psi_0|^2}}}$$

'63 P. Anderson. This may be used in HE Physics to obtain massive gauge bosons

'64 Brout - Englert
Guralnik - Hagen - Kibble
Higgs

SSB + Gauge Field in a QFT setting

space (A_μ, ϕ)

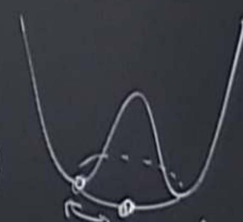
gauge transf. $\phi(x) \to \phi(x) e^{i\alpha(x)}$

$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$

ϕ complex, A_μ $U(1)$ gauge field

$$\int dx \; \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 + \frac{1}{2} D_\mu\phi \cdot \overline{D^\mu \phi} + V(\phi)$$

$$D_\mu = \partial_\mu - i e A_\mu$$



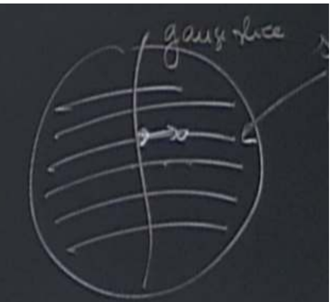
classically $\phi = \phi_0 e^{i\alpha(x)}$

classical vacuum ϕ_0 real

A_μ = pure gauge = $\partial_\mu \alpha(x)$

not really a degeneracy of the classical gr. st.

$$\boxed{\lambda_L = \sqrt{\frac{m^*}{4e^2\mu_0|\psi_0|^2}}}$$



Fix the gauge on ϕ, not on A

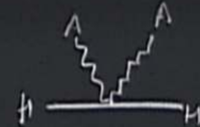
Unitary Gauge $\phi = \begin{pmatrix}\Phi\\ 0\end{pmatrix}$ $\phi = \Phi_0 + H$

$\pi = 0$ (Gauge invariance) ϕ_0 minimum of the potential

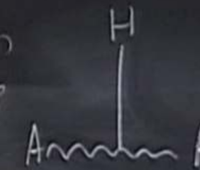
but coupling between ϕ and A_μ

mass for H

Photon A_μ gets a mass $\boxed{m_A^2 = e^2\phi_0^2}$

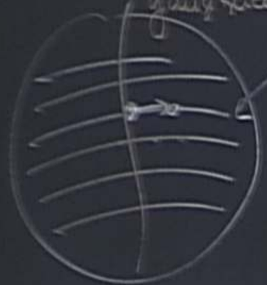


Mass Term for the Gauge Field!



$$\boxed{\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2} + \frac{1}{2}\partial_\mu H \partial^\mu H + \left(\frac{M^2}{2}H^2 + \frac{M^2}{\Phi_0}H^3 + \frac{M^4}{8\Phi_0^2}H^4\right) + \frac{1}{2}e^2H^2A_\mu^2 + \boxed{\frac{1}{2}e^2\phi_0^2A_\mu^2} + e^2\phi_0 H A_\mu A^\mu$$

Maxwell term kinetic term for H H coupling in the N.JS model gauge slice



space (A_μ, ϕ)

gauge transf. $\phi(x) \to \phi(x) e^{i\alpha(x)}$

$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$

classical vacuum $A_\mu = \text{pure gauge} = \partial_\mu \alpha(x)$

not really a degeneracy of the classical gr. st.

$= e^2 \phi_0^2$

Propagator $G_{\mu\nu}(p) = \frac{-i}{k^2 + M_A^2 - i\epsilon_+}\left(h_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)$

mass

Term

Gauge

eld!

H

A A

$\phi_0^2 A_\mu^2 + e^2 \phi_0 H A_\mu A^\mu$

gauge transf. $\phi(x) \to \phi(x) e^{i\alpha(x)}$

$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$

not really a degeneracy of the classical gr. st.

Propagator $G_{\mu\nu}(p) = \frac{-i}{k^2 + M_A^2 - i\epsilon_+}\left(h_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)$

$\subset$ mass

photon is massive, 3 polarizations

it can be at rest, still transverse

$K_{rest} = (M_A, \vec{0})$ $\quad \epsilon = (0, \vec{\epsilon})$

massless photon 2 states 2 + 2 uncoupled

charged boson H Π

massive Goldst.

massive photon 3 states 3 + 1 coupled

1 massive boson H (Higgs Boson)

Propagator $G_{\mu\nu}(p) = \frac{-i}{k^2+M_A^2-i\epsilon_+}\left(h_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)$

mass

photon is massive, 3 polarizations
it can be at rest still transverse

$K_{rest} = (M_A, \vec{0})$ $\quad \epsilon = (0, \vec{\epsilon})$

massless photon 2 states 2 + 2 uncoupled
charged boson H massive Π Goldstone

massive photon 3 states 3 + 1 coupled
1 massive boson H | Higgs Boson

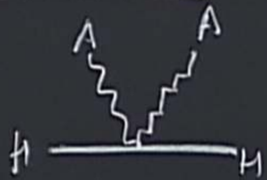
The Goldstone Boson has been "eaten" by the gauge field

$\phi = \phi_0 + H$

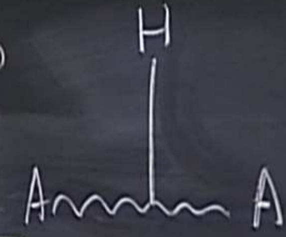
ϕ_0 minimum of the potential

mass for H

Photon A_μ gets a mass $\boxed{m_A^2 = e^2 \phi_0^2}$ = nothing but $\boxed{E^2_{\text{plasmon}}}$



Mass Term for the Gauge Field!



Propagator

photon is

it can be

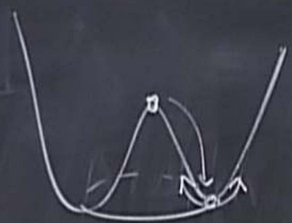
$K_{rest} =$

$$\dots H + \left(\frac{M^2}{2} H^2 + \frac{M^2}{\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right) + \frac{1}{2} e^2 H^2 A_\mu^2 + \boxed{\frac{1}{2} e^2 \phi_0^2 A_\mu^2} + e^2 \phi_0 H A_\mu A^\mu$$

for H

H coupling in the N-JS model

$B=0$ $\qquad \frac{d^2y}{dt^2} = -\frac{e^2\rho}{\epsilon_0 m} y \quad E \sim \frac{dy}{dt}$ $\qquad$ 3 polarizations

*  $\Rightarrow$ Neutral condensate of charged particles + gauge field $\Longleftrightarrow$ Higgs Mechanism. "Plasmons" New massive gauge fields 3 polarizations

* "physically" Higgs Mech $\Leftarrow$ SSB

* mathematically one vaccum $|\Omega\rangle$ $\quad \varphi|\Omega\rangle = 0$

no breaking of the symmetry

no SSB for a local gauge invariance!

n of $E \sim \frac{dy}{dt}$

3 polarizations

Plasmon $E_{pl} \approx \hbar\omega_p$ $M_{pl} = \frac{\hbar\omega_p}{c^2}$

te of
les $\Longleftrightarrow$ Higgs Mechanism

"Plasmons"
New massive gauge fields
3 polarization

+

Higgs Boson
fluctuation in the density of the condensate
"phonon"

$|\Omega\rangle = 0$

ce!

Standard Model of E.W interactions

$\underbrace{U(1) \times SU(2)}_{\Downarrow}$ symmetry + SU(2) Scalar

$U(1)_{QED}$ symmetry

Higgs Mechanism — "Plasmons" — Higgs Boson

$\Leftrightarrow$

New massive gauge fields 3 polarization + fluctuation in the density of the condensate "phonon"

Standard Model of E.W interactions

A_μ: $\underbrace{U(1)\times SU(2)}$ symmetry + $SU(2)$ Scalar ϕ

$\Downarrow$ $\qquad\qquad\qquad\qquad \Downarrow$

$U(1)_{QED}$ symmetry massless γ EM — 3 massive vector bosons Z, W_+, W_- + Higgs