Title: Quantum Field Theory II -15

Date: Nov 28, 2014 09:00 AM

URL: http://pirsa.org/14110024

Abstract:

1] Spontaneous Continuous Symmetry Breaking & Nambu-Goldston

Abelian $U(1)$ sym

Scalar Complex $\phi = \phi_1 + i\phi_2$

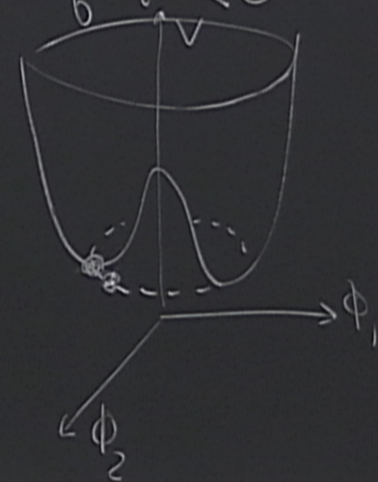
$\bar{\phi} = \phi_1 - i\phi_2$

Euclidean Signature (Lorentz)

$$S = \int d^D x \, \frac{1}{2} \partial_\mu \phi \, \partial^\mu \bar{\phi} + V(\phi)$$

$$V(\phi) = \frac{t}{2} \phi\bar{\phi} + \frac{g}{8} (\phi\bar{\phi})^2$$

if $t<0$



$\langle \phi \rangle = \phi_0 e^{i\alpha}$ $\quad \alpha$ a p

degenerate ground s

$$\phi_0 = \sqrt{\frac{-2t}{g}}$$

$$V = (|\phi| - \phi_0)^2 \cdot \frac{g}{2} \phi_0^2 +$$

$\alpha_0 = 0 \qquad \phi = \begin{pmatrix} \phi_0 + H \\ \pi \end{pmatrix} \qquad V=$

Goldstone Boson

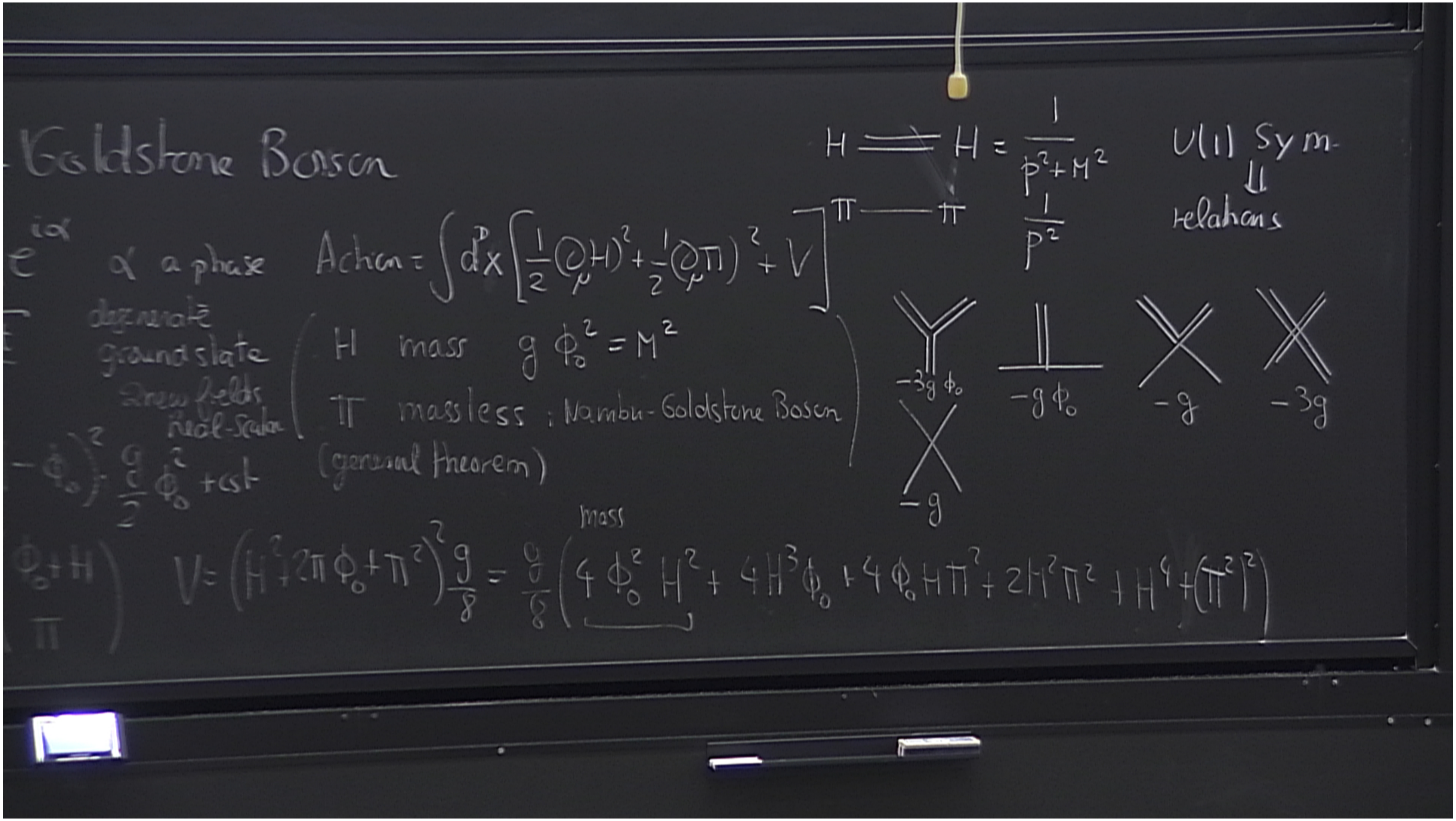
$e^{i\alpha}$ α a phase Action $= \int d^D y \; \frac{1}{2}(\partial H)^2 + \frac{1}{2}(\partial \pi)^2 + V$

degenerate ground state H mass $g\,\phi_0^2$

π massless

$(\ldots - \phi_0)^2 - \frac{g}{2}\phi_0^2 + cst$

$\begin{pmatrix} \phi_0 + H \\ \pi \end{pmatrix}$ $V = (H^2 + 2\pi\phi_0 + \pi^2)^2 \frac{g}{8} = \frac{g}{8}\Big(\underbrace{4\phi_0^2 H^2}_{\text{mass}} + 4H^3\phi_0 + 4\phi_0 H \pi^2 + 2H^2\pi^2 + H^4\Big)$


Goldstone Boson
e^{iα}
α a phase
degenerate ground state
2 new fields Real-Scalar
Action = ∫d^D x [1/2 (∂_μ H)^2 + 1/2 (∂_μ π)^2 + V]
H mass g φ_0^2 = M^2
π massless : Nambu-Goldstone Boson
(general theorem)
g/2 φ_0^2 + cst
mass
V = (H^2 + 2π φ_0 + π^2)^2 g/8 = g/8 (4 φ_0^2 H^2 + 4H^3 φ_0 + 4 φ_0 H π^2 + 2H^2 π^2 + H^4 + (π^2)^2)
H = H = 1/(p^2+M^2)
π — π 1/p^2
U(1) Sym.
relations
−3g φ_0
−g φ_0
−g
−3g
−g

$U(1)$ 1 Goldstone Boson. $U(1) \to Id.$

$SU(2)$ several Goldstone Bosons

$SU(2) \to U(1)$ different patterns of S.S.B.

$G \to H$ subgroup

$\dim[G/H] = \#$ Goldstone Bosons

Linear - Sigma - Model

Nambu - Jona Lasinio model

Quantum Theory ; the charge operator $Q = \int d^3\vec{x} \cdot J(x)$

SSB several vacua $|\Omega_\alpha\rangle$ $Q|\Omega_\alpha\rangle \neq 0$

no SSB : one vacuum $Q|\Omega\rangle = 0$

Φ charged scalar field

1 particle $(+e) \neq$ antiparticle $(-e)$

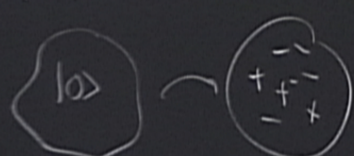
coupling to $U(1)$ gauge field

A_μ gauge potential

$t<0$ gain energy by creating pair.

$p, \bar{p}$ particles

classical vacuum is unstable



finite density of $p, \bar{p}$

Plasma physics (elementary)

Plasma waves or Langmuir Waves

Langmuir, Tonks 28'

free electrons with density ρ

fixed ion background $+e$ density ρ

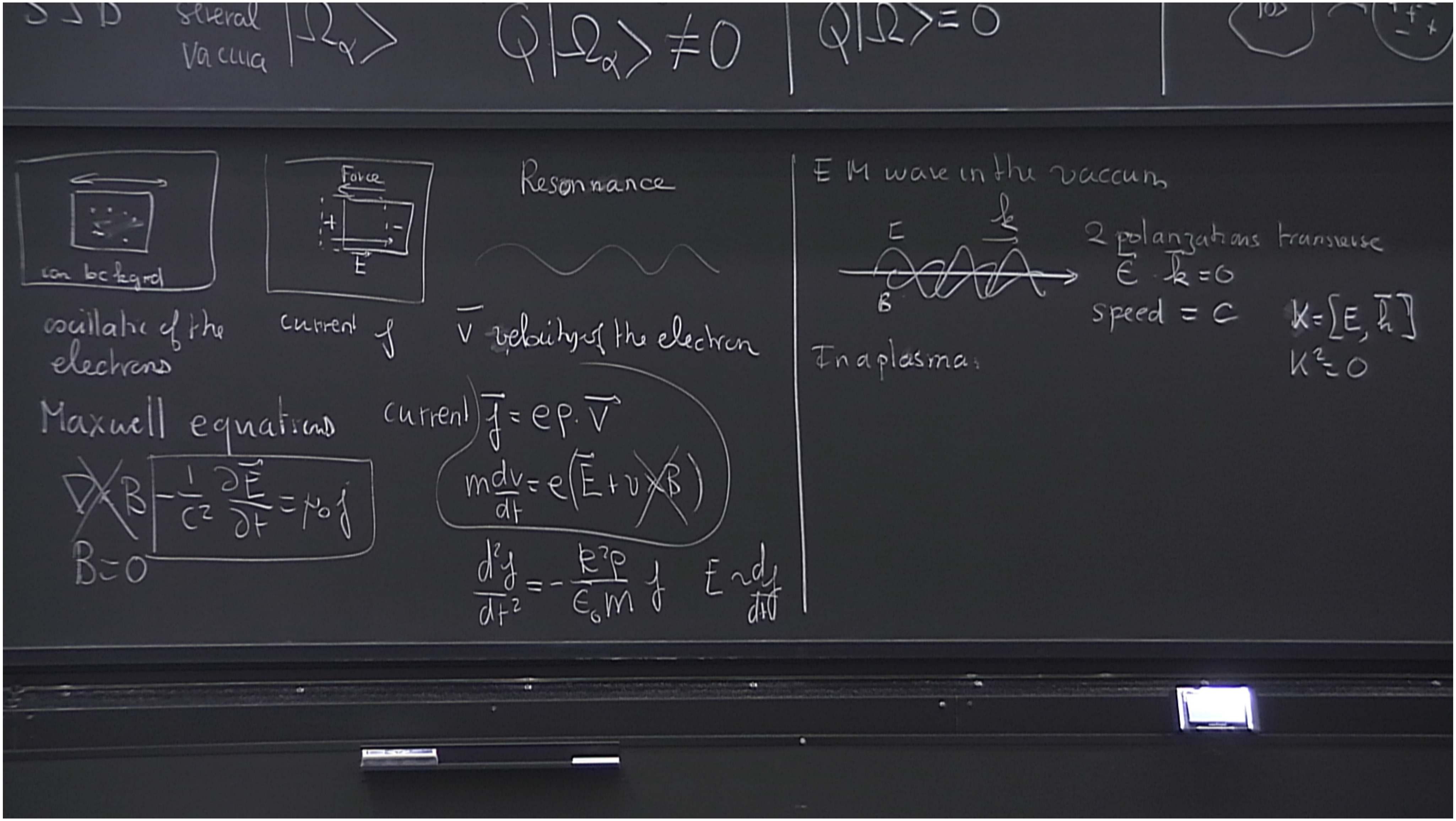
electrons charge e

mass m

density ρ

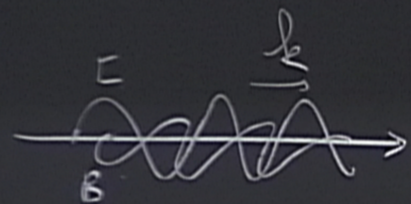
neglect temperature

waves oscillating with frequency $\omega_p^2 = \dfrac{\rho e^2}{m \varepsilon_0}$


several vacua
Q|Ωα> ≠ 0
Q|Ω> = 0
ion bckgrd
Force
E
oscillatic of the electrons
current j
Resonnance
v velocity of the electron
Maxwell equations
current j = eρ·v
B = 0
EM wave in the vaccum
E
k
B
2 polarizations transverse
E·k = 0
speed = c
K = [E, h]
K² = 0
In a plasma:

finite density of p, p

EM wave in the vacuum



2 polarizations transverse

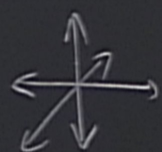
$\vec{E}\cdot\vec{k}=0$

speed = c $K=[E,\vec{h}]$

$K^2=0$

In a plasma:

you can have EM waves at rest !



3 directions (polarizations)

"photon" massive   quantum of plasma oscill.

3 polarizations   Plasmon $E_{pl} \simeq \hbar\omega_p$ $M_{pl}=\frac{\hbar\omega_p}{c^2}$

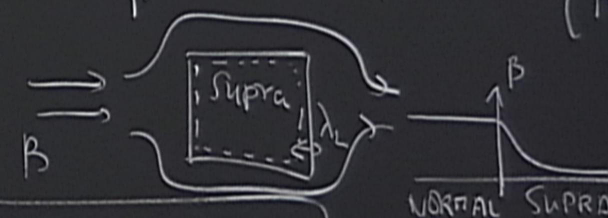
Plasmons in Supracond

Cooper Pairs ± 2e , e

condensate ≈ plasma · d

$\frac{c}{\omega_{pl}} = \lambda_L$ London penetrati

(Meiss



finite density of $p, \bar{p}$

...anzations transverse

$k = 0$

$d = c$ $\quad K = [E, \bar{h}]$

$K^2 = 0$

...est !

...polarizations)

...massive quantum of plasma oscill.

...ons Plasmon $E_{pl} \simeq \hbar\omega_p$ $\quad M_{pl} = \frac{\hbar\omega_p}{c^2}$

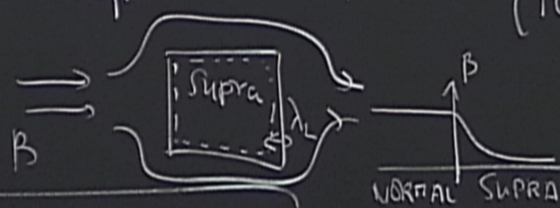
Plasmons in Supraconductors

Cooper Pairs $\pm 2e$, effective mass m^*

condensate $\approx$ plasma: density ρ $\quad \omega_p^2 = \frac{\rho(2e)^2}{m^* \varepsilon_0}$

$\frac{c}{\omega_{pl}} = \lambda_L$ London penetration length

(Meissner Effect)



Landau-Ginzburg phenomenological theory

charged condensate (BE type)

Wave function $\Psi(x)$

$$F_{LG} = \frac{1}{2m^*}\left|\left(-i\hbar\nabla - 2eA\right)\psi\right|^2 + \underbrace{\alpha|\psi|^2 + \frac{\beta}{2}|\psi^4|}_{\text{phenomenenenological potential}} + \frac{|B^2|}{2\mu_0}$$

covariant derivative

$F^2_{\mu\nu}$

$E = 0;\ B \neq 0$

no time dependence

finite density potential $\rho = |\psi|^2 \neq 0$

finite density of $p, \bar{p}$

Plasmons in Supraconductors

$\boxed{\frac{1}{c^2} = \varepsilon_0 \mu_0}$

Cooper Pairs $\pm 2e$, effective mass m^*

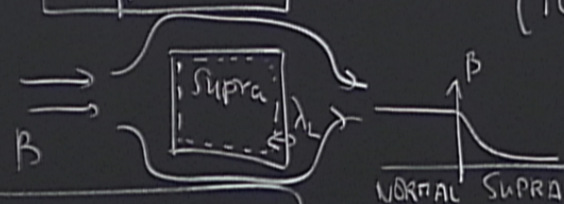
condensate $\approx$ plasma · density ρ

$$\boxed{\omega_p^2 = \frac{\rho (2e)^2}{m^* \varepsilon_0}}$$

$$\boxed{\frac{c}{\omega_{pl}} = \lambda_L}$$

London penetration length

(Meissner Effect)



Landau-Ginzburg phenomenological theory

charged condensate (BE type)

Wave function $\Psi(x)$

...zations transverse

$k = 0$

$= c$ $\quad K = [E, \frac{\hbar}{\ }]$

$K^2 = 0$

...larizations)

...ssive $\quad$ quantum of plasma oscill.

Plasmon $\quad E_{pl} \approx \hbar \omega_p \quad M_{pl} = \frac{\hbar \omega_p}{c^2}$

$$F_{LG} = \frac{1}{2m^*}\left|\left(-i\hbar\nabla - 2eA\right)\psi\right|^2 + \alpha|\psi|^2 + \frac{\beta}{2}|\psi^4| + \frac{|B^2|}{2\mu_0}$$

covariant derivative

phenomenenenological potential

$F_{\mu\nu}^2$

'63 P. Anderson.

$E=0;\ B\neq 0$

no time dependence

finite density potential

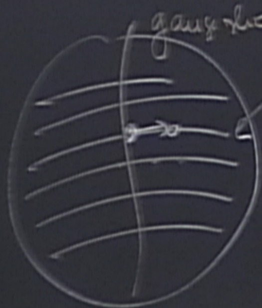
$\rho = |\psi|^2 \neq 0$

$$\boxed{\text{Penetration length} \quad \lambda_L = \sqrt{\frac{m^*}{4e^2\mu_0|\psi_0|^2}}}$$

'63 P. Anderson. This may be used in HE Physics to obtain massive gauge bosons

'64 Brout-Englert
Guralnik-Hagen-Kibble
Higgs

SSB + Gauge Field in a QFT setting



space (A_μ, ϕ)

gauge transf $\phi(x) \to \phi(x) e^{i\alpha(x)}$

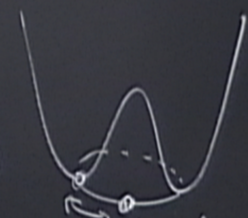
$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$

ϕ complex, A_μ $U(1)$ gauge field

$$\int dx \; \frac{1}{4}\left(\partial_\mu A_\nu - \partial_\nu A_\mu\right)^2 + \frac{1}{2} D_\mu \phi \cdot \overline{D^\mu \phi} + V(\phi)$$

$$D_\mu = \partial_\mu - i e A_\mu$$

classically $\phi = \phi_0 e^{i\alpha(x)}$

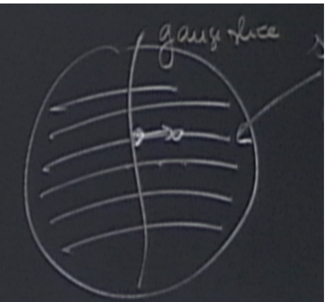


classical vacuum ϕ_0 real

A_μ = pure gauge = $\partial_\mu \alpha(x)$

not really a degeneracy of the classical g.st

$$\boxed{\lambda_L = \sqrt{\frac{m^*}{4e^2\mu_0|\phi_0|^2}}}$$

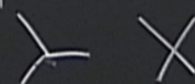


Fix the gauge on ϕ, not on A

Unitary Gauge $\phi = \begin{pmatrix}\phi \\ 0\end{pmatrix}$ $\quad \phi = \Phi_0 + H$

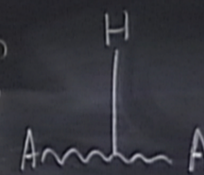
$\pi = 0$ (Gauge invariance) $\quad \phi_0$ minimum of the potential

but coupling between ϕ and A_μ $\quad$ mass for H

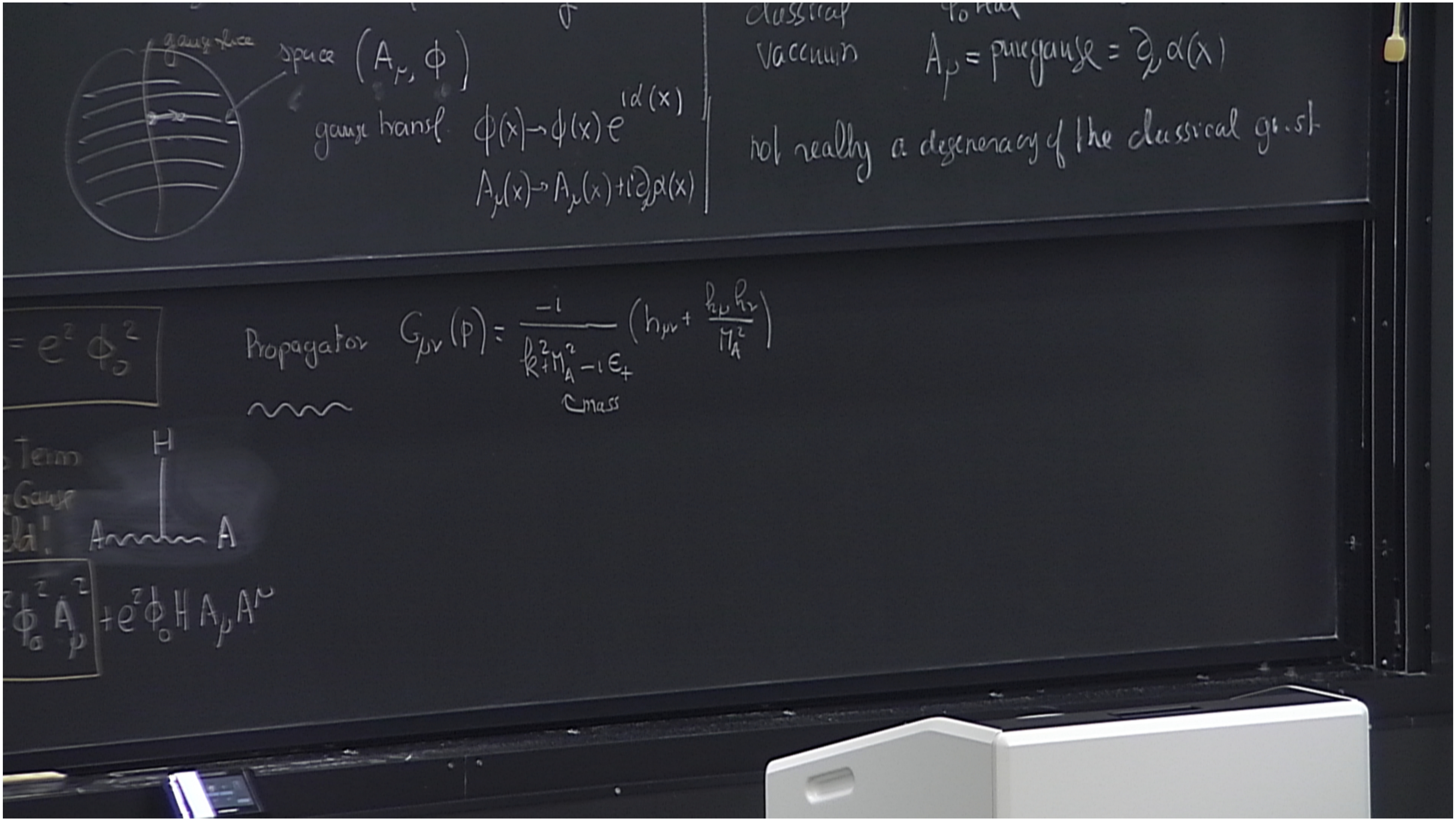


Photon A_μ gets a mass $\boxed{m_A^2 = e^2\phi_0^2}$

Mass Term for the Gauge Field!



$$\underbrace{\boxed{\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2}}_{\text{Maxwell term}} + \underbrace{\frac{1}{2}\partial_\mu H\partial^\mu H}_{\text{kinetic term for H}} + \underbrace{\left(\frac{M^2}{2}H^2 + \frac{M^2}{\Phi_0}H^3 + \frac{M^4}{8\phi_0^2}H^4\right)}_{\text{H coupling in the N.J.S model}} + \frac{1}{2}e^2H^2A_\mu^2 + \boxed{\frac{1}{2}e^2\phi_0^2A_\mu^2} + e^2\phi_0 H A_\mu A^\mu$$


gauge slice
space (A_μ, ϕ)
gauge transf $\phi(x) \to \phi(x) e^{i\alpha(x)}$
$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$
classical vacuum $A_\mu = \text{pure gauge} = \partial_\mu \alpha(x)$
not really a degeneracy of the classical gr. st.
$= e^2 \phi_0^2$
Propagator $G_{\mu\nu}(p) = \frac{-i}{k^2 + M_A^2 - i\epsilon_+} \left(h_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2} \right)$
mass
Term
Gauge
H
A A
$+ e^2 \phi_0 H A_\mu A^\mu$

gauge transf $\phi(x) \to \phi(x) e^{i\alpha(x)}$

$A_\mu(x) \to A_\mu(x) + i\partial_\mu \alpha(x)$

not really a degeneracy of the classical g.s.

Propagator $G_{\mu\nu}(p) = \frac{-i}{k^2 + M_A^2 - i\epsilon_+}\left(h_{\mu\nu} + \frac{k_\mu k_\nu}{M_A^2}\right)$

mass

photon is massive, 3 polarizations

it can be at rest, still transverse

$K_{rest} = (M_A, \vec{0})$ $\quad \epsilon = (0, \vec{\epsilon})$

massless photon 2 states 2

charged boson H massive, Π Goldstone + 2

uncoupled

massive photon 3 states 3

1 massive boson H (Higgs Boson) +

coupled

Propagator $G_{\mu\nu}(p) = \dfrac{-i}{k^2+M_A^2-i\epsilon_+}\left(h_{\mu\nu}+\dfrac{k_\mu k_\nu}{M_A^2}\right)$

$\underbrace{M_A^2}_{\text{mass}}$

photon is massive, 3 polarizations
it can be at rest, still transverse

$K_{rest} = (M_A, \vec{0}) \qquad \epsilon = (0, \vec{\epsilon})$

massless photon 2 states $\left.\begin{matrix} 2 \\ + \\ 2 \end{matrix}\right)$ uncoupled
charged boson H (massive) Π (Goldstone)

massive photon 3 states $\left.\begin{matrix} 3 \\ + \\ 1 \end{matrix}\right)$ coupled
1 massive boson H | Higgs Boson

The Goldstone Boson has been "eaten" by the gauge field

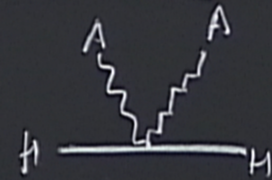
Photon A_μ gets a mass $\boxed{m_A^2 = e^2 \phi_0^2}$

nothing but $\boxed{E^2_{plasmon}}$ $=$

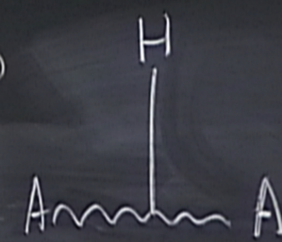
$\phi = \phi_0 + H$

ϕ_0 minimum of the potential

mass for H



Mass Term for the Gauge Field!



Propagator

photon is

it can be

$K_{rest} =$

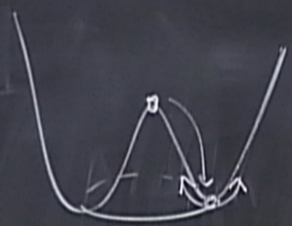
$$H + \left(\frac{M^2}{2} H^2 + \frac{M^2}{\phi_0} H^3 + \frac{M^4}{8\phi_0^2} H^4\right) + \frac{1}{2} e^2 H^2 A_\mu^2 + \boxed{\frac{1}{2} e^2 \phi_0^2 A_\mu^2} + e^2 \phi_0 H A_\mu A^\mu$$

for H   H coupling in the N-JS model

$B=0$

$$\frac{d^2y}{dt^2} = -\frac{e^2\rho}{\epsilon_0 m}\, y \qquad E \sim \frac{dy}{dt}$$

3 polarizations

*  $\Longrightarrow$ Neutral condensate of charged particles + gauge field $\Longleftrightarrow$ Higgs Mechanism.

"Plasmons"
New massive gauge fields
3 polarizations

* "physically" Higgs Mech $\Longleftarrow$ SSB

* mathematically one vaccum $|\Omega\rangle$ $\quad \varphi|\Omega\rangle = 0$

no breaking of the symmetry

no SSB for a local gauge invariance!

$E \sim \frac{dy}{d\nu}$ | 3 polarizations | Plasmon $E_{pl} \approx \hbar\omega_p$ $M_{pl} = \frac{\hbar\omega_p}{c^2}$

...te of ...les $\Longleftrightarrow$ Higgs Mechanism "Plasmons" Higgs Boson

New massive gauge fields 3 polarization + fluctuation in the density of the condensate "phonon"

$|\Omega\rangle = 0$

...ce!

Standard Model of E.W interactions

$\underbrace{U(1) \times SU(2)}$ symmetry + SU(2) Scalar

$\Downarrow$

$U(1)_{QED}$ symmetry

Higgs Mechanism

"Plasmons"

New massive gauge fields

& polarization

+ Higgs Boson

fluctuation in the density of the condensate

"phonon"

Standard Model of E.W interactions

A_μ: $U(1)\times SU(2)$ symmetry + SU(2) Scalar ϕ

$\Downarrow$ $U(1)_{QED}$ symmetry massless γ EM

$\Downarrow$ 3 massive vector bosons Z, W_+, W_- + Higgs