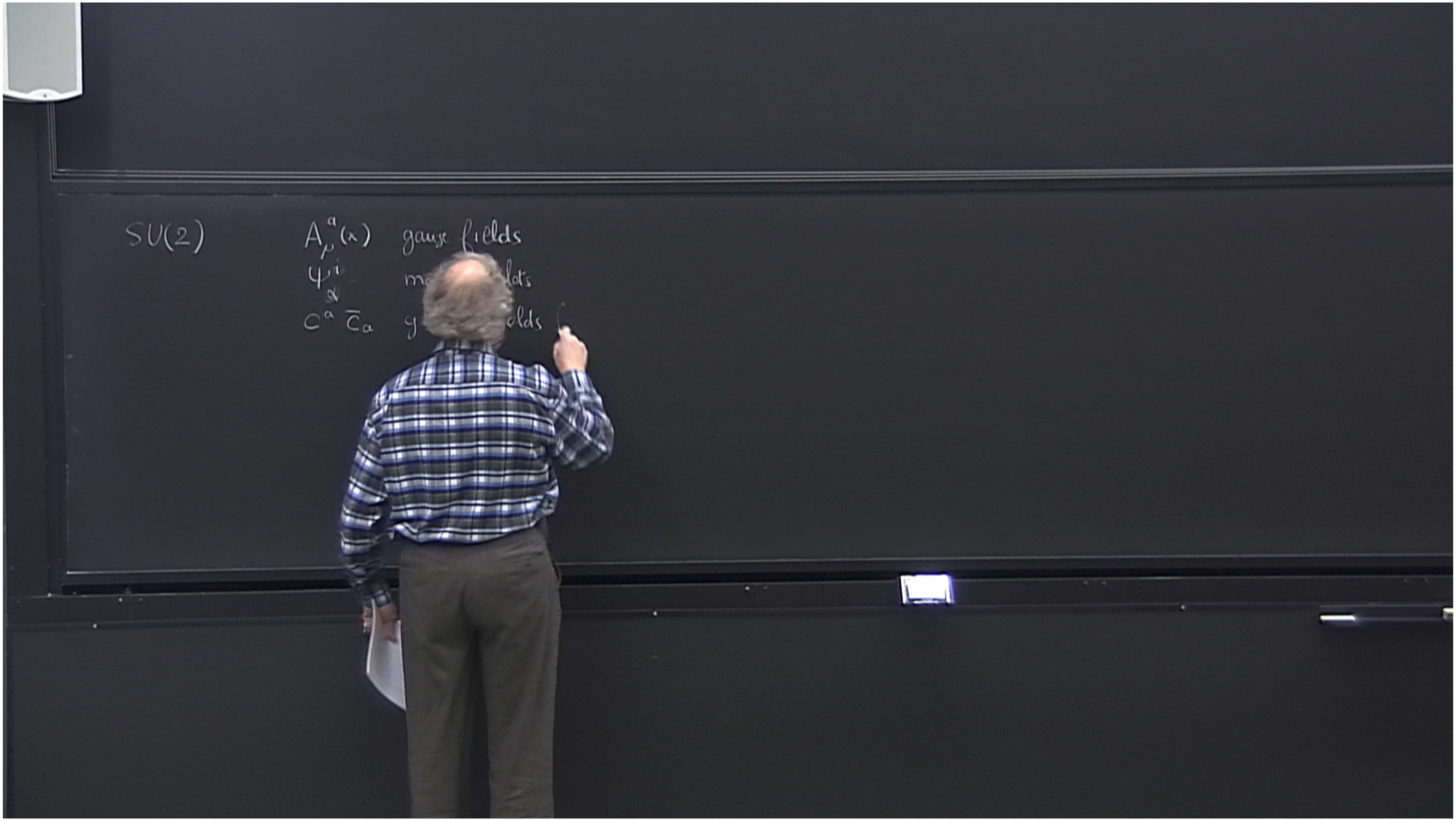
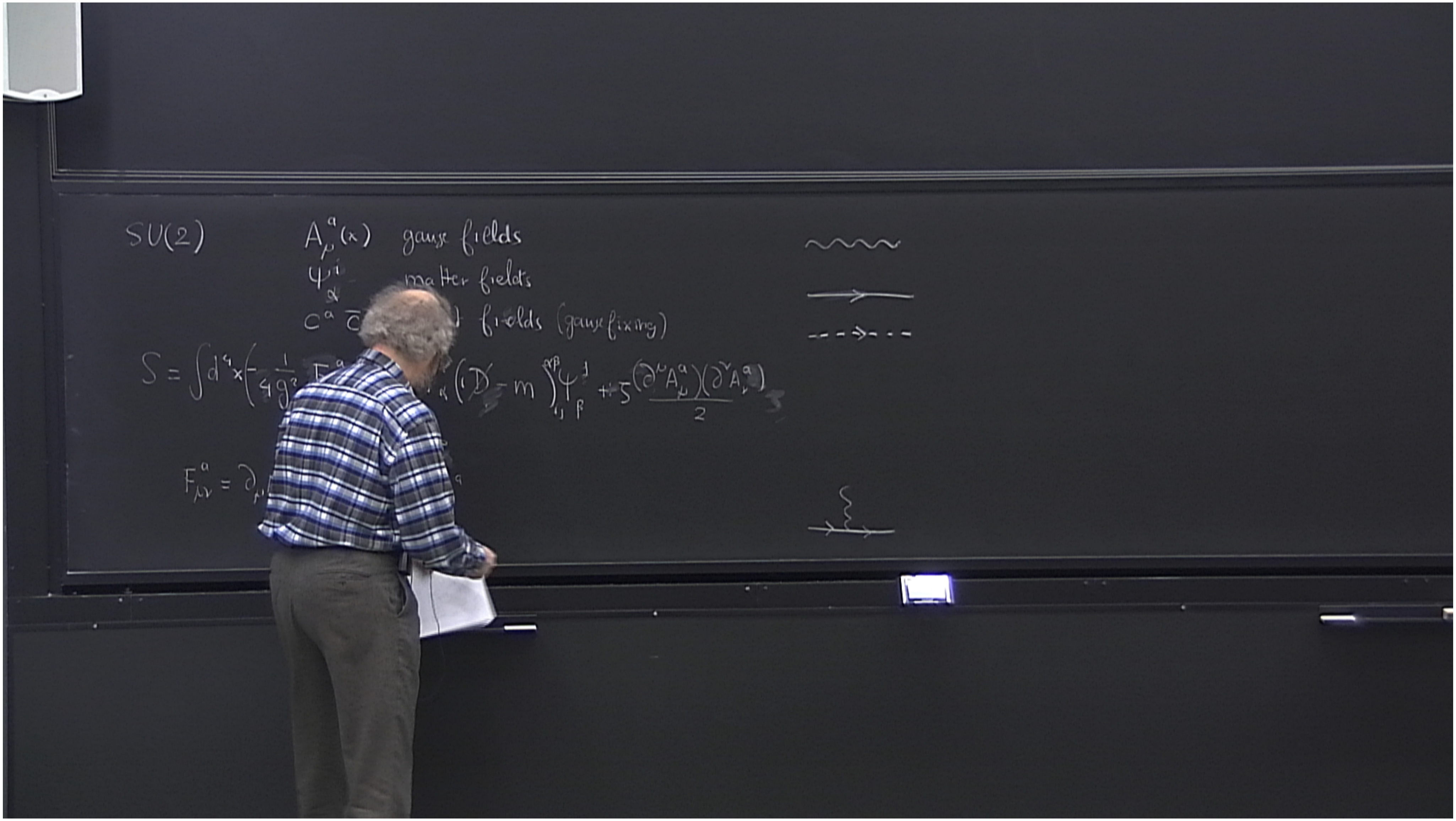
Title: Quantum Field Theory II -14

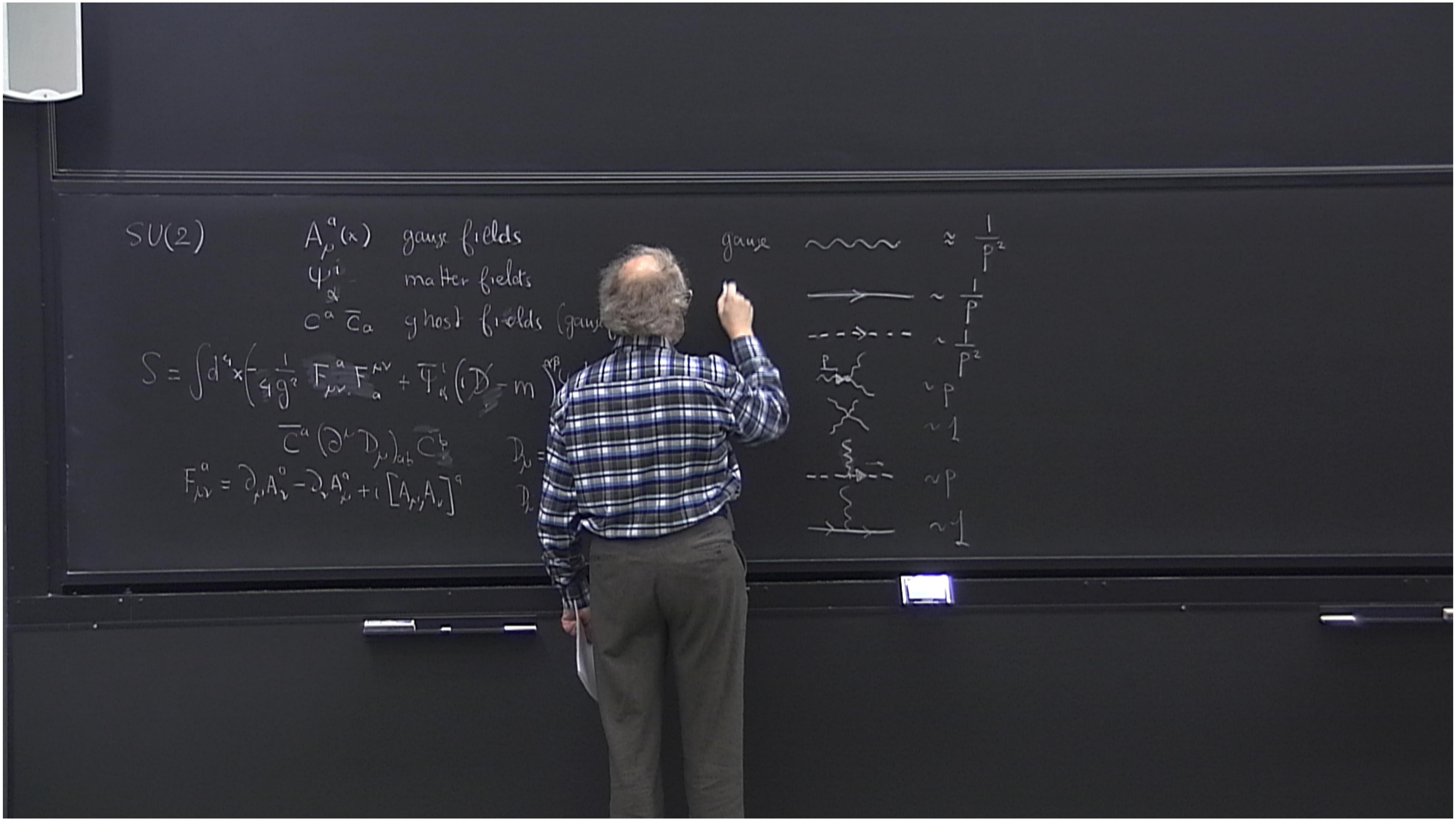
Date: Nov 27, 2014 09:00 AM

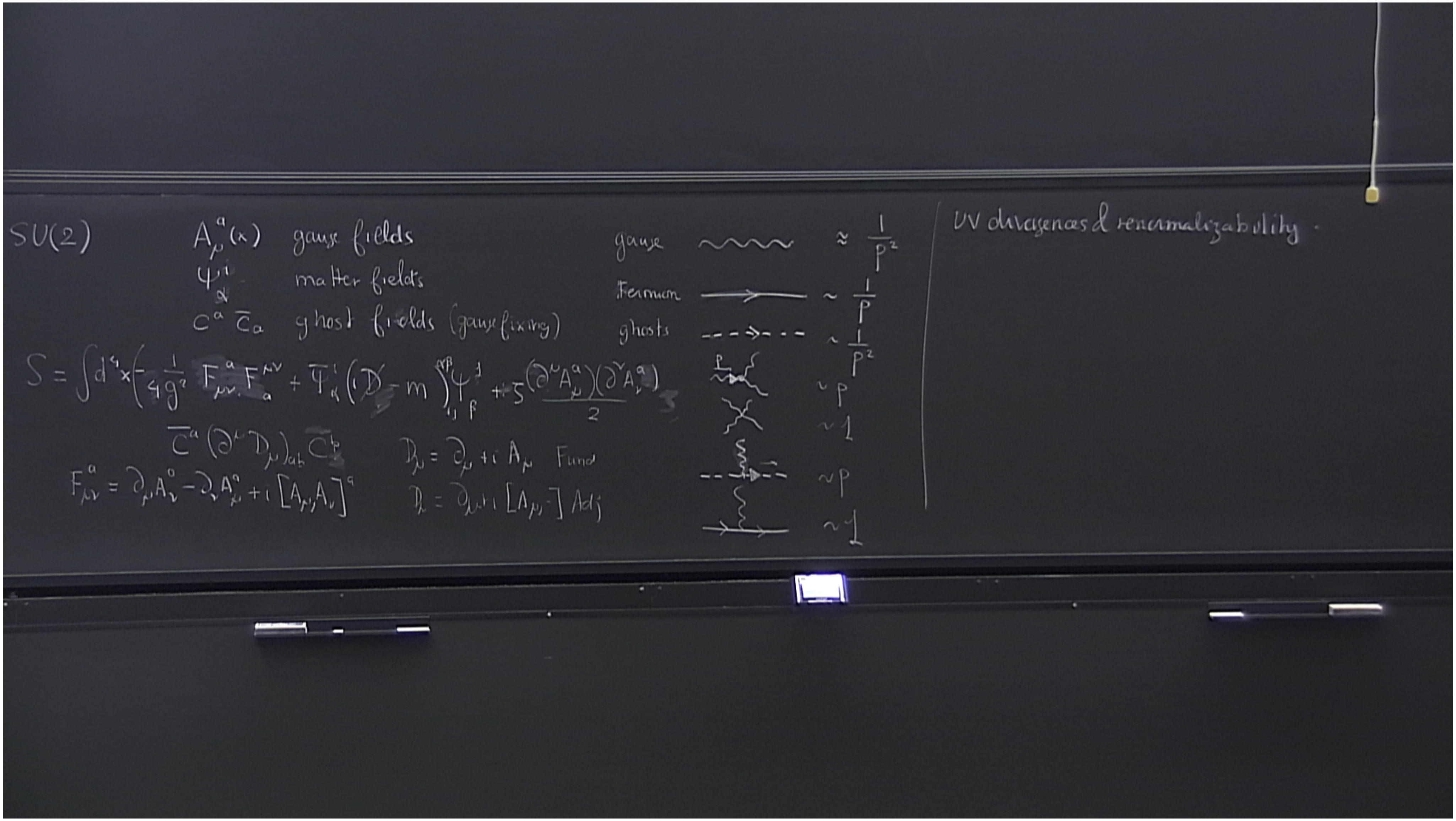
URL: http://pirsa.org/14110023

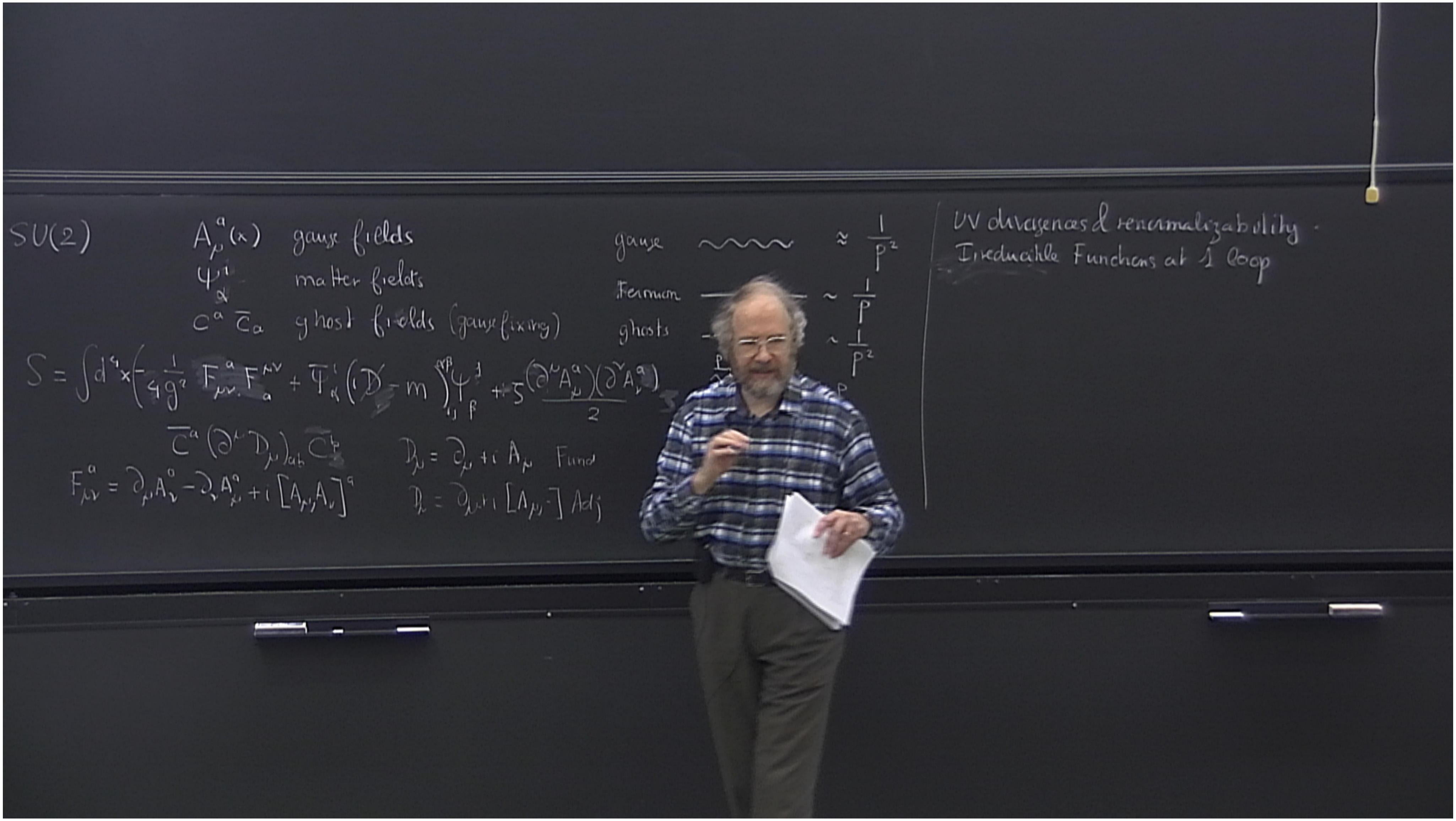
Abstract:


SU(2)
$A_\mu^a(x)$ gauge fields
ψ^i_α
$c^a \; \bar{c}_a$


SU(2)
gauge fields
matter fields
fields (gauge fixing)


SU(2)
gauge fields
matter fields
ghost fields
gauge


SU(2)
gauge fields
matter fields
ghost fields (gauge fixing)
gauge
Fermion
ghosts
Fund
Adj
UV divergences & renormalizability.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Irreducible Functions at 1 loop
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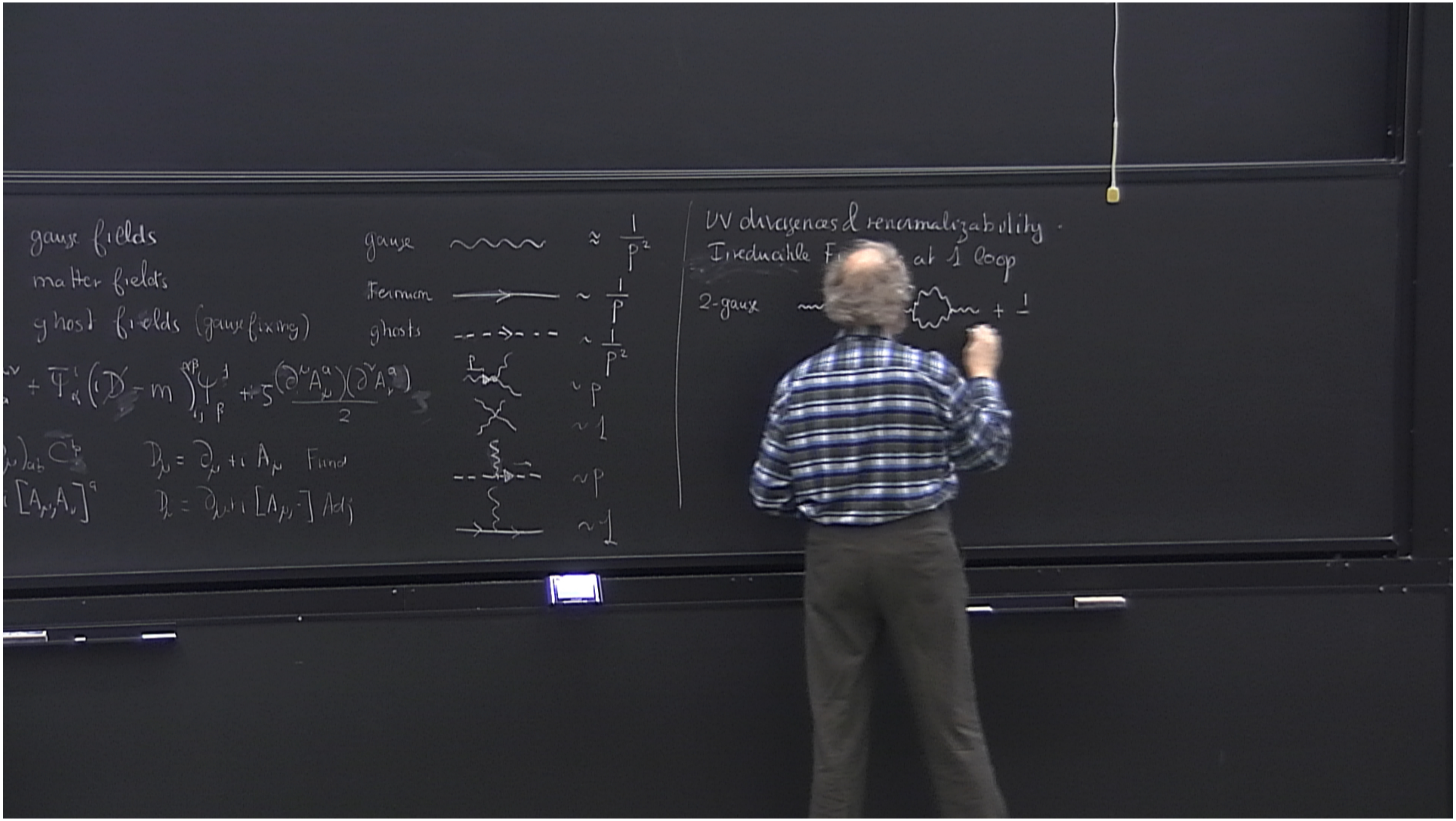
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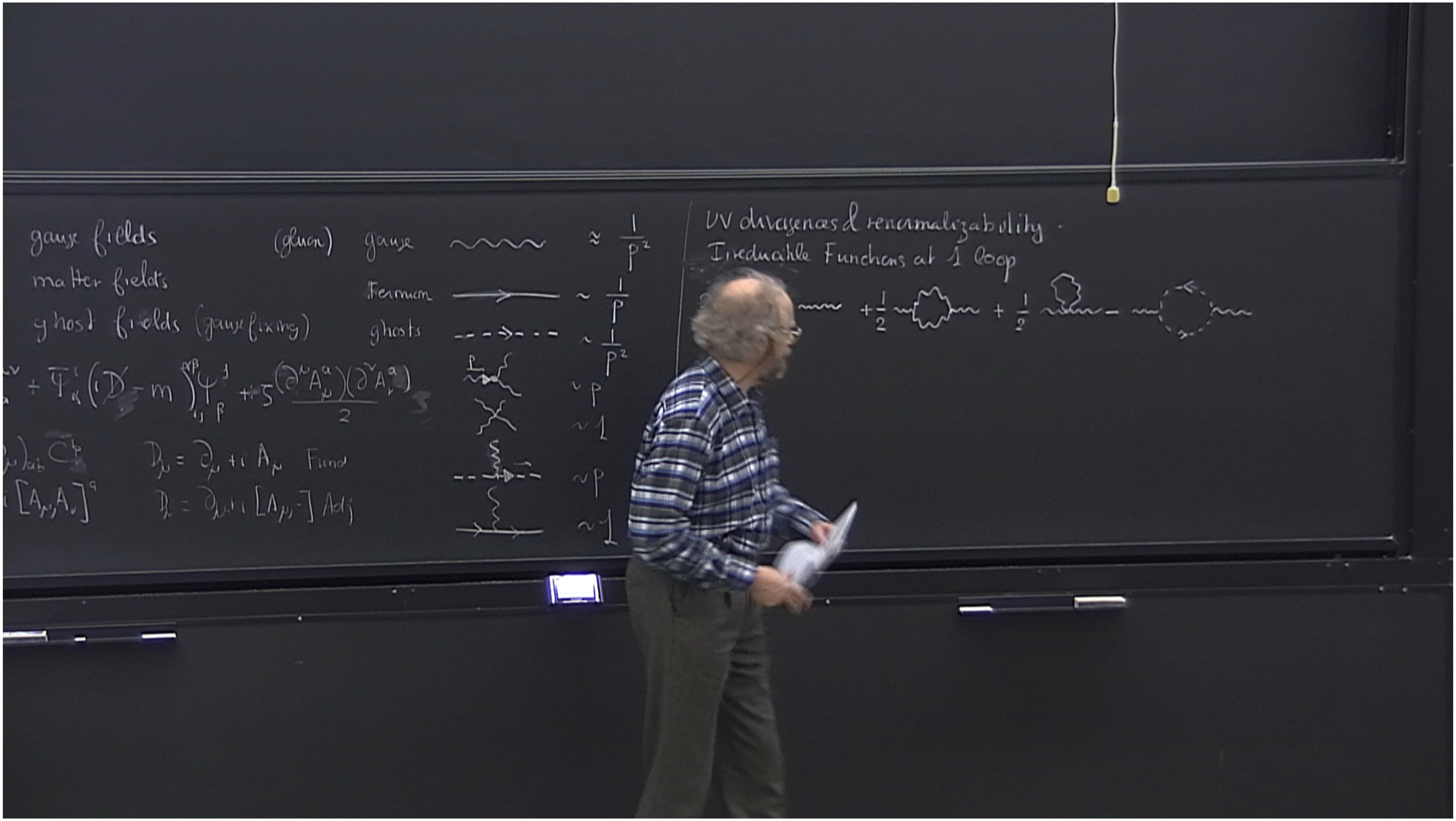
2-gauge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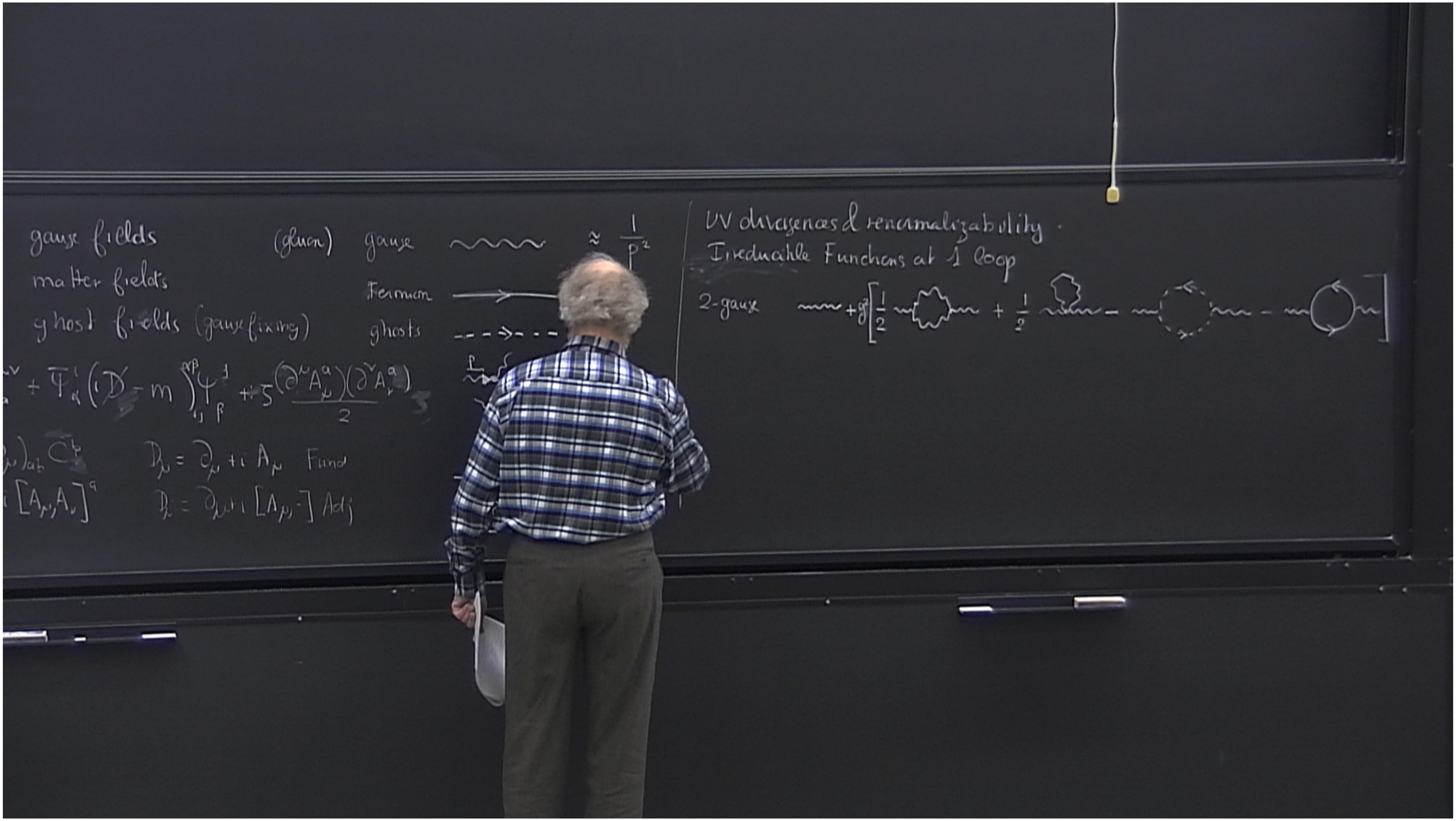
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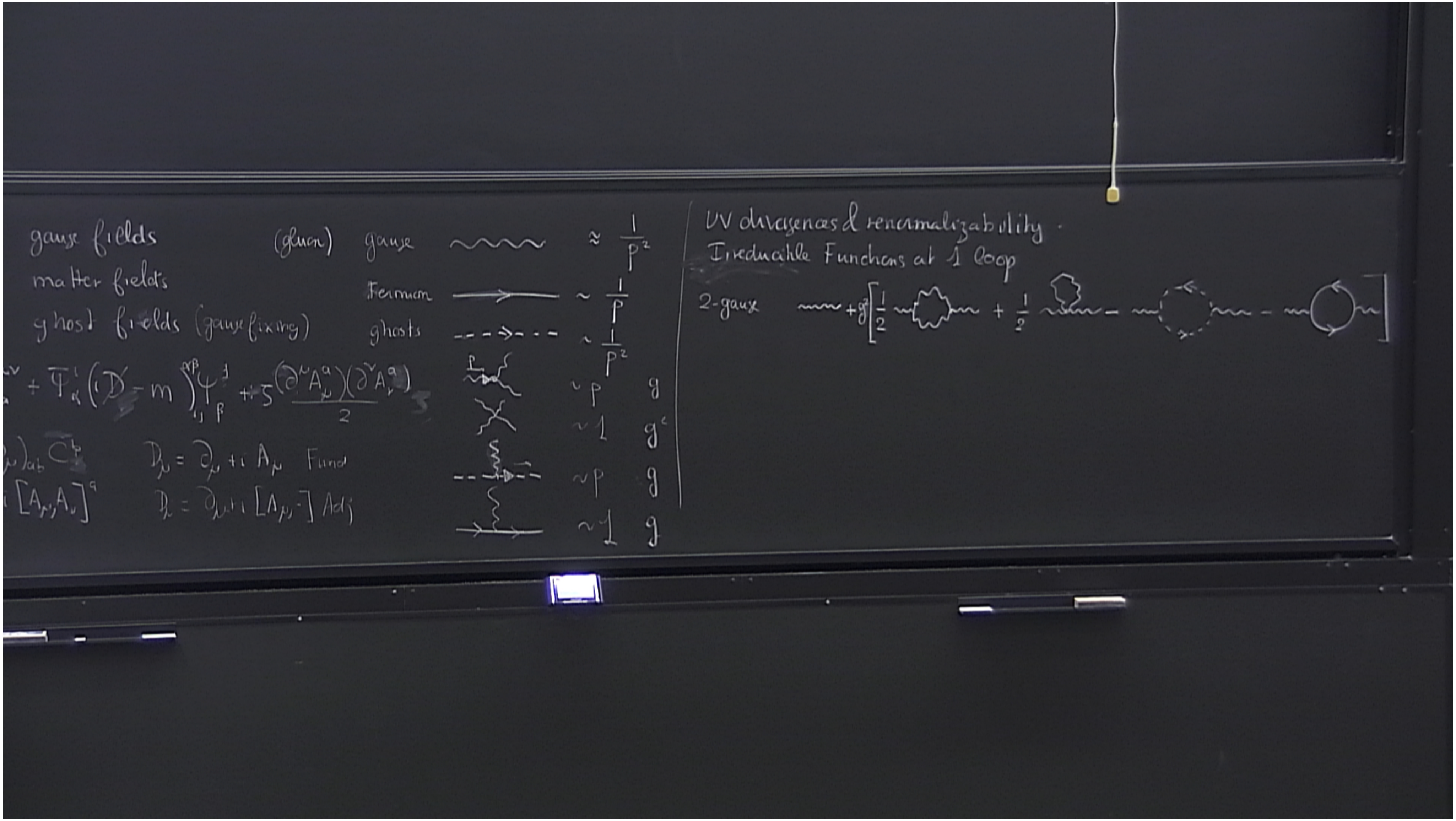
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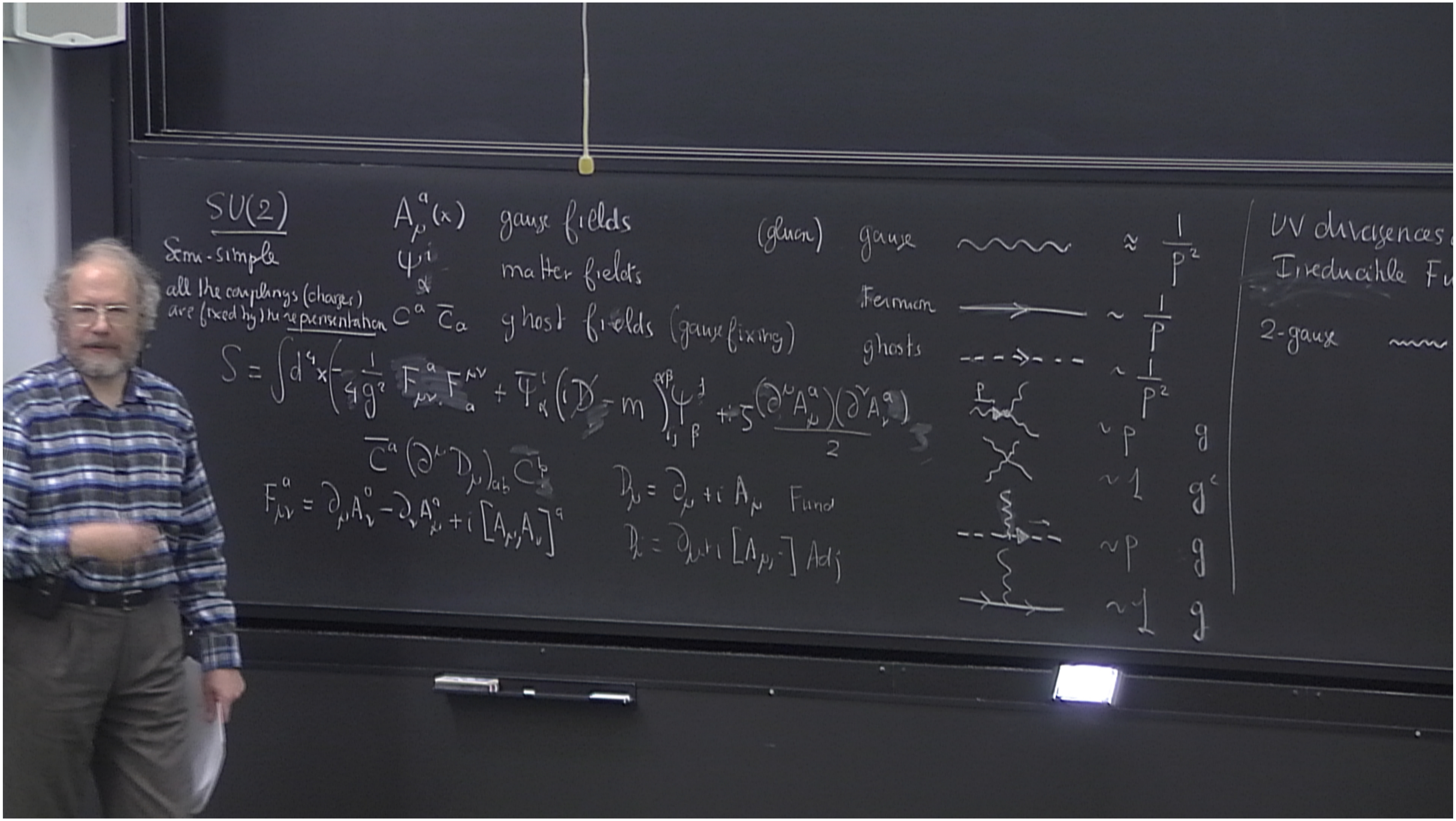
SU(2)
Semi-simple
all the couplings (charges) are fixed by the representation
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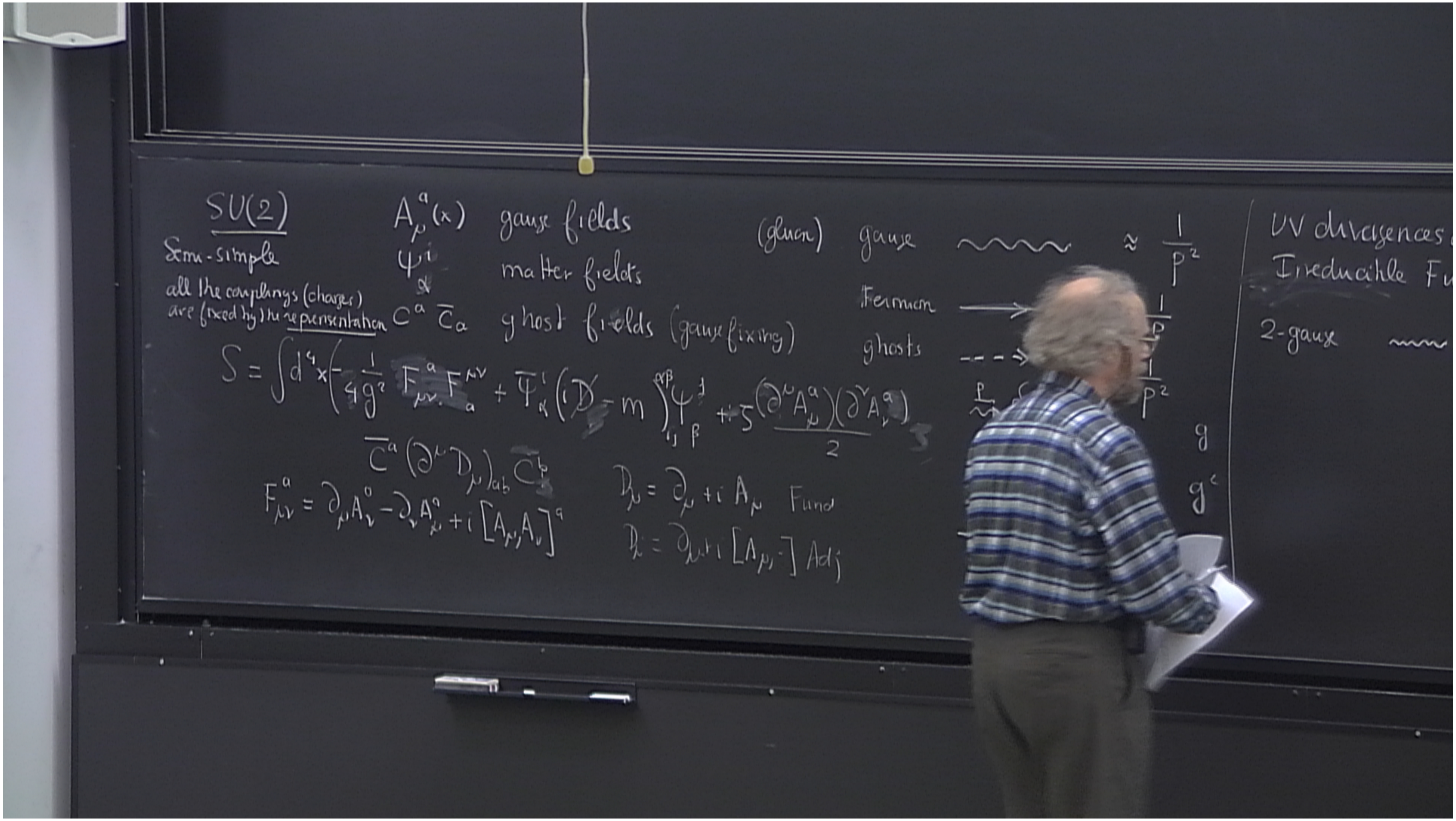
ψ^i_α matter fields
$c^a\ \bar{c}_a$ ghost fields (gauge fixing)
(gluon) gauge
Fermion
ghosts
UV divergences
Irreducible
2-gauge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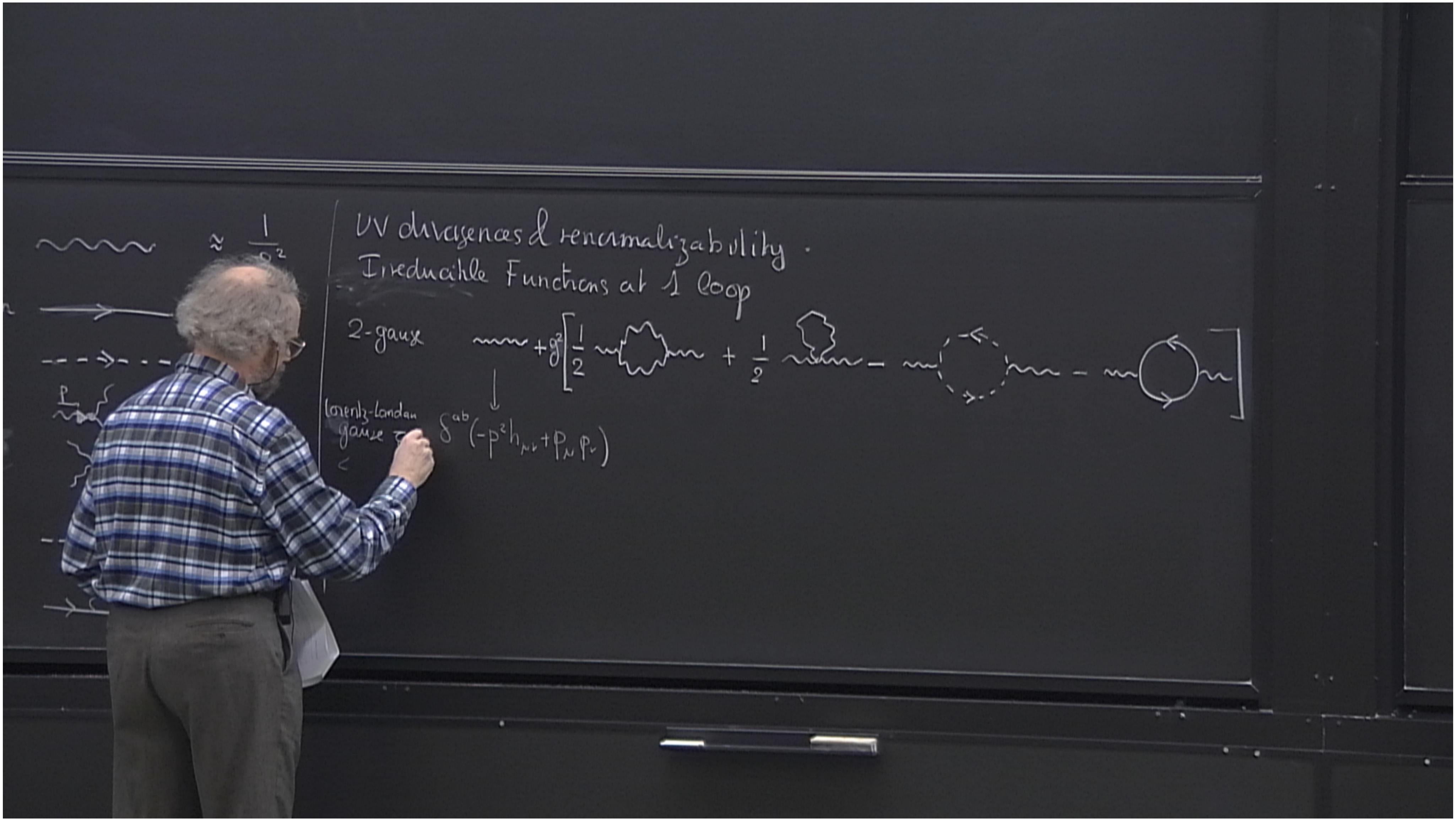
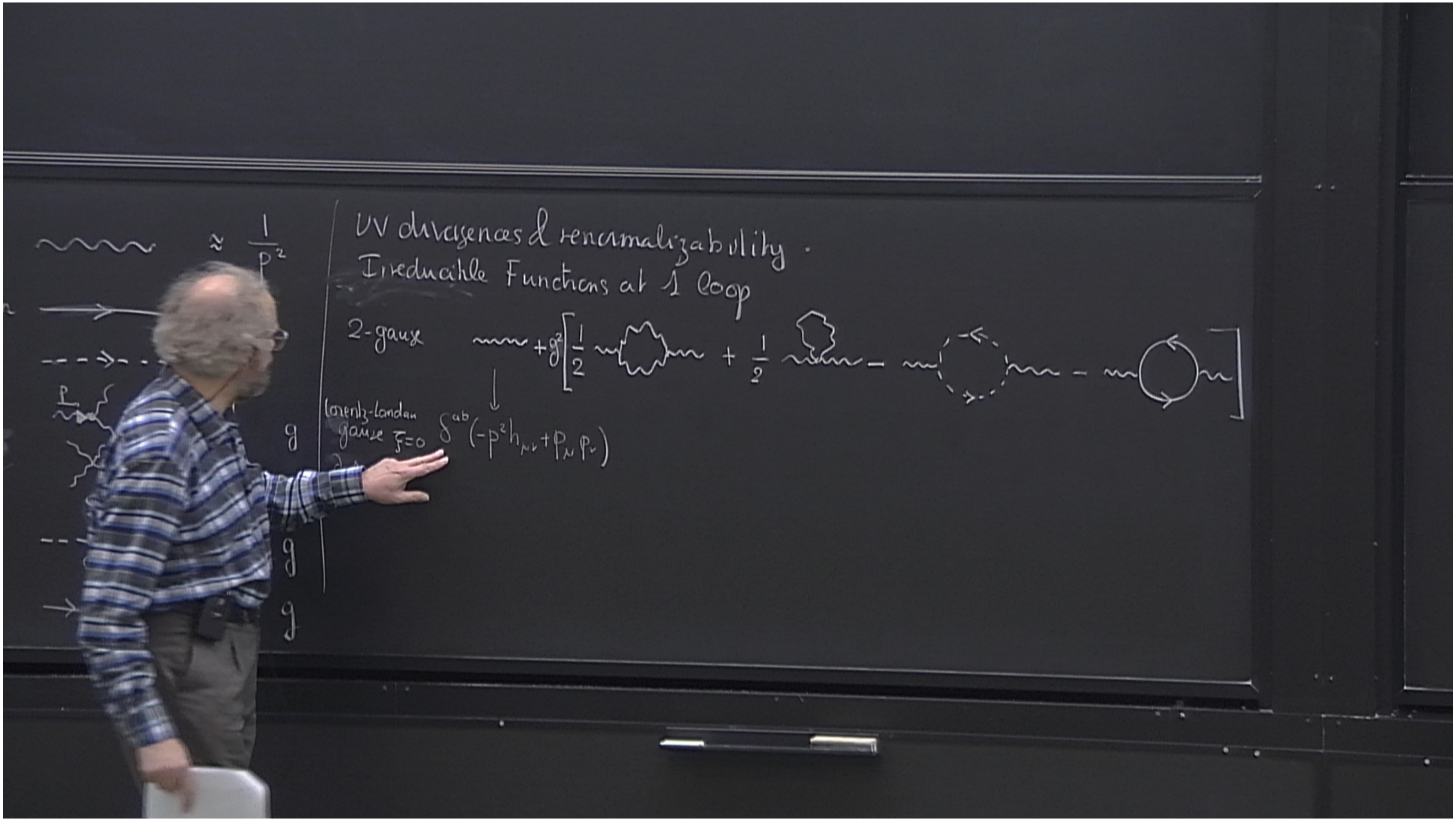
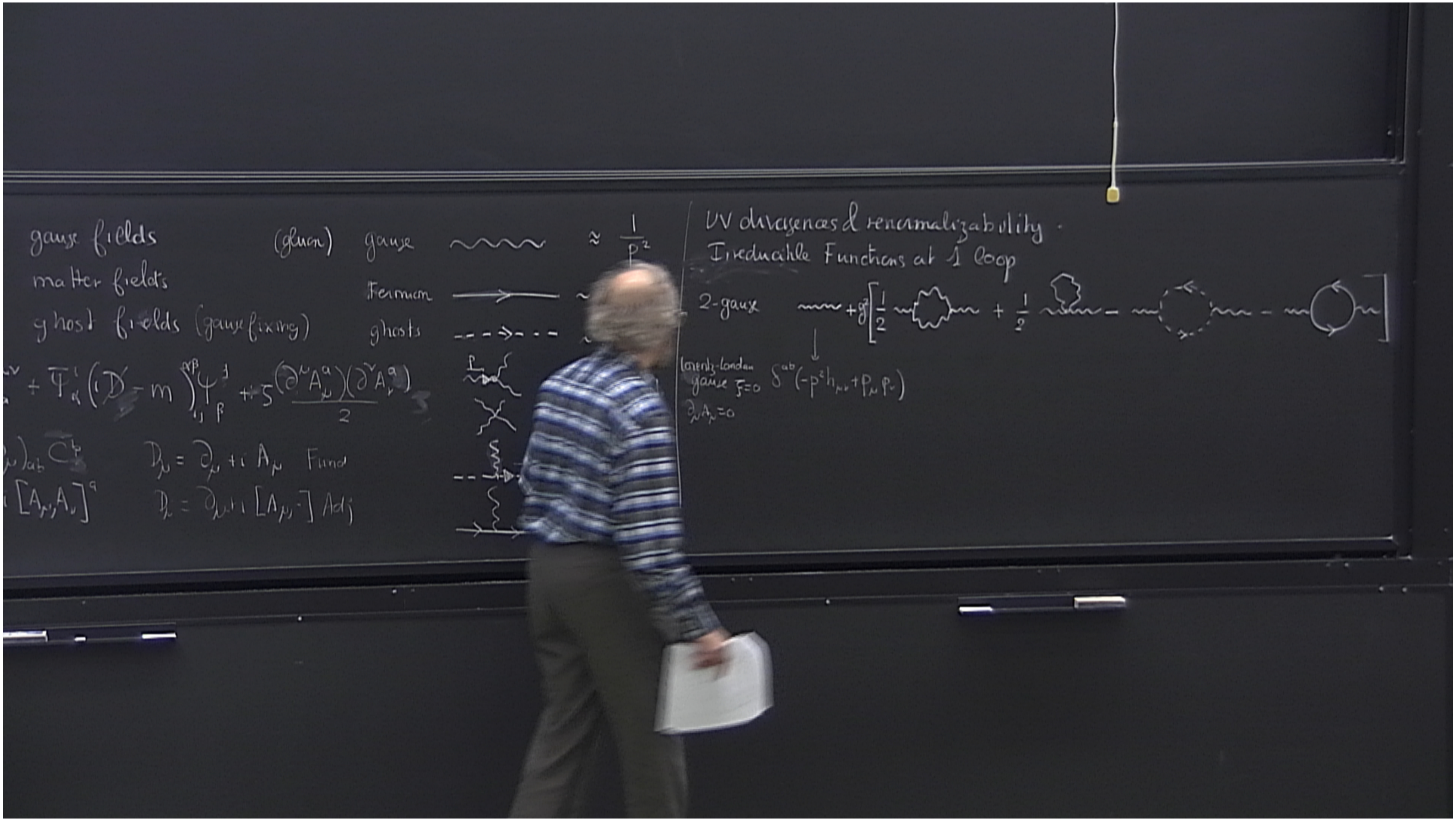

UV divergences & renormalizability.
Irreducible Functions at 1 loop
2-gauge
Lorentz-Landau gauge
δ^ab(-p²h_μν + p_μ p_ν)


UV divergences & renormalizability.
Irreducible Functions at 1 loop
2-gauge
Lorentz-Landau
gauge ξ=0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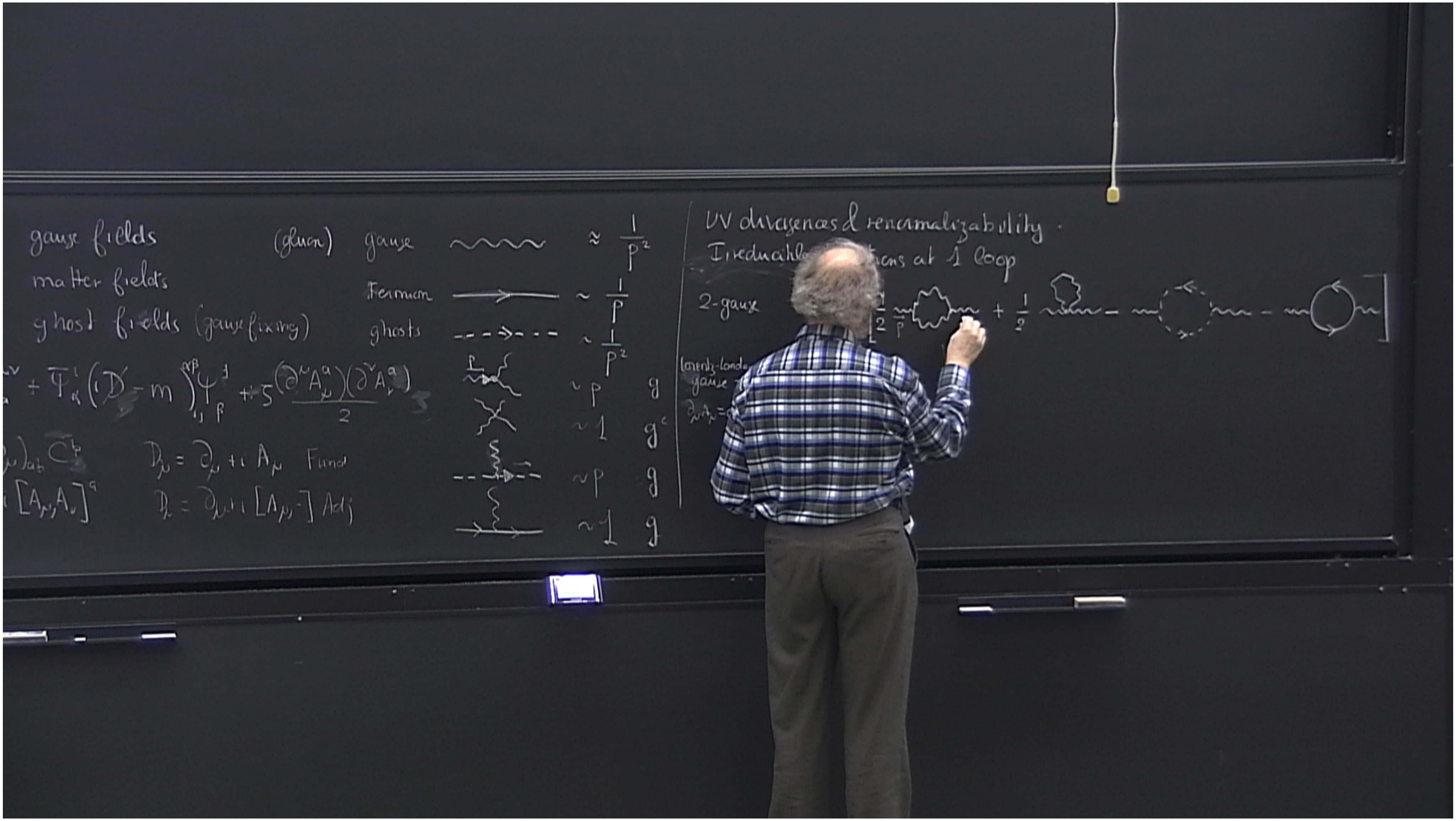

gauge fields
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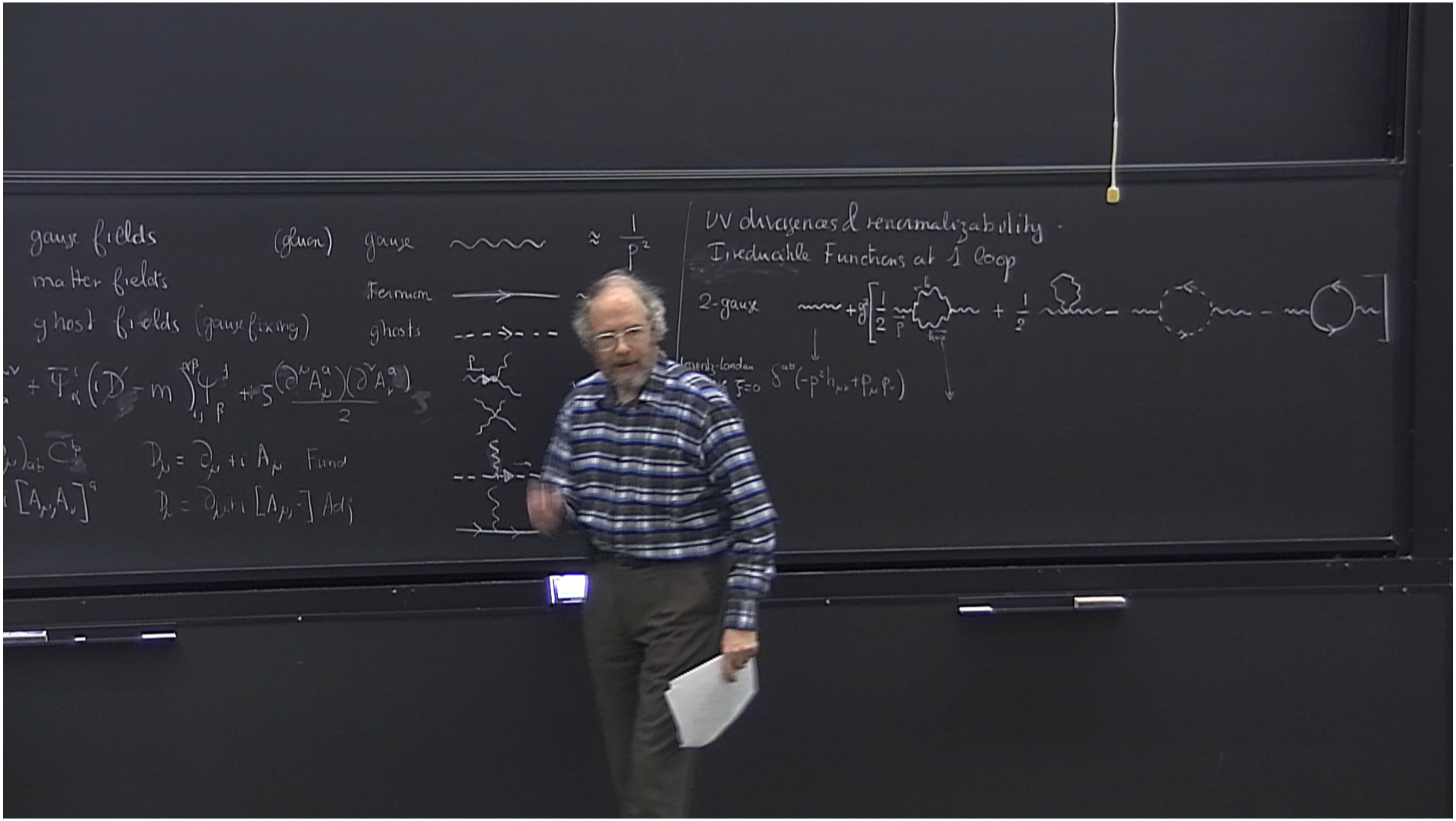
2-gauge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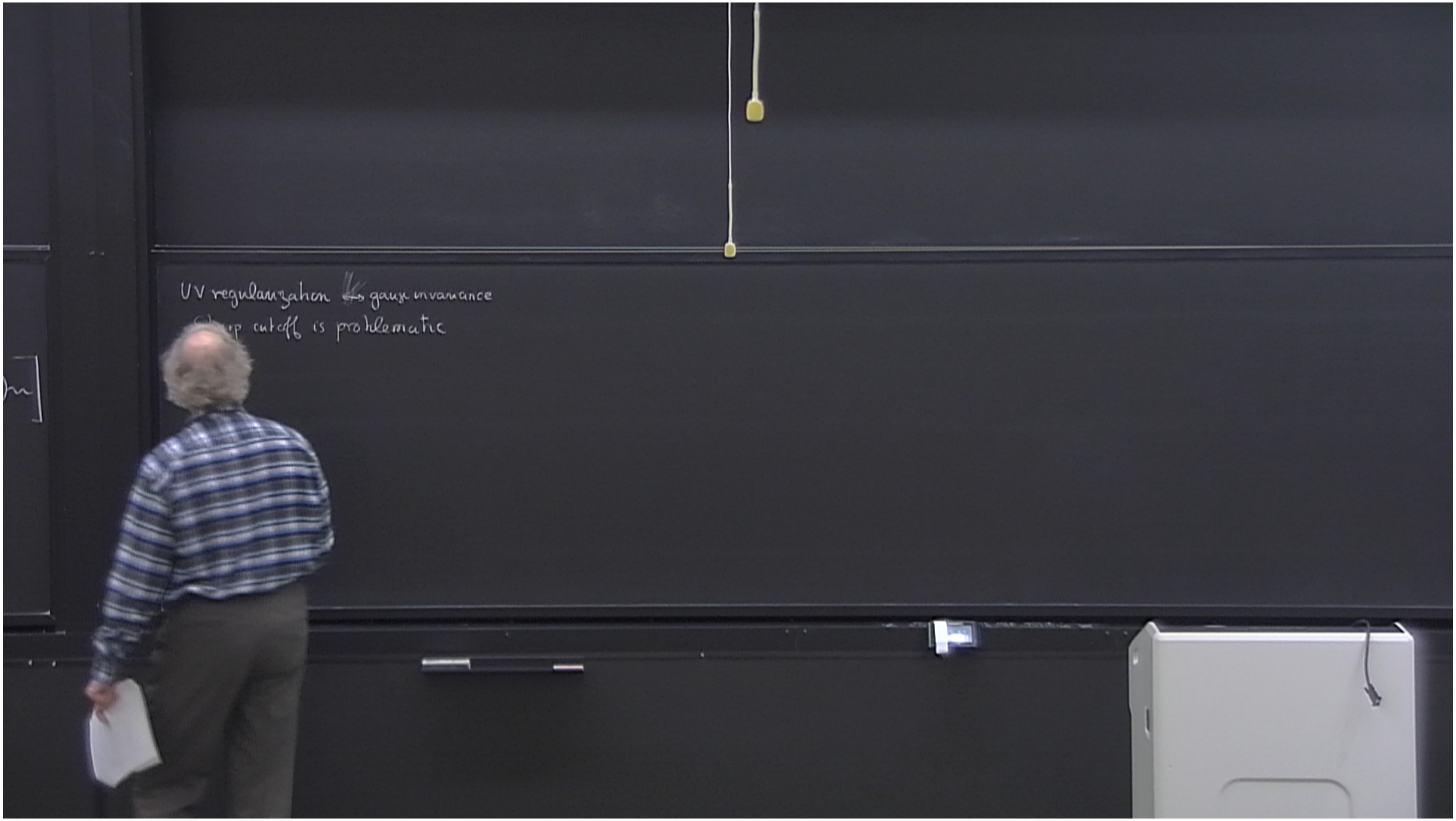
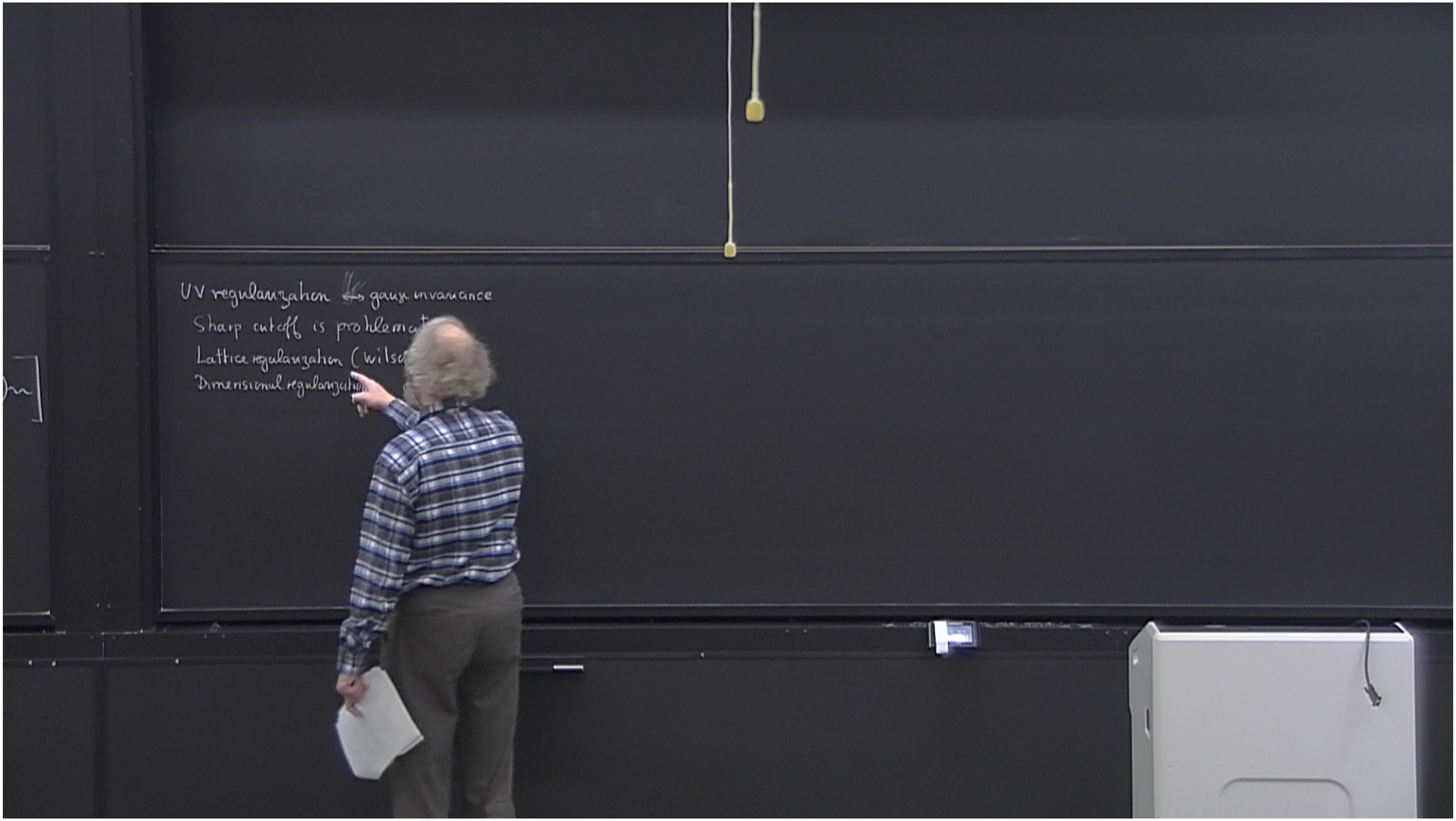
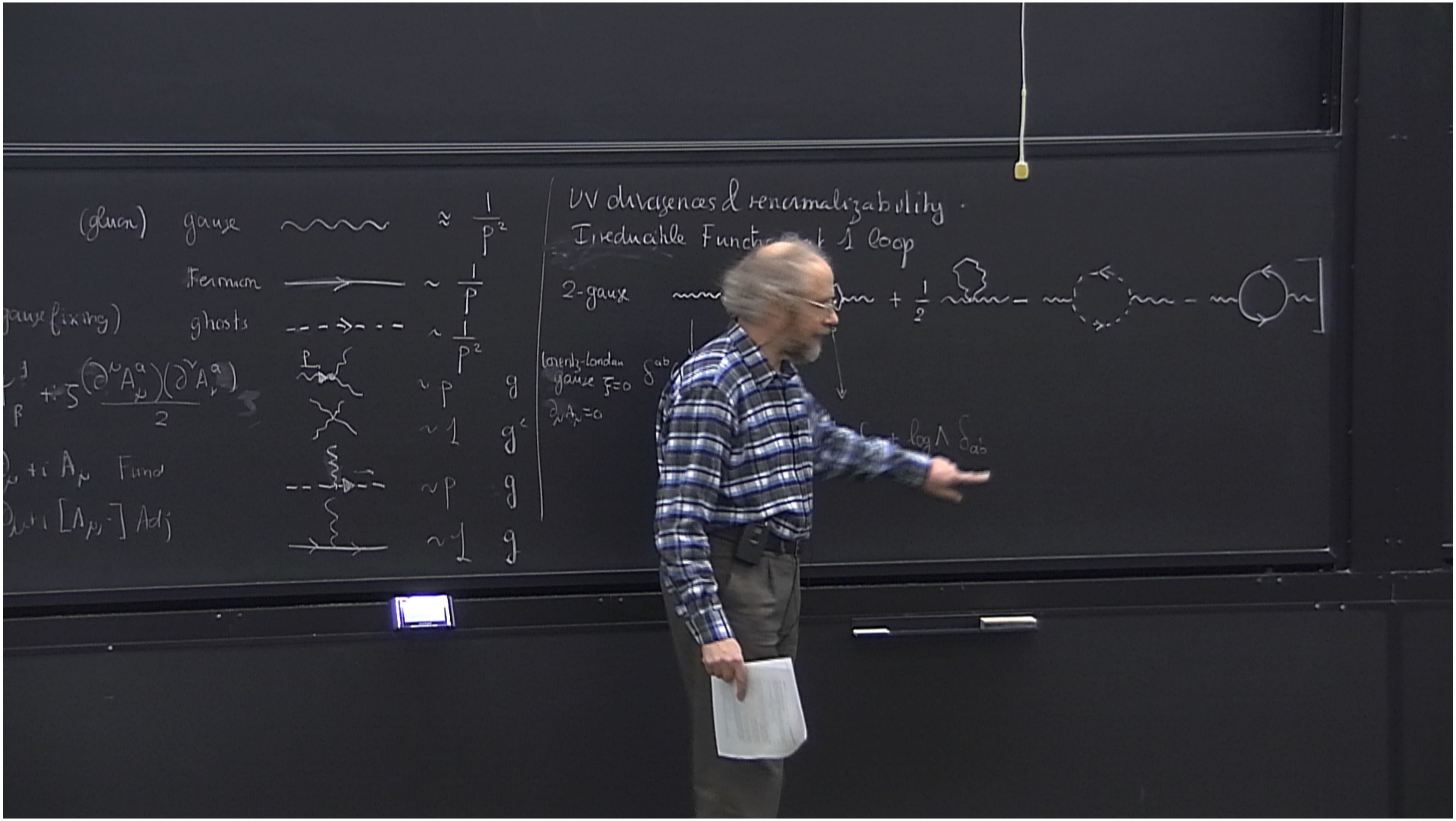

UV regularization $\not\Leftrightarrow$ gauge invariance
cutoff is problematic


UV regularization gauge invariance
Sharp cutoff is problemat
Lattice regularization (Wilso
Dimensional regularizatio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Irreducible Functions at 1 loop
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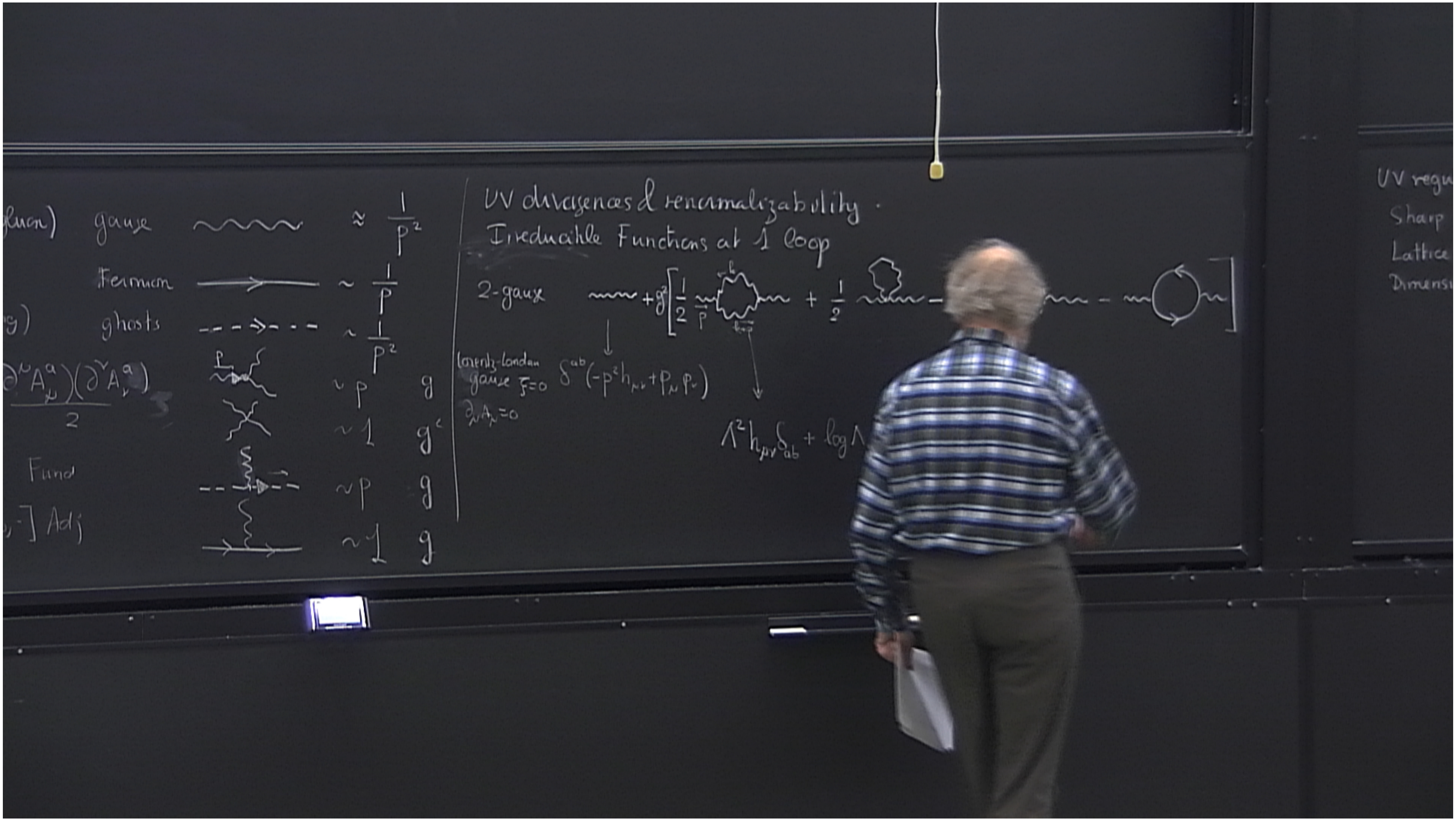
UV divergences & renormalizability .
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gauge ξ=0
gauge
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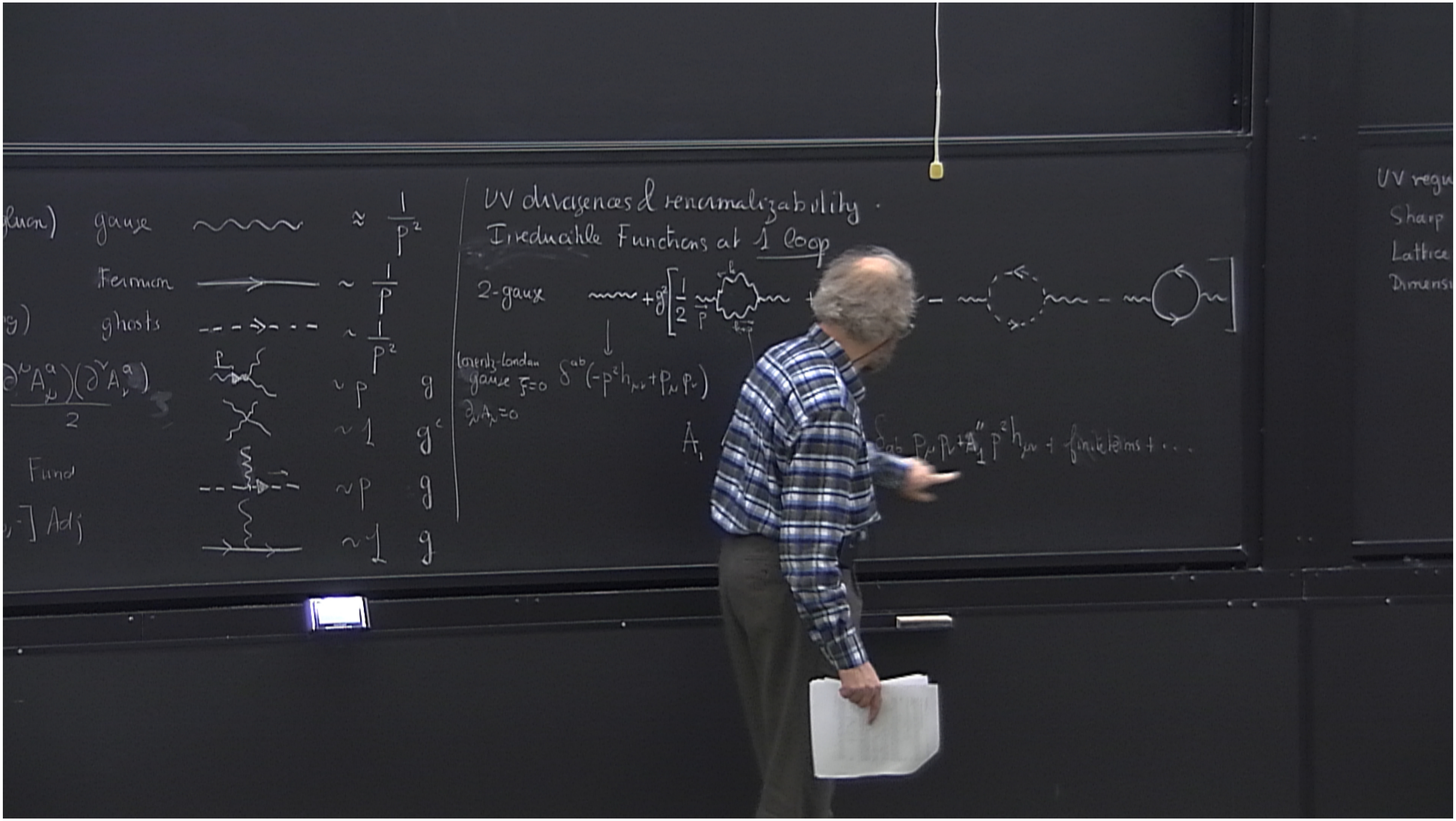
Sharp
Lattice
Dimensi


UV divergences & renormalizability
Irreducible Functions at 1 loop
2-gauge
Lorentz-Landau gauge ξ=0
∂A=0
gauge
Fermion
ghosts
Fund
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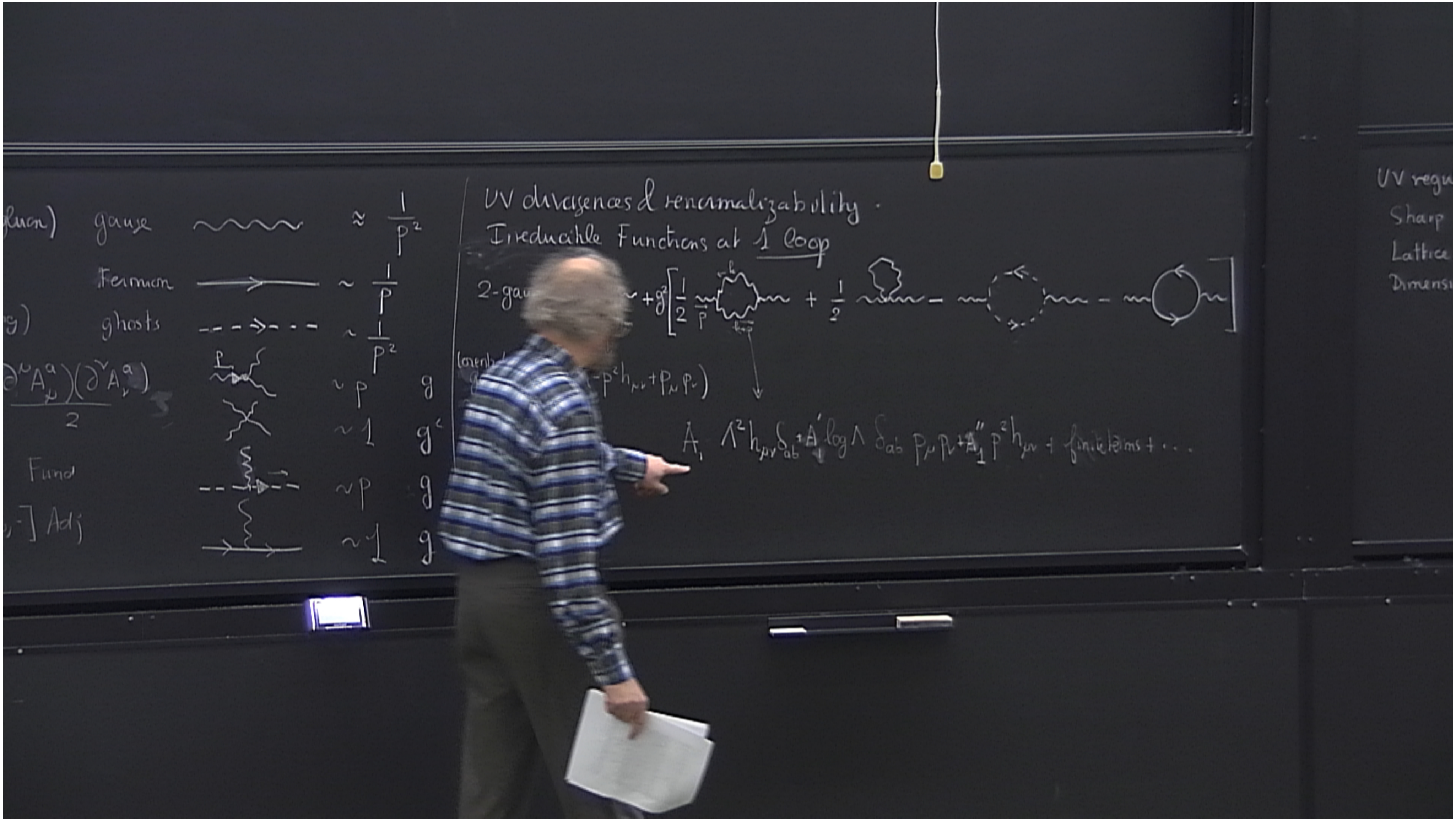
UV divergences & renormalizability.
Irreducible Functions at 1 loop
gauge
Fermion
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Fund
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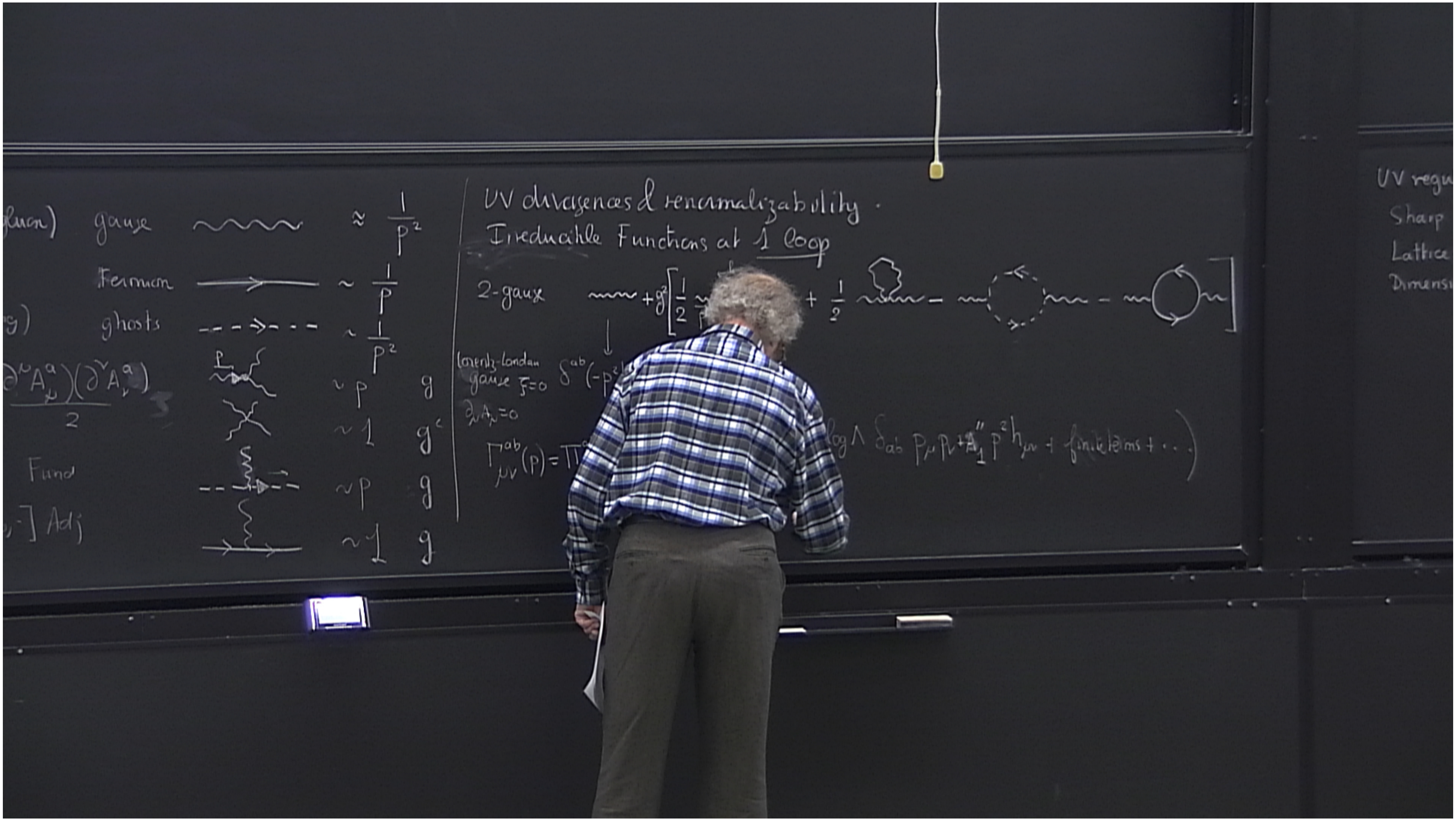
UV divergences & renormalizability.
Irreducible Functions at 1 loop
2-gauge
Lorentz-Landau gauge ξ=0
gauge
Fermion
ghosts
Fund
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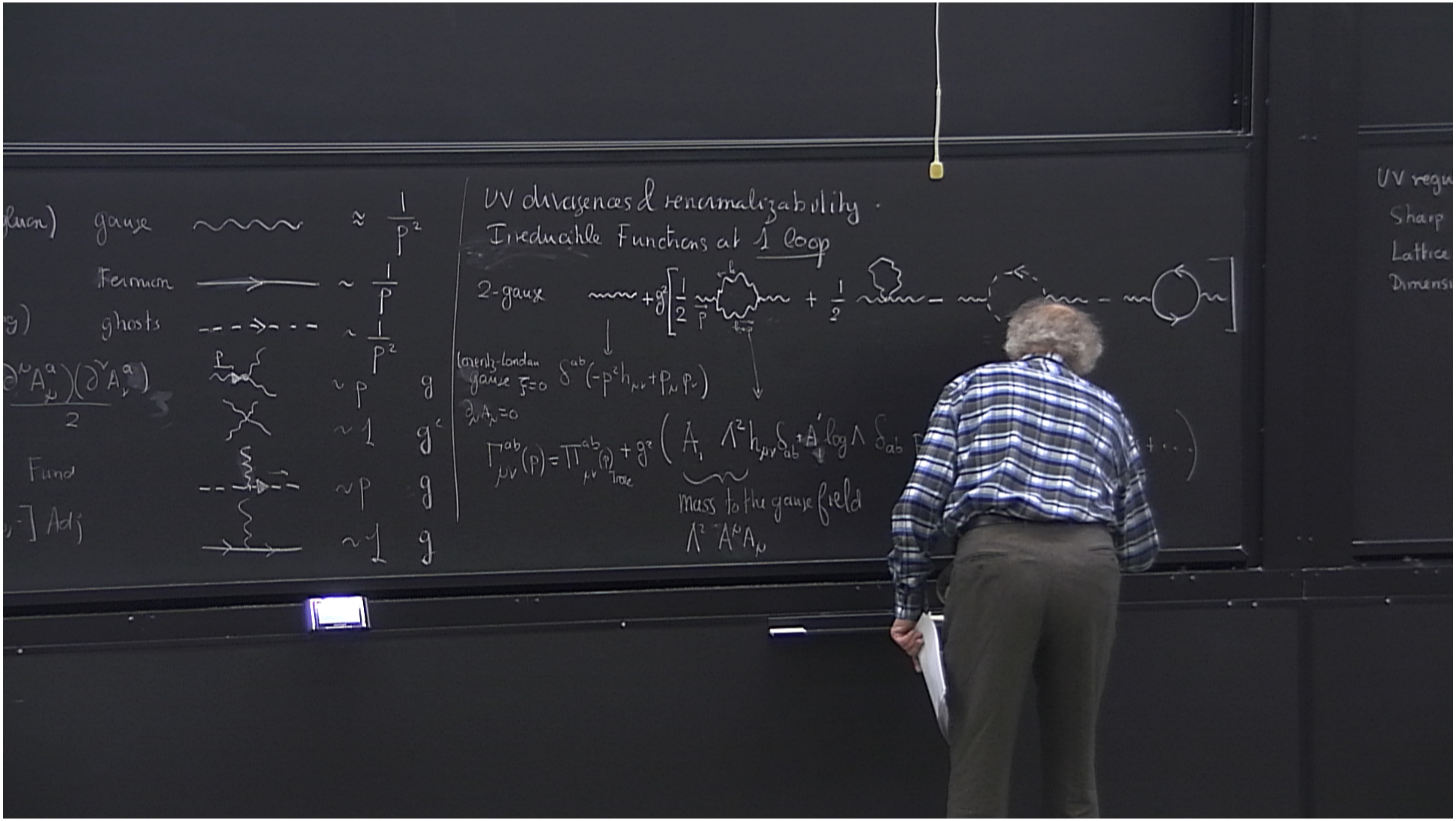
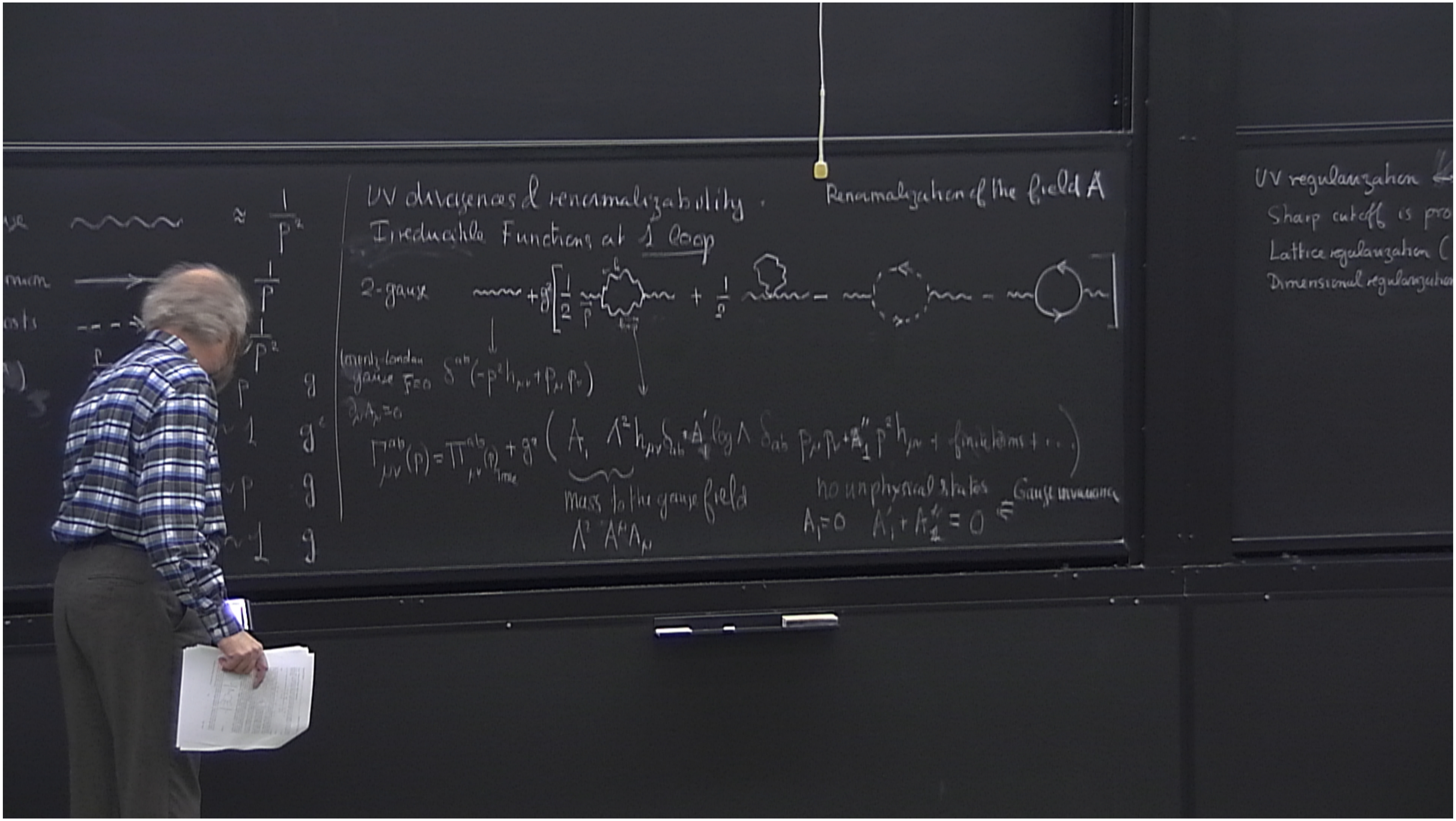
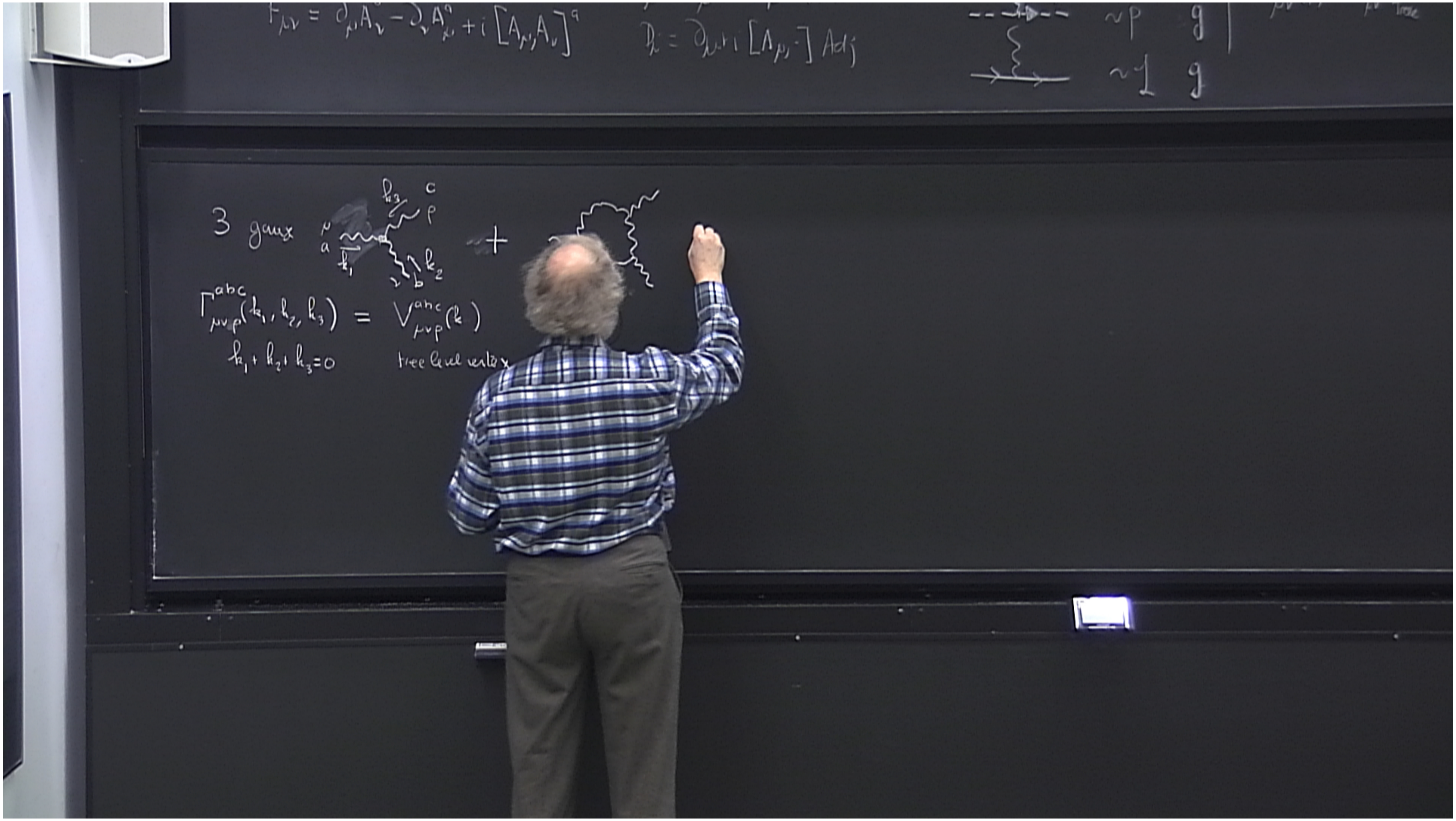
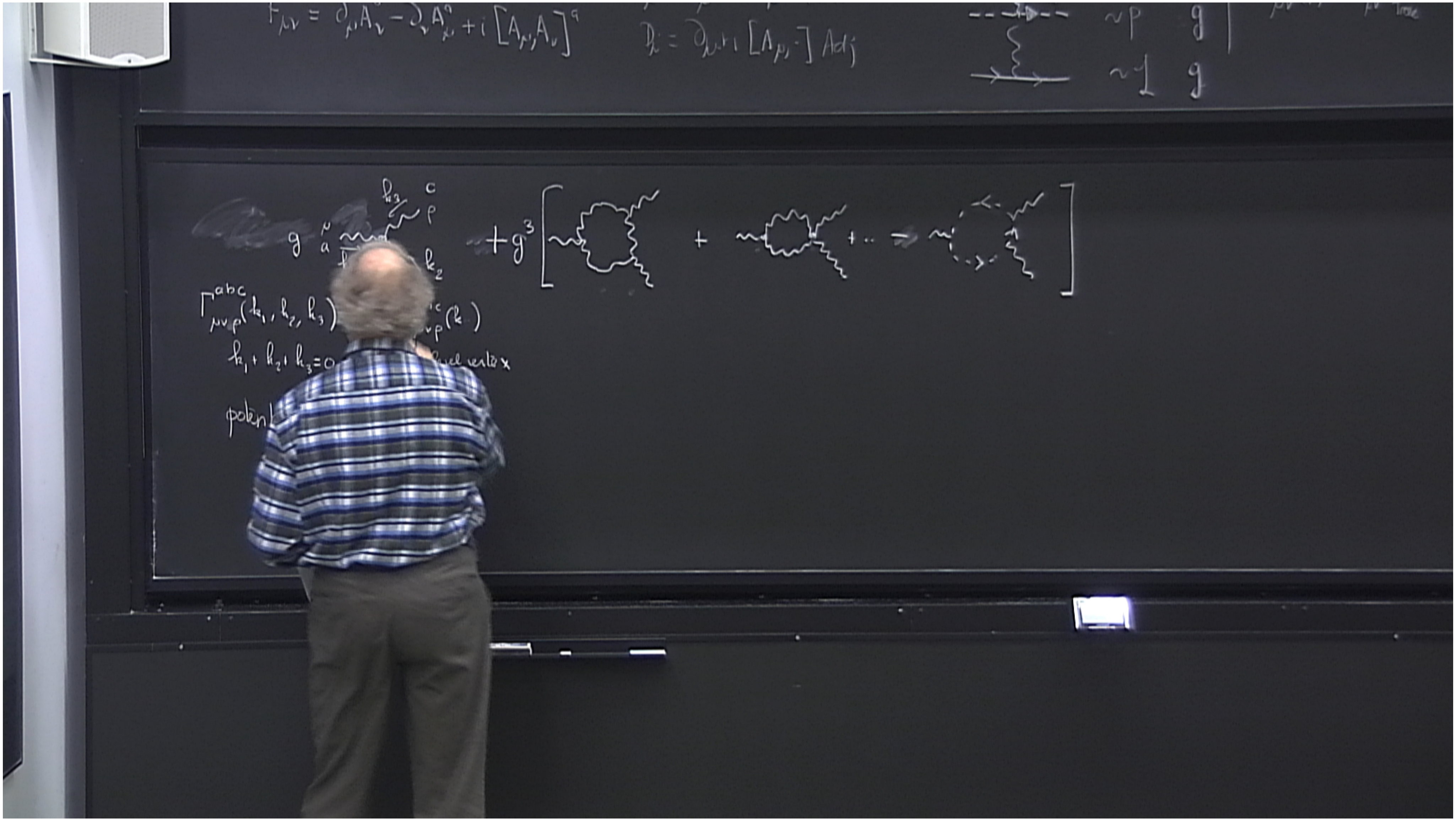
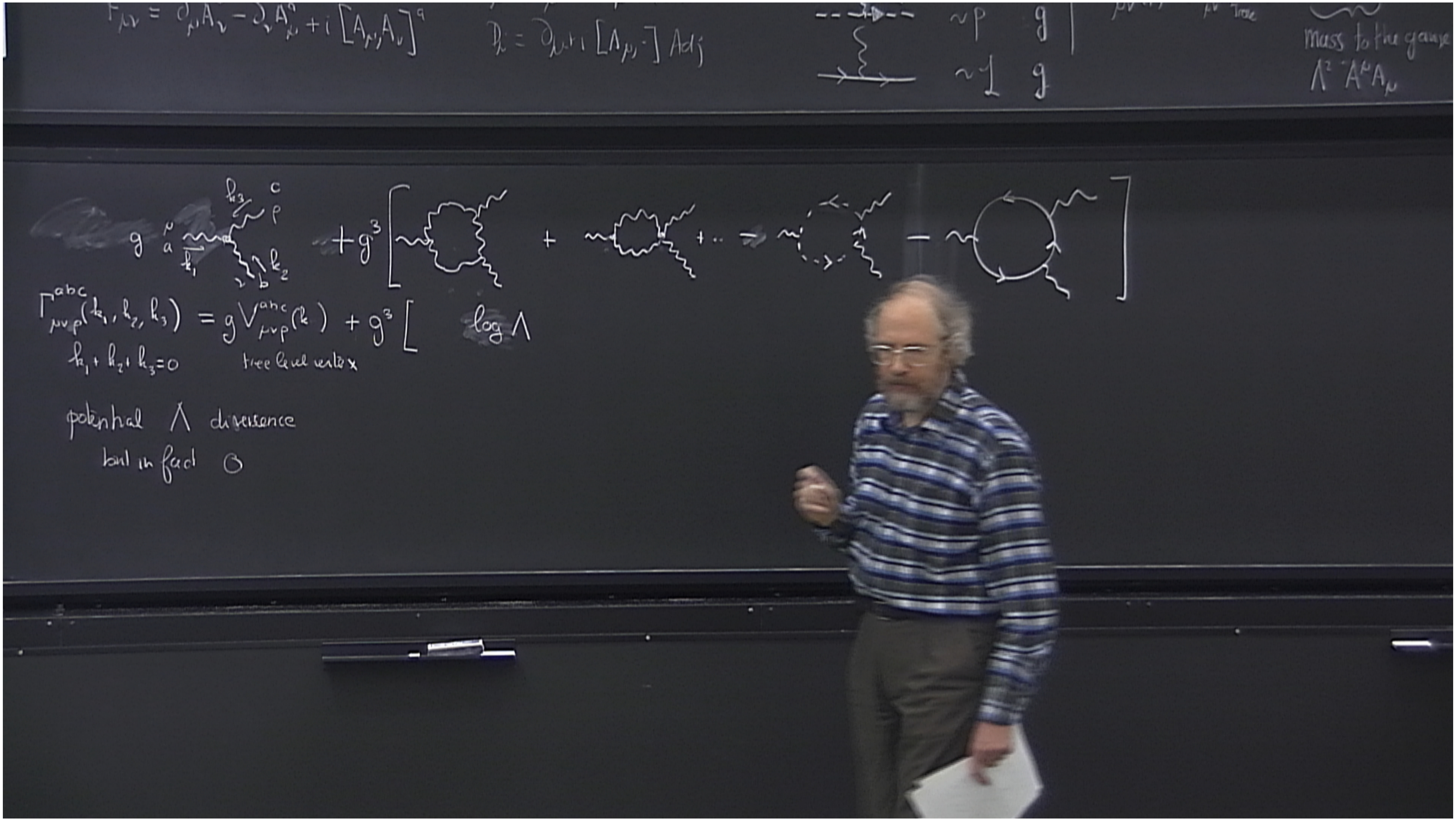

UV divergences & renormalizability
Irreducible Functions at 1 loop
2-gauge
gauge
Fermion
ghosts
Fund
Adj
Lorentz-Landau gauge ξ=0
mass to the gauge field
UV regu
Sharp
Lattice
Dimensi

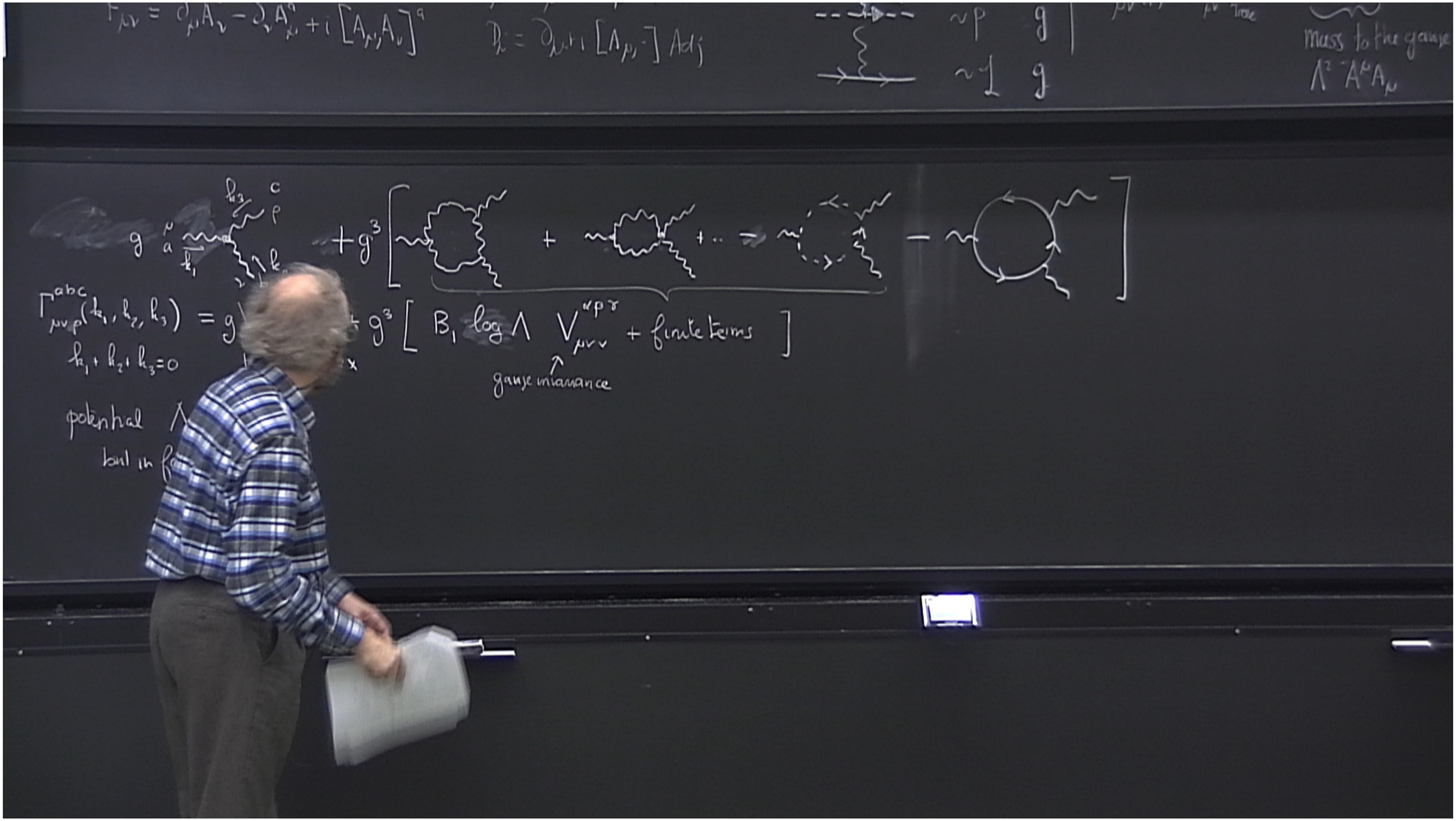

UV divergences & renormalizability
Irreducible Functions at 1 loop
Renormalization of the field A
2-gauge
Lorentz-Landau gauge ξ=0
mass to the gauge field
no unphysical states
Gauge invariance
UV regularization
Sharp cutoff is pro
Lattice regularization (
Dimensional regularization

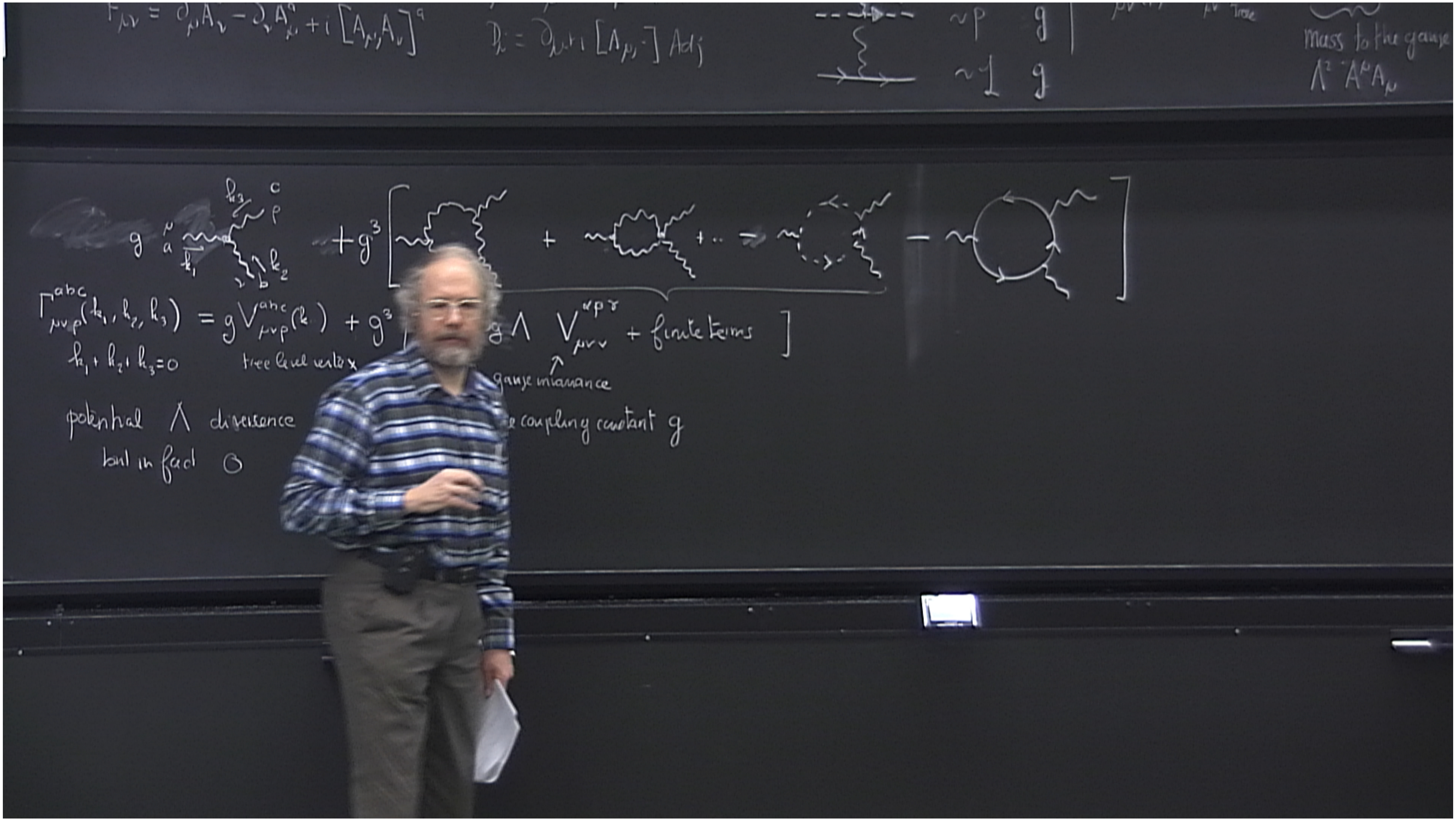


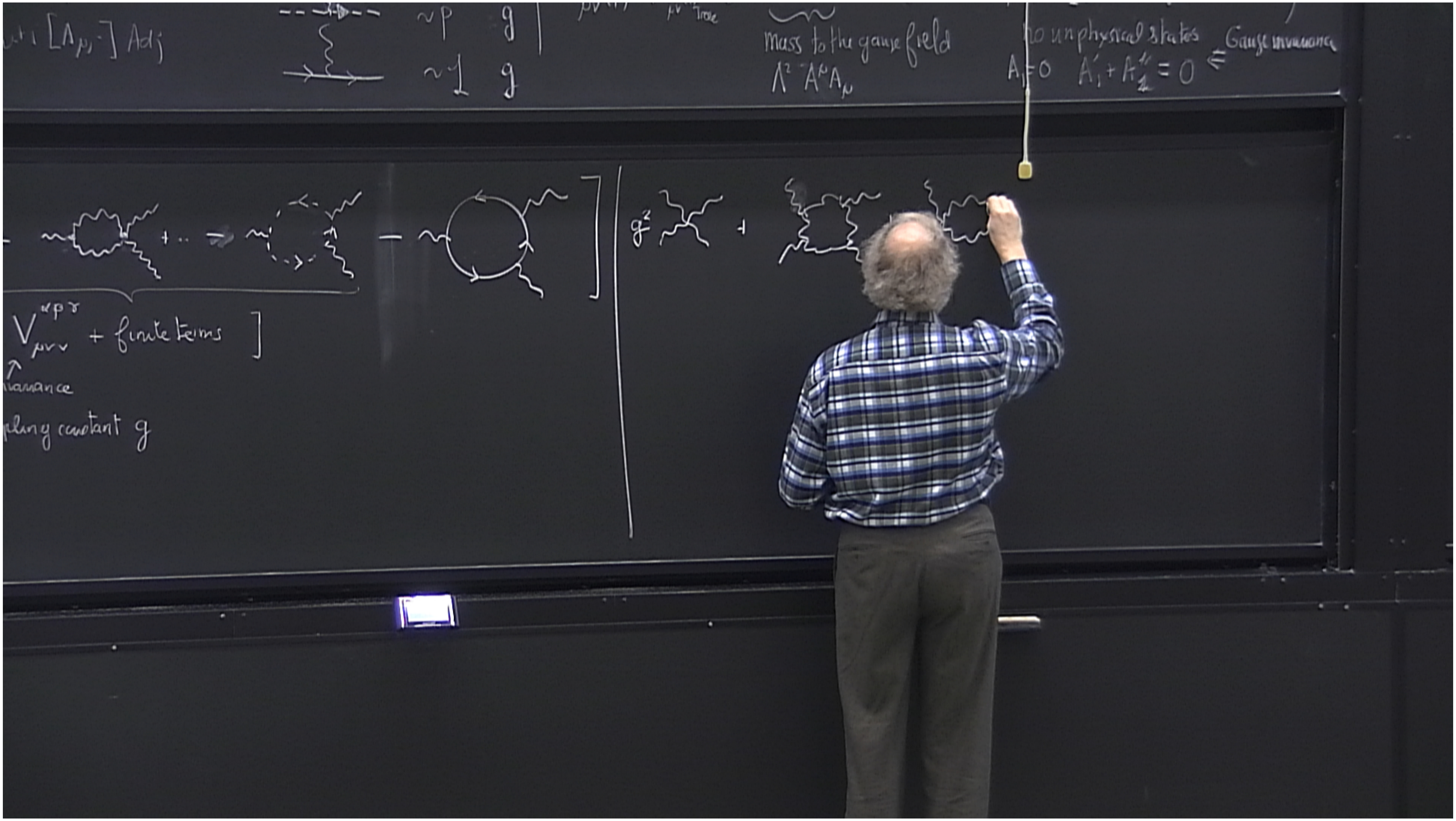
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]^a$
$D_\mu = \partial_\mu + i[A_\mu, \cdot]$ Adj
$\Gamma^{abc}_{\mu\nu\rho}(k_1, k_2, k_3) = V^{abc}_{\mu\nu\rho}(k)$
$k_1 + k_2 + k_3 = 0$
tree level vertex

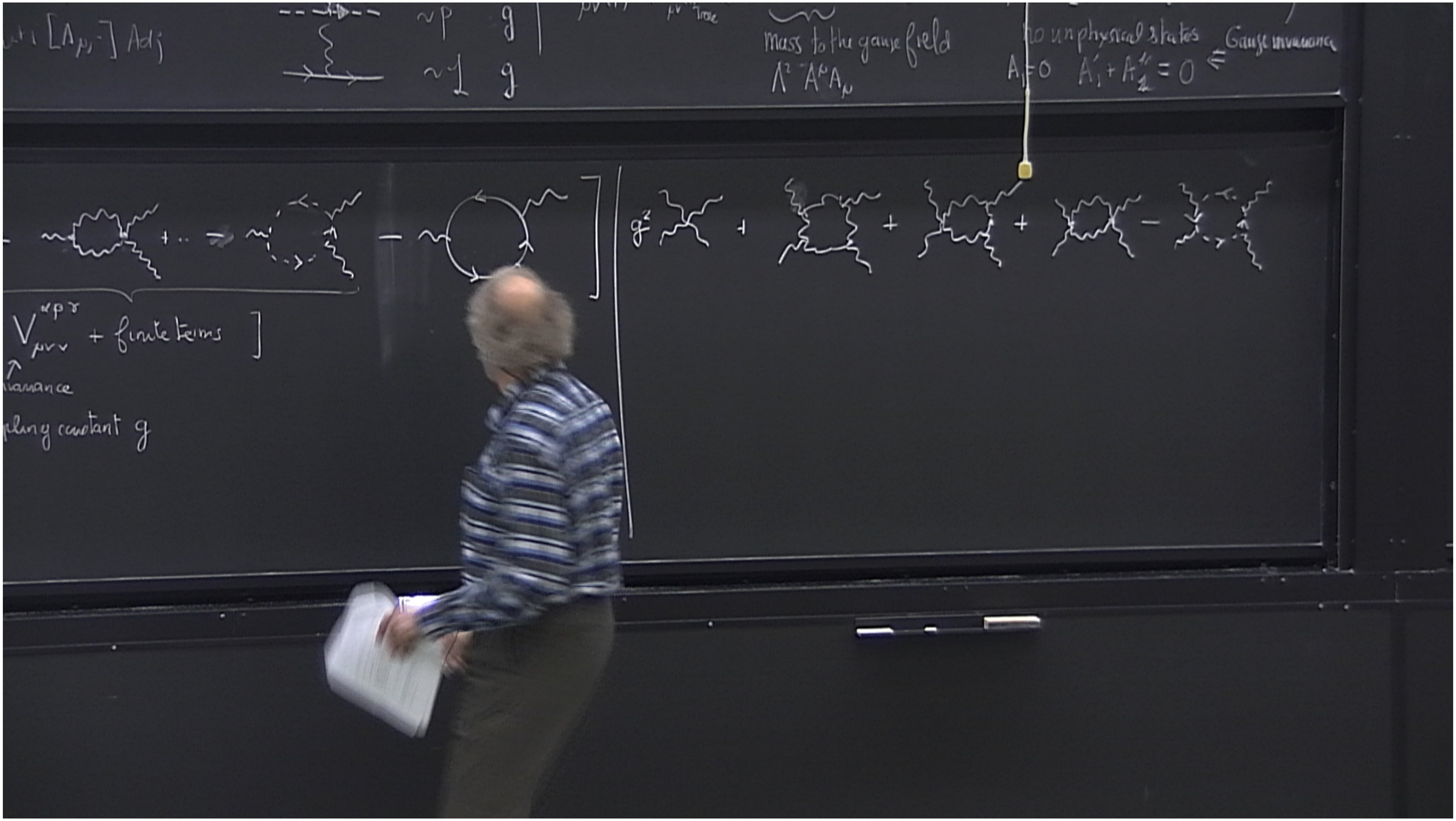



tree level vertex
potential Λ divergence
but in fact 0
mass to the gauge


mass to the gauge
+ finite terms
gauge invariance
potential
but in




Mass to the gauge field
no unphysical states
Gauge invariance
+ finite terms


Mass to the gauge field
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Gauge invariance
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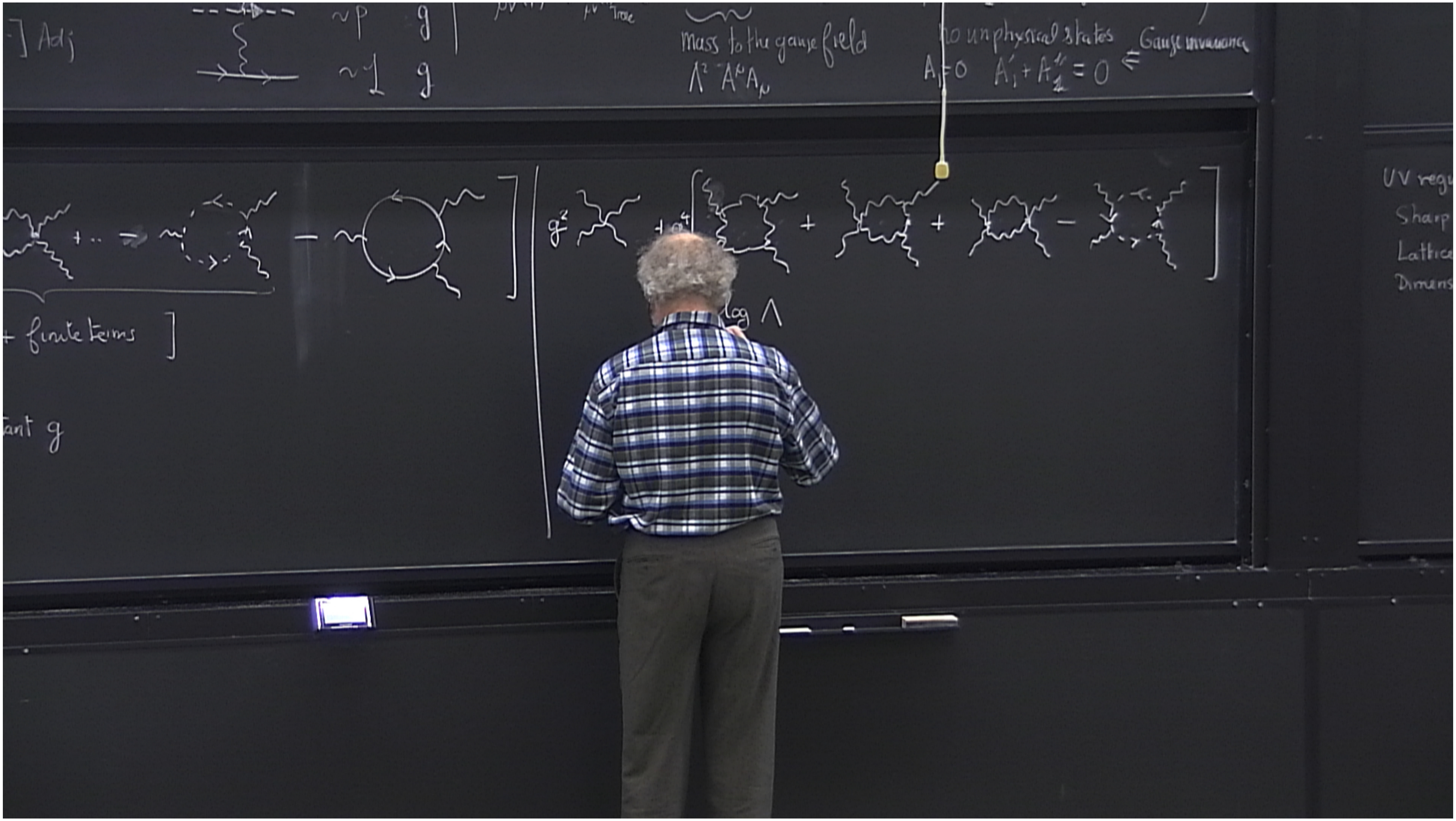
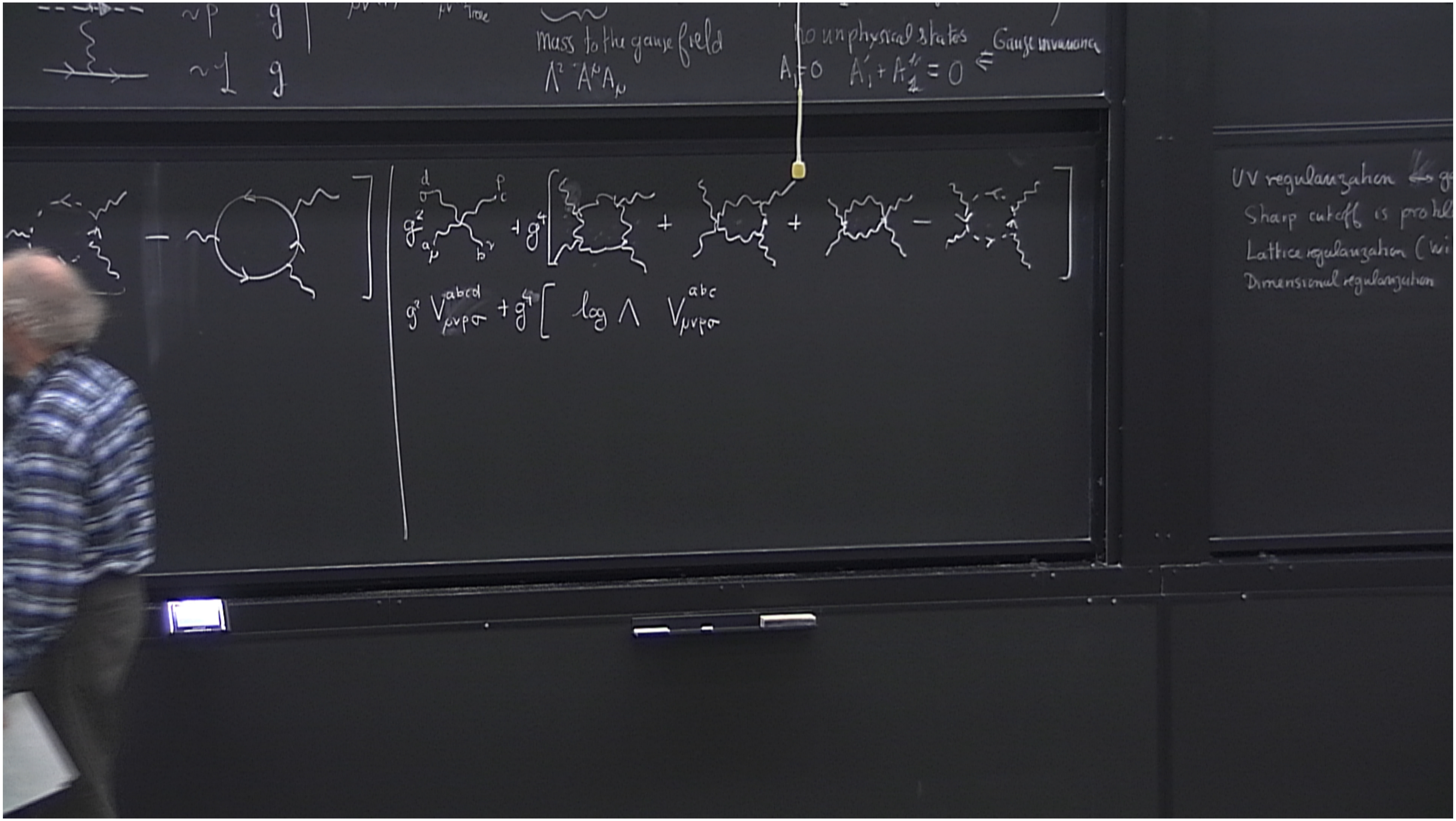

Adj
mass to the gauge field
no unphysical states
Gauge invariance
+ finite terms
log Λ
UV regu
Sharp
Lattice
Dimens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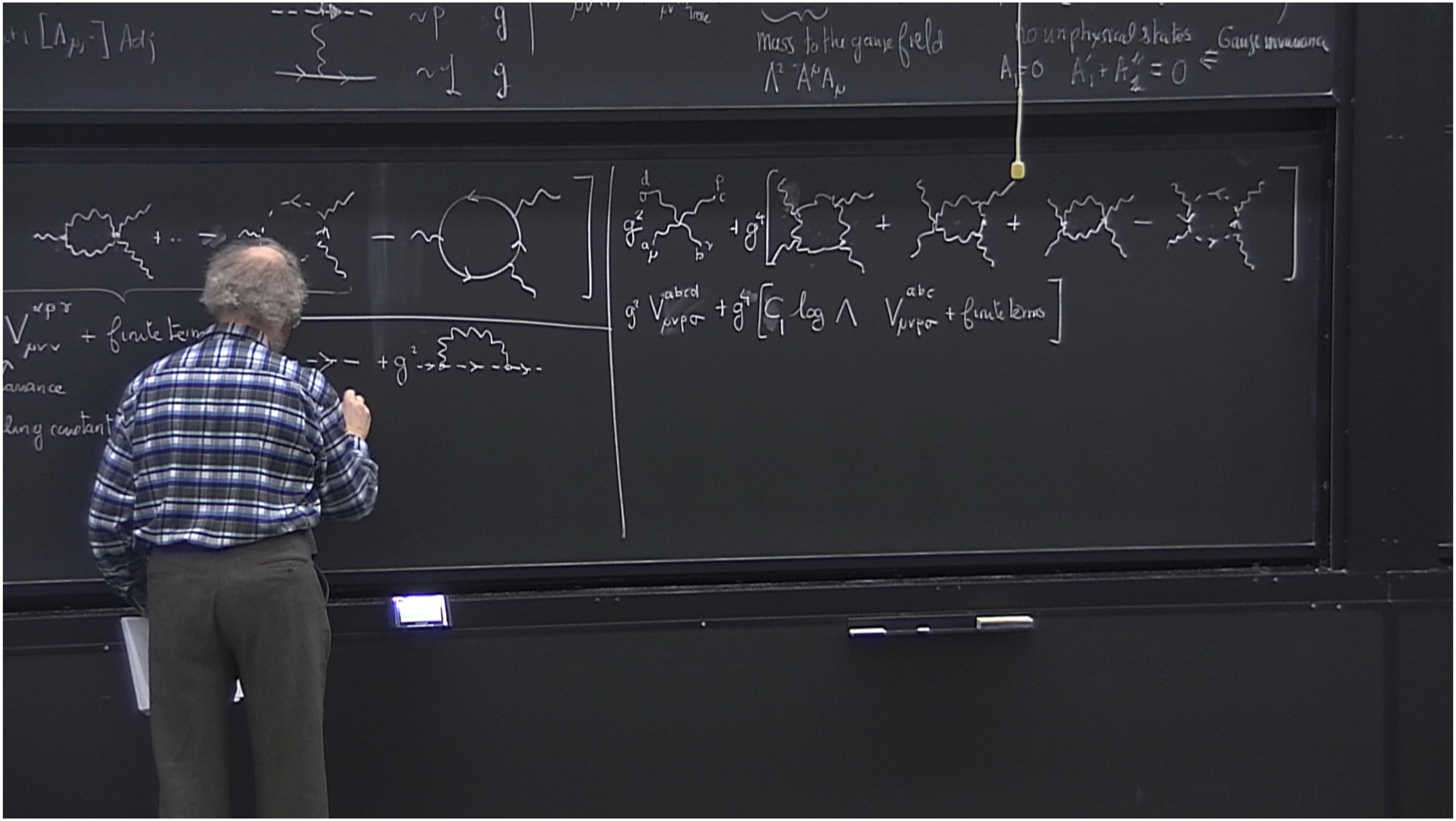
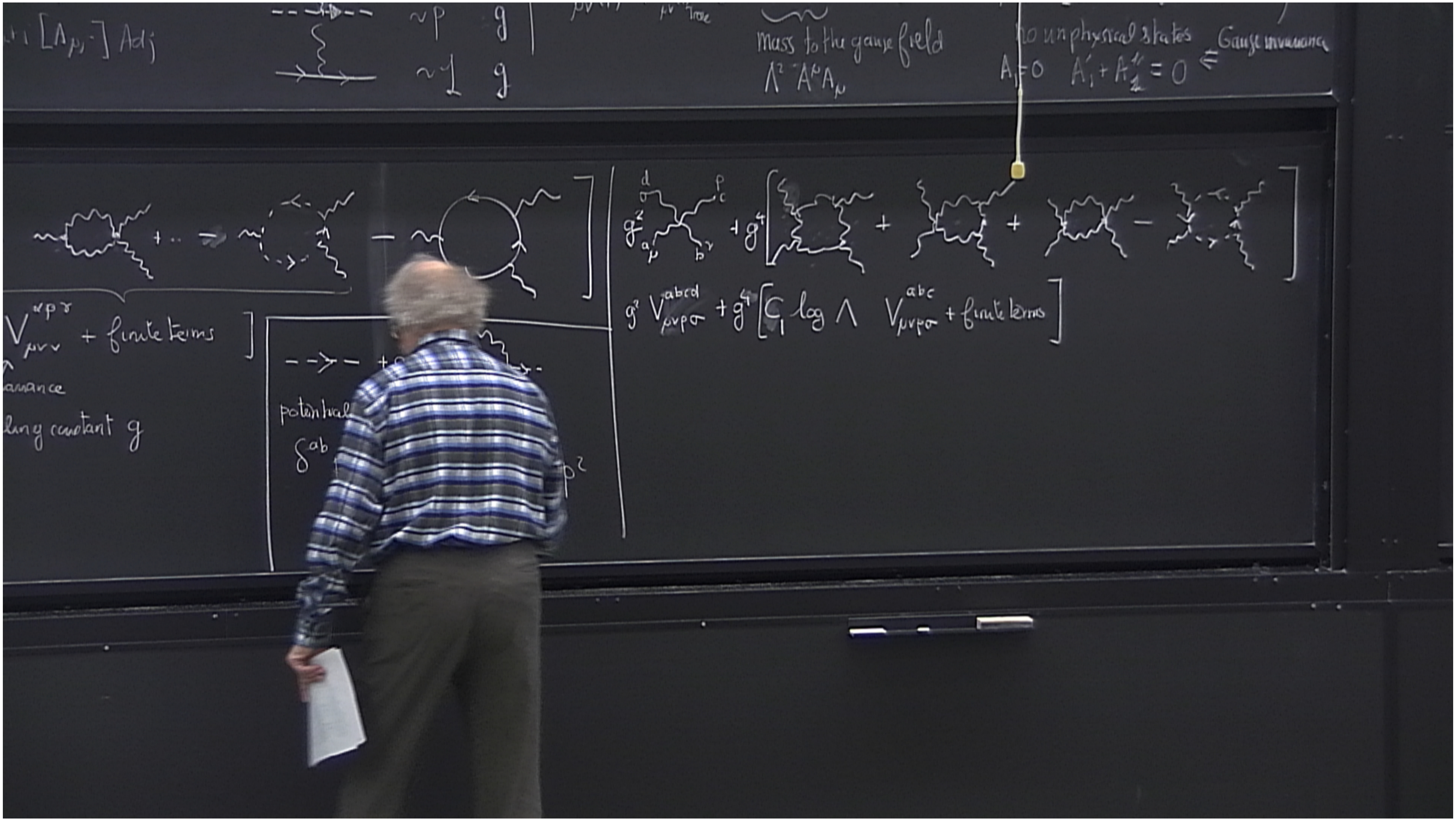
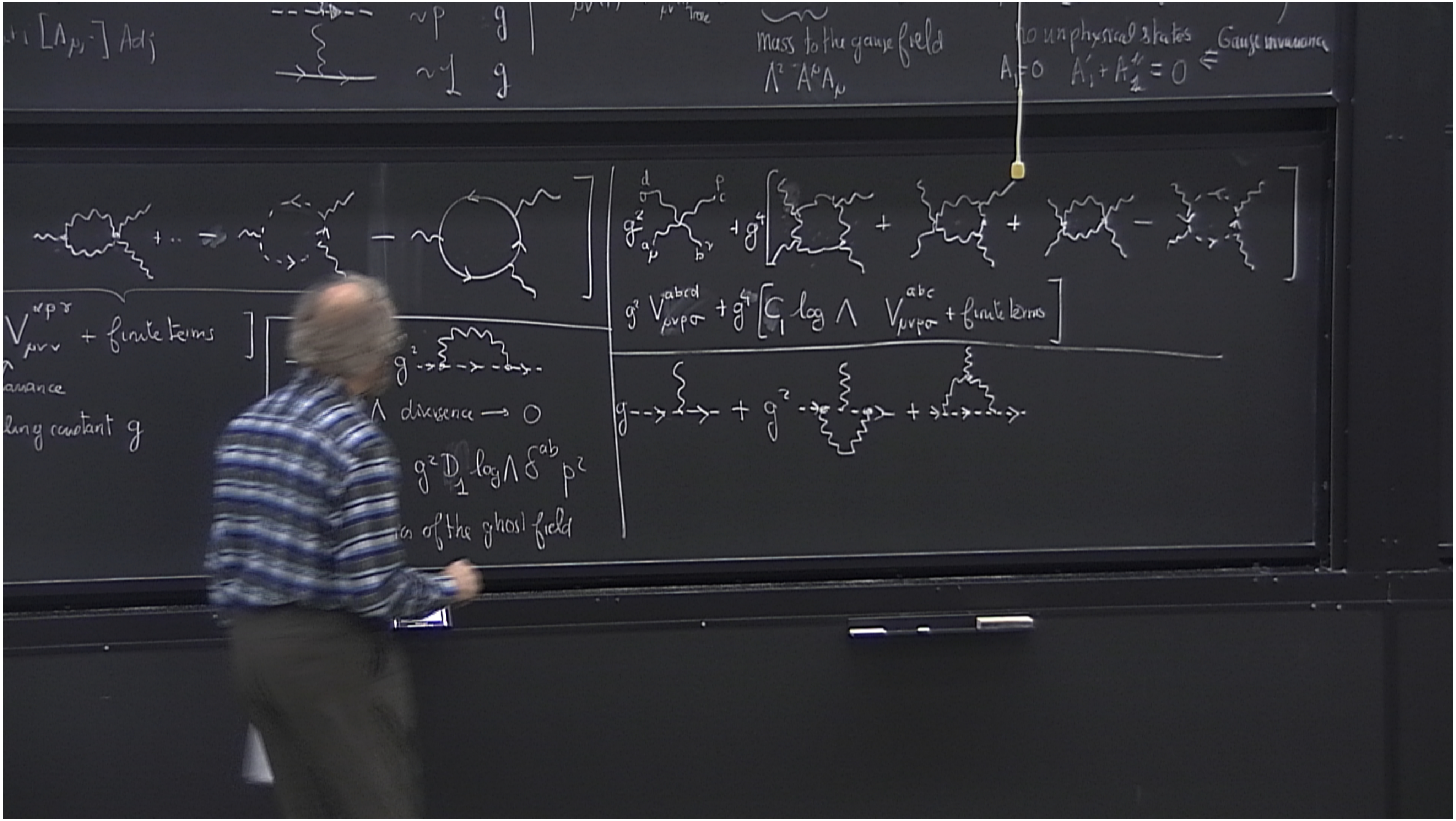
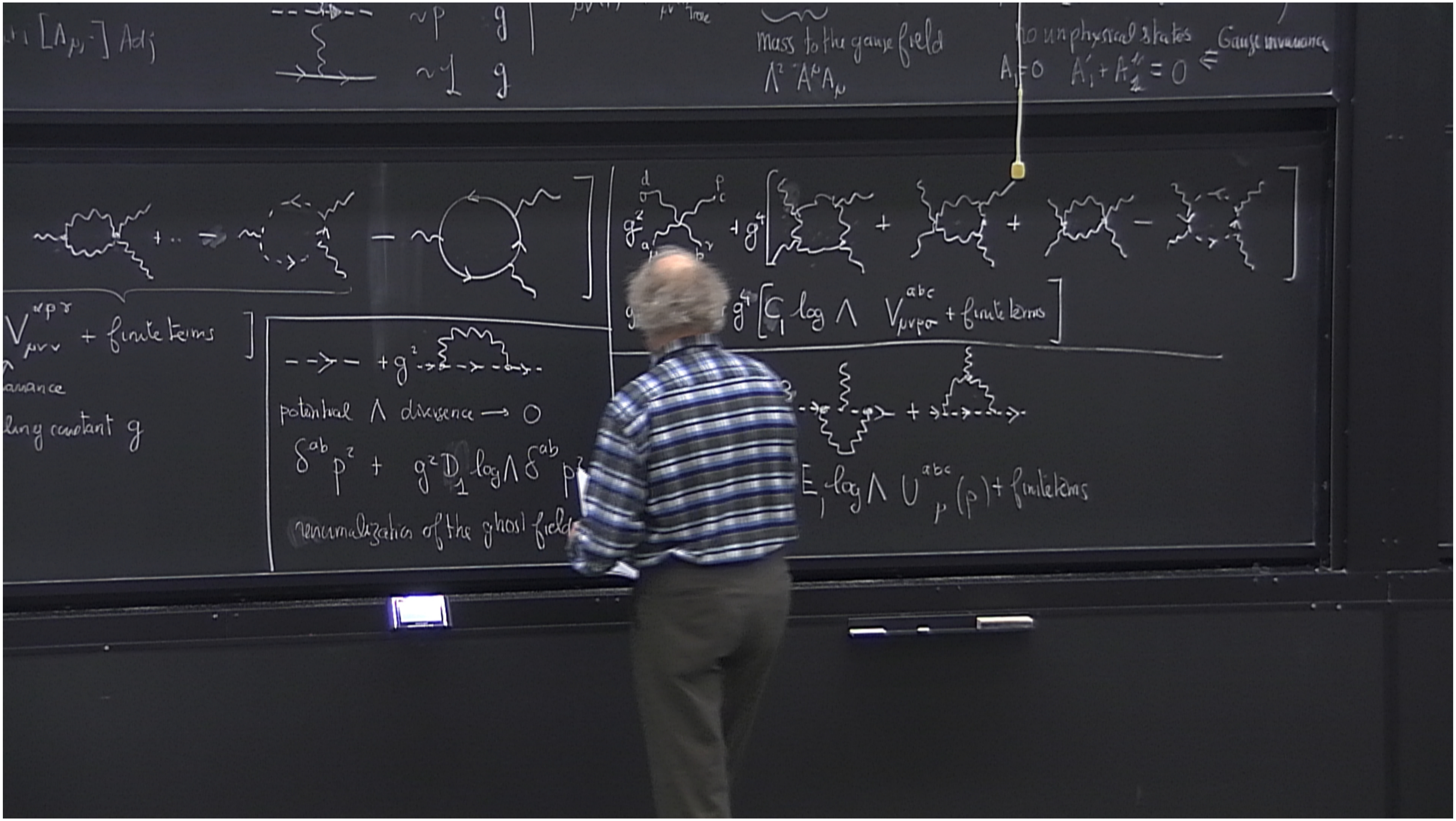
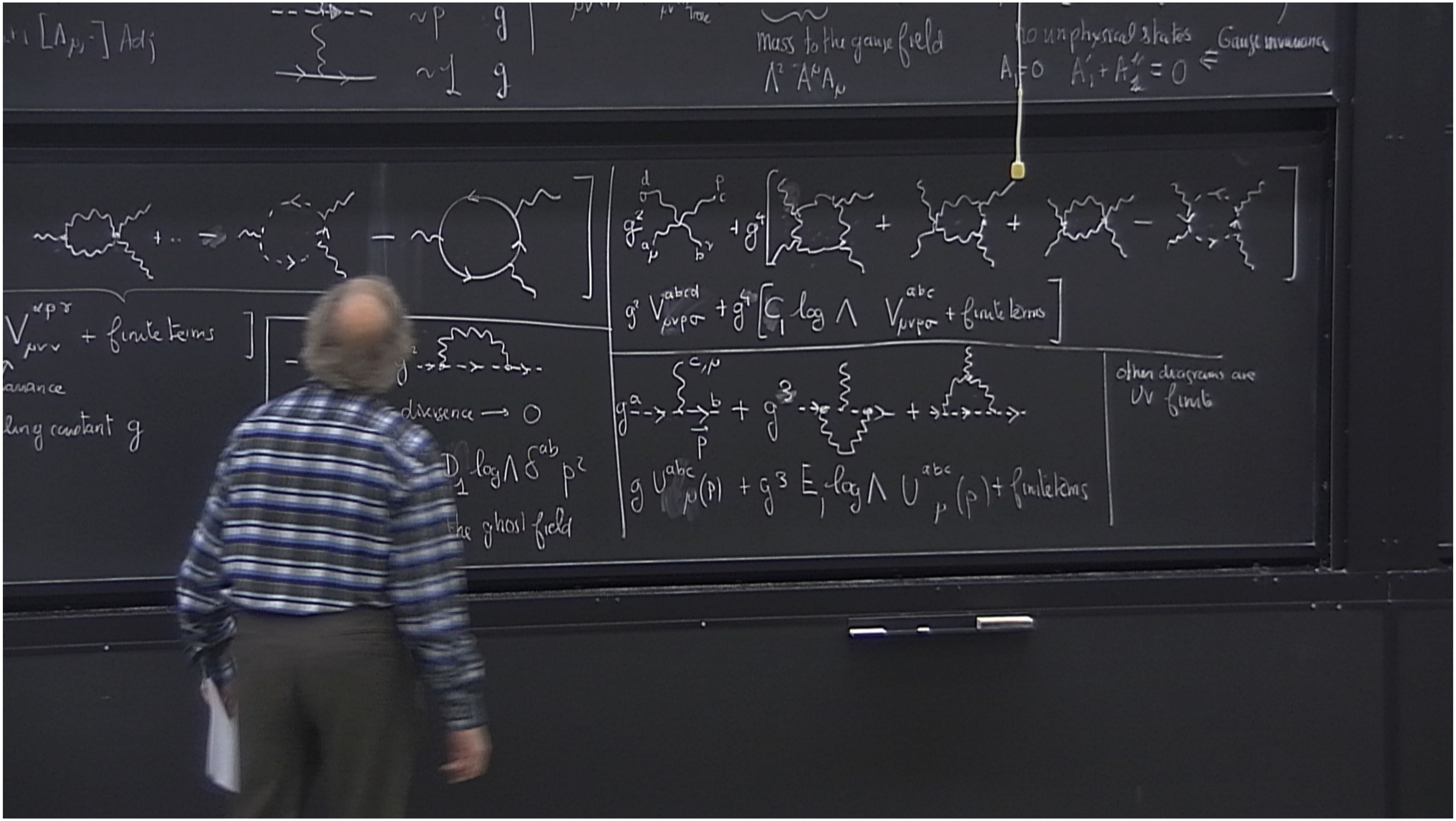



Mass to the gauge field
no unphysical states
Gauge invariance
+ finite terms
coupling constant g
potential
+ finite terms


Mass to the gauge field
no unphysical states
Gauge invariance
+ finite terms
divergence → 0
of the ghost field


Mass to the gauge field
no unphysical states
Gauge invariance
+ finite terms
coupling constant g
potential Λ divergence → 0
renormalization of the ghost field
+ finite terms
+ finite terms



Mass to the gauge field
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Gauge invariance
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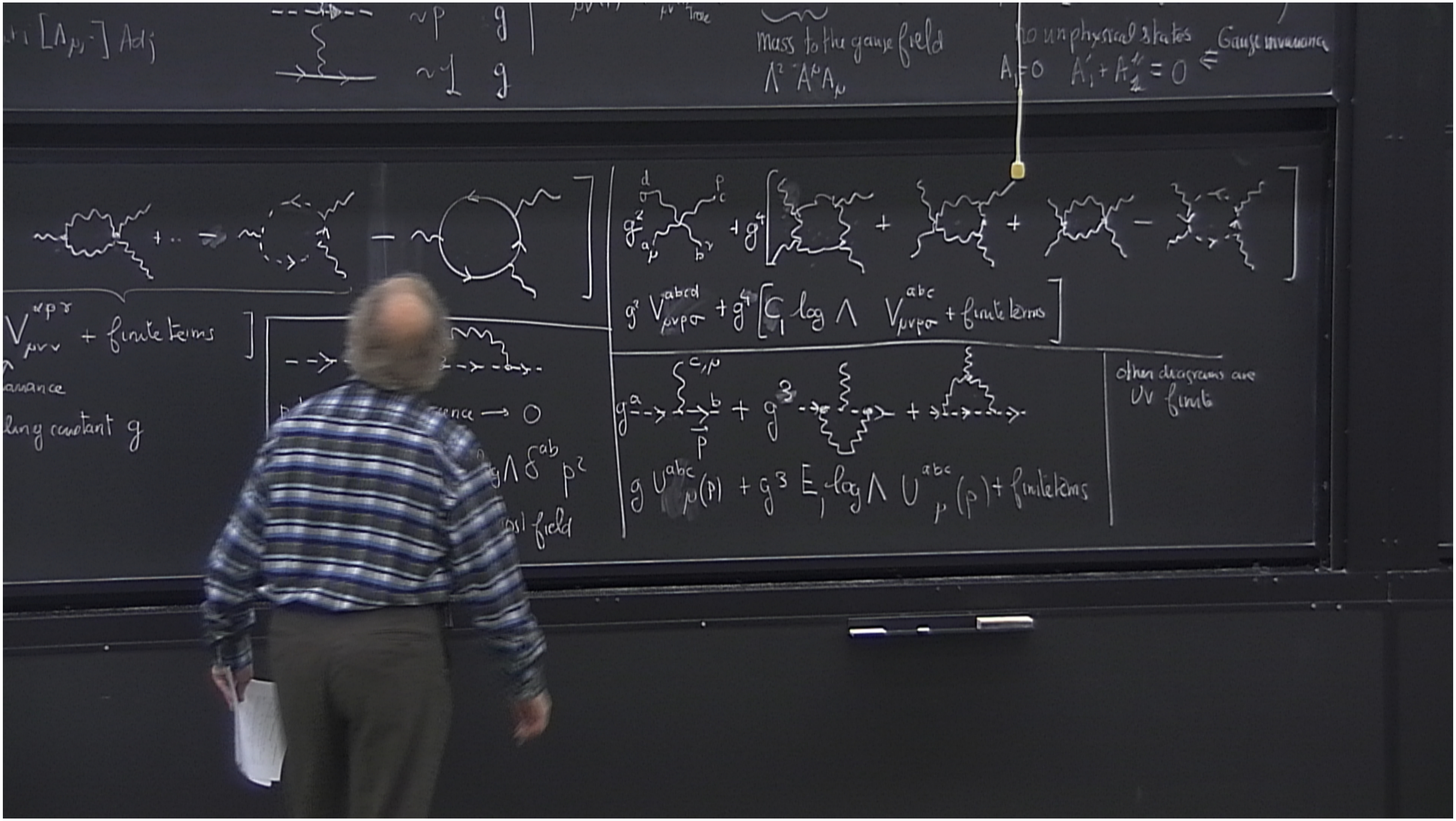
divergence → 0
ghost field
other diagrams are
UV finite


Mass to the gauge field
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Gauge invariance
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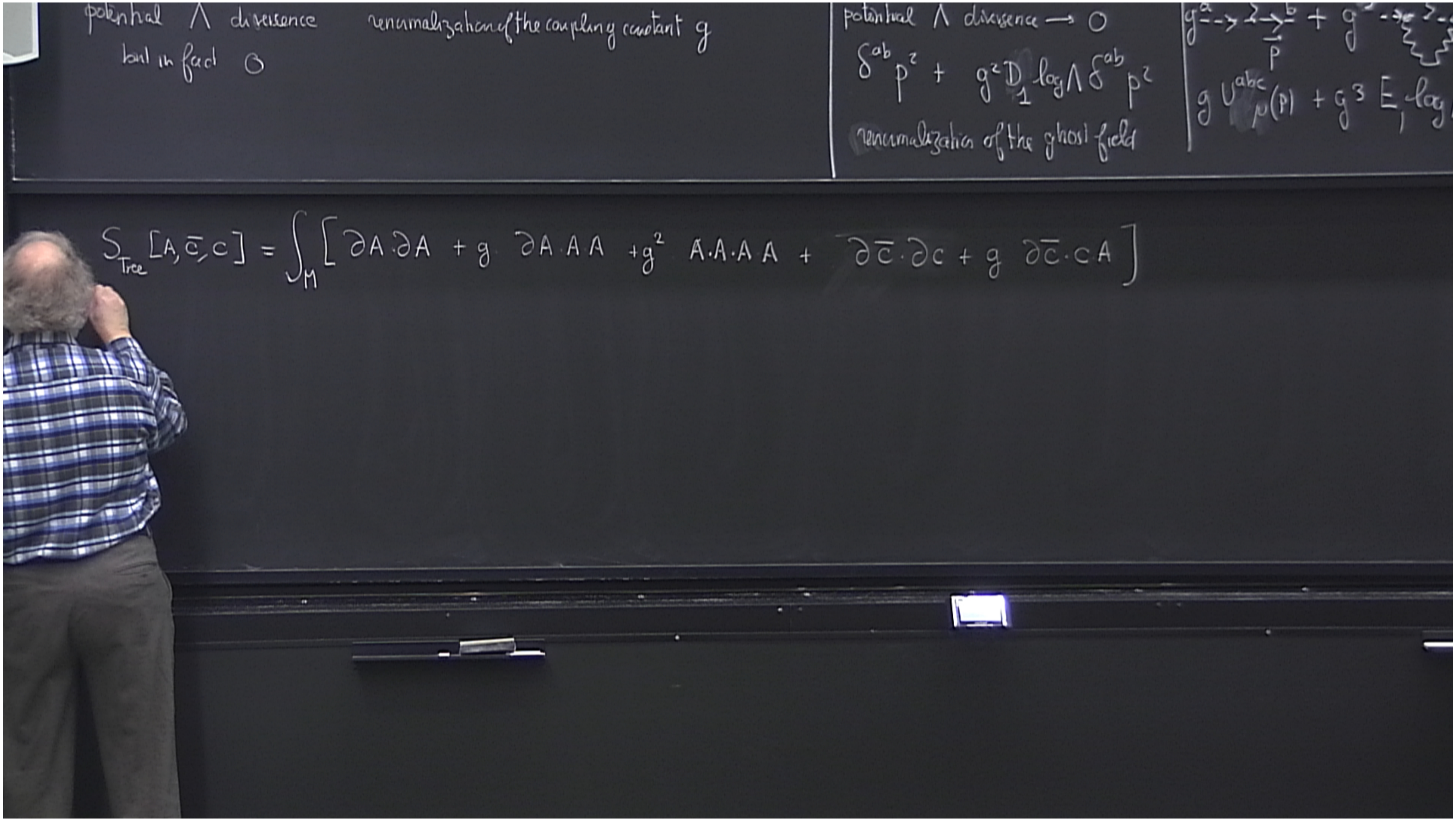
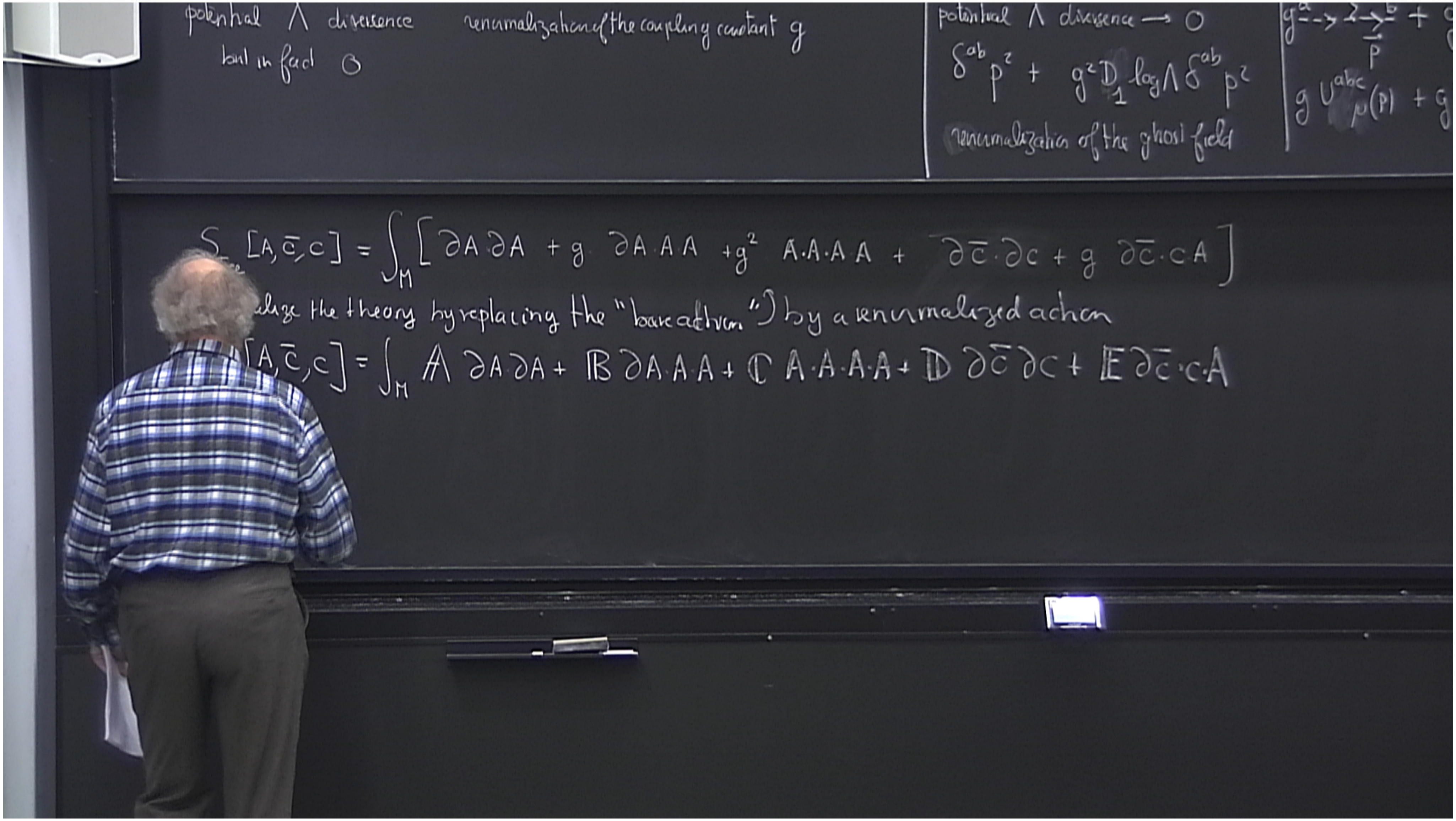

potential Λ divergence
but in fact 0
renormalization of the coupling constant g
potential Λ divergence → 0
δ^ab p² + g² D_1 log Λ δ^ab p²
renormalization of the ghost field
g V^abc_μ(p) + g³ E_1 log
S_Tree [A, c̄, c] = ∫_M [∂A·∂A + g ∂A·A·A + g² A·A·A·A + ∂c̄·∂c + g ∂c̄·c A]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renormalization of the ghost field
S [A, c̄, c] = ∫_M [∂A ∂A + g ∂A·A A + g^2 A·A·A A + ∂c̄·∂c + g ∂c̄·c A]
alize the theory by replacing the "bare action" S by a renormalized action
[A, c̄, c] = ∫_M 𝔸 ∂A∂A + 𝔹 ∂A·A·A + ℂ A·A·A·A + 𝔻 ∂c̄ ∂c + 𝔼 ∂c̄·c·A

potential Λ divergence   renormalization of the coupling constant g

but in fact 0

potential Λ divergence → 0

$\delta^{ab} p^2 + g^2 D_1 \log\Lambda\, \delta^{ab} p^2$

renormalization of the ghost field

$g\, V^{abc}_{\mu}(p) + g$

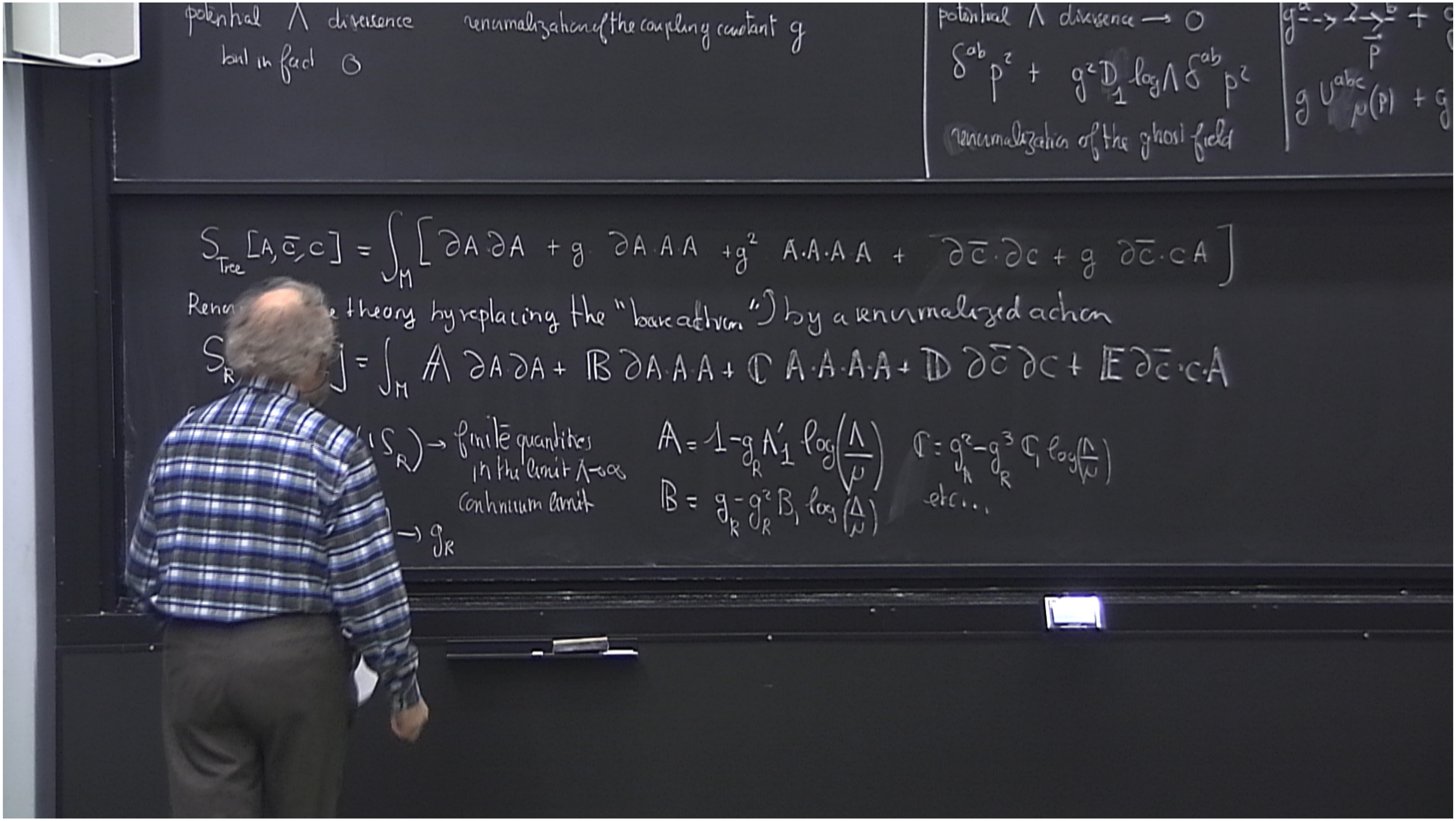
$$S_{Tree}[A,\bar{c},c] = \int_M \left[\partial A \partial A + g\, \partial A \cdot A\, A + g^2\, A \cdot A \cdot A\, A + \partial\bar{c}\cdot\partial c + g\, \partial\bar{c}\cdot c\, A \right]$$

Renormalize the theory by replacing the "bare action" S by a renormalized action

$$S_R[A,\bar{c},c] = \int_M \mathbb{A}\, \partial A \partial A + \mathbb{B}\, \partial A \cdot A \cdot A + \mathbb{C}\, A\cdot A\cdot A\cdot A + \mathbb{D}\, \partial\bar{c}\, \partial c + \mathbb{E}\, \partial\bar{c}\cdot c \cdot A$$

$\int D[A] D[\bar{c}, c] \exp(i S_R) \to$ finite quantities in the limit $\Lambda \to \infty$ $\mathbb{A} = 1 - g A'_1$

Renormalized theory   continuum limit



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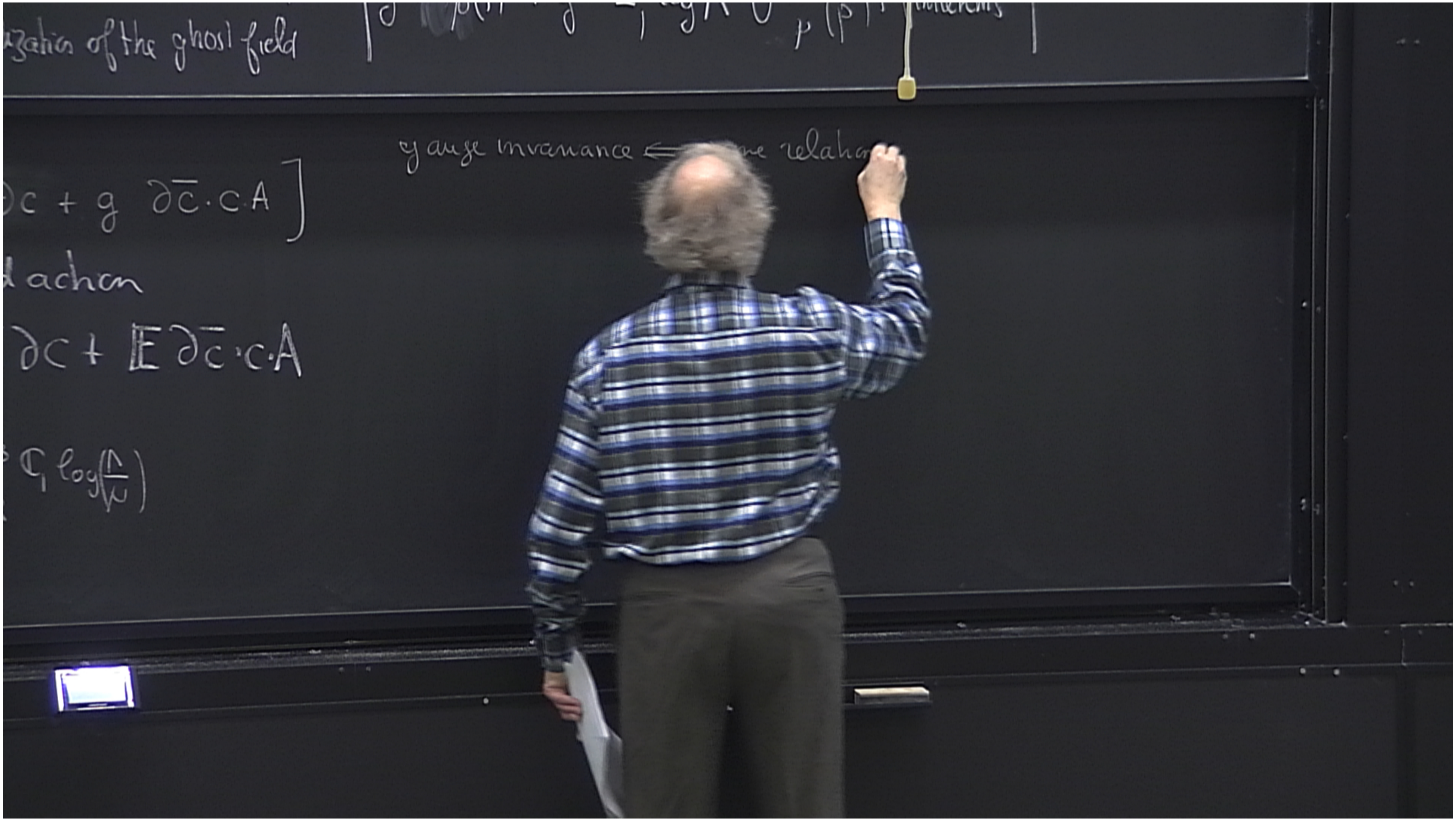
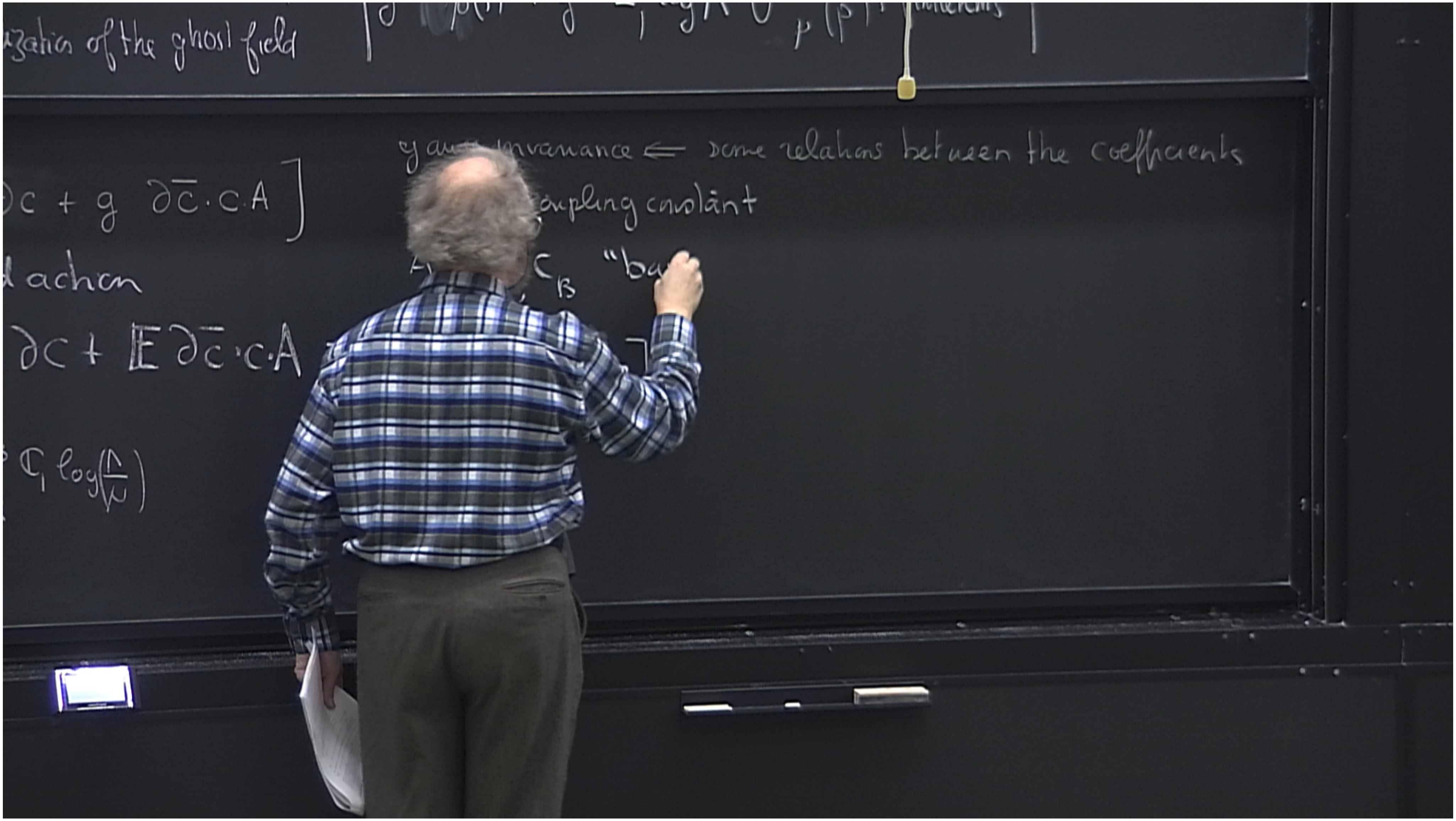
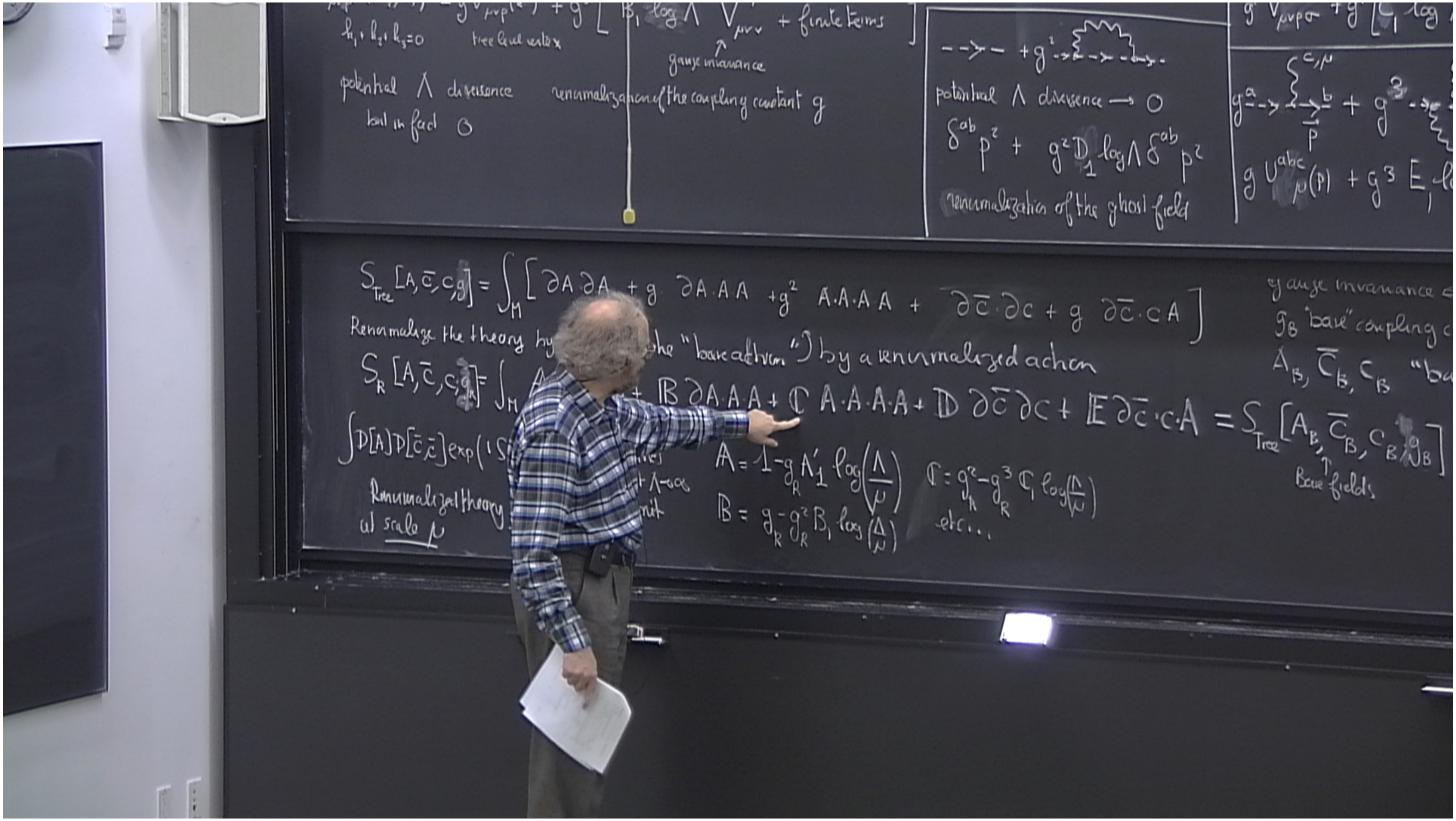
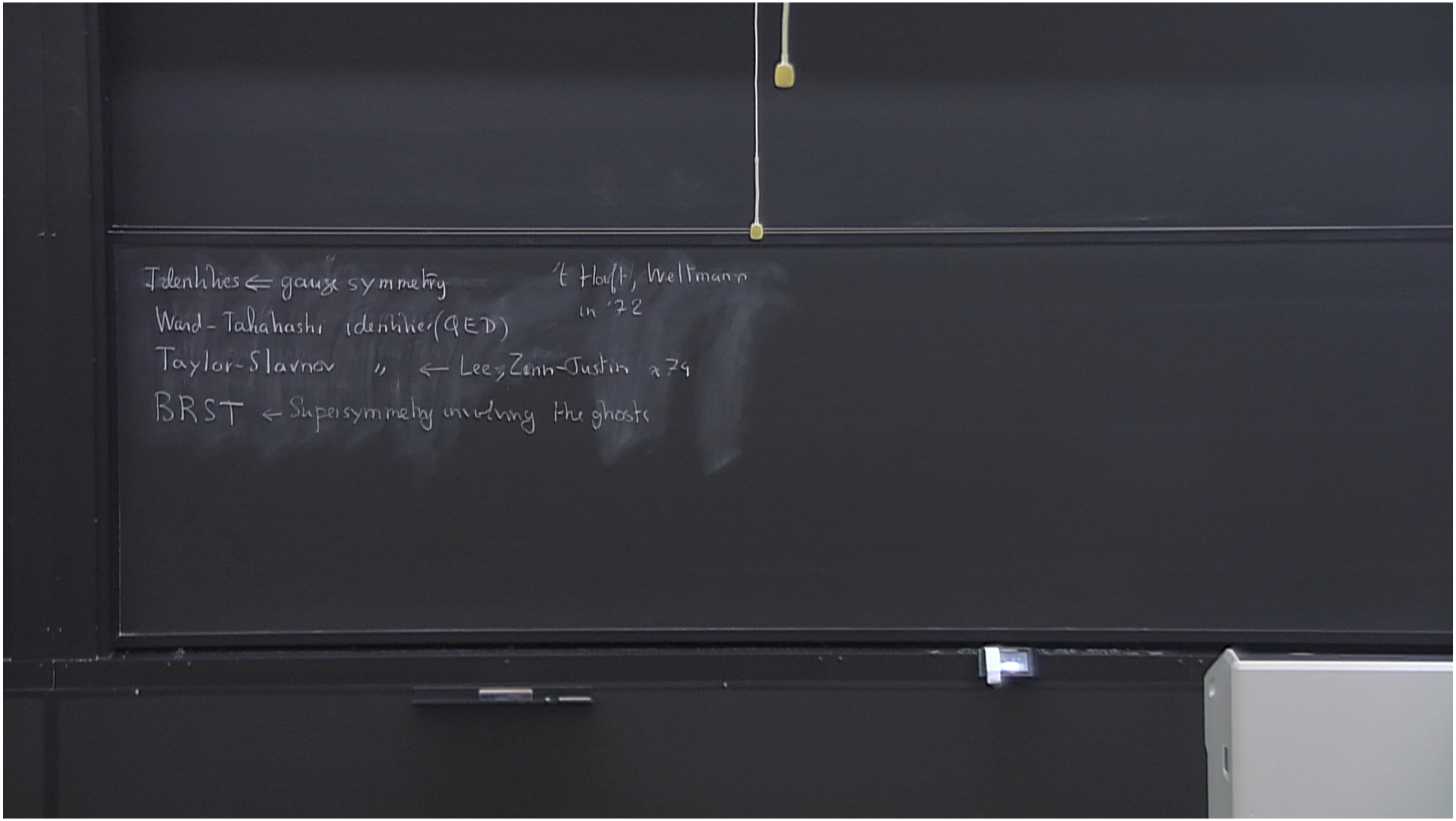
S_Tree [A, c̄, c] = ∫_M [∂A ∂A + g ∂A·A A + g² A·A·A A + ∂c̄·∂c + g ∂c̄·c A]
theory by replacing the "bare action" by a renormalized action
= ∫_M 𝔸 ∂A ∂A + 𝔹 ∂A·A·A + ℂ A·A·A·A + 𝔻 ∂c̄ ∂c + 𝔼 ∂c̄·c·A
finite quantities in the limit Λ→∞ continuum limit
A = 1 − g_R A'_1 log(Λ/μ)
B = g_R − g_R² B_1 log(Λ/μ)
C = g_R² − g_R³ C_1 log(Λ/μ)
etc...
→ g_R

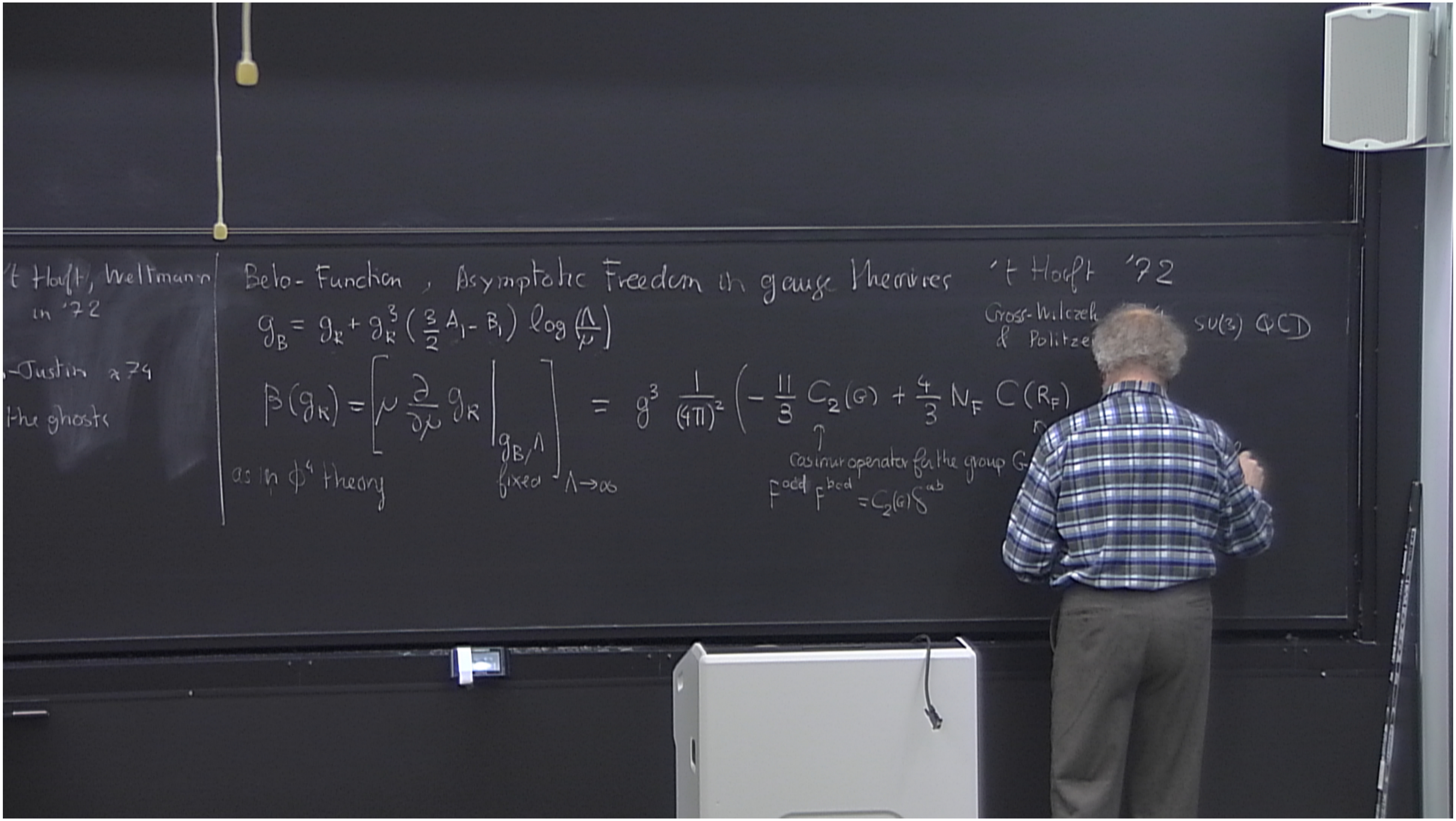

of the ghost field
gauge invariance ⟸
achon

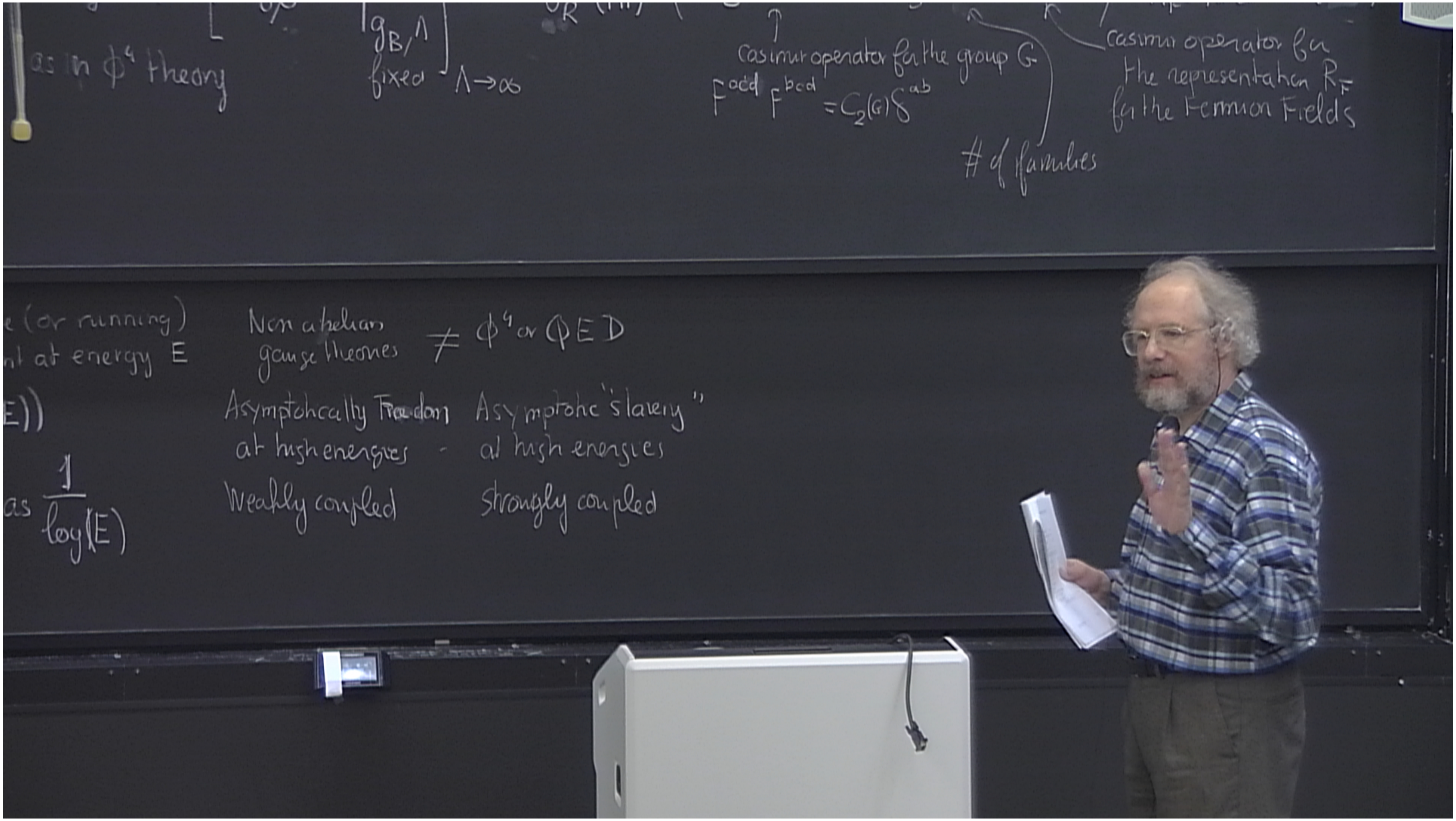

of the ghost field
invariance ⇐ some relations between the coefficients
coupling constant
"ba

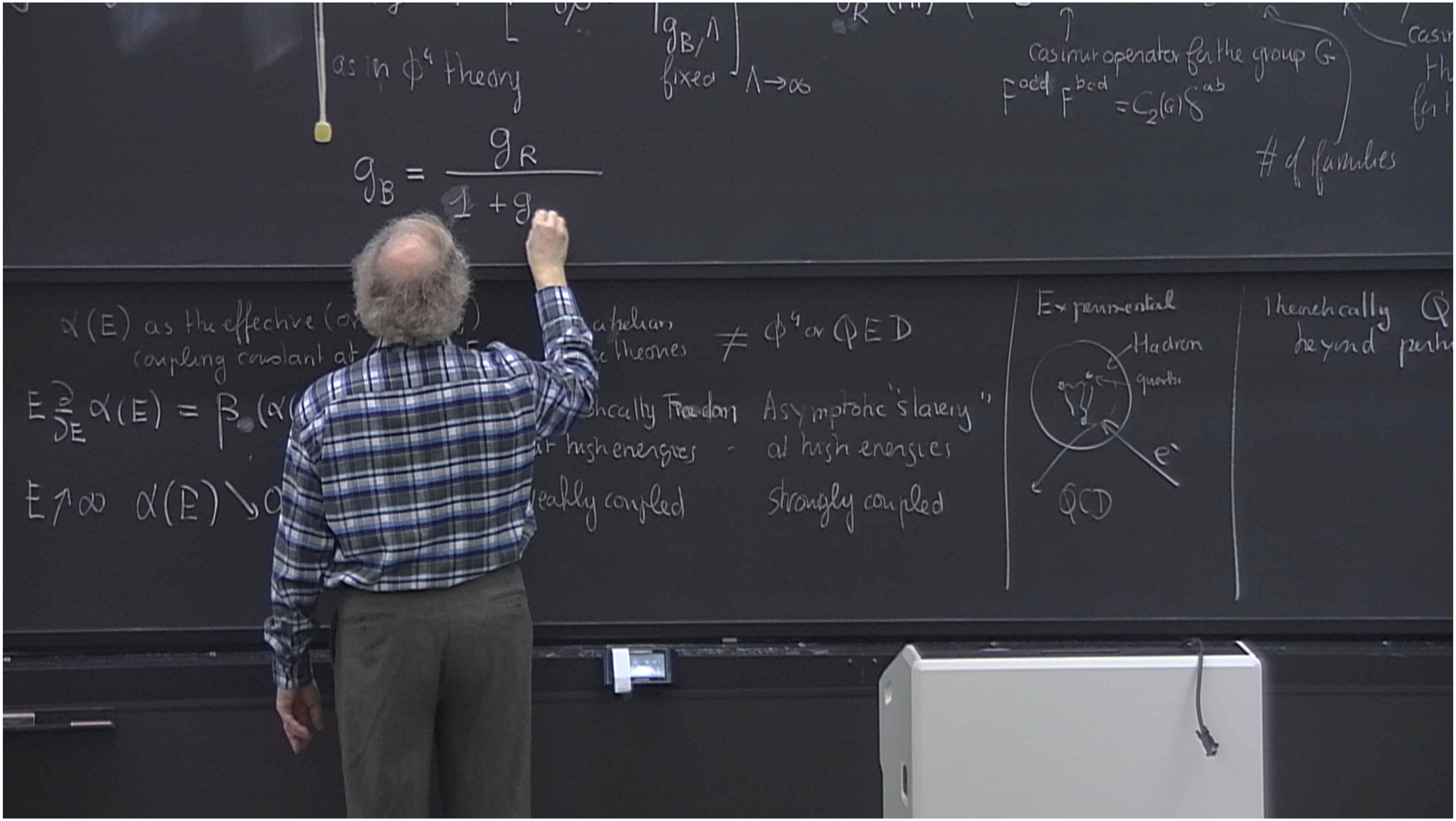


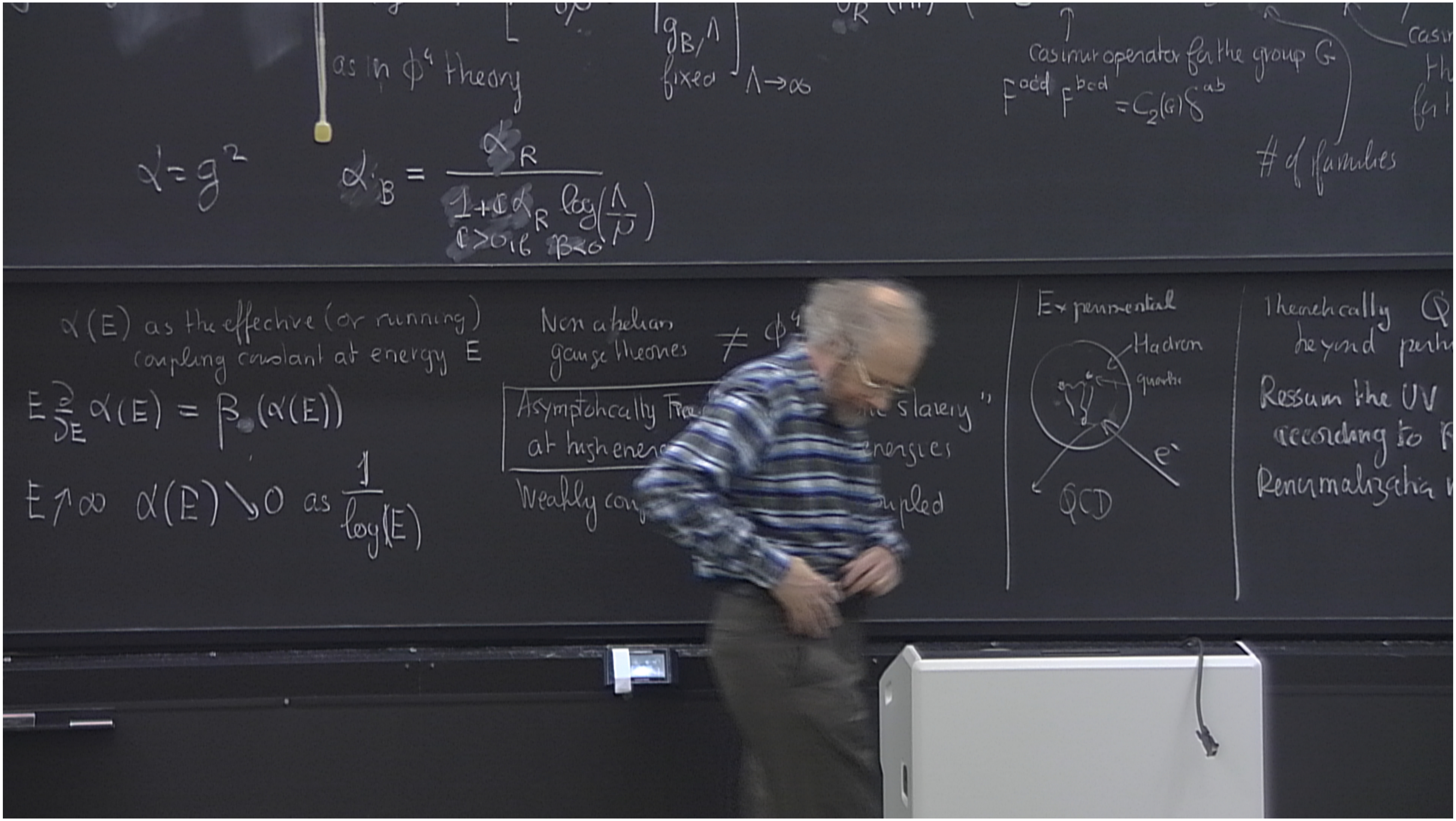
+ finite terms
tree level vertex
gauge invariance
potential Λ divergence
but in fact 0
renormalization of the coupling constant g
potential Λ divergence → 0
renormalization of the ghost field
Renormalize the theory by
"bare action"
by a renormalized action
Renormalized theory
at scale μ
etc...
gauge invariance
g_B "bare" coupling
Bare fields


Identities ⇐ gauge symmetry
't Hooft, Veltman in '72
Ward-Takahashi identities (QED)
Taylor-Slavnov " ← Lee, Zinn-Justin ≈ 74
BRST ← Supersymmetry involving the ghosts


't Hooft, Veltman in '72
-Justin
the ghosts
Beta-Function , Asymptotic Freedom in gauge theories 't Hooft '72
Gross-Wilczek & Politzer
SU(3) QCD
$g_B = g_R + g_R^3 \left(\frac{3}{2} A_1 - B_1 \right) \log \left(\frac{\Lambda}{\mu} \right)$
$\beta(g_R) = \left[\mu \frac{\partial}{\partial \mu} g_R \Big|_{g_B, \Lambda \text{ fixed}} \right]_{\Lambda \to \infty} = g^3 \frac{1}{(4\pi)^2} \left(-\frac{11}{3} C_2(G) + \frac{4}{3} N_F C(R_F) \right.$
as in ϕ^4 theory
Casimir operator for the group G
$F^{acd} F^{bcd} = C_2(G) \delta^{ab}$


as in ϕ^4 theory
g_B/Λ fixed
$\Lambda \to \infty$
casimir operator for the group G
$F^{acd} F^{bcd} = C_2(G)\delta^{ab}$
casimir operator for the representation R_F for the Fermion Fields
# of families
(or running)
at energy E
Non abelian gauge theories $\neq$ ϕ^4 or QED
Asymptotically Freedom at high energies
Asymptotic "slavery" at high energies
as $\frac{1}{\log(E)}$
Weakly coupled
strongly coupled


as in φ⁴ theory
fixed
Λ→∞
Casimir operator for the group G
# of families
g_B = g_R / 1 + g
coupling constant at
≠ φ⁴ or QED
Asymptotic "slavery"
at high energies
strongly coupled
Experimental
Hadron
quarks
QCD
Theoretically
beyond


as in ϕ⁴ theory
fixed
Λ→∞
casimir operator for the group G
# of families
α = g²
α(E) as the effective (or running) coupling constant at energy E
Non abelian gauge theories
Asymptotically Free
at high energ
Weakly co
Experimental
Hadron
quarks
QCD
Theoretically
beyond
Ressum the UV
according to
Renormalization