

Title: Quantum Field Theory II -14

Date: Nov 27, 2014 09:00 AM

URL: <http://pirsa.org/14110023>

Abstract:

$SU(2)$

$A_\mu^a(x)$  gauge fields

$\psi^i$  matter fields

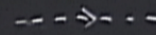
$c^a, \bar{c}_a$  ghost fields

SU(2)

$A_\mu^a(x)$  gauge fields

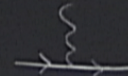
$\psi^i$  matter fields

$c^a, \bar{c}^a$  ghost fields (gauge fixing)



$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} (i \not{D} - m) \psi + S \frac{(\partial^\mu A_\mu^a)^2}{2} \right)$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$



SU(2)

$A_\mu^a(x)$  gauge fields

$\Psi_\alpha^i$  matter fields

$c^a, \bar{c}_a$  ghost fields (gauge)

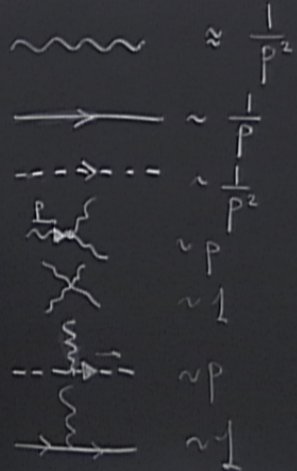
$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F_a^{\mu\nu} + \bar{\Psi}_\alpha^i (i \not{D} - m) \Psi_\alpha^i \right)$$

$$\bar{c}^a (\not{\partial} \not{D}_\mu)_{ab} c^b$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i [A_\mu, A_\nu]^a$$

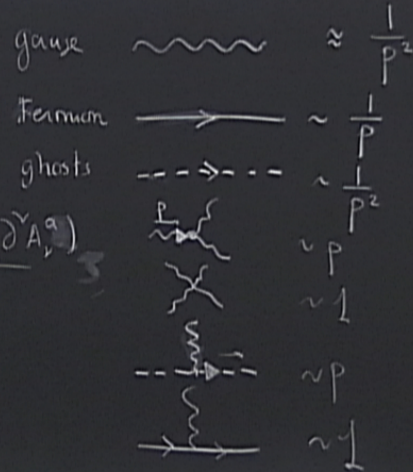
$$D_\mu = \partial_\mu + i g A_\mu^a T^a$$

gauge



SU(2)

$A_\mu^a(x)$  gauge fields  
 $\Psi^i$  matter fields  
 $c^a, \bar{c}_a$  ghost fields (gauge fixing)



UV divergences & renormalizability

$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu}_a + \bar{\Psi}^i (i \not{D} - m) \Psi^i + S \frac{(\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a)}{2} \right)$$

$$\bar{c}^a (\partial^\mu D_\mu)_{ab} c^b$$




$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i [A_\mu, A_\nu]^a$$

SU(2)

$A_\mu^a(x)$  gauge fields  
 $\Psi_\alpha^i$  matter fields  
 $c^a, \bar{c}_a$  ghost fields (gauge fixing)

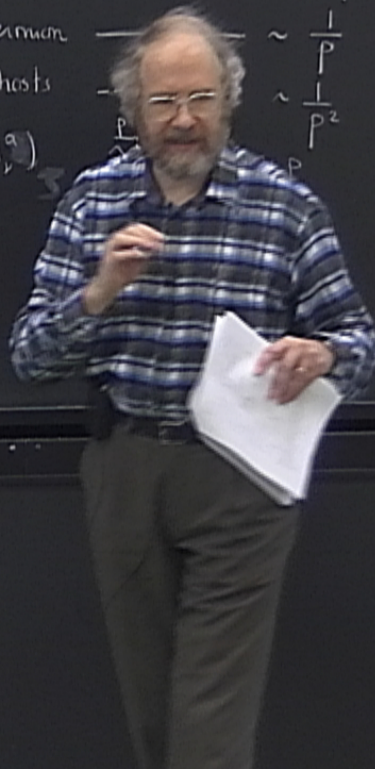
gauge   $\approx \frac{1}{p^2}$   
 Fermion   $\sim \frac{1}{p}$   
 ghosts   $\sim \frac{1}{p^2}$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop

$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu}_a + \bar{\Psi}_\alpha^i (i\mathcal{D} - m)_{ij} \Psi_\alpha^j + S \frac{(\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a)}{2} \right)$$

$$\bar{c}^a (\partial^\mu \mathcal{D}_\mu)_{ab} c^b \quad \mathcal{D}_\mu = \partial_\mu + i A_\mu \text{ (Fund)}$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i [A_\mu, A_\nu]^a \quad \mathcal{D}_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ (Adj)}$$



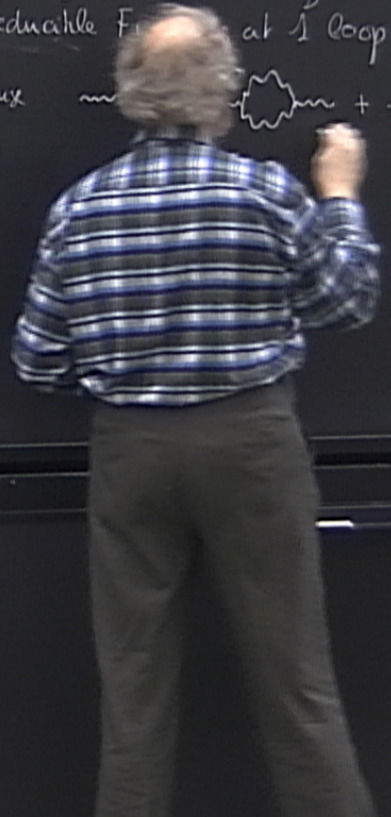
gauge fields  $\approx \frac{1}{p^2}$   
 matter fields  $\sim \frac{1}{p}$   
 ghost fields (gauge fixing)  $\sim \frac{1}{p^2}$

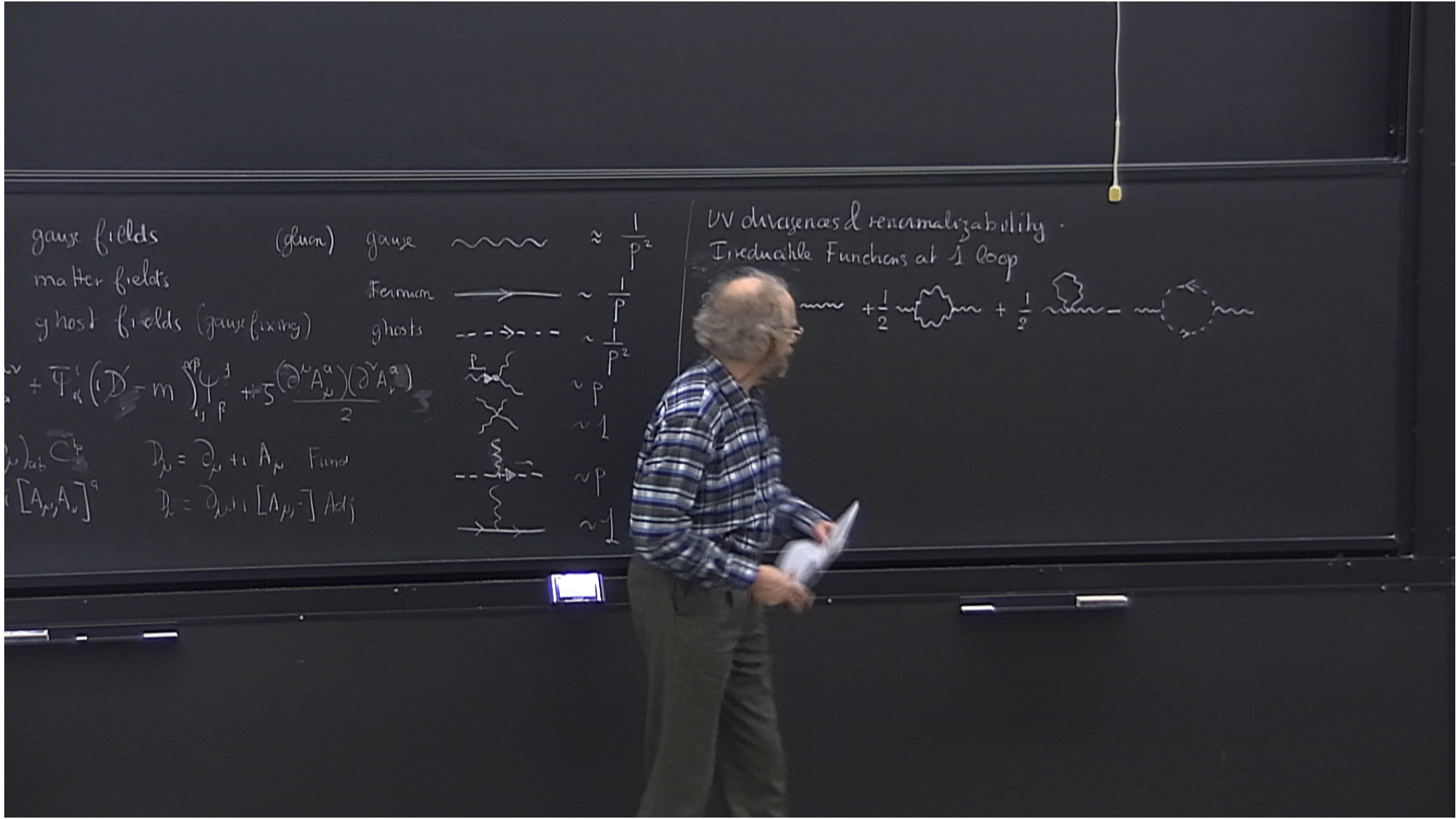
$\Psi^\dagger + \bar{\Psi} \left( i \not{D} - m \right) \Psi + S \frac{(\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a)}{2}$   
 $D_\mu = \partial_\mu + i A_\mu$  Fund  
 $D_\mu = \partial_\mu + i [A_\mu, \cdot]$  Adj

gauge  $\sim \frac{1}{p^2}$   
 Fermion  $\sim \frac{1}{p}$   
 ghosts  $\sim \frac{1}{p^2}$

$\sim p$   
 $\sim 1$   
 $\sim p$   
 $\sim 1$

UV divergences & renormalizability  
 Irreducible F at 1 loop





gauge fields

(gluon)

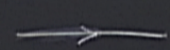
gauge



$$\approx \frac{1}{p^2}$$

matter fields

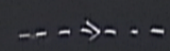
Feynman



$$\sim \frac{1}{p}$$

ghost fields (gauge fixing)

ghosts

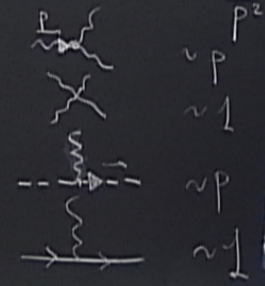


$$\sim \frac{1}{p}$$

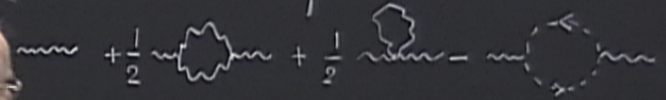
$$\mathcal{L} = \bar{\Psi} \gamma_\mu (i \not{D} - m) \Psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$$

$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + [A_\mu, -] \text{ Adj}$$



UV divergences & renormalizability  
 Irreducible Functions at 1 loop





gauge fields

(gluon)

gauge



$$\approx \frac{1}{p^2}$$

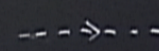
matter fields

Fermion



ghost fields (gauge fixing)

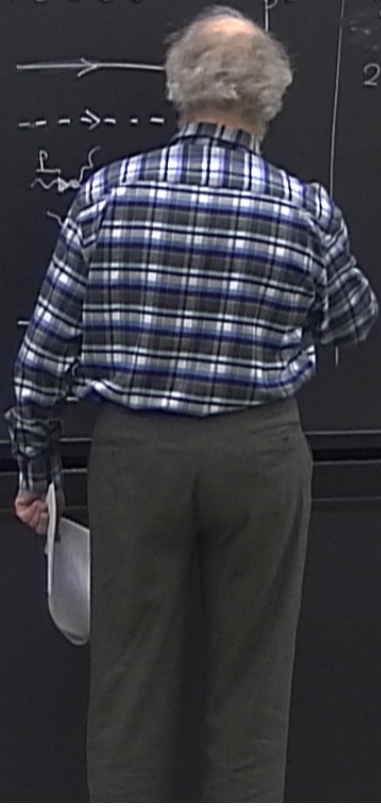
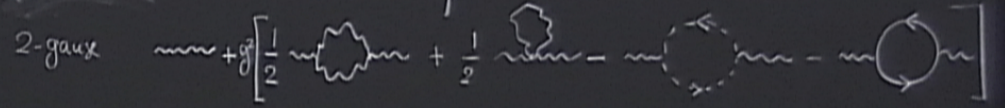
ghosts



$$\Psi^\dagger + \bar{\Psi} \left( i \not{D} - m \right) \Psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$$

$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$
$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$

UV divergences & renormalizability  
Irreducible Functions at 1 loop



gauge fields

(gluon)

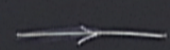
gauge



$$\approx \frac{1}{p^2}$$

matter fields

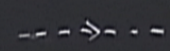
Feynman



$$\sim \frac{1}{p}$$

ghost fields (gauge fixing)

ghosts

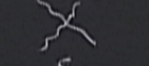


$$\sim \frac{1}{p^2}$$

$$\bar{\psi} + \bar{\psi} \gamma_\mu (i \not{D} - m) \psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$$



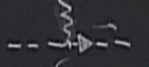
$$\sim p \quad g$$



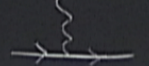
$$\sim 1 \quad g^c$$

$$D_\mu = \partial_\mu + i A_\mu \quad \text{Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \quad \text{Adj}$$

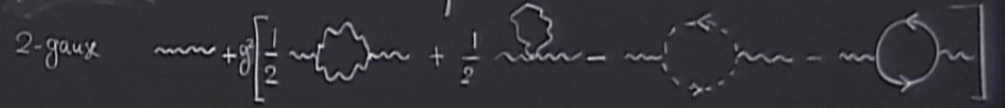


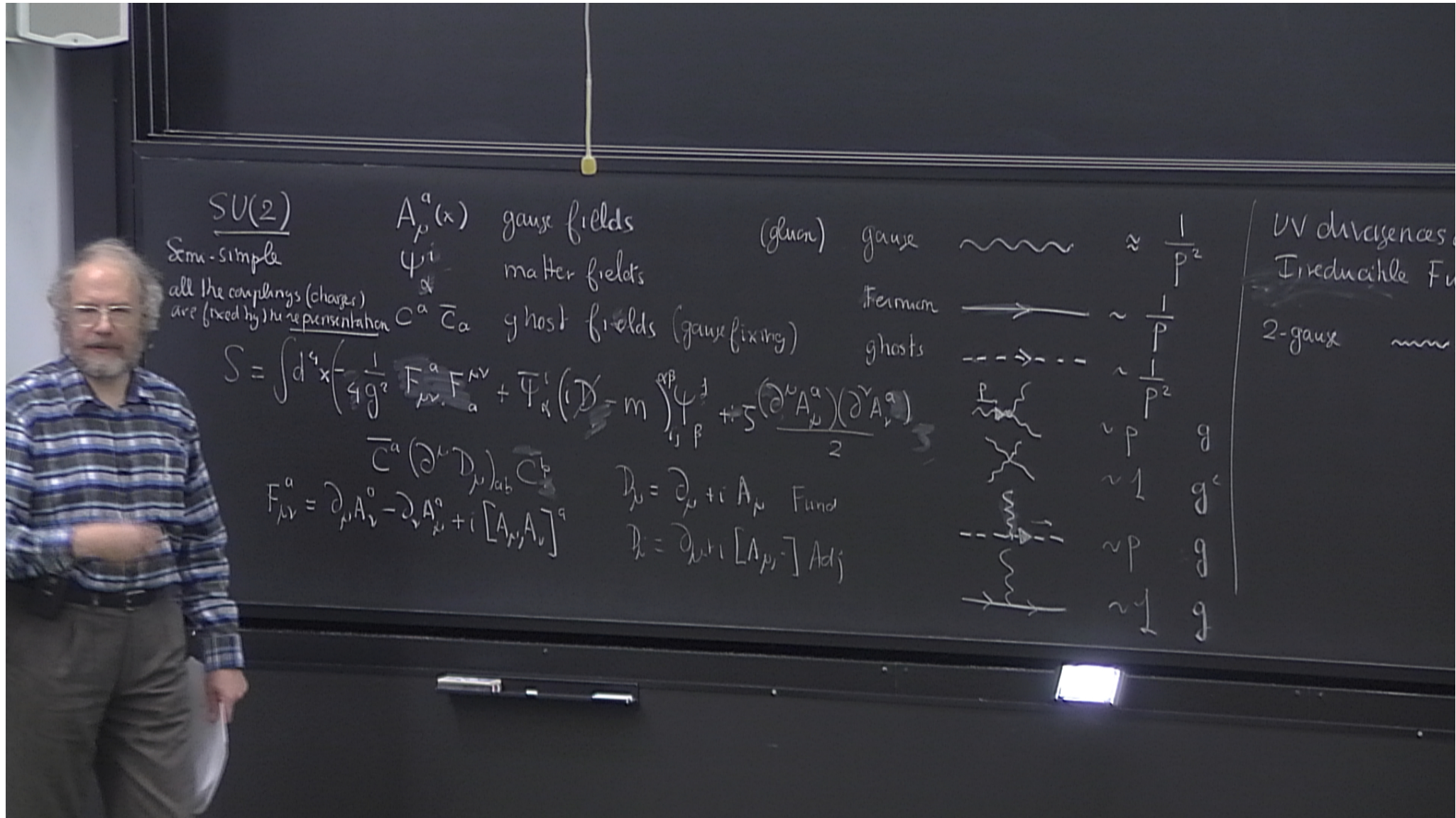
$$\sim p \quad g$$



$$\sim 1 \quad g$$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop



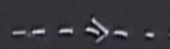


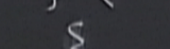
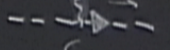


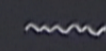


SU(2)

Sem-simple  
all the couplings (charges)  
are fixed by the representation

$A_\mu^a(x)$  gauge fields (gluon)  
 $\Psi^i$  matter fields  
 $C^a, \bar{C}^a$  ghost fields (gauge fixing)

gauge		$\sim \frac{1}{p^2}$
Fermion		$\sim \frac{1}{p}$
ghosts		$\sim \frac{1}{p}$
		$\sim p$
		$\sim 1$
		$\sim p$
		$\sim 1$

UV divergences  
Irreducible F  
2-gauge 

$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu}_a + \bar{\Psi}^i (i \not{D} - m)_{ij} \Psi^j + S \frac{(\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a)}{2} \right)$$

$$\bar{C}^a (\partial^\mu D_\mu)_{ab} C^b$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i [A_\mu, A_\nu]^a$$

$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$

SU(2)

Sem-simple

all the couplings (charges) are fixed by the representation

$A_\mu^a(x)$  gauge fields

$\Psi^i$  matter fields

$c^a, \bar{c}_a$  ghost fields (gauge fixing)

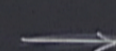
(gluon)

gauge



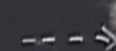
$$\approx \frac{1}{p^2}$$

Fermion



$$\frac{1}{p}$$

ghosts



$$\frac{1}{p^2}$$

UV divergences

Irreducible F

2-gauge



$$S = \int d^4x \left( -\frac{1}{4g^2} F_{\mu\nu}^a F^{\mu\nu}_a + \bar{\Psi}^i (i \not{D} - m) \Psi^j + S \frac{(\partial^\mu A_\mu^a)(\partial^\nu A_\nu^a)}{2} \right)$$

$$\bar{c}^a (\partial^\mu D_\mu)_{ab} c^b$$

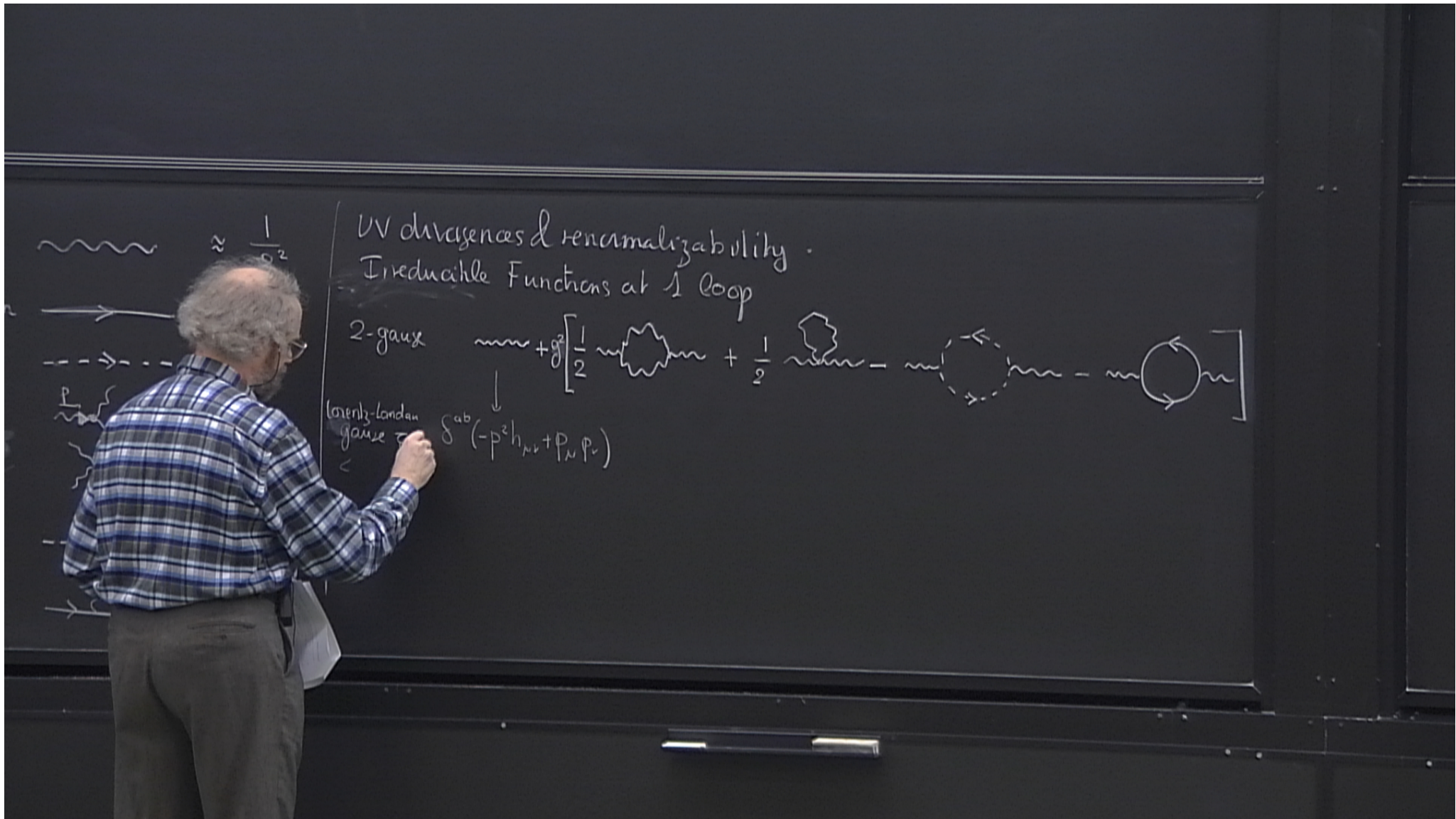
$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + i [A_\mu, A_\nu]^a$$

$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$

g

g'



UV divergences & renormalizability.  
 Irreducible Functions at 1 loop

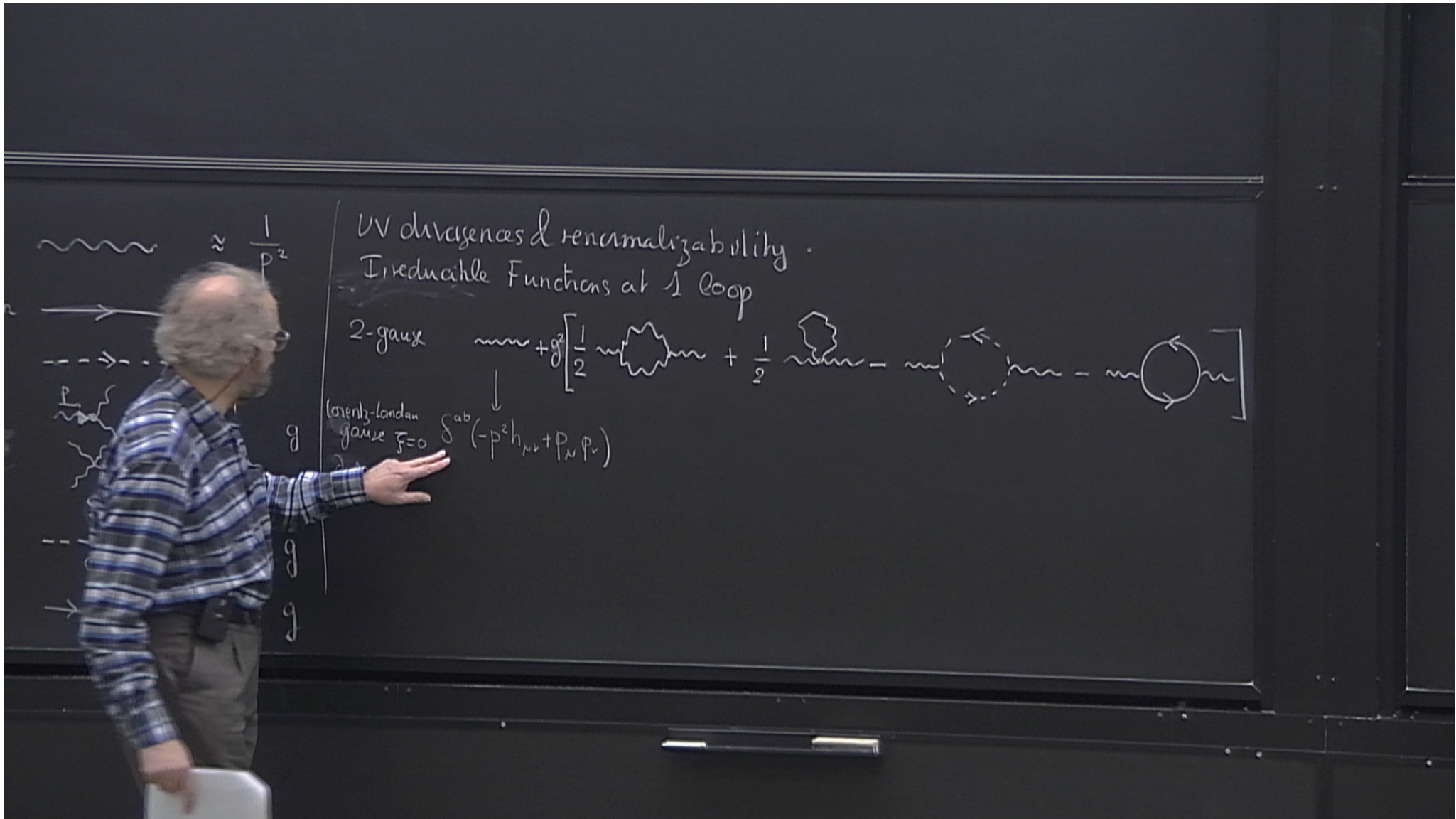
2-gauge  $\approx \frac{1}{\epsilon^2}$

$$+ g^2 \left[ \frac{1}{2} \text{ (triangle diagram) } + \frac{1}{2} \text{ (bubble diagram) } \right]$$

lorentz-landau gauge  $\tau$

$$\delta^{ab} (-p^2 h_{\mu\nu} + p_\mu p_\nu)$$

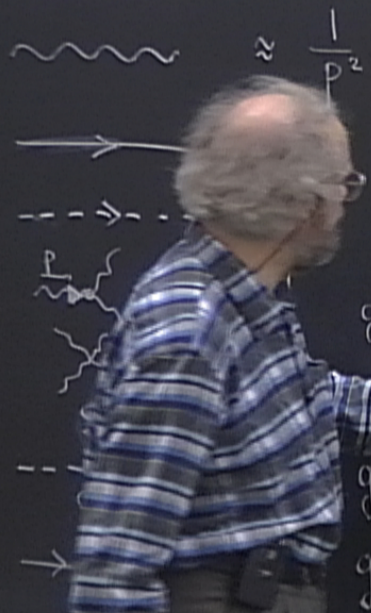




UV divergences & renormalizability.  
 Irreducible Functions at 1 loop

2-gauge  $\text{---} + g^2 \left[ \frac{1}{2} \text{---} + \frac{1}{2} \text{---} - \text{---} - \text{---} \right]$

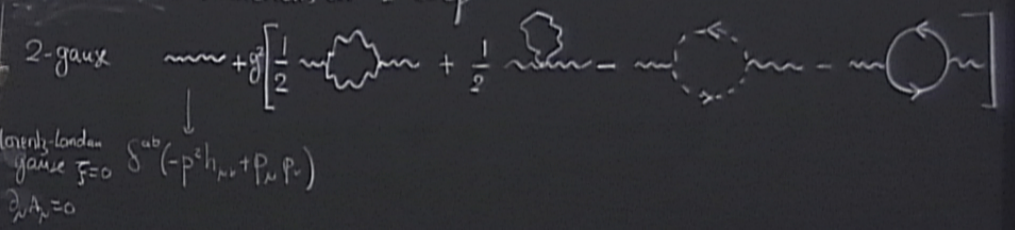
lorentz-landau gauge  $\vec{\nabla} \cdot \vec{A} = 0$   
 $\delta^{ab} (-p^2 h_{\mu\nu} + p_\mu p_\nu)$

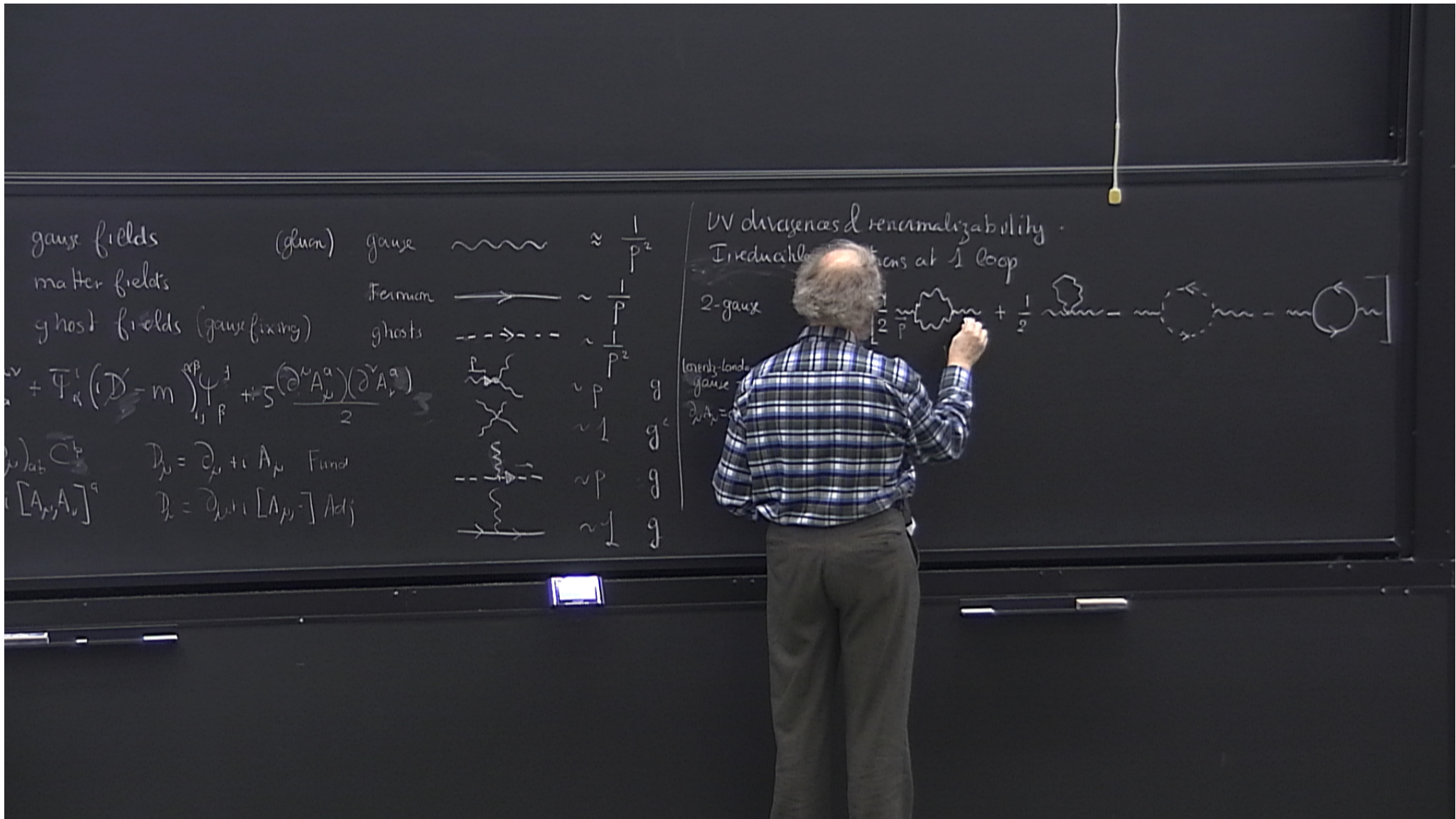


gauge fields (gluon) gauge  $\approx \frac{1}{p^2}$   
 matter fields Fermion  $\approx \frac{1}{p}$   
 ghost fields (gauge fixing) ghosts  $\approx \frac{1}{p}$

$\mathcal{L} = \bar{\psi} (i \not{D} - m) \psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$   
 $D_\mu = \partial_\mu + i A_\mu$  Fund  
 $D_\mu = \partial_\mu + i [A_\mu, \cdot]$  Adj

UV divergences & renormalizability  
 Irreducible Functions at 1 loop





gauge fields

(gluon)

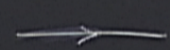
gauge



$$\approx \frac{1}{p^2}$$

matter fields

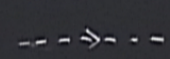
Fermion



$$\sim \frac{1}{p}$$

ghost fields (gauge fixing)

ghosts



$$\sim \frac{1}{p^2}$$

$$\bar{\psi} + \bar{\Psi}_K^L (iD - m) \psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$$

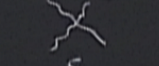
$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$



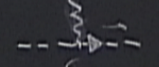
$$\sim p$$

$$g$$



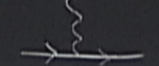
$$\sim 1$$

$$g^2$$



$$\sim p$$

$$g$$

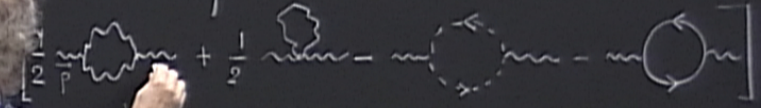


$$\sim 1$$

$$g$$

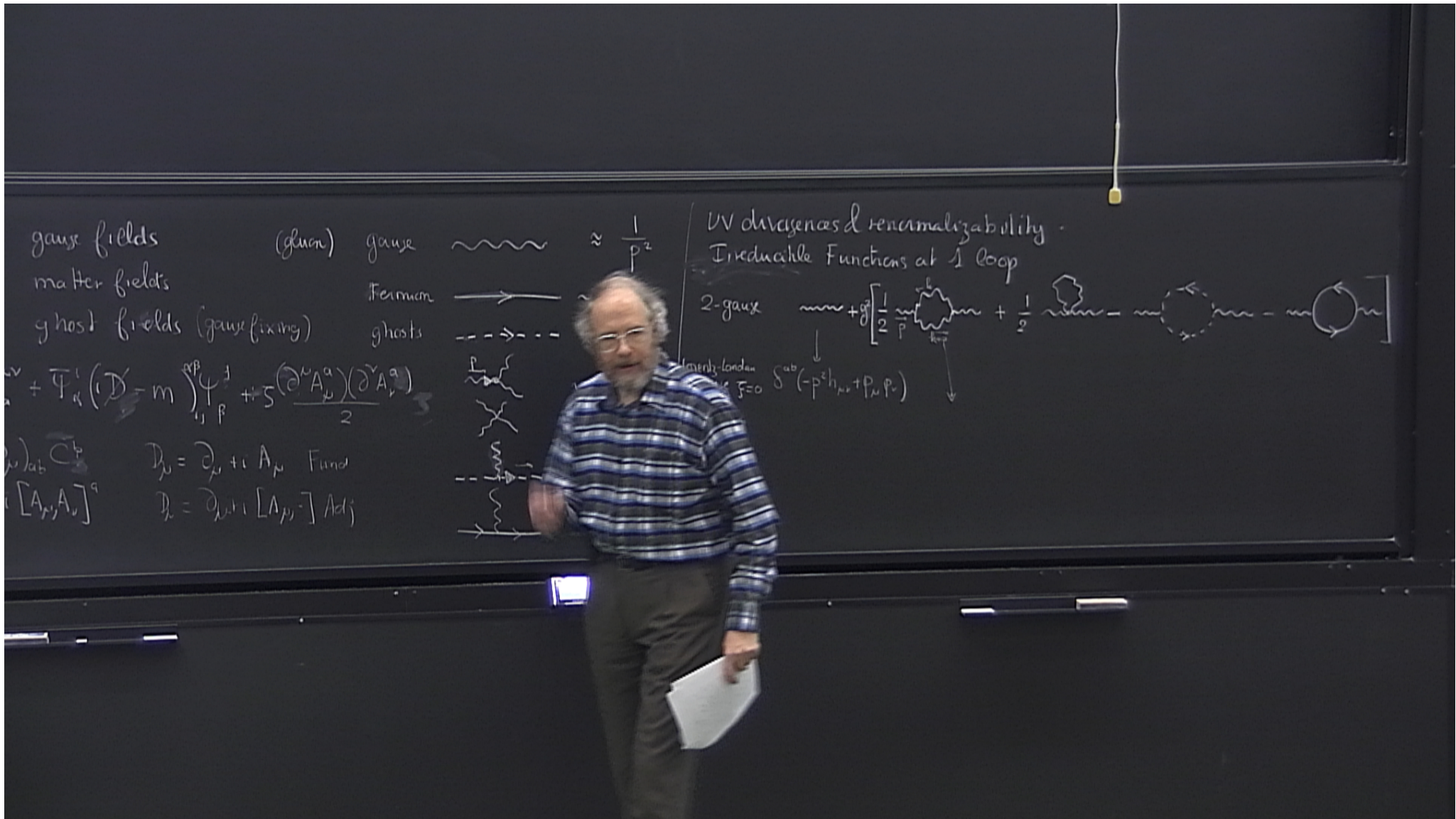
UV divergences & renormalizability  
 Irreducible diagrams at 1 loop

2-gauge



Landau-Landau gauge  
 $\partial_\mu A_\mu = 0$





gauge fields

(gluon)

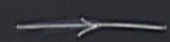
gauge



$$\approx \frac{1}{p^2}$$

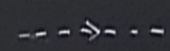
matter fields

Fermion



ghost fields (gauge fixing)

ghosts



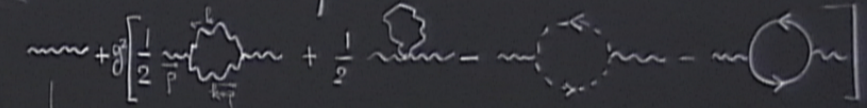
$$\mathcal{L} = \bar{\Psi} \gamma^\mu (iD_\mu - m) \Psi + S \frac{(\partial^\mu A_\nu^a)(\partial^\nu A_\mu^a)}{2}$$

$$D_\mu = \partial_\mu + i A_\mu \text{ Fund}$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot] \text{ Adj}$$

UV divergences & renormalizability  
Irreducible Functions at 1 loop

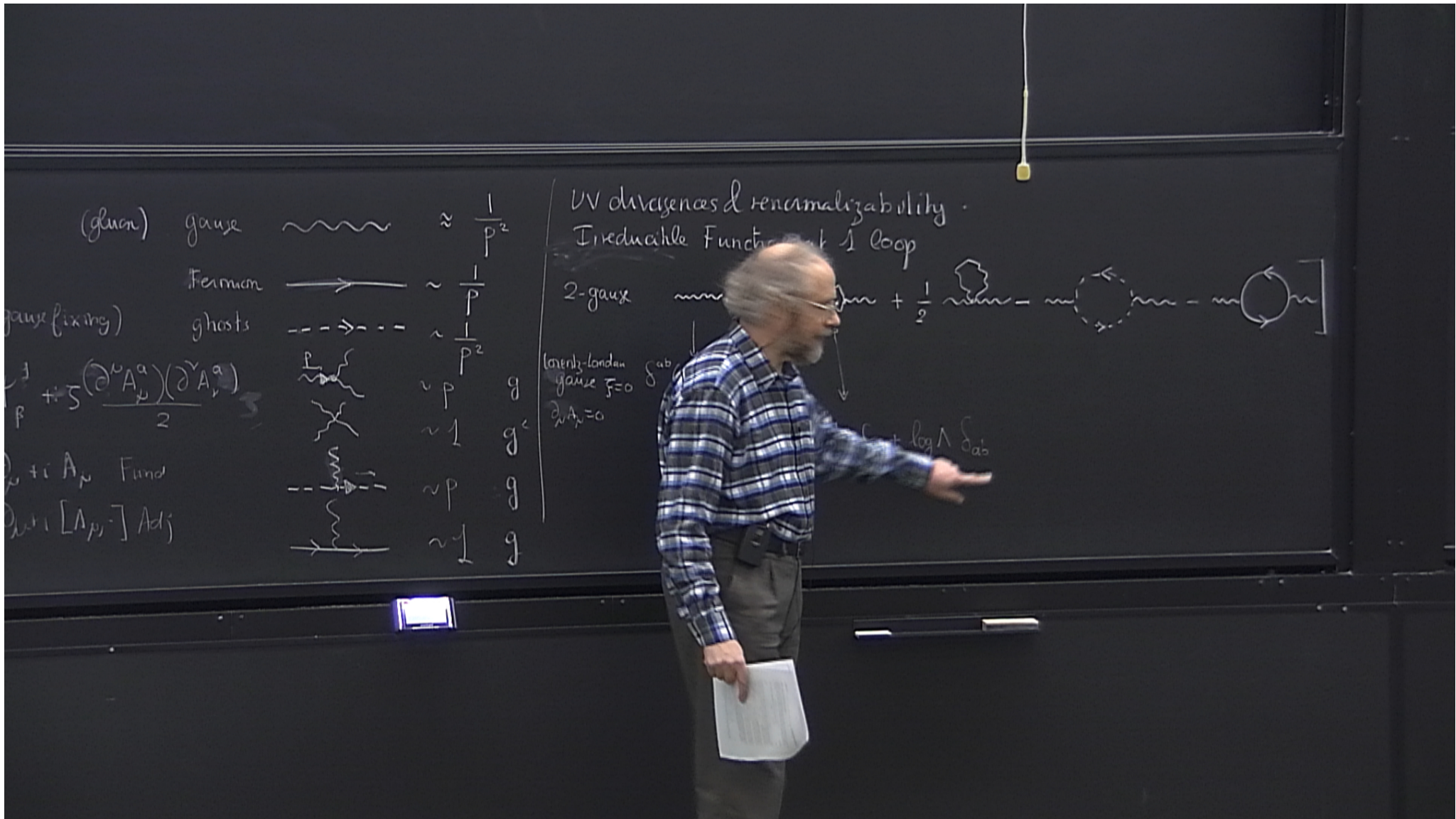
2-gluon



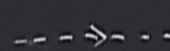


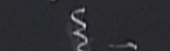
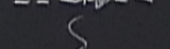


$$S^{ab}(-p^2 h_{\mu\nu} + p_\mu p_\nu)$$

UV regularization  $\leftrightarrow$  gauge invariance  
Sharp cutoff is problematic

UV regularization  $\leftrightarrow$  gauge invariance  
Sharp cutoff is problematic  
Lattice regularization (Wilson)  
Dimensional regularization



(gluon)	gauge		$\approx \frac{1}{p^2}$
	Fermion		$\sim \frac{1}{p}$
(gauge fixing)	ghosts		$\sim \frac{1}{p^2}$
			$\sim p$ g
			$\sim 1$ g <sup>c</sup>
			$\sim p$ g
			$\sim 1$ g

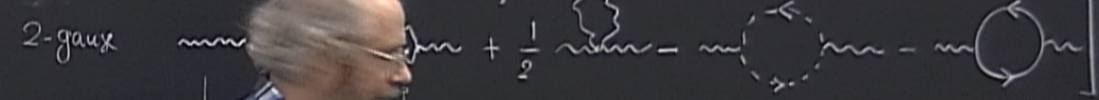
$$+\frac{1}{2} \int d^4x (\partial^\mu A_\mu^a)^2$$

$$+ i \int d^4x \bar{\psi} \gamma^\mu \partial_\mu \psi$$

$$+ \int d^4x \bar{\psi} \psi$$

$$+ \int d^4x \bar{\psi} \psi$$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop



Lorentz-Landau gauge  $F=0$   
 $\partial_\mu A_\mu = 0$

$\int d^4x \log \Lambda \delta_{ab}$

func)	gauge		$\approx \frac{1}{p^2}$
	Fermion		$\sim \frac{1}{p}$
g)	ghosts		$\sim \frac{1}{p^2}$
			$\sim p$ $g$
			$\sim 1$ $g^2$
	Fund		$\sim p$ $g$
	Adj		$\sim 1$ $g$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop

2-gauge  $\left[ \frac{1}{2} \frac{1}{p^2} \left( \text{loop diagrams} \right) + \frac{1}{2} \left( \text{ghost loop} \right) \right]$

$\downarrow$   
 Lorentz-Landau gauge  $\vec{\nabla} \cdot \vec{A} = 0$   
 $\delta^{ab} (-p^2 h_{\mu\nu} + p_\mu p_\nu)$   
 $\Lambda^2 h_{\mu\nu} \delta_{ab} + \log \Lambda$

UV regu  
 Sharp  
 Lattice  
 Dimensi

gauge		$\approx \frac{1}{p^2}$
Fermion		$\sim \frac{1}{p}$
ghosts		$\sim \frac{1}{p^2}$
$\frac{\partial^\mu A_\nu^a}{2} (\partial^\nu A_\mu^a)$		$\sim p$
Fund		$\sim 1$
Adj		$\sim p$
		$\sim 1$



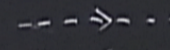
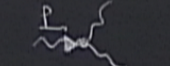

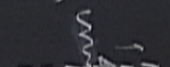
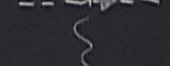
UV divergences & renormalizability  
 Irreducible Functions at 1 loop



lorentz landau gauge  $\vec{\nabla} \cdot \vec{A} = 0$   
 $\delta^{ab} (-p^2 h_{\mu\nu} + p_\mu p_\nu)$   
 $A_i$

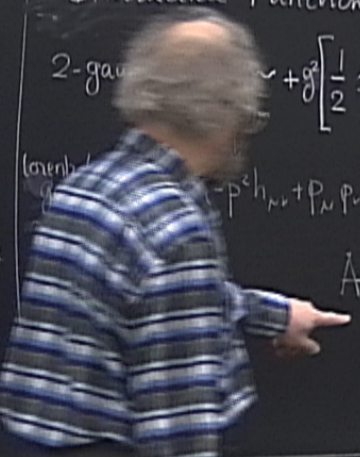
$\delta^{ab} p_\mu p_\nu + A_1'' p^2 h_{\mu\nu} + \text{finite terms} + \dots$

UV regu  
 Sharp  
 Lattice  
 Dimensi

gauge		$\approx \frac{1}{p^2}$
Fermion		$\sim \frac{1}{p}$
ghosts		$\sim \frac{1}{p^2}$
$\frac{\partial^\mu A_\nu^a}{2} (\partial^\nu A_\mu^a)$		$\sim p$
Fund		$\sim 1$
Adj		$\sim p$
		$\sim 1$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop

2-gau  $\sim +g^2 \left[ \frac{1}{2} \frac{1}{p} \left( \text{loop diagrams} \right) + \frac{1}{2} \left( \text{loop diagrams} \right) \right]$



$A_\mu \Lambda^2 h_{\mu\nu} \delta_{ab} + A'_\mu \log \Lambda \delta_{ab} p_\mu p_\nu + A''_\mu p^2 h_{\mu\nu} + \text{finite terms} + \dots$

UV regu  
 Sharp  
 Lattice  
 Dimensi

gauge		$\approx \frac{1}{p^2}$
Fermion		$\sim \frac{1}{p}$
ghosts		$\sim \frac{1}{p^2}$
$\frac{\partial^\mu A_\nu^a}{2} (\partial^\nu A_\mu^a)$		$\sim p$
Fund		$\sim 1$
Adj		$\sim p$
		$\sim 1$

UV divergences & renormalizability  
 Irreducible Functions at 1 loop

2-gauge  $\sim \frac{1}{2} \frac{1}{p^2} + \frac{1}{2} \left[ \text{loop diagrams} \right]$

$\downarrow$   
 Lorentz-Landau gauge  $\vec{\nabla} \cdot \vec{A} = 0$   
 $\delta^{ab} (-p^2 \delta_{\mu\nu} + p_\mu p_\nu)$   
 $\Gamma_{\mu\nu}^{ab}(p) = \Pi_{\mu\nu}^{ab}(p)$   
 $\log \Lambda \delta_{ab} p_\mu p_\nu + A_1 p^2 h_{\mu\nu} + \text{finite terms} + \dots$

UV regu  
 Sharp  
 Lattice  
 Dimensi



gauge		$\approx \frac{1}{p^2}$
Fermion		$\sim \frac{1}{p}$
ghosts		$\sim \frac{1}{p^2}$
$\frac{\partial^\mu A_\nu^a}{2} (\partial^\nu A_\mu^a)$		$\sim p$
Fund		$\sim 1$
Adj		$\sim p$
		$\sim 1$
		$\sim 1$

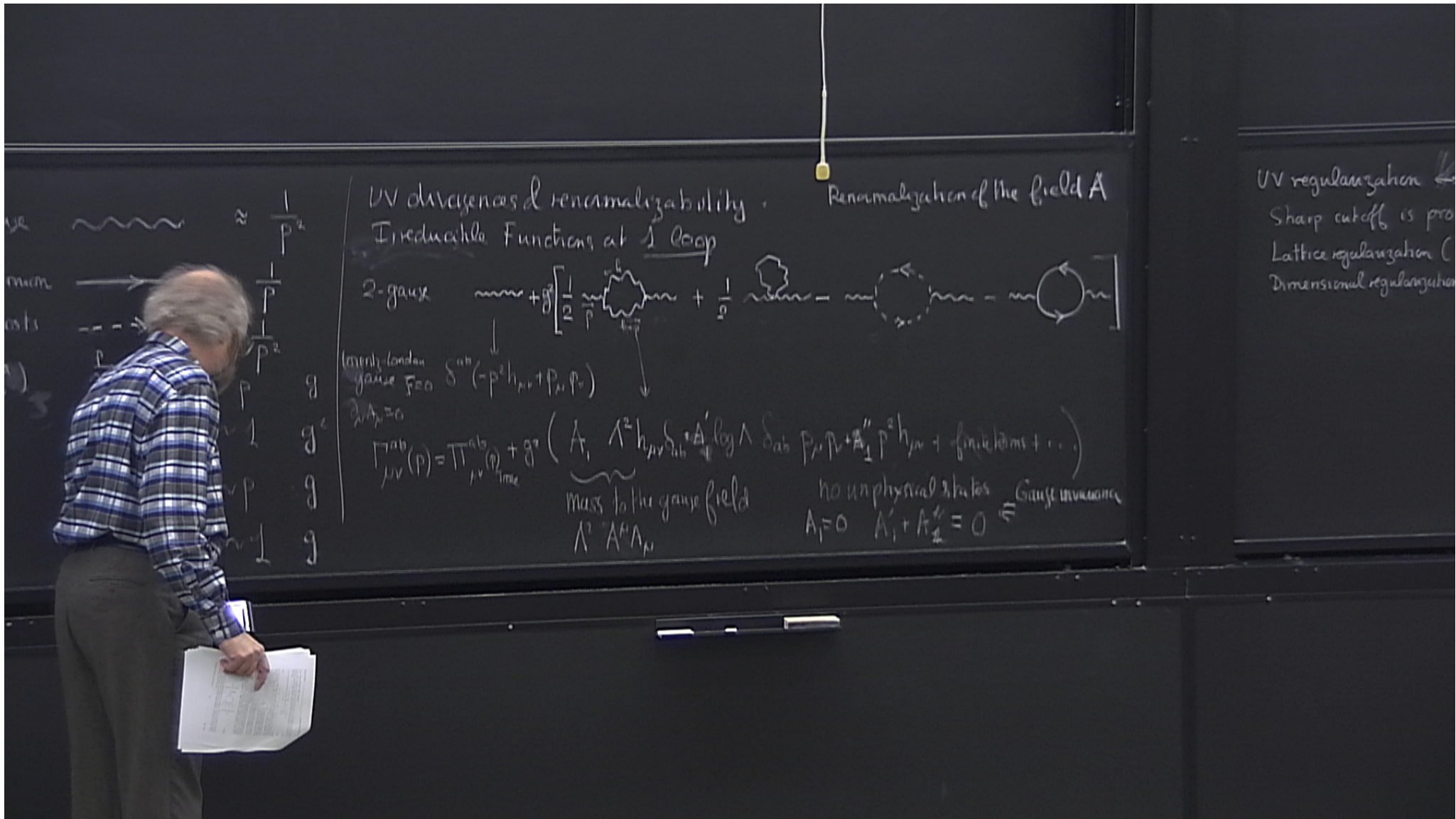
UV divergences & renormalizability  
 Irreducible Functions at 1 loop

2-gauge  $\left[ \frac{1}{2} \frac{1}{p^2} + g^2 \left( \frac{1}{2} \frac{1}{p^2} \right) + \frac{1}{2} \left( \text{loop diagrams} \right) \right]$

lorentz-landau gauge  $\vec{\nabla} \cdot \vec{A} = 0$   
 $\delta^{ab} (-p^\mu h_{\mu\nu} + p_\mu p_\nu)$

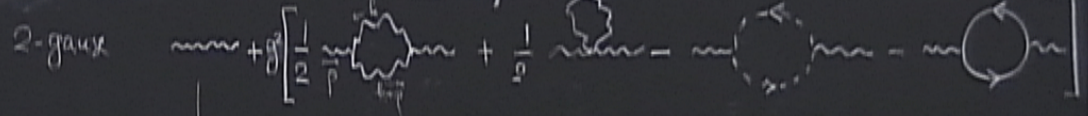
$\Gamma_{\mu\nu}^{ab}(p) = \Pi_{\mu\nu}^{ab}(p) + g^2 \left( \Lambda^2 h_{\mu\nu} \delta_{ab} + \Lambda \log \Lambda \delta_{ab} + \dots \right)$   
 mass to the gauge field  
 $\Lambda^2 A^\mu A_\mu$

UV regu  
 Sharp  
 Lattice  
 Dimensi



UV divergences & renormalizability  
 Irreducible Functions at 1 loop

Renormalization of the field A



(maxwell-london gauge  $F_{\mu\nu} = 0$ )  $\delta^{ab} (-p^2 h_{\mu\nu} + p_\mu p_\nu)$

$\partial_\mu A_\mu = 0$

$$\Gamma_{\mu\nu}^{ab}(p) = \Pi_{\mu\nu}^{ab}(p) + g^2 \left( A_1 \Lambda^2 h_{\mu\nu} \delta_{ab} + A_2 \log \Lambda \delta_{ab} p_\mu p_\nu + A_3 p^2 h_{\mu\nu} + \dots \right)$$

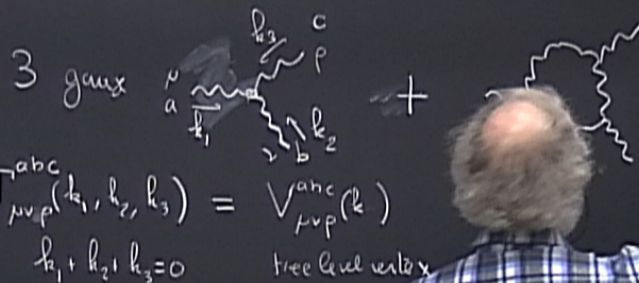
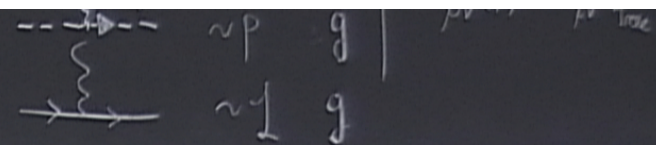
mass to the gauge field  $\Lambda^2 A^{\mu\nu} A_\mu$

no unphysical states  $A_1 = 0 \quad A_1 + A_2 = 0$   $\Rightarrow$  Gauge invariance

UV regularization  
 Sharp cutoff is pro  
 Lattice regularization  
 Dimensional regularization

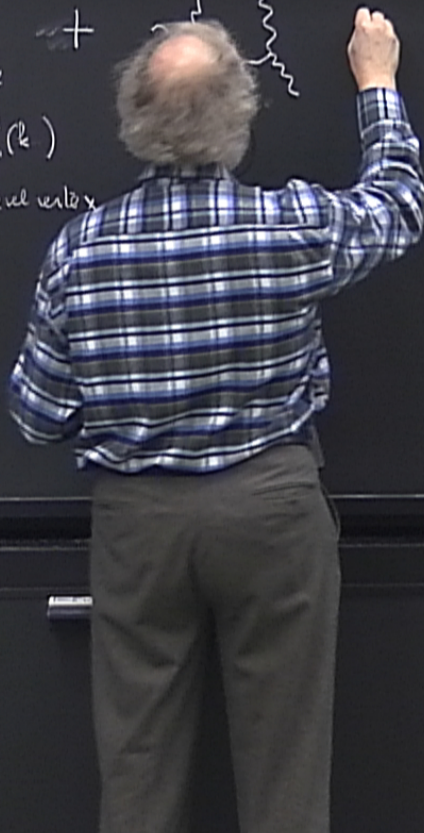
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]^a$$

$$D_\mu = \partial_\mu + i[A_\mu, \cdot]_{Adj}$$



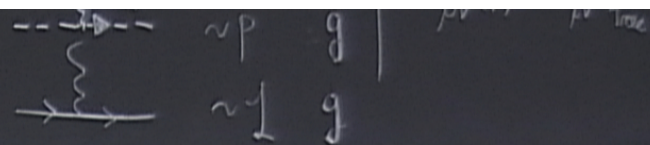
$$\Gamma_{\mu\nu\rho}^{abc}(k_1, k_2, k_3) = V_{\mu\nu\rho}^{abc}(k)$$

$k_1 + k_2 + k_3 = 0$  tree level vertex



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i[A_\mu, A_\nu]^a$$

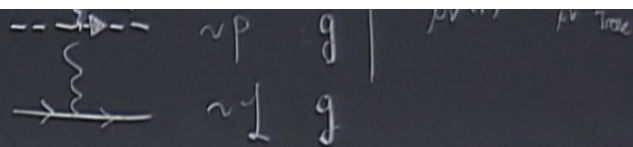
$$D_\mu = \partial_\mu + i[A_\mu, \cdot]_{Adj}$$



$\Gamma_{\mu\nu\rho}^{abc}(k_1, k_2, k_3)$   
 $k_1 + k_2 + k_3 = 0$   
 vertex  
 potential

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu]^a$$

$$D_\mu = \partial_\mu + i [A_\mu, \cdot]_{Adj}$$

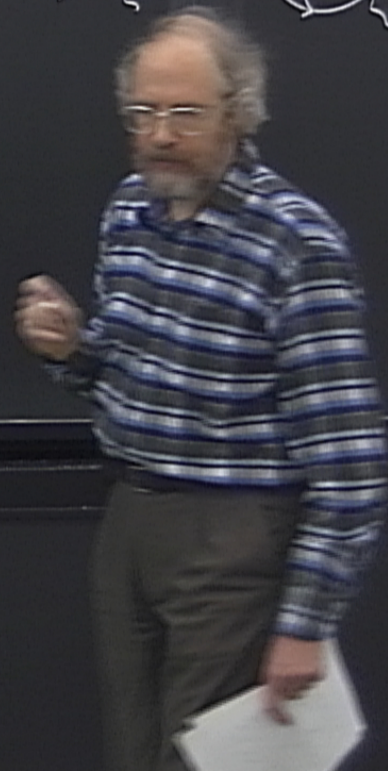


mass to the gauge  
 $\Lambda^2 A^\mu A_\mu$

$$\Gamma_{\mu\nu\rho}^{abc}(k_1, k_2, k_3) = g V_{\mu\nu\rho}^{abc}(k) + g^3 \left[ \text{tree level vertex} + \text{loop diagrams} \right]$$

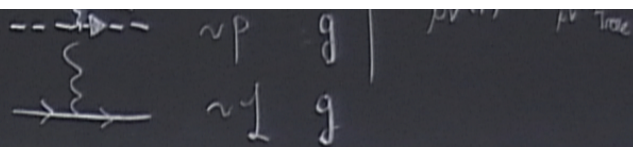
$k_1 + k_2 + k_3 = 0$   
 tree level vertex  
 log  $\Lambda$

potential  $\Lambda$  divergence  
 but in fact 0



$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu]^a$$

$$D_\mu = \partial_\mu + i [A_\mu, -] \text{Adj}$$



mass to the gauge  
 $\Lambda^2 A^\mu A_\mu$

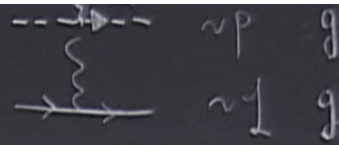
$$\Gamma_{\mu\nu\rho}^{abc}(k_1, k_2, k_3) = g^3 \left[ \text{diagrams} + g^3 \left[ B_1 \log \Lambda V_{\mu\nu\rho}^{abc} + \text{finite terms} \right] \right]$$

↑  
gauge invariance

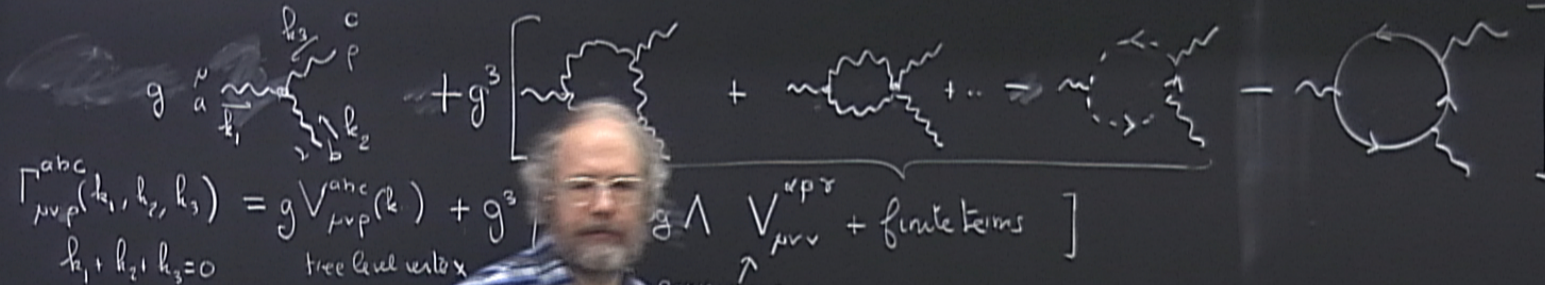
potential  $\Lambda$   
 but in f

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + i [A_\mu, A_\nu]^a$$

$$D_i = \partial_{i+1} [A_{\mu\nu}]^a$$



mass to the gauge  
 $\Lambda^2 A^\mu A_\mu$



$$\Gamma_{\mu\nu\rho}^{abc}(k_1, k_2, k_3) = g V_{\mu\nu\rho}^{abc}(k) + g^3 \left[ g \Lambda V_{\mu\nu\rho}^{abc} + \text{finite terms} \right]$$

$k_1 + k_2 + k_3 = 0$   
 tree level vertex

gauge invariance

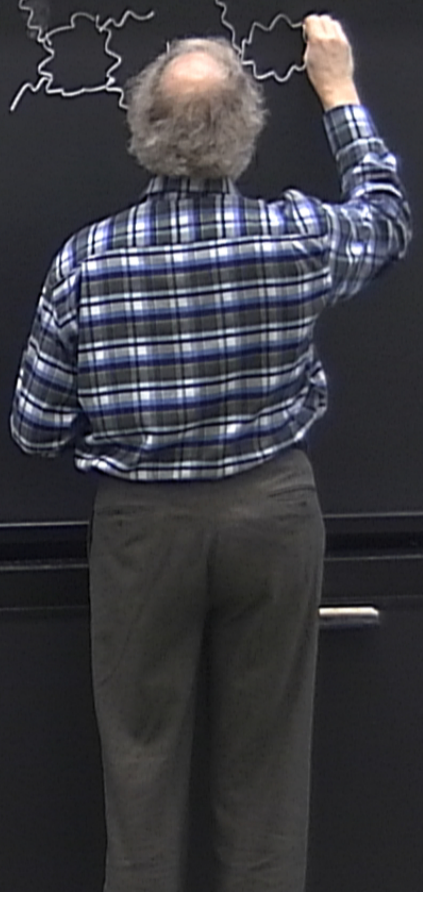
coupling constant  $g$

potential  $\Lambda$  divergence  
 but in fact 0

$[A_{\mu\nu}]^{\text{Adj}}$        $\sim p$        $g$        $\Lambda^2 A^\mu A_\mu$       mass to the gauge field  
 $\sim \perp$        $g$        $\Lambda^2 A^\mu A_\mu$

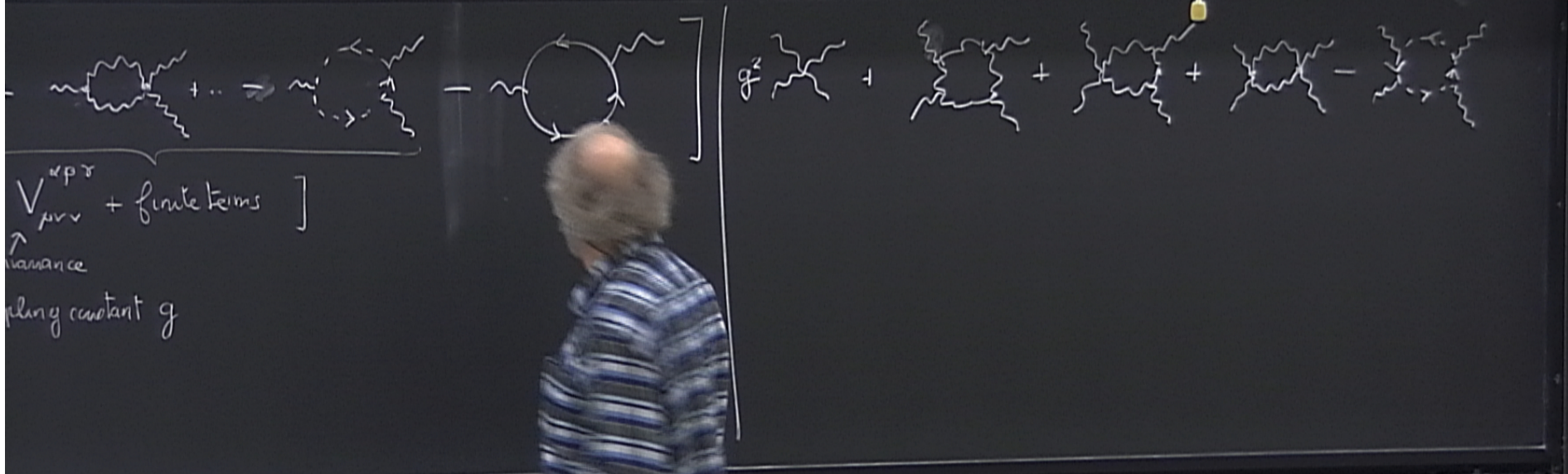
$V_{\mu\nu\rho\sigma}^{abcd} + \text{finite terms}$   
 gauge invariance  
 coupling constant  $g$

no unphysical states      Gauge invariance  
 $A_0 = 0$        $A_1 + A_2 = 0$

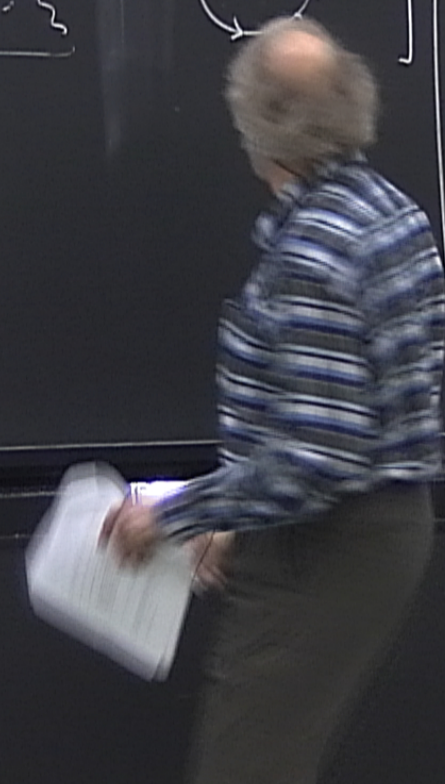




$[A_{\mu\nu}]^2 A_{\mu\nu}$       $\sim p$       $g$       $\sim p^2$       $\Lambda^2 A^\mu A_\mu$      mass to the gauge field  
 no unphysical states     Gauge invariance  
 $A_0 = 0$       $A_1 + A_2 = 0$



$V_{\mu\nu}^{\alpha\beta\gamma} + \text{finite terms}$   
 ↑  
 gauge invariance  
 coupling constant  $g$



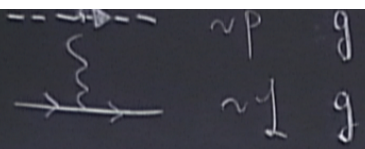
$\Gamma^2 A^\mu A_\mu$  mass to the gauge field  
 no unphysical states  $\Leftrightarrow$  Gauge invariance  
 $A_0 = 0 \quad A_1 + A_2 = 0$

$\sim p \quad g$   
 $\sim 1 \quad g$

UV regu  
 Sharp  
 Lattice  
 Dimens

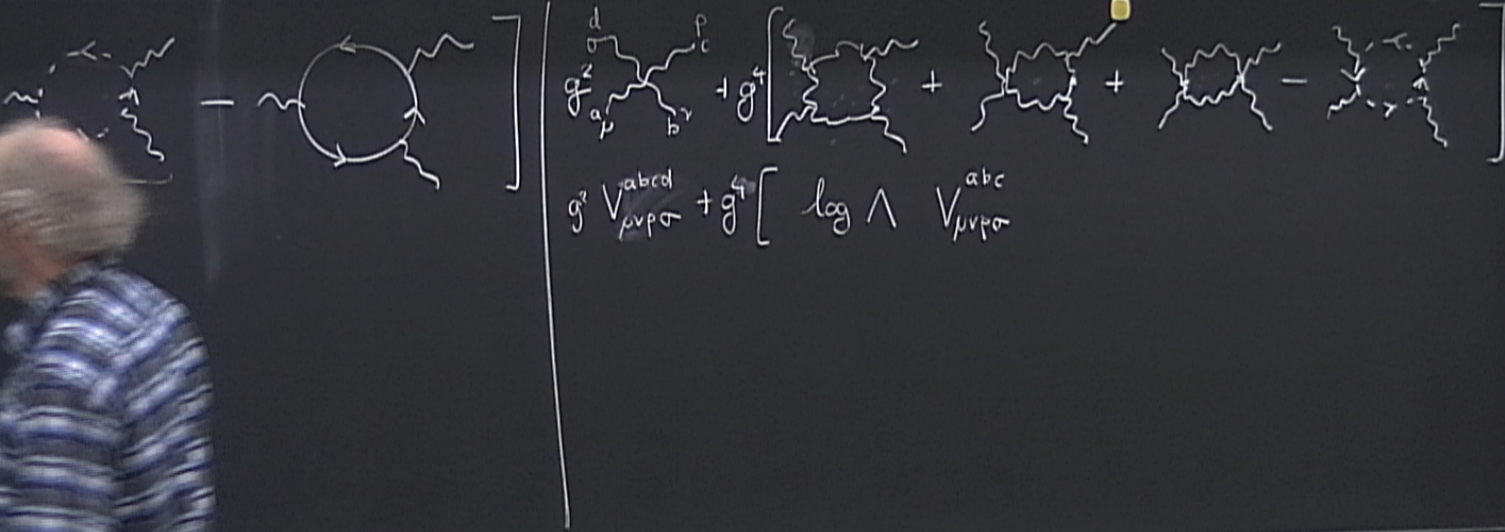
$g^2$   
 $\log \Lambda$

+ finite terms ]  
 ant g



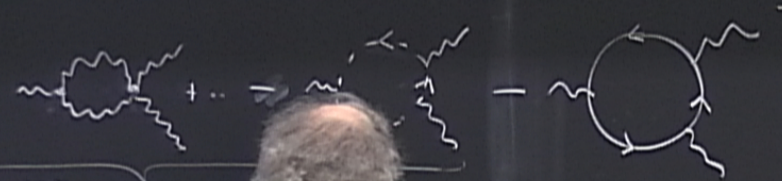
mass to the gauge field  
 $\Lambda^2 A^\mu A_\mu$

no unphysical states  
 $A_1 = 0 \quad A'_1 + A'_2 = 0 \iff$  Gauge invariance




UV regularization  $\rightarrow g$   
 Sharp cutoff is probl  
 Lattice regularization (wi  
 Dimensional regularization

$[A_\mu, A_\nu] \text{ Adj}$        $\sim p \quad g$        $\sim 1 \quad g$        $\mu \nu \text{ trace}$        $\mu \nu \text{ trace}$        $\Lambda^2 A^\mu A_\mu$        $\text{mass to the gauge field}$        $A_\mu = 0 \quad A'_i + A''_i = 0 \Leftrightarrow \text{Gauge invariance}$

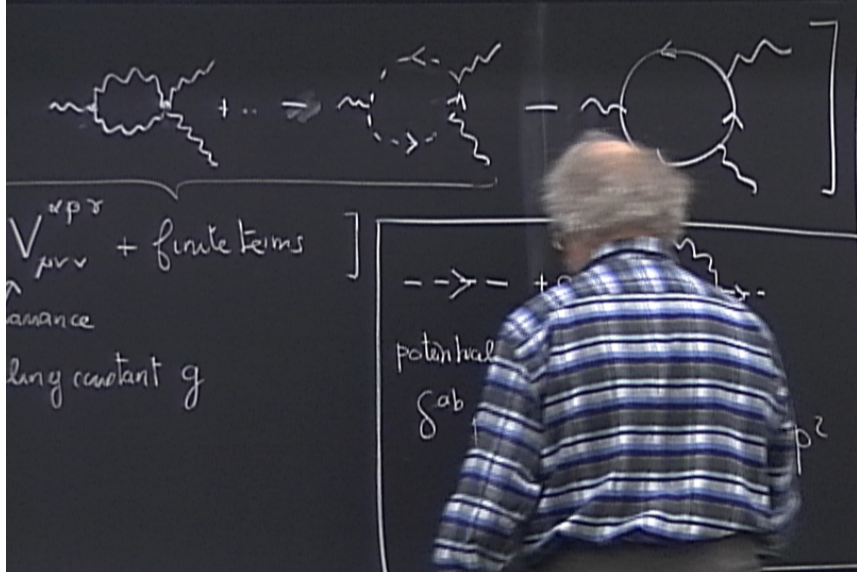


$V_{\mu\nu}^{\alpha\beta\gamma} + \text{finite terms}$   
 $\text{gauge invariance}$   
 $\text{long constant}$




$g^2 V_{\mu\nu\rho\sigma}^{abcd} + g^4 \left[ C_1 \log \Lambda \quad V_{\mu\nu\rho\sigma}^{abc} + \text{finite terms} \right]$

$[A_\mu, A_\nu] = A_\mu A_\nu - A_\nu A_\mu$   
 $\sim p \quad g$   
 $\sim 1 \quad g$   
 mass to the gauge field  
 $\Lambda^2 A^\mu A_\mu$   
 no unphysical states  $\Leftrightarrow$  Gauge invariance  
 $A_0 = 0 \quad A_1 + A_2 = 0$

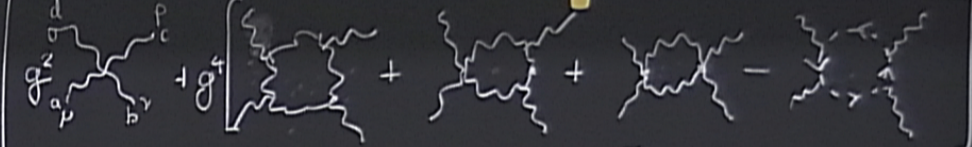


$$g^2 V_{\mu\nu\rho\sigma}^{abcd} + g^4 \left[ C_1 \log \Lambda V_{\mu\nu\rho\sigma}^{abc} + \text{finite terms} \right]$$

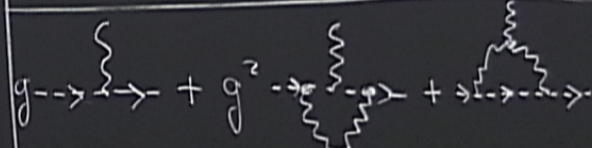
$[A_\mu, \dots] Ad_j$    
 $\sim p \quad g$    
 $\sim 1 \quad g$    
 Mass to the gauge field  $\Lambda^2 A^\mu A_\mu$    
 no unphysical states  $A_i=0 \quad A'_1 + A'_2 = 0 \iff$  Gauge invariance



$V_{\mu\nu}^{\alpha\beta\gamma} + \text{finite terms}$    
 $\Lambda$  divergence  $\rightarrow 0$    
 $g^2 D_1 \log \Lambda \delta^{ab} p^2$    
 renormalization of the ghost field

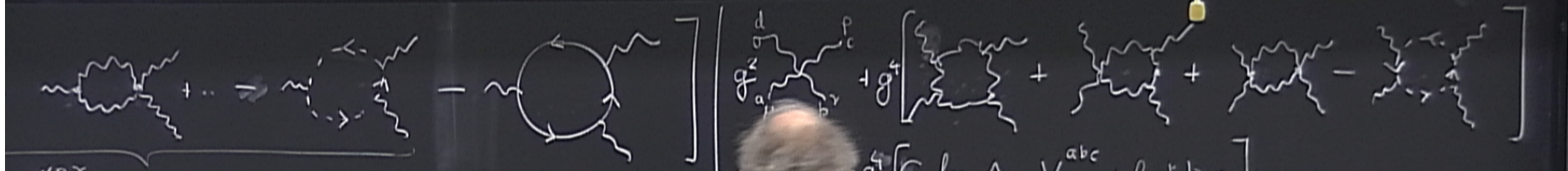


$g^2 V_{\mu\nu\rho\sigma}^{abcd} + g^4 [C_1 \log \Lambda V_{\mu\nu\rho\sigma}^{abc} + \text{finite terms}]$



$g \rightarrow \dots + g^2 \rightarrow \dots + \dots$

$[A_\mu]_{Adj}$   $\sim p$   $g$   $\mu$  trace  $\mu$  trace  
 $\Lambda^2 A^\mu A_\mu$  no unphysical states  $\Rightarrow$  Gauge invariance  
 $A_i=0 \quad A'_i + A''_i = 0$

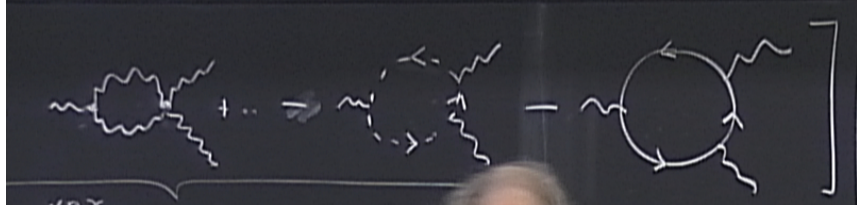


$V_{\mu\nu}^{abc} + \text{finite terms}$   
 gauge invariance  
 long constant  $g$

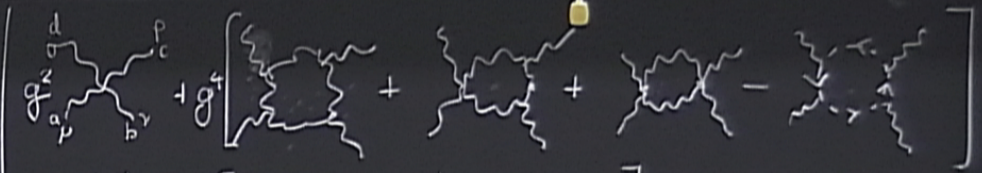
$\text{potential } \Lambda \text{ divergence } \rightarrow 0$   
 $\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$   
 renormalization of the ghost field

$g^4 [C_1 \log \Lambda V_{\mu\nu\rho\sigma}^{abc} + \text{finite terms}]$   
 $E_1 \log \Lambda U_\mu^{abc}(p) + \text{finite terms}$

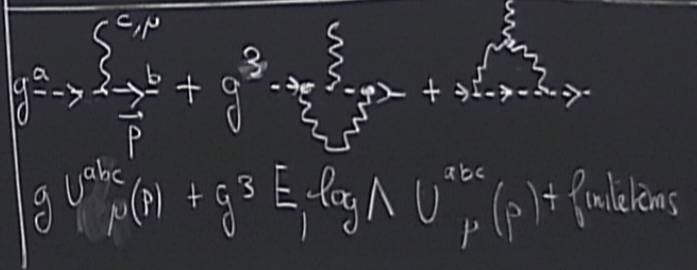
$[A_\mu, \cdot]_{Adj}$       $\sim p \quad g$       $\sim \frac{1}{p} \quad g$       $\mu \text{ tree}$       $\mu \text{ tree}$   
 mass to the gauge field      $\Lambda^2 A^\mu A_\mu$      no unphysical states      $A_\mu = 0 \quad A'_1 + A''_2 = 0 \Leftrightarrow$  Gauge invariance



$V_{\mu\nu}^{abc}$  + finite terms  
 divergence  $\rightarrow 0$   
 $D \frac{1}{\Lambda} \log \Lambda \delta^{ab} p^c$   
 the ghost field



$g^2 V_{\mu\nu\rho\sigma}^{abcd} + g^4 \left[ C_1 \log \Lambda V_{\mu\nu\rho\sigma}^{abc} + \text{finite terms} \right]$

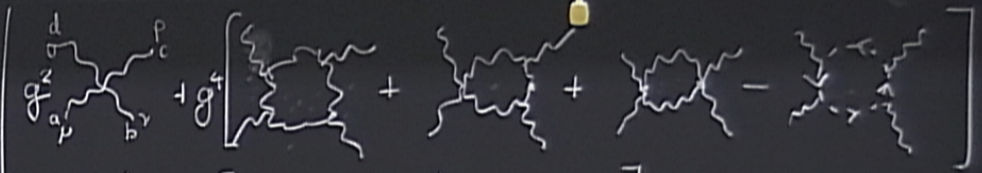


$g U_{\mu}^{abc}(p) + g^3 E_1 \log \Lambda U_{\mu}^{abc}(p) + \text{finite terms}$

other diagrams are UV finite



$[A_\mu, \dots] \text{Adj}$    
 $\sim p \quad g$    
 $\sim 1 \quad g$    
 Mass to the gauge field  $\Lambda^2 A^\mu A_\mu$    
 no unphysical states  $\Leftrightarrow$  Gauge invariance  $A_\mu = 0 \quad A'_1 + A''_2 = 0$



$V_{\mu\nu}^{\alpha\beta\gamma} + \text{finite terms}$    
 gauge invariance  $\rightarrow \circ$    
 $g \Lambda \delta^{ab} p^c$    
 ghost field

$g^2 V_{\mu\nu\alpha}^{abcd} + g^4 [C_1 \log \Lambda V_{\mu\nu\alpha}^{abc} + \text{finite terms}]$    
 $g^3 U_{\mu}^{abc}(p) + g^3 E_1 \log \Lambda U_{\mu}^{abc}(p) + \text{finite terms}$

other diagrams are UV finite

potential  $\Lambda$  divergence  
but in fact 0

renormalization of the coupling constant  $g$

potential  $\Lambda$  divergence  $\rightarrow 0$

$$\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$$

renormalization of the ghost field

$$g^2 \rightarrow 2 \rightarrow \frac{b}{p} + g^2 \rightarrow \dots$$

$$g U_{\rho}^{abc} + g^3 E \log$$

$$S_{\text{Tree}}[A, \bar{c}, c] = \int_M \left[ \partial A \cdot \partial A + g \partial A \cdot A A + g^2 A \cdot A \cdot A A + \partial \bar{c} \cdot \partial c + g \partial \bar{c} \cdot c A \right]$$

potential  $\Lambda$  divergence  
but in fact 0

renormalization of the coupling constant  $g$

potential  $\Lambda$  divergence  $\rightarrow 0$

$$\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$$

renormalization of the ghost field

$$g^a \rightarrow 2 \frac{g^a}{p} + g^a$$

$$g U_{\mu}^{abc}(p) + g$$

$$S[A, \bar{c}, c] = \int_M \left[ \partial A \partial A + g \partial A \cdot A A + g^2 A \cdot A \cdot A A + \partial \bar{c} \cdot \partial c + g \partial \bar{c} \cdot c A \right]$$

regularize the theory by replacing the "bare action" by a renormalized action

$$S[A, \bar{c}, c] = \int_M \left[ A \partial A \partial A + B \partial A \cdot A A + C A \cdot A \cdot A A + D \partial \bar{c} \partial c + E \partial \bar{c} \cdot c A \right]$$

potential  $\Lambda$  divergence  
but in fact 0 renormalization of the coupling constant  $g$

potential  $\Lambda$  divergence  $\rightarrow 0$   
 $\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$   
renormalization of the ghost field

$g^a \rightarrow 2 \frac{g^a}{p} + g$   
 $g U_{\rho}^{abc}(p) + g$

$$S_{\text{Tree}}[A, \bar{c}, c] = \int_M \left[ \partial A \partial A + g \partial A \cdot A A + g^2 A \cdot A \cdot A A + \partial \bar{c} \cdot \partial c + g \partial \bar{c} \cdot c A \right]$$

Renormalize the theory by replacing the "bare action" by a renormalized action

$$S_R[A, \bar{c}, c] = \int_M \left[ A \partial A \partial A + B \partial A \cdot A A + C A \cdot A \cdot A A + D \partial \bar{c} \partial c + E \partial \bar{c} \cdot c A \right]$$

$$\int D[A] D[\bar{c}, c] \exp(i S_R) \rightarrow \text{finite quantities in the limit } \Lambda \rightarrow \infty$$

Renormalized theory      Continuum limit

$$A = 1 - g A'_1$$

potential  $\Lambda$  divergence  
but in fact 0

renormalization of the coupling constant  $g$

potential  $\Lambda$  divergence  $\rightarrow 0$

$$\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$$

renormalization of the ghost field

$$g^2 \rightarrow 2 \frac{g^2}{P} + g^2$$

$$g U_{\mu}^{abc}(P) + g^2$$

$$S_{\text{Tree}}[A, \bar{c}, c] = \int_M \left[ \partial A \partial A + g \partial A \cdot A A + g^2 A \cdot A \cdot A A + \partial \bar{c} \cdot \partial c + g \partial \bar{c} \cdot c A \right]$$

Renormalize the theory by replacing the "bare action" by a renormalized action

$$S_R[A] = \int_M \left[ A \partial A \partial A + B \partial A \cdot A A + C A \cdot A \cdot A A + D \partial \bar{c} \partial c + E \partial \bar{c} \cdot c A \right]$$

$(S_R) \rightarrow$  finite quantities  
in the limit  $\Lambda \rightarrow \infty$   
continuum limit  
 $\rightarrow g_R$

$$A = 1 - g_R A'_1 \log\left(\frac{\Lambda}{\mu}\right)$$

$$B = g_R - g_R^2 B_1 \log\left(\frac{\Lambda}{\mu}\right)$$

$$D = g_R^2 - g_R^3 C_1 \log\left(\frac{\Lambda}{\mu}\right)$$

etc...

ization of the ghost field  $\int \mathcal{D}c \mathcal{D}\bar{c} \exp(-\int \bar{c} \partial \cdot c)$   $\int \mathcal{D}A \exp(-\int \frac{1}{2} A \cdot \square A)$   $\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-\int \bar{\psi} \not{\partial} \psi)$

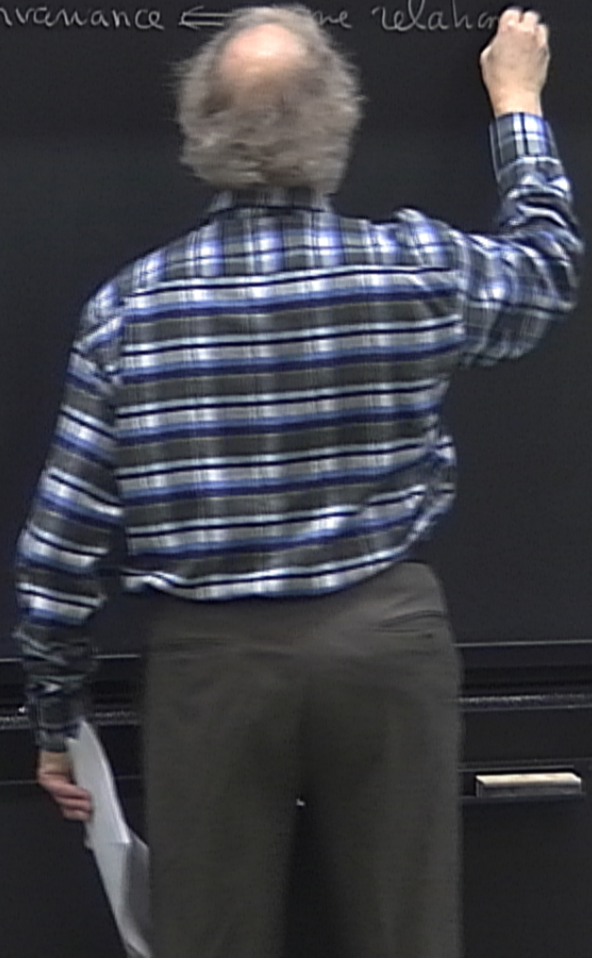
$$\partial c + g \partial \bar{c} \cdot c \cdot A$$

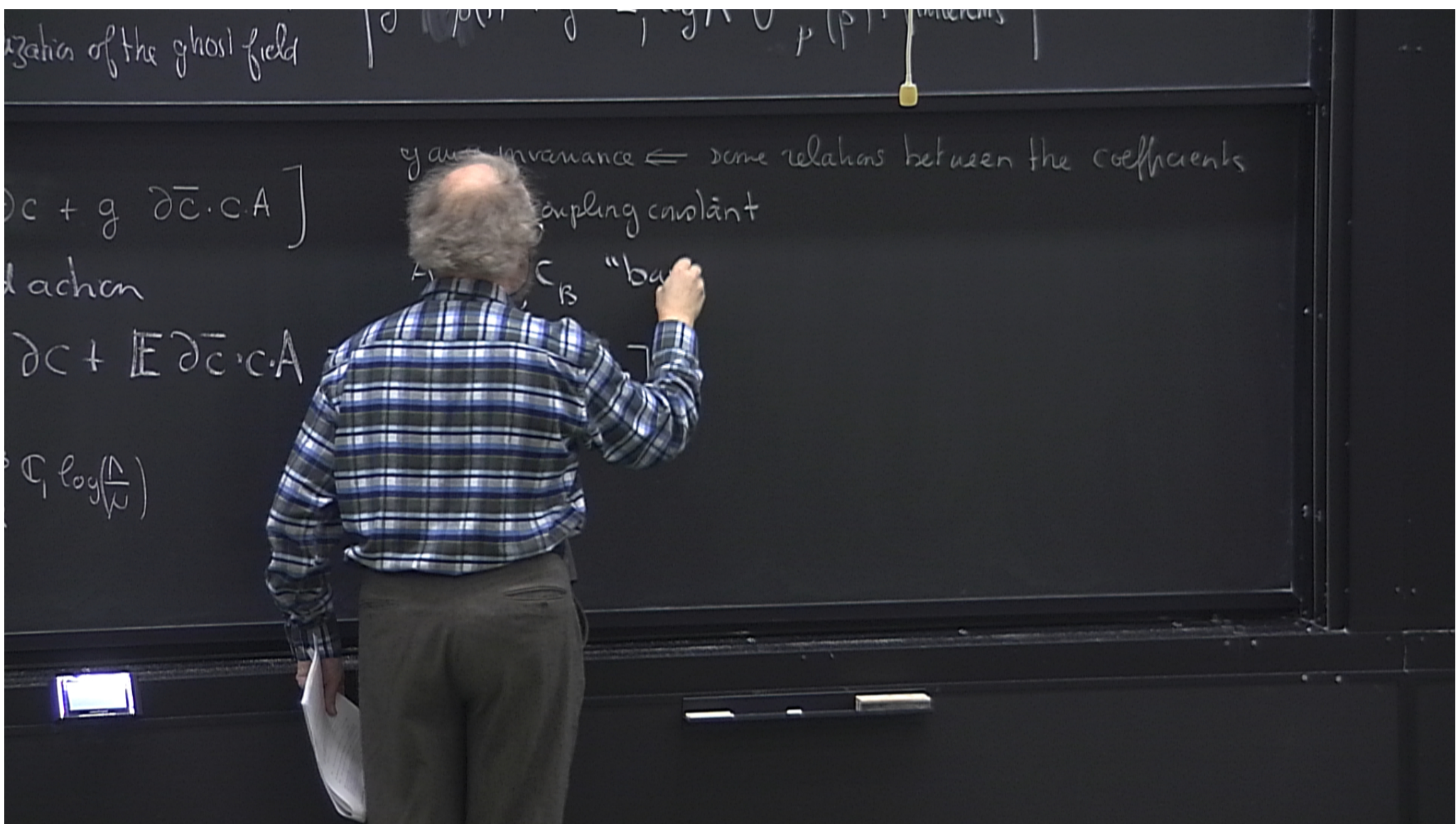
action

$$\partial c + \int \partial \bar{c} \cdot c \cdot A$$

$$C_1 \log\left(\frac{\Lambda}{\mu}\right)$$

gauge invariance  $\leftarrow$  the relation





$h_1, h_2, h_3 = 0$  tree level vertex  
 $g \int V_{\mu\nu\rho} + g \int [B_1 \log \Lambda V_{\mu\nu} + \text{finite terms}]$   
 gauge invariance  
 potential  $\Lambda$  divergence but in fact 0  
 renormalization of the coupling constant  $g$   
 $\delta^{ab} p^2 + g^2 D_1 \log \Lambda \delta^{ab} p^2$   
 renormalization of the ghost field  
 $g^2 V_{\mu\nu\rho} + g \int [C_1 \log \Lambda \dots]$   
 $g^2 \frac{c, \mu}{p} + g^3 \dots$   
 $g U_{\mu}^{abc} p + g^3 E_1 \dots$

$S_{\text{Tree}}[A, \bar{c}, c, g] = \int_M [\partial A \partial A + g \partial A \cdot A A + g^2 A \cdot A \cdot A A + \partial \bar{c} \cdot \partial c + g \partial \bar{c} \cdot c A]$   
 Renormalize the theory by the "bare action" by a renormalized action  
 $S_R[A, \bar{c}, c, g] = \int_M [A \partial A \partial A + B \partial A A A + C A \cdot A \cdot A A + D \partial \bar{c} \partial c + E \partial \bar{c} \cdot c A] = S_{\text{Tree}}[A_B, \bar{c}_B, c_B, g_B]$   
 $\int D[A] D[\bar{c}] D[c] \exp(i S_R) = \int D[A_B] D[\bar{c}_B] D[c_B] \exp(i S_{\text{Tree}})$   
 Renormalize theory at scale  $\mu$   
 $A = 1 - g_R \Lambda_1 \log(\frac{\Lambda}{\mu})$   
 $B = g_R - g_R^2 B_1 \log(\frac{\Lambda}{\mu})$   
 $C = g_R^2 - g_R^3 C_1 \log(\frac{\Lambda}{\mu})$   
 etc...  
 gauge invariance  $\Rightarrow$   
 $g_B$  "bare" coupling c  
 $A_B, \bar{c}_B, c_B$  "bare fields"



Identities  $\leftarrow$  gauge symmetry 't Hooft, Veltmann  
in '72  
Ward-Takahashi identities (QED)  
Taylor-Slavnov "  $\leftarrow$  Lee, Zinn-Justin '79  
BRST  $\leftarrow$  Supersymmetry involving the ghosts

't Hooft, Veltmann  
in '72

-Justin '74  
the ghosts

Beta-Function, Asymptotic Freedom in gauge theories

't Hooft '72

Gross-Wilczek & Politzer  
SU(3) QCD

$$g_B = g_R + g_R^3 \left( \frac{3}{2} A_1 - B_1 \right) \log \left( \frac{\Lambda}{\mu} \right)$$

$$\beta(g_R) = \left[ \mu \frac{\partial}{\partial \mu} g_R \right]_{g_B/\Lambda \text{ fixed}} = g^3 \frac{1}{(4\pi)^2} \left( -\frac{11}{3} C_2(G) + \frac{4}{3} N_F C(R_F) \right)$$

as in  $\phi^4$  theory

Casimir operator for the group G  
 $f^{acd} f^{bcd} = C_2(G) \delta^{ab}$

as in  $\phi^4$  theory

$\left. \begin{matrix} g_B/\Lambda \\ \text{fixed} \end{matrix} \right\} \Lambda \rightarrow \alpha_s$

casimir operator for the group  $G$   
 $f^{acd} f^{bcd} = C_2(G) \delta^{ab}$

casimir operator for the representation  $R_F$  for the Fermion Fields

# of families

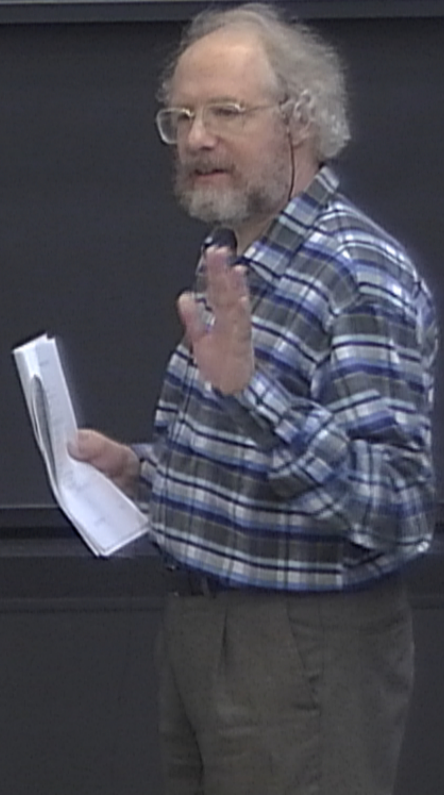
(or running)  
 at energy  $E$

Non abelian gauge theories  $\neq \phi^4$  or  $\phi ED$

Asymptotically Freedom at high energies - Weakly coupled

Asymptotic "slavery" at high energies - Strongly coupled

as  $\frac{1}{\log(E)}$



as in  $\phi^4$  theory

$g_B/\Lambda$   
fixed  $\Lambda \rightarrow \infty$

Casimir operator for the group  $G$   
 $F^{abcd} = C_2(G) \delta^{ab}$

# of families

$$g_B = \frac{g_R}{1 + g}$$

$\alpha(E)$  as the effective (or running) coupling constant at energy  $E$

$$E \frac{d}{dE} \alpha(E) = \beta(\alpha)$$

$$E \rightarrow \infty \quad \alpha(E) \rightarrow 0$$

abelian theories

$\neq \phi^4$  or  $\phi^4$  ED

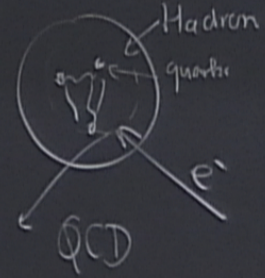
Asymptotic Freedom at high energies

"Asymptotic 'slavery'" at high energies

Weakly coupled

Strongly coupled

Experimental



Theoretically  $\alpha$  beyond perturbation theory

$$\alpha = g^2$$

$$\alpha_B = \frac{\alpha_R}{1 + \beta \alpha_R \log(\frac{\Lambda}{\mu})}$$

$\beta > 0$  IR  $\beta < 0$  UV

as in  $\phi^4$  theory

$g_B/\Lambda$   
fixed  $\Lambda \rightarrow \infty$

Casimir operator for the group  $G$   
 $F^{abcd} = C_2(G) \delta^{ab}$

# of families

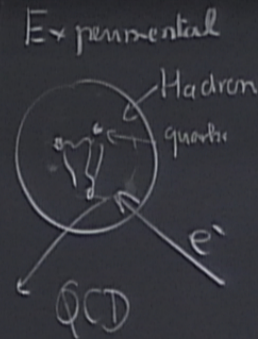
$\alpha(E)$  as the effective (or running) coupling constant at energy  $E$

$$E \frac{d}{dE} \alpha(E) = \beta(\alpha(E))$$

$$E \uparrow \infty \quad \alpha(E) \rightarrow 0 \text{ as } \frac{1}{\log(E)}$$

Non abelian gauge theories  $\neq \phi^4$

Asymptotically Free at high energies  
"the slavery" at high energies  
Weakly coupled



Theoretically  $\alpha$  beyond perturbation theory  
Resum the UV according to Borel summation