Title: Quantum Field Theory II -13

Date: Nov 26, 2014 09:00 AM

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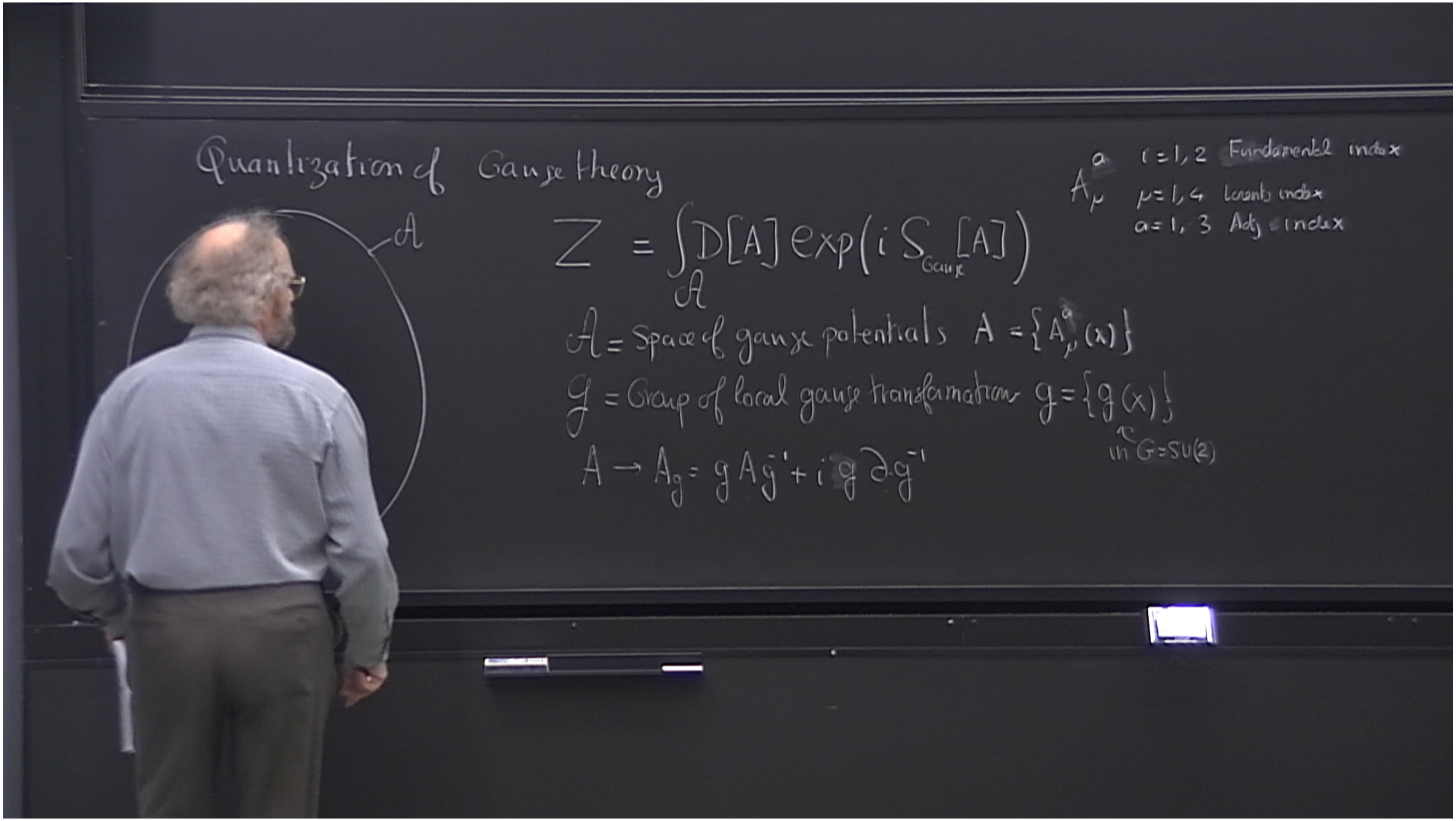
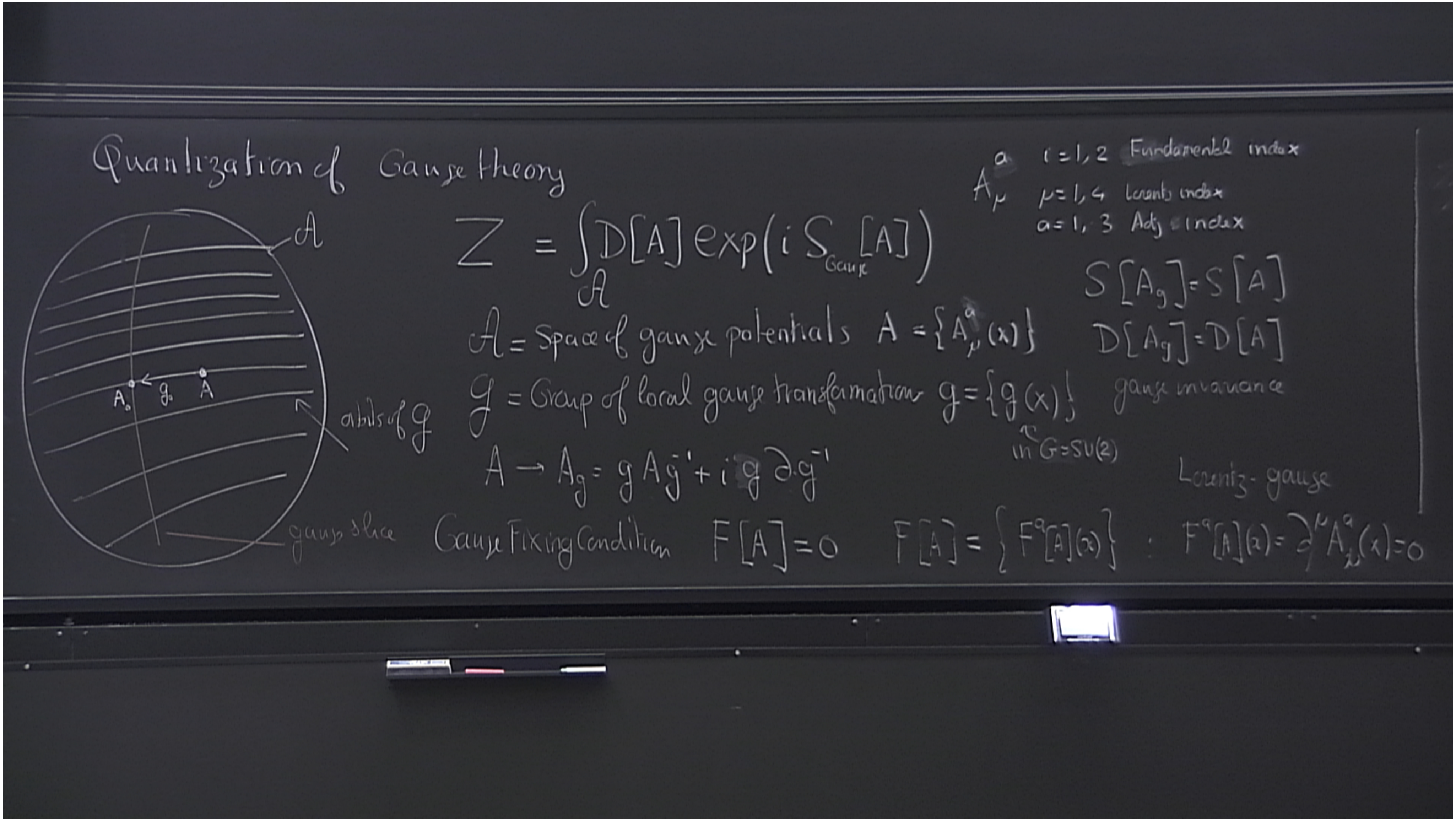
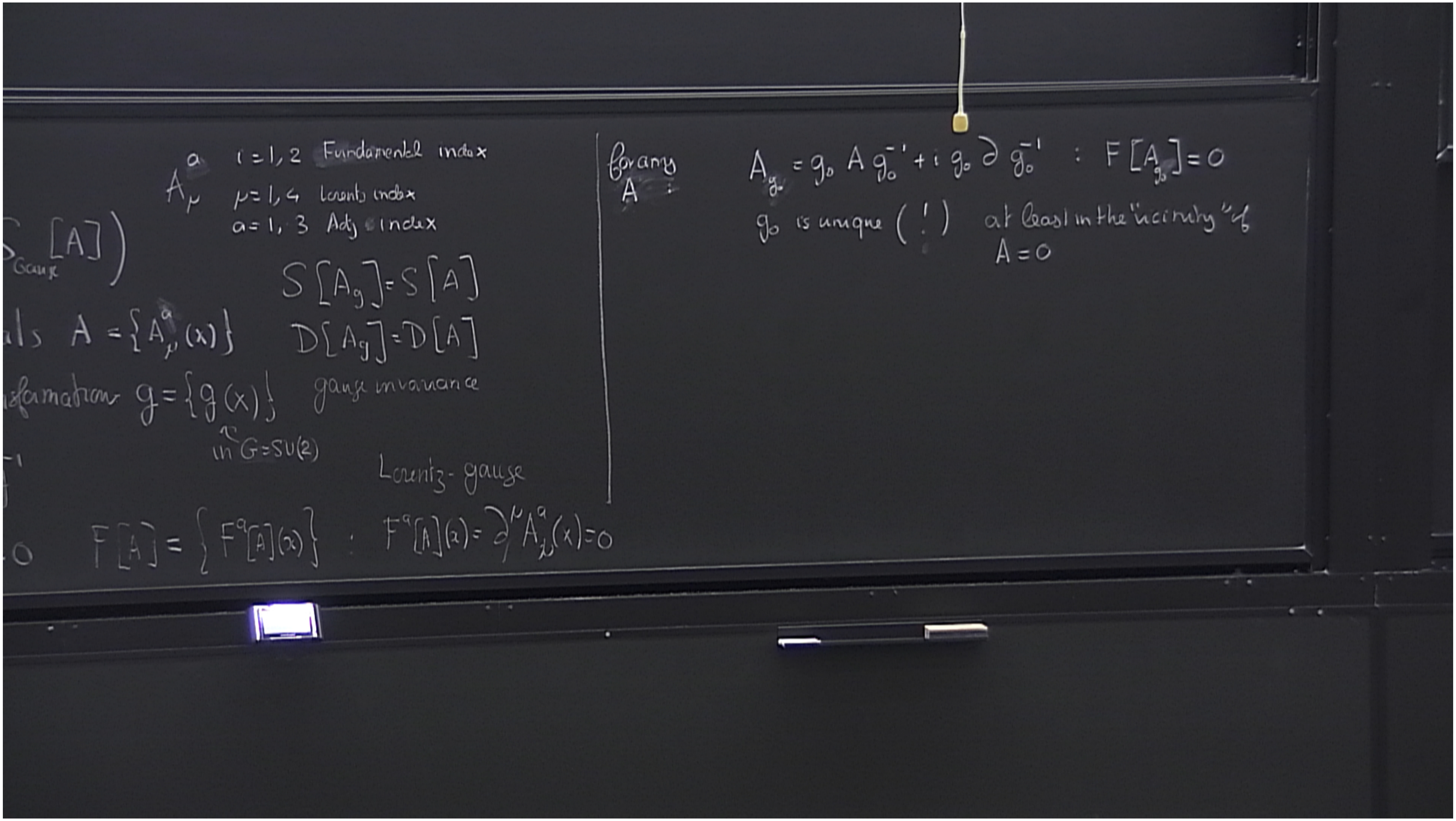
Abstract:


Quantization of Gauge theory
A
i=1,2 Fundamental index
μ=1,4 Lorentz index
a=1,3 Adj index
Z = ∫ D[A] exp(i S_Gauge [A])
A = Space of gauge potentials A = {A^a_μ(x)}
G = Group of local gauge transformation g = {g(x)}
in G=SU(2)
A → A_g = g A g^-1 + i g ∂ g^-1


Quantization of Gauge theory
$Z = \int_{\mathcal{A}} D[A] \exp(i S_{Gauge}[A])$
$\mathcal{A}$ = Space of gauge potentials $A = \{A^a_\mu(x)\}$
$\mathcal{G}$ = Group of local gauge transformation $g = \{g(x)\}$
in G=SU(2)
$A \rightarrow A_g = g A g^{-1} + i g \partial g^{-1}$
Gauge Fixing Condition $F[A]=0$ $F[A] = \{F^a[A](x)\}$: $F^a[A](x) = \partial^\mu A^a_\mu(x) = 0$
A^a_μ
i = 1, 2 Fundamental index
μ = 1, 4 Lorentz index
a = 1, 3 Adj index
$S[A_g] = S[A]$
$D[A_g] = D[A]$
gauge invariance
Lorentz-gauge
$\mathcal{A}$
A_0
g
A
orbits of $\mathcal{G}$
gauge slice



A^a_μ
i = 1, 2 Fundamental index
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a = 1, 3 Adj index
$S_{Gauge}[A]$)
$S[A_g] = S[A]$
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gauge invariance
in G = SU(2)
Lorentz-gauge
$F[A] = \{F^a[A](x)\}$: $F^a[A](x) = \partial^\mu A^a_\mu(x) = 0$
for any A :
$A_{g_0} = g_0 A g_0^{-1} + i g_0 \partial g_0^{-1}$: $F[A_{g_0}] = 0$
g_0 is unique (!) at least in the "vicinity" of A = 0

for any A : $A_{g_0} = g_0 A g_0^{-1} + i\, g_0 \partial g_0^{-1}$: $F[A_{g_0}] = 0$

g_0 is unique (!) at least in the "vicinity" of $A = 0$

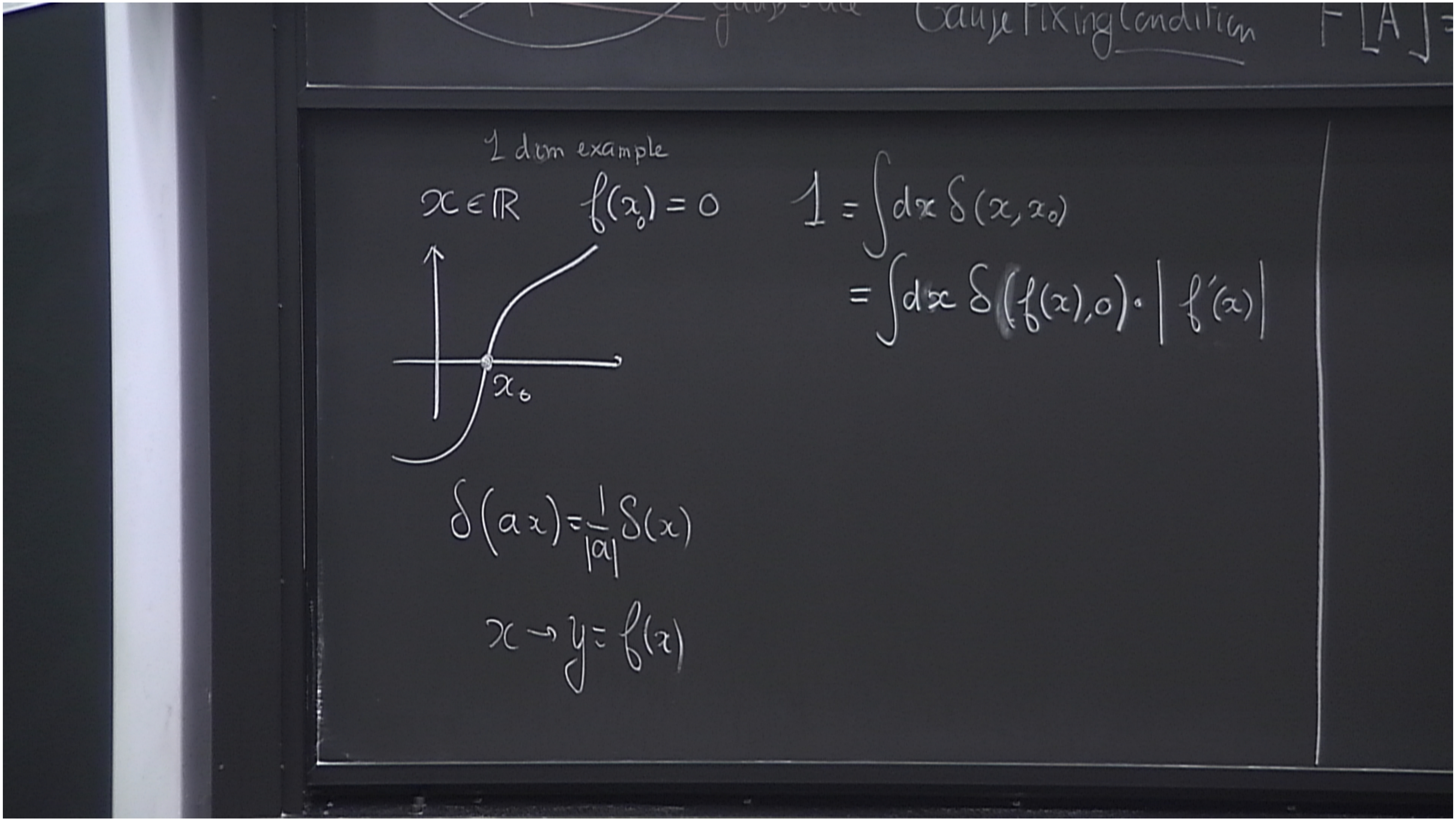
$g_0 = g_F[A]$ it depends on A and on F

$$Z = \int_{\mathcal{A}} D[A] \exp(i S[A]) \cdot \int_{\mathcal{G}} D[g]\, \delta[g - g_F[A]]$$

$D[g]$: Haar Measure; δ: Dirac δ-function

$=$

$(x) = 0$


Gauge fixing condition
F[A] =
1 dim example
x ∈ ℝ
f(x₀) = 0
1 = ∫dx δ(x, x₀)
= ∫dx δ(f(x), 0) · |f'(x)|
x₀
δ(ax) = 1/|a| δ(x)
x → y = f(x)

...index

...ts index

...j index

$[A_g] = S[A]$

$[A_g] = D[A]$

...ge invariance

Lorentz-gauge

$F^a[A](x) = \partial^\mu A^a_\mu(x) = 0$

for any A : $A_{g_0} = g_0 A g_0^{-1} + i g_0 \partial g_0^{-1}$: $F[A_{g_0}] = 0$

g_0 is unique (!) at least in the "vicinity" of $A = 0$

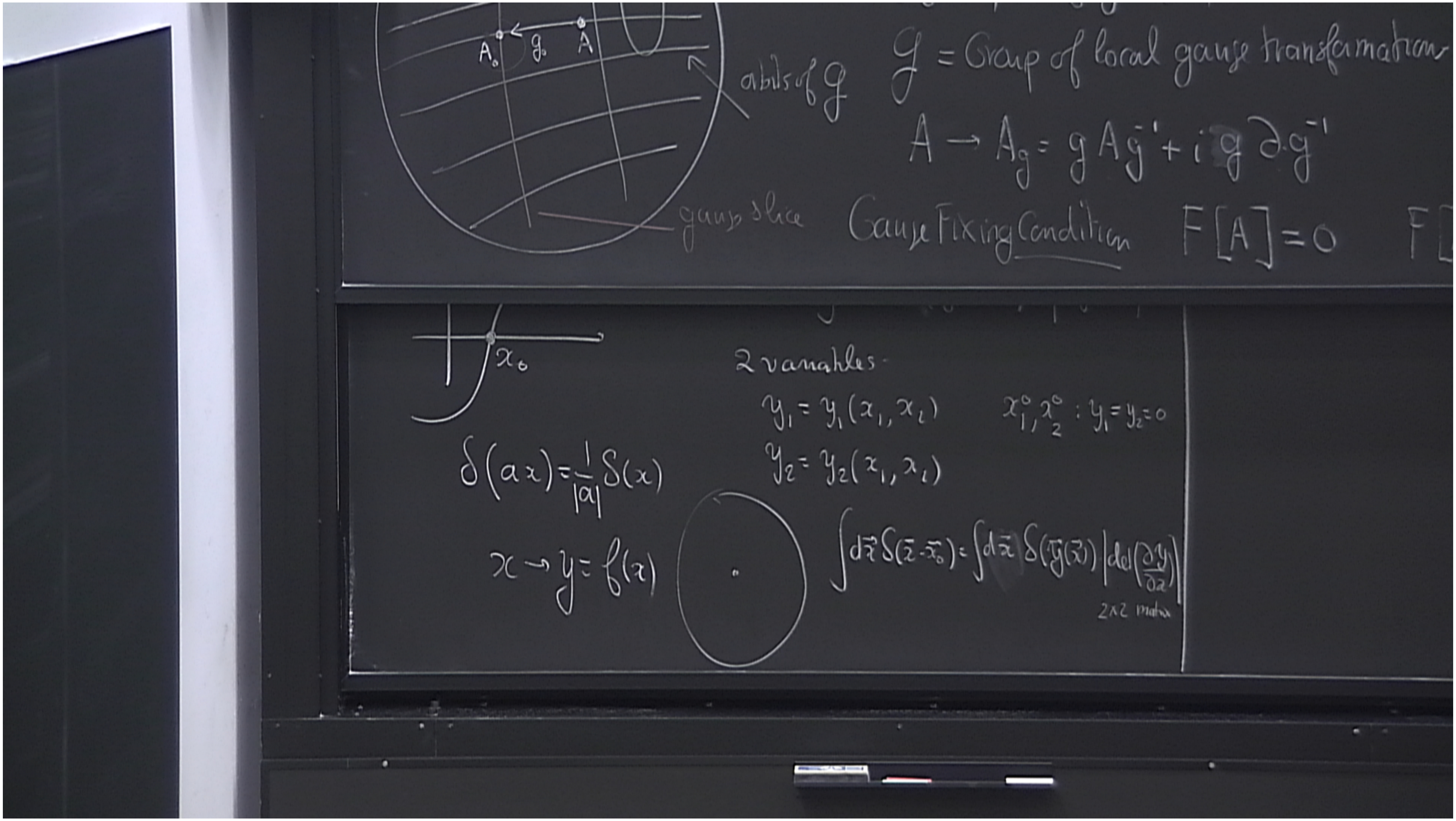
$g_0 = g_F[A]$ it depends on A and on F

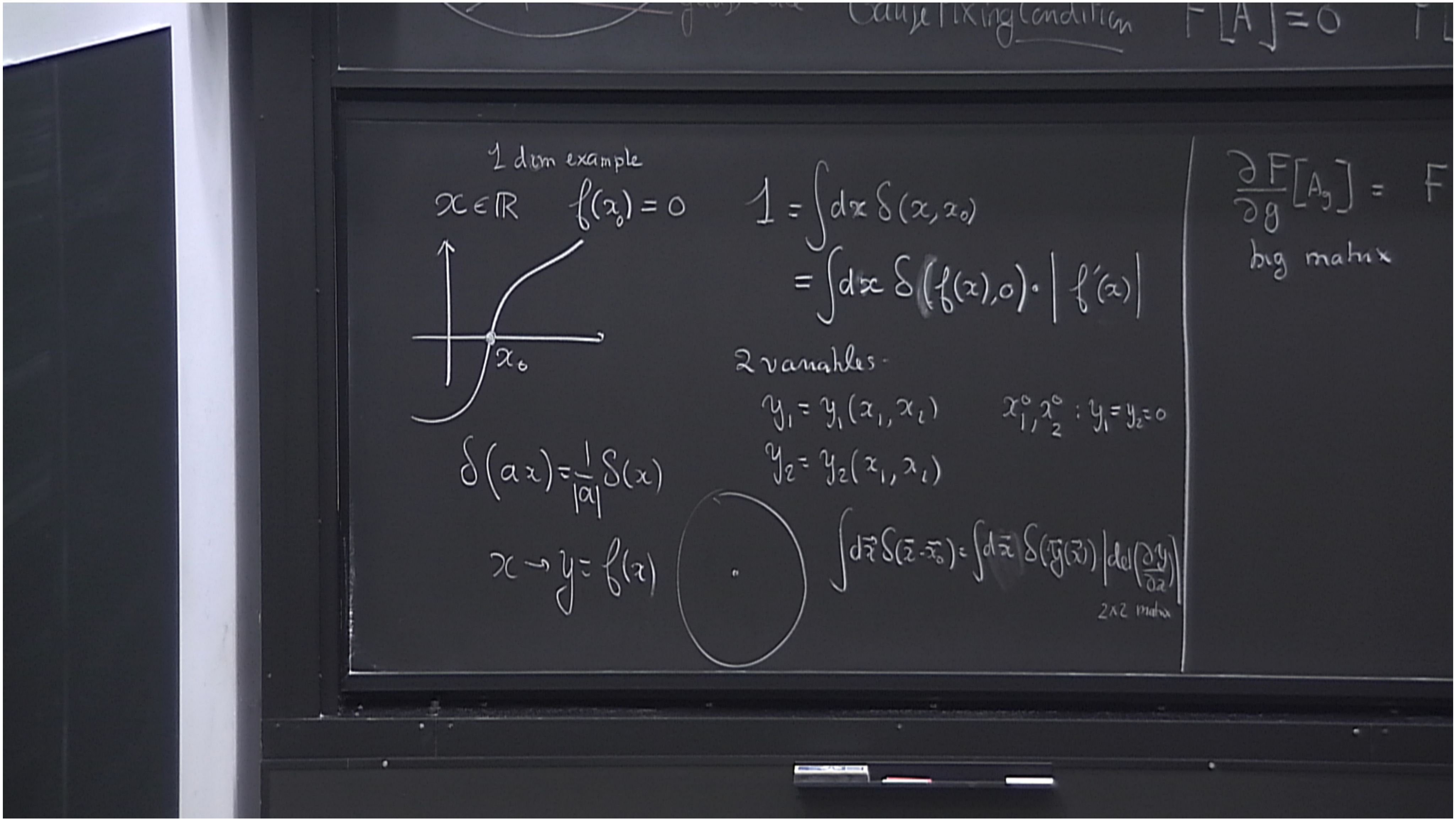
$$Z = \int_{\mathcal{A}} D[A] \exp(i S[A]) \int_{\mathcal{G}} D[g]\, \delta[g, g_F[A]]$$

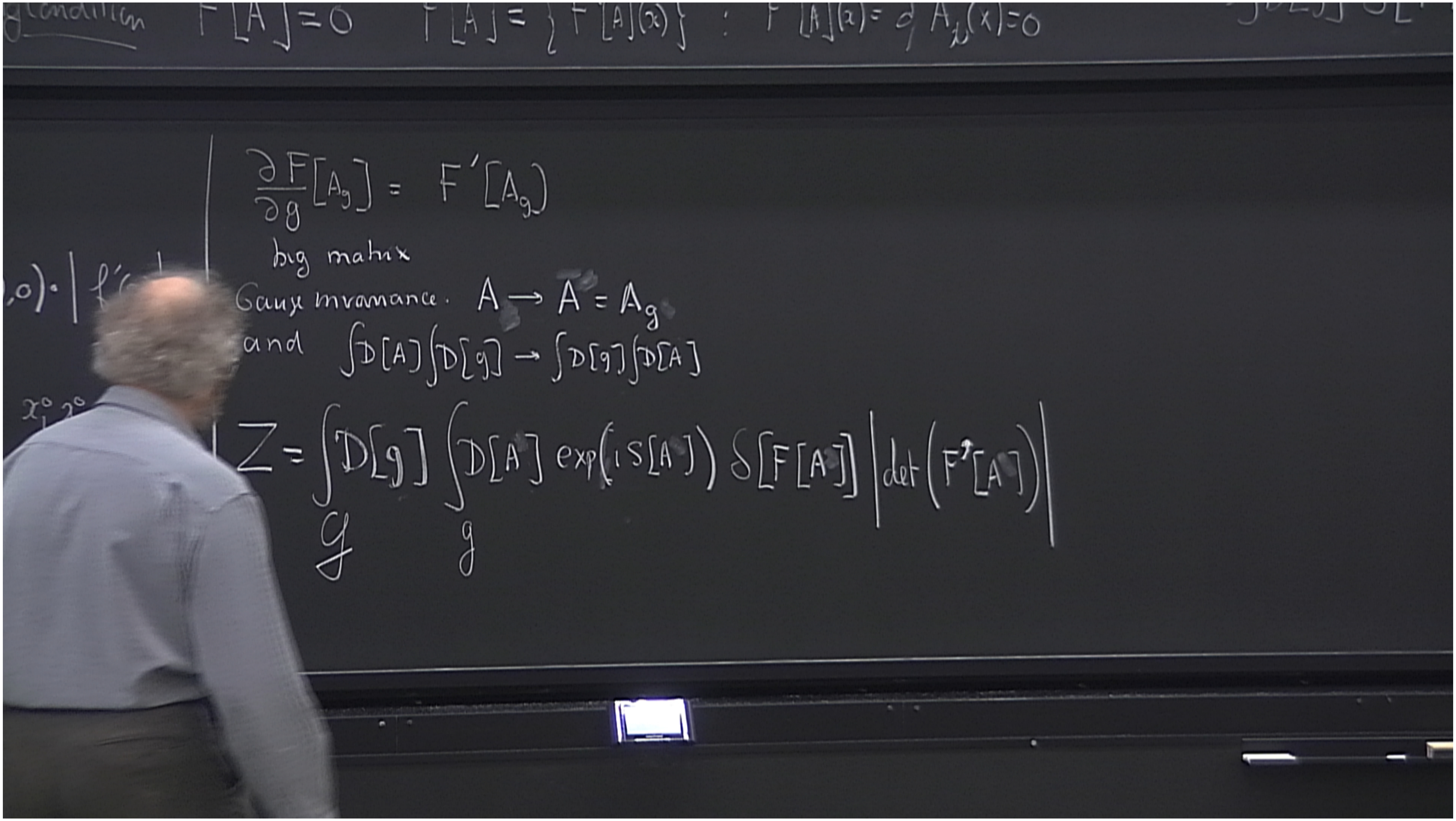
Haar Measure

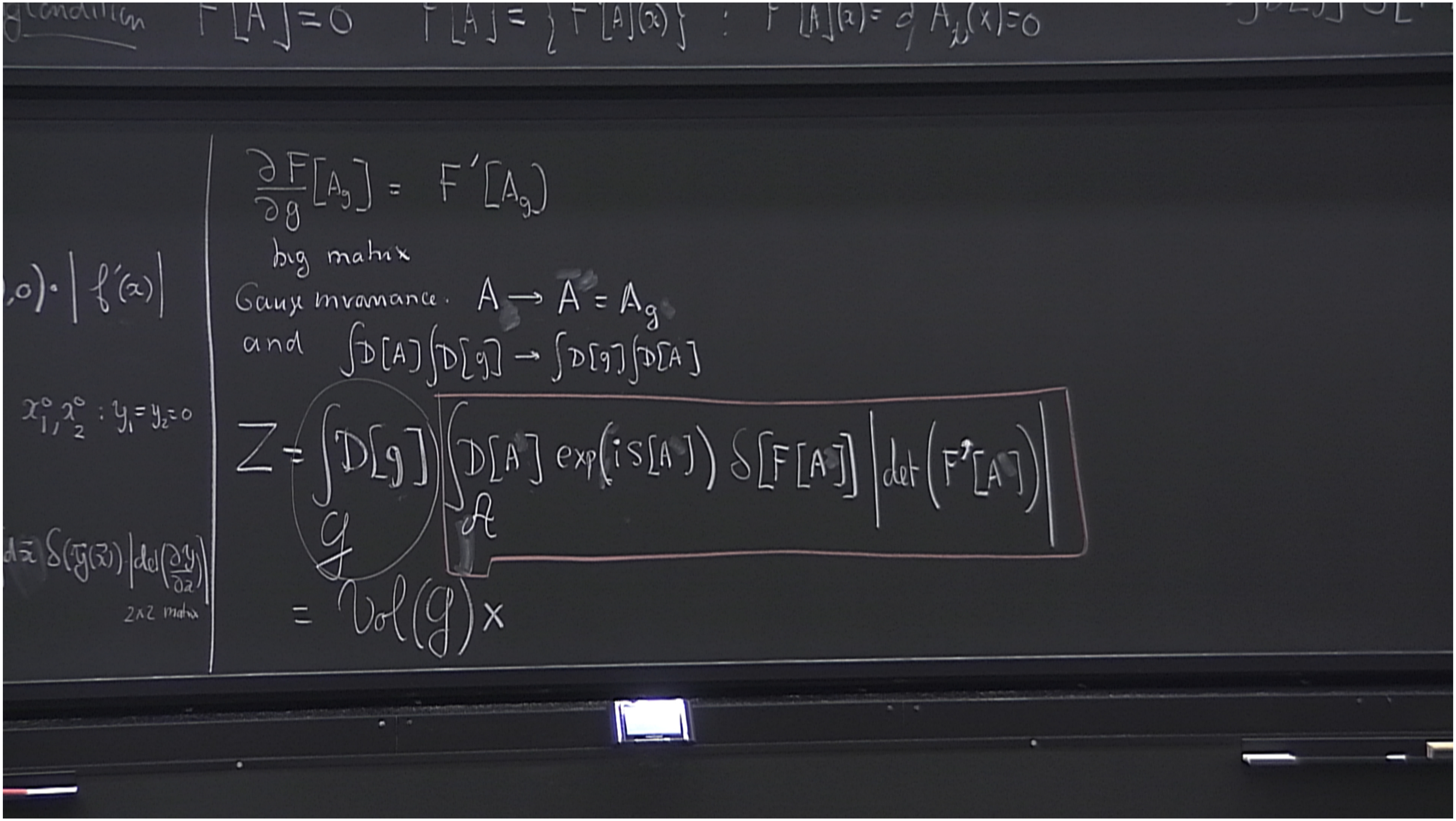
Dirac δ-function

$$= \int D[g]\, \delta[F[A_g]]\, |\det$$


A₀ g₀ A
orbits of g
g = Group of local gauge transformations
A → A_g = g A g⁻¹ + i g ∂ g⁻¹
gauge slice
Gauge Fixing Condition F[A] = 0
x₀
δ(ax) = 1/|a| δ(x)
x → y = f(x)
2 variables
y₁ = y₁(x₁, x₂)
y₂ = y₂(x₁, x₂)
x₁⁰, x₂⁰ : y₁ = y₂ = 0
∫ dx⃗ δ(x⃗ − x⃗₀) = ∫ dx⃗ δ(y⃗(x⃗)) |det(∂y/∂x)|
2×2 matrix


Gauge Fixing condition
$F[A] = 0$
1 dim example
$x \in \mathbb{R}$
$f(x_0) = 0$
x_0
$\delta(ax) = \frac{1}{|a|}\delta(x)$
$x \to y = f(x)$
$1 = \int dx\, \delta(x, x_0)$
$= \int dx\, \delta(f(x), 0) \cdot |f'(x)|$
2 variables
$y_1 = y_1(x_1, x_2)$
$x_1^0, x_2^0 : y_1 = y_2 = 0$
$y_2 = y_2(x_1, x_2)$
$\int d\vec{x}\, \delta(\vec{x} - \vec{x}_0) = \int d\vec{x}\, \delta(\vec{y}(\vec{x})) \left|\det\left(\frac{\partial y}{\partial x}\right)\right|$
2x2 matrix
$\frac{\partial F}{\partial g}[A_g] =$
big matrix


condition F[A] = 0 F[A] = {F[A](x)} : F[A](x) = ∂ A(x) = 0
∂F/∂g [A_g] = F′[A_g]
big matrix
Gauge invariance. A → A′ = A_g
and ∫D[A] ∫D[g] → ∫D[g] ∫D[A]
Z = ∫D[g] ∫D[A] exp(i S[A]) δ[F[A]] |det(F′[A])|


F[A] = 0   F[A] = {F[A](x)} : F[A](x) = ∂ A(x) = 0
∂F/∂g [A_g] = F′[A_g]
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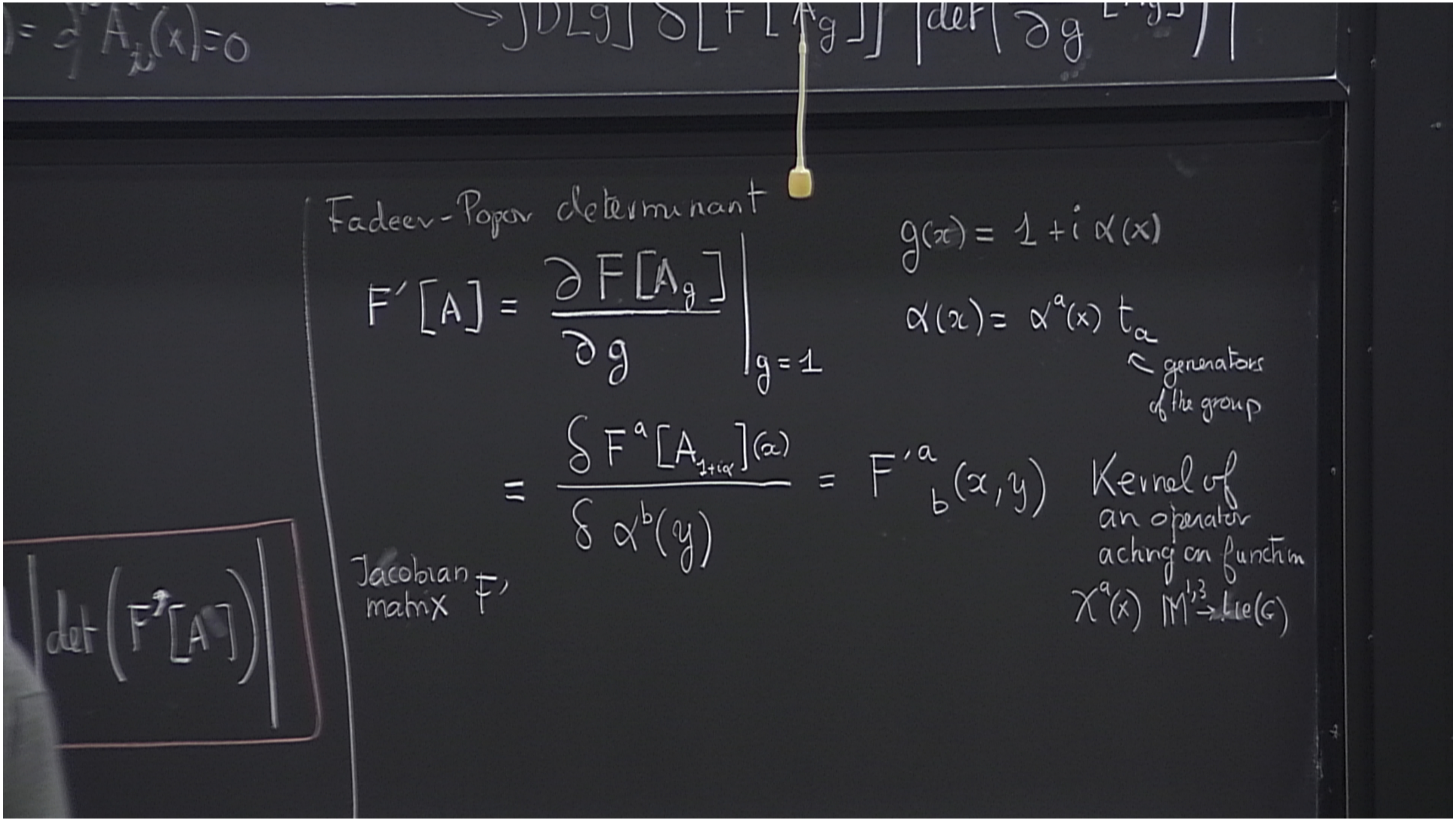
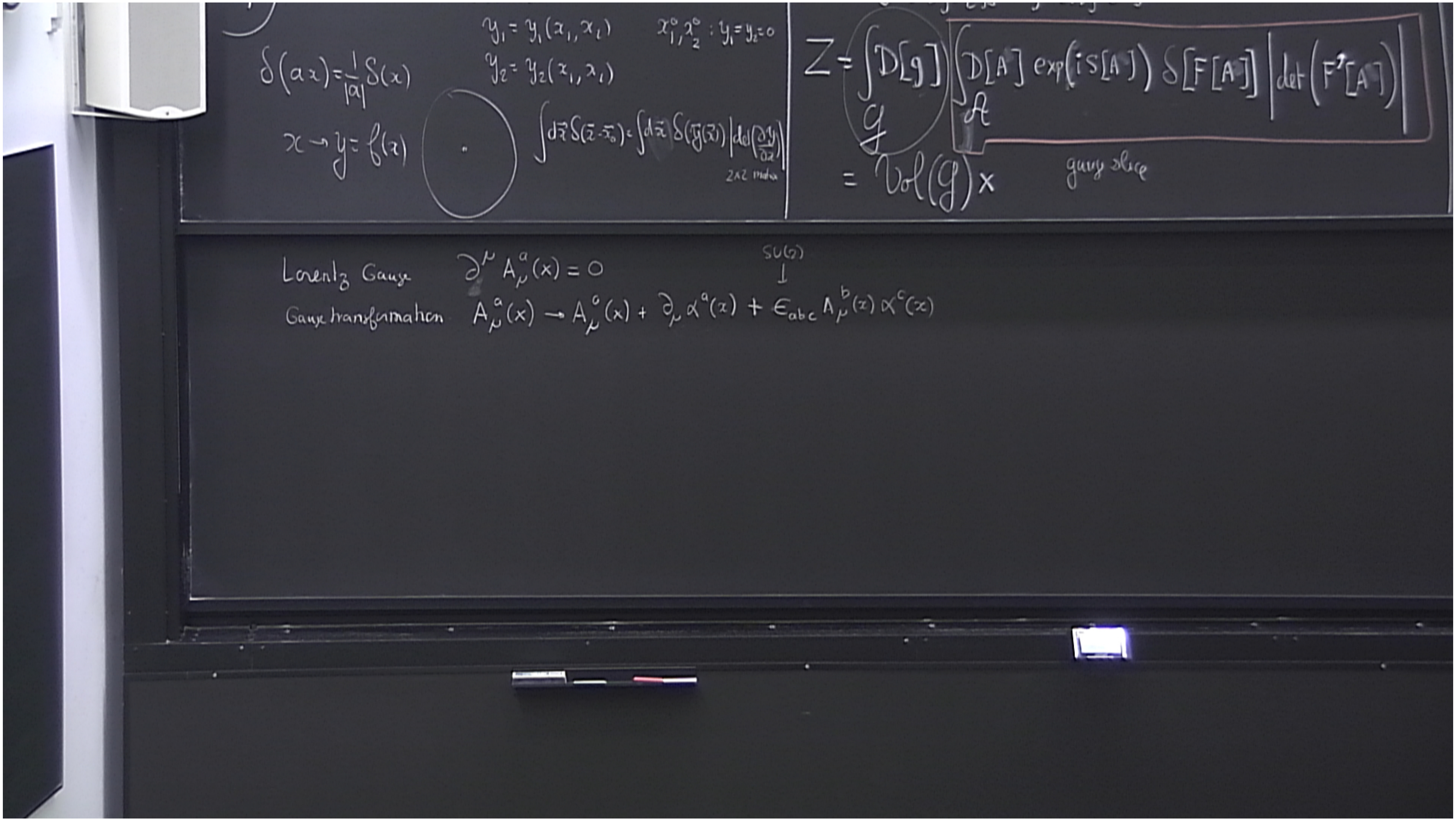
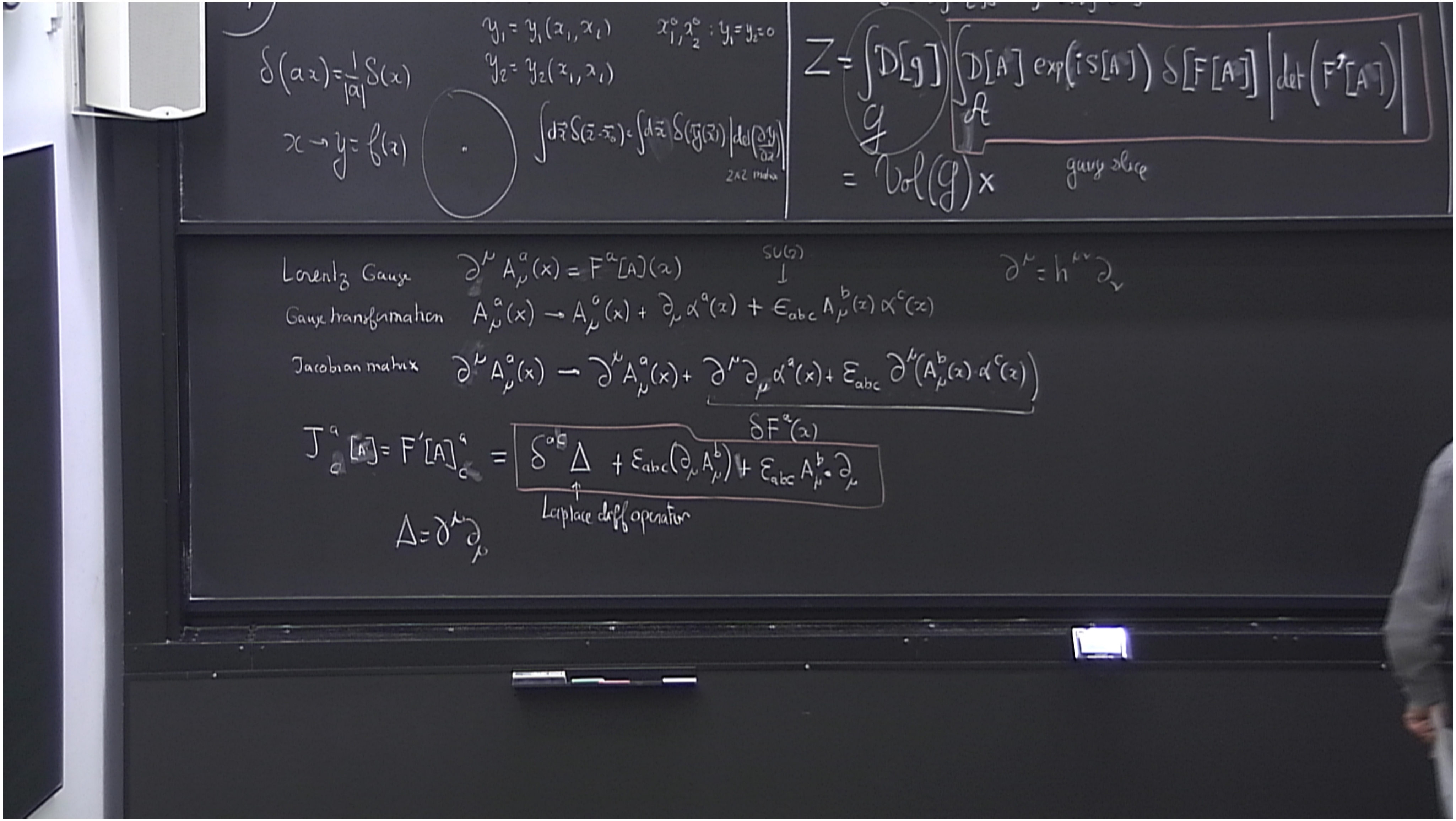
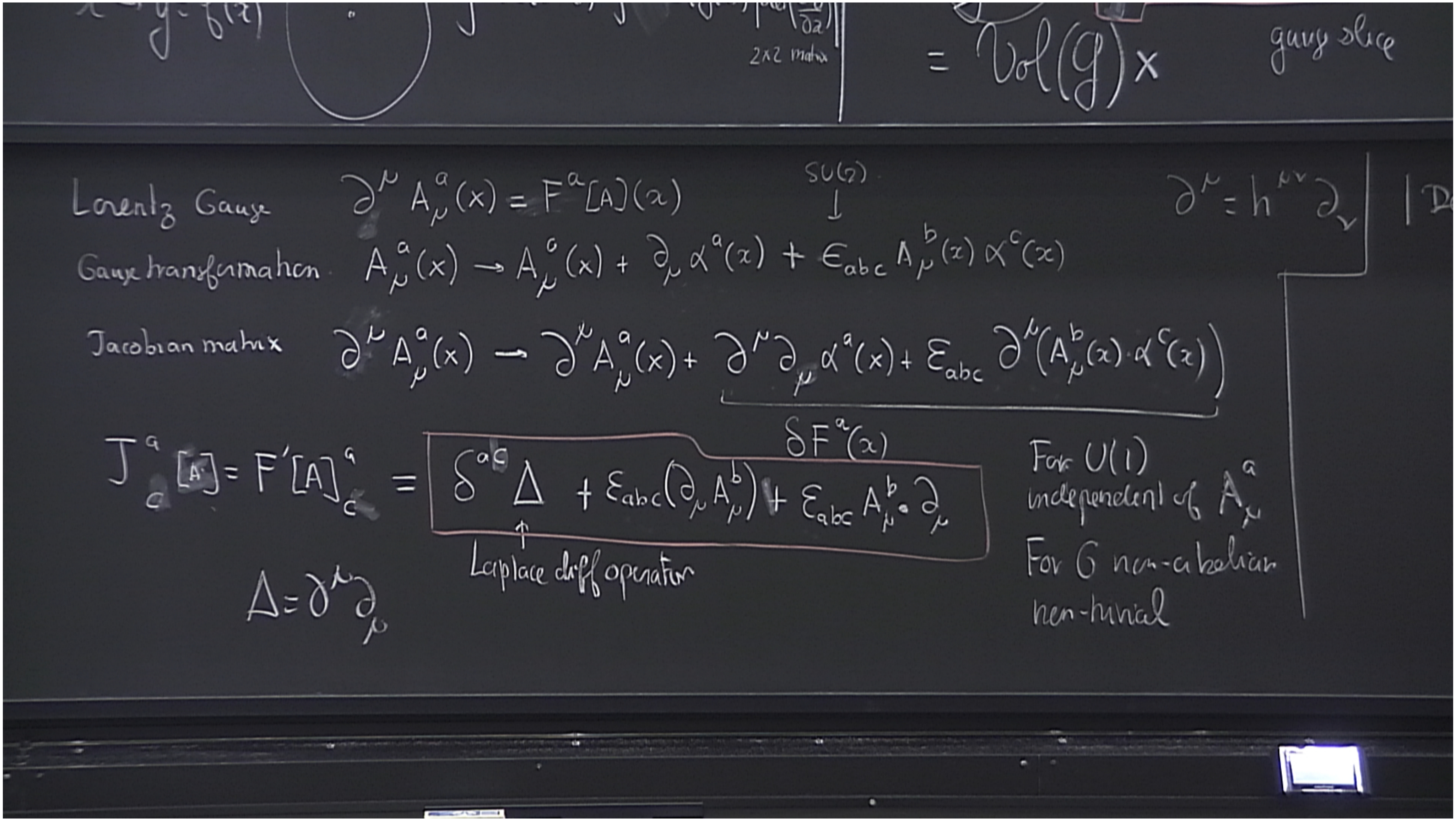
Gauge invariance. A → A′ = A_g
and ∫D[A]∫D[g] → ∫D[g]∫D[A]
Z = ∫_g D[g] ∫_A D[A] exp(iS[A]) δ[F[A]] |det(F′[A])|
= Vol(g) ×
f′(x)
x°1, x°2 : y1 = y2 = 0
2x2 matrix

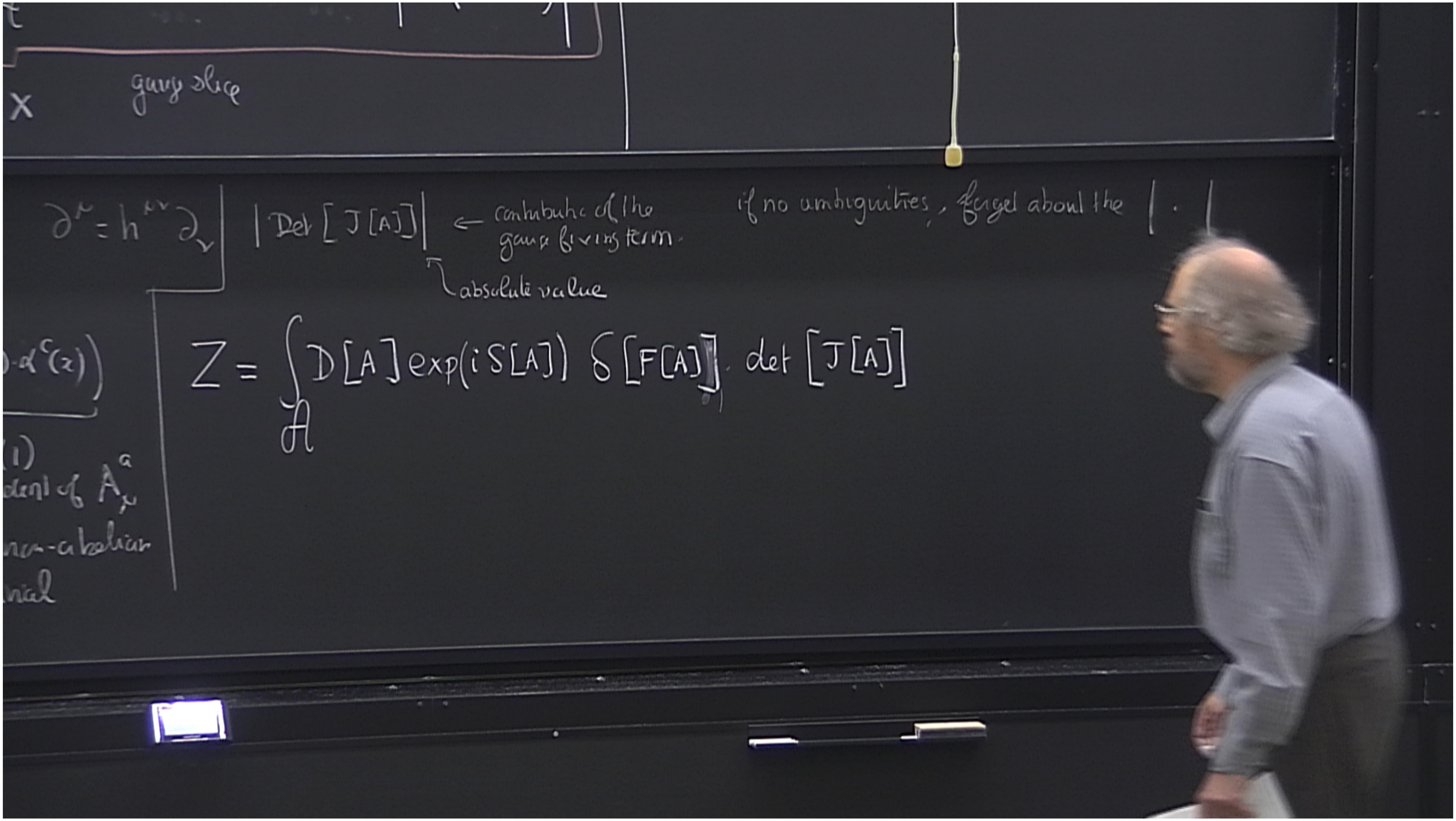

Fadeev-Popov determinant
$F'[A] = \frac{\partial F[A_g]}{\partial g}\Big|_{g=1}$
$= \frac{\delta F^a[A_{1+i\alpha}](x)}{\delta \alpha^b(y)} = F'^a{}_b(x,y)$
Kernel of an operator acting on function
$\chi^a(x)$ $M^{1,3} \to Lie(G)$
$g(x) = 1 + i\,\alpha(x)$
$\alpha(x) = \alpha^a(x)\, t_a$
generators of the group
Jacobian matrix F'
$|\det(F'[A])|$
$\int D[g]\, \delta[F[A_g]]\, |\det(\partial g\, ...)|$



δ(ax) = 1/|a| δ(x)
y1 = y1(x1, x2)
y2 = y2(x1, x2)
x1°, x2° : y1 = y2 = 0
x → y = f(x)
∫dx⃗ δ(x⃗ − x⃗0) = ∫dx⃗ δ(y⃗(x⃗)) |det(∂y/∂x)|
2x2 matrix
Z = ∫D[g] ∫D[A] exp(iS[A]) δ[F[A]] |det(F'[A])|
= Vol(g) x
gauge slice
Lorentz Gauge ∂^μ A_μ^a(x) = 0
SU(2)
Gauge transformation A_μ^a(x) → A_μ^a(x) + ∂_μ α^a(x) + ε_abc A_μ^b(x) α^c(x)


$y_1 = y_1(x_1, x_2)$ $x_1^0, x_2^0 : y_1 = y_2 = 0$
$\delta(ax) = \frac{1}{|a|}\delta(x)$
$y_2 = y_2(x_1, x_2)$
$x \to y = f(x)$
$\int d\vec{x}\, \delta(\vec{x} - \vec{x}_0) = \int d\vec{x}\, \delta(\vec{y}(\vec{x})) \left|\det\left(\frac{\partial y}{\partial x}\right)\right|$
2x2 matrix
$Z = \int_{\mathcal{G}} D[g] \int_{\mathcal{A}} D[A] \exp(iS[A])\, \delta[F[A]] \left|\det(F'[A])\right|$
$= \mathrm{Vol}(\mathcal{G}) \times$
gauge slice
Lorentz Gauge $\partial^\mu A_\mu^a(x) = F^a[A](x)$
SU(2)
$\partial^\mu = h^{\mu\nu}\partial_\nu$
Gauge transformation $A_\mu^a(x) \to A_\mu^a(x) + \partial_\mu \alpha^a(x) + \epsilon_{abc} A_\mu^b(x)\alpha^c(x)$
Jacobian matrix $\partial^\mu A_\mu^a(x) \to \partial^\mu A_\mu^a(x) + \partial^\mu \partial_\mu \alpha^a(x) + \epsilon_{abc}\partial^\mu (A_\mu^b(x)\alpha^c(x))$
$\delta F^a(x)$
$J^a_c[A] = F'[A]^a_c = \delta^{ac}\Delta + \epsilon_{abc}(\partial_\mu A_\mu^b) + \epsilon_{abc} A_\mu^b \partial_\mu$
Laplace diff operator
$\Delta = \partial^\mu \partial_\mu$


2x2 matrix
= Vol(G) x
gauge slice
Lorentz Gauge ∂^μ A_μ^a(x) = F^a[A](x)
SU(2)
∂^μ = η^μν ∂_ν
Gauge transformation A_μ^a(x) → A_μ^a(x) + ∂_μ α^a(x) + ε_abc A_μ^b(x) α^c(x)
Jacobian matrix ∂^μ A_μ^a(x) → ∂^μ A_μ^a(x) + ∂^μ ∂_μ α^a(x) + ε_abc ∂^μ(A_μ^b(x) · α^c(x))
δF^a(x)
J^a_c[A] = F′[A]^a_c = δ^ac Δ + ε_abc(∂_μ A_μ^b) + ε_abc A_μ^b · ∂_μ
Laplace diff operator
Δ = ∂^μ ∂_μ
For U(1) independent of A_μ^a
For G non-abelian non-trivial


gauge slice
∂^μ = h^{μν} ∂_ν
|Det [J[A]]| ← contribution of the gauge fixing term.
absolute value
if no ambiguities, forget about the | · |
Z = ∫_A D[A] exp(i S[A]) δ[F[A]] · det [J[A]]
non-abelian

gauge slice

$\partial^\mu = h^{\mu\nu}\partial_\nu$

$\left|\mathrm{Det}\left[J[A]\right]\right|$ ← contribution of the gauge fixing term.

└ absolute value

if no ambiguities, forget about the $|\cdot|$

expand in powers of A and compute.

$$Z = \int_{\mathcal{A}} D[A]\exp(iS[A])\,\delta\left[F[A]\right]\cdot\det\left[J[A]\right]$$

$$\det(J) = \int D[\bar{c}, c]\exp(i\,\bar{c}\cdot J\cdot c)$$

Berezin calculus and represent this determinant as an integration over anticommuting fields

Ghosts and anti-Ghosts

Fadeev Popov

$\cdot\alpha^c(x))$

(1)

dent of A^a_μ

non-abelian

nal

gauge slice

$h^{\mu\nu}\partial_\nu$

$|\mathrm{Det}[J[A]]|$ ← contribution of the gauge fixing term.

absolute value

if no ambiguities, forget about the $|\cdot|$

$$Z = \int_{\mathcal{A}} D[A]\exp(iS[A])\,\delta[F[A]]\cdot\det[J[A]]$$

$$\det(J) \propto \int D[\bar{c},c]\exp(i\,\bar{c}\cdot J\cdot c)$$

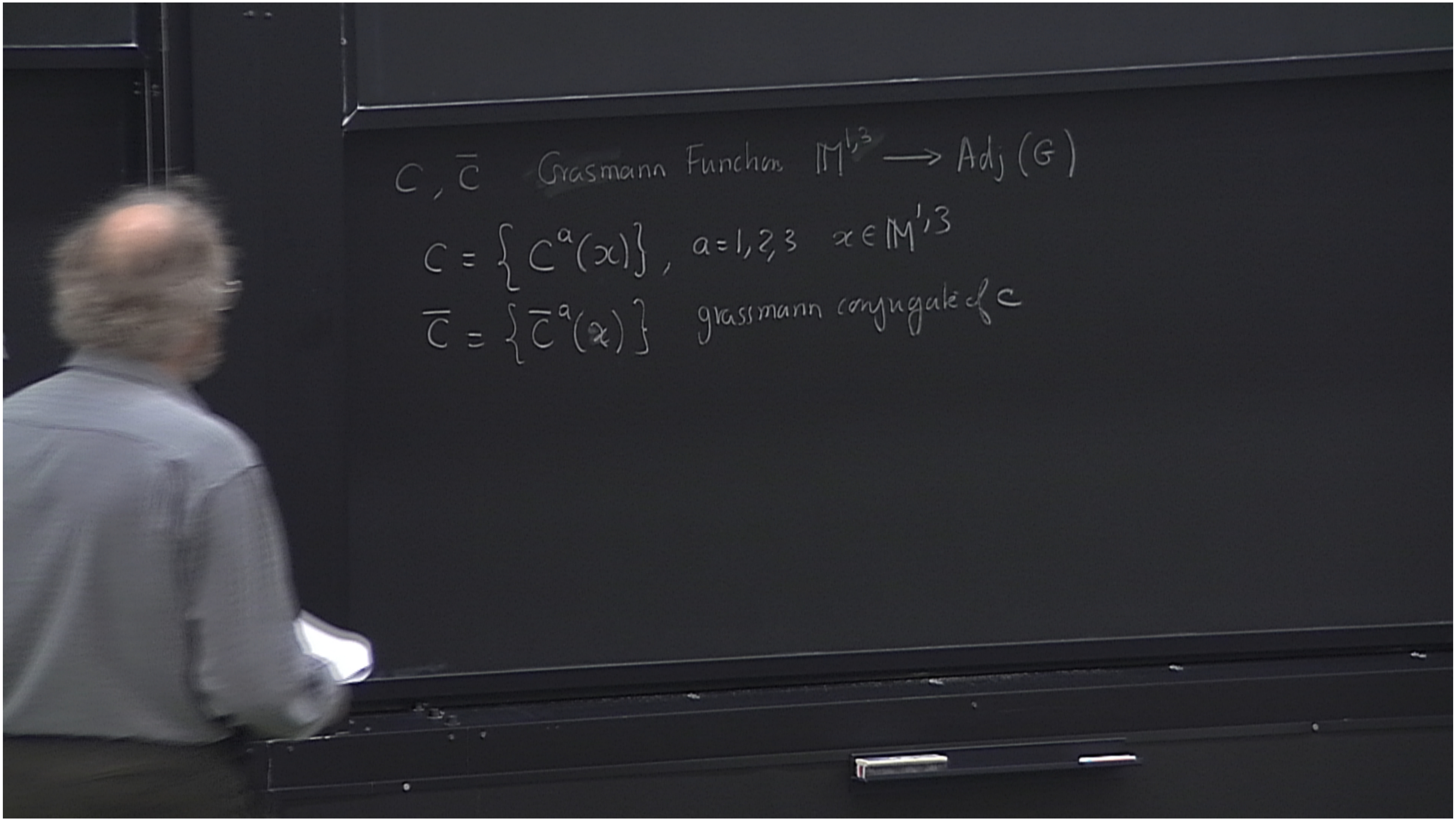
$c, \bar{c}$ ghosts

expand in powers of A and compute.

Berezin calculus and represent this determinant as an integration over anticommuting fields

Ghosts and anti-Ghosts

Fadeev Popov



$C, \bar{C}$ Grasmann Functions $\mathbb{M}^{1,3} \longrightarrow Adj(G)$
$C = \{C^a(x)\}$, $a = 1,2,3$ $x \in \mathbb{M}^{1,3}$
$\bar{C} = \{\bar{C}^a(x)\}$ grassmann conjugate of C

$C, \bar{C}$ Grasmann Functions $\mathbb{M}^{1,3} \longrightarrow \mathrm{Adj}(G)$

$C = \{ C^a(x) \}$, $a = 1,2,3$ $\quad x \in \mathbb{M}^{1,3}$ $\Big)$ anticommute

$\bar{C} = \{ \bar{C}^a(x) \}$ grassmann conjugate of c

Action for the ghosts

$$\bar{C} \cdot J c = S_{ghosts} = \int d^D x \; \bar{C}^a(x) [$$

$c, \bar{c}$ Grassmann Function $\mathbb{M}^{1,3} \longrightarrow Adj(G)$

$c = \{ c^a(x) \}$, $a = 1,2,3$ $x \in \mathbb{M}^{1,3}$) anticommute

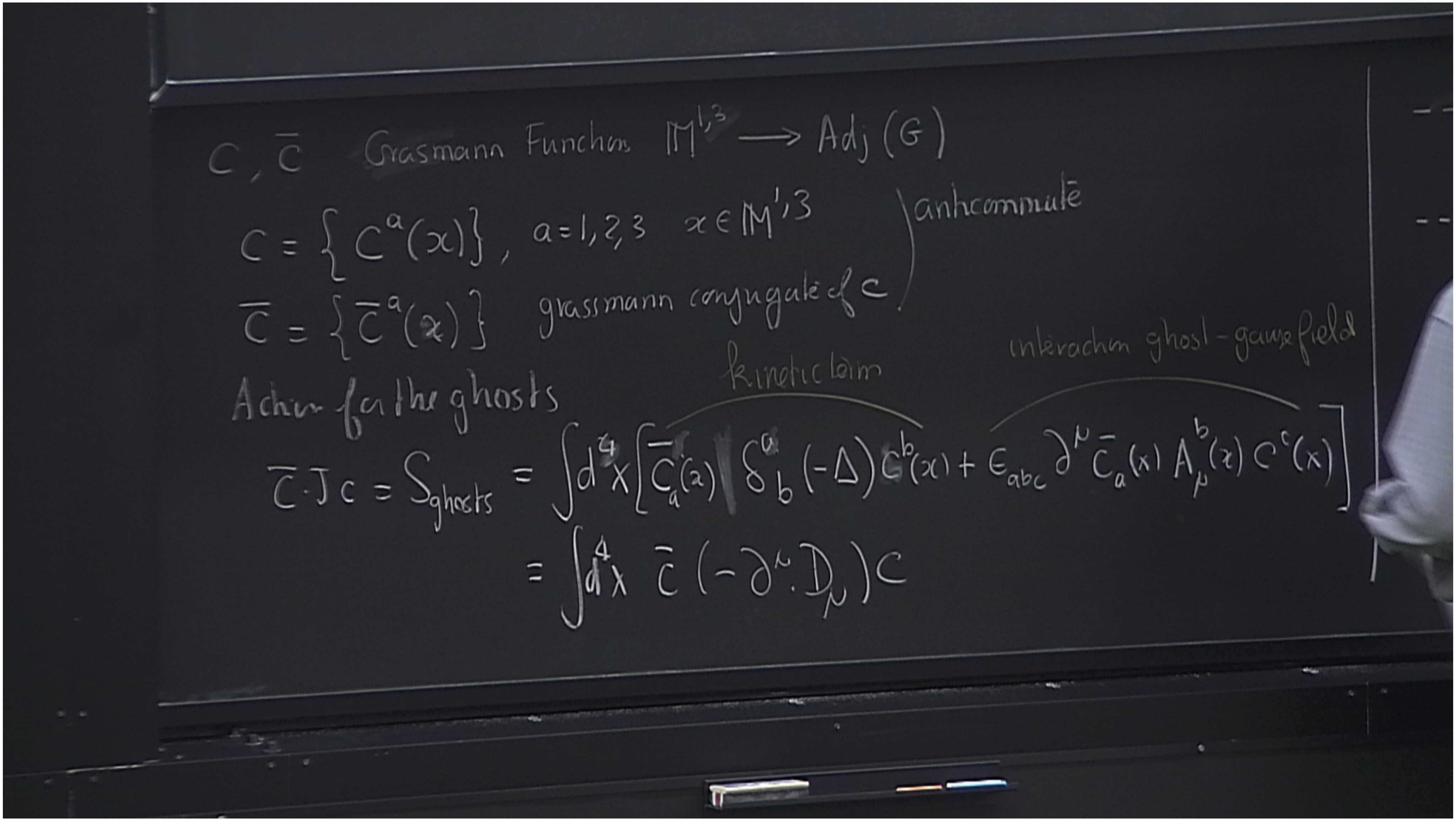
$\bar{c} = \{ \bar{c}^a(x) \}$ grassmann conjugate of c

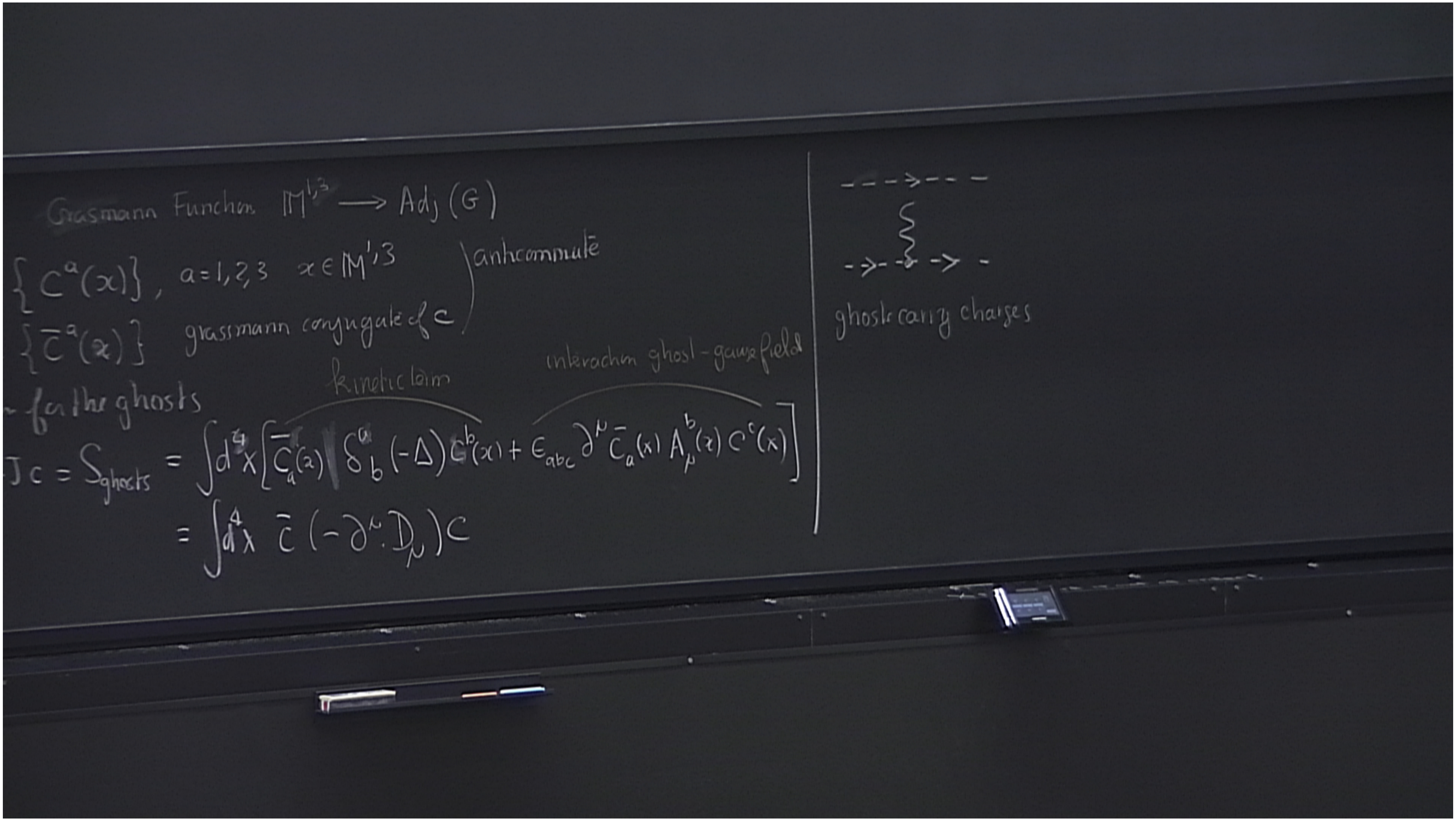
Action for the ghosts

kinetic term — interaction ghost-gauge field

$$\bar{c} \cdot J c = S_{ghosts} = \int d^4x \left[\bar{c}_a(x) \, \delta^a_b (-\Delta) c^b(x) + \epsilon_{abc} \partial^\mu \bar{c}_a(x) A^b_\mu(x) c^c(x) \right]$$

$$= \int d^4x \; \bar{c} \left(-\partial^\mu \cdot D_\mu \right) c$$


c, c̄ Grasmann Function M^{1,3} ⟶ Adj(G)
c = { c^a(x) }, a = 1,2,3 x ∈ M^{1,3}
c̄ = { c̄^a(x) } grassmann conjugate of c
anticommute
Action for the ghosts
kinetic term
interaction ghost - gauge field
c̄·J c = S_ghosts = ∫d^4x [c̄_a(x) δ^a_b (−Δ) c^b(x) + ε_abc ∂^μ c̄_a(x) A^b_μ(x) c^c(x)]
= ∫d^4x c̄ (−∂^μ D_μ) c


Grasmann Function M^{1,3} ⟶ Adj (G)
{C^a(x)}, a=1,2,3 x ∈ M^{1,3}
anticommute
{C̄^a(x)} grassmann conjugate of c
for the ghosts
kinetic term
interaction ghost - gauge field
S_ghosts = ∫d^4x [C̄_a(x) δ^a_b (−Δ) C^b(x) + ε_abc ∂^μ C̄_a(x) A^b_μ(x) C^c(x)]
= ∫d^4x c̄ (−∂^μ D_μ) c
ghosts carry charges

$\lambda(x)$ $M \to Lie(G)$

s of A

lus and

s determinant

ther over

g fields

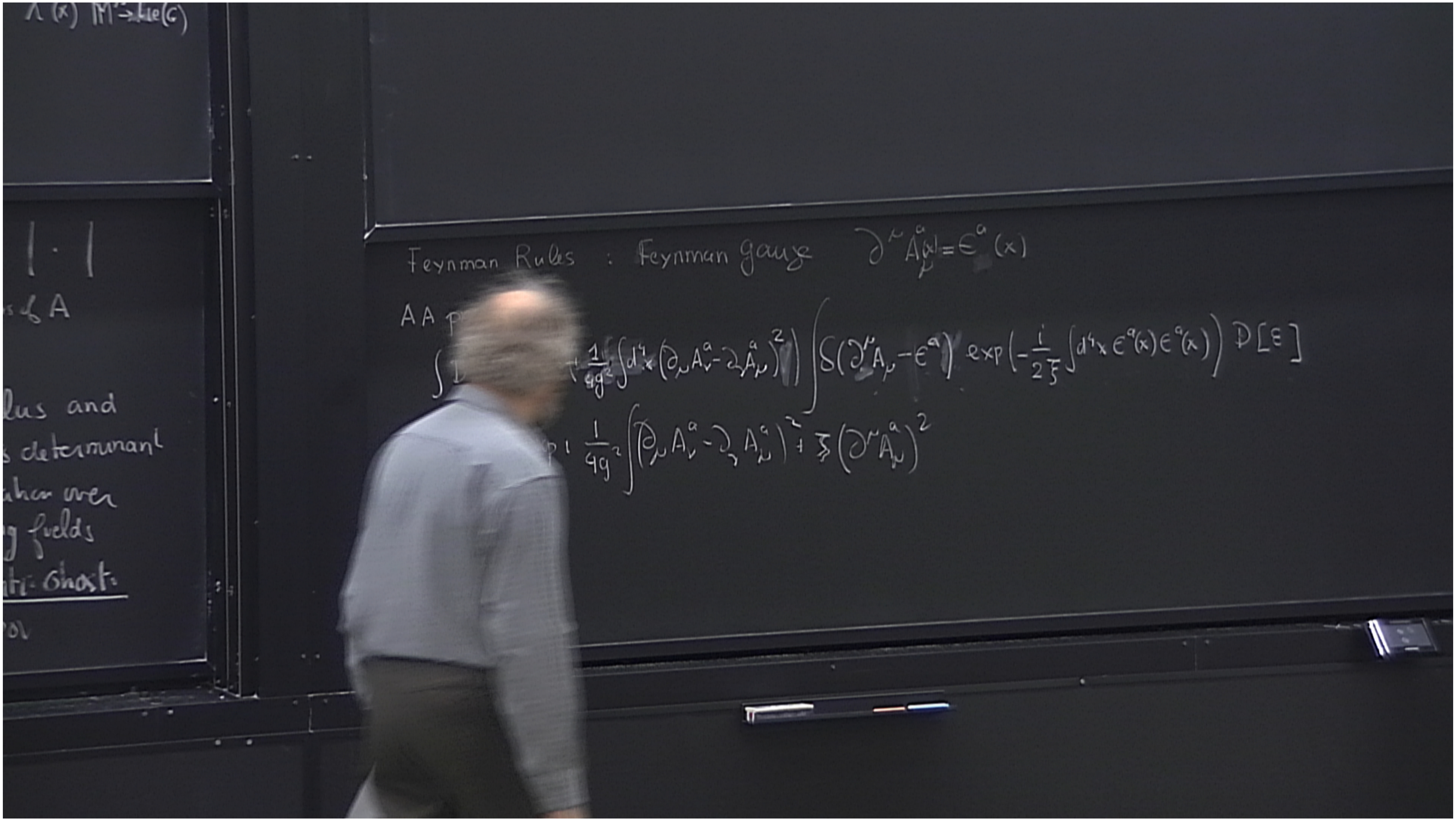
nti-Ghost

or

Feynman Rules : Feynman gauge $\partial^\mu A_\mu(x) = \epsilon_\mu(x)$

AA propagator

$$\int D[A] \exp\left(i \frac{1}{4g^2}\int d^4x \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2\right) \delta(\partial^\mu A_\mu)$$


Feynman Rules : Feynman gauge $\partial^\mu A^a_\mu(x) = \epsilon^a(x)$
AA
$\exp\left(-\frac{i}{2\xi}\int d^4x\, \epsilon^a(x)\epsilon^a(x)\right) \mathcal{D}[\epsilon]$
$\frac{1}{4g^2}\int (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \xi(\partial^\mu A^a_\mu)^2$
lus and
determinant
ther over
fields
Ghost

Feynman Rules : Feynman gauge $\partial^\mu A^a_\mu(x) = \epsilon^a(x)$ depends on a parameter ξ

$\xi = 0$ Lorentz gauge

AA propagator

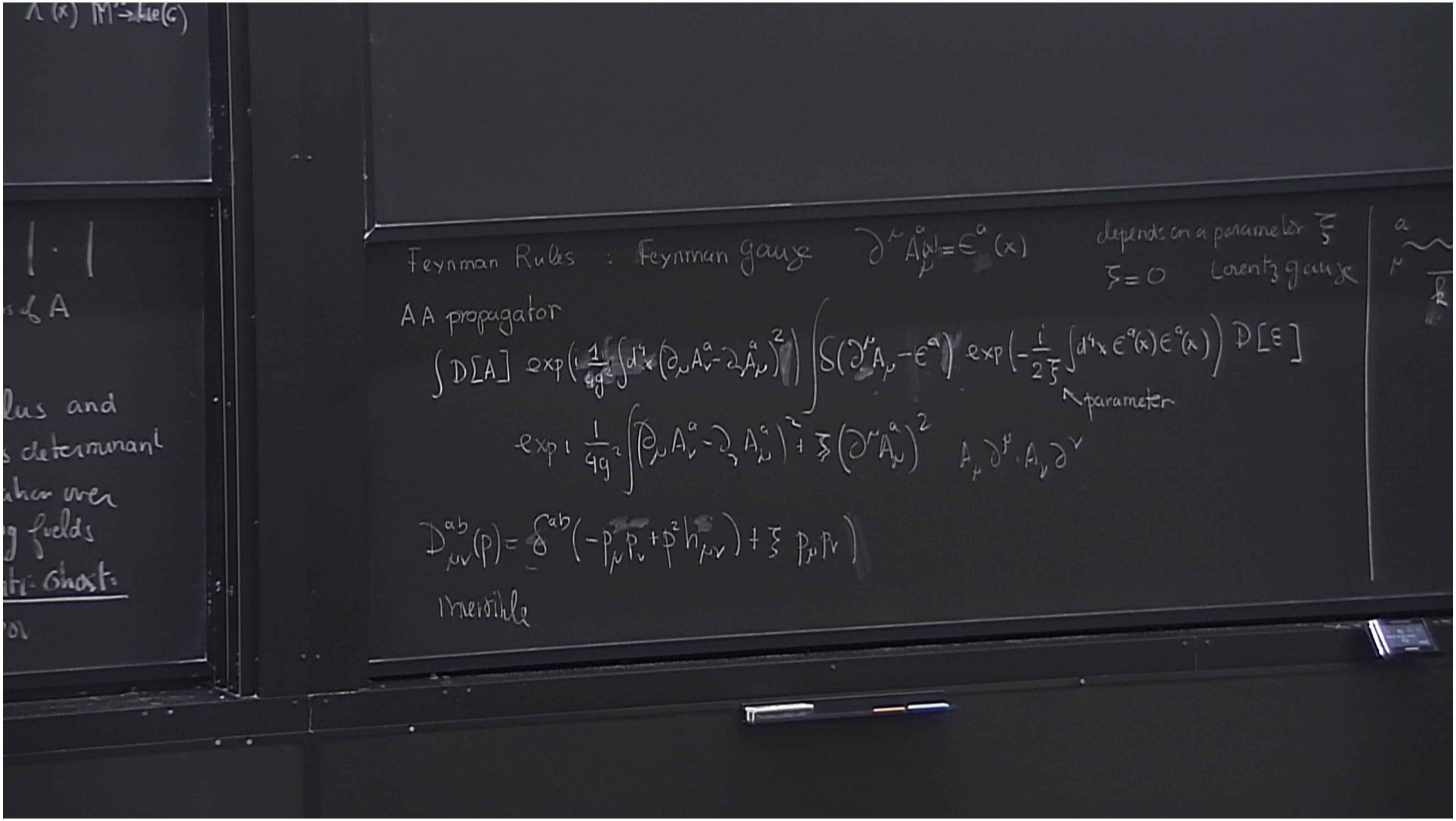
$$\int D[A]\, \exp\left(i \frac{1}{4g^2}\int d^4x \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2\right) \int \delta\left(\partial^\mu A_\mu - \epsilon^a\right) \exp\left(-\frac{i}{2\xi}\int d^4x\, \epsilon^a(x)\epsilon^a(x)\right) D[\epsilon]$$

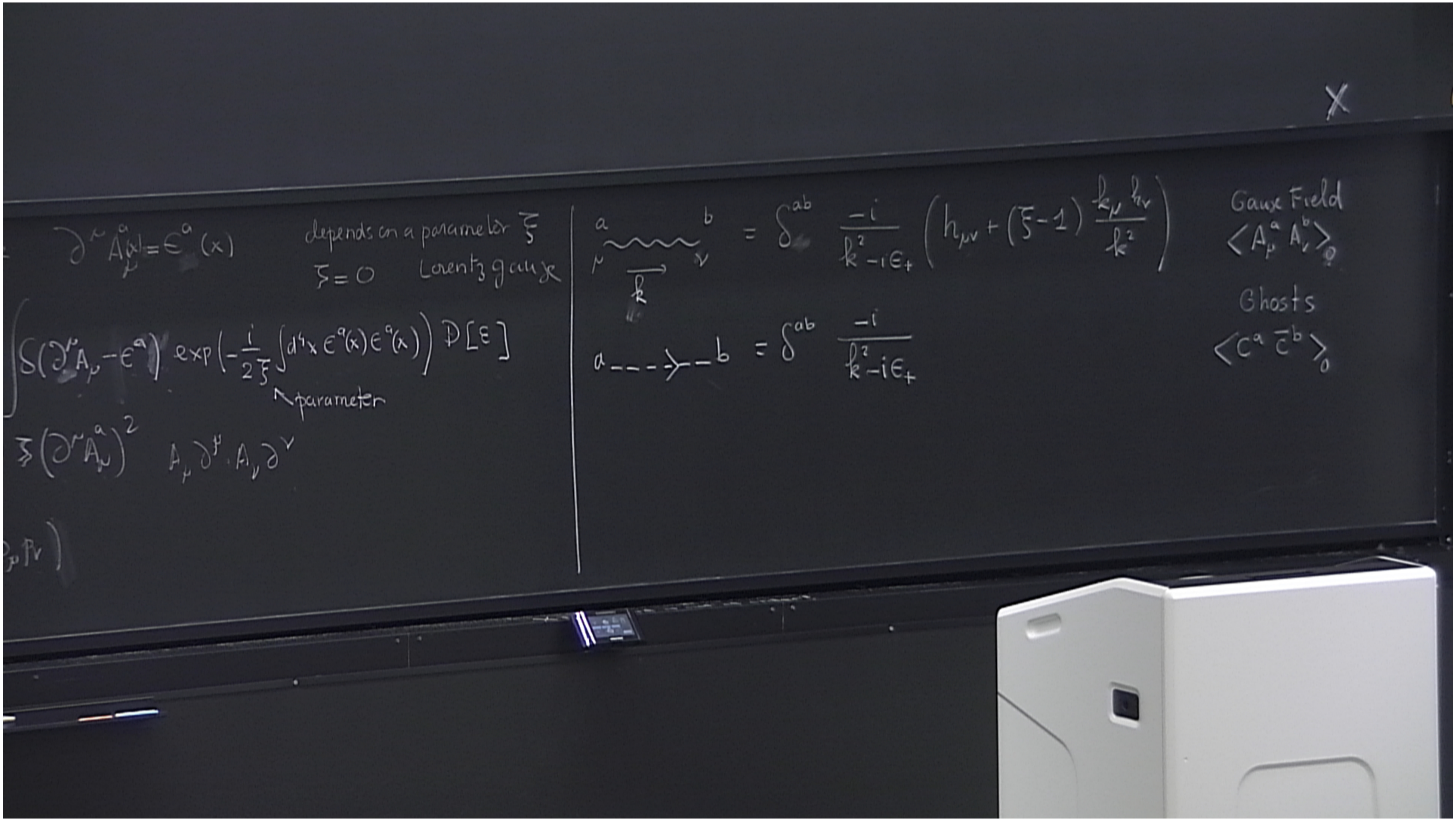
↖ parameter

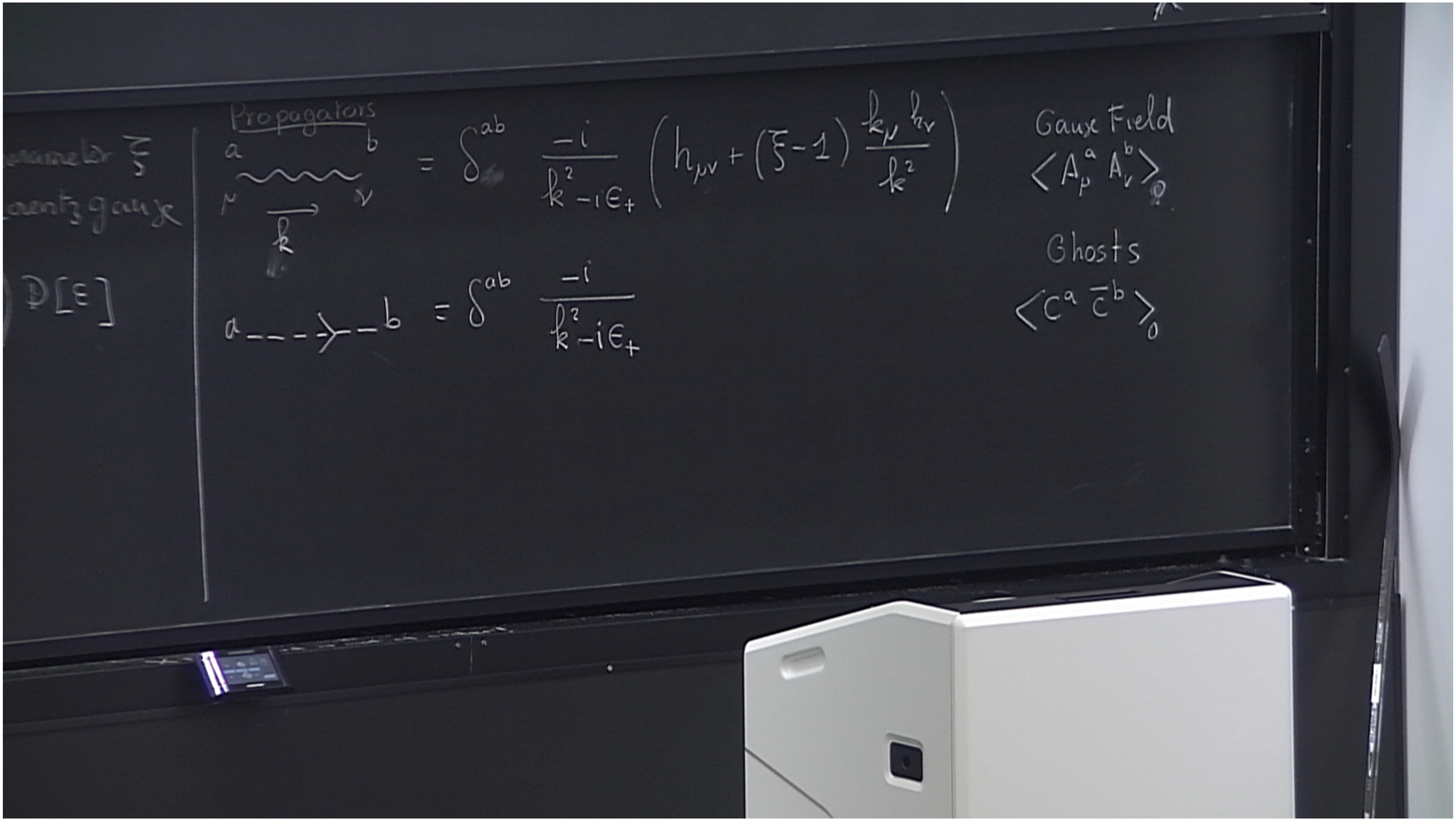
$$\exp i \frac{1}{4g^2}\int \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \xi \left(\partial^\mu A^a_\mu\right)^2$$

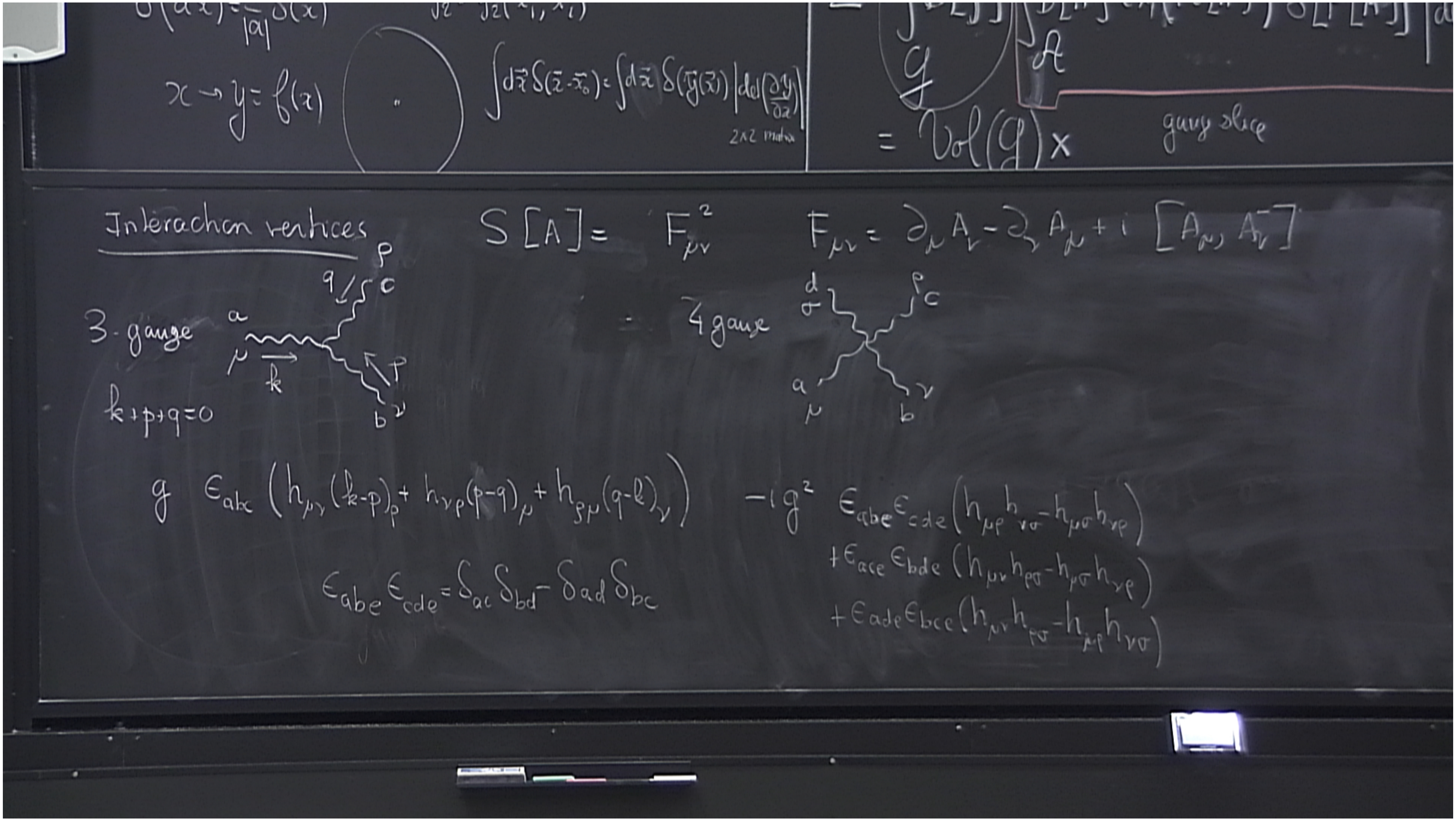
$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(-p_\mu p_\nu + p^2 h_{\mu\nu}\right) + \xi\, p_\mu p_\nu)$$

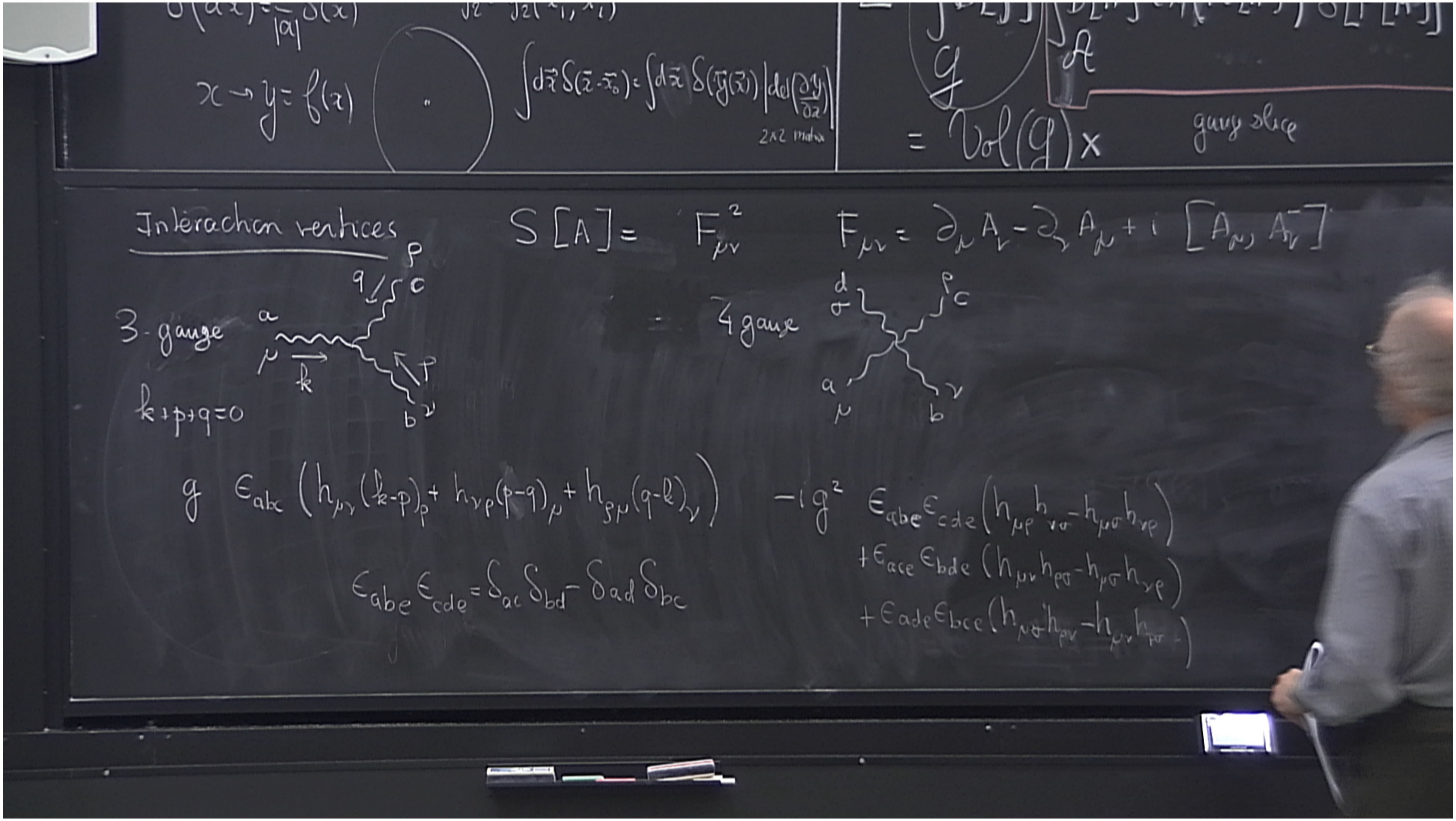
invertible

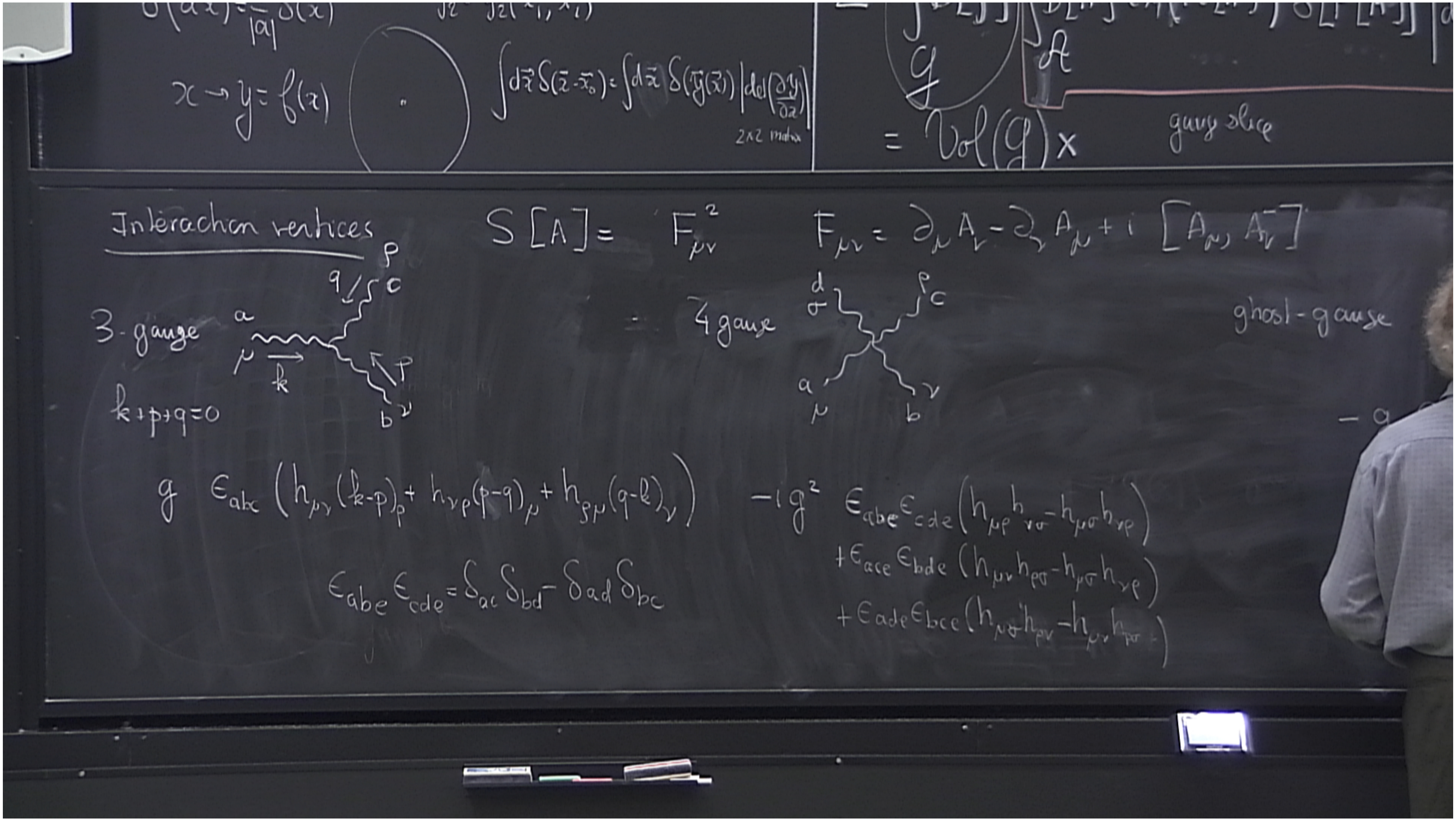

Feynman Rules : Feynman gauge $\partial^\mu A^a_\mu(x) = \epsilon^a(x)$
depends on a parameter ξ
$\xi = 0$ Lorentz gauge
AA propagator
$\int D[A] \exp\left(i \frac{1}{4g^2} \int d^4x \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2\right) \int \delta(\partial^\mu A_\mu - \epsilon^a) \exp\left(-\frac{i}{2\xi} \int d^4x\, \epsilon^a(x) \epsilon^a(x)\right) D[\epsilon]$
parameter
$\exp i \frac{1}{4g^2} \int \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \xi \left(\partial^\mu A^a_\mu\right)^2$ $A_\mu \partial^\mu A_\nu \partial^\nu$
$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(-p_\mu p_\nu + p^2 h_{\mu\nu}\right) + \xi\, p_\mu p_\nu)$
invertible

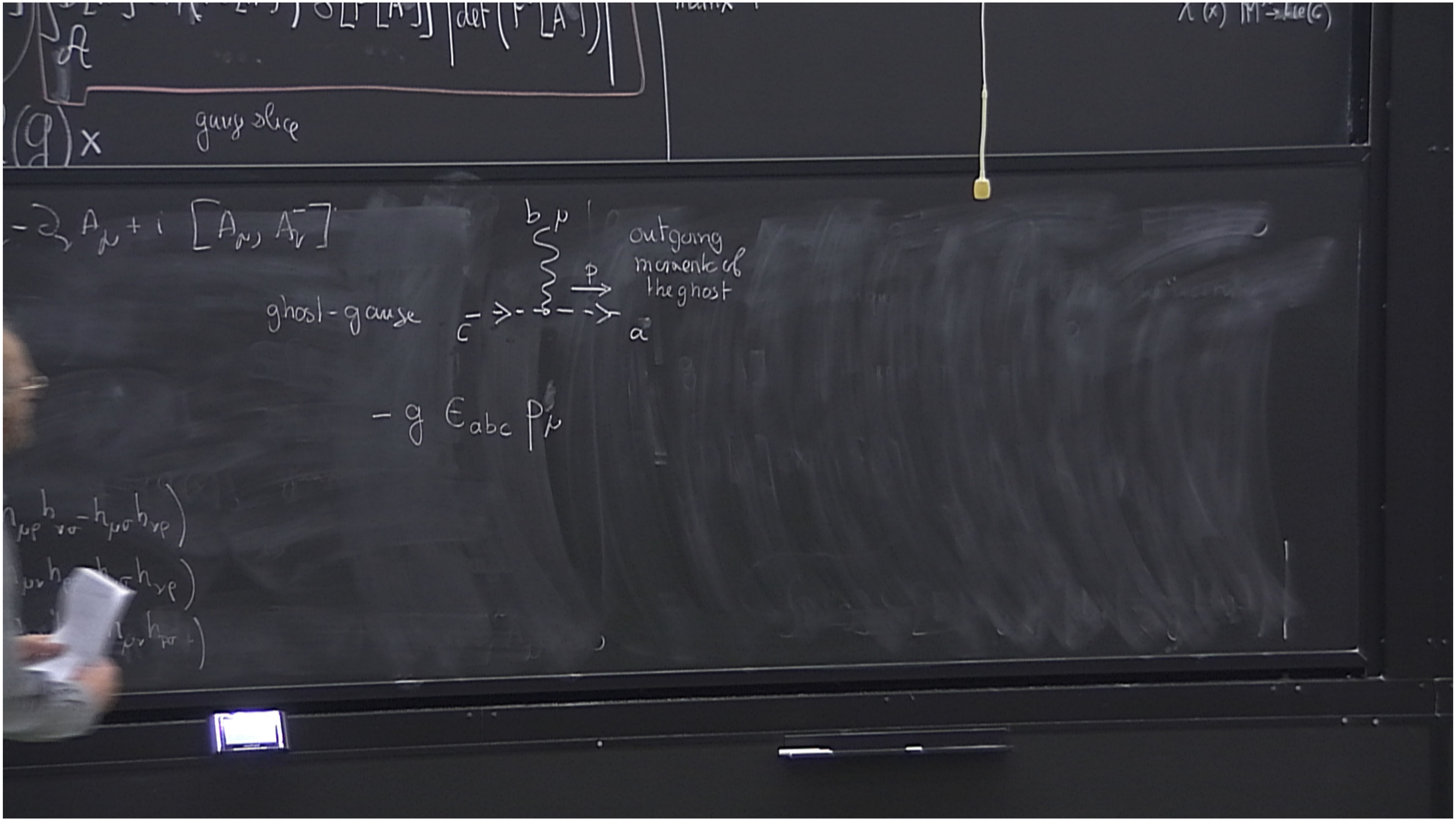

$\partial^\mu A^a_\mu(x) = \epsilon^a(x)$
depends on a parameter ξ
$\xi = 0$ Lorentz gauge
$\int \delta(\partial^\mu A_\mu - \epsilon^a) \exp\left(-\frac{i}{2\xi}\int d^4x\, \epsilon^a(x)\epsilon^a(x)\right) \mathcal{D}[\epsilon]$
parameter
$\xi(\partial^\mu A^a_\nu)^2$ $A_\mu \partial^\mu A_\nu \partial^\nu$
$= \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}\left(h_{\mu\nu} + (\xi - 1)\frac{k_\mu k_\nu}{k^2}\right)$
$= \delta^{ab} \frac{-i}{k^2 - i\epsilon_+}$
Gauge Field
$\langle A^a_\mu A^b_\nu \rangle_0$
Ghosts
$\langle c^a \bar{c}^b \rangle_0$

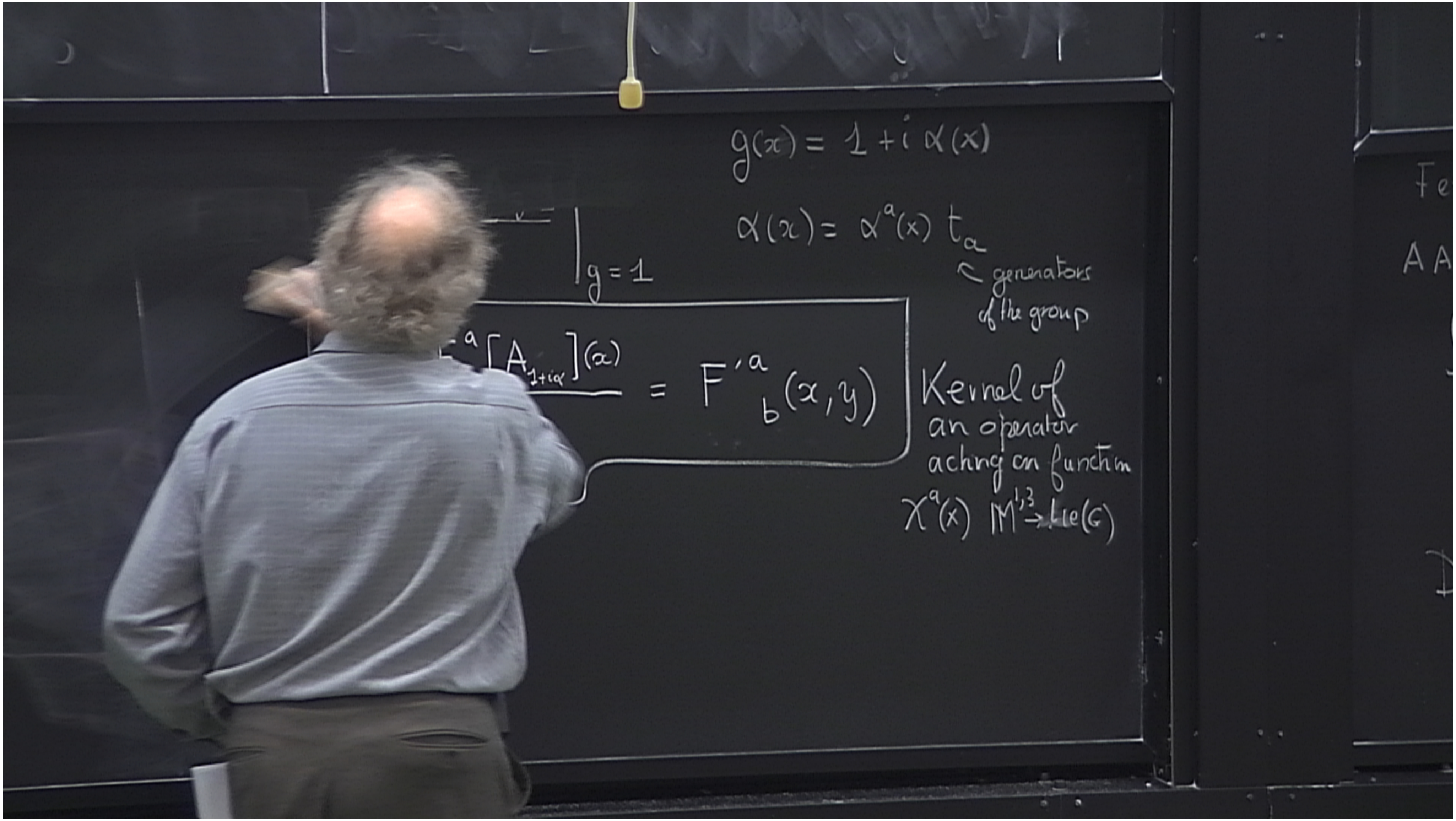

Propagators
= δ^ab −i / (k² − iε₊) (h_μν + (ξ − 1) k_μ k_ν / k²)
Gauge Field
⟨A^a_μ A^b_ν⟩₀
Ghosts
⟨c^a c̄^b⟩₀
a - - - > - - b = δ^ab −i / (k² − iε₊)
D[ε]

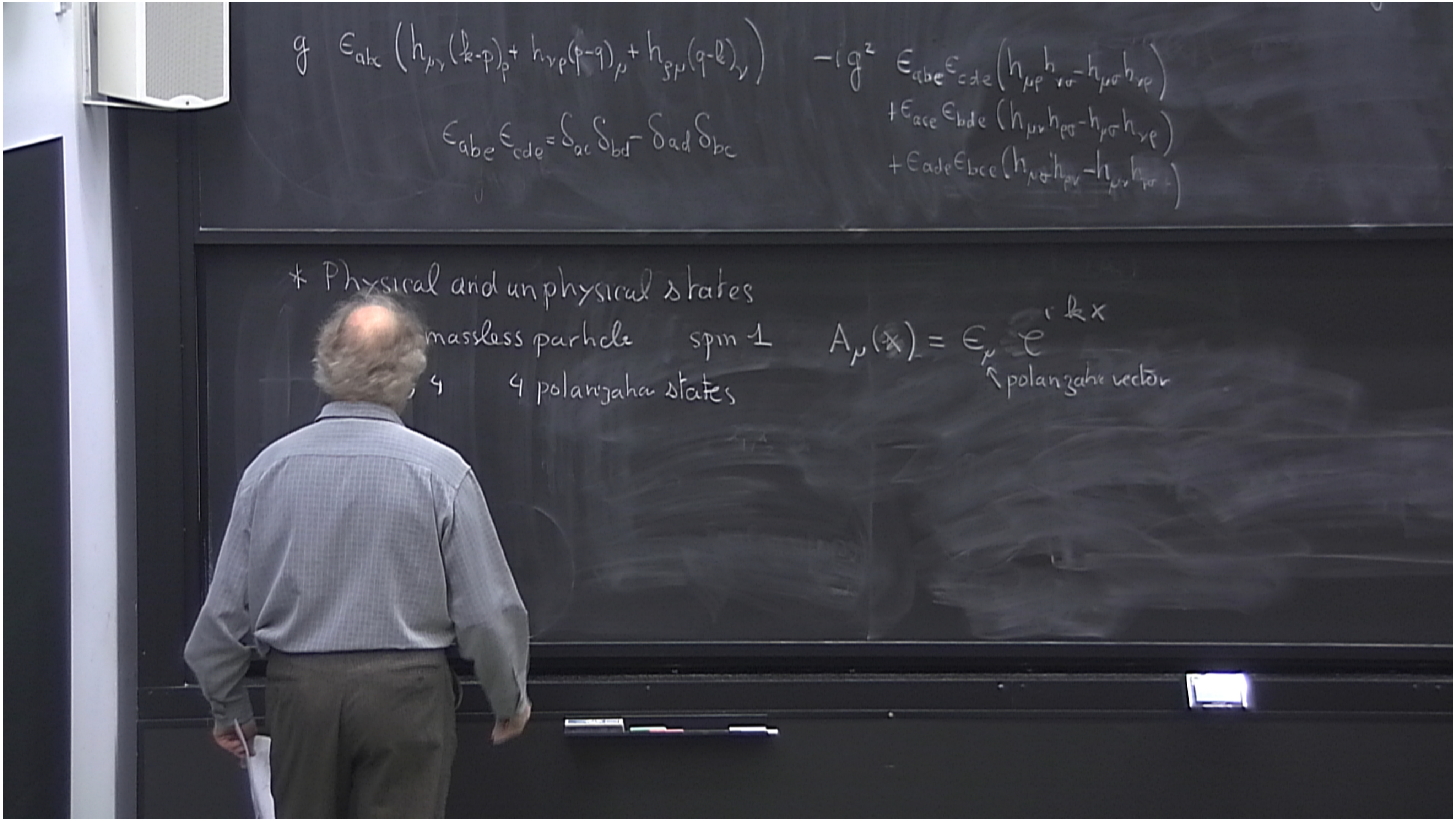

x → y = f(x)
2x2 matrix
= Vol(G) x
gauge slice
Interaction vertices
S [A] = F_{μν}^2
F_{μν} = ∂_μ A_ν − ∂_ν A_μ + i [A_μ, A_ν]
3-gauge
4 gauge
k+p+q=0
g ε_{abc} (h_{μν}(k−p)_ρ + h_{νρ}(p−q)_μ + h_{ρμ}(q−k)_ν)
ε_{abe} ε_{cde} = δ_{ac} δ_{bd} − δ_{ad} δ_{bc}
−i g^2 ε_{abe} ε_{cde} (h_{μρ} h_{νσ} − h_{μσ} h_{νρ})
+ ε_{ace} ε_{bde} (h_{μν} h_{ρσ} − h_{μσ} h_{νρ})
+ ε_{ade} ε_{bce} (h_{μν} h_{ρσ} − h_{μρ} h_{νσ})


x → y = f(x)
2x2 matrix
= Vol(G) ×
gauge slice
Interaction vertices
S [A] = F_{μν}^2
F_{μν} = ∂_μ A_ν − ∂_ν A_μ + i [A_μ, A_ν]
3-gauge
4 gauge
k+p+q=0
g ε_{abc} (h_{μν}(k−p)_ρ + h_{νρ}(p−q)_μ + h_{ρμ}(q−k)_ν)
ε_{abe} ε_{cde} = δ_{ac} δ_{bd} − δ_{ad} δ_{bc}
−i g² ε_{abe} ε_{cde} (h_{μρ} h_{νσ} − h_{μσ} h_{νρ})
+ ε_{ace} ε_{bde} (h_{μν} h_{ρσ} − h_{μσ} h_{νρ})
+ ε_{ade} ε_{bce} (h_{μσ} h_{ρν} − h_{μν} h_{ρσ})


Interaction vertices
S[A] = F_{μν}^2
F_{μν} = ∂_μ A_ν − ∂_ν A_μ + i [A_μ, A_ν]
3-gauge
k+p+q=0
4 gauge
ghost-gauge
g ε_abc (h_μν (k-p)_ρ + h_νρ (p-q)_μ + h_ρμ (q-k)_ν)
−i g^2 ε_abe ε_cde (h_μρ h_νσ − h_μσ h_νρ)
+ ε_ace ε_bde (h_μν h_ρσ − h_μσ h_νρ)
+ ε_ade ε_bce (h_μσ h_ρν − h_μν h_ρσ)
ε_abe ε_cde = δ_ac δ_bd − δ_ad δ_bc
x → y = f(x)
2x2 matrix
= Vol(G) x
gauge slice


gauge slice
ghost - gauge
outgoing momenta of the ghost
b μ
p
c
a
− g ε abc p μ


g(x) = 1 + i α(x)
α(x) = α^a(x) t_a
generators of the group
|g=1
= F'^a_b(x,y)
Kernel of an operator acting on function
χ^a(x) M^{1,3} → Lie(G)


$g\,\epsilon_{abc}\left(h_{\mu\nu}(k-p)_\rho + h_{\nu\rho}(p-q)_\mu + h_{\rho\mu}(q-\ell)_\nu\right)$
$-ig^2\,\epsilon_{abe}\epsilon_{cde}\left(h_{\mu\rho}h_{\nu\sigma} - h_{\mu\sigma}h_{\nu\rho}\right)$
$+\epsilon_{ace}\epsilon_{bde}\left(h_{\mu\nu}h_{\rho\sigma} - h_{\mu\sigma}h_{\nu\rho}\right)$
$+\epsilon_{ade}\epsilon_{bce}\left(h_{\mu\sigma}h_{\rho\nu} - h_{\mu\nu}h_{\rho\sigma}\right)$
$\epsilon_{abe}\epsilon_{cde} = \delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}$
* Physical and unphysical states
massless particle spin 1
$A_\mu(x) = \epsilon_\mu e^{ikx}$
polarization vector
4 polarization states

$$g \, \epsilon_{abc} \left(h_{\mu\nu} (k-p)_\rho + h_{\nu\rho} (p-q)_\mu + h_{\rho\mu} (q-k)_\nu \right)$$

$$-ig^2 \, \epsilon_{abe} \epsilon_{cde} \left(h_{\mu\rho} h_{\nu\sigma} - h_{\mu\sigma} h_{\nu\rho} \right)$$
$$+ \epsilon_{ace} \epsilon_{bde} \left(h_{\mu\nu} h_{\rho\sigma} - h_{\mu\sigma} h_{\nu\rho} \right)$$
$$+ \epsilon_{ade} \epsilon_{bce} \left(h_{\mu\sigma} h_{\rho\nu} - h_{\mu\nu} h_{\rho\sigma} \right)$$

$$\epsilon_{abe} \epsilon_{cde} = \delta_{ac} \delta_{bd} - \delta_{ad} \delta_{bc}$$

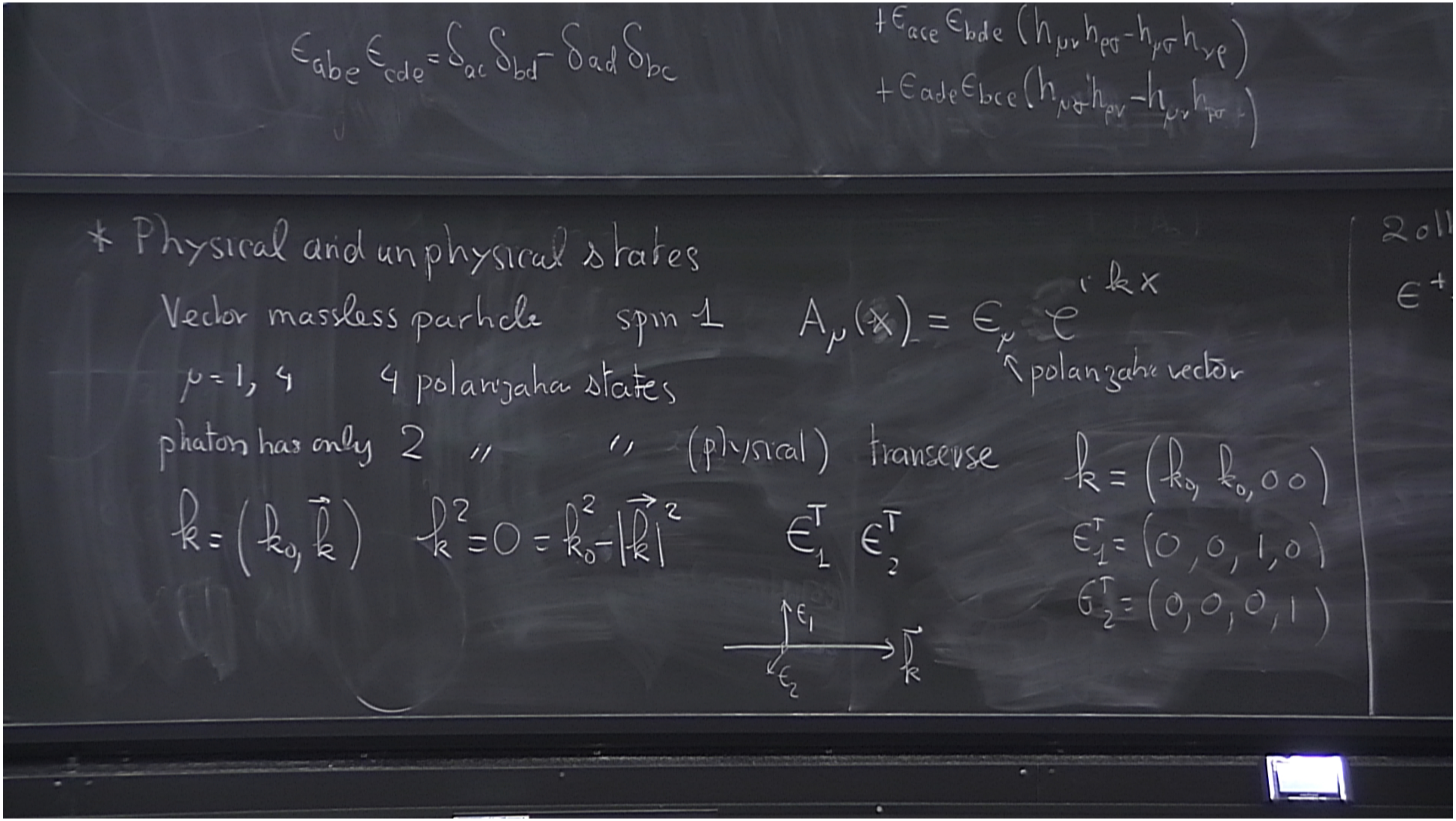
* Physical and unphysical states

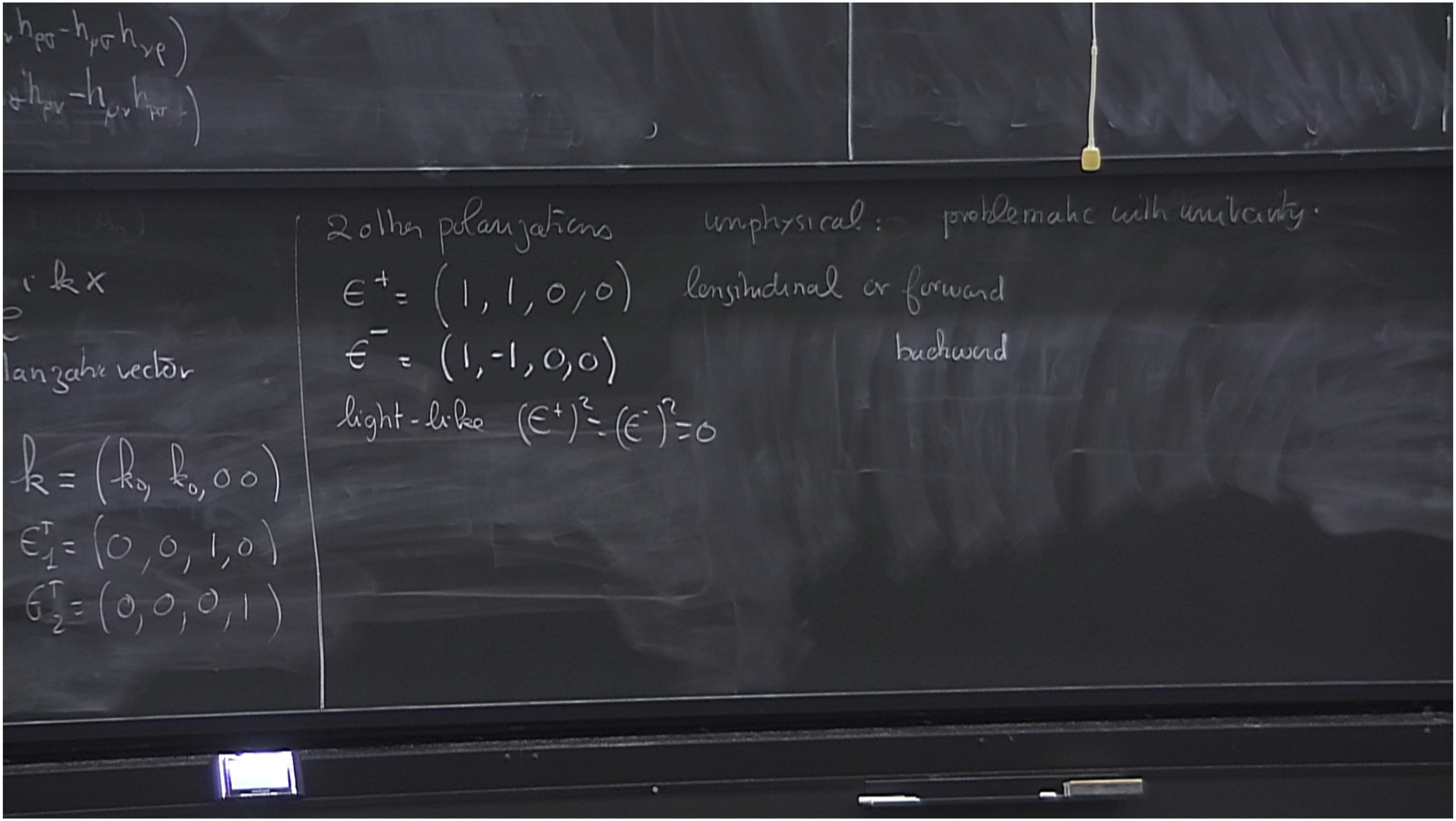
Vector massless particle spin 1 $\quad A_\mu(x) = \epsilon_\mu \, e^{ikx}$

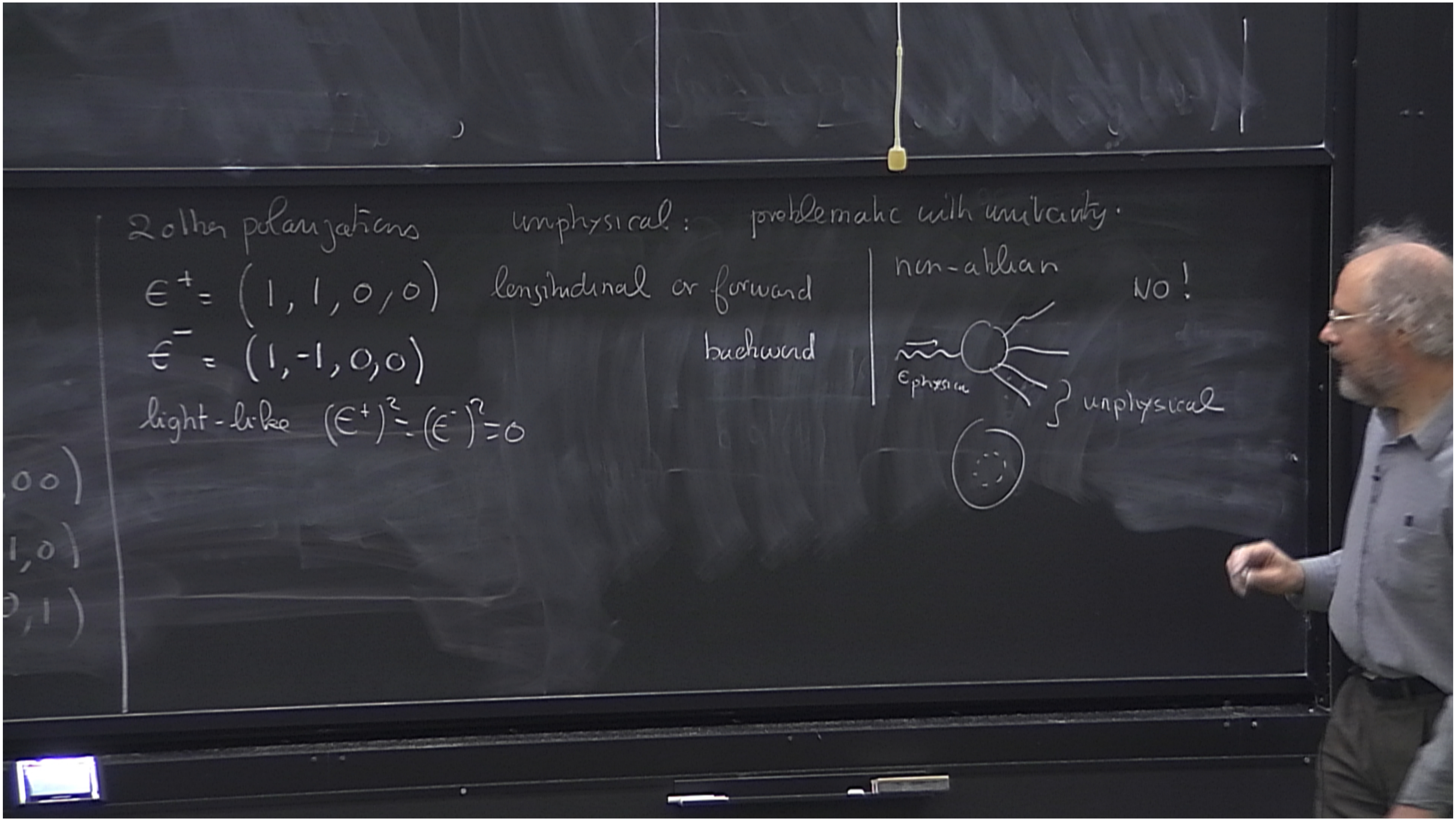
$\uparrow$ polarization vector

$\mu = 1, 4$ 4 polarization states

photon has only 2 " " (physical)


ε_abe ε_cde = δ_ac δ_bd − δ_ad δ_bc
+ε_ace ε_bde (h_μν h_ρσ − h_μσ h_νρ)
+ε_ade ε_bce (h_μσ h_ρν − h_μν h_ρσ)
* Physical and unphysical states
Vector massless particle spin 1
A_μ(x) = ε_μ e^{ikx}
polarization vector
μ = 1, 4 4 polarization states
photon has only 2 " " (physical) transverse
k = (k_0, k⃗) k² = 0 = k_0² − |k⃗|²
ε_1^T ε_2^T
k = (k_0, k_0, 0 0)
ε_1^T = (0, 0, 1, 0)
ε_2^T = (0, 0, 0, 1)
ε_1
ε_2
k⃗


2 other polarizations
unphysical: problematic with unitarity.
ε+ = (1, 1, 0, 0) longitudinal or forward
ε− = (1, −1, 0, 0) backward
light-like (ε+)² = (ε−)² = 0
k = (k0, k0, 0 0)
ε1T = (0, 0, 1, 0)
ε2T = (0, 0, 0, 1)


2 other polarizations
unphysical : problematic with unitarity.
ε+ = (1, 1, 0, 0) longitudinal or forward
ε− = (1, -1, 0, 0) backward
light-like (ε+)² = (ε−)² = 0
non-abelian
NO!
εphysical
unphysical