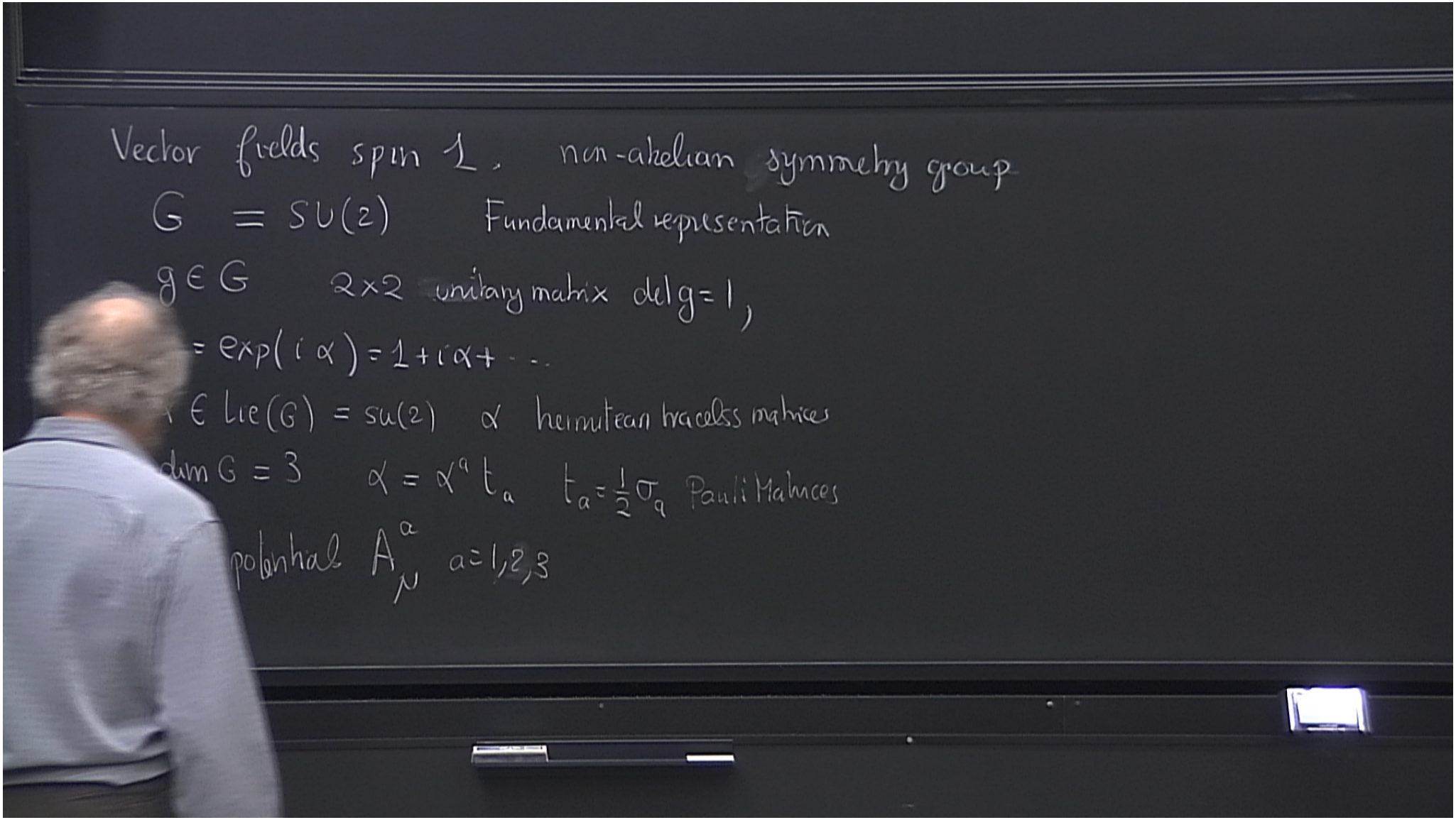
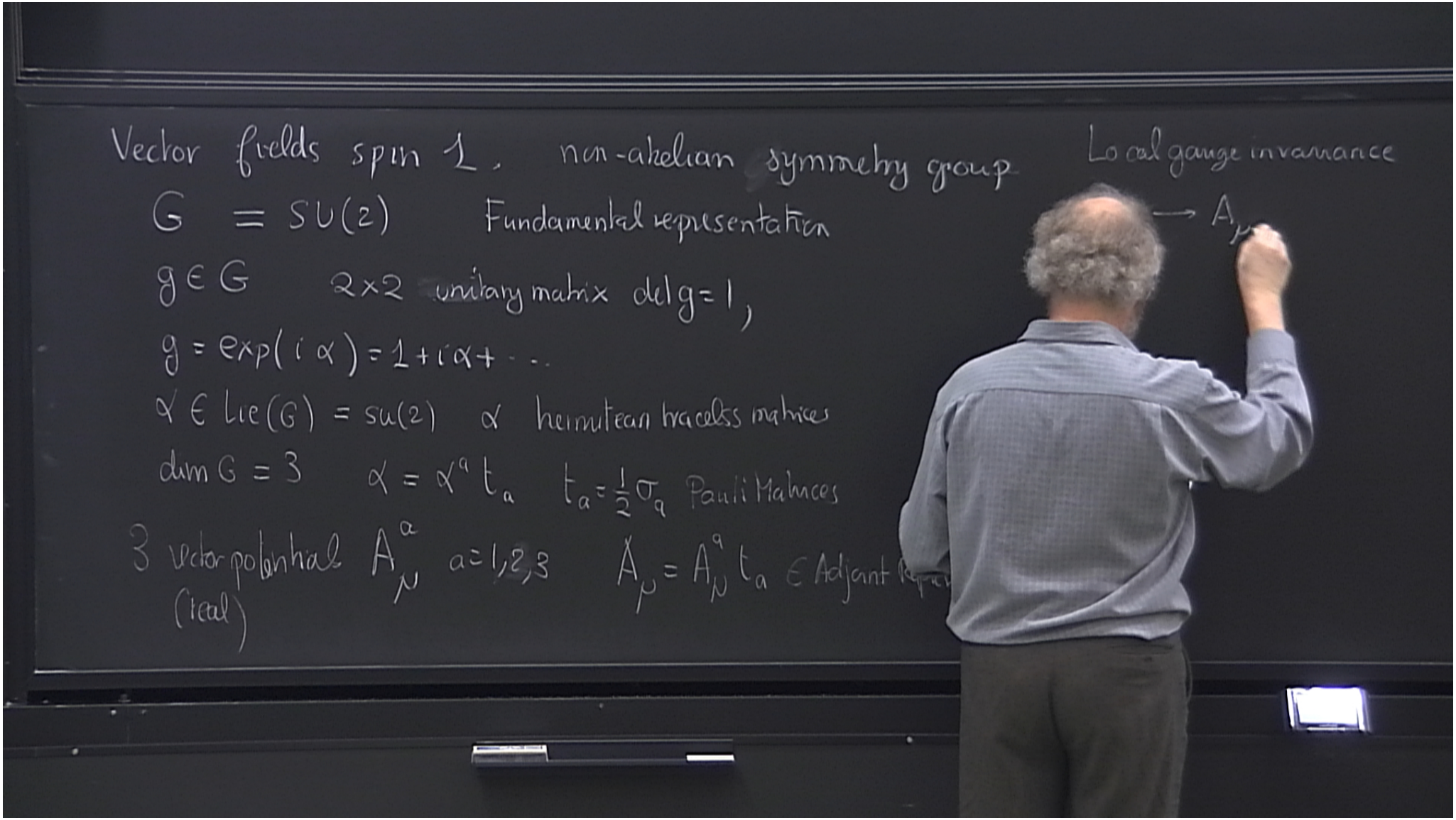
Title: Quantum Field Theory II -12

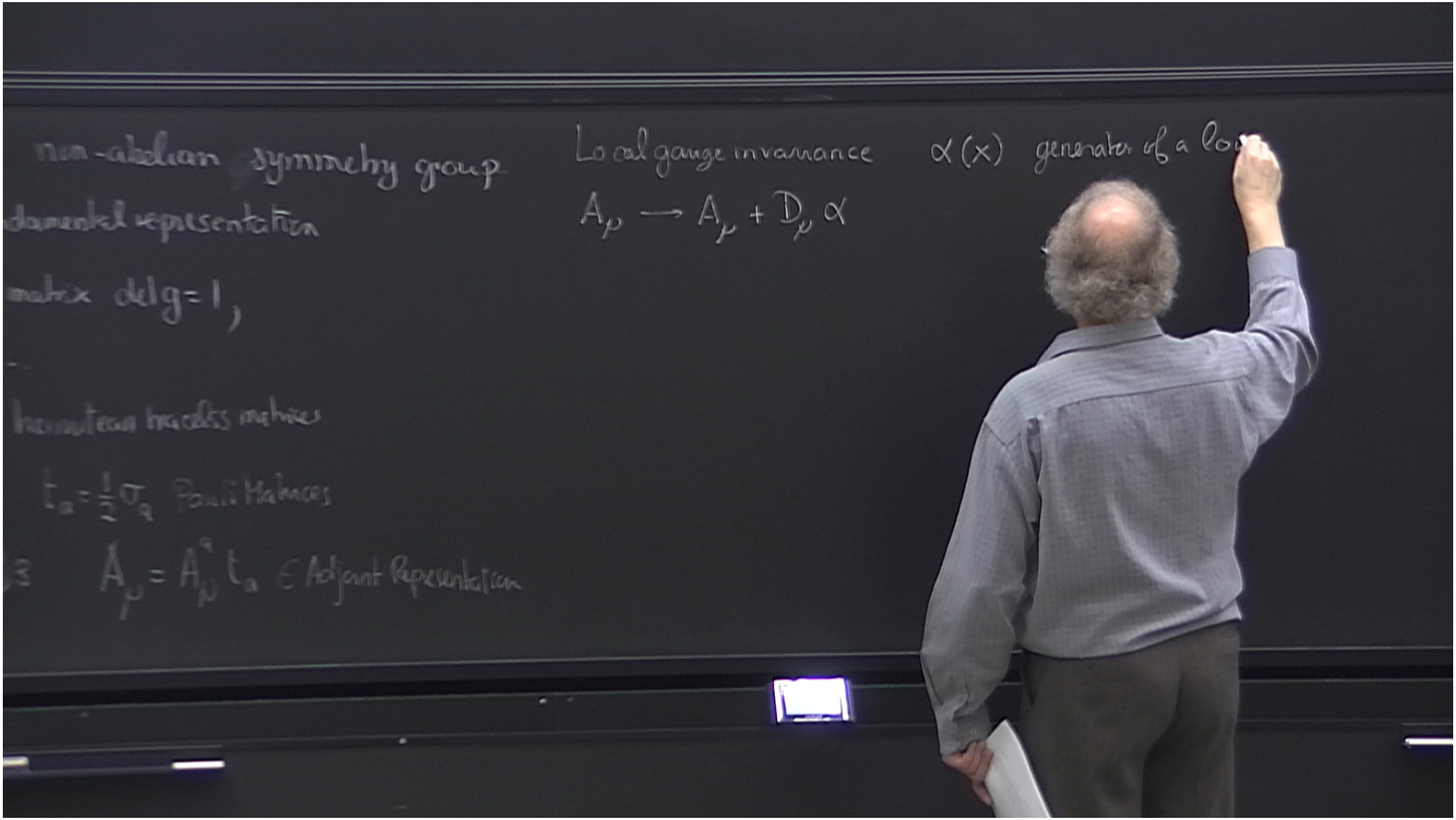
Date: Nov 25, 2014 09:00 AM

URL: http://pirsa.org/14110021

Abstract:


Vector fields spin 1. non-abelian symmetry group
G = SU(2) Fundamental representation
g ∈ G 2×2 unitary matrix det g = 1,
= exp(iα) = 1 + iα + ...
∈ Lie(G) = su(2) α hermitean traceless matrices
dim G = 3 α = α^a t_a t_a = ½ σ_a Pauli Matrices
potential A^a_μ a = 1,2,3


Vector fields spin 1. non-abelian symmetry group
Local gauge invariance
G = SU(2) Fundamental representation
g ∈ G 2×2 unitary matrix det g = 1,
g = exp(iα) = 1 + iα + ...
α ∈ Lie(G) = su(2) α hermitean traceless matrices
dim G = 3 α = α^a t_a t_a = ½ σ_a Pauli Matrices
3 vector potential A^a_μ a = 1,2,3 (real)
A_μ = A^a_μ t_a ∈ Adjant


non-abelian symmetry group
damental representation
matrix detg=1,
Hermitean traceless matrices
$t_a = \frac{1}{2}\sigma_a$ Pauli Matrices
$A_\mu = A_\mu^a t_a \in$ Adjoint Representation
Local gauge invariance $\alpha(x)$ generator of a
$A_\mu \rightarrow A_\mu + D_\mu \alpha$

Local gauge invariance $\alpha(x)$ generator of a local gauge transformation

$A_\mu \longrightarrow A_\mu + D_\mu \alpha$ $\quad D_\mu \alpha = \partial_\mu \alpha - i\,[A_\mu, \alpha]$

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Field-strength Tensor

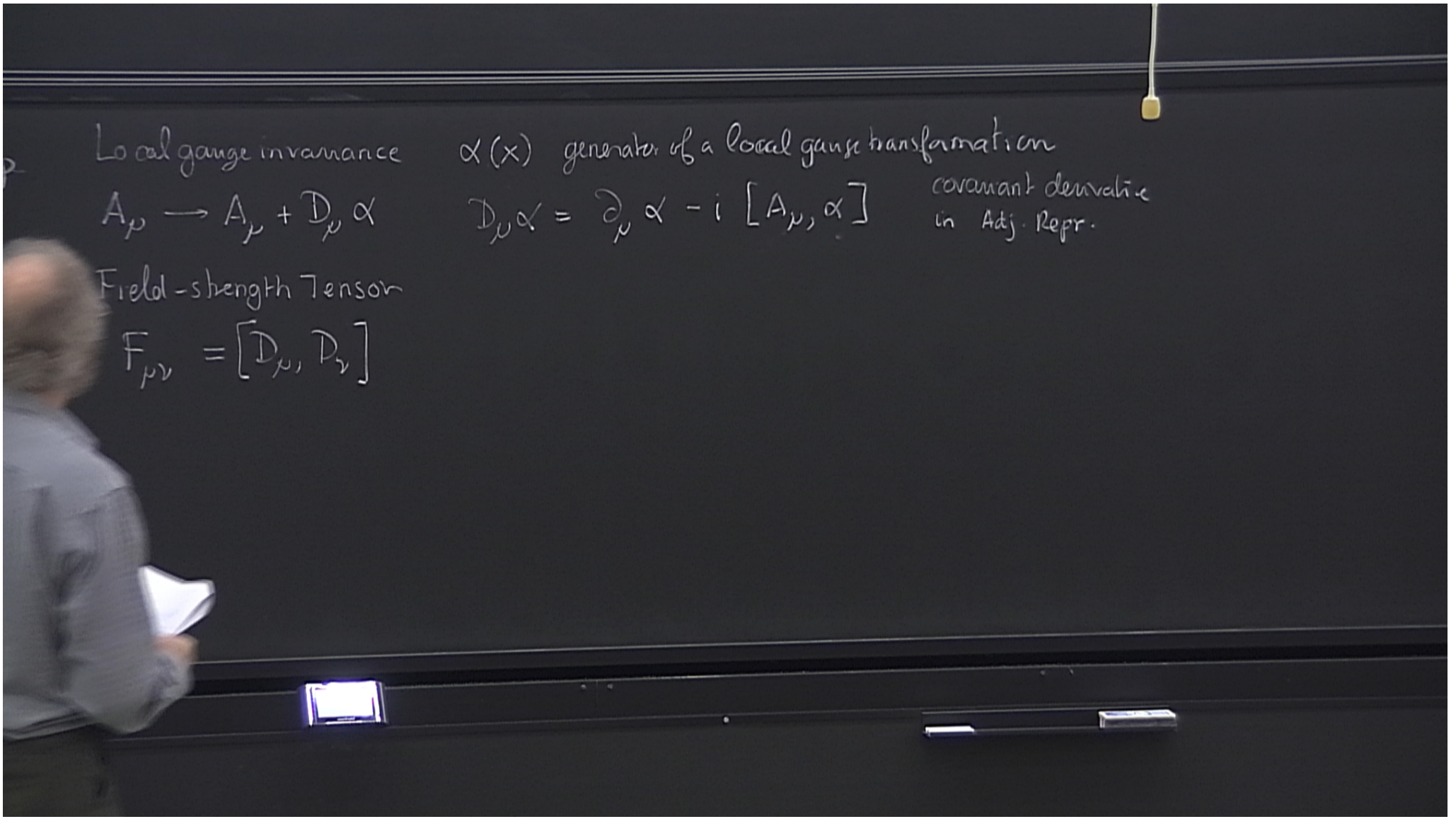
$F_{\mu\nu}$

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Field-strength Tensor

$F_{\mu\nu} = [D_\mu, D_\nu]$



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Field-strength Tensor

$$F_{\mu\nu} = [D_\mu, D_\nu] = F^a_{\mu\nu}\, t_a$$

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Field-strength Tensor

$$F_{\mu\nu} = [D_\mu, D_\nu] = F^a_{\mu\nu}\, t_a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu$$

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$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \varepsilon^a{}_{bc} A^b_\mu A^c_\nu$$

↑ structure constant of SU(2)

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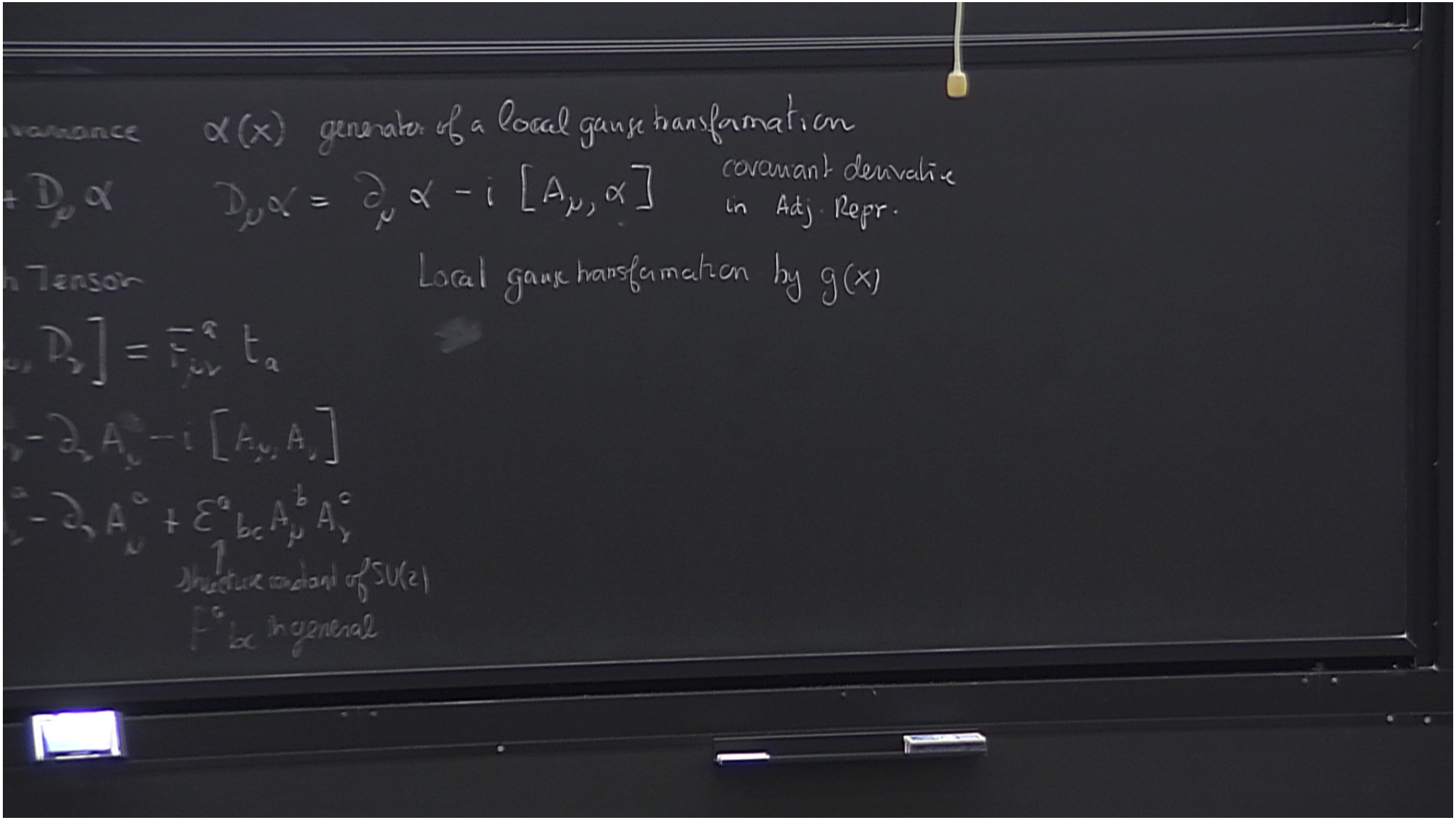
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$f^a{}_{bc}$ in general


variance
$\alpha(x)$ generator of a local gauge transformation
$+ D_\rho \alpha$
$D_\mu \alpha = \partial_\mu \alpha - i [A_\mu, \alpha]$
covariant derivative in Adj. Repr.
n Tensor
Local gauge transformation by $g(x)$
$, D_\nu] = F^a_{\mu\nu} t_a$
$- \partial_\nu A^a_\mu - i [A_\mu, A_\nu]$
$- \partial_\nu A^a_\mu + \varepsilon^a{}_{bc} A^b_\mu A^c_\nu$
Structure constant of SU(2)
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$- \partial_\nu A_\mu - i [A_\mu, A_\nu]$ $F^a_{\mu\nu} \rightarrow (\vec{E}^a, \vec{B}^a)$

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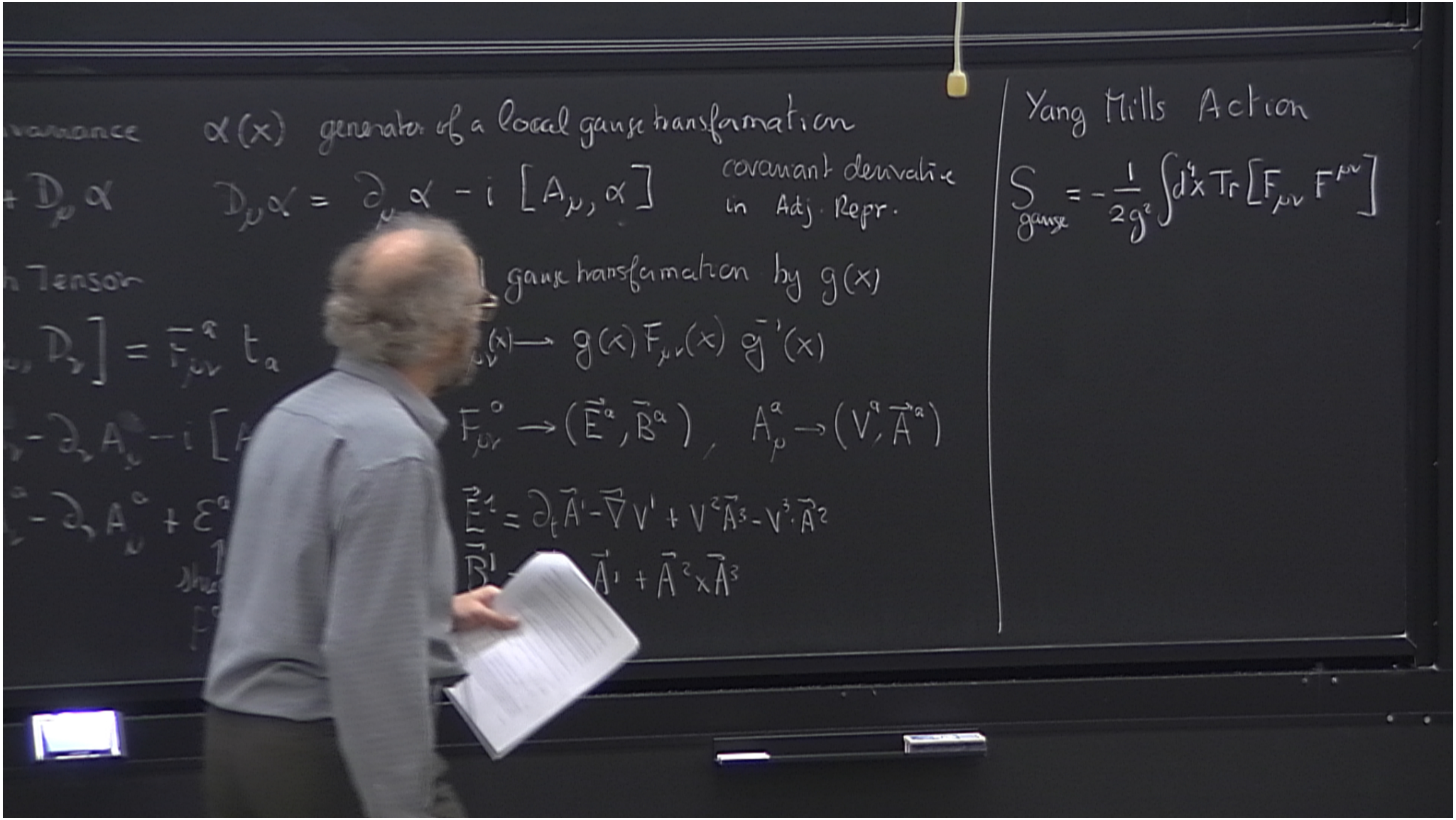
$\partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu]$ $F^0_{\mu\nu} \to (\vec{E}^a, \vec{B}^a)$, $A^a_\mu \to (V^a, \vec{A}^a)$

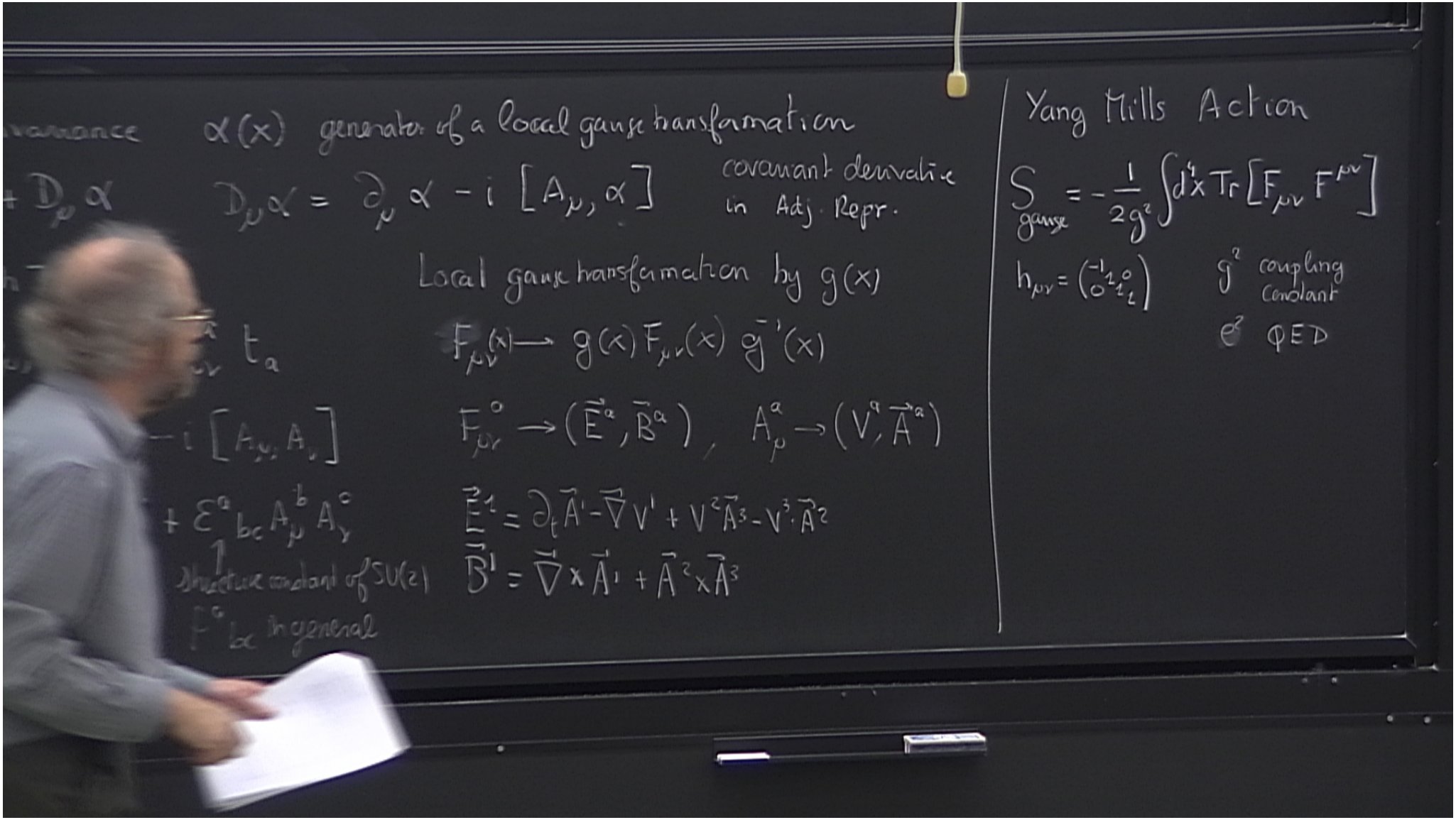
$\partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \varepsilon^a{}_{bc} A^b_\mu A^c_\nu$ $\vec{E}^1 = \partial_t \vec{A}^1 - \vec{\nabla} V^1 + V^2 \vec{A}^3 - V^3 \vec{A}^2$

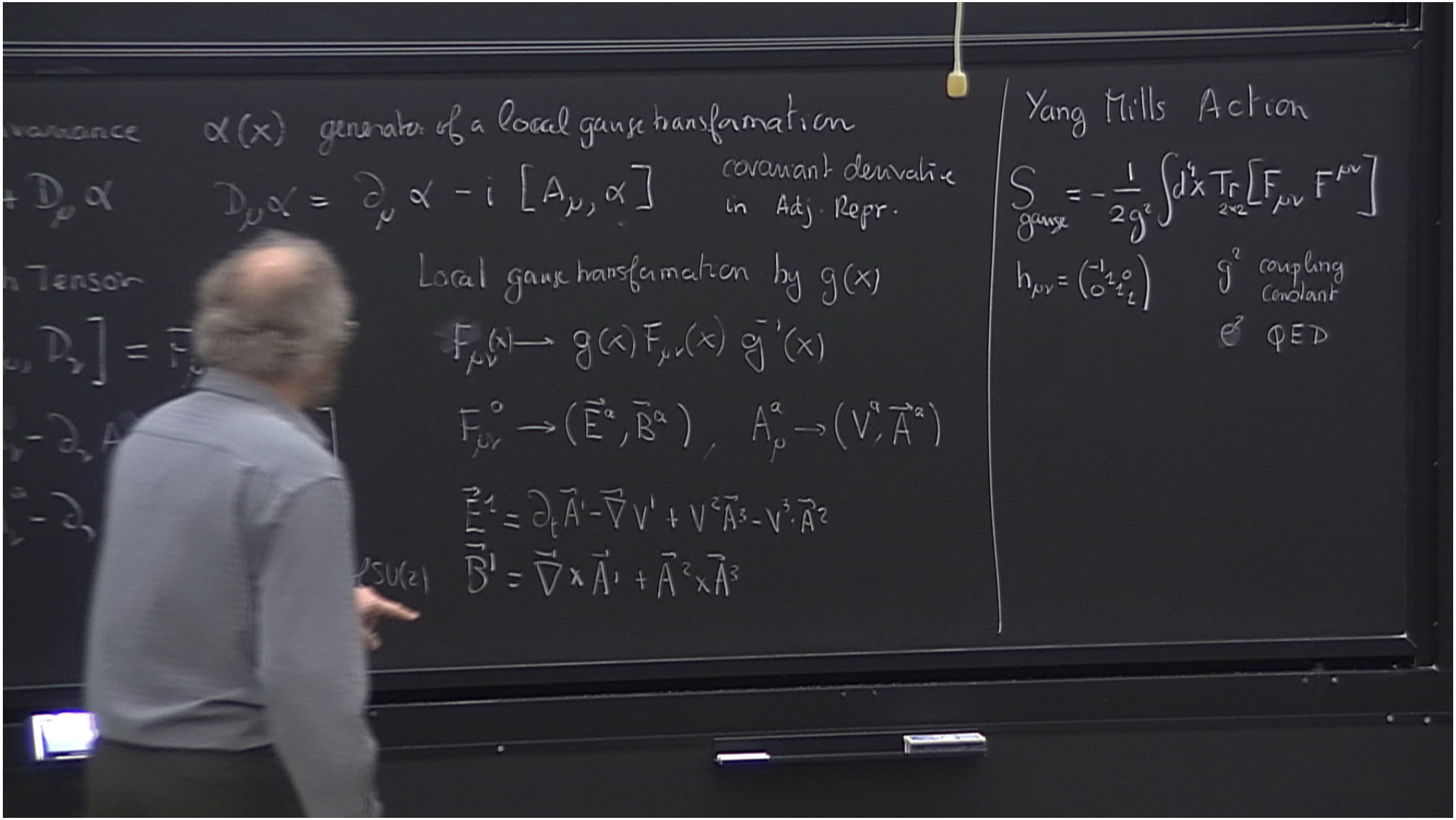
$\vec{B}^1 = \vec{\nabla} \times \vec{A}^1 + \vec{A}^2 \times \vec{A}^3$

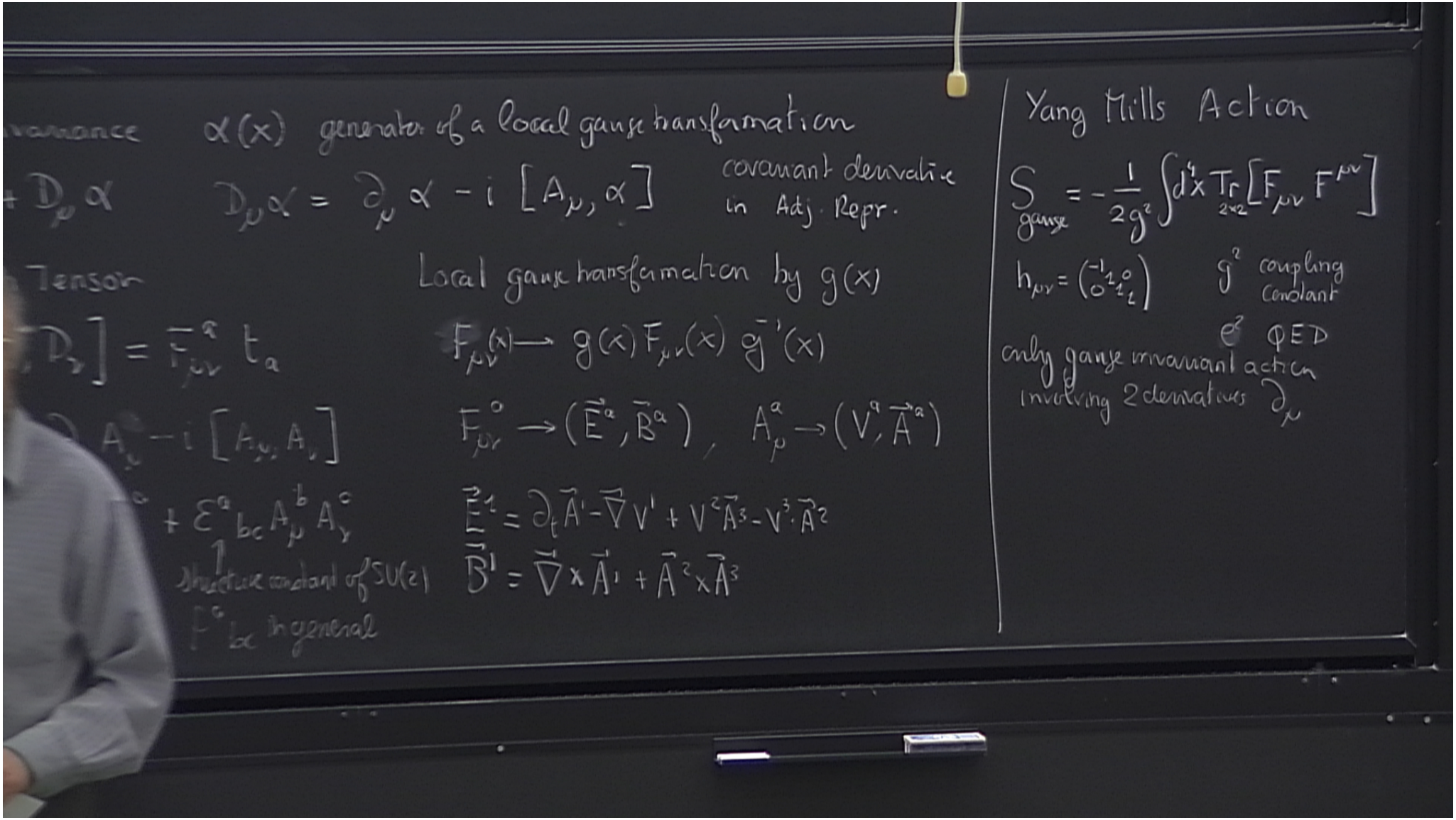
1 Structure constant of SU(2)

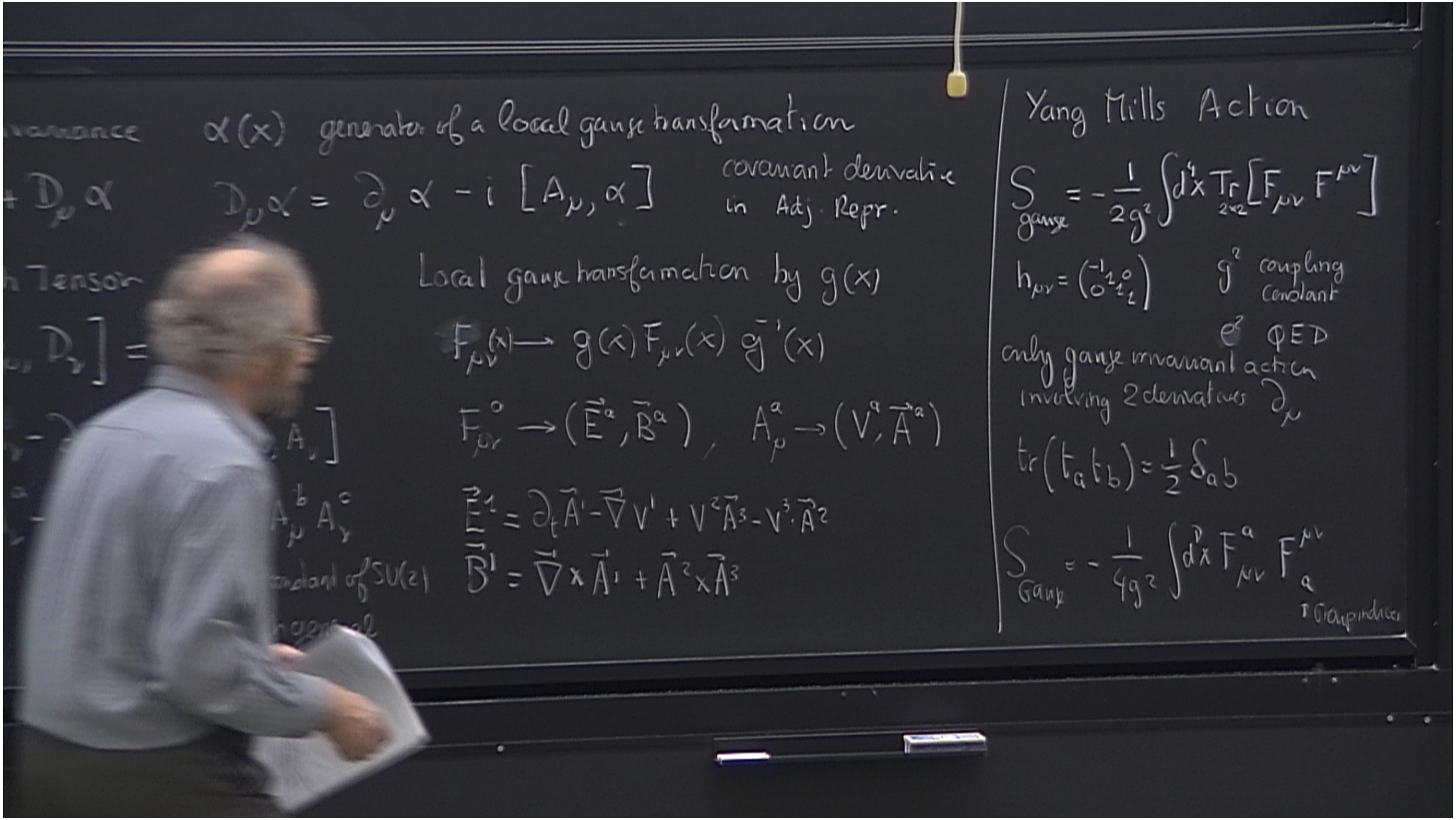
f^a_{bc} in general

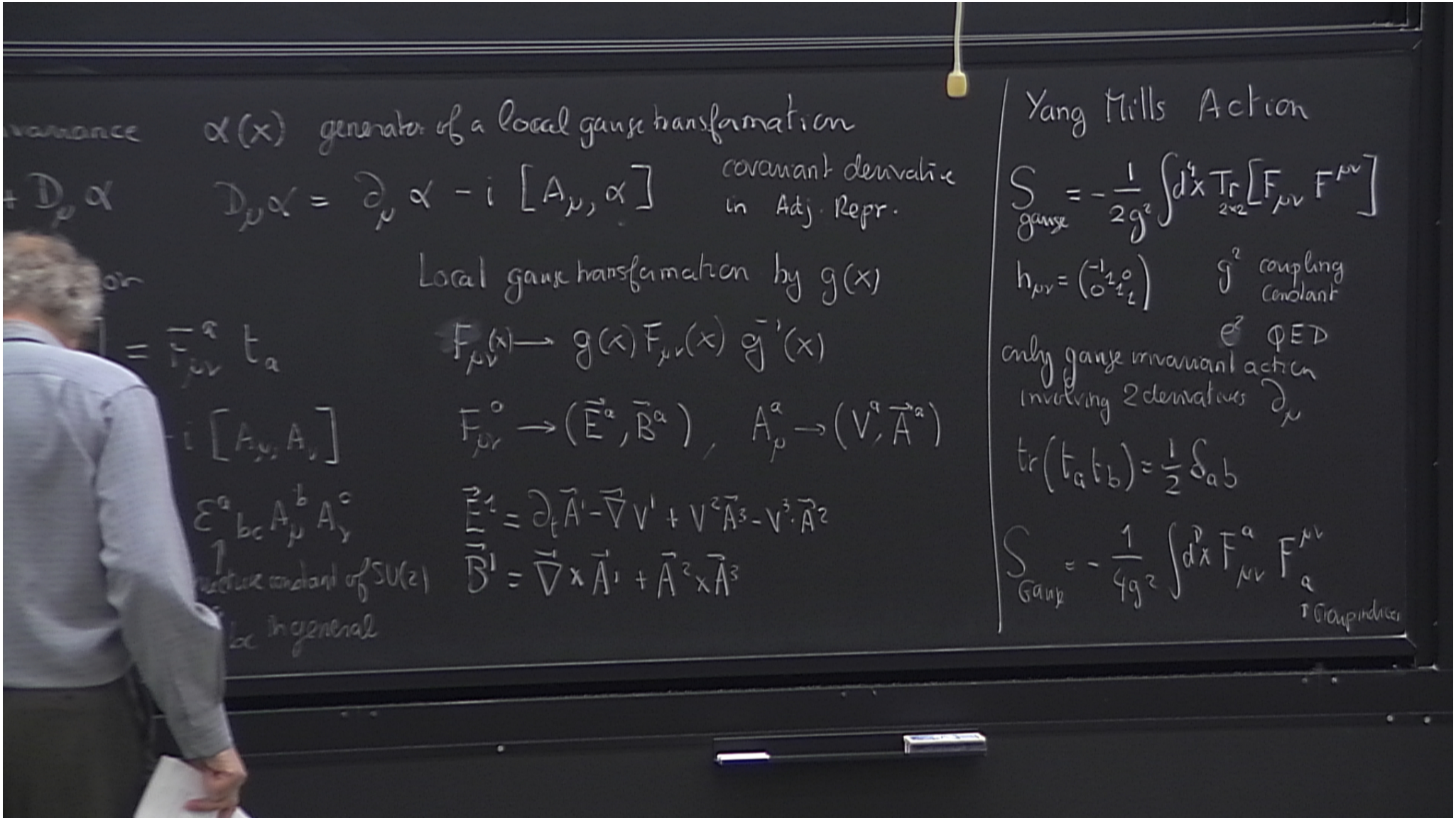

Yang Mills Action
$S_{gauge} = -\frac{1}{2g^2}\int d^4x\, Tr[F_{\mu\nu} F^{\mu\nu}]$
$\alpha(x)$ generator of a local gauge transformation
$D_\mu \alpha = \partial_\mu \alpha - i\,[A_\mu, \alpha]$
covariant derivative in Adj. Repr.
gauge transformation by $g(x)$
$F_{\mu\nu}(x) \longrightarrow g(x) F_{\mu\nu}(x)\, g^{-1}(x)$
$F^a_{\mu\nu} \to (\vec{E}^a, \vec{B}^a)$, $A^a_\mu \to (V^a, \vec{A}^a)$
$\vec{E}^1 = \partial_t \vec{A}^1 - \vec{\nabla} V^1 + V^2 \vec{A}^3 - V^3 \vec{A}^2$
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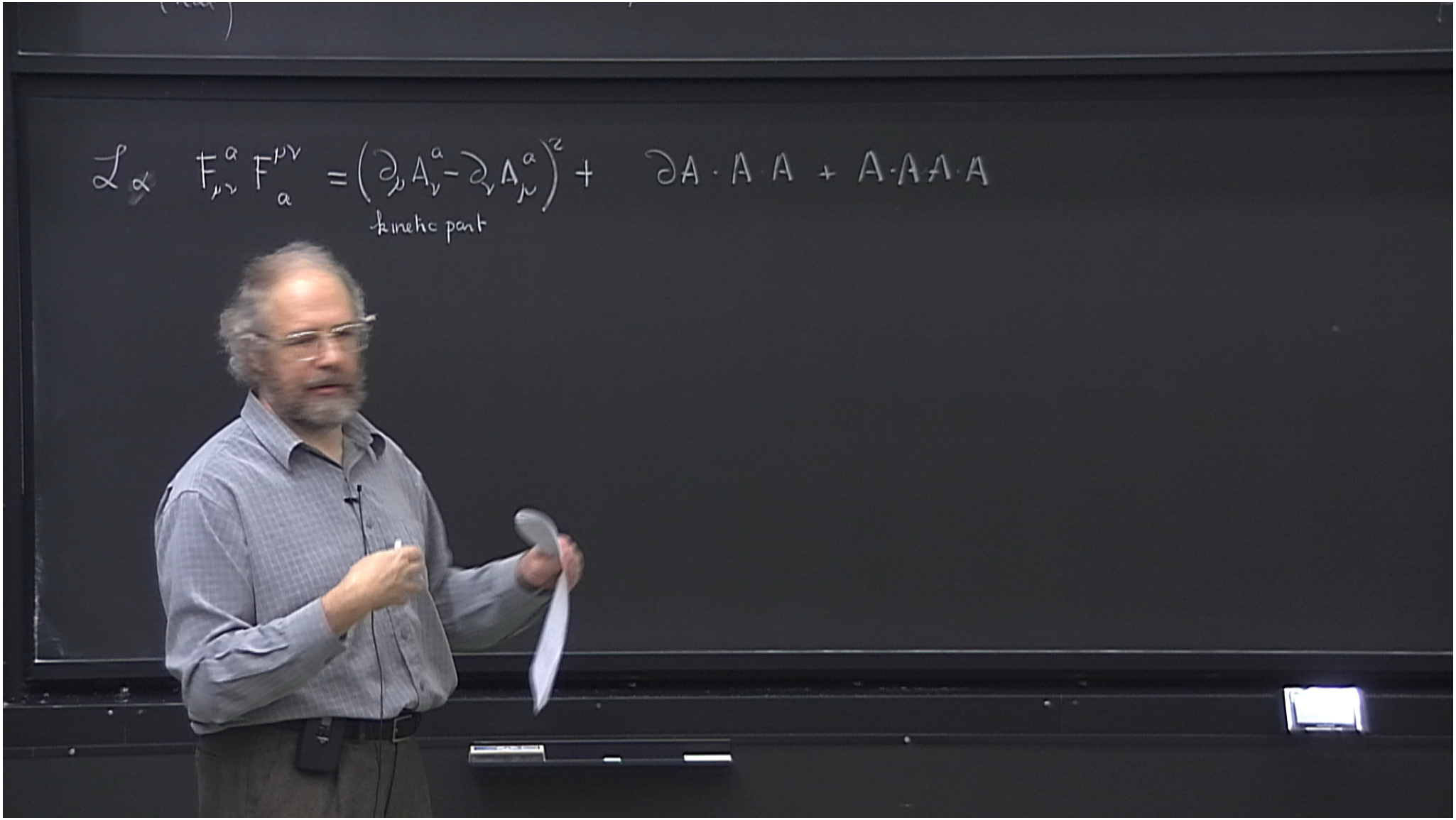

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$S_{gauge} = -\frac{1}{2g^2} \int d^4x \, Tr[F_{\mu\nu} F^{\mu\nu}]$
$h_{\mu\nu} = (-1, 1, 1, 1)$
g^2 coupling constant
e^2 QED


α(x) generator of a local gauge transformation
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Yang Mills Action
$S_{gauge} = -\frac{1}{2g^2} \int d^4x \, Tr_{2\times 2} [F_{\mu\nu} F^{\mu\nu}]$
$h_{\mu\nu} = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$
g^2 coupling constant
e^2 QED


variance
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only gauge invariant action involving 2 derivatives ∂_μ


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a Group index


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Group index


$\mathcal{L} \propto F^a_{\mu\nu} F^{\mu\nu}_a = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \partial A \cdot A\, A + A \cdot A A \cdot A$
kinetic part

$$\mathcal{L} \propto -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu}_a = \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \partial A \cdot A\, A + A \cdot A\, A \cdot A$$

kinetic part as in Maxwell

interaction between the gauge fields

3 currents $J^a_\mu = \frac{1}{g^2} \epsilon_{abc} F^b_{\mu\nu} A^c_\nu \neq 0$ the gauge bosons (vector) are charged

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Coupling with matter :

Dirac Fermions (quarks & leptons)

gauge transf $\psi(x) \to g(x)\psi(x)$

$g(x)$: 2×2 matrix; $\psi(x)$: 2-vector

$\psi = \begin{pmatrix} \psi^1 \\ \psi_2 \end{pmatrix} = (\psi^i)$ $i = 1,2$ fundamental representation of $SU(2)$

$$\mathcal{L} \propto -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu}_a = \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)^2 + \partial A \cdot A\, A + A \cdot A\, A \cdot A$$

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fundamental representation of $SU(2)$

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kinetic part as in Maxwell

interaction between the gauge fields

3 currents $J^a_\mu = \frac{1}{g^2} \epsilon_{abc} F^b_{\mu\nu} A^c_\nu$ ... gauge bosons (vector) are charged

Coupling with matter :

Dirac Fermions (quarks & ...

$\psi(x) \to g(x)\psi(x)$ ... 2-vector

2×2 matrix

$$S_{Fermion} = \int d^3x\, \bar{\psi}(i\not{D} - m)\psi$$

$$= \int d^D x\, \bar{\psi}^\alpha_i(x)$$

$$\psi = \begin{pmatrix} \psi^1 \\ \psi_2 \end{pmatrix} = (\psi^1) \; ...$$

be in general

A·A A·A

the gauge fields

(vector) are charged

$$S_{\text{Fermion}} = \int d^4x\, \bar{\Psi}(i\not{D} - m)\Psi$$

$$= \int d^D x\, \bar{\Psi}^{\alpha}_{i}(x)\left[i(\gamma^{\mu})^{\beta}_{\alpha}\left(\delta^{i}_{j}\partial_{\mu} - iA^{a}_{\mu}(x)\tfrac{1}{2}(\sigma_a)^{i}_{j}\right) - m\delta^{\beta}_{\alpha}\delta^{i}_{j}\right]\Psi^{j}_{\beta}(x)$$

$\Psi(x)\Psi(x)$ 2-vector

$i = 1,2$ Group index

$\alpha = 1,\dots,4$ Dirac index

$a = 1,3$ Group indices Adj Representation

Pauli matrix

$A \cdot A\, A + A \cdot A A \cdot A$ interaction between the gauge fields

the gauge bosons (vector) are charged

interaction $\bar{\psi} A \psi$

transf $\psi(x) \to g(x)\psi(x)$ (2×2 matrix, 2-vector)

$$S_{\text{Fermion}} = \int d^4x\, \bar{\psi}(i\not{D} - m)\psi$$

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$i = 1, 2$ Group index

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kinetic part as in Maxwell

interaction between the gauge fields

interaction $\bar{\psi} A$

3 currents $J^a_\mu = \frac{1}{g^2} \epsilon_{abc} F^b_{\mu\nu} A^c_\nu \neq 0$ the gauge bosons (vector) are charged

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Coupling with matter:

gauge transf $\psi(x) \to g(x)\psi(x)$ (2×2 matrix, 2-vector)

$$S_{\text{Fermion}} = \int d^4x\, \bar{\psi}(i\not{D} - m)\psi$$

$$= \int d^D x\, \bar{\psi}^\alpha_i(x) \left[i (\gamma^\mu)^\beta_\alpha \right. ($$

Dirac Fermions (quarks & leptons)

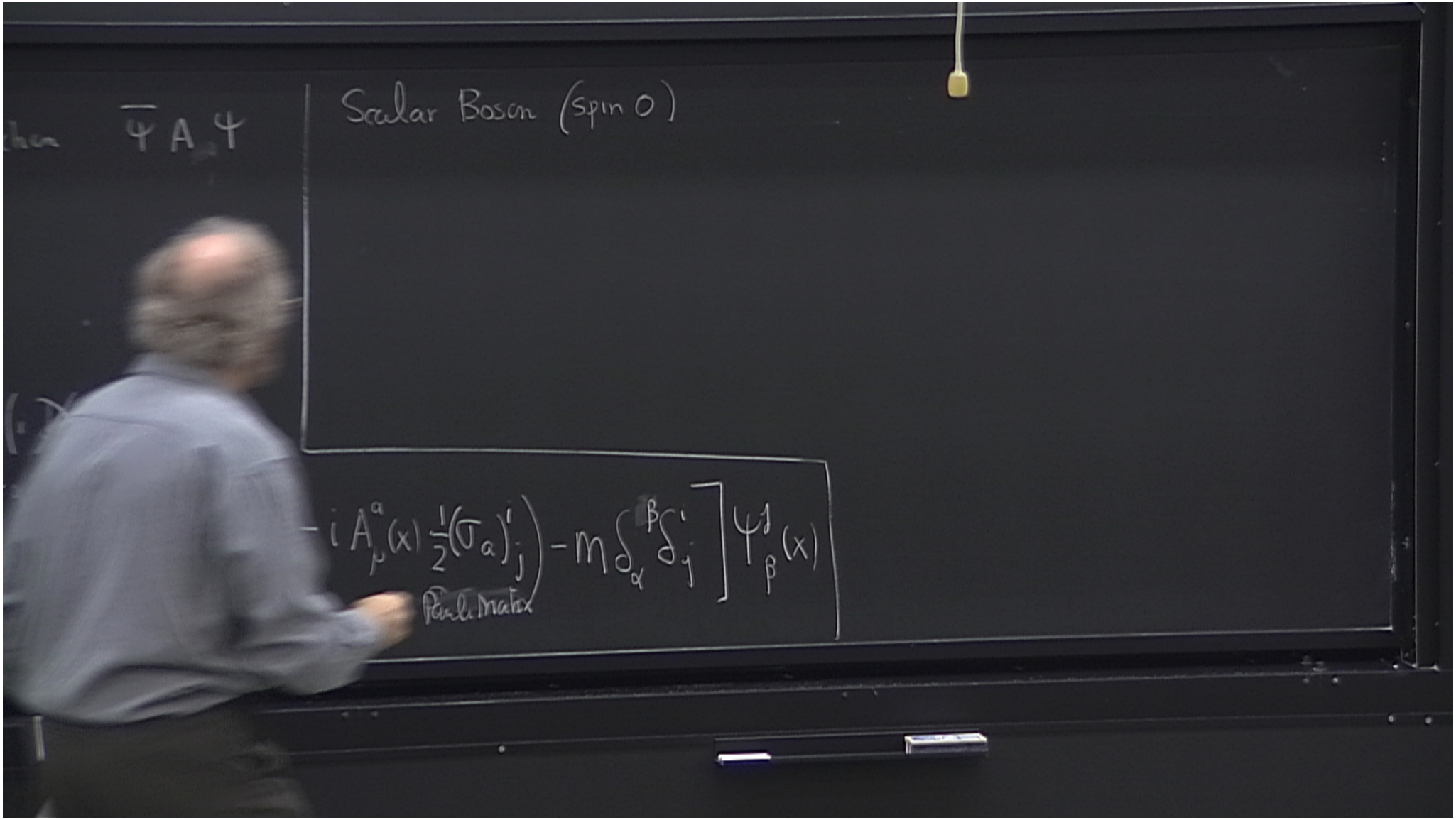
$$\psi = \begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix} = (\psi^i) \quad i = 1,2$$

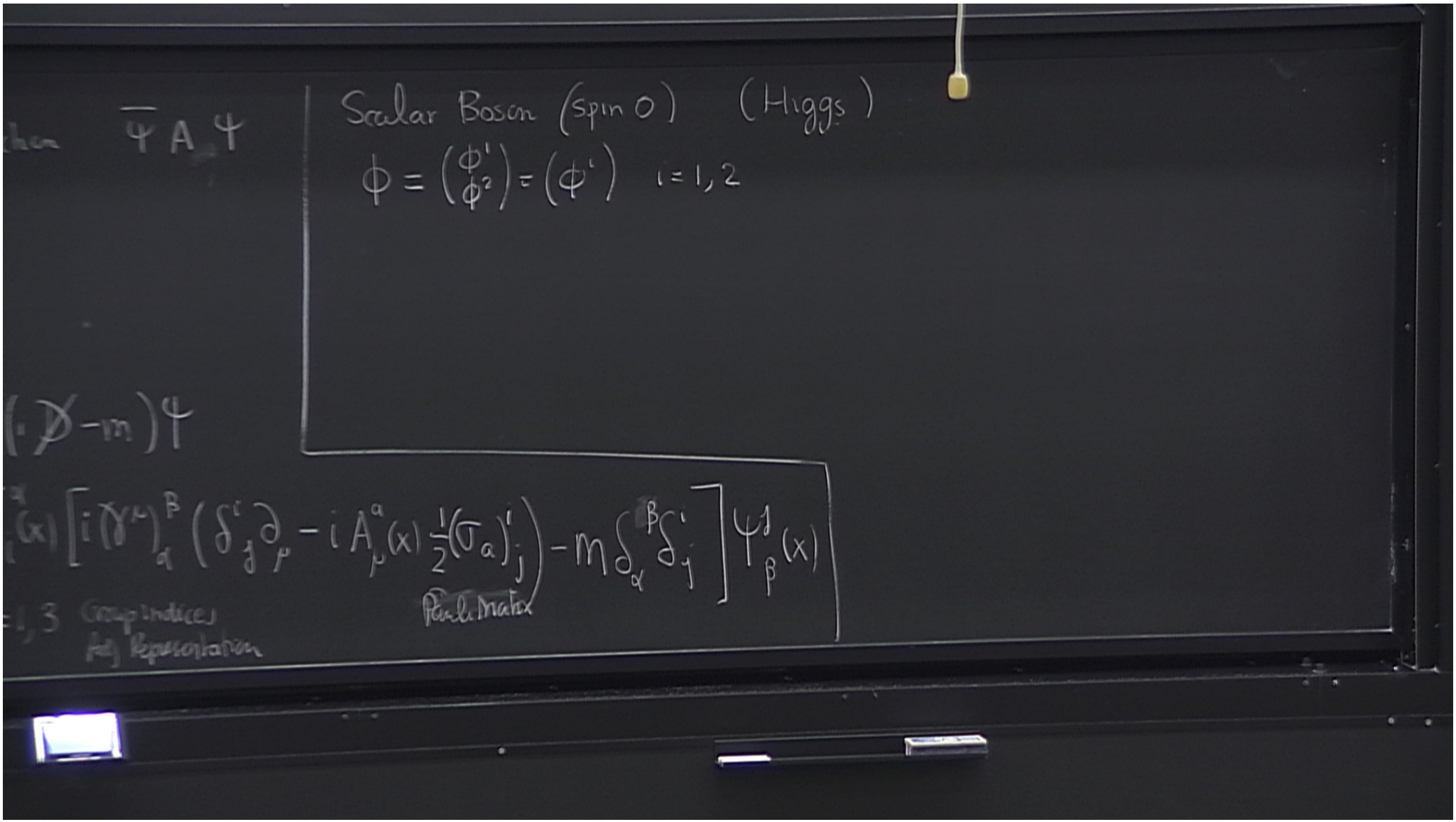
fundamental representation of $SU(2)$

$i = 1,2$ Group index

$\alpha = 1,4$ Dirac index

$a = 1,3$ Group indices Adj Representa


$\bar{\psi} A \psi$
Scalar Boson (spin 0)
$-i A^a_\mu(x) \frac{1}{2}(\sigma_a)^i_j \Big) - m \delta^\beta_\alpha \delta^i_j \Big] \psi^j_\beta(x)$
Pauli matrix


$\bar{\psi} A \psi$
Scalar Boson (spin 0) (Higgs)
$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i) \quad i = 1, 2$
$(i\not{\partial} - m)\psi$
$\left[i(\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \frac{1}{2}(\sigma_a)^i_j \right) - m \delta^\beta_\alpha \delta^i_j \right] \psi^j_\beta(x)$
Pauli matrix
1, 3 (Group indices)
Adj Representation

$\bar{\psi} A \psi$

Scalar Boson (spin 0) (Higgs)

$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i)$ $i = 1, 2$ complex fields ϕ^i

$(i\not{\partial} - m)\psi$

$$\bar{\psi}^{\alpha}_{i}(x)\left[i(\gamma^{\mu})^{\beta}_{\alpha}\left(\delta^{i}_{j}\partial_{\mu} - i A^{a}_{\mu}(x)\tfrac{1}{2}(\sigma_a)^{i}_{j}\right) - m\delta^{\beta}_{\alpha}\delta^{i}_{j}\right]\psi^{j}_{\beta}(x)$$

Pauli matrix

$= 1, 3$ (Group indices)
Adj Representation

$\bar{\psi} A \psi$

Scalar Boson (spin 0) (Higgs)

$\bar{\Phi}\Phi = \sum_i \bar{\Phi}^i \Phi^i$

$\Phi = \begin{pmatrix} \Phi^1 \\ \Phi^2 \end{pmatrix} = (\Phi^i) \quad i = 1, 2$ complex fields Φ^i

$$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2} \overline{D_\mu \Phi} \, D^\mu \Phi + \frac{m^2}{2} \bar{\Phi} \Phi + \frac{g}{8} (\bar{\Phi} \Phi)^2$$

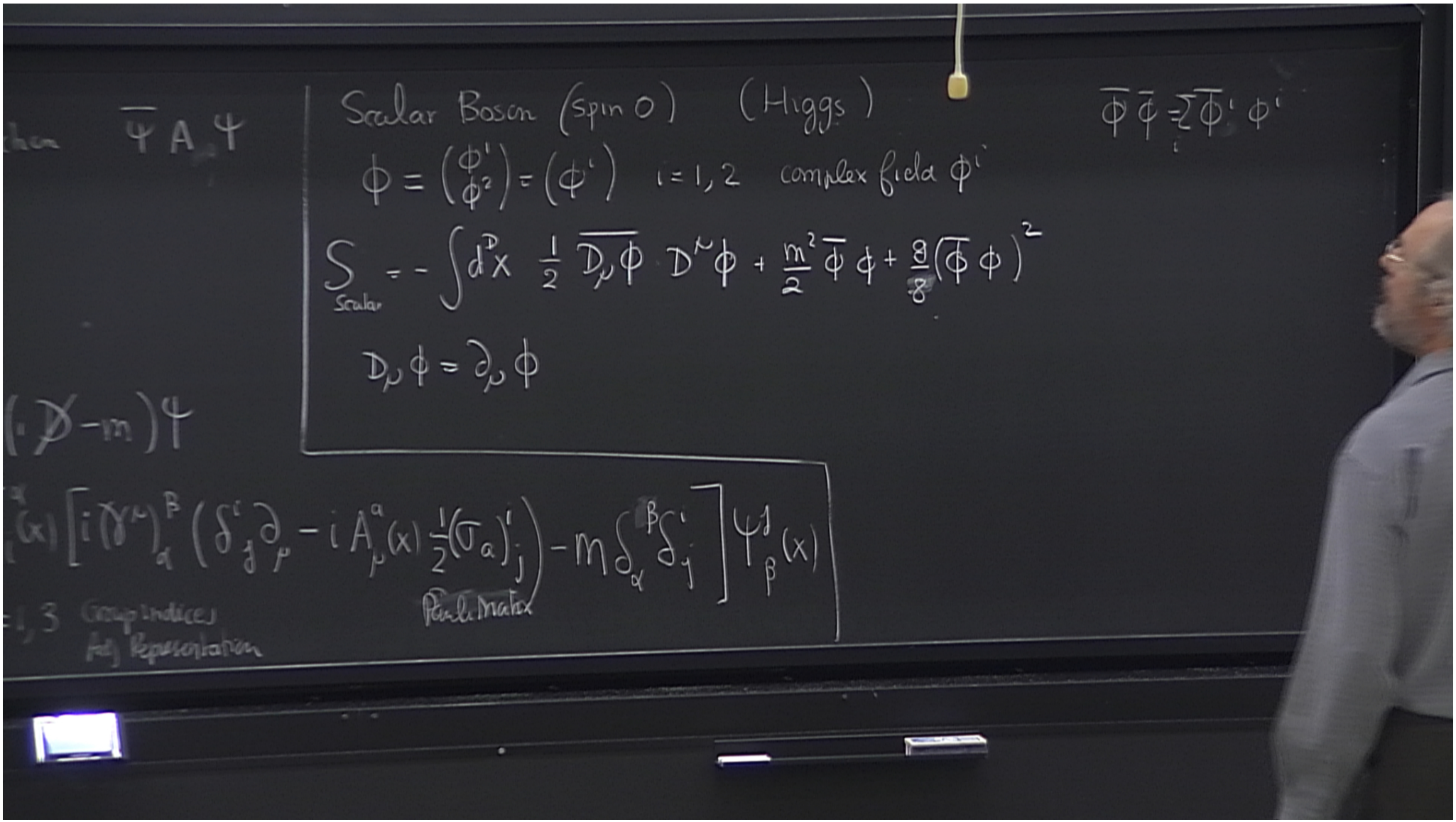
$(i \not{\partial} - m)\psi$

$$\bar{\psi}^\alpha_i(x) \left[i (\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \tfrac{1}{2} (\sigma_a)^i_j \right) - m \delta^\beta_\alpha \delta^i_j \right] \psi^j_\beta(x)$$

Pauli matrix

$i = 1, 3$ Group indices

Adj Representation


$\bar{\psi} A \psi$
Scalar Boson (spin 0) (Higgs)
$\bar{\Phi}\Phi = \sum_i \bar{\phi}^i \phi^i$
$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i)$ $i = 1, 2$ complex fields ϕ^i
$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \frac{m^2}{2} \bar{\phi} \phi + \frac{g}{8} (\bar{\phi} \phi)^2$
$D_\mu \phi = \partial_\mu \phi$
$(i \not{D} - m) \psi$
$\bar{\psi}_\alpha^i(x) \left[i (\gamma^\mu)_\alpha^\beta \left(\delta_j^i \partial_\mu - i A_\mu^a(x) \frac{1}{2} (\sigma_a)_j^i \right) - m \delta_\alpha^\beta \delta_j^i \right] \psi_\beta^j(x)$
Pauli matrix
$i, j = 1, 3$ (Group indices)
Adj Representation

$\bar{\psi} A \psi$

Scalar Boson (spin 0) (Higgs)

$\bar{\Phi}\Phi = \sum_i \bar{\Phi}^i \Phi^i$

$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i) \quad i = 1, 2$ complex fields ϕ^i

$$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \frac{m^2}{2} \bar{\phi} \phi + \frac{g}{8} (\bar{\phi} \phi)^2$$

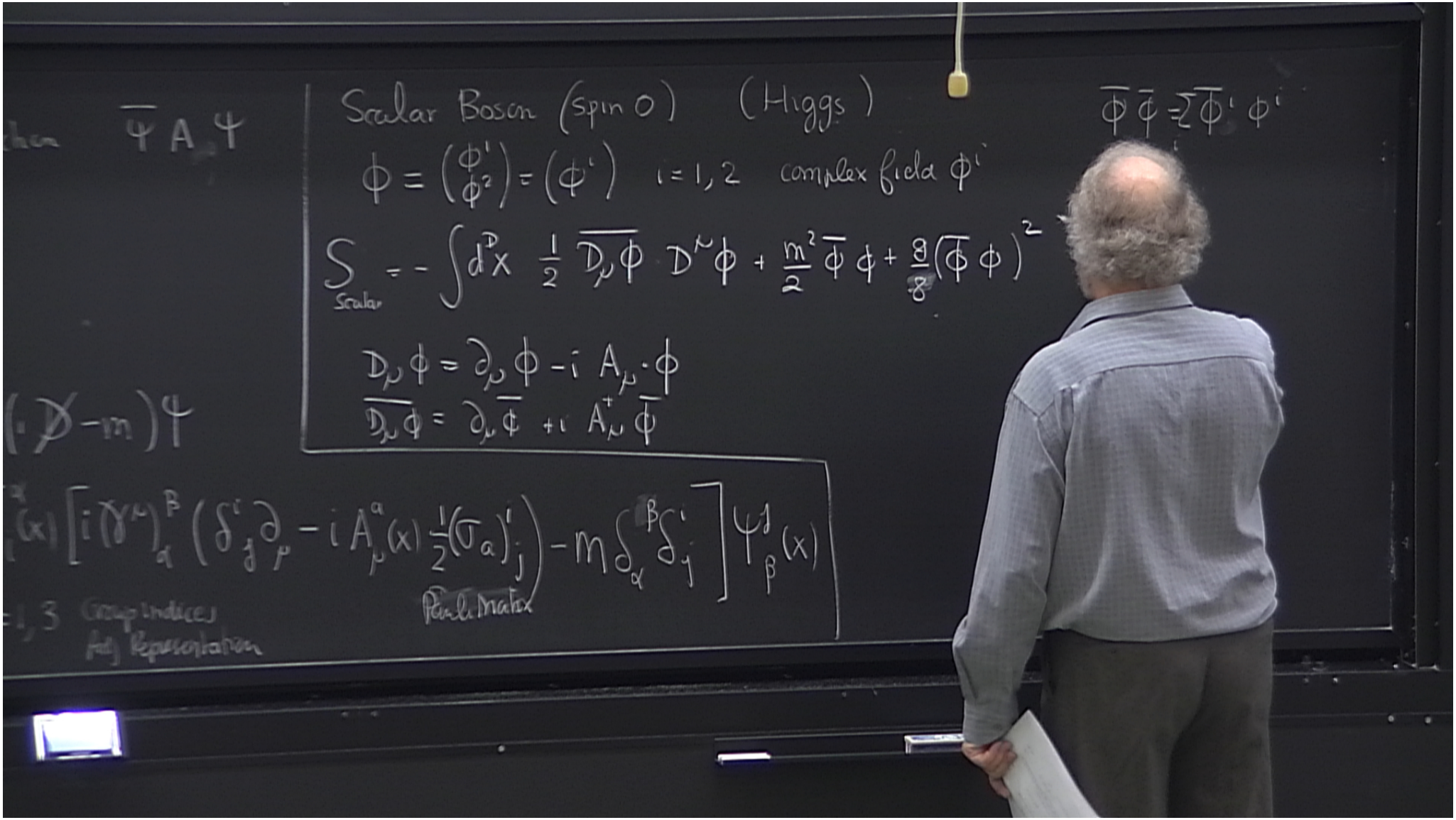
$$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$$

$(\not{\partial} - m)\psi$

$$\bar{\psi}_\alpha^i(x) \left[i (\gamma^\mu)_\alpha^\beta \left(\delta_j^i \partial_\mu - i A_\mu^a(x) \tfrac{1}{2} (\sigma_a)_j^i \right) - m \delta_\alpha^\beta \delta_j^i \right] \psi_\beta^j(x)$$

Pauli matrix

$i, j = 1, 3$ Group indices, Adj Representation


Scalar Boson (spin 0) (Higgs)
$\phi = \begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix} = (\phi^i)$ $i=1,2$ complex fields ϕ^i
$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2}\overline{D_\mu \phi}\, D^\mu \phi + \frac{m^2}{2}\bar{\phi}\phi + \frac{g}{8}(\bar{\phi}\phi)^2$
$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$
$\overline{D_\mu \phi} = \partial_\mu \bar{\phi} + i A_\mu^+ \bar{\phi}$
$\bar{\psi} A \psi$
$(\not{\partial} - m)\psi$
$\left[i(\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \frac{1}{2}(\sigma_a)^i_j \right) - m \delta^\beta_\alpha \delta^i_j \right] \psi^j_\beta(x)$
Pauli matrix
1, 3 Group indices
Adj Representation
$\bar{\Phi}\Phi = \sum \bar{\Phi}^i \Phi^i$

$\bar{\psi} A \psi$

Scalar Boson (spin 0) (Higgs)

$\bar{\phi}\phi = \sum_i \bar{\phi}^i \phi^i$

$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i) \quad i = 1, 2$ complex fields ϕ^i

$$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \frac{m^2}{2} \bar{\phi} \phi + \frac{g}{8} (\bar{\phi} \phi)^2$$

gauge invariant

$$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$$

$$\overline{D_\mu \phi} = \partial_\mu \bar{\phi} + i A_\mu^+ \bar{\phi}$$

$(\not{\partial} - m)\psi$

$$\left[i (\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \frac{1}{2} (\sigma_a)^i_j \right) - m \delta^\beta_\alpha \delta^i_j \right] \psi^j_\beta(x)$$

Pauli matrix

$i, j = 1, 3$ (Group indices) Adj Representation

$\partial A \cdot A \cdot A + A \cdot A \cdot A \cdot A$

interaction between the gauge fields

the gauge bosons (vector) are changed

interaction $\bar{\psi} A \psi$

$\not{D}\psi \to g(x) \not{D}\psi$

$\psi(x) \to g(x)\psi(x)$

g: 2×2 matrix, 2-vector

$$S_{Fermion} = \int d^4x \, \bar{\psi}(i\not{D} - m)\psi$$

$$= \int d^D x \, \bar{\psi}^{\alpha}_{i}(x) \left[i\gamma^{\mu}{}^{\beta}_{\alpha} \left(\delta^{i}_{j}\partial_{\mu} - i A^{a}_{\mu}(x) \tfrac{1}{2}(\sigma_a)^{i}_{j} \right) - m\delta^{\beta}_{\alpha}\delta^{i}_{j} \right] \psi^{j}_{\beta}(x)$$

$i = 1, 2$ Group index

$\alpha = 1, 4$ Dirac index

$a = 1, 3$ Group indices, Adj Representation

σ_a: Pauli matrix

Scalar Boson (spin 0) (H

$$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i) \quad i = 1, 2$$

$$S_{Scalar} = -\int d^D x \, \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \frac{m}{2}$$

$$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$$

$$\overline{D_\mu \phi} = \partial_\mu \bar{\phi} + i A^{+}_{\mu} \bar{\phi}$$

$\partial A \cdot A \cdot A + A \cdot A \cdot A \cdot A$

interaction between the gauge fields

the gauge bosons (vector) are changed

interaction $\bar{\psi} A \psi$

$\not{D}\psi \to g(x) \not{D} \psi$

gauge invariant

$\psi(x) \to g(x)\psi(x)$

g: 2×2 matrix, ψ: 2-vector

$$S_{Fermion} = \int d^4x \, \bar{\psi}(i \not{D} - m)\psi$$

$$= \int d^D x \, \bar{\psi}^{\alpha}_{i}(x) \left[i\gamma^{\mu}{}^{\beta}_{\alpha} \left(\delta^{i}_{j} \partial_{\mu} - i A^{a}_{\mu}(x) \tfrac{1}{2} (\sigma_a)^{i}_{j} \right) - m \delta^{\beta}_{\alpha} \delta^{i}_{j} \right] \psi^{j}_{\beta}$$

$i = 1, 2$ Group index

$\alpha = 1, \ldots 4$ Dirac index

$a = 1, 3$ Group indices Adj Representation

σ_a: Pauli matrix

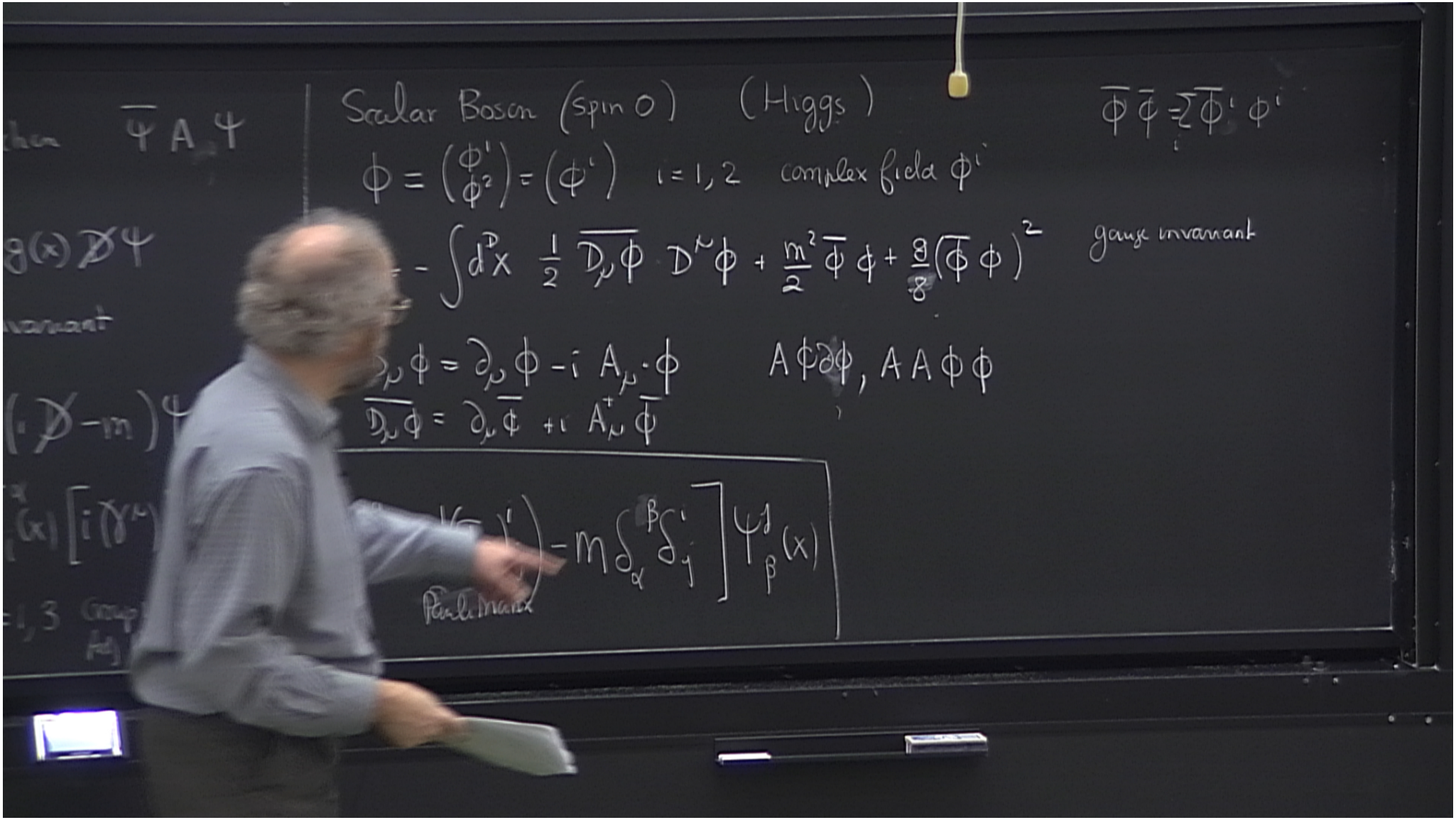
Scalar Boson (spin 0)

$$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i) \quad i = 1, 2$$

$$S_{Scalar} = -\int d^D x \, \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \ldots$$

$$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$$

$$\overline{D_\mu \phi} = \partial_\mu \bar{\phi} + i A^{+}_{\mu} \bar{\phi}$$


Scalar Boson (spin 0) (Higgs)
gauge invariant
Pauli matrix

$\bar{\psi} A \psi$

$g(x) \not{D} \psi$

$(\not{D} - m)\psi$

Scalar Boson (spin 0) (Higgs)

$\bar{\phi}\phi = \sum_i \bar{\phi}^i \phi^i$

$\phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix} = (\phi^i)$ $i = 1, 2$ complex fields ϕ^i

$$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2} \overline{D_\mu \phi} \, D^\mu \phi + \frac{m^2}{2} \bar{\phi} \phi + \frac{g}{8} (\bar{\phi} \phi)^2$$

gauge invariant

$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$

$\overline{D_\mu \phi} = \partial_\mu \bar{\phi} + i A_\mu^+ \bar{\phi}$

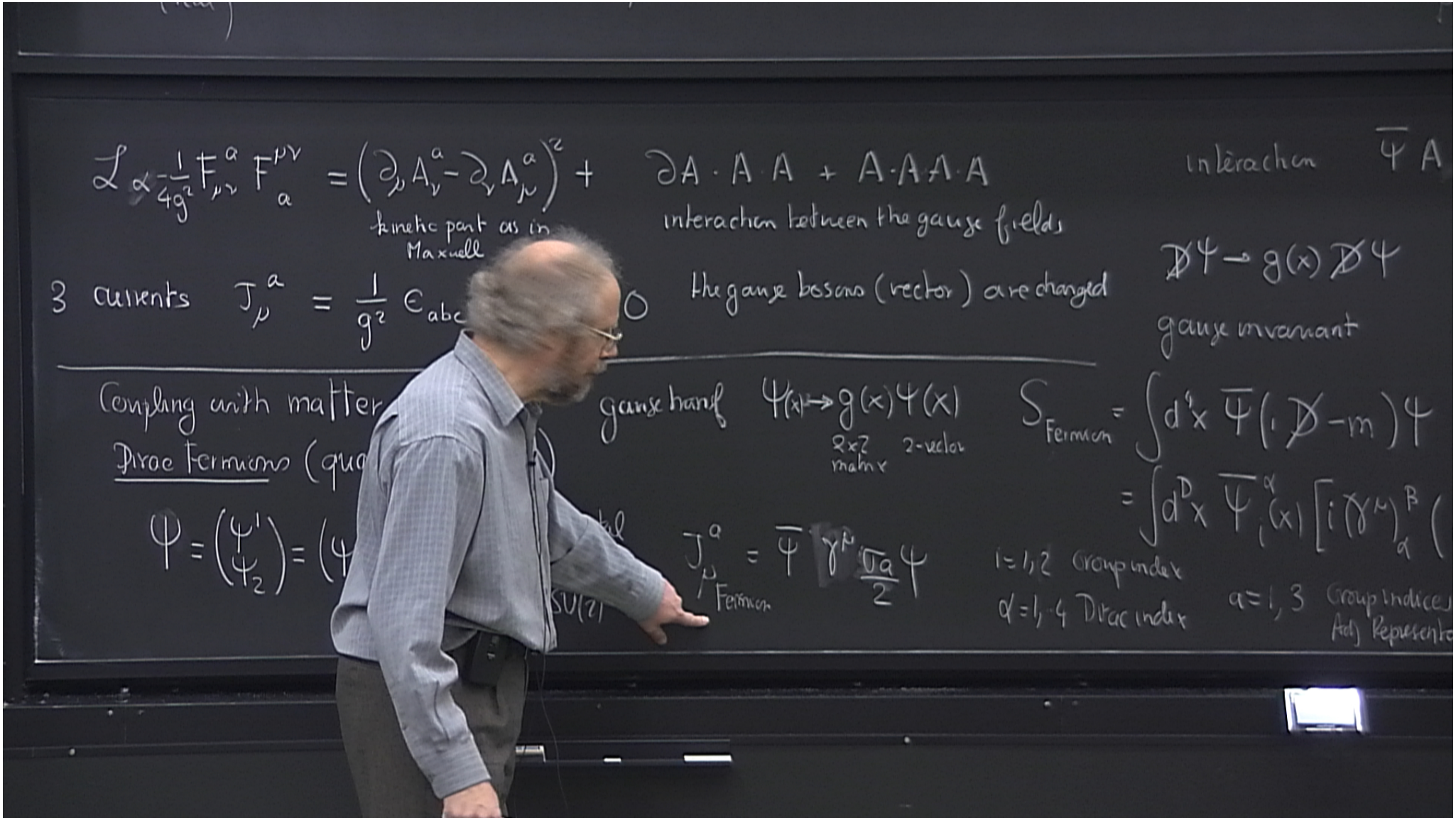
$A \phi \partial \phi, \; A A \phi \phi$

$$\psi^a_\alpha(x) \left[i (\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \tfrac{1}{2} (\sigma_a)^i_j \right) - m \delta^\beta_\alpha \delta^i_j \right] \psi^j_\beta(x)$$

Pauli matrix

$J^a_\mu = J^a_{\mu\,\text{matter}} + J^a_{\mu\,\text{gauge field}}$

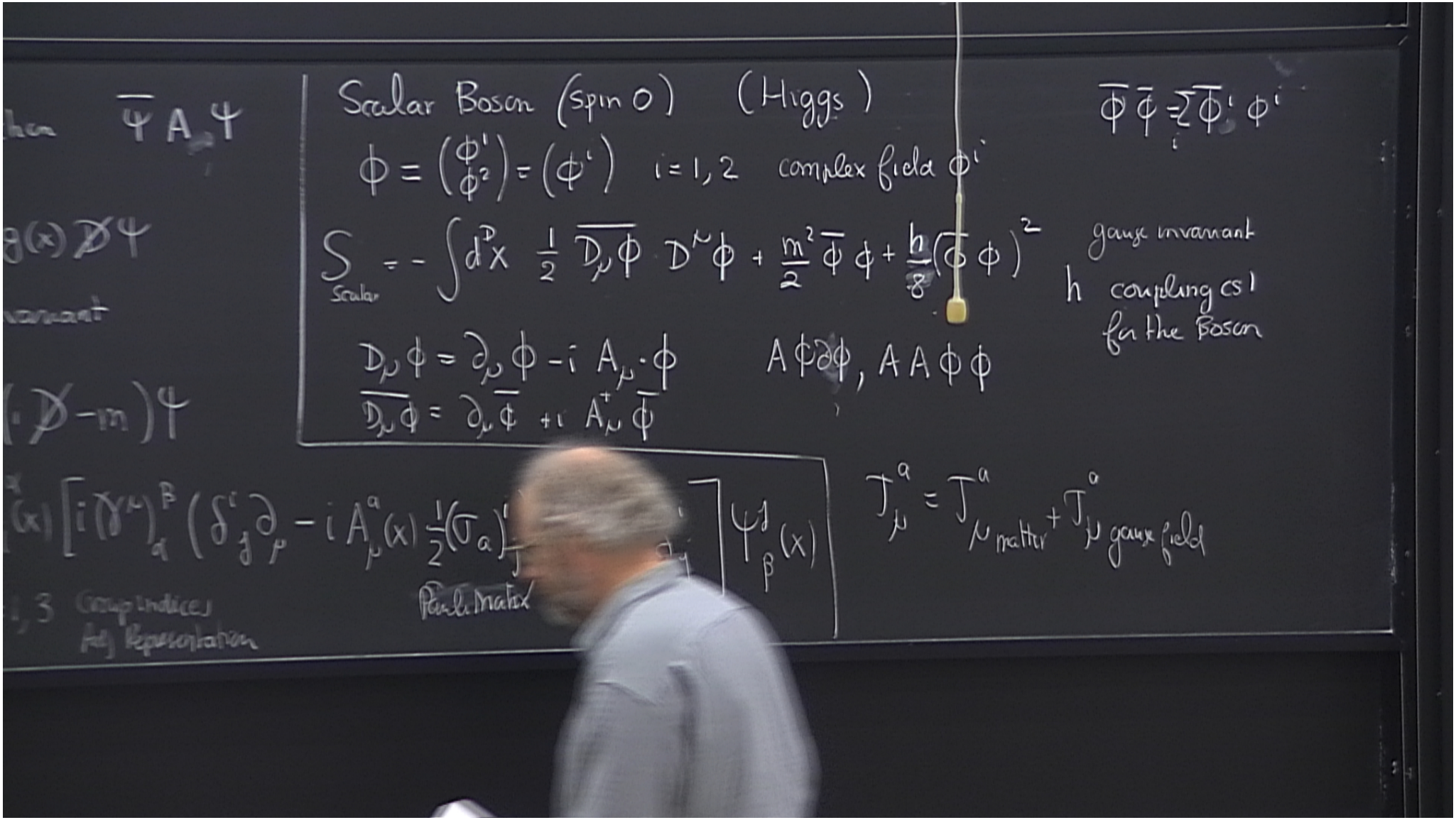
$i, 3$ Group indices
Adj Representation


$\mathcal{L} \propto -\frac{1}{4g^2} F^a_{\mu\nu} F^{\mu\nu}_a = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)^2 + \partial A \cdot A \cdot A + A \cdot A \cdot A \cdot A$
kinetic part as in Maxwell
interaction between the gauge fields
interaction $\bar{\psi} A$
3 currents $J^a_\mu = \frac{1}{g^2} \epsilon_{abc}$ 0
the gauge bosons (vector) are charged
$\not{D}\psi \to g(x) \not{D}\psi$
gauge invariant
Coupling with matter
Dirac Fermions (qua
gauge transf $\psi(x) \to g(x)\psi(x)$
2x2 matrix
2-vector
$S_{Fermion} = \int d^3x \, \bar{\psi}(i\not{D} - m)\psi$
$= \int d^D x \, \bar{\psi}^\alpha_i(x) [i(\gamma^\mu)^\beta_\alpha$ (
$\psi = \begin{pmatrix} \psi^1 \\ \psi_2 \end{pmatrix} = (\psi$
$J^a_\mu = \bar{\psi} \gamma^\mu \frac{\sigma^a}{2} \psi$
Fermion
SU(2)
$i = 1, 2$ Group index
$\alpha = 1, 4$ Dirac index
$a = 1, 3$ Group indices Adj Representa

$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = (\psi^i)$ $i=1,2$ representation of $SU(2)$ $J^\mu = \bar{\psi} \gamma^\mu \frac{\sigma^a}{2} \psi$ Fermion $i=1,2$ Group index $\alpha=1,4$ Dirac index $a=1,3$ Group indices Adj Representation

Feynman Diagrams & perturbative expansion: charge = coupling constant g

AA 


Scalar Boson (spin 0) (Higgs)
$\phi = \begin{pmatrix}\phi^1\\ \phi^2\end{pmatrix} = (\phi^i)$ $i = 1, 2$ complex field ϕ^i
$\overline{\phi}\,\phi = \sum_i \overline{\phi}^i \phi^i$
$S_{\text{Scalar}} = -\int d^D x \; \frac{1}{2}\overline{D_\mu \phi}\, D^\mu \phi + \frac{m^2}{2}\overline{\phi}\,\phi + \frac{h}{8}(\overline{\phi}\,\phi)^2$
gauge invariant
h coupling cst for the Boson
$D_\mu \phi = \partial_\mu \phi - i A_\mu \cdot \phi$
$\overline{D_\mu \phi} = \partial_\mu \overline{\phi} + i A_\mu^+ \overline{\phi}$
$A\phi\partial\phi,\ AA\phi\phi$
$J^a_\mu = J^a_{\mu\ \text{matter}} + J^a_{\mu\ \text{gauge field}}$
Pauli matrix

Fermion

$\alpha = 1, 4$ Dirac index $\quad a = 1, 3$ Group indices Adj Representation

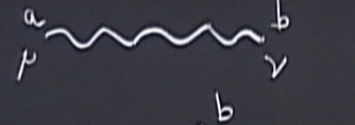
Pauli matrix

xpansion: charge = coupling constant g of YM

$$A_\mu \to g A_\mu \qquad \mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\, \partial A\, A\, A + g^2\, A\, A\, A\, A$$

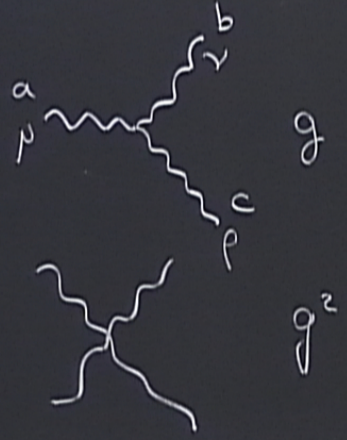
$\alpha = 1, 4$ Dirac index $a = 1, 3$ Group indices Adj Representation

Feynman Diagrams & perturbative expansion: charge = coupling constant g of YM



$\langle A^a_\mu A^b_\nu \rangle_{g=0}$

$A_\mu \to g A_\mu$ $\mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\, \partial A$

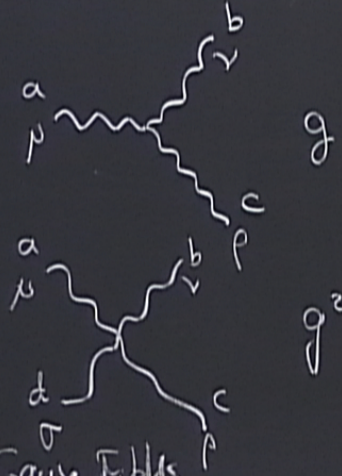


of SU(2) Fermion $\frac{1}{2}$ $\alpha = 1, 4$ Dirac index $a = 1, 3$ Group indices Adj Representa

Feynman Diagrams & perturbative expansion: charge = coupling constant g of YM

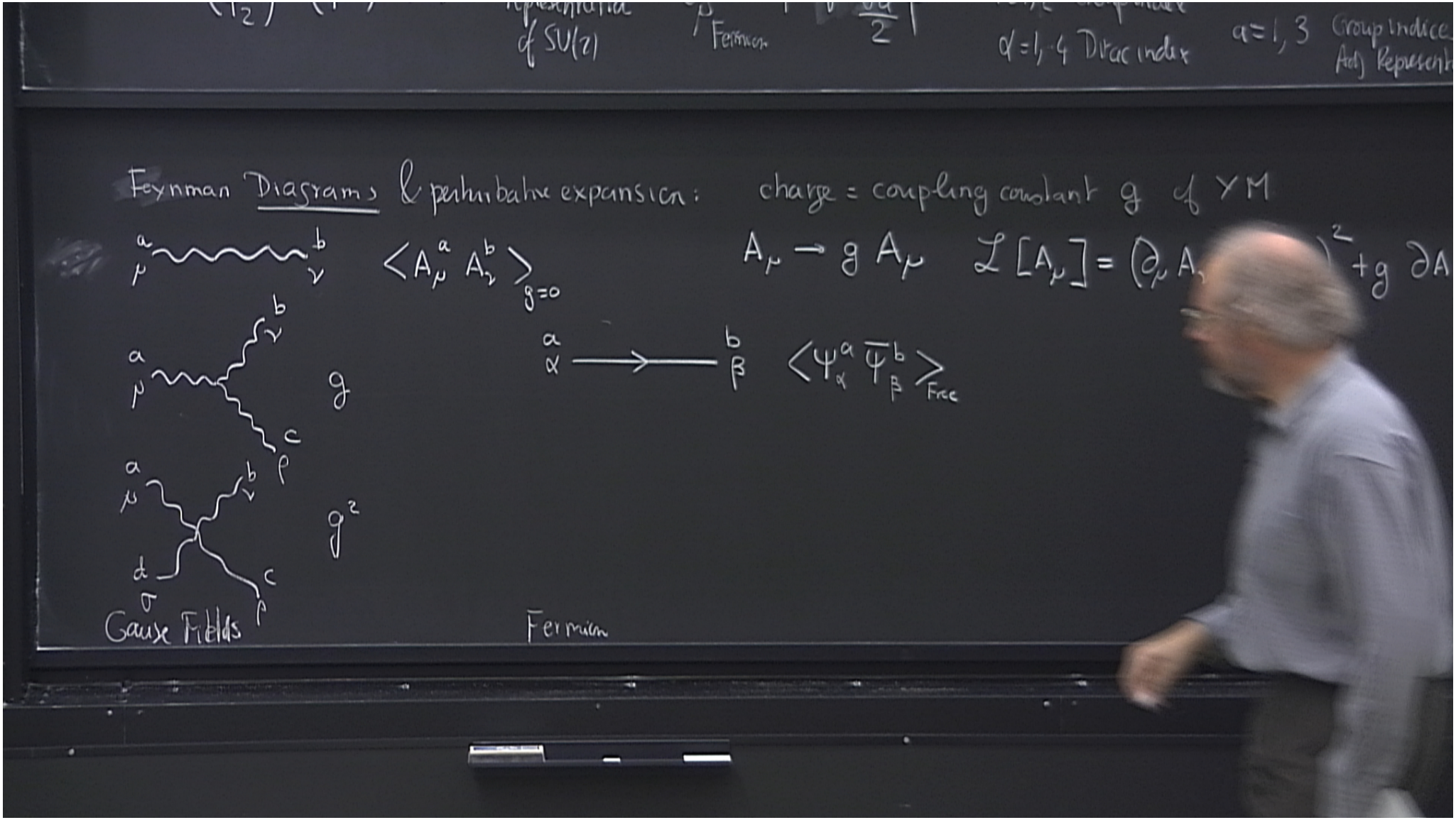


$\langle A^a_\mu A^b_\nu \rangle_{g=0}$ $\quad A_\mu \to g A_\mu \quad \mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g$



Gauge Fields

Fermion


Fermion
$\alpha = 1, 4$ Dirac index
$a = 1, 3$ Group indices
Adj Representation
Feynman Diagrams & perturbative expansion:
charge = coupling constant g of YM
$\langle A^a_\mu A^b_\nu \rangle_{g=0}$
$A_\mu \to g A_\mu$
$\mathcal{L}[A_\mu] = (\partial_\mu A_\nu)^2 + g\, \partial A$
$\langle \psi^a_\alpha \bar{\psi}^b_\beta \rangle_{Free}$
g
g^2
Gauge Fields
Fermion

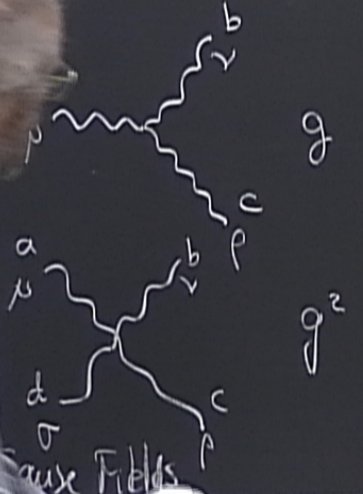
of SU(2) Fermion $\alpha = 1, 4$ Dirac index $a = 1, 3$ Group indices Adj Representa

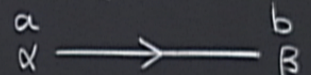
Feynman Diagrams & perturbative expansion: charge = coupling constant g of YM

$A_\mu \to g A_\mu$ $\mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\, \partial A$

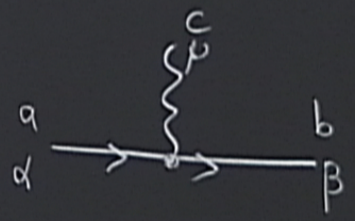


$\langle A^a_\mu A^b_\nu \rangle_{g=0}$



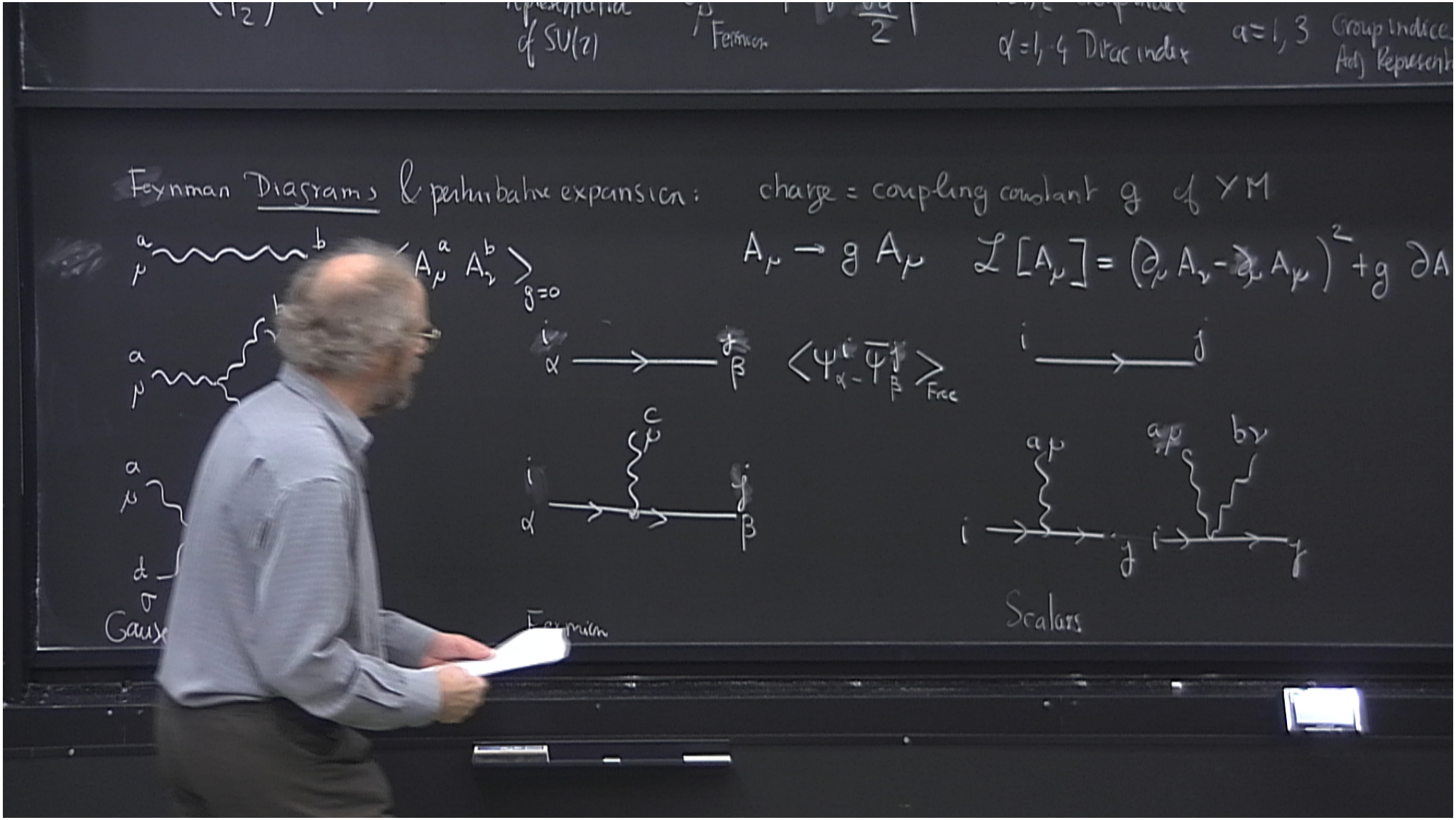


$\langle \psi^a_\alpha \bar{\psi}^b_\beta \rangle_{Free}$



Gauge Fields

Fermion


Feynman Diagrams & perturbative expansion:
charge = coupling constant g of YM
$A_\mu \to g A_\mu$
$\mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\,\partial A$
$\langle A^a_\mu A^b_\nu \rangle_{g=0}$
$\langle \psi^i_\alpha \bar{\psi}^j_\beta \rangle_{Free}$
Fermion
Scalars
Group indices
Adj Representation
$\alpha = 1, 4$ Dirac index
$a = 1, 3$

of SU(2) Fermion $\frac{\sigma_a}{2}$ $\alpha = 1, 4$ Dirac index $a = 1, 3$ Group indices Adj Representa

Feynman Diagrams & perturbative expansion: charge = coupling constant g of YM

$A_\mu \to g A_\mu$ $\mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\, \partial A$

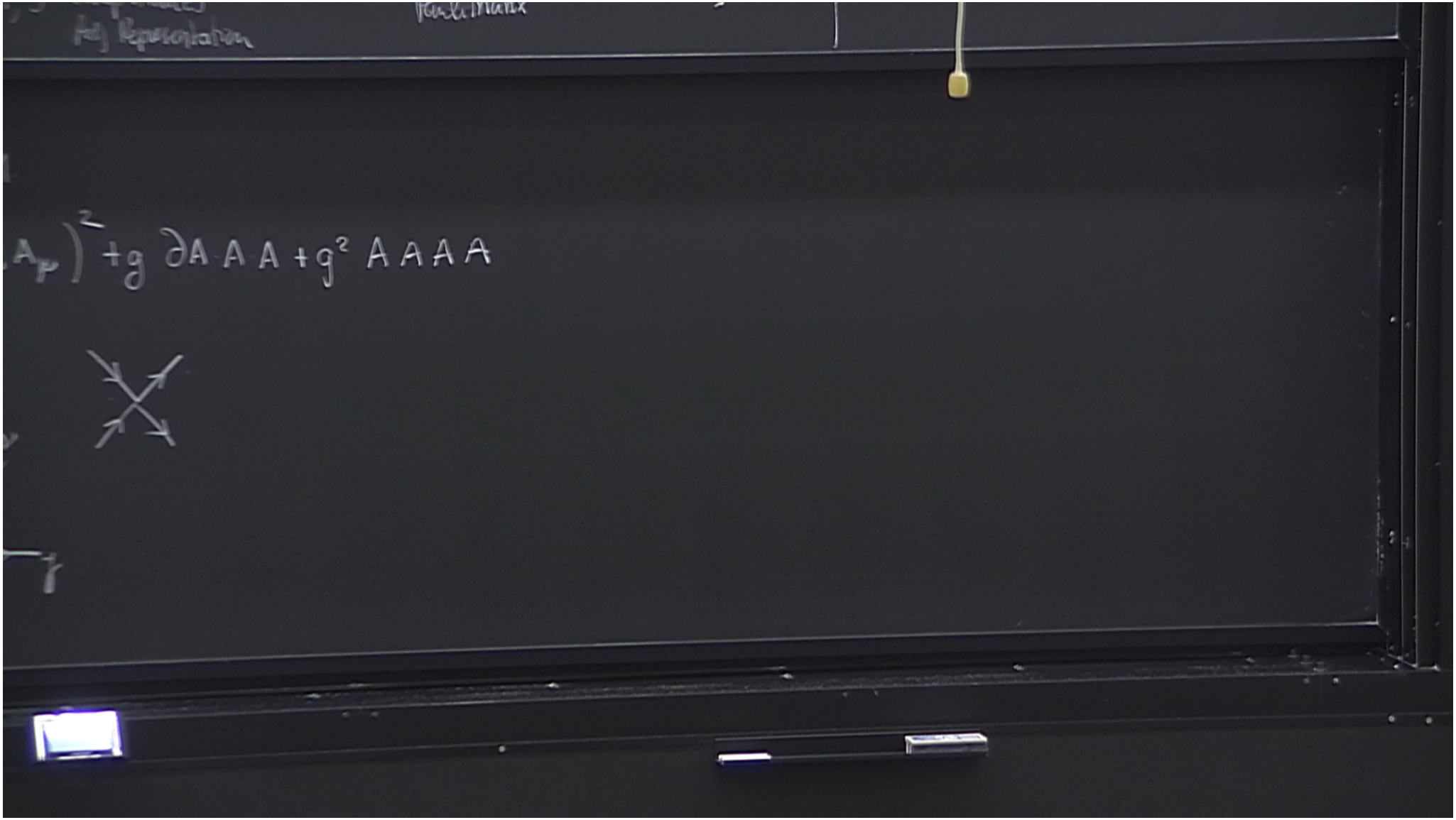
$\langle A^a_\mu A^b_\nu \rangle_{g=0}$

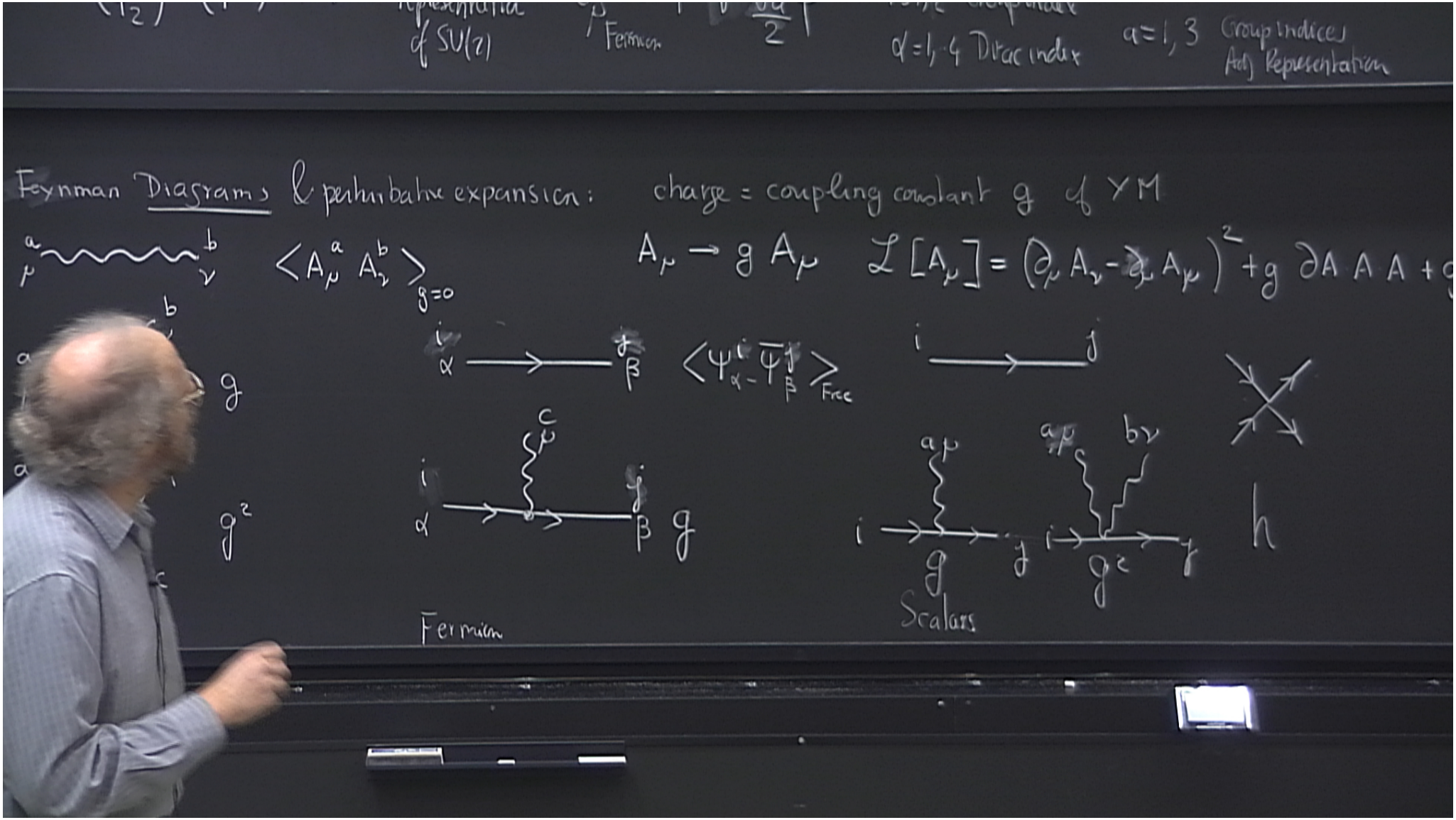
$\langle \psi^i_\alpha \bar{\psi}^j_\beta \rangle_{Free}$

Gauge Fields

Fermion

Scalars


$A_\mu)^2 + g\,\partial A\,A\,A + g^2\,A\,A\,A\,A$


Feynman Diagrams & perturbative expansion:
charge = coupling constant g of YM
$A_\mu \to g A_\mu$
$\mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\,\partial A\,A\,A + g$
$\langle A^a_\mu A^b_\nu \rangle_{g=0}$
$\langle \psi^i_\alpha \bar{\psi}^j_\beta \rangle_{Free}$
Fermion
Scalars
Group indices
Adj Representation
Dirac index

$\Psi = \begin{pmatrix}\psi^1\\ \psi^2\end{pmatrix} = (\psi^i)$ $i=1,2$ fundamental representation of $SU(2)$

$J^a_\mu = \bar\psi \gamma^\mu \frac{\sigma^a}{2} \psi$ Fermion

$= \int d^4x\, \psi_i(x) \left[i(\gamma^\mu)^\beta_\alpha \left(\delta^i_j \partial_\mu - i A^a_\mu(x) \tfrac{1}{2}(\sigma_a)^i_j \right) \right.$

$i=1,2$ Group index

$\alpha=1,\dots 4$ Dirac index

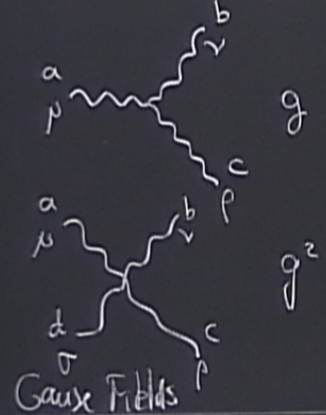
$a=1,3$ Group indices Adj Representation

Pauli matrices

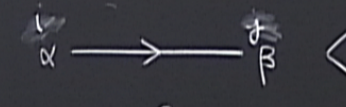
Feynman Diagrams & perturbative expansion: charge = coupling constant g of YM

$A_\mu \to g A_\mu$ $\quad \mathcal{L}[A_\mu] = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + g\, \partial A\, A\, A + g^2\, A\, A\, A\, A$

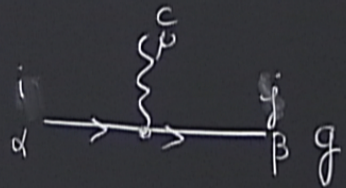
$\langle A^a_\mu A^b_\nu \rangle_{g=0}$



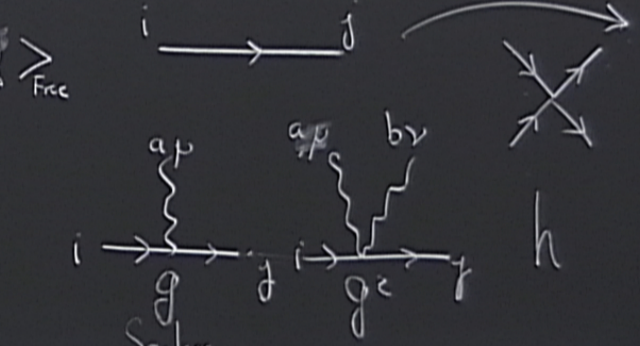
Gauge Fields



$\langle \psi^i_\alpha \bar\psi^j_\beta \rangle_{Free}$



Fermion



Scalars

$\langle \phi^i \bar\phi^j \rangle$

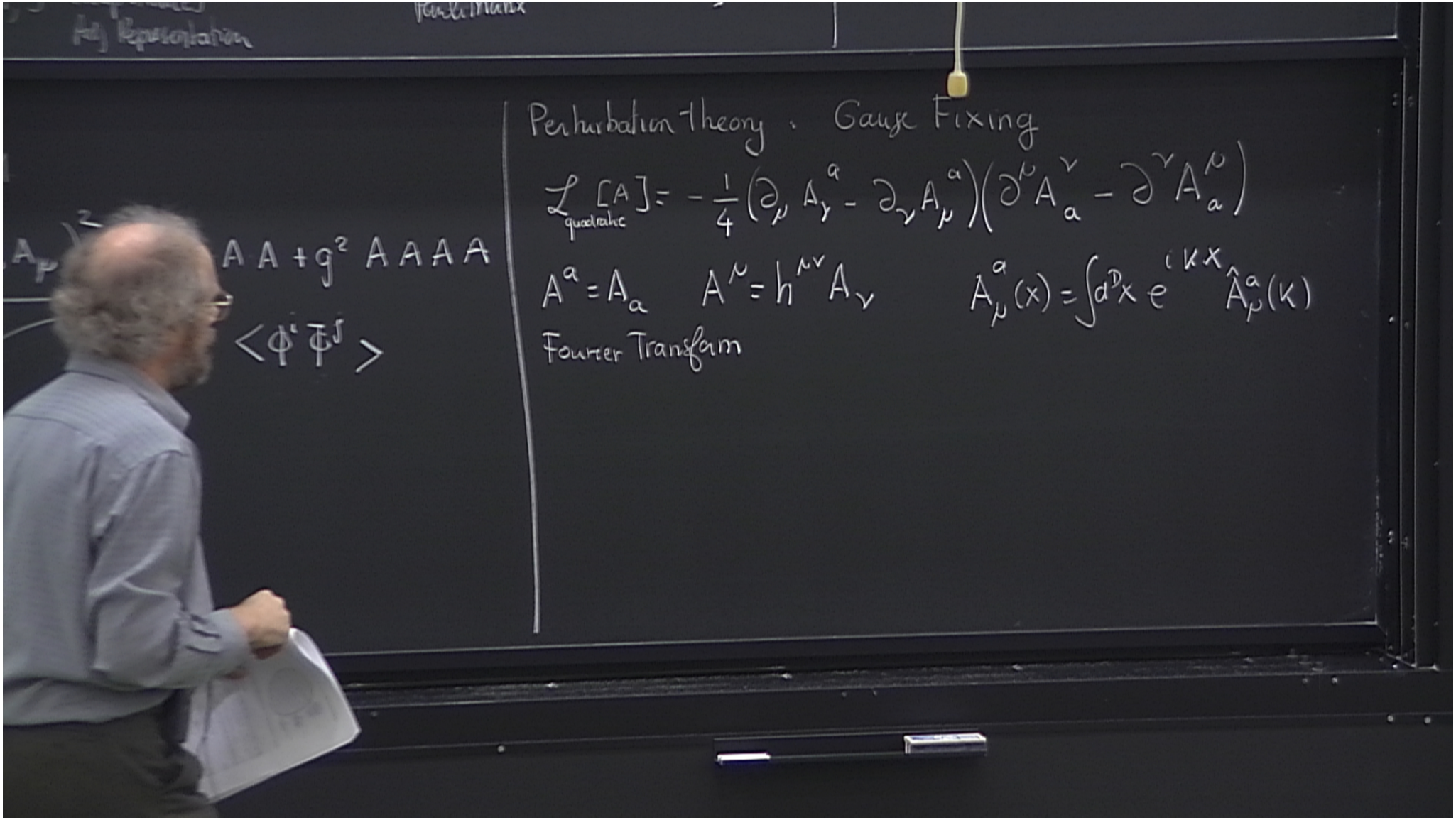
Perturbation Theory : Gauge Fixing

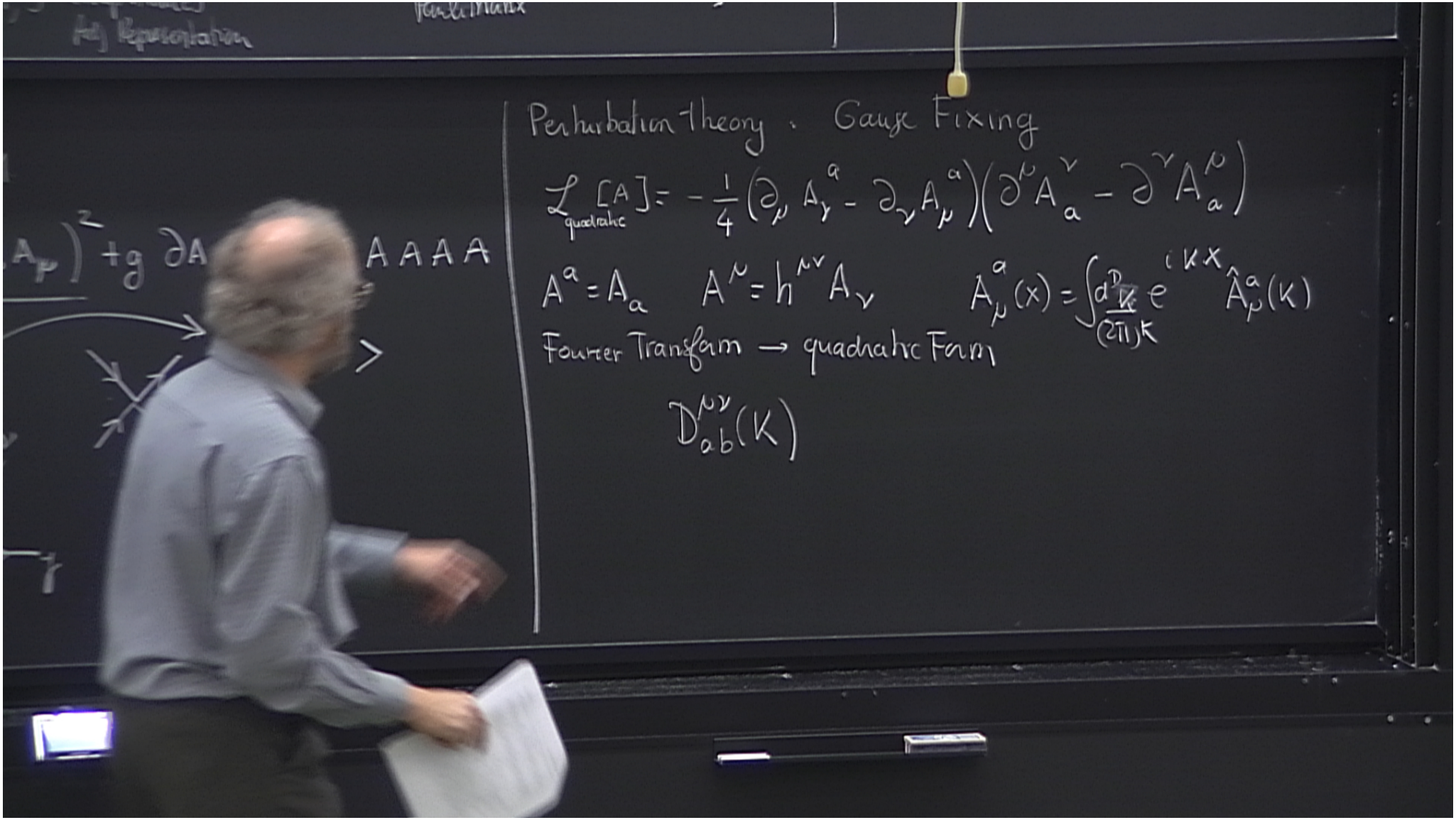
$A_\mu)^2 + g\,\partial A\,A\,A + g^2\,A\,A\,A\,A$

$\langle \phi^i \bar{\phi}^j \rangle$



h


Adj Representation
$A_\mu)^2$
$AA + g^2 AAAA$
$\langle \phi^i \bar{\phi}^j \rangle$
Perturbation Theory . Gauge Fixing
$\mathcal{L}_{\text{quadratic}}[A] = -\frac{1}{4}(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a)$
$A^a = A_a \quad A^\mu = h^{\mu\nu} A_\nu \quad A^a_\mu(x) = \int d^D x\, e^{ikx} \hat{A}^a_\mu(k)$
Fourier Transform


Perturbation Theory : Gauge Fixing
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Fourier Transform → quadratic Form
$D^{\mu\nu}_{ab}(K)$

$(A_\mu)^2 + g\,\partial A\,A\,A + g^2\,A\,A\,A\,A$

$\langle \phi^i \bar{\phi}^j \rangle$



h

Perturbation Theory . Gauge Fixing

$$\mathcal{L}_{\text{quadratic}}[A] = -\frac{1}{4}\left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right)\left(\partial^\mu A^\nu_a - \partial^\nu A^\mu_a\right)$$

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Fourier Transform → quadratic Form

$$D^{\mu\nu}_{ab}(K) = \delta_{ab}\left(-K^2 h^{\mu\nu} + K^\mu K^\nu\right)$$

$A_\mu)^2 + g\,\partial A\,A\,A + g^2\,A\,A\,A\,A$

$\langle \phi^i \bar{\phi}^j \rangle$



h

Perturbation Theory : Gauge Fixing

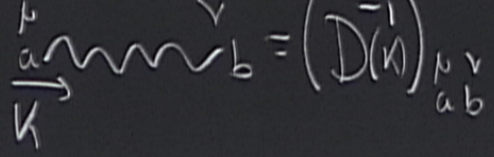
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propagator



$= \left(D^{-1}(k)\right)^{\mu\nu}_{ab}$

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$\langle \phi^i \phi^j \rangle$



h

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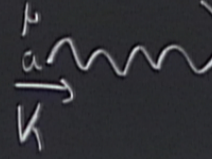
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Fourier Transform → quadratic Form

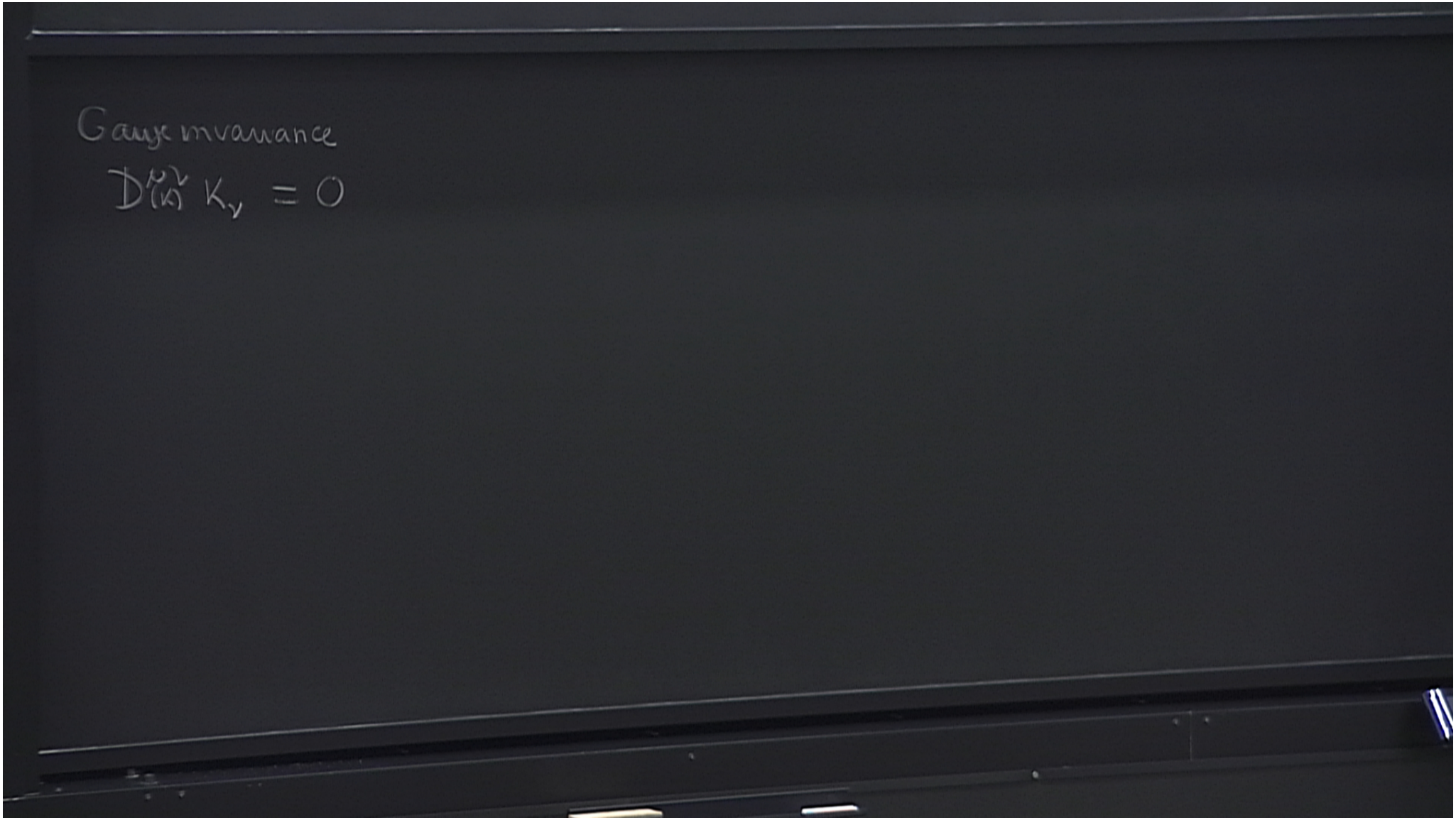
$$D^{\mu\nu}_{ab}(K) = \delta_{ab}\left(-K^2 h^{\mu\nu} + K^\mu K^\nu\right)$$

4×4 matrix

propagator



$= \left(D^{-1}(K)\right)^{\mu\nu}_{ab} = \delta_{ab}$


Gauge invariance
$D^{\mu\nu}_{(k)} K_\nu = 0$

Gauge invariance : as in QED

$D^{\mu\nu}(k) K_\nu = 0$

$A^a_\mu(k) = K_\mu \alpha^a(k) \longrightarrow A^a_\mu(x) = \partial_\mu \alpha^a(x)$

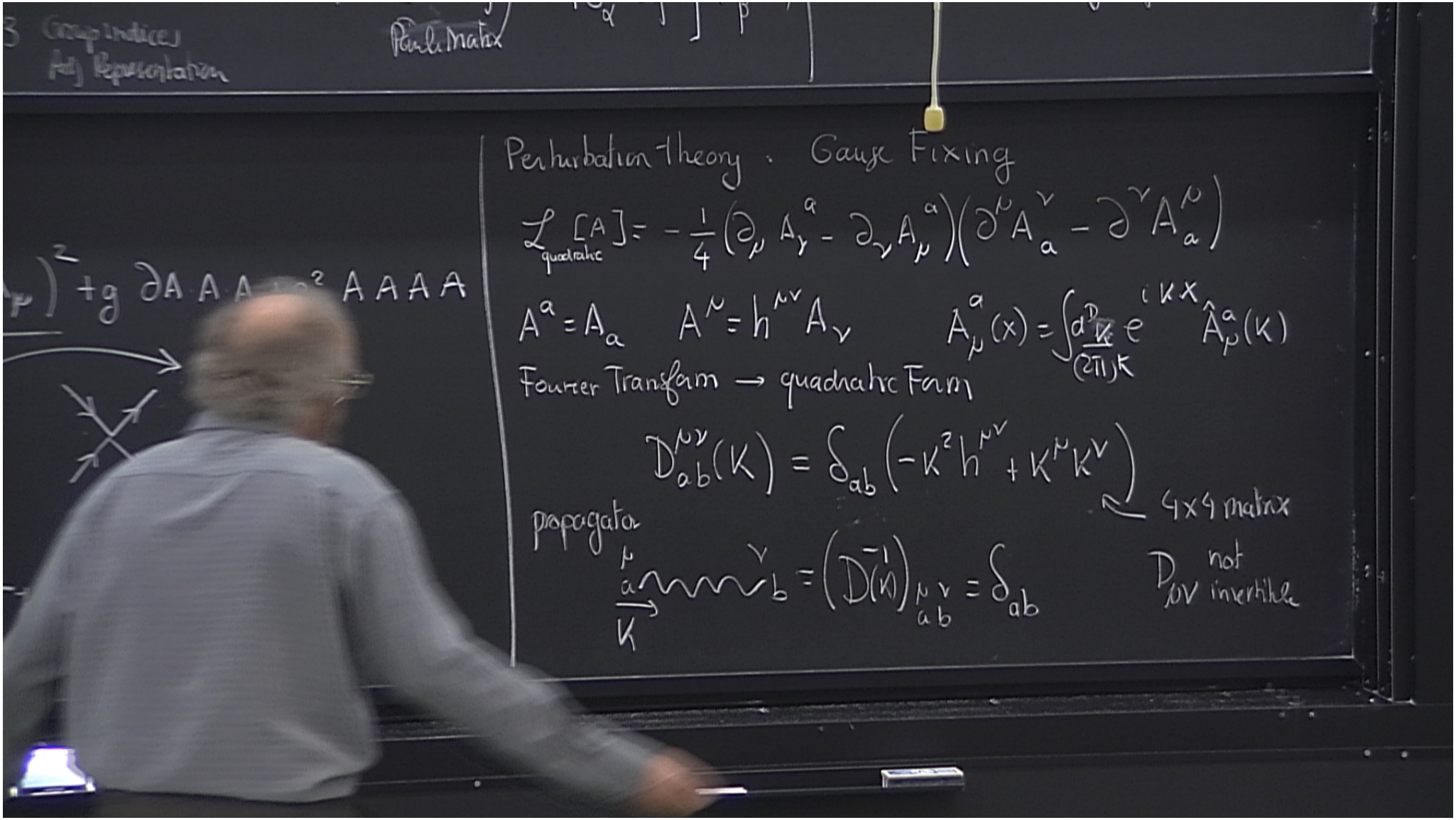
$D^{\mu\nu} A_\nu = 0$ pure gauge

Gauge invariance : as in QED

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Fourier Transform → quadratic Form
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4×4 matrix
not invertible
propagator
$\overrightarrow{K}$
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$A = (A_0, \vec{A})$

Gauge fixing :

Axial Gauge $A_0 = 0$] not Lorentz Invariant

Coulomb Gauge $\vec{\nabla} \cdot \vec{A} = 0$]

Lorentz-Landau Gauge

$\partial^\mu A_\mu = 0$] Lorentz invariant

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YM: not so simple

Gauge fixing (QED: enough) $A = (A_0, \vec{A})$

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General geometrical Picture

Gauge invariance : as in QED

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tz-Landau Gauge

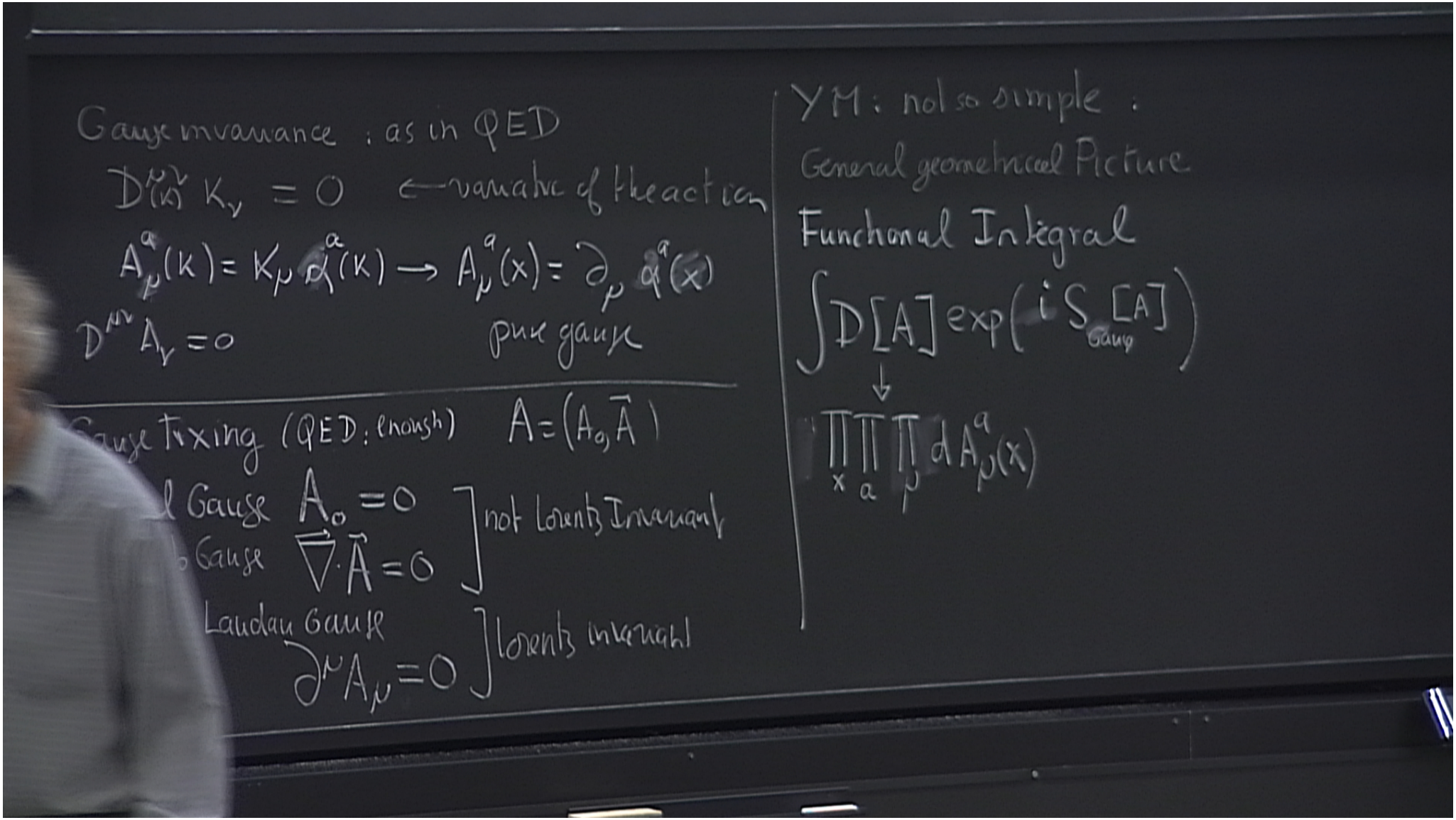
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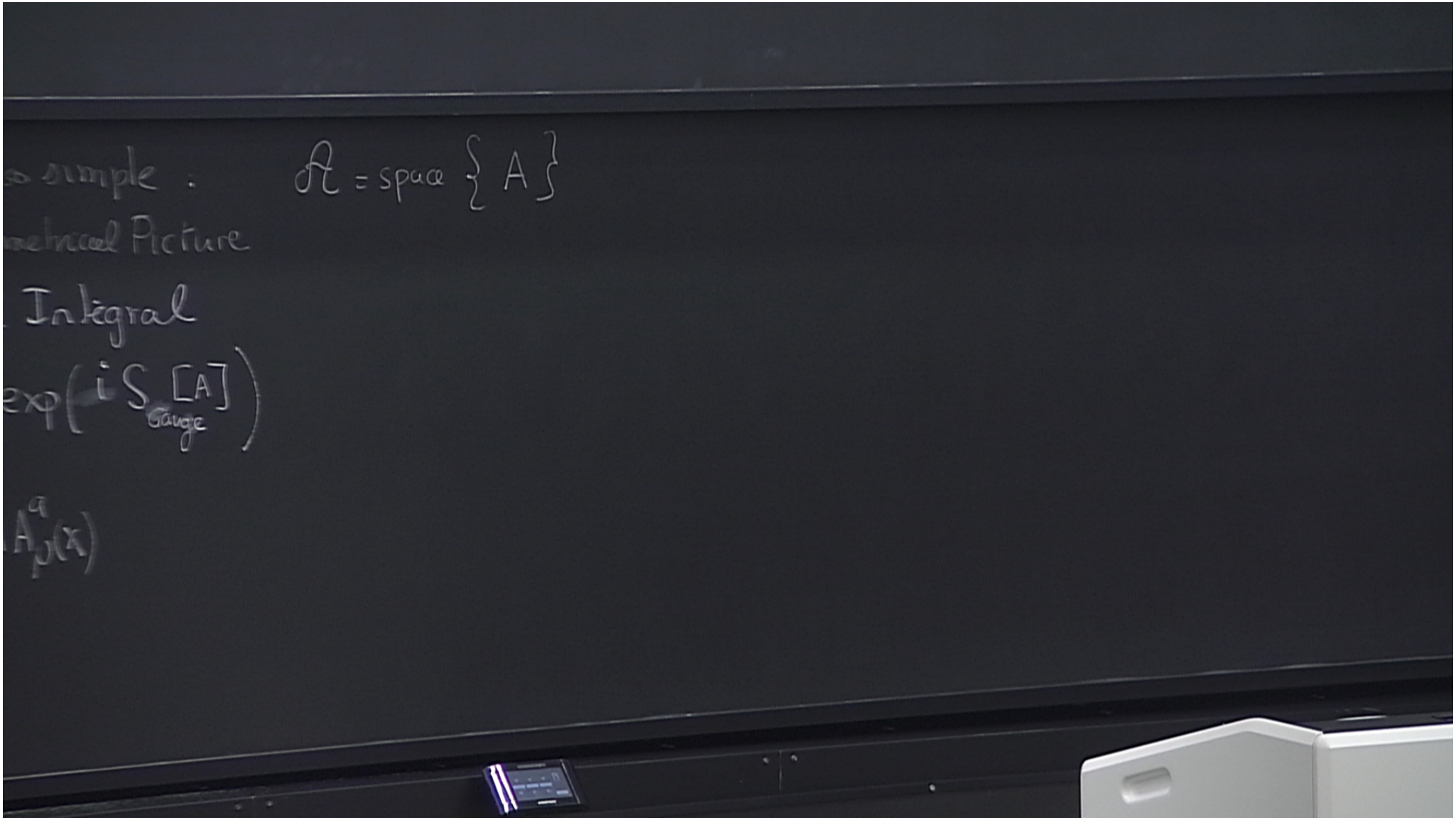
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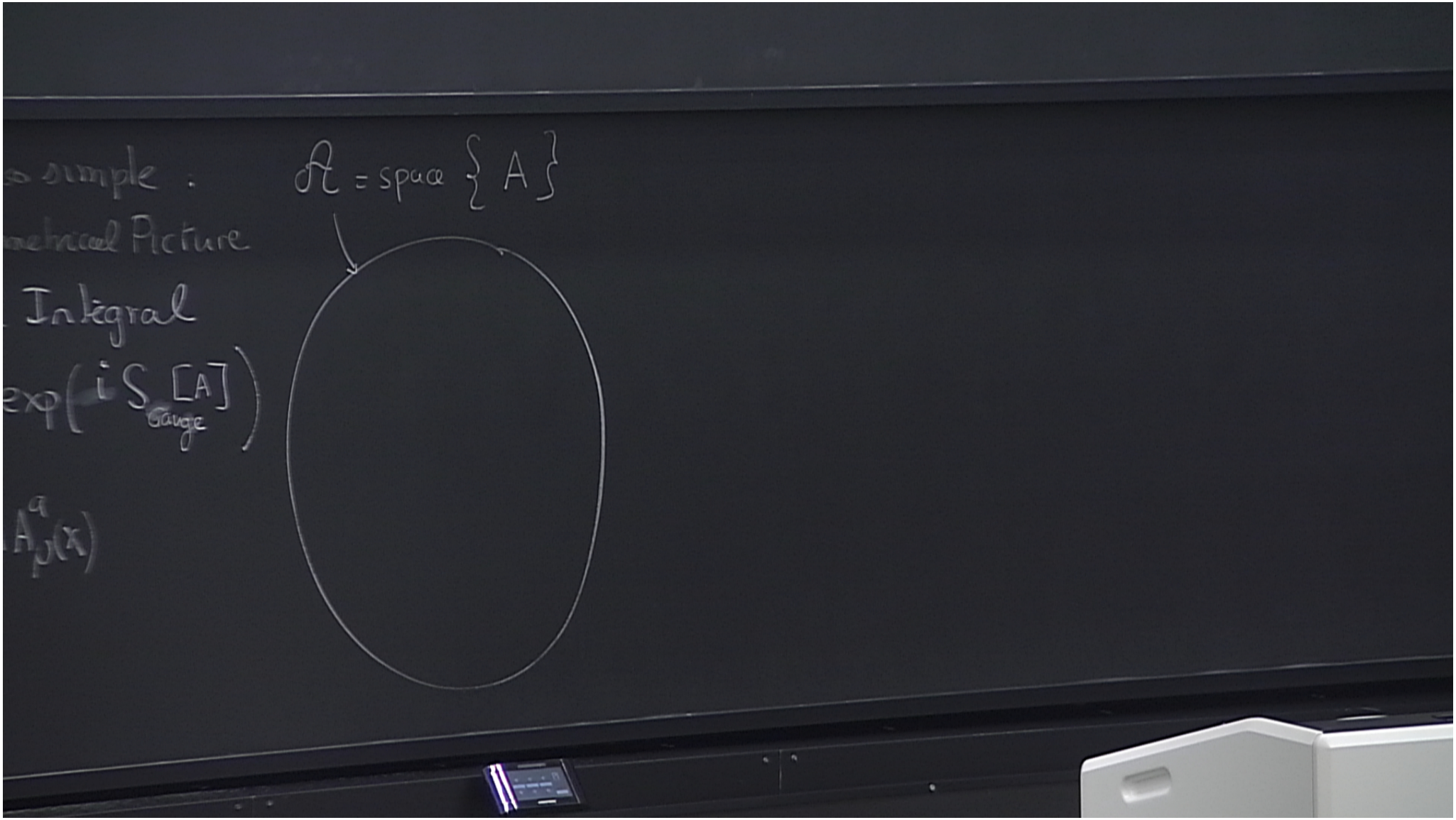
General geometrical Picture

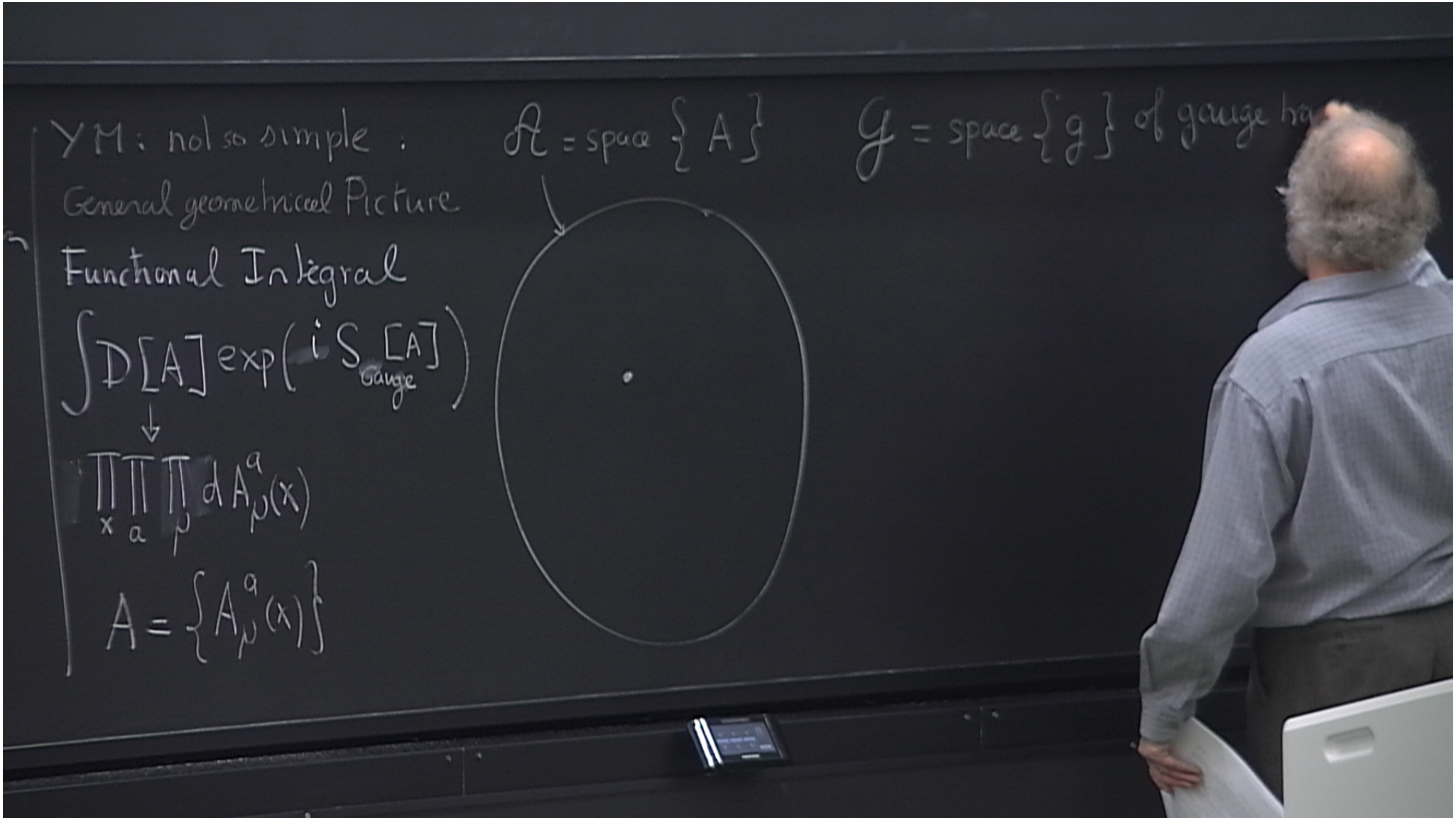
Functional Integral

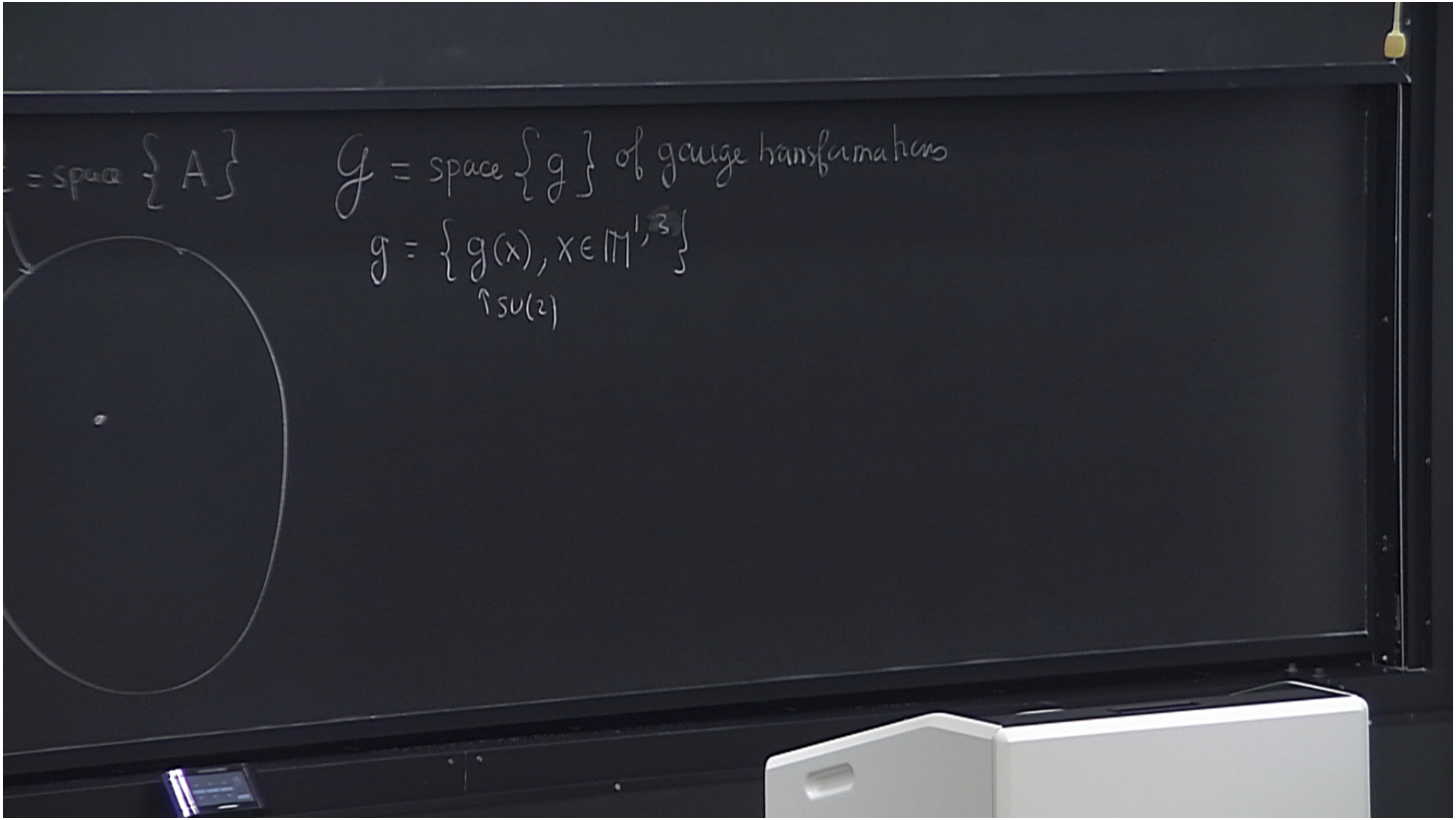
$$\int \mathcal{D}[A] \exp\left(i S_{Gauge}[A] \right)$$


Gauge invariance : as in QED
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Functional Integral
$\int D[A] \exp\left(i S_{Gauge}[A] \right)$
$\prod_x \prod_a \prod_\mu dA^a_\mu(x)$


simple :
$\mathcal{A}$ = space $\{A\}$
metrical Picture
Integral
$\exp\left(iS_{\text{Gauge}}[A]\right)$
$A^a_\mu(x)$


simple .
metrical Picture
Integral
exp(i S [A])
Gauge
A^a_μ(x)
A = space { A }


YM: not so simple :
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$\prod_x \prod_a \prod_\mu dA^a_\mu(x)$
$A = \{A^a_\mu(x)\}$
$\mathcal{A}$ = space $\{A\}$
$\mathcal{G}$ = space $\{g\}$ of gauge


= space { A }
$\mathcal{G}$ = space { g } of gauge transformations
g = { g(x), x ∈ M^{1,3} }
↑ SU(2)

YM: not so simple :

General geometrical Picture

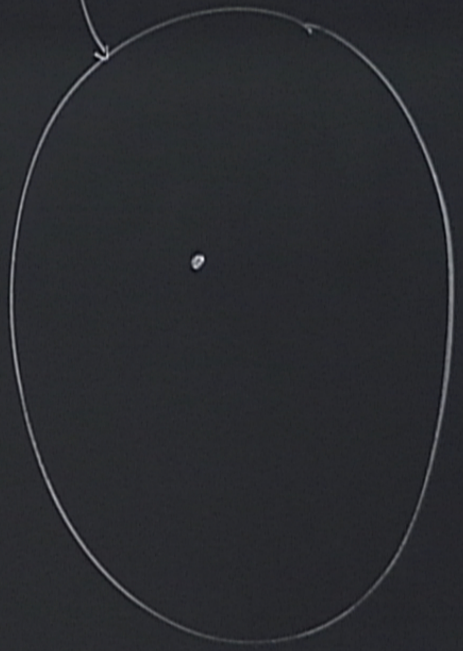
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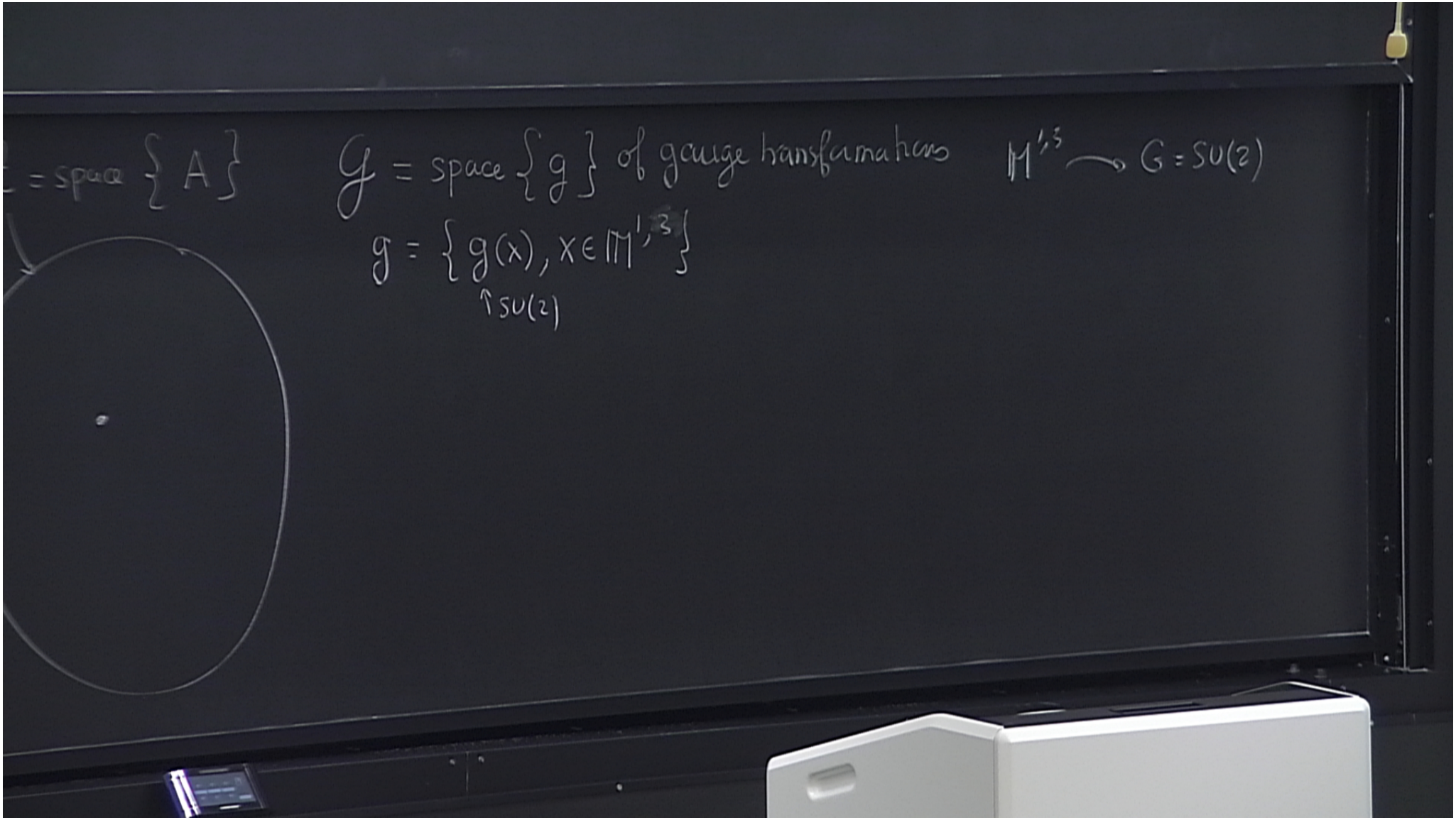
$\mathcal{A}$ = space $\{A\}$

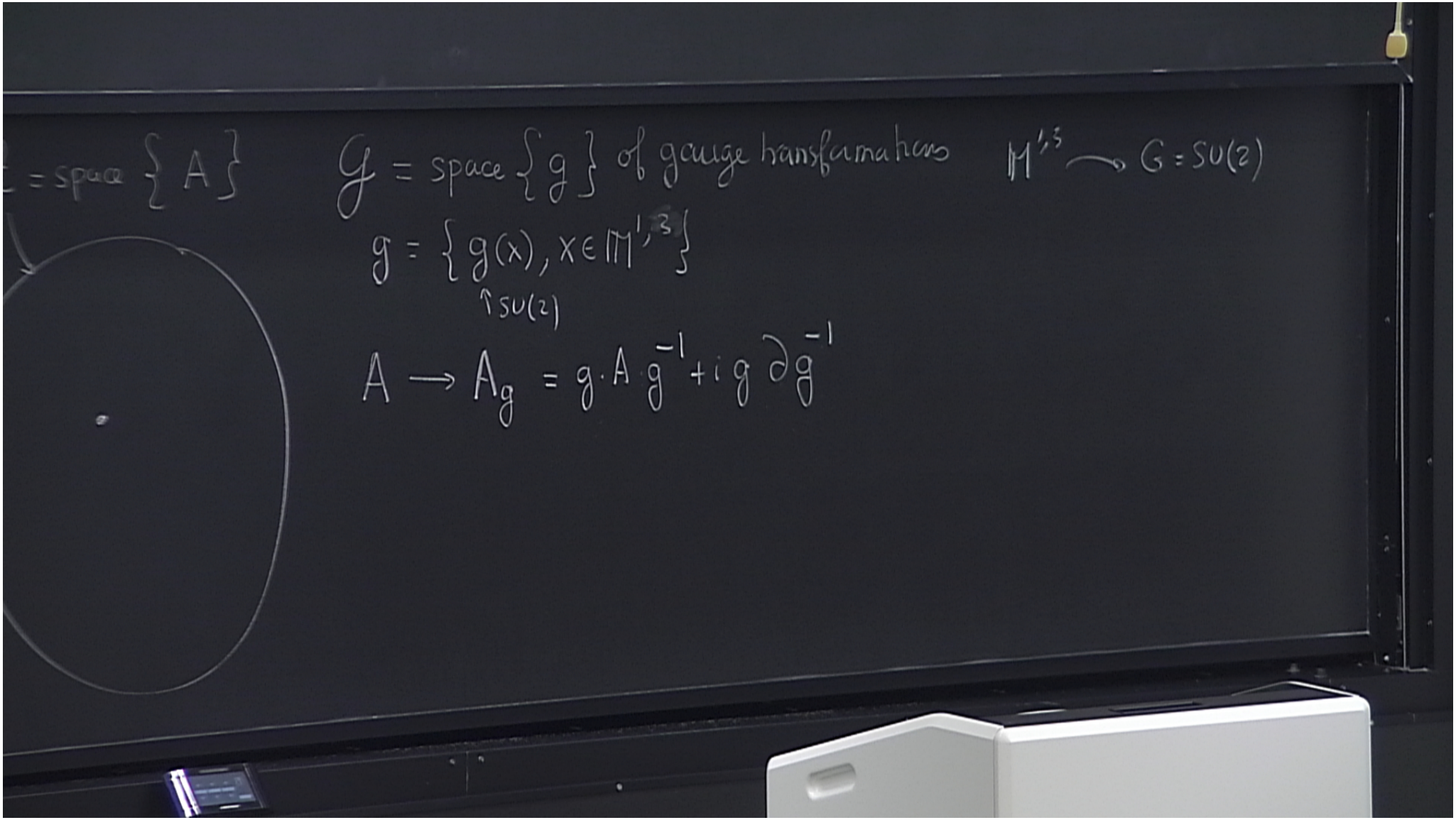


$\mathcal{G}$ = space $\{g\}$ of gauge transf

$$g = \{ g(x), x \in \mathbb{M}^{1,3} \}$$

$\uparrow SU(2)$


= space { A }
𝒢 = space { g } of gauge transformations
M^{1,3} → G = SU(2)
g = { g(x), x ∈ M^{1,3} }
↑ SU(2)


= space { A }
$\mathcal{G}$ = space { g } of gauge transformations $M^{1,3} \to G = SU(2)$
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$\uparrow SU(2)$
$A \to A_g = g \cdot A\, g^{-1} + i\, g\, \partial g^{-1}$

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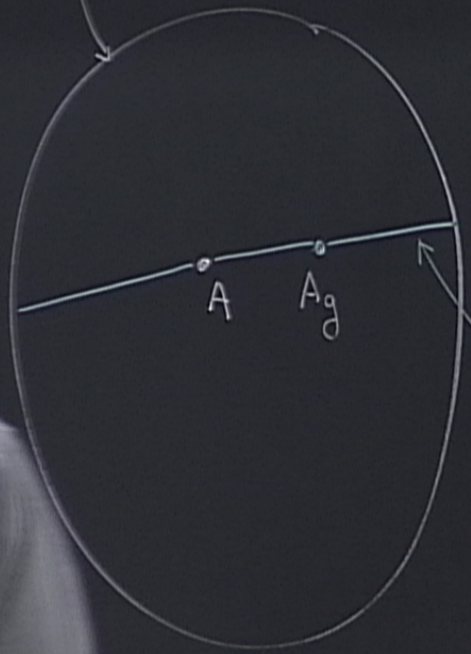
$\mathcal{G}$ = space $\{g\}$ of gauge transformations $\quad M^{1,3} \longrightarrow G = SU(2)$

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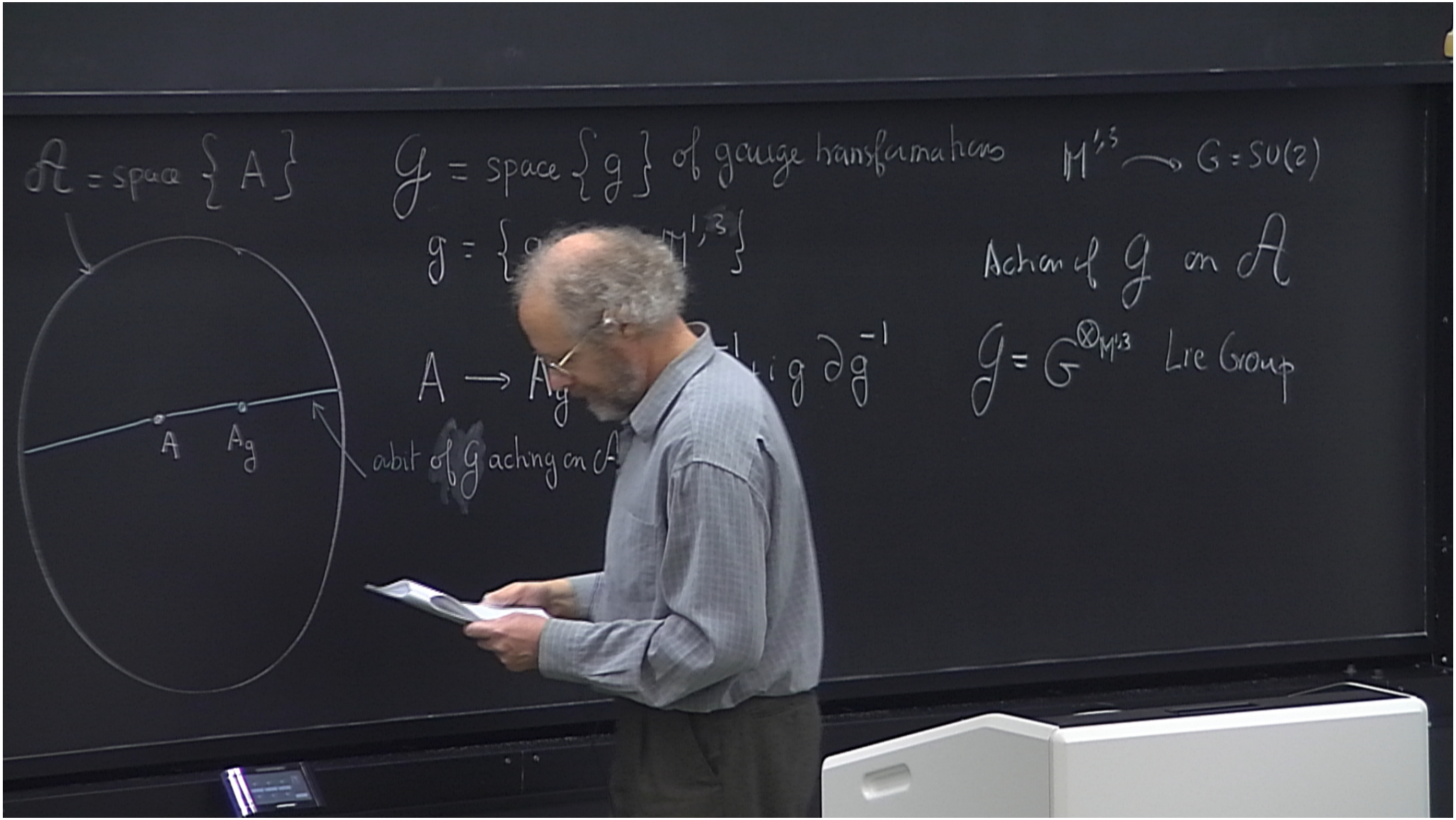
$\uparrow SU(2)$

Action of g on $\mathcal{A}$

$$A \longrightarrow A_g = g \cdot A\, g^{-1} + i\, g\, \partial g^{-1}$$



orbit of g acting on $\mathcal{A}$


$\mathcal{A}$ = space $\{A\}$
$\mathcal{G}$ = space $\{g\}$ of gauge transformations
$M^{1,3} \to G = SU(2)$
$g = \{ \ldots M^{1,3} \}$
Action of $\mathcal{G}$ on $\mathcal{A}$
$A \to A_g$
$\mathcal{G} = G^{\otimes_{M^{1,3}}}$ Lie Group
A
A_g
orbit of $\mathcal{G}$ acting on $\mathcal{A}$

$\mathcal{A}$ = space $\{A\}$

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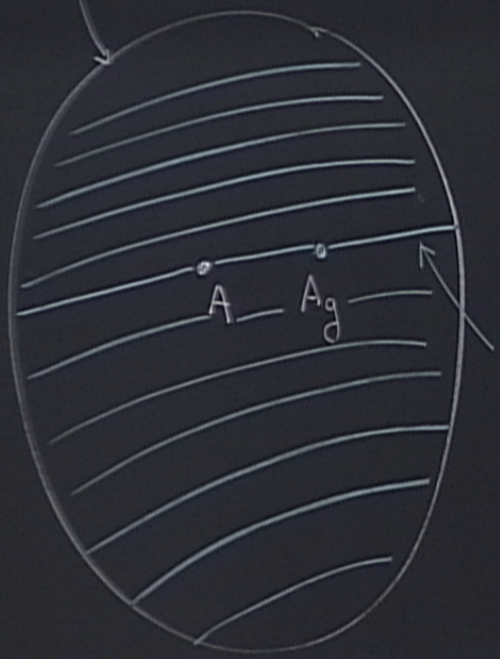
$g = \{g(x), x \in M^{1,3}\}$

$\uparrow SU(2)$

Action of $\mathcal{G}$ on $\mathcal{A}$

$$A \longrightarrow A_g = g \cdot A \cdot g^{-1} + i\, g\, \partial g^{-1}$$

$\mathcal{G} = G^{\otimes_{M^{1,3}}}$ Lie Group



$\mathcal{A}$ = space $\{A\}$

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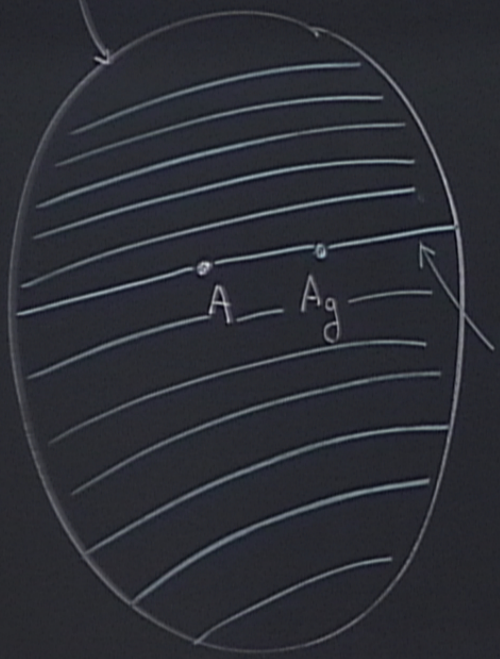
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$\uparrow SU(2)$

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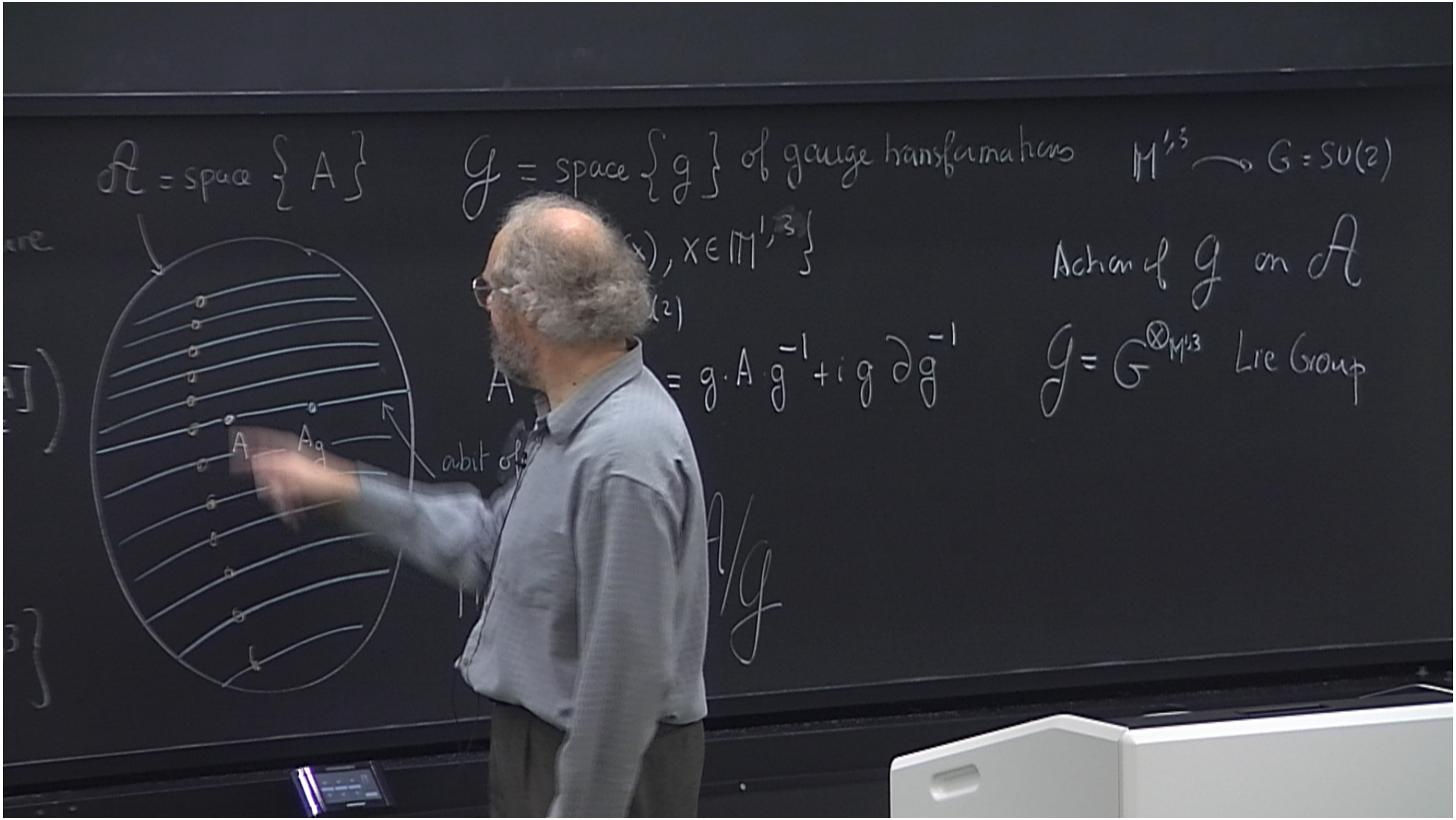
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orbit of $\mathcal{G}$ acting on $\mathcal{A}$

Physical Config = $\mathcal{A}/\mathcal{G}$


$\mathcal{A}$ = space $\{A\}$
$\mathcal{G}$ = space $\{g\}$ of gauge transformations
$M^{1,3} \to G = SU(2)$
Action of $\mathcal{G}$ on $\mathcal{A}$
$\mathcal{G} = G^{\otimes_{M^{1,3}}}$ Lie Group
$A^g = g \cdot A \cdot g^{-1} + i\, g\, \partial g^{-1}$
A
Ag
orbit of
$\mathcal{A}/\mathcal{G}$

$\mathcal{A}$ = space $\{A\}$

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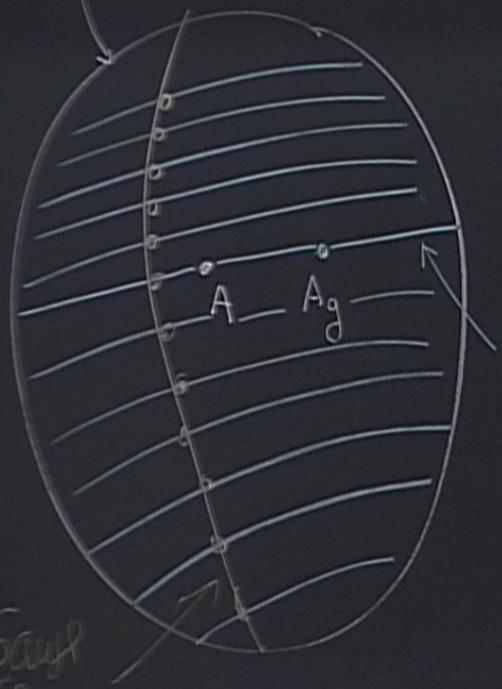
$g = \{g(x), x \in M^{1,3}\}$

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orbit of $\mathcal{G}$ acting on $\mathcal{A}$

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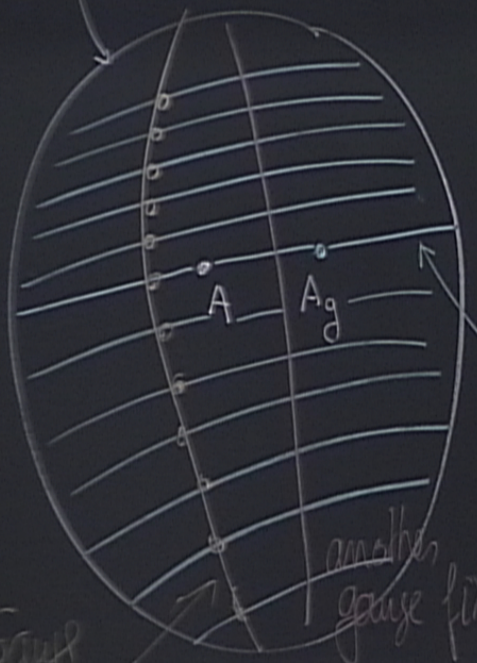
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Action of $\mathcal{G}$ on $\mathcal{A}$

$A \longrightarrow A_g = g \cdot A \, g^{-1} + i g \, \partial g^{-1}$

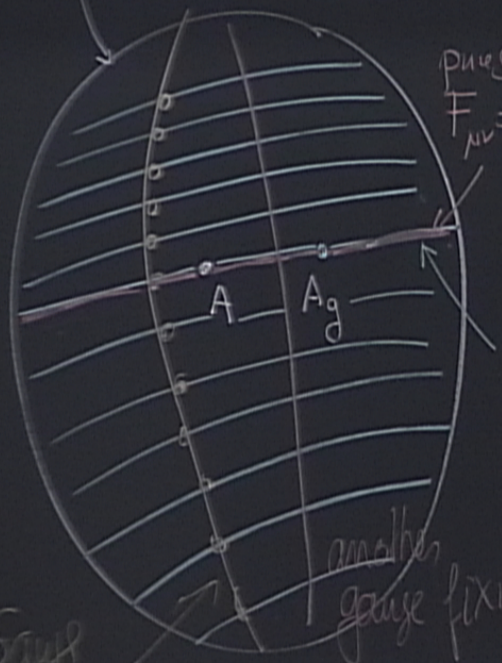
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orbit of $\mathcal{G}$ acting on $\mathcal{A}$

Physical Config = $\mathcal{A}/\mathcal{G}$

$\mathcal{A}$ = space $\{A\}$ $\quad$ $\mathcal{G}$ = space $\{g\}$ of gauge transformations $\quad \mathbb{M}^{1,3} \longrightarrow G = SU(2)$



puissaux $F_{\mu\nu} = 0$

$g = \{ g(x), x \in \mathbb{M}^{1,3} \}$
$\uparrow SU(2)$

Action of $\mathcal{G}$ on $\mathcal{A}$

$A \longrightarrow A_g = g \cdot A \, g^{-1} + i \, g \, \partial g^{-1}$

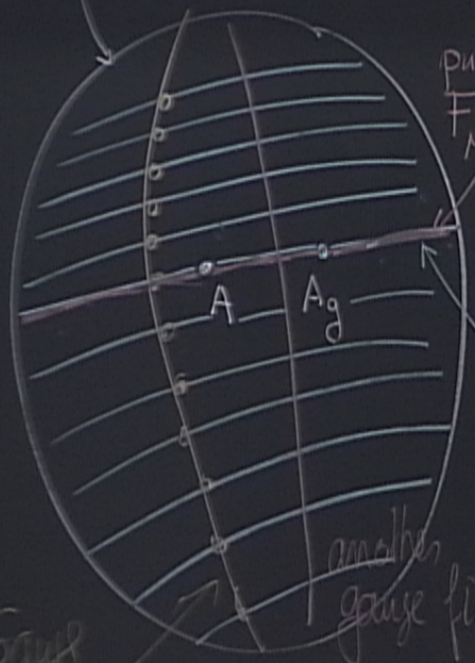
$\mathcal{G} = G^{\otimes_{\mathbb{M}^{1,3}}}$ Lie Group

orbit of $\mathcal{G}$ acting on $\mathcal{A}$

Physical Config = $\mathcal{A}/\mathcal{G}$

Gauge fixing

$\mathcal{A}$ = space $\{A\}$ $\quad$ $\mathcal{G}$ = space $\{g\}$ of gauge transformations $\quad M^{1,3} \longrightarrow G = SU(2)$



puregauge $F_{\mu\nu} = 0$

$g = \{ g(x), x \in M^{1,3} \}$

$\uparrow SU(2)$

Action of $\mathcal{G}$ on $\mathcal{A}$

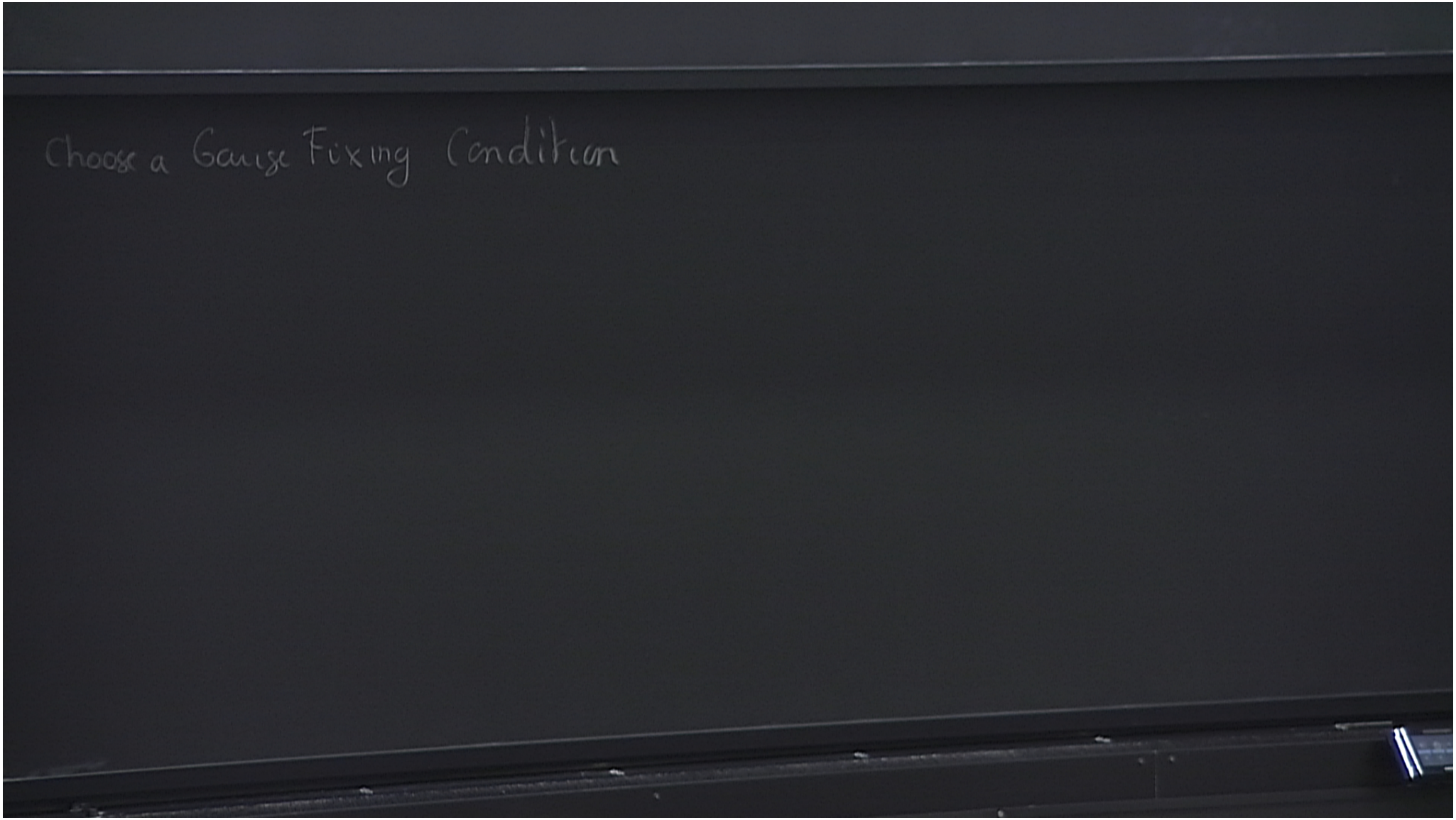
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$\mathcal{G} = G^{\otimes_{M^{1,3}}}$ Lie Group

orbit of $\mathcal{G}$ acting on $\mathcal{A}$

Physical Config = $\mathcal{A}/\mathcal{G}$

another gauge fixing


Choose a Gauge Fixing Condition

Choose a Gauge Fixing Condition = choose a slice

$$F[A] = 0$$

Choose a Gauge Fixing Condition = choose a slice

$F[A] = 0$ choose a function $F[A] = \{F^a(A, \partial A, \ldots)(x)\}$

Lorentz Gauge $F^a[A] = \partial^\mu A^a_\mu(x)$

Choose a Gauge Fixing Condition = choose a slice

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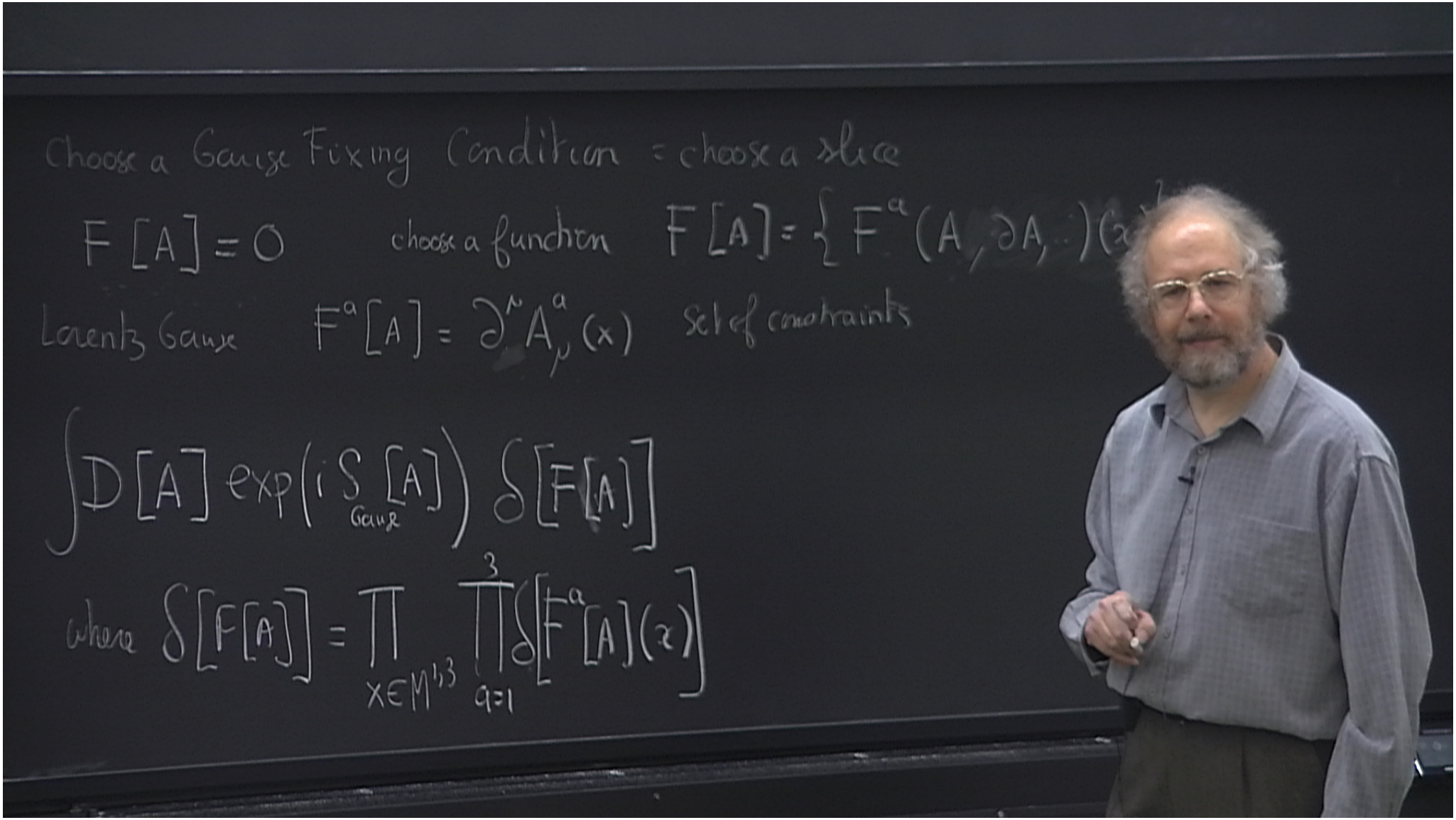
Lorentz Gauge $F^a[A] = \partial^\mu A^a_\mu(x)$ Set of constraints

Choose a Gauge Fixing Condition = choose a slice

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$$\int \mathcal{D}[A] \exp\left(i S_{\text{Gauge}}[A] \right)$$


Choose a Gauge Fixing Condition = choose a slice
F[A] = 0 choose a function F[A] = { F^a(A, ∂A, ...)(x)
Lorentz Gauge F^a[A] = ∂^μ A^a_μ(x) Set of constraints
∫ D[A] exp(i S_Gauge[A]) δ[F[A]]
where δ[F[A]] = Π_{x∈M^{1,3}} Π_{a=1}^{3} δ[F^a[A](x)]

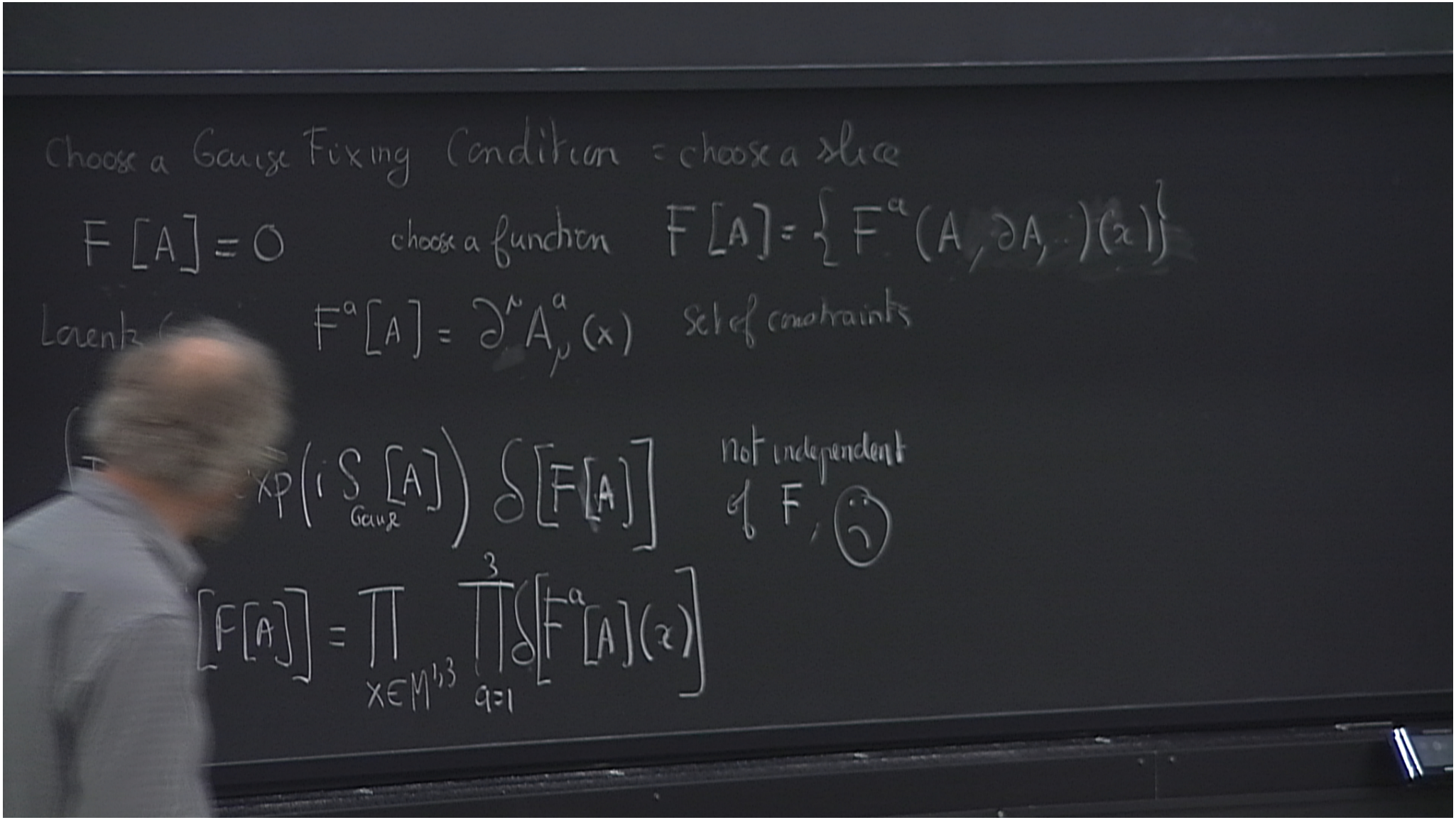
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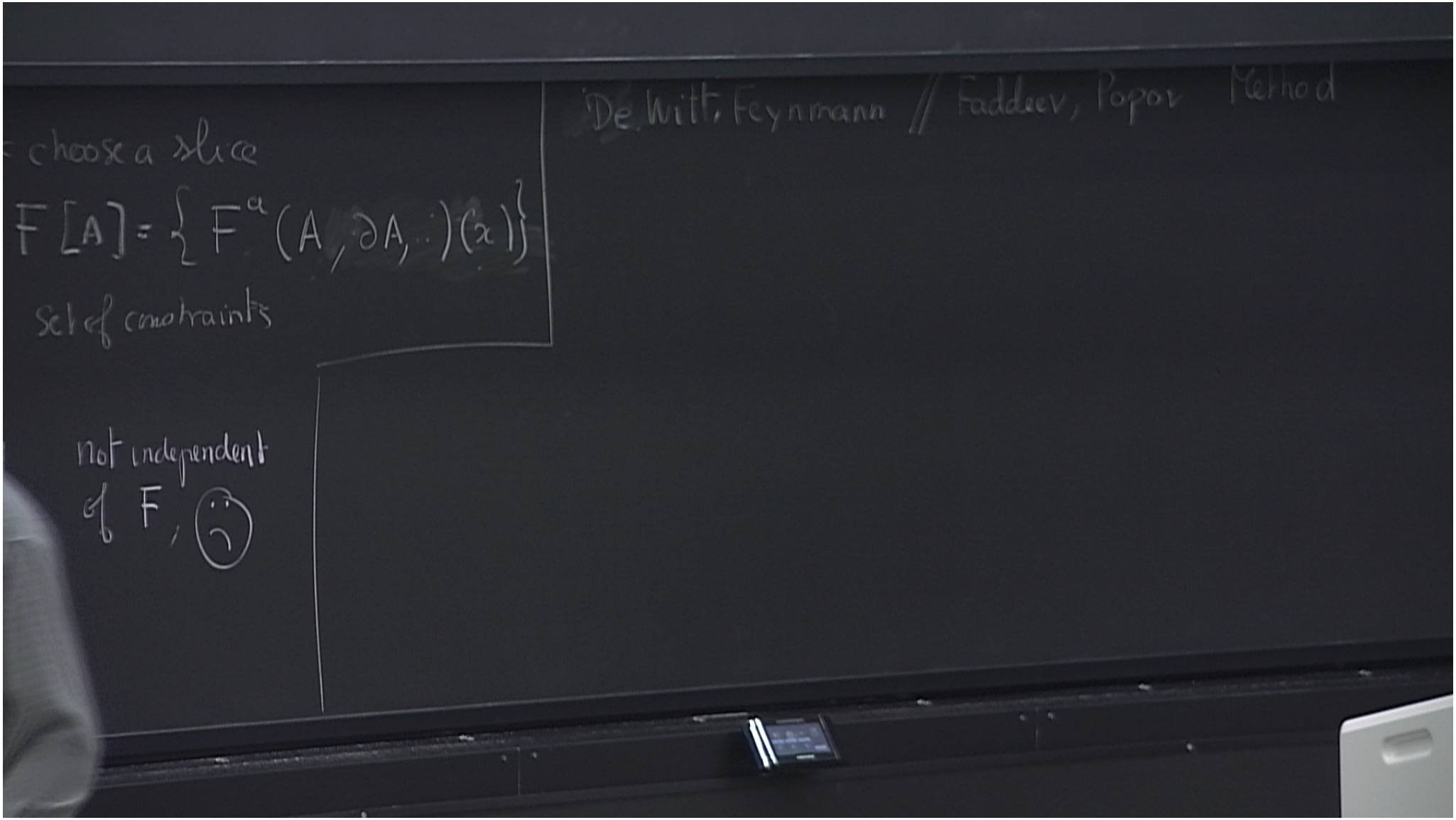
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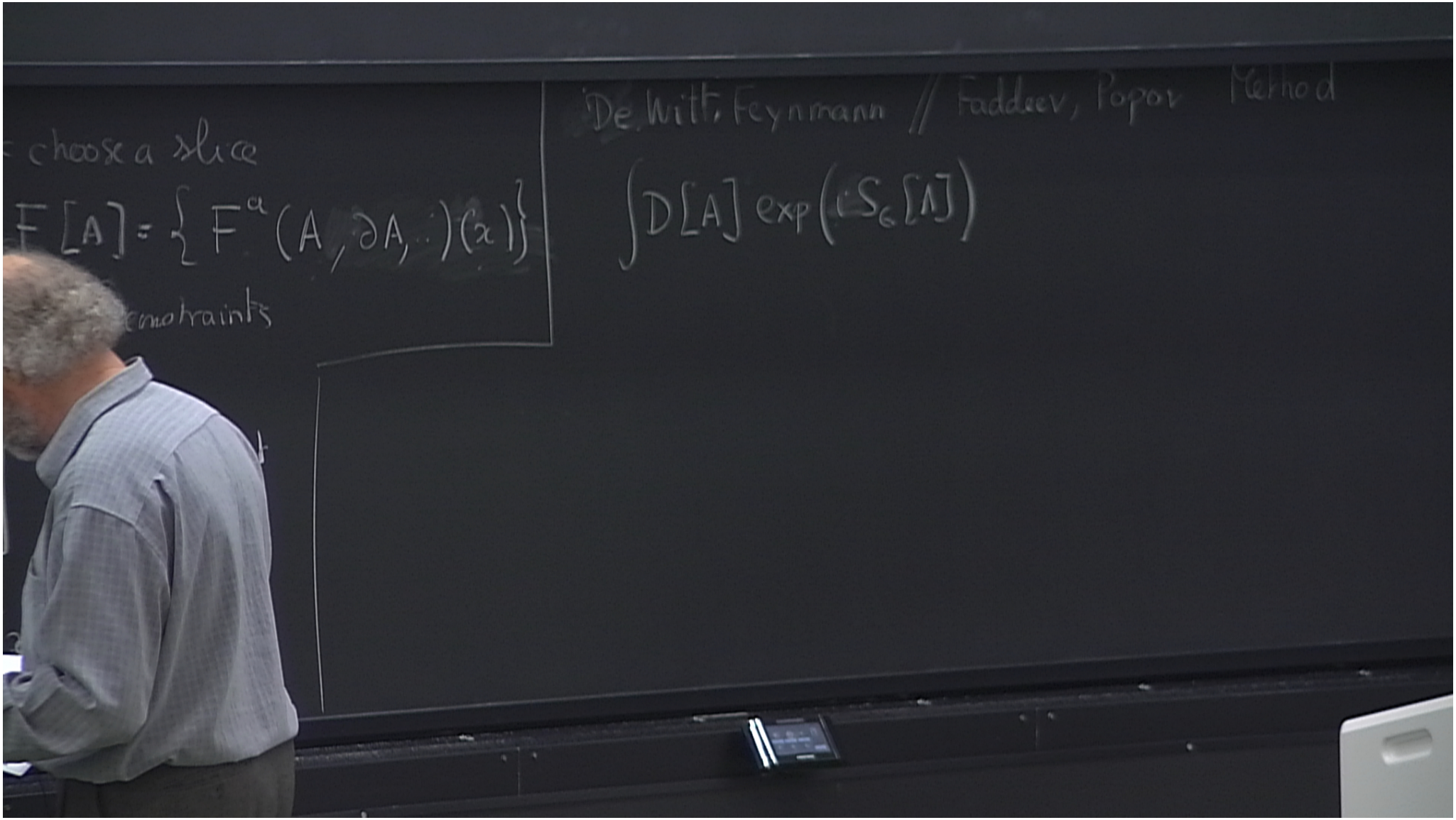
Lorentz Gauge $F^a[A] = \partial^\nu A^a_\mu(x)$ Set of constraints

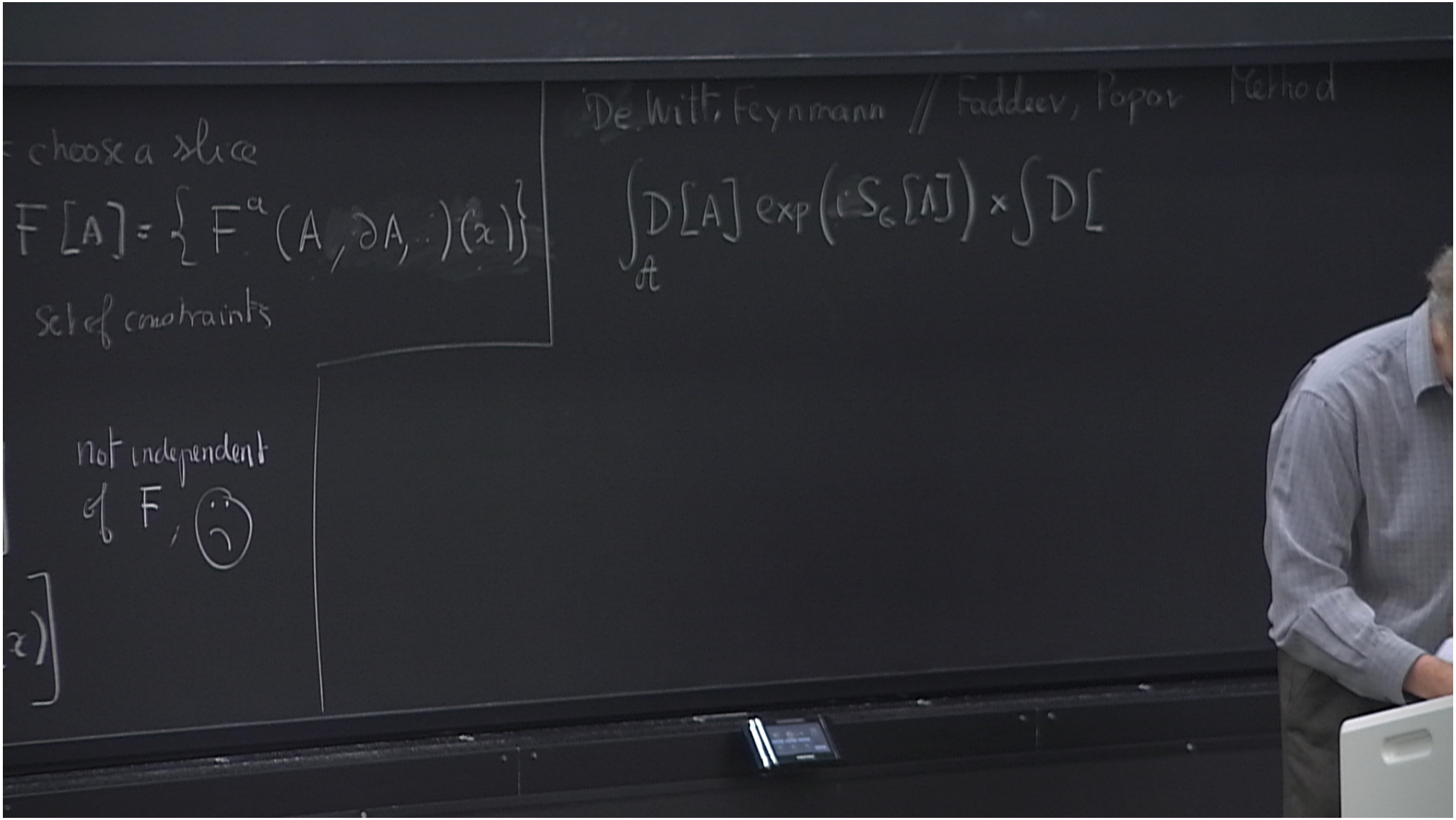
$$\int \mathcal{D}[A] \exp\left(i S_{\text{Gauge}}[A]\right) \delta[F[A]]$$ not independent of F,

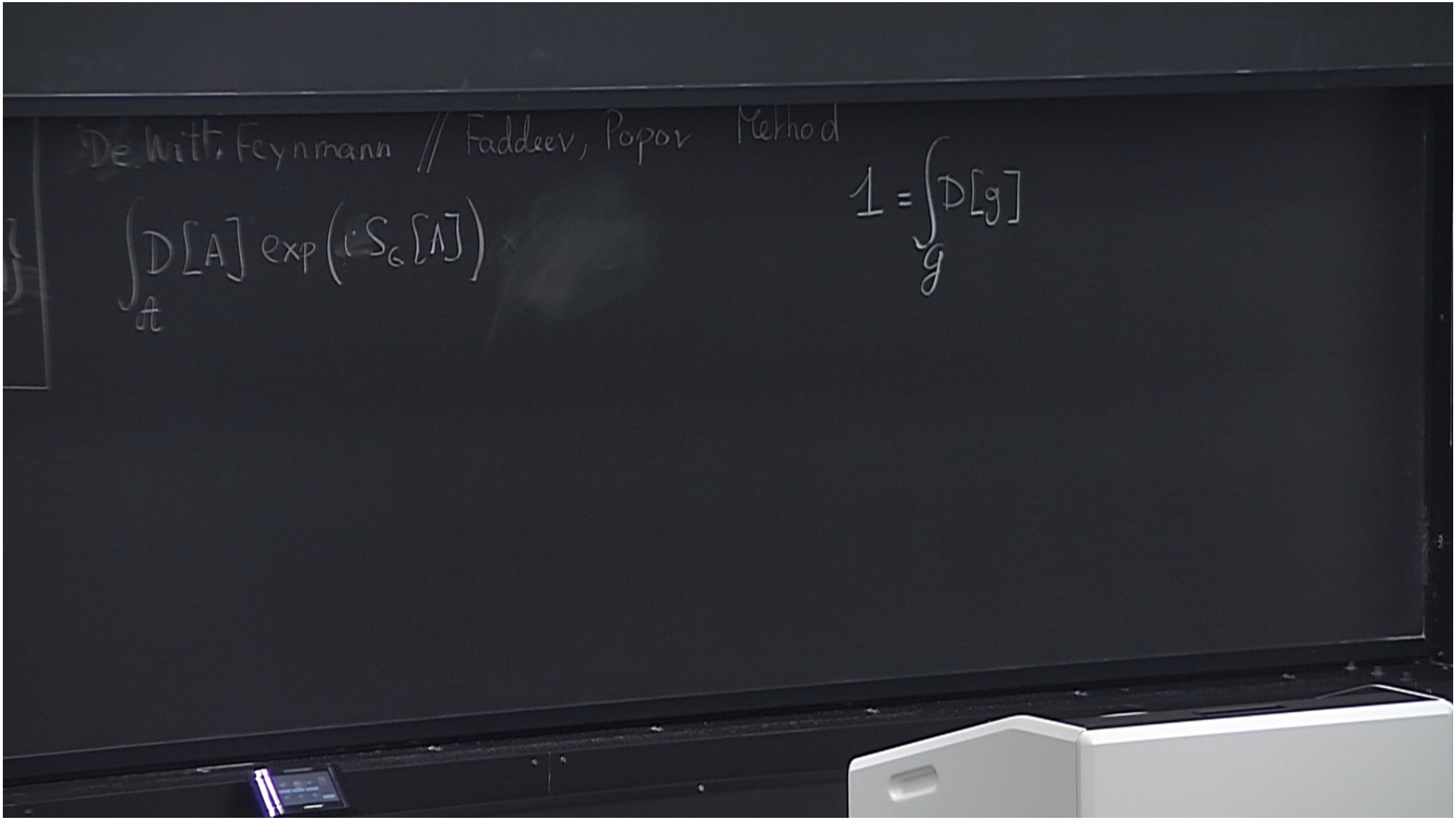
where $\delta[F[A]] = \prod_{x \in M^{1,3}} \prod_{a=1}^{3} \delta[F^a[A](x)]$

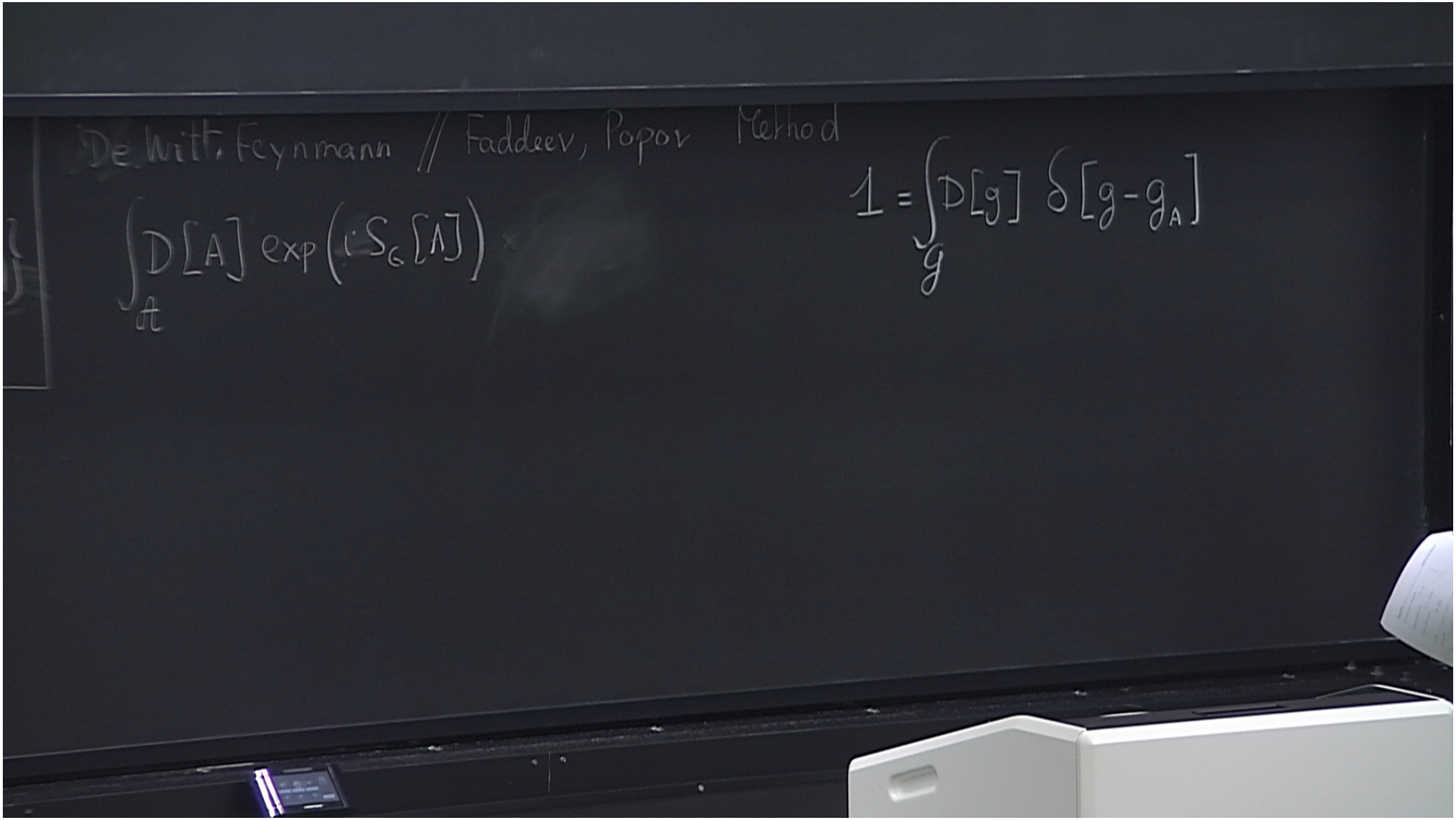

Choose a Gauge Fixing Condition = choose a slice
F[A] = 0
choose a function
F[A] = {F^a(A, ∂A, ...)(x)}
Lorentz
F^a[A] = ∂^ν A^a_μ(x)
set of constraints
not independent
of F, ☹
[F[A]] = ∏_{x∈M^{1,3}} ∏_{a=1}^{3} δ[F^a[A](x)]

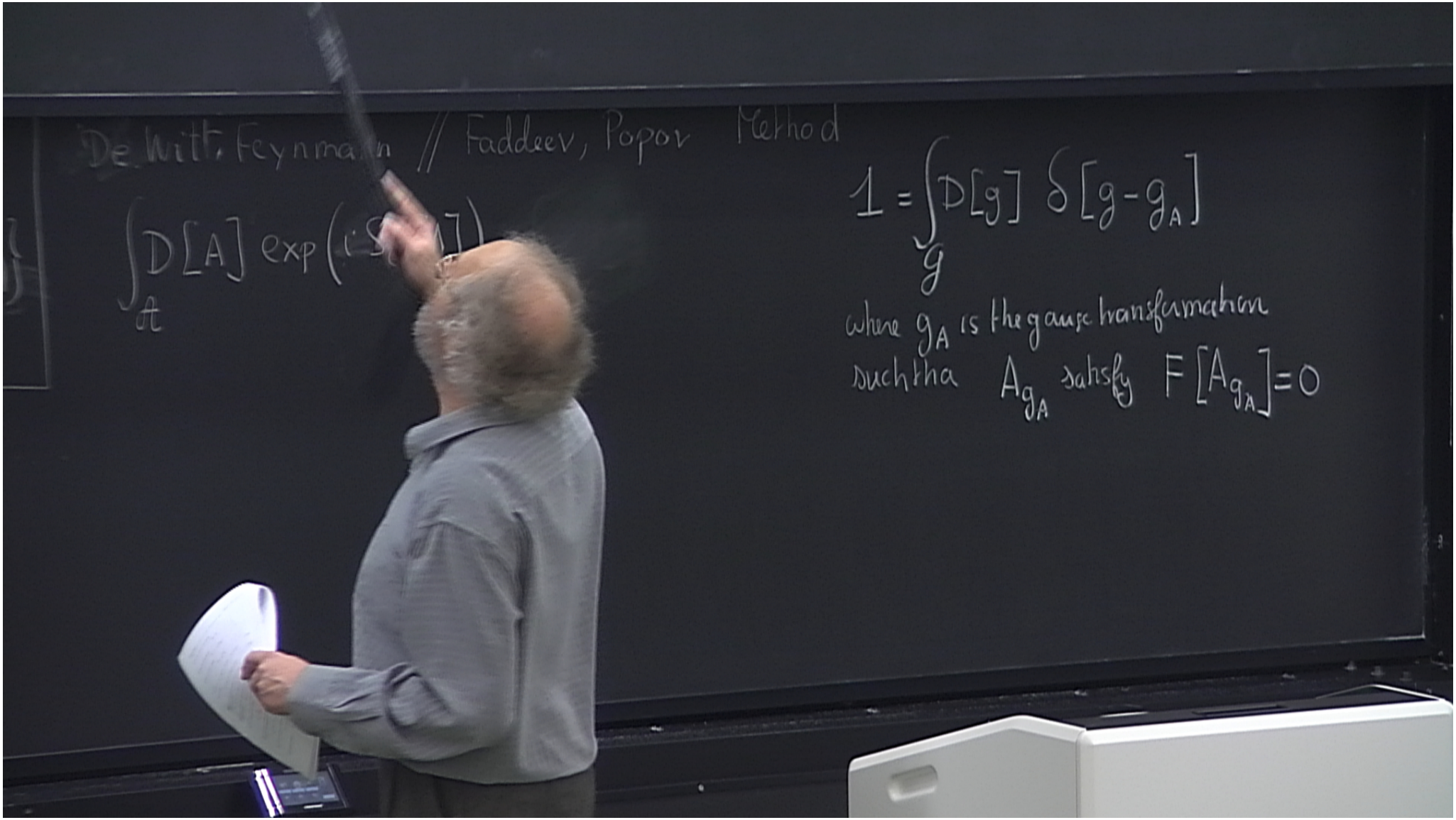

choose a slice
$F[A] = \{F^a(A, \partial A, \ldots)(x)\}$
Set of constraints
not independent of F,
De Witt, Feynmann // Faddeev, Popov Method

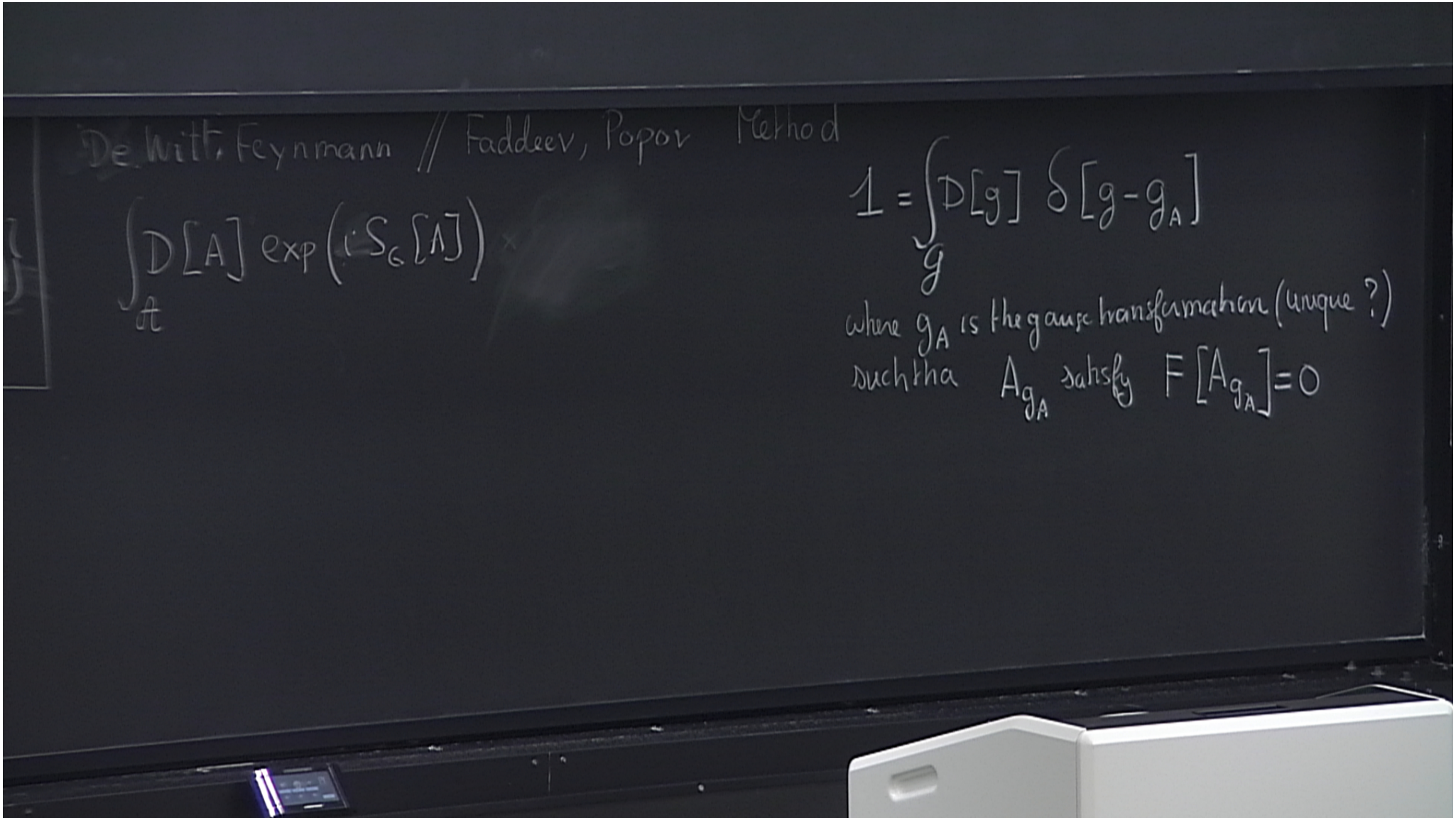

choose a slice
$F[A] = \{F^a(A, \partial A, \ldots)(x)\}$
constraints
De Witt, Feynmann // Faddeev, Popov Method
$\int D[A] \exp(iS_{GF}[A])$

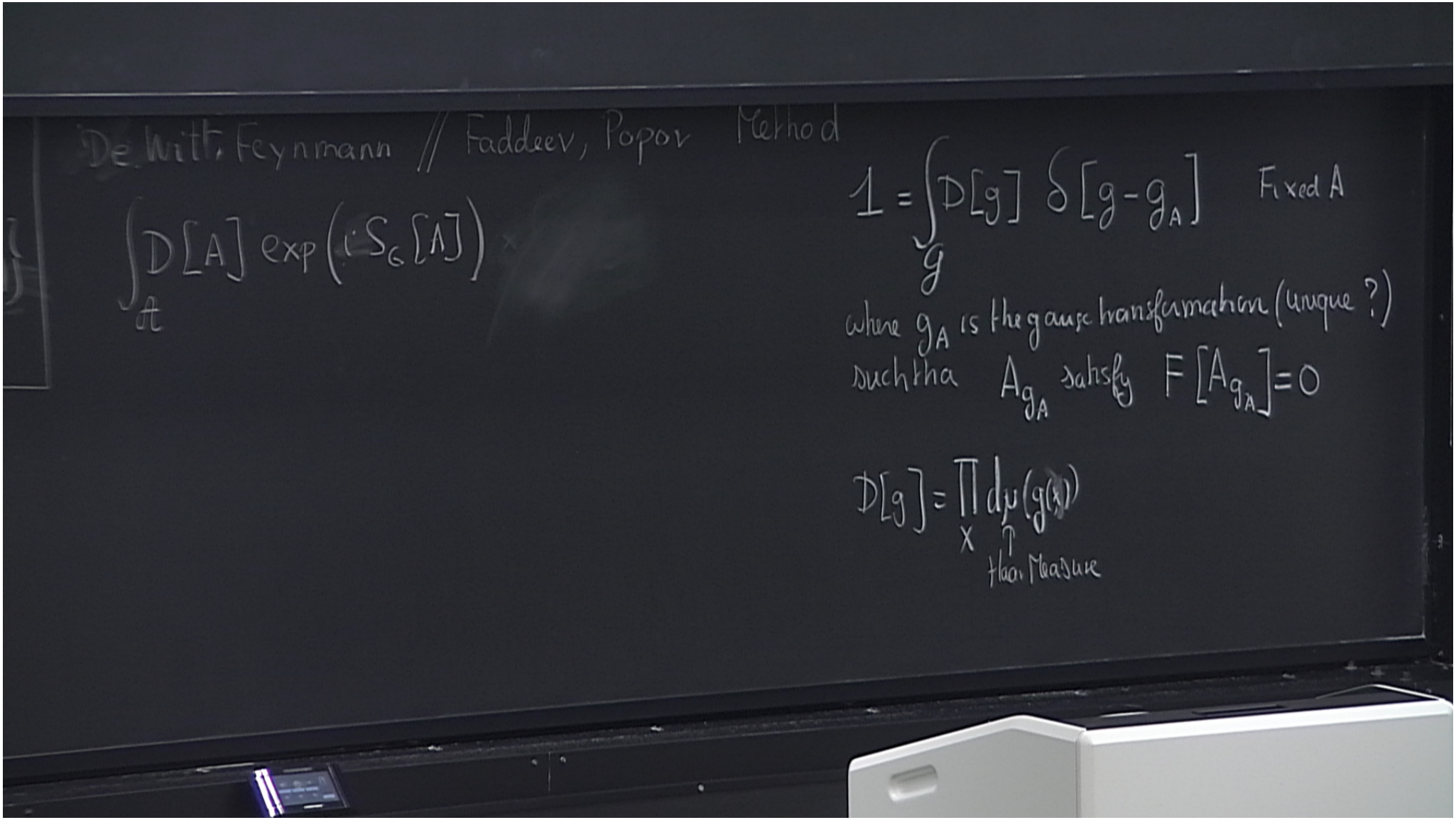

choose a slice
$F[A] = \{F^a(A, \partial A, \ldots)(x)\}$
Set of constraints
not independent of F,
De Witt, Feynmann // Faddeev, Popov Method
$\int_{\mathcal{A}} \mathcal{D}[A] \exp(iS_{\text{eff}}[A]) \times \int \mathcal{D}[$

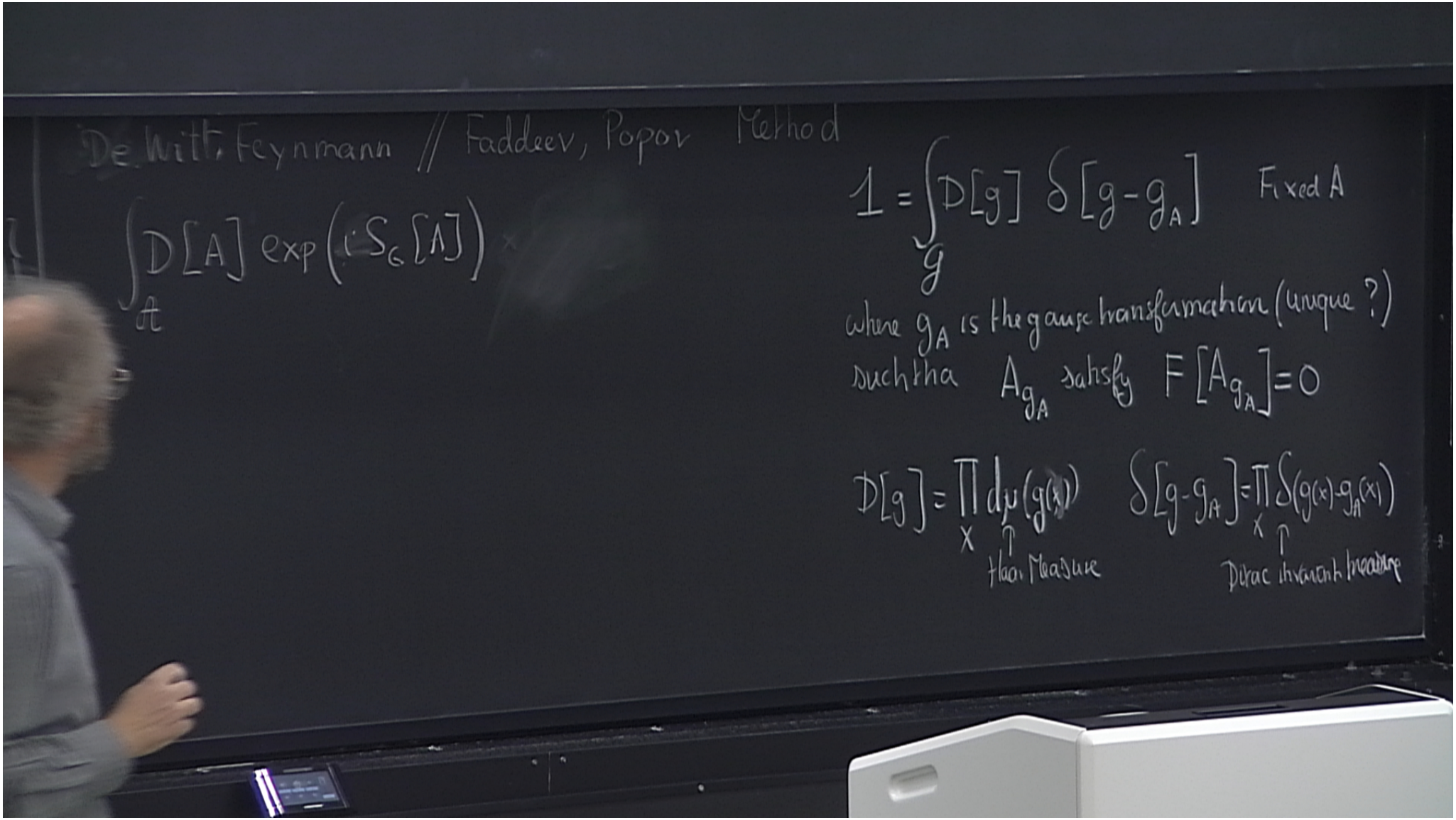

De Witt, Feynmann // Faddeev, Popov Method
$\int_A D[A] \exp\left(i S_G[A]\right) \times$
$1 = \int_g D[g]$

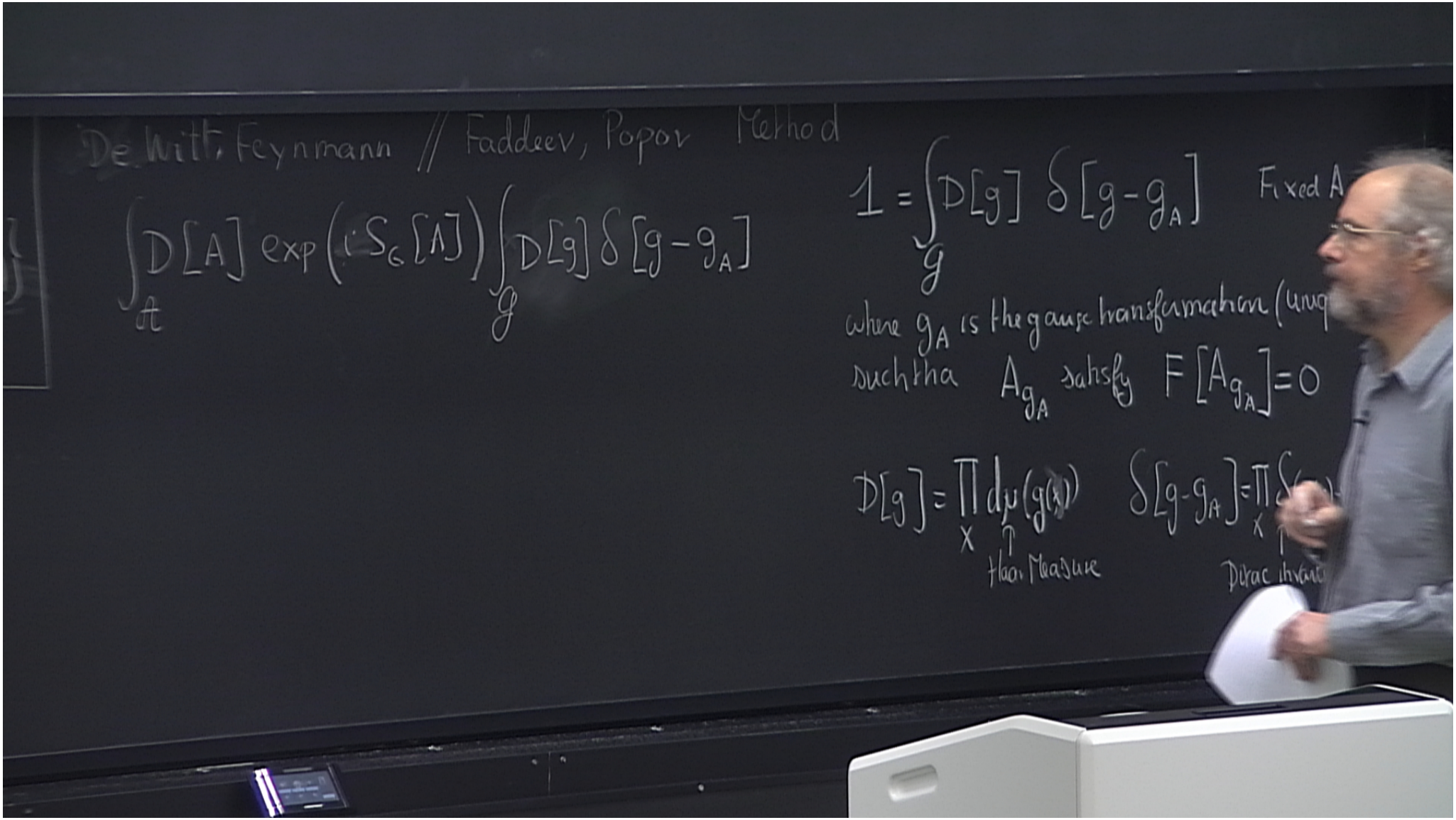

De Witt, Feynmann // Faddeev, Popov Method
$\int_{\mathcal{A}} \mathcal{D}[A] \exp\left(i S_G[A]\right) \times$
$1 = \int_{\mathcal{G}} \mathcal{D}[g]\, \delta[g - g_A]$

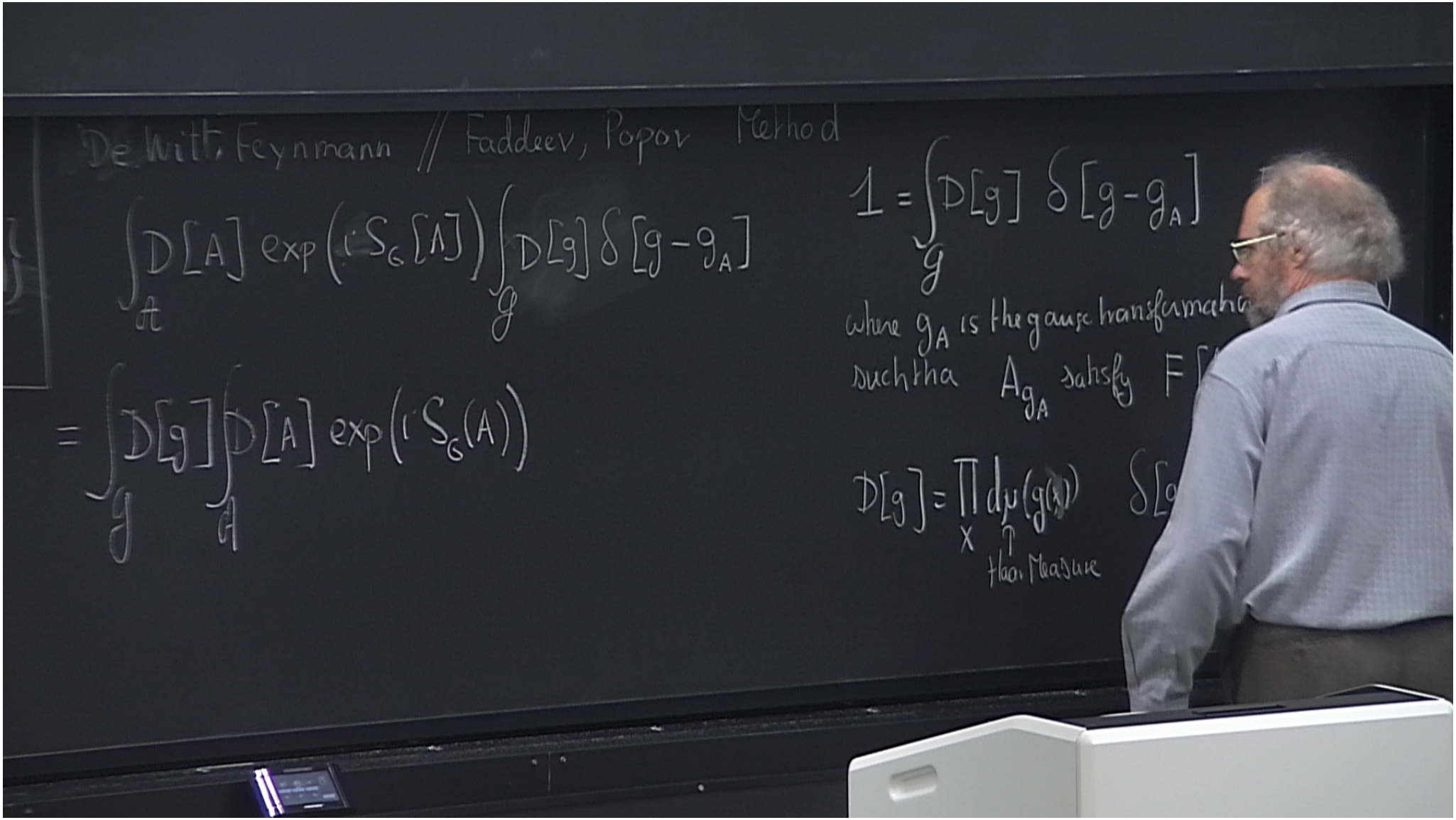

De Witt, Feynman // Faddeev, Popov Method
$\int_{\mathcal{A}} \mathcal{D}[A] \exp(iS[A])$
$1 = \int_{\mathcal{G}} \mathcal{D}[g]\, \delta[g - g_A]$
where g_A is the gauge transformation
such that A_{g_A} satisfy $F[A_{g_A}] = 0$

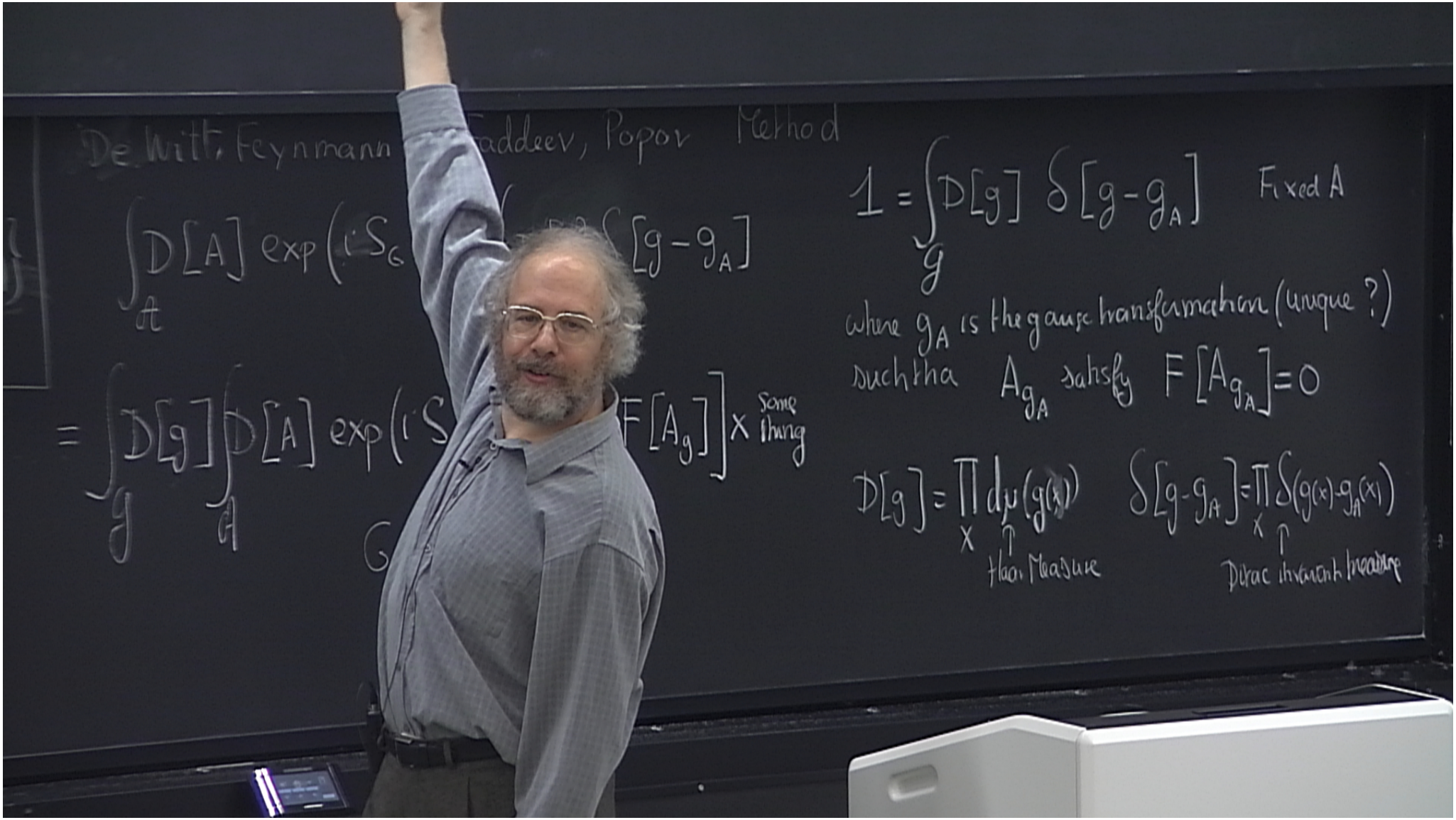

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$\int_{\mathcal{A}} D[A] \exp\left(i S_G[A]\right) \times$
$1 = \int_{\mathcal{G}} D[g]\, \delta[g - g_A]$
where g_A is the gauge transformation (unique ?)
such tha A_{g_A} satisfy $F[A_{g_A}] = 0$

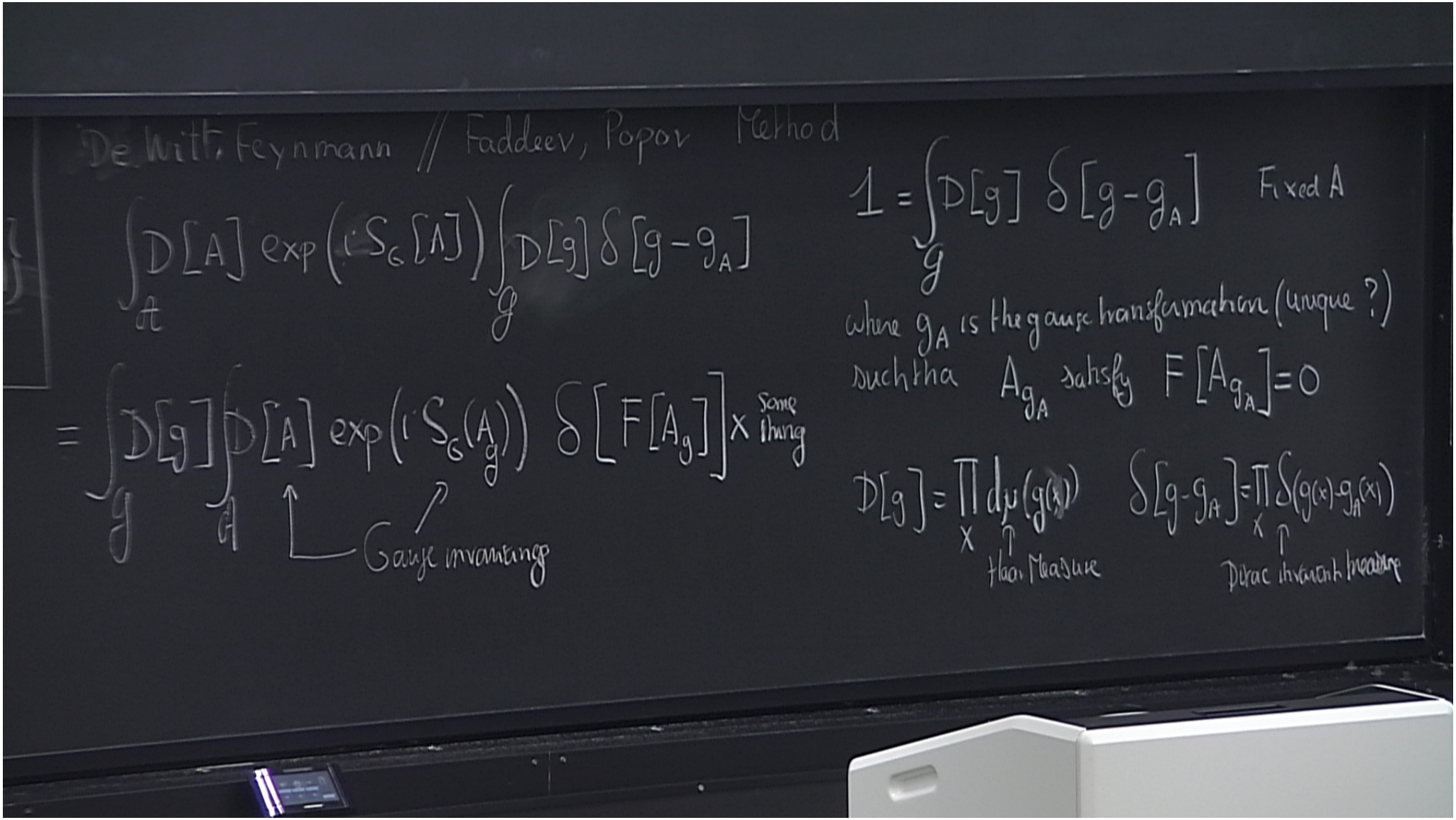

De Witt, Feynmann // Faddeev, Popov Method
$\int_{\mathcal{A}} \mathcal{D}[A] \exp\left(i S_G[A]\right) \times$
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where g_A is the gauge transformation (unique ?)
such tha A_{g_A} satisfy $F[A_{g_A}] = 0$
$\mathcal{D}[g] = \prod_x d\mu(g(x))$
Haar Measure

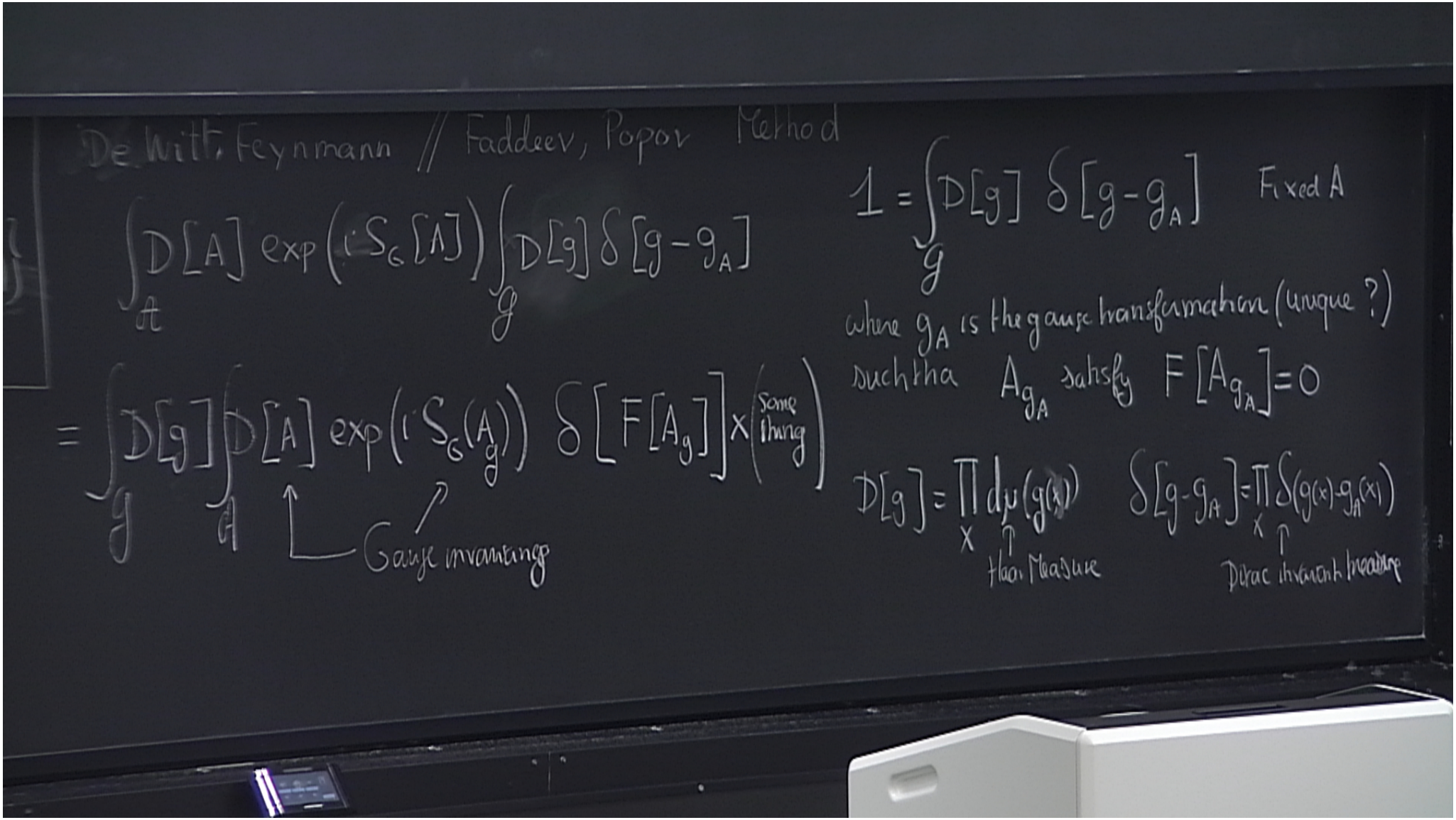

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$D[g] = \prod_x d\mu(g(x))$
Haar Measure
$\delta[g - g_A] = \prod_x \delta(g(x) - g_A(x))$
Dirac invariant measure

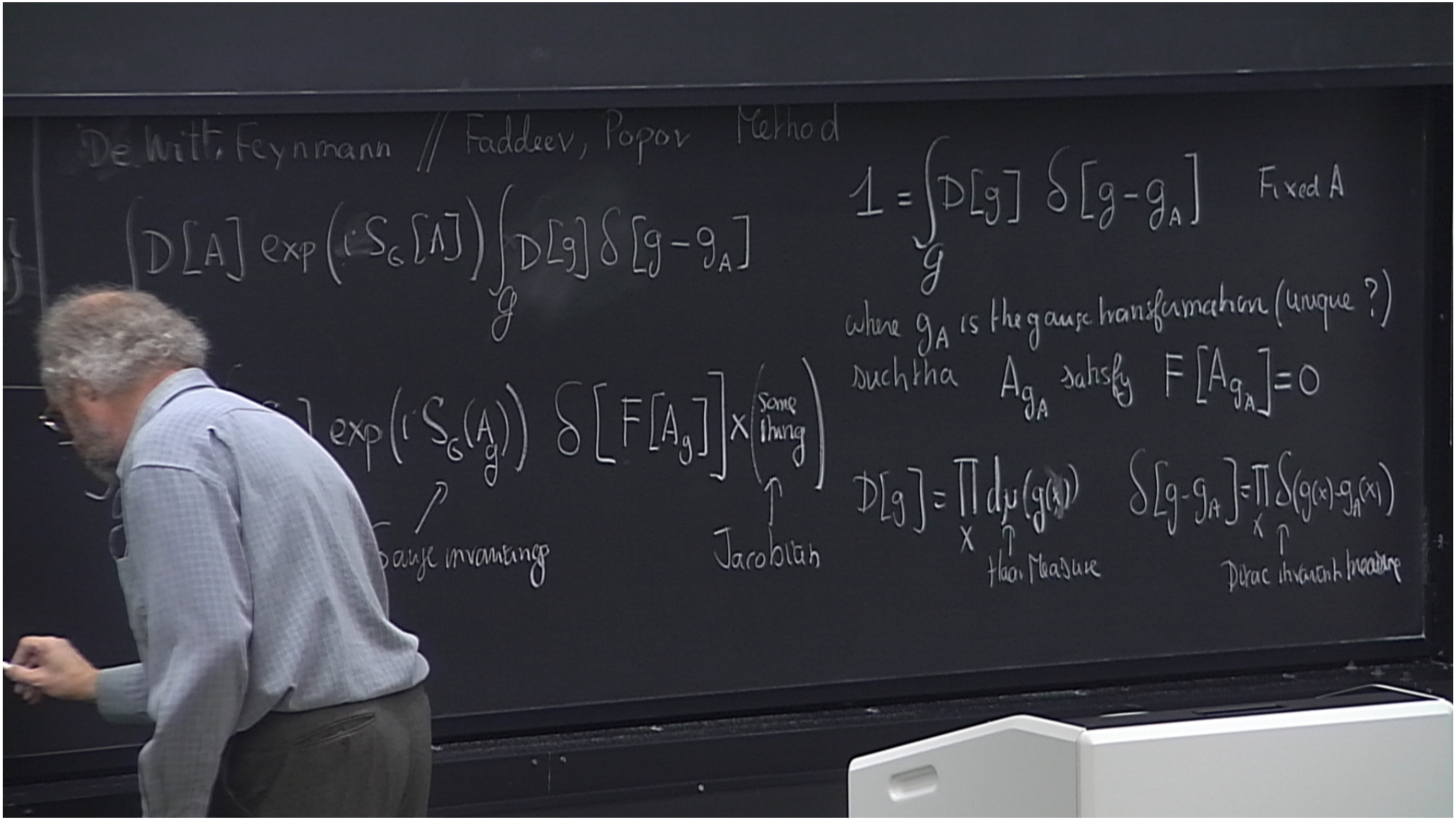

De Witt, Feynmann // Faddeev, Popov Method
$\int_{\mathcal{A}} D[A] \exp\left(i S_G[A]\right) \int_{\mathcal{G}} D[g]\, \delta[g - g_A]$
$1 = \int_{\mathcal{G}} D[g]\, \delta[g - g_A]$
Fixed A
where g_A is the gauge transformation (unq
such tha A_{g_A} satisfy $F[A_{g_A}] = 0$
$D[g] = \prod_x d\mu(g(x))$
Haar Measure
$\delta[g - g_A] = \prod_x \delta($
Dirac

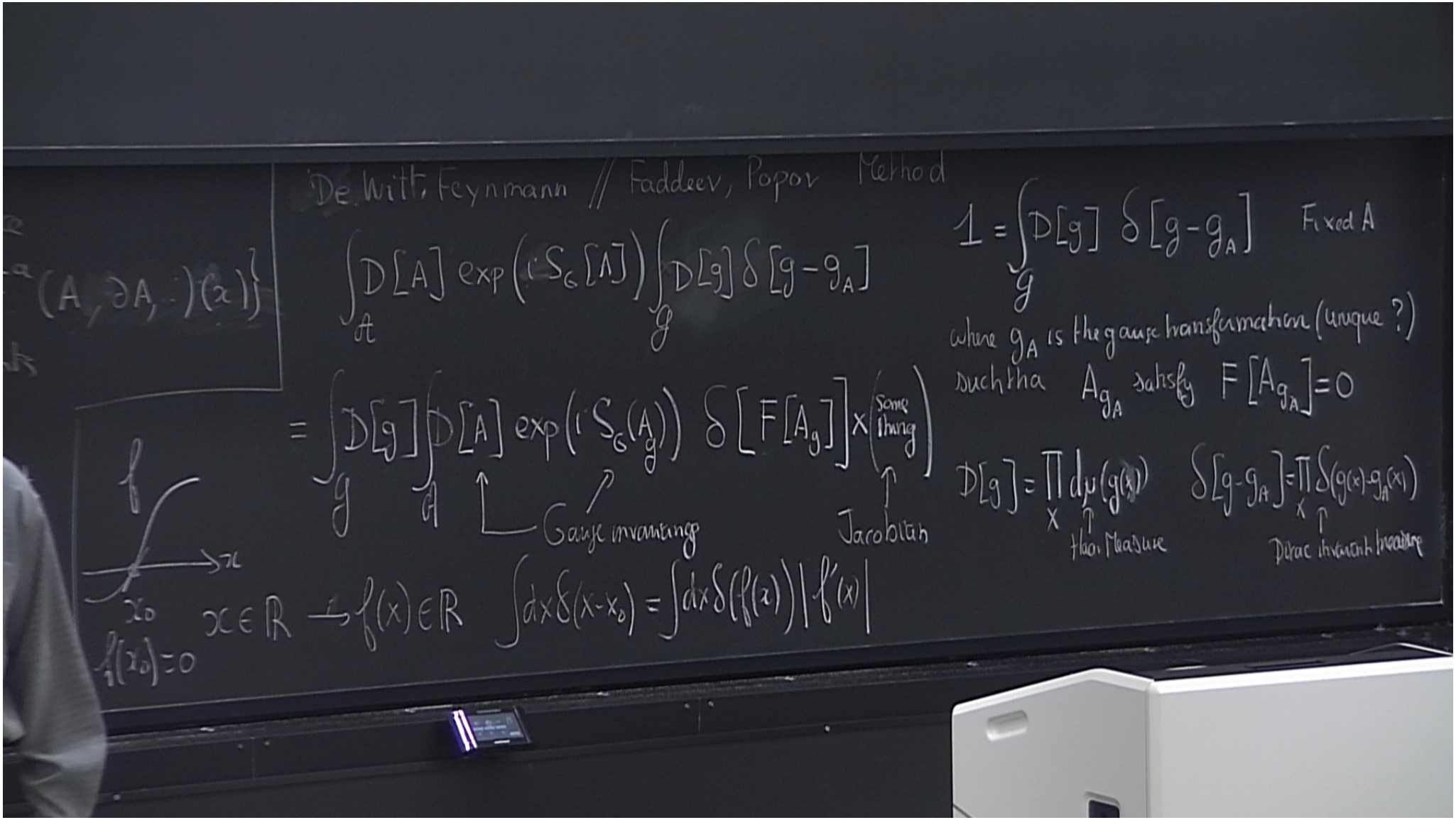

De Witt, Feynmann // Faddeev, Popov Method
$\int_A \mathcal{D}[A] \exp(iS_G[A]) \int_g \mathcal{D}[g]\, \delta[g - g_A]$
$= \int_g \mathcal{D}[g] \int_A \mathcal{D}[A] \exp(iS_G(A))$
$1 = \int_g \mathcal{D}[g]\, \delta[g - g_A]$
where g_A is the gauge transformation
such that A_{g_A} satisfy
$\mathcal{D}[g] = \prod_x d\mu(g(x))$
Haar Measure

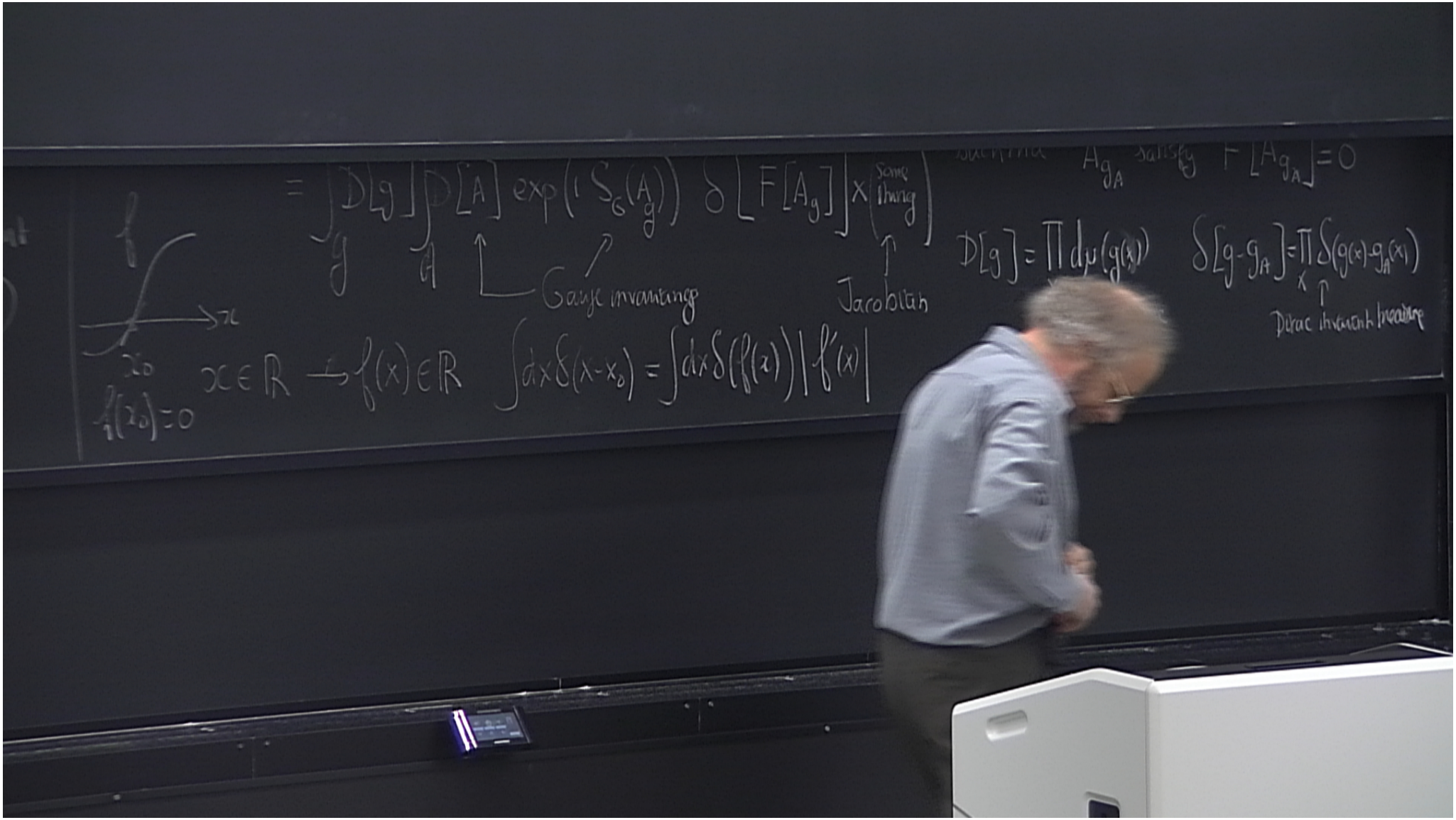

De Witt, Feynmann
Faddeev, Popov
Method
$\int_A D[A] \exp(iS_G$
$[g-g_A]$
$= \int_g D[g] \int_A D[A] \exp(iS$
$F[A_g]]$ x Some thing
G
$1 = \int_g D[g]\, \delta[g-g_A]$ Fixed A
where g_A is the gauge transformation (unique ?)
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Haar Measure
$\delta[g-g_A] = \prod_x \delta(g(x)-g_A(x))$
Dirac invariant measure


De Witt, Feynmann // Faddeev, Popov Method
$\int_A \mathcal{D}[A] \exp(iS_G[A]) \int_g \mathcal{D}[g]\, \delta[g - g_A]$
$= \int_g \mathcal{D}[g] \int_A \mathcal{D}[A] \exp(iS_G(A_g))\, \delta[F[A_g]] \times$ some thing
Gauge invariance
$1 = \int_g \mathcal{D}[g]\, \delta[g - g_A]$ Fixed A
where g_A is the gauge transformation (unique ?)
such tha A_{g_A} satisfy $F[A_{g_A}] = 0$
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Haar Measure
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De Witt, Feynmann // Faddeev, Popov Method
$\int_A \mathcal{D}[A] \exp(iS_G[A]) \int_g \mathcal{D}[g]\, \delta[g - g_A]$
$= \int_g \mathcal{D}[g] \int_A \mathcal{D}[A] \exp(iS_G(A_g))\, \delta[F[A_g]] \times (\text{Some thing})$
Gauge invariance
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De Witt, Feynmann // Faddeev, Popov Method
$\int D[A] \exp(iS_G[A]) \int_g D[g]\, \delta[g - g_A]$
$1 = \int_g D[g]\, \delta[g - g_A]$ Fixed A
where g_A is the gauge transformation (unique ?)
such tha A_{g_A} satisfy $F[A_{g_A}] = 0$
$\exp(iS_G(A_g))\, \delta[F[A_g]] \times (\text{Some thing})$
$D[g] = \prod_x d\mu(g(x))$
$\delta[g - g_A] = \prod_x \delta(g(x) - g_A(x))$
gauge invariance
Jacobian
Haar Measure
Dirac invariant measure


De Witt, Feynmann // Faddeev, Popov Method
Fixed A
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Some thing
Gauge invariance
Jacobian
Haar Measure
Dirac invariant measure


Gauge invariance
Jacobian
Dirac invariant measure