Title: 14/15 PSI - Quantum Field Theory II-Lecture 10

Date: Nov 21, 2014 09:00 AM

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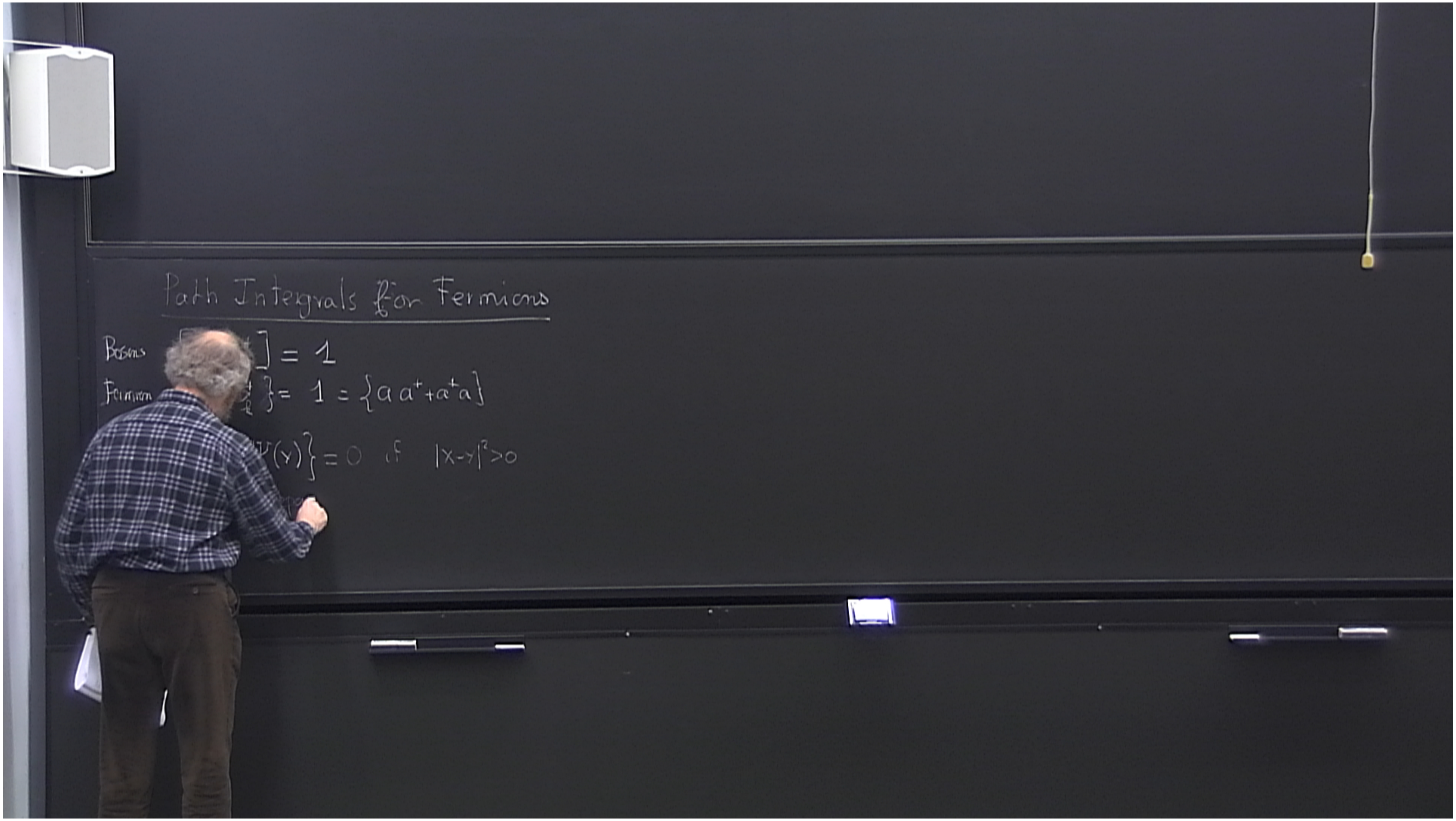
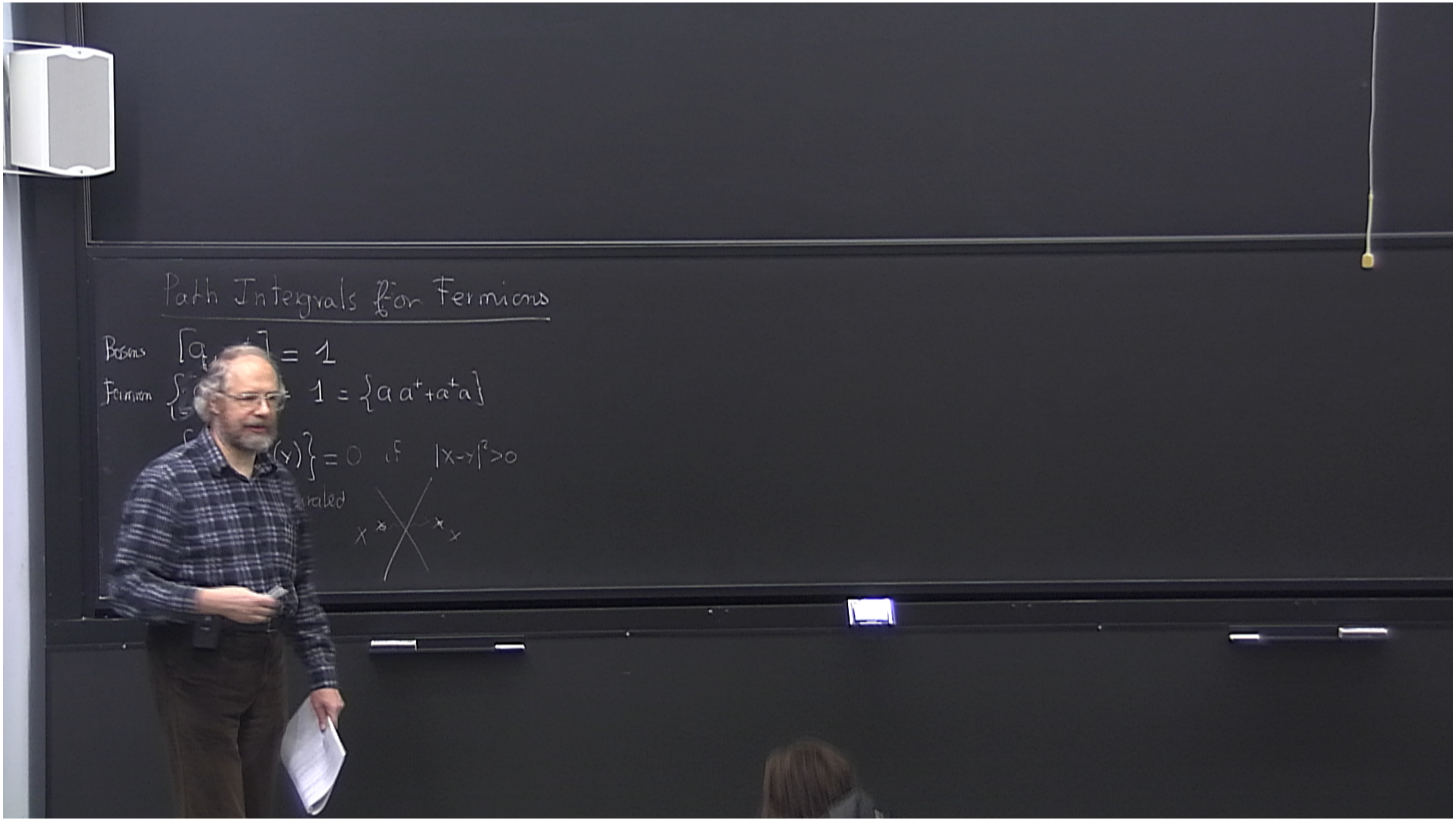
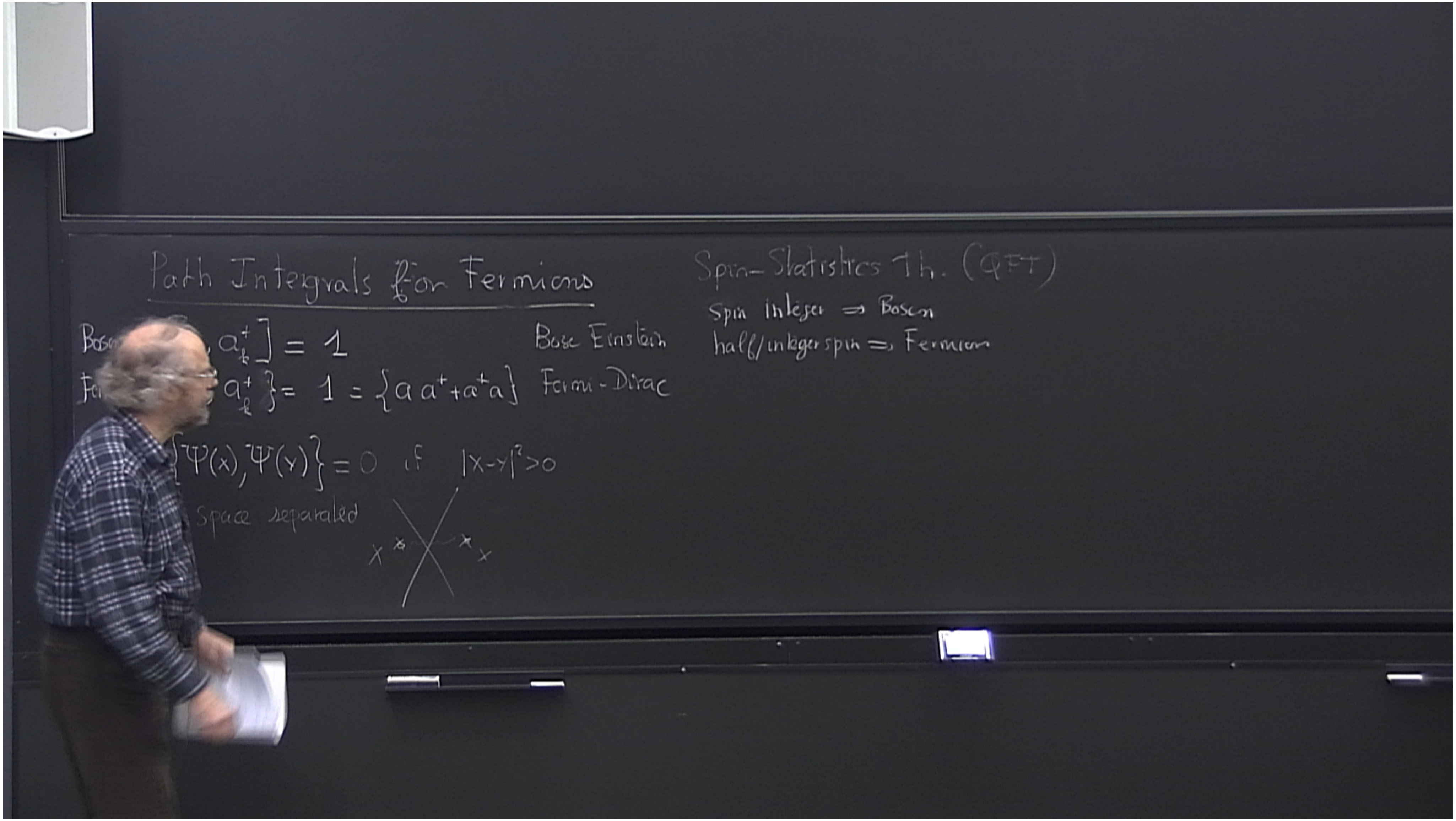
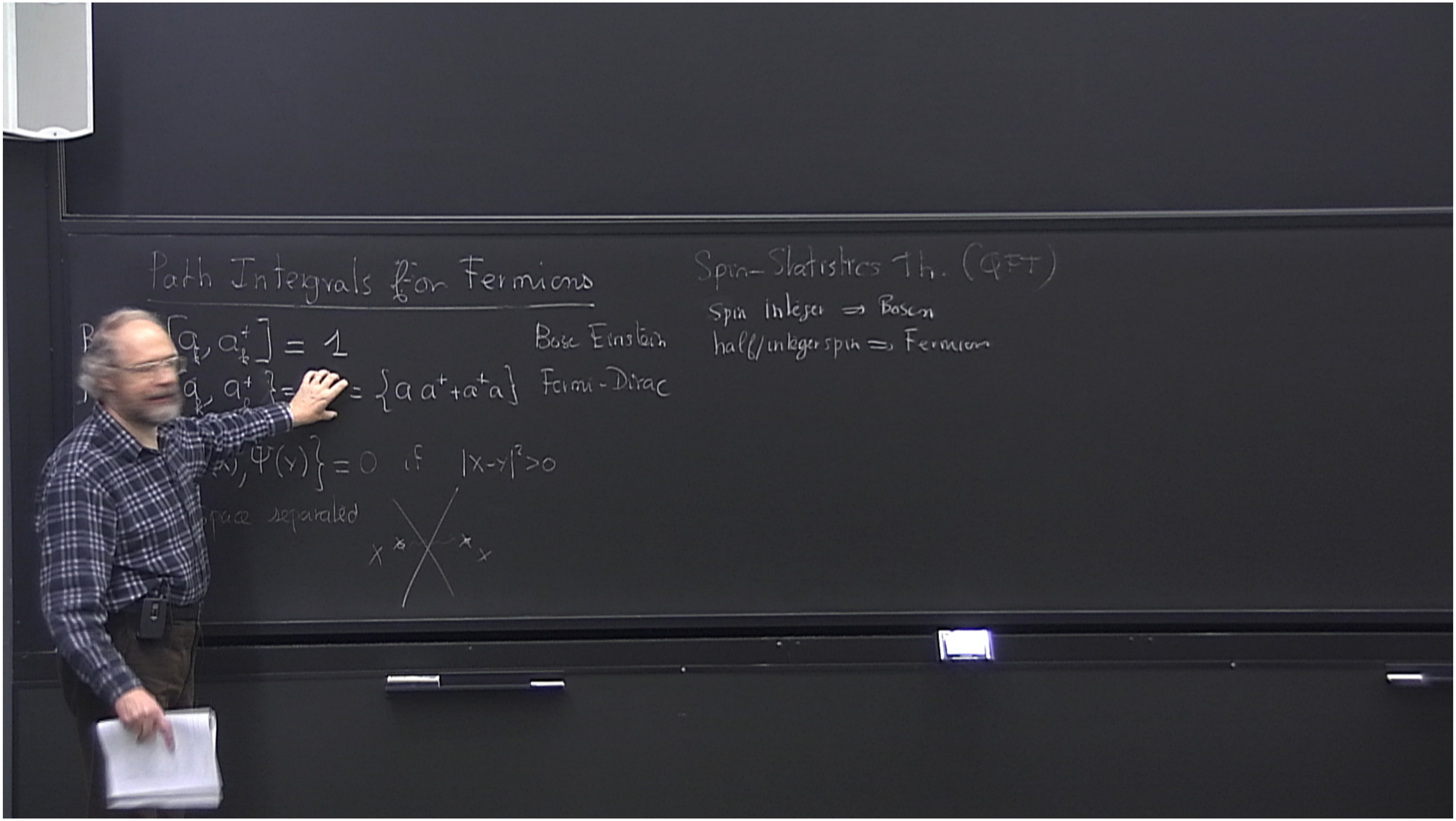
Abstract:


Path Integrals for Fermions
[a, a†]


Path Integrals for Fermions
Bosons] = 1
Fermion }= 1 = {a a⁺+a⁺a}
Ψ(y)} = 0 if |x-y|² > 0


Path Integrals for Fermions
Bosons
= 1
Fermions
1 = {a a⁺+a⁺a}
= 0 if |x-y|²>0


Path Integrals for Fermions
Spin-Statistics Th. (QFT)
spin integer ⇒ Boson
half/integer spin ⇒ Fermion
Bose Einstein
Fermi-Dirac
= 1 = {a a+ + a+ a}
{Ψ(x), Ψ(y)} = 0 if |x-y|² > 0
space separated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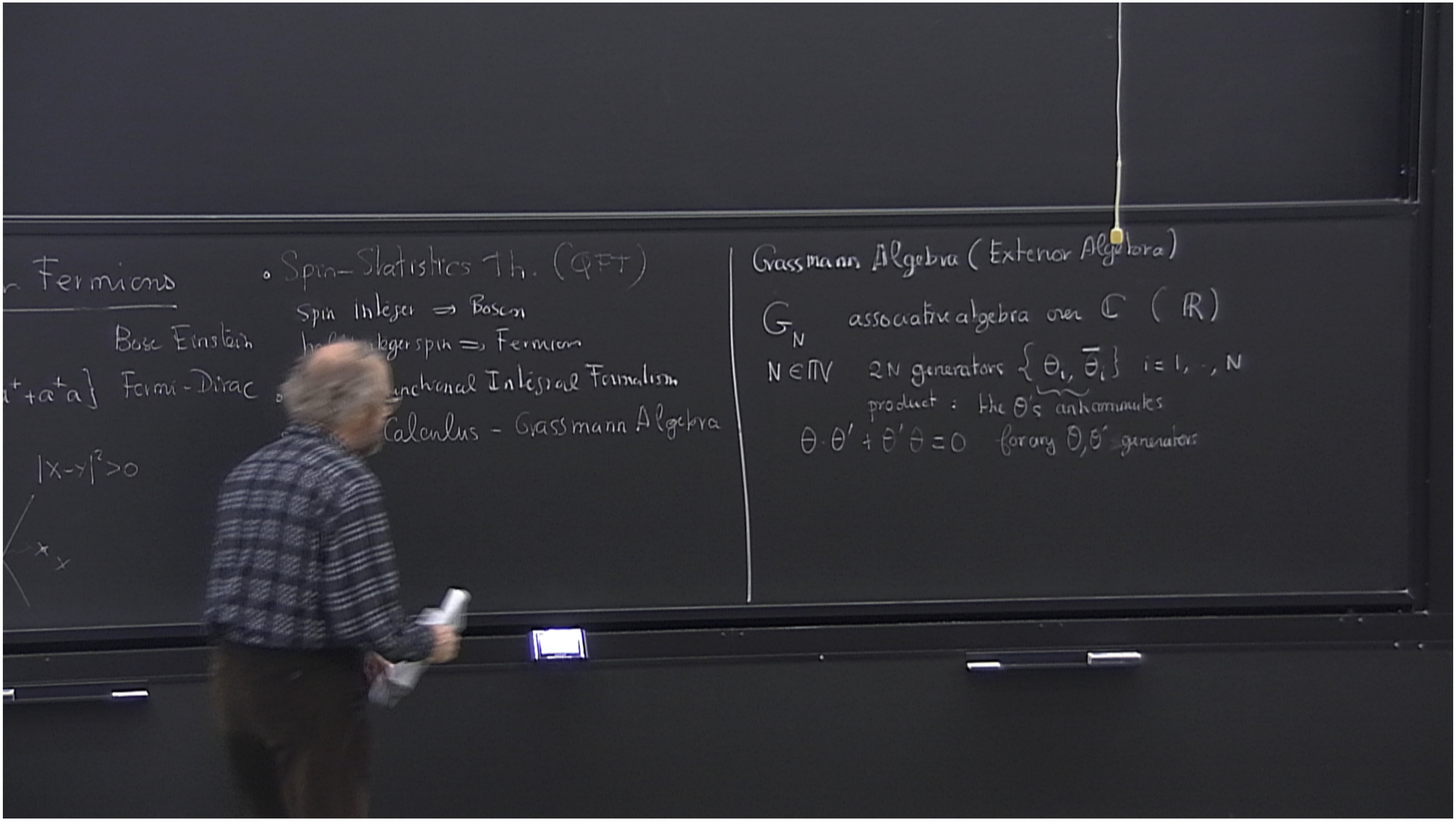
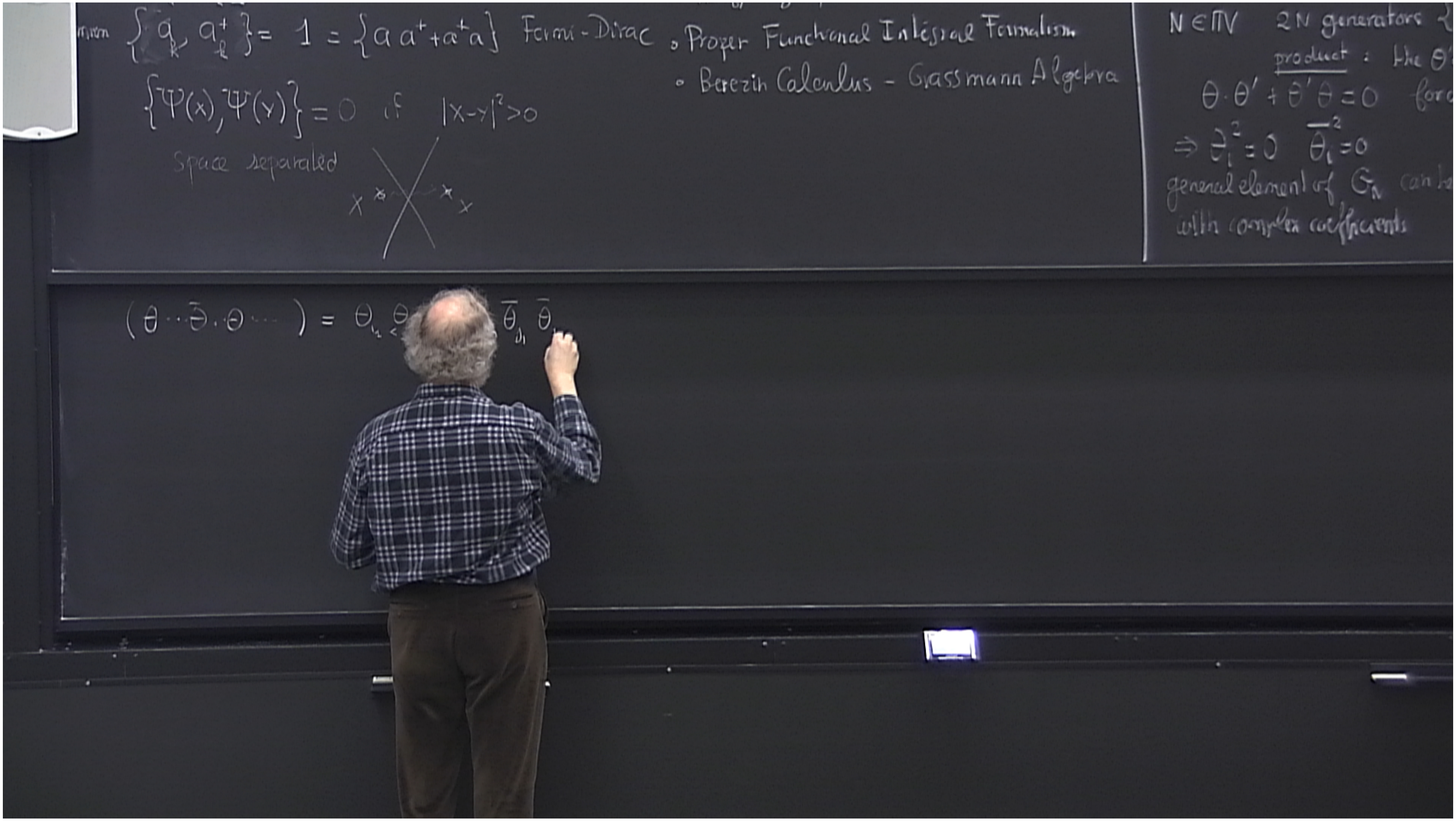

Fermions
Bose Einstein
Fermi-Dirac
$|x-y|^2 > 0$
• Spin-Statistics Th. (QFT)
Spin integer ⇒ Boson
eger spin ⇒ Fermion
nctional Integral Formalism
Calculus - Grassmann Algebra
Grassmann Algebra (Exterior Algebra)
G_N associative algebra over $\mathbb{C}$ ($\mathbb{R}$)
$N \in \mathbb{N}$ 2N generators $\{\theta_i, \bar{\theta}_i\}$ $i = 1, .., N$
product: the θ's anticommute
$\theta \cdot \theta' + \theta' \theta = 0$ for any θ, θ' generators


{a_k, a_k^+} = 1 = {a a^+ + a^+ a} Fermi-Dirac
{Ψ(x), Ψ(y)} = 0 if |x-y|^2 > 0
Space separated
∘ Proper Functional Integral Formalism
∘ Berezin Calculus - Grassmann Algebra
N ∈ ℕ 2N generators
product : the θ
θ·θ' + θ'θ = 0
⇒ θ_i^2 = 0 θ̄_i^2 = 0
general element of G_N can be
with complex coefficients
(θ ··· θ̄ · θ ···) = θ θ θ̄ θ̄

$\{a_k, a_\ell^+\} = 1 = \{a a^+ + a^+ a\}$ Fermi-Dirac

- Proper Functional Integral Formalism
- Berezin Calculus - Grassmann Algebra

$\{\Psi(x), \bar\Psi(y)\} = 0$ if $|x-y|^2 > 0$

space separated

$N \in \mathbb{N}$ 2N generators

product : the θ

$\theta \cdot \theta' + \theta' \theta = 0$ for

$\Rightarrow \theta_i^2 = 0 \quad \bar\theta_i^2 = 0$

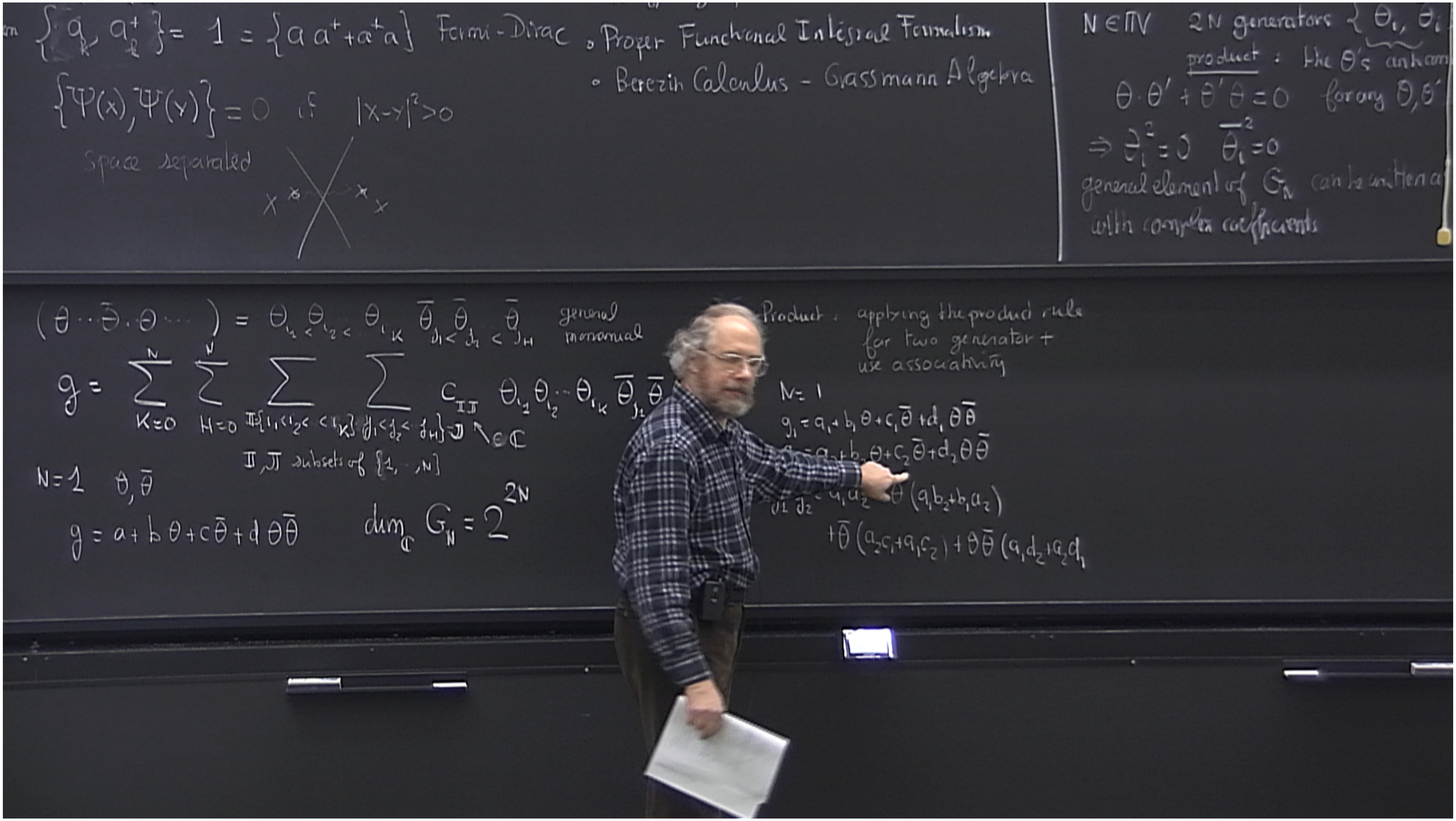
general element of G_N can

with complex coefficients

$(\theta \cdots \bar\theta \cdot \theta \cdots) = \theta_{i_1} \theta_{i_2} \cdots \theta_{i_K} \bar\theta_{j_1} \bar\theta_{j_2} \cdots \bar\theta_{j_H}$ $\quad i_1 < i_2 < \cdots$, $j_1 < j_2 < \cdots$ general monomial

$$\sum_{K=0}^{N} \sum_{H=0}^{N} \sum_{I = \{i_1 < i_2 < \cdots < i_K\}} \sum_{\{j_1 < j_2 < \cdots j_H\} = J} C_{IJ}\, \theta_{i_1} \theta_{i_2} \cdots \theta_{i_K} \bar\theta_{j_1} \bar\theta_{j_2} \cdots \bar\theta_{j_K} \qquad C_{IJ} \in \mathbb{C}$$

I, J subsets of $\{1, \ldots, N\}$


$\{a_k, a_k^+\} = 1 = \{a a^+ + a^+ a\}$ Fermi-Dirac
$\{\Psi(x), \Psi(y)\} = 0$ if $|x-y|^2 > 0$
Space separated
∘ Proper Functional Integral Formalism
∘ Berezin Calculus - Grassmann Algebra
$N \in \mathbb{N}$ 2N generators $\{\theta_i, \bar\theta_i$
product : the θ's anticom
$\theta \cdot \theta' + \theta' \theta = 0$ for any θ, θ'
$\Rightarrow \theta_i^2 = 0 \quad \bar\theta_i^2 = 0$
general element of G_N can be written a
with complex coefficients
$(\theta \cdots \bar\theta \cdot \theta \cdots) = \theta_{i_1 <} \theta_{i_2 <} \theta_{i_K} \bar\theta_{j_1 <} \bar\theta_{j_2 <} \bar\theta_{j_H}$ general monomial
$g = \sum_{K=0}^{N} \sum_{H=0}^{N} \sum_{\mathbb{I}=\{i_1<i_2<\cdots<i_K\}} \sum_{\{j_1<j_2<\cdots<j_H\}=\mathbb{J}} C_{\mathbb{I}\mathbb{J}} \, \theta_{i_1} \theta_{i_2} \cdots \theta_{i_K} \bar\theta_{j_1} \bar\theta$
$\in \mathbb{C}$
$\mathbb{I}, \mathbb{J}$ subsets of $\{1, \ldots, N\}$
$N = 1 \quad \theta, \bar\theta$
$g = a + b\theta + c\bar\theta + d\theta\bar\theta$
$\dim_{\mathbb{C}} G_N = 2^{2N}$
Product : applying the product rule for two generator + use associativity
$N = 1$
$g_1 = a_1 + b_1\theta + c_1\bar\theta + d_1\theta\bar\theta$
$+\bar\theta(a_2 c_1 + a_1 c_2) + \theta\bar\theta(a_1 d_2 + a_2 d_1$

a} Fermi-Dirac

- Proper Functional Integral Formalism
- Berezin Calculus - Grassmann Algebra

$|x-y|^2 > 0$

$N \in \mathbb{N}$ 2N generators $\{\theta_i, \bar\theta_i\}$ $i = 1, \dots, N$

product : the θ's anticommute

$\theta \cdot \theta' + \theta' \theta = 0$ for any θ, θ' generators

$\Rightarrow \theta_i^2 = 0 \quad \bar\theta_i^2 = 0$

general element of G_N can be written as a Polynomial in the $\theta, \bar\theta$'s with complex coefficients

$\theta_{i_K} \; \bar\theta_{j_1} \bar\theta \dots \bar\theta$ general monomial

$\underbrace{\dots \theta_{i_K} \bar\theta_{j_1} \bar\theta_{j_2} \dots \bar\theta_{j_H}}_{\text{degree } H+K}$

$j_1 < j_2 < \dots j_H\}$

Product : applying the product rule for two generator + use associativity

$N = 1$

$g_1 = a_1 + b_1 \theta + c_1 \bar\theta + d_1 \theta\bar\theta$

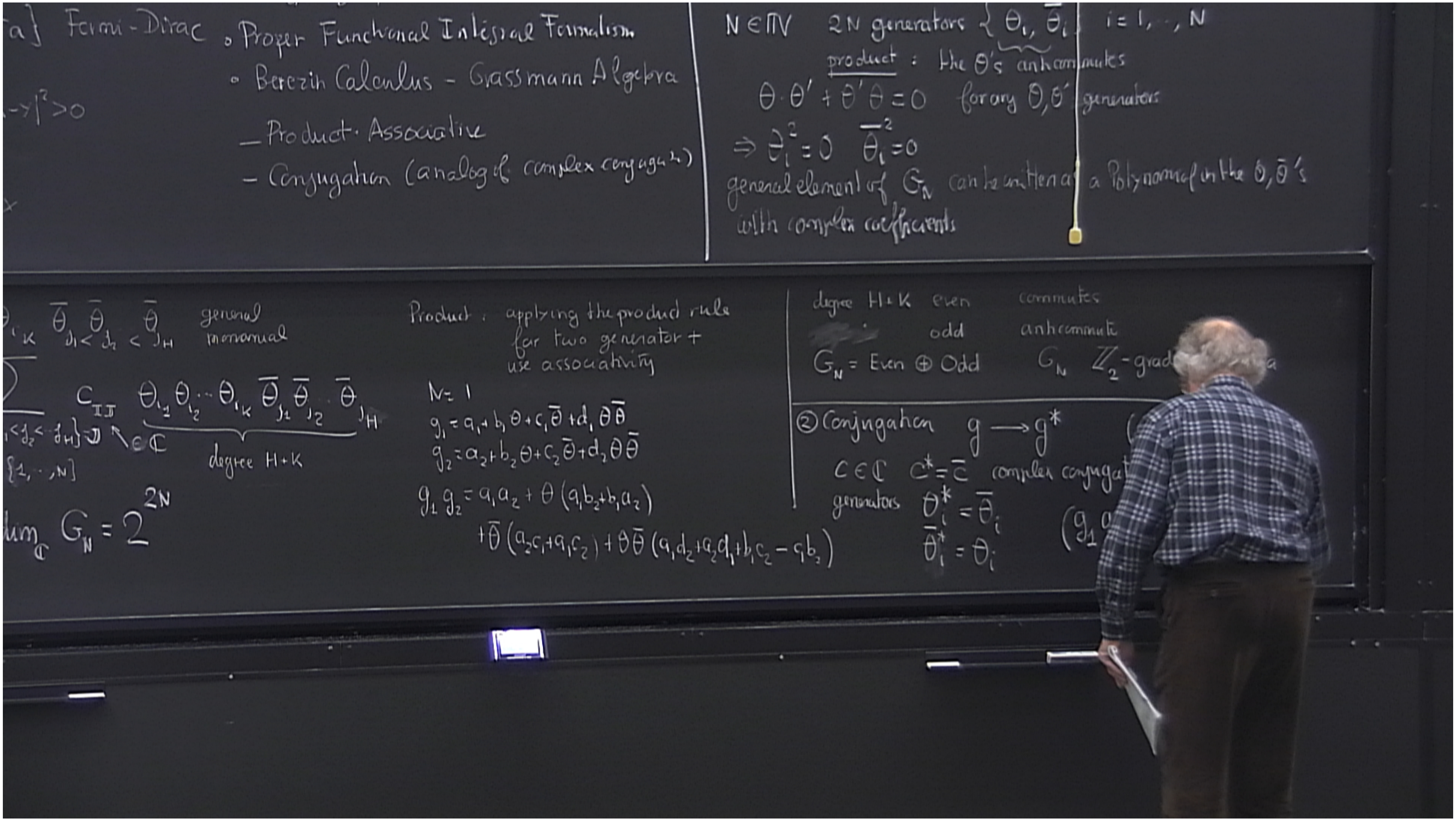
$g_2 = a_2 + b_2 \theta + c_2 \bar\theta + d_2 \theta\bar\theta$

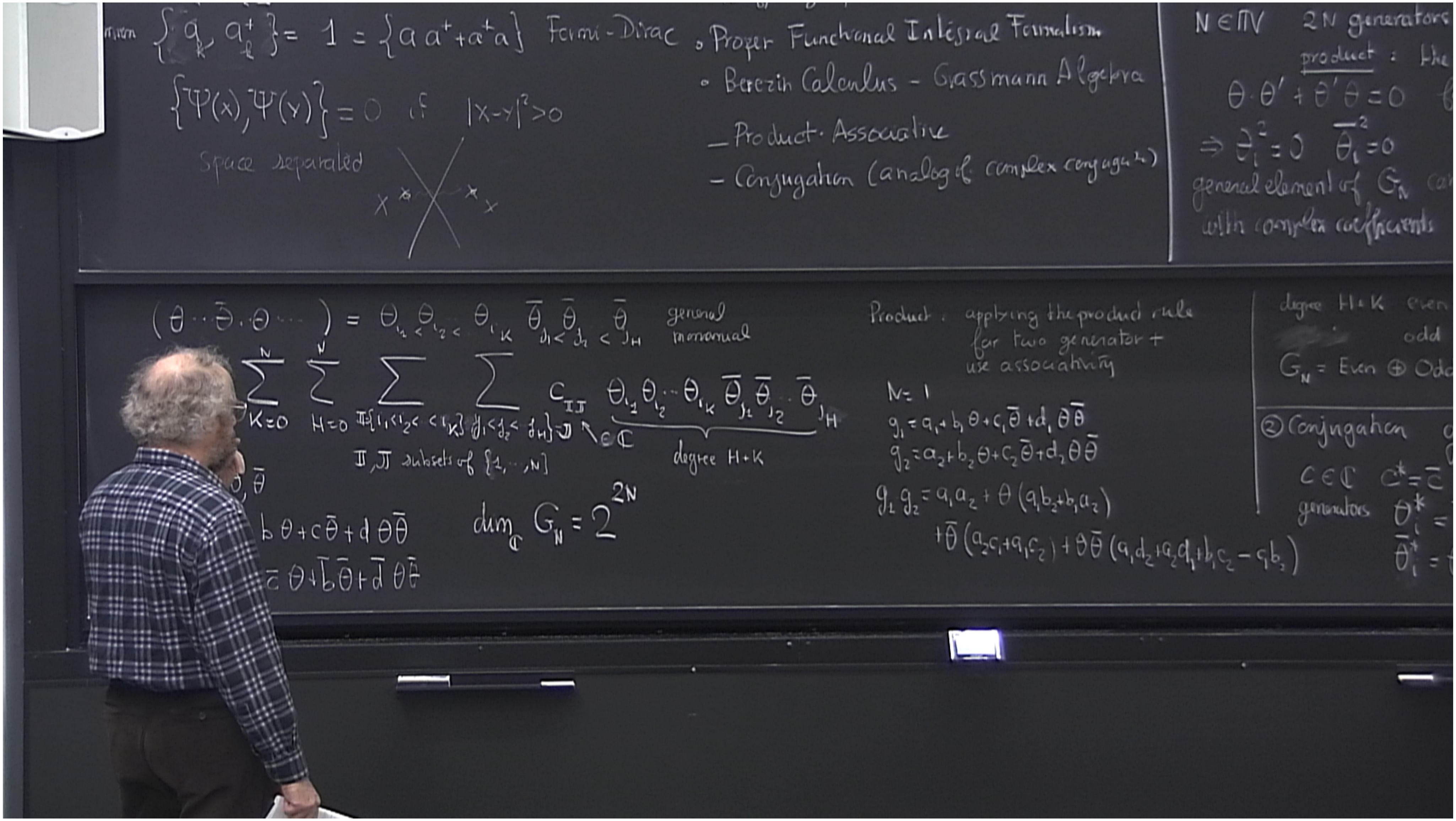
$g_1 g_2 = a_1 a_2 + \theta (a_1 b_2 + b_1 a_2)$
$+ \bar\theta (a_2 c_1 + a_1 c_2) + \theta\bar\theta (a_1 d_2 + a_2 d_1 + b_1 c_2 - c_1 b_2)$

degree H+K even commutes

odd anticommute

$G_N = \text{Even} \oplus \text{Odd}$ $\quad G_N$ $\mathbb{Z}_2$-graded algebra


Fermi-Dirac
Proper Functional Integral Formalism
Berezin Calculus - Grassmann Algebra
— Product. Associative
— Conjugation (analog of complex conjugation)
N ∈ ℕ 2N generators {θ_i, θ̄_i} i = 1, .., N
product : the θ's anticommutes
θ·θ′ + θ′θ = 0 for any θ, θ′ generators
⇒ θ_i² = 0 θ̄_i² = 0
general element of G_N can be written as a Polynomial in the θ, θ̄'s
with complex coefficients
general monomial
degree H+K
dim_ℂ G_N = 2^{2N}
Product : applying the product rule for two generator + use associativity
N = 1
g_1 = a_1 + b_1θ + c_1θ̄ + d_1θθ̄
g_2 = a_2 + b_2θ + c_2θ̄ + d_2θθ̄
g_1g_2 = a_1a_2 + θ(a_1b_2 + b_1a_2)
+θ̄(a_2c_1 + a_1c_2) + θθ̄(a_1d_2 + a_2d_1 + b_1c_2 − c_1b_2)
degree H+K even commutes
odd anticommute
G_N = Even ⊕ Odd
② Conjugation g ⟶ g*
c ∈ ℂ c* = c̄ complex conjugat
generators θ_i* = θ̄_i
θ̄_i* = θ_i


{a_k, a_ℓ^+} = 1 = {a a^+ + a^+ a} Fermi-Dirac
{Ψ(x), Ψ(y)} = 0 if |x-y|^2 > 0
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Proper Functional Integral Formalism
Berezin Calculus - Grassmann Algebra
Product. Associative
Conjugation (analog of complex conjugation)
N ∈ ℕ 2N generators
product : the
θ·θ' + θ'θ = 0
⇒ θ_i^2 = 0 θ̄_i^2 = 0
general element of G_N can
with complex coefficients
general monomial
c_IJ ∈ ℂ
degree H+K
I, J subsets of {1, ..., N}
dim_ℂ G_N = 2^{2N}
Product: applying the product rule for two generator + use associativity
N = 1
g_1 = a_1 + b_1 θ + c_1 θ̄ + d_1 θθ̄
g_2 = a_2 + b_2 θ + c_2 θ̄ + d_2 θθ̄
degree H+K even
odd
G_N = Even ⊕ Odd
② Conjugation
c ∈ ℂ c* = c̄
generators

...mial in the $\theta, \bar\theta$'s

...raded algebra

$(g^*)^* = g$

$= g_2^* \, g_1^*$

- Think of $g \in G_N$ as a polynomial in $(\theta, \bar\theta)$

$\left\{ \frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\theta_j} \right\} = 0$ derivations anticommutes

- Derivation and Integration

- $\frac{\partial}{\partial\theta_i} g = ?$ $\quad \frac{\partial}{\partial\theta_i} 1 = 0 \quad \frac{\partial}{\partial\theta_i}\theta_j = \delta_{ij}, \ \frac{\partial}{\partial\theta_i}\bar\theta_j = 0 \quad \left\{ \frac{\partial}{\partial\bar\theta_i}, \frac{\partial}{\partial\bar\theta_j} \right\} = 0$

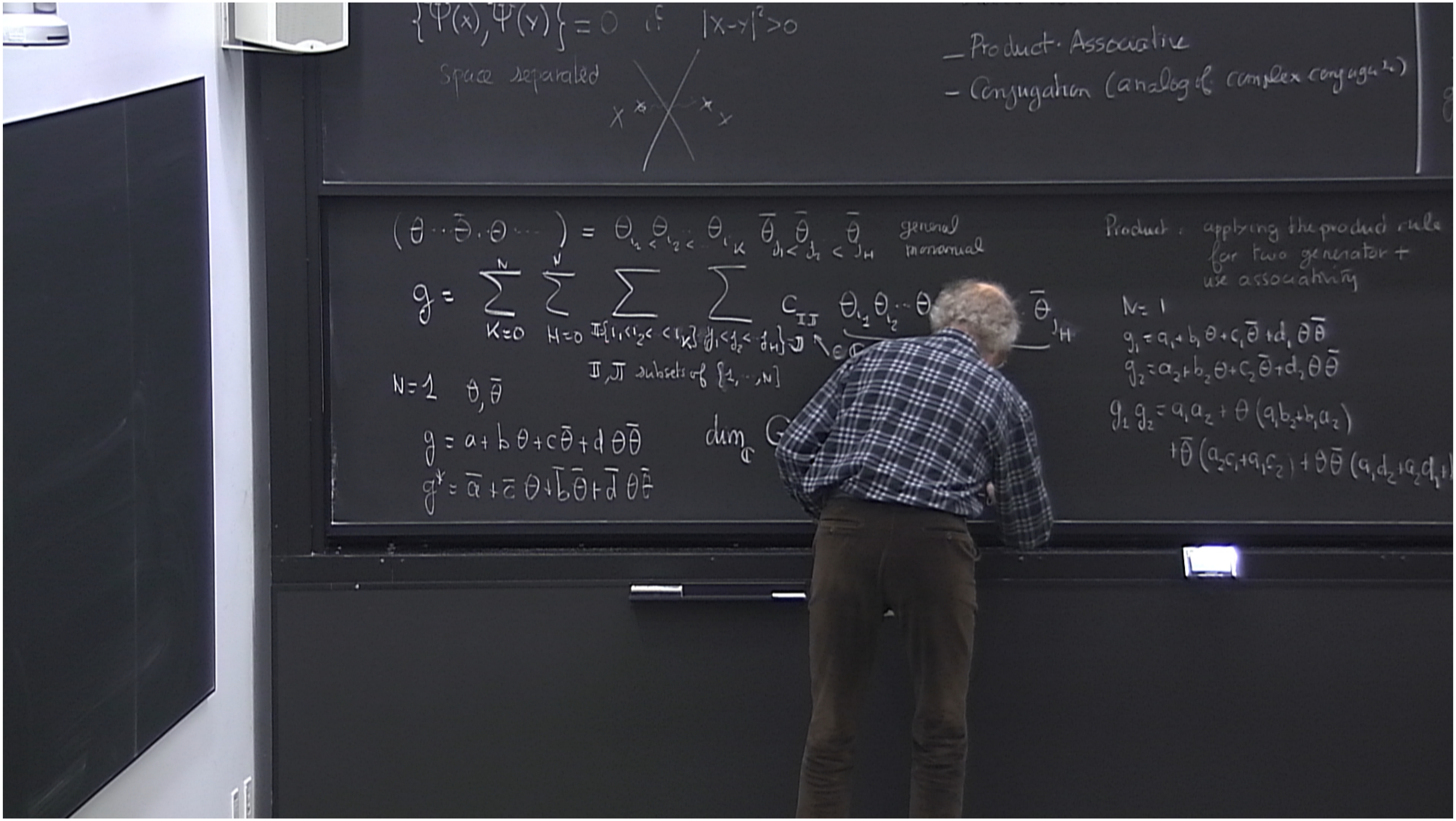
$\frac{\partial}{\partial\bar\theta_i} 1 = 0 \quad \frac{\partial}{\partial\bar\theta_i}\bar\theta_j = \delta_{ij}, \ \frac{\partial}{\partial\bar\theta_i}\theta_j = 0 \quad \left\{ \frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\bar\theta_j} \right\} = 0$

$\frac{\partial}{\partial\theta_i} g = \sum c \ \theta\theta\theta \ \bar\theta \ \bar\theta$ $\quad$ same for $\frac{\partial}{\partial\bar\theta_i}$

0 if θ_i not in the monomial

if θ_i is in the monomial move it to the left and remove it

$\cancel{\theta_i}(\theta \, \theta \, \bar\theta \, \bar\theta)$



{Ψ(x), Ψ(y)} = 0 if |x-y|² > 0
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– Product · Associative
– Conjugation (analog of complex conjugation)
(θ · · θ̄ · θ · · ·) = θ_{i_1} θ_{i_2} … θ_{i_K} θ̄_{j_1} θ̄_{j_2} … θ̄_{j_H}
general monomial
Product: applying the product rule for two generator + use associativity
I, J subsets of {1, …, N}
N = 1 θ, θ̄
g = a + b θ + c θ̄ + d θ θ̄
g* = ā + c̄ θ + b̄ θ̄ + d̄ θ θ̄
N = 1
g₁ = a₁ + b₁ θ + c₁ θ̄ + d₁ θ θ̄
g₂ = a₂ + b₂ θ + c₂ θ̄ + d₂ θ θ̄
g₁ g₂ = a₁a₂ + θ (a₁b₂ + b₁a₂)
+ θ̄ (a₂c₁ + a₁c₂) + θ θ̄ (a₁d₂ + a₂d₁ +

$$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\theta_j}\right\} = 0$$

$$\left\{\frac{\partial}{\partial\bar\theta_i}, \frac{\partial}{\partial\bar\theta_j}\right\} = 0$$

$$\left\{\frac{\partial}{\partial\theta_i}, \frac{\partial}{\partial\bar\theta_j}\right\} = 0$$

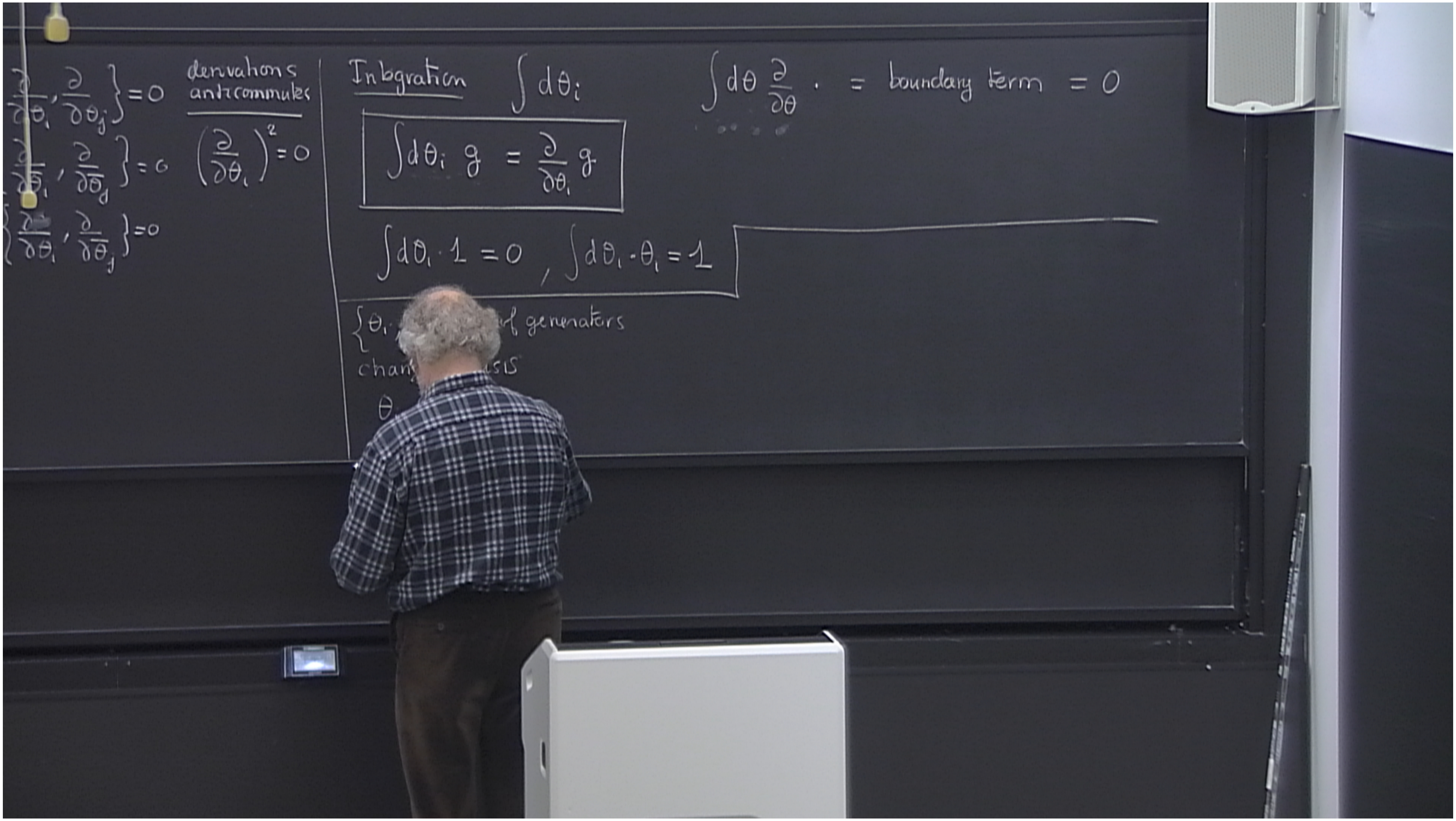
derivations anticommutes

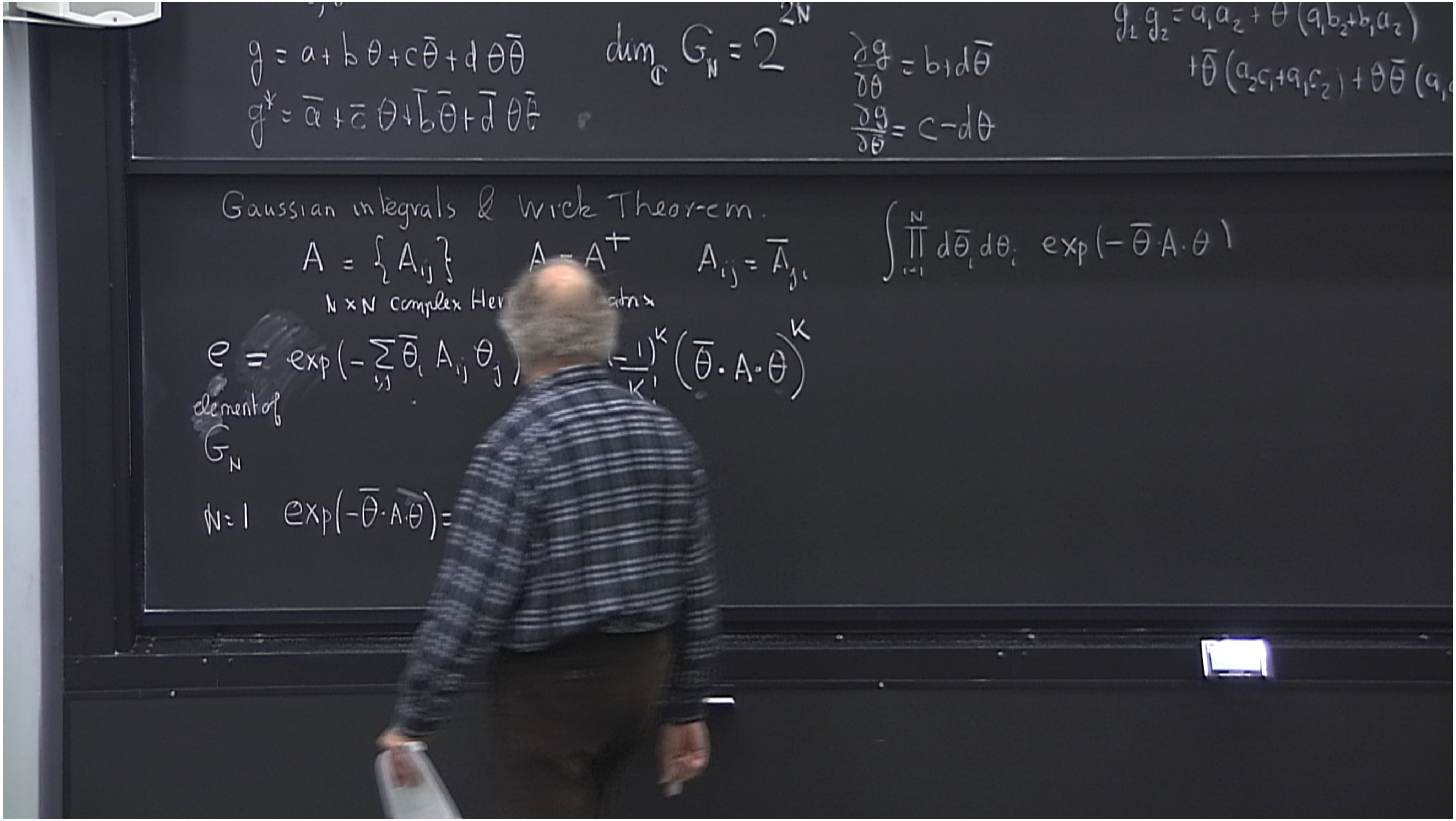
$$\left(\frac{\partial}{\partial\theta_i}\right)^2 = 0$$

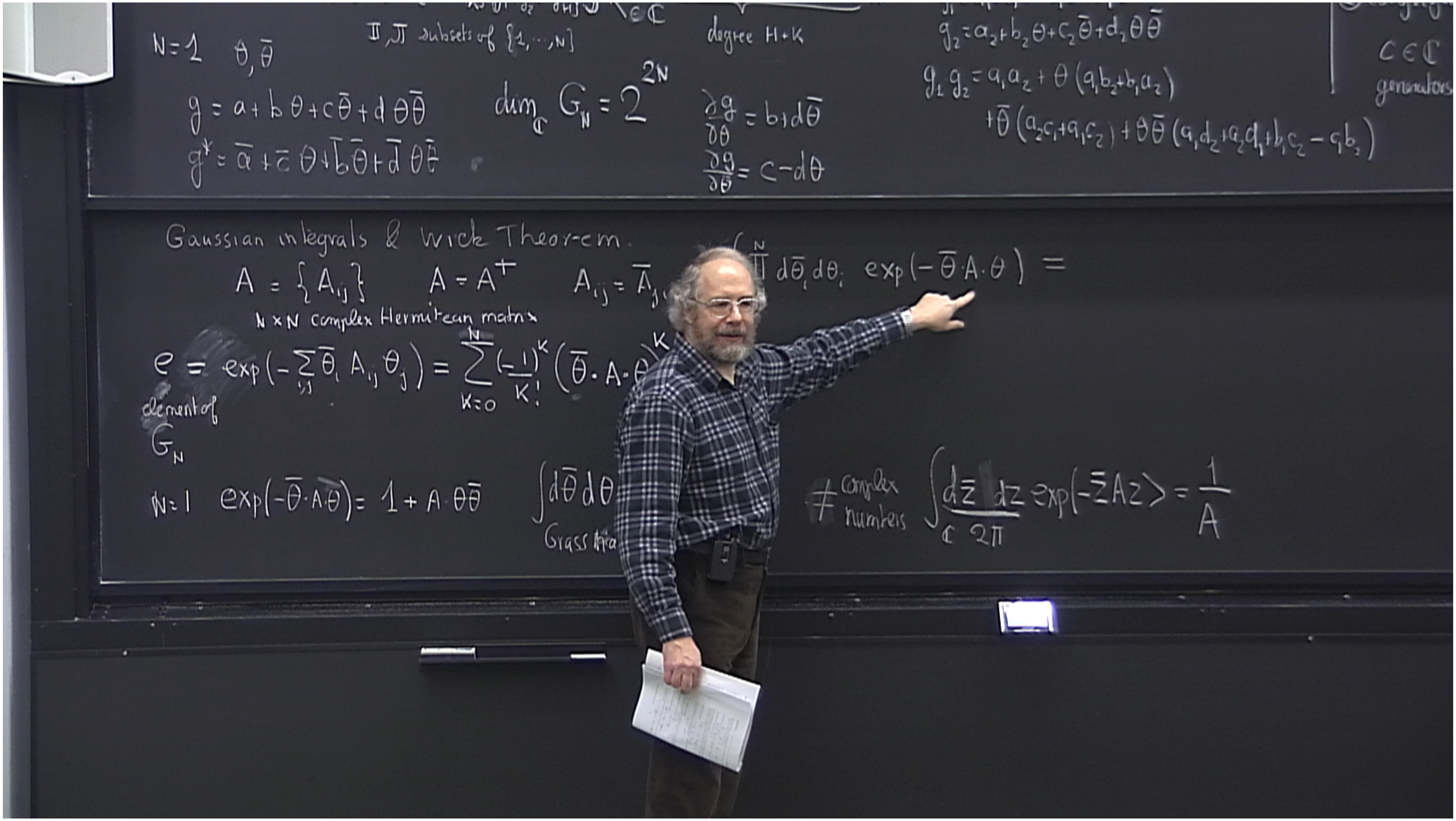
Integration $\int d\theta_i$

$$\boxed{\int d\theta_i \, g = \frac{\partial}{\partial\theta_i} g}$$

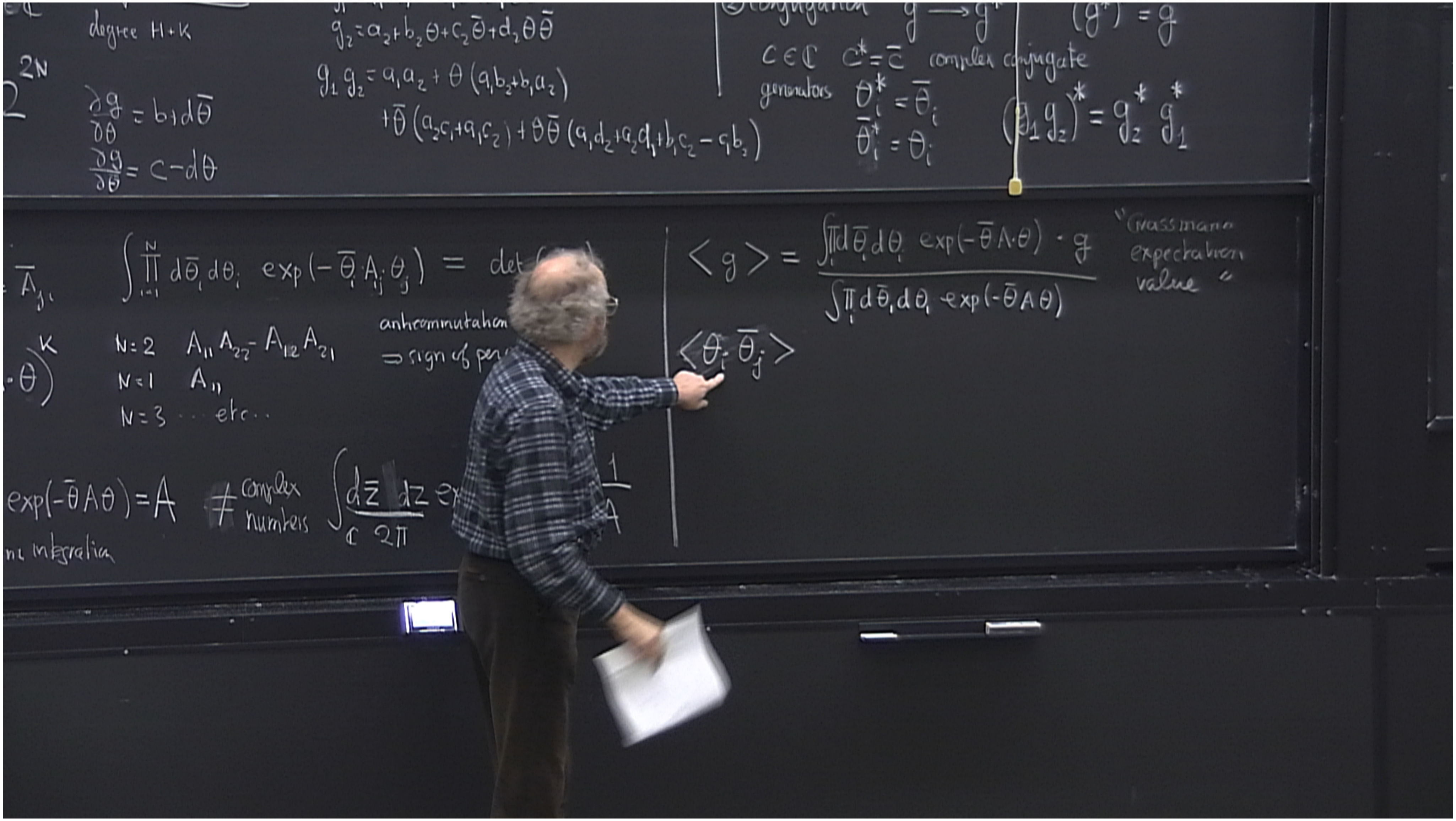
$$\int d\theta \frac{\partial}{\partial\theta} \cdot \; = \text{boundary term} = 0$$

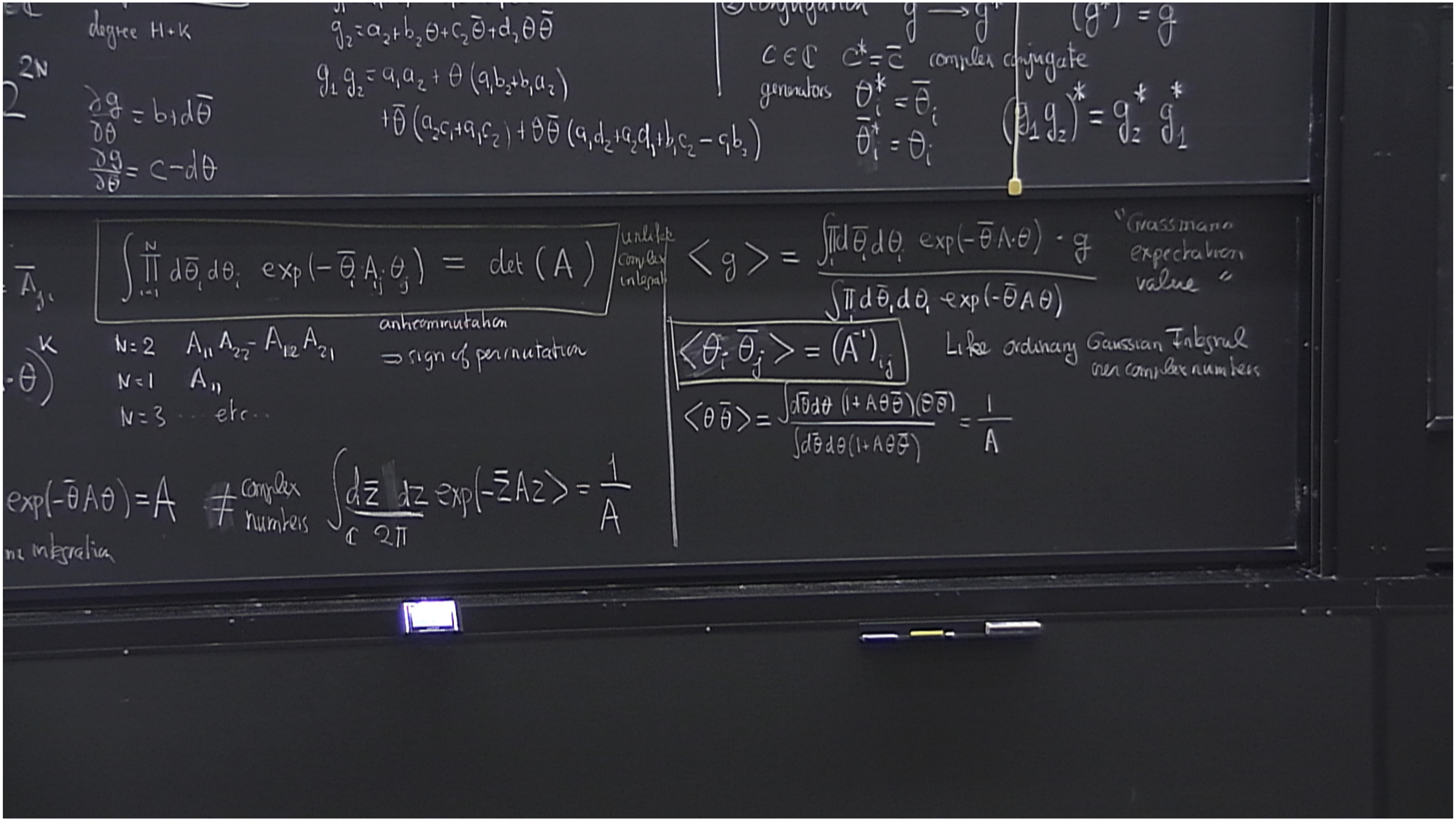

derivations anticommutes
$\left(\frac{\partial}{\partial\theta_i}\right)^2 = 0$
Integration $\int d\theta_i$
$\int d\theta_i \, g = \frac{\partial}{\partial\theta_i} g$
$\int d\theta_i \cdot 1 = 0$, $\int d\theta_i \cdot \theta_i = 1$
$\int d\theta \frac{\partial}{\partial\theta} \cdot$ = boundary term = 0
generators


g = a + b θ + c θ̄ + d θ θ̄
g* = ā + c̄ θ + b̄ θ̄ + d̄ θ θ̄
dim_ℂ G_N = 2^2N
∂g/∂θ = b + d θ̄
∂g/∂θ̄ = c − d θ
g_1 g_2 = a_1 a_2 + θ (a_1 b_2 + b_1 a_2)
+ θ̄ (a_2 c_1 + a_1 c_2) + θ θ̄ (a_1
Gaussian integrals & Wick Theorem.
A = {A_ij}
A = A^†
A_ij = Ā_ji
N × N complex Her
atrix
∫ ∏_{i=1}^N dθ̄_i dθ_i exp(− θ̄ · A · θ)
e = exp(− Σ_{i,j} θ̄_i A_ij θ_j)
(−1)^K / K! (θ̄ · A · θ)^K
element of
G_N
N = 1 exp(− θ̄ · A · θ) =



I, J subsets of {1, ..., N}
degree H+K
N=1 θ, θ̄
g = a + bθ + cθ̄ + dθθ̄
g* = ā + c̄θ + b̄θ̄ + d̄θθ̄
dim_C G_N = 2^2N
∂g/∂θ = b + dθ̄
∂g/∂θ̄ = c − dθ
g2 = a2 + b2θ + c2θ̄ + d2θθ̄
g1g2 = a1a2 + θ(a1b2 + b1a2)
+θ̄(a2c1 + a1c2) + θθ̄(a1d2 + a2d1 + b1c2 − c1b2)
c ∈ C
generators
Gaussian integrals & Wick Theorem.
A = {Aij}   A = A†   Aij = Āji
N×N complex Hermitean matrix
∫ ∏ dθ̄i dθi exp(−θ̄·A·θ) =
e = exp(−Σ θ̄i Aij θj) = Σ (−1)^K/K! (θ̄·A·θ)^K
element of G_N
N=1 exp(−θ̄·A·θ) = 1 + A·θθ̄
∫dθ̄ dθ
Grassmann
complex numbers ∫ dz̄ dz/2π exp(−z̄Az) = 1/A


degree N+K
2^{2N}
$\frac{\partial g}{\partial \theta} = b + d\bar{\theta}$
$\frac{\partial g}{\partial \bar{\theta}} = c - d\theta$
$g_2 = a_2 + b_2\theta + c_2\bar{\theta} + d_2\theta\bar{\theta}$
$g_1 g_2 = a_1 a_2 + \theta(a_1 b_2 + b_1 a_2)$
$+\bar{\theta}(a_2 c_1 + a_1 c_2) + \theta\bar{\theta}(a_1 d_2 + a_2 d_1 + b_1 c_2 - c_1 b_2)$
$c \in \mathbb{C}$ $c^* = \bar{c}$ complex conjugate
generators $\theta_i^* = \bar{\theta}_i$
$\bar{\theta}_i^* = \theta_i$
$(g^*)^* = g$
$(g_1 g_2)^* = g_2^* g_1^*$
$\int \prod_{i=1}^{N} d\bar{\theta}_i d\theta_i \exp(-\bar{\theta}_i A_{ij} \theta_j) = \det$
$N=2$ $A_{11}A_{22} - A_{12}A_{21}$
$N=1$ A_{11}
$N=3$ … etc…
anticommutation
$\Rightarrow$ sign of per
$\exp(-\bar{\theta}A\theta) = A$
$\neq$ complex numbers
$\int_{\mathbb{C}} \frac{d\bar{z}\, dz}{2\pi}$
$\frac{1}{A}$
integration
$\langle g \rangle = \frac{\int \prod_i d\bar{\theta}_i d\theta_i \exp(-\bar{\theta}A\theta) \cdot g}{\int \prod_i d\bar{\theta}_i d\theta_i \exp(-\bar{\theta}A\theta)}$
"Grassmann expectation value"
$\langle \theta_i \bar{\theta}_j \rangle$


degree H+K
$g_2 = a_2 + b_2\theta + c_2\bar\theta + d_2\theta\bar\theta$
$g_1 g_2 = a_1 a_2 + \theta(a_1 b_2 + b_1 a_2)$
$+\bar\theta(a_2 c_1 + a_1 c_2) + \theta\bar\theta(a_1 d_2 + a_2 d_1 + b_1 c_2 - c_1 b_2)$
$\frac{\partial g}{\partial \theta} = b + d\bar\theta$
$\frac{\partial g}{\partial \bar\theta} = c - d\theta$
$c \in \mathbb{C}$ $c^* = \bar c$ complex conjugate
generators $\theta_i^* = \bar\theta_i$
$\bar\theta_i^* = \theta_i$
$(g_1 g_2)^* = g_2^* g_1^*$
$\int \prod_{i=1}^{N} d\bar\theta_i d\theta_i \exp(-\bar\theta_i A_{ij} \theta_j) = \det(A)$
unlike complex integral
$\langle g \rangle = \frac{\int \prod_i d\bar\theta_i d\theta_i \exp(-\bar\theta A \theta)\, g}{\int \prod d\bar\theta_i d\theta_i \exp(-\bar\theta A \theta)}$
"Grassmann expectation value"
N=2 $A_{11}A_{22} - A_{12}A_{21}$
N=1 A_{11}
N=3 ... etc..
anticommutation ⇒ sign of permutation
$\langle \theta_i \bar\theta_j \rangle = (A^{-1})_{ij}$
Like ordinary Gaussian Integral over complex numbers
$\langle \theta\bar\theta \rangle = \frac{\int d\bar\theta d\theta (1 + A\theta\bar\theta)(\theta\bar\theta)}{\int d\bar\theta d\theta (1 + A\theta\bar\theta)} = \frac{1}{A}$
$\exp(-\bar\theta A \theta) = A$
≠ complex numbers $\int_{\mathbb{C}} \frac{d\bar z\, dz}{2\pi} \exp(-\bar z A z) = \frac{1}{A}$

$$\int_{\mathbb{C}} \frac{d\bar z\, dz}{2\pi} \exp(-\bar z A z) = \frac{1}{A}$$

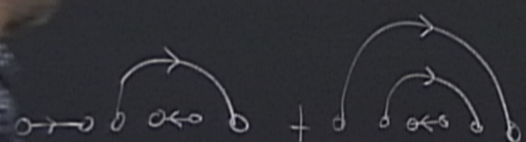
$\int d\theta\, d\bar\theta\, (1 + A\theta\bar\theta)$ A

$$\boxed{\langle \bar\theta_i\, \theta_j \rangle = -(A^{-1})_{ji} = -(A^{-1T})_{ij}}$$ unlike $\mathbb{C}$

- Derivative
- $\frac{\partial}{\partial \theta_i} g$

$\frac{\partial}{\partial \theta_i} g =$

Theorem For Fermions



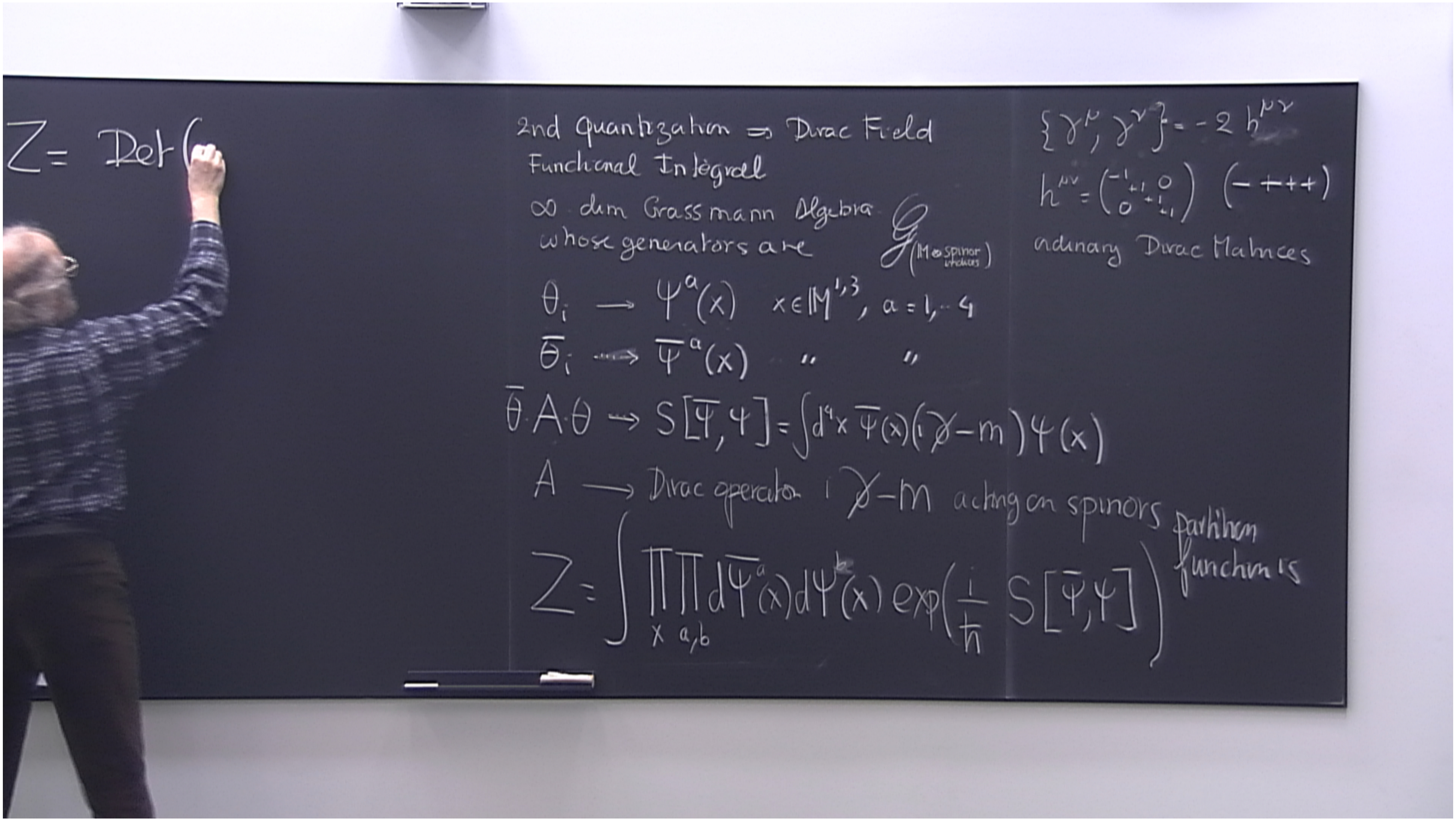
Dirac Field : Dirac Equation in 1+3 dim

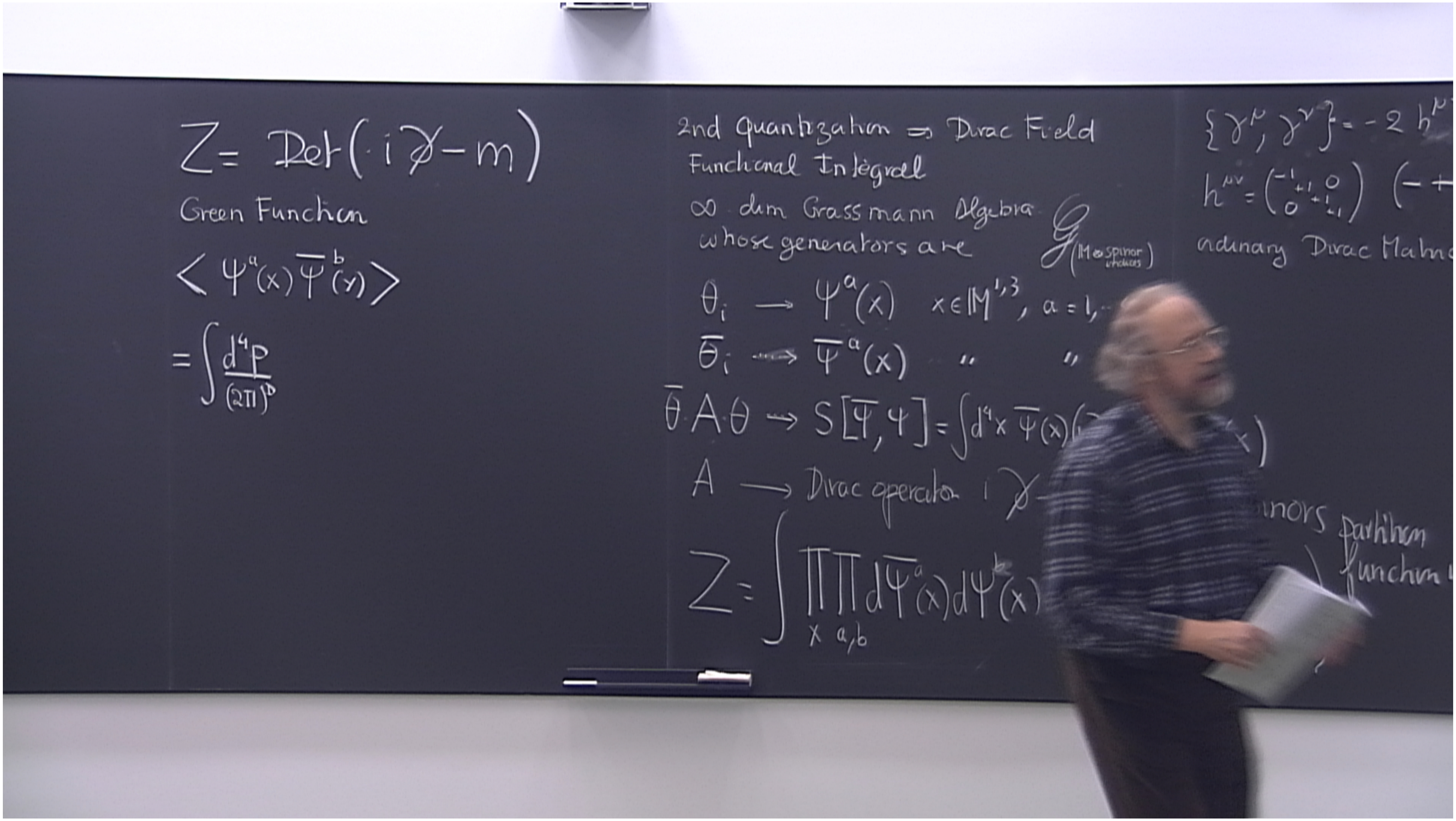
$\psi = \psi^a,\ a = 1, \ldots 4$ a Dirac Indices

$\bar\psi = \bar\psi^a,$

Action

$$S[\bar\psi, \psi] = \int d^4x\, \bar\psi(x) (i \partial\!\!\!/ - m) \psi(x)$$
$$= \int d^4x\, \bar\psi^a(x) \left(i \gamma^\mu_{ab} \frac{\partial}{\partial x_\mu} - m\delta_{ab}\right) \psi^b(x)$$


$Z = \mathrm{Det}($
2nd Quantization $\Rightarrow$ Dirac Field
Functional Integral
∞ dim Grassmann Algebra $\mathcal{G}$ $(\mathbb{M} \otimes$ spinor indices$)$
whose generators are
$\theta_i \longrightarrow \psi^a(x) \quad x \in \mathbb{M}^{1,3},\ a = 1, \dots 4$
$\bar{\theta}_i \longrightarrow \bar{\psi}^a(x)$ " "
$\bar{\theta} \cdot A \cdot \theta \rightsquigarrow S[\bar{\psi}, \psi] = \int d^4x\, \bar{\psi}(x)(i\not{\partial} - m)\psi(x)$
$A \longrightarrow$ Dirac operator $i\not{\partial} - m$ acting on spinors
$Z = \int \prod_x \prod_{a,b} d\bar{\psi}^a(x)\, d\psi^b(x)\, \exp\left(\frac{i}{\hbar} S[\bar{\psi}, \psi]\right)$
partition function
$\{\gamma^\mu, \gamma^\nu\} = -2\, h^{\mu\nu}$
$h^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix} \quad (-+++)$
ordinary Dirac Matrices


Z = Det(i ∂̸ − m)
Green Function
⟨ ψ^a(x) ψ̄^b(y) ⟩
= ∫ d^4p / (2π)^D
2nd quantization ⇒ Dirac Field
Functional Integral
∞ dim Grassmann Algebra
whose generators are
𝒢(M ⊗ spinor choices)
θ_i → ψ^a(x)   x ∈ M^{1,3}, a = 1,
θ̄_i → ψ̄^a(x)   "   "
θ̄ A θ → S[ψ̄, ψ] = ∫d^4x ψ̄(x)
A → Dirac operator i ∂̸
Z = ∫ ∏_x ∏_{a,b} dψ̄^a(x) dψ^b(x)
partition
function
{γ^μ, γ^ν} = −2 h^μν
h^μν = (−1 +1 0; 0 +1 +1)
ordinary Dirac Matrices

$$Z = \mathrm{Det}(i\partial\!\!\!/ - m)$$

Green Function

$$\langle \psi^a(x)\, \bar{\psi}^b(y) \rangle$$

$$= \int \frac{d^4p}{(2\pi)^D}$$

2nd Quantization $\Rightarrow$ Dirac Field

Functional Integral

∞ dim Grassmann Algebra $\mathcal{G}$ whose generators are $\mathcal{G}_{(\mathbb{M} \otimes \text{spinor choices})}$

$$\theta_i \longrightarrow \psi^a(x) \quad x \in \mathbb{M}^{1,3},\ a = 1, \dots 4$$

$$\bar{\theta}_i \longrightarrow \bar{\psi}^a(x) \quad \text{"} \quad \text{"}$$

$$\bar{\theta} \cdot A \cdot \theta \rightsquigarrow S[\bar{\psi}, \psi] = \int d^4x\, \bar{\psi}(x)(i\partial\!\!\!/ - m)\psi(x)$$

$A \longrightarrow$ Dirac operator $i\partial\!\!\!/ - m$ acting on spinors

$$Z = \int \prod_x \prod_{a,b} d\bar{\psi}^a(x)\, d\psi^b(x) \exp\left(\frac{i}{\hbar} S[\bar{\psi}, \psi]\right)$$

partition function

$$\{\gamma^\mu, \gamma^\nu\} = -2h^{\mu\nu}$$

$$h^{\mu\nu} = \begin{pmatrix} -1 & & & 0 \\ & +1 & & \\ & & +1 & \\ 0 & & & +1 \end{pmatrix} \quad (- + \dots$$

ordinary Dirac Matrices

$$Z = \mathrm{Det}(i\not{\partial} - m)$$

Green Function ; Dirac Field Operator

$$\langle \Psi^a(x)\bar{\Psi}^b(y)\rangle = \langle 0|T[\Psi(x)\bar{\Psi}(y)]|0\rangle$$

$$= \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)}\left(\frac{i}{-\not{p} - m - i\epsilon_+}\right)_{ab}$$

2nd Quantization $\Rightarrow$ Dirac Field

Functional Integral

∞ dim Grassmann Algebra $\mathcal{G}$

whose generators are $\mathcal{G}_{(M \otimes \text{spinor choices})}$

$$\theta_i \longrightarrow \Psi^a(x) \quad x \in M^{1,3},\ a = 1, \dots 4$$

$$\bar{\theta}_i \longrightarrow \bar{\Psi}^a(x) \quad '' \quad ''$$

$$\bar{\theta}\cdot A\cdot\theta \rightsquigarrow S[\bar{\Psi},\Psi] = \int d^4x\, \bar{\Psi}(x)(i\not{\partial} - m)\Psi(x)$$

$A \longrightarrow$ Dirac operator $i\not{\partial} - m$ acting on spinors

$$Z = \int \prod_x \prod_{a,b} d\bar{\Psi}^a(x)\, d\Psi^b(x)\, \exp\left(\frac{i}{\hbar} S[\bar{\Psi},\Psi]\right)$$

partition function

$$\{\gamma^\mu, \gamma^\nu\} = -2h^{\mu\nu}$$

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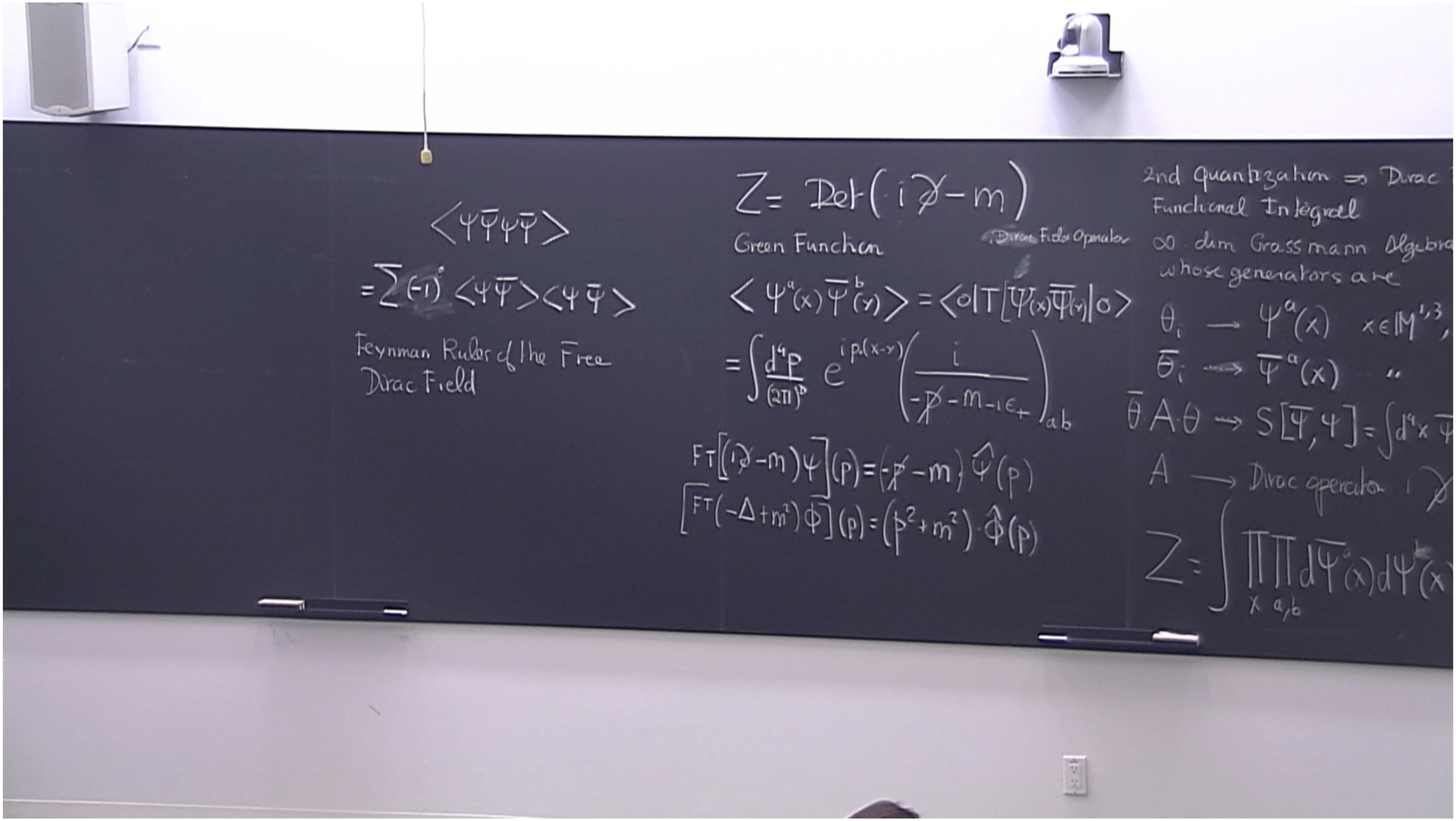
ordinary Dirac Matri


$\langle \psi\bar\psi\psi\bar\psi \rangle$
$= \sum (-1)^{\sigma} \langle \psi\bar\psi \rangle \langle \psi\bar\psi \rangle$
Feynman Rules of the Free Dirac Field
$Z = \mathrm{Det}(i\not\partial - m)$
Green Function
Dirac Field Operator
$\langle \psi^a(x)\bar\psi^b(y) \rangle = \langle 0| T[\Psi(x)\bar\Psi(y)] |0\rangle$
$= \int \frac{d^4p}{(2\pi)^4} e^{ip\cdot(x-y)} \left(\frac{i}{-\not p - m - i\epsilon_+} \right)_{ab}$
$FT[(i\not\partial - m)\psi](p) = (-\not p - m)\hat\psi(p)$
$[FT(-\Delta + m^2)\phi](p) = (p^2 + m^2)\cdot\hat\phi(p)$
2nd Quantization ⇒ Dirac
Functional Integral
∞ dim Grassmann Algebra whose generators are
$\theta_i \longrightarrow \psi^a(x) \quad x \in M^{1,3}$,
$\bar\theta_i \rightsquigarrow \bar\psi^a(x)$ "
$\bar\theta \cdot A \cdot \theta \rightsquigarrow S[\bar\psi, \psi] = \int d^4x\, \bar\psi$
$A \longrightarrow$ Dirac operator $i\not\partial$
$Z = \int \prod_x \prod_{a,b} d\bar\psi^a(x)\, d\psi^b(x)$