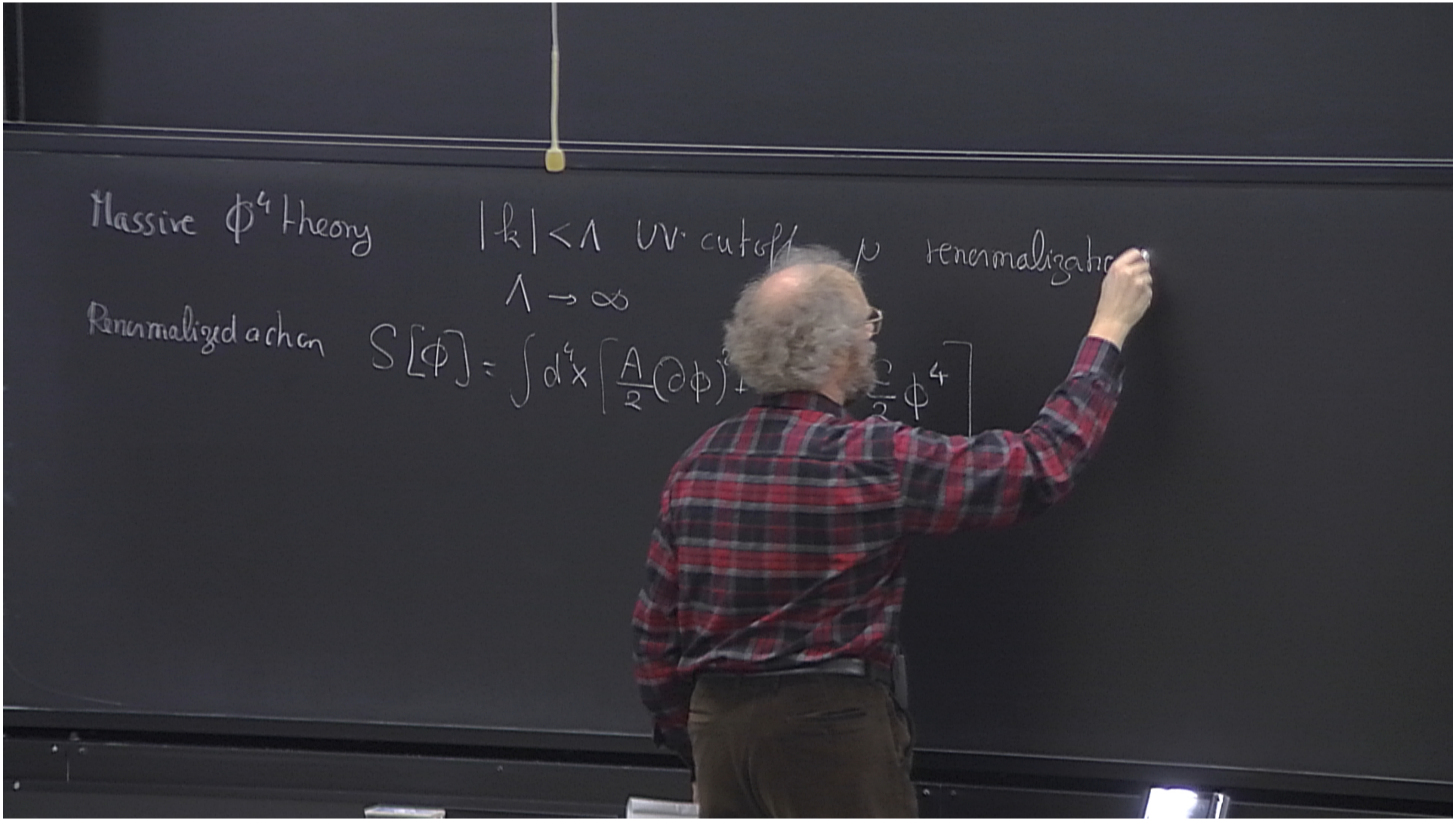
Title: 14/15 PSI - Quantum Field Theory II-Lecture 8

Date: Nov 19, 2014 09:00 AM

URL: http://pirsa.org/14110015

Abstract:

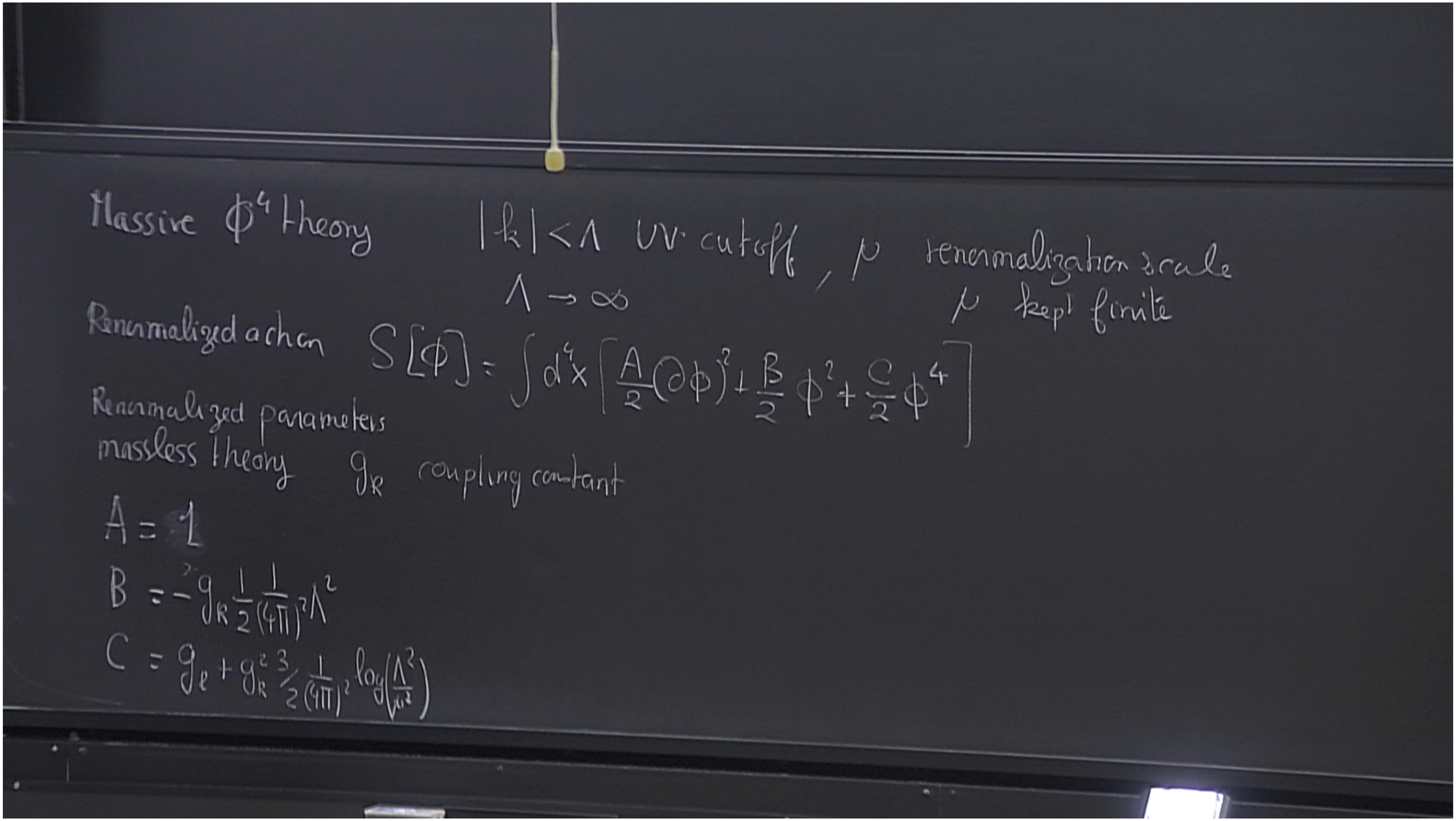

Massive φ4 theory
|k| < Λ UV cutoff
μ renormalizatio
Λ → ∞
Renormalized action
S[φ] = ∫d4x [A/2 (∂φ)2
φ4]

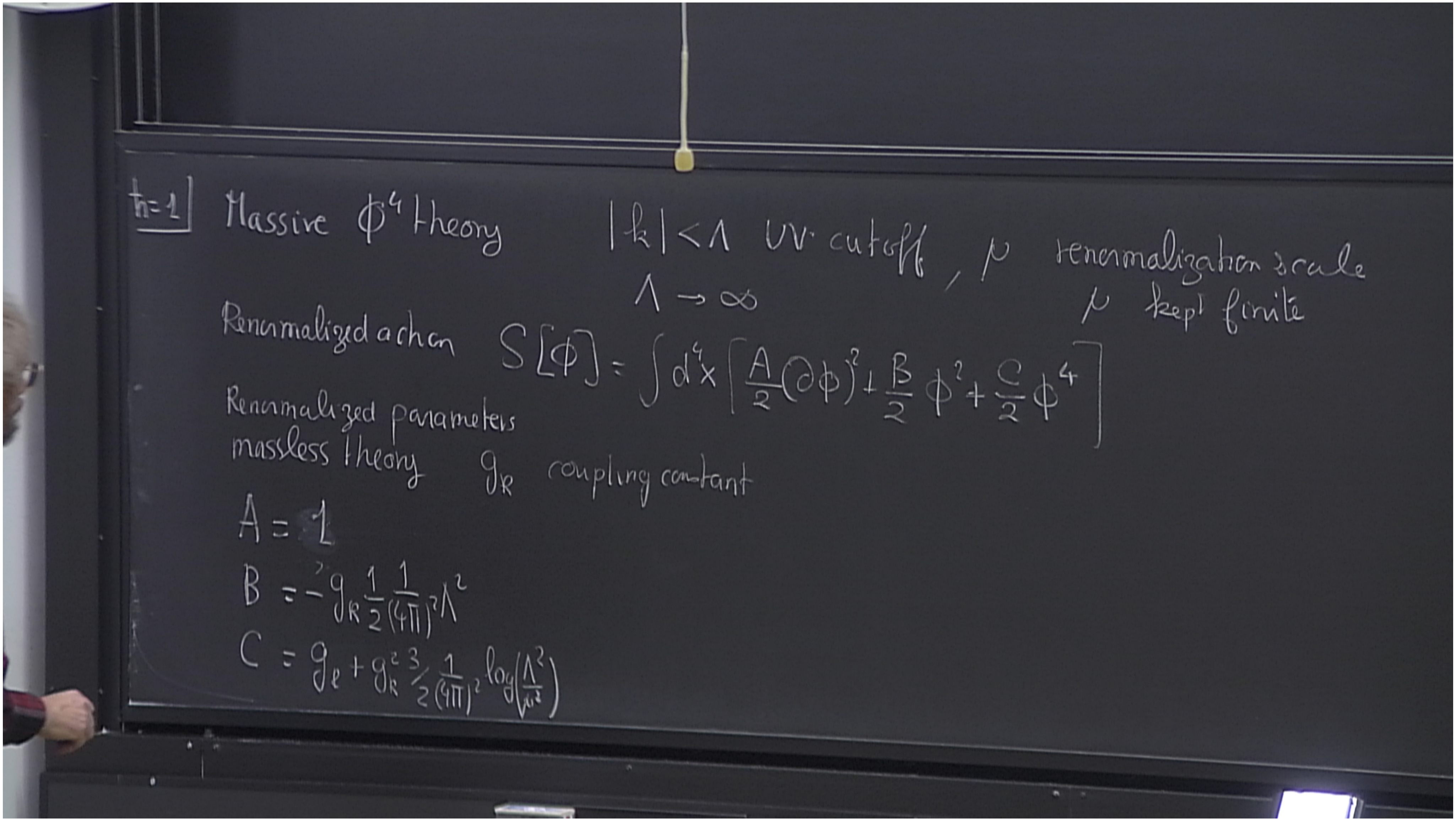
Massive ϕ^4 theory

$|k| < \Lambda$ UV cutoff, μ renormalization scale

$\Lambda \to \infty$ μ kept finite

Renormalized action $S[\phi] = \int d^4x \left[\frac{A}{2}(\partial\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \right]$


Massive ϕ^4 theory
$|k| < \Lambda$ UV cutoff, μ renormalization scale
$\Lambda \to \infty$
μ kept finite
Renormalized action $S[\phi] = \int d^4x \left[\frac{A}{2}(\partial\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \right]$
Renormalized parameters
massless theory g_R coupling constant
$A = 1$
$B = -g_R \frac{1}{2} \frac{1}{(4\pi)^2} \Lambda^2$
$C = g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{\mu^2}\right)$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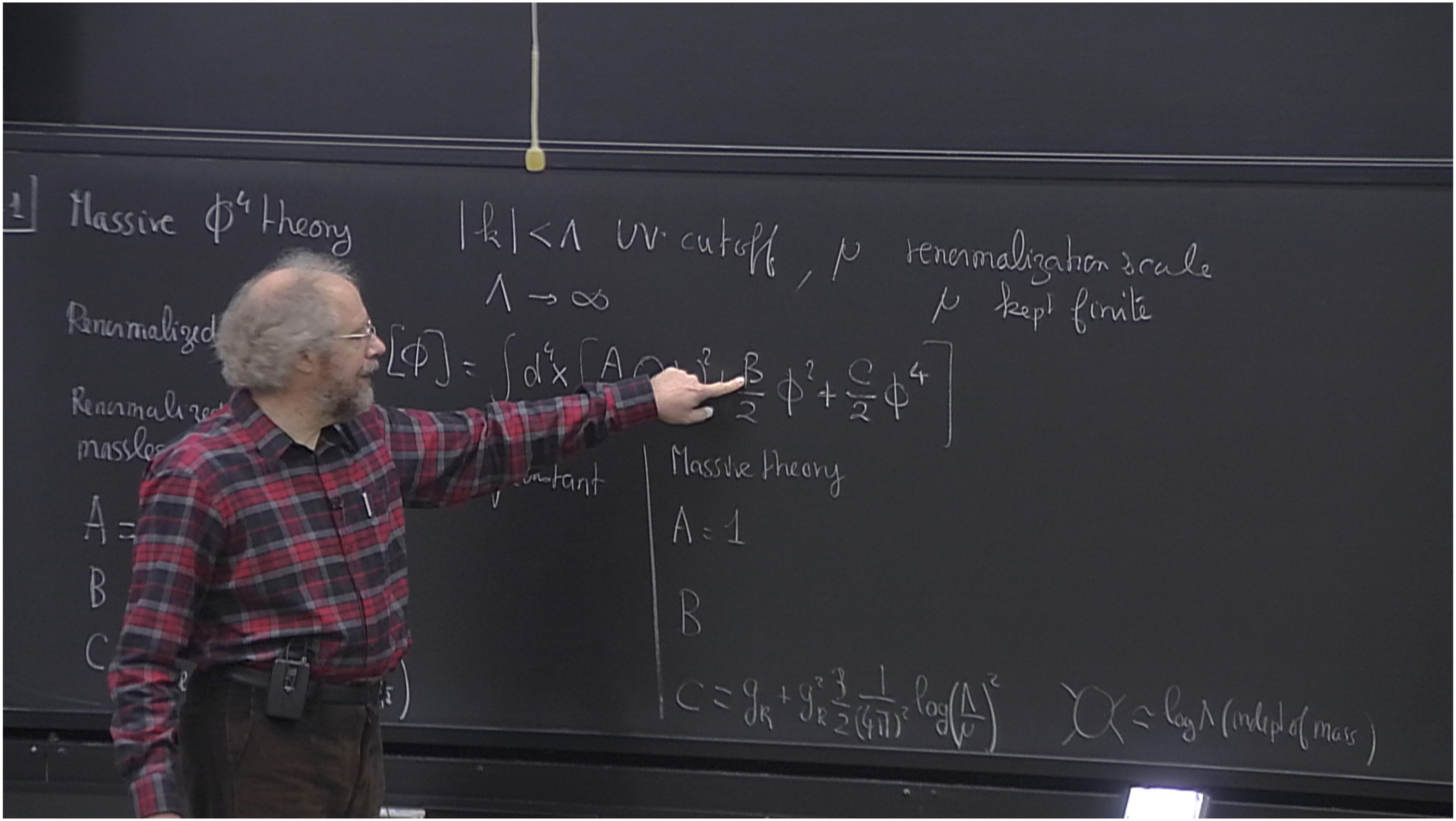

Massive φ⁴ theory
|k| < Λ UV cutoff , μ renormalization scale
Λ → ∞
μ kept finite
Renormalized
B/2 φ² + C/2 φ⁴
Massive theory
A = 1
B
C = g_R + g_R² 3/2 1/(4π)² log(Λ/μ)²
≃ log Λ (indep of mass)
constant

...utoff, μ renormalization scale

μ kept finite

$$= T(B, \Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B}$$

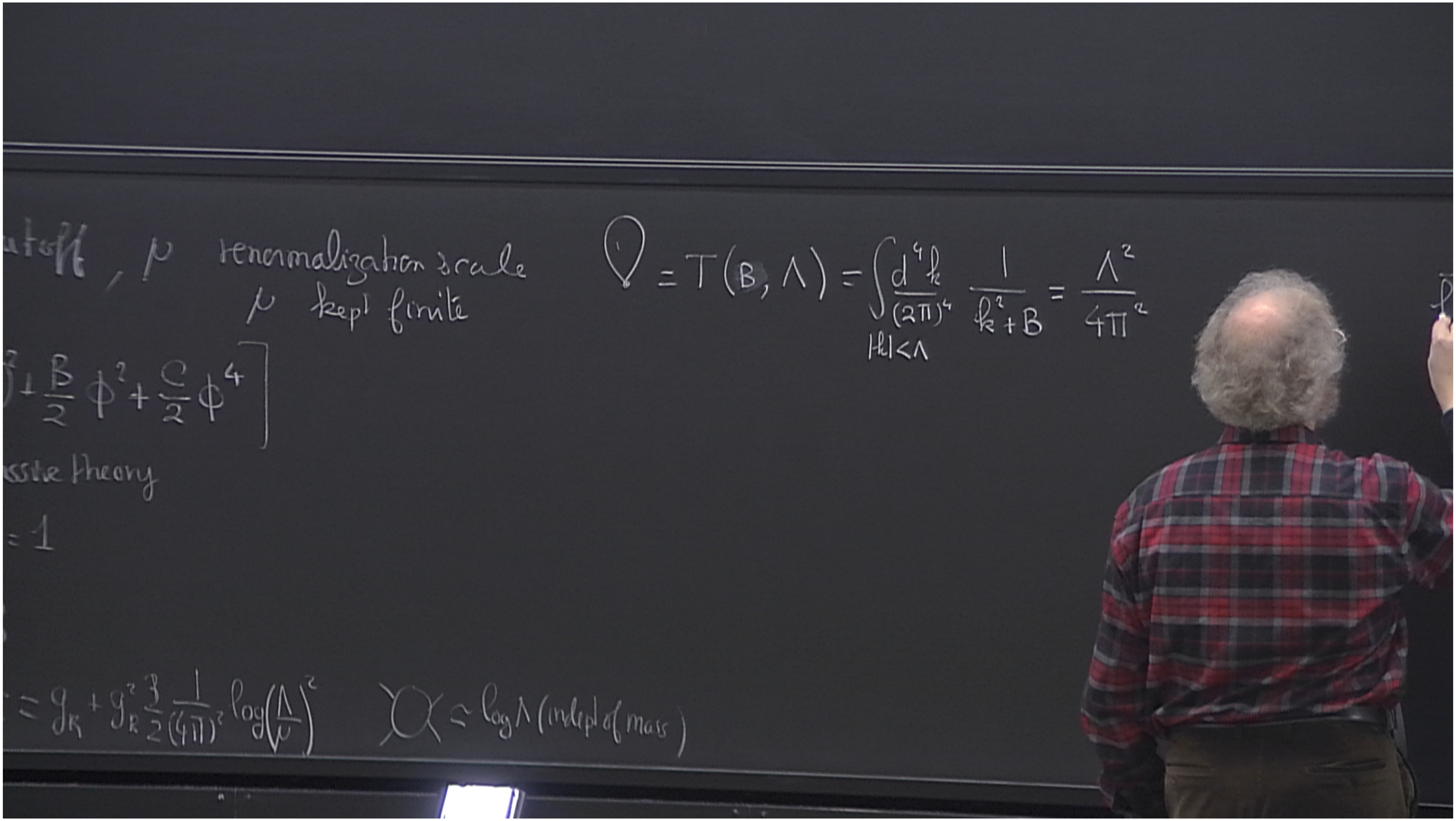
$$\ldots + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \Big]$$

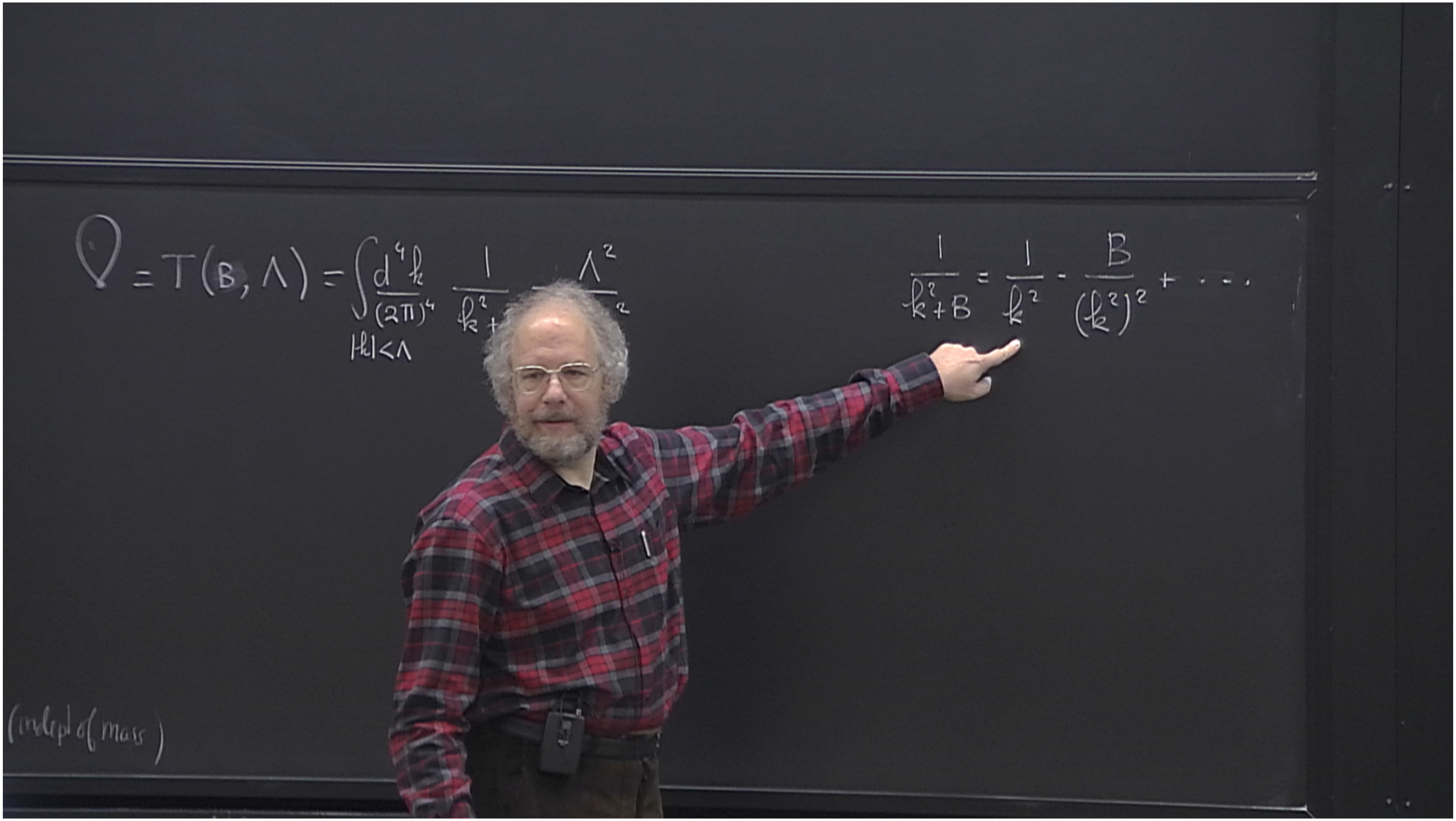
...ssive theory

$= 1$

$$= g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda}{\mu}\right)^2$$

$\simeq \log\Lambda$ (indep't of mass)


, μ renormalization scale
μ kept finite
$\frac{B}{2}\phi^2 + \frac{C}{2}\phi^4$
theory
$= T(B, \Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{4\pi^2}$
$= g_R + g_R^2 \frac{3}{2} \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda}{\mu}\right)^2$
$\simeq \log\Lambda$ (indep't of mass)


= T(B, Λ) = ∫ d⁴k/(2π)⁴
|k|<Λ
1/(k²+B) = 1/k² - B/(k²)² + . . .
(indep of mass)

$$= T(B,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2} \log \Lambda^2 + \cdots \qquad \frac{1}{k^2+B} = \frac{1}{k^2} - \frac{B}{(k^2)^2} + \cdots$$

(indep of mass)

$\hbar = 1$ Massive ϕ^4 theory

$|k| < \Lambda$ UV cutoff, μ renormalization scale $=$

$\Lambda \to \infty$ μ kept finite

Renormalized action $S[\phi] = \int d^4x \left[\frac{A}{2}(\partial\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \right]$

Renormalized parameters

massless theory g_R coupling constant

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$C = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$

Massive theory g_R

m_R renormalized mass

$A = 1$

$B = m_R^2$

$C = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda}{\mu}\right)^2$

$\simeq \log\Lambda$ (indep. of mass)

$\boxed{\hbar = 1}$ Massive ϕ^4 theory $\quad |k| < \Lambda$ UV cutoff, μ renormalization scale

$\Lambda \to \infty \qquad \mu$ kept finite

Renormalized action $S[\phi] = \int d^4x \left[\frac{A}{2}(\partial\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \right]$

Renormalized parameters

massless theory $\quad g_R$ coupling constant

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Massive theory $\quad g_R$

m_R renormalized mass

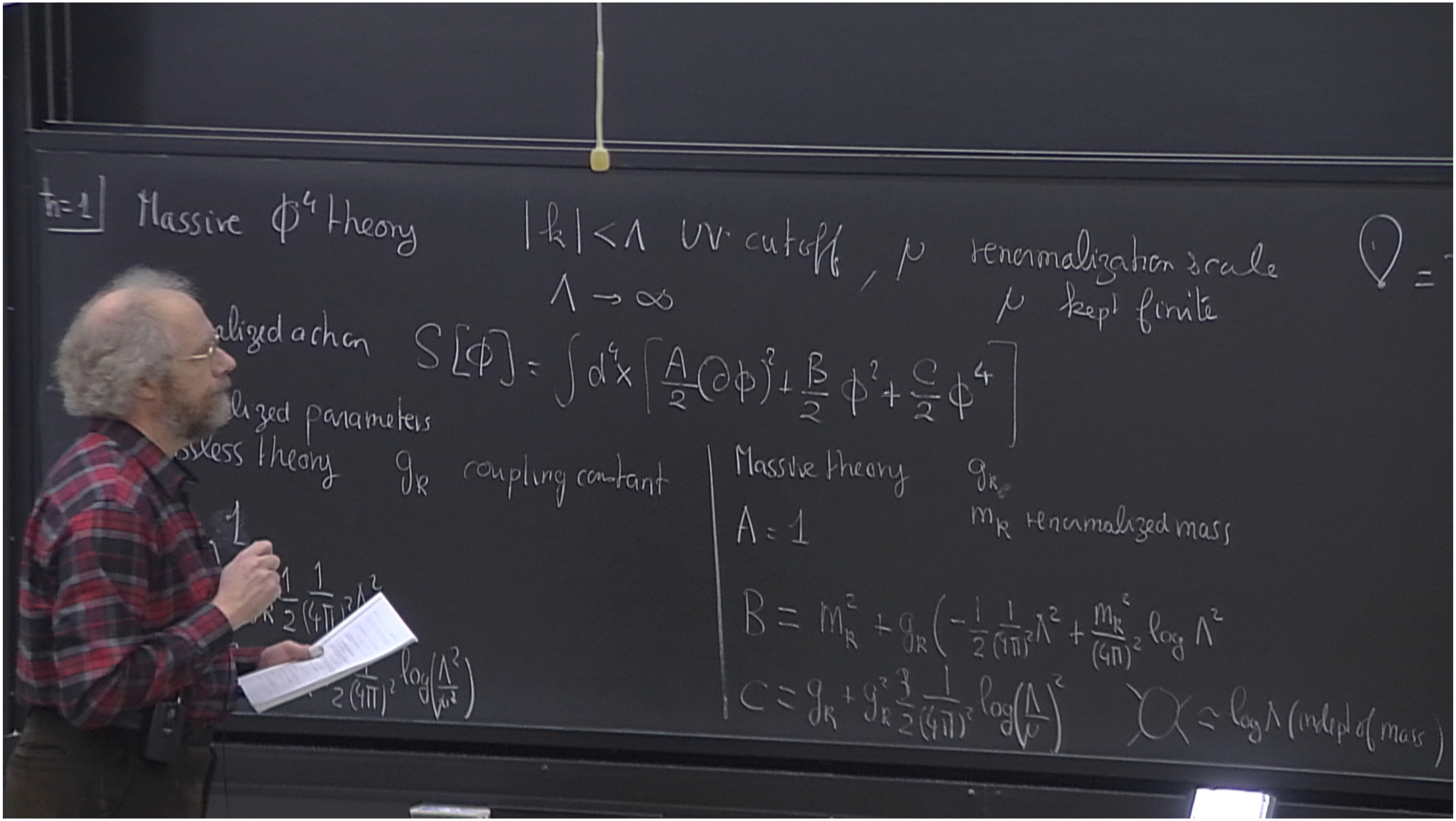
$A = 1$

$B = m_R^2 + g_R\left(-\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2\right.$

$C = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda}{\mu}\right)^2$

$\simeq \log\Lambda$ (indep't of mass)

=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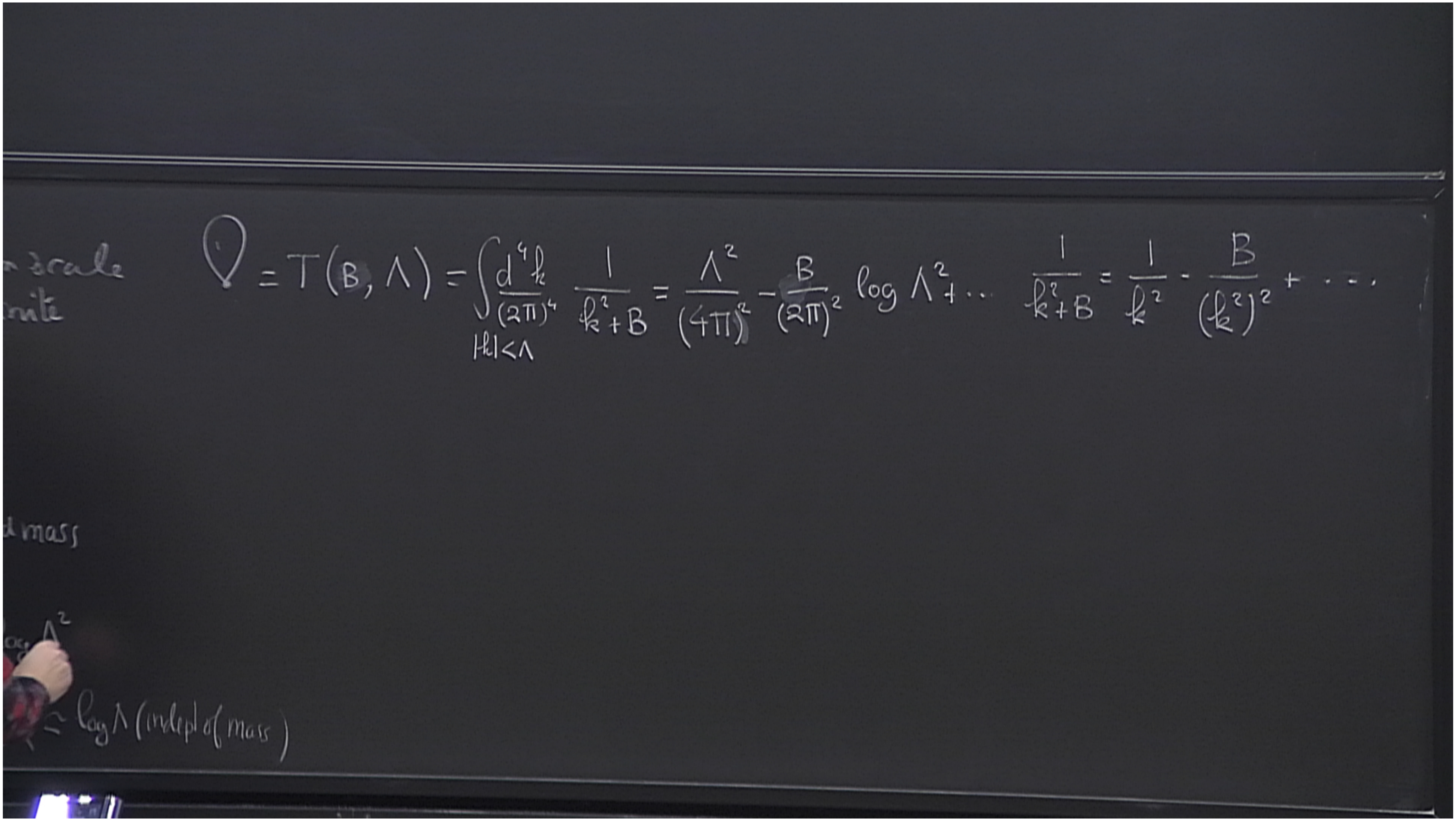
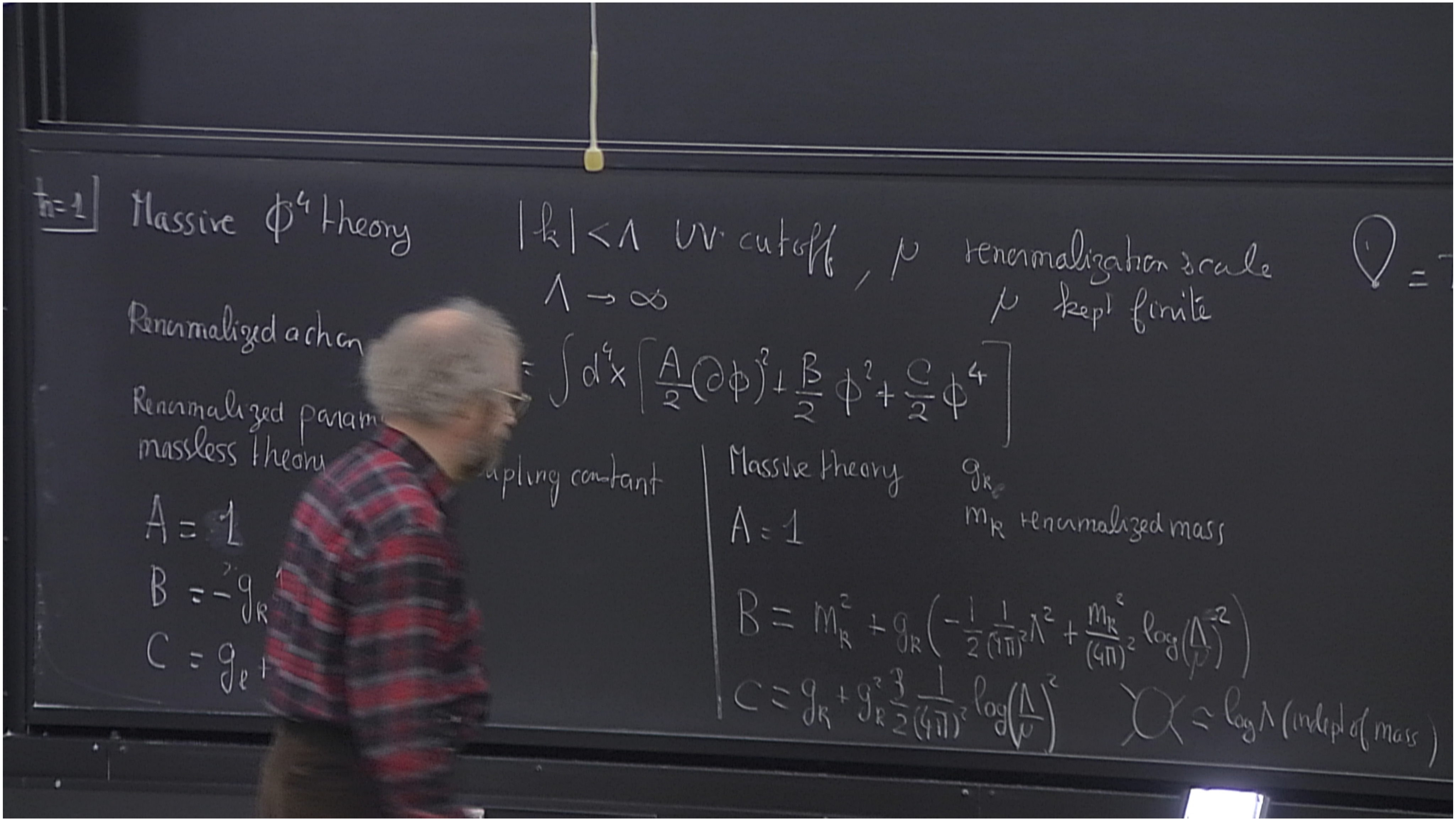
|k| < Λ UV cutoff , μ renormalization scale
Λ → ∞
μ kept finite
S[φ] = ∫ d^4x [A/2 (∂φ)^2 + B/2 φ^2 + C/2 φ^4]
parameters
theory g_R coupling constant
Massive theory g_R
m_R renormalized mass
A = 1
B = m_R^2 + g_R(−1/2 1/(4π)^2 Λ^2 + m_R^2/(4π)^2 log Λ^2
C = g_R + g_R^2 3/2 1/(4π)^2 log(Λ/μ)^2
≃ log Λ (indep't of mass)



scale
nite
$= T(B, \Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2} \log \Lambda^2 + \dots$
$\frac{1}{k^2+B} = \frac{1}{k^2} - \frac{B}{(k^2)^2} + \dots$
d mass
$\propto \Lambda^2$
$\simeq \log \Lambda$ (indep't of mass)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Renormalized action
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Massive theory
g_R
m_R renormalized mass
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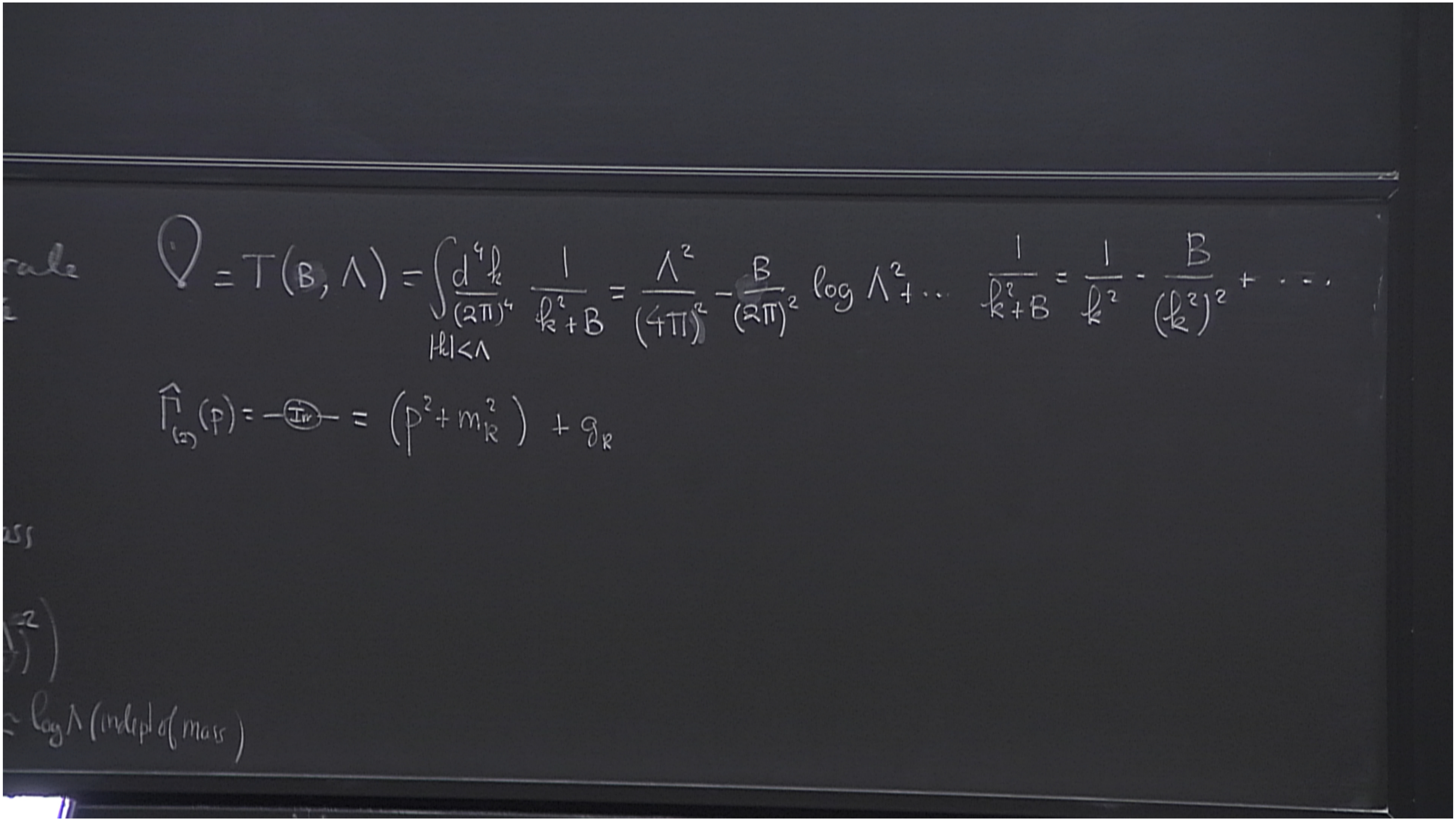
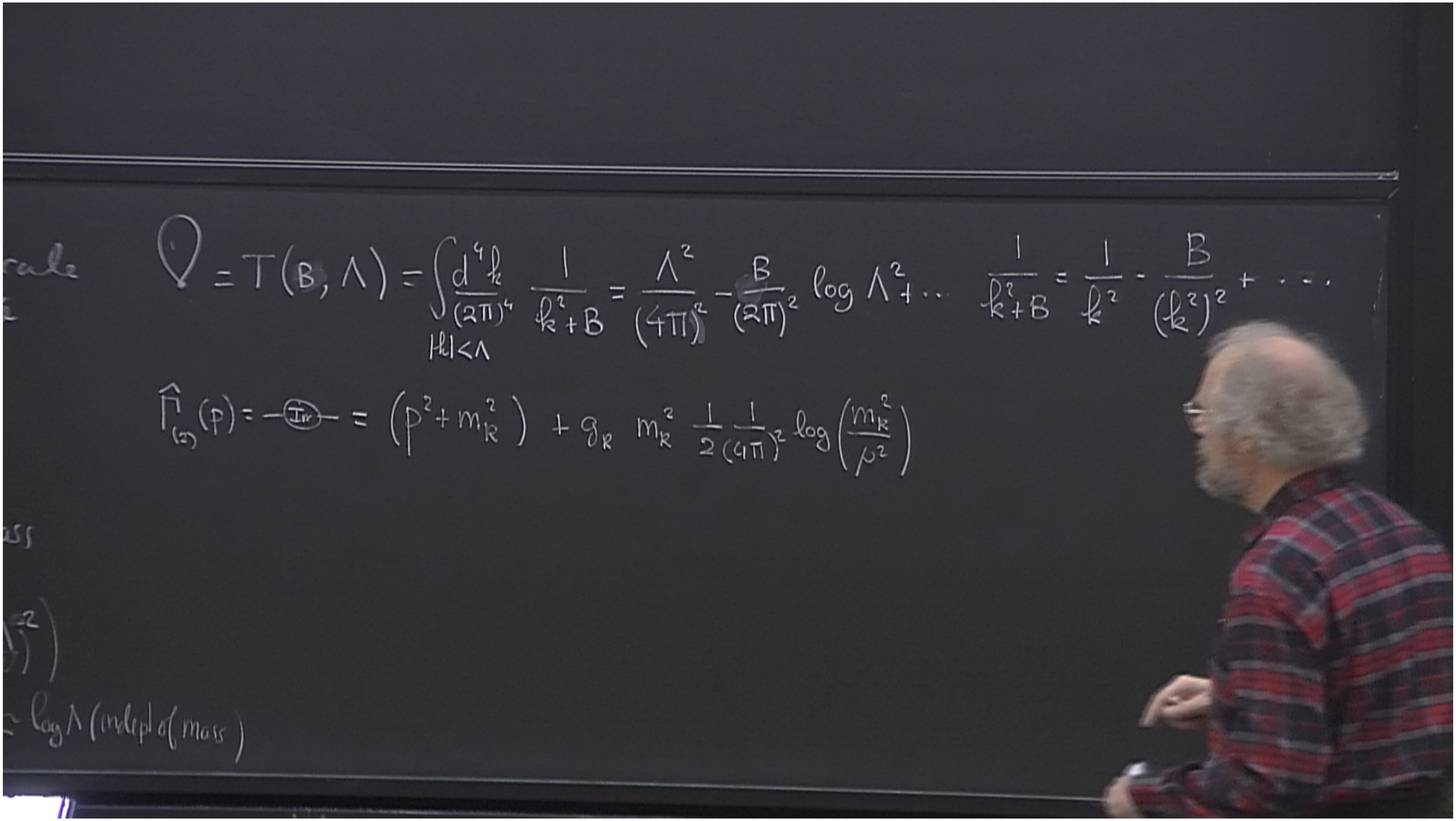
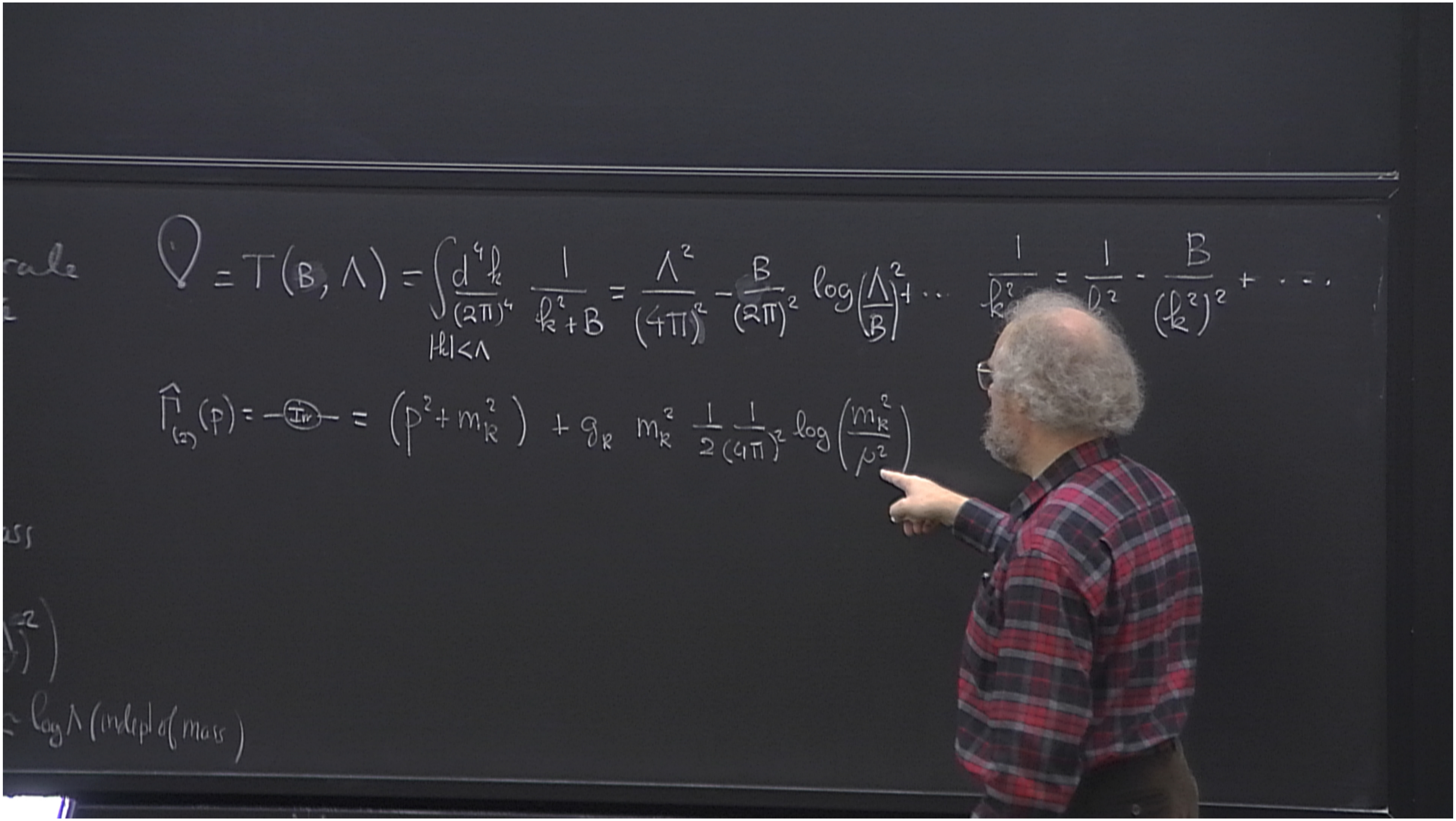
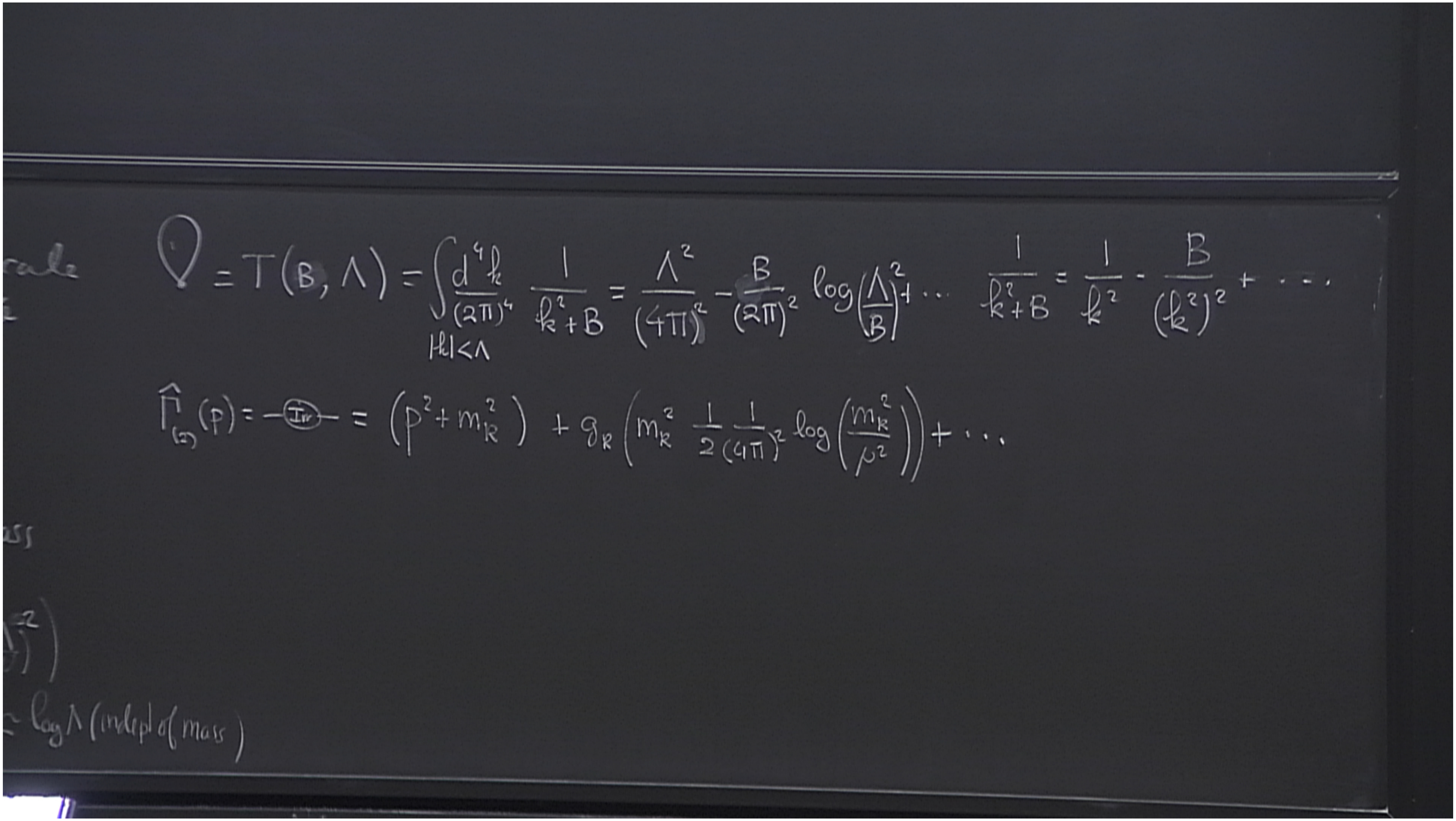


rale
$= T(B, \Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2} \log \Lambda^2 + \cdots$
$\frac{1}{k^2+B} = \frac{1}{k^2} - \frac{B}{(k^2)^2} + \cdots$
$\hat{\Gamma}_{(2)}(p) = $ Irr $= (p^2 + m_R^2) + g_R$
ass
$\log \Lambda$ (indep. of mass)


= T(B, Λ) = ∫_{|k|<Λ} d⁴k/(2π)⁴ 1/(k² + B) = Λ²/(4π)² − B/(2π)² log Λ² + … 1/(k² + B) = 1/k² − B/(k²)² + …
Γ̂_(2)(p) = (p² + m_R²) + g_R m_R² (1/2) 1/(4π)² log(m_R²/μ²)
log Λ (indep't of mass)


$= T(B,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2} \log\left(\frac{\Lambda}{B}\right)^2 + \cdots$
$\frac{1}{k^2} = \frac{1}{k^2} - \frac{B}{(k^2)^2} + \cdots$
$\hat{\Gamma}_{(2)}(p) = -(\text{Irr})- = (p^2 + m_R^2) + g_R\, m_R^2 \frac{1}{2} \frac{1}{(4\pi)^2} \log\left(\frac{m_R^2}{\mu^2}\right)$
$\sim \log\Lambda$ (indep of mass)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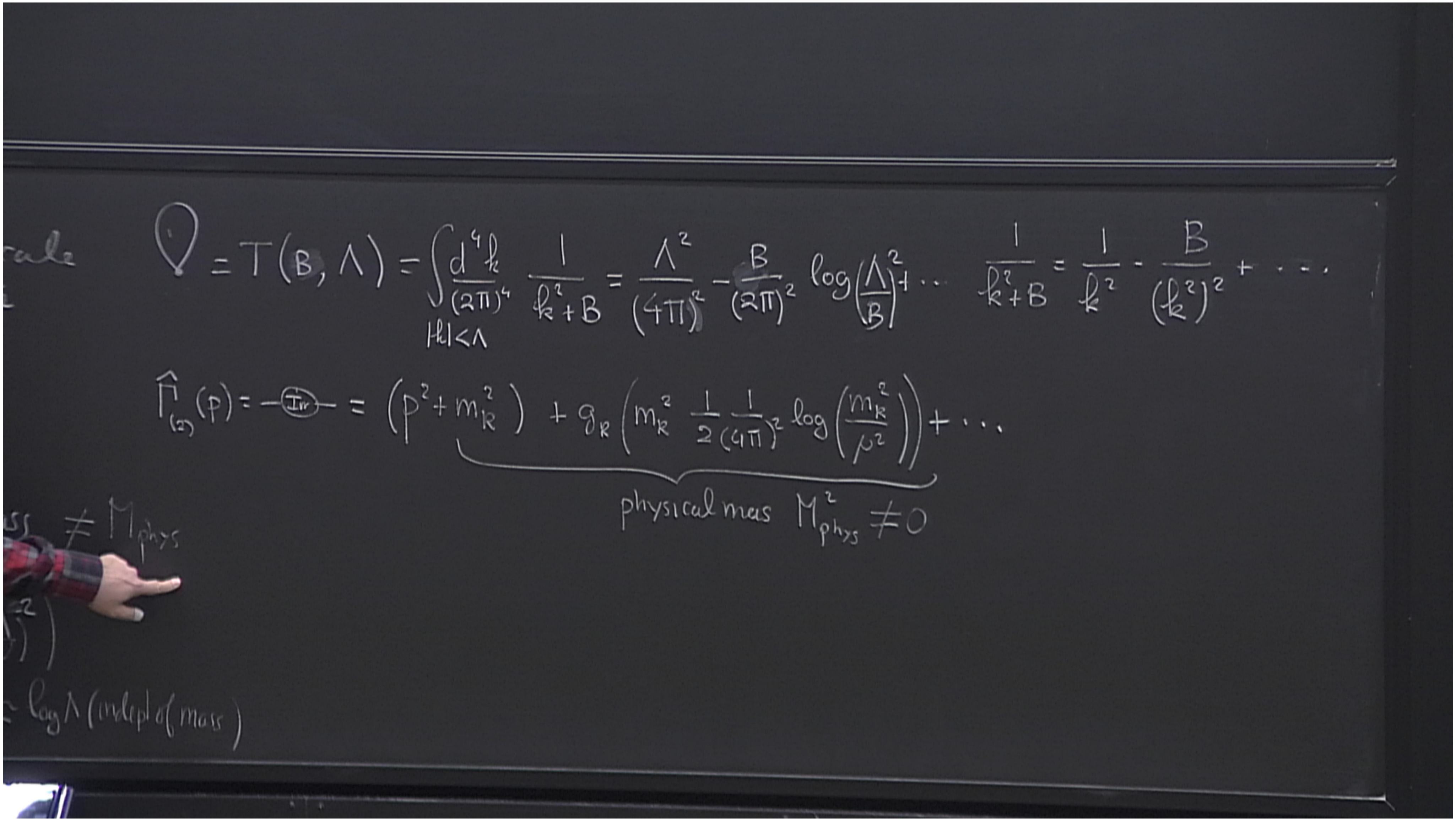
~ log Λ (indep of mass)



cale
$= T(B,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2} \log\left(\frac{\Lambda}{B}\right)^2 + \cdots$
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physical mass $M_{phys}^2 \neq 0$
ss $\neq M_{phys}$
$\log\Lambda$ (indep of mass)

cutoff , μ renormalization scale
μ kept finite

$\phi)^2 + \frac{B}{2}\phi^2 + \frac{C}{2}\phi^4 \Big]$

Massive theory $\quad g_R$

$A = 1 \quad$ m_R renormalized mass $\neq M_{phys}$

$$\bigcirc = T(B,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4}\frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2}\log\left(\frac{\Lambda}{B}\right)^2 + \dots$$

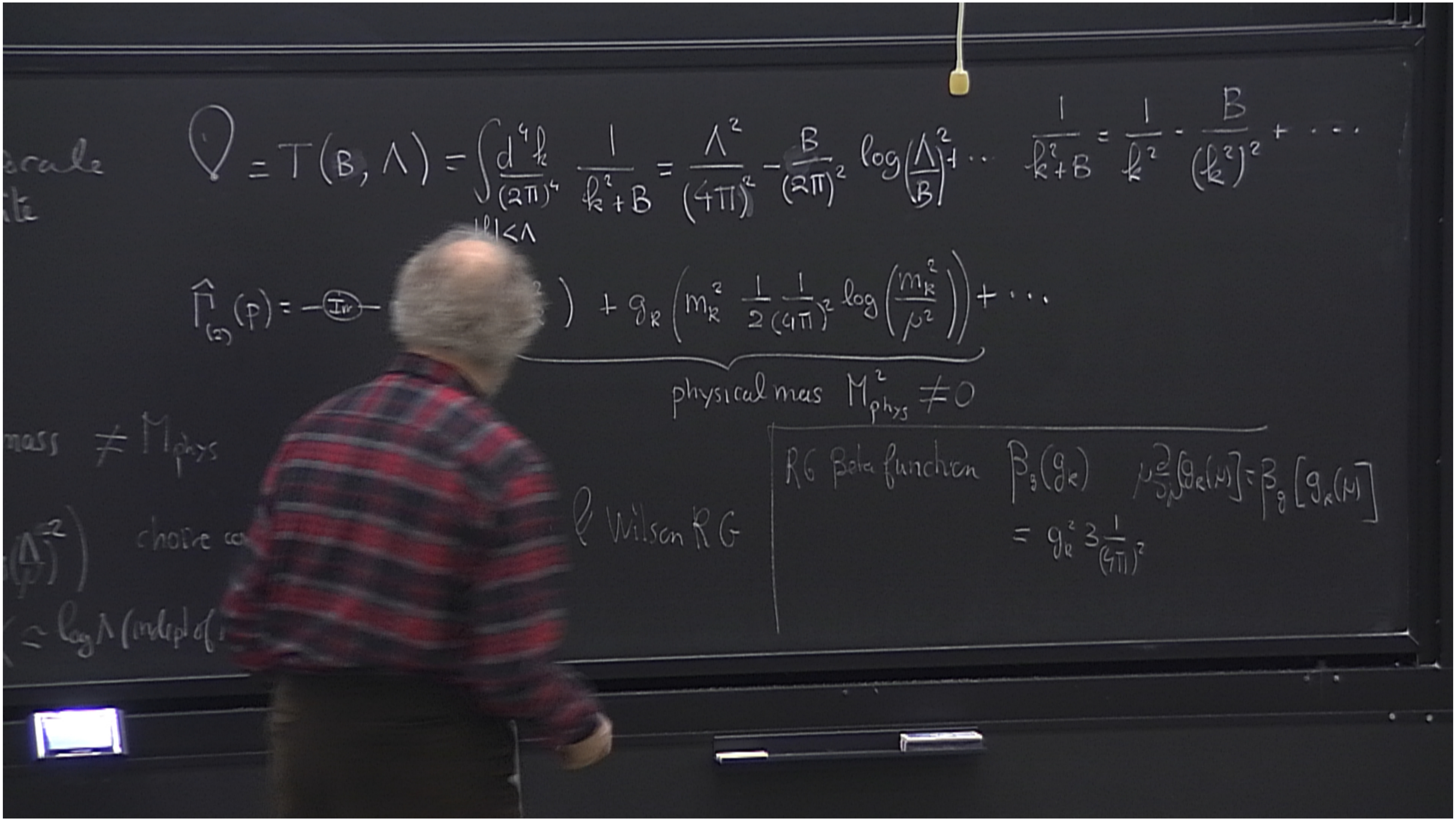
$$\hat{\Gamma}_{(2)}(p) = -\text{(Ir)}- = \underbrace{(p^2 + m_R^2) + g_R\left(m_R^2 \frac{1}{2}\frac{1}{(4\pi)^2}\log\left(\frac{m_R^2}{\mu^2}\right)\right)}_{\text{physical mass } M^2_{phys} \neq 0} +$$

$$B = m_R^2 + g_R\left(-\frac{1}{2}\frac{1}{(4\pi)^2}\Lambda^2 + \frac{m_R^2}{(4\pi)^2}\log\left(\frac{\Lambda}{\mu}\right)^2\right)$$

choice convenient to study limit $m_R \to 0$ & Wilson RG

$$C = g_R + g_R^2 \frac{3}{2}\frac{1}{(4\pi)^2}\log\left(\frac{\Lambda}{\mu}\right)^2$$

$\rangle\!\langle \simeq \log\Lambda$ (indep of mass)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|k|<Λ
1/(k²+B) = 1/k² − B/(k²)² + ···
physical mas M²phys ≠ 0
mass ≠ Mphys
Wilson RG
RG Beta function βg(gR)
= gR² 3 1/(4π)²

$$\text{(loop)} = T(B,\Lambda) = \int_{|k|<\Lambda} \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2+B} = \frac{\Lambda^2}{(4\pi)^2} - \frac{B}{(2\pi)^2}\log\left(\frac{\Lambda}{B}\right)^2 + \cdots \qquad \frac{1}{k^2+B} = \frac{1}{k^2} - \frac{B}{(k^2)^2} + \cdots$$

$$\hat{\Gamma}_{(2)}(p) = -\text{(Ir)}- = \underbrace{(p^2 + m_R^2) + g_R\left(m_R^2\,\frac{1}{2}\,\frac{1}{(4\pi)^2}\log\left(\frac{m_R^2}{\mu^2}\right)\right)}_{\text{physical mass } M^2_{phys} \neq 0} + \cdots$$

...scale

...ite

...mass $\neq M_{phys}$

...$\left(\frac{\Lambda}{\mu}\right)^2$)

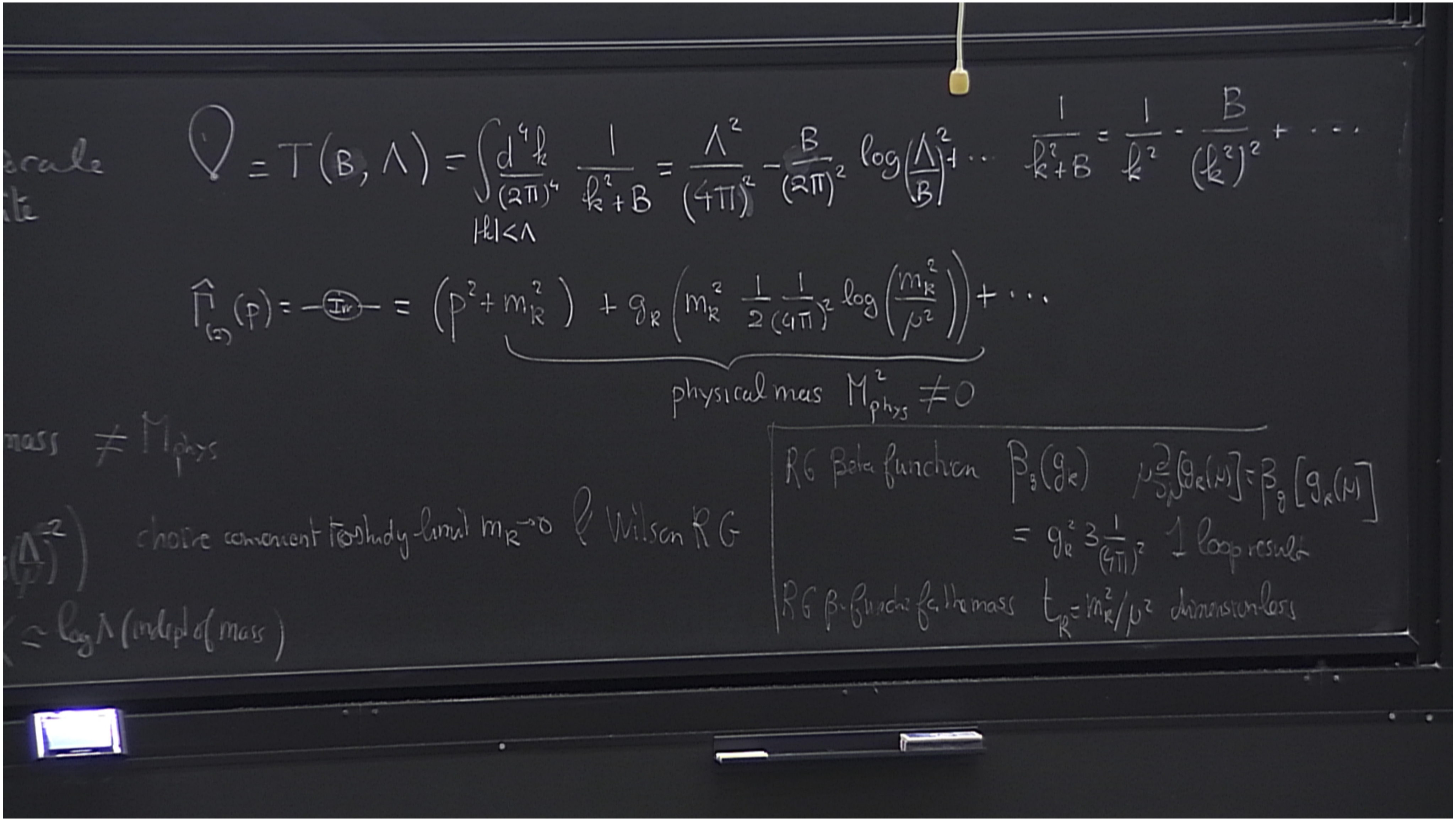
choose convenient to study limit $m_R \to 0$ & Wilson RG

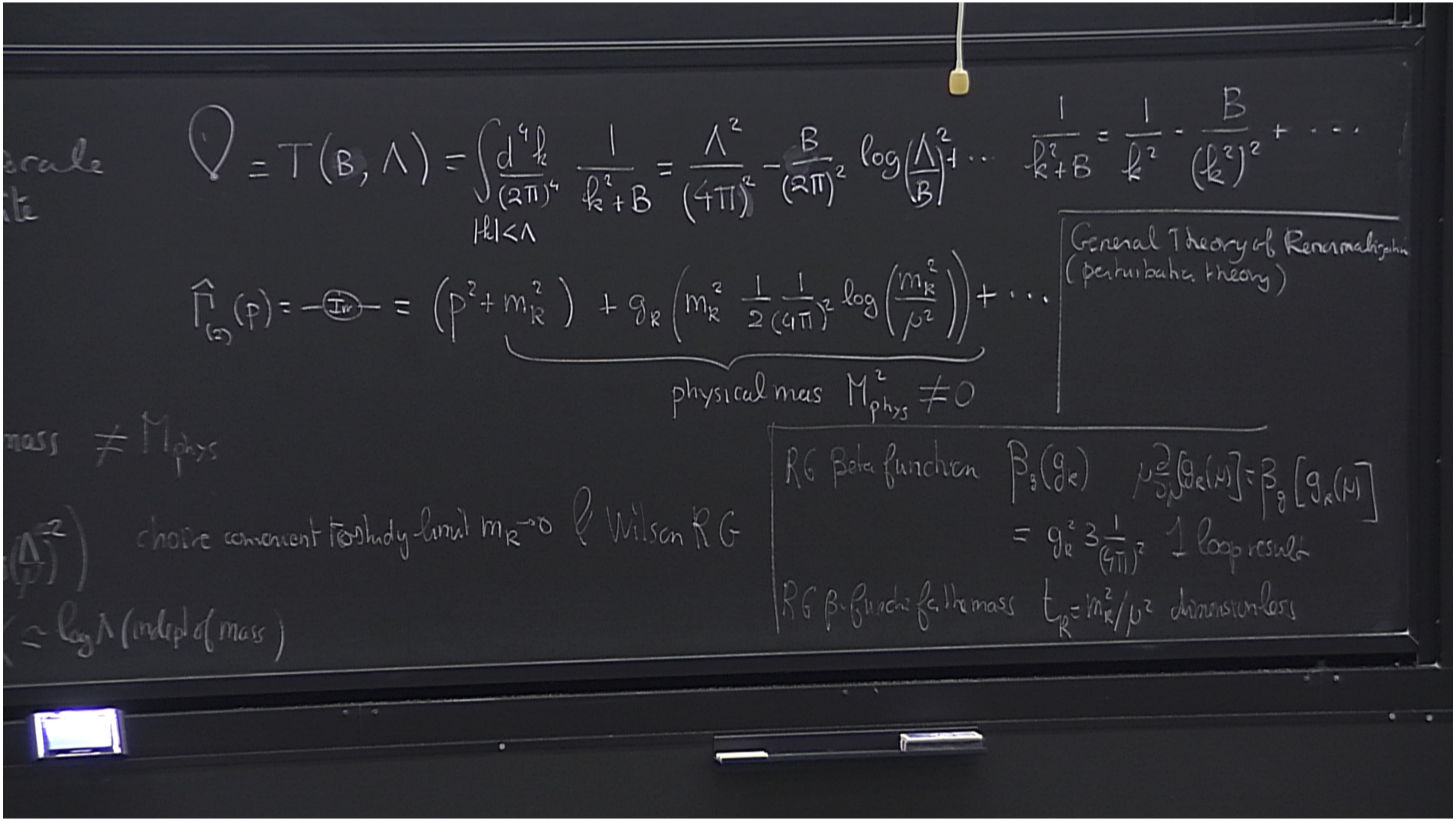
...$\simeq \log\Lambda$ (indept of mass)

RG Beta function $\beta_g(g_R)$ $\quad \mu\frac{\partial}{\partial\mu}[g_R(\mu)] = \beta_g[g_R(\mu)]$

$= g_R^2\, 3\, \frac{1}{(4\pi)^2}$

RG β-function for the mass $\quad t_R = m_R^2/\mu^2$ dimensionless


= T(B, Λ) = ∫ d^4k/(2π)^4 1/(k^2+B) = Λ^2/(4π)^2 − B/(2π)^2 log(Λ/B)^2 + ...
|k|<Λ
1/(k^2+B) = 1/k^2 − B/(k^2)^2 + ...
Γ̂_(2)(p) = (p^2 + m_R^2) + g_R (m_R^2 1/2 1/(4π)^2 log(m_R^2/μ^2)) + ...
physical mass M^2_phys ≠ 0
mass ≠ M_phys
choose convenient to study limit m_R → 0 & Wilson RG
≃ log Λ (indep of mass)
RG Beta function β_g(g_R) μ ∂/∂μ [g_R(μ)] = β_g [g_R(μ)]
= g_R^2 3 1/(4π)^2 1 loop result
RG β-function for the mass t_R = m_R^2/μ^2 dimensionless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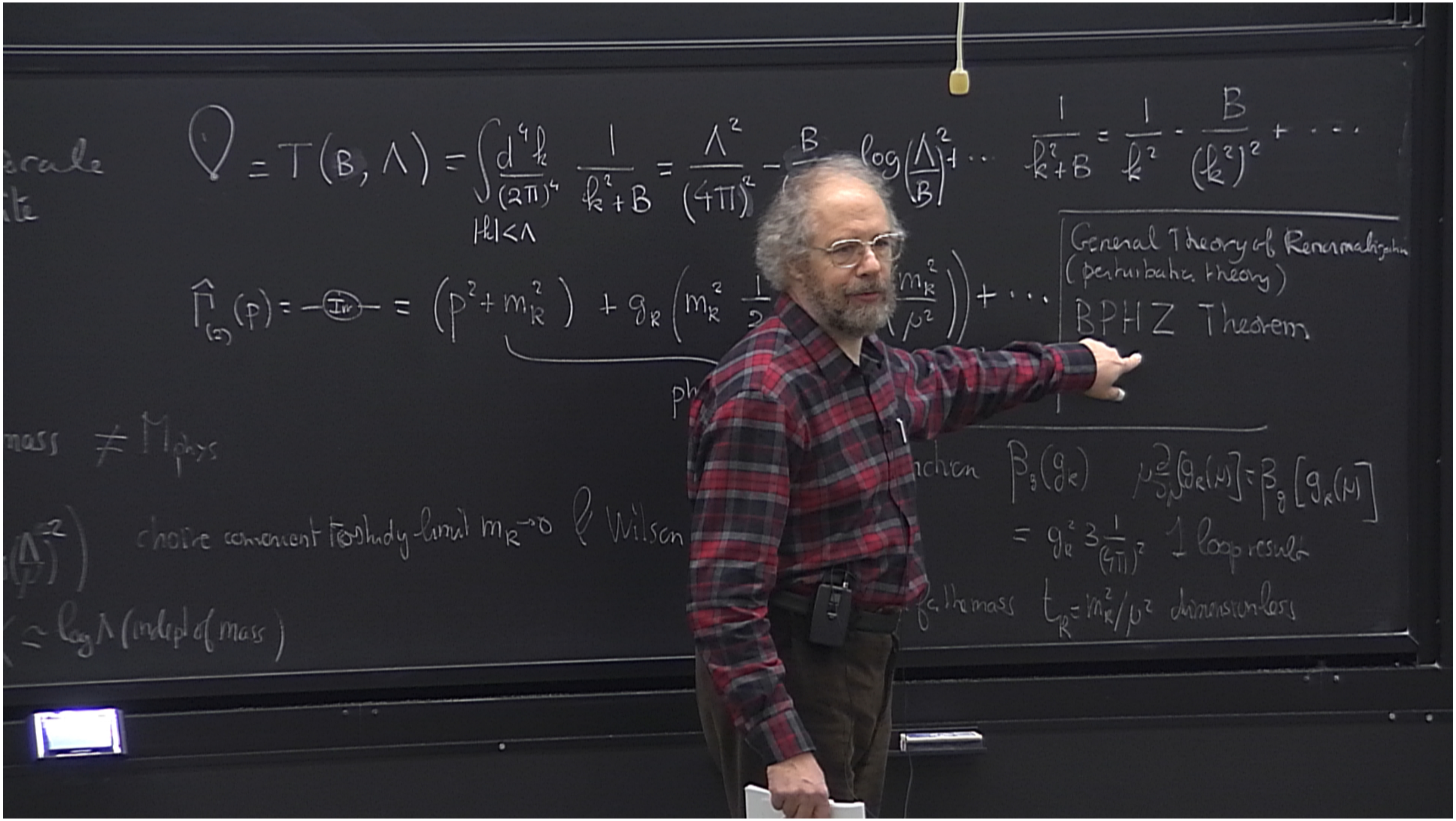
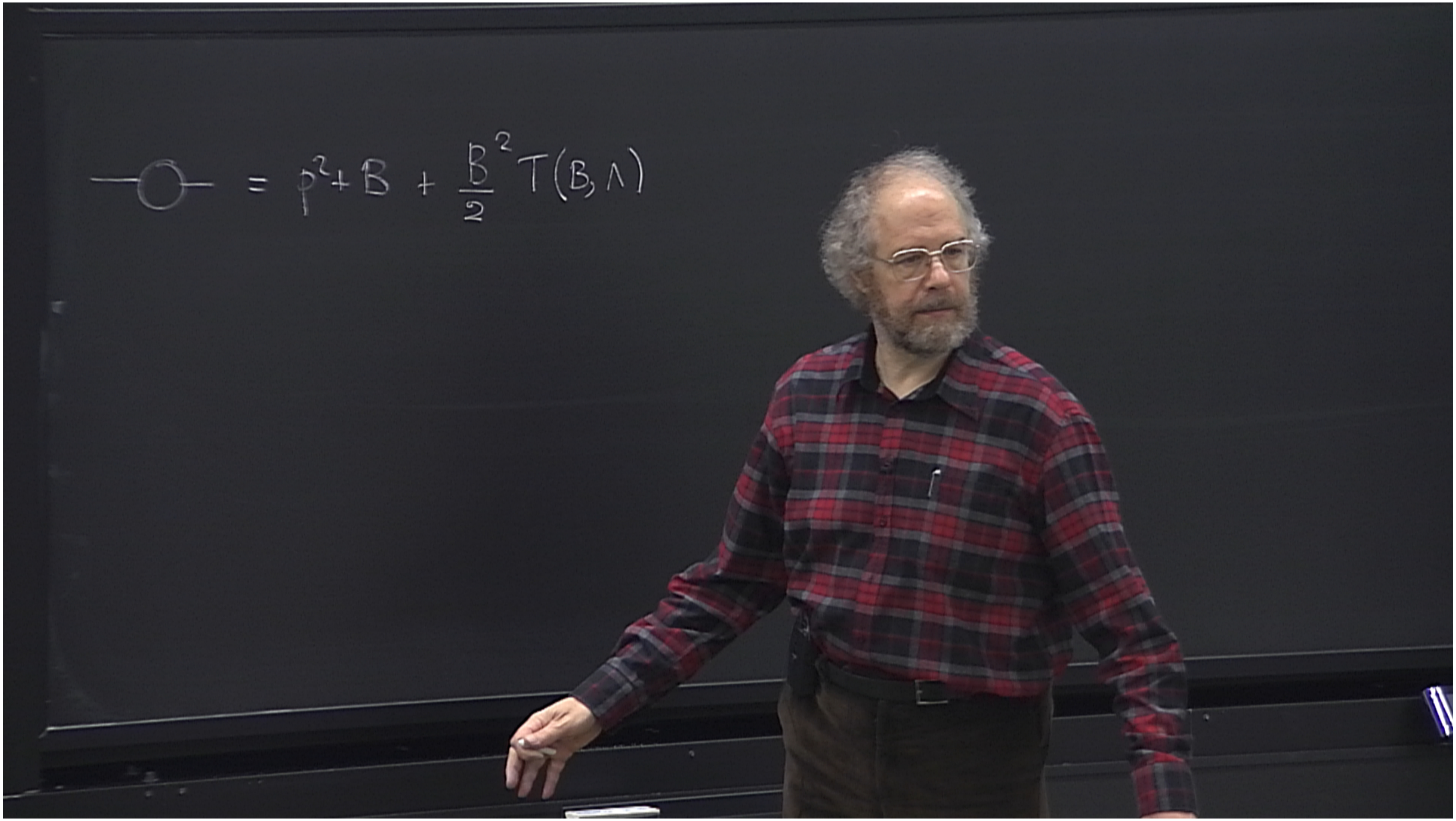
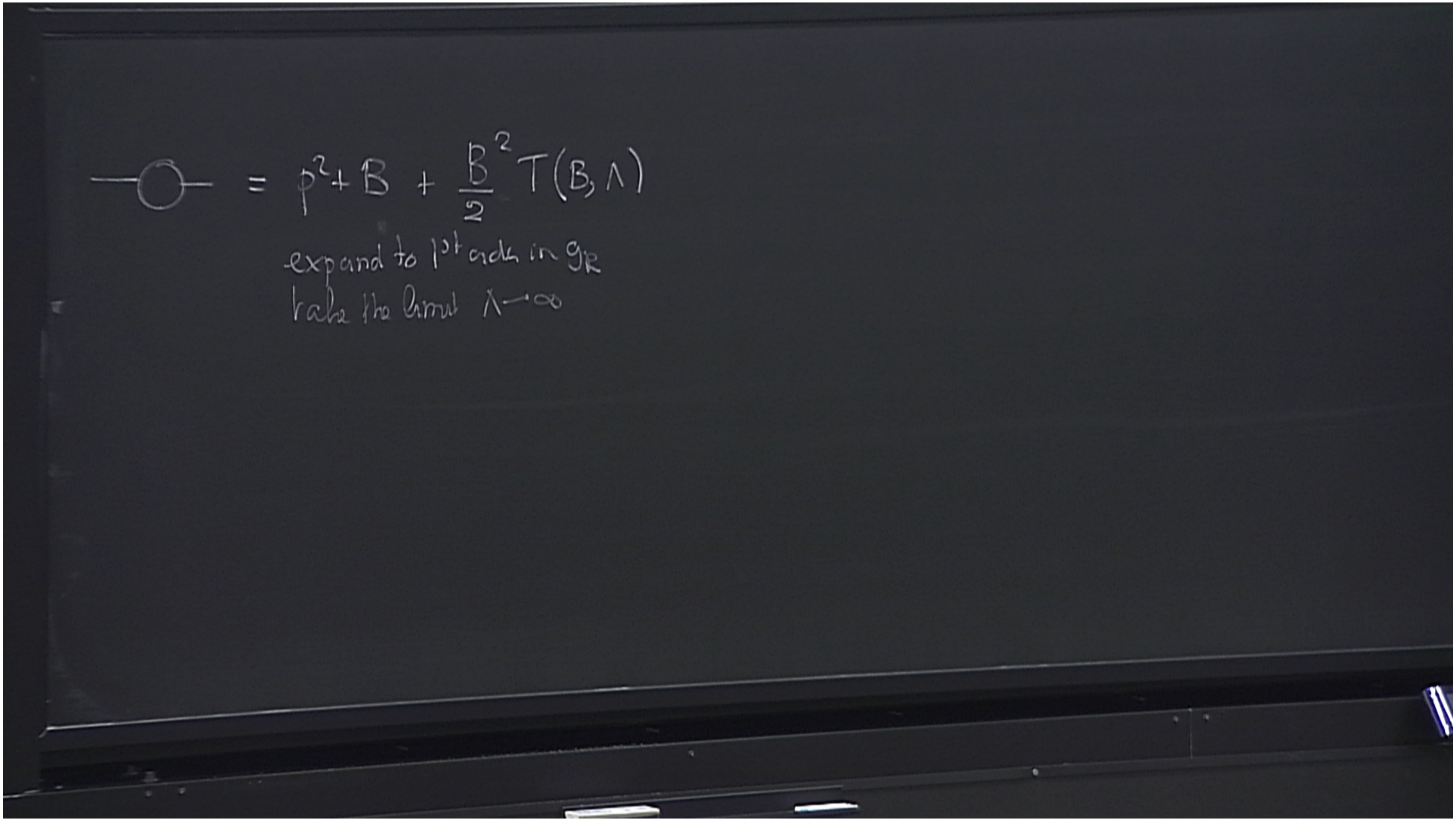
|k|<Λ
1/(k^2+B) = 1/k^2 − B/(k^2)^2 + ···
General Theory of Renormalization
(perturbation theory)
Γ̂(2)(p) = −(Irr)− = (p^2 + m_R^2) + g_R (m_R^2 1/2 1/(4π)^2 log(m_R^2/μ^2)) + ···
physical mass M_phys^2 ≠ 0
mass ≠ M_phys
choose convenient to study limit m_R → 0 & Wilson RG
≃ log Λ (indep of mass)
RG Beta function β_g(g_R) μ ∂/∂μ [g_R(μ)] = β_g [g_R(μ)]
= g_R^2 3 1/(4π)^2 1 loop result
RG β-function for the mass t_R = m_R^2/μ^2 dimensionless


General Theory of Renormalization
(perturbation theory)
BPHZ Theorem
choose convenient to study limit mR→0 & Wilson
1 loop result
dimensionless

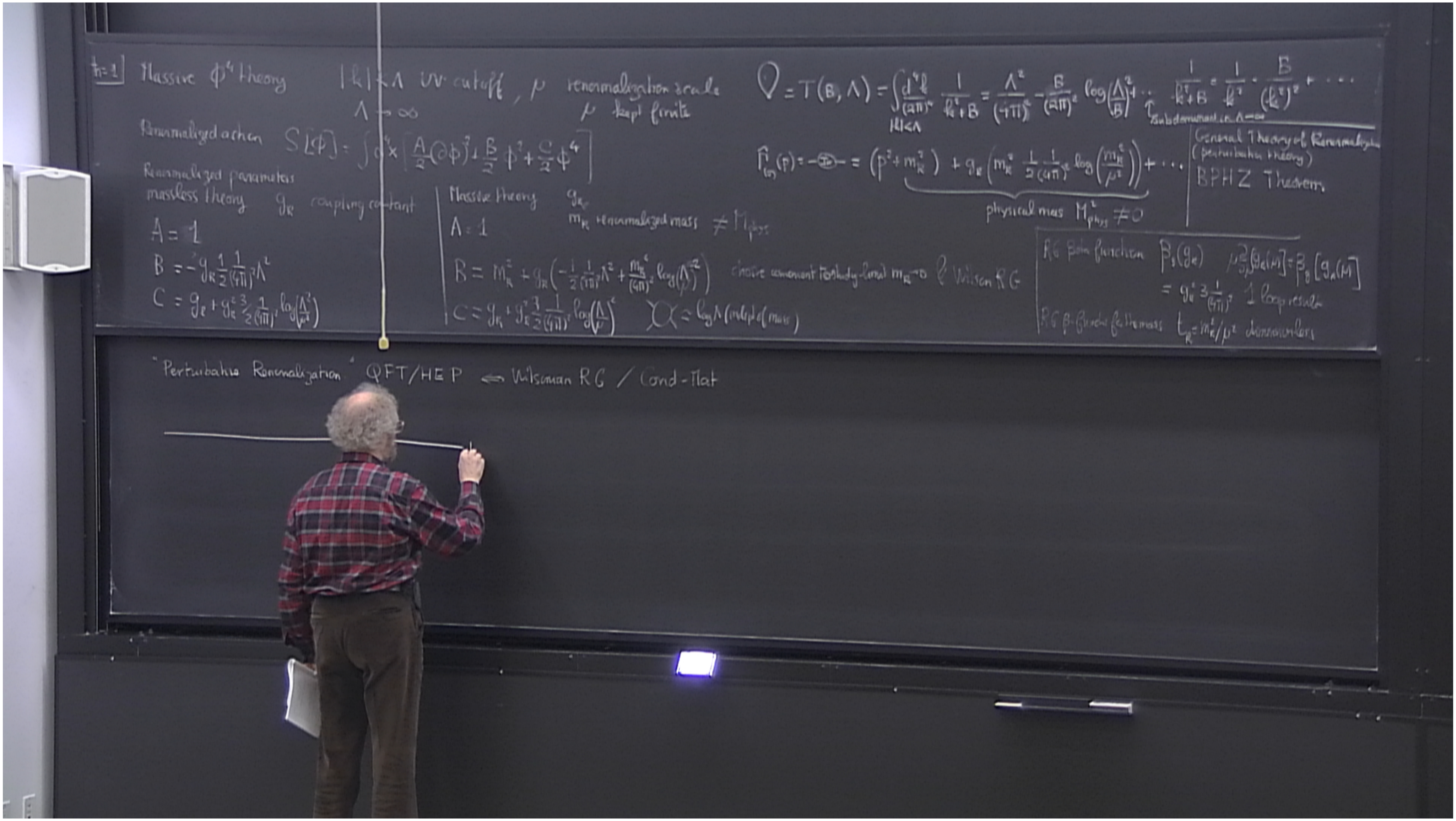

= p² + B + B²/2 T(B, Λ)

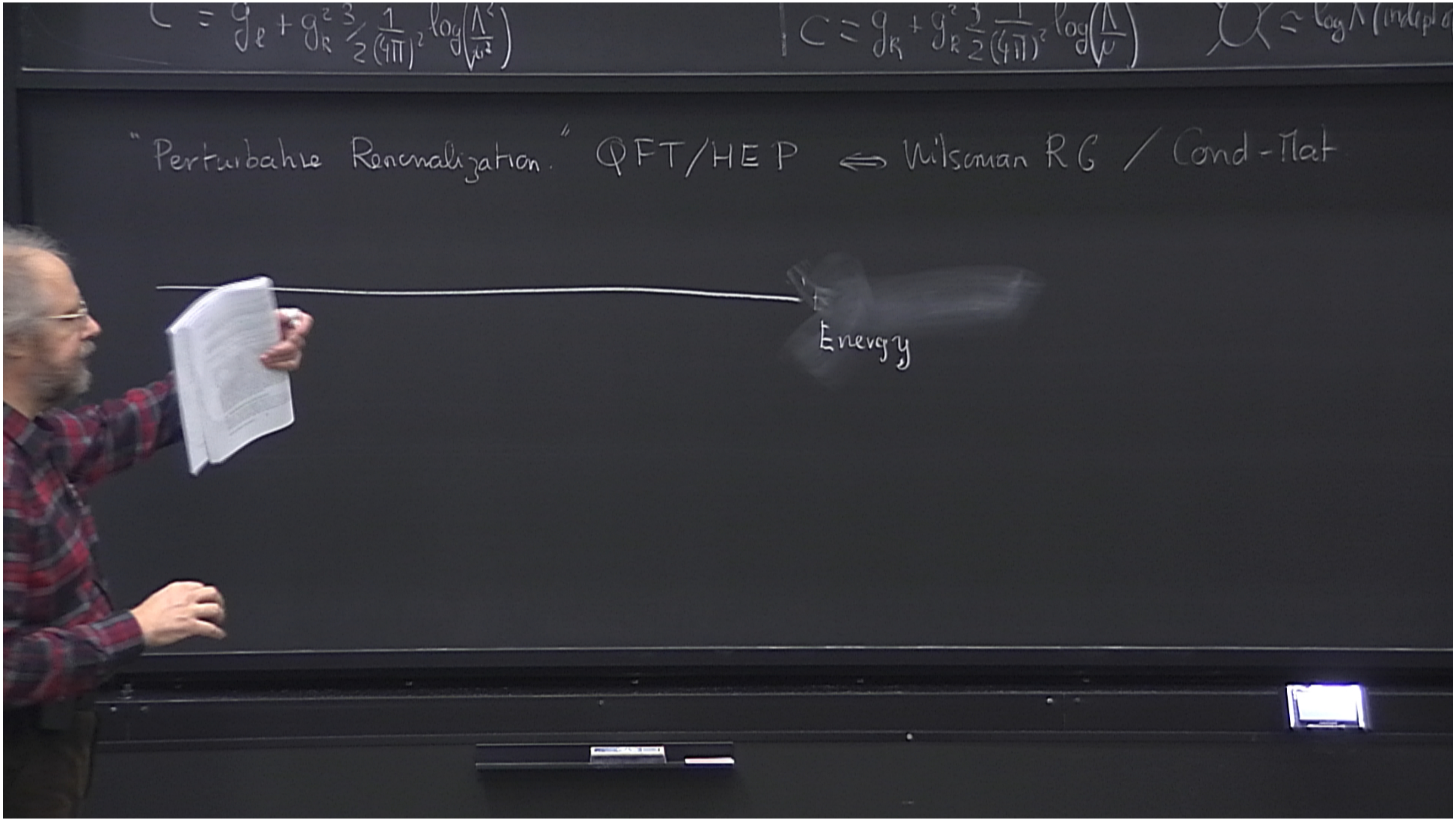
 $= p^2 + B + \frac{B^2}{2} T(B, \Lambda)$

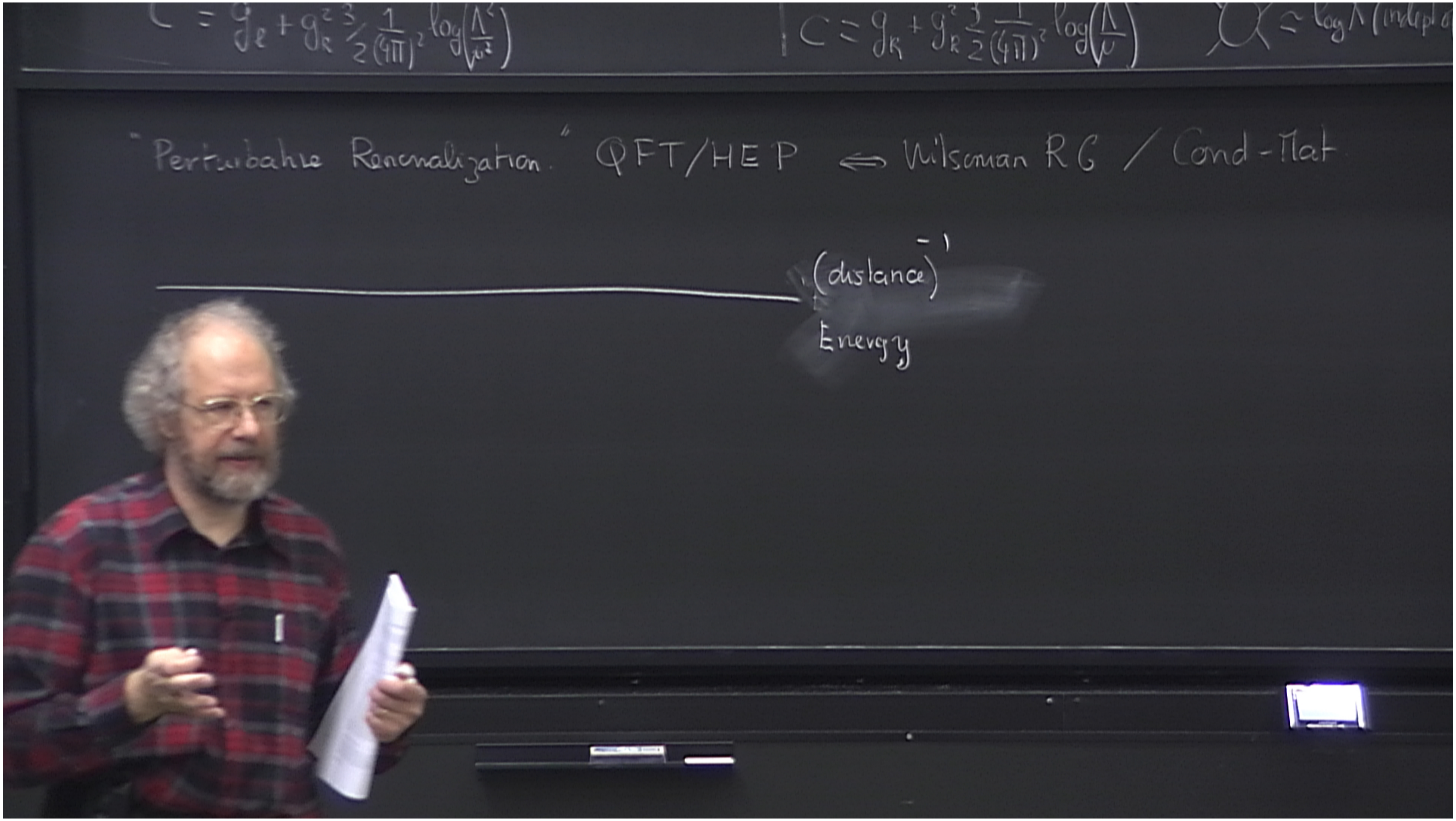
expand to 1st order in g_R

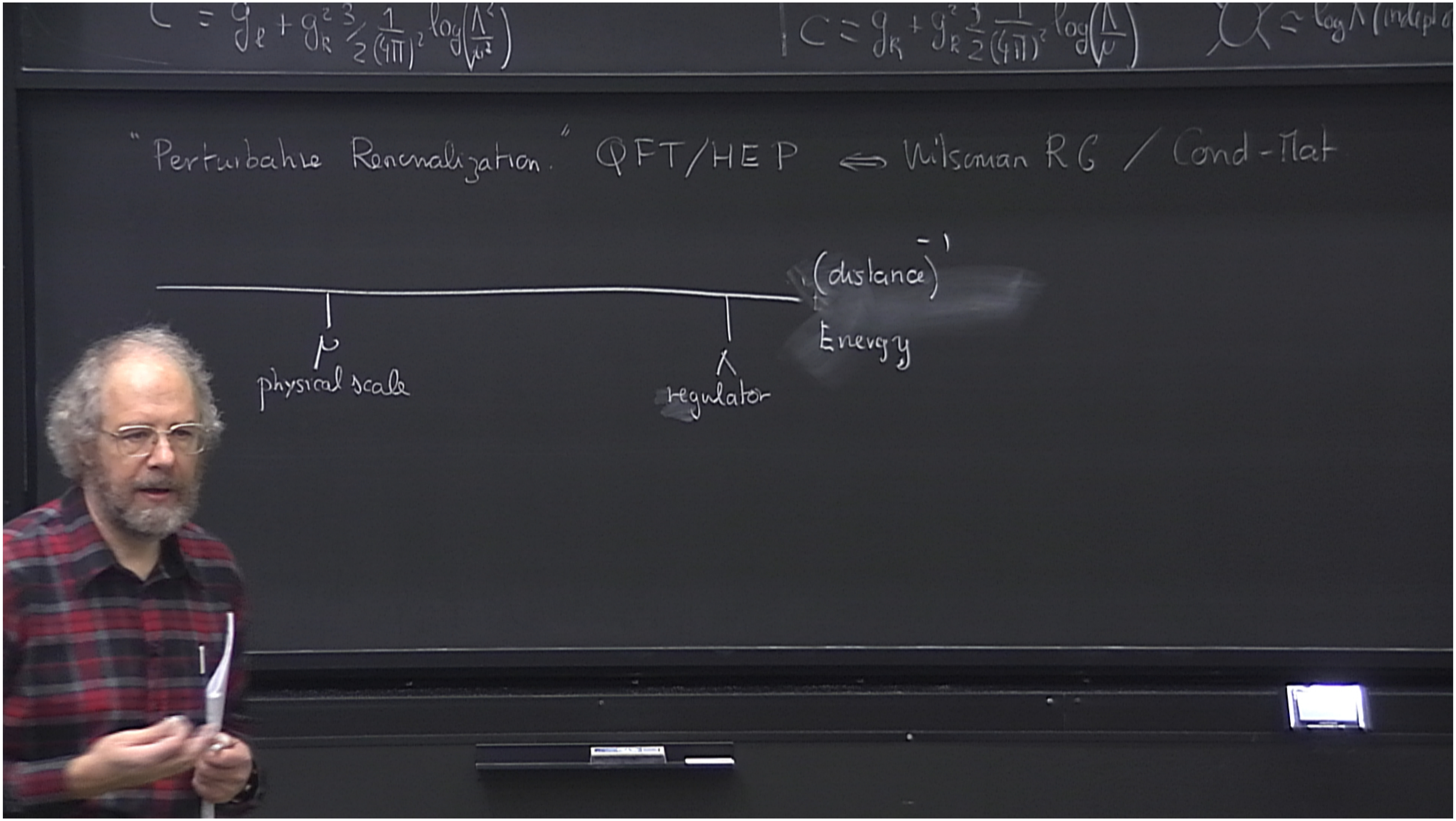


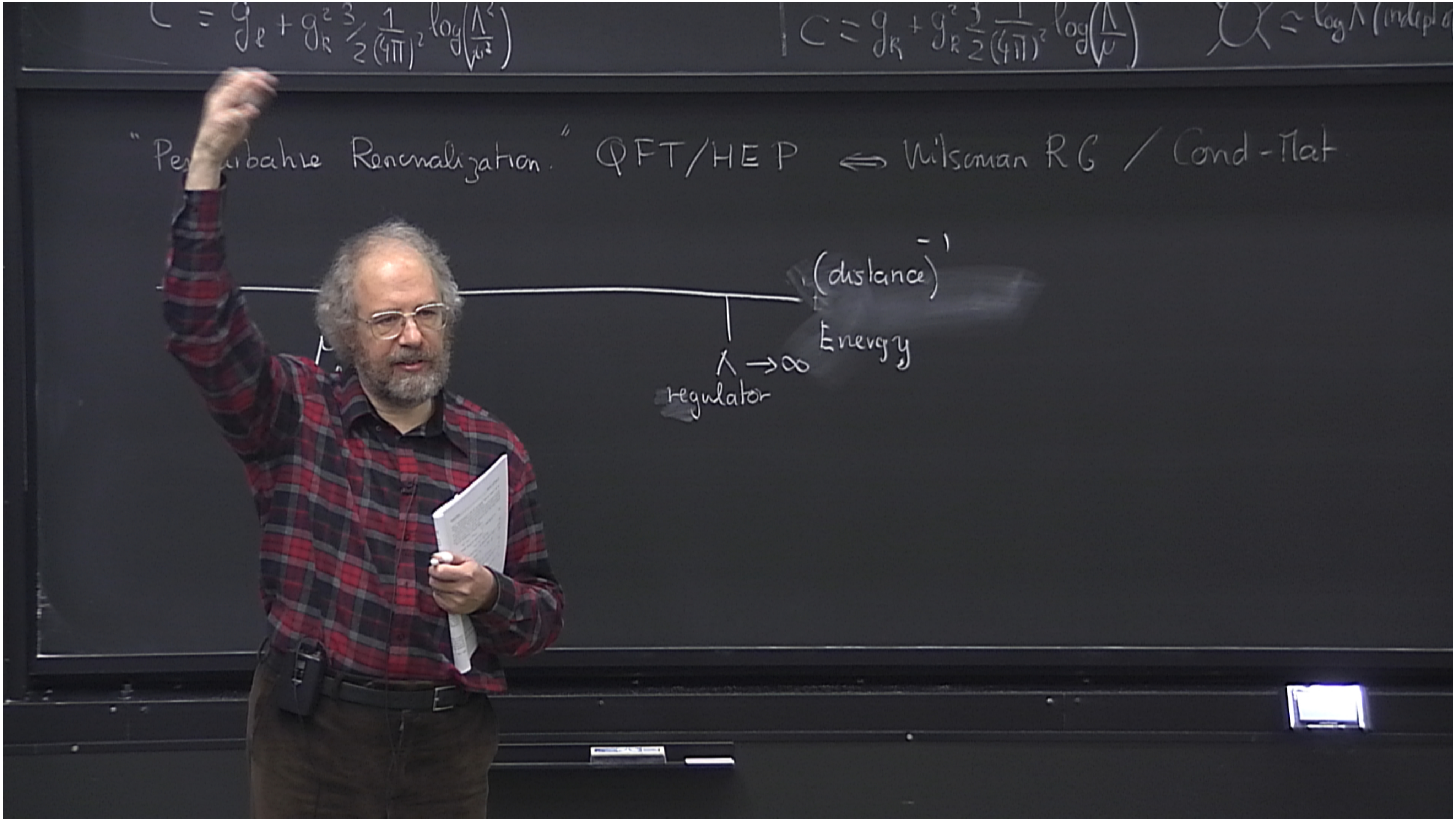
$= p^2 + B + \frac{B}{2}^2 T(B, \Lambda)$
expand to 1st order in g_R
take the limit $\Lambda \to \infty$

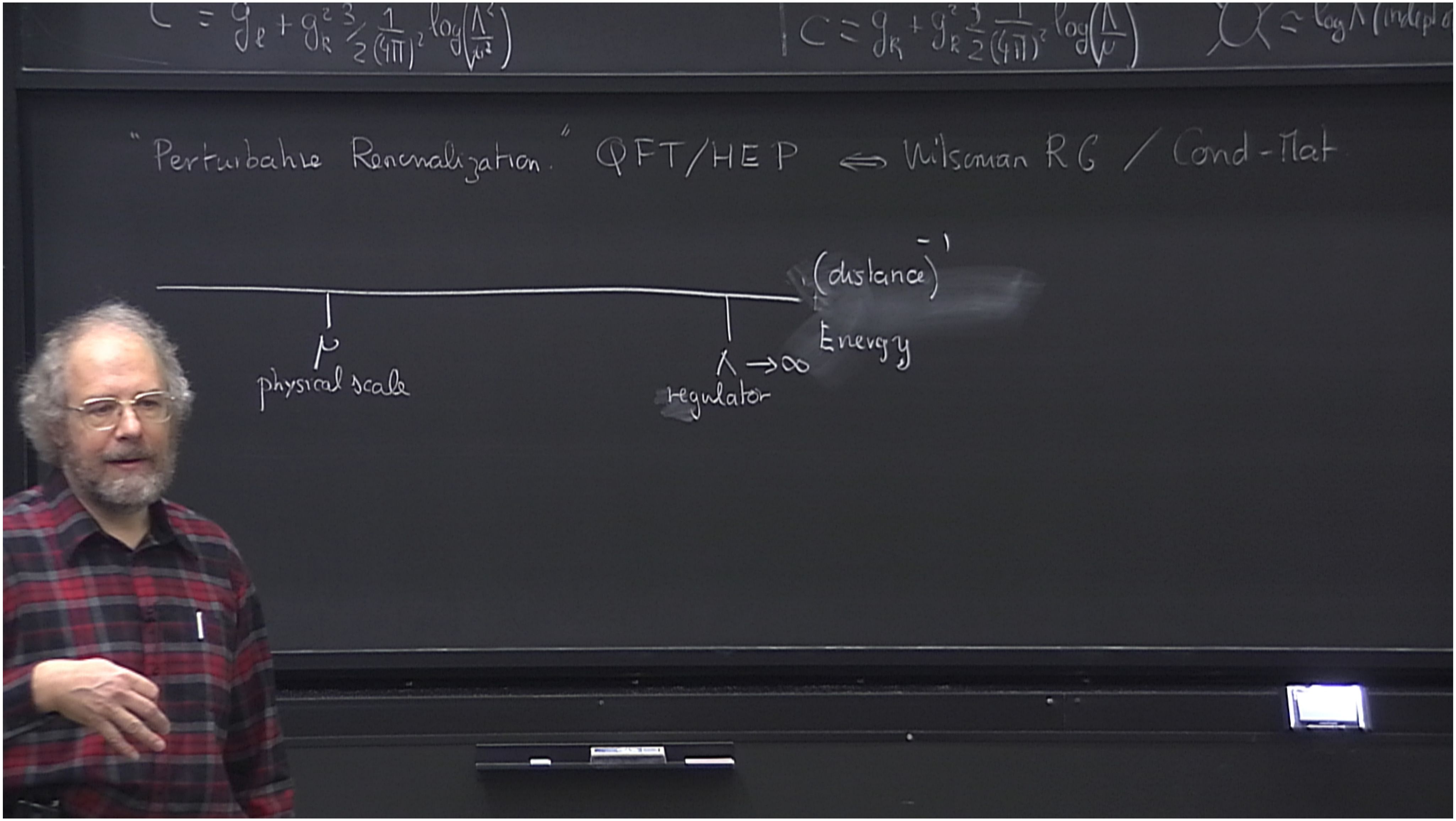

Massive φ4 theory
UV cutoff
renormalization scale
Renormalized action
Renormalized parameters
massless theory
coupling constant
Massive theory
renormalized mass
General Theory of Renormalization
(perturbative theory)
BPHZ Theorem
physical mass
RG Beta function
1 loop result
"Perturbative Renormalization" QFT/HEP
Wilsonian RG / Cond-Mat

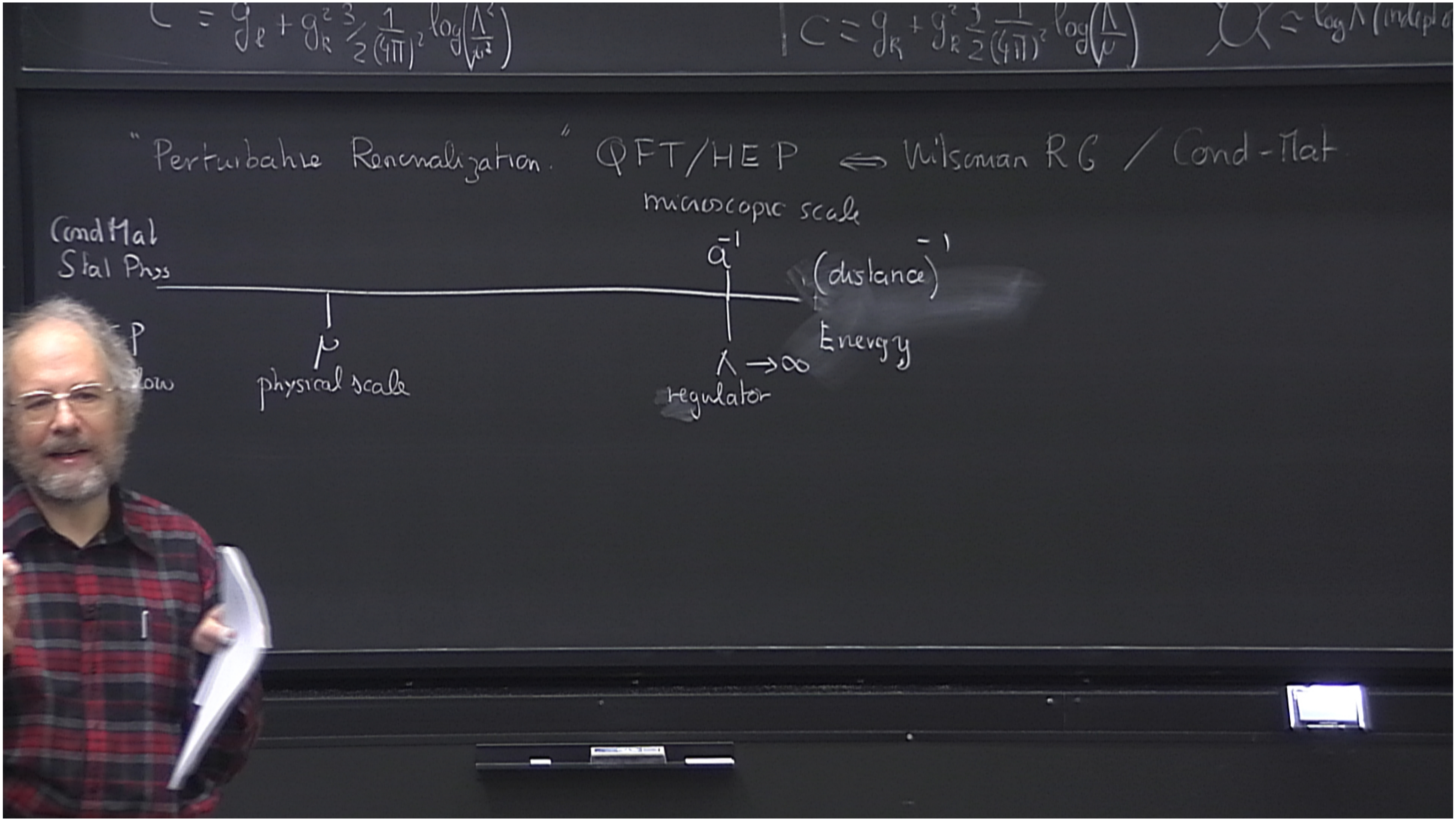

"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
Energy

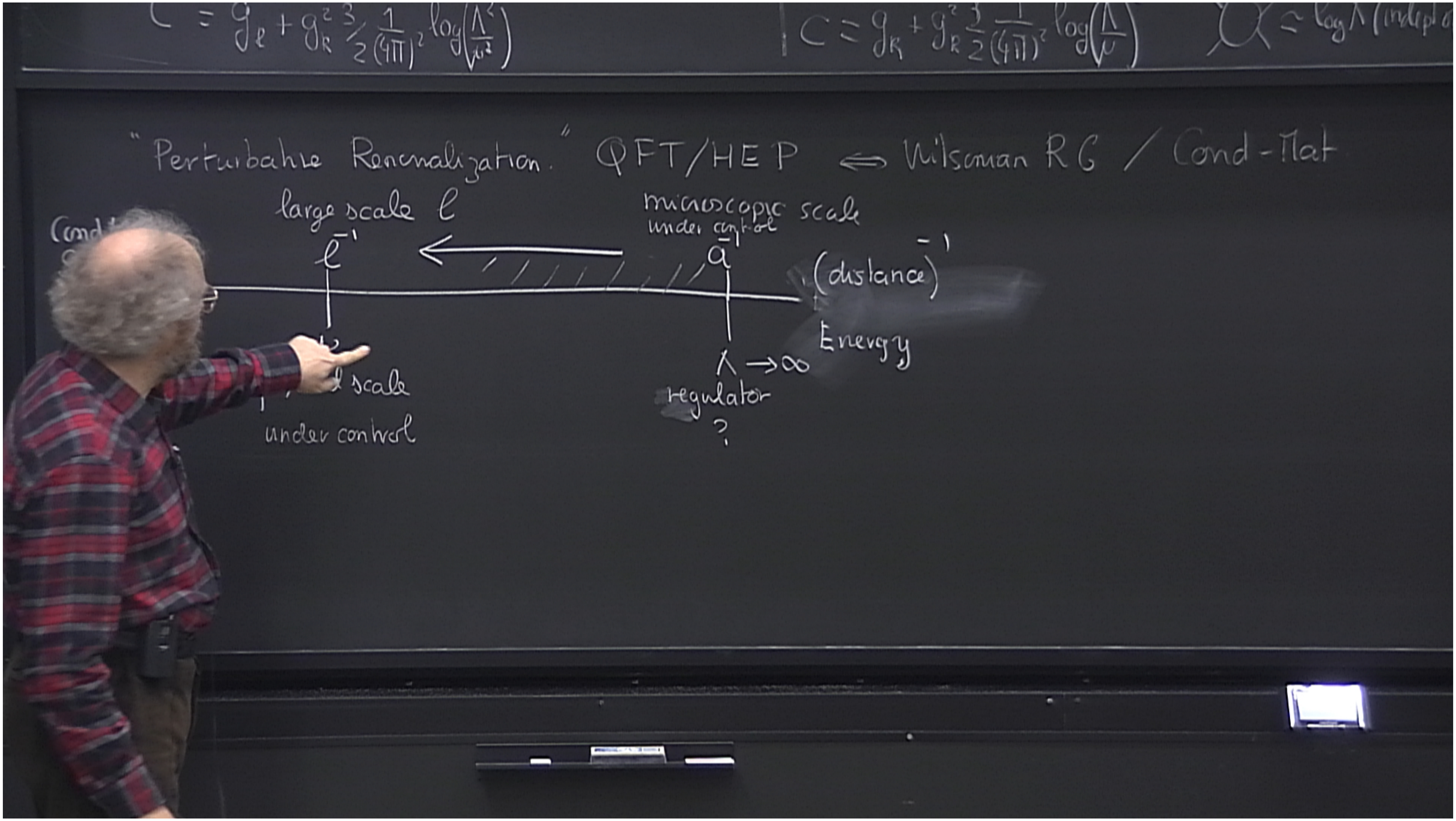

"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
(distance)⁻¹
Energy


"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
(distance)⁻¹
Energy
μ
physical scale
Λ
regulator

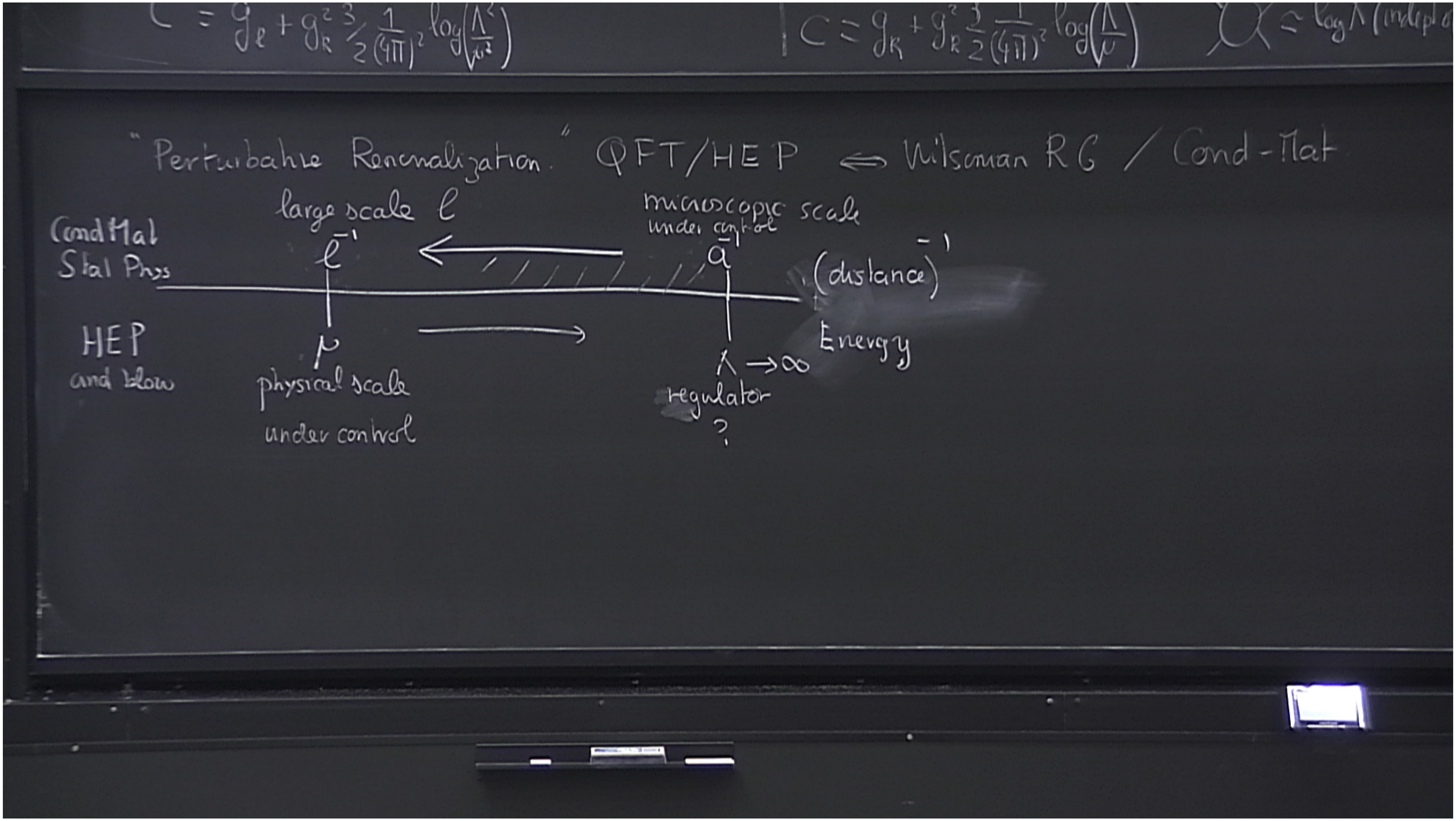

"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
(distance)^-1
Energy
Λ → ∞
regulator


"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
(distance)⁻¹
Energy
μ
physical scale
Λ → ∞
regulator


"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
microscopic scale
Cond Mat
Stat Phys
(distance)$^{-1}$
Energy
physical scale
regulator
$\Lambda \to \infty$



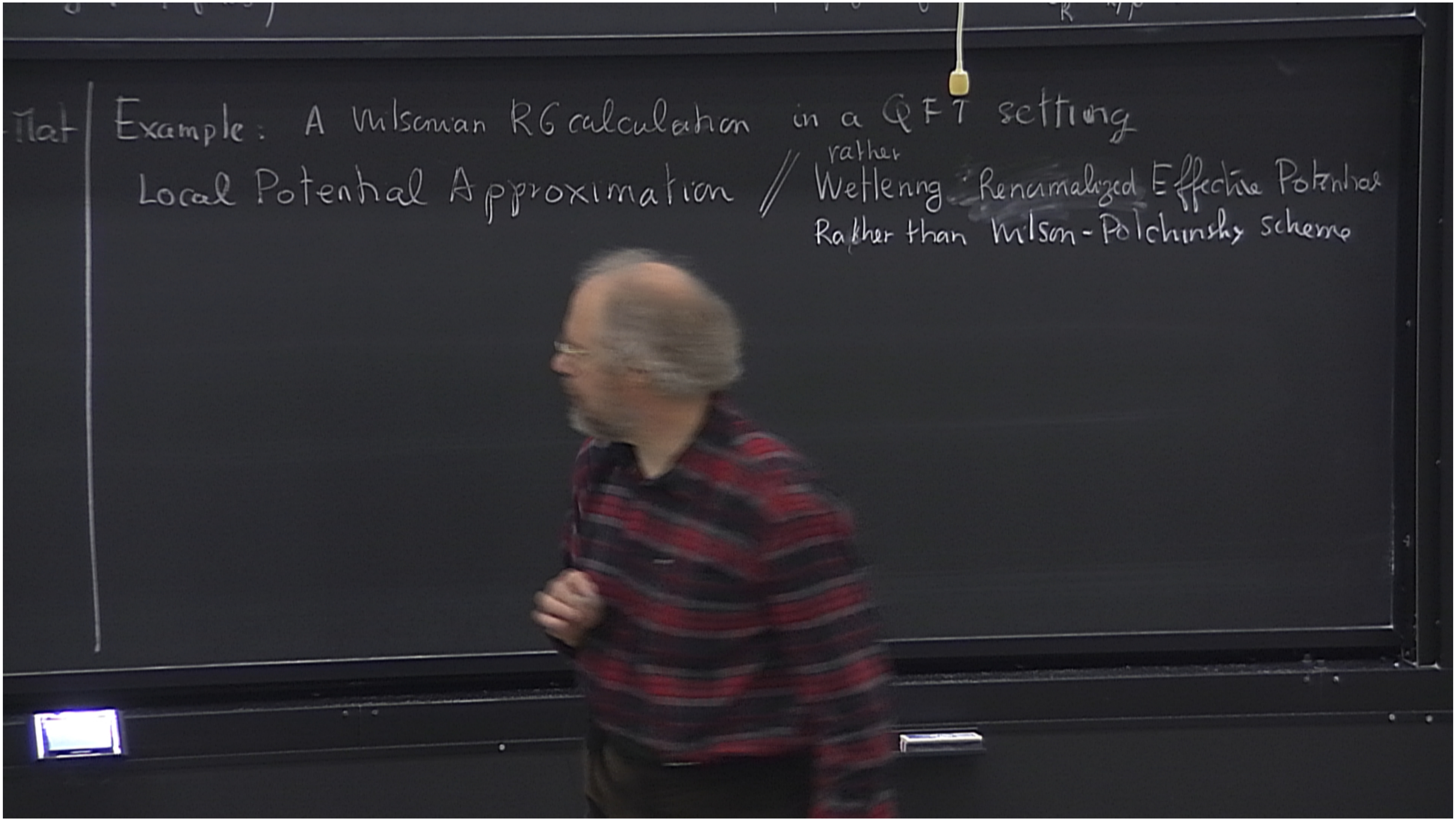
"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
large scale ℓ
microscopic scale
under control
(distance)⁻¹
Energy
Λ → ∞
regulator
?
scale
under control

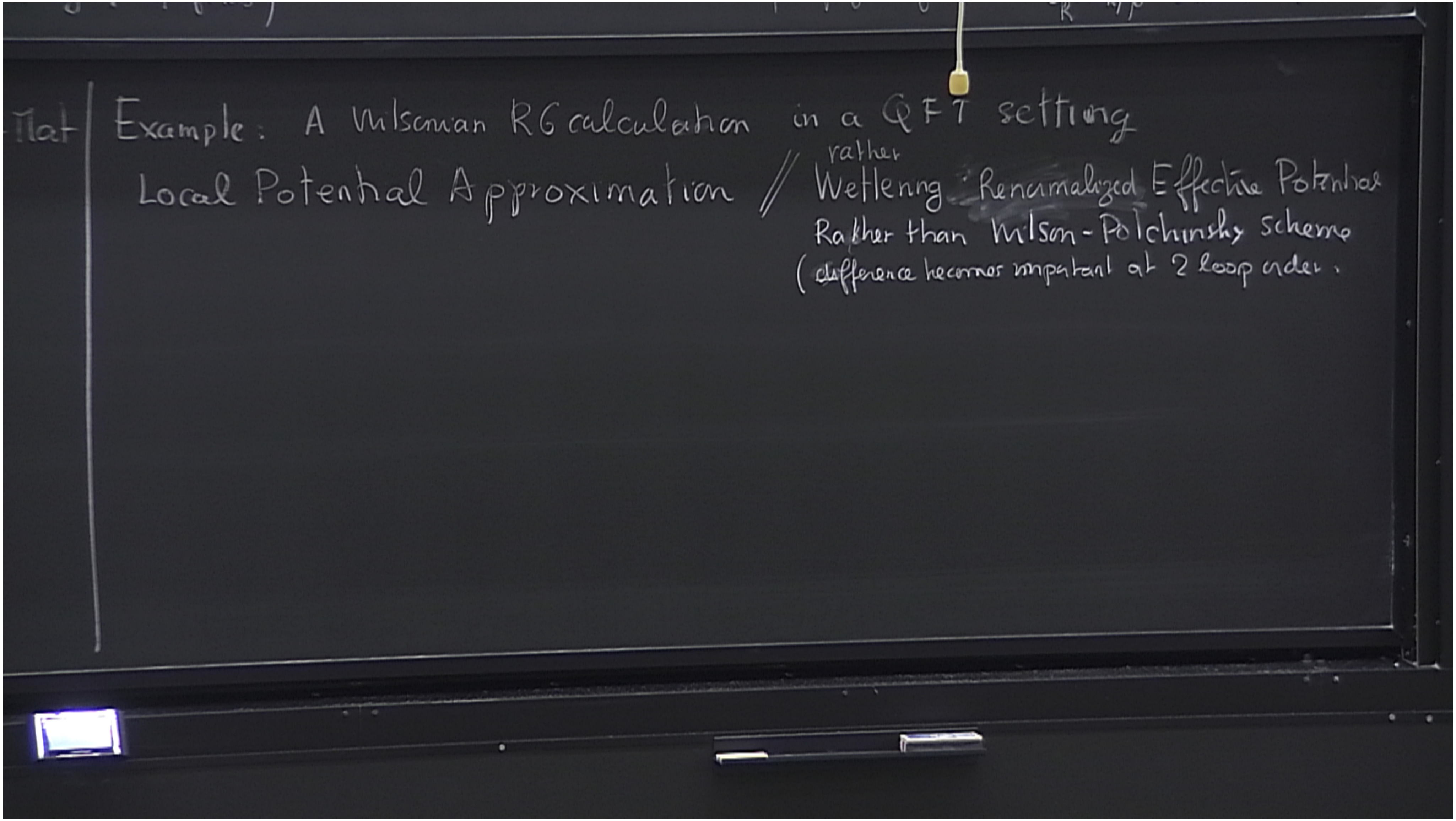

"Perturbative Renormalization" QFT/HEP ⟺ Wilsonian RG / Cond-Mat
large scale ℓ
microscopic scale
under control
Cond Mat
Stat Phys
(distance)⁻¹
HEP
and below
physical scale
under control
Λ → ∞
regulator
?
Energy

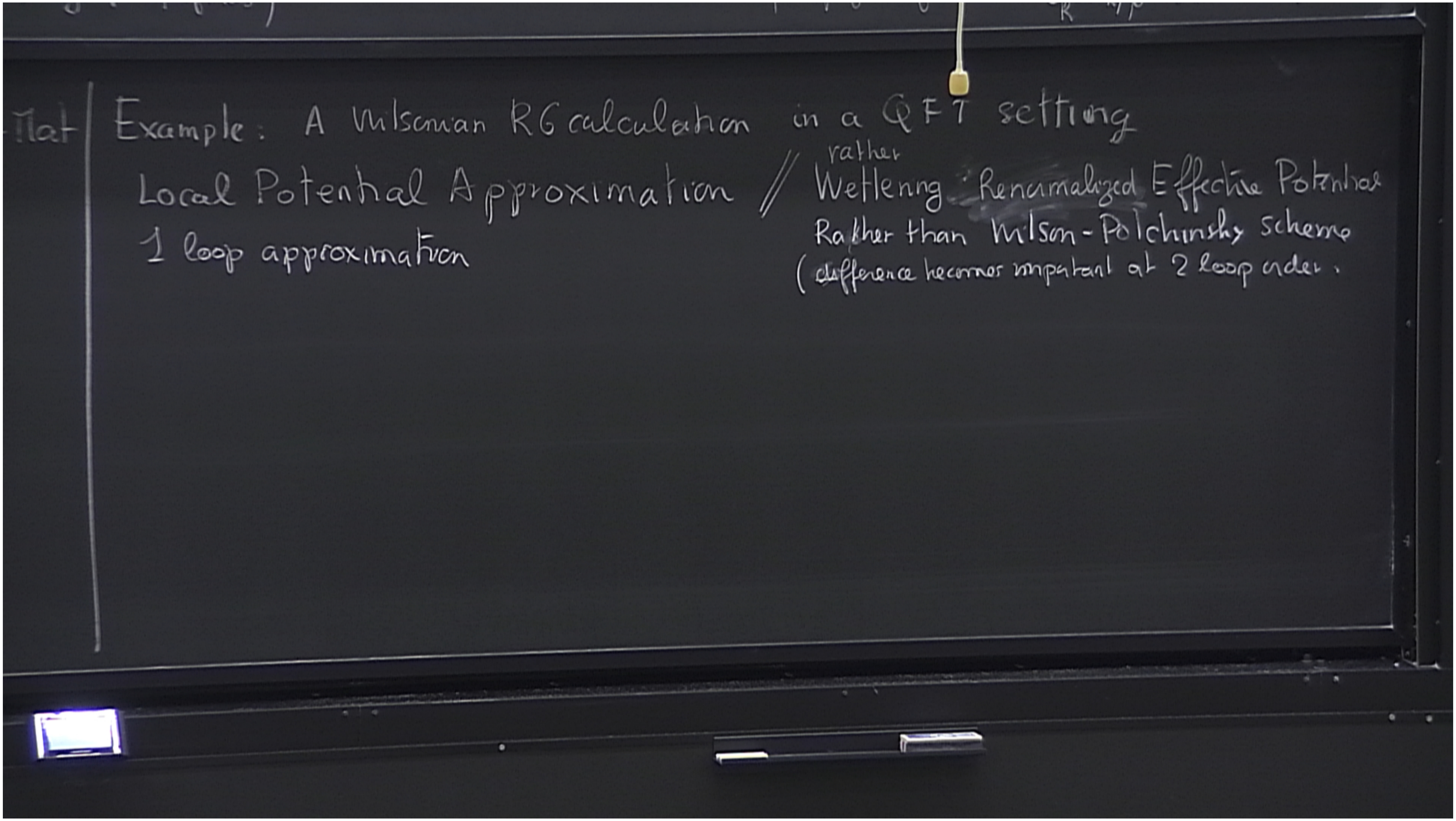
-flat | Example: A Wilsonian RG calculation in a QFT setting

Example: A Wilsonian RG calculation in a QFT setting

Local Potential Approximation


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
rather
Wetterich: Renormalized Effective Potential
Rather than Wilson-Polchinski scheme


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation // rather Wetterich Renormalized Effective Potential
Rather than Wilson-Polchinski scheme
(difference becomes important at 2 loop order.


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation // Wetterich: Renormalized Effective Potential
rather
1 loop approximation
Rather than Wilson-Polchinsky scheme
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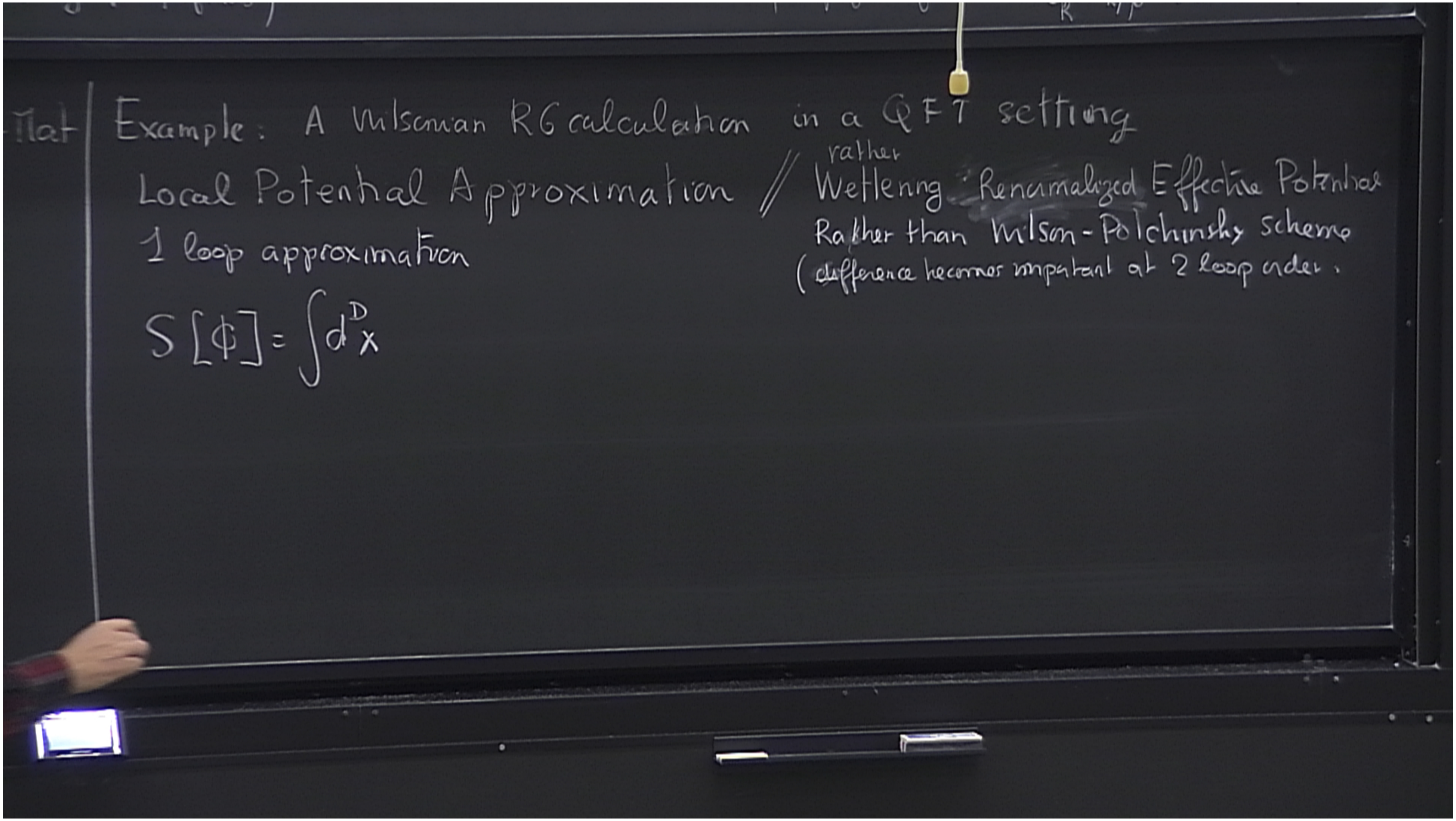
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1 loop approximation
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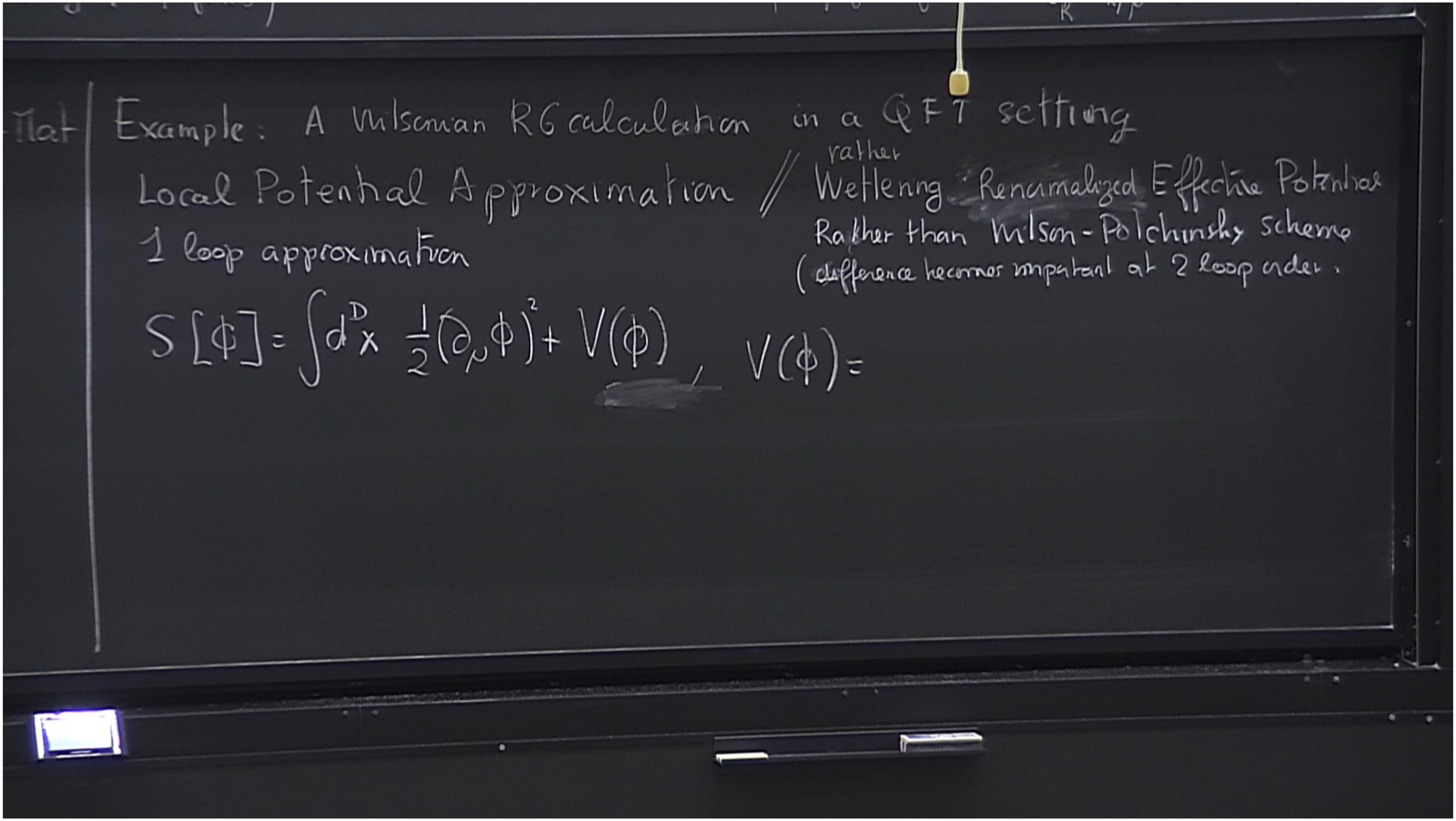
S[ϕ] = ∫d^D x


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
1 loop approximation
rather
Wetterich: Renormalized Effective Potential
Rather than Wilson-Polchinski scheme
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$S[\phi] = \int d^D x \, \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$
$V(\phi) =$

Example: A Wilsonian RG calculation in a QFT setting

Local Potential Approximation // rather Wetterich: Renormalized Effective Potential

Rather than Wilson-Polchinski scheme

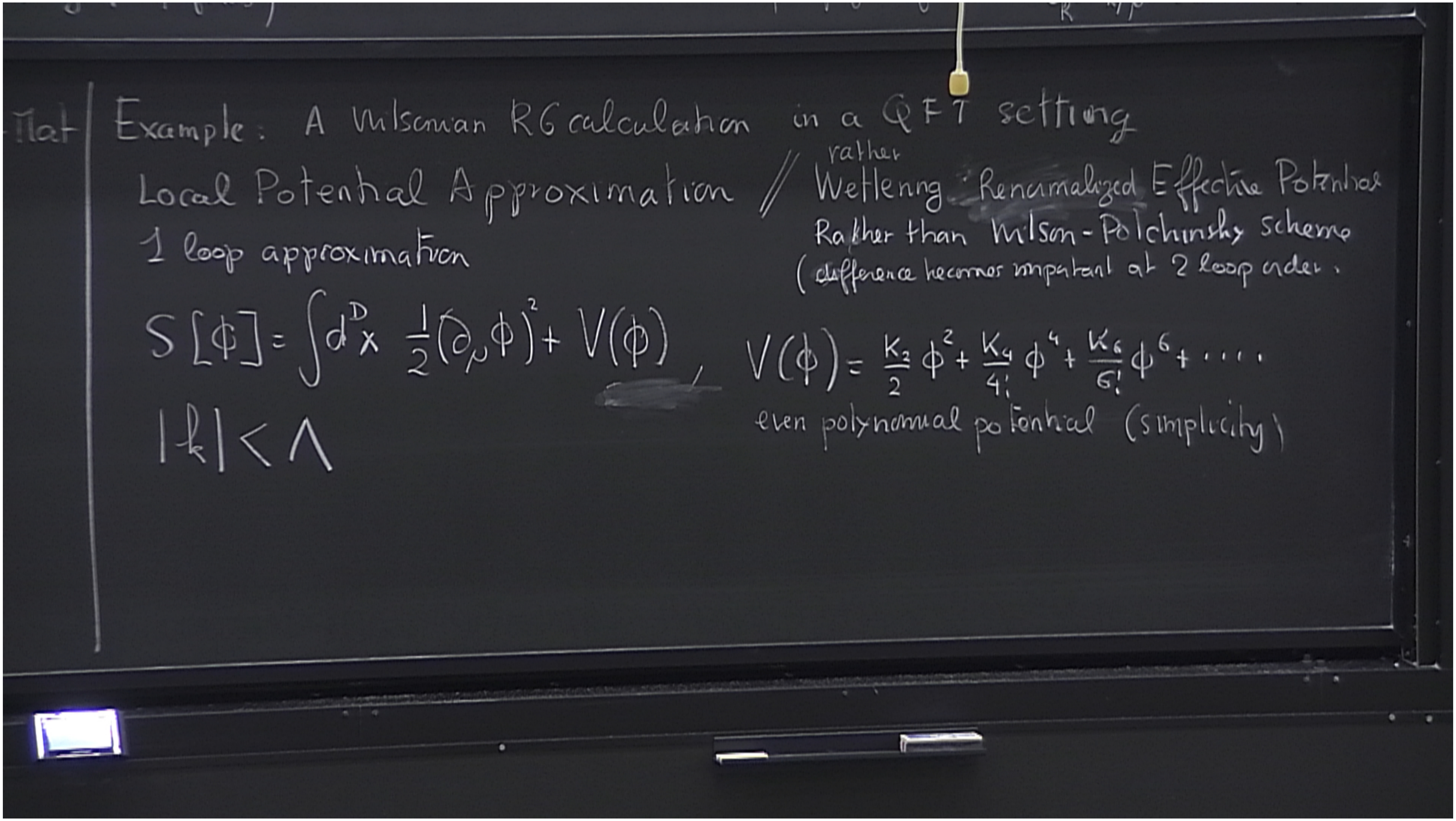
(difference becomes important at 2 loop order.

1 loop approximation

$$S[\phi] = \int d^D x \; \frac{1}{2}(\partial_\mu \phi)^2 + V(\phi)$$

$$V(\phi) = \frac{K_2}{2}\phi^2 + \frac{K_4}{4!}\phi^4 + \frac{K_6}{6!}\phi^6 + \cdots$$

even polynomial potential (simplicity)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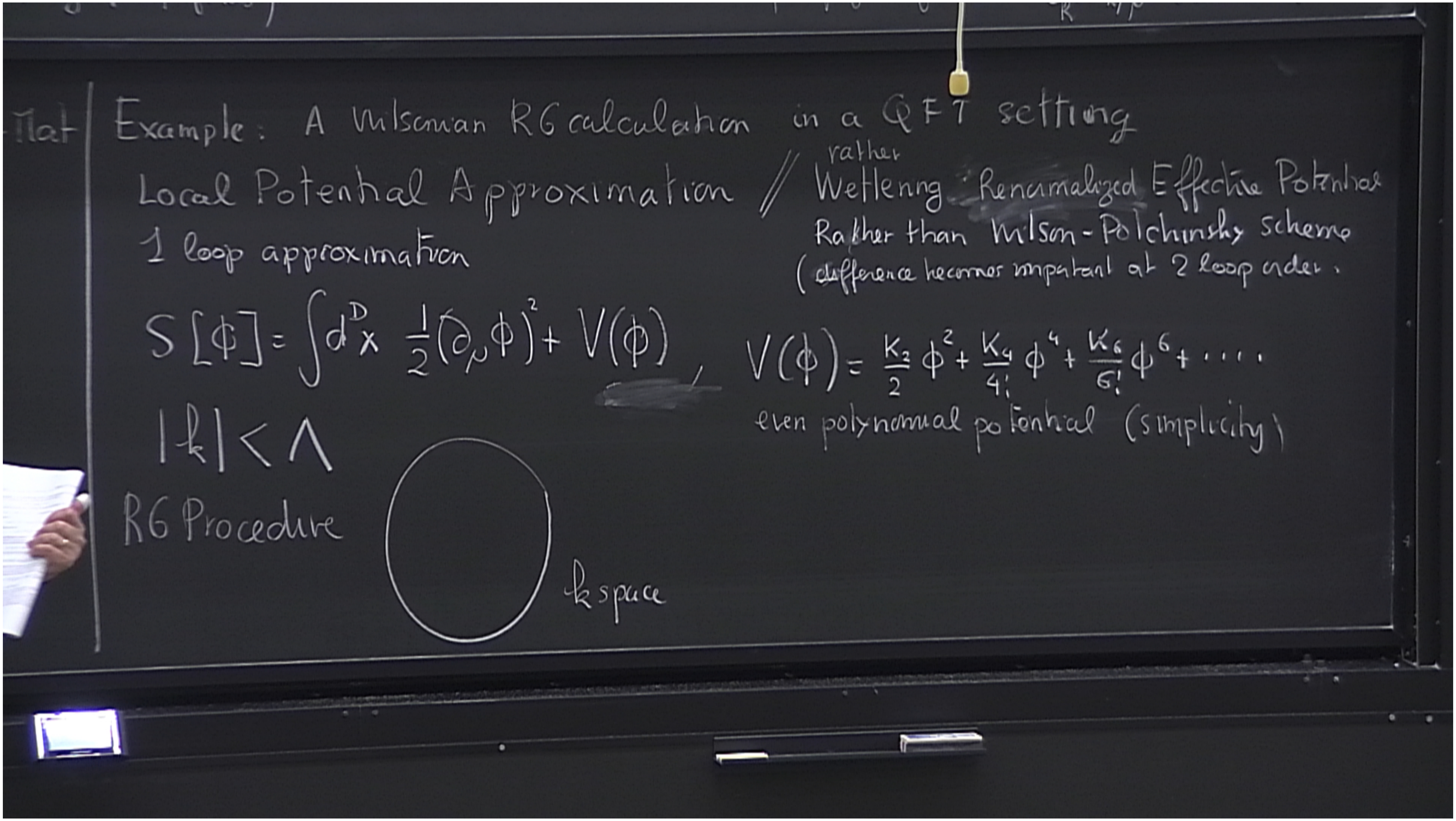
even polynomial potential (simplicity)
$|k| < \Lambda$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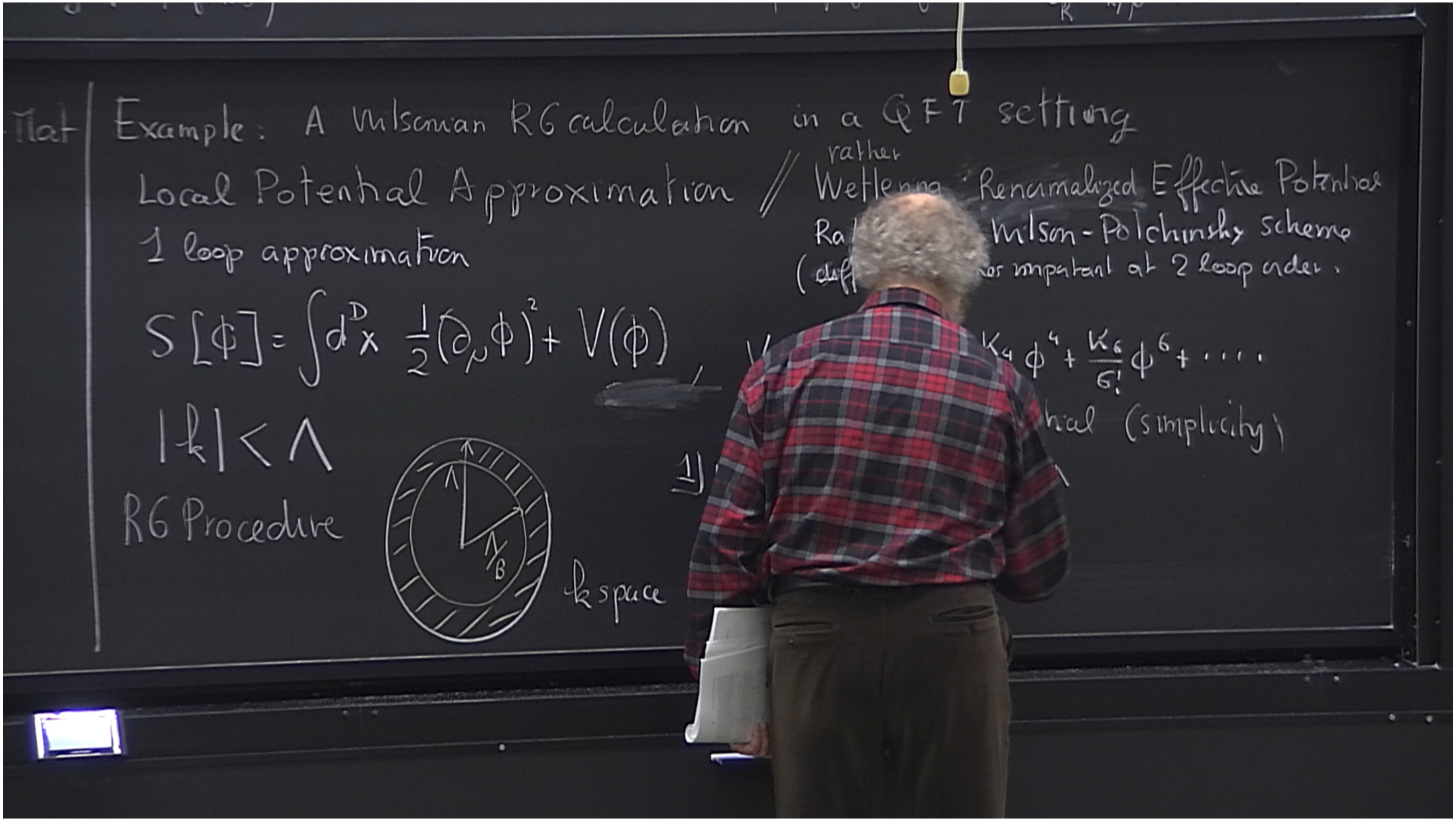
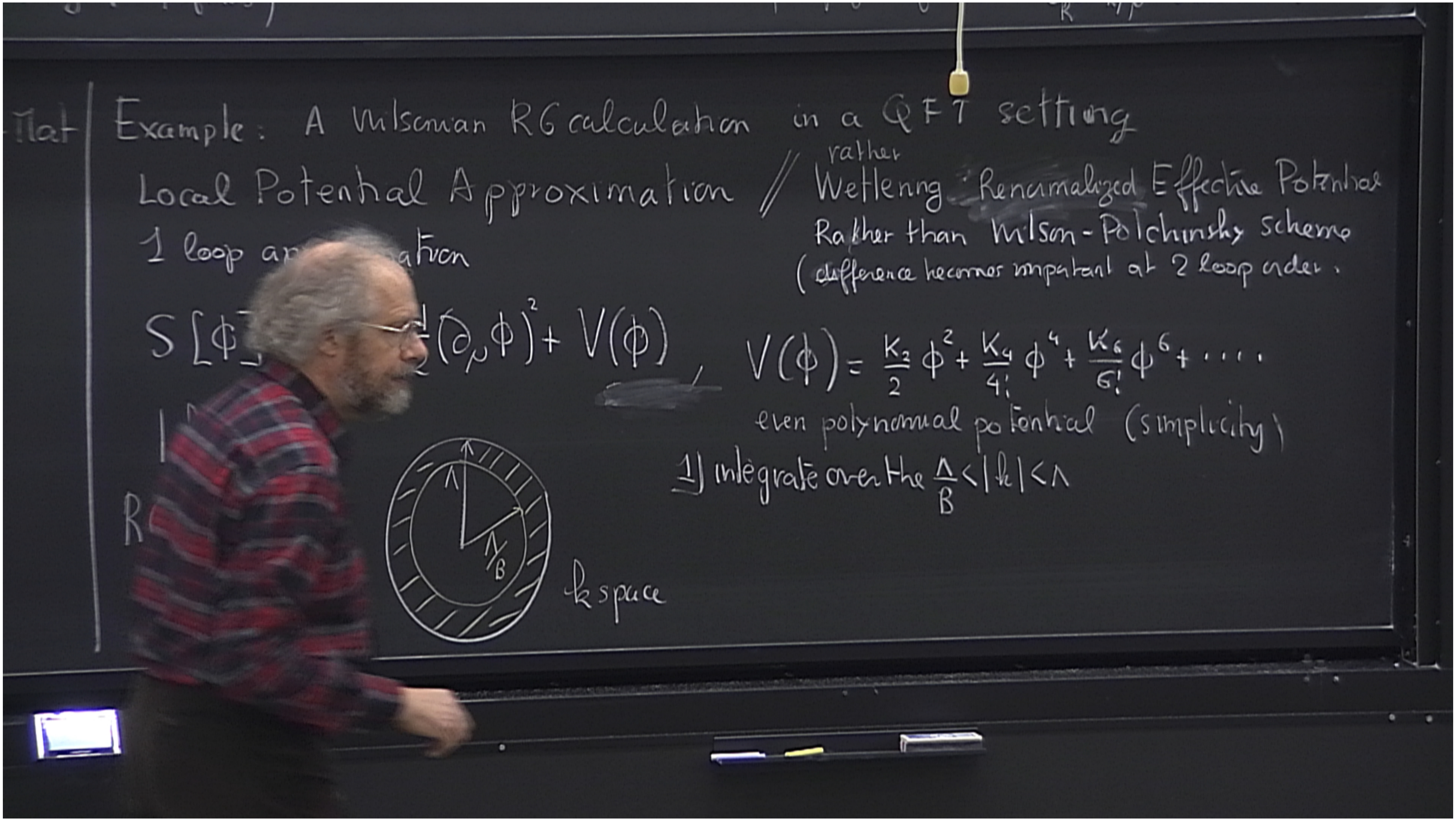
RG Procedure
k space


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
rather
Renormalized Effective Potential
Wilson-Polchinsky scheme
important at 2 loop order.
1 loop approximation
S[ϕ] = ∫d^D x 1/2 (∂_μ ϕ)^2 + V(ϕ)
|k| < Λ
RG Procedure
Λ
Λ/B
k space
(simplicity)


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
1 loop approximation
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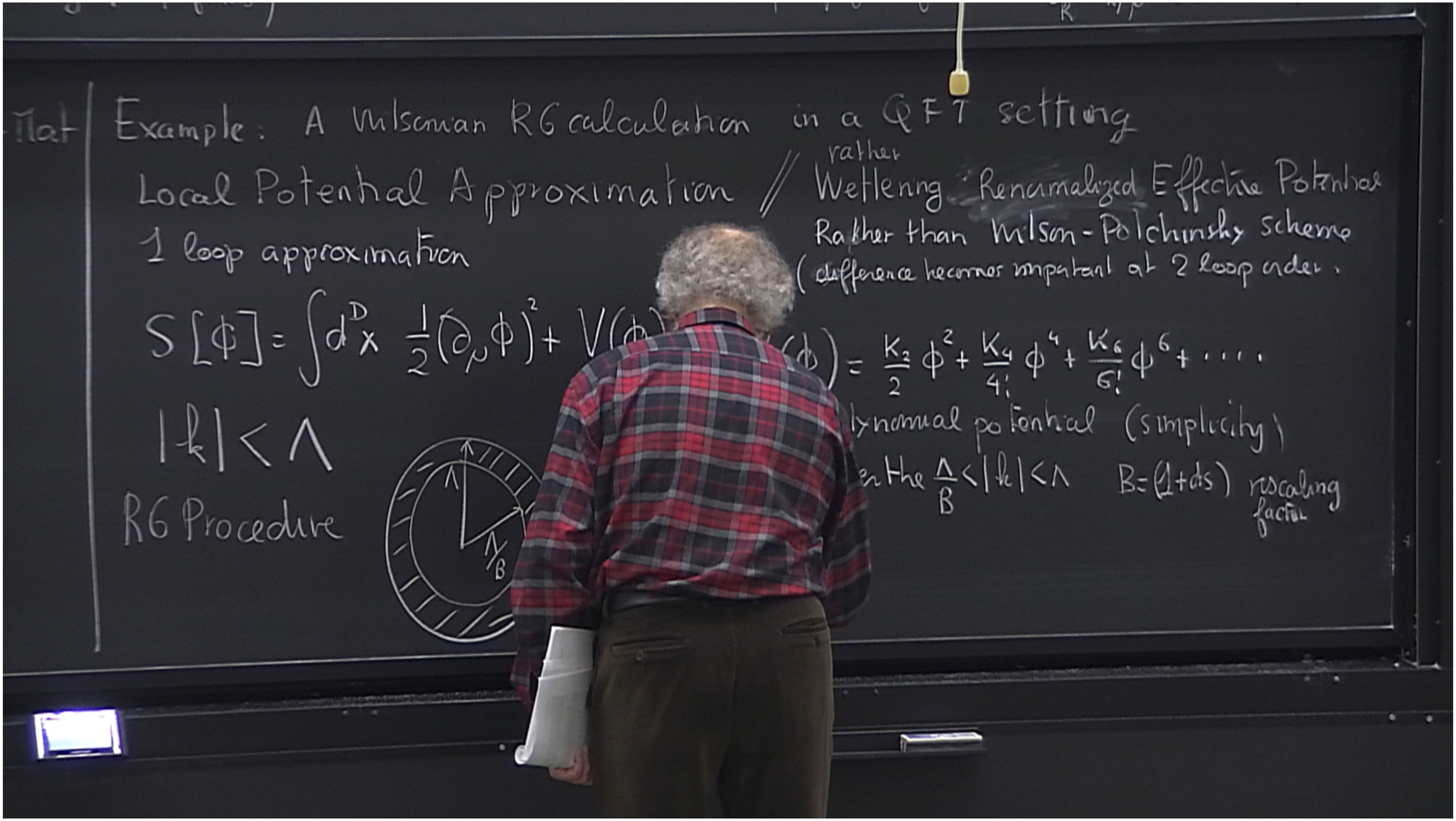
$V(\phi) = \frac{K_2}{2}\phi^2 + \frac{K_4}{4!}\phi^4 + \frac{K_6}{6!}\phi^6 + \cdots$
even polynomial potential (simplicity)
1) integrate over the $\frac{\Lambda}{B} < |k| < \Lambda$
k space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Local Potential Approximation // rather Wetterich Renormalized Effective Potential
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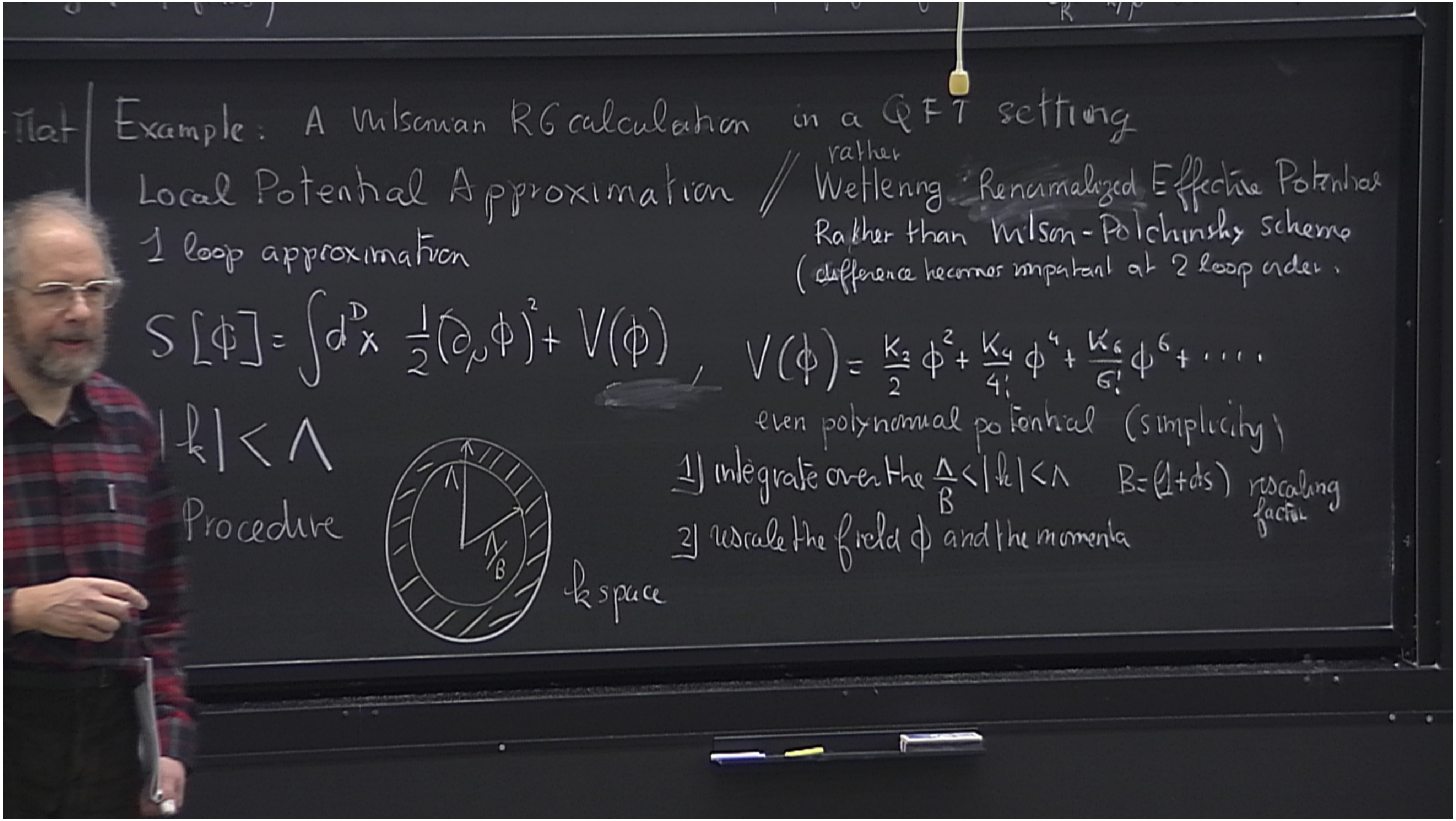
S[ϕ] = ∫d^D x 1/2 (∂_μ ϕ)^2 + V(ϕ)
= K_2/2 ϕ^2 + K_4/4! ϕ^4 + K_6/6! ϕ^6 + ...
|k| < Λ
polynomial potential (simplicity)
RG Procedure
Λ/B < |k| < Λ
B = (1+ds) rescaling factor


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
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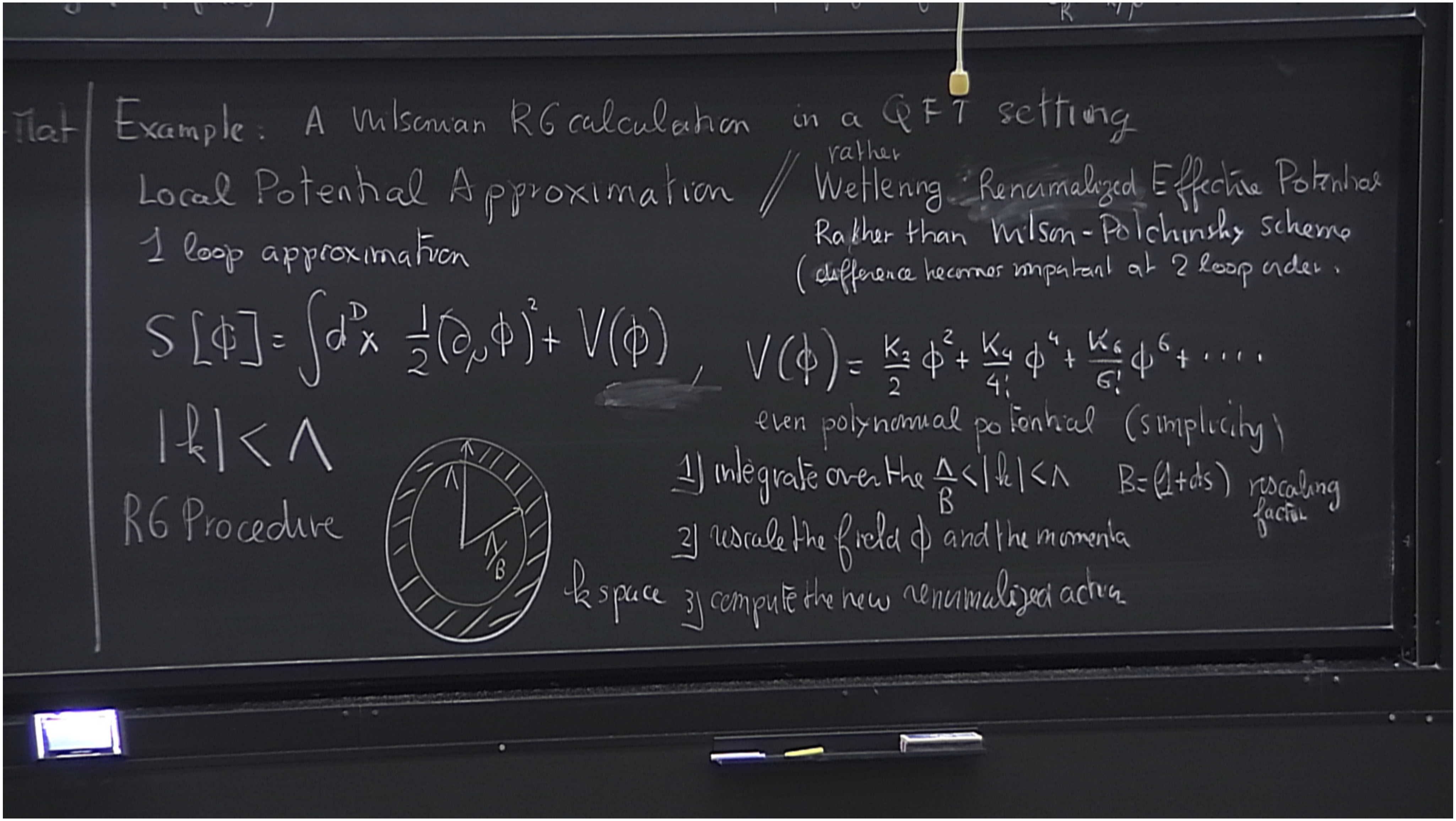
V(φ) = K_2/2 φ^2 + K_4/4! φ^4 + K_6/6! φ^6 + ...
even polynomial potential (simplicity)
|k| < Λ
Procedure
1) integrate over the Λ/B < |k| < Λ B = (1+ds) rescaling factor
2) rescale the field φ and the momenta
k space


Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
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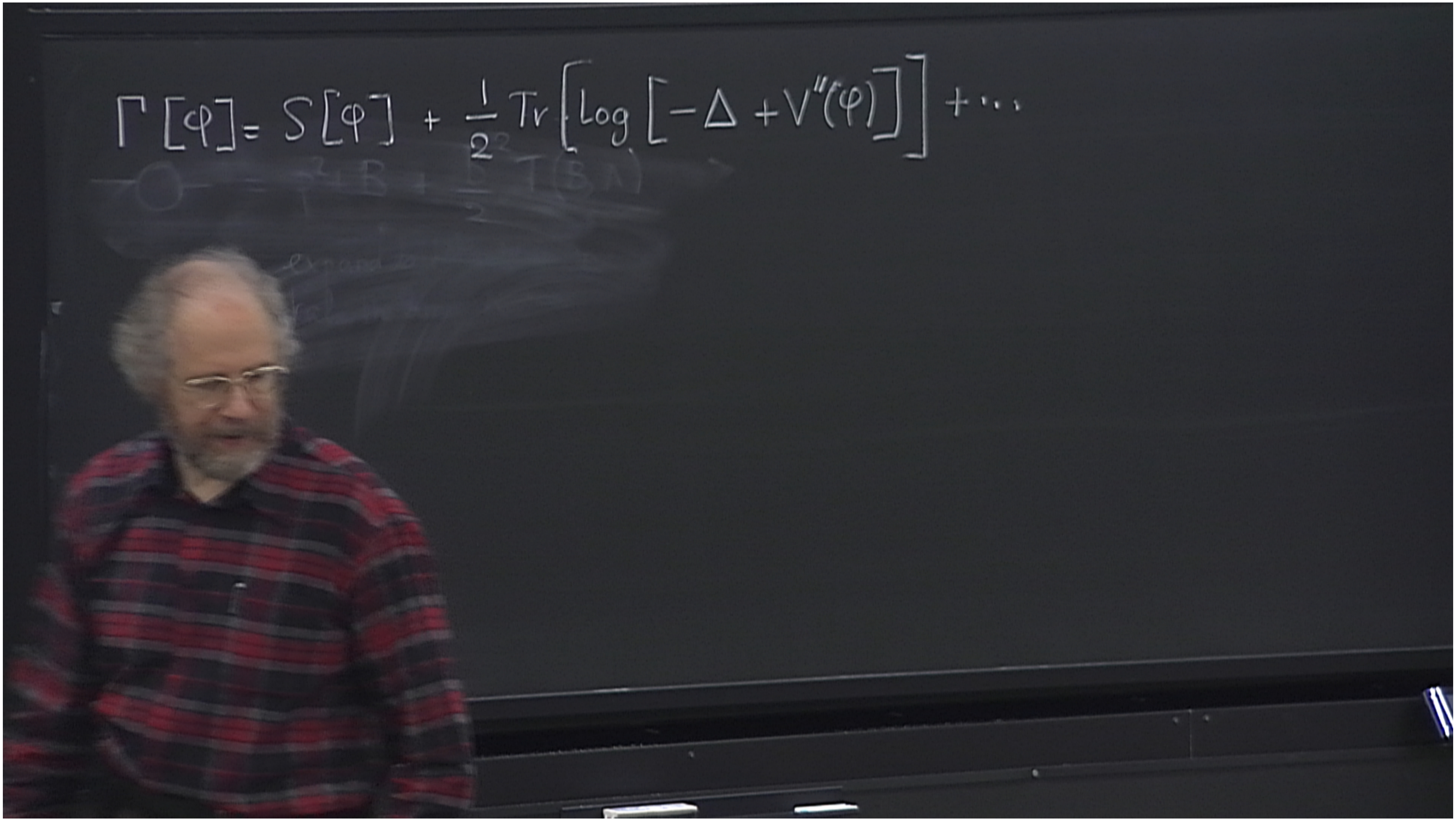
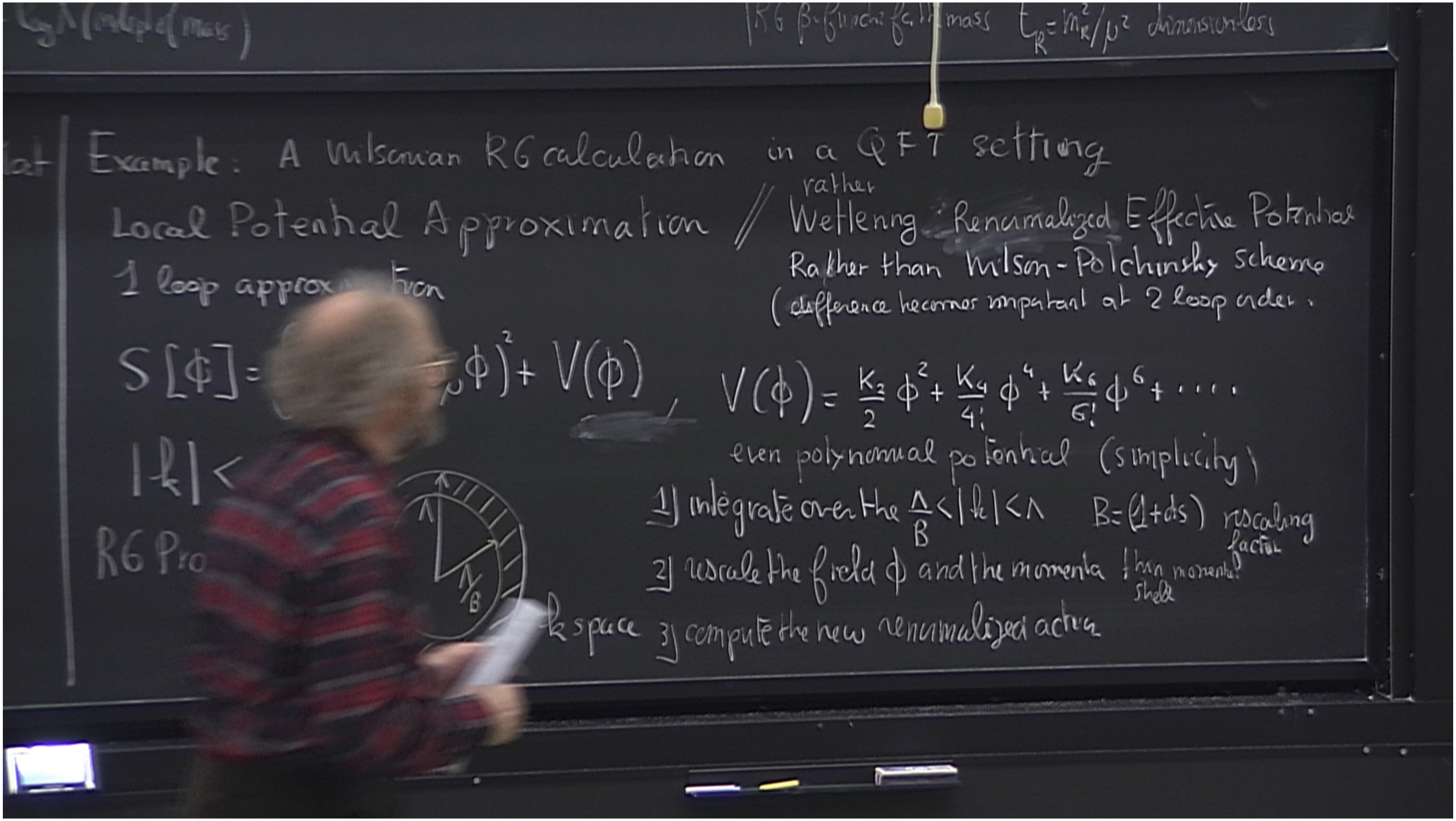
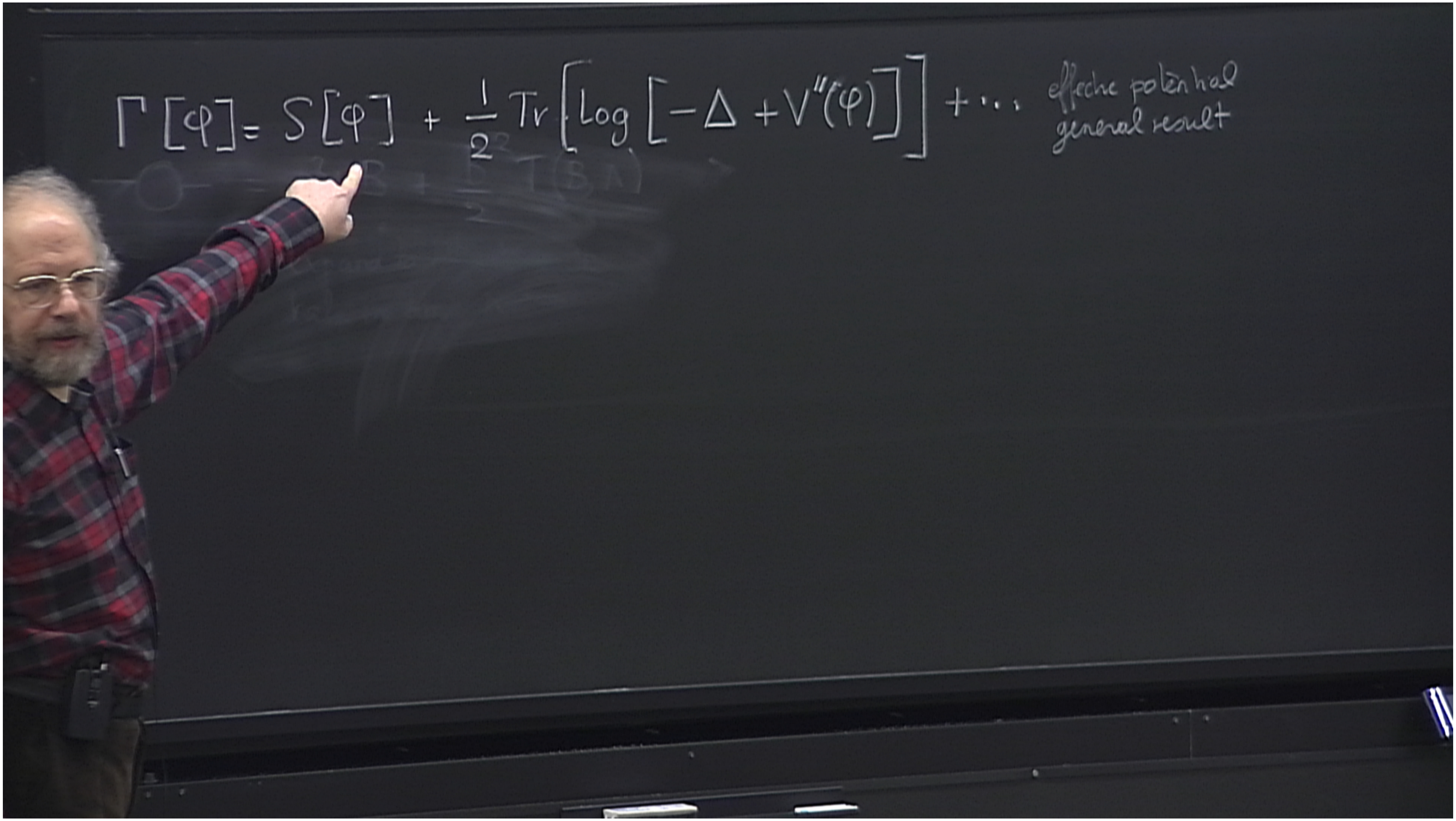
even polynomial potential (simplicity)
$|k| < \Lambda$
RG Procedure
Λ
Λ/B
k space
1) integrate over the $\frac{\Lambda}{B} < |k| < \Lambda$
$B = (1 + ds)$ rescaling factor
2) rescale the field ϕ and the momenta
3) compute the new renormalized action


Γ[φ] = S[φ] + 1/2 Tr[Log[−Δ + V''(φ)]] + ...


RG β-function for the mass $t_R = m_R^2/\mu^2$ dimensionless
Example: A Wilsonian RG calculation in a QFT setting
Local Potential Approximation
rather Wetterich: Renormalized Effective Potential
Rather than Wilson-Polchinski scheme
(difference becomes important at 2 loop order.
1 loop approx tion
$S[\phi] =$
$+ V(\phi)$
$V(\phi) = \frac{K_2}{2}\phi^2 + \frac{K_4}{4!}\phi^4 + \frac{K_6}{6!}\phi^6 + \dots$
even polynomial potential (simplicity)
$|k| <$
RG Pro
1) integrate over the $\frac{\Lambda}{B} < |k| < \Lambda$
$B = (1 + ds)$ rescaling factor
2) rescale the field ϕ and the momenta
thin momentum shell
Λ
Λ / B
k space
3) compute the new renormalized action


Γ[φ] = S[φ] + 1/2 Tr[Log[−Δ + V''(φ)]] + . . .
effective potential
general result

- $\Gamma_{\Lambda}[\varphi] = S[\varphi] + \frac{1}{2}\mathrm{Tr}\left[\mathrm{Log}\left[-\Delta + V''(\varphi)\right]\right]_{\Lambda} + \dots$ effective potential general result

- φ varies "slowly" with respect to Λ (and the $\frac{1}{m_{phys}}$)

$\varphi(x) \simeq$ constant : treat it locally

$$\mathrm{Tr}\left[\mathrm{Log}\left[-\Delta + V''(\varphi)\right]\right] = \int d^D x_0 \langle x_0 | \mathrm{Log}\left[-\Delta + V''(\varphi)\right] | x_0 \rangle \simeq \int d^D x_0 \langle x_0 | \mathrm{Log}\left(-\Delta_x + V''(\varphi(x_0))\right) | x_0 \rangle$$

$$- \ \Gamma_{\Lambda}[\varphi] = S[\varphi] + \frac{1}{2}\,\mathrm{Tr}\left[\mathrm{Log}\left[-\Delta + V''(\varphi)\right]\right]_{\Lambda} + \dots$$

effective potential
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$$= \int d^D x_0 \int \frac{d^D k}{(2\pi)^D} \,\mathrm{Log}\left(k^2 + V''(\varphi(x_0))\right)$$

- $\Gamma_\Lambda[\varphi] = S[\varphi] + \frac{1}{2}\mathrm{Tr}\left[\mathrm{Log}\left[-\Delta + V''(\varphi)\right]\right]_\Lambda + \cdots$ effective potential general result

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$\varphi(x) \simeq$ constant : treat it locally

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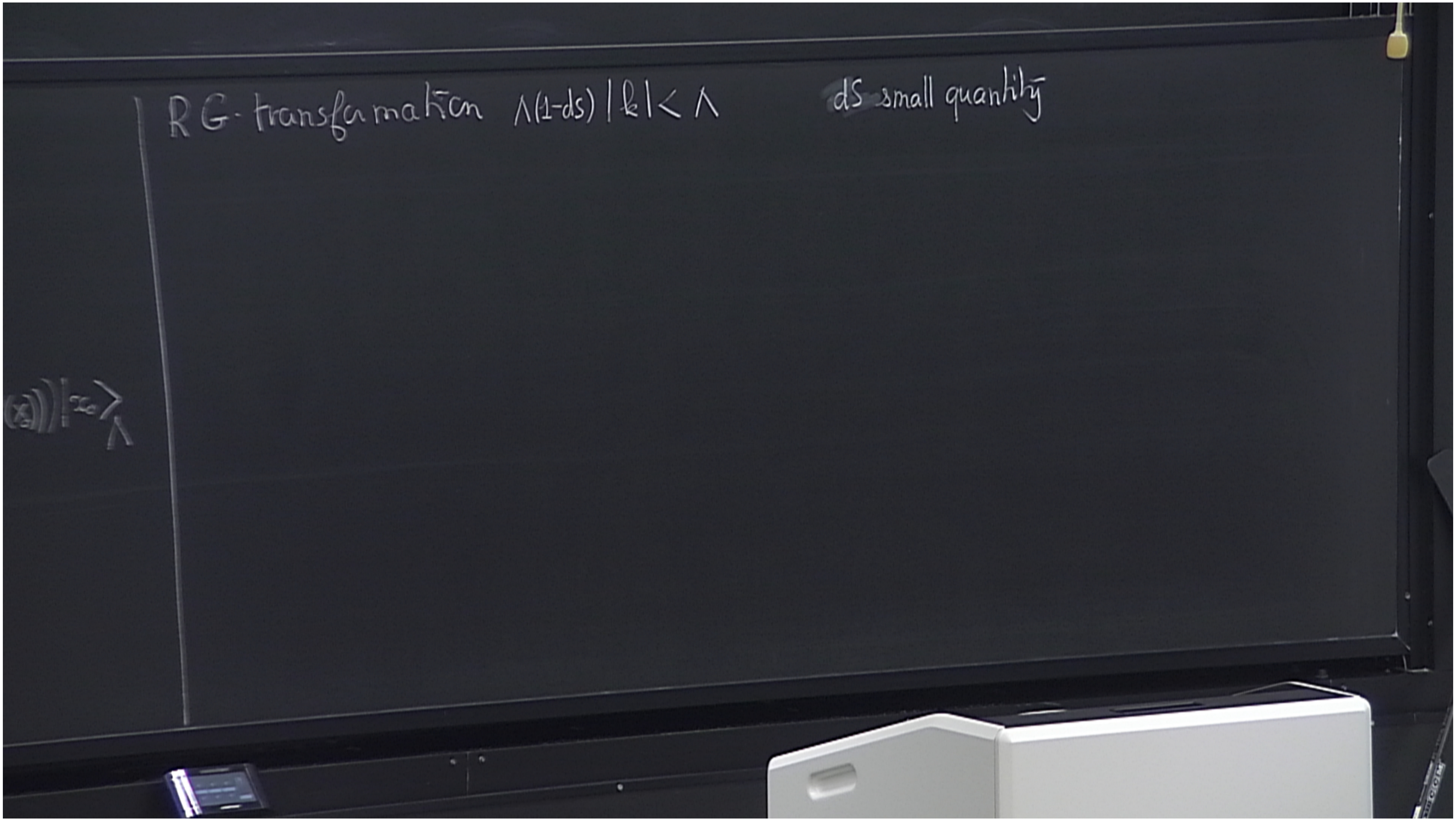
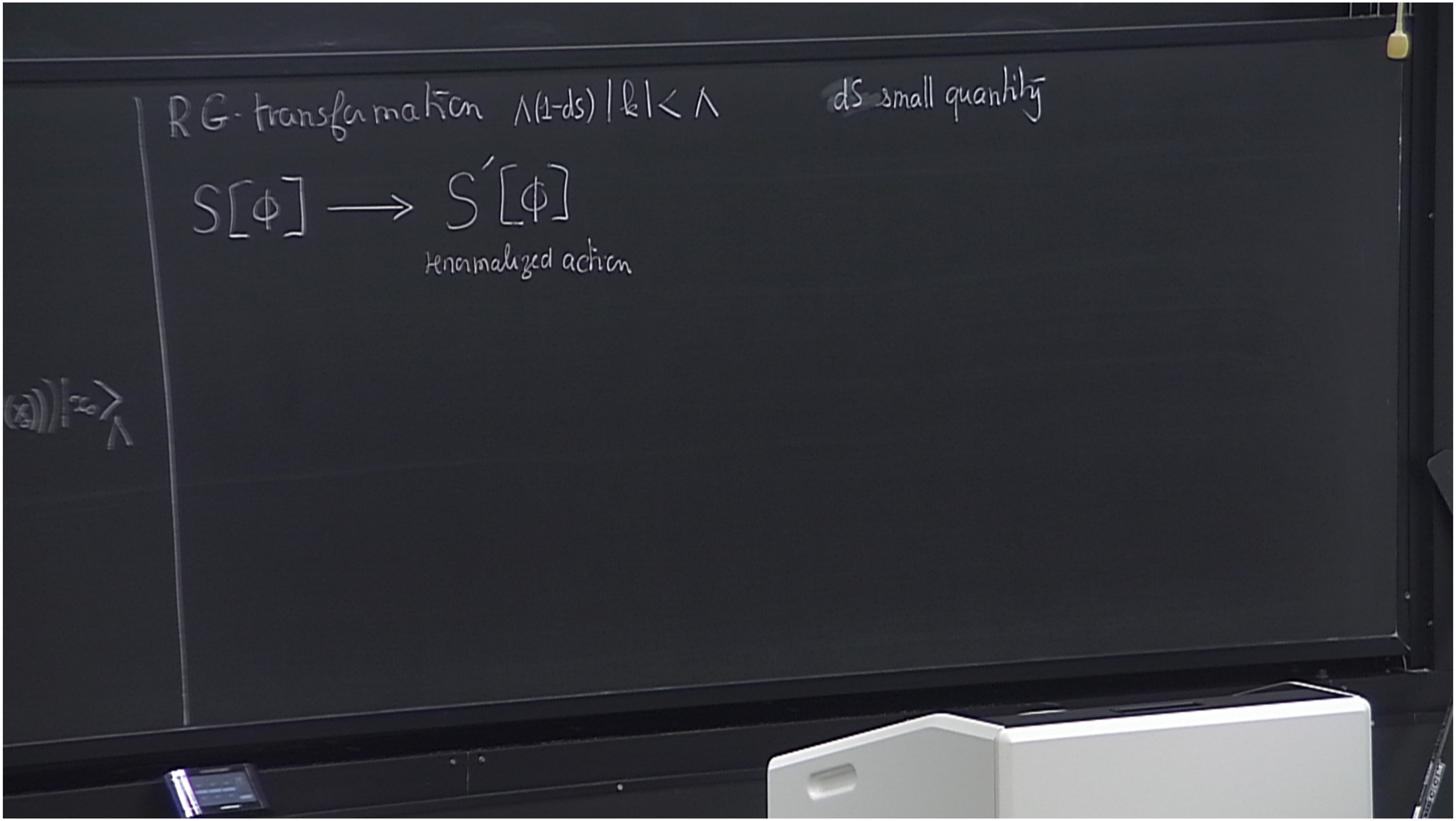
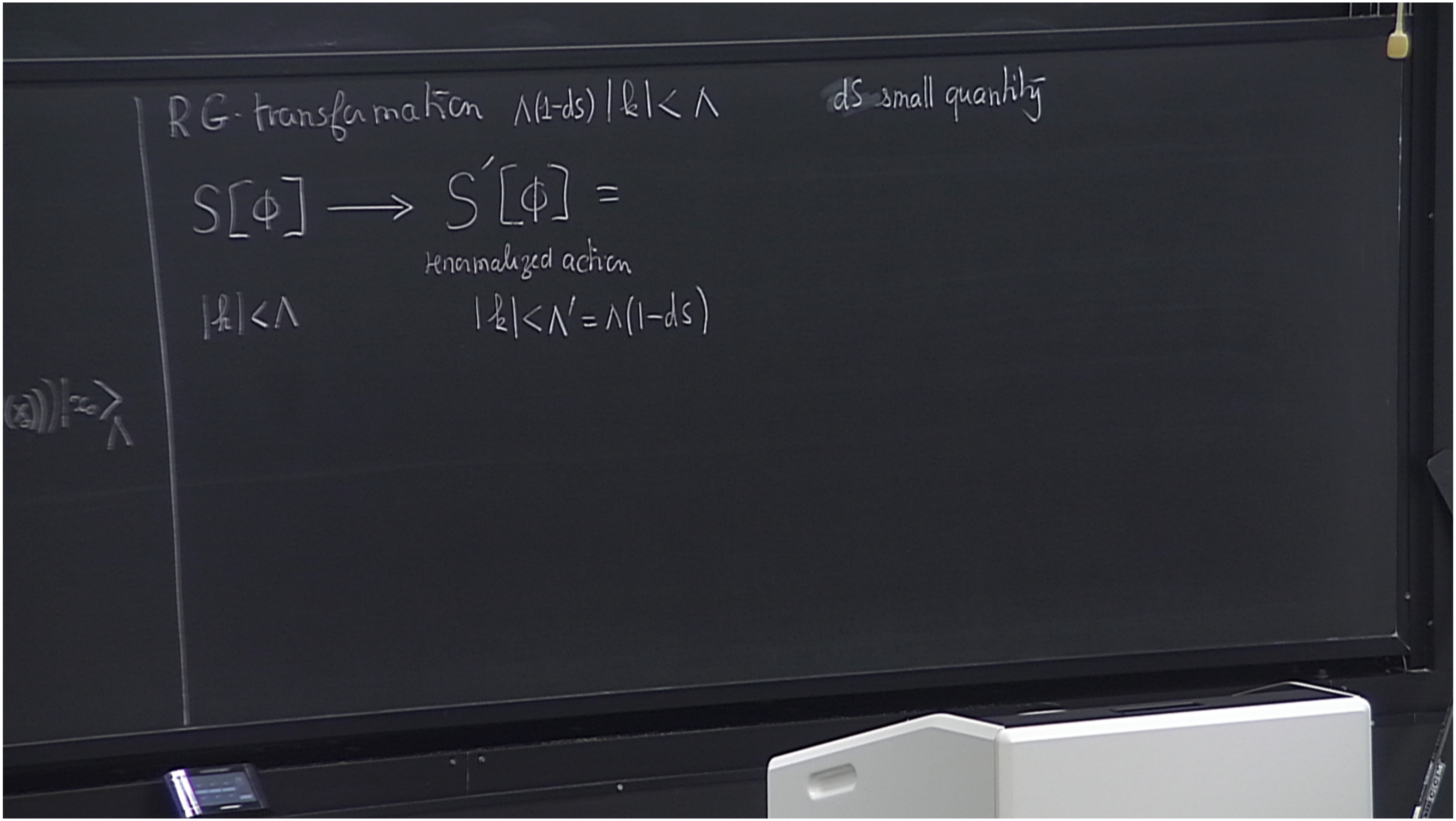
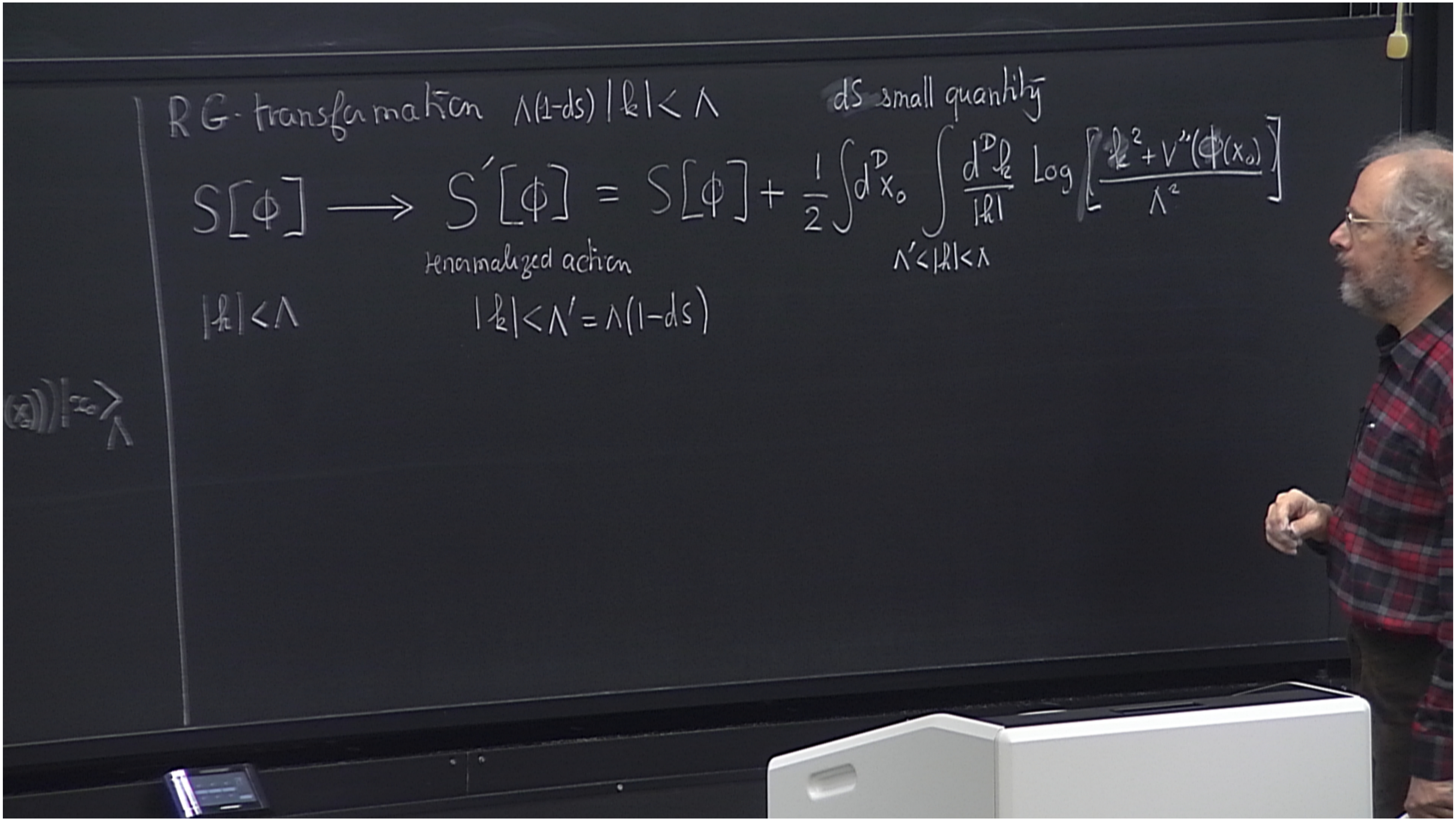
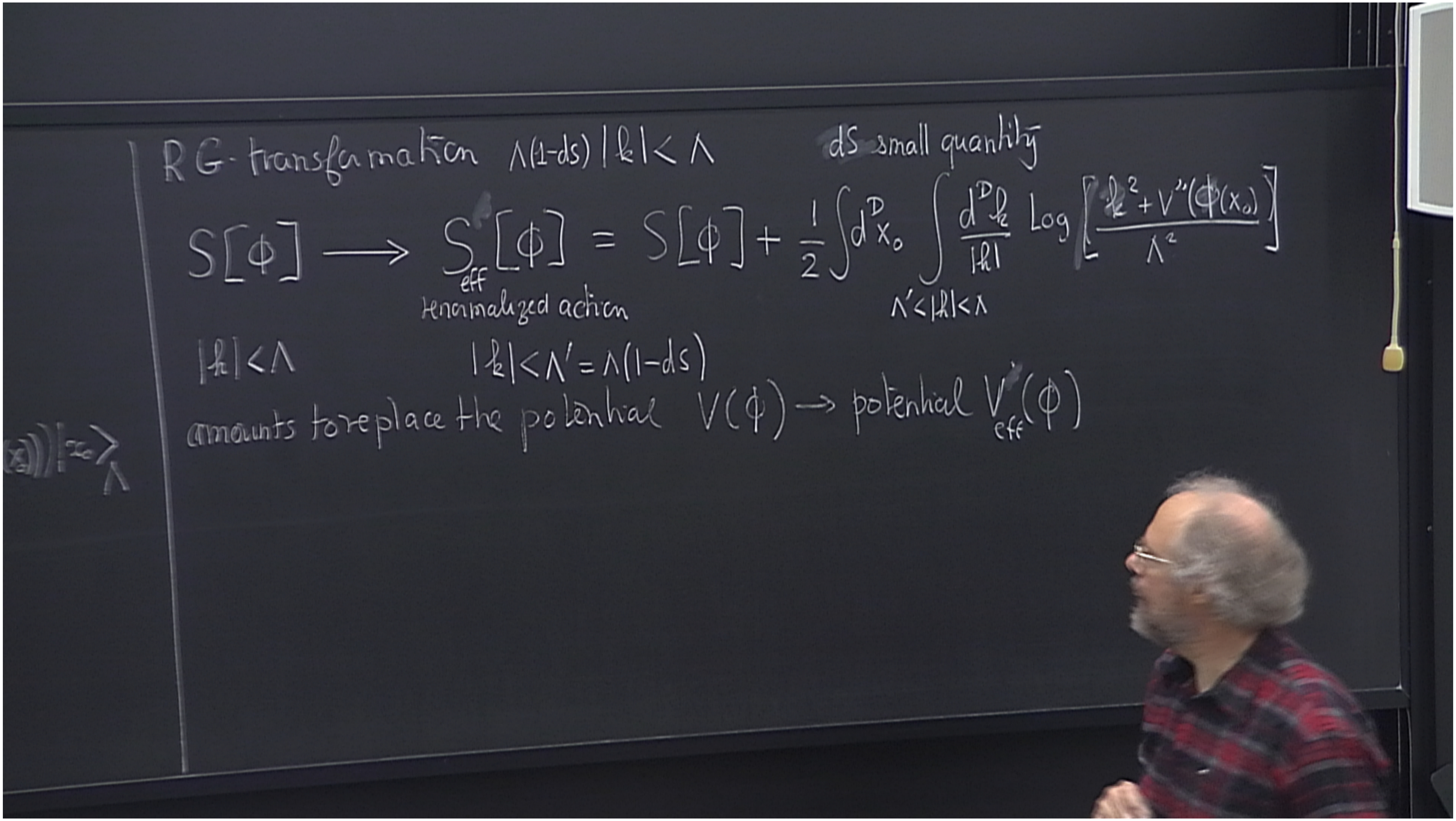
Full Quantum Theory


R G. transformation Λ(1-ds) | k | < Λ
dS small quantity


RG-transfomation Λ(1-ds) | k | < Λ
ds small quantity
S[ϕ] ⟶ S´[ϕ]
renormalized action


RG-transformation Λ(1-ds) |k| < Λ
dS small quantity
S[φ] ⟶ S′[φ] =
renormalized action
|k| < Λ
|k| < Λ′ = Λ(1-ds)


RG-transformation Λ(1-ds) |k| < Λ
ds small quantity
S[φ] ⟶ S′[φ] = S[φ] + 1/2 ∫d^D x₀ ∫ d^D k/|k| Log[(k² + V″(φ(x₀)))/Λ²]
renormalized action
Λ′<|k|<Λ
|k| < Λ
|k| < Λ′ = Λ(1-ds)


RG-transformation $\Lambda(1-ds) < |k| < \Lambda$
ds small quantity
$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{|k|} \operatorname{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$
renormalized action
$|k| < \Lambda$
$|k| < \Lambda' = \Lambda(1-ds)$
amounts to replace the potential $V(\phi) \rightarrow$ potential $V_{eff}(\phi)$

RG-transformation $\Lambda(1-ds) < |k| < \Lambda$ $\qquad$ ds small quantity

$$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{|k|} \operatorname{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$$

renormalized action

$|k| < \Lambda$ $\qquad$ $|k| < \Lambda' = \Lambda(1-ds)$

amounts to replace the potential $V(\phi) \to$ potential $V_{eff}(\phi)$

$$V_{eff}(\phi) = V(\phi) + \frac{1}{2} S_D$$

S_D = "volume" of the unit sphere in $\mathbb{R}^D$

RG-transformation $\Lambda(1-ds) < |k| < \Lambda$ $\quad$ ds small quantity

$$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{|k|} \,\mathrm{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$$

renormalized action

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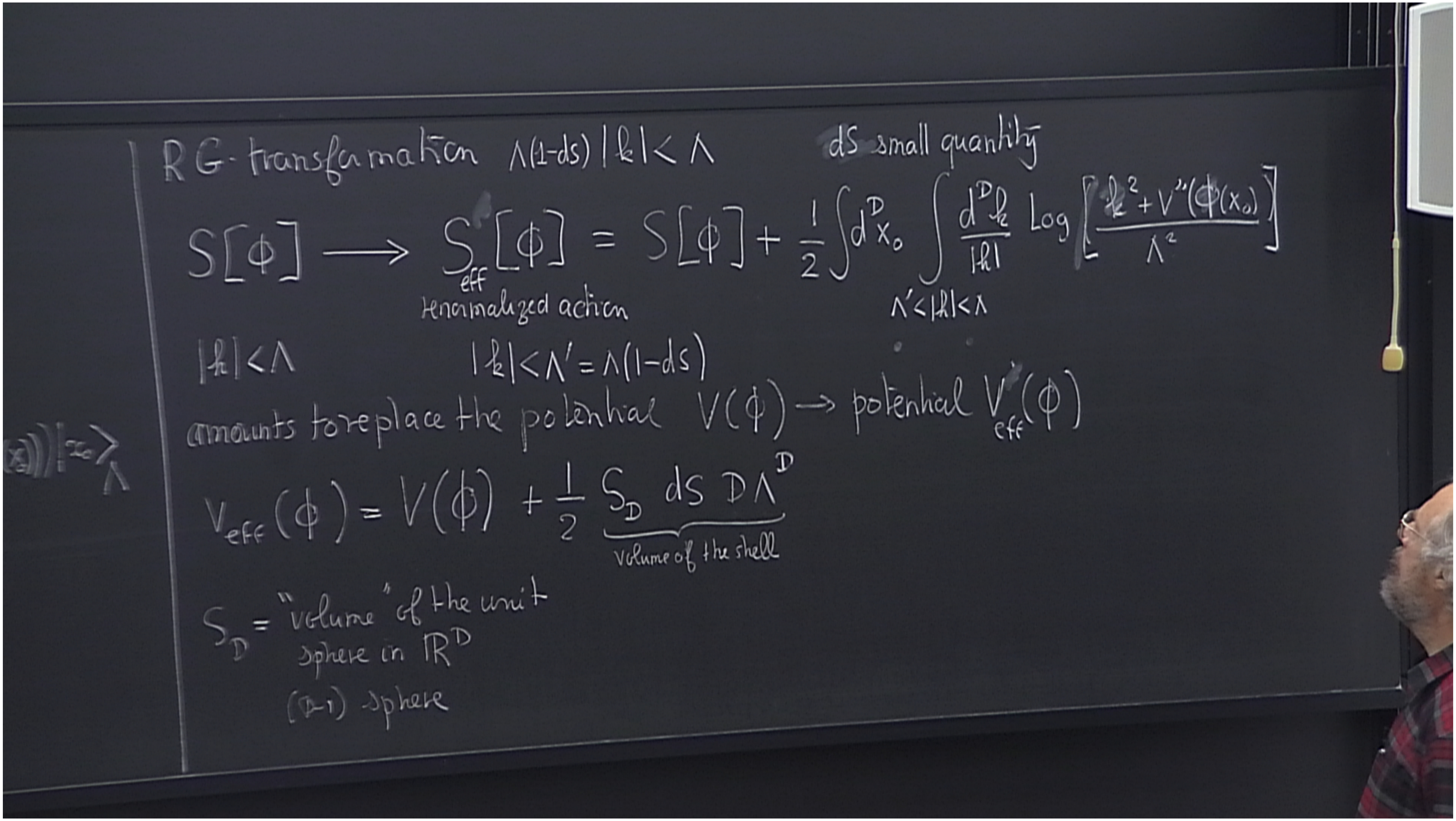
$(D-1)$ sphere


RG-transformation $\Lambda(1-ds) < |k| < \Lambda$
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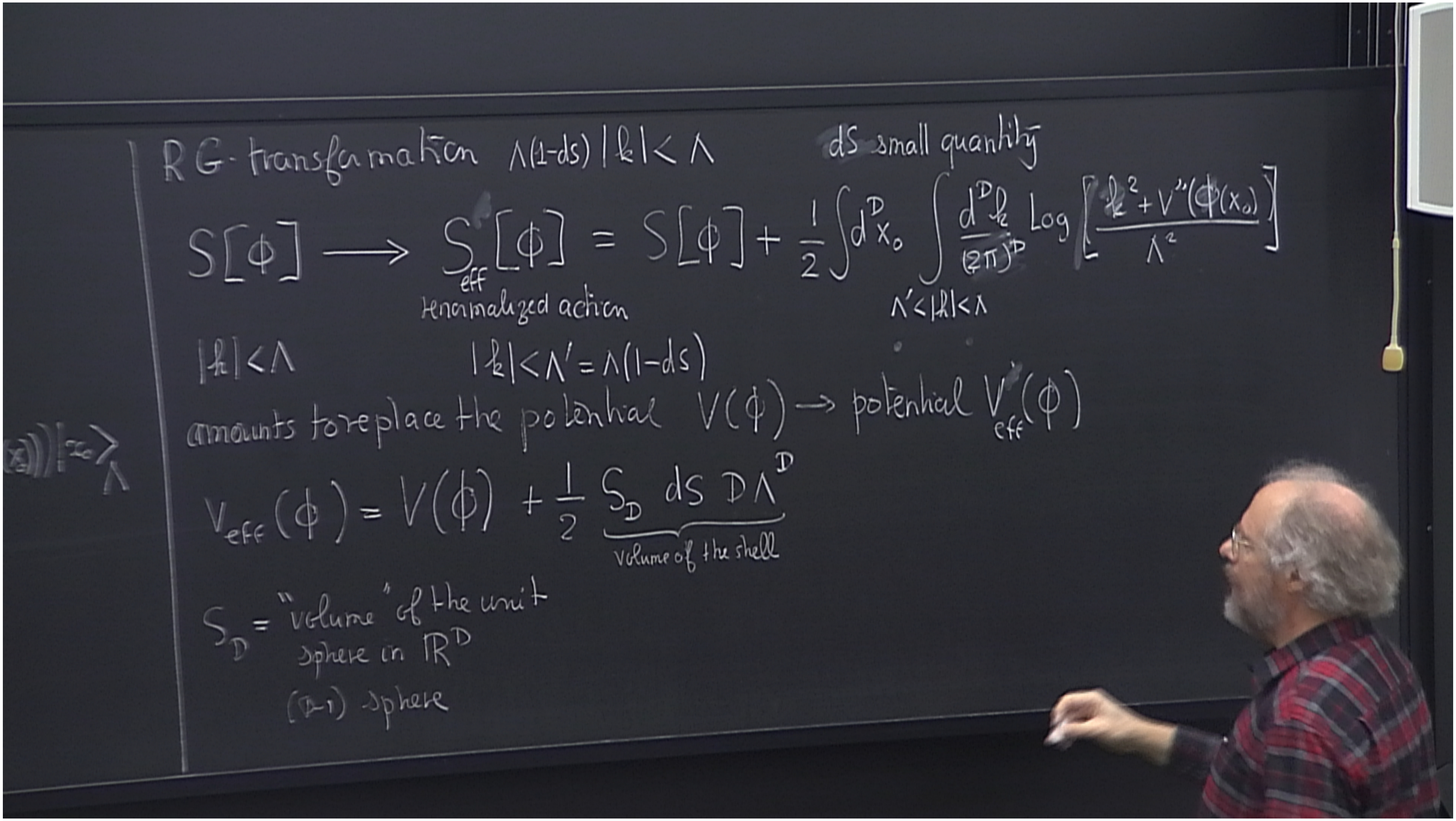
renormalized action
$|k| < \Lambda$
$|k| < \Lambda' = \Lambda(1-ds)$
amounts to replace the potential $V(\phi) \rightarrow$ potential $V_{eff}(\phi)$
$V_{eff}(\phi) = V(\phi) + \frac{1}{2} S_D \, ds \, D \Lambda^D$
volume of the shell
S_D = "volume" of the unit sphere in $\mathbb{R}^D$
(D-1) sphere



RG-transformation $\Lambda(1-ds) < |k| < \Lambda$
ds small quantity
$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \operatorname{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$
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(D-1) sphere

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renormalized action

$|k| < \Lambda$ $|k| < \Lambda' = \Lambda(1-ds)$

amounts to replace the potential $V(\phi) \rightarrow$ potential $V_{eff}(\phi)$

$$V_{eff}(\phi) = V(\phi) + \frac{1}{2} \underbrace{S_D \, ds \, D \Lambda^D}_{\text{volume of the shell}} \frac{1}{(2\pi)^D} \, \mathrm{Log}\left[1 + \Lambda^{-2} V''(\phi)\right]$$

S_D = "volume" of the unit sphere in $\mathbb{R}^D$

(D-1) sphere

RG-transformation $\Lambda(1-ds) < |k| < \Lambda$ ds small quantity

$$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \operatorname{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$$

renormalized action

$|k| < \Lambda$ $\qquad |k| < \Lambda' = \Lambda(1-ds)$

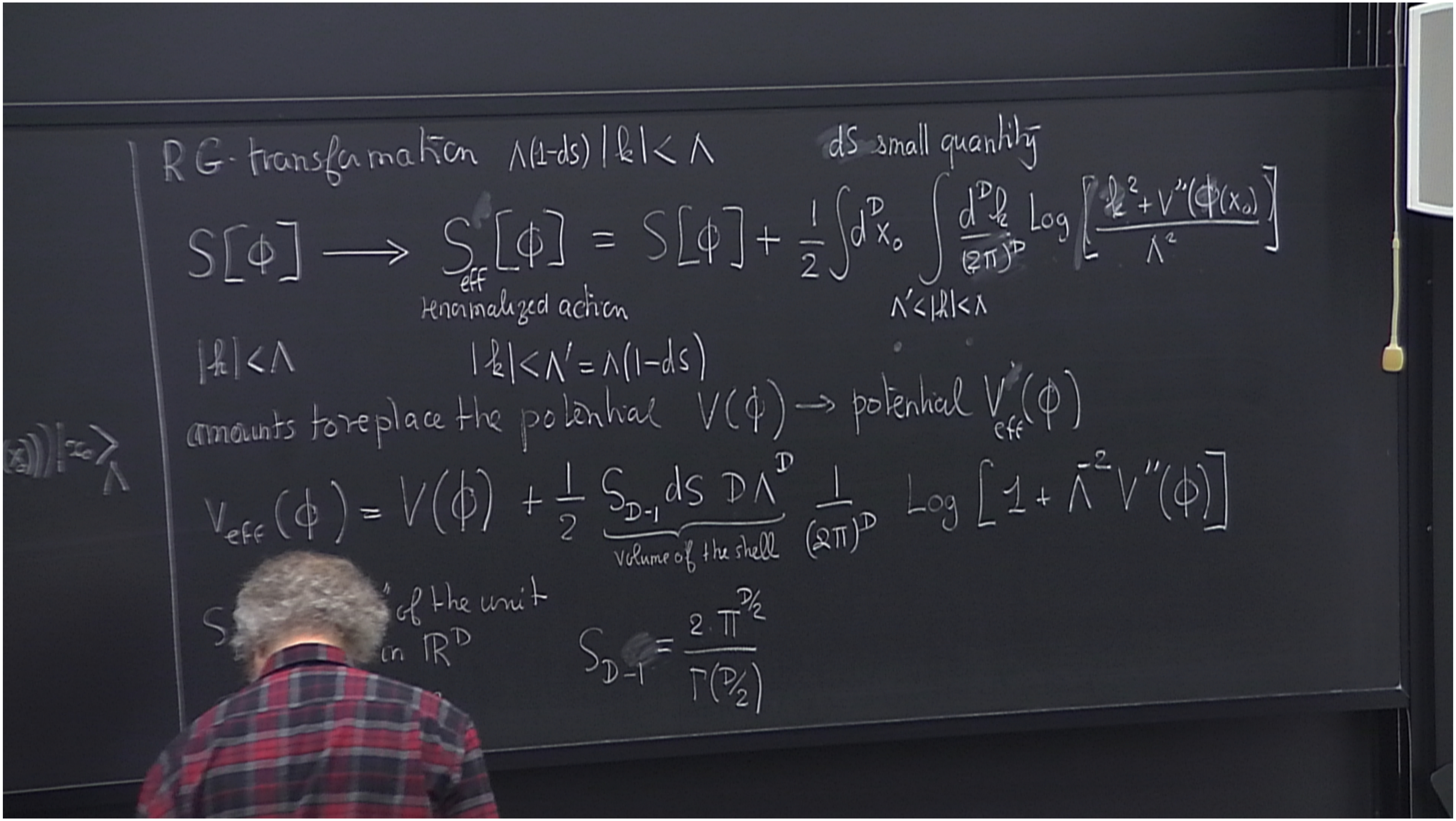
amounts to replace the potential $V(\phi) \longrightarrow$ potential $V_{eff}(\phi)$

$$V_{eff}(\phi) = V(\phi) + \frac{1}{2} \underbrace{S_{D-1}\, ds\, D \Lambda^D}_{\text{Volume of the shell}} \frac{1}{(2\pi)^D} \operatorname{Log}\left[1 + \Lambda^{-2} V''(\phi)\right]$$

S_{D-1} = "volume" of the unit sphere in $\mathbb{R}^D$

(D-1) sphere

$$S_{D-1} = \frac{2\,\pi^{D/2}}{\Gamma(D/2)}$$


RG-transformation Λ(1-ds) | k | < Λ
dS small quantity
S[φ] ⟶ S_eff[φ] = S[φ] + 1/2 ∫ d^D x_0 ∫ d^D k/(2π)^D Log [(k^2 + V''(φ(x_0))) / Λ^2]
renormalized action
Λ' < | k | < Λ
| k | < Λ
| k | < Λ' = Λ(1-ds)
amounts to replace the potential V(φ) ⟶ potential V'_eff(φ)
V_eff(φ) = V(φ) + 1/2 S_{D-1} ds D Λ^D 1/(2π)^D Log [1 + Λ^{-2} V''(φ)]
volume of the shell
of the unit
in R^D
S_{D-1} = 2 π^{D/2} / Γ(D/2)

RG-transformation $\Lambda(1-ds) \, |k| < \Lambda$ $\quad$ ds small quantity

$$S[\phi] \longrightarrow S_{eff}[\phi] = S[\phi] + \frac{1}{2}\int d^D x_0 \int_{\Lambda' < |k| < \Lambda} \frac{d^D k}{(2\pi)^D} \operatorname{Log}\left[\frac{k^2 + V''(\phi(x_0))}{\Lambda^2}\right]$$

renormalized action

$|k| < \Lambda$ $\qquad$ $|k| < \Lambda' = \Lambda(1-ds)$

$|x_0\rangle$ Λ

amounts to replace the potential $V(\phi) \longrightarrow$ potential $V_{eff}(\phi)$

$$V_{eff}(\phi) = V(\phi) + \frac{1}{2} \underbrace{S_{D-1}\, ds\, D \Lambda^D}_{\text{volume of the shell}} \frac{1}{(2\pi)^D} \operatorname{Log}\left[1 + \Lambda^{-2} V''(\phi)\right]$$

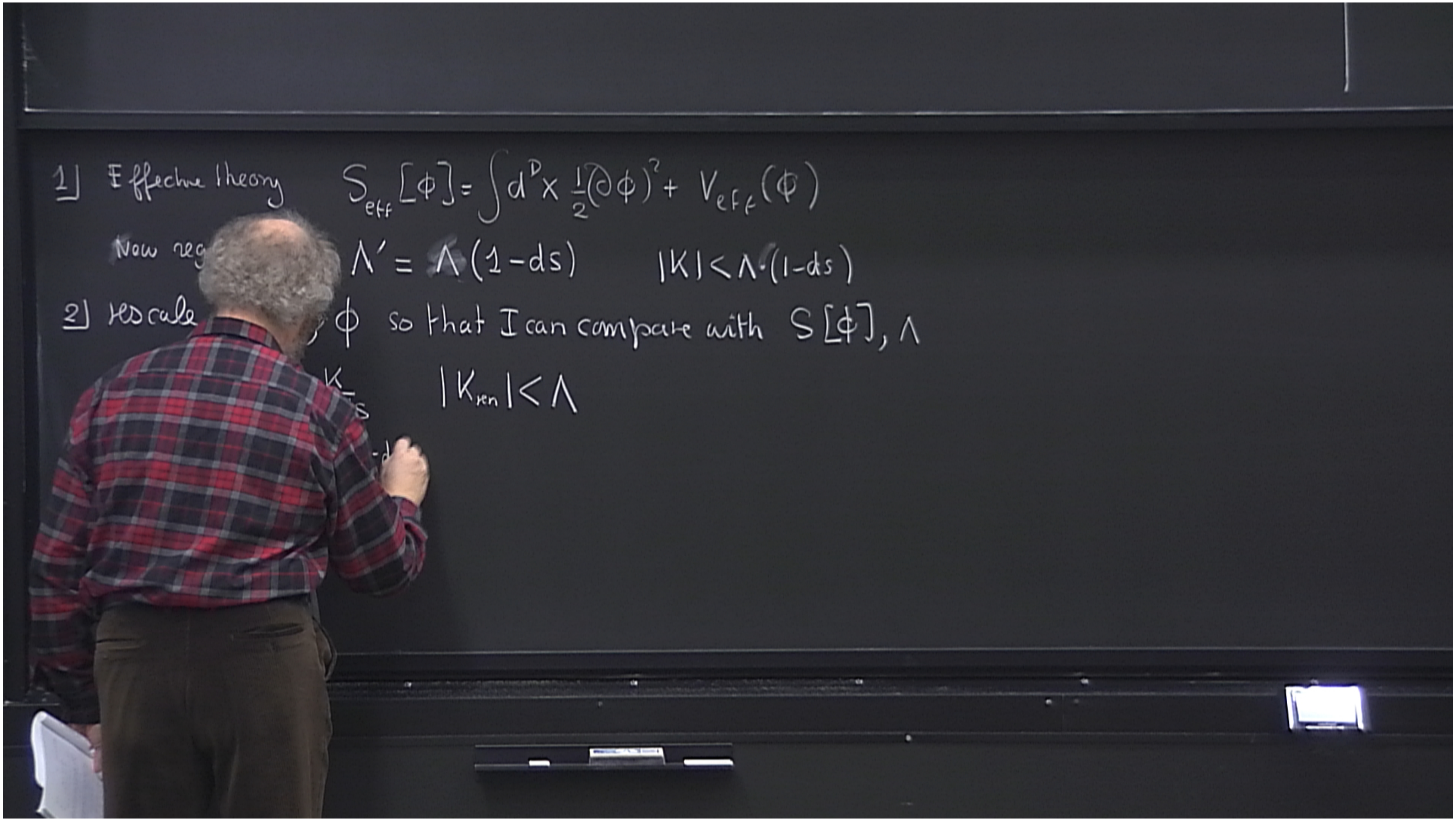
S_{D-1} = "volume" of the unit sphere in $\mathbb{R}^D$

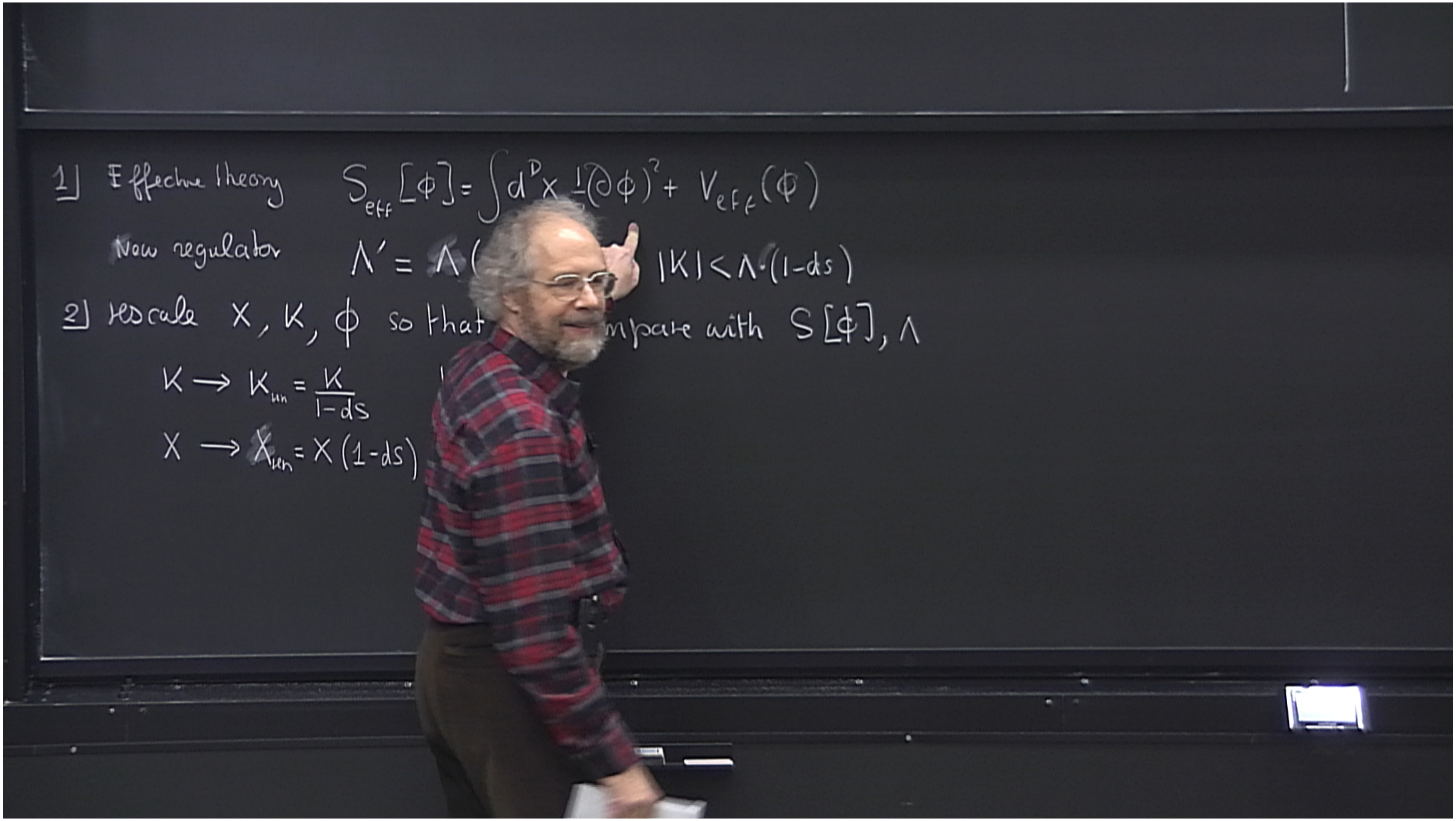
$$S_{D-1} = \frac{2 \cdot \pi^{D/2}}{\Gamma(D/2)}$$

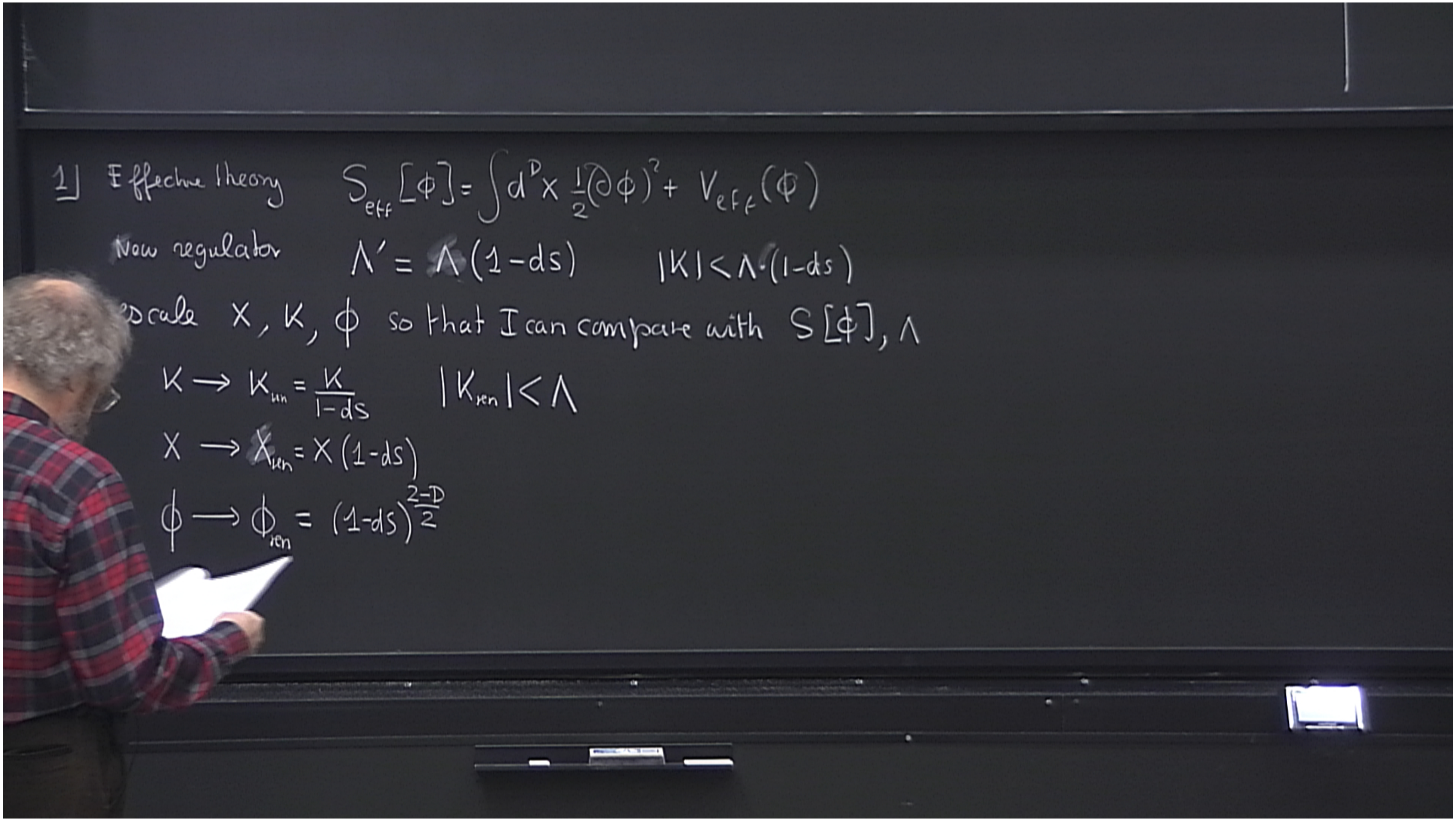
$\bigcirc$ $(D-1)$ sphere

Effective theory $S_{eff}[\phi] = \int d^D x \, \frac{1}{2}(\partial\phi)^2 + V_{eff}(\phi)$

New regulator $\Lambda' = \Lambda(1-ds)$


1] Effective theory $S_{eff}[\phi] = \int d^Dx \frac{1}{2}(\partial\phi)^2 + V_{eff}(\phi)$
Now reg
$\Lambda' = \Lambda(1-ds)$
$|K| < \Lambda\cdot(1-ds)$
2] rescale
ϕ so that I can compare with $S[\phi]$, Λ
$|K_{ren}| < \Lambda$


1] Effective theory $S_{eff}[\phi] = \int d^Dx \, \frac{1}{2}(\partial\phi)^2 + V_{eff}(\phi)$
New regulator $\Lambda' = \Lambda($ $|K| < \Lambda \cdot (1-ds)$
2] rescale x, K, ϕ so that
mpare with $S[\phi], \Lambda$
$K \longrightarrow K_{un} = \frac{K}{1-ds}$
$X \longrightarrow X_{un} = X(1-ds)$


1] Effective theory $S_{eff}[\phi] = \int d^D x \, \frac{1}{2}(\partial\phi)^2 + V_{eff}(\phi)$
New regulator $\Lambda' = \Lambda(1-ds)$ $|K| < \Lambda\cdot(1-ds)$
escale x, K, ϕ so that I can compare with $S[\phi], \Lambda$
$K \longrightarrow K_{ren} = \frac{K}{1-ds}$ $|K_{ren}| < \Lambda$
$X \longrightarrow X_{ren} = X(1-ds)$
$\phi \longrightarrow \phi_{ren} = (1-ds)^{\frac{2-D}{2}}$

1] Effective theory $S_{eff}[\phi] = \int d^D x \, \frac{1}{2}(\partial\phi)^2 + V_{eff}(\phi)$

New regulator $\Lambda' = \Lambda(1-ds)$ $\quad |K| < \Lambda\cdot(1-ds)$

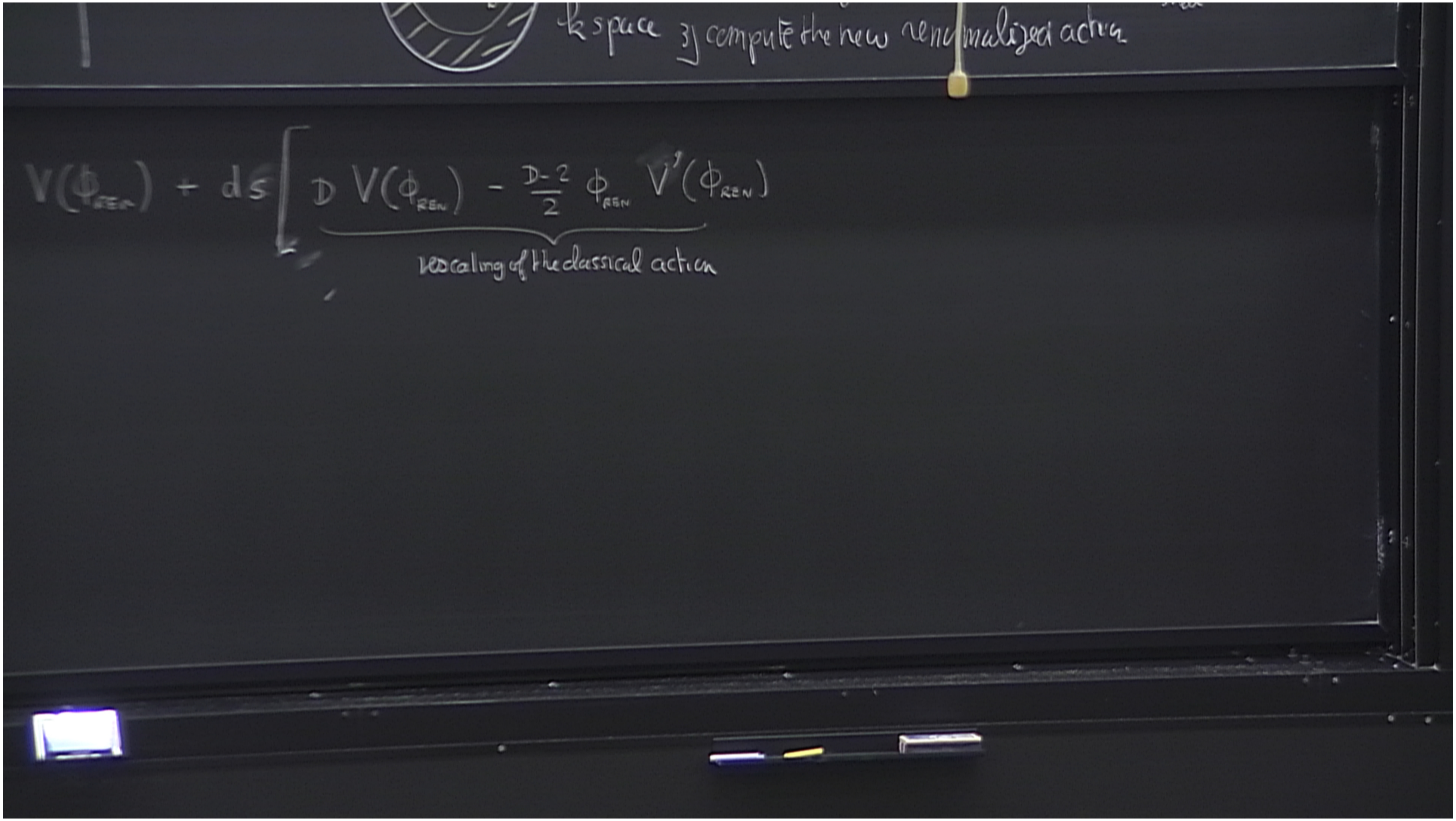
2] rescale X, K, ϕ so that I can compare with $S[\phi]$, Λ

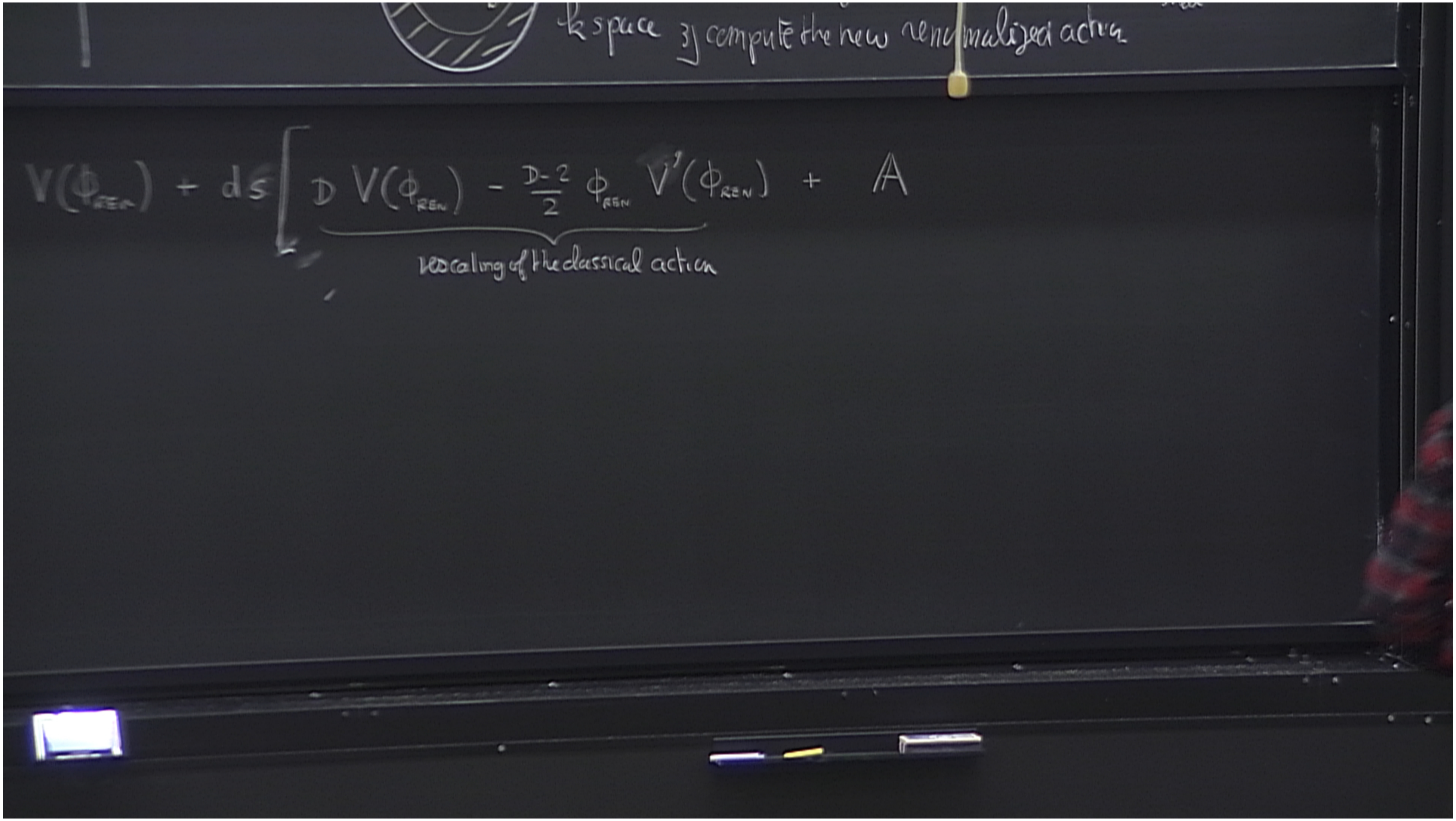
$$K \longrightarrow K_{ren} = \frac{K}{1-ds} \qquad |K_{ren}| < \Lambda$$

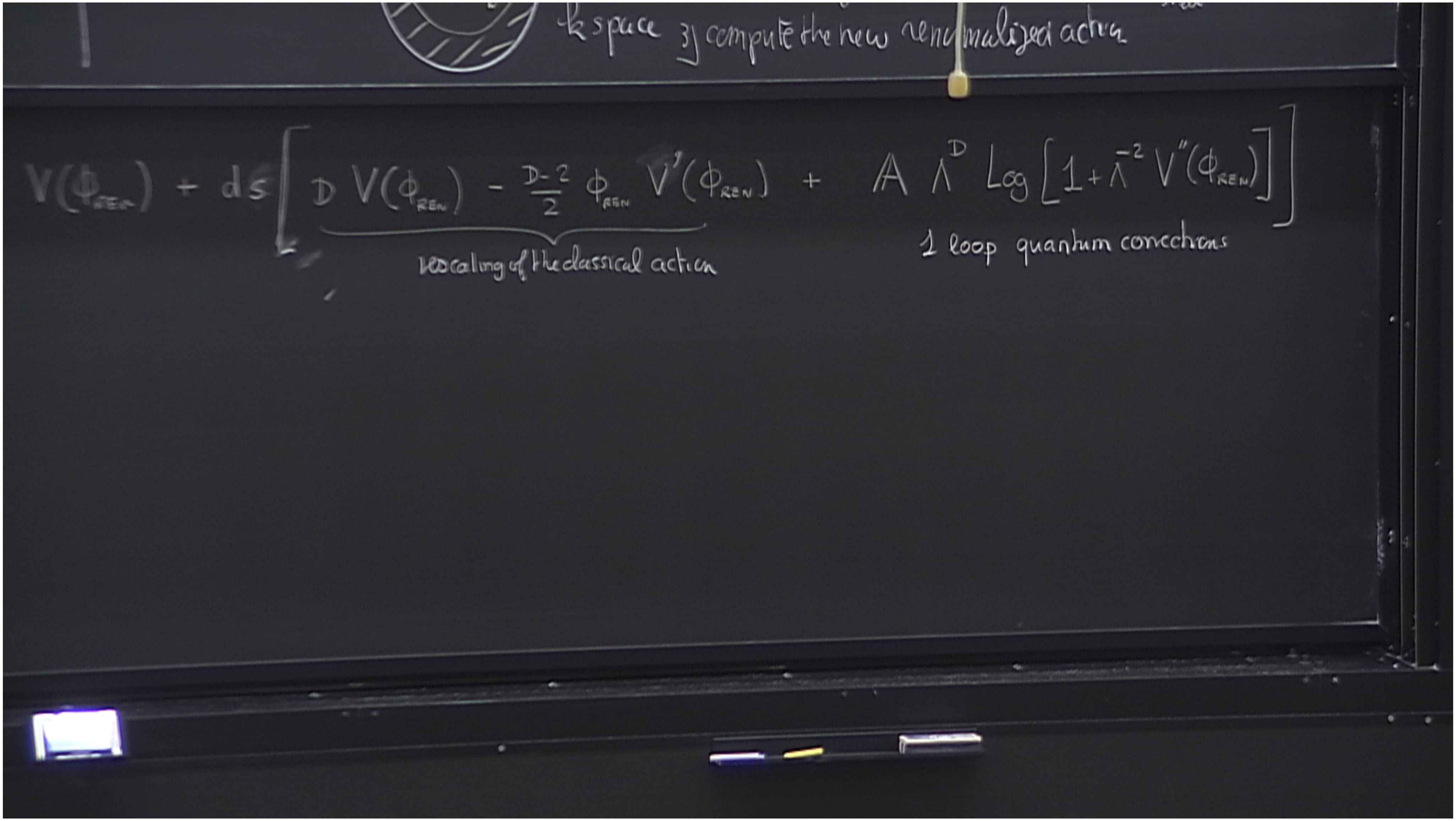
$$X \longrightarrow X_{ren} = X(1-ds)$$

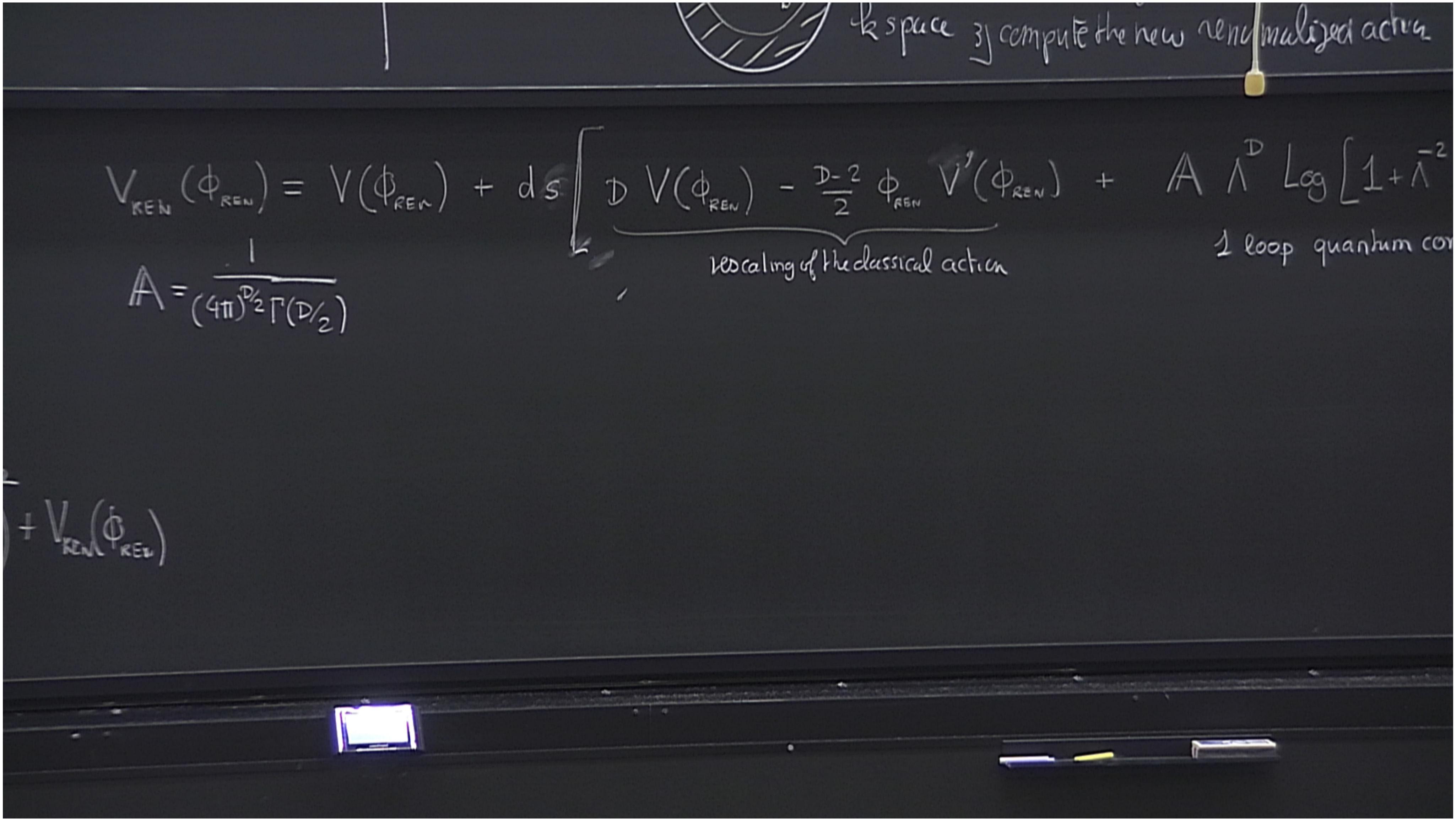
$$\phi \longrightarrow \phi_{ren} = (1+ds)^{-\frac{D-2}{2}}\,\phi$$

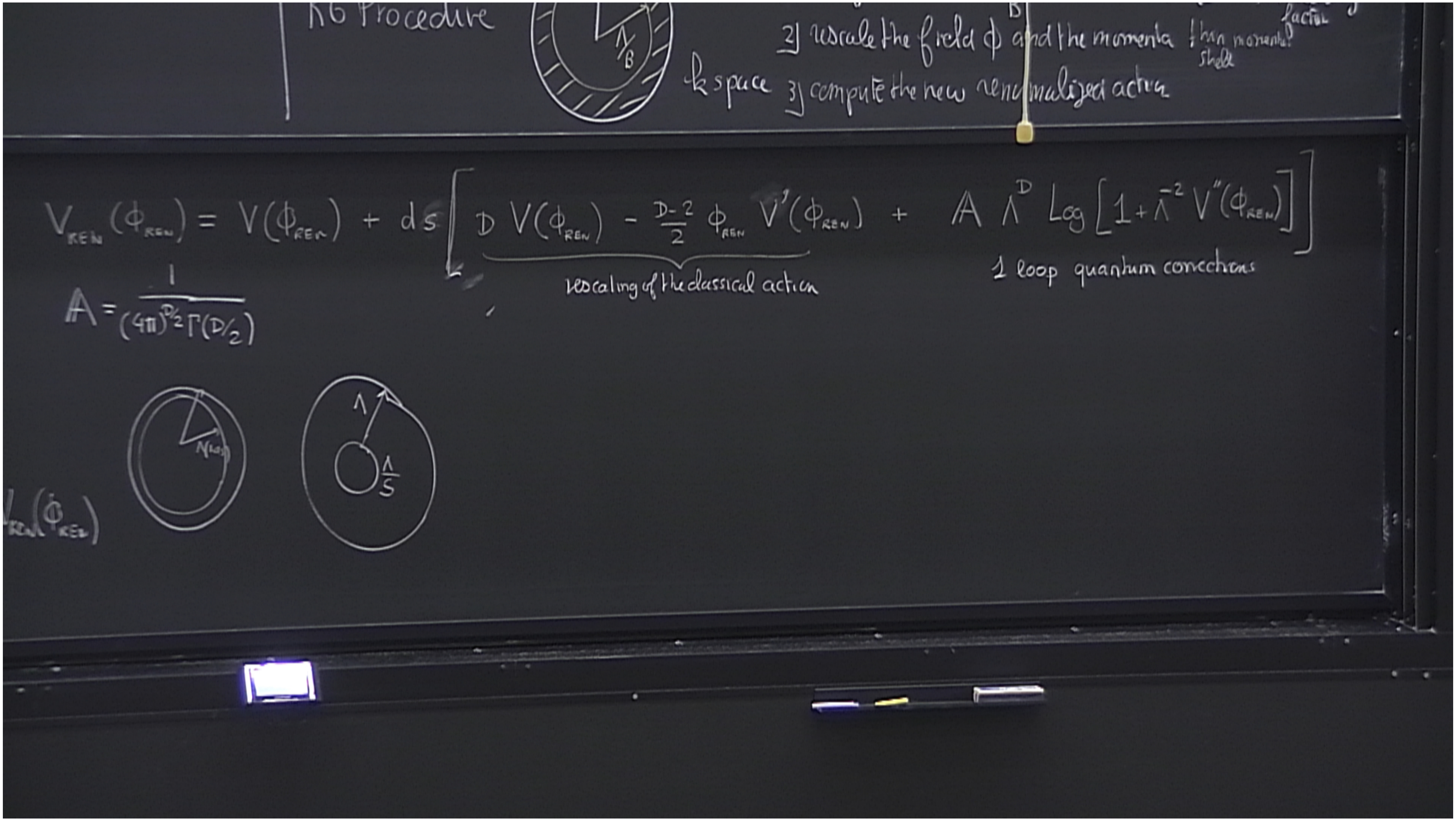
$$S_{eff}[\phi] \equiv S_R[\phi_{REN}] = \int d^D x_{REN} \, \frac{1}{2}\left(\frac{\partial\phi_{REN}}{\partial x_{REN}}\right)^2 + V_{REN}(\phi_{REN})$$

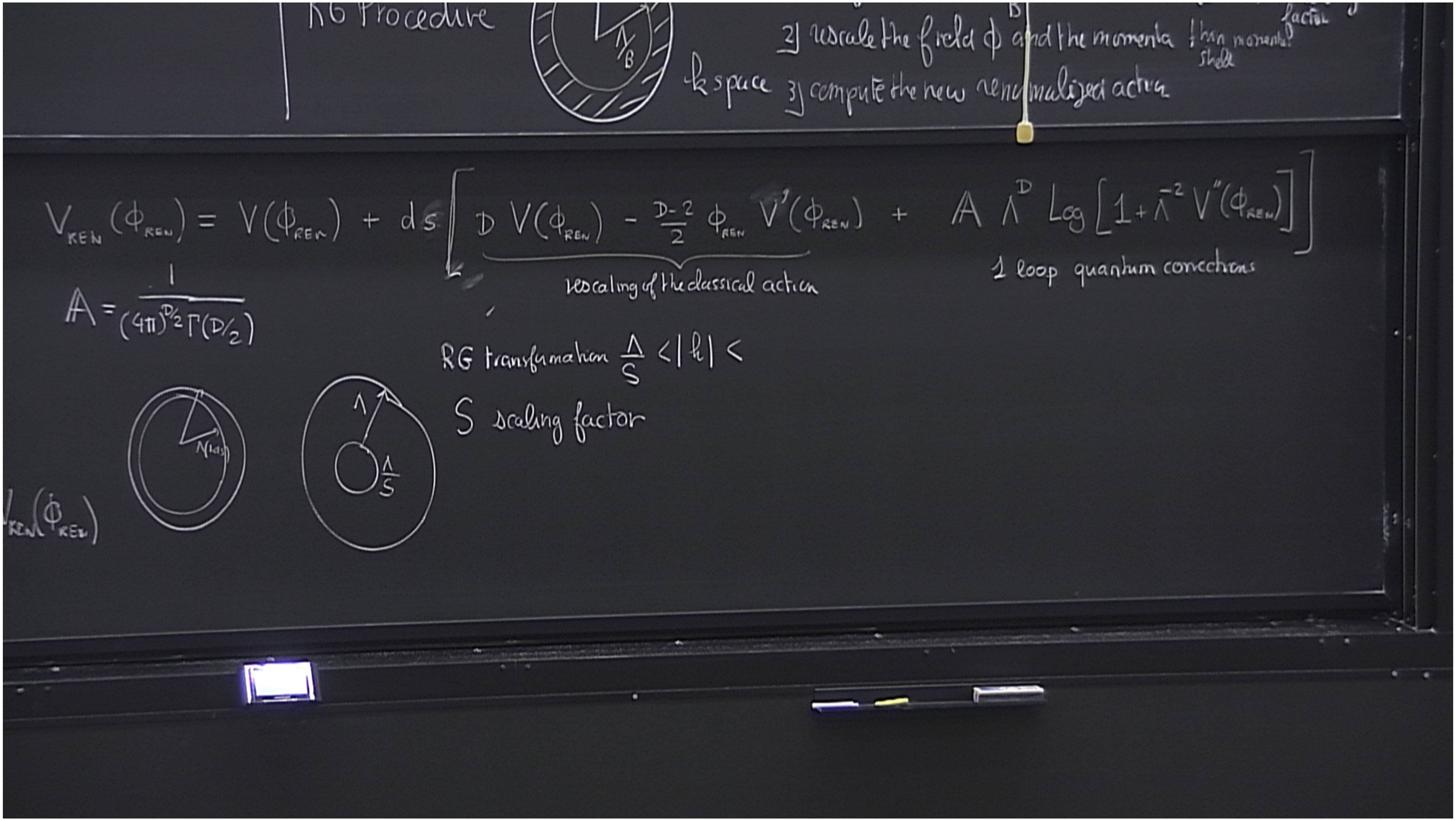

k space 3) compute the new renormalized action
V(ϕ_REN) + ds [D V(ϕ_REN) - (D-2)/2 ϕ_REN V'(ϕ_REN)]
rescaling of the classical action

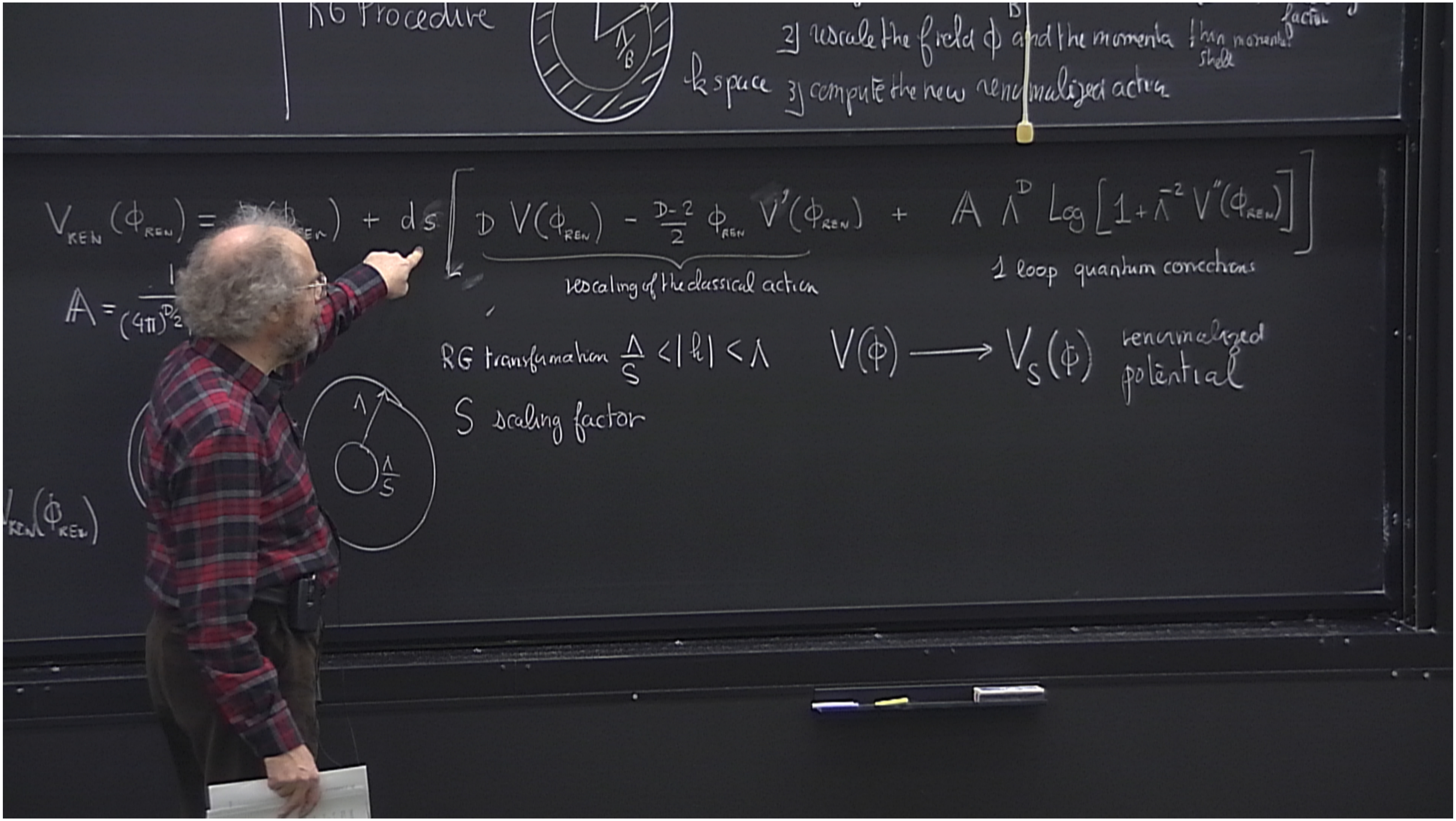

k space 3) compute the new renormalized action
V(φ_REN) + ds [D V(φ_REN) − (D−2)/2 φ_REN V'(φ_REN) + A
rescaling of the classical action

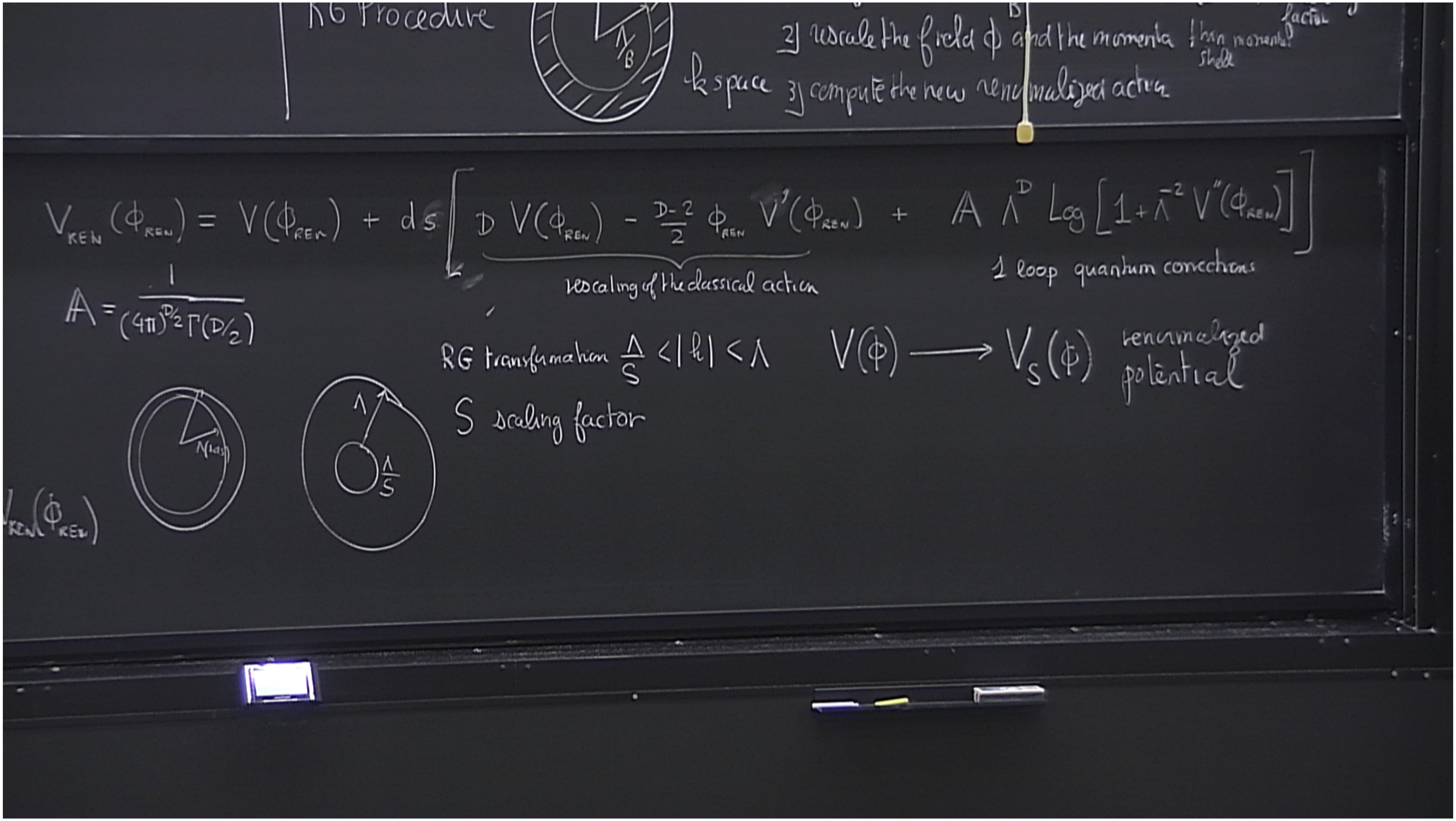

k space 3) compute the new renormalized action
$V(\phi_{REN}) + ds\left[D\, V(\phi_{REN}) - \frac{D-2}{2}\phi_{REN} V'(\phi_{REN}) + A\, \Lambda^{D} \operatorname{Log}\left[1+\Lambda^{-2} V''(\phi_{REN})\right]\right]$
rescaling of the classical action
1 loop quantum corrections

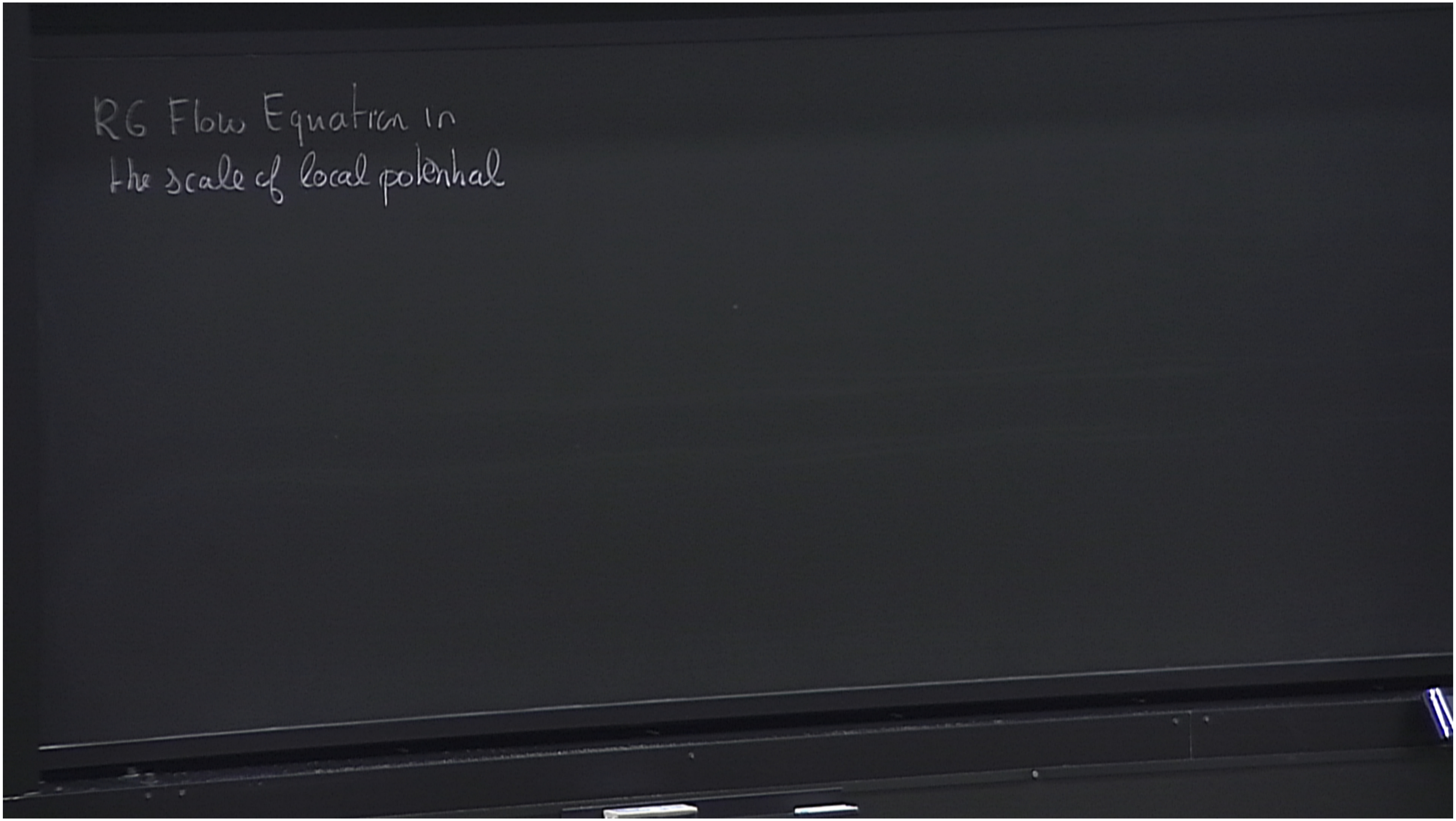

k space 3) compute the new renormalized action
$V_{REN}(\phi_{REN}) = V(\phi_{REN}) + ds \left[D\, V(\phi_{REN}) - \frac{D-2}{2} \phi_{REN} V'(\phi_{REN}) + A \Lambda^{D} \mathrm{Log}\left[1 + \Lambda^{-2}\right.\right.$
rescaling of the classical action
1 loop quantum cor
$A = \frac{1}{(4\pi)^{D/2}\, \Gamma(D/2)}$
$+ V_{REN}(\phi_{REN})$


RG Procedure
2] rescale the field ϕ and the momenta
3] compute the new renormalized action
k space
$V_{REN}(\phi_{REN}) = V(\phi_{REN}) + ds\left[D\,V(\phi_{REN}) - \frac{D-2}{2}\phi_{REN}V'(\phi_{REN}) + A\,\Lambda^{D}\,Log\left[1+\Lambda^{-2}V''(\phi_{REN})\right]\right]$
rescaling of the classical action
1 loop quantum corrections
$A = \frac{1}{(4\pi)^{D/2}\Gamma(D/2)}$


RG Procedure
k space
2] rescale the field ϕ and the momenta
3] compute the new renormalized action
V_REN(ϕ_REN) = V(ϕ_REN) + ds [D V(ϕ_REN) − (D−2)/2 ϕ_REN V′(ϕ_REN) + A Λ^D Log[1 + Λ^−2 V″(ϕ_REN)]]
rescaling of the classical action
1 loop quantum corrections
A = 1 / ((4π)^(D/2) Γ(D/2))
RG transformation Λ/S < |k| <
S scaling factor
V_REN(ϕ_REN)


RG Procedure
2] rescale the field ϕ and the momenta
thin momentum shell
k space
3] compute the new renormalized action
rescaling of the classical action
1 loop quantum corrections
RG transformation Λ/S < |k| < Λ
V(ϕ) ⟶ V_S(ϕ) renormalized potential
S scaling factor

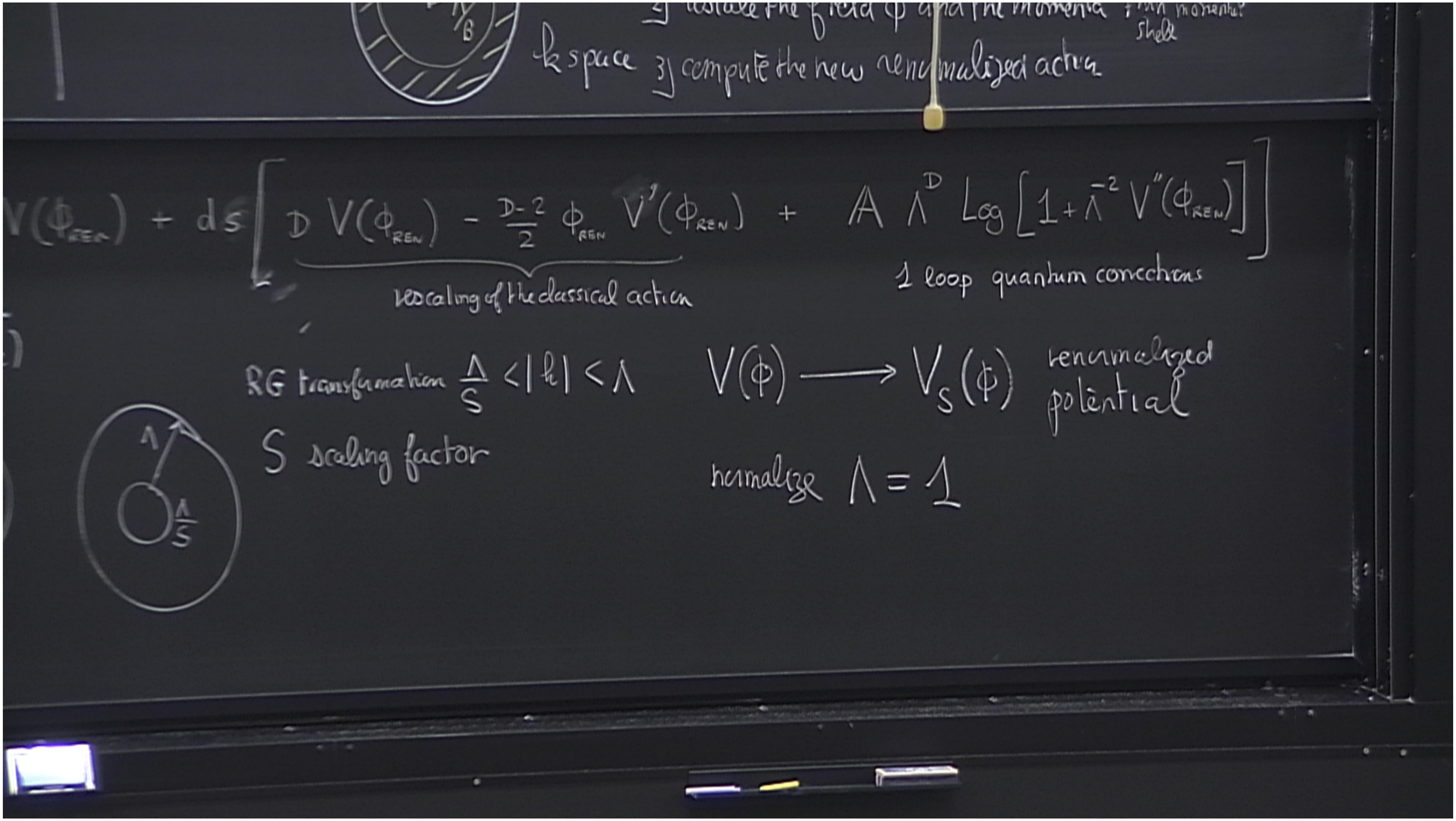

RG Procedure
Λ/B
k space
2] rescale the field ϕ and the momenta
thin momenta shell
factor
3] compute the new renormalized action
$V_{REN}(\phi_{REN}) = V(\phi_{REN}) + ds\left[D\, V(\phi_{REN}) - \frac{D-2}{2}\phi_{REN} V'(\phi_{REN}) + A\, \Lambda^{D} \operatorname{Log}\left[1+\Lambda^{-2} V''(\phi_{REN})\right]\right]$
rescaling of the classical action
1 loop quantum corrections
$A = \frac{1}{(4\pi)^{D/2}\Gamma(D/2)}$
RG transformation $\frac{\Lambda}{S} < |k| < \Lambda$
$V(\phi) \longrightarrow V_S(\phi)$ renormalized potential
S scaling factor
Λ
Λ/S
$V_{REN}(\phi_{REN})$


RG Flow Equation in
the scale of local potential

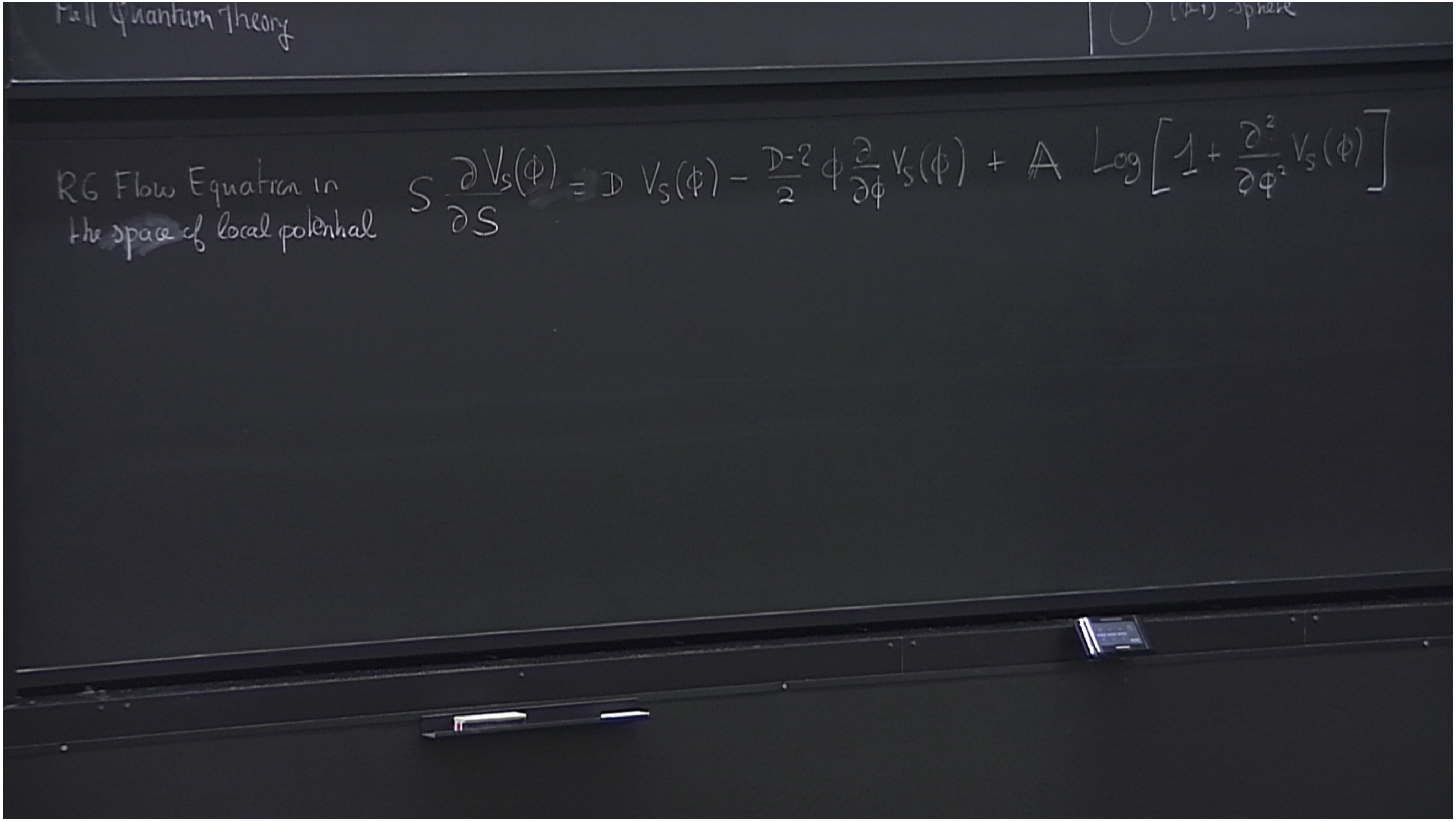
RG Flow Equation in the scale of local potential $S\frac{\partial}{\partial S}$

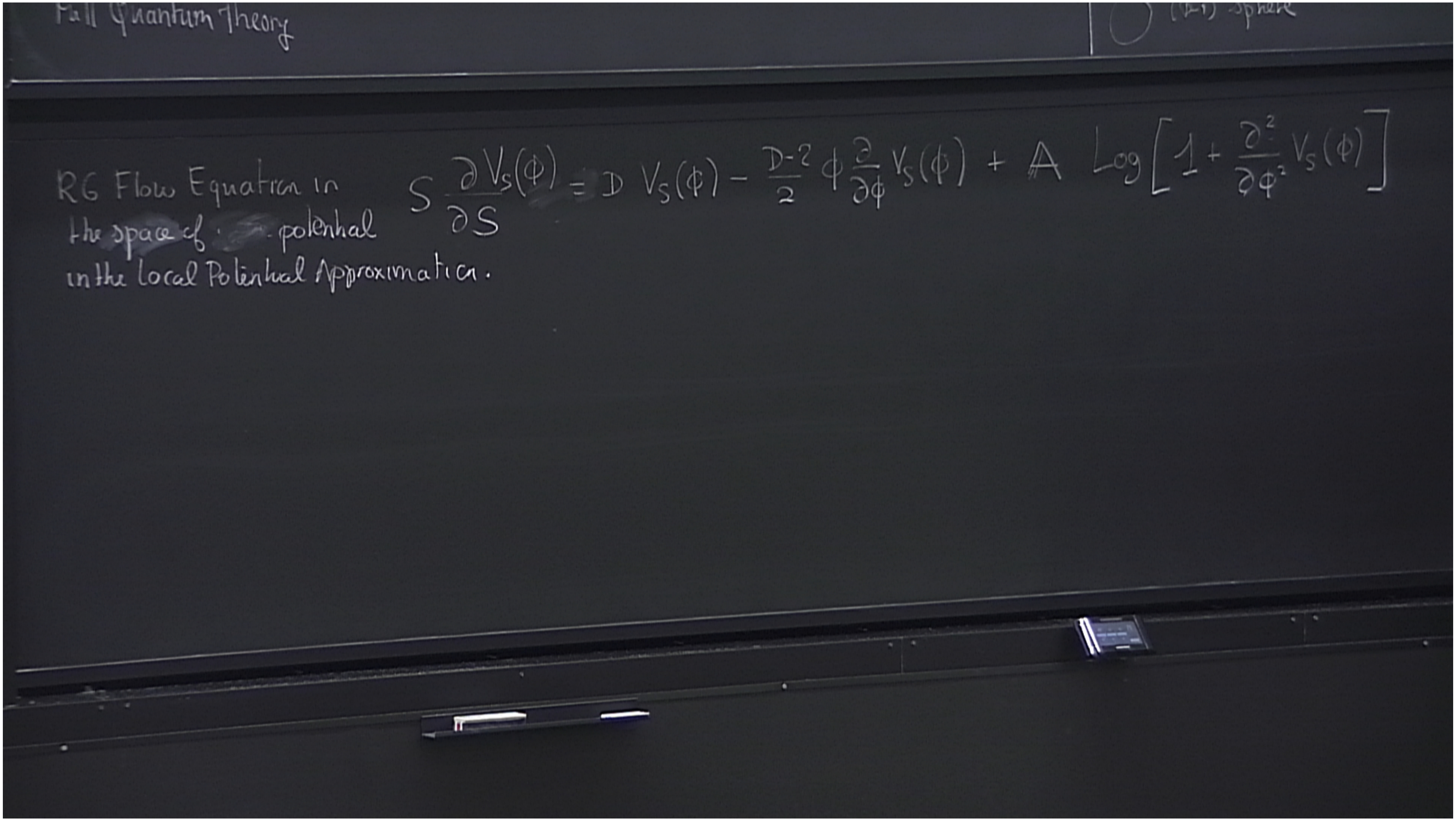
RG Flow Equation in the scale of local potential

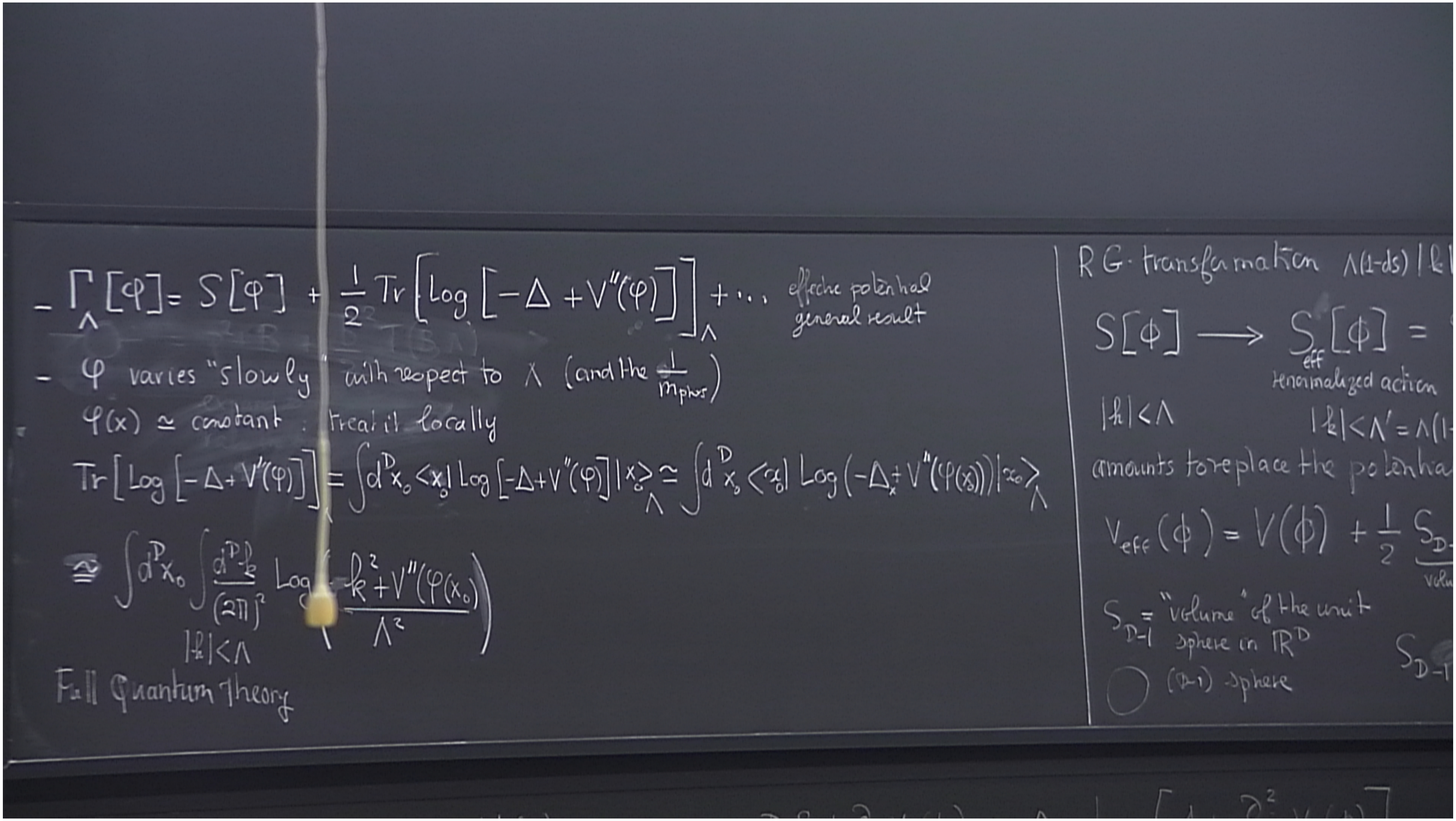
$$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A$$

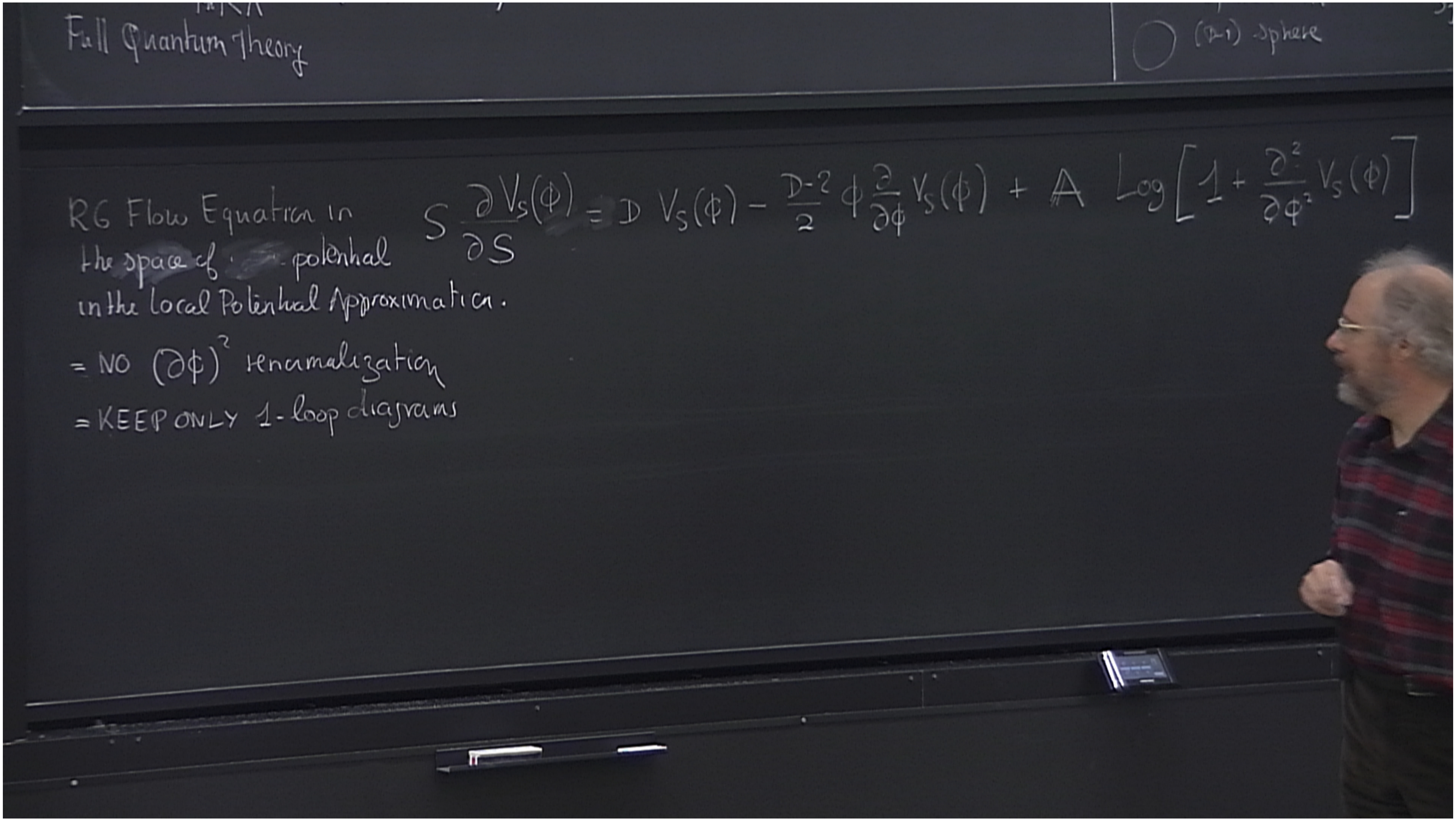


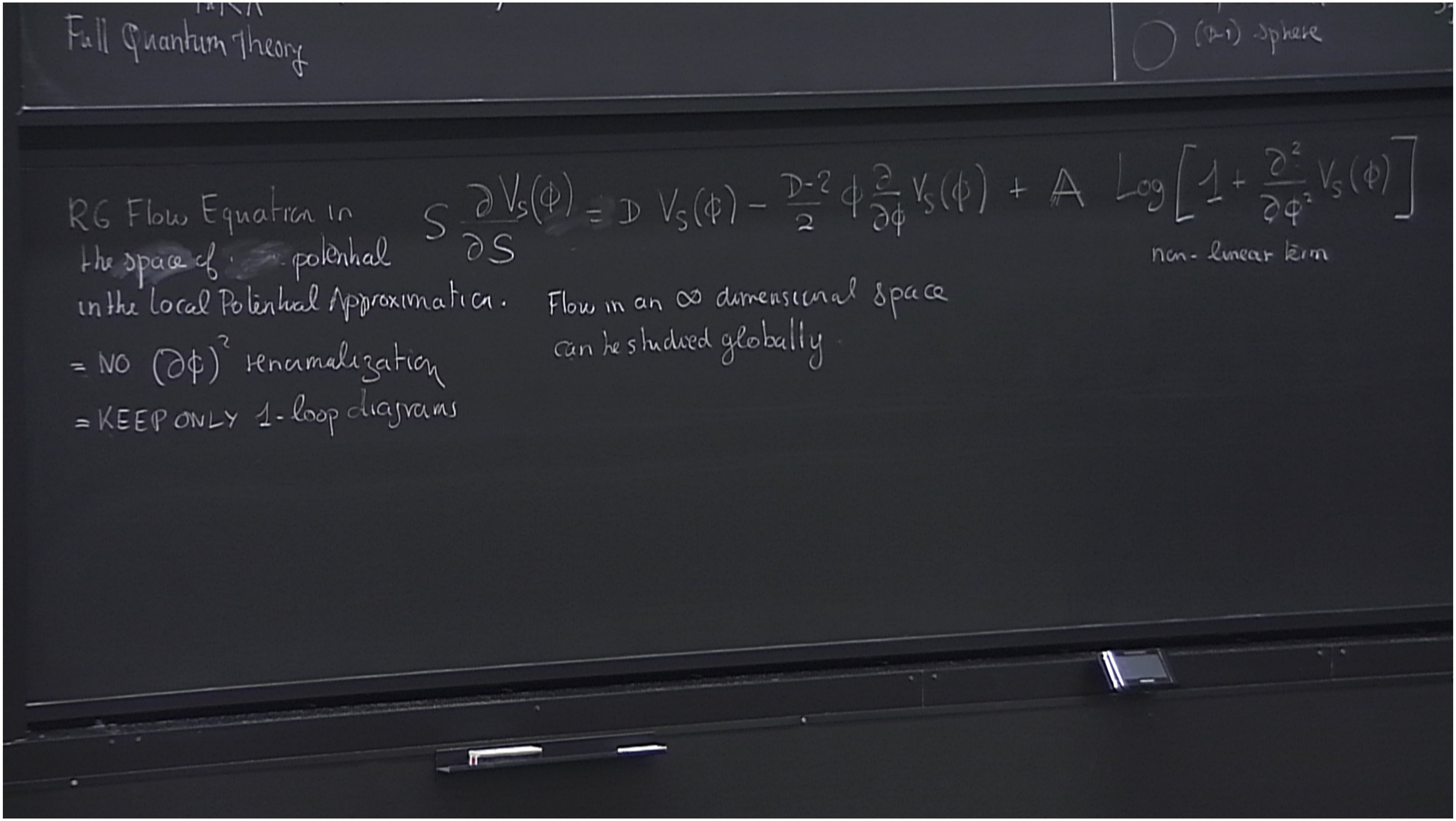
shell
k space 3) compute the new renormalized action
$V(\phi_{REN}) + ds\left[D\, V(\phi_{REN}) - \frac{D-2}{2}\phi_{REN} V'(\phi_{REN}) + A\,\Lambda^{D} \mathrm{Log}\left[1+\Lambda^{-2}V''(\phi_{REN})\right]\right]$
rescaling of the classical action
1 loop quantum corrections
RG transformation $\frac{\Lambda}{S} < |k| < \Lambda$
$V(\phi) \longrightarrow V_S(\phi)$ renormalized potential
S scaling factor
normalize $\Lambda = 1$
Λ
$\frac{\Lambda}{S}$


Full Quantum Theory
(D-1) sphere
RG Flow Equation in the space of local potential
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \operatorname{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$


Full Quantum Theory
RG Flow Equation in
the space of potential
in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \,\mathrm{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$


- Γ_Λ[φ] = S[φ] + 1/2 Tr[Log[−Δ + V''(φ)]]_Λ + ...
effective potential
general result
- φ varies "slowly" with respect to Λ (and the 1/m_phys)
φ(x) ≃ constant : treat it locally
Tr[Log[−Δ + V''(φ)]] = ∫d^D x_0 <x_0| Log[−Δ + V''(φ)] |x_0> ≃ ∫d^D x_0 <x_0| Log(−Δ_x + V''(φ(x_0))) |x_0>_Λ
≅ ∫d^D x_0 ∫ d^D k/(2π)^D Log(k^2 + V''(φ(x_0)))/Λ^2
|k|<Λ
Full Quantum Theory
RG transformation Λ(1−ds) | k |
S[ϕ] ⟶ S_eff[ϕ] =
renormalized action
|k|<Λ
|k|<Λ' = Λ(1−
amounts to replace the potentia
V_eff(ϕ) = V(ϕ) + 1/2 S_D−
S_D−1 = "volume" of the unit sphere in ℝ^D
(D−1) sphere
S_D−1


Full Quantum Theory
(D-1) sphere
RG Flow Equation in
the space of potential
in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \, \mathrm{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams


Full Quantum Theory
(D-1) sphere
RG Flow Equation in the space of potential in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \operatorname{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
non-linear term
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space can be studied globally.

Full Quantum Theory

(D-1) sphere

RG Flow Equation in the space of potential in the Local Potential Approximation.

$$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \,\mathrm{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$$

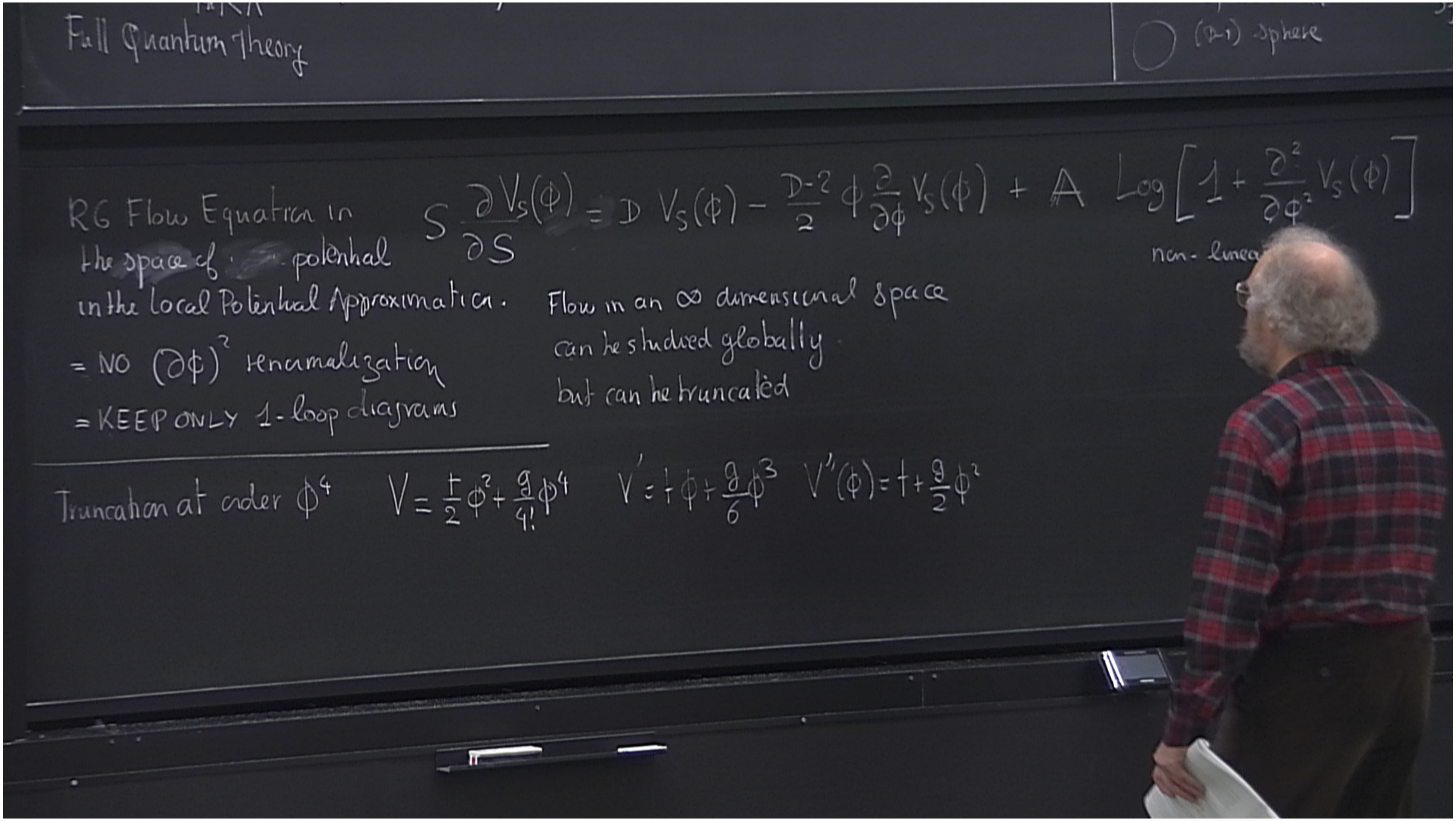
non-linear term

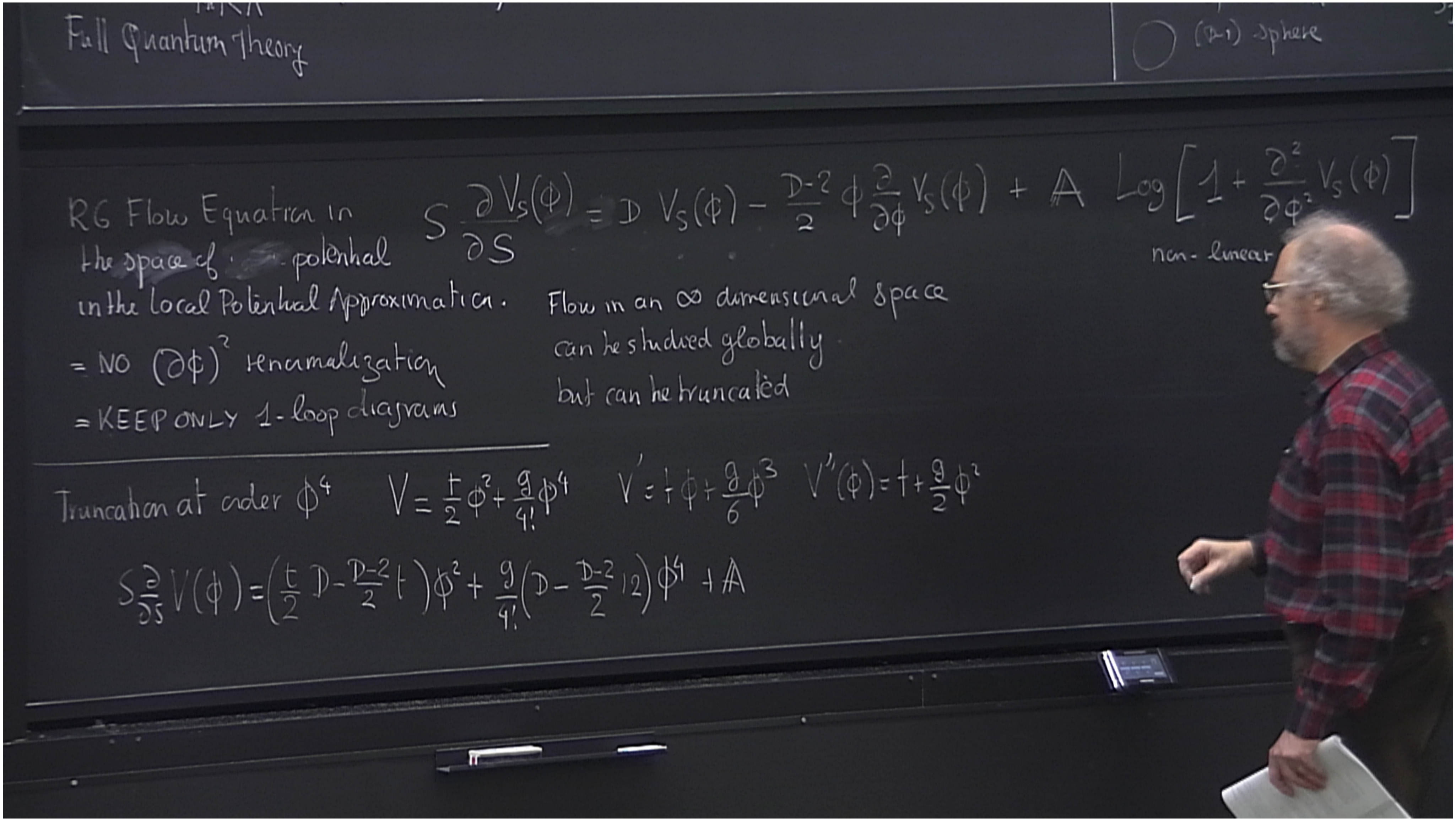
= NO $(\partial\phi)^2$ renormalization

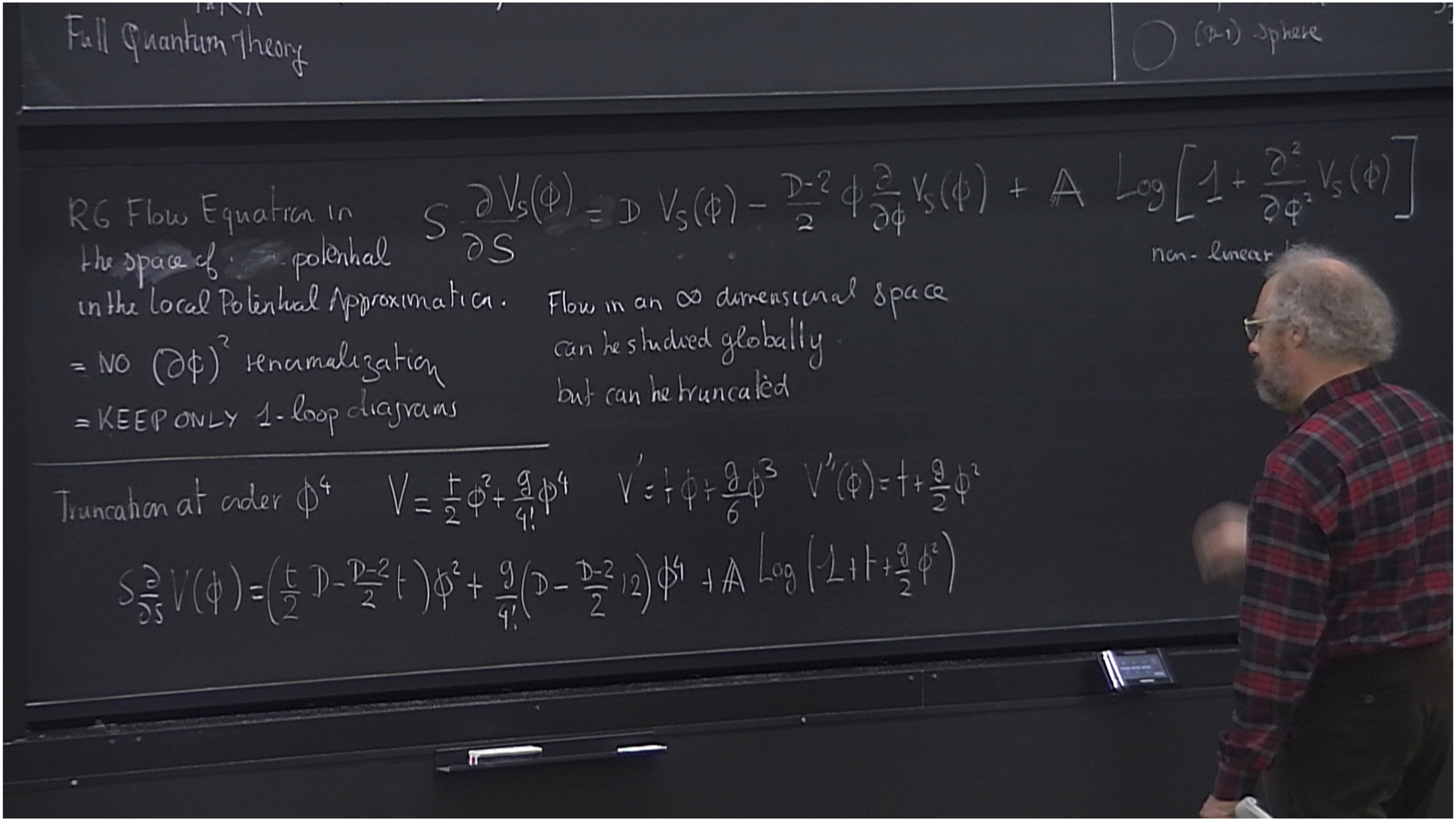
= KEEP ONLY 1-loop diagrams

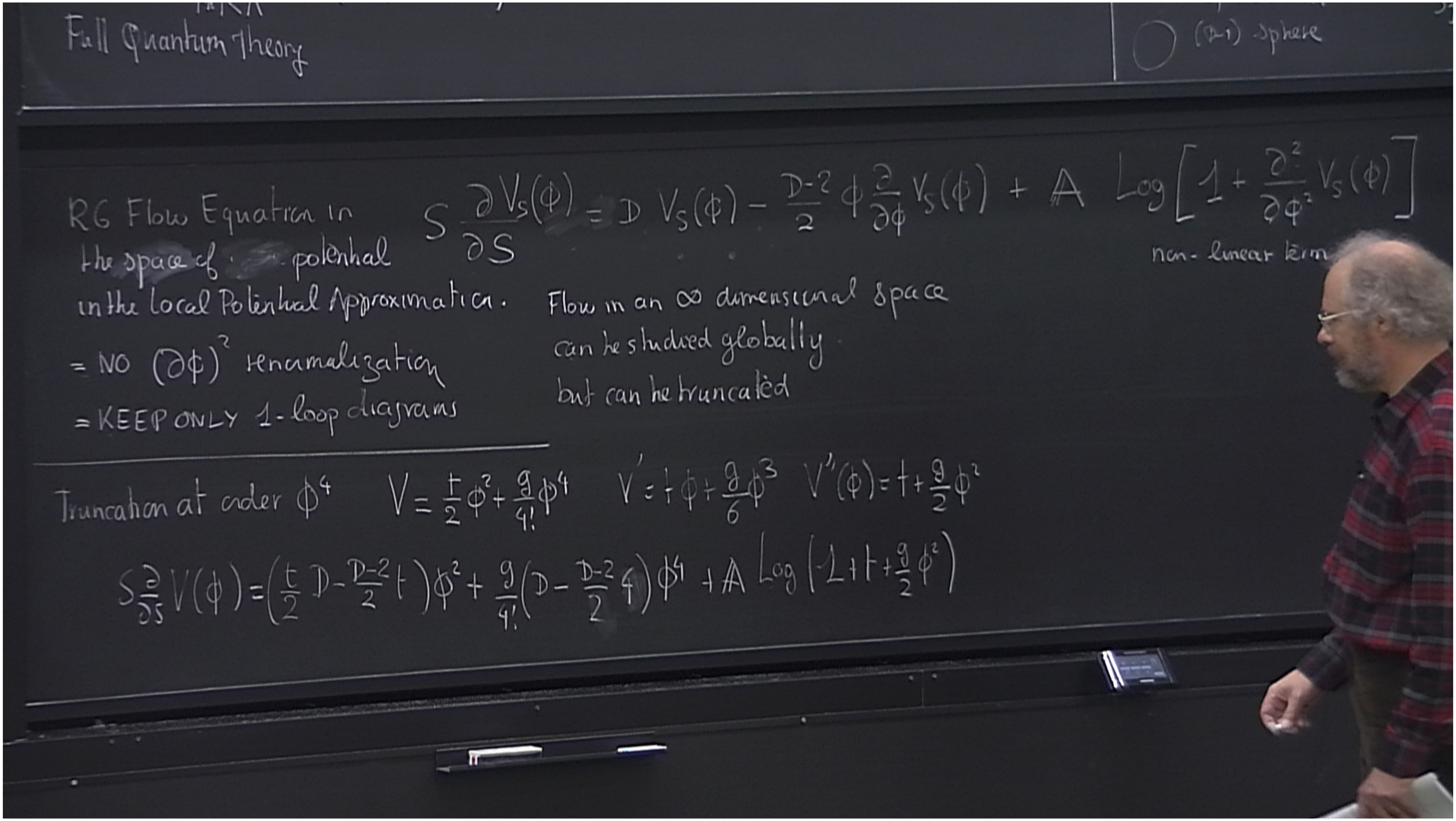
Flow in an ∞ dimensional space can be studied globally but can be truncated

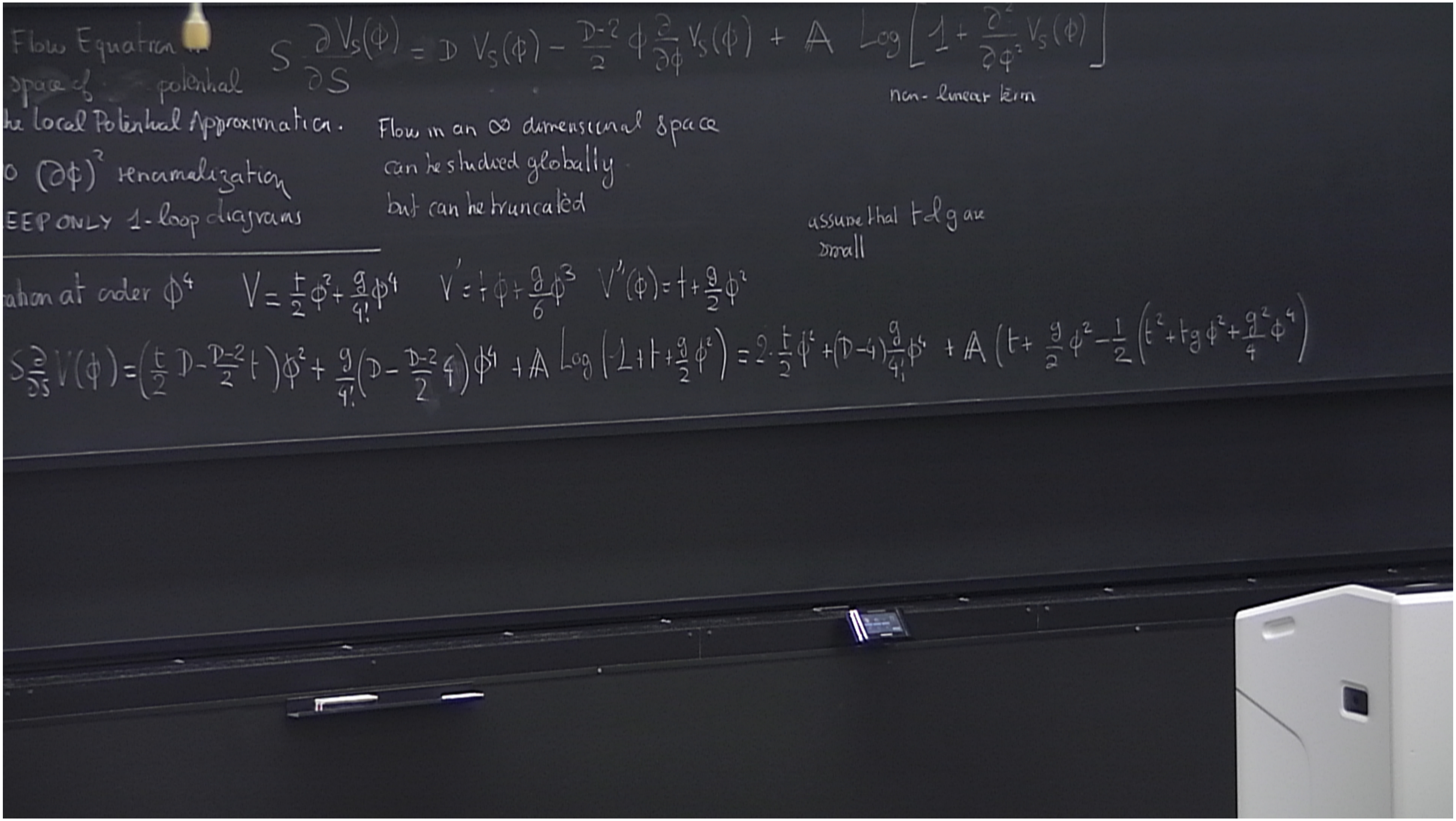
Truncation at order ϕ^4 $\quad V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$

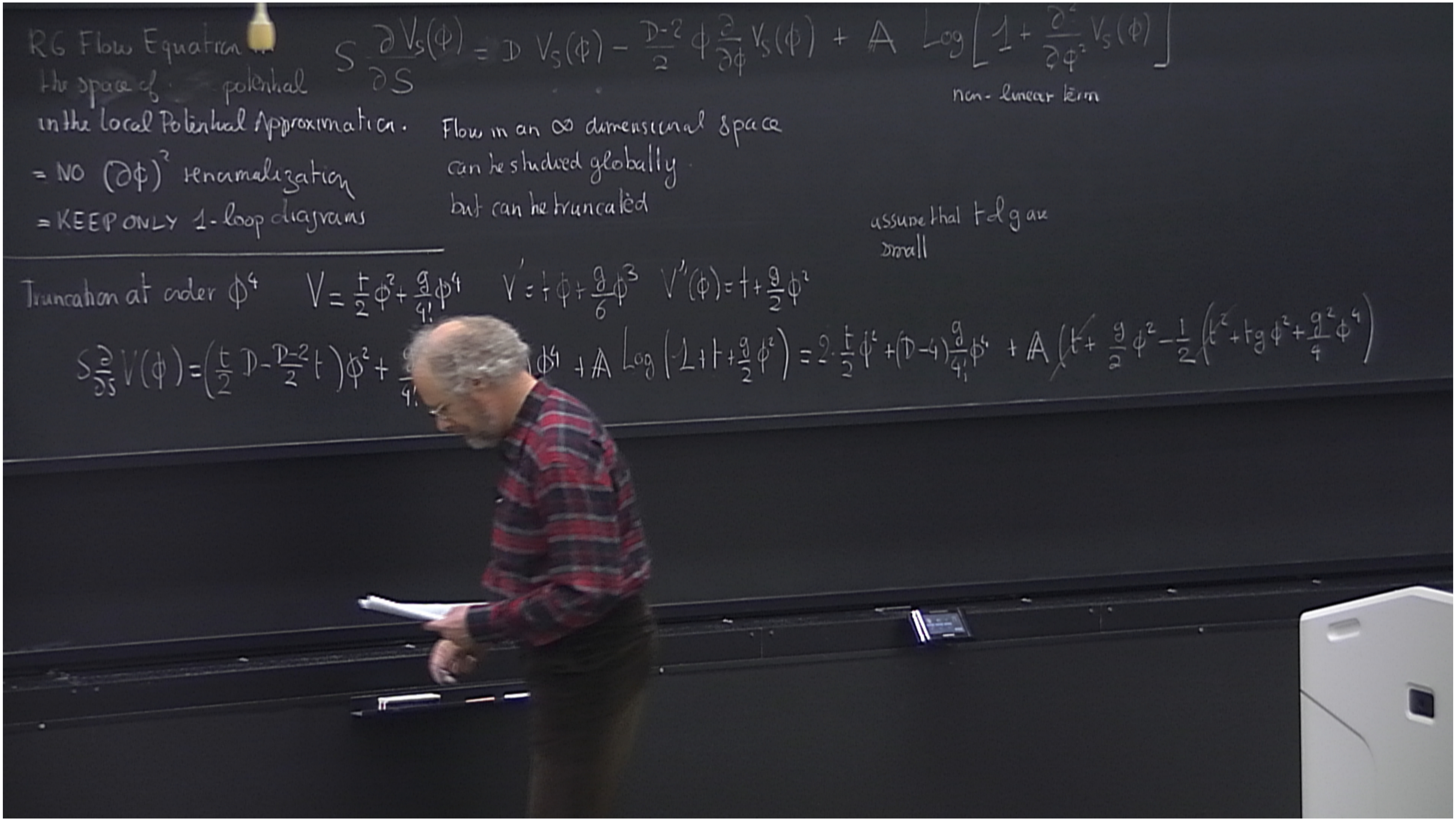

Full Quantum Theory
(D-1) sphere
RG Flow Equation in the space of potential in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \operatorname{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
non-linear
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
Truncation at order ϕ^4
$V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$
$V' = t\phi + \frac{g}{6}\phi^3$
$V''(\phi) = t + \frac{g}{2}\phi^2$


Full Quantum Theory
(D-1) sphere
RG Flow Equation in
the space of potential
in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \, \mathrm{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
non-linear
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
Truncation at order ϕ^4
$V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$
$V' = t\phi + \frac{g}{6}\phi^3$
$V''(\phi) = t + \frac{g}{2}\phi^2$
$S\frac{\partial}{\partial S} V(\phi) = \left(\frac{t}{2} D - \frac{D-2}{2} t\right)\phi^2 + \frac{g}{4!}\left(D - \frac{D-2}{2} 12\right)\phi^4 + A$

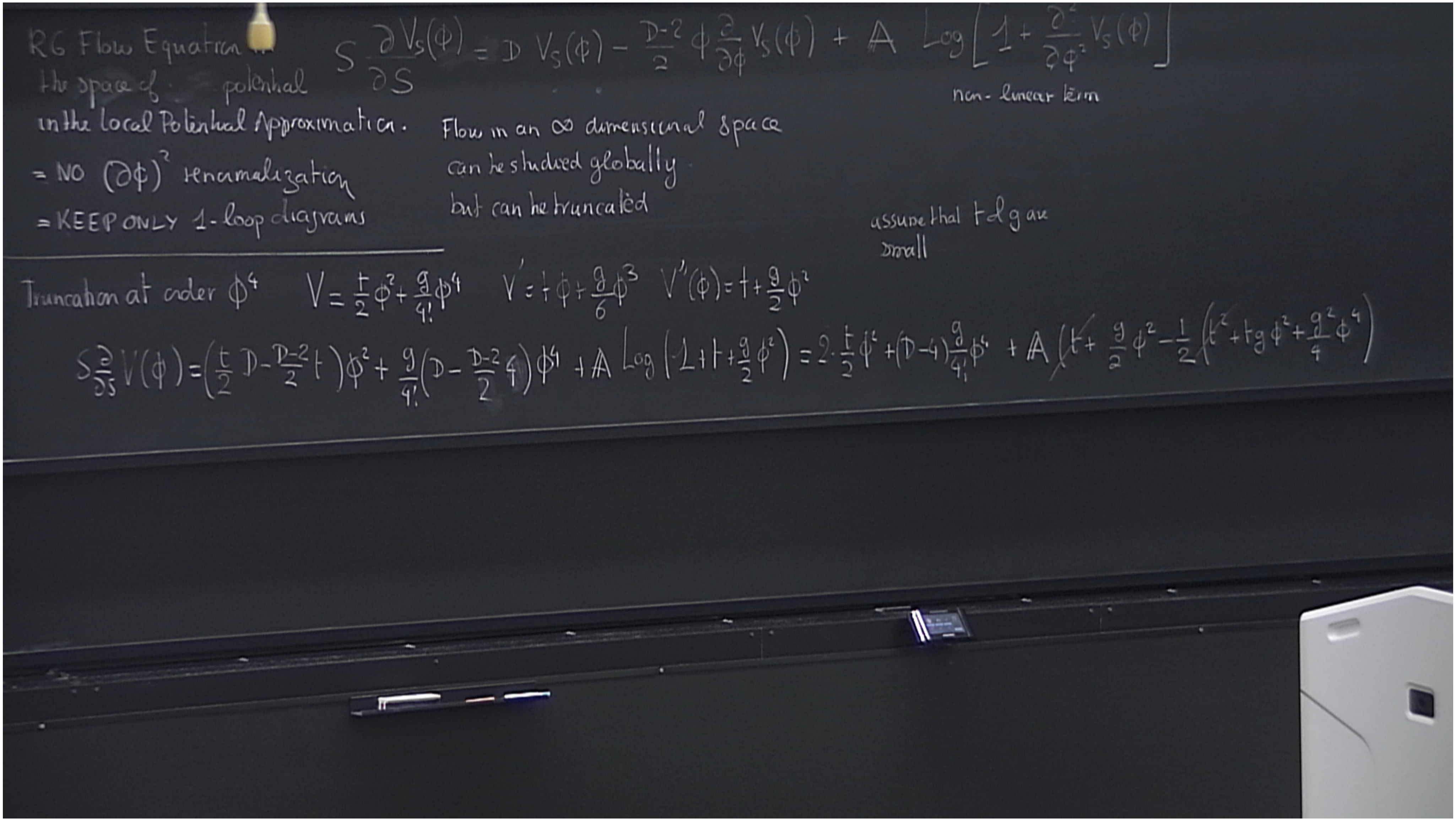

Full Quantum Theory
(D-1) sphere
RG Flow Equation in the space of potential in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \operatorname{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
non-linear
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
Truncation at order ϕ^4
$V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$
$V' = t\phi + \frac{g}{6}\phi^3$
$V''(\phi) = t + \frac{g}{2}\phi^2$
$S\frac{\partial}{\partial S} V(\phi) = \left(\frac{t}{2} D - \frac{D-2}{2} t\right)\phi^2 + \frac{g}{4!}\left(D - \frac{D-2}{2} 12\right)\phi^4 + A \operatorname{Log}\left(1 + t + \frac{g}{2}\phi^2\right)$


Full Quantum Theory
(D-1) -sphere
RG Flow Equation in the space of [illegible] potential in the Local Potential Approximation.
$S \frac{\partial V_S(\phi)}{\partial S} = D\, V_S(\phi) - \frac{D-2}{2} \phi \frac{\partial}{\partial \phi} V_S(\phi) + A \, \mathrm{Log}\left[1 + \frac{\partial^2}{\partial \phi^2} V_S(\phi)\right]$
non-linear term
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
Truncation at order ϕ^4
$V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$
$V' = t\phi + \frac{g}{6}\phi^3$
$V''(\phi) = t + \frac{g}{2}\phi^2$
$S\frac{\partial}{\partial S} V(\phi) = \left(\frac{t}{2} D - \frac{D-2}{2} t\right)\phi^2 + \frac{g}{4!}\left(D - \frac{D-2}{2} 4\right)\phi^4 + A \, \mathrm{Log}\left(1 + t + \frac{g}{2}\phi^2\right)$


Flow Equation
space of potential
$S \frac{\partial V_s(\phi)}{\partial S} = D\, V_s(\phi) - \frac{D-2}{2}\phi \frac{\partial}{\partial \phi} V_s(\phi) + A \, Log\left[1 + \frac{\partial^2}{\partial \phi^2} V_s(\phi)\right]$
non-linear term
he local Potential Approximation.
o $(\partial\phi)^2$ renormalization
EEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
assume that t & g are small
ation at order ϕ^4
$V = \frac{t}{2}\phi^2 + \frac{g}{4!}\phi^4$
$V' = t\phi + \frac{g}{6}\phi^3$
$V''(\phi) = t + \frac{g}{2}\phi^2$
$S\frac{\partial}{\partial S} V(\phi) = \left(\frac{t}{2} D - \frac{D-2}{2} t\right)\phi^2 + \frac{g}{4!}\left(D - \frac{D-2}{2} 4\right)\phi^4 + A \, Log\left(1 + t + \frac{g}{2}\phi^2\right) = 2\cdot\frac{t}{2}\phi^2 + (D-4)\frac{g}{4!}\phi^4 + A\left(t + \frac{g}{2}\phi^2 - \frac{1}{2}\left(t^2 + tg\phi^2 + \frac{g^2}{4}\phi^4\right)\right)$



RG Flow Equation
the space of potential
in the local Potential Approximation.
= NO (∂φ)² renormalization
= KEEP ONLY 1-loop diagrams
S ∂V_S(φ)/∂S = D V_S(φ) − (D−2)/2 φ ∂/∂φ V_S(φ) + A Log[1 + ∂²/∂φ² V_S(φ)]
non-linear term
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
assume that t & g are small
Truncation at order φ⁴
V = t/2 φ² + g/4! φ⁴
V' = tφ + g/6 φ³
V''(φ) = t + g/2 φ²


RG Flow Equation
the space of potential
in the Local Potential Approximation.
= NO $(\partial\phi)^2$ renormalization
= KEEP ONLY 1-loop diagrams
$S\frac{\partial V_S(\phi)}{\partial S} = D\,V_S(\phi) - \frac{D-2}{2}\phi\frac{\partial}{\partial\phi}V_S(\phi) + A\,\mathrm{Log}\left[1+\frac{\partial^2}{\partial\phi^2}V_S(\phi)\right]$
non-linear term
Flow in an ∞ dimensional space
can be studied globally
but can be truncated
assume that t & g are
small
Truncation at order ϕ^4 $V=\frac{t}{2}\phi^2+\frac{g}{4!}\phi^4$ $V'=t\phi+\frac{g}{6}\phi^3$ $V''(\phi)=t+\frac{g}{2}\phi^2$
$S\frac{\partial}{\partial S}V(\phi)=\left(\frac{t}{2}D-\frac{D-2}{2}t\right)\phi^2+\frac{g}{4!}\left(D-\frac{D-2}{2}4\right)\phi^4+A\,\mathrm{Log}\left(1+t+\frac{g}{2}\phi^2\right)=2\cdot\frac{t}{2}\phi^2+(D-4)\frac{g}{4!}\phi^4+A\left(t+\frac{g}{2}\phi^2-\frac{1}{2}\left(t^2+tg\phi^2+\frac{g^2}{4}\phi^4\right)\right)$

$$V_s(\phi) = \frac{t_s}{2}\phi^2 + \frac{g_s}{4!}\phi^4$$

$$s\frac{\partial}{\partial s} t_s = 2\, t_s + A\left(g - t_g\right)$$

$$s\frac{\partial}{\partial s} g_s = (D-4)\, g_s + A\,(-3)\, g_s^2$$

$$V_s(\phi) = \frac{t_s}{2}\phi^2 + \frac{g_s}{4!}\phi^4$$

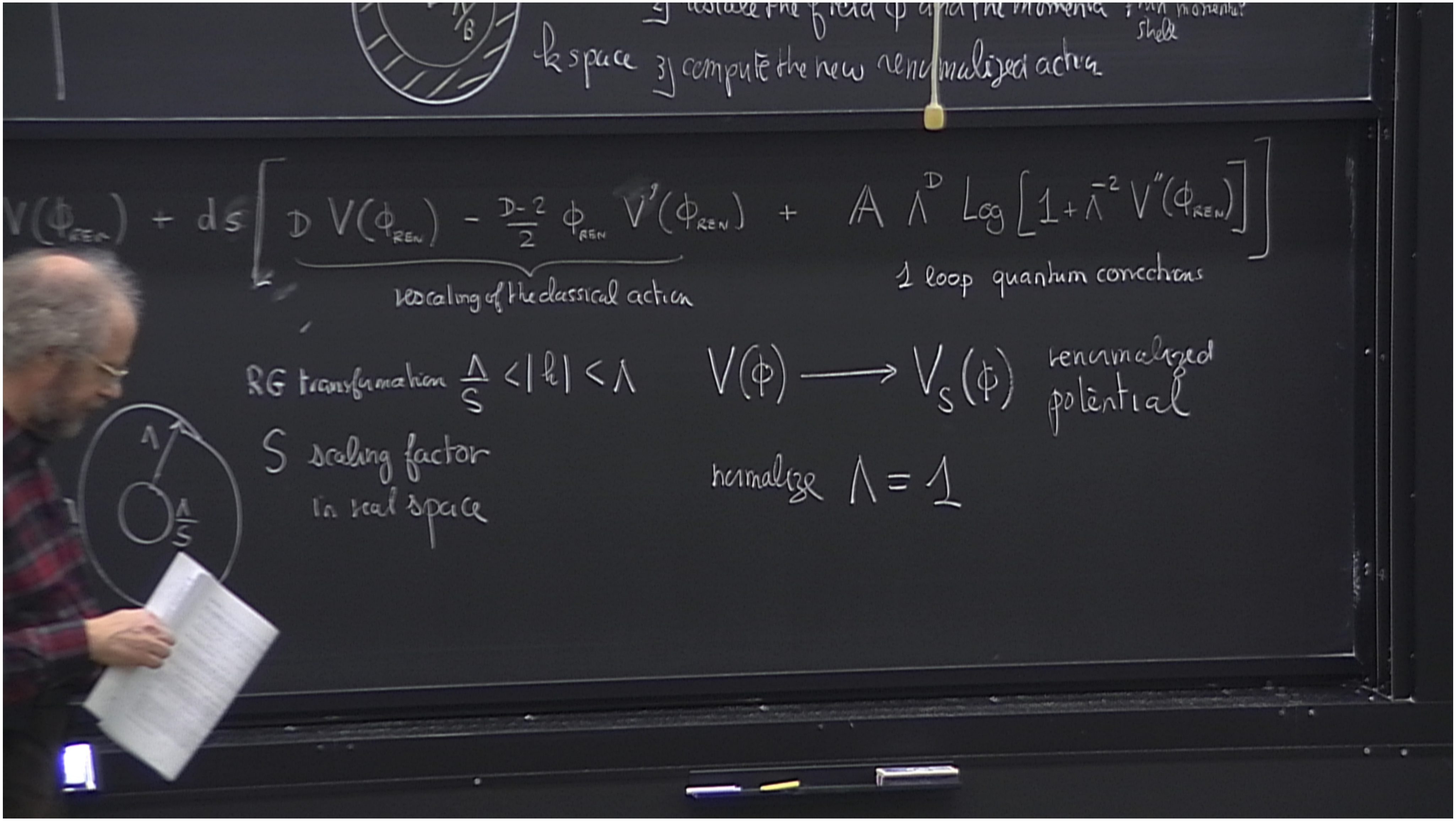
Wilson "Beta" Function: $W_{\text{wilson}} = -\beta_{\text{Callan Symanzik}}$

$$W_{\text{wilson}} = s\frac{\partial}{\partial s} \qquad \beta_{cs} = \mu\frac{\partial}{\partial\mu}$$

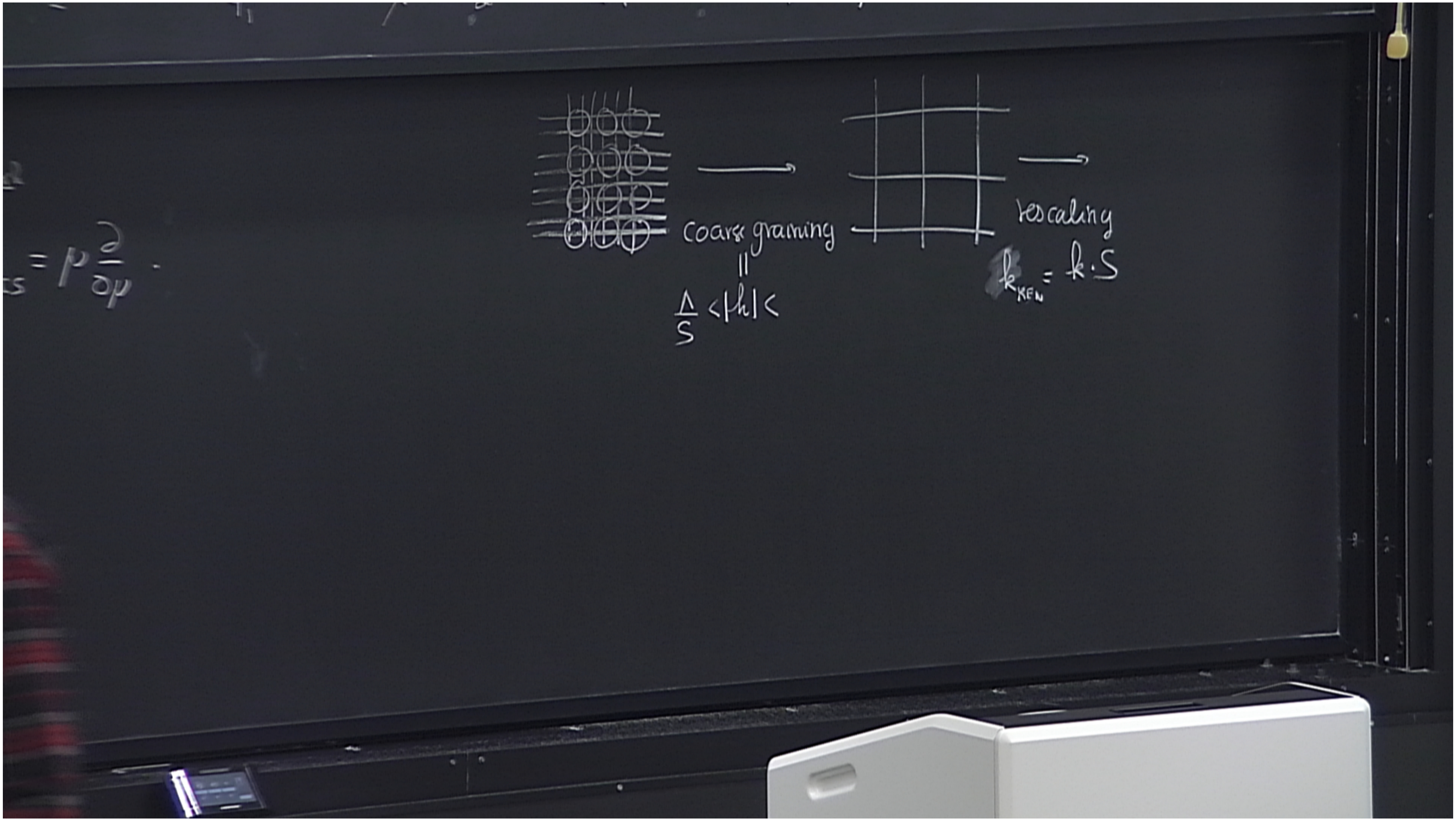
$$s\frac{\partial}{\partial s}t_s = 2\,t_s + A\left(g_s - t_s g_s\right) = W_t(t_s, g_s)$$

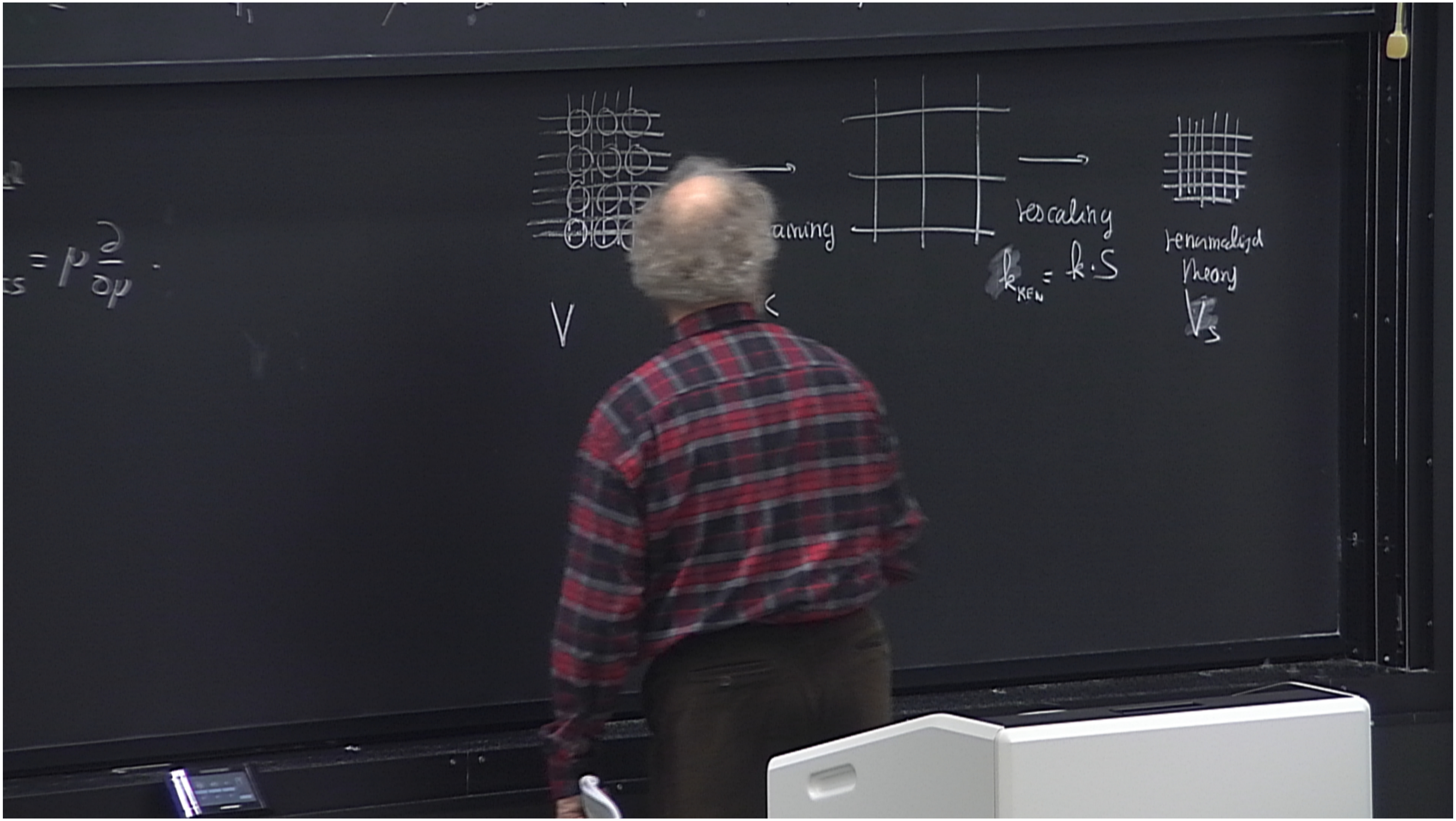
$$s\frac{\partial}{\partial s}g_s = (D-4)g_s + A(-3)g_s^2 = W_g(t_s, g_s)$$

$A = \frac{1}{(4\pi)^2}$ at $D=4$

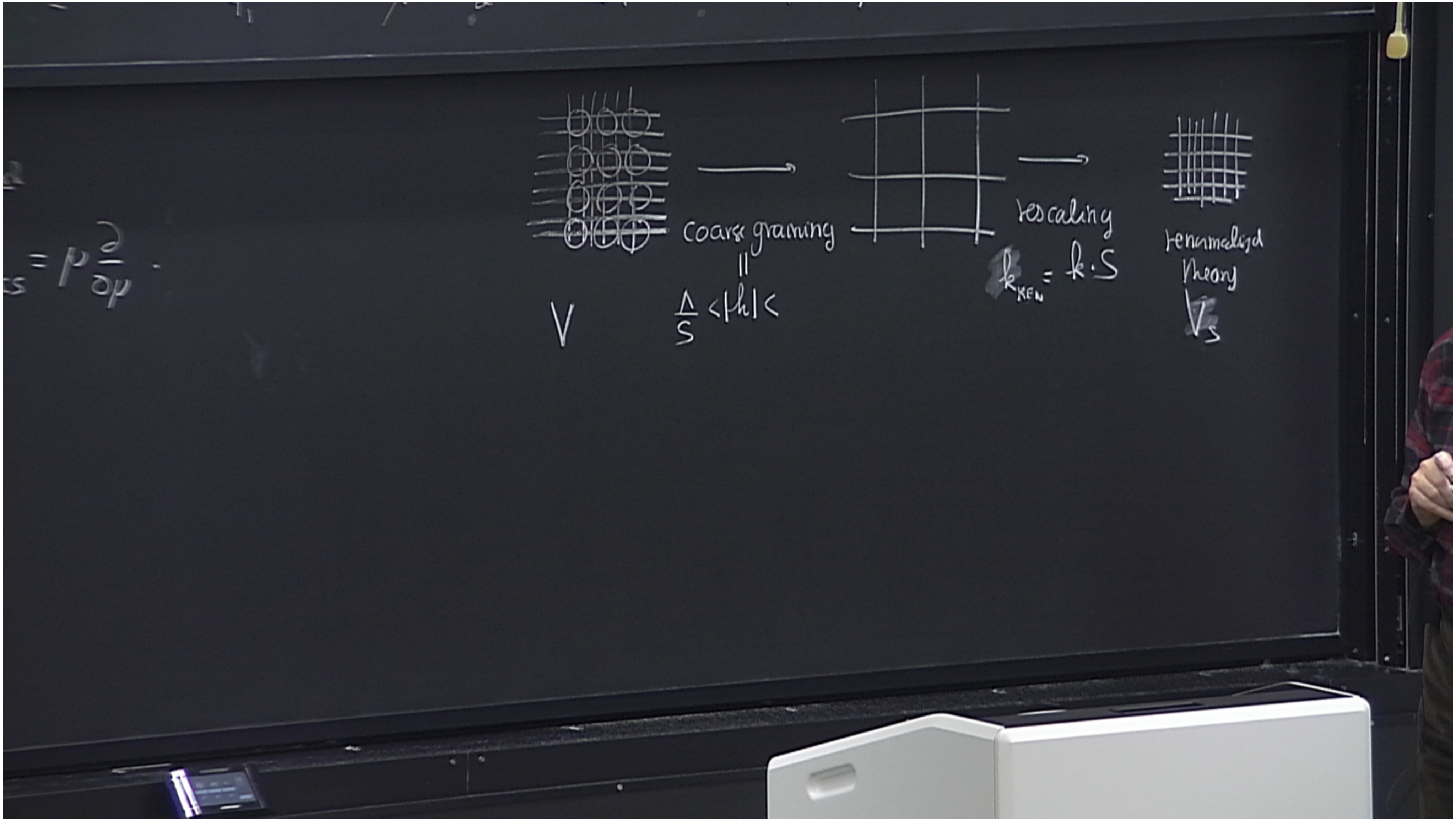


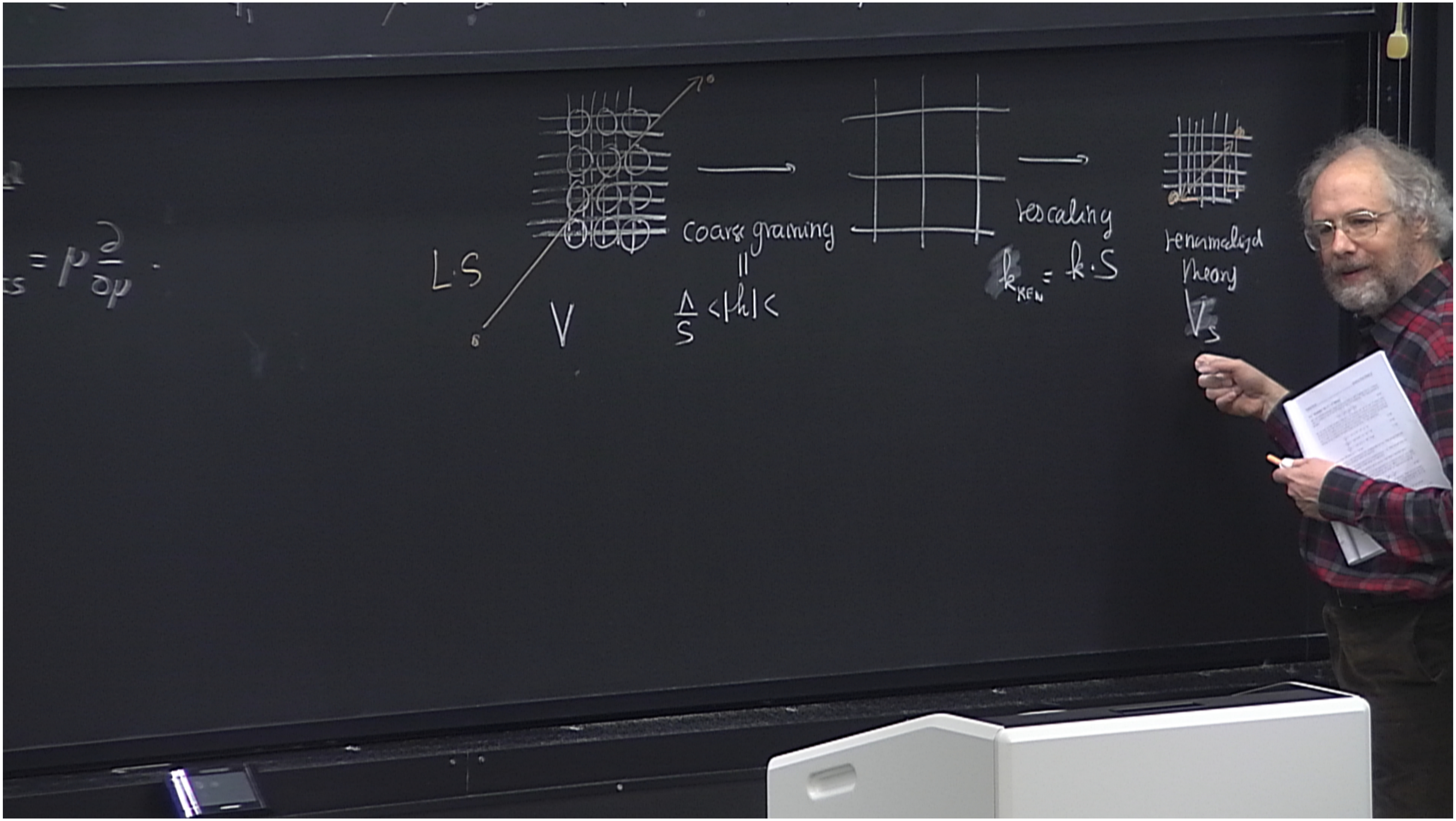
k space
3) compute the new renormalized action
$V(\phi_{REN}) + ds \left[D\, V(\phi_{REN}) - \frac{D-2}{2} \phi_{REN} V'(\phi_{REN}) + A \Lambda^{D} \mathrm{Log}\left[1 + \Lambda^{-2} V''(\phi_{REN})\right]\right]$
rescaling of the classical action
1 loop quantum corrections
RG transformation $\frac{\Lambda}{S} < |k| < \Lambda$
S scaling factor in real space
$V(\phi) \longrightarrow V_S(\phi)$ renormalized potential
normalize $\Lambda = 1$

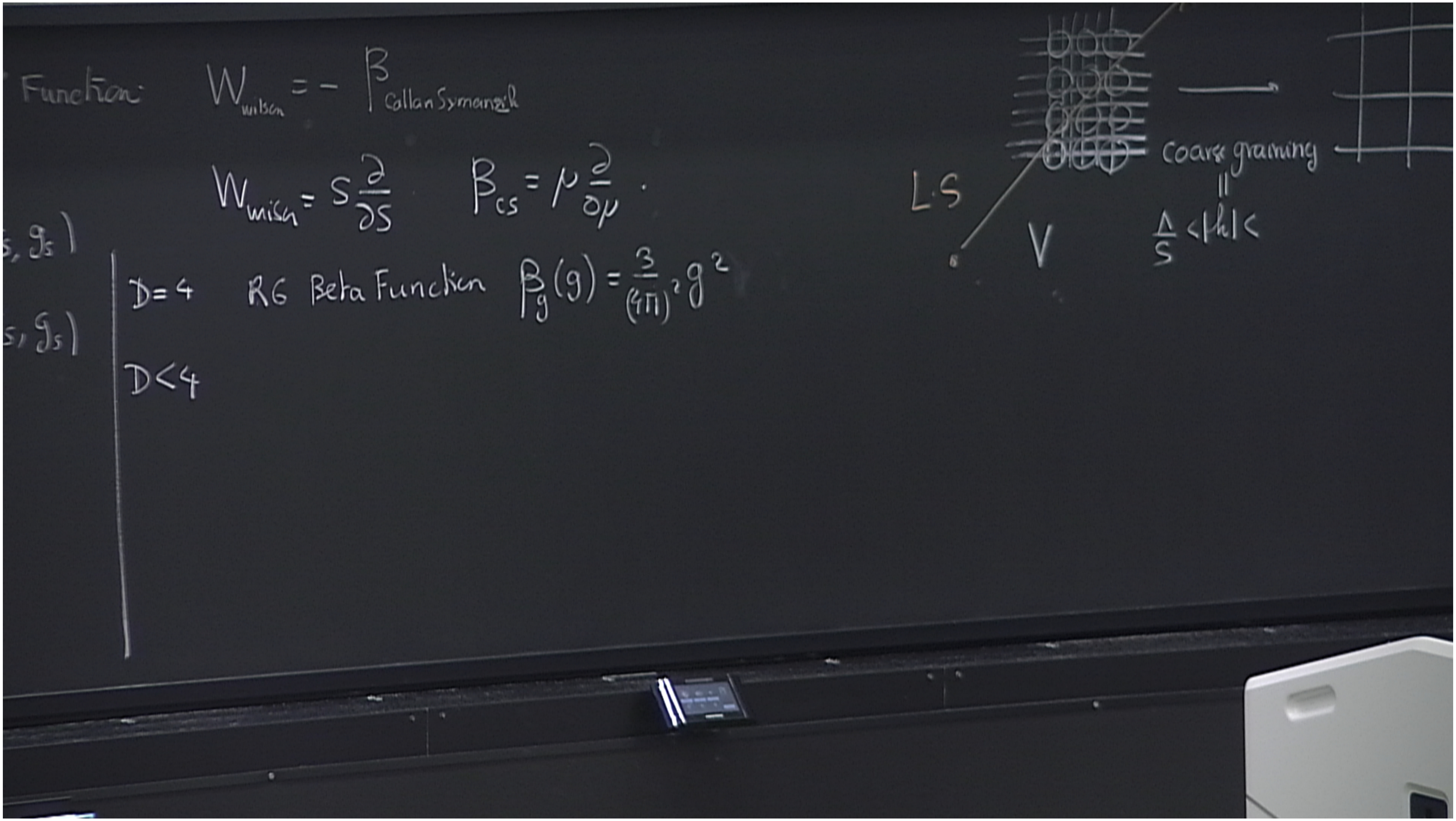

coarse graining
rescaling
$\frac{\Lambda}{S} < |k| <$
$k_{new} = k \cdot S$

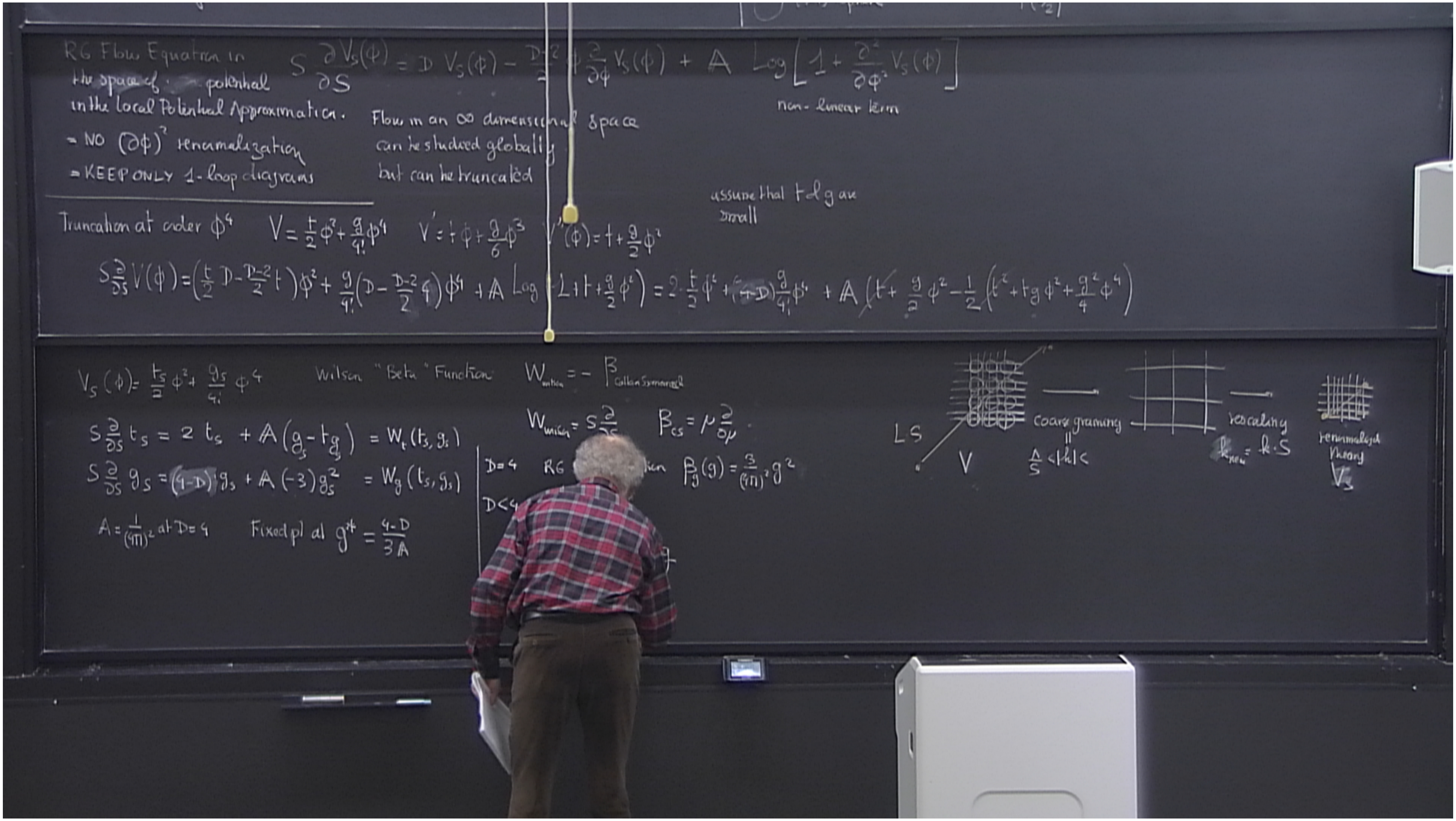


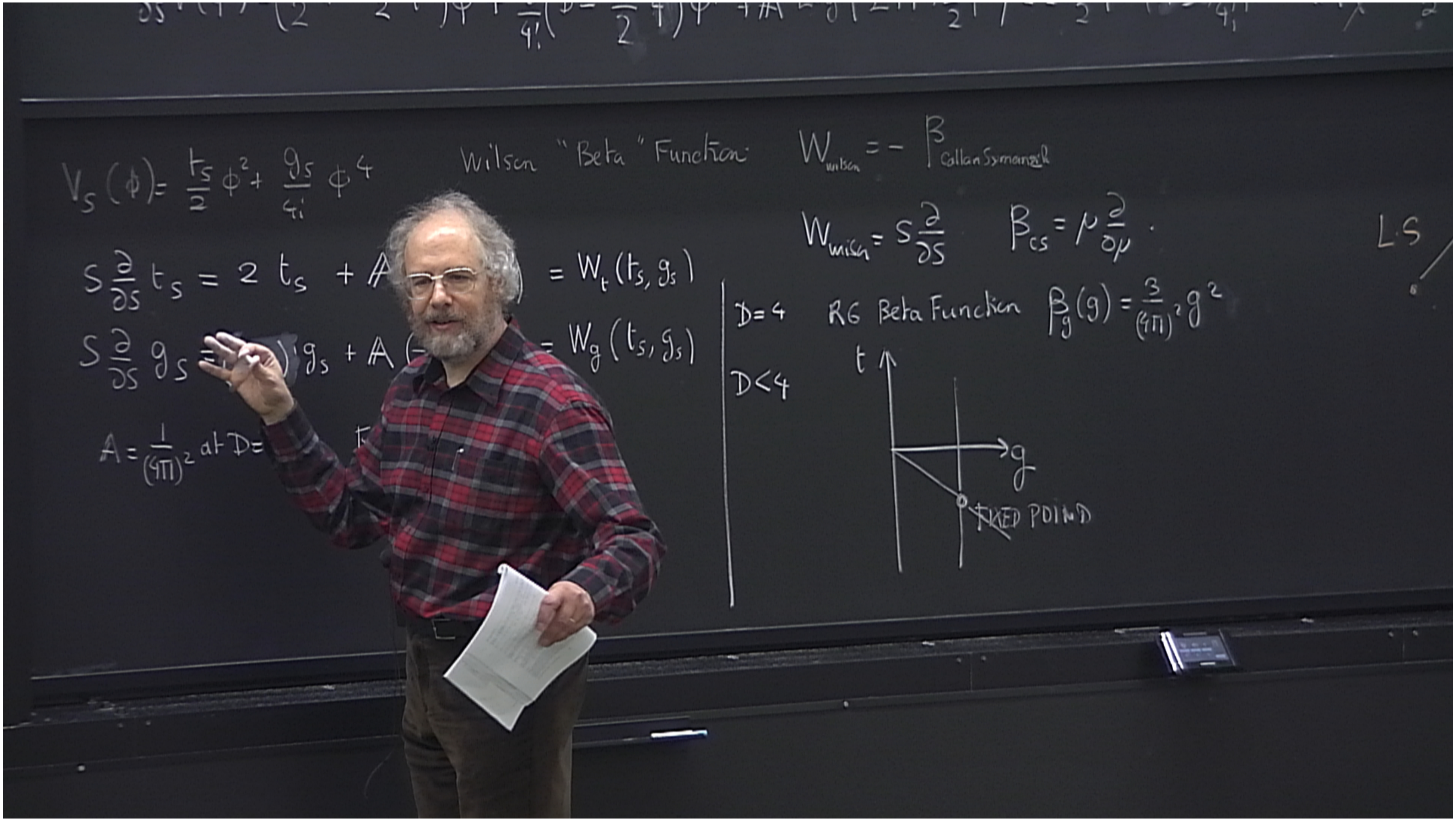
rescaling
renormalized theory


coarse graining
rescaling
renormalized theory
k·S
V
Vs

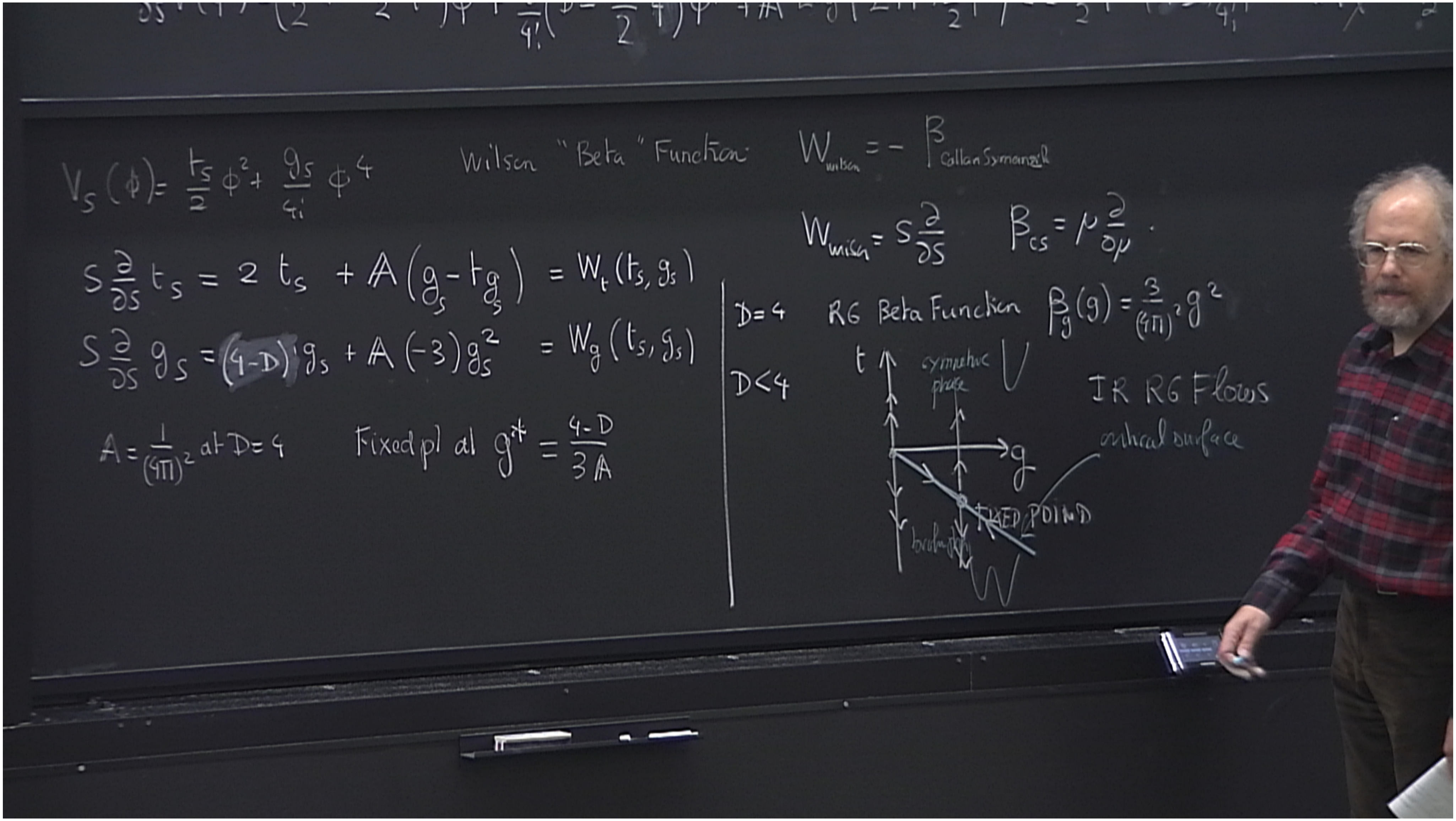

L·S
V
coarse graining
rescaling
renormalized theory
k·S


Function:
W_wilson = − β_Callan Symanzik
W_wilson = s ∂/∂s
β_CS = μ ∂/∂μ
D=4
RG Beta Function β_g(g) = 3/(4π)^2 g^2
D<4
L.S
V
coarse graining
Δ/s < |k| <

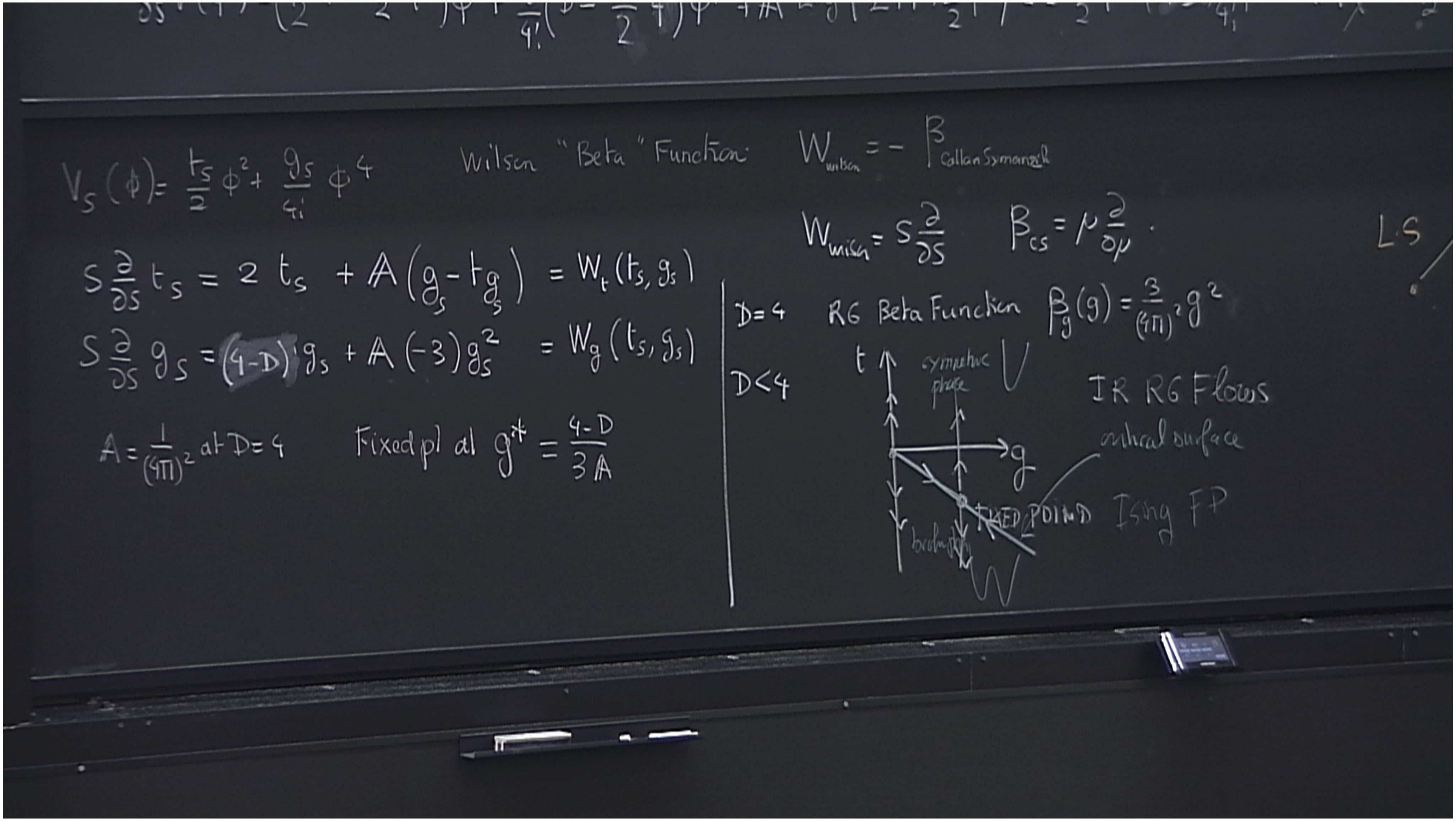

RG Flow Equation in the space of potential in the Local Potential Approximation.
= NO (∂φ)² renormalization
= KEEP ONLY 1-loop diagrams
Flow in an ∞ dimensional space can be studied globally but can be truncated
non-linear term
assume that t & g are small
Truncation at order φ⁴
Wilson "Beta" Function
D=4
D<4
Fixed pt at
coarse graining
rescaling
renormalized theory
LS
V



Wilson "Beta" Function
$W_{wilson} = -\beta_{Callan Symanzik}$
$V_s(\phi) = \frac{t_s}{2}\phi^2 + \frac{g_s}{4!}\phi^4$
$W_{wilson} = s\frac{\partial}{\partial s}$
$\beta_{cs} = \mu\frac{\partial}{\partial\mu}$
$s\frac{\partial}{\partial s}t_s = 2t_s + A$
$= W_t(t_s, g_s)$
$s\frac{\partial}{\partial s}g_s =$
$g_s + A$
$= W_g(t_s, g_s)$
$A = \frac{1}{(4\pi)^2}$ at D=
D=4
RG Beta Function $\beta_g(g) = \frac{3}{(4\pi)^2}g^2$
D<4
t
g
FIXED POINT
L.S



Vs(φ) = ts/2 φ² + gs/4! φ⁴
Wilson "Beta" Function
W wilson = − β Callan Symanzik
W wilson = s ∂/∂s
β cs = μ ∂/∂μ
s ∂/∂s ts = 2 ts + A(gs − tgs) = Wt(ts, gs)
s ∂/∂s gs = (4−D) gs + A(−3) gs² = Wg(ts, gs)
A = 1/(4π)² at D=4
Fixed pt at g* = (4−D)/3A
D=4
RG Beta Function βg(g) = 3/(4π)² g²
D<4
IR RG Flows
critical surface
FIXED POINT
t
g


Wilson "Beta" Function
W_wilson = − β_Callan Symanzik
V_s(φ) = t_s/2 φ² + g_s/4! φ⁴
W_wilson = s ∂/∂s
β_cs = μ ∂/∂μ
L.S
s ∂/∂s t_s = 2 t_s + A(g_s − t g_s) = W_t(t_s, g_s)
s ∂/∂s g_s = (4−D) g_s + A(−3) g_s² = W_g(t_s, g_s)
D=4
RG Beta Function β_g(g) = 3/(4π)² g²
D<4
A = 1/(4π)² at D=4
Fixed pt at g* = (4−D)/3A
IR RG Flows
critical surface
FIXED POINT
Ising FP
t
g