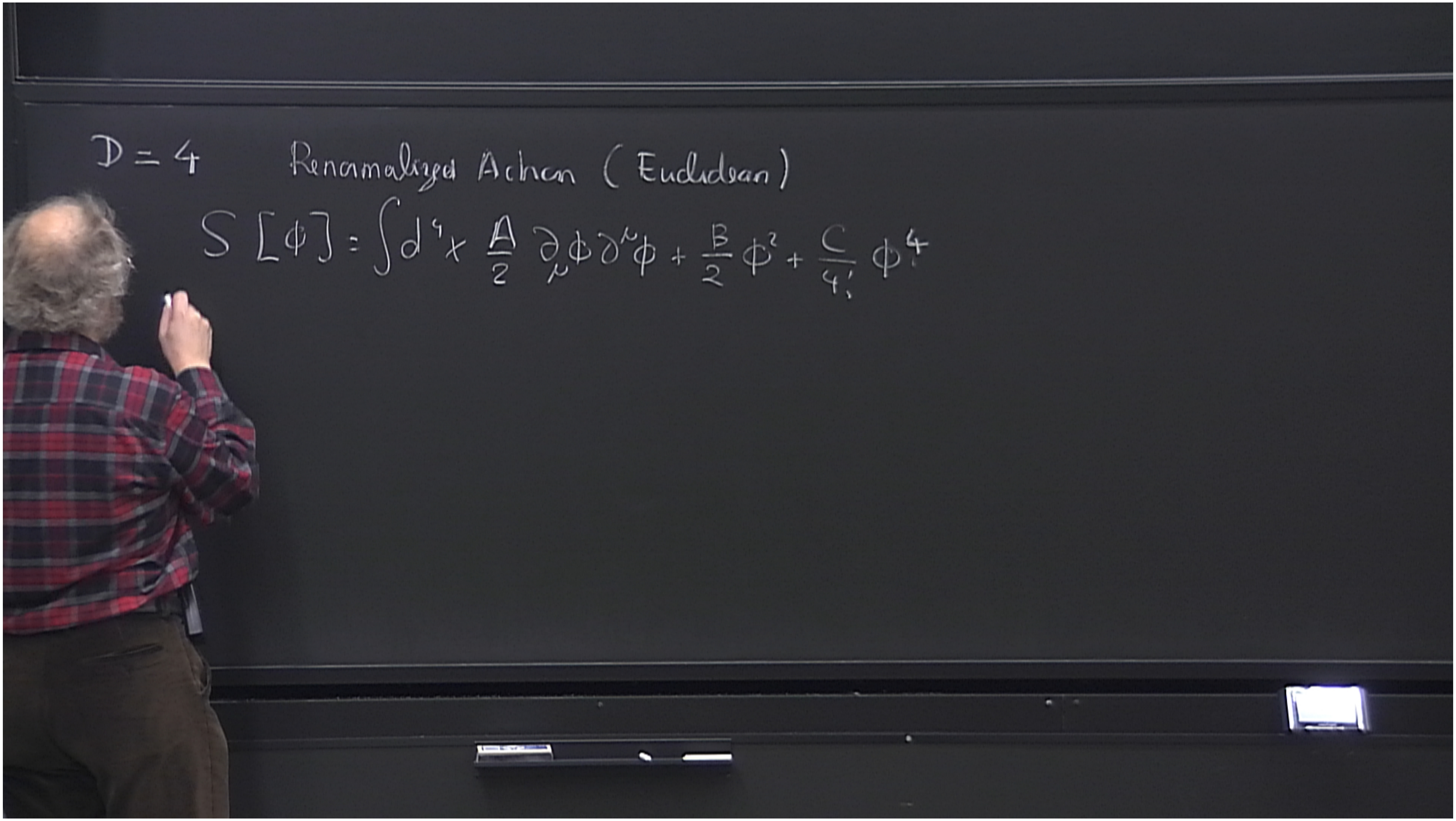
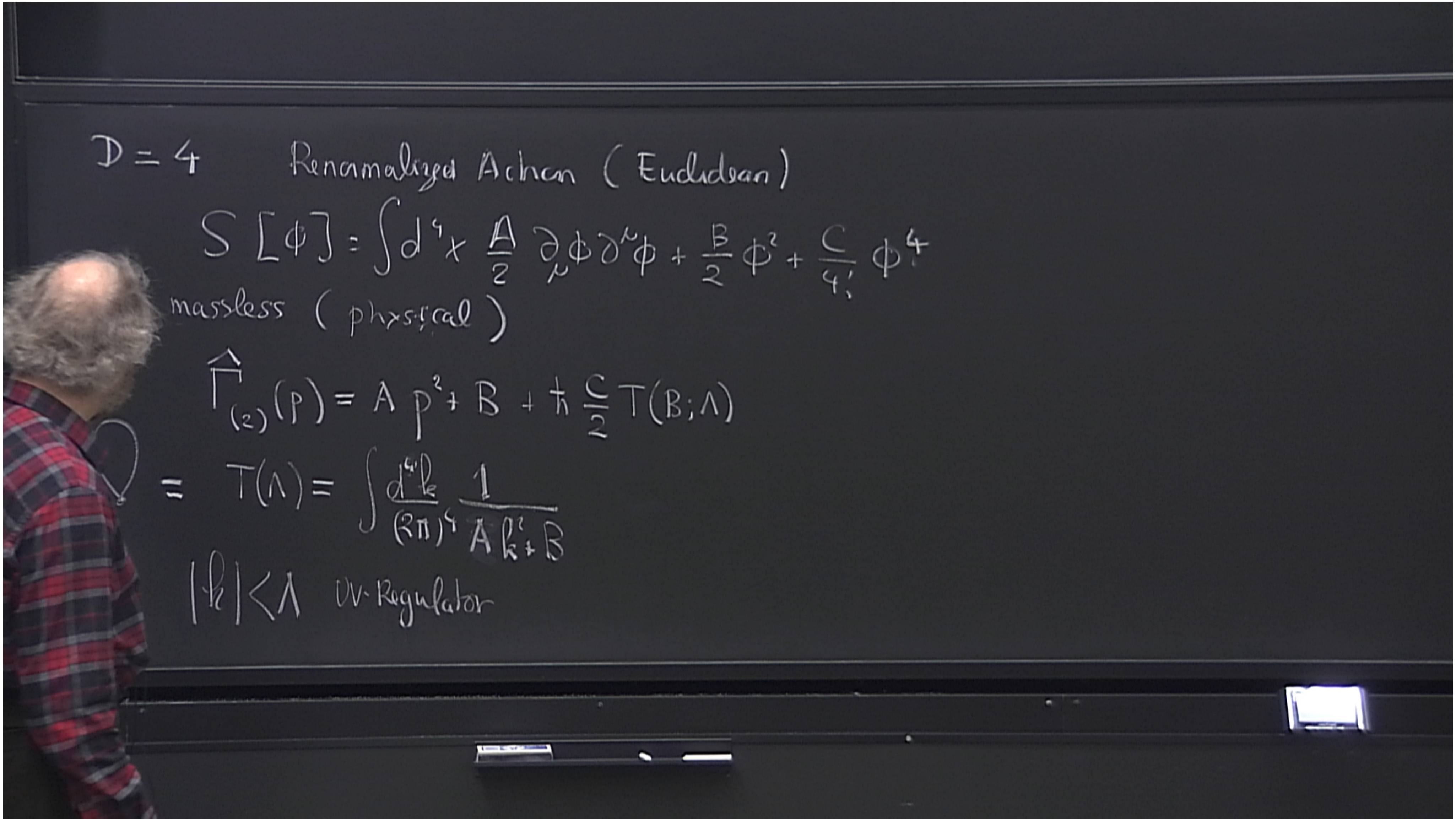
Title: 14/15 PSI - Quantum Field Theory II-Lecture 7

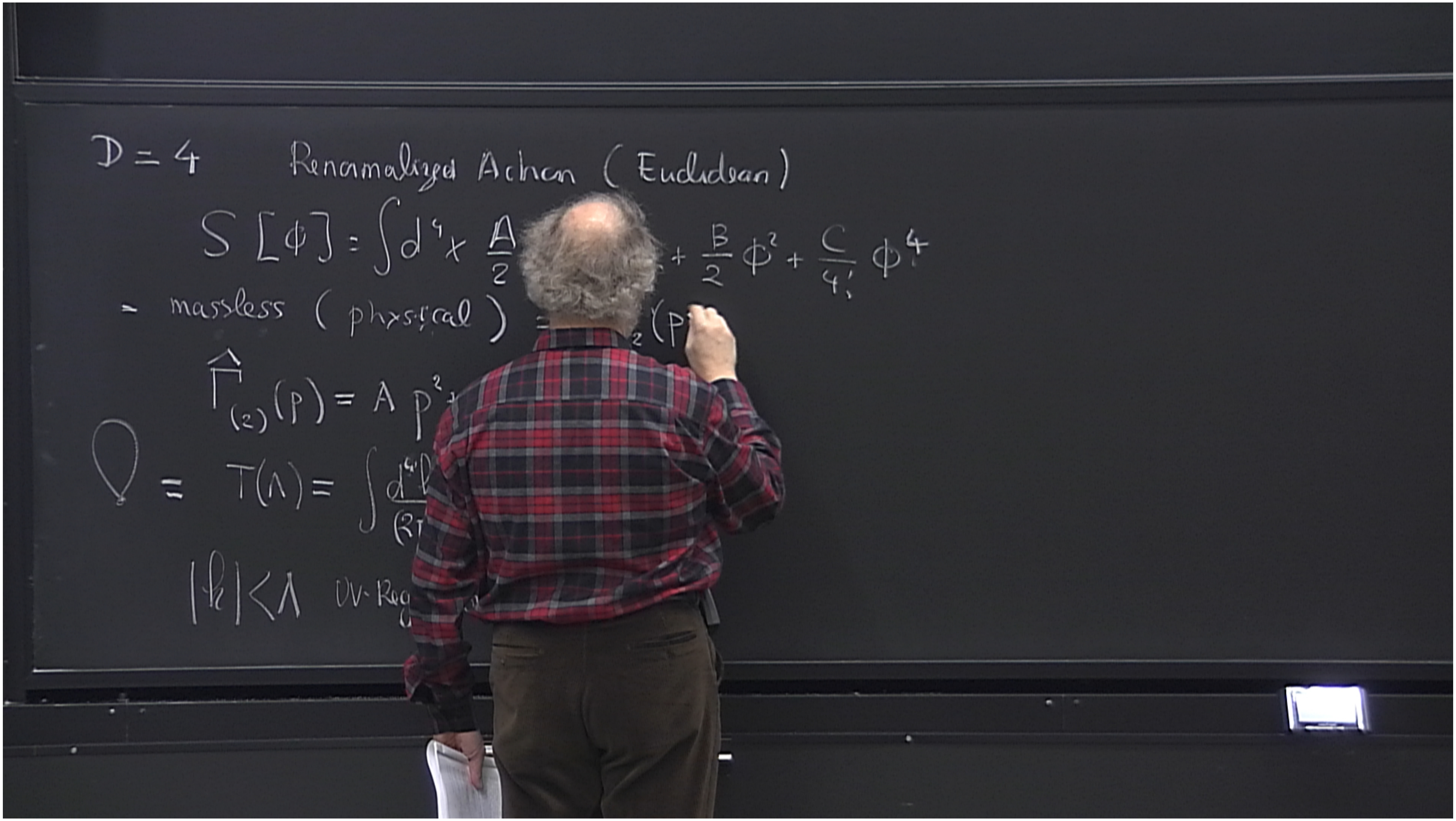
Date: Nov 18, 2014 09:00 AM

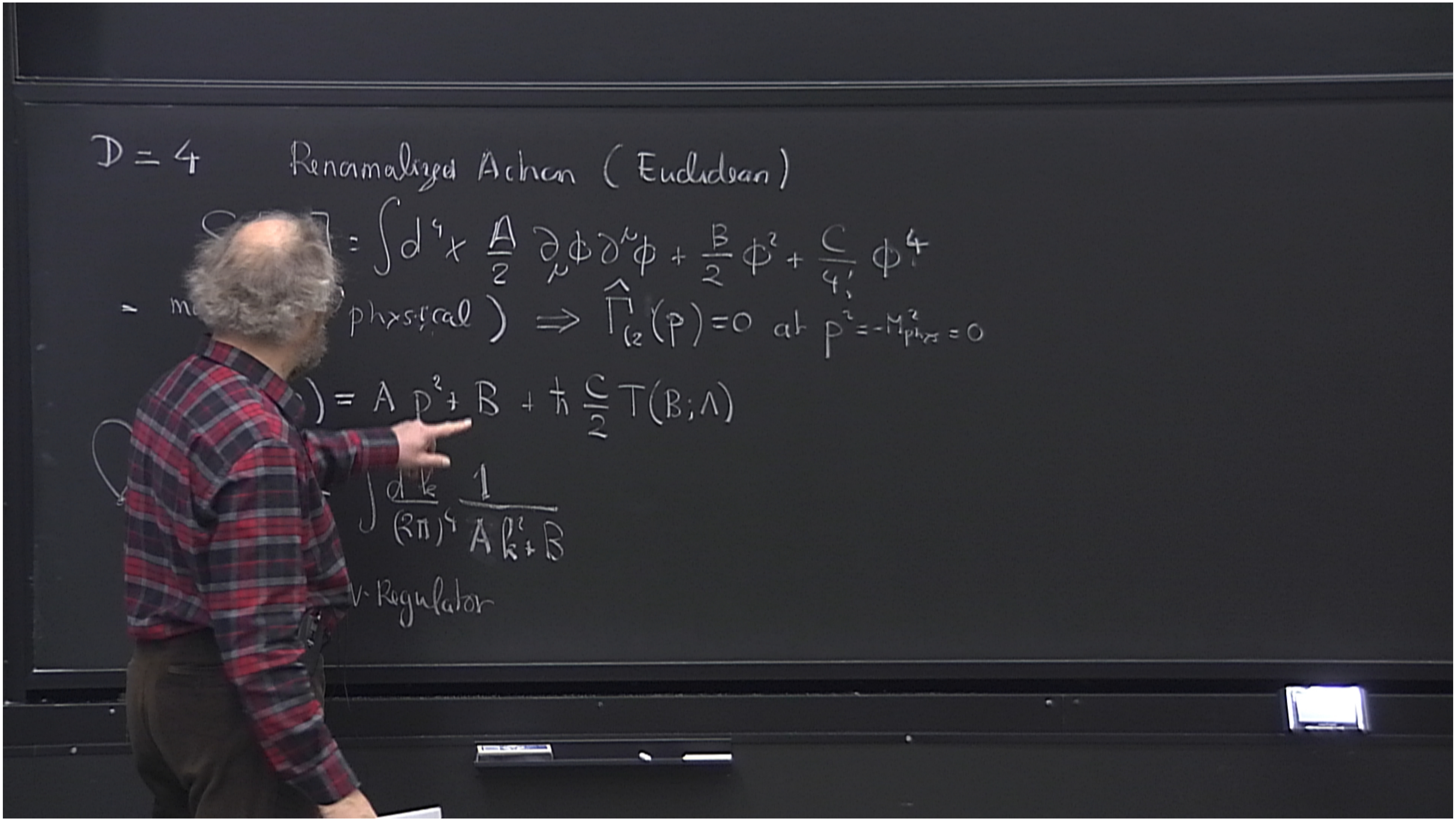
URL: http://pirsa.org/14110014

Abstract:

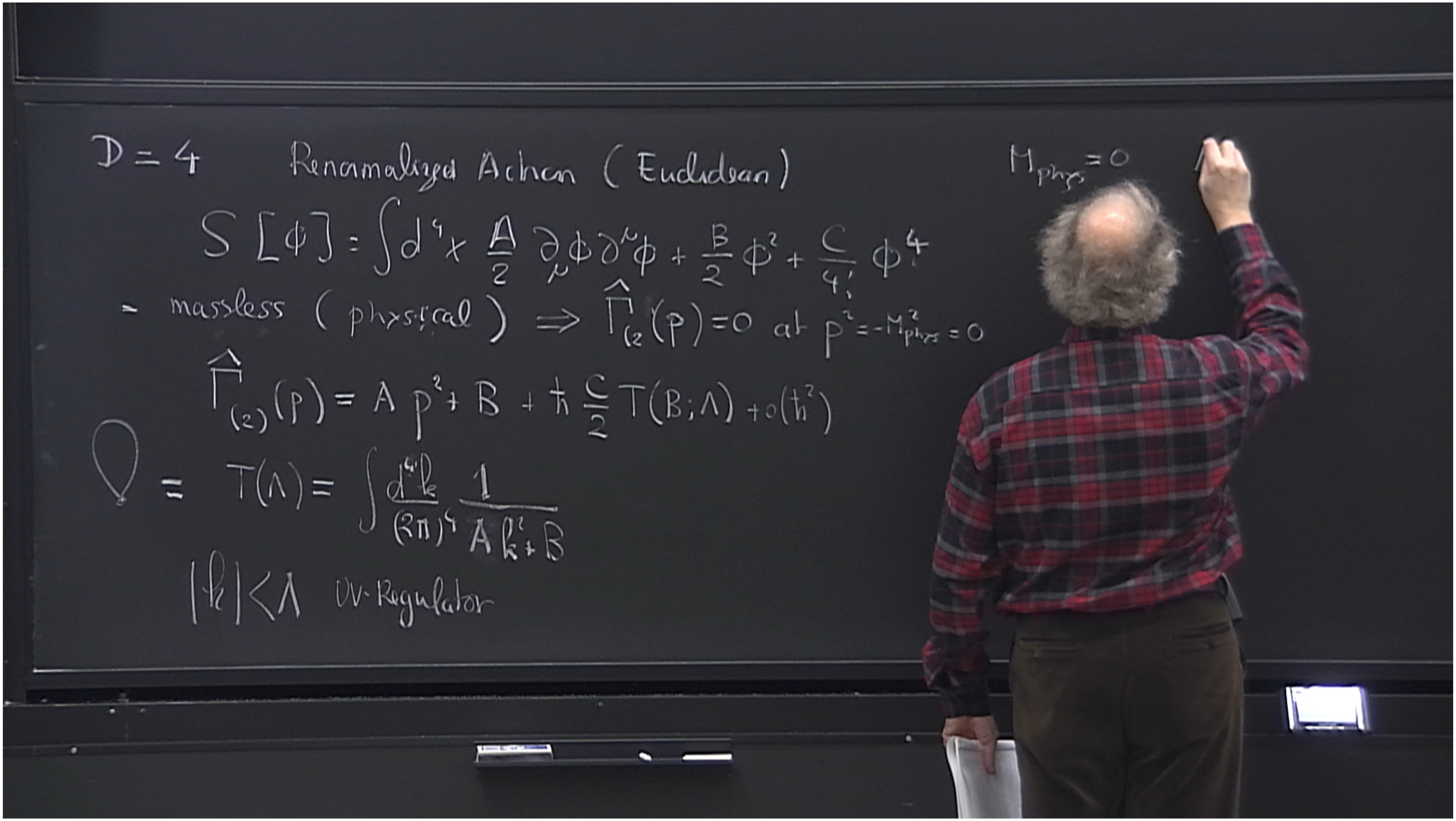

D = 4
Renormalized Action (Euclidean)
$S[\phi] = \int d^4x \, \frac{A}{2} \partial_\mu \phi \partial^\mu \phi + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4$

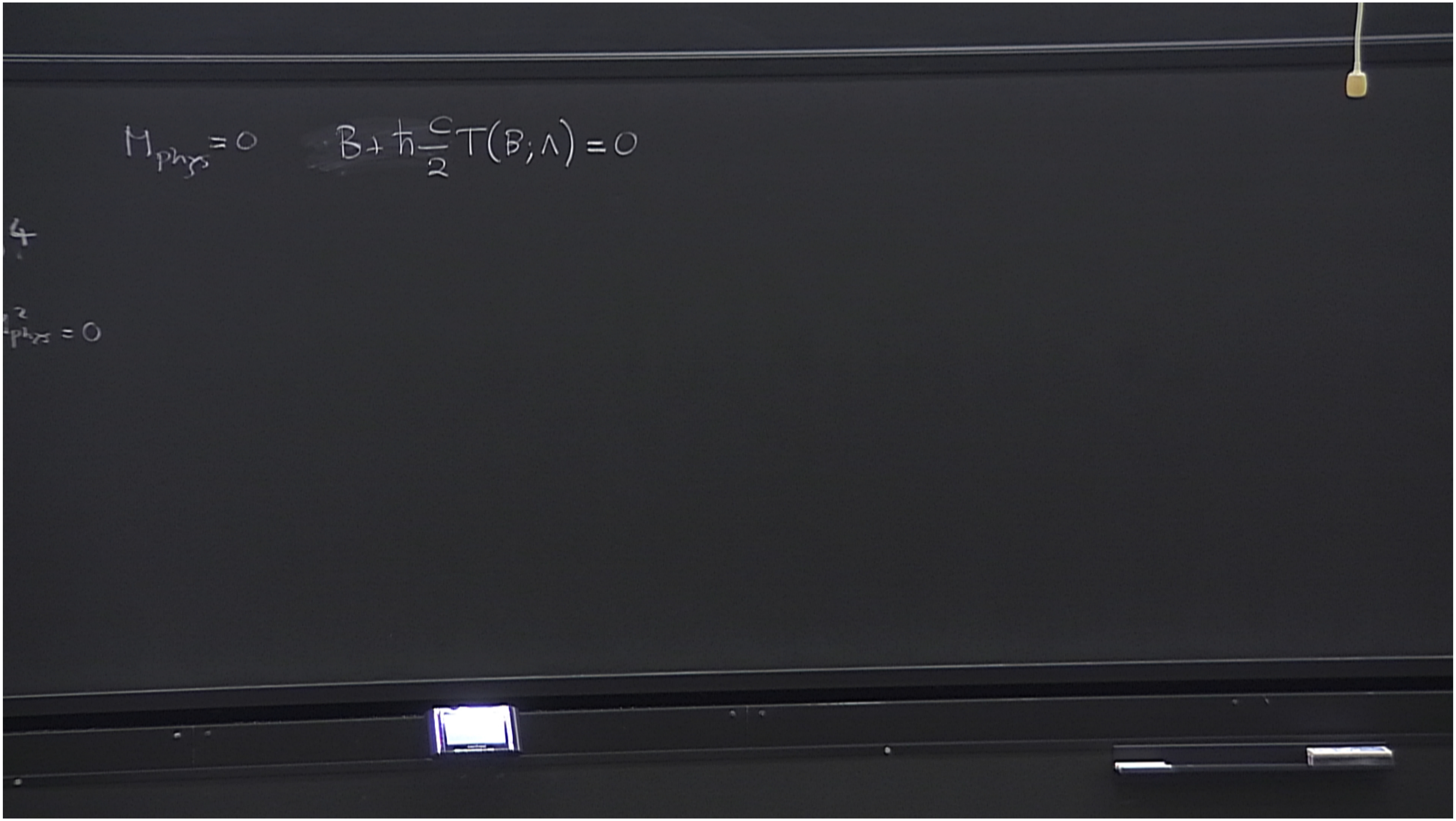

D = 4 Renormalized Action (Euclidean)
$S[\phi] = \int d^4x \, \frac{A}{2} \partial_\mu \phi \partial^\mu \phi + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4$
massless (physical)
$\hat{\Gamma}_{(2)}(p) = A p^2 + B + \hbar \frac{C}{2} T(B;\Lambda)$
$= T(\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{A k^2 + B}$
$|k| < \Lambda$ UV-Regulator


D = 4
Renormalized Action (Euclidean)
S [φ] = ∫d⁴x A/2
+ B/2 φ² + C/4! φ⁴
- massless (physical)
Γ̂(2)(p) = A p²
T(Λ) =
|k| < Λ
UV-Reg



$D = 4$
Renormalized Action (Euclidean)
$= \int d^4x \, \frac{A}{2} \partial_\mu\phi \partial^\mu\phi + \frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$
physical) $\Rightarrow \hat{\Gamma}_{(2)}(p) = 0$ at $p^2 = -M^2_{phys} = 0$
$) = A p^2 + B + \hbar \frac{C}{2} T(B;\Lambda)$
$\int \frac{d^4k}{(2\pi)^4} \frac{1}{Ak^2+B}$
Regulator


D = 4
Renormalized Action (Euclidean)
$M_{phys} = 0$
$S[\phi] = \int d^4x \, \frac{A}{2} \partial_\mu \phi \partial^\mu \phi + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4$
- massless (physical) $\Rightarrow \hat{\Gamma}_{(2)}(p) = 0$ at $p^2 = -M^2_{phys} = 0$
$\hat{\Gamma}_{(2)}(p) = A p^2 + B + \hbar \frac{C}{2} T(B;\Lambda) + o(\hbar^2)$
$= T(\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{Ak^2 + B}$
$|k| < \Lambda$ UV-Regulator


$M_{phys} = 0$
$B + \hbar \frac{c}{2} T(B; \Lambda) = 0$

$$M_{phys} = 0 \qquad B + \hbar \frac{c}{2} T(B;\Lambda) = 0$$

or (up to $O(\hbar)$) $$B + \hbar \frac{c}{2} T(0;\Lambda) = 0$$

4

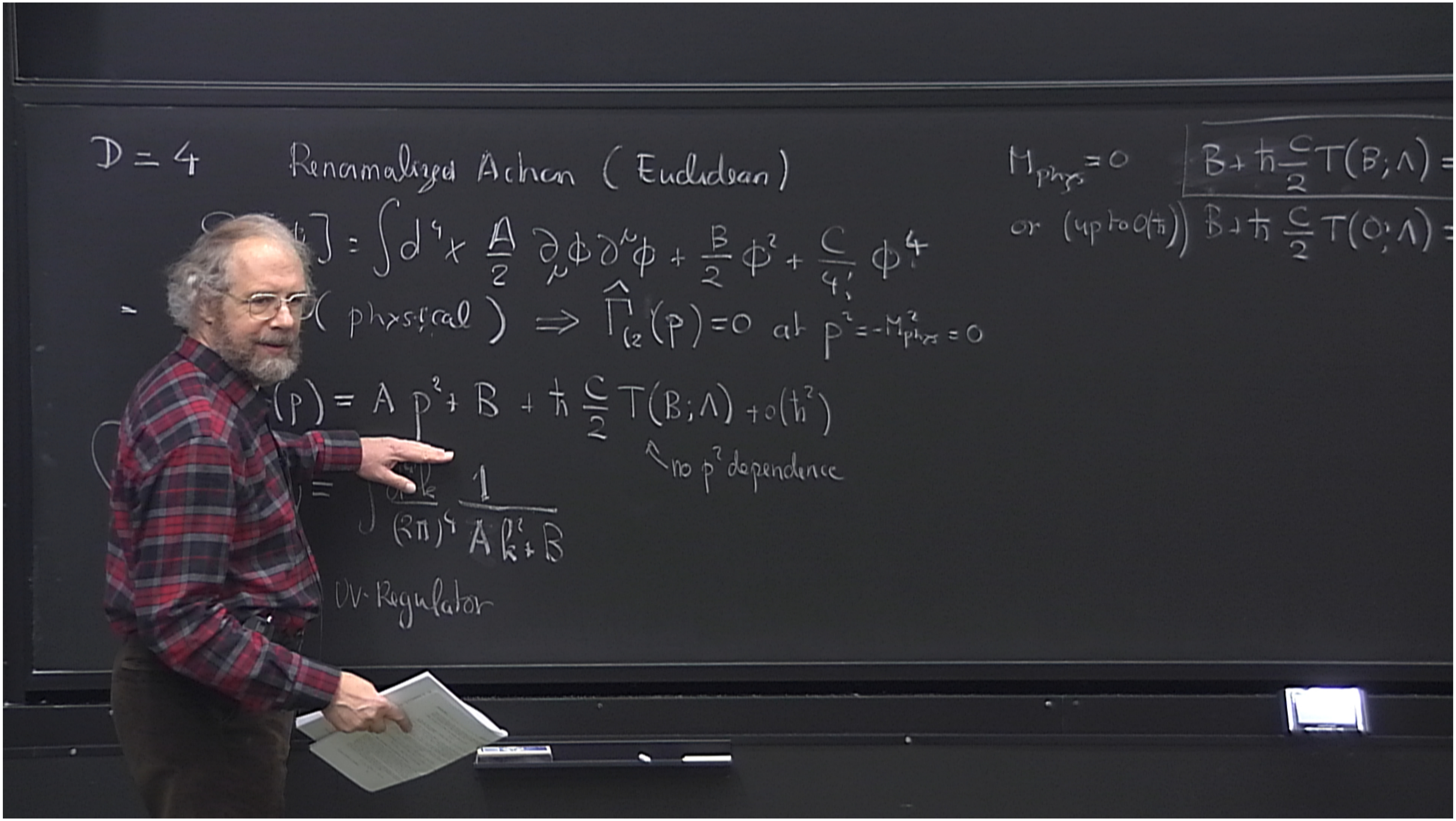
$M^2_{phys} = 0$

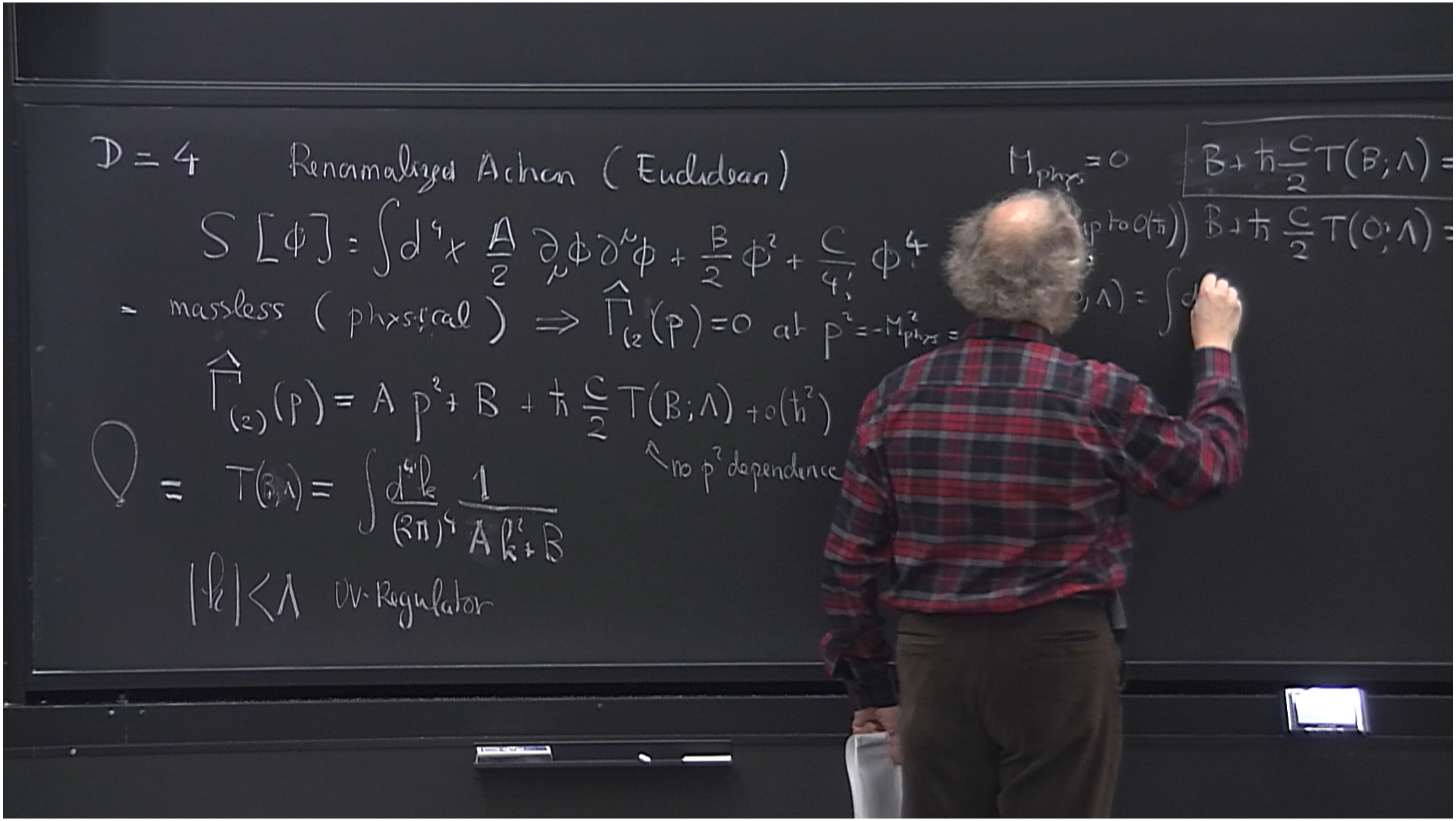
$M_{phys} = 0$ $\boxed{B + \hbar \frac{c}{2} T(B; \Lambda) = 0}$ $\boxed{A = 1}$

or (up to $O(\hbar)$) $B + \hbar \frac{c}{2} T(0; \Lambda) = 0$

4

$M^2_{phys} = 0$


D = 4
Renormalized Action (Euclidean)
M_phys = 0
or (up to O(ħ))
(physical) ⇒ Γ̂_(2)(p) = 0 at p² = -M²_phys = 0
no p² dependence
UV-Regulator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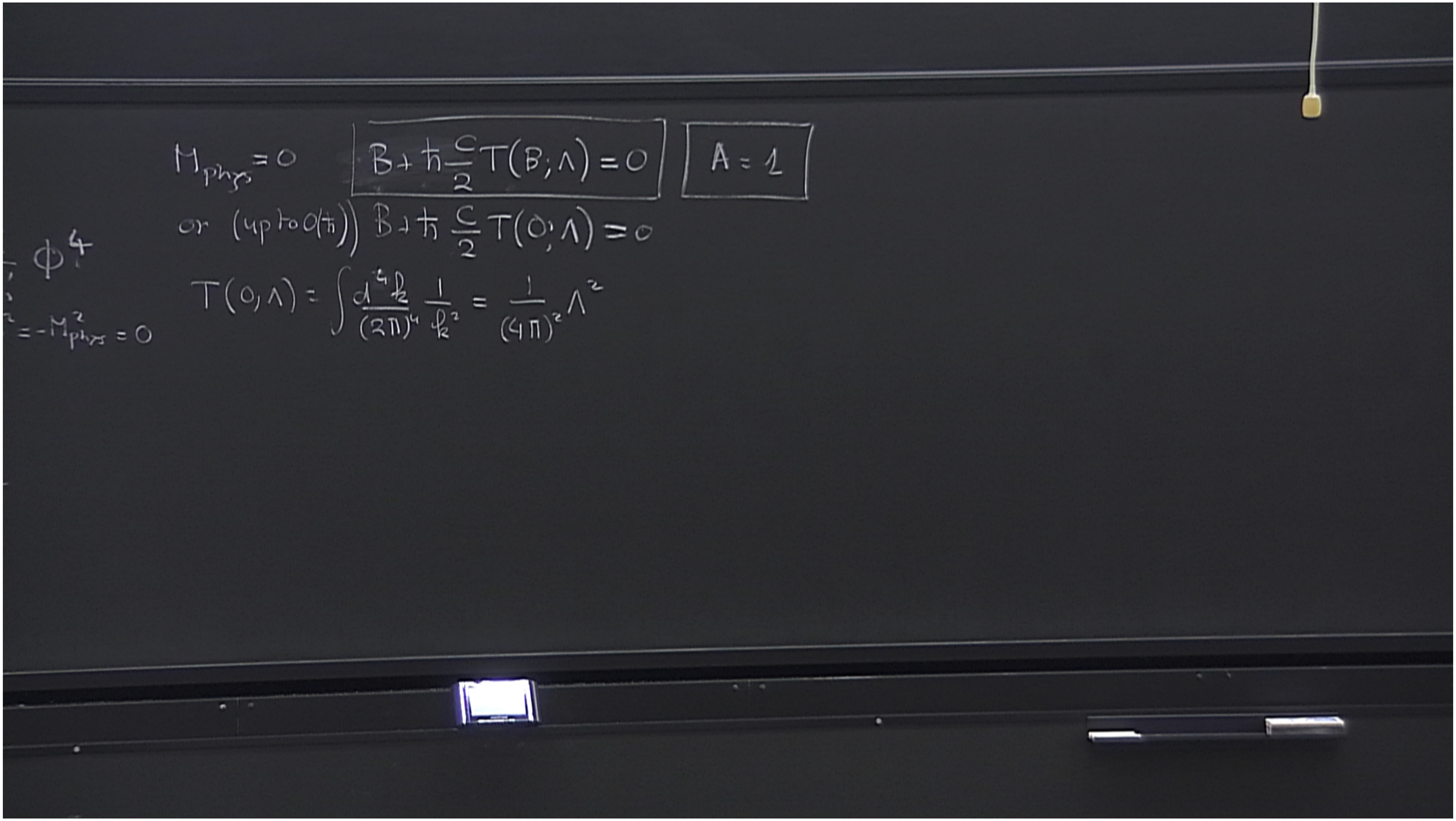
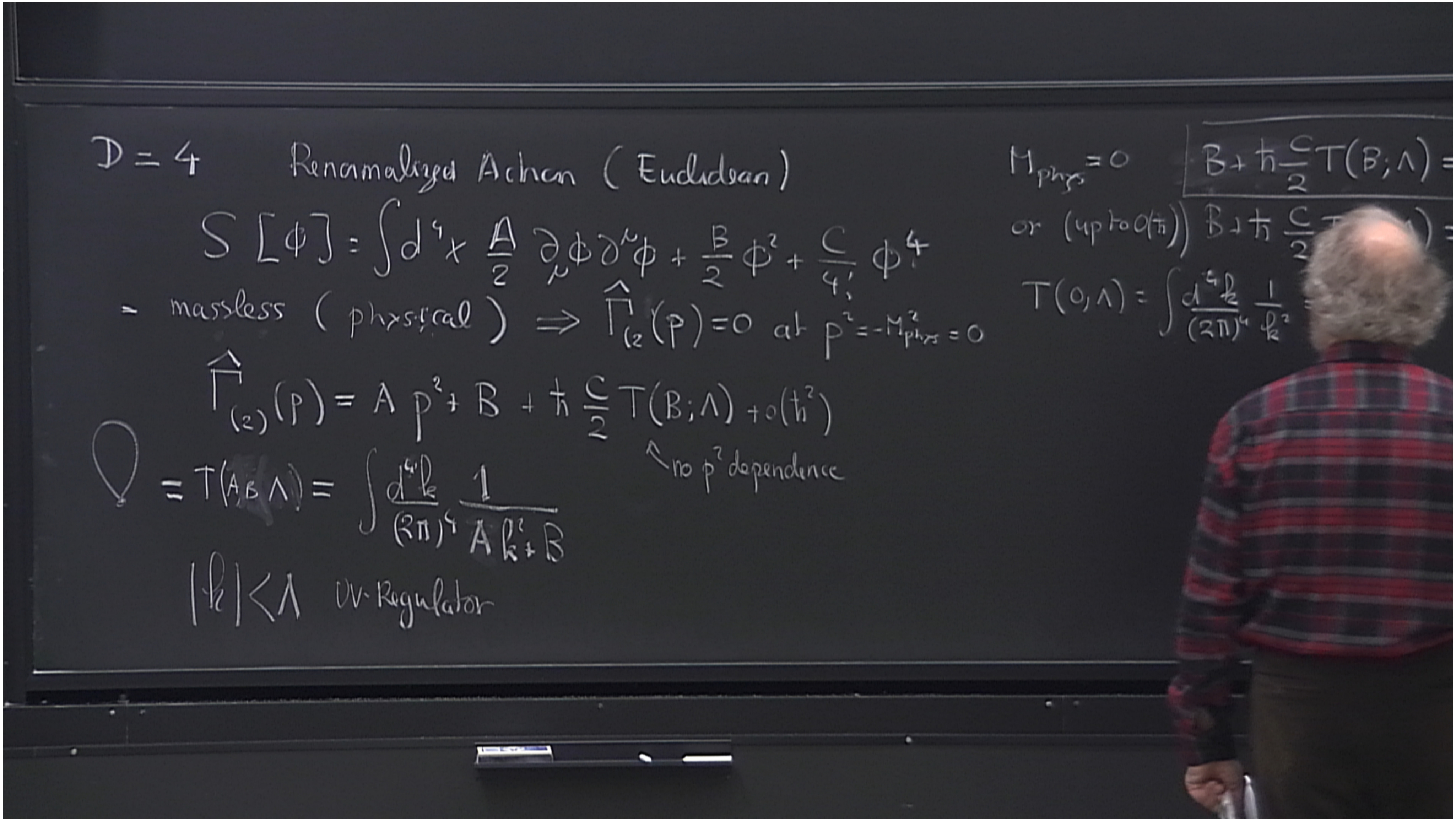
S [φ] = ∫d⁴x A/2 ∂μφ∂^μφ + B/2 φ² + C/4! φ⁴
- massless (physical) ⇒ Γ̂(2)(p) = 0 at p² = -M²phys =
Γ̂(2)(p) = A p² + B + ħ C/2 T(B;Λ) + o(ħ²)
no p² dependence
= T(B;Λ) = ∫ d⁴k/(2π)⁴ 1/(A k² + B)
|k| < Λ UV-Regulator
Mphys = 0
B + ħ C/2 T(B;Λ) =
B + ħ C/2 T(0;Λ) =


$M_{phys} = 0$
$B + \hbar \frac{c}{2} T(B;\Lambda) = 0$
$A = 1$
or (up to $O(\hbar)$) $B + \hbar \frac{c}{2} T(0;\Lambda) = 0$
ϕ^4
$T(0,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = \frac{1}{(4\pi)^2} \Lambda^2$
$= -M_{phys}^2 = 0$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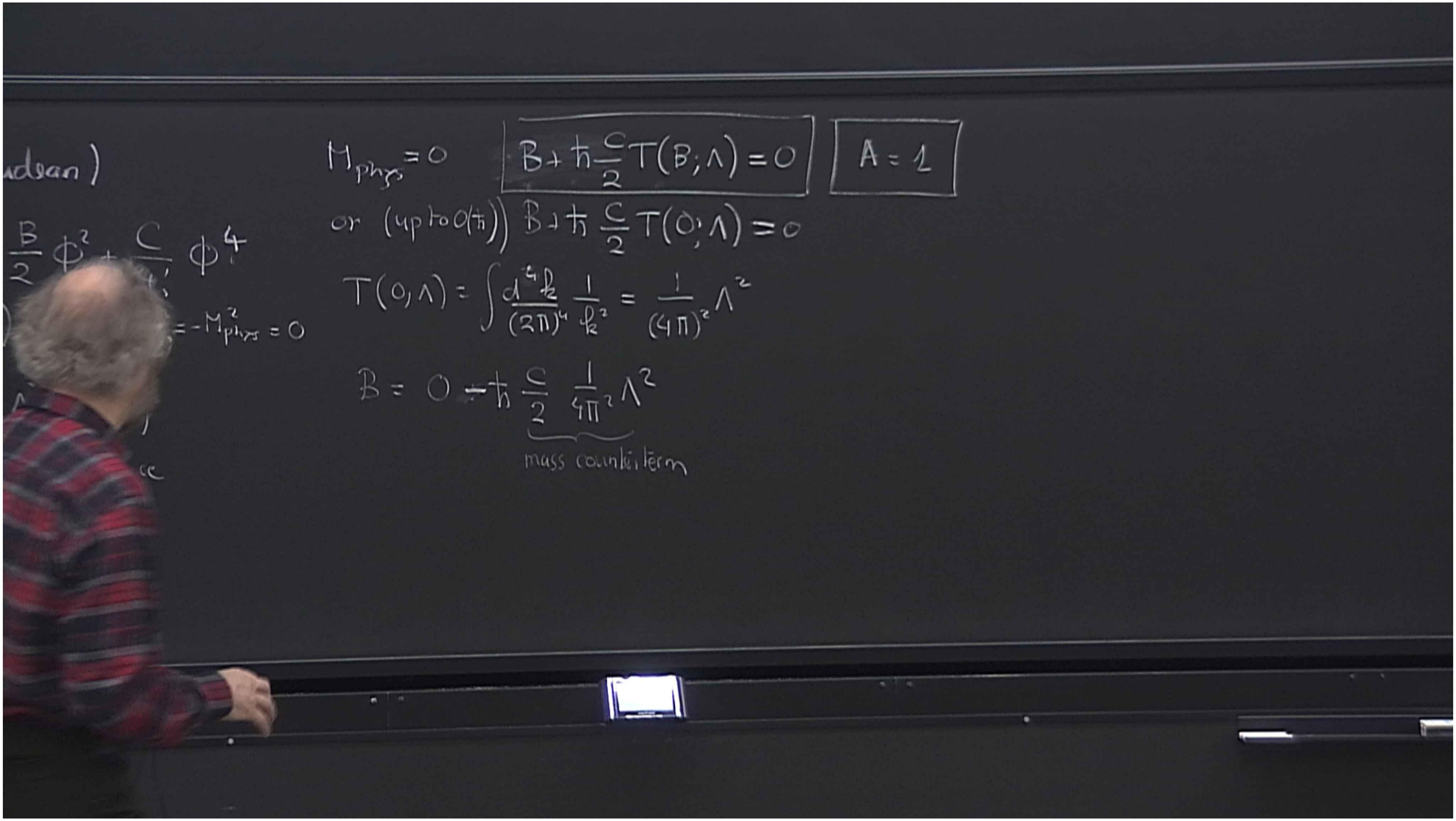
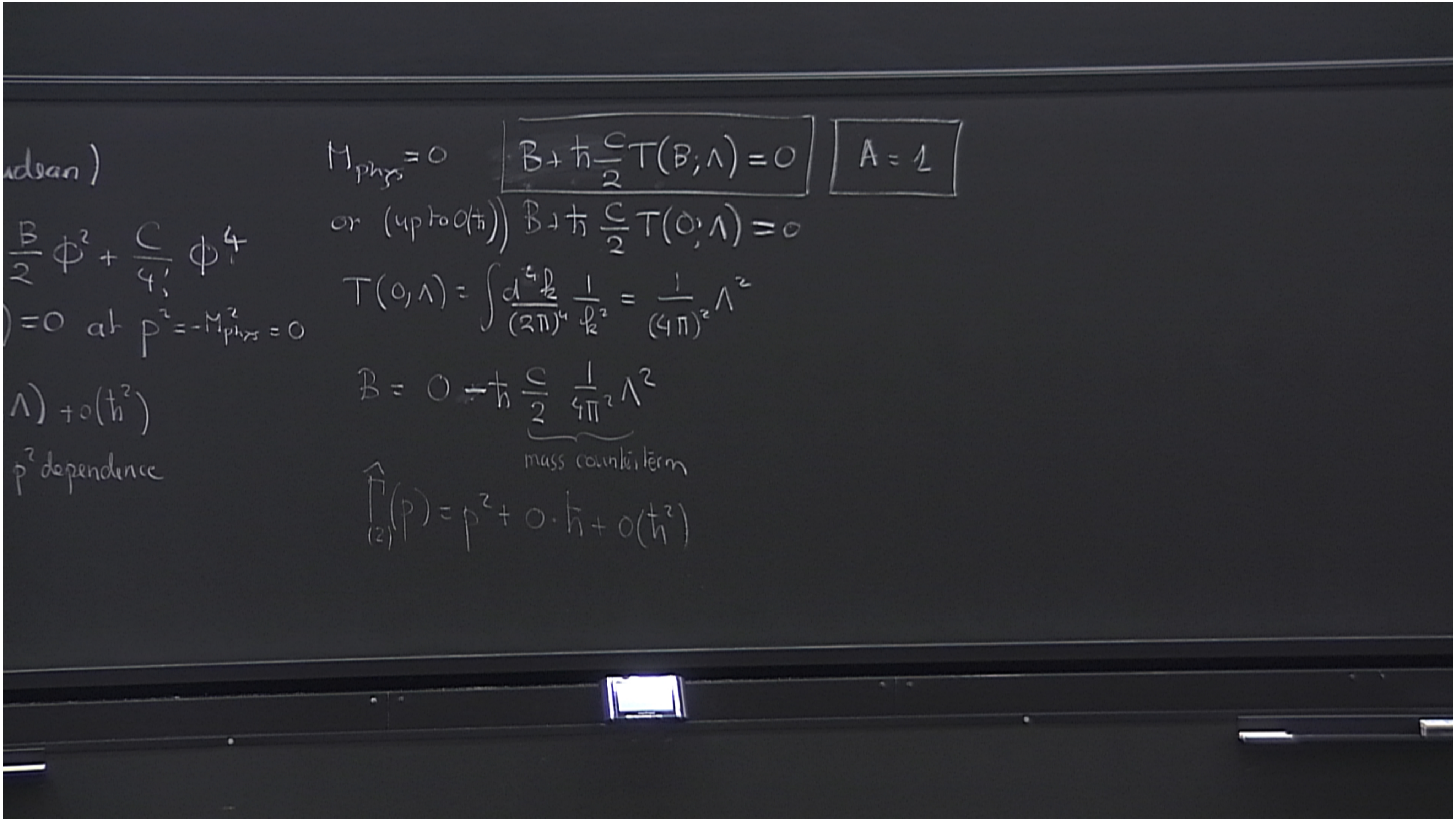
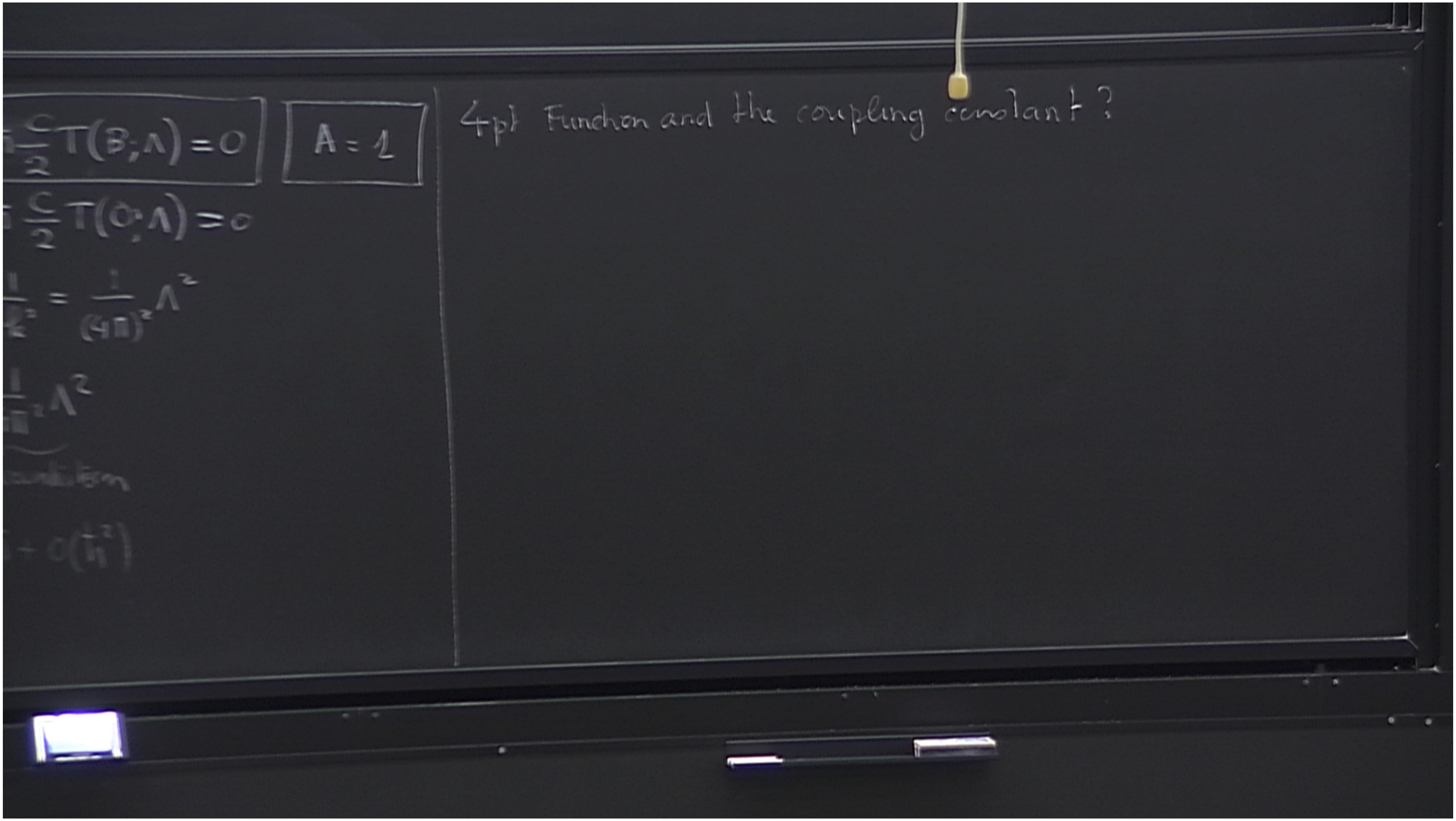
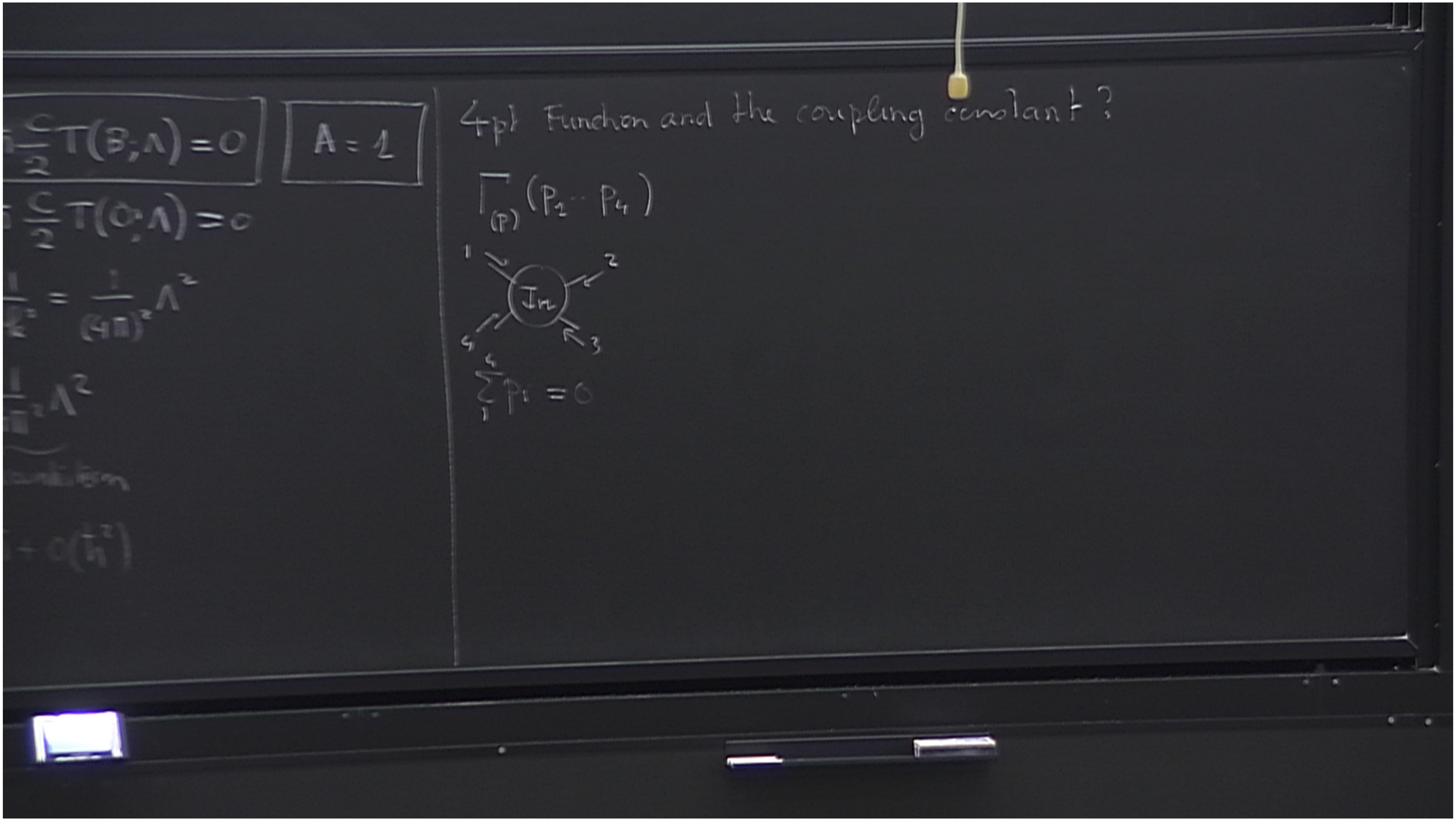
$S[\phi] = \int d^4x \, \frac{A}{2} \partial_\mu \phi \partial^\mu \phi + \frac{B}{2} \phi^2 + \frac{C}{4!} \phi^4$
- massless (physical) $\Rightarrow \hat{\Gamma}_{(2)}(p) = 0$ at $p^2 = -M^2_{phys} = 0$
$\hat{\Gamma}_{(2)}(p) = A p^2 + B + \hbar \frac{C}{2} T(B;\Lambda) + o(\hbar^2)$
no p^2 dependence
$= T(A, B, \Lambda) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{A k^2 + B}$
$|k| < \Lambda$ UV-Regulator
$M_{phys} = 0$
$B + \hbar \frac{C}{2} T(B;\Lambda) =$
or (up to $O(\hbar)$) $B + \hbar \frac{C}{2}$
$T(0,\Lambda) = \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2}$

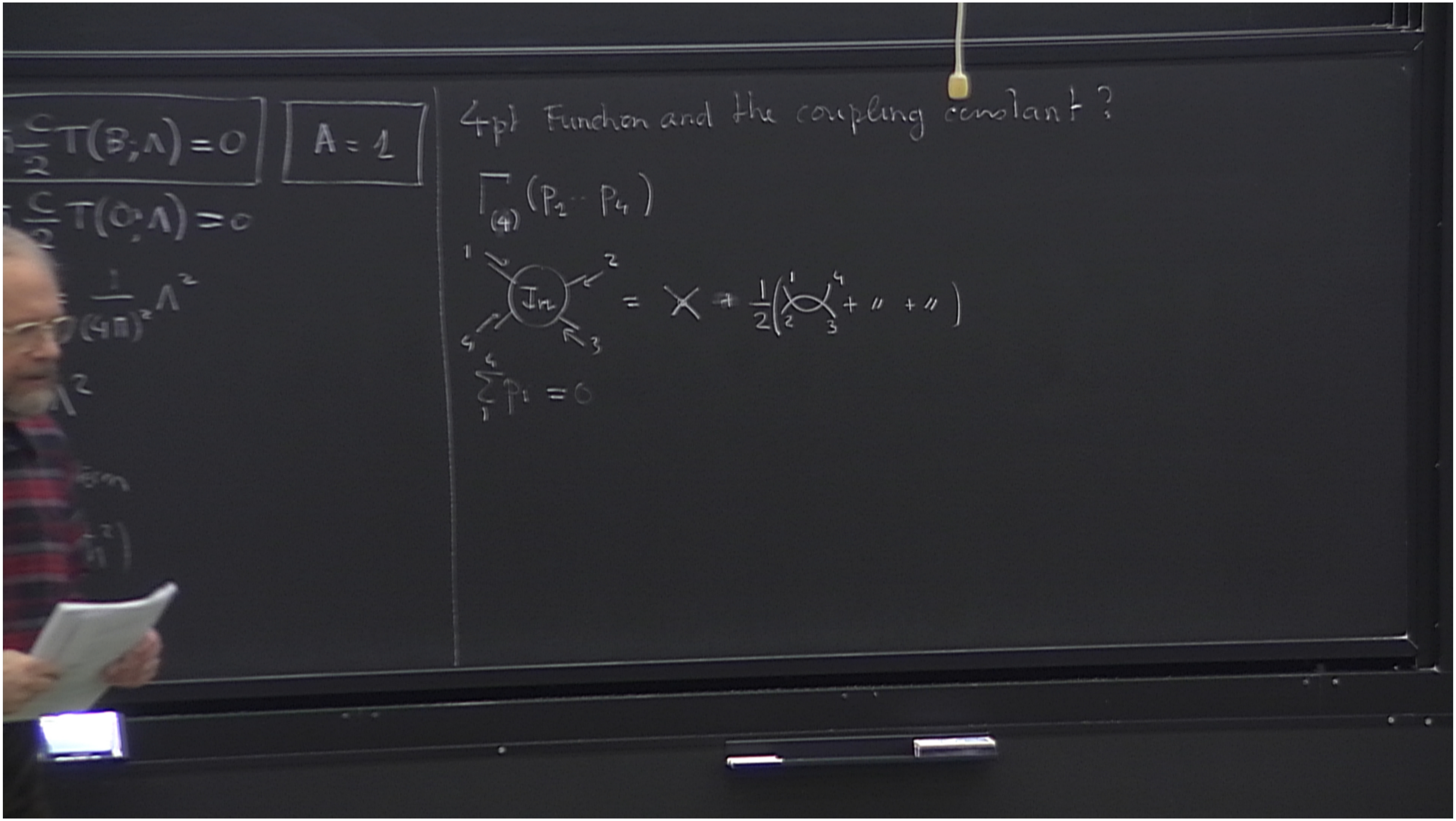

M_phys = 0
B + ħ C/2 T(B; Λ) = 0
A = 1
or (up to O(ħ)) B + ħ C/2 T(0; Λ) = 0
B/2 φ² + C/4! φ⁴
= −M²_phys = 0
T(0, Λ) = ∫ d⁴k/(2π)⁴ 1/k² = 1/(4π)² Λ²
B = 0 − ħ C/2 1/4π² Λ²
mass counterterm

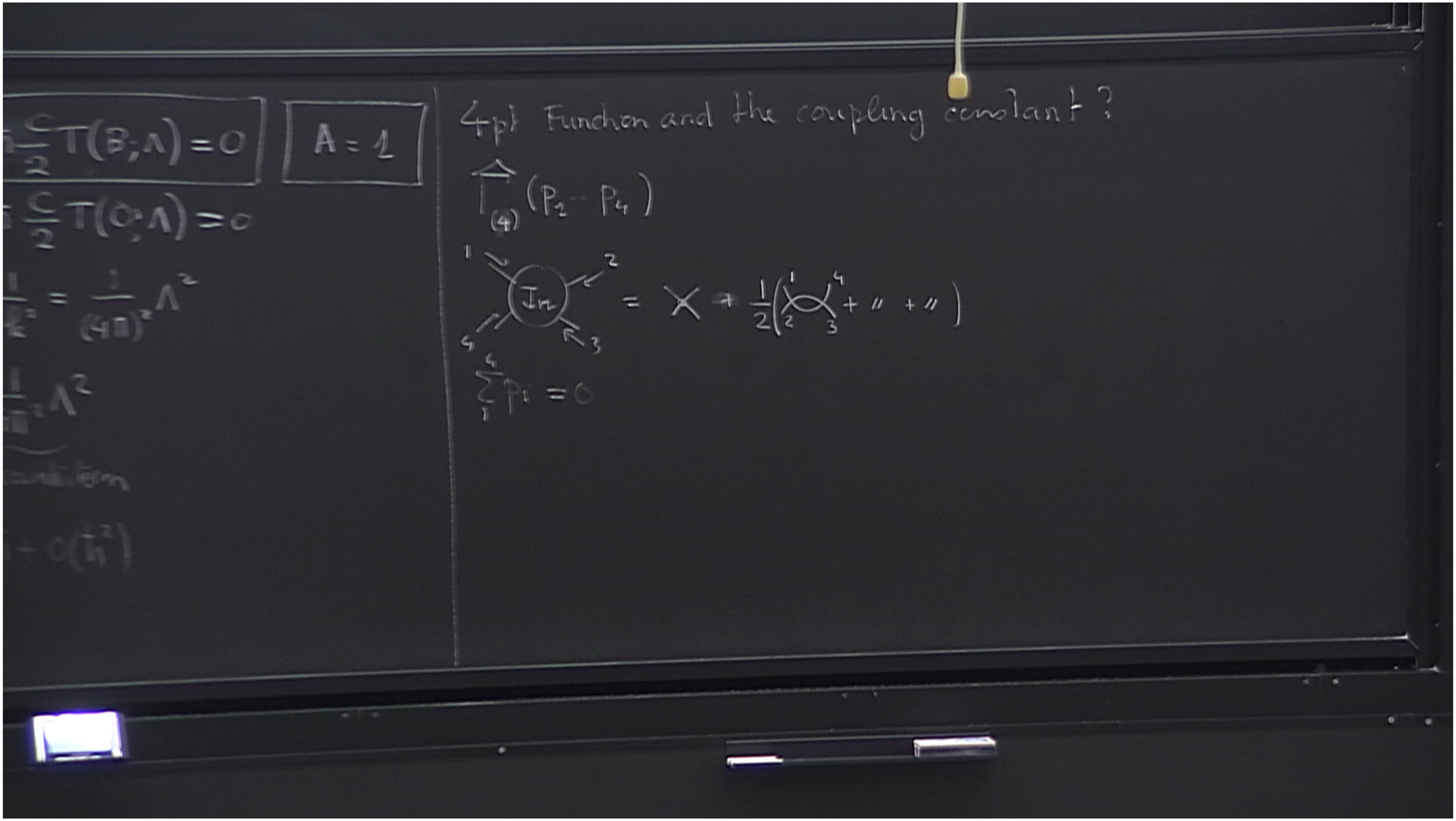


...idean)
$\frac{B}{2}\phi^2 + \frac{C}{4!}\phi^4$
$) = 0$ at $p^2 = -M_{phys}^2 = 0$
$\Lambda) + O(\hbar^2)$
p^2 dependence
$M_{phys} = 0$
$B + \hbar \frac{C}{2} T(B;\Lambda) = 0$
$A = 1$
or (up to $O(\hbar)$) $B + \hbar \frac{C}{2} T(0;\Lambda) = 0$
$T(0,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} = \frac{1}{(4\pi)^2}\Lambda^2$
$B = 0 - \hbar \frac{C}{2} \frac{1}{4\pi^2}\Lambda^2$
mass counterterm
$\hat{\Gamma}_{(2)}(p) = p^2 + 0 \cdot \hbar + O(\hbar^2)$


4pt Function and the coupling constant ?
A = 1


4pt Function and the coupling constant ?
A = 1
Γ(p) (p1 .. p4)
Σ pi = 0


4pt Function and the coupling constant ?
A = 1
Γ(4) (p1 ·· p4)
= X + 1/2 (+ 〃 + 〃)
Σ pi = 0


4pt Function and the coupling constant ?
Γ(4)(p1 .. p4)
= X + 1/2(+ " + ")
Σ pi = 0
A = 1

$\frac{c}{2} T(B;\Lambda) = 0$

$A = 1$

$\frac{c}{2} T(0;\Lambda) = 0$

$\frac{1}{k^2} = \frac{1}{(4\pi)^2}\Lambda^2$

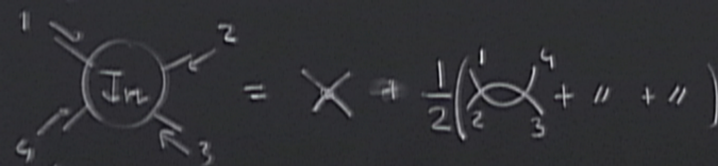
Λ^2

condition

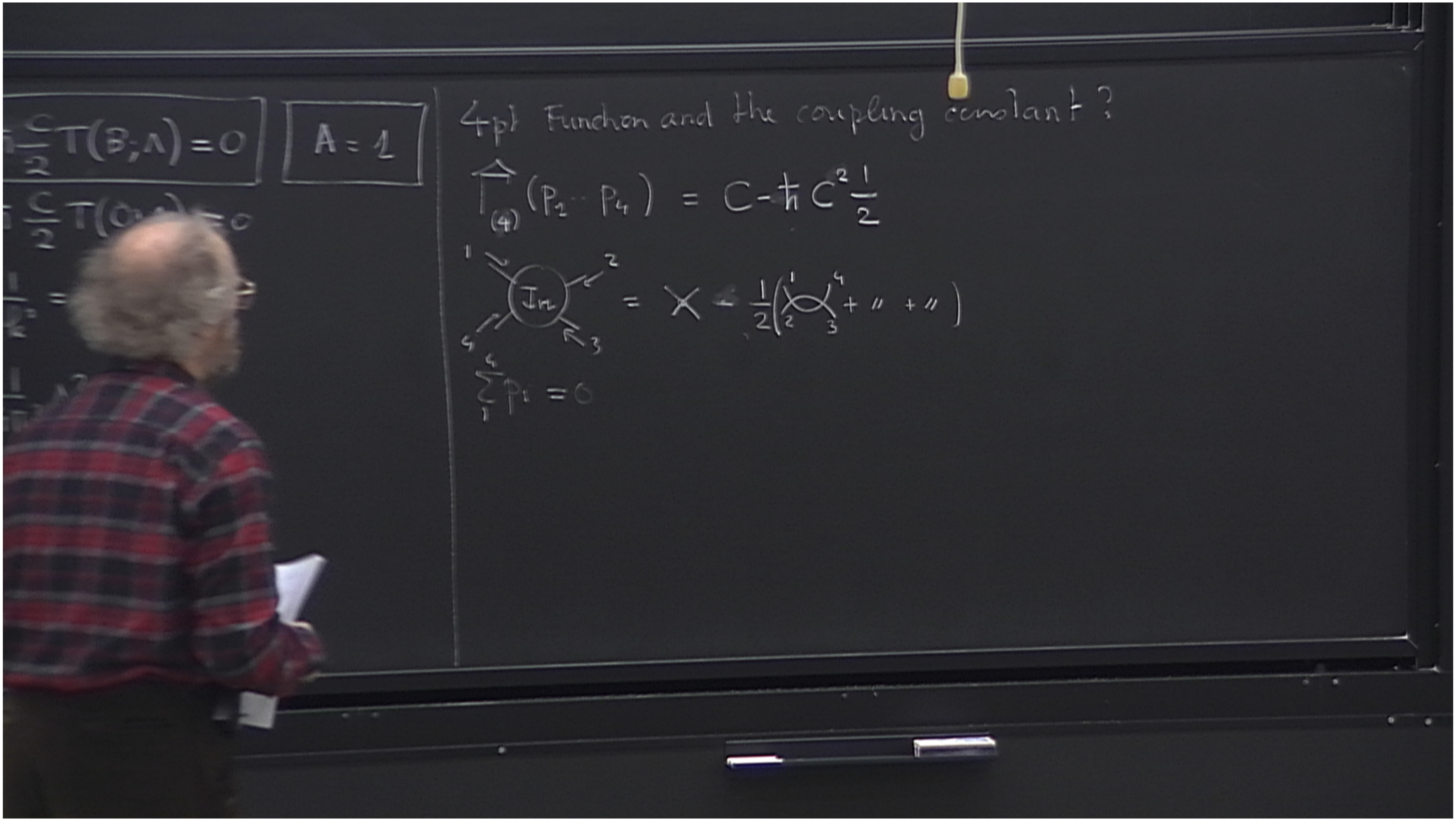
$- O(\hbar^2)$

4pt Function and the coupling constant?

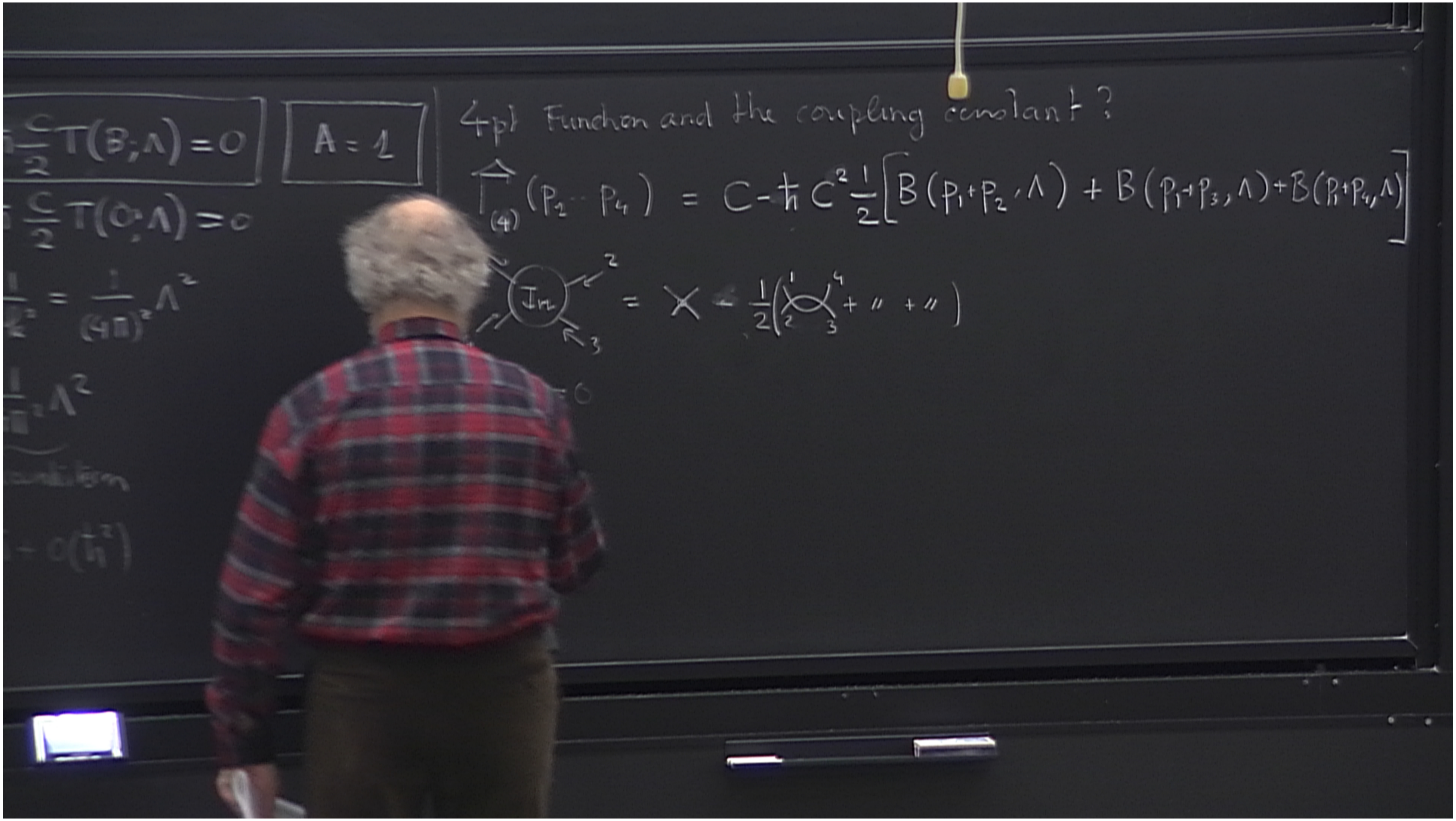
$\hat{\Gamma}_{(4)}(p_1 \cdots p_4) = C$



$\sum_{i=1}^{4} p_i = 0$



A = 1
4pt Function and the coupling constant ?
Γ(4)(p1 .. p4) = C - ħ C² 1/2
= X - 1/2 (+ " + ")
Σ pi = 0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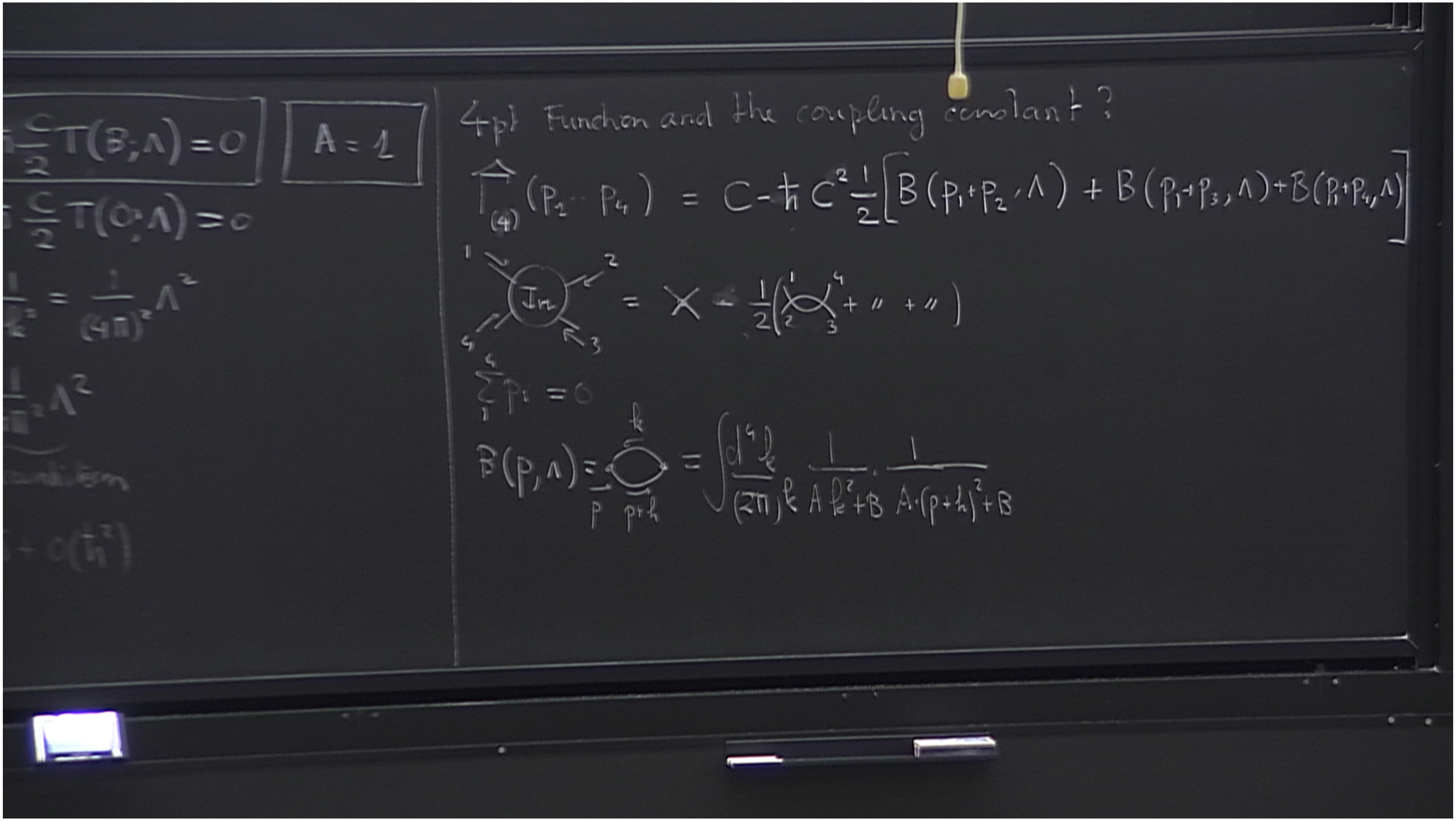
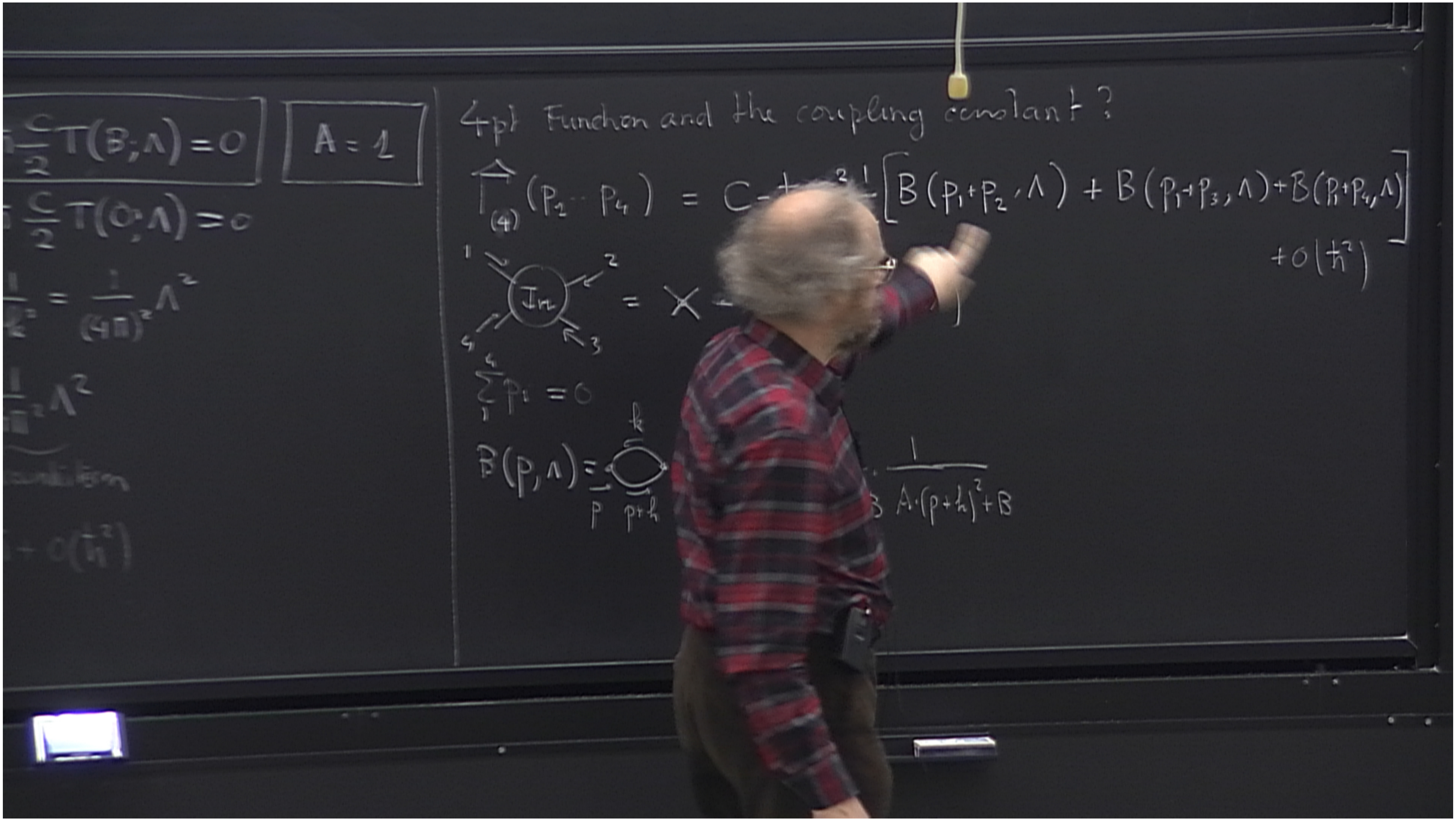
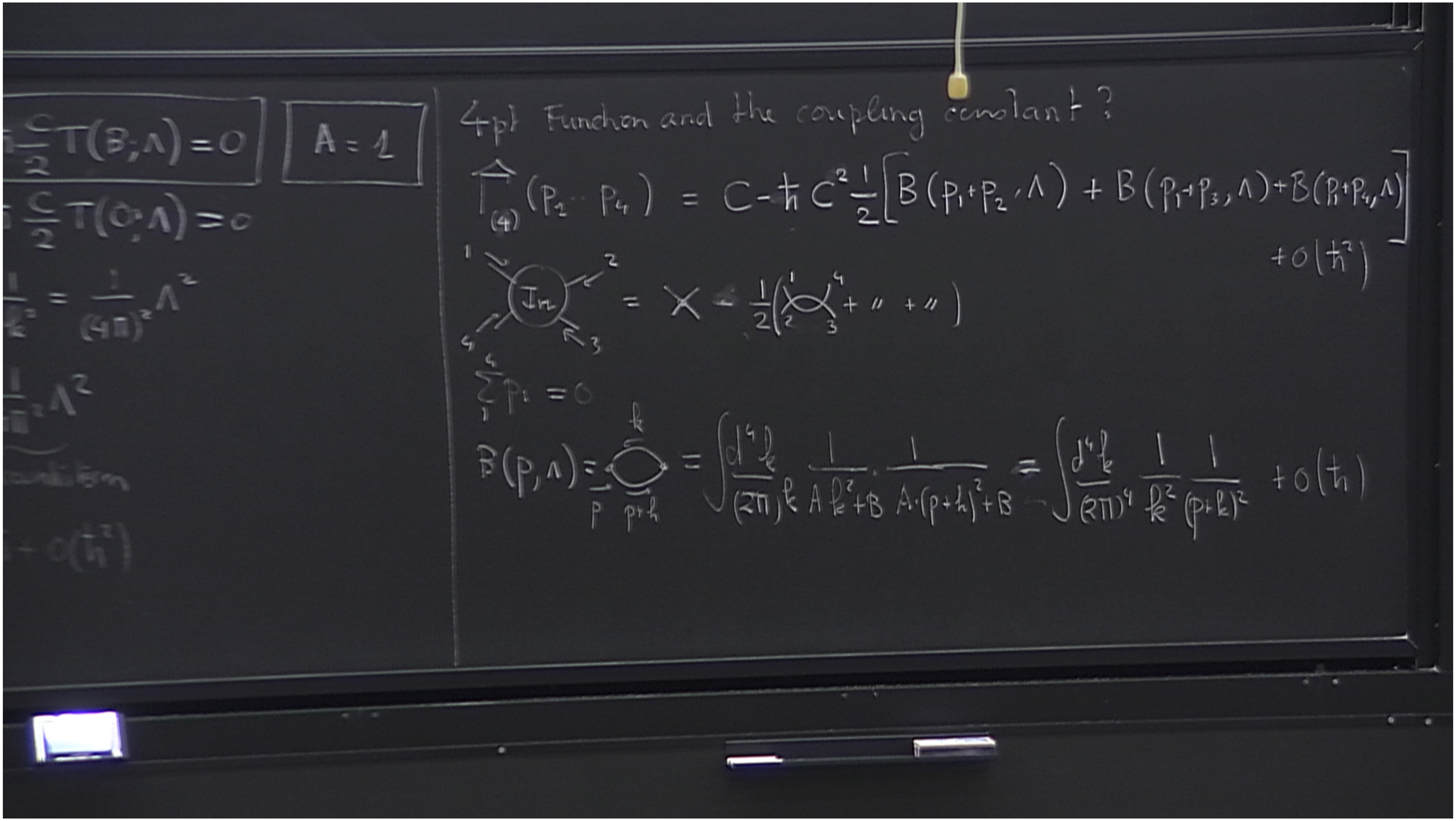
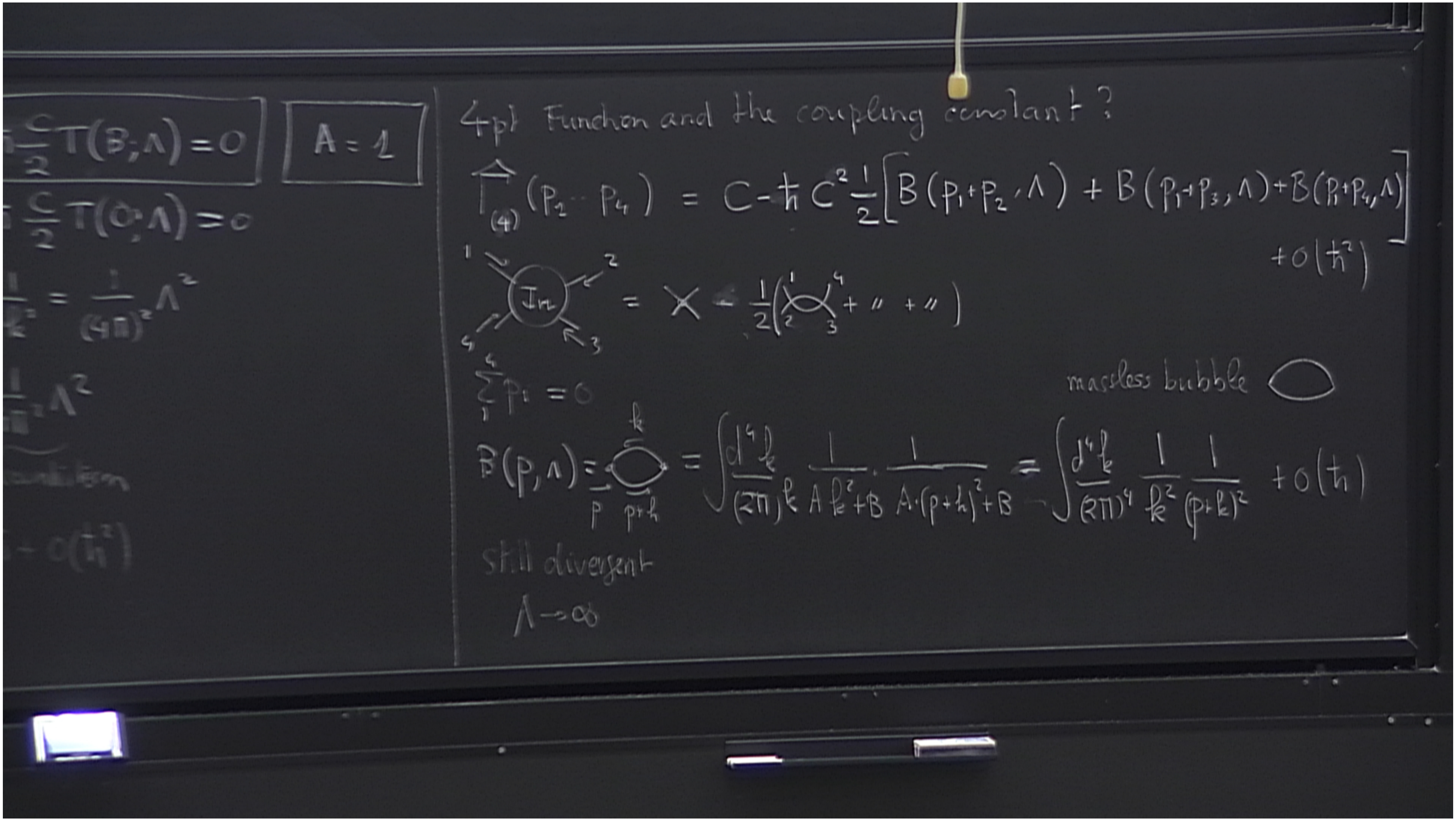
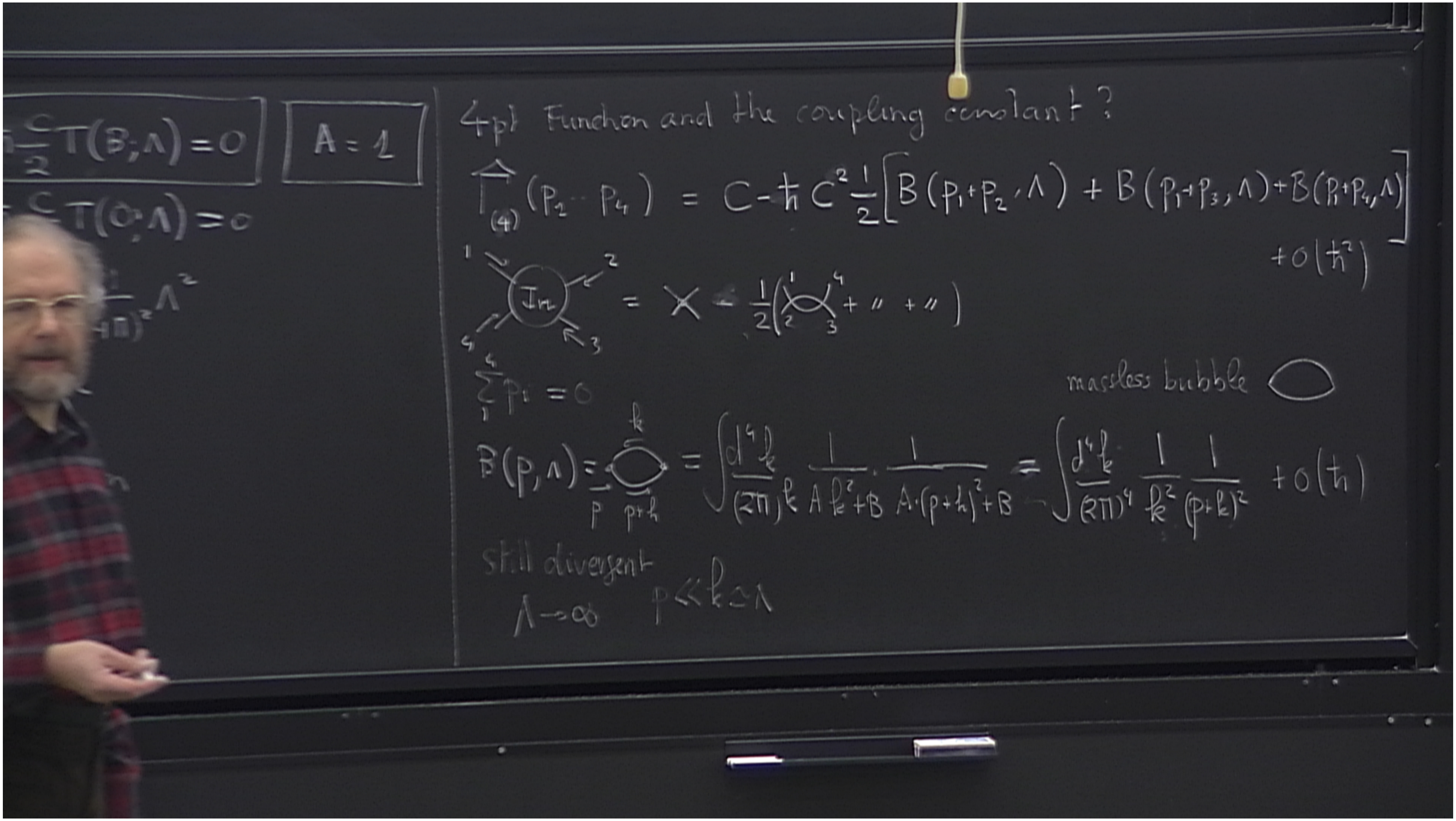

A = 1
4pt Function and the coupling constant ?
Γ(4)(p1 .. p4) = C − ħ C² 1/2 [B(p1+p2, Λ) + B(p1+p3, Λ) + B(p1+p4, Λ)]
Σ pi = 0
B(p, Λ) = ∫ d⁴k/(2π)⁴ · 1/(A k² + B) · 1/(A·(p+k)² + B)

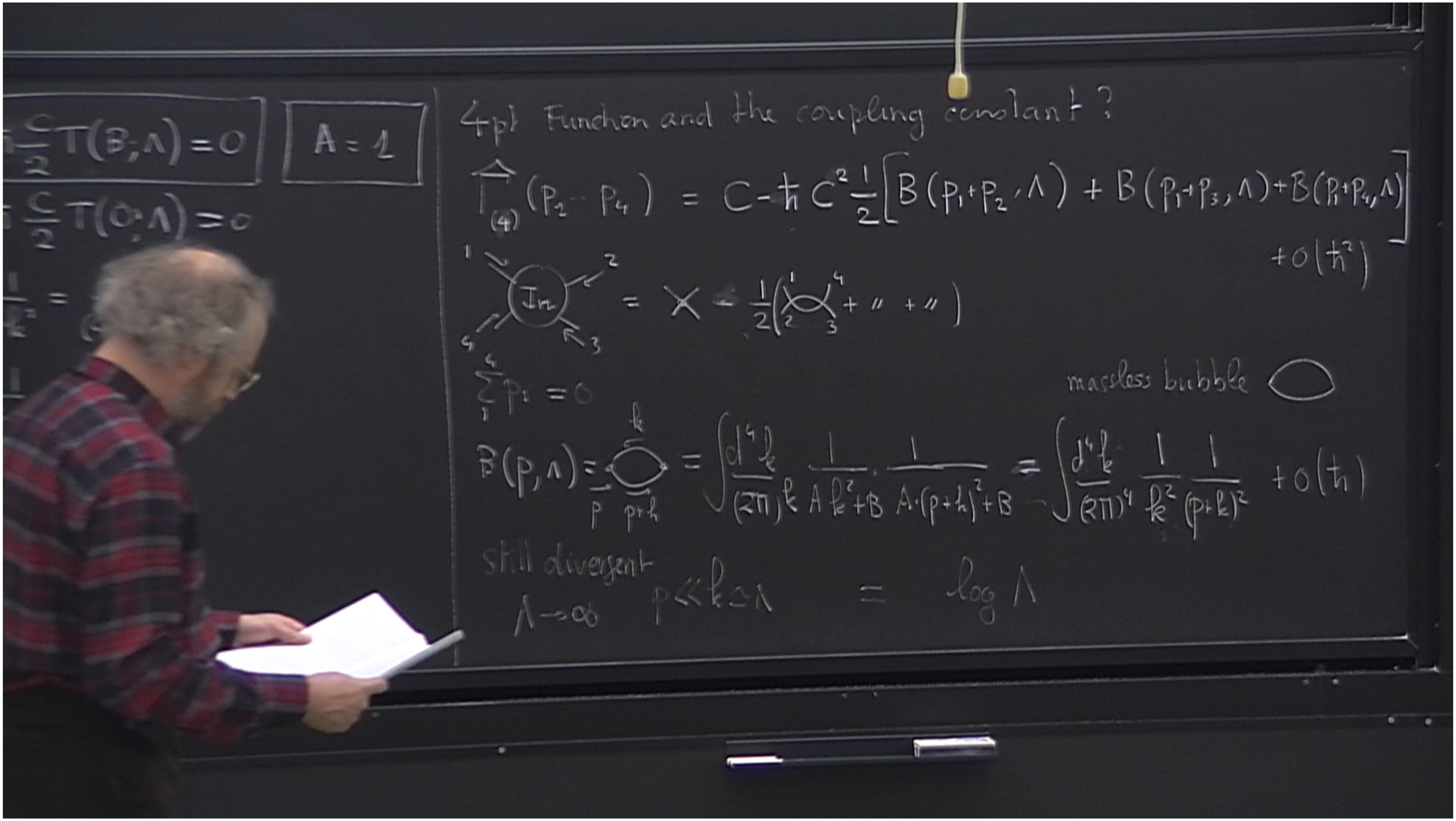

4pt Function and the coupling constant ?
A = 1
+ O(ħ²)

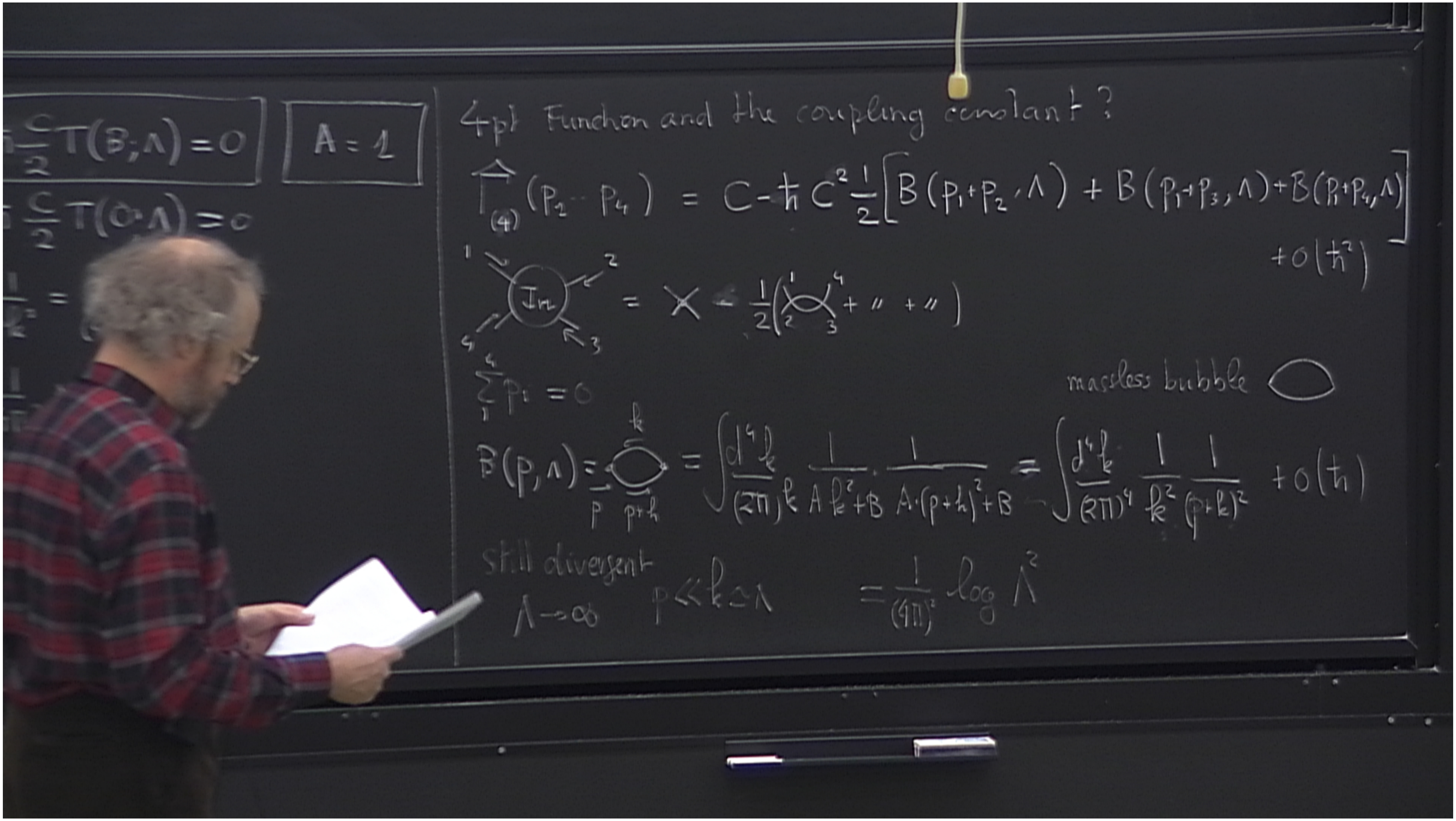

4pt Function and the coupling constant?
Γ(4)(p1 ... p4) = C − ħ C² 1/2 [B(p1+p2, Λ) + B(p1+p3, Λ) + B(p1+p4, Λ)] + O(ħ²)
= X + 1/2 (+ " + ")
Σ pi = 0
B(p, Λ) = ∫ d⁴k/(2π)⁴ 1/(A k² + B) · 1/(A·(p+k)² + B) = ∫ d⁴k/(2π)⁴ 1/k² 1/(p+k)² + O(ħ)
A = 1

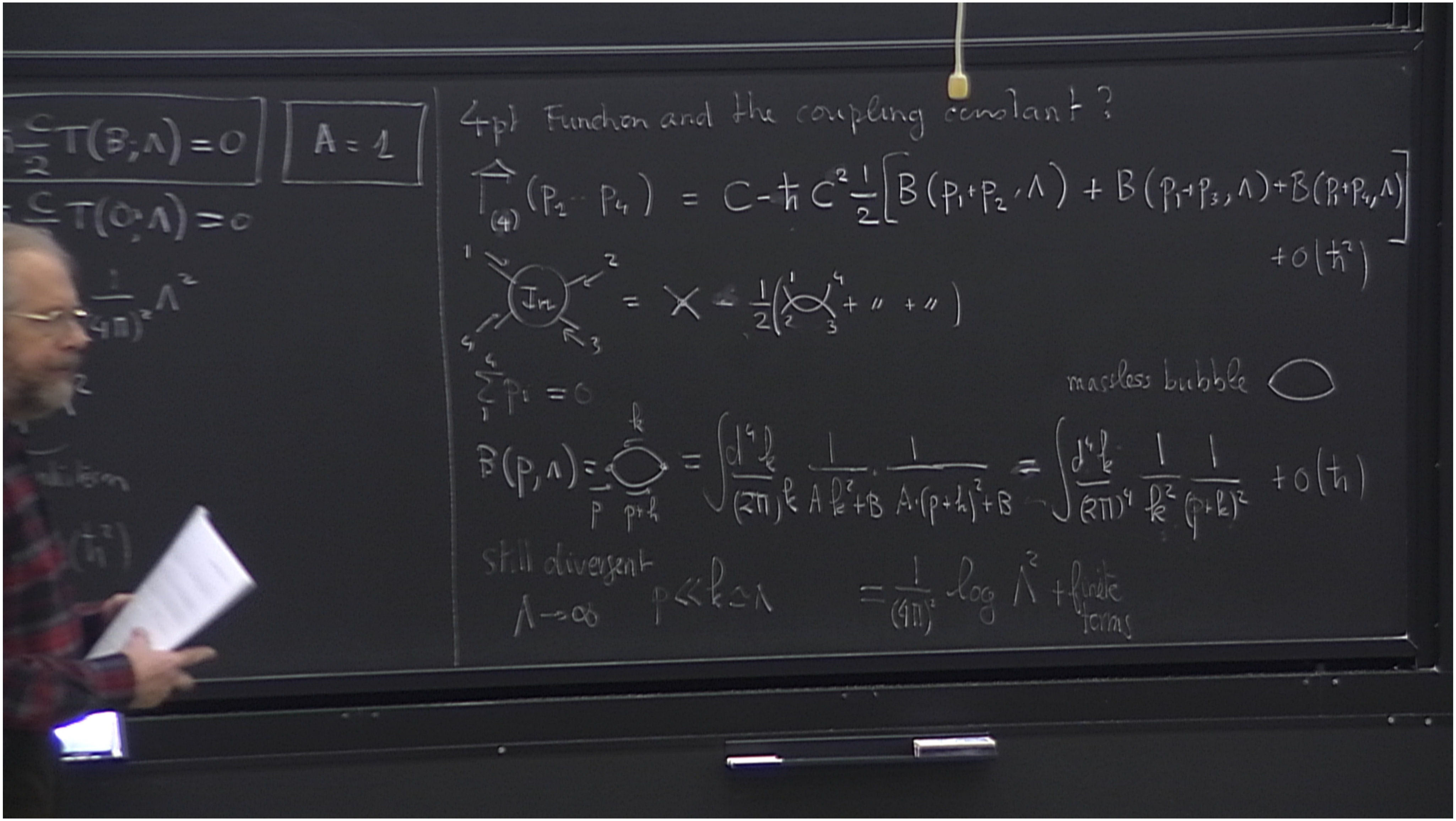

A = 1
4pt Function and the coupling constant ?
masless bubble
still divergent
Λ → ∞



4pt Function and the coupling constant ?
A = 1
$\Gamma_{(4)}(p_1 \dots p_4) = C - \hbar C^2 \frac{1}{2}\left[B(p_1+p_2,\Lambda) + B(p_1+p_3,\Lambda) + B(p_1+p_4,\Lambda)\right] + O(\hbar^2)$
$\sum_1^4 p_i = 0$
massless bubble
$B(p,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{Ak^2+B} \cdot \frac{1}{A\cdot(p+k)^2+B} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2} + O(\hbar)$
still divergent
$\Lambda \to \infty$ $p \ll k \simeq \Lambda$


4pt Function and the coupling constant ?
$\Gamma_{(4)}(p_1 \dots p_4) = C - \hbar C^2 \frac{1}{2}\left[B(p_1+p_2, \Lambda) + B(p_1+p_3, \Lambda) + B(p_1+p_4, \Lambda)\right] + O(\hbar^2)$
$\sum_1^4 p_i = 0$
massless bubble
$B(p,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{A k^2 + B} \cdot \frac{1}{A \cdot (p+k)^2 + B} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2} + O(\hbar)$
still divergent
$\Lambda \to \infty$ $p \ll k \simeq \Lambda$ = log Λ
A = 1


4pt Function and the coupling constant ?
A = 1
massless bubble
still divergent
Λ → ∞
p ≪ k ≃ Λ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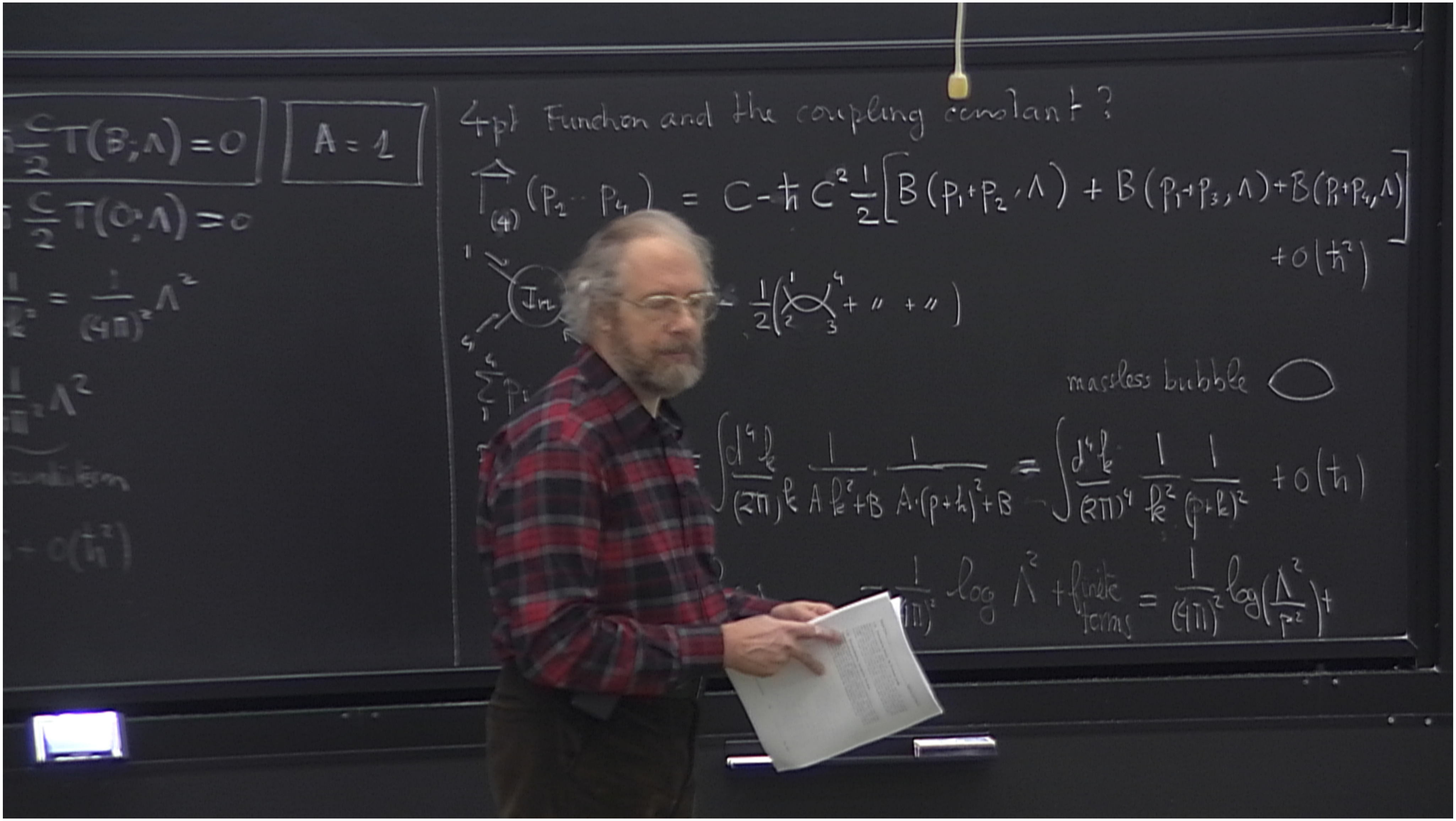
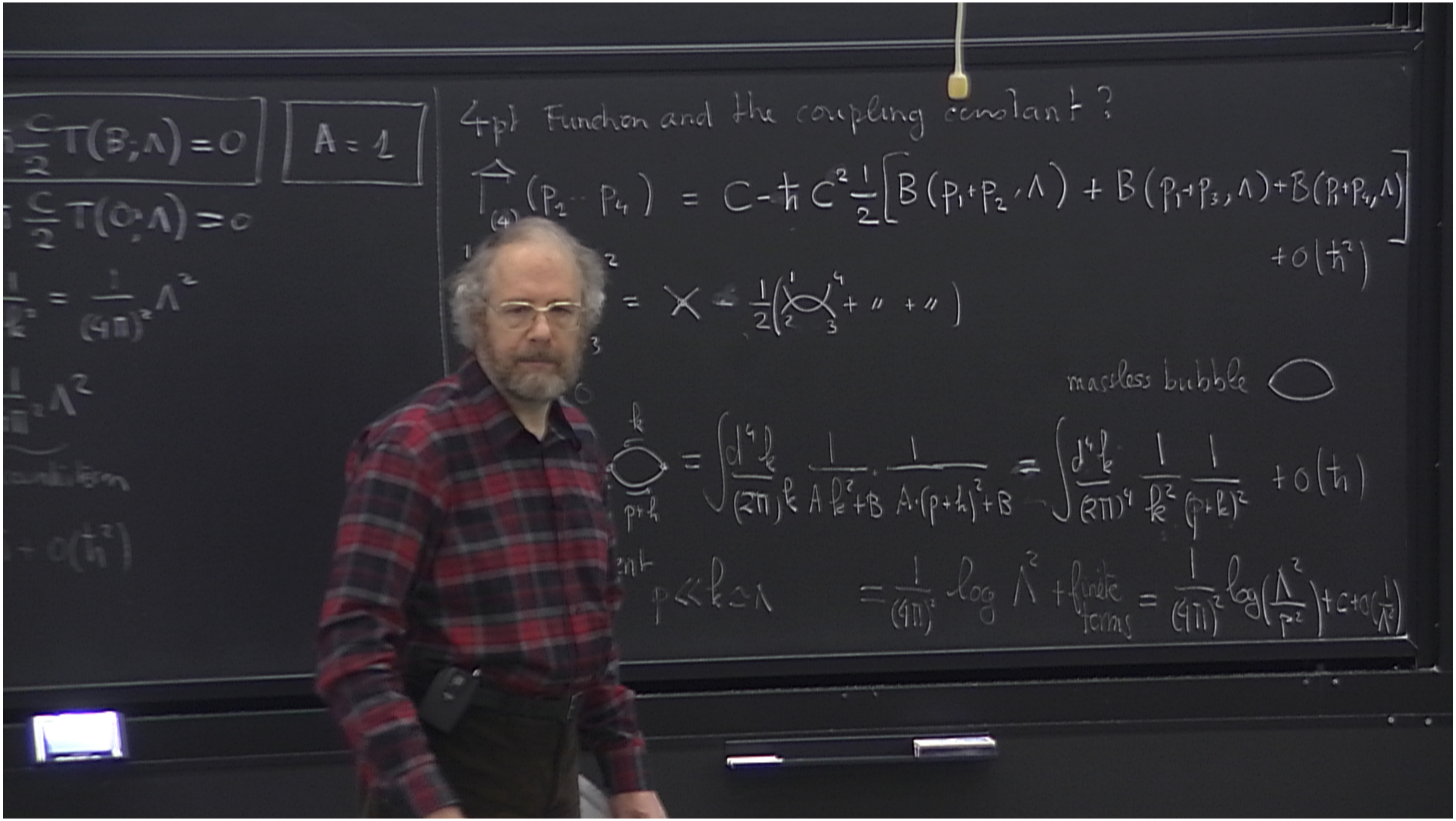
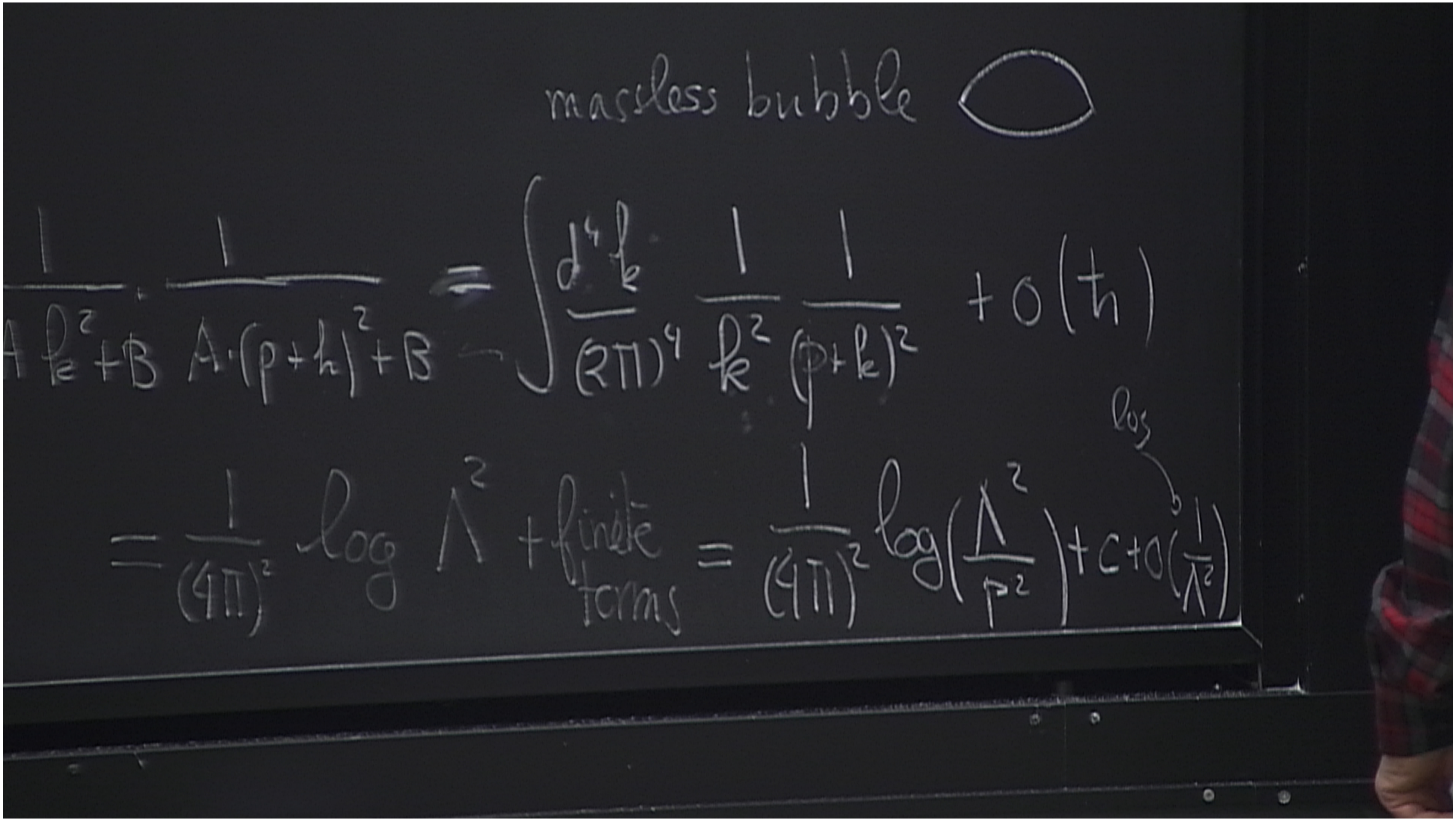
$\sum_1^4 p_i = 0$
massless bubble
$B(p,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{Ak^2+B} \cdot \frac{1}{A(p+k)^2+B} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2} + O(\hbar)$
still divergent
$\Lambda \to \infty$ $p \ll k \simeq \Lambda$ $= \frac{1}{(4\pi)^2} \log \Lambda^2$ + finite terms


4pt Function and the coupling constant ?
A = 1
massless bubble



4pt Function and the coupling constant ?
A = 1
massless bubble
+ finite terms


massless bubble
$\frac{1}{A k^2 + B} \frac{1}{A \cdot (p+k)^2 + B} \to \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2} + O(\hbar)$
$= \frac{1}{(4\pi)^2} \log \Lambda^2$ + finite terms $= \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + c + O\left(\frac{1}{\Lambda^2}\right)$
log

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log \Lambda}{\Lambda^2}\right)$

subdominant terms

$\searrow 0 \quad \Lambda \to \infty$

$$= B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log\Lambda}{\Lambda^2}\right)$$

↑ finite nb. depends on the regularization

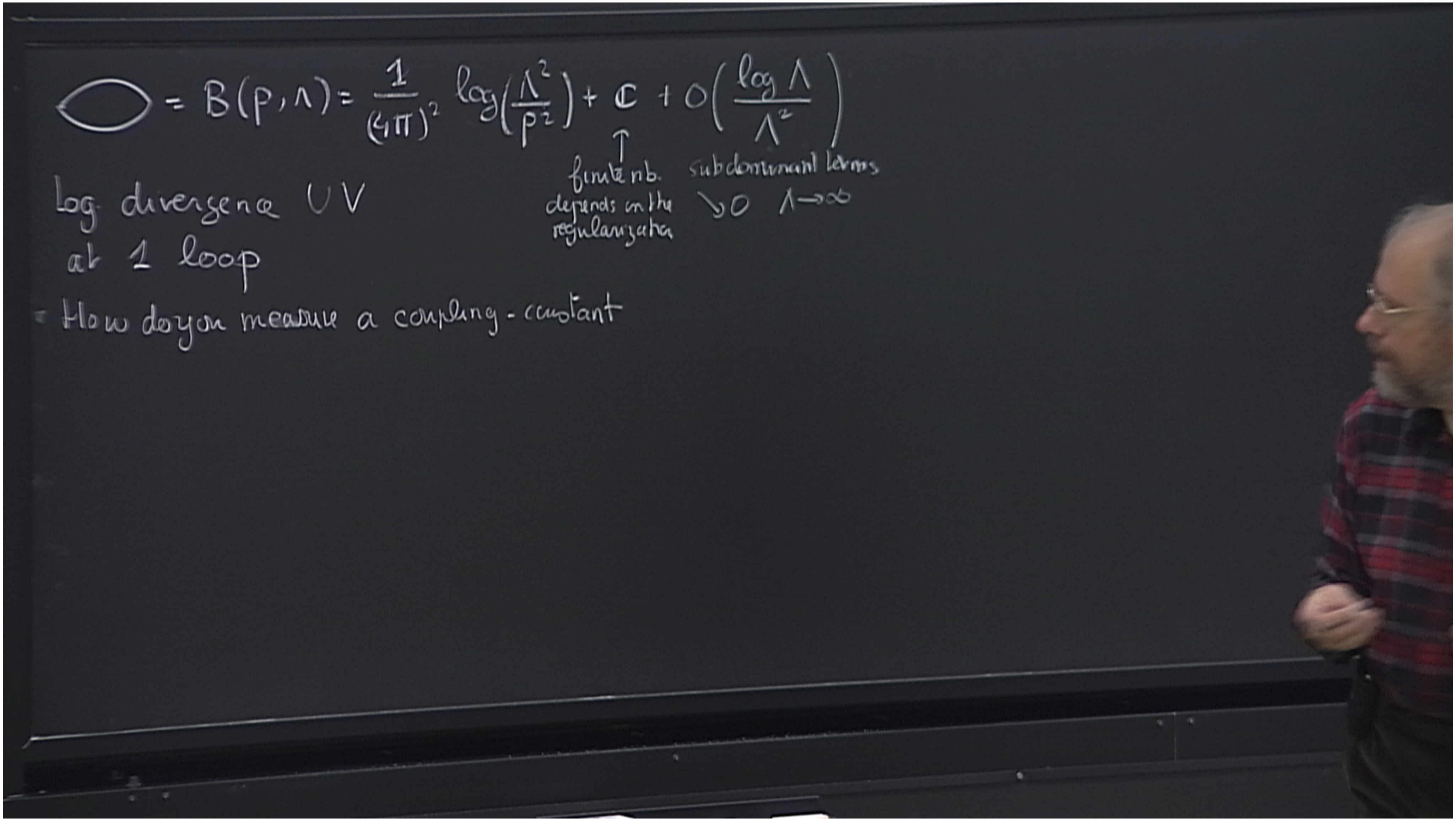
subdominant terms $\searrow 0 \quad \Lambda \to \infty$

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log\Lambda}{\Lambda^2}\right)$

$\mathbb{C}$: finite nb. depends on the regularization

$O\left(\frac{\log\Lambda}{\Lambda^2}\right)$: subdominant terms $\searrow 0$ $\Lambda \to \infty$

log divergence UV
at 1 loop


= B(p, Λ) = 1/(4π)^2 log(Λ^2/p^2) + C + O(log Λ / Λ^2)
finite nb. depends on the regularization
subdominant terms ↘0 Λ→∞
Log divergence UV at 1 loop
How do you measure a coupling constant

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + C + O\left(\frac{\log \Lambda}{\Lambda^2}\right)$

C ↑ finite nb. depends on the regularization

$O\left(\frac{\log \Lambda}{\Lambda^2}\right)$: subdominant terms $\searrow 0$ $\Lambda \to \infty$

log divergence UV at 1 loop

- How do you measure a coupling-constant?

scattering experiments

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log \Lambda}{\Lambda^2}\right)$

$\mathbb{C}$: finite nb. depends on the regularization

subdominant terms $\searrow 0$ $\Lambda \to \infty$

Log divergence UV at 1 loop

- How do you measure a coupling-constant ?

Scattering experiments

Differential cross-section

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + C + O\left(\frac{\log\Lambda}{\Lambda^2}\right)$

finite nb. depends on the regularization (under C)

subdominant terms $\searrow 0$ $\Lambda \to \infty$

Log divergence UV at 1 loop

- How doyou measure a coupling-constant ?

Scattering experiments

Differential cross-section

↓

S-matrix elements

$$\bigcirc = B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log\Lambda}{\Lambda^2}\right)$$

↑ finite nb. depends on the regularization

subdominant terms $\searrow 0$ $\Lambda \to \infty$

Log divergence UV at 1 loop

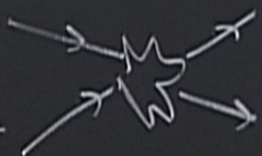
- How do you measure a coupling-constant ?

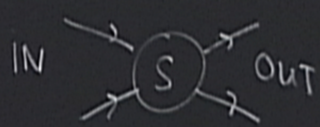
Scattering experiments

Differential cross-section

↓

S-matrix elements





 $= B(p,\Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log \Lambda}{\Lambda^2}\right)$

finite nb. depends on the regularization

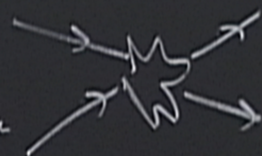
subdominant terms $\searrow 0 \quad \Lambda \to \infty$

Log divergence UV at 1 loop

- How do you measure a coupling-constant ?

Scattering experiments

Differential cross-section



↓

S-matrix elements

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LS2 reduction formula

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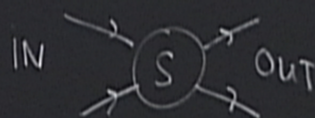

$$\bigcirc = B(p,\Lambda) = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + C + O\left(\frac{\log\Lambda}{\Lambda^2}\right)$$

C ↑ finite nb. depends on the regularization

$O(\ldots)$: subdominant terms $\searrow 0$ $\Lambda \to \infty$

Log divergence UV at 1 loop

- How do you measure a coupling-constant?

Scattering experiments

Differential cross-section

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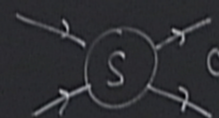
S-matrix elements



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LSZ reduction formula

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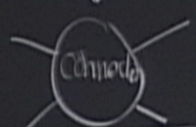
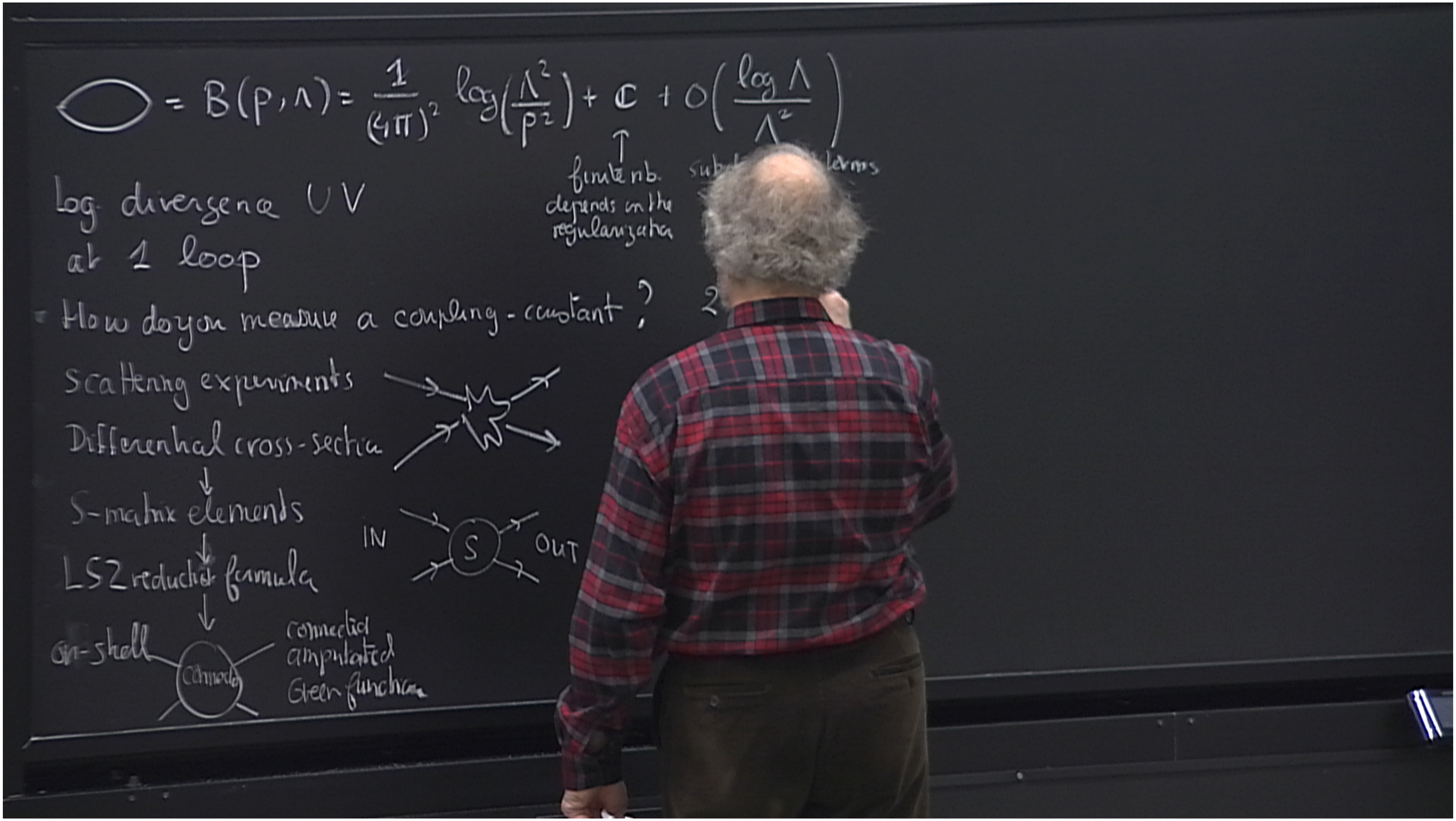


connected amputated Green function


= B(p, Λ) = 1/(4π)² log(Λ²/p²) + C + O(log Λ / Λ²)
finite nb.
depends on the
regularization
log divergence UV
at 1 loop
How do you measure a coupling-constant?
Scattering experiments
Differential cross-section
S-matrix elements
LSZ reduction formula
IN
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OUT
on-shell
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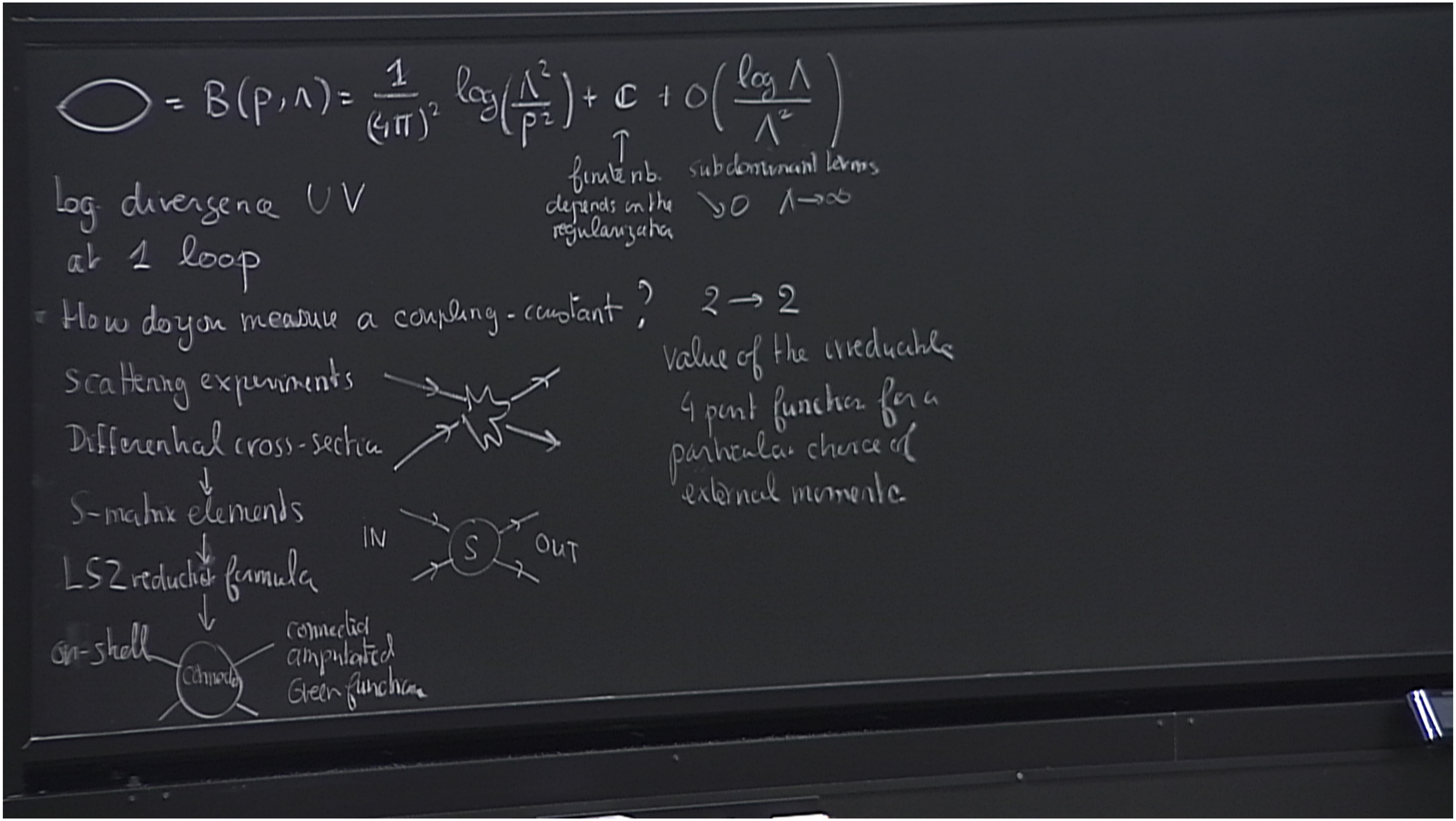
Green function


= B(p, Λ) = 1/(4π)² log(Λ²/p²) + C + O(log Λ/Λ²)
finite nb. depends on the regularization
subdominant terms ↘0 Λ→∞
log divergence UV
at 1 loop
How do you measure a coupling-constant ?
2 → 2
Value of the irreducible 4 point function for a particular choice of external momenta
Scattering experiments
Differential cross-section
S-matrix elements
LSZ reduction formula
IN
S
OUT
on-shell
connected amputated Green functions

 $= B(p,\Lambda) = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + \mathbb{C} + O\left(\frac{\log \Lambda}{\Lambda^2}\right)$

$\mathbb{C}$: finite nb. depends on the regularization

$O\left(\frac{\log \Lambda}{\Lambda^2}\right)$: subdominant terms $\searrow 0$ $\Lambda \to \infty$

log divergence UV
at 1 loop

- How do you measure a coupling-constant? $2 \to 2$

Value of the irreducible 4 point function for a particular choice of external momenta (experimental choice)

Scattering experiments

Differential cross-section

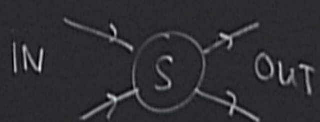
$\downarrow$

S-matrix elements

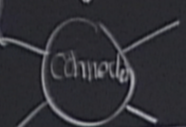
$\downarrow$

LSZ reduction formula

$\downarrow$



on-shell



connected amputated Green function

choice of reference or renormalization momenta.

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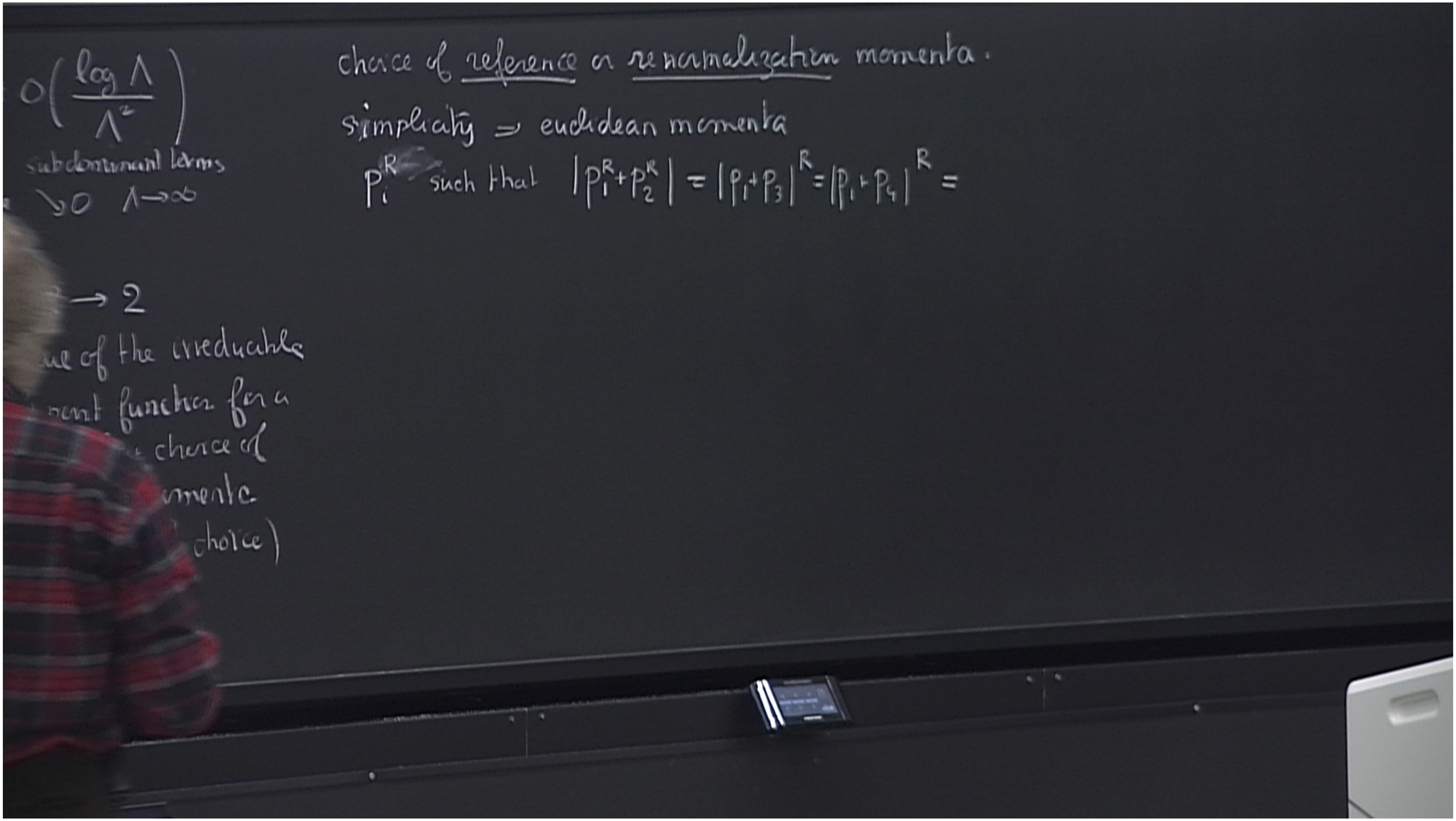
choice of reference or renormalization momenta.

simplicity $\Rightarrow$ euclidean momenta


choice of reference or renormalization momenta.
simplicity ⇒ euclidean momenta
p_i^R such that $|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R =$
$O\left(\frac{\log \Lambda}{\Lambda^2}\right)$
subdominant terms
$\searrow 0 \quad \Lambda \to \infty$
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function for a
choice of
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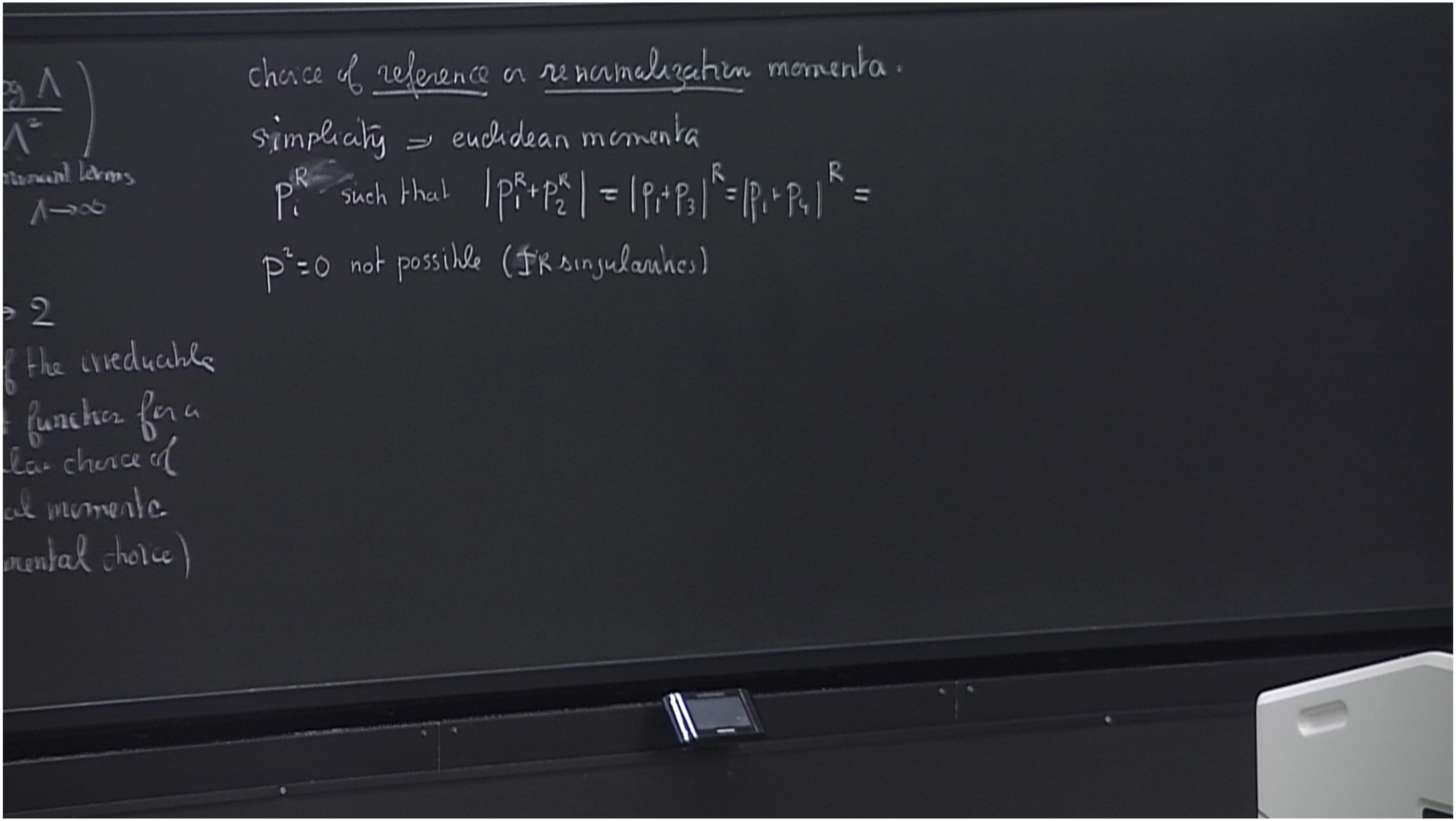
choice of reference or renormalization momenta.
simplicity ⇒ euclidean momenta
p_i^R such that $|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R =$
$p^2=0$ not possible (IR singularities)
$\Lambda \to \infty$
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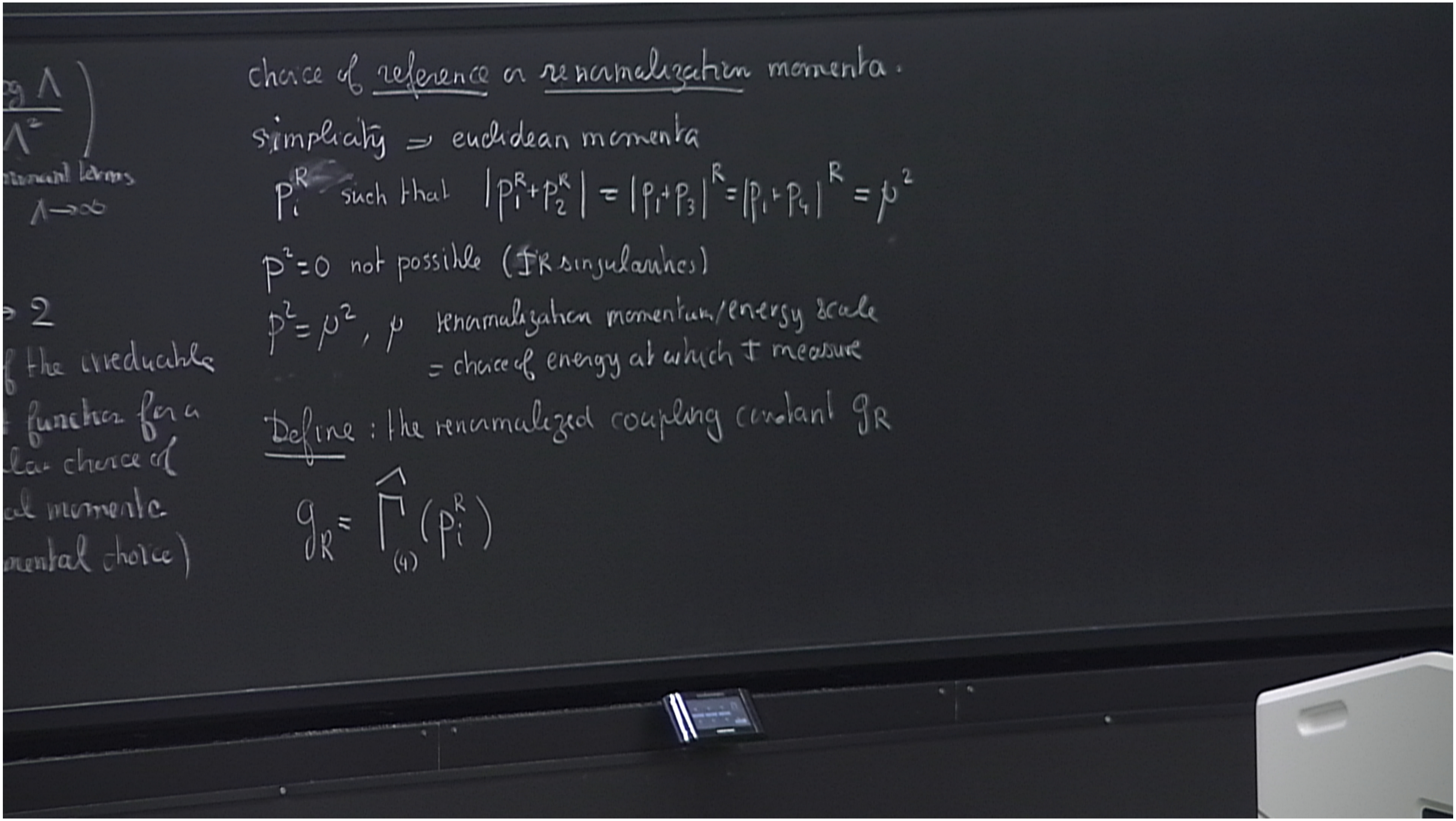
choice of <u>reference</u> or <u>renormalization</u> momenta.

simplicity ⇒ euclidean momenta

p_i^R such that $|p_1^R + p_2^R| = |p_1 + p_3|^R = |p_1 + p_4|^R = \mu^2$

$p^2 = 0$ not possible (IR singularities)

$p^2 = \mu^2$ μ renormalization momentum/energy scale
= choice of energy at which I measure


choice of reference or renormalization momenta.
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p_i^R such that $|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$
$p^2=0$ not possible (IR singularities)
$p^2=\mu^2$, μ renormalization momentum/energy scale
= choice of energy at which I measure
Define : the renormalized coupling constant g_R
$g_R = \hat{\Gamma}_{(4)}(p_i^R)$
the irreducible
function for a
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$\Lambda \to \infty$

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simplicity ⇒ euclidean momenta

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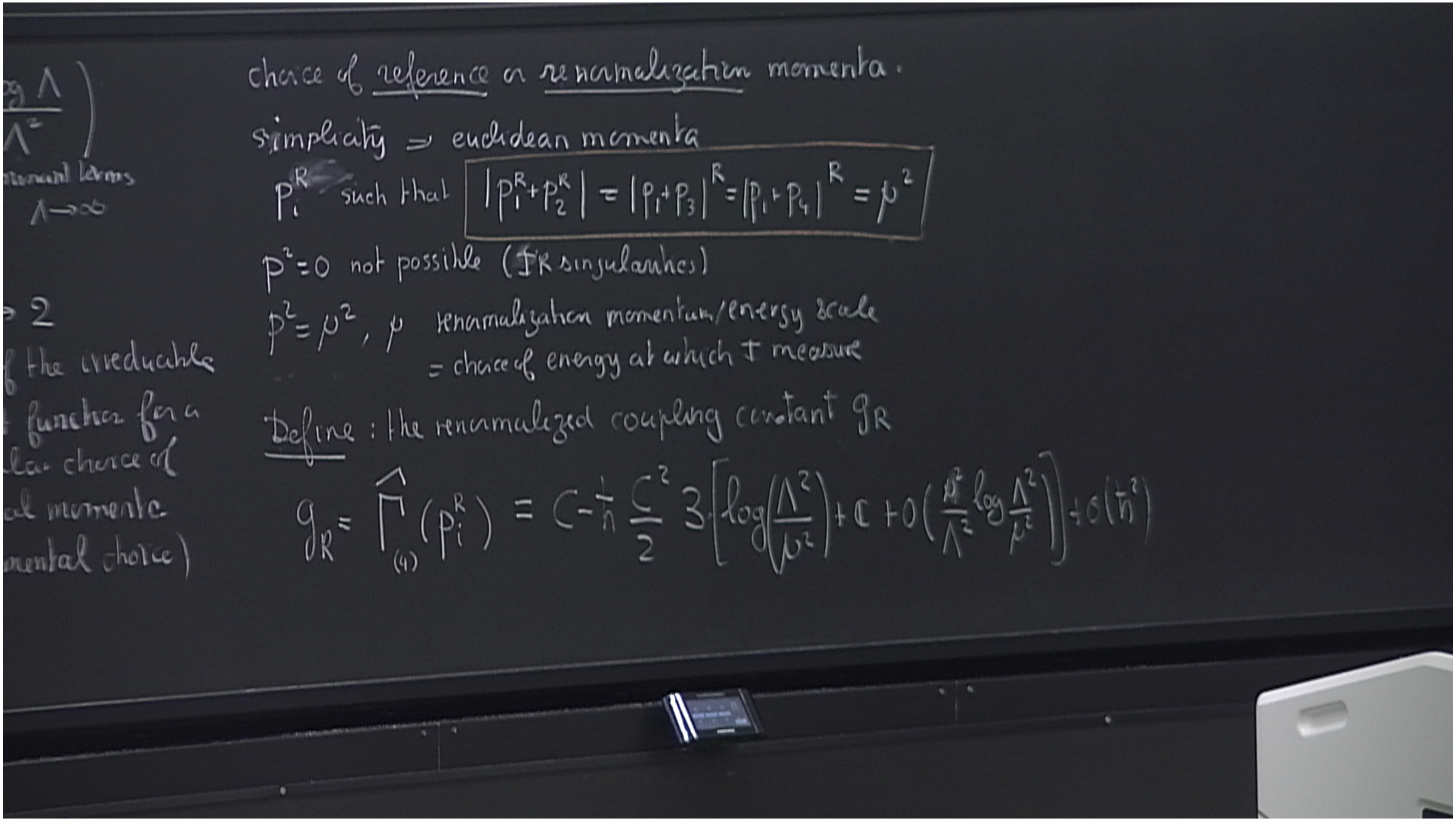
Define: the renormalized coupling constant g_R

$$g_R = \hat{\Gamma}_{(4)}(p_i^R) =$$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Define : the renormalized coupling constant g_R
$g_R = \hat{\Gamma}_{(4)}(p_i^R) = C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + C + O\left(\frac{\mu^2}{\Lambda^2} \log \frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$
the irreducible
function for a
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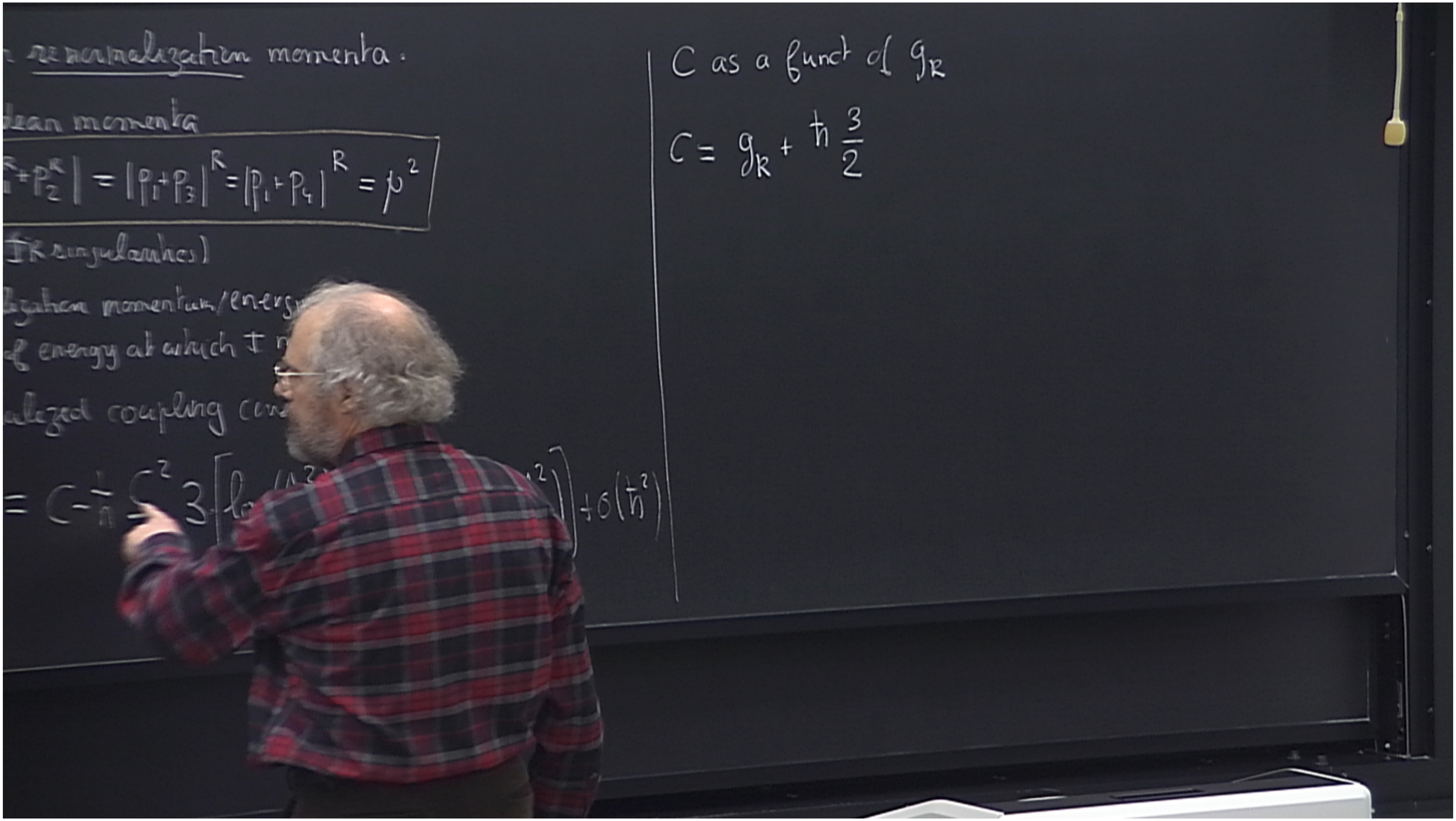
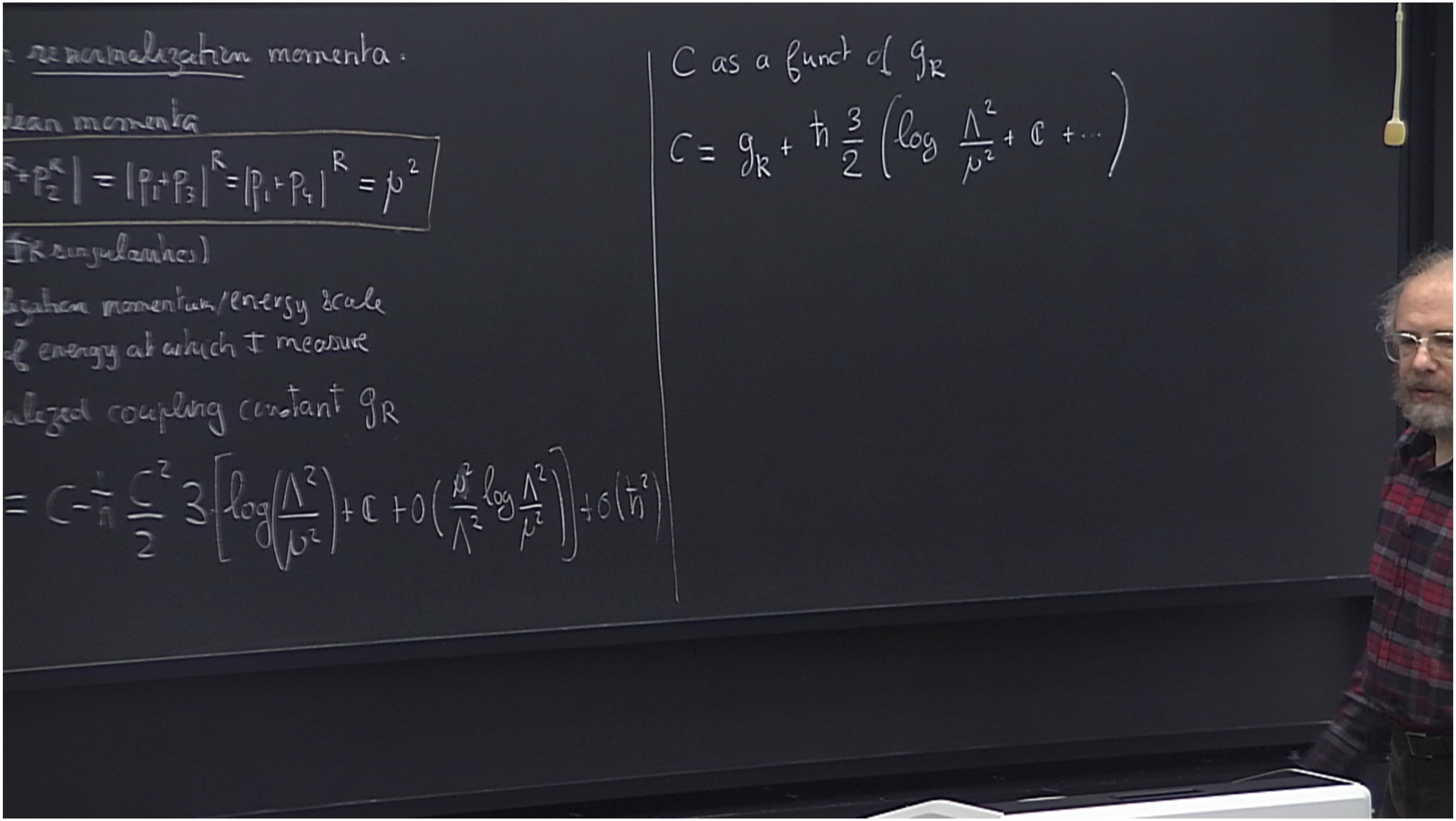
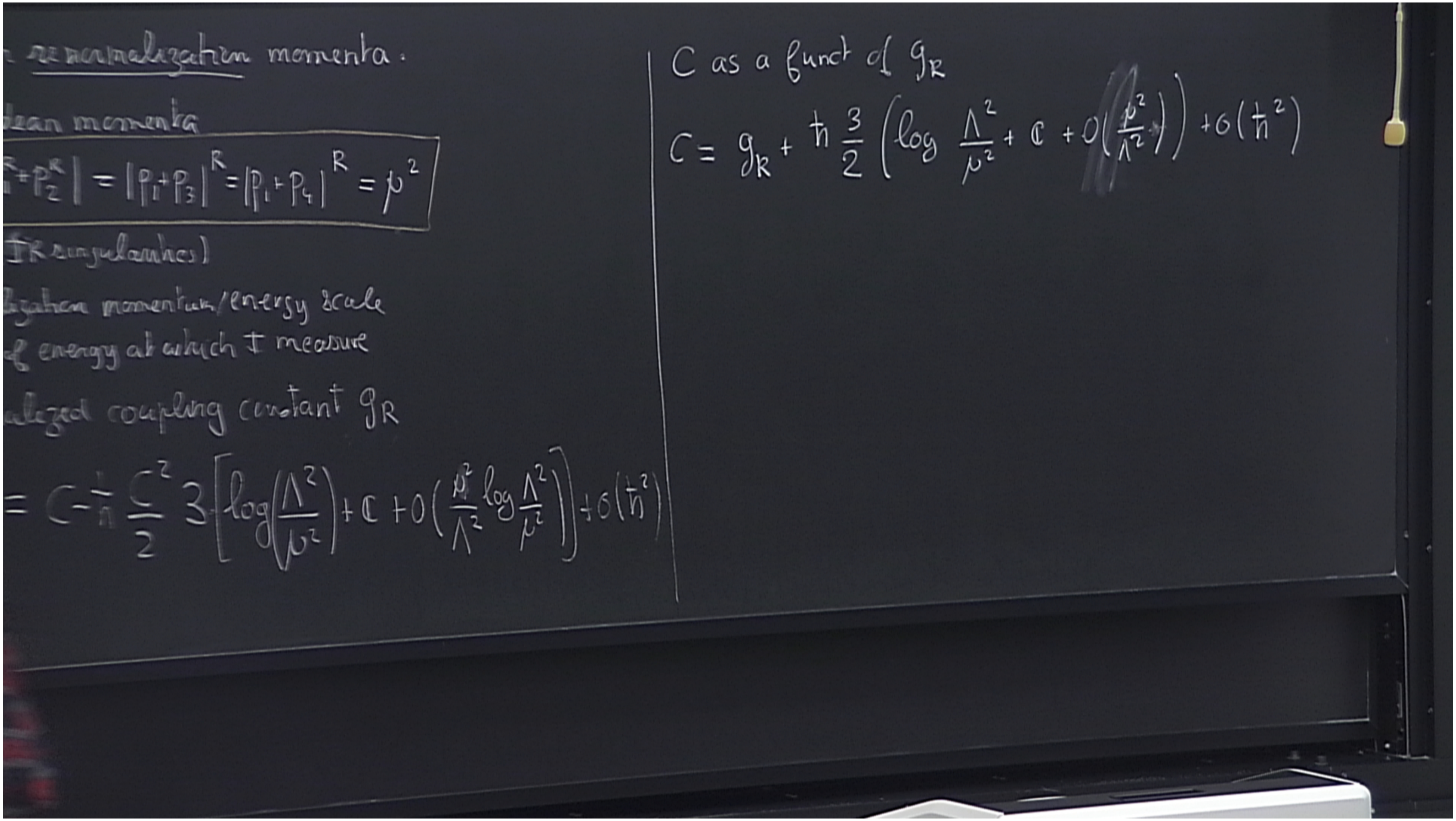

renormalization momenta.
C as a funct of g_R
C = g_R + ħ 3/2


renormalization momenta.
dean momenta
$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$
IR singularities)
lization momentum/energy scale
of energy at which I measure
alized coupling constant g_R
$= C - \frac{1}{\hbar}\frac{C^2}{2}3\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$
C as a funct of g_R
$C = g_R + \hbar\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + \cdots\right)$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C as a funct of g_R
$C = g_R + \hbar\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$

... renormalization momenta.

...dean momenta

$$|p_1^R + p_2^R| = |p_1 + p_3|^R = |p_1 + p_4|^R = \mu^2$$

(IR singularities)

...lization momentum/energy scale

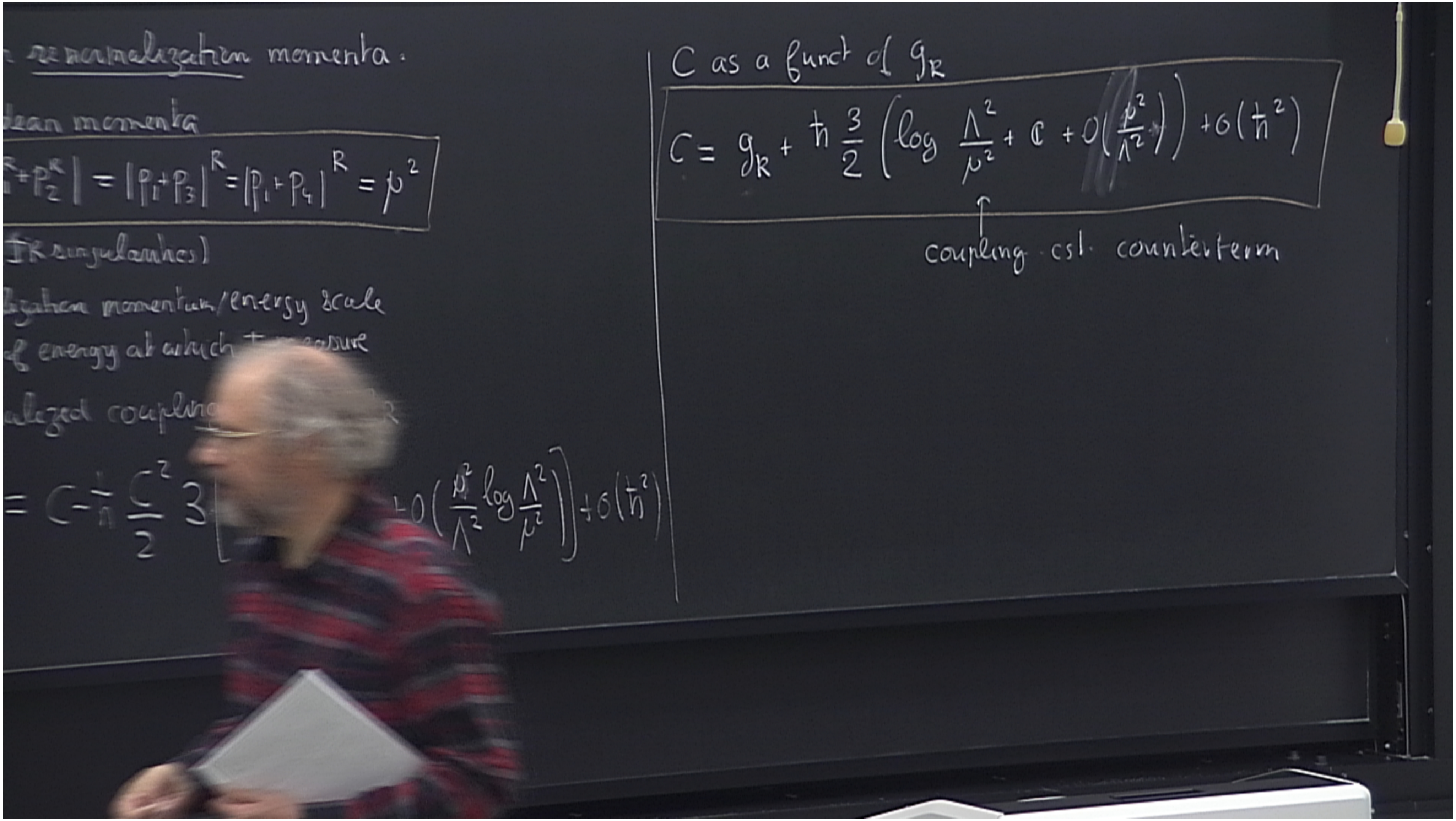
...of energy at which I measure

...alized coupling constant g_R

$$= C - \frac{1}{\hbar}\frac{C^2}{2}\, 3\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar\,\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$


renormalization momenta.
C as a funct of g_R
$C = g_R + \hbar \frac{3}{2}\left(\log \frac{\Lambda^2}{\mu^2} + c + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$
coupling cst. counterterm
(IR singularities)
momentum/energy scale
of energy at which
coupling

choice of reference or renormalization momenta.

simplicity ⇒ euclidean momenta

p_i^R such that $\boxed{|p_1^R + p_2^R| = |p_1 + p_3|^R = |p_1 + p_4|^R = \mu^2}$

$p^2 = 0$ not possible (IR singularities)

$p^2 = \mu^2$, μ renormalization momentum/energy scale
= choice of energy at which I measure

Define: the renormalized coupling constant g_R

$$g_R = \hat{\Gamma}_{(4)}(p_i^R) = C - \frac{1}{\hbar}\frac{C^2}{2}3\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + c + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

C as a funct of g_R

$$\boxed{C = g_R + \hbar\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2}\right.}$$

couplin

renormalization momenta:

...dean momenta

$$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$$

(IR singularities)

...ization momentum/energy scale

of energy at which I measure

...alized coupling constant g_R

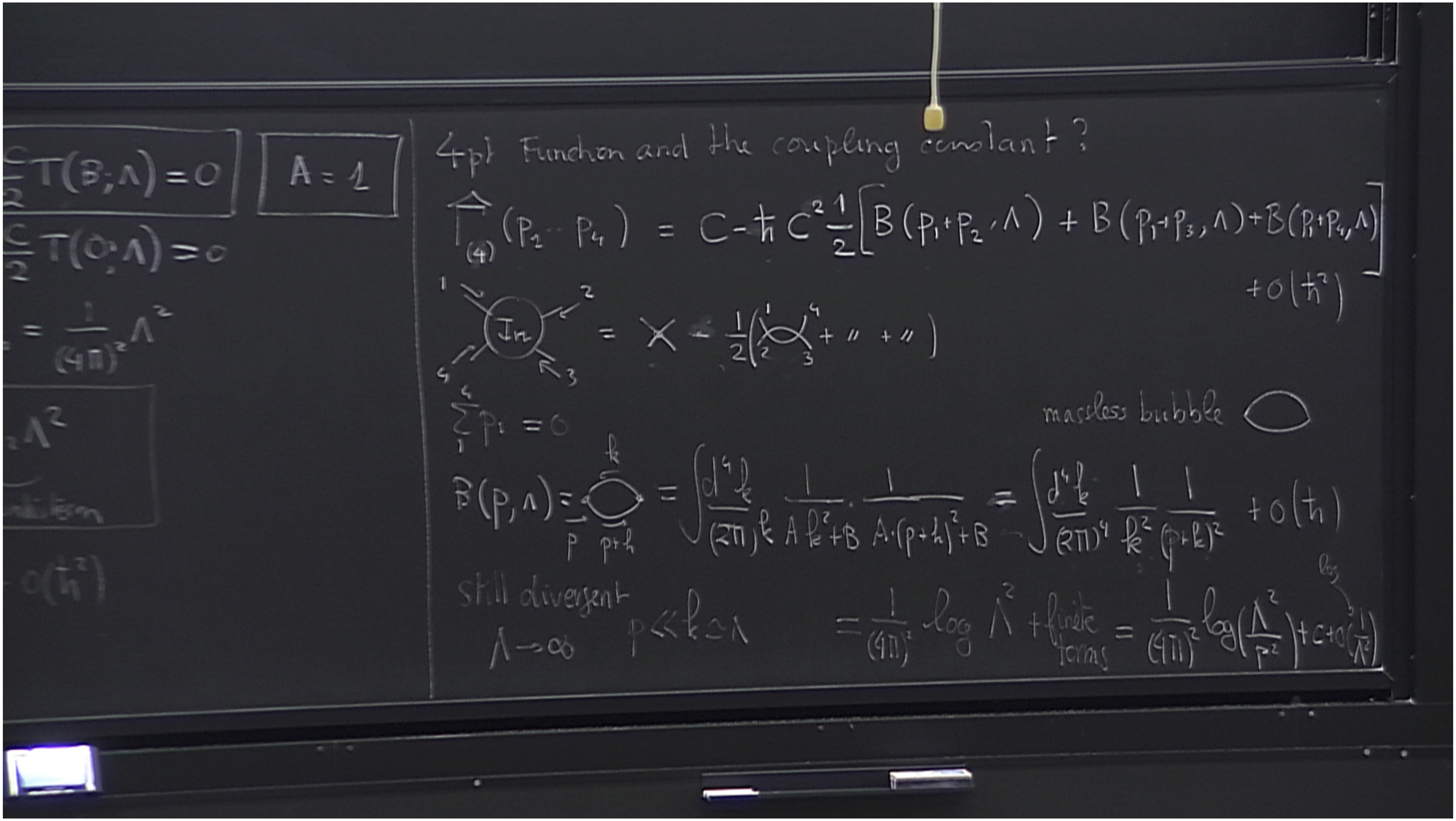
$$= C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2} \log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar \frac{3}{2} \left(\log \frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$

↑ coupling cst. counterterm

reexpress the 4-pt. irreducible function in term of g_R


4pt Function and the coupling constant ?
$\hat{\Gamma}_{(4)}(p_1 \cdots p_4) = C - \hbar C^2 \frac{1}{2}\left[B(p_1+p_2,\Lambda) + B(p_1+p_3,\Lambda) + B(p_1+p_4,\Lambda)\right] + O(\hbar^2)$
$\sum_1^4 p_i = 0$
massless bubble
$B(p,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{A k^2 + B} \cdot \frac{1}{A\cdot(p+k)^2 + B} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} \frac{1}{(p+k)^2} + O(\hbar)$
still divergent
$\Lambda \to \infty$
$p \ll k \simeq \Lambda$
$= \frac{1}{(4\pi)^2} \log \Lambda^2 + \text{finite terms} = \frac{1}{(4\pi)^2} \log\left(\frac{\Lambda^2}{p^2}\right) + c + O\left(\frac{1}{\Lambda^2}\right)$
A = 1
$\frac{c}{2} T(B;\Lambda) = 0$
$\frac{c}{2} T(0;\Lambda) = 0$
$= \frac{1}{(4\pi)^2}\Lambda^2$
$O(\hbar^2)$

renormalization momenta.

Euclidean momenta

$$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$$

(IR singularities)

renormalization momentum/energy scale

of energy at which I measure

coupling constant g_R

$$3\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

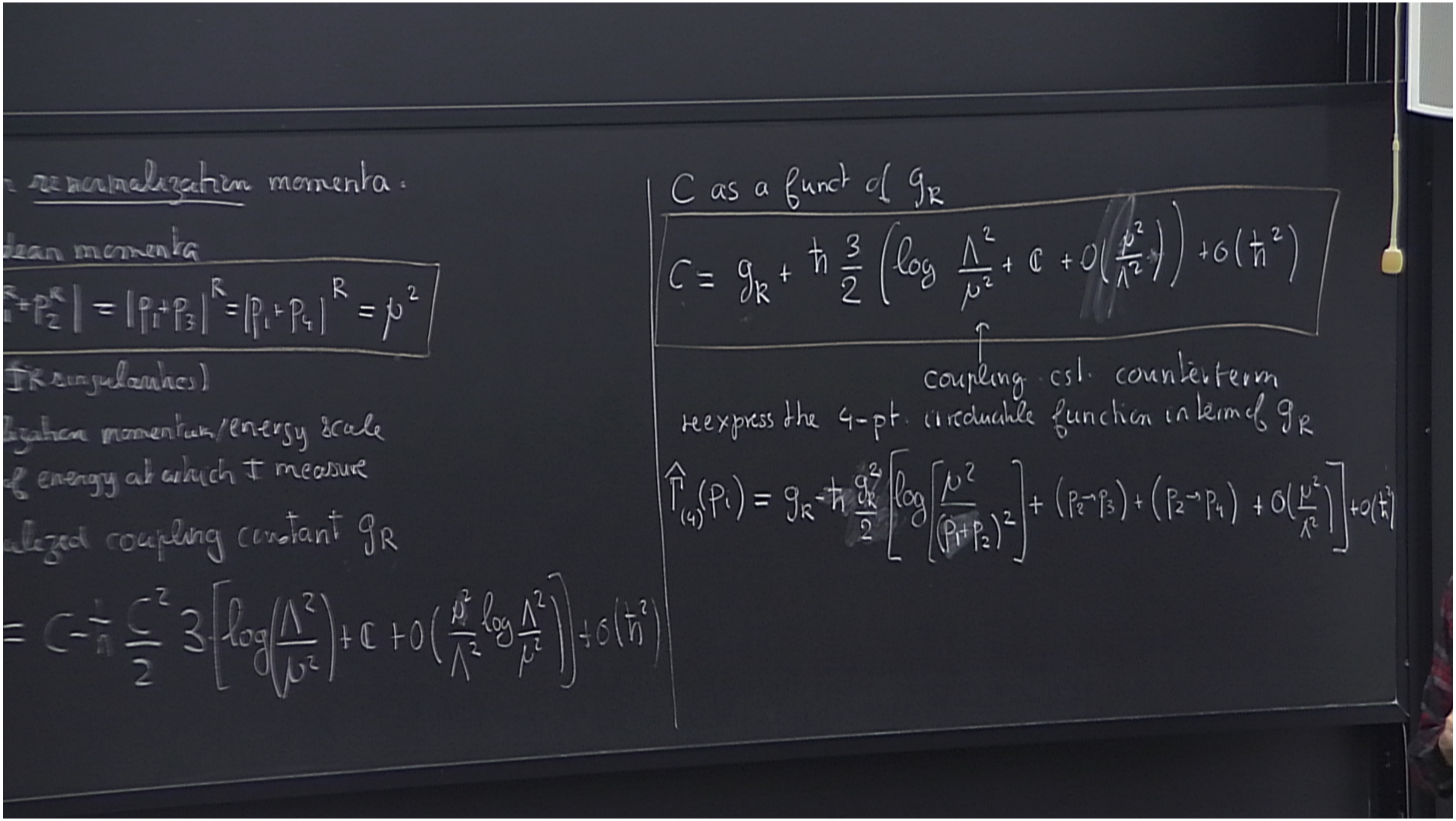
C as a funct of g_R

$$C = g_R + \hbar\,\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$

coupling cst. counterterm

reexpress the 4-pt. irreducible function in terms of g_R

$$\hat{\Gamma}_{(4)}(p_i) = g_R$$



renormalization momenta:
dean momenta
$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$
(IR singularities)
zation momentum/energy scale
of energy at which I measure
alized coupling constant g_R
$= C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$
C as a funct of g_R
$C = g_R + \hbar \frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$
coupling cst. counterterm
reexpress the 4-pt. irreducible function in term of g_R
$\hat{\Gamma}_{(4)}(p_i) = g_R - \hbar\frac{g_R^2}{2}\left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2\to p_3) + (p_2\to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$

choice of reference or renormalization momenta.

simplicity ⇒ euclidean momenta

p_i^R such that $|p_1^R + p_2^R| = |p_1 + p_3|^R = |p_1 + p_4|^R = \mu^2$

$p^2 = 0$ not possible (IR singularities)

$p^2 = \mu^2$, μ renormalization momentum/energy scale
= choice of energy at which I measure

Define: the renormalized coupling constant g_R

$$g_R = \hat{\Gamma}_{(4)}(p_i^R) = C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + c + O\left(\frac{\mu^2}{\Lambda^2} \log \frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

$$C = g_R + \hbar * C^2 = g_R + \hbar * g_R^2 + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar \frac{3}{2} \left(\log \frac{\Lambda^2}{\mu^2} + \right.$$

coupling

reexpress the 4-pt. irreducib

$$\hat{\Gamma}_{(4)}(p_i) = g_R - \hbar \frac{g_R^2}{2} \left[\log \left[\frac{\mu^2}{(p_1 + p_2)^2} \right. \right.$$

$\boxed{\frac{c}{2} T(B;\Lambda) = 0}$ $\boxed{A = 1}$

$\frac{c}{2} T(0;\Lambda) = 0$

$= \frac{1}{(4\pi)^2}\Lambda^2$

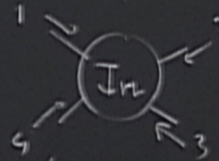
Λ^2

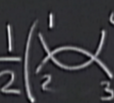
condition

$o(\hbar^2)$

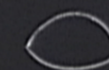
4pt Function and the coupling constant?

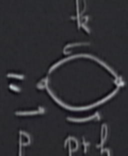
$$\hat{\Gamma}_{(4)}(p_1 \dots p_4; c, \Lambda) = c - \hbar c^2 \frac{1}{2}\left[B(p_1+p_2,\Lambda) + B(p_1+p_3,\Lambda) + B(p_1+p_4,\Lambda)\right] + o(\hbar^2)$$



$= \times - \frac{1}{2}\Big($  $+ \text{''} + \text{''}\Big)$

$\sum_{i=1}^{4} p_i = 0$

massless bubble 

$B(p,\Lambda) =$ 

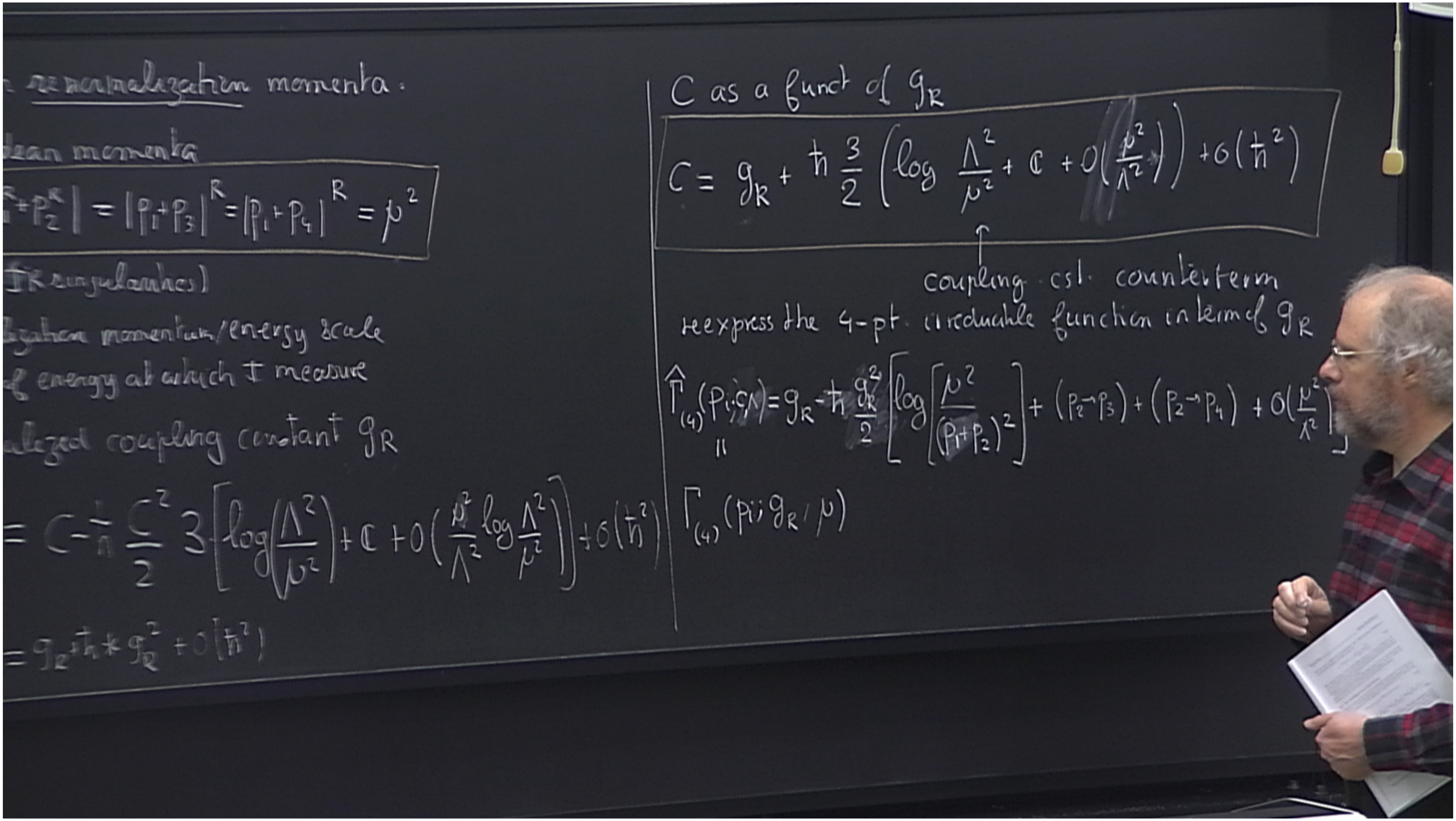
$$= \int \frac{d^4k}{(2\pi)^4} \frac{1}{Ak^2+B}\cdot\frac{1}{A(p+k)^2+B} = \int \frac{d^4k}{(2\pi)^4}\frac{1}{k^2}\frac{1}{(p+k)^2} + o(\hbar)$$

still divergent

$\Lambda \to \infty$ $\quad p \ll k \simeq \Lambda$

$$= \frac{1}{(4\pi)^2}\log\Lambda^2 + \text{finite terms} = \frac{1}{(4\pi)^2}\log\left(\frac{\Lambda^2}{p^2}\right) + c + o\left(\frac{1}{\Lambda^2}\right)$$

log


renormalization momenta.
C as a funct of g_R
$C = g_R + \hbar \frac{3}{2} \left(\log \frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right) \right) + O(\hbar^2)$
coupling cst. counterterm
reexpress the 4-pt. irreducible function in term of g_R
$\Gamma_{(4)}(p_i; g_R, \mu)$
energy at which I measure
coupling constant g_R
$= g_R + \hbar * g_R^2 + O(\hbar^2)$

renormalization momenta.

...dean momenta

$$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$$

(IR singularities)

...zation momentum/energy scale

...of energy at which I measure

...lized coupling constant g_R

$$= C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

$$* g_R^2 + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar \frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$

↑ coupling cst. counterterm

reexpress the 4-pt. irreducible function in terms of g_R

$$\hat{\Gamma}_{(4)}(p_i; C, \Lambda) = g_R - \hbar\frac{g_R^2}{2}\left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2 \to p_3) + (p_2 \to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$$

$$\| $$

$$\hat{\Gamma}^R_{(4)}(p_i; g_R, \mu)$$ renormalized 4-pt function

renormalization momenta.

...dean momenta

$$|p_1^R + p_2^R| = |p_1 + p_3|^R = |p_1 + p_4|^R = \mu^2$$

(IR singularities)

...ization momentum/energy scale

...of energy at which I measure

...alized coupling constant g_R

$$= C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2} \log \frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

$$= g_R + \hbar * g_R^2 + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar \frac{3}{2} \left(\log \frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$

↑ coupling cst. counterterm

reexpress the 4-pt. irreducible function in term of g_R

$$\hat{\Gamma}_{(4)}(p_i; C, \Lambda) = g_R - \hbar \frac{g_R^2}{2} \left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2 \to p_3) + (p_2 \to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$$

$$\parallel$$

$$\hat{\Gamma}^R_{(4)}(p_i; g_R, \mu)$$ renormalized 4-pt function

... renormalization momenta.

... lean momenta

$$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$$

(IR singularities)

...ization momentum/energy scale

... of energy at which I measure

...alized coupling constant g_R

$$= C - \hbar \frac{C^2}{2}\, 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$$

$$= g_R + \hbar * g_R^2 + O(\hbar^2)$$

C as a funct of g_R

$$C = g_R + \hbar \frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$$

↑ coupling cst. counterterm

reexpress the 4-pt. irreducible function in term of g_R

$$\hat{\Gamma}_{(4)}(p_i; g, \Lambda) = g_R - \hbar \frac{g_R^2}{2}\left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2 \to p_3) + (p_2 \to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$$

$$\| $$

$$\hat{\Gamma}^R_{(4)}(p_i; g_R, \mu) \quad \text{renormalized 4-pt function}$$

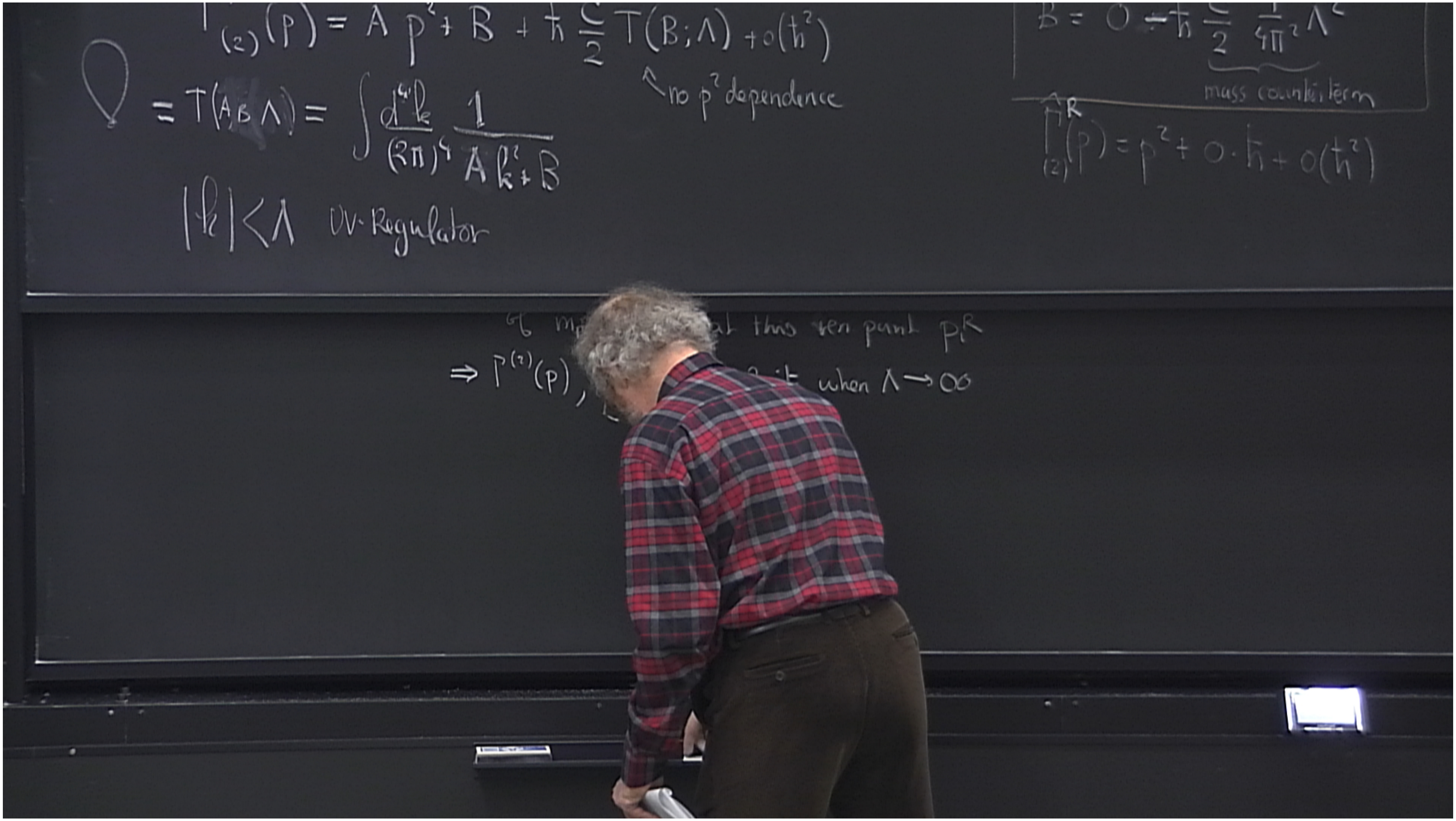
finite UV limit $\Lambda \to \infty$, g_R as fixed regulator

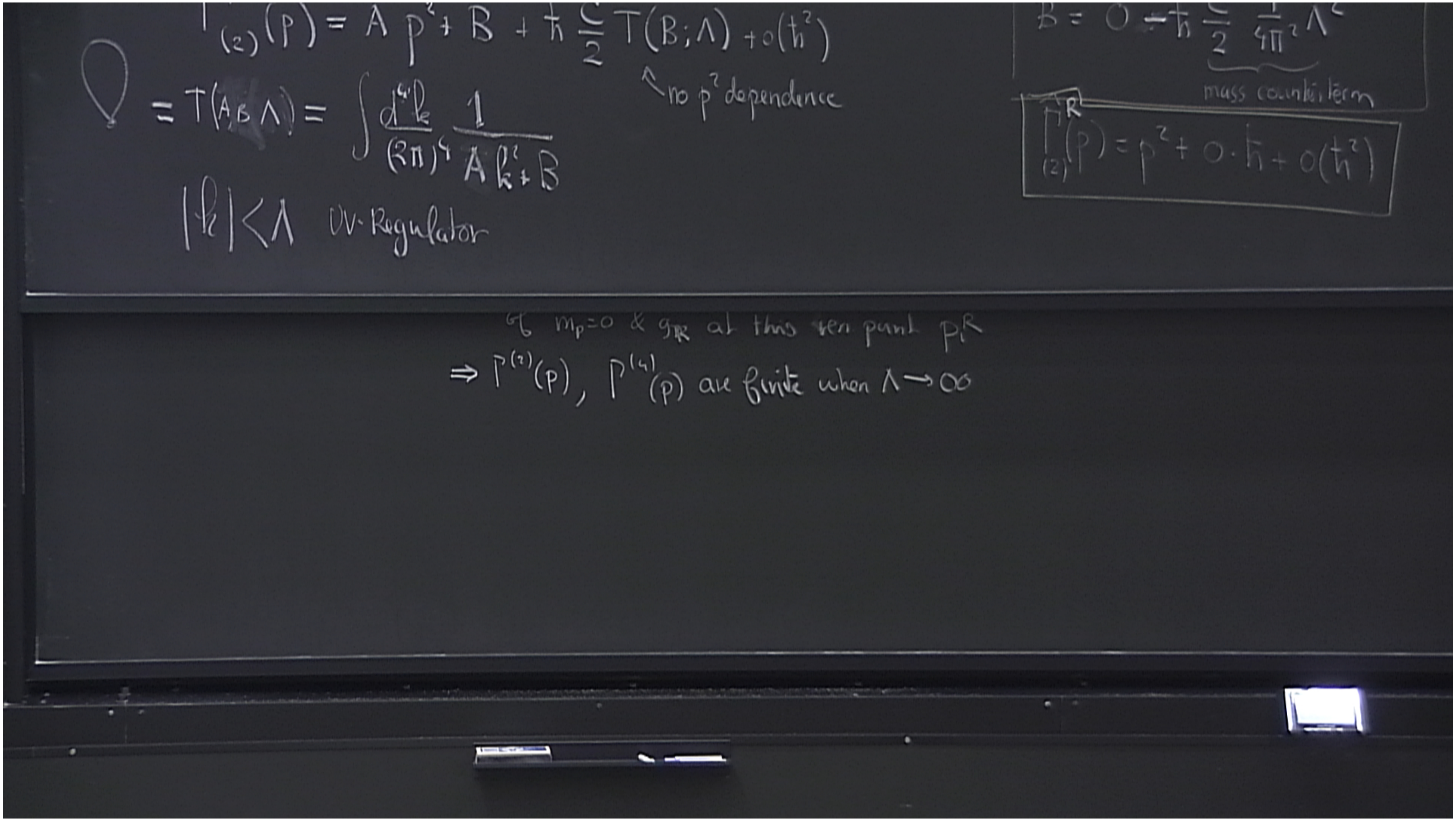
Renormalization recipe: - chose a renorm. scale μ, g_R

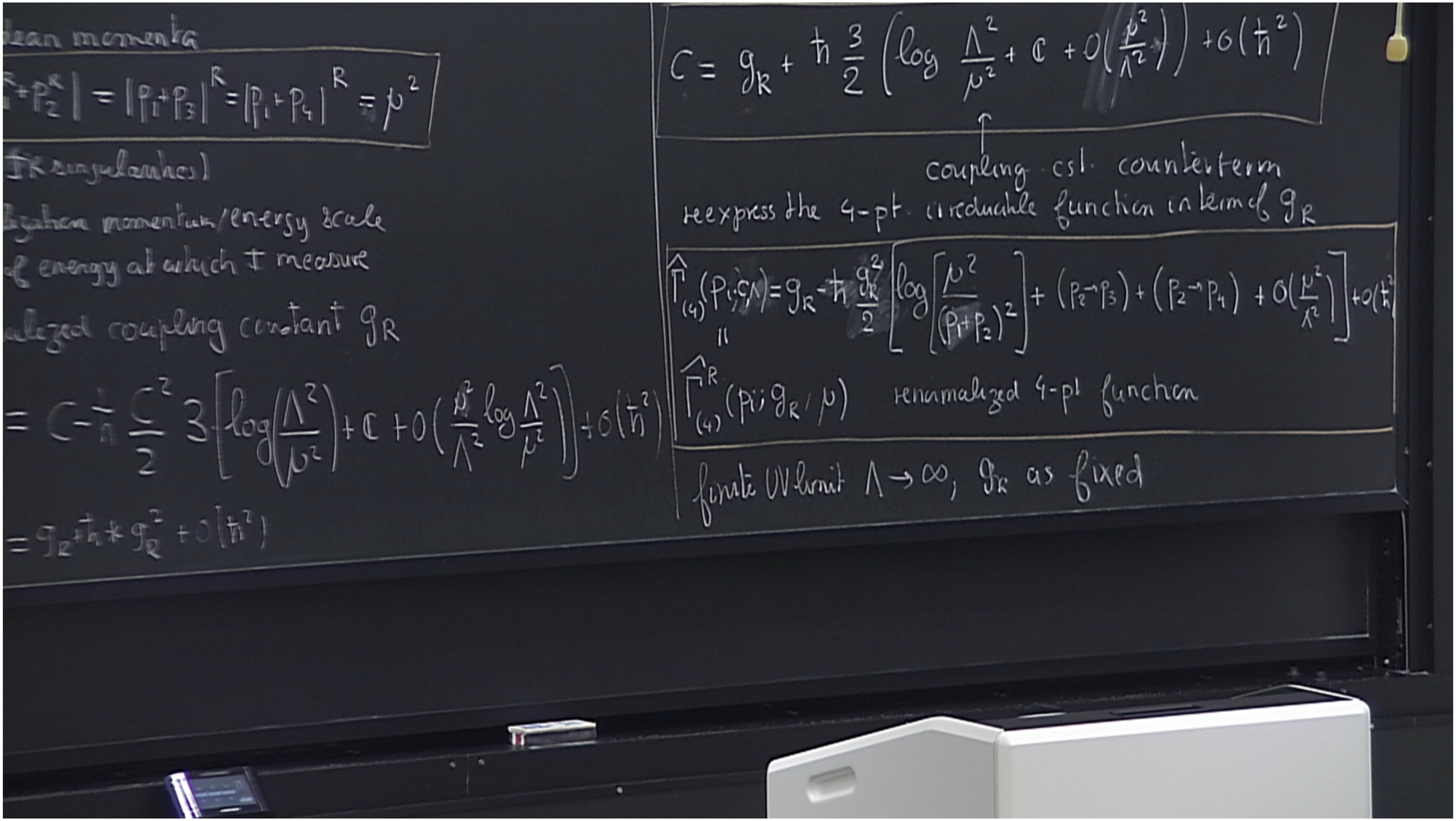
chose a physical mass $m_{phys} = 0$

- adjust the A, B, C in $S[\phi]$ so that $\Gamma^{(2)}$ and $\Gamma^{(4)}$ are defined as a function of $m_p = 0$ & g_R at this ren. point p_i^R

$\Rightarrow \Gamma^{(2)}(p)$, $\Gamma^{(4)}(p)$ are finite when $\Lambda \to \infty$


$\Gamma_{(2)}(p) = A p^2 + B + \hbar \frac{c}{2} T(B;\Lambda) + o(\hbar^2)$
no p^2 dependence
$= T(A,B,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{A k^2 + B}$
$|k| < \Lambda$ UV-Regulator
mass counterterm
$\Gamma^R_{(2)}(p) = p^2 + 0 \cdot \hbar + o(\hbar^2)$
when $\Lambda \to \infty$


$\Gamma_{(2)}(p) = A\,p^2 + B + \hbar \frac{c}{2} T(B;\Lambda) + o(\hbar^2)$
no p^2 dependence
$= T(A,B,\Lambda) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{A k^2 + B}$
$|k| < \Lambda$ UV-Regulator
$B = 0 - \hbar \frac{c}{2} \frac{1}{4\pi^2} \Lambda^2$
mass counterterm
$\Gamma^R_{(2)}(p) = p^2 + 0 \cdot \hbar + o(\hbar^2)$
of $m_R = 0$ & g_R at this ren. point p_i^R
$\Rightarrow \Gamma^{(2)}(p)$, $\Gamma^{(4)}(p)$ are finite when $\Lambda \to \infty$


lean momenta
$|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$
IR singularities)
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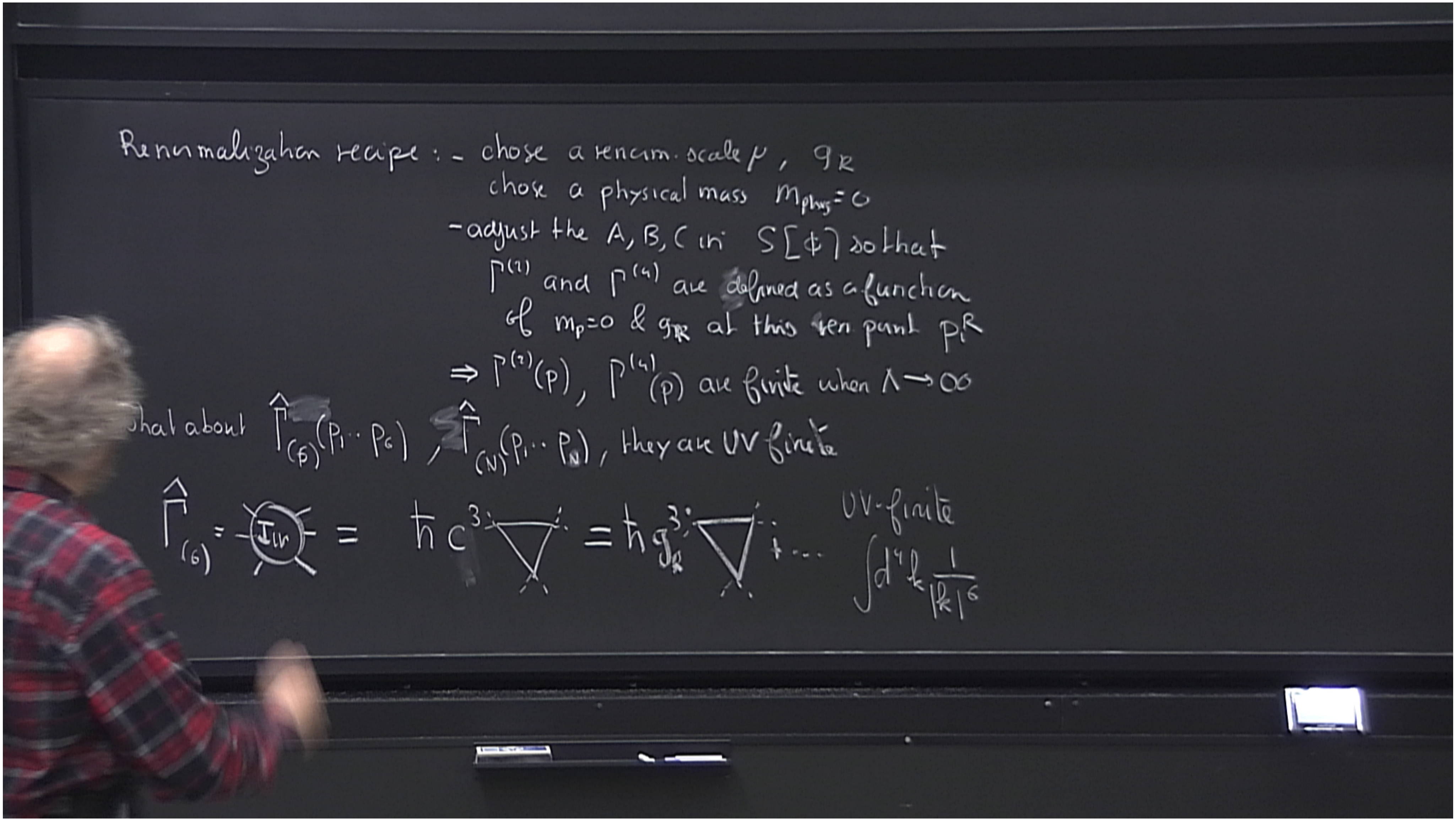
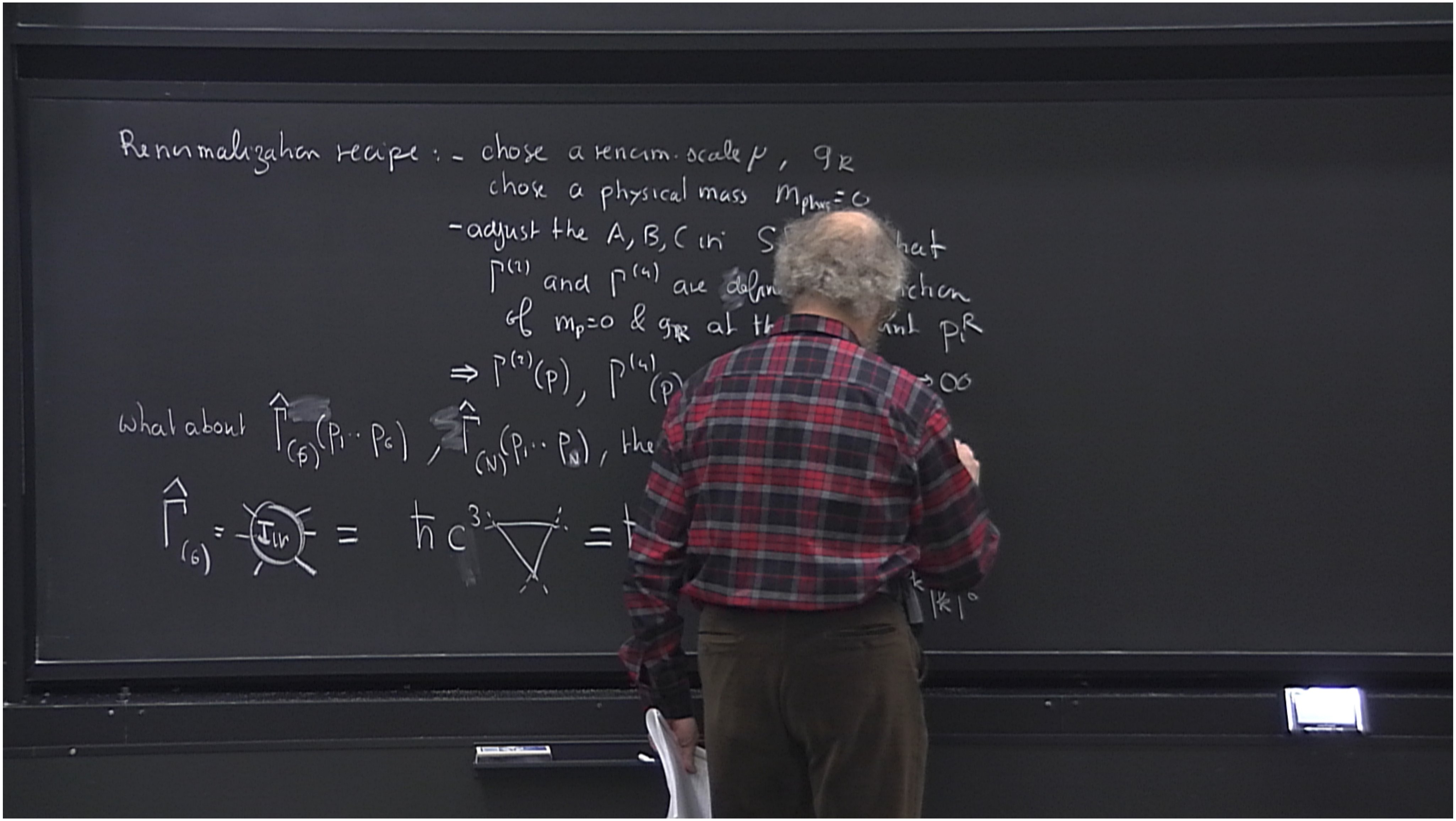
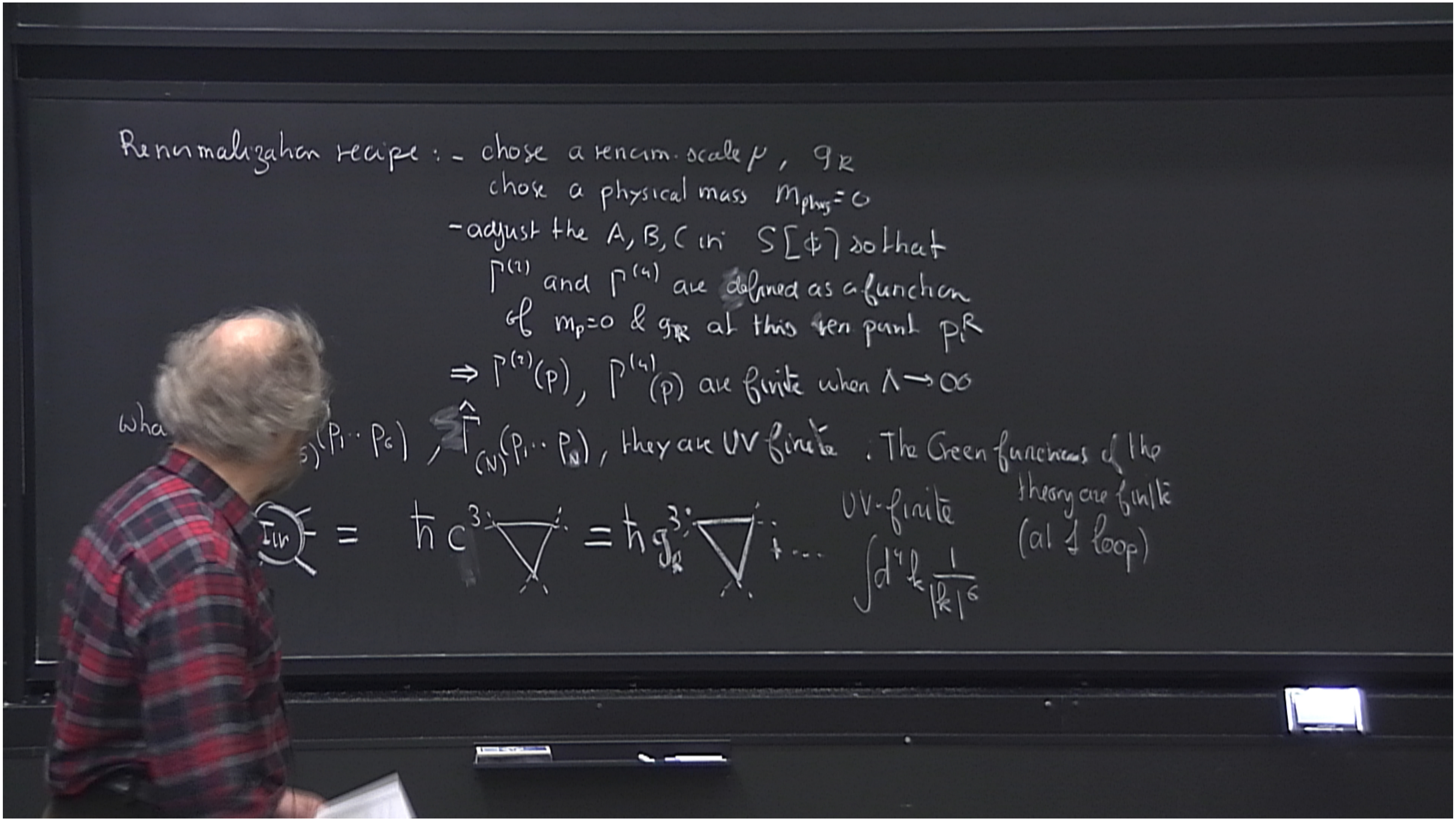
$= C - \hbar \frac{C^2}{2} 3 \left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$
$= g_R + \hbar * g_R^2 + O(\hbar^2)$
$C = g_R + \hbar \frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$
coupling cst. counterterm
reexpress the 4-pt. irreducible function in terms of g_R
$\hat{\Gamma}_{(4)}(p_i; C, \Lambda) = g_R - \hbar \frac{g_R^2}{2}\left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2 \to p_3) + (p_2 \to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$
$\hat{\Gamma}^R_{(4)}(p_i; g_R, \mu)$ renormalized 4-pt function
finite UV limit $\Lambda \to \infty$, g_R as fixed


Renormalization recipe: - chose a renorm. scale μ, g_R
chose a physical mass m_phys = 0
- adjust the A, B, C in S[Φ] so that
Γ^(2) and Γ^(4) are defined as a function
of m_p = 0 & g_R at this ren. point p_i^R
⇒ Γ^(2)(p), Γ^(4)(p) are finite when Λ → ∞
What about Γ̂_(6)(p_1 .. p_6), Γ̂_(N)(p_1 .. p_N), they are UV finite
Γ̂_(6) =
= ħ c^3
= ħ g_R^3
+ ...
UV-finite
∫d^4k 1/|k|^6


Renormalization recipe: - chose a renorm. scale μ, g_R
chose a physical mass m_phys = 0
- adjust the A, B, C in S
Γ^(2) and Γ^(4) are
of m_p = 0 & g_R at the
⇒ Γ^(2)(p), Γ^(4)(p)
what about Γ̂_(6)(p_1 .. p_6), Γ̂_(N)(p_1 .. p_N), the
Γ̂_(6) = = ħ c^3 =


Renormalization recipe: - chose a renorm. scale μ, g_R
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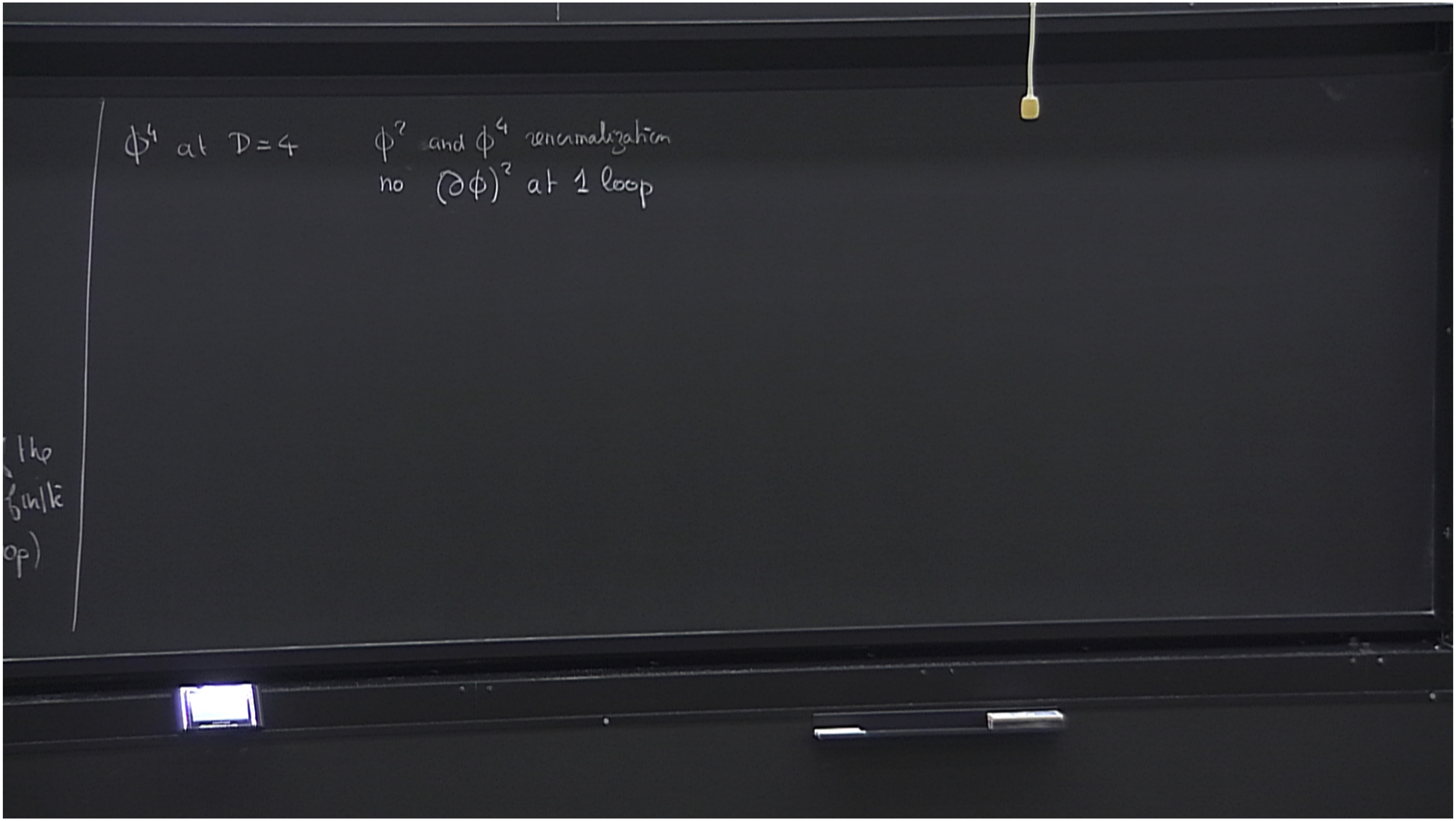
- adjust the A, B, C in S[ϕ] so that
Γ^(2) and Γ^(4) are defined as a function
of m_p = 0 & g_R at this ren. point p_i^R
⇒ Γ^(2)(p), Γ^(4)(p) are finite when Λ → ∞
wha
(p_1 .. p_6), Γ̂_(N)(p_1 .. p_N), they are UV finite. The Green functions of the theory are finite (at 1 loop)
= ħ c^3 = ħ g_R^3 + ...
UV-finite
∫d^4k 1/|k|^6

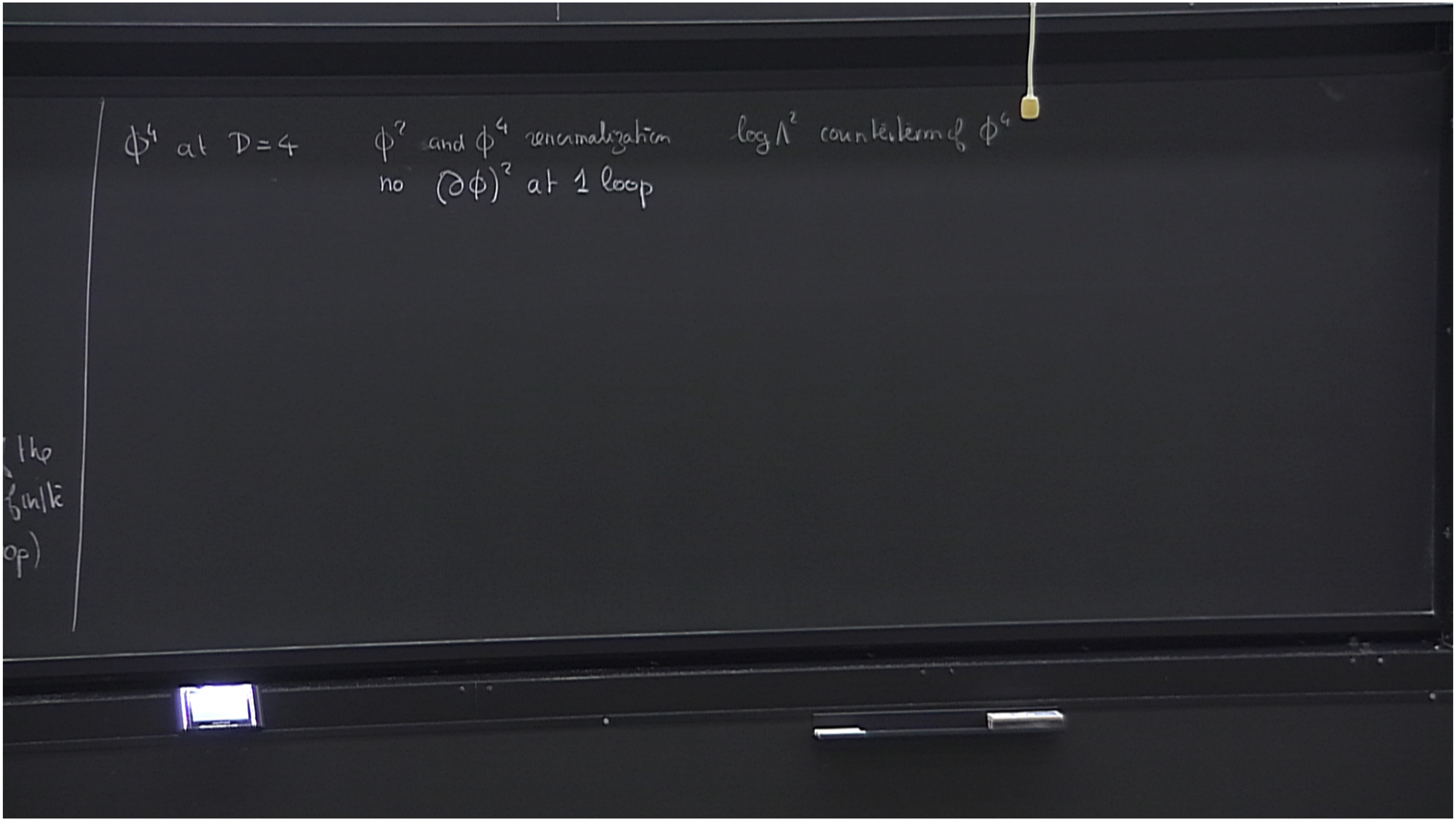
ϕ^4 at $D=4$ ϕ^2 and ϕ^4 renormalization

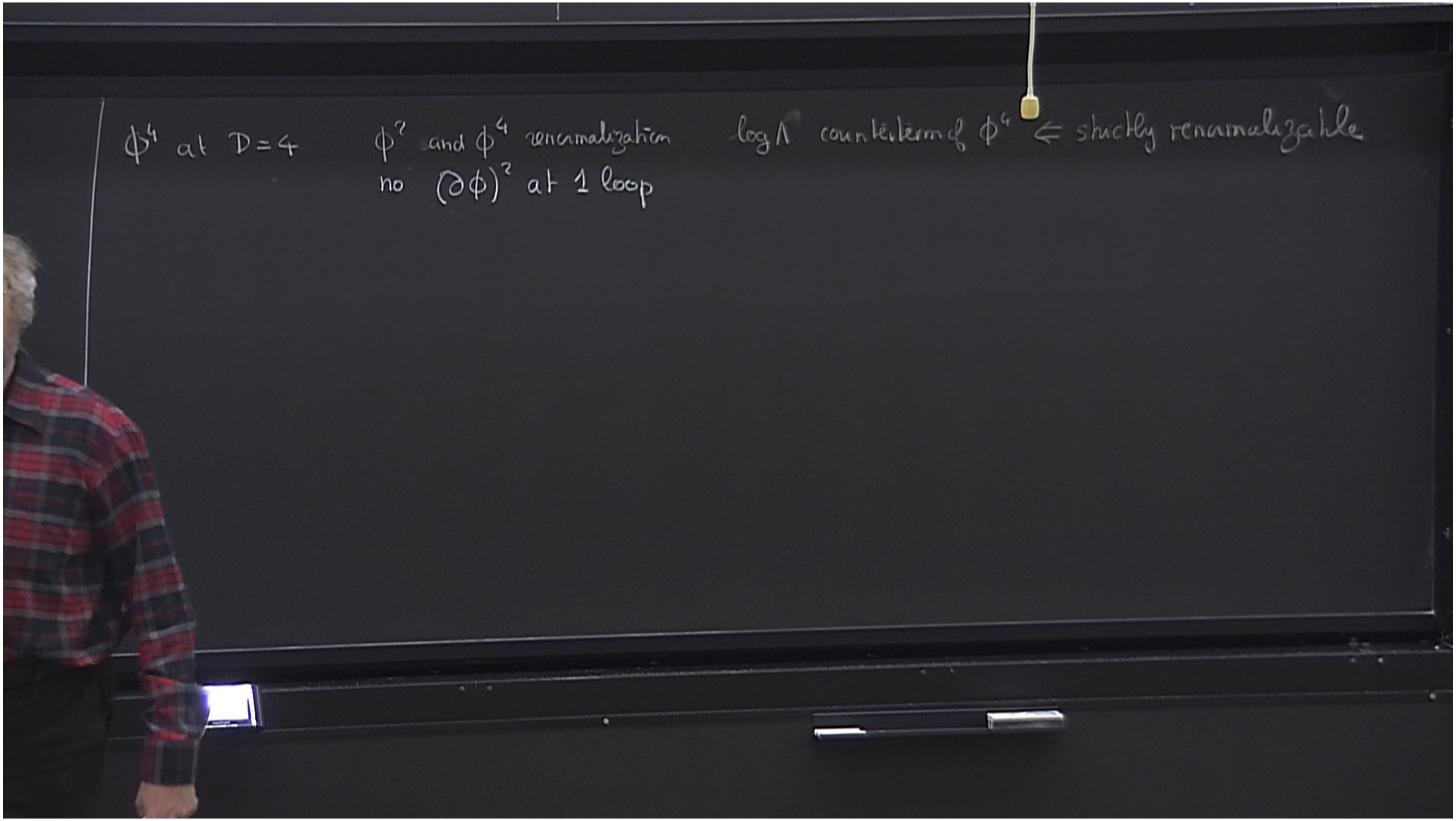
the

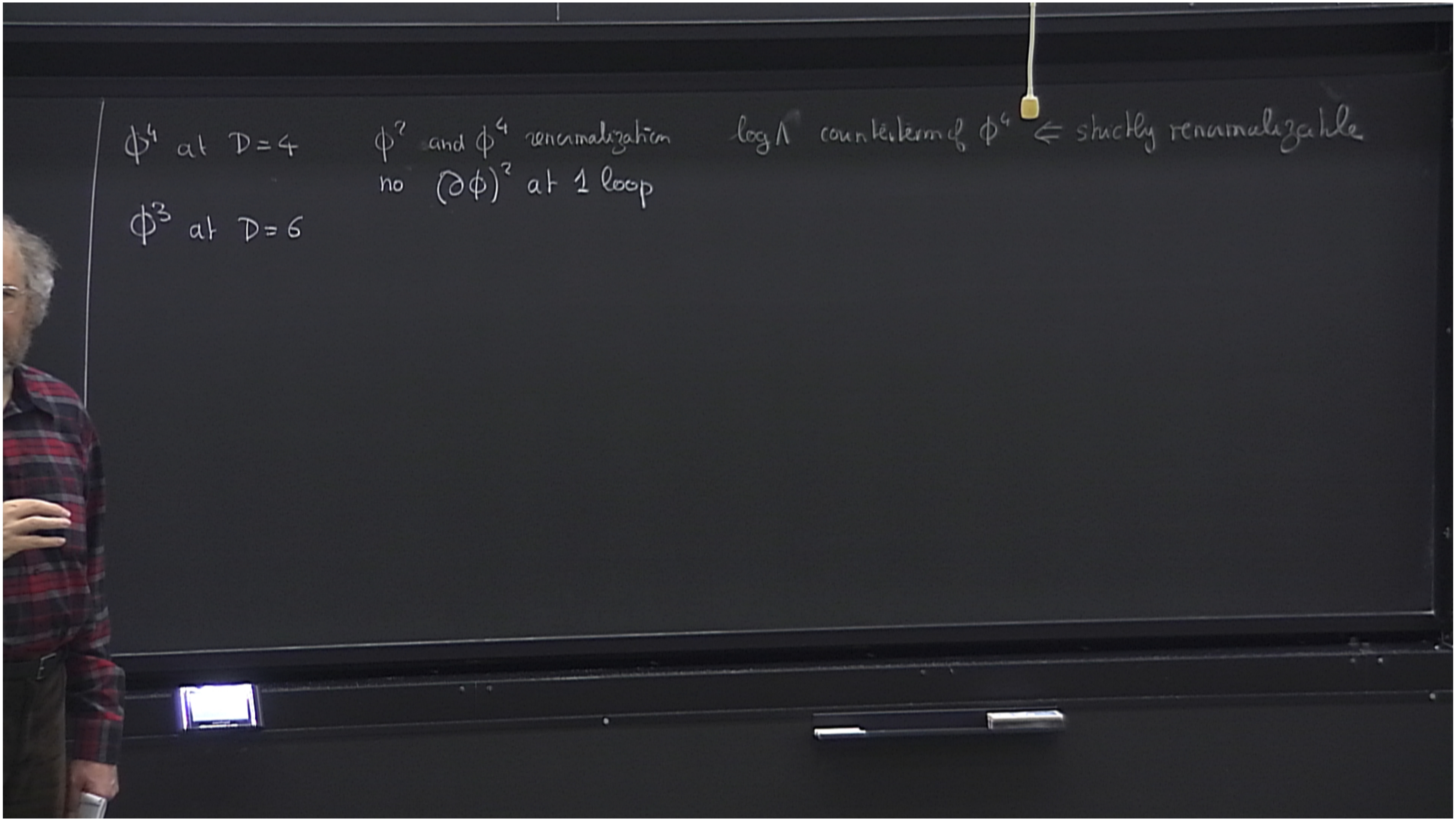
finite

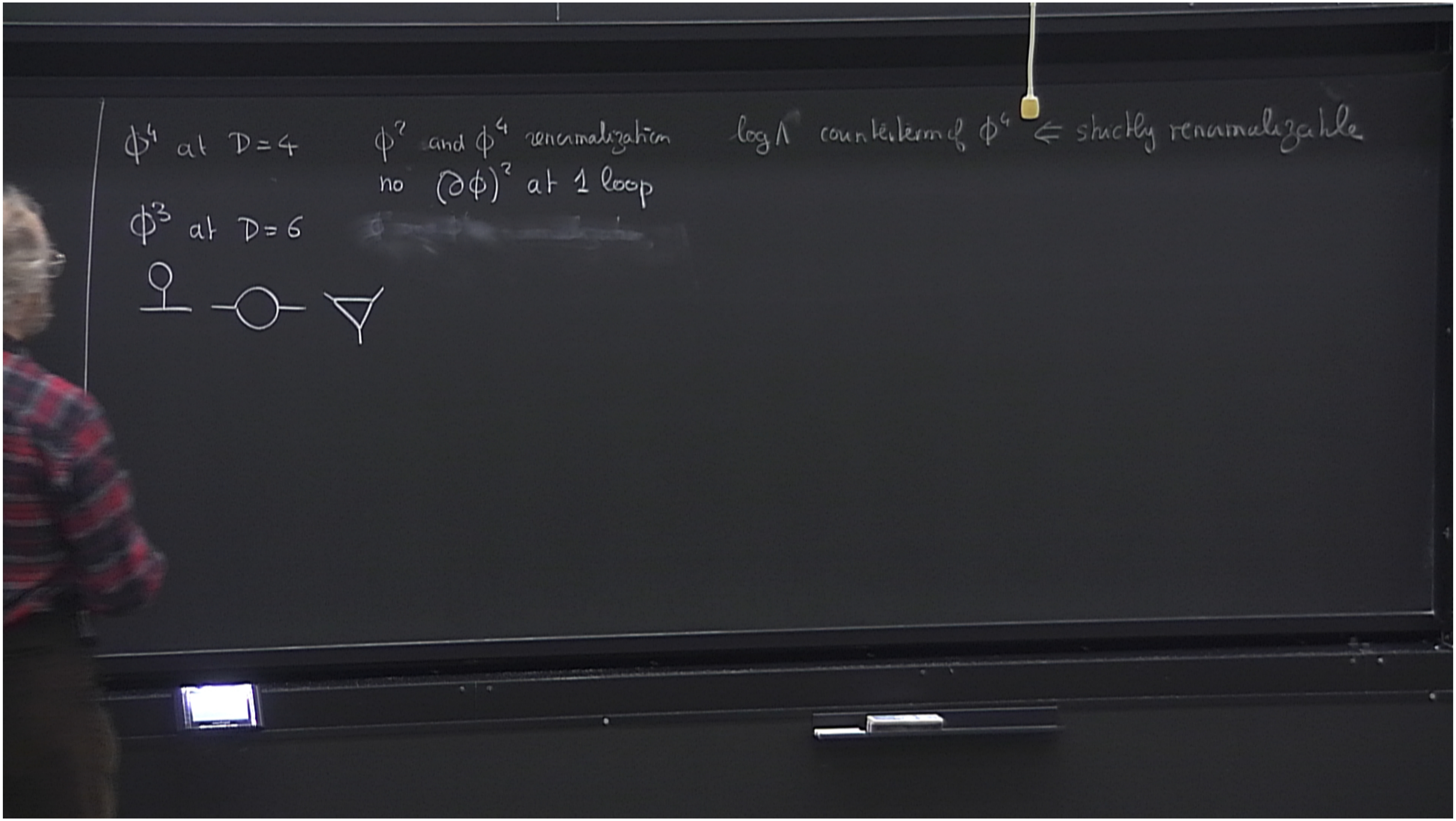
op)

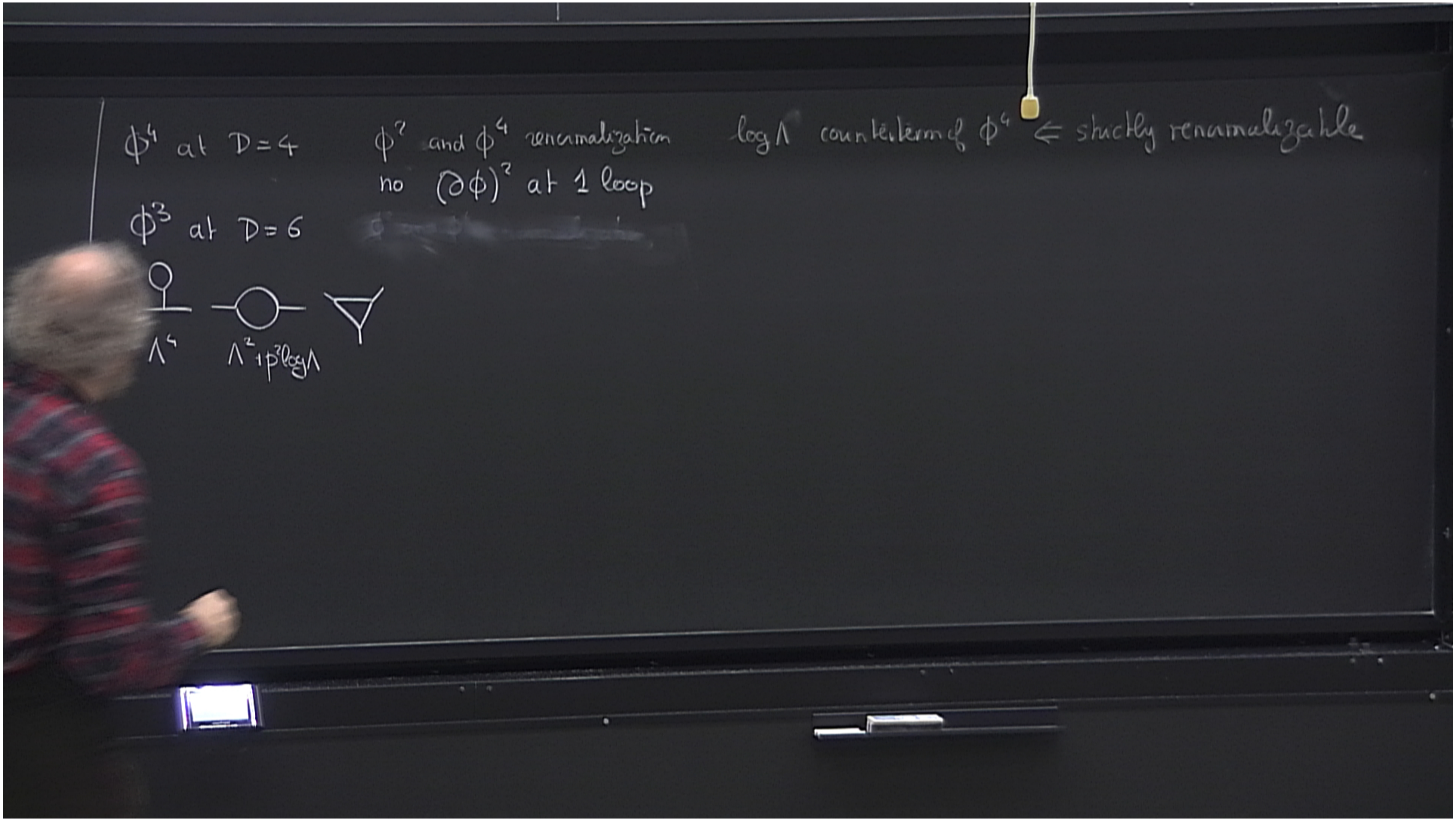

ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
no $(\partial\phi)^2$ at 1 loop

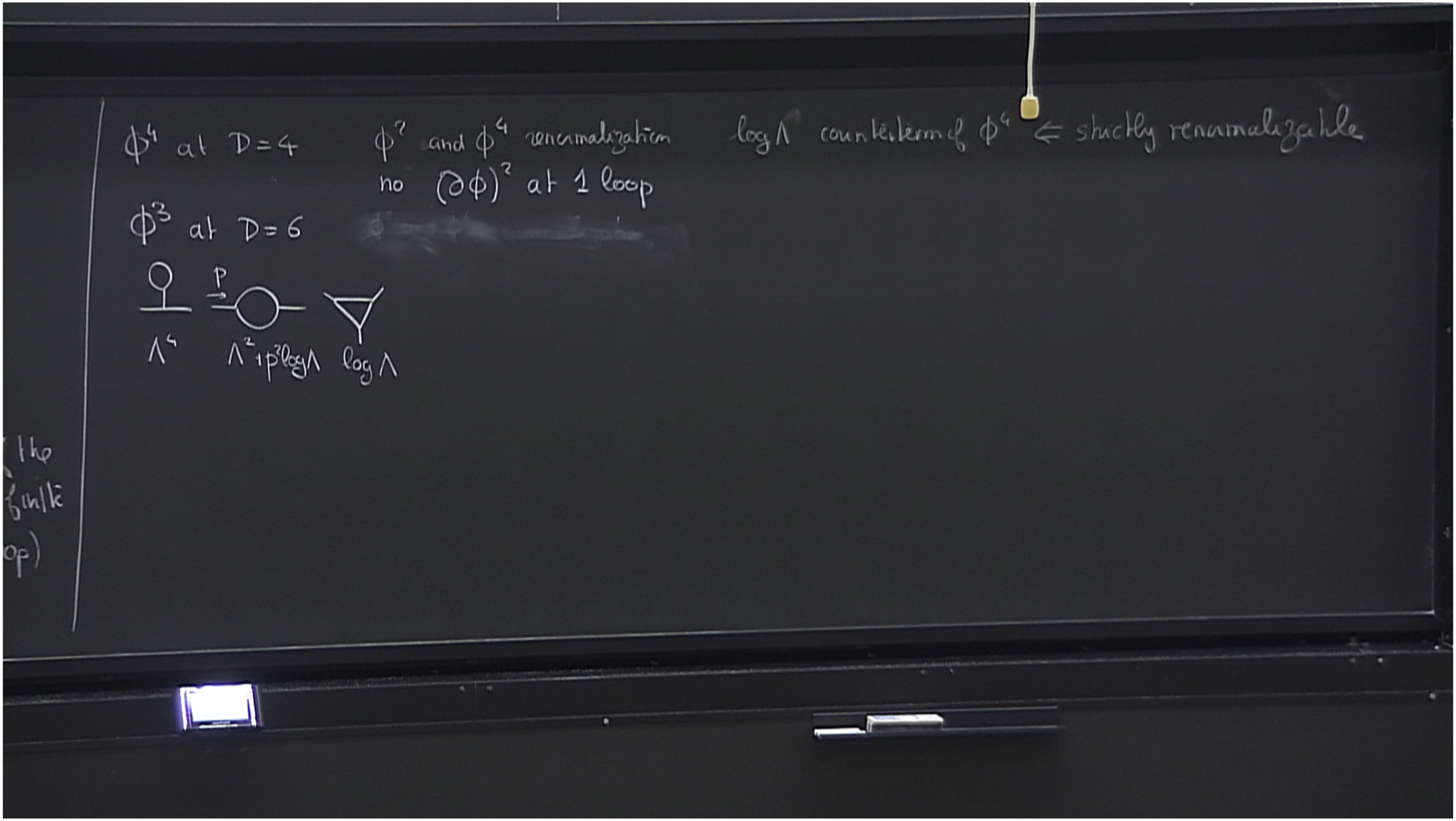

ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log \Lambda^2$ counterterm of ϕ^4
no $(\partial\phi)^2$ at 1 loop

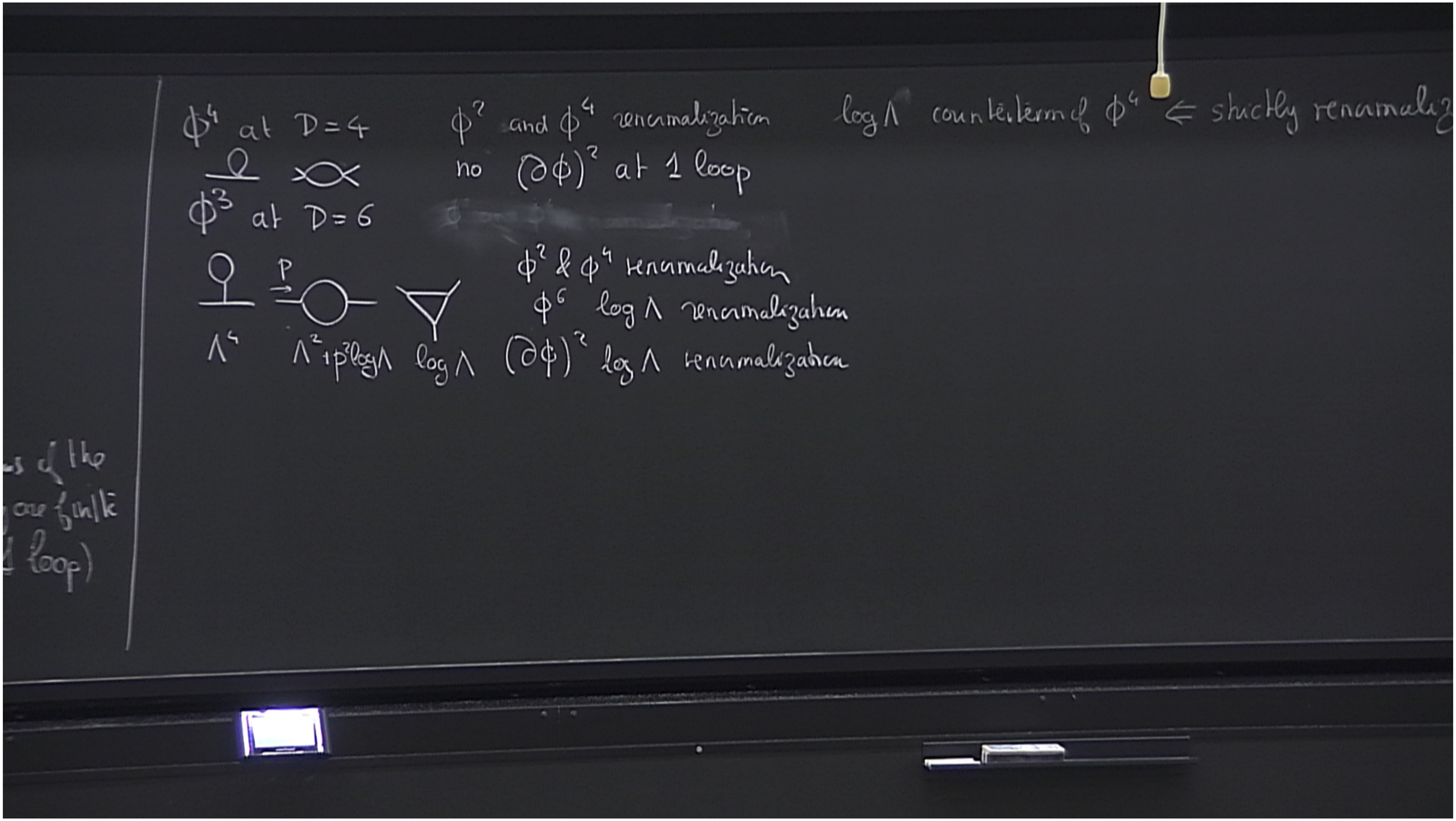

φ4 at D = 4
φ2 and φ4 renormalization
no (∂φ)2 at 1 loop
log Λ counterterm of φ4 ⇐ strictly renormalizable


ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
log Λ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$

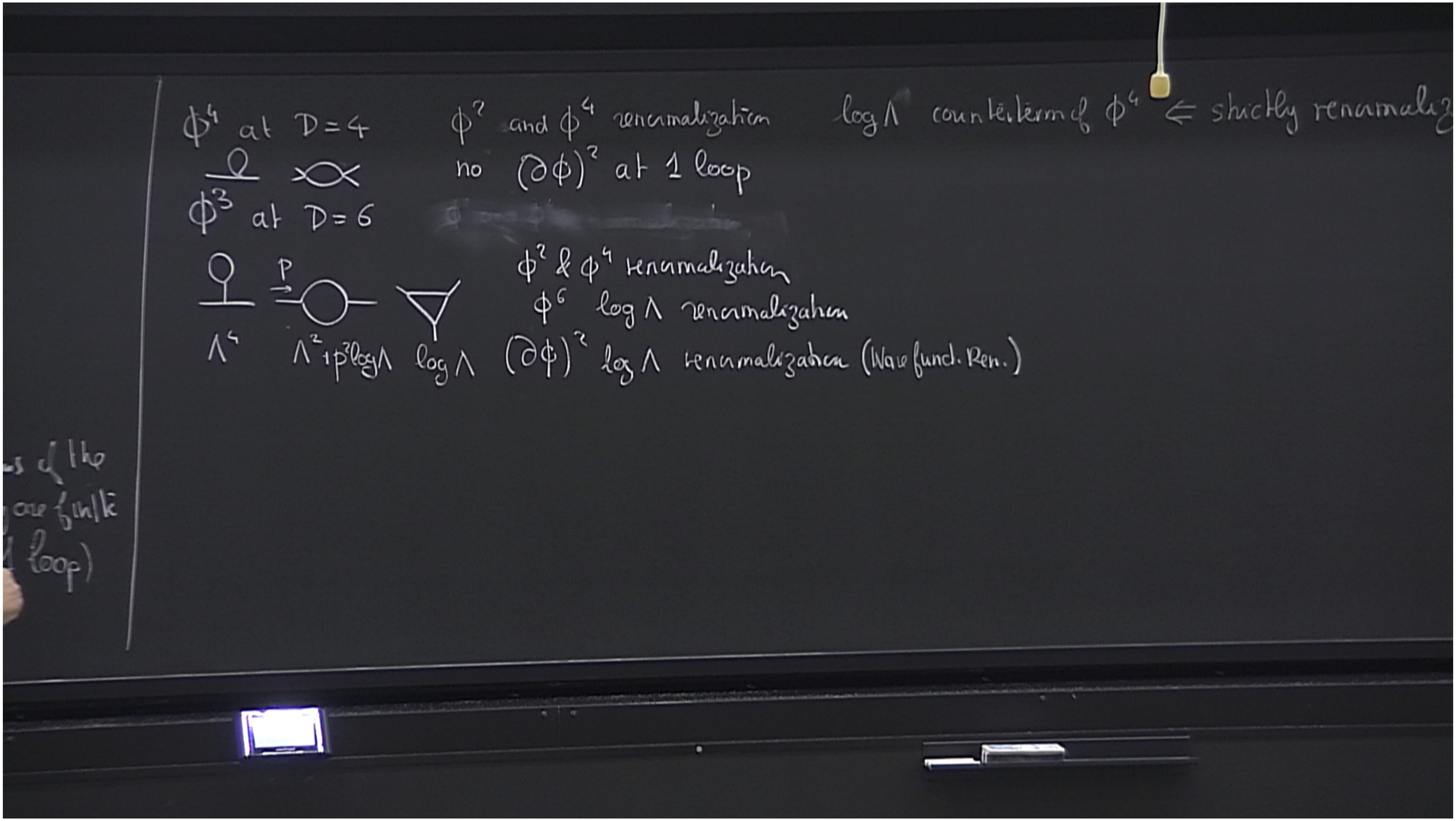

ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log\Lambda$ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$


ϕ4 at D=4
ϕ2 and ϕ4 renormalization
log Λ counterterm of ϕ4 ⇐ strictly renormalizable
no (∂ϕ)2 at 1 loop
ϕ3 at D=6
Λ4
Λ2 + p2 log Λ


ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
no $(\partial\phi)^2$ at 1 loop
$\log\Lambda$ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
ϕ^3 at $D=6$
p
Λ^4
$\Lambda^2 + p^2\log\Lambda$
$\log\Lambda$



ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log\Lambda$ counterterm of ϕ^4 ⇐ strictly renormaliz
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$
p
ϕ^2 & ϕ^4 renormalization
ϕ^6 $\log\Lambda$ renormalization
Λ^4
$\Lambda^2 + p^2\log\Lambda$
$\log\Lambda$
$(\partial\phi)^2$ $\log\Lambda$ renormalization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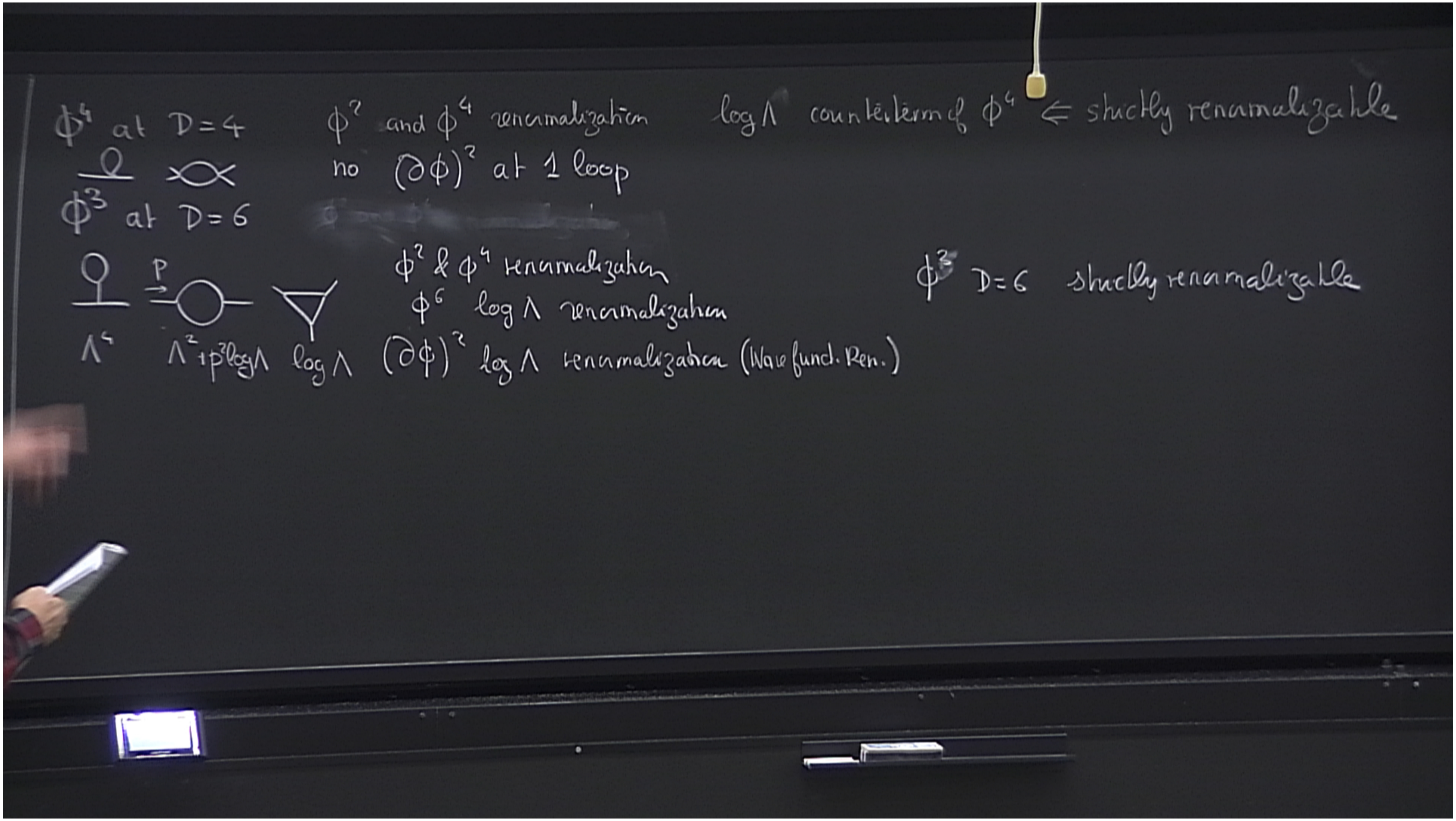
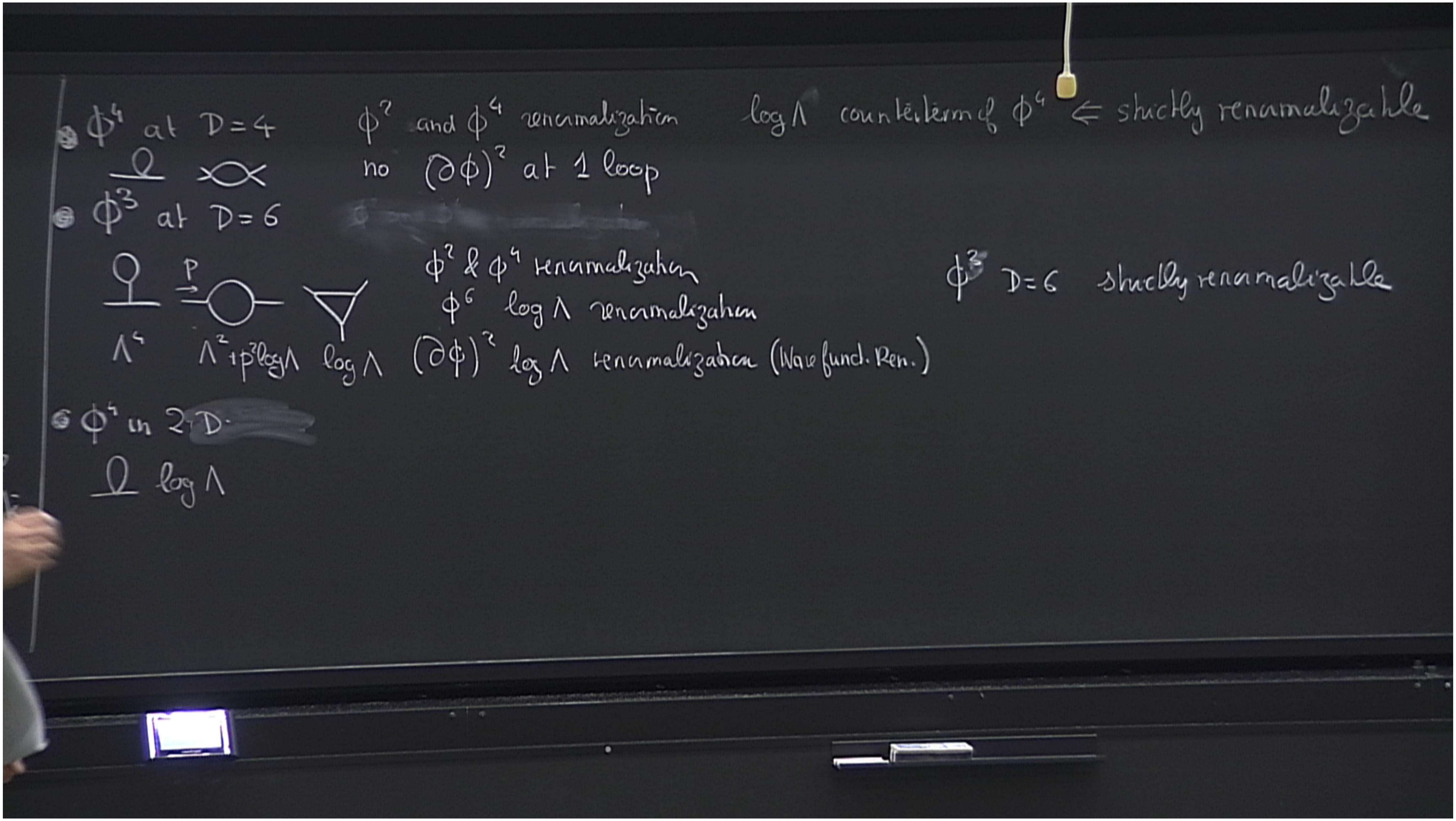
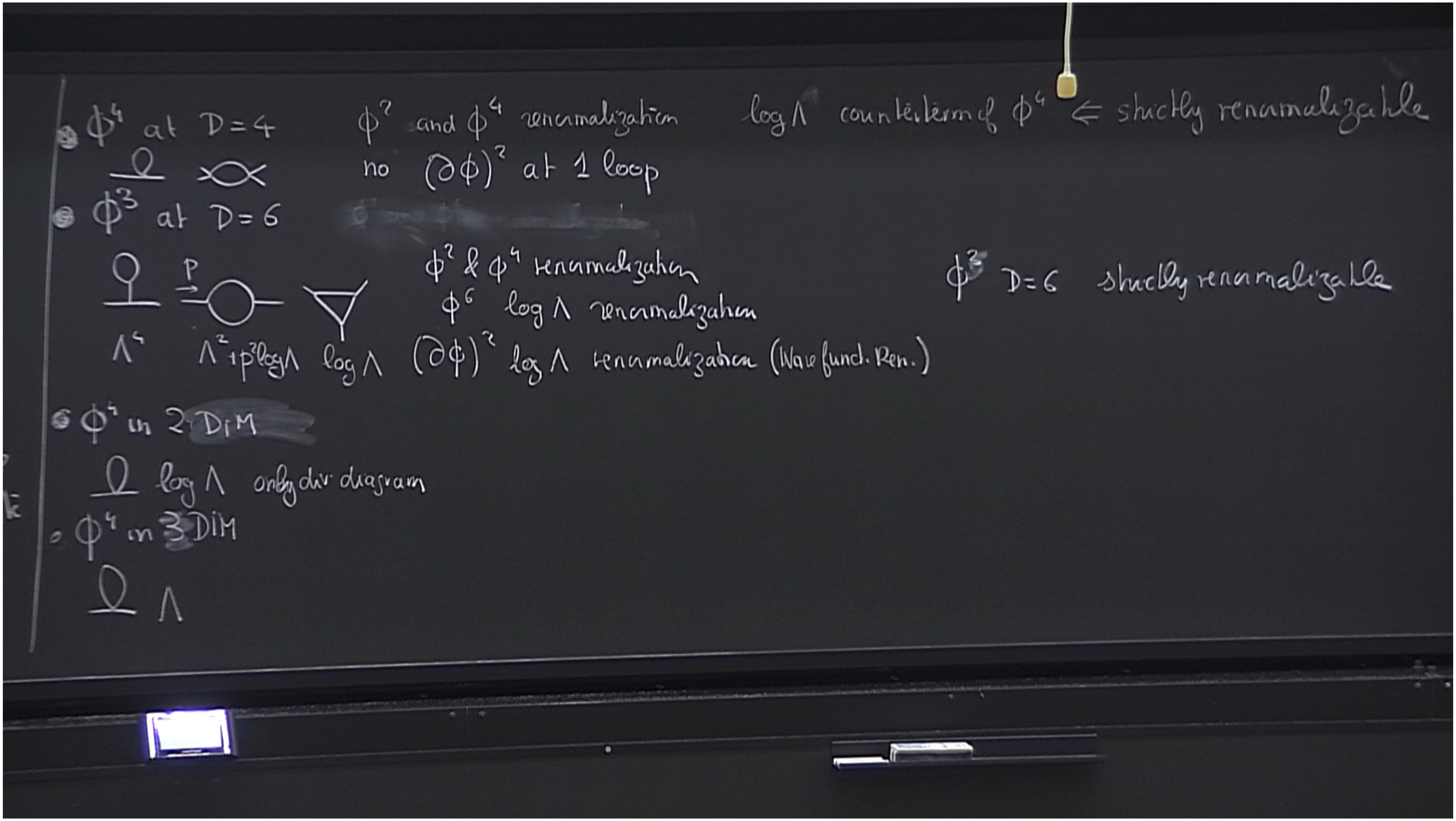
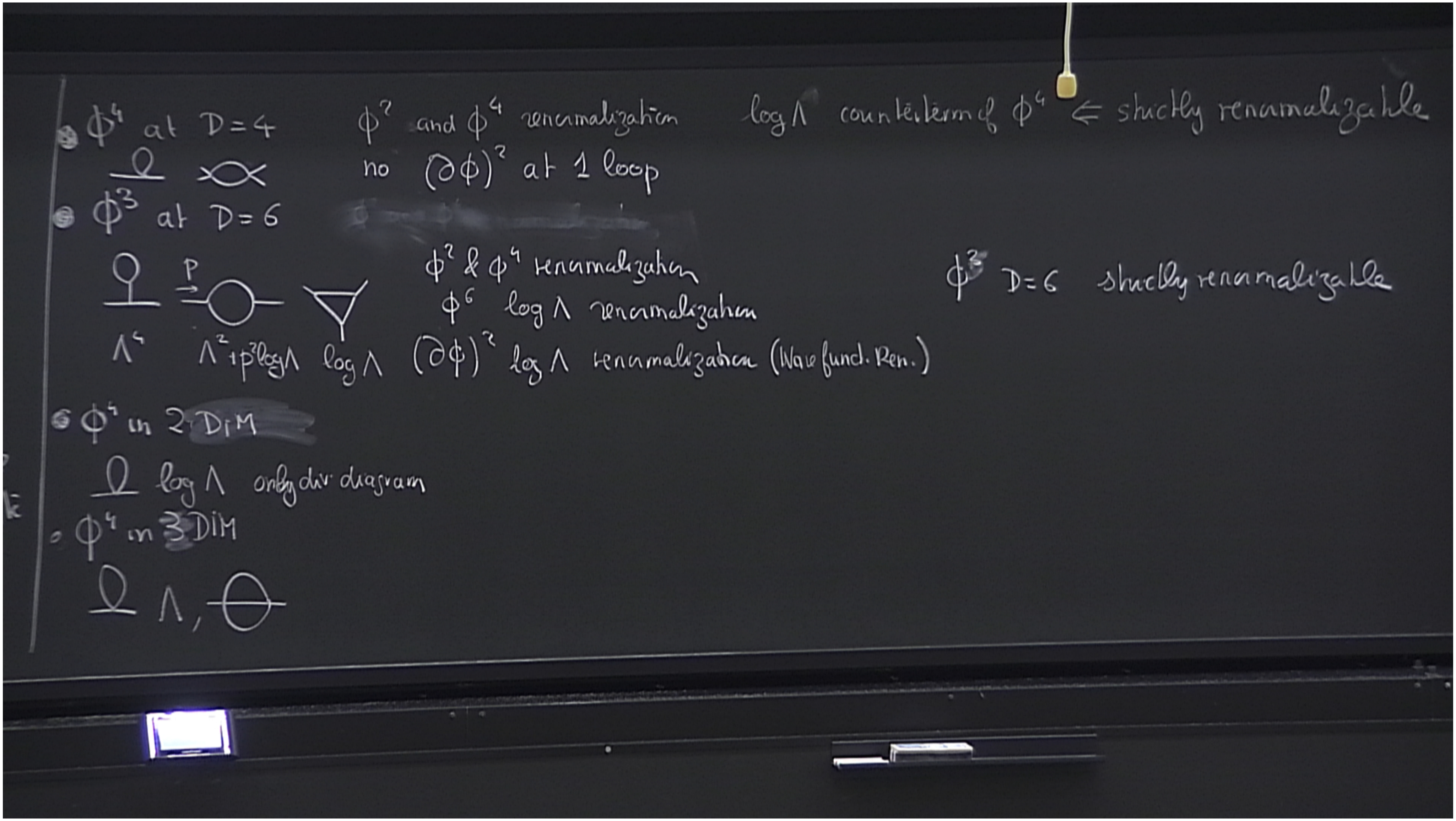
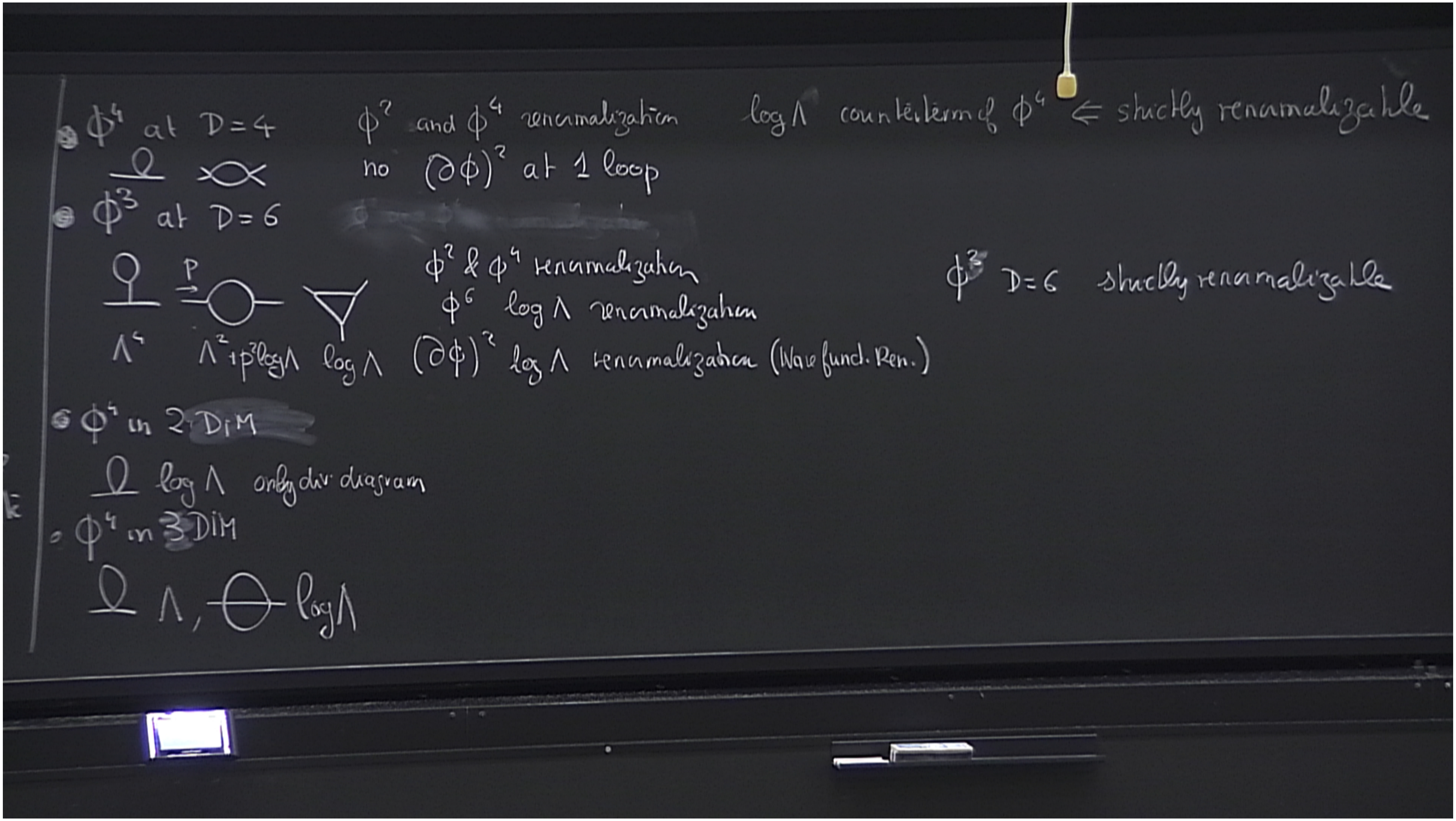
ϕ^2 & ϕ^4 renormalization
ϕ^6 $\log\Lambda$ renormalization
Λ^4
$\Lambda^2 + p^2\log\Lambda$
$\log\Lambda$
$(\partial\phi)^2$ $\log\Lambda$ renormalization (Wave funct. Ren.)
s of the
are finite
loop)


ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log\Lambda$ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$
ϕ^2 & ϕ^4 renormalization
ϕ^6 $\log\Lambda$ renormalization
ϕ^3 $D=6$ strictly renormalizable
P
Λ^4
$\Lambda^2 + p^2\log\Lambda$
$\log\Lambda$
$(\partial\phi)^2$ $\log\Lambda$ renormalization (Wave funct. Ren.)

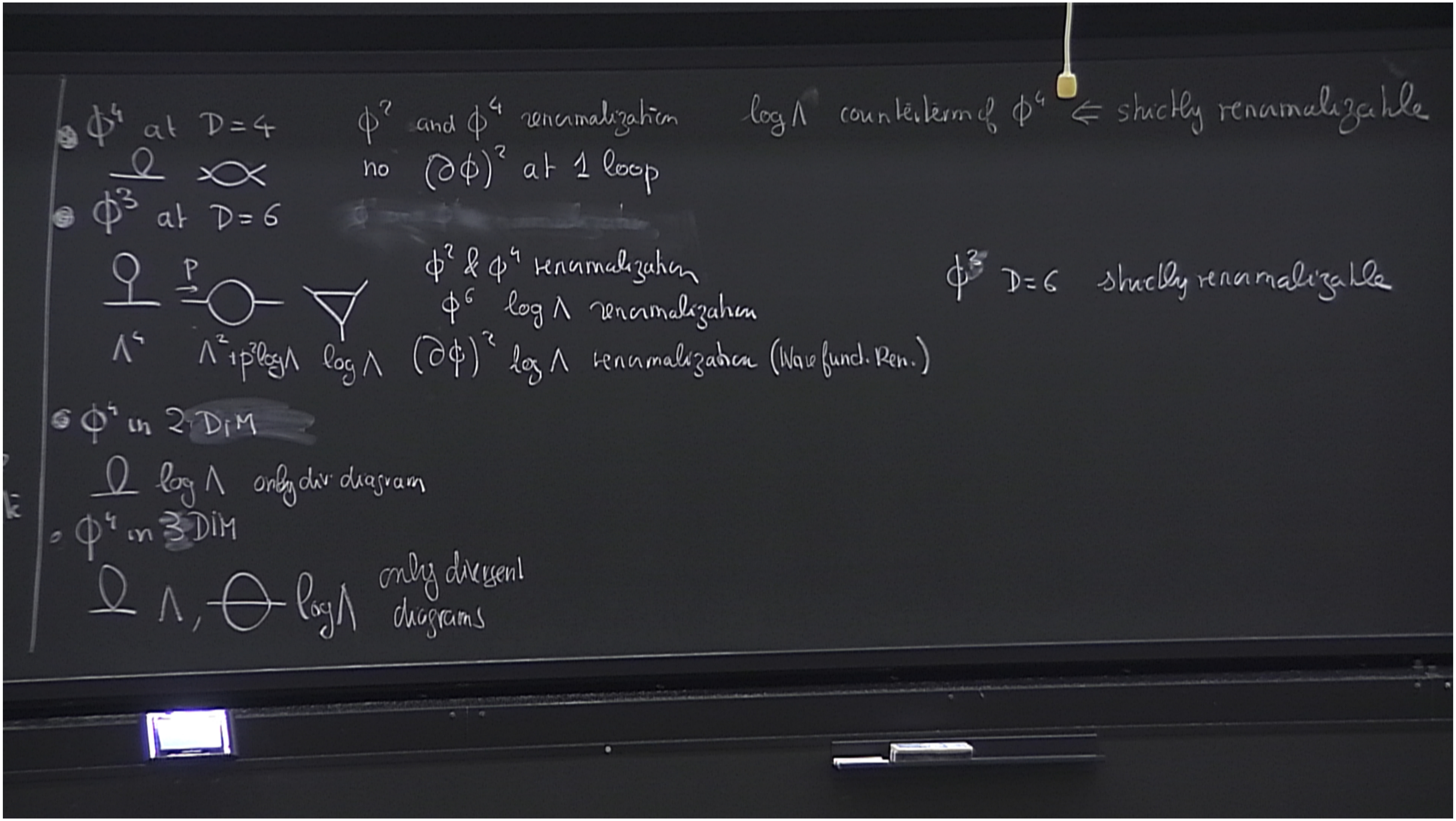

φ⁴ at D=4
φ² and φ⁴ renormalization
log Λ counterterm of φ⁴ ⇐ strictly renormalizable
no (∂φ)² at 1 loop
φ³ at D=6
φ² & φ⁴ renormalization
φ³ D=6 strictly renormalizable
p
φ⁶ log Λ renormalization
Λ⁴
Λ² + p² log Λ
log Λ
(∂φ)² log Λ renormalization (Wave funct. Ren.)
φ⁴ in 2 D
log Λ

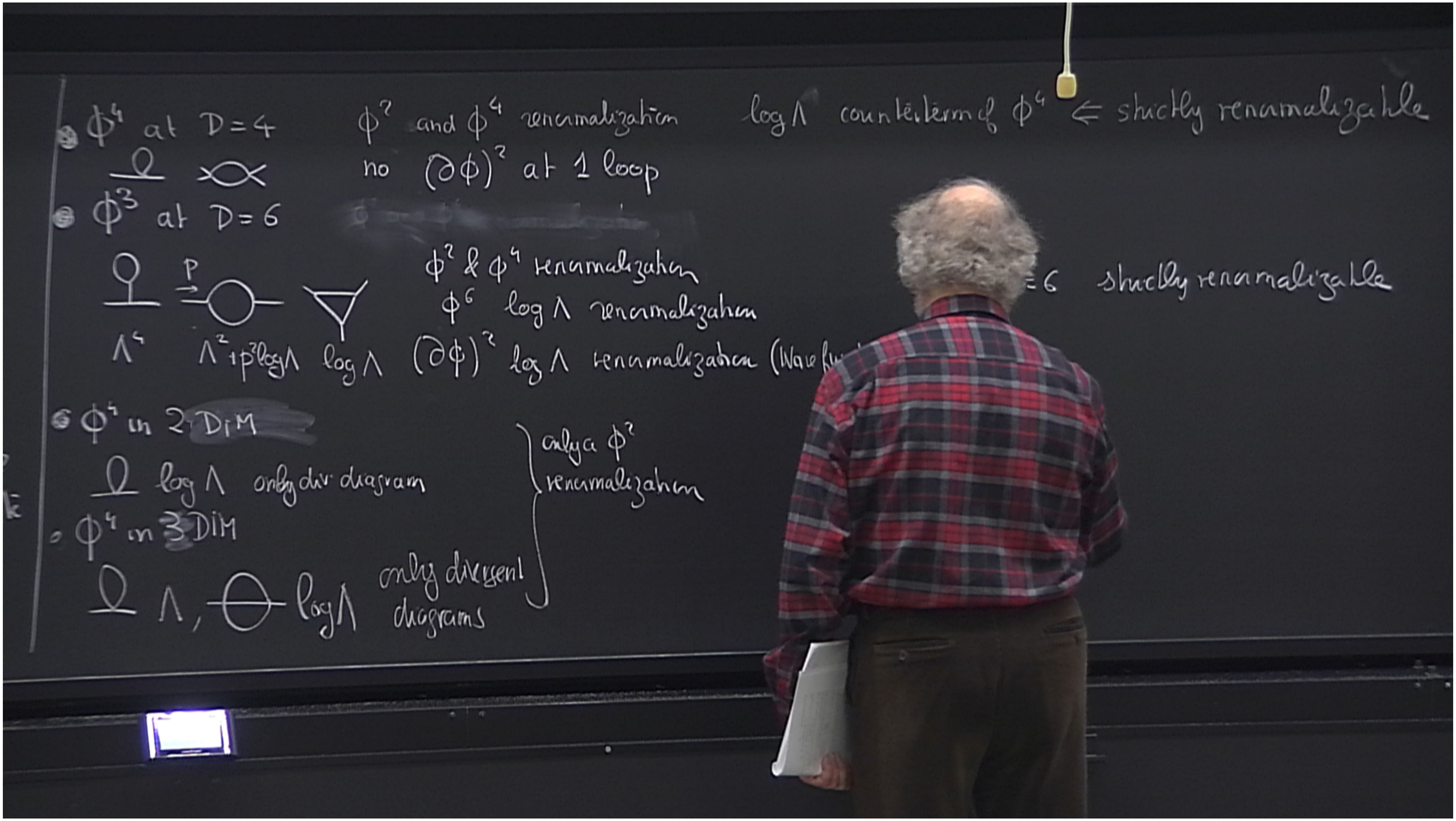

ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log\Lambda$ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$
P
ϕ^2 & ϕ^4 renormalization
ϕ^3 $D=6$ strictly renormalizable
ϕ^6 $\log\Lambda$ renormalization
Λ^4
$\Lambda^2+p^2\log\Lambda$
$\log\Lambda$
$(\partial\phi)^2$ $\log\Lambda$ renormalization (Wave funct. Ren.)
ϕ^4 in 2 DiM
$\log\Lambda$ only div. diagram
ϕ^4 in 3 DiM
Λ


• φ⁴ at D=4   φ² and φ⁴ renormalization   log Λ counterterm of φ⁴ ⇐ strictly renormalizable
no (∂φ)² at 1 loop
• φ³ at D=6
φ² & φ⁴ renormalization   φ³ D=6 strictly renormalizable
p
φ⁶ log Λ renormalization
Λ⁴   Λ²+p² log Λ   log Λ   (∂φ)² log Λ renormalization (Wave funct. Ren.)
• φ⁴ in 2 DIM
log Λ only div. diagram
• φ⁴ in 3 DIM
Λ,



φ⁴ at D=4
φ² and φ⁴ renormalization
log Λ counterterm of φ⁴ ⇐ strictly renormalizable
no (∂φ)² at 1 loop
φ³ at D=6
p
φ² & φ⁴ renormalization
φ⁶ log Λ renormalization
φ³ D=6 strictly renormalizable
Λ⁴ Λ²+p²logΛ log Λ (∂φ)² log Λ renormalization (Wave funct. Ren.)
φ⁴ in 2 DiM
log Λ only div. diagram
φ⁴ in 3 DiM
Λ, log Λ


φ⁴ at D=4
φ² and φ⁴ renormalization
log Λ counterterm of φ⁴ ⇐ strictly renormalizable
no (∂φ)² at 1 loop
φ³ at D=6
P
φ² & φ⁴ renormalization
φ⁶ log Λ renormalization
φ³ D=6 strictly renormalizable
Λ⁴
Λ²+p²logΛ
log Λ
(∂φ)² log Λ renormalization (Wave funct. Ren.)
φ⁴ in 2 DiM
log Λ only div. diagram
φ⁴ in 3 DiM
Λ,
logΛ
only divergent diagrams


φ⁴ at D=4
φ² and φ⁴ renormalization
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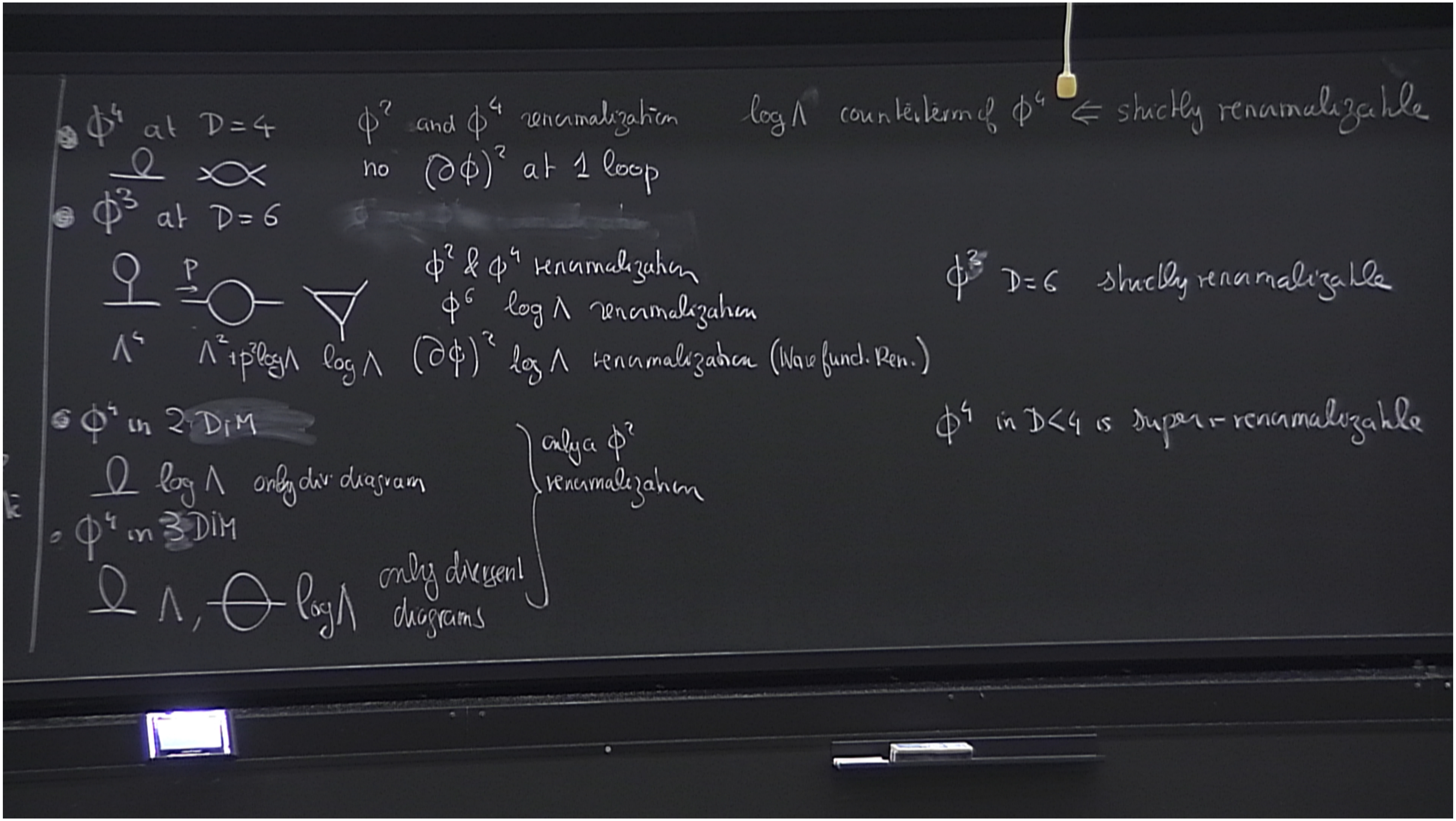
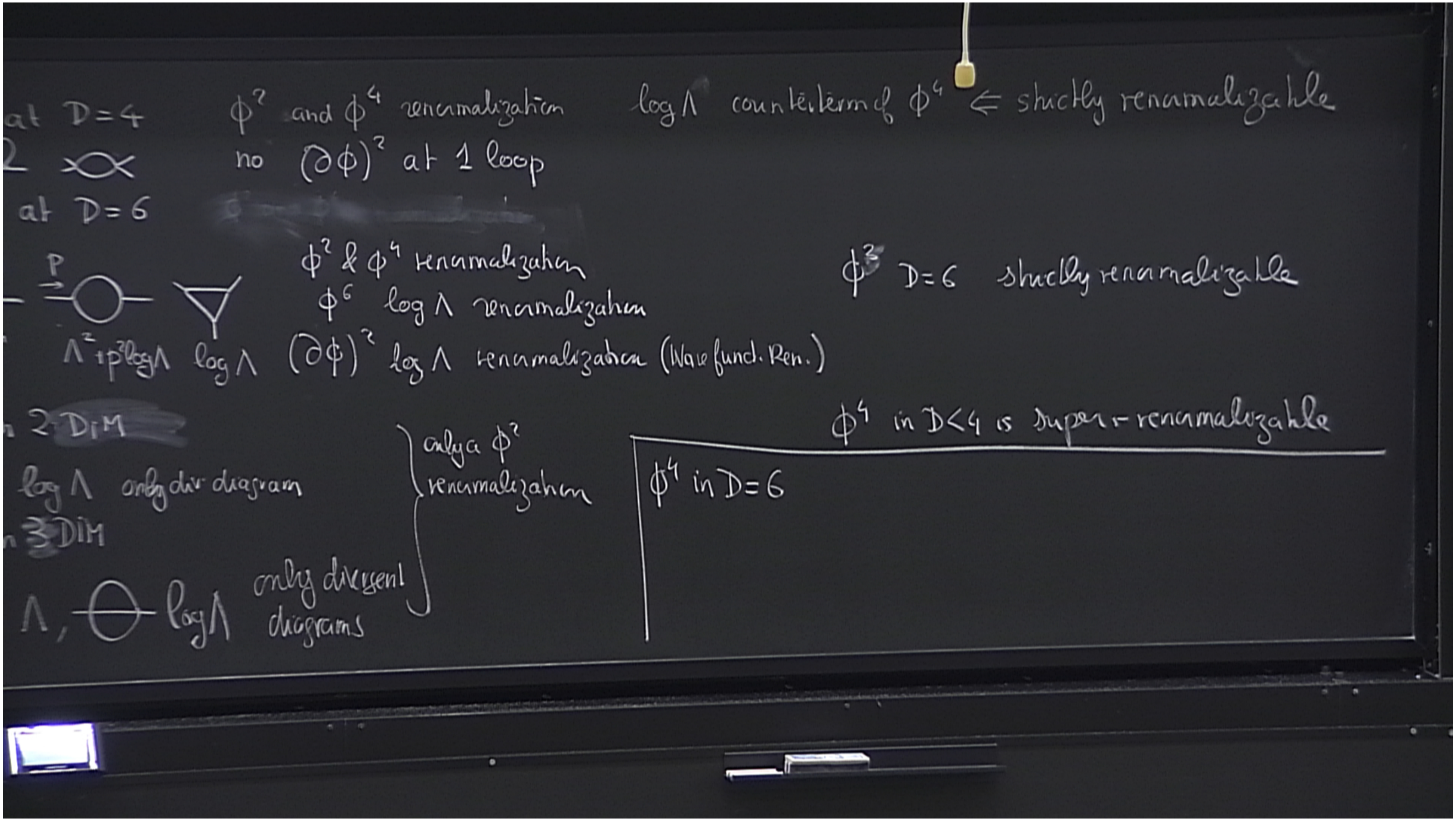
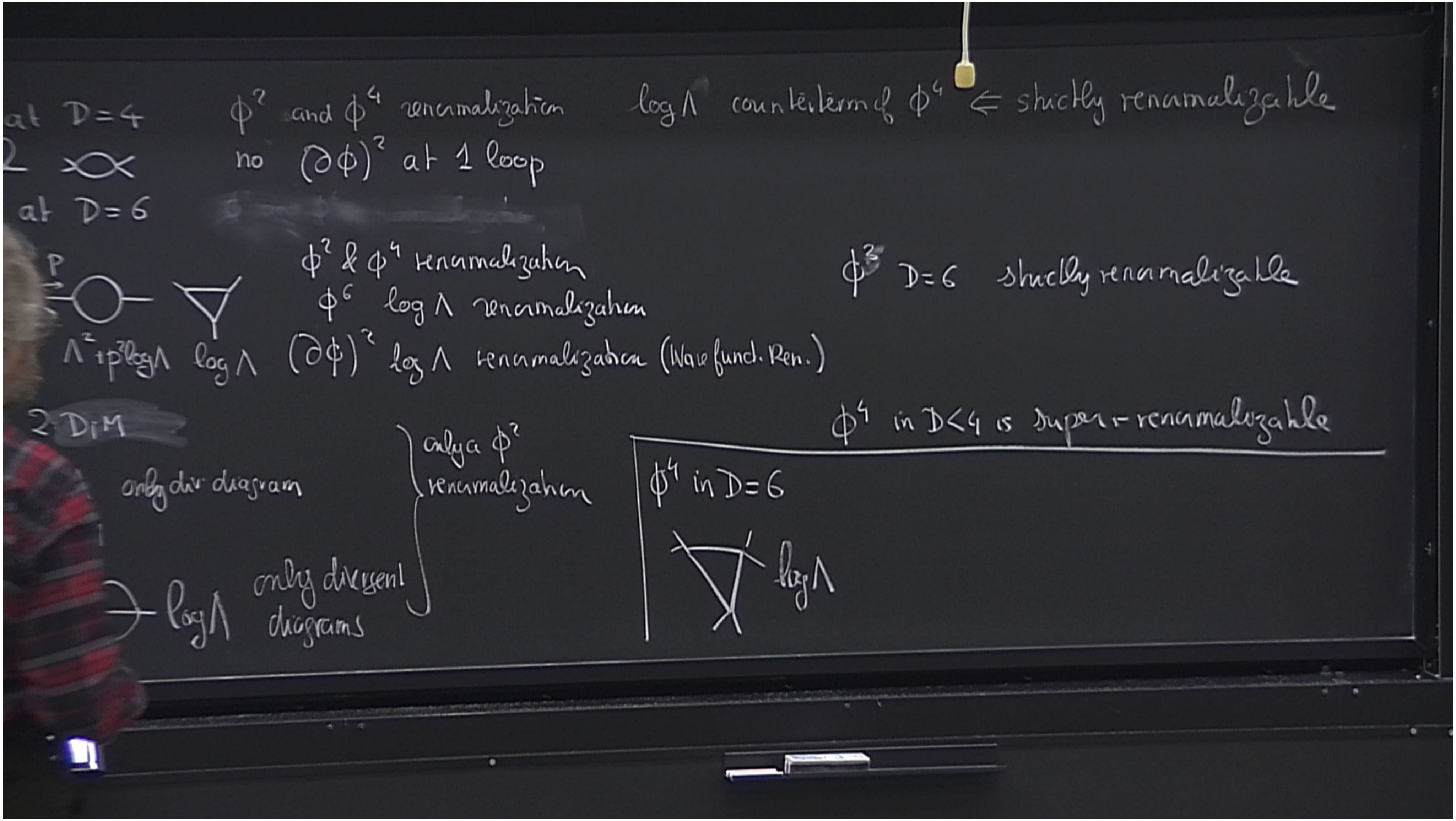
φ² & φ⁴ renormalization
φ⁶ log Λ renormalization
Λ⁴
Λ²+p²logΛ
log Λ
(∂φ)² log Λ renormalization
strictly renormalizable
φ⁴ in 2 DIM
log Λ only div diagram
only a φ² renormalization
φ⁴ in 3 DIM
Λ, log Λ
only divergent diagrams


ϕ^4 at $D=4$
ϕ^2 and ϕ^4 renormalization
$\log\Lambda$ counterterm of ϕ^4 $\Leftarrow$ strictly renormalizable
no $(\partial\phi)^2$ at 1 loop
ϕ^3 at $D=6$
p
ϕ^2 & ϕ^4 renormalization
ϕ^6 $\log\Lambda$ renormalization
Λ^4
$\Lambda^2 + p^2 \log\Lambda$
$\log\Lambda$
$(\partial\phi)^2$ $\log\Lambda$ renormalization (Wave funct. Ren.)
ϕ^3 $D=6$ strictly renormalizable
ϕ^4 in 2 DIM
$\log\Lambda$ only div. diagram
only a ϕ^2 renormalization
ϕ^4 in 3 DIM
Λ, $\log\Lambda$ only divergent diagrams
ϕ^4 in $D<4$ is super-renormalizable


at D=4
ϕ² and ϕ⁴ renormalization
log Λ counterterm of ϕ⁴ ⇐ strictly renormalizable
no (∂ϕ)² at 1 loop
at D=6
ϕ² & ϕ⁴ renormalization
ϕ³ D=6 strictly renormalizable
ϕ⁶ log Λ renormalization
Λ²+p²logΛ log Λ (∂ϕ)² log Λ renormalization (Wave funct. Ren.)
2 DIM
ϕ⁴ in D<4 is super-renormalizable
only a ϕ² renormalization
log Λ only div diagram
ϕ⁴ in D=6
3 DIM
Λ, log Λ only divergent diagrams


at D=4
ϕ² and ϕ⁴ renormalization
log Λ counterterm of ϕ⁴ ⇐ strictly renormalizable
no (∂ϕ)² at 1 loop
at D=6
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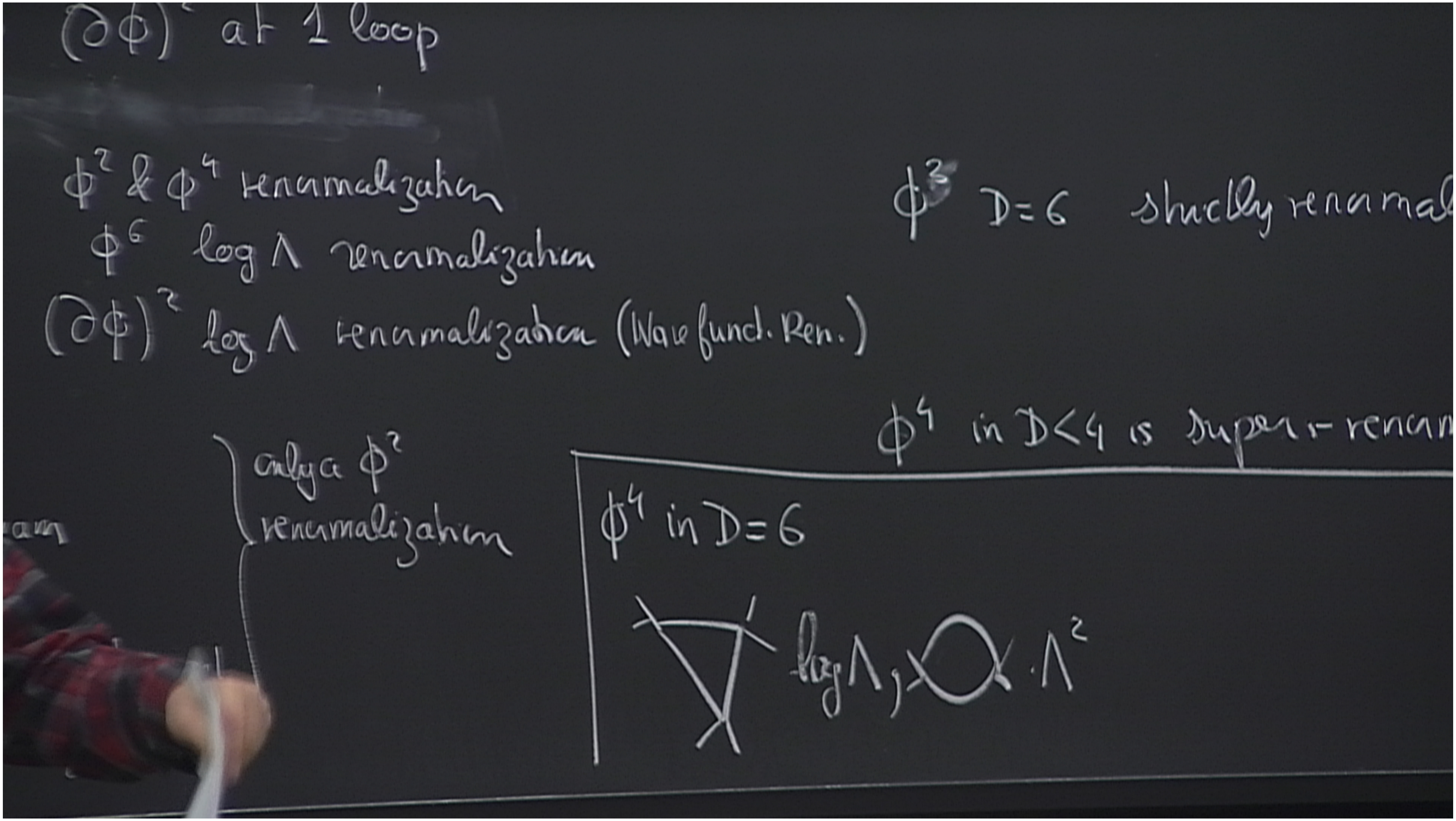
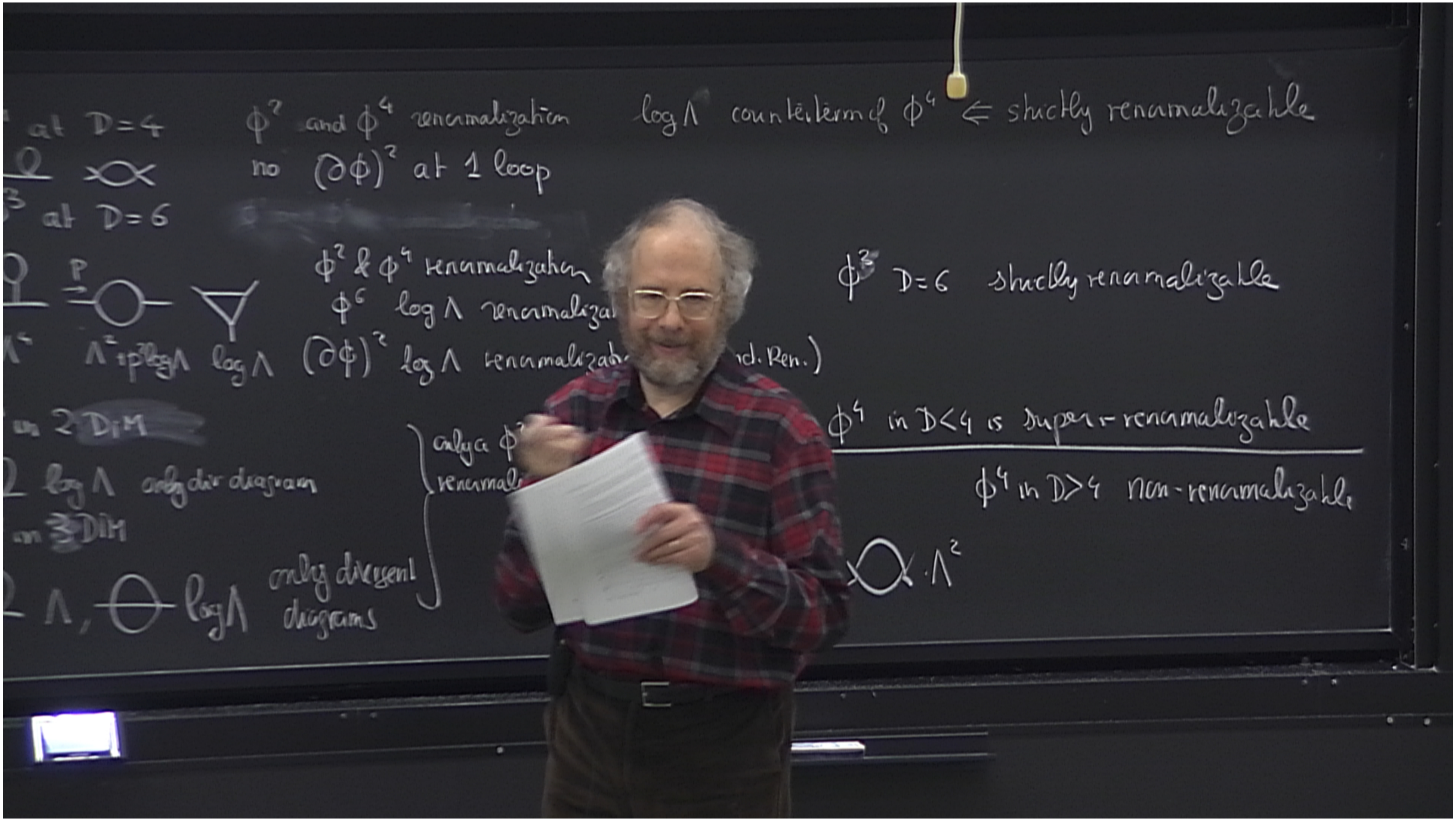
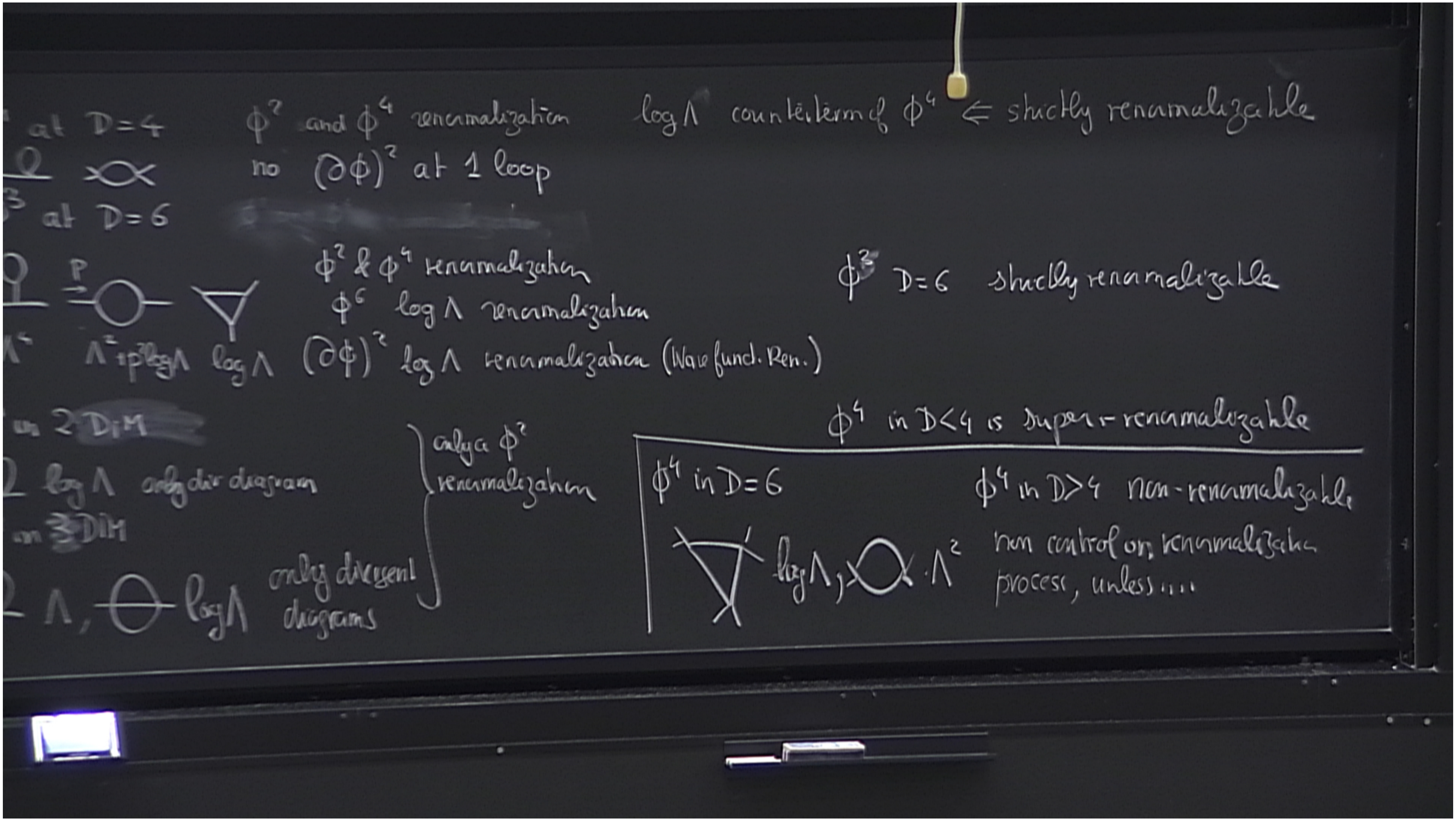
Λ²+p²logΛ log Λ (∂ϕ)² log Λ renormalization (Wave funct. Ren.)
2 DIM
ϕ⁴ in D<4 is super-renormalizable
only a ϕ² renormalization
only div diagram
ϕ⁴ in D=6
log Λ
log Λ
only divergent diagrams


(∂φ)² at 1 loop
φ² & φ⁴ renormalization
φ⁶ log Λ renormalization
(∂φ)² log Λ renormalization (Wave funct. Ren.)
only a φ² renormalization
φ³ D=6 strictly renormal
φ⁴ in D<4 is super-renorm
φ⁴ in D=6
log Λ, · Λ²


at D=4
at D=6
in 2 DIM
log Λ only div diagram
in 3 DIM
Λ, log Λ only divergent diagrams
only a φ renormal
φ² and φ⁴ renormalization
no (∂φ)² at 1 loop
φ² & φ⁴ renormalization
φ⁶ log Λ renormaliza
(∂φ)² log Λ renormaliza
log Λ counterterm of φ⁴ ⇐ strictly renormalizable
φ³ D=6 strictly renormalizable
φ⁴ in D<4 is super-renormalizable
φ⁴ in D>4 non-renormalizable
· Λ²


at D=4
φ² and φ⁴ renormalization
log Λ counterterm of φ⁴ ⇐ strictly renormalizable
no (∂φ)² at 1 loop
at D=6
φ² & φ⁴ renormalization
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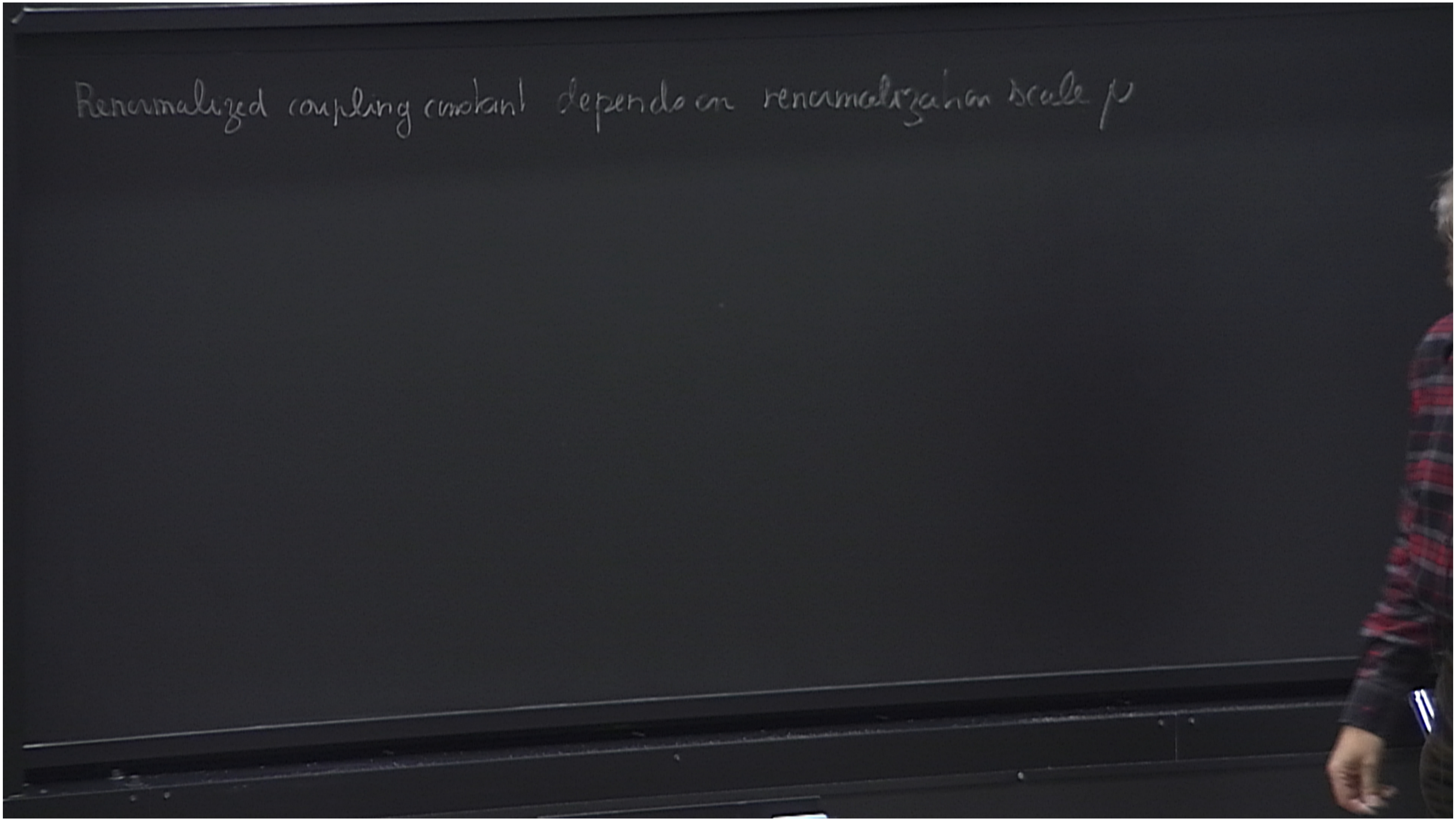
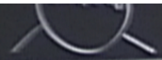
Λ²+p²logΛ logΛ (∂φ)² log Λ renormalization (Wave funct. Ren.)
φ⁴ in D<4 is super-renormalizable
in 2 DIM
only a φ² renormalization
log Λ only div diagram
in 3 DIM
Λ, log Λ only divergent diagrams
φ⁴ in D=6
φ⁴ in D>4 non-renormalizable
log Λ, · Λ²
non control on renormalization process, unless....


Renormalized coupling constant depends on renormalization scale μ

Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimension full parameter μ



Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

Physical Renormalized theory

μ, g_R

Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

Physical Renormalized theory

μ, g_R

$\mu' \neq \mu$, g_R'

Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

...me Physical Renormalized theory

different energies $\begin{cases} \mu , & g_R \\ \mu' \neq \mu , & g_R' \neq g_R \end{cases}$ different results

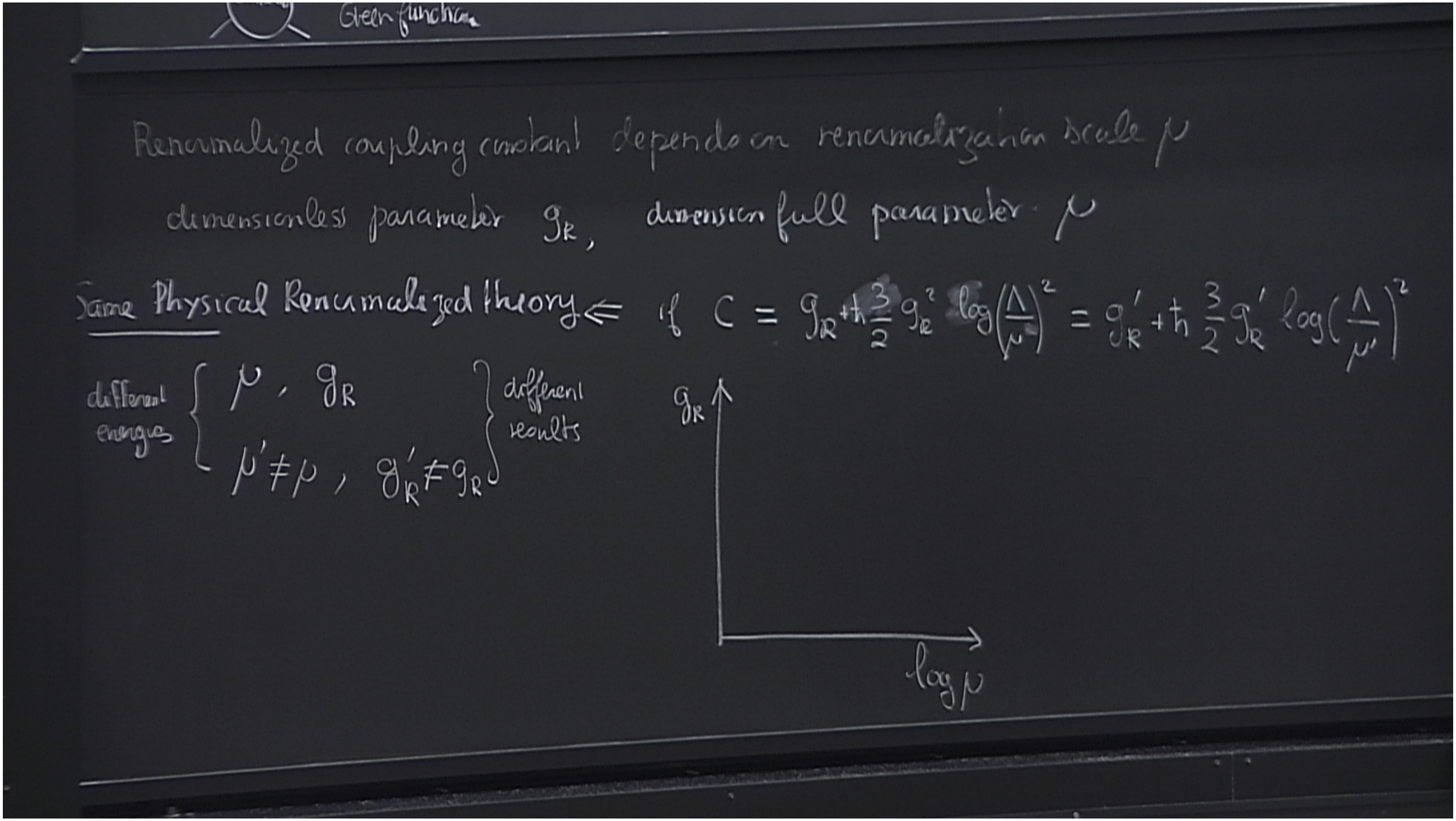
Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

Same Physical Renormalized theory $\Leftarrow$ if $C = g_R + \hbar \frac{3}{2} g_R^2 \log\left(\frac{\Lambda}{\mu}\right)^2 = g_R' + \hbar \frac{3}{2} g_R' \log\left(\frac{\Lambda}{\mu'}\right)^2$

different energies $\left\{ \begin{array}{l} \mu, \; g_R \\ \mu' \neq \mu, \; g_R' \neq g_R \end{array} \right\}$ different results


Green function
Renormalized coupling constant depends on renormalization scale μ
dimensionless parameter g_R, dimensionfull parameter μ
Same Physical Renormalized theory ⇐ if C = g_R + ħ 3/2 g_R² log(Λ/μ)² = g'_R + ħ 3/2 g'_R log(Λ/μ')²
different energies
μ, g_R
μ' ≠ μ, g'_R ≠ g_R
different results
g_R
log μ

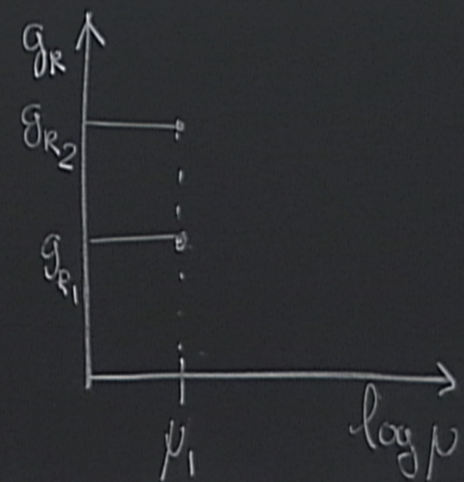
Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

Same Physical Renormalized theory $\Leftarrow$ if $C = g_R + \hbar \frac{3}{2} g_R^2 \log\left(\frac{\Lambda}{\mu}\right)^2 = g_R' + \hbar \frac{3}{2} g_R' \log\left(\frac{\Lambda}{\mu'}\right)^2$

different energies $\begin{cases} \mu, \; g_R \\ \mu' \neq \mu, \; g_R' \neq g_R \end{cases}$ different results



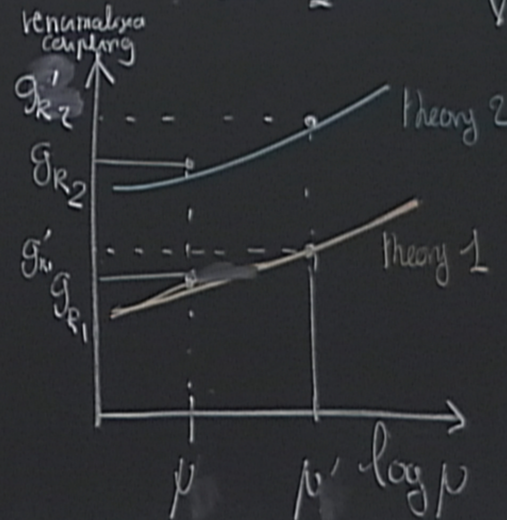
Green function

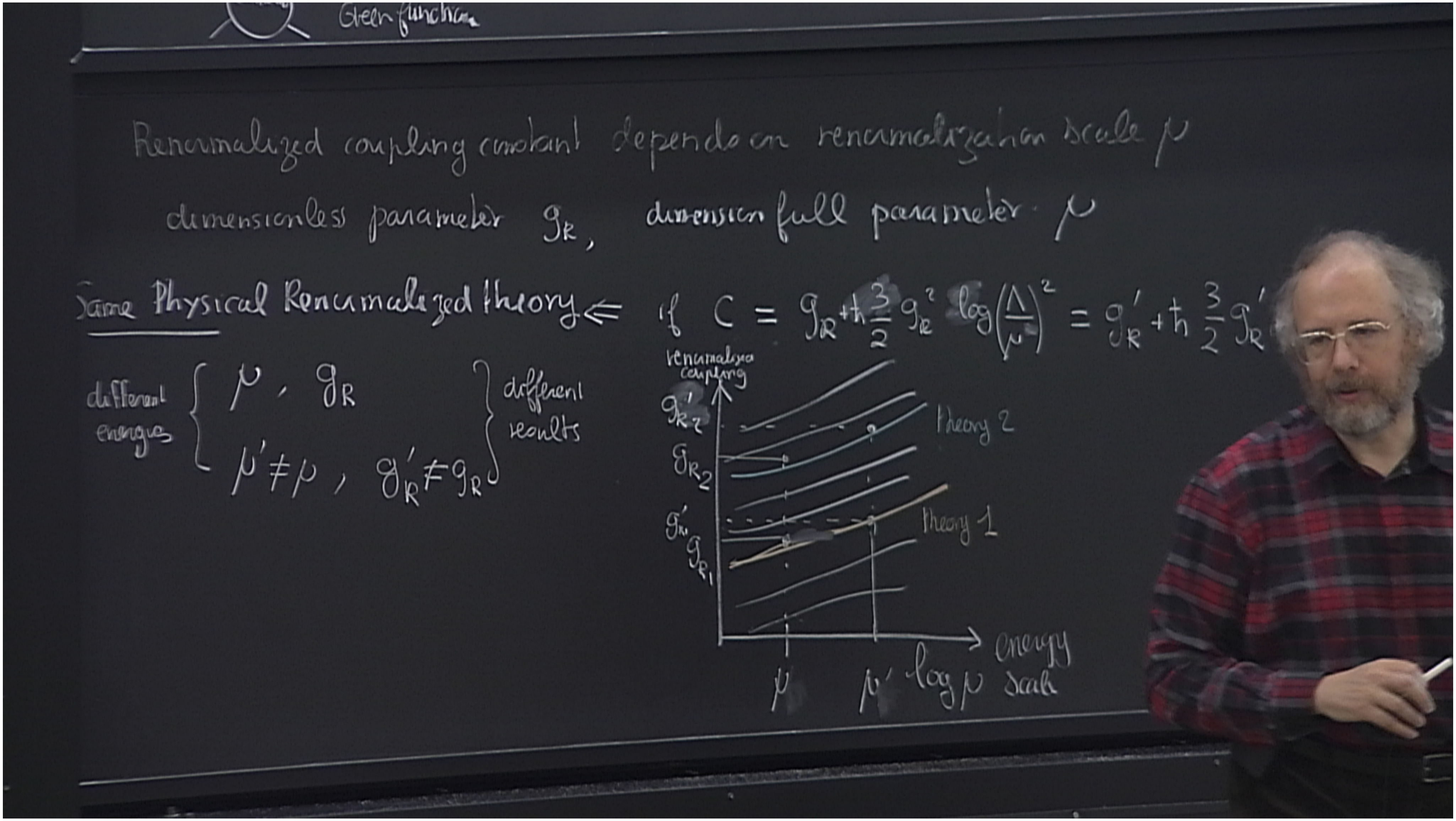
Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

Same Physical Renormalized theory $\Leftarrow$ if $C = g_R + \hbar \frac{3}{2} g_R^2 \log\left(\frac{\Lambda}{\mu}\right)^2 = g_R' + \hbar \frac{3}{2} g_R' \log\left(\frac{\Lambda}{\mu'}\right)^2$

different energies $\left\{ \begin{array}{l} \mu, \; g_R \\ \mu' \neq \mu, \; g_R' \neq g_R \end{array} \right\}$ different results




Green function
Renormalized coupling constant depends on renormalization scale μ
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Same Physical Renormalized theory ⇐ if C = g_R + ħ 3/2 g_R^2 log(Λ/μ)^2 = g_R' + ħ 3/2 g_R'
different energies
μ , g_R
μ' ≠ μ , g_R' ≠ g_R
different results
renormalized coupling
Theory 2
Theory 1
energy scale
log μ

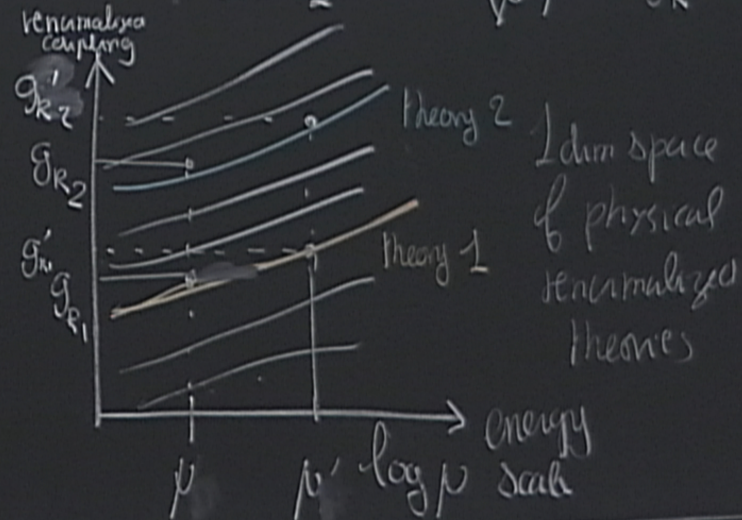
Green function

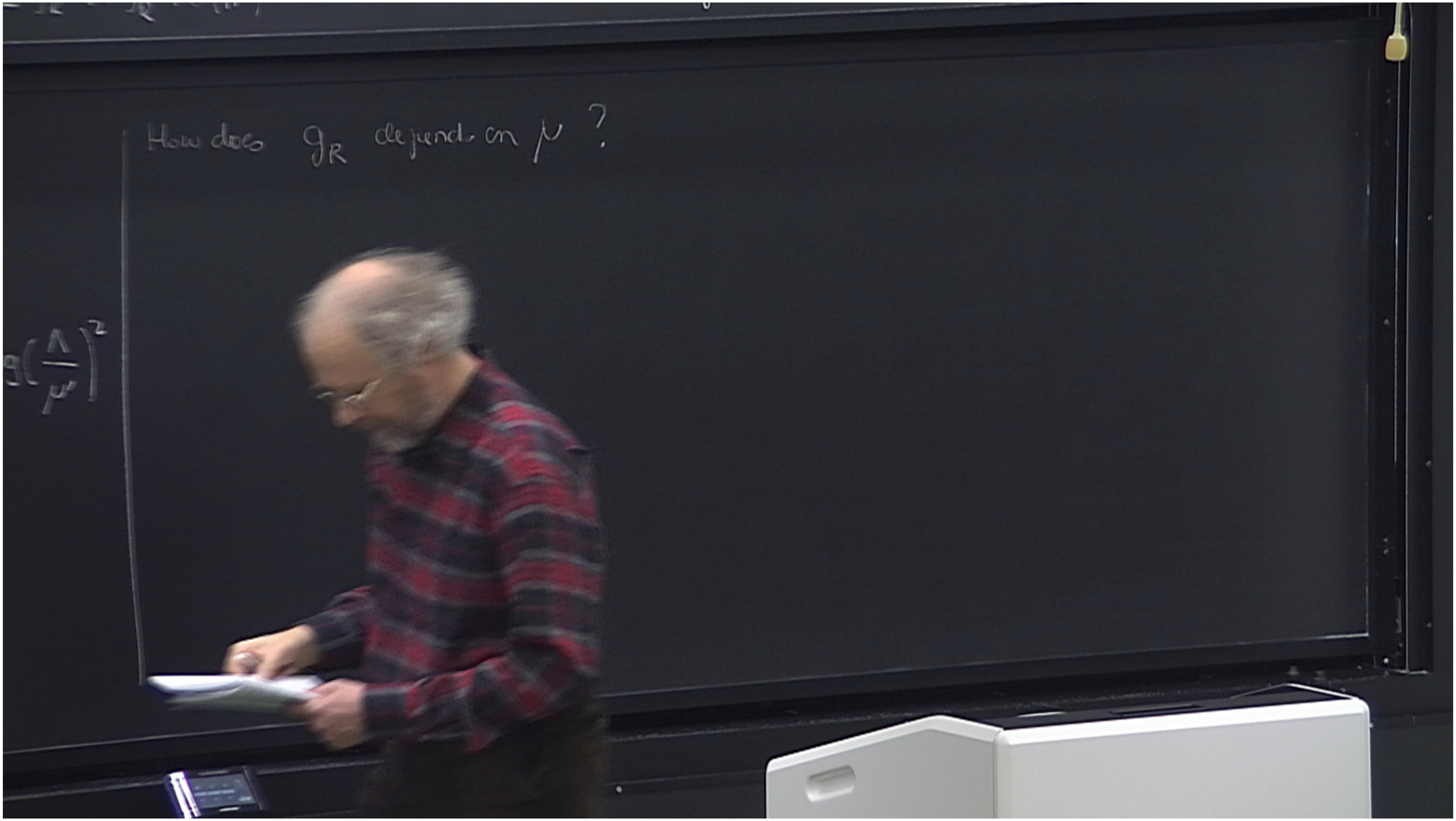
Renormalized coupling constant depends on renormalization scale μ

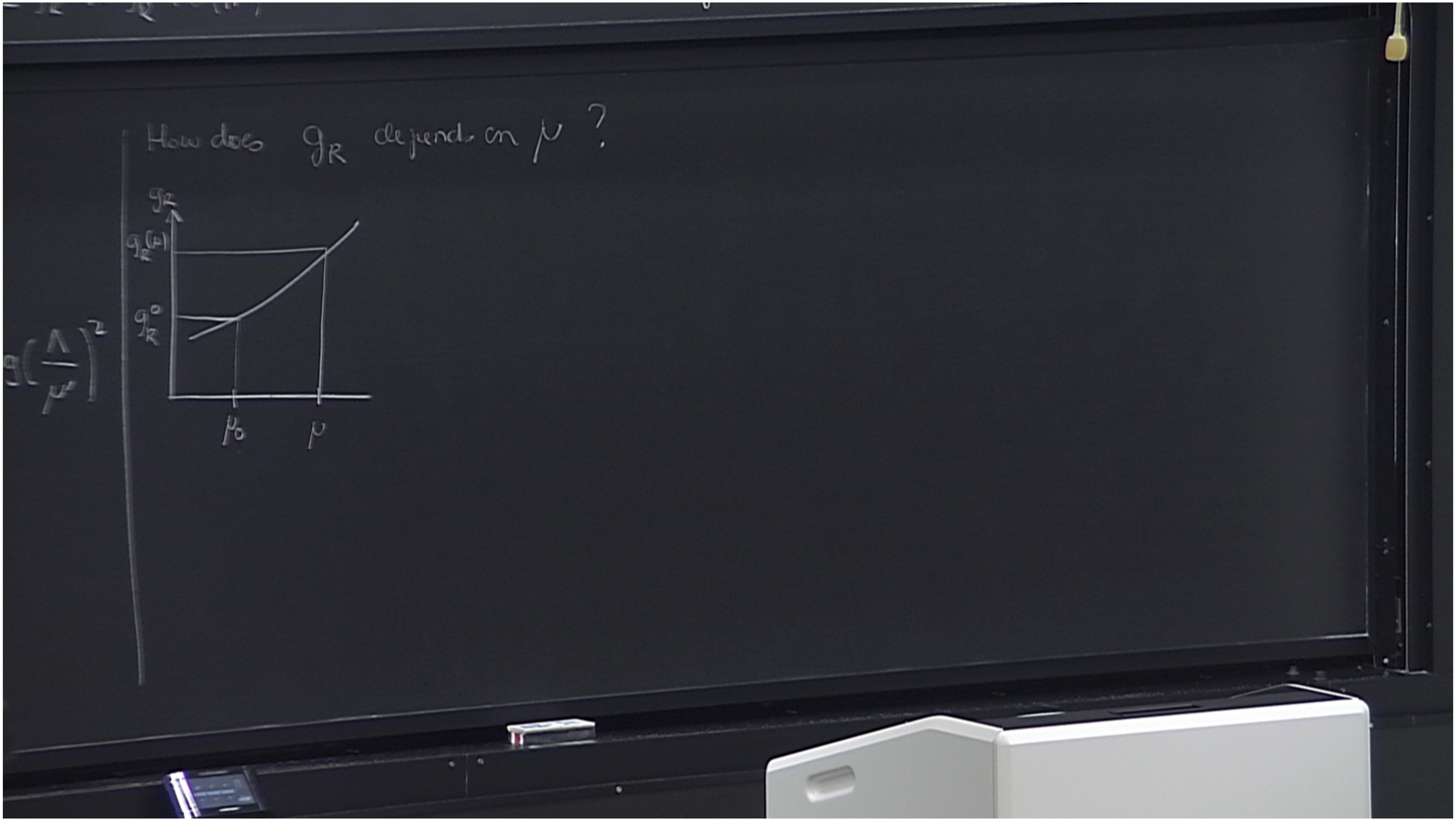
dimensionless parameter g_R, dimensionfull parameter μ

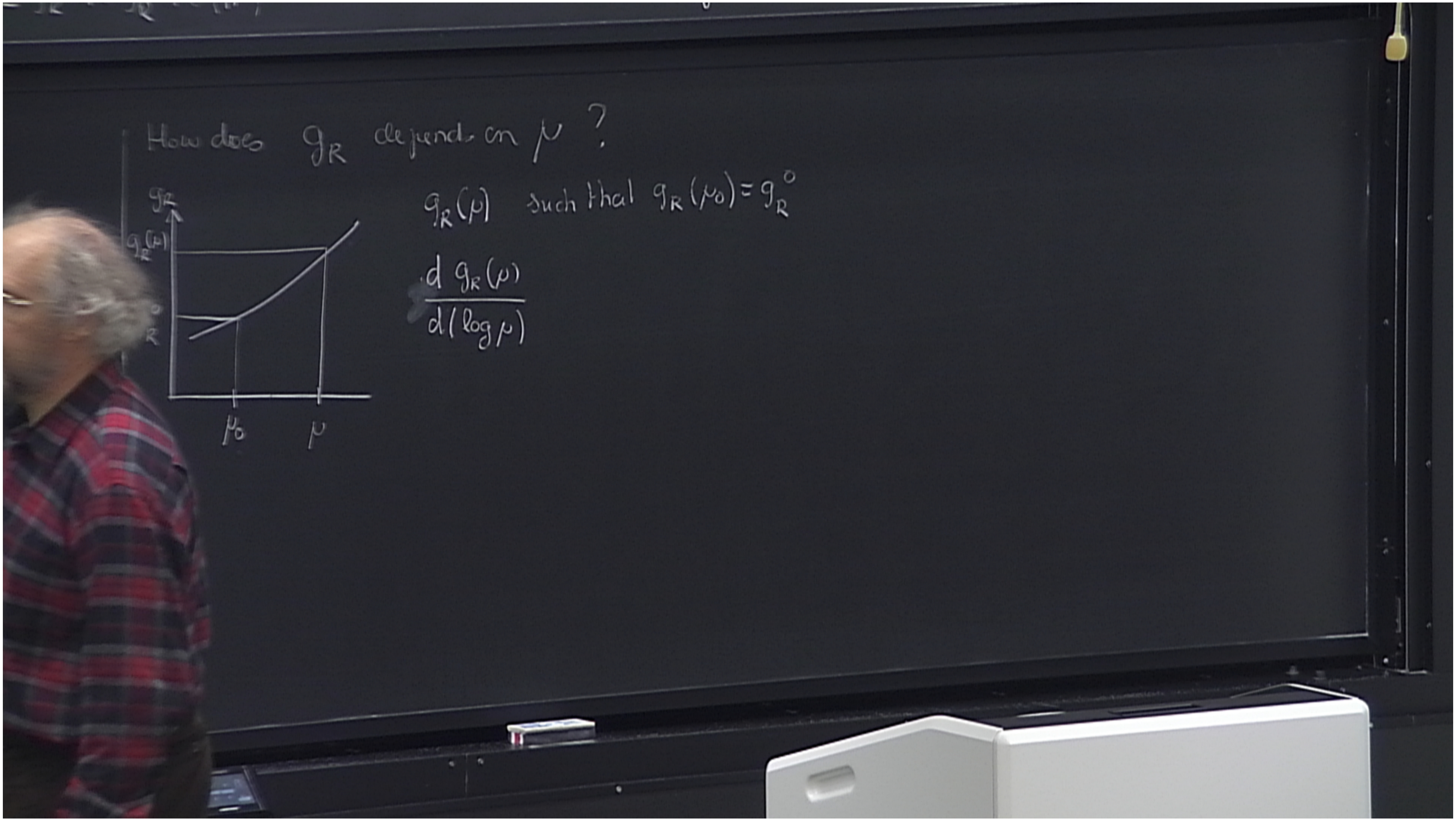
__Same Physical__ Renormalized theory $\Leftarrow$ if $C = g_R + \hbar \frac{3}{2} g_R^2 \log\left(\frac{\Lambda}{\mu}\right)^2 = g_R' + \hbar \frac{3}{2} g_R' \log\left(\frac{\Lambda}{\mu'}\right)^2$

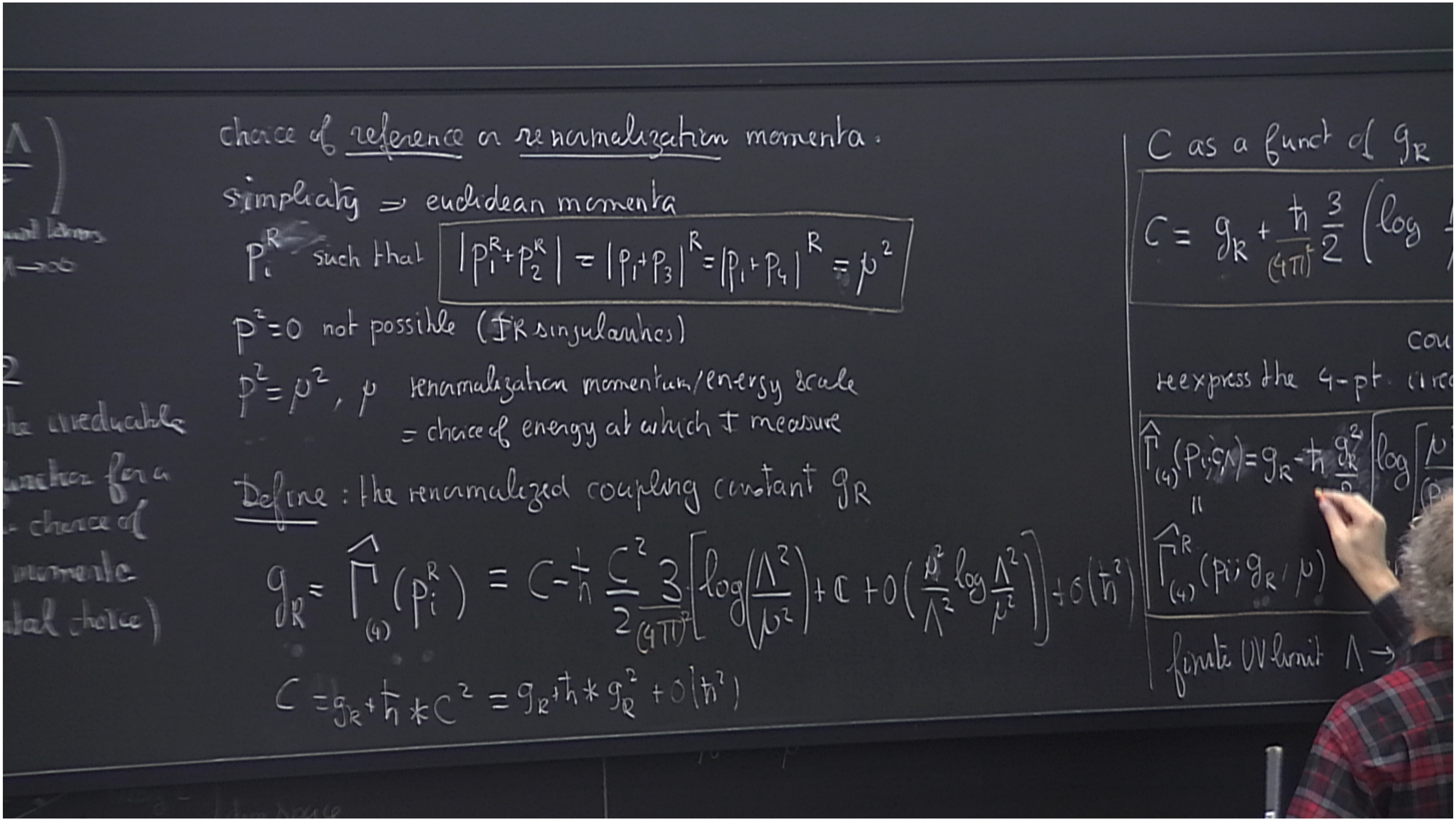
different energies $\left\{ \begin{array}{l} \mu, \; g_R \\ \mu' \neq \mu, \; g_R' \neq g_R \end{array} \right\}$ different results

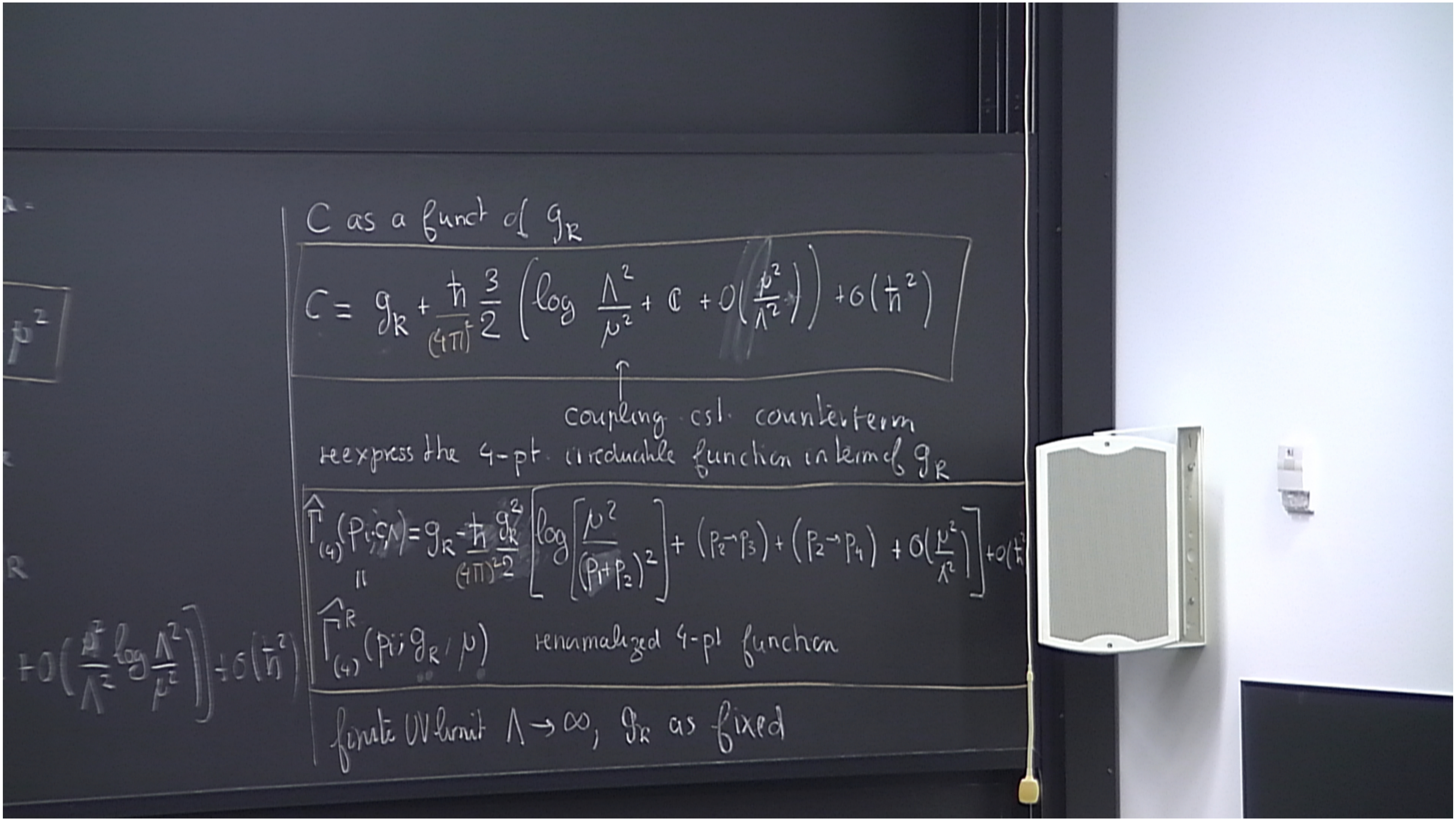



How does g_R depend on μ ?
$g\left(\frac{\Lambda}{\mu}\right)^2$


How does g_R depend on μ ?
g_R
$g_R(\mu)$
g_R^0
μ_0
μ
$g\left(\frac{\Lambda}{\mu}\right)^2$


How does g_R depend on μ ?
g_R(μ) such that g_R(μ_0) = g_R^0
d g_R(μ) / d(log μ)
g_R
g_R(μ)
μ_0
μ


choice of reference or renormalization momenta.
simplicity ⇒ euclidean momenta
p_i^R such that $|p_1^R+p_2^R| = |p_1+p_3|^R = |p_1+p_4|^R = \mu^2$
$p^2=0$ not possible (IR singularities)
$p^2=\mu^2$, μ renormalization momentum/energy scale
= choice of energy at which I measure
Define : the renormalized coupling constant g_R
$g_R = \hat{\Gamma}_{(4)}(p_i^R) = C - \hbar \frac{C^2}{2} \frac{3}{(4\pi)^2}\left[\log\left(\frac{\Lambda^2}{\mu^2}\right) + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\right] + O(\hbar^2)$
$C = g_R + \hbar * C^2 = g_R + \hbar * g_R^2 + O(\hbar^2)$
C as a funct of g_R
reexpress the 4-pt.
finite UV limit $\Lambda \to$


C as a funct of g_R
$C = g_R + \frac{\hbar}{(4\pi)^2}\frac{3}{2}\left(\log\frac{\Lambda^2}{\mu^2} + \mathbb{C} + O\left(\frac{\mu^2}{\Lambda^2}\right)\right) + O(\hbar^2)$
coupling cst. counterterm
reexpress the 4-pt. irreducible function in terms of g_R
$\hat{\Gamma}_{(4)}(p_i; C, \Lambda) = g_R - \frac{\hbar}{(4\pi)^2}\frac{g_R^2}{2}\left[\log\left[\frac{\mu^2}{(p_1+p_2)^2}\right] + (p_2 \to p_3) + (p_2 \to p_4) + O\left(\frac{\mu^2}{\Lambda^2}\right)\right] + O(\hbar^2)$
$\hat{\Gamma}^R_{(4)}(p_i; g_R, \mu)$ renormalized 4-pt function
finite UV limit $\Lambda \to \infty$, g_R as fixed
$+ O\left(\frac{\mu^2}{\Lambda^2}\log\frac{\Lambda^2}{\mu^2}\right)\Bigg] + O(\hbar^2)$

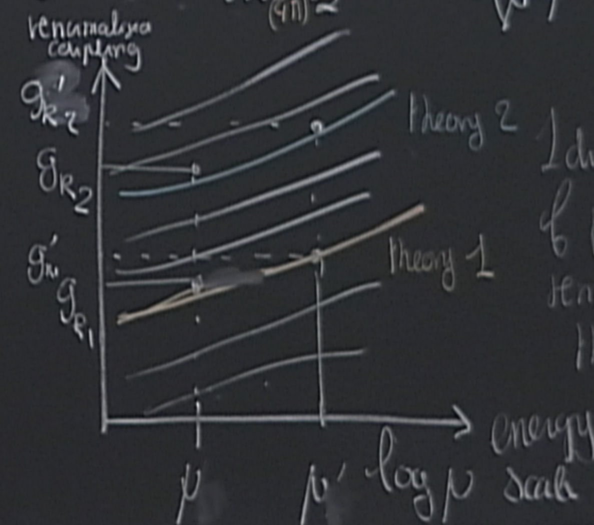
Green function

Renormalized coupling constant depends on renormalization scale μ

dimensionless parameter g_R, dimensionfull parameter μ

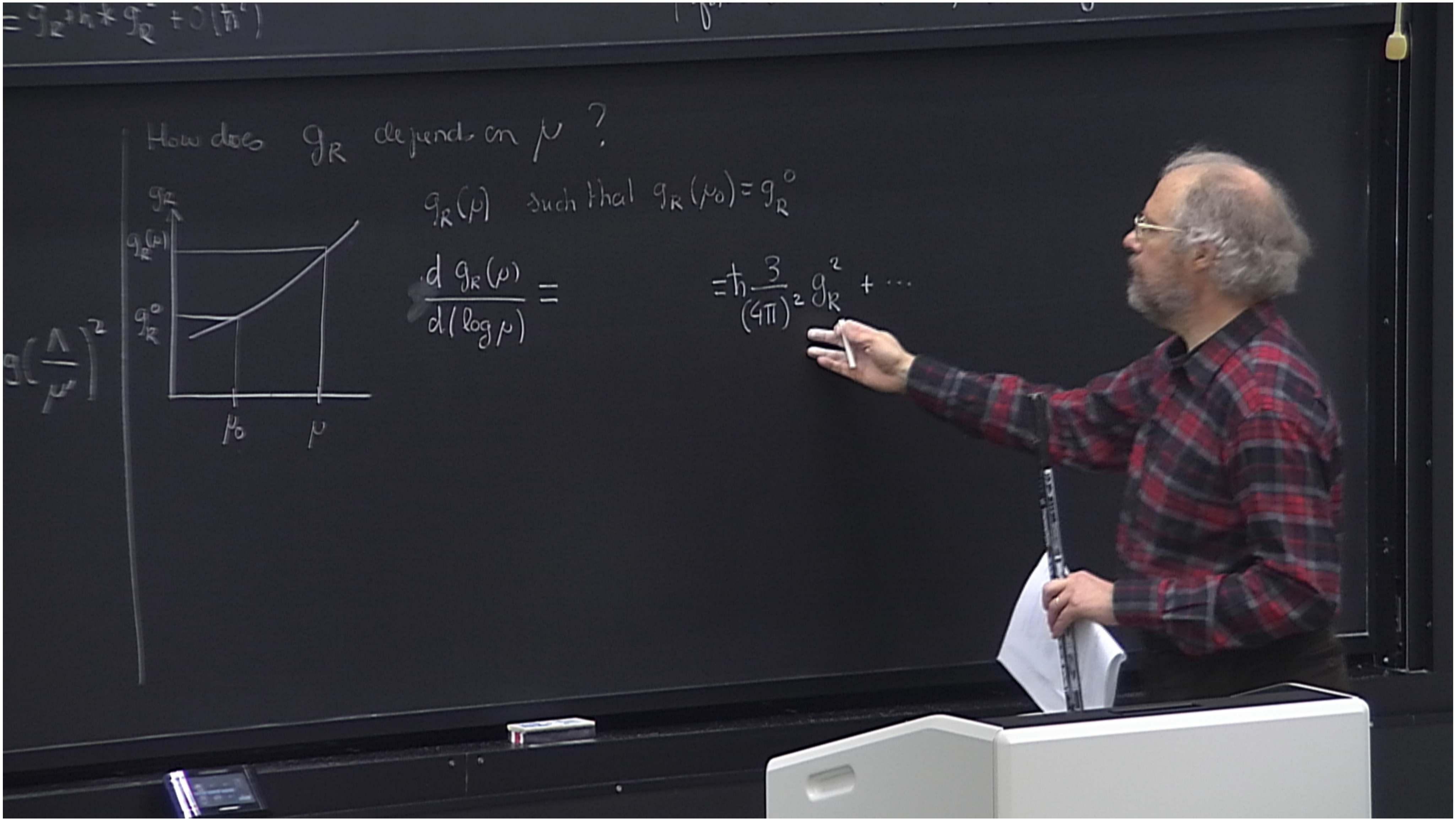
Same Physical Renormalized theory $\Leftarrow$ if $C = g_R + \frac{\hbar}{(4\pi)^2}\frac{3}{2} g_R^2 \log\left(\frac{\Lambda}{\mu}\right)^2 = g_R' + \frac{\hbar}{(4\pi)^2}\frac{3}{2} g_R' \log\left(\frac{\Lambda}{\mu'}\right)^2$

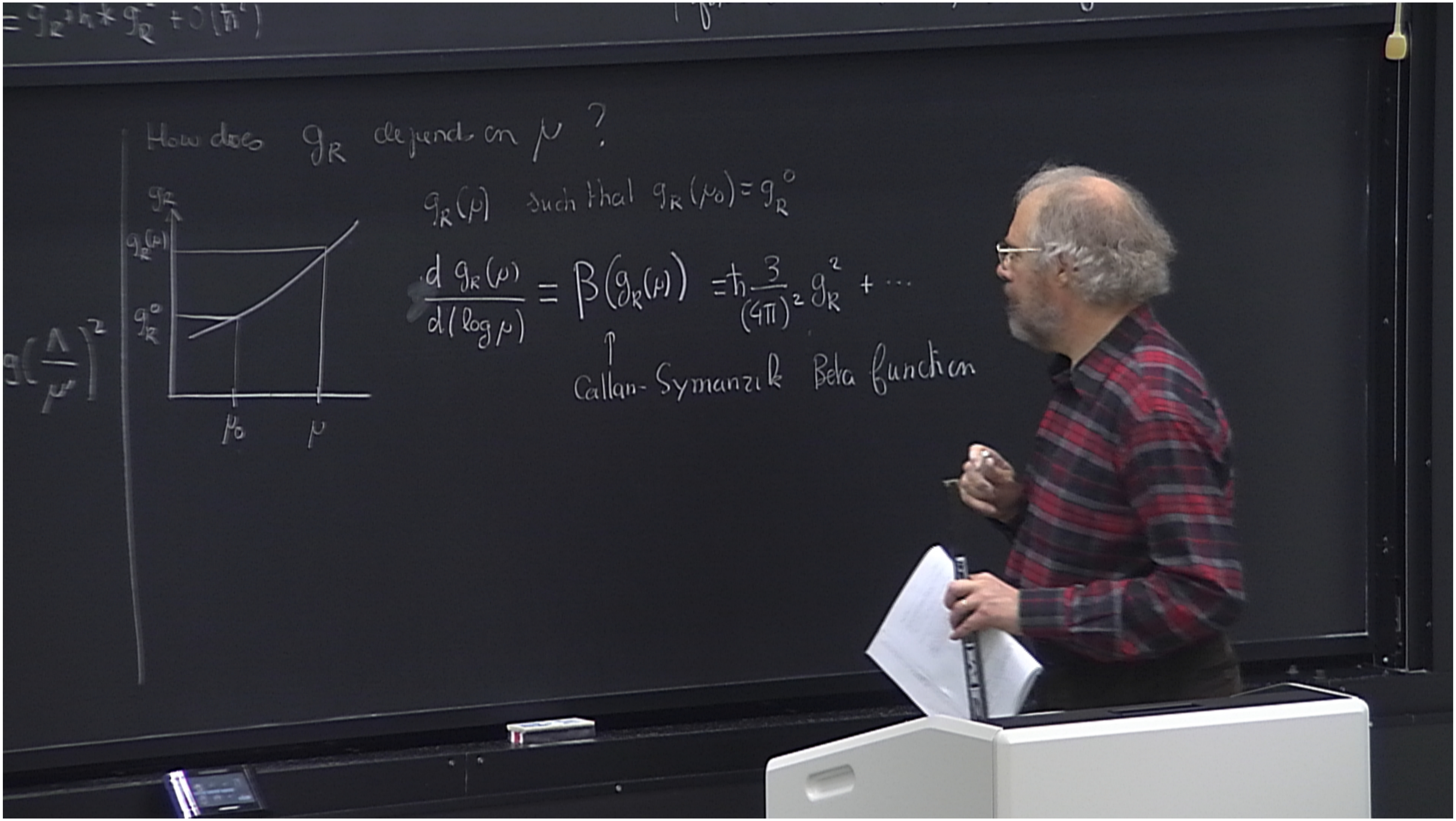
different energies $\left\{ \begin{array}{l} \mu , \; g_R \\ \mu' \neq \mu , \; g_R' \neq g_R \end{array} \right\}$ different results

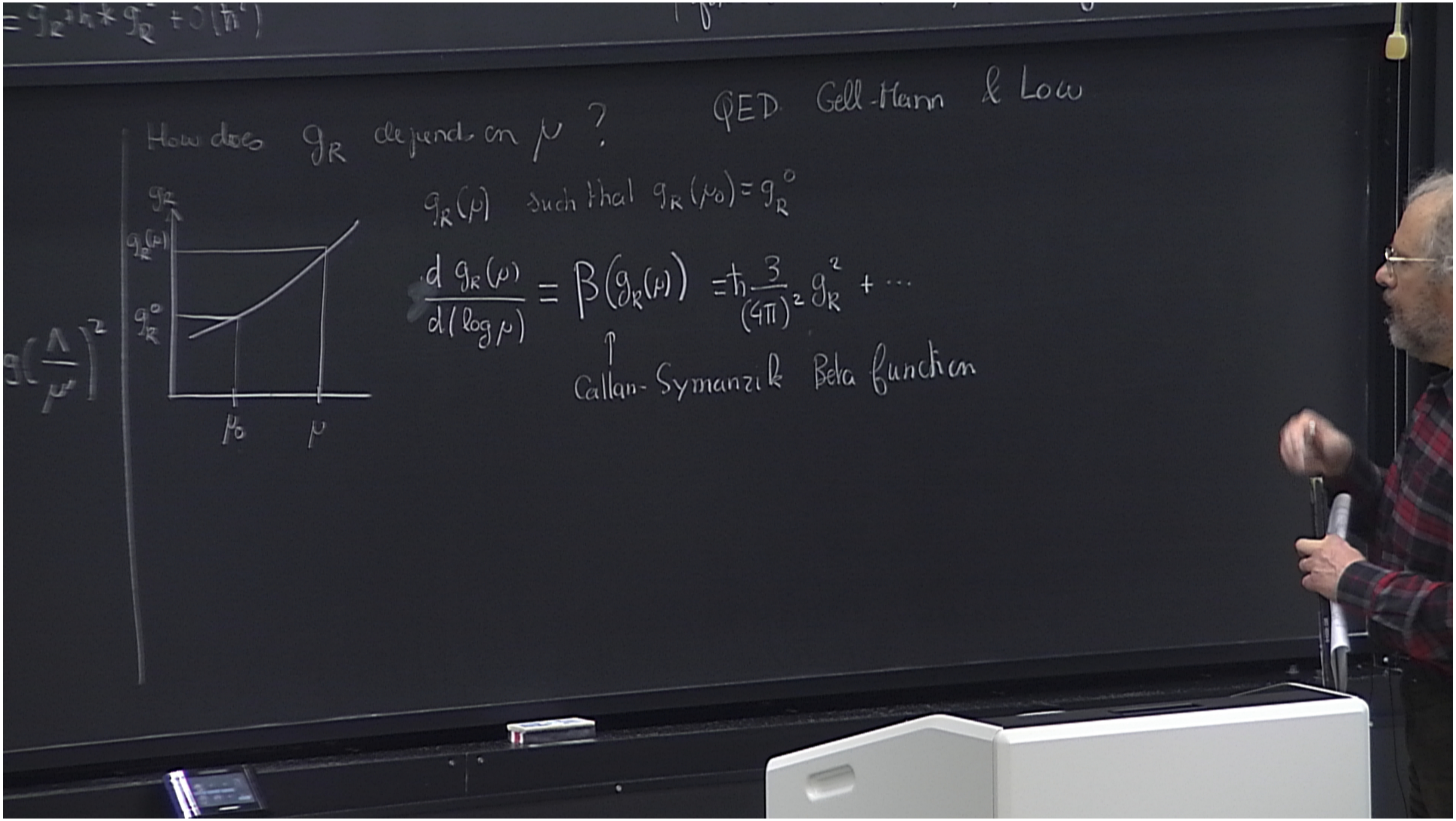


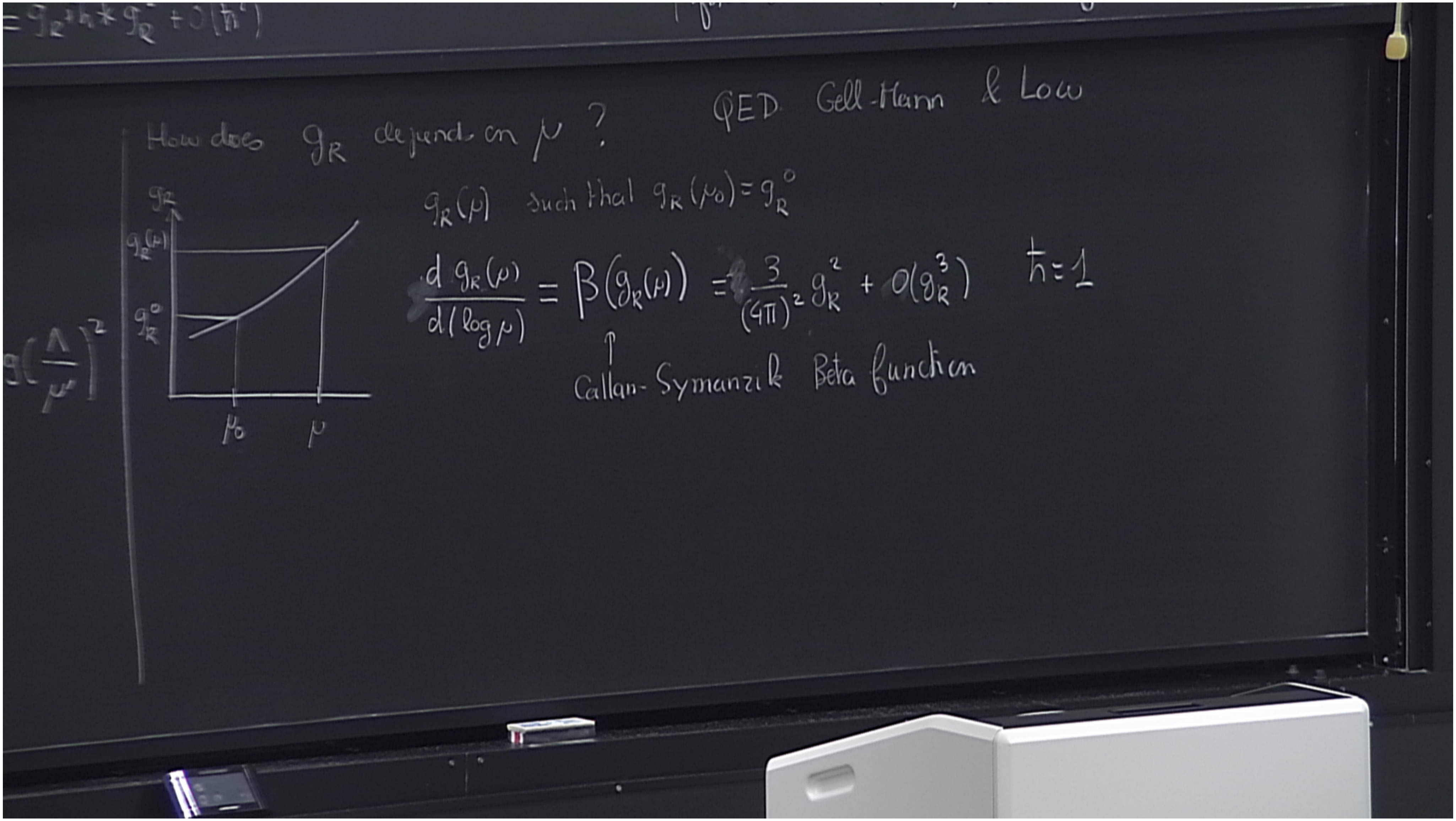
1 dim space of physical renormalized theories

How d


How does g_R depend on μ ?
g_R(μ) such that g_R(μ_0) = g_R^0
d g_R(μ) / d(log μ) =
= ħ 3/(4π)^2 g_R^2 + ···
g_R
g_R(μ)
g_R^0
μ_0
μ


How does g_R depend on μ ?
$g_R(\mu)$ such that $g_R(\mu_0) = g_R^0$
$\frac{d\, g_R(\mu)}{d(\log \mu)} = \beta(g_R(\mu)) = \hbar \frac{3}{(4\pi)^2} g_R^2 + \cdots$
Callan-Symanzik Beta function
g_R
$g_R(\mu)$
g_R^0
μ_0
μ


How does g_R depend on μ ?
QED Gell-Mann & Low
g_R(μ) such that g_R(μ_0) = g_R^0
d g_R(μ) / d(log μ) = β(g_R(μ)) = ħ 3/(4π)^2 g_R^2 + ···
Callan-Symanzik Beta function
g_R
g_R(μ)
g_R^0
μ_0
μ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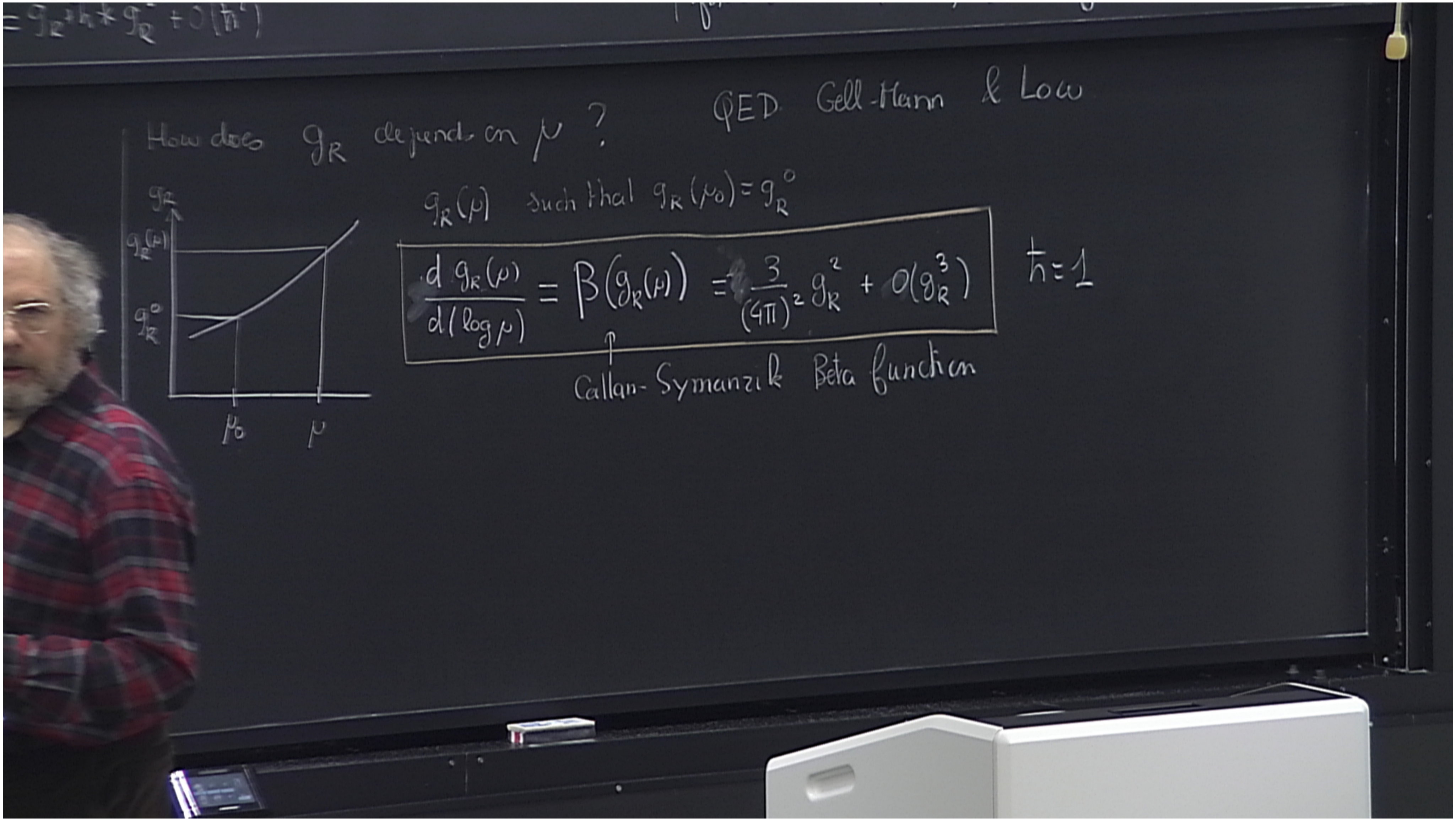
$\frac{d\, g_R(\mu)}{d(\log \mu)} = \beta(g_R(\mu)) = \frac{3}{(4\pi)^2} g_R^2 + O(g_R^3)$
$\hbar = 1$
Callan-Symanzik Beta function
g_R
$g_R(\mu)$
g_R^0
μ_0
μ


QED Gell-Mann & Low
How does g_R depend on μ ?
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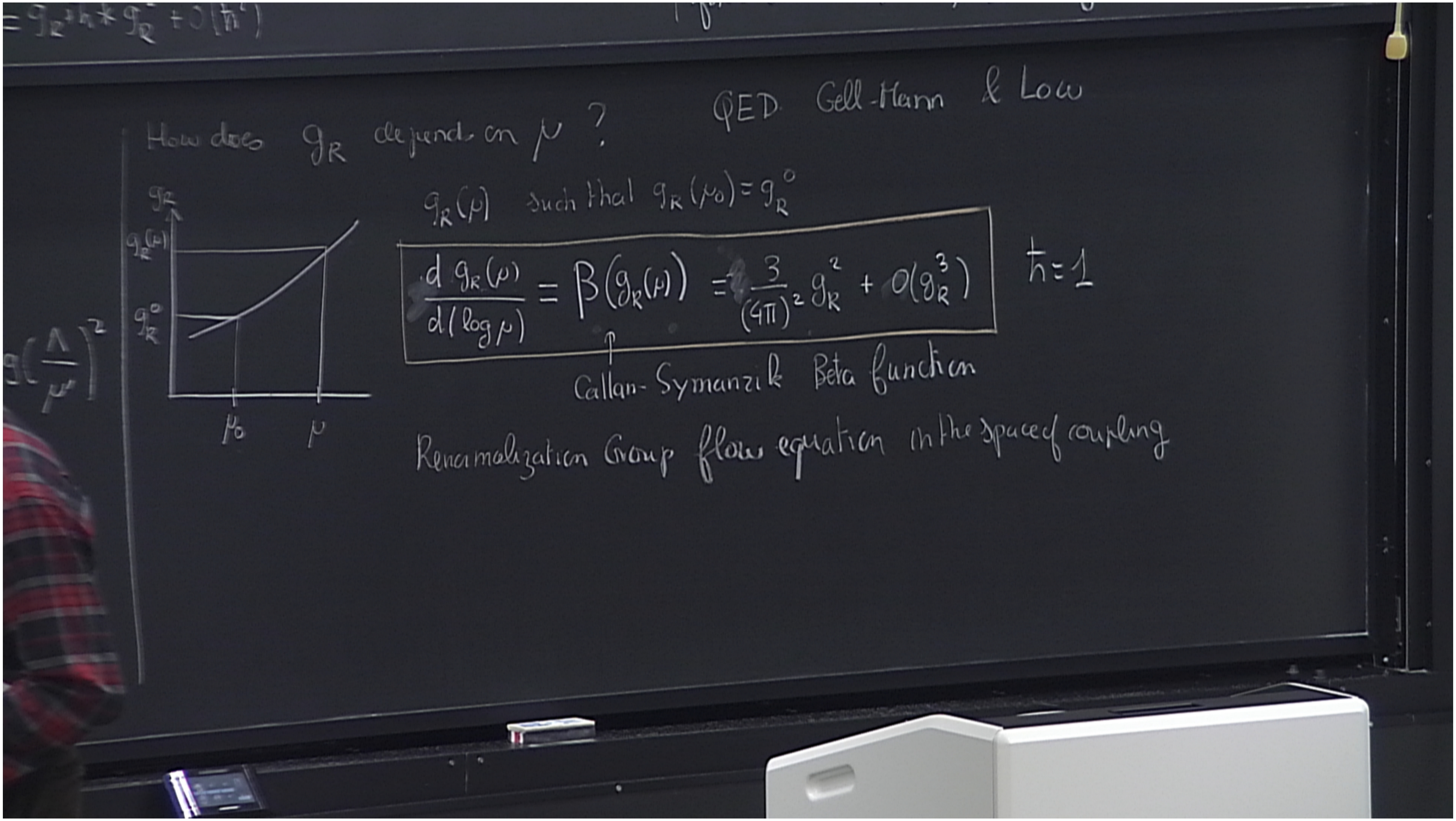
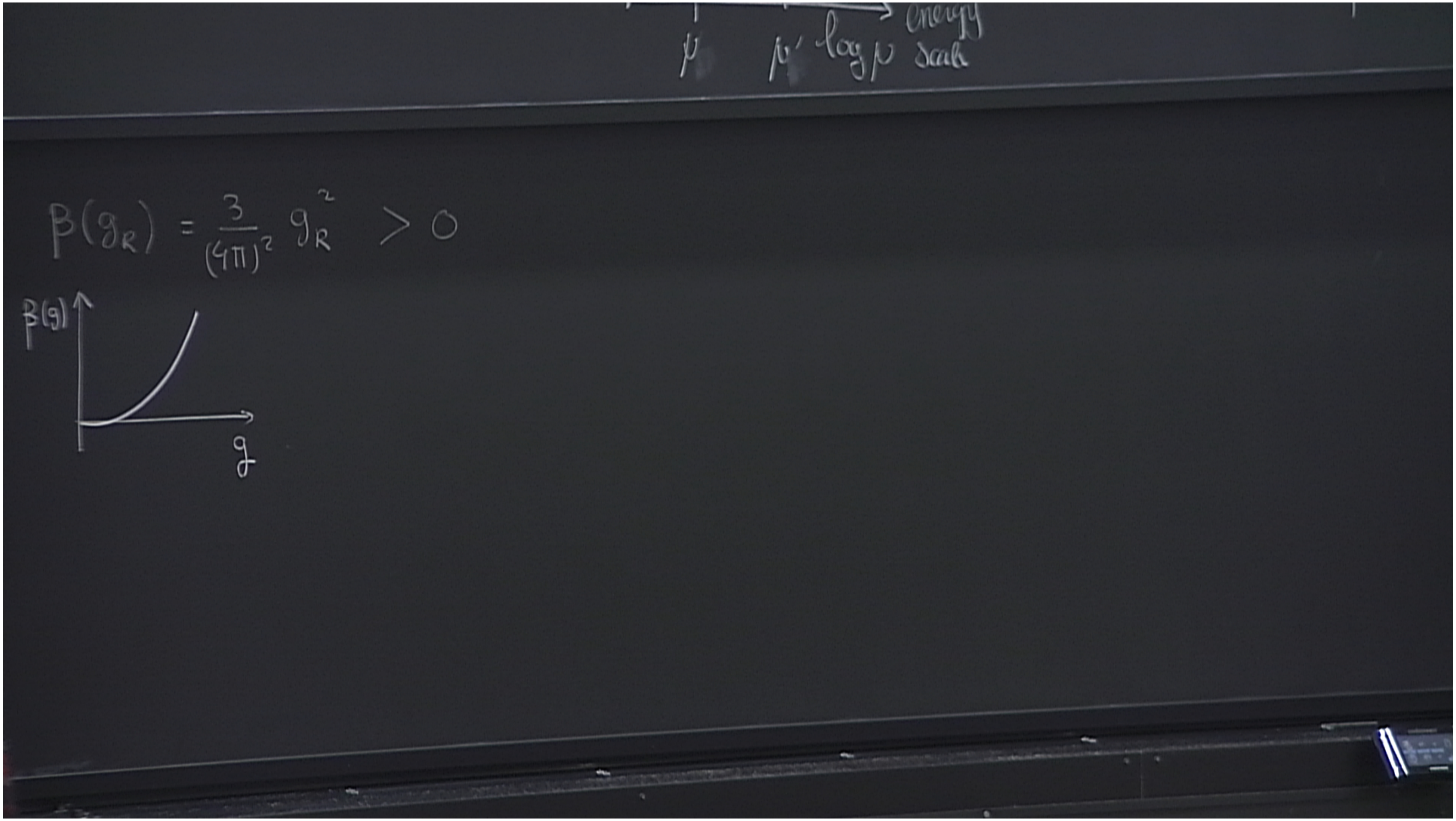
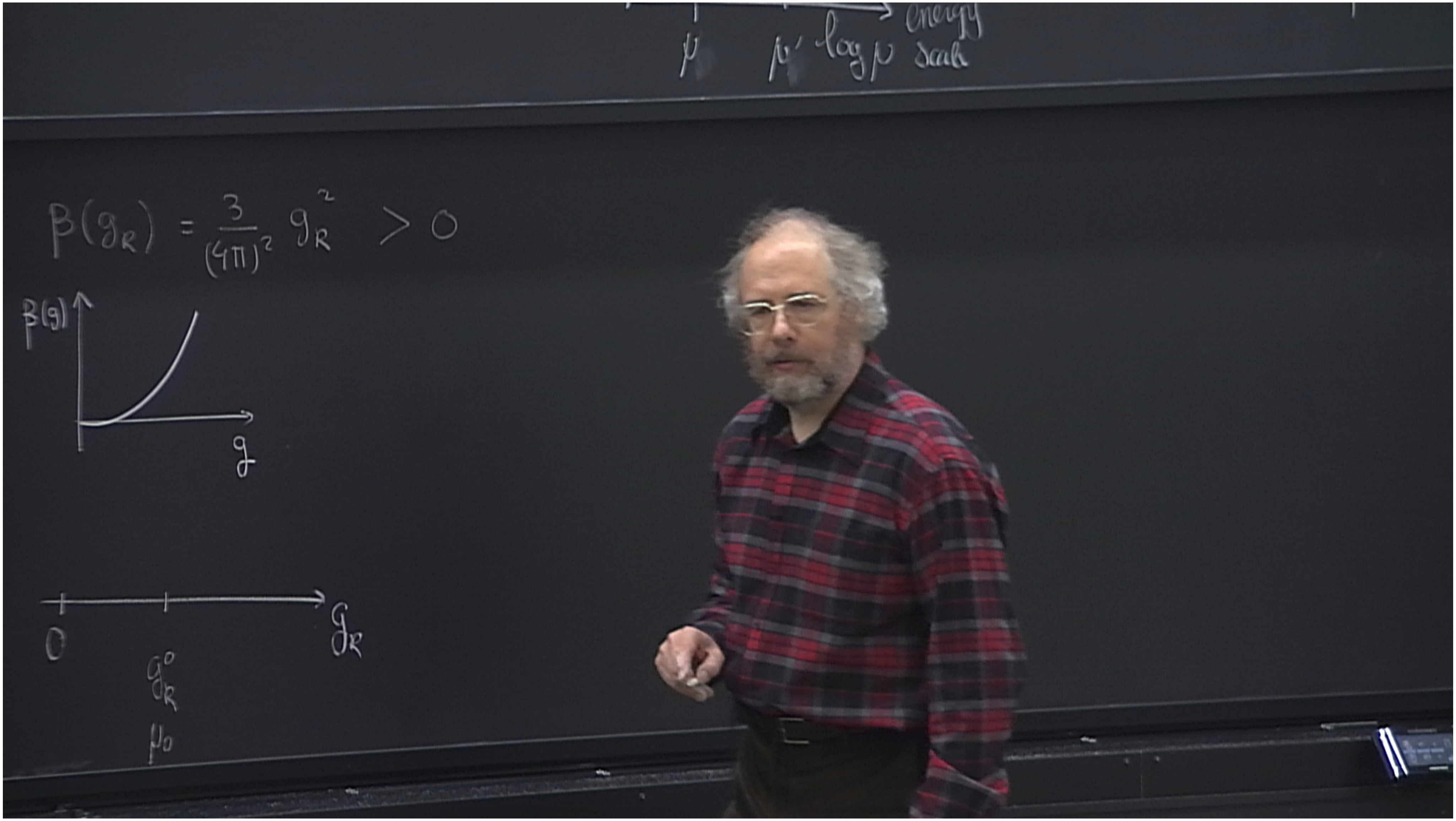
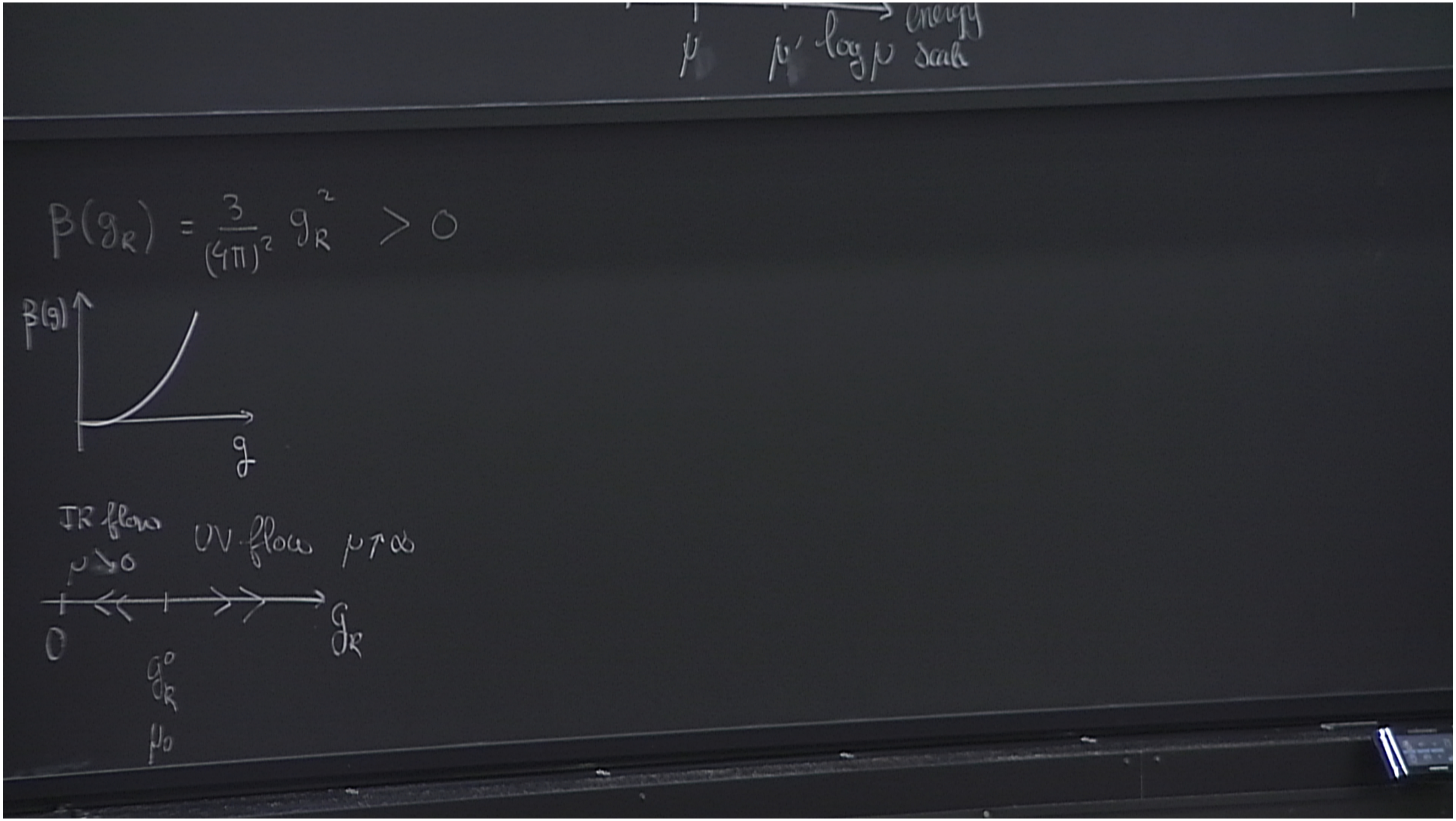
$\frac{d\, g_R(\mu)}{d(\log \mu)} = \beta(g_R(\mu)) = \frac{3}{(4\pi)^2} g_R^2 + O(g_R^3)$
$\hbar = 1$
Callan-Symanzik Beta function
g_R
$g_R(\mu)$
g_R^0
μ_0
μ

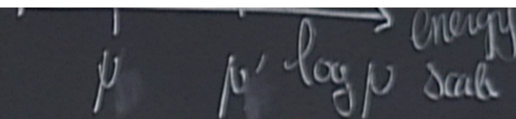

How does g_R depend on μ ?
QED Gell-Mann & Low
g_R(μ) such that g_R(μ_0) = g_R^0
d g_R(μ) / d(log μ) = β(g_R(μ)) = 3/(4π)^2 g_R^2 + O(g_R^3)
ħ = 1
Callan-Symanzik Beta function
Renormalization Group flow equation in the space of coupling
g_R
g_R(μ)
g_R^0
μ_0
μ


energy scale
μ μ' log μ
β(g_R) = 3/(4π)² g_R² > 0
β(g)
g

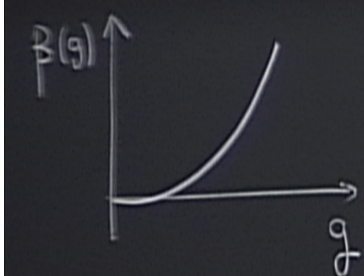


energy scale
μ
μ'
$\log \mu$
$\beta(g_R) = \frac{3}{(4\pi)^2} g_R^2 > 0$
$\beta(g)$
g
0
g_R
g_R^0
μ_0


μ μ' log μ energy scale
β(g_R) = 3/(4π)^2 g_R^2 > 0
β(g)
g
IR flow
μ→0
UV flow μ↑∞
0
g_R
g_R^0
μ_0



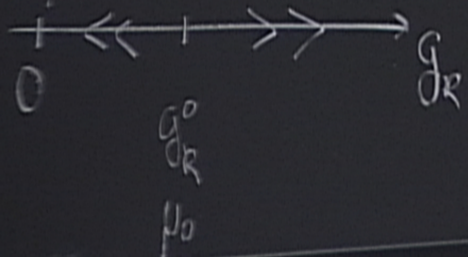
$$\beta(g_R) = \frac{3}{(4\pi)^2}\, g_R^2 > 0$$

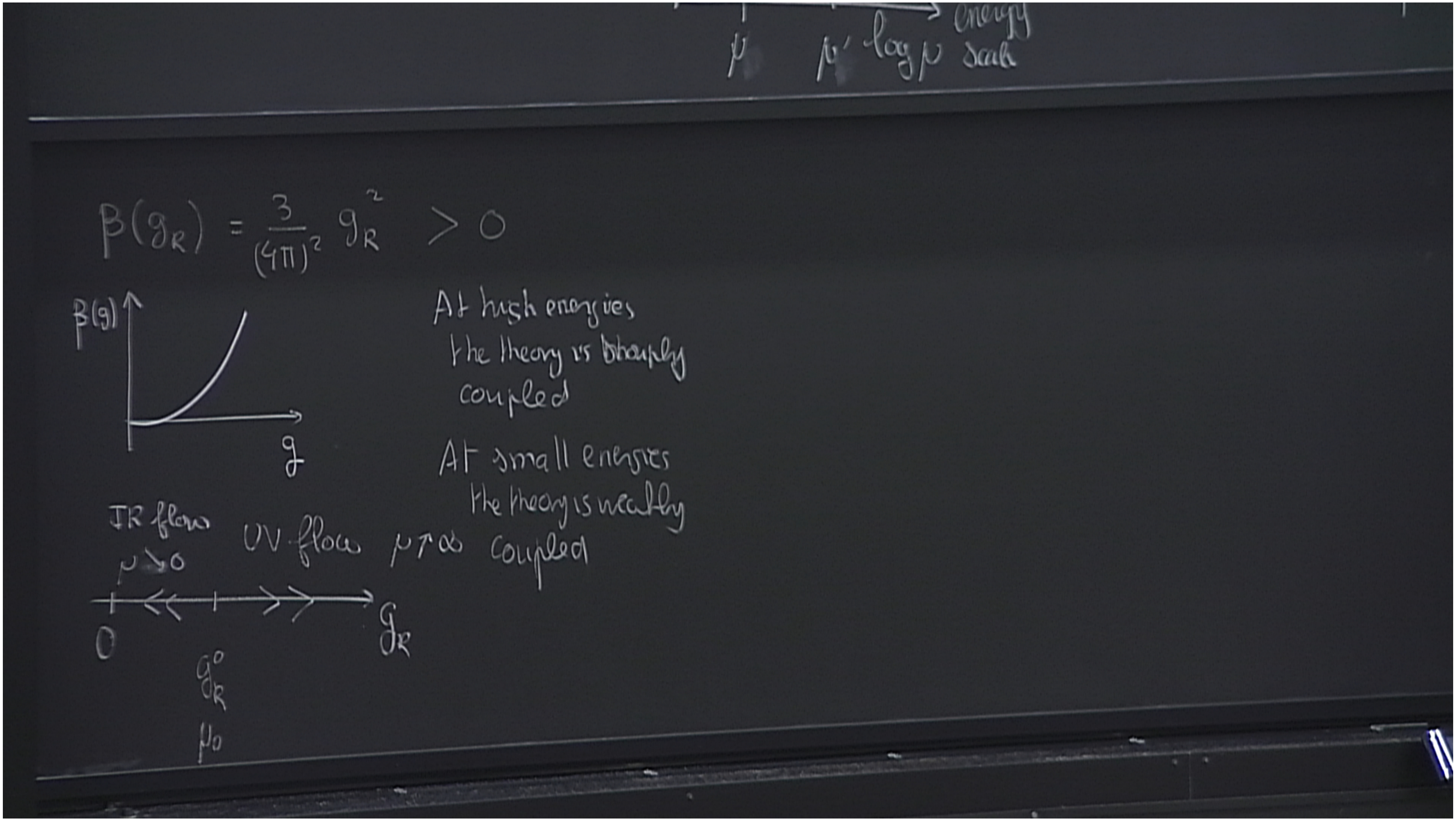


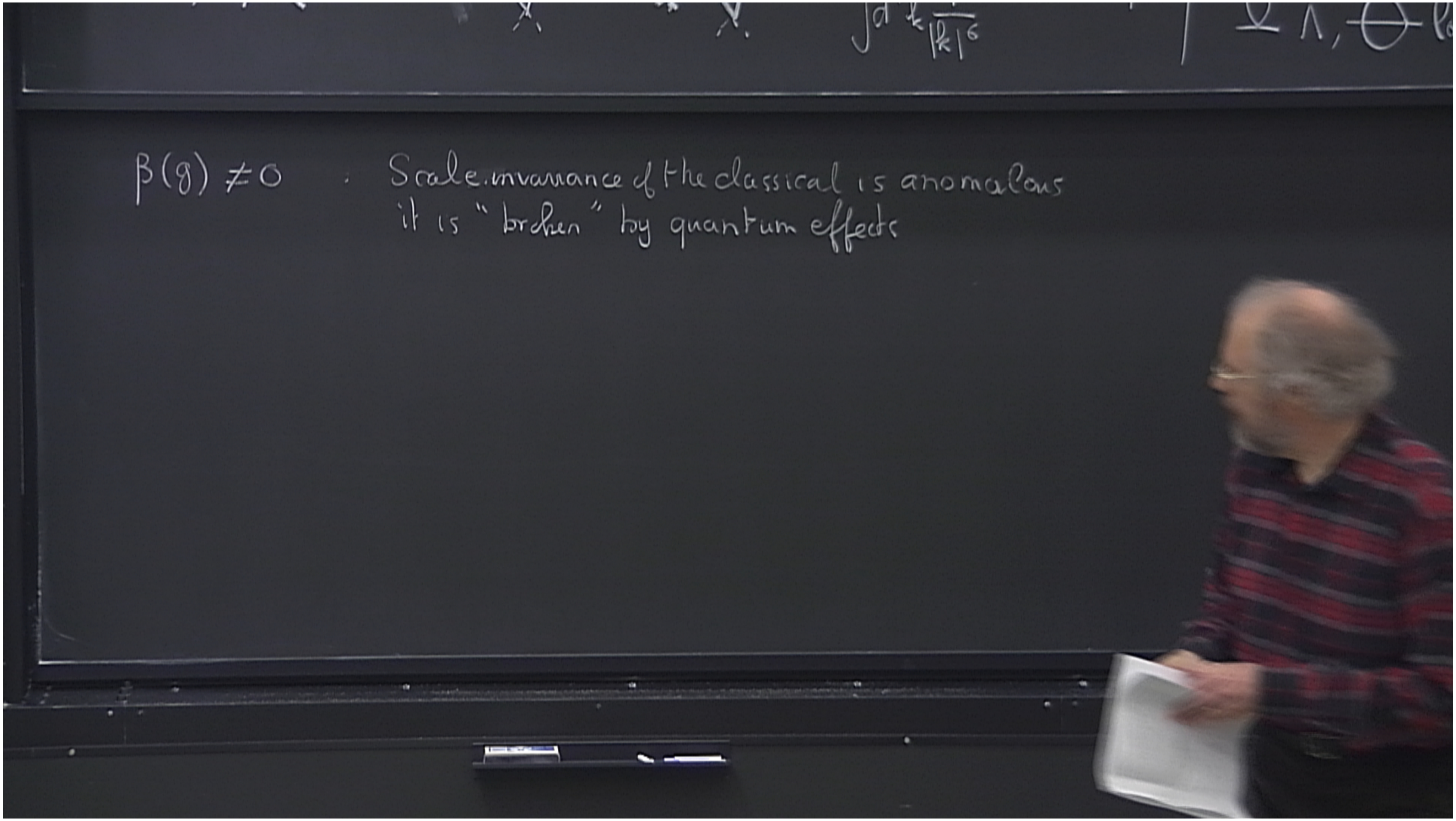
At high energies
the theory is strongly
coupled

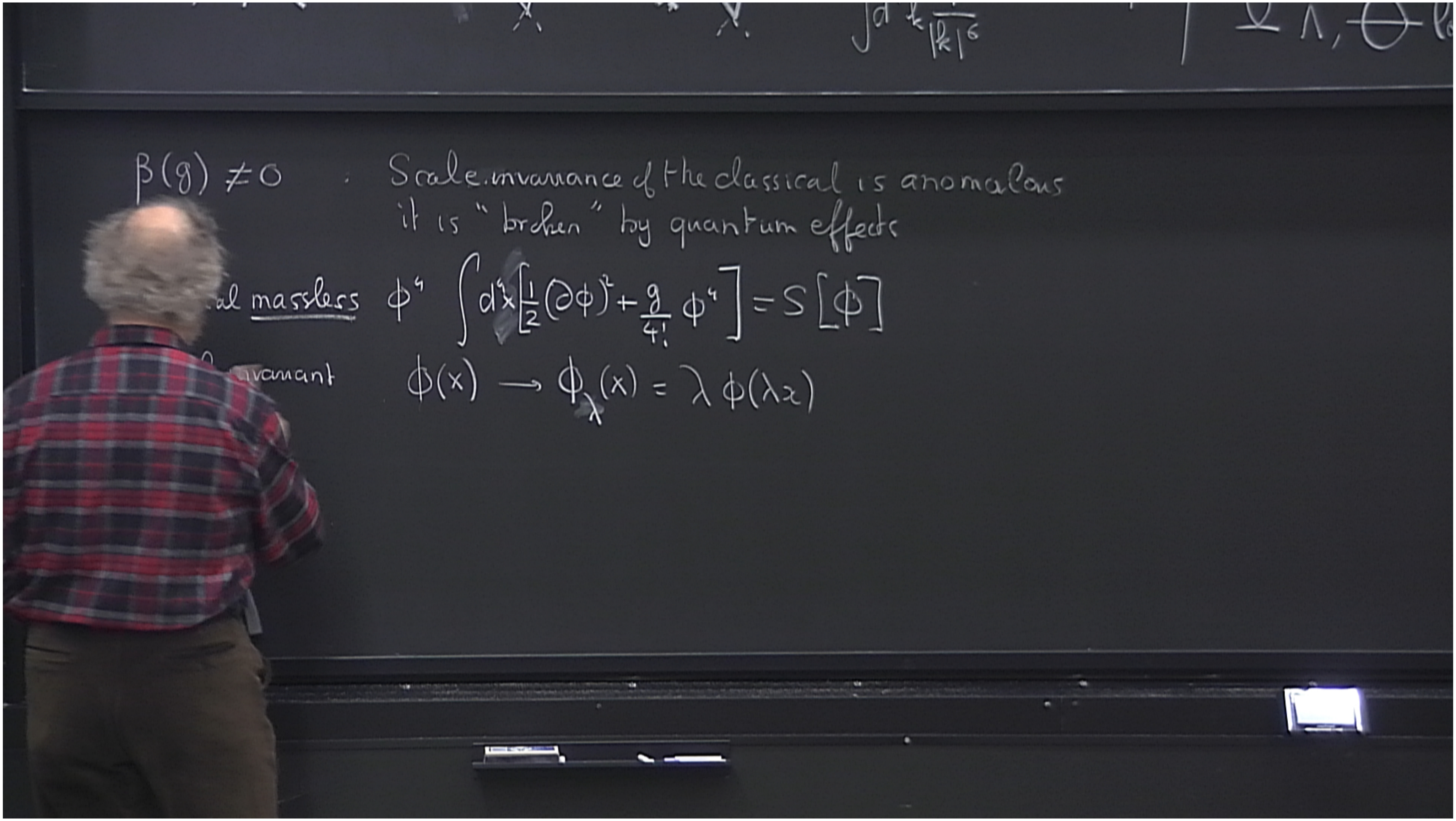
At small energies
the theory is weakly
coupled

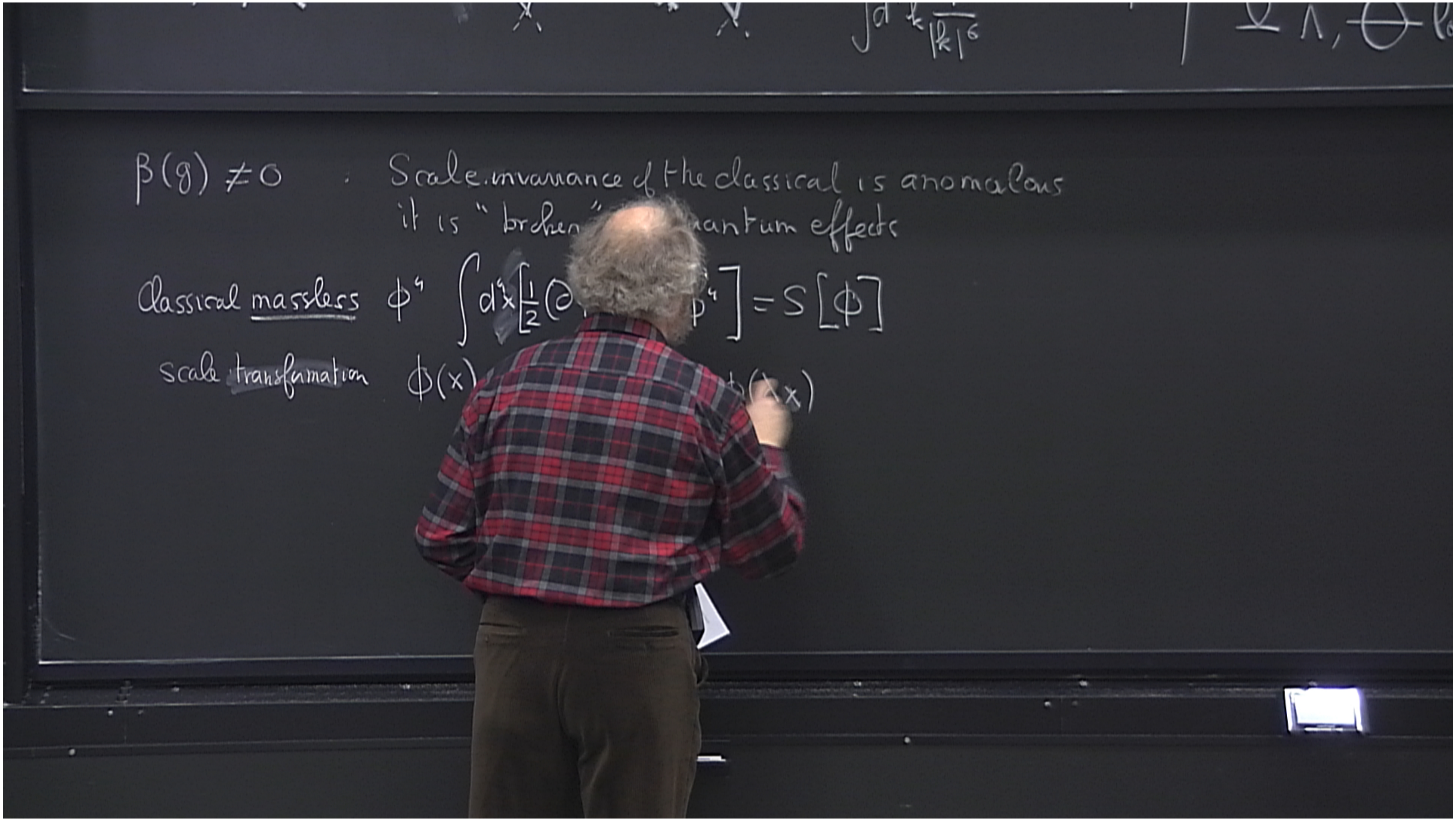
IR flow $\mu \to 0$ UV flow $\mu \uparrow \infty$




μ μ' log μ
energy scale
β(g_R) = 3/(4π)^2 g_R^2 > 0
β(g)
g
At high energies
the theory is strongly
coupled
At small energies
the theory is weakly
coupled
IR flow
μ ↘ 0
UV flow μ ↗ ∞
0
g_R
g_R^0
μ_0


β(g) ≠ 0 : Scale invariance of the classical is anomalous
it is "broken" by quantum effects


β(g) ≠ 0 : Scale invariance of the classical is anomalous
it is "broken" by quantum effects
massless φ⁴ ∫d⁴x [½(∂φ)² + g/4! φ⁴] = S[φ]
invariant φ(x) → φ_λ(x) = λ φ(λx)


β(g) ≠ 0 : Scale invariance of the classical is anomalous
it is "broken"
quantum effects
Classical massless φ⁴
= S[Φ]
scale transformation φ(x)

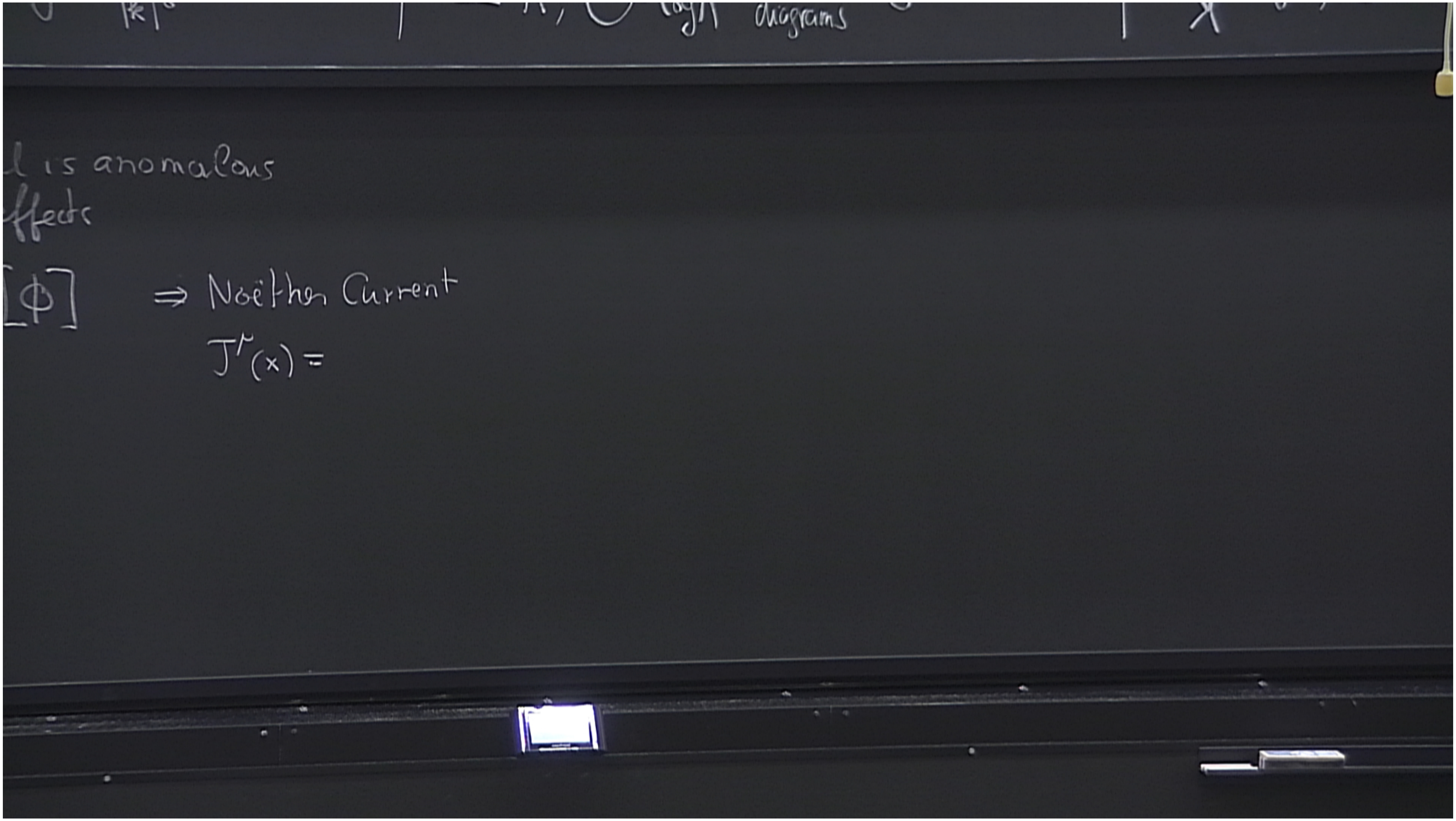
$\beta(g) \neq 0$: Scale invariance of the classical is anomalous
it is "broken" by quantum effects

Classical massless ϕ^4 $\int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4\right] = S[\phi]$

scale transformation $\phi(x) \longrightarrow \phi_\lambda(x) = \lambda\,\phi(\lambda x)$

scale invariance $S[\phi_\lambda] = S[\phi]$
$x \to \lambda x$

conformal invariance $x \to \frac{1}{x}$


diagrams
l is anomalous
ffects
⇒ Noether Current
$J^\mu(x) =$

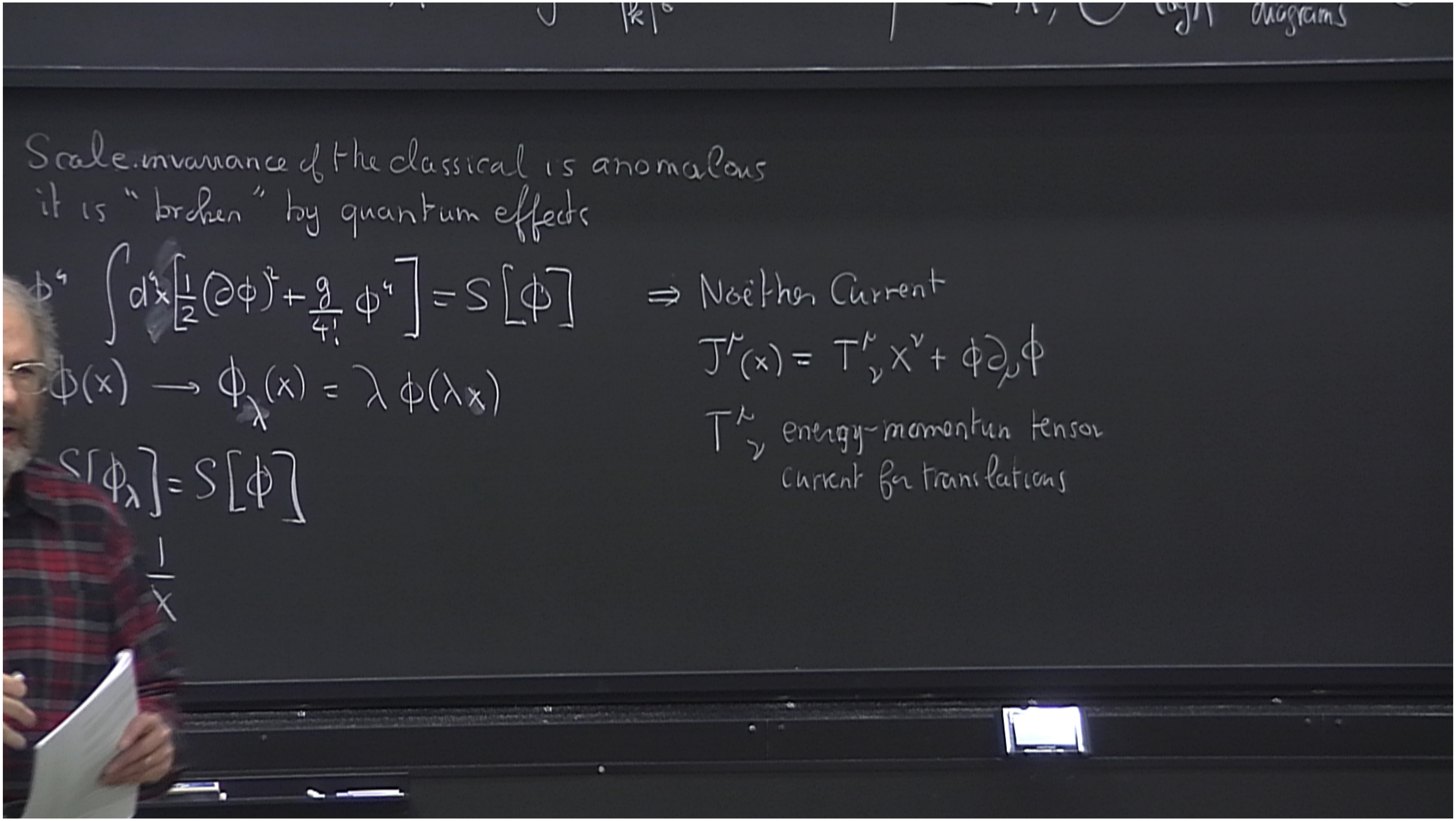
diagrams

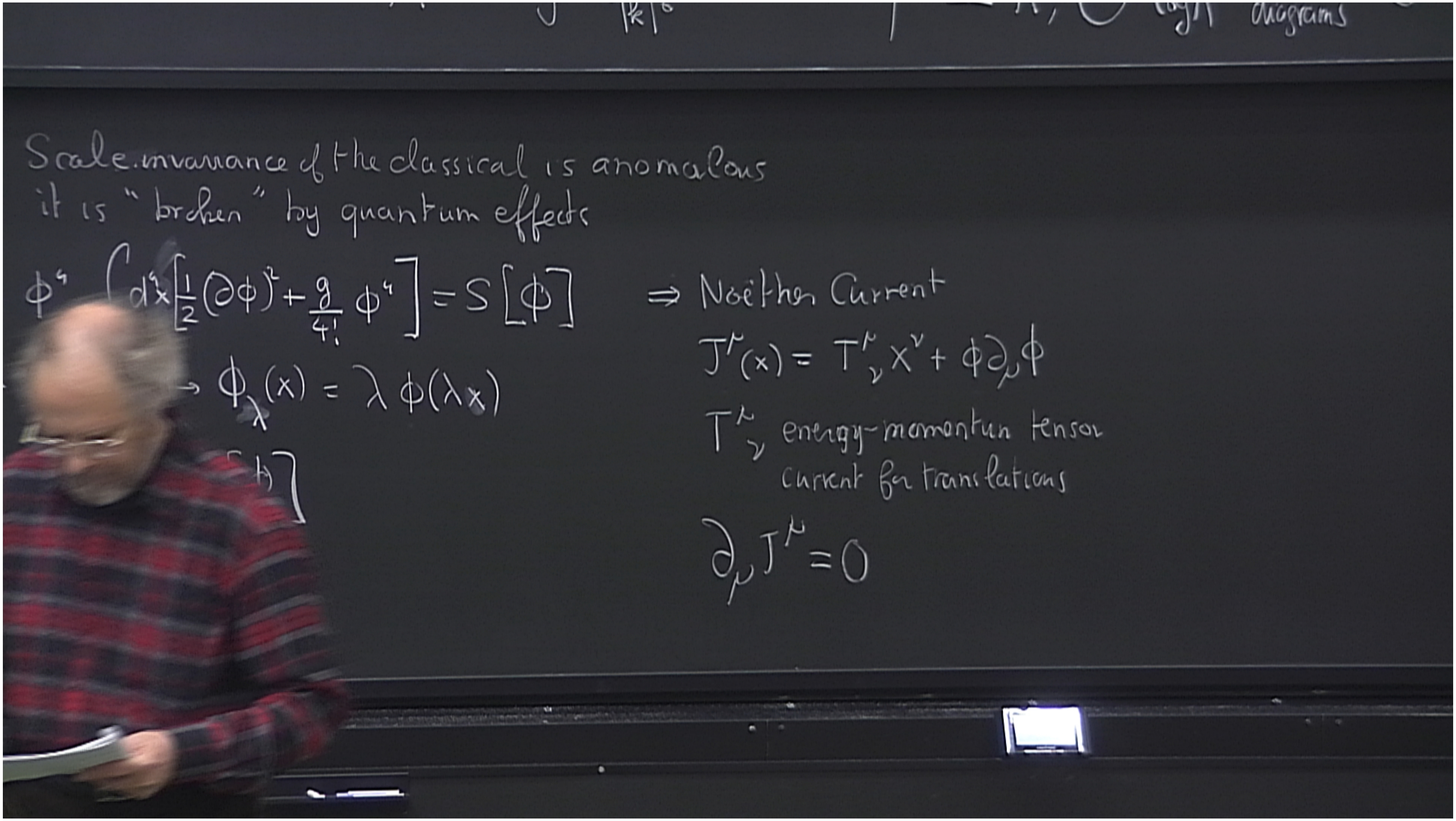
l is anomalous

ffects

$[\phi]$ $\Rightarrow$ Noether Current

$$J^\mu(x) = T^\mu{}_\nu x^\nu + \phi\partial_\mu\phi$$


diagrams
Scale invariance of the classical is anomalous
it is "broken" by quantum effects
ϕ^4 $\int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4\right] = S[\phi]$
$\phi(x) \longrightarrow \phi_\lambda(x) = \lambda\,\phi(\lambda x)$
$S[\phi_\lambda] = S[\phi]$
$\frac{1}{x}$
$\Rightarrow$ Noether Current
$J^\mu(x) = T^\mu{}_\nu x^\nu + \phi\,\partial_\nu\phi$
$T^\mu{}_\nu$ energy-momentum tensor
current for translations


diagrams
Scale invariance of the classical is anomalous
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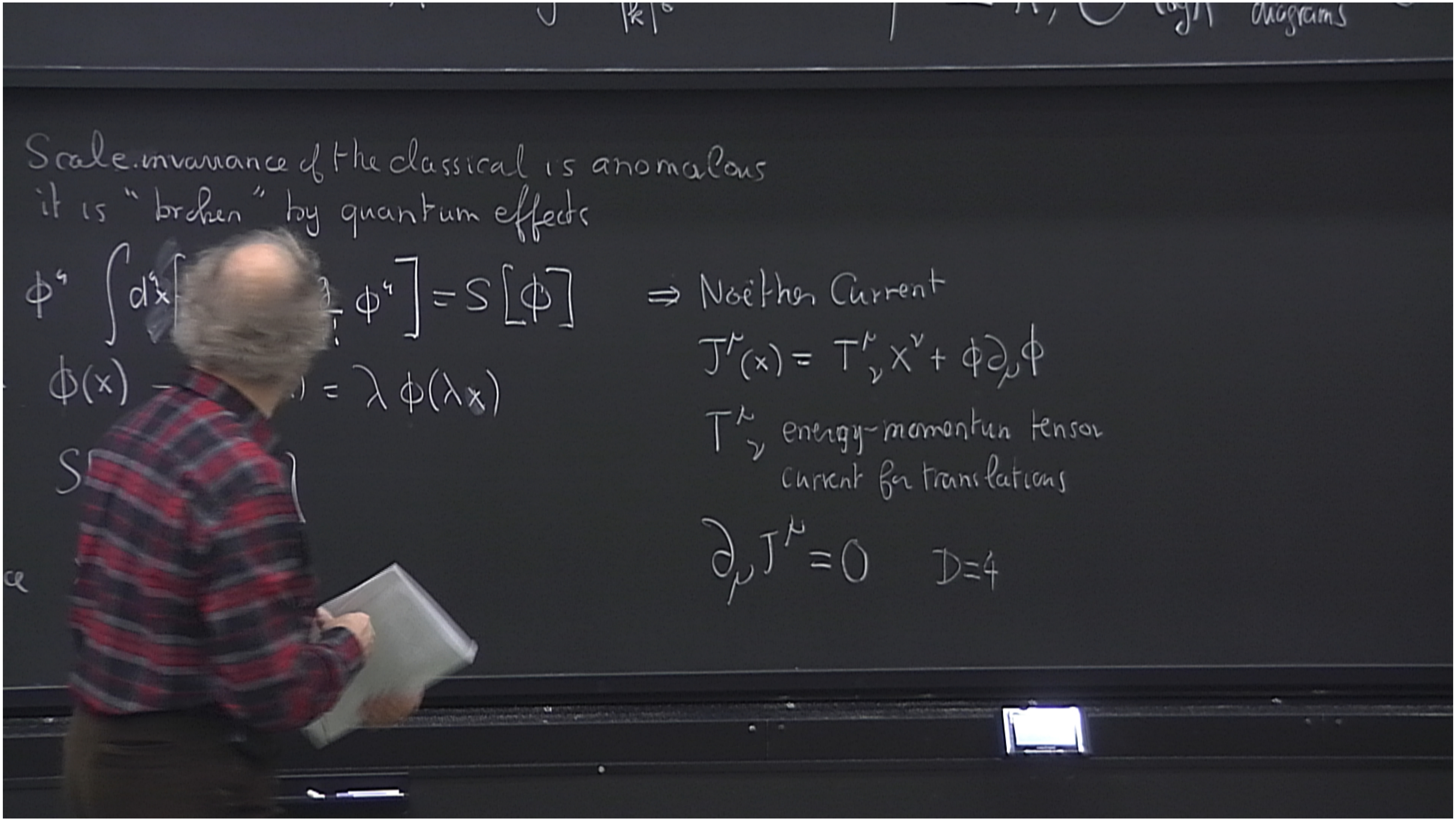
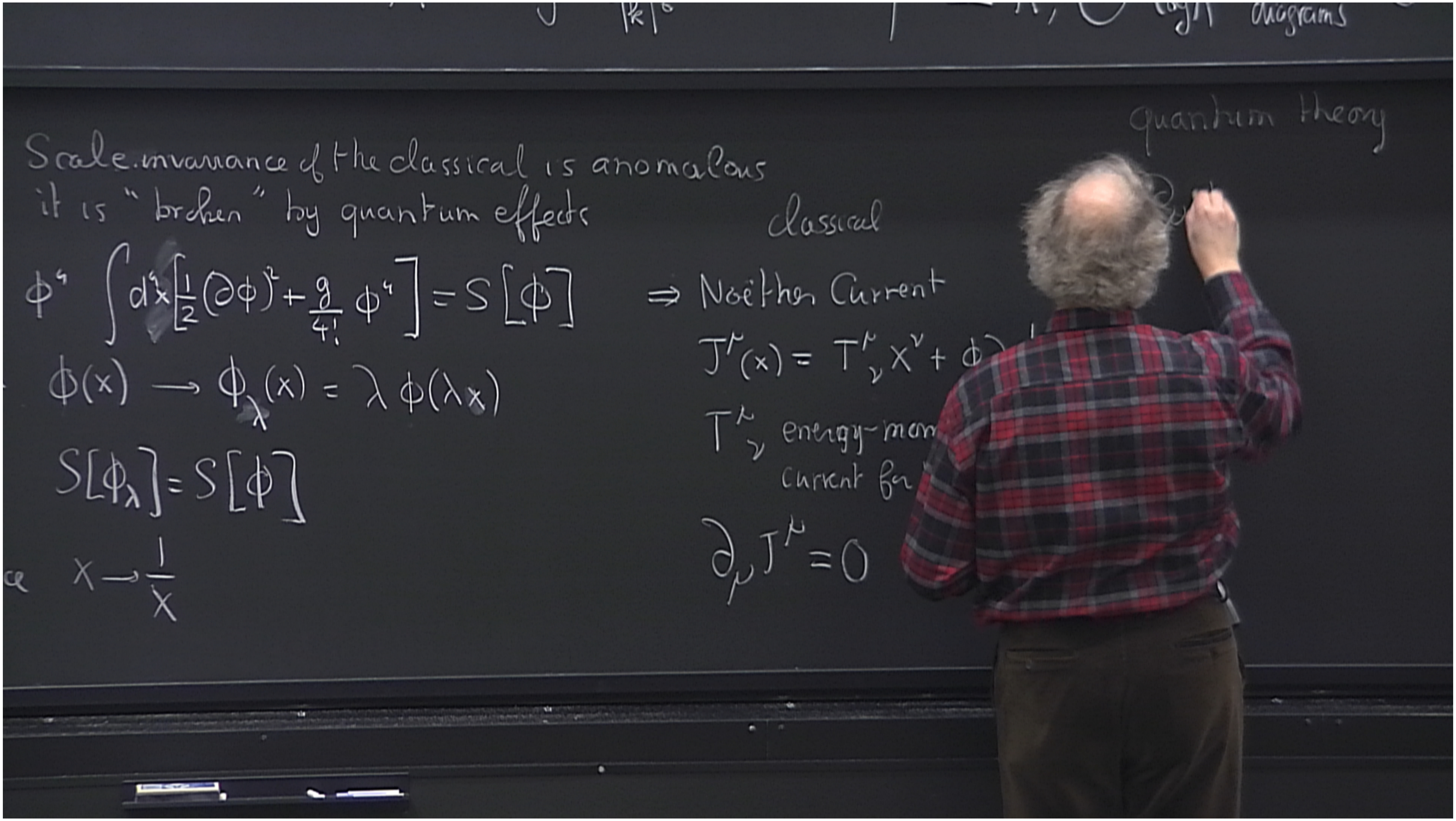
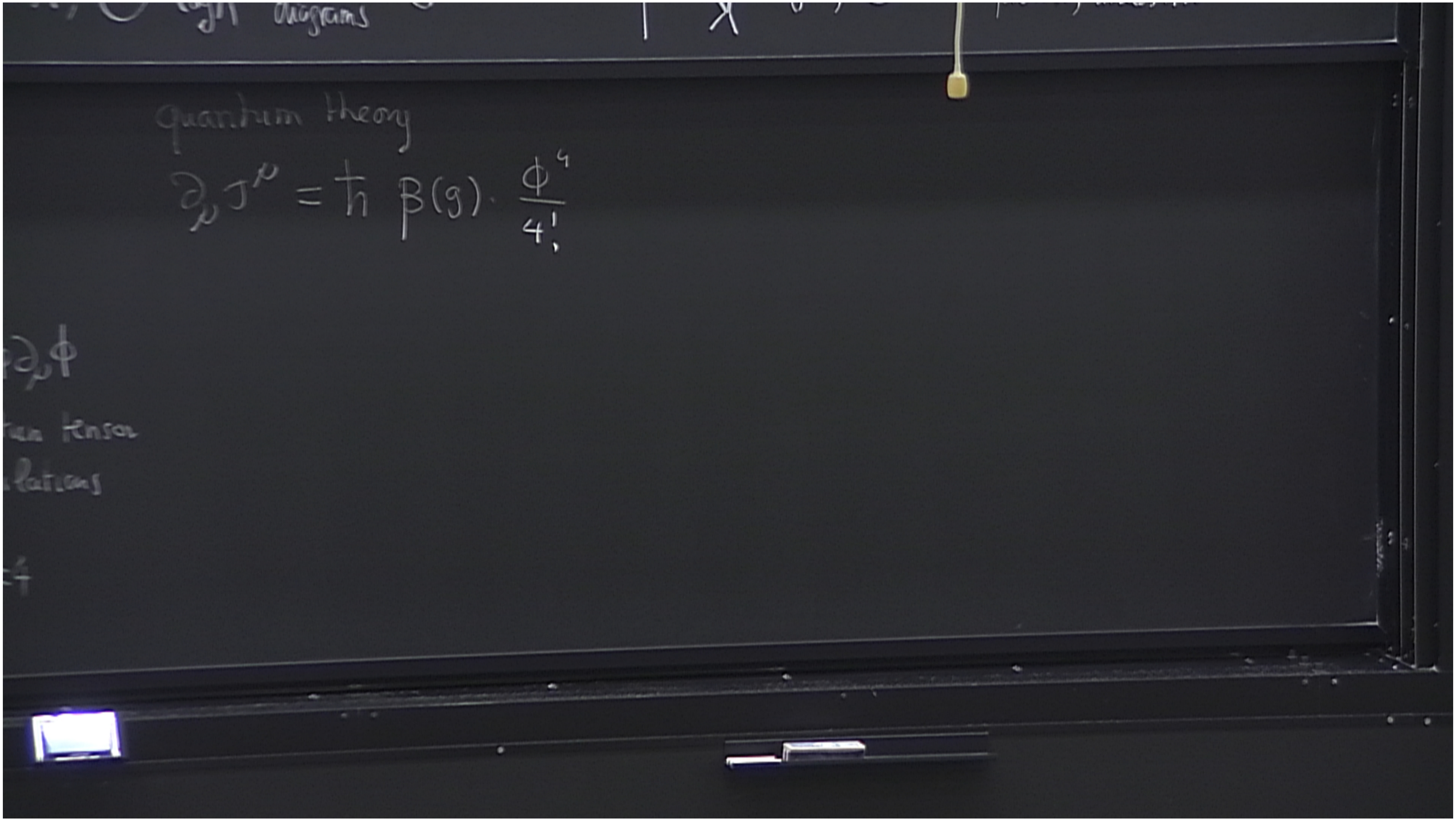
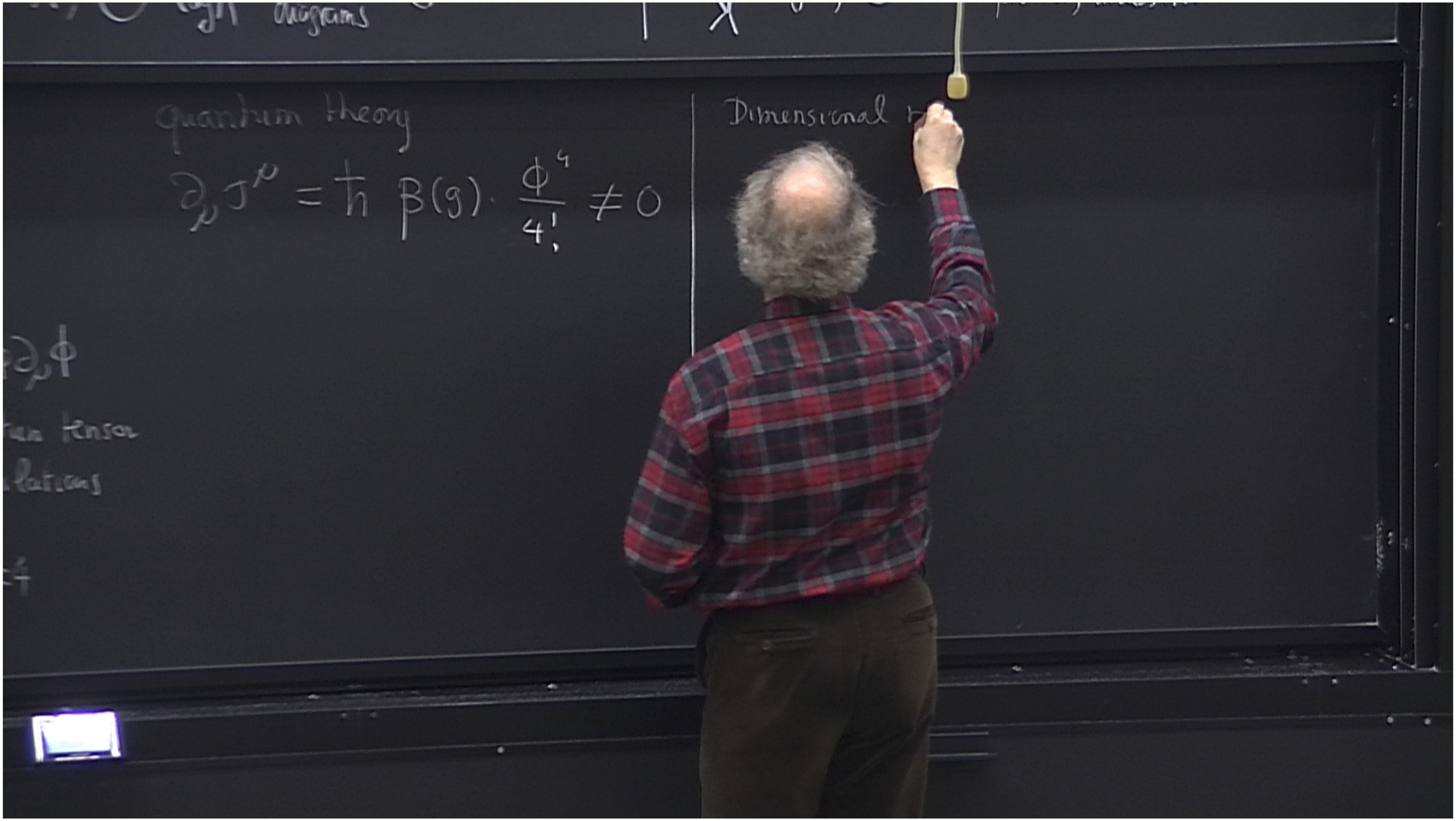
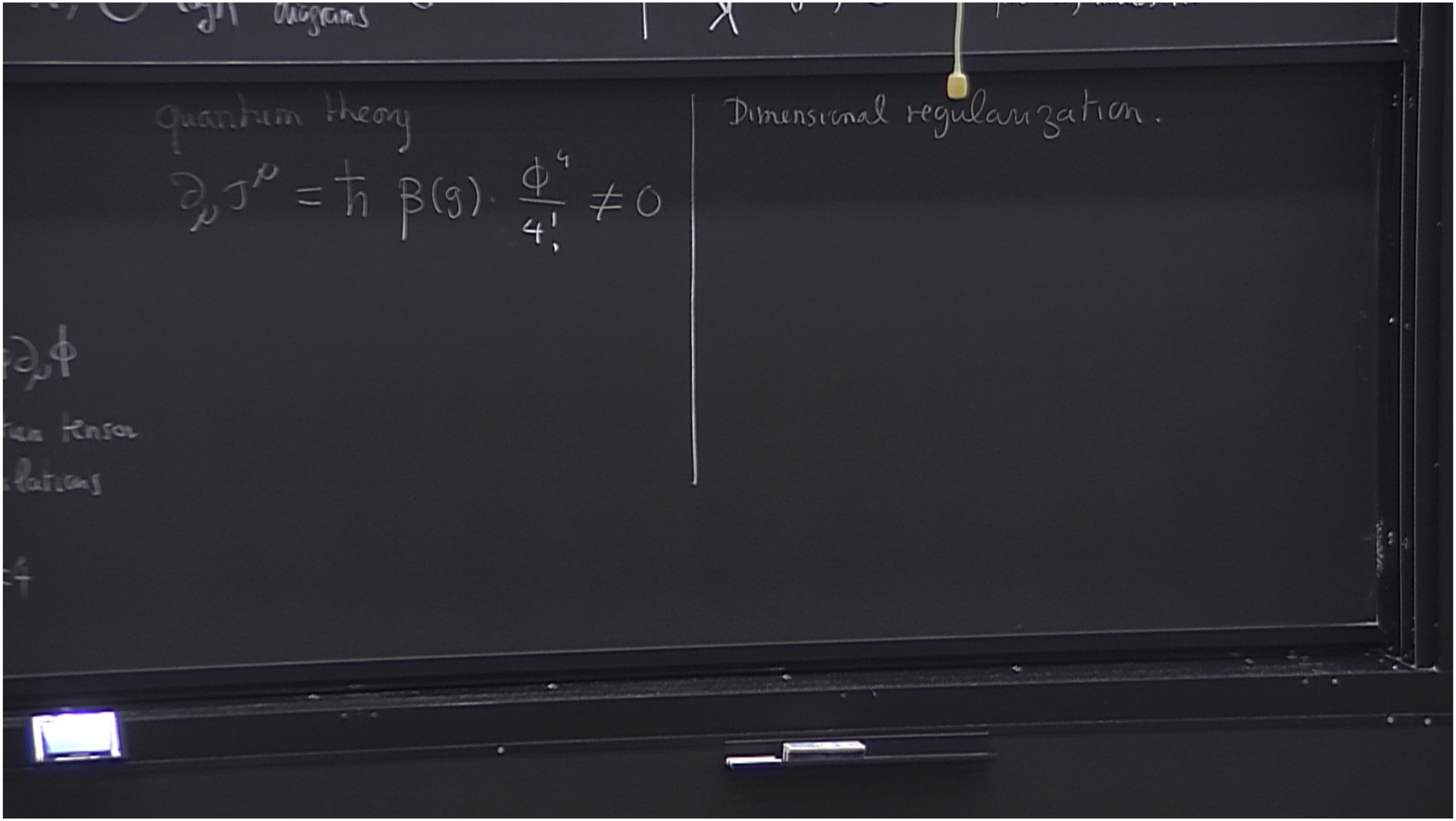
ϕ^4 $\int d^4x \left[\frac{1}{2}(\partial\phi)^2 + \frac{g}{4!}\phi^4\right] = S[\phi]$
$\phi_\lambda(x) = \lambda\,\phi(\lambda x)$
⇒ Noether Current
$J^\mu(x) = T^\mu{}_\nu x^\nu + \phi\partial_\mu\phi$
$T^\mu{}_\nu$ energy-momentum tensor
current for translations
$\partial_\mu J^\mu = 0$


diagrams
Scale invariance of the classical is anomalous
it is "broken" by quantum effects
= S [φ]
⇒ Noether Current
J^μ(x) = T^μ_ν x^ν + φ∂_μφ
T^μ_ν energy-momentum tensor
current for translations
∂_μ J^μ = 0
D=4


quantum theory
Scale invariance of the classical is anomalous
it is "broken" by quantum effects
classical
⇒ Noether Current
energy-mom
current for
diagrams


diagrams
quantum theory
$\partial_\mu J^\mu = \hbar \beta(g) \cdot \frac{\phi^4}{4!}$
tensor


quantum theory
$\partial_\mu J^\mu = \hbar\, \beta(g) \cdot \frac{\phi^4}{4!} \neq 0$
Dimensional
diagrams
tensor


quantum theory
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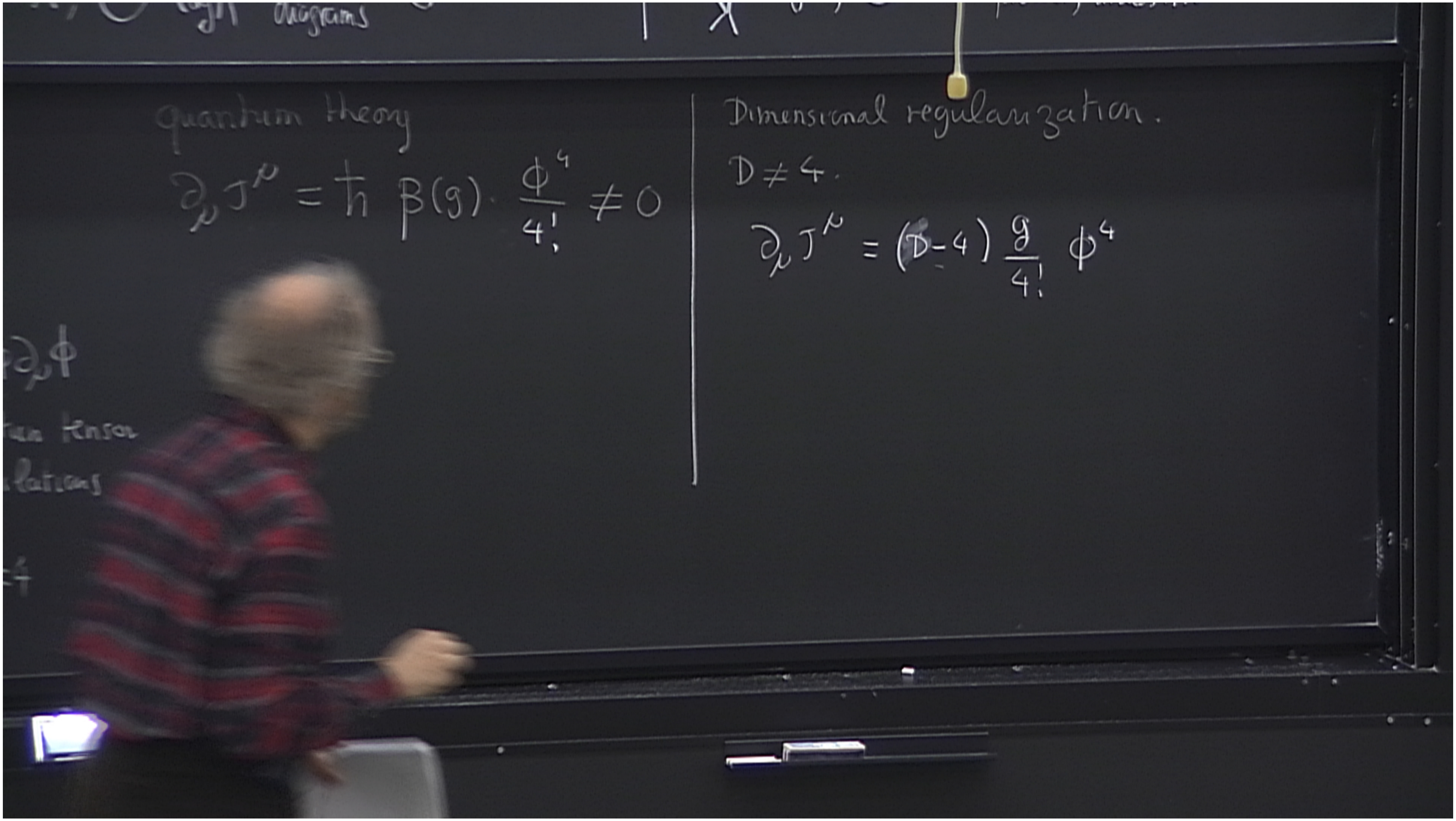
Dimensional regularization.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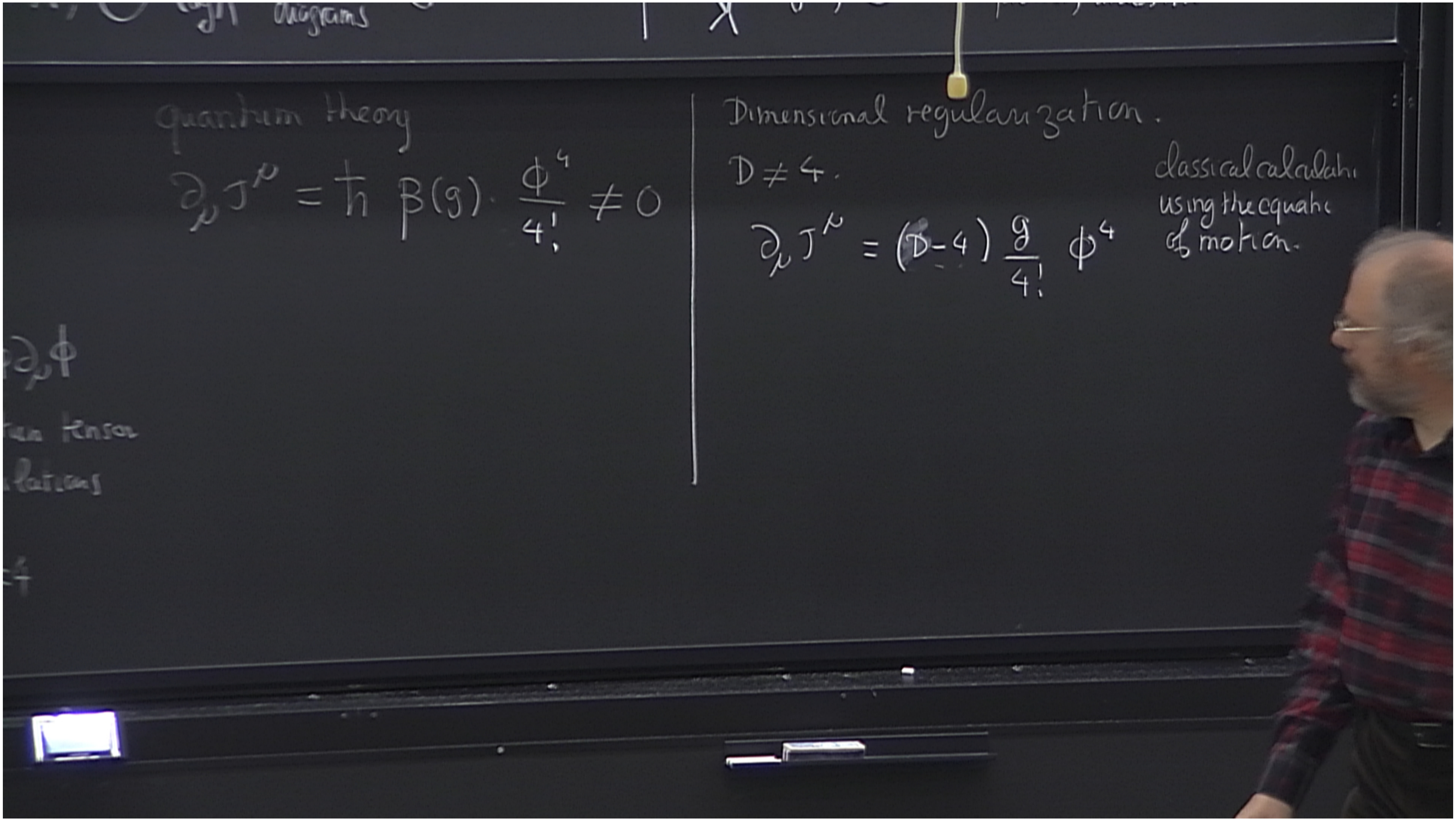
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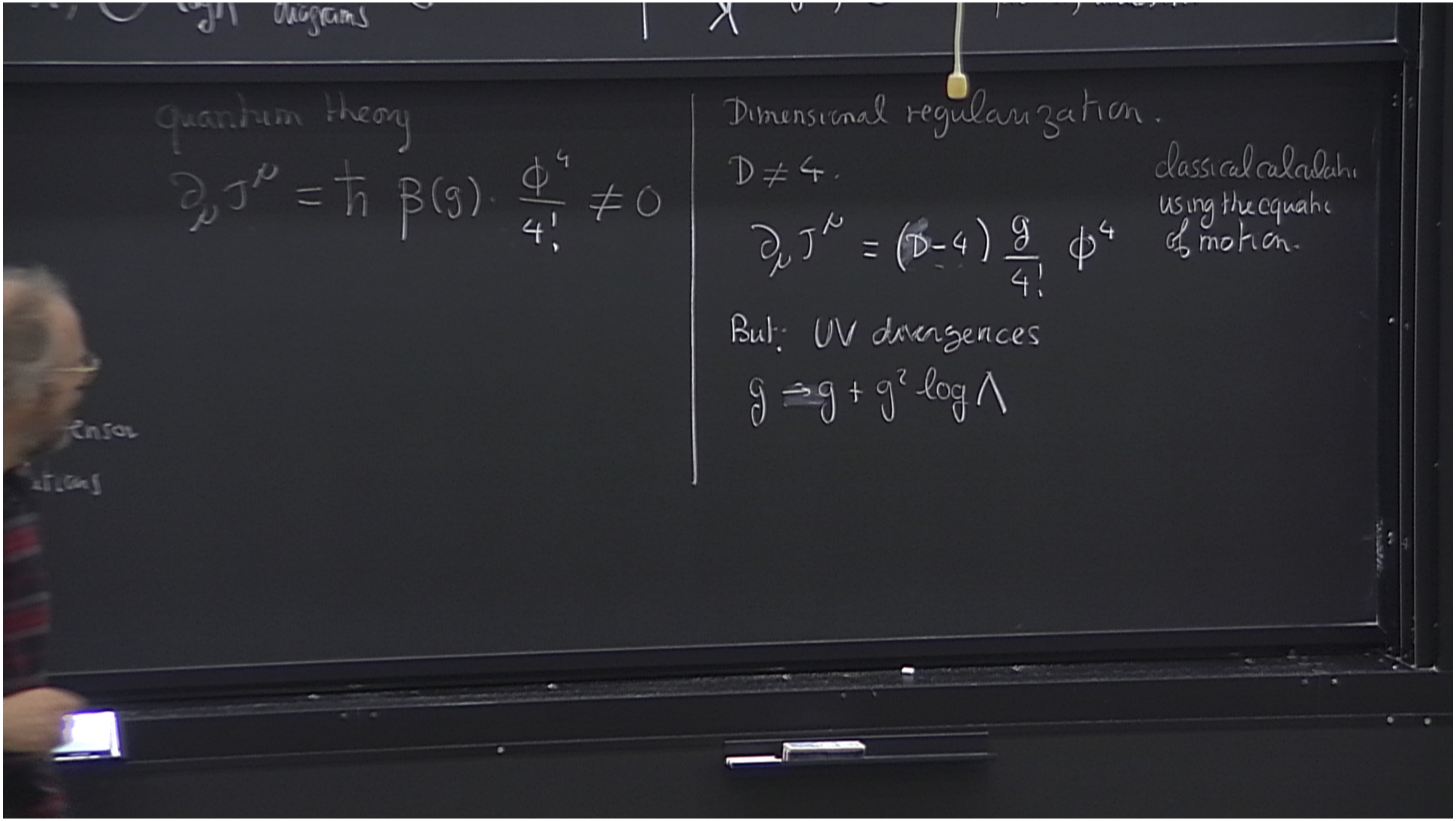
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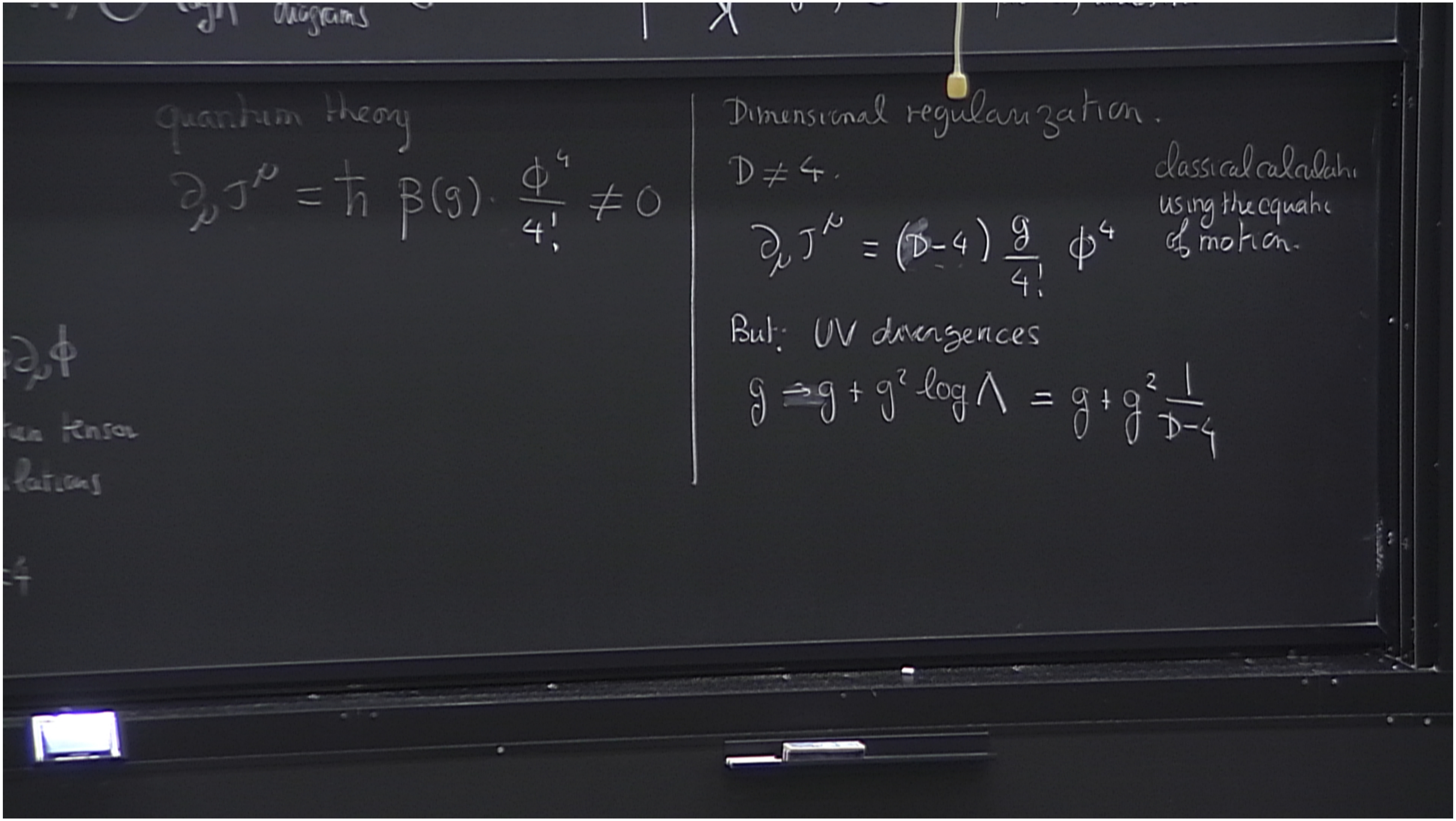
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Scale anomaly

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classical calculation using the equation of motion.

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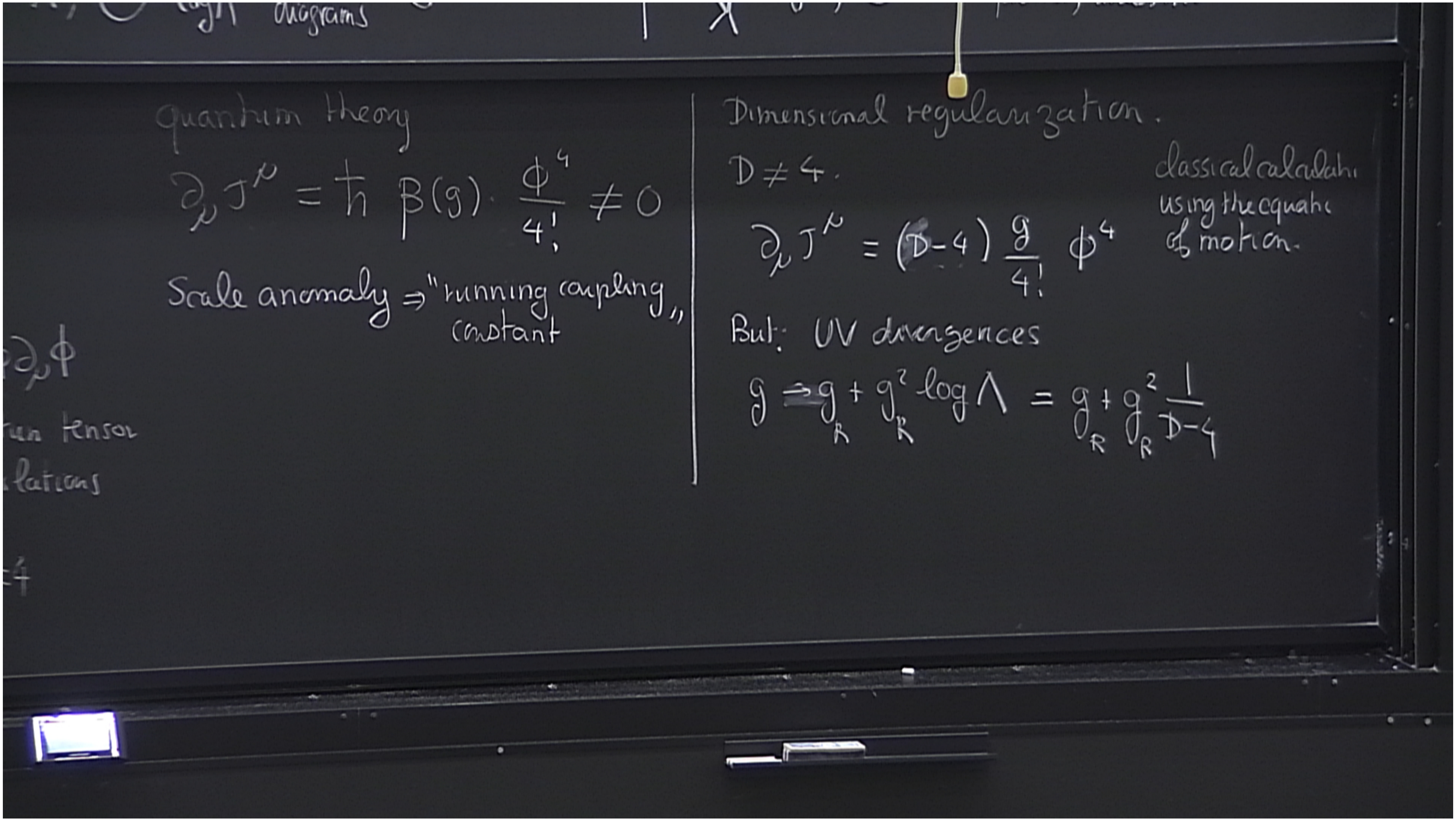
$$g \to g_R + g_R^2 \log \Lambda = g_R + g_R^2 \frac{1}{D-4}$$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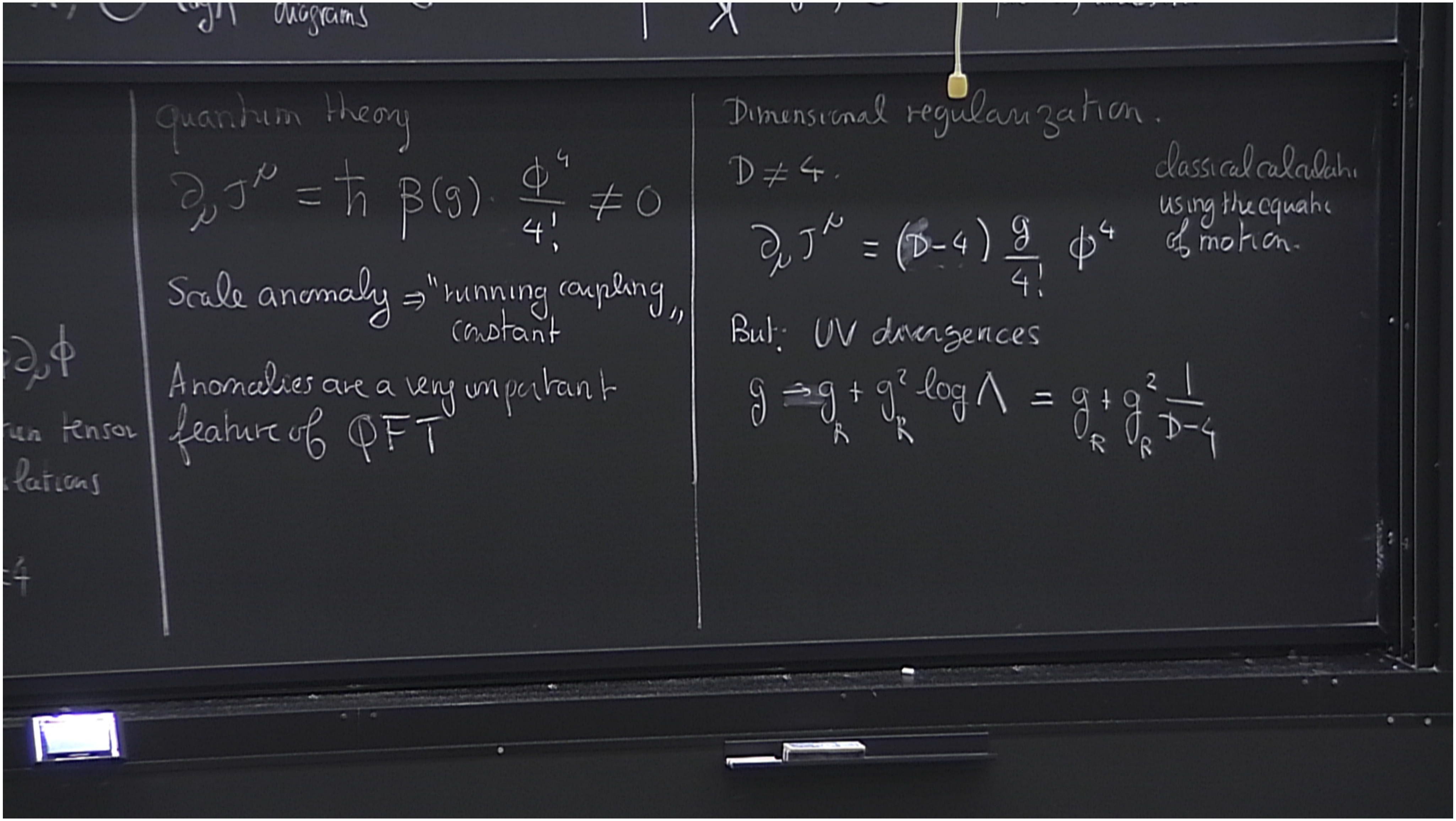
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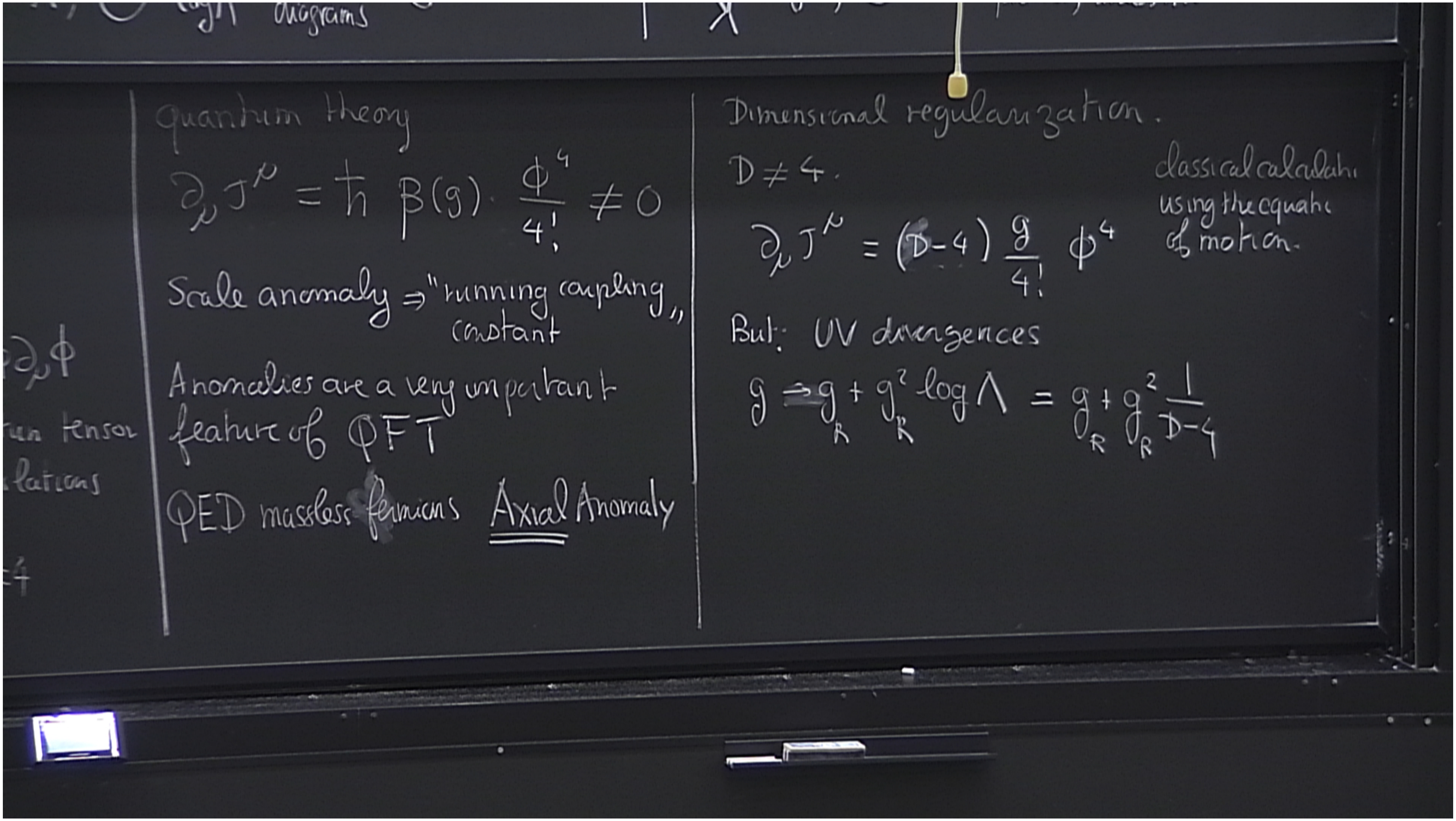
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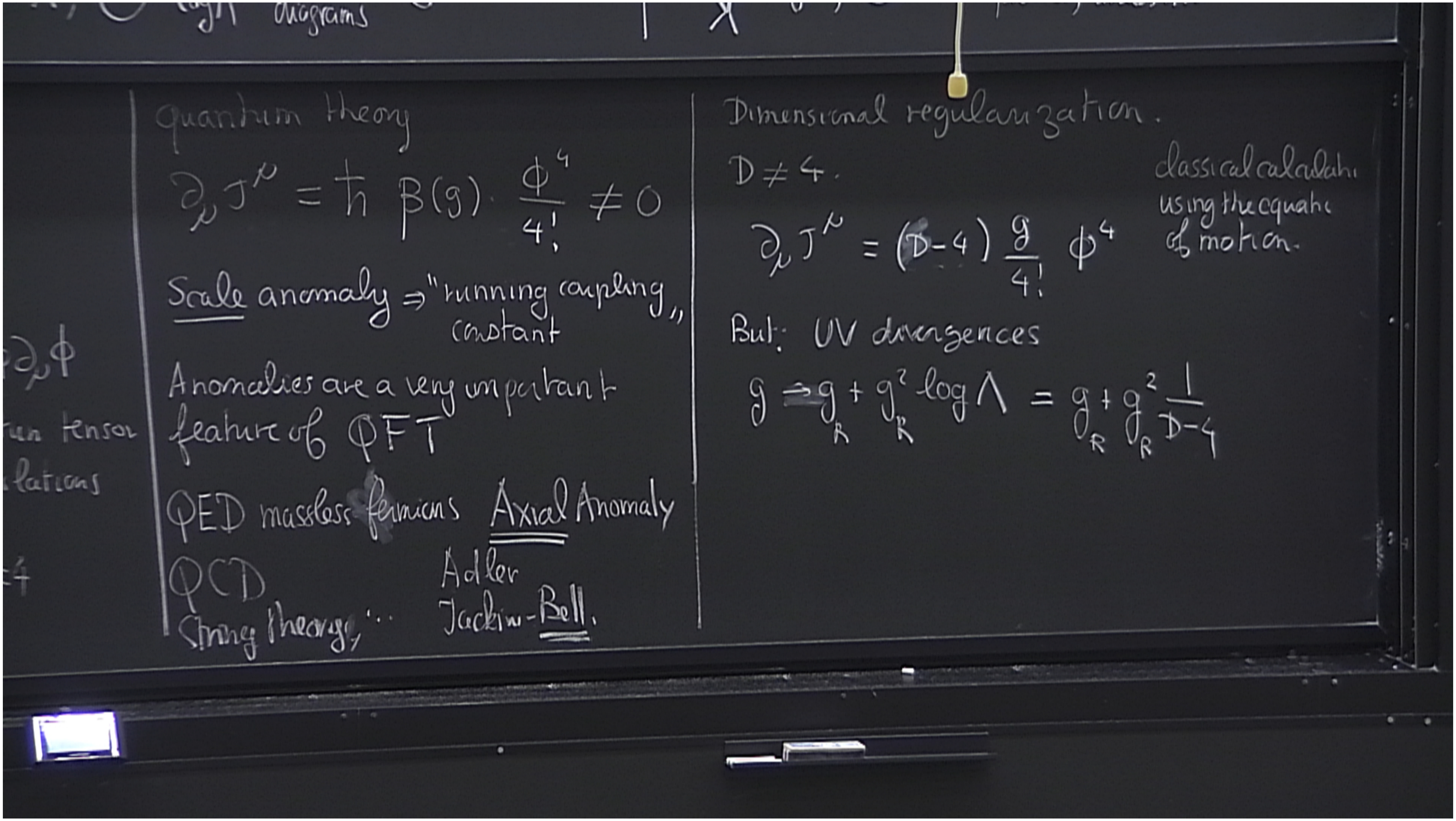
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