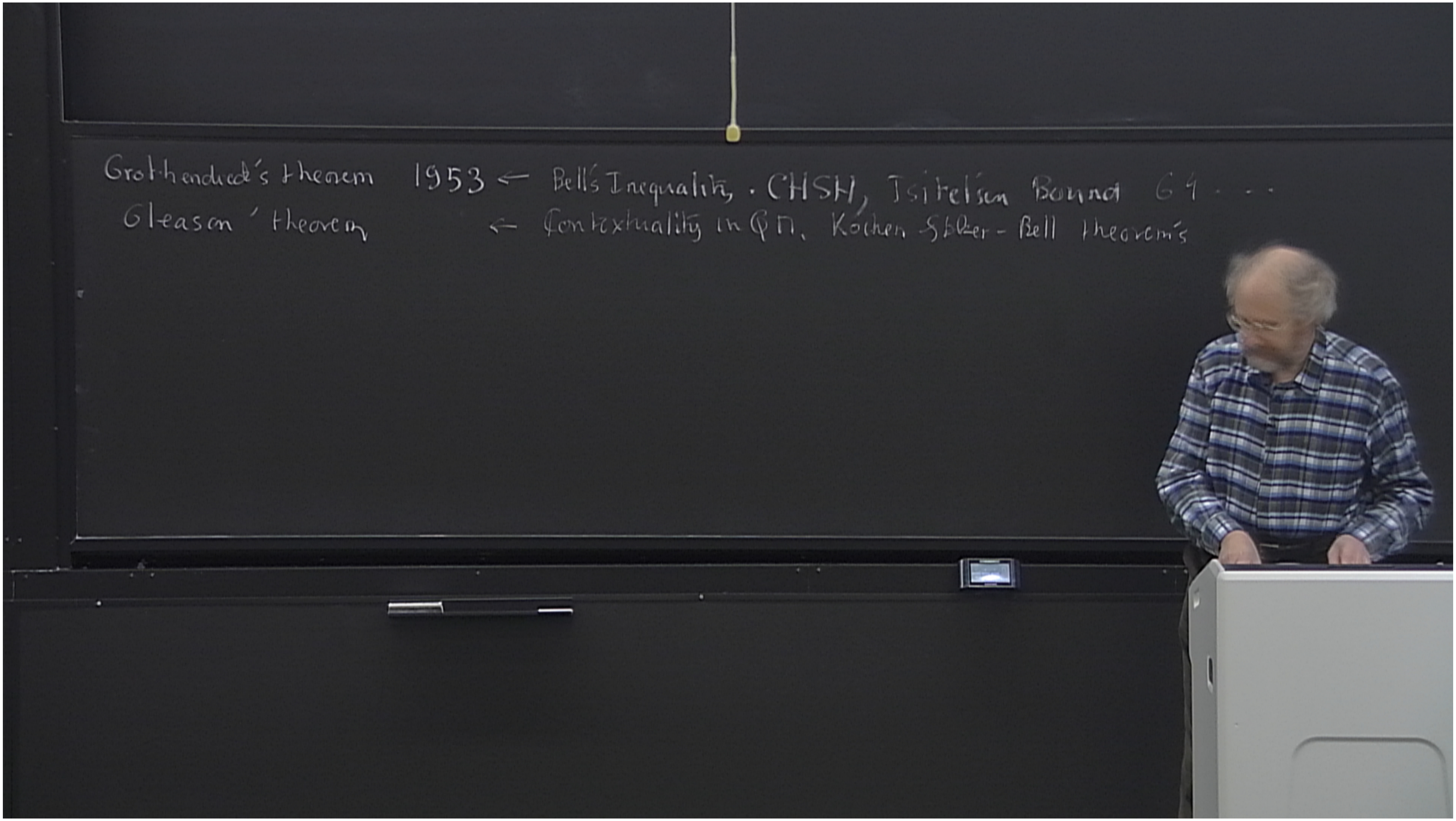
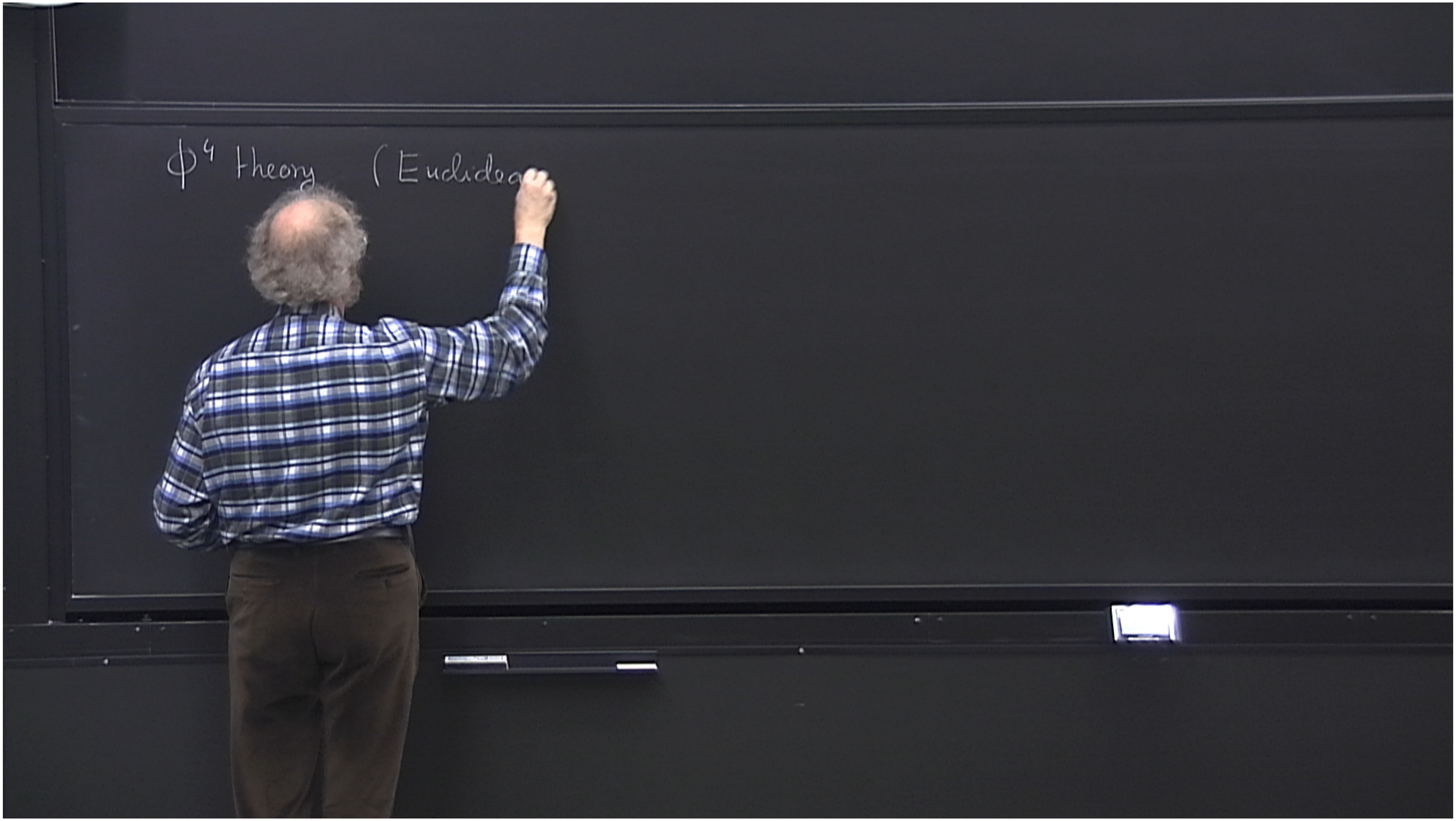
Title: 14/15 PSI - Quantum Field Theory II-Lecture 5

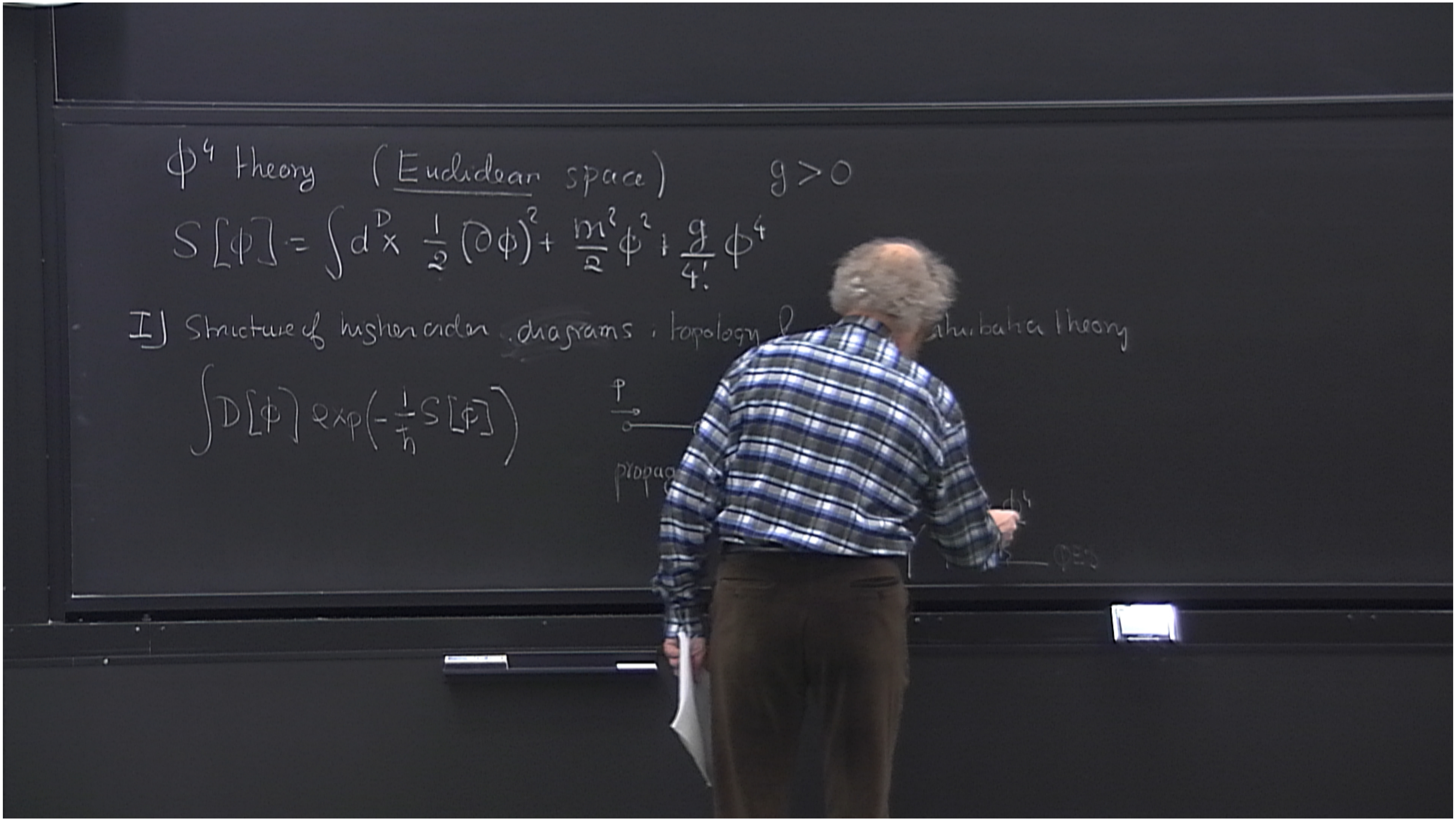
Date: Nov 14, 2014 09:00 AM

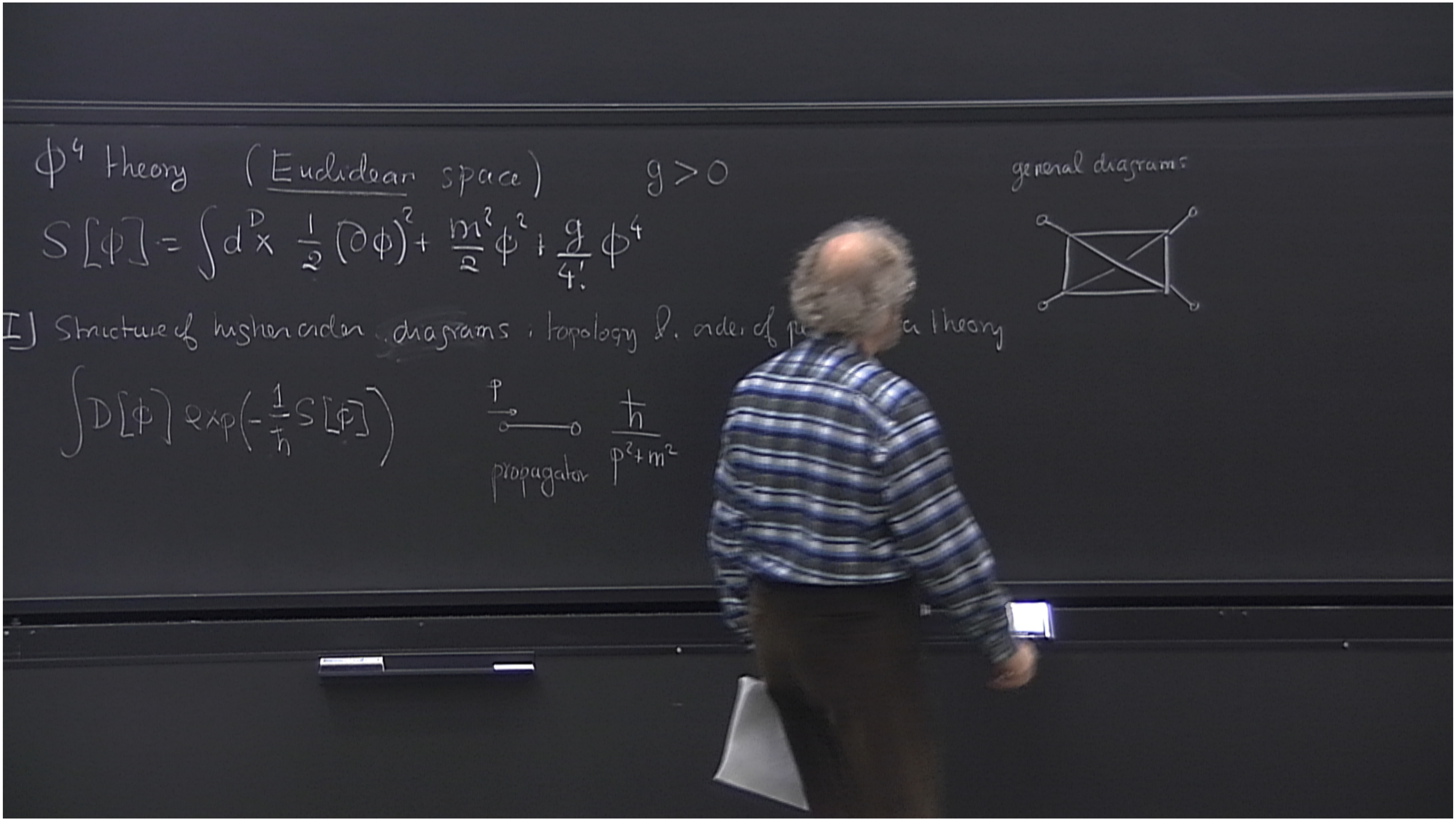
URL: http://pirsa.org/14110010

Abstract:


Grothendieck's theorem 1953 ← Bell's Inequality. CHSH, Tsirelson Bound 64 . . .
Gleason' theorem ← Contextuality in QM. Kochen-Specker-Bell theorems


ϕ^4 theory (Euclidea


ϕ^4 theory (Euclidean space) $g>0$
$S[\phi] = \int d^Dx \ \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$
I] Structure of higher order diagrams : topology
theory
$\int D[\phi] \exp\left(-\frac{1}{\hbar}S[\phi]\right)$
p
ϕ^4
QED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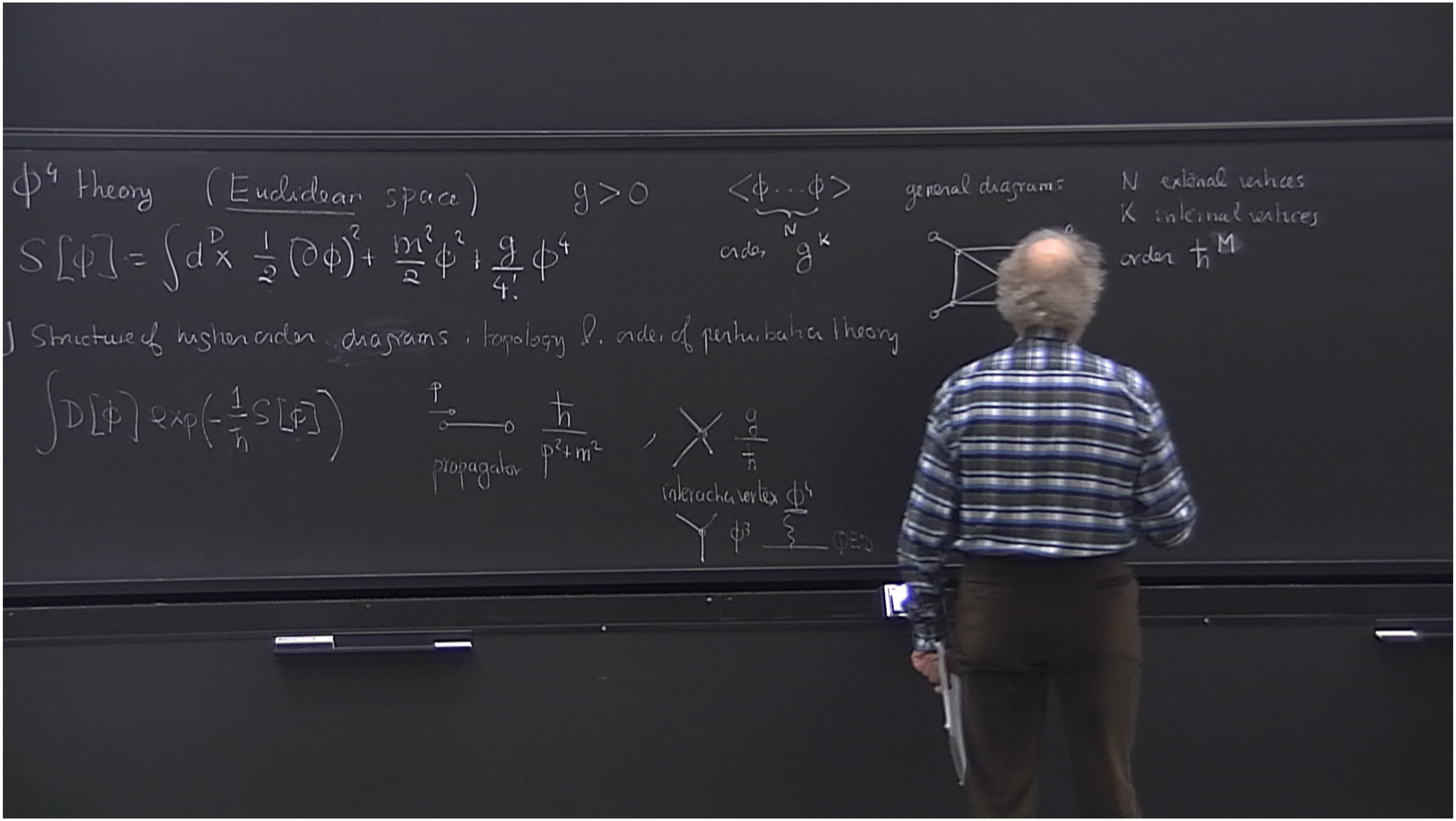
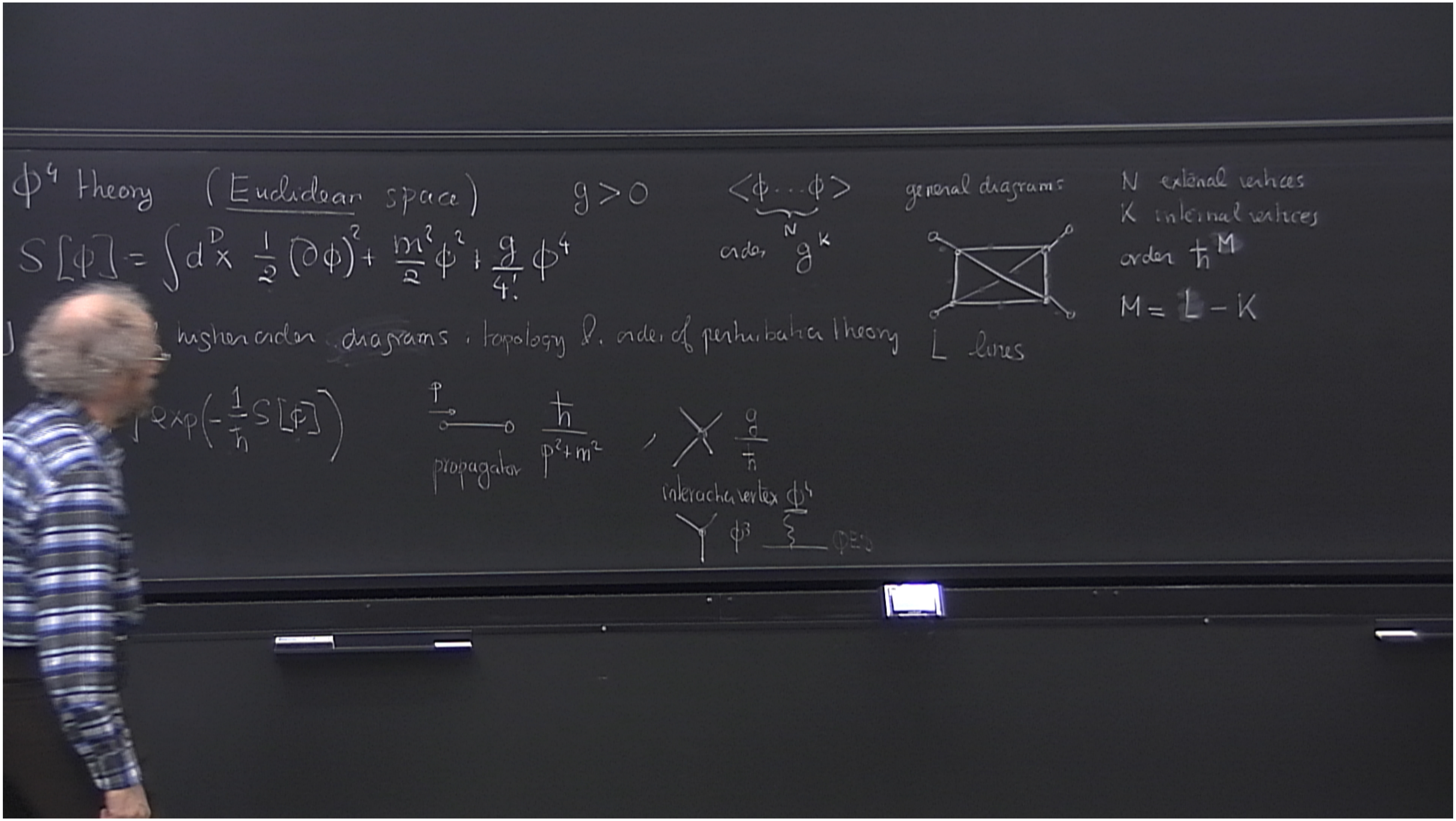
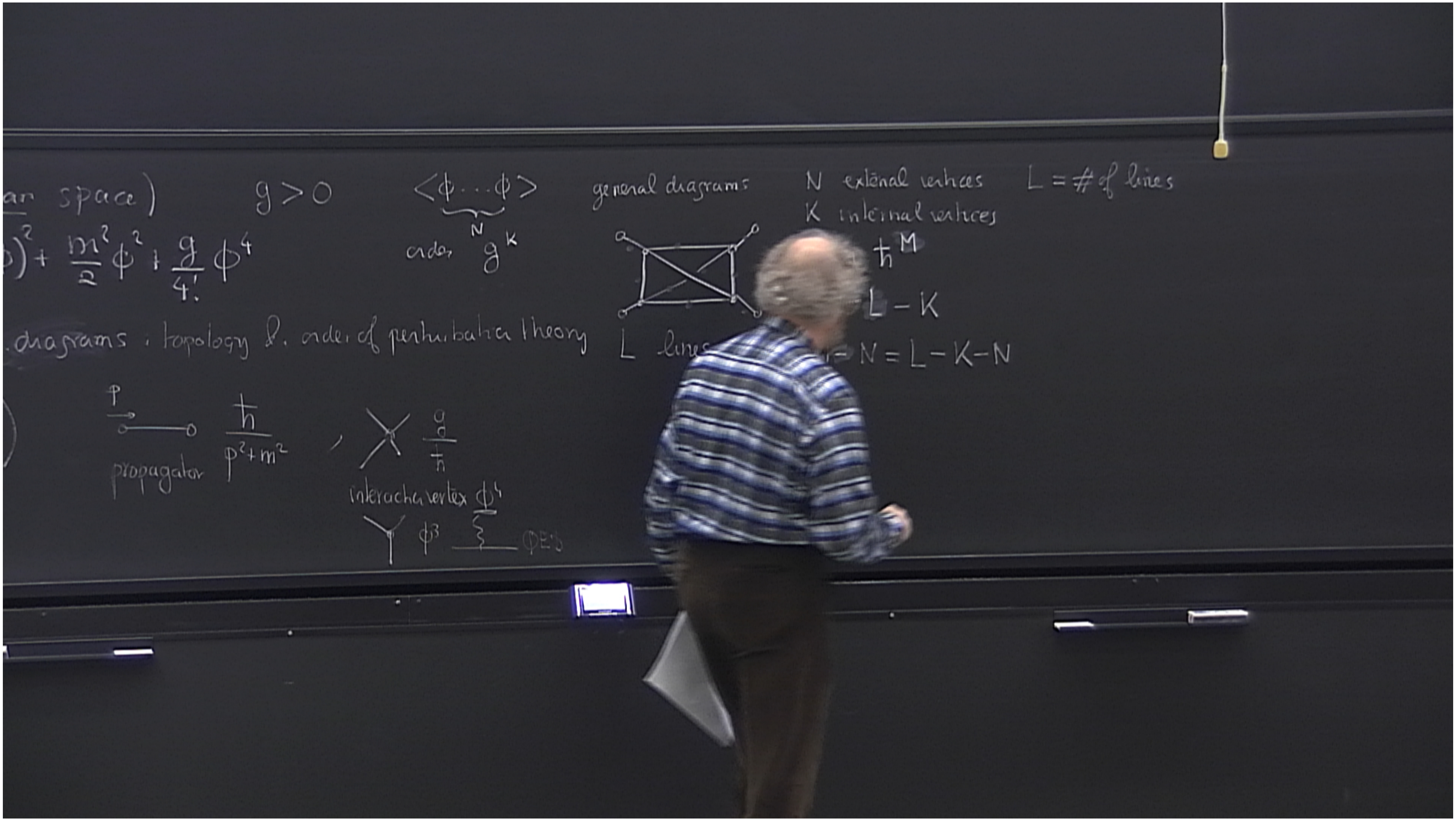
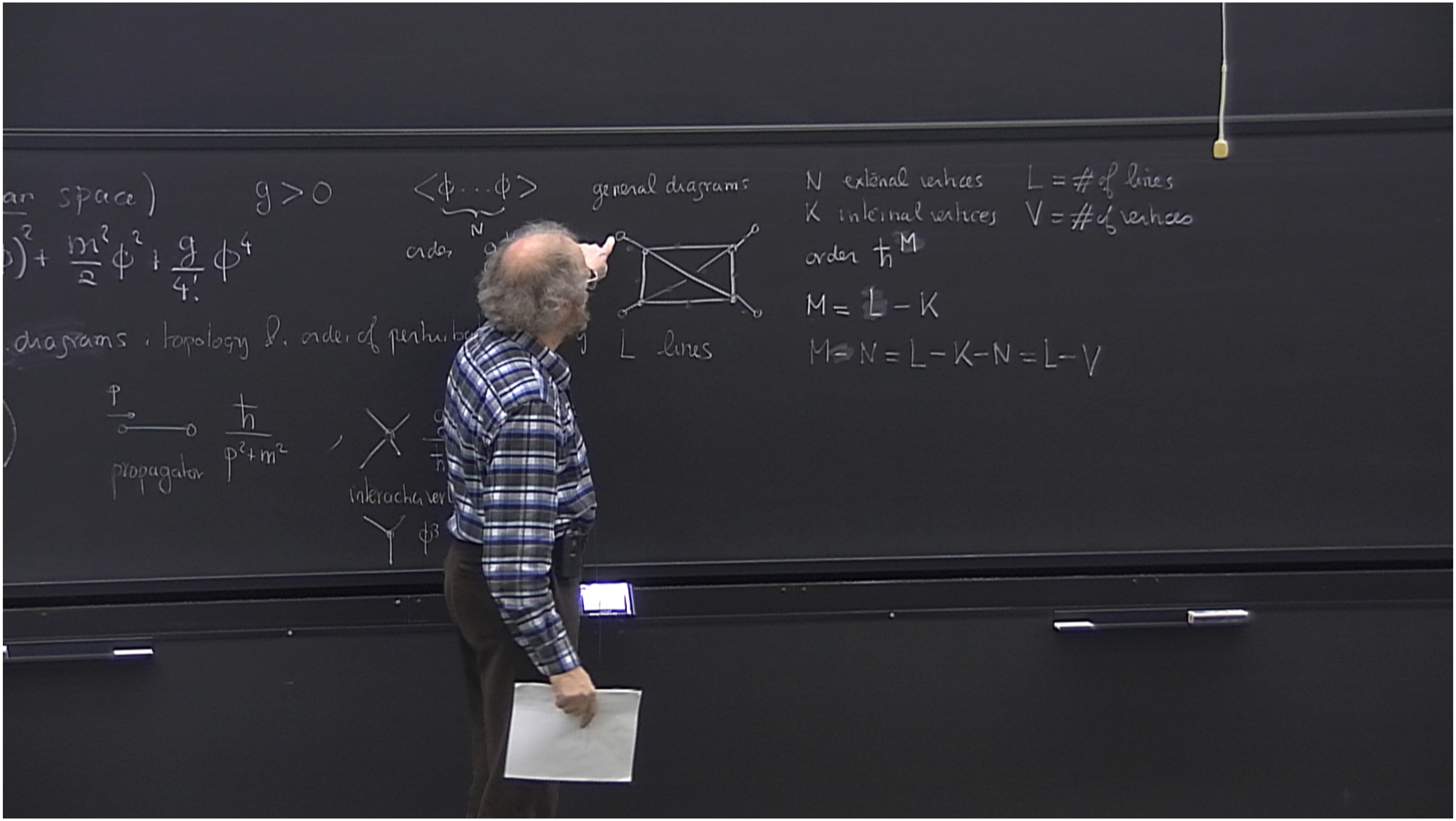
general diagrams:
I] Structure of higher order diagrams : topology & order of
theory
$\int D[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right)$
p
$\frac{\hbar}{p^2+m^2}$
propagator


φ⁴ theory (Euclidean space) g > 0
S[φ] = ∫dᴰx ½(∂φ)² + m²/2 φ² + g/4! φ⁴
⟨φ…φ⟩ N
order g^K
general diagrams
N external vertices
K internal vertices
order ħ^M
Structure of higher order diagrams : topology & order of perturbation theory
∫D[φ] exp(−1/ħ S[φ])
p
propagator ħ/(p²+m²)
g/ħ
interaction vertex φ⁴
φ³
QED



ϕ^4 theory (Euclidean space) $g>0$
$S[\phi] = \int d^D x \; \frac{1}{2}(\partial\phi)^2 + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4$
$\langle \underbrace{\phi \ldots \phi}_{N} \rangle$
order g^K
general diagrams:
N external vertices
K internal vertices
order $\hbar^M$
$M = L - K$
L lines
higher order diagrams : topology & order of perturbation theory
$\exp\left(-\frac{1}{\hbar} S[\phi]\right)$
p
propagator $\frac{\hbar}{p^2+m^2}$
$\frac{g}{\hbar}$
interaction vertex ϕ^4
ϕ^3


space)
g > 0
⟨φ...φ⟩
N
order g^K
general diagrams:
N external vertices
K internal vertices
L = # of lines
ħ^M
L − K
N = L − K − N
L lines
diagrams : topology & order of perturbation theory
propagator
ħ / (p² + m²)
g / ħ
interaction vertex φ⁴
φ³


g > 0
⟨φ . . . φ⟩
N
general diagrams:
N external vertices
K internal vertices
L = # of lines
V = # of vertices
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M = L − K
M = N = L − K − N = L − V
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diagrams : topology & order of perturb
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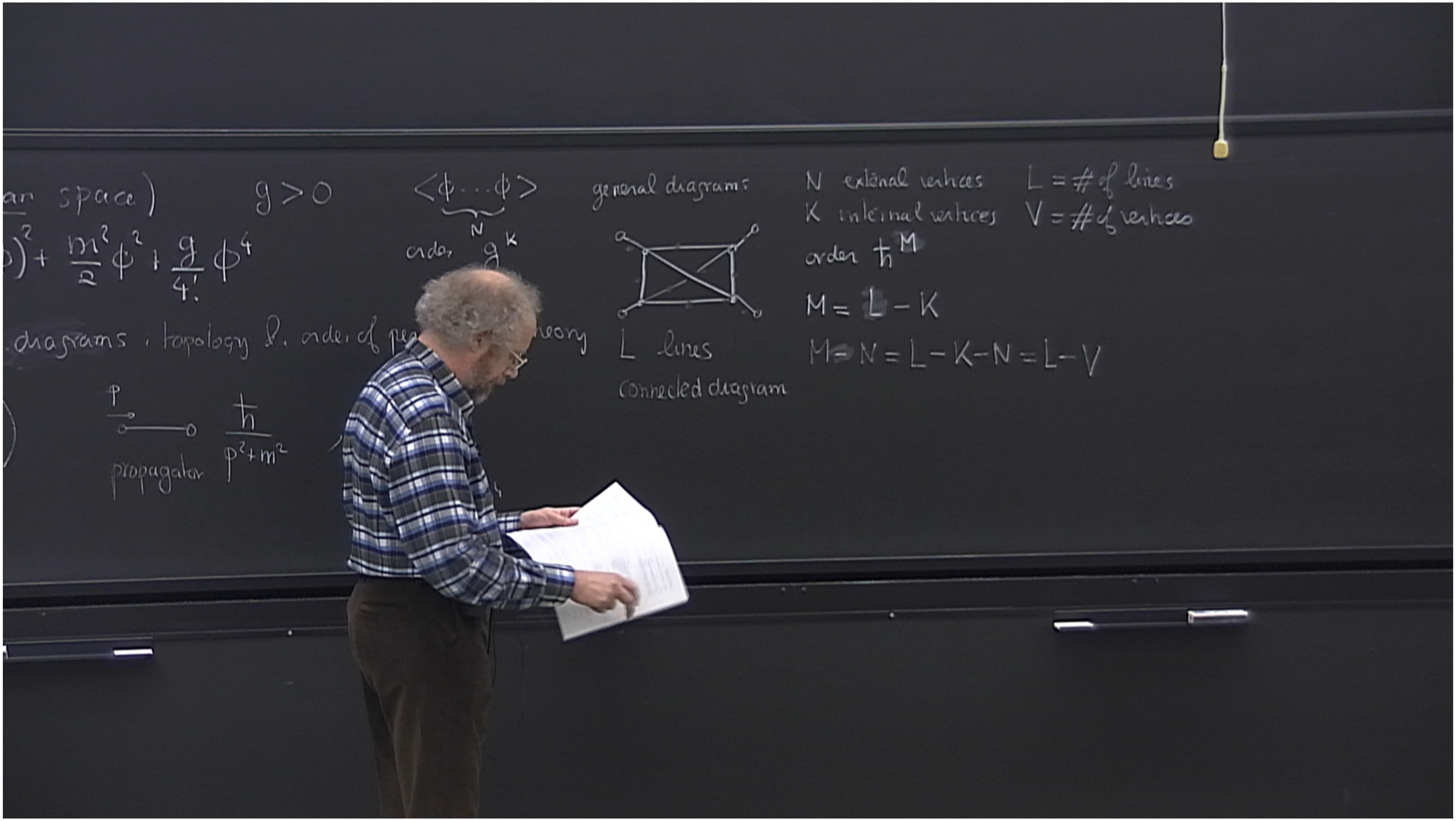
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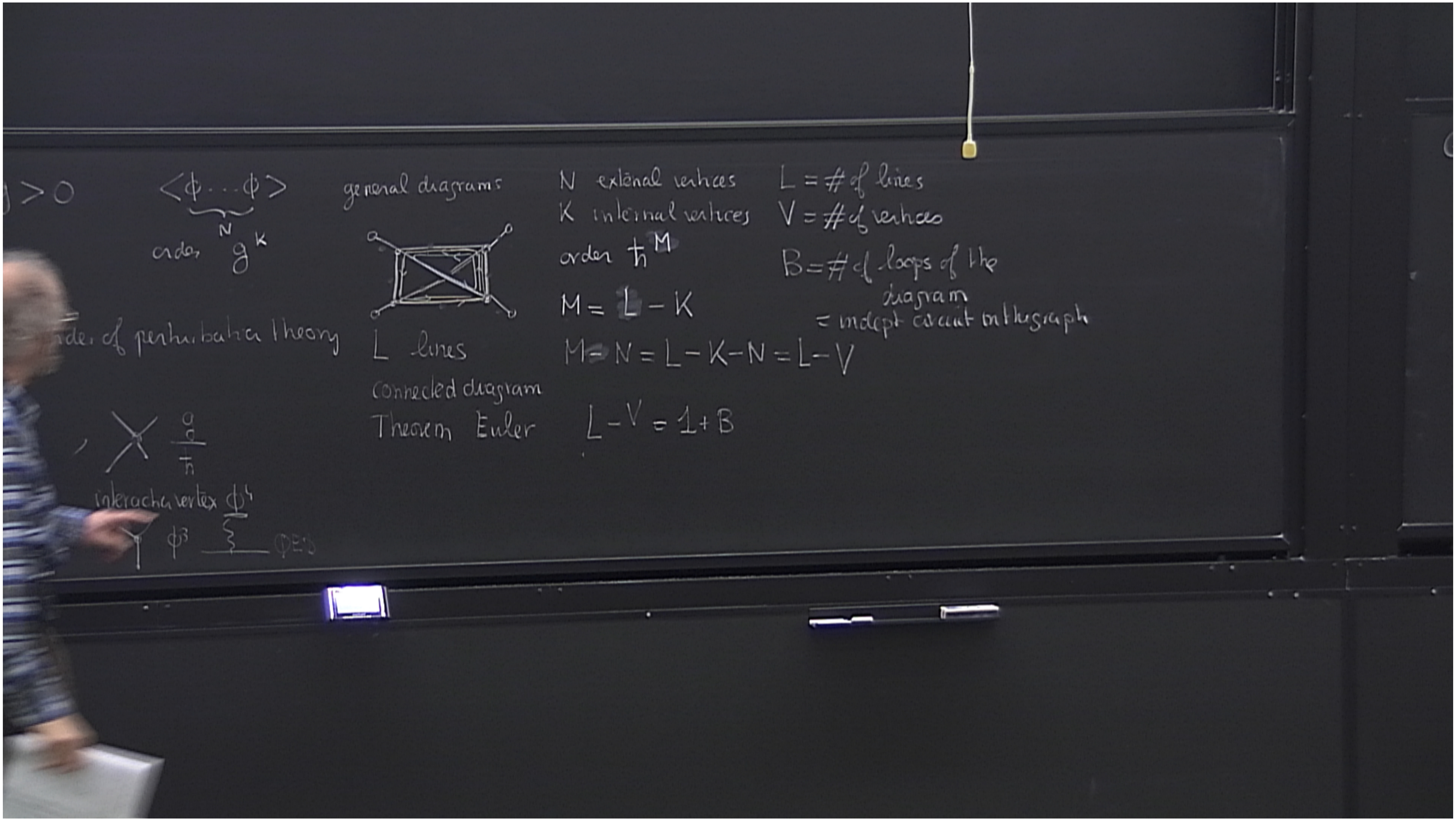
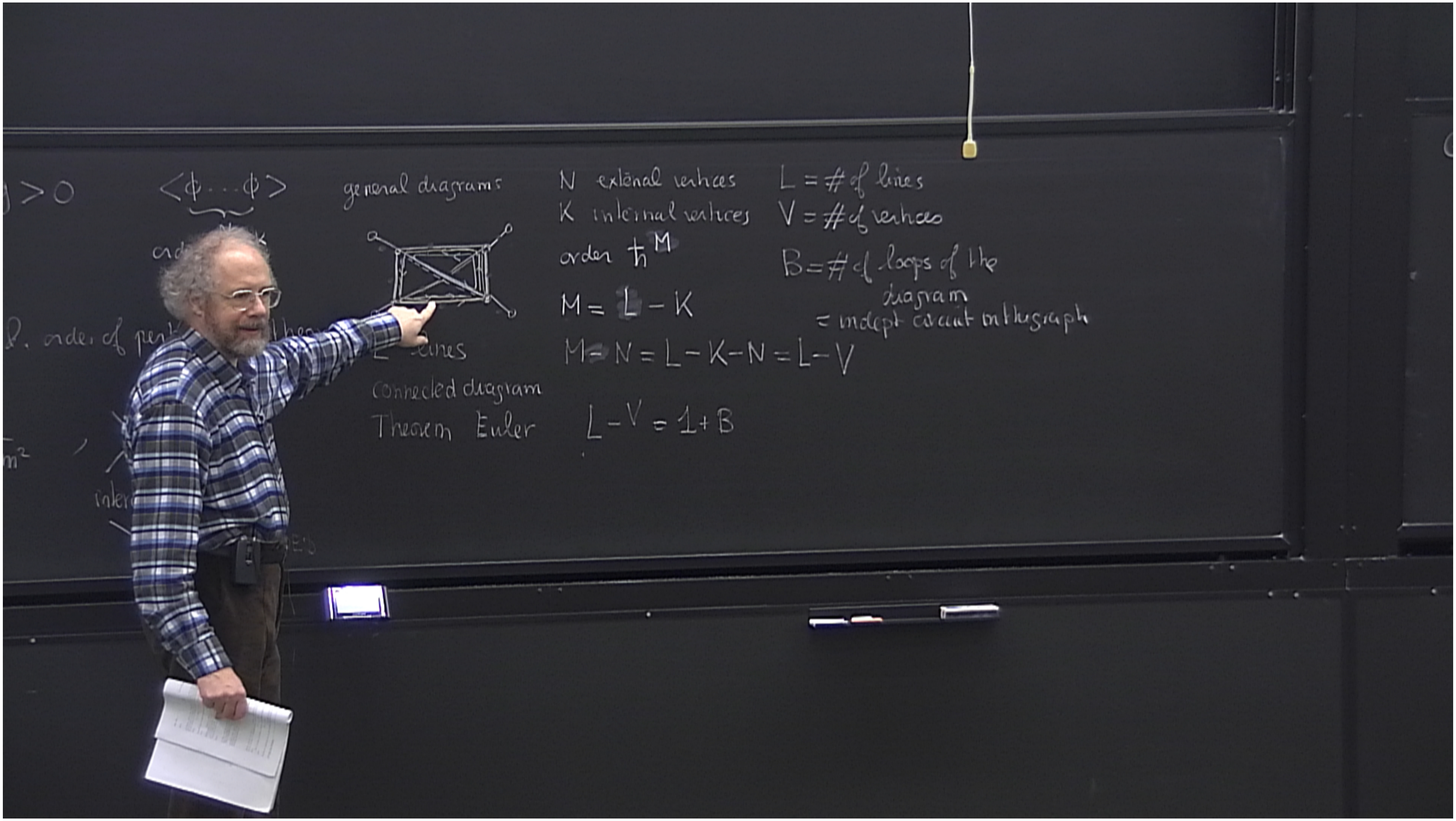
connected diagram
diagrams : topology & order of pe
propagator
ħ / (p² + m²)


$g > 0$
$\langle \phi \dots \phi \rangle$
N
order g^K
order of perturbation theory
interaction vertex ϕ^4
ϕ^3
QED
general diagrams
L lines
connected diagram
Theorem Euler
N external vertices
K internal vertices
order $\hbar^M$
$M = L - K$
$M - N = L - K - N = L - V$
$L - V = 1 + B$
L = # of lines
V = # of vertices
B = # of loops of the diagram
= indept circuit on the graph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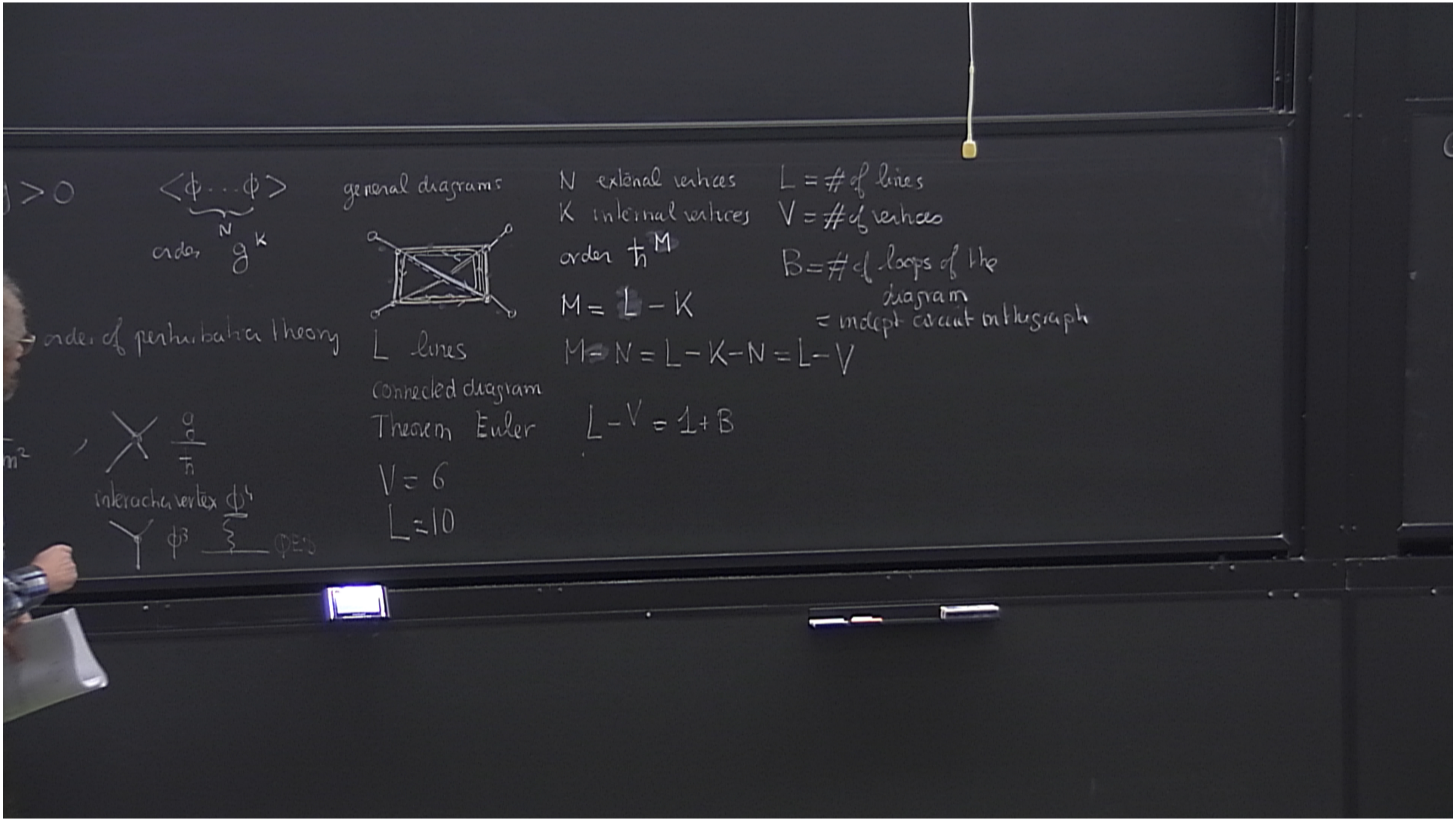
connected diagram
Theorem Euler L − V = 1 + B


<φ ... φ>
order g^K
order of perturbation theory
interaction vertex φ^4
φ^3
QED
general diagrams
L lines
connected diagram
Theorem Euler
V = 6
L = 10
N external vertices
K internal vertices
order ħ^M
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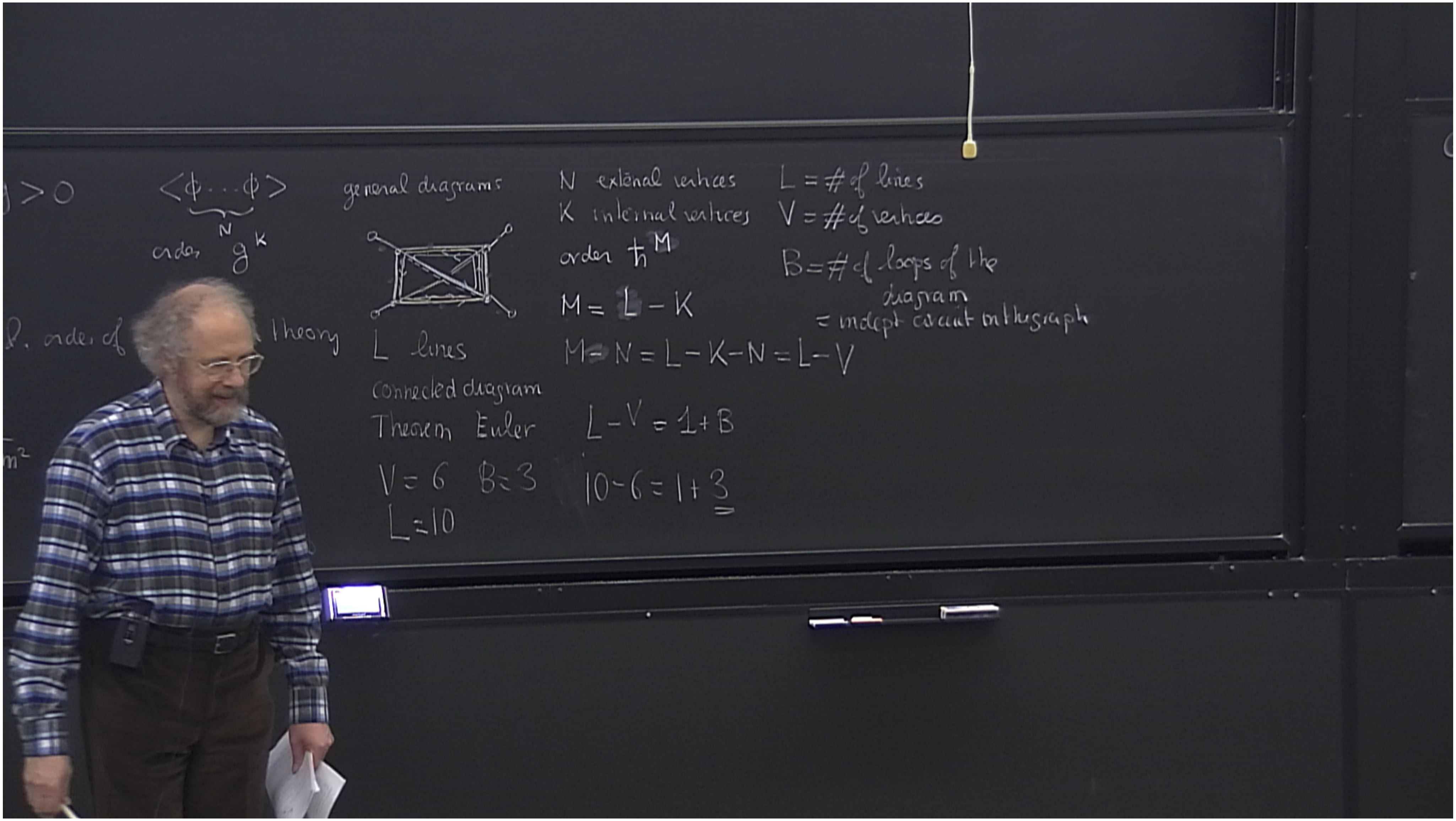
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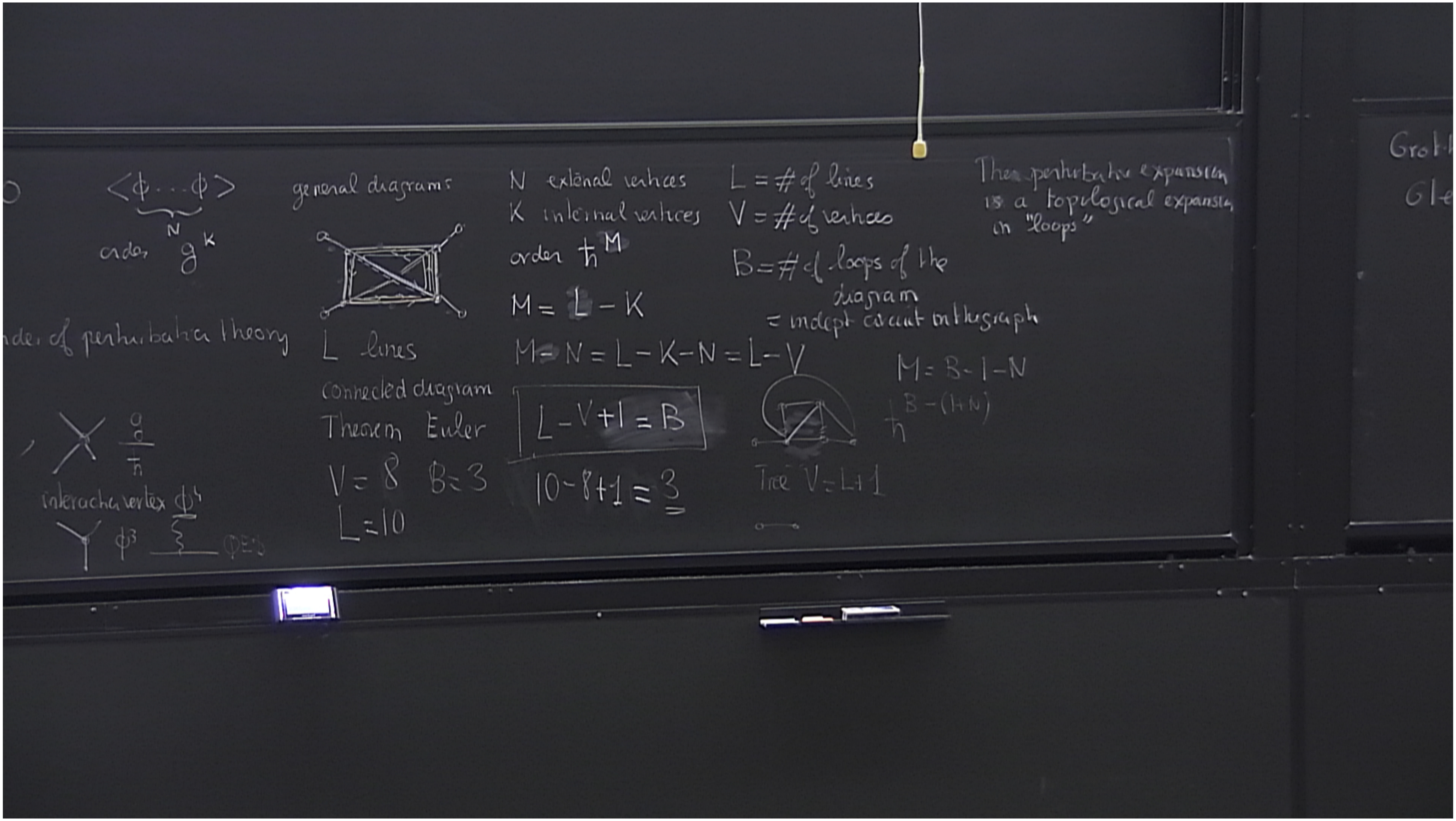
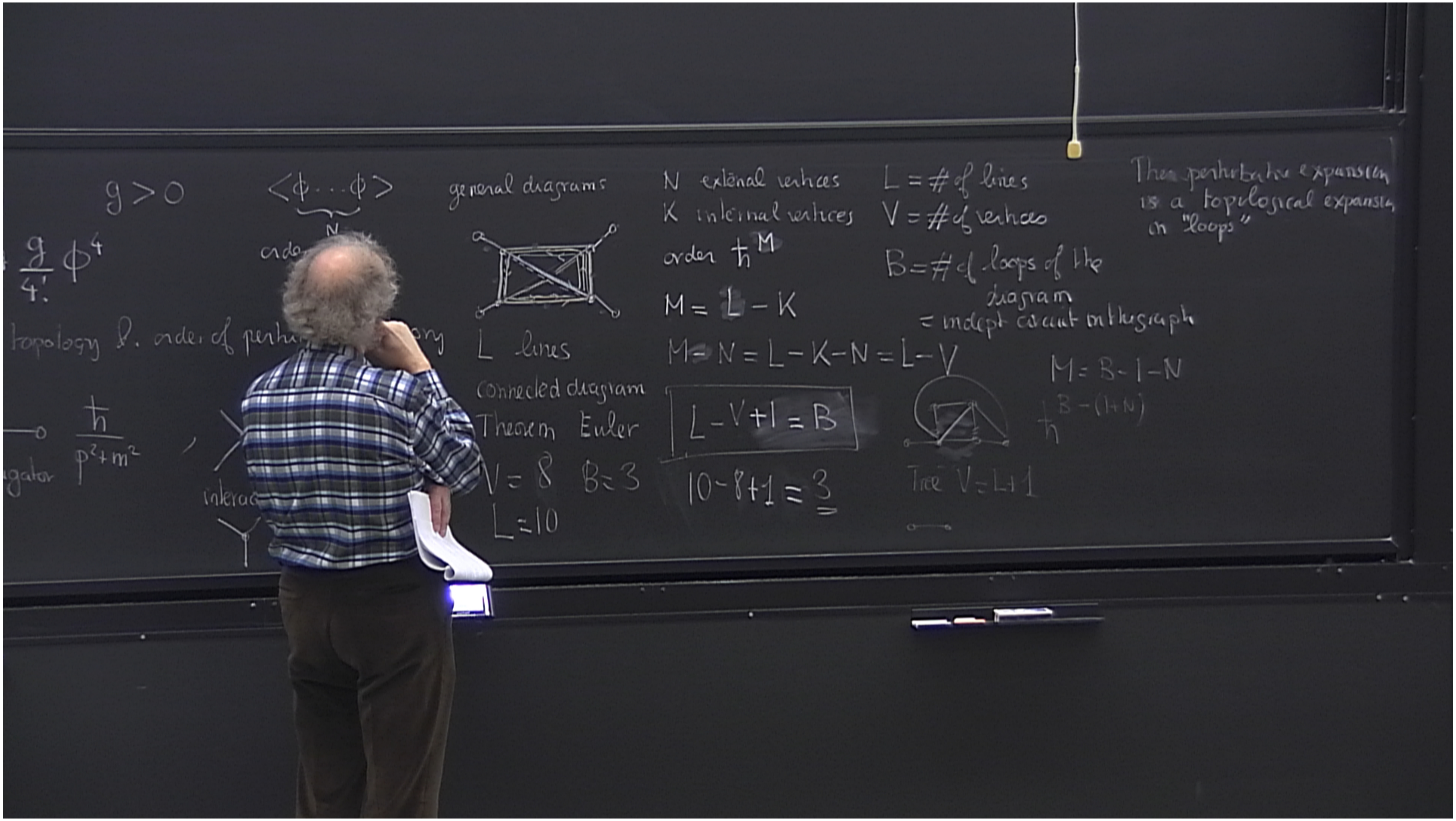
connected diagram
Theorem Euler
L − V = 1 + B
V = 6 B = 3
L = 10
10 − 6 = 1 + 3
order g^K
theory


⟨φ . . . φ⟩
N
order g^K
order of perturbation theory
interaction vertex φ⁴
φ³
general diagrams
L lines
connected diagram
Theorem Euler
V = 8 B = 3
L = 10
N external vertices
K internal vertices
order ħ^M
M = L − K
M − N = L − K − N = L − V
L − V + 1 = B
10 − 8 + 1 = 3
L = # of lines
V = # of vertices
B = # of loops of the diagram
= indept circuit on the graph
M = B − 1 − N
ħ^{B − (1+N)}
Tree V = L + 1
The perturbative expansion is a topological expansion in "loops"



g > 0
general diagrams
N external vertices
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L lines
connected diagram
Theorem Euler
L − V + 1 = B
V = 8 B = 3
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10 − 8 + 1 = 3
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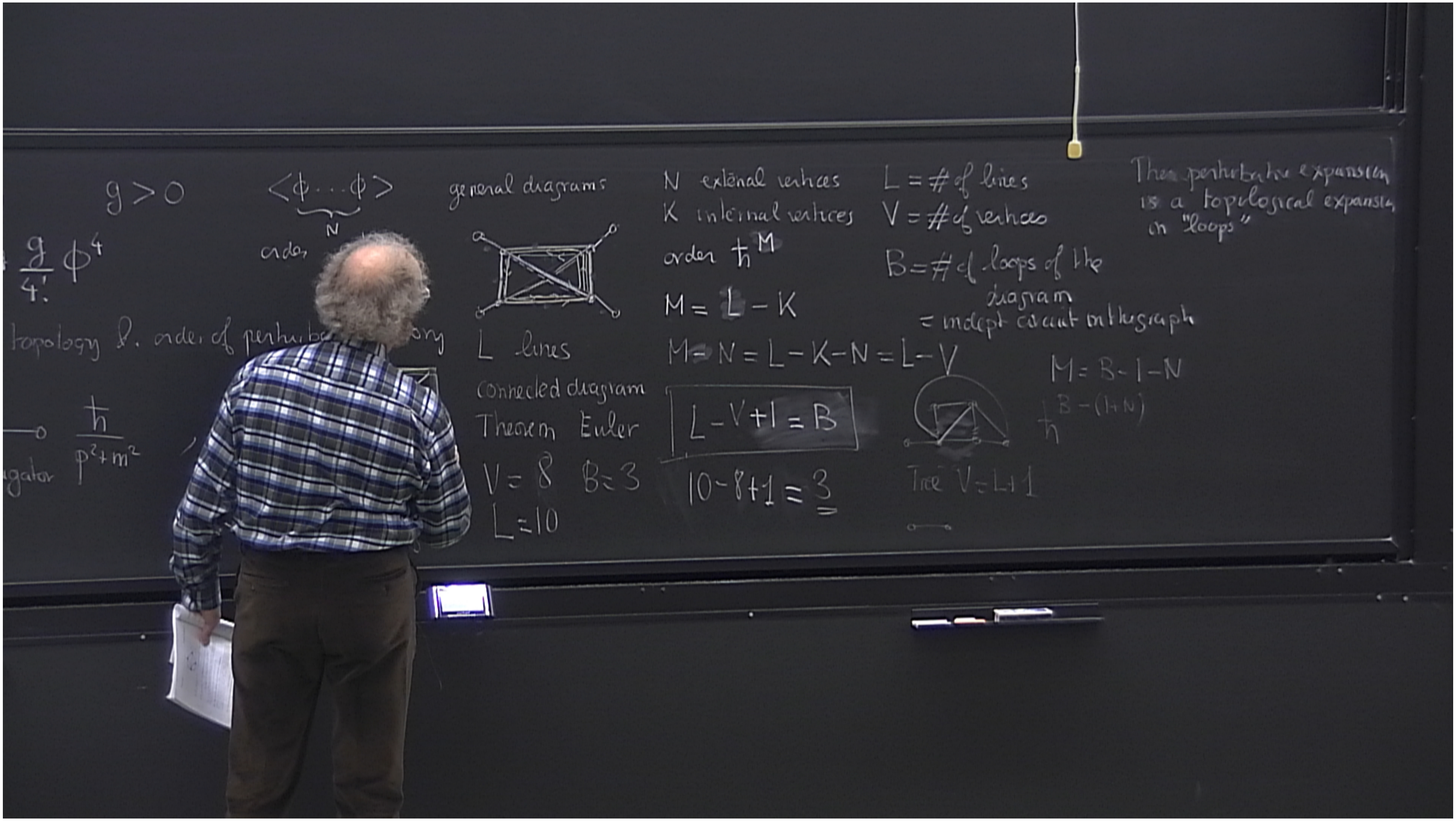
g > 0
$\frac{g}{4!}\phi^4$
topology & order of perturbation theory
$\frac{\hbar}{p^2+m^2}$
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Theorem Euler
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$\boxed{L - V + 1 = B}$
10 − 8 + 1 = 3
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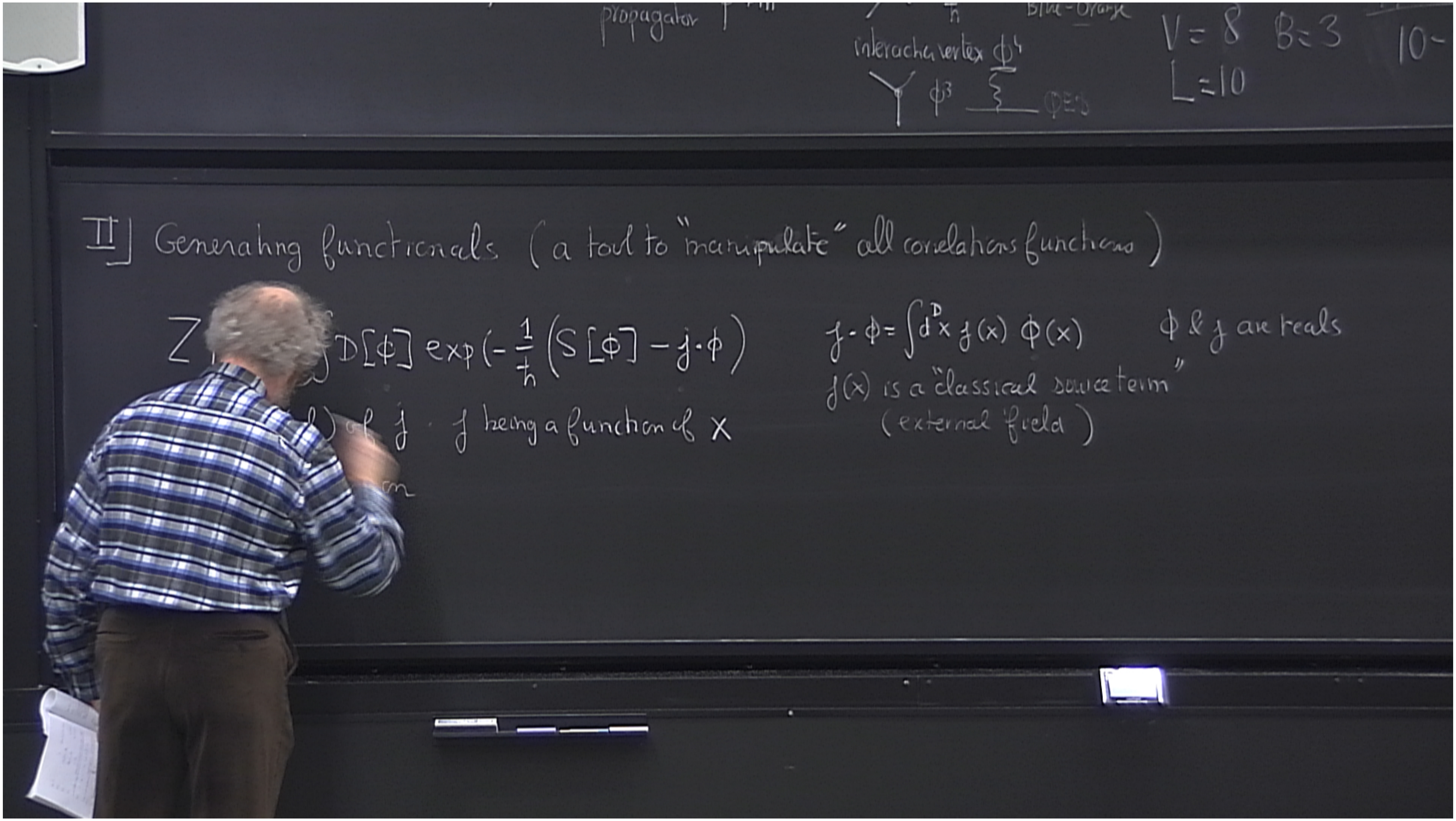
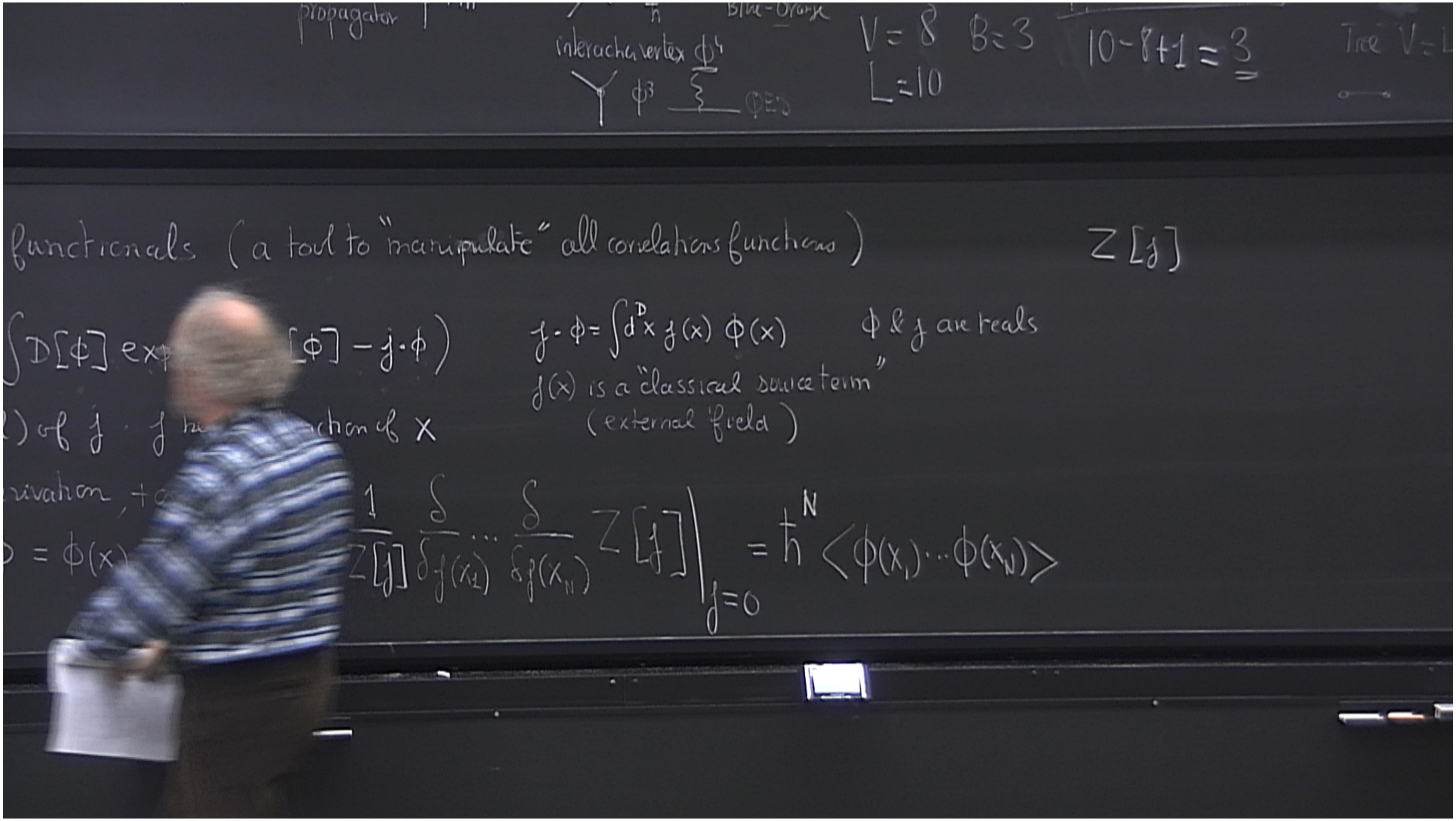
M = B − 1 − N
$\hbar^{B-(1+N)}$
Tree V = L + 1
The perturbative expansion is a topological expansion in "loops"


propagator
Blue-Orange
interaction vertex ϕ⁴
V = 8 B = 3
L = 10
II] Generating functionals (a tool to "manipulate" all correlations functions)
D[ϕ] exp(- 1/ħ (S[ϕ] - j·ϕ))
j·ϕ = ∫d^D x j(x) ϕ(x)
ϕ & j are reals
j(x) is a "classical source term"
(external field)
j being a function of x

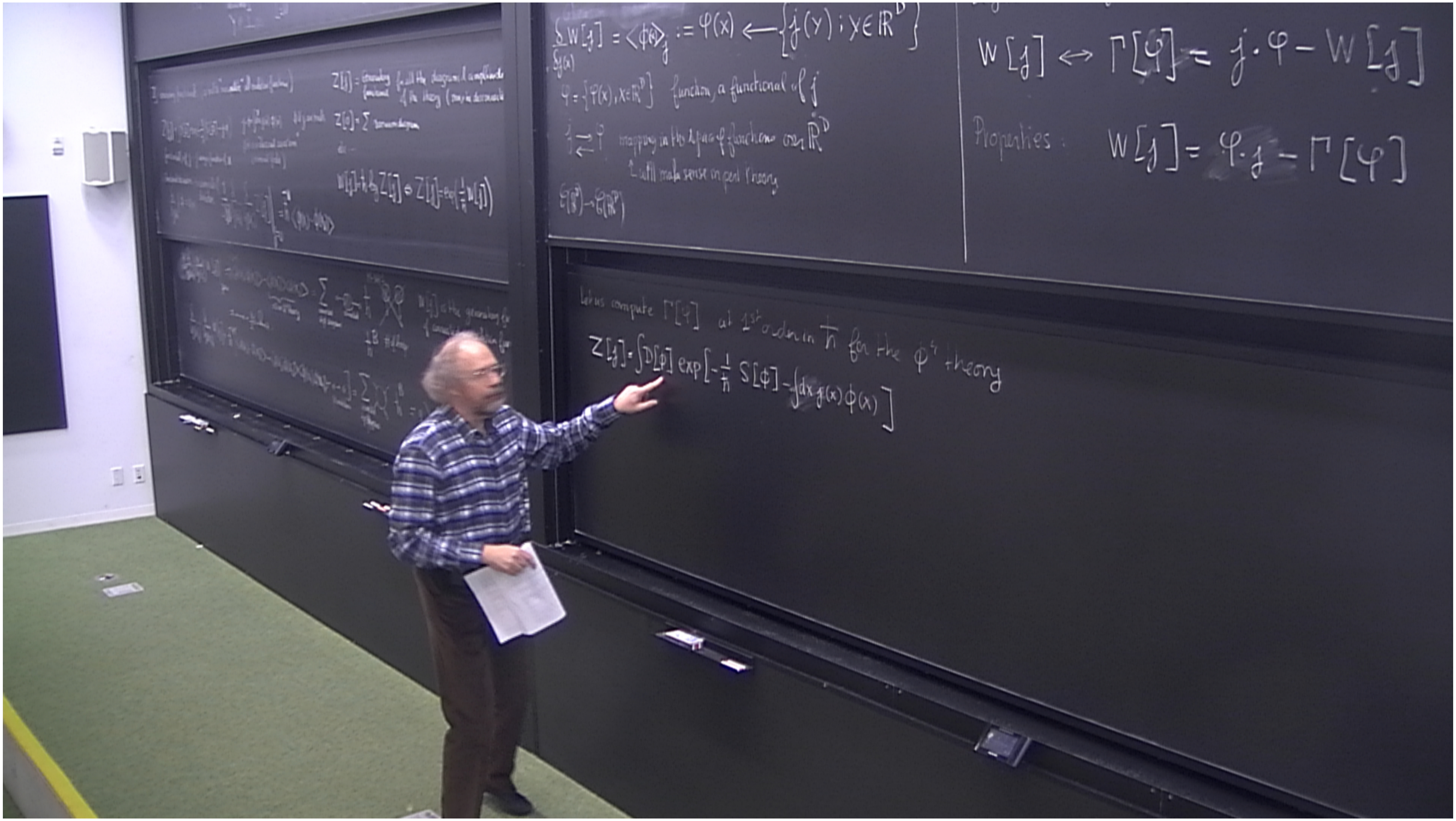

propagator
interaction vertex ϕ^4
ϕ^3
$V = 8$ $B = 3$
$L = 10$
$10 - 8 + 1 = 3$
Tree
functionals (a tool to "manipulate" all correlations functions)
$Z[j]$
$j \cdot \phi = \int d^D x \, j(x) \, \phi(x)$
ϕ & j are reals
$j(x)$ is a "classical source term"
(external field)
$= \hbar^N \langle \phi(x_1) \cdots \phi(x_N) \rangle$

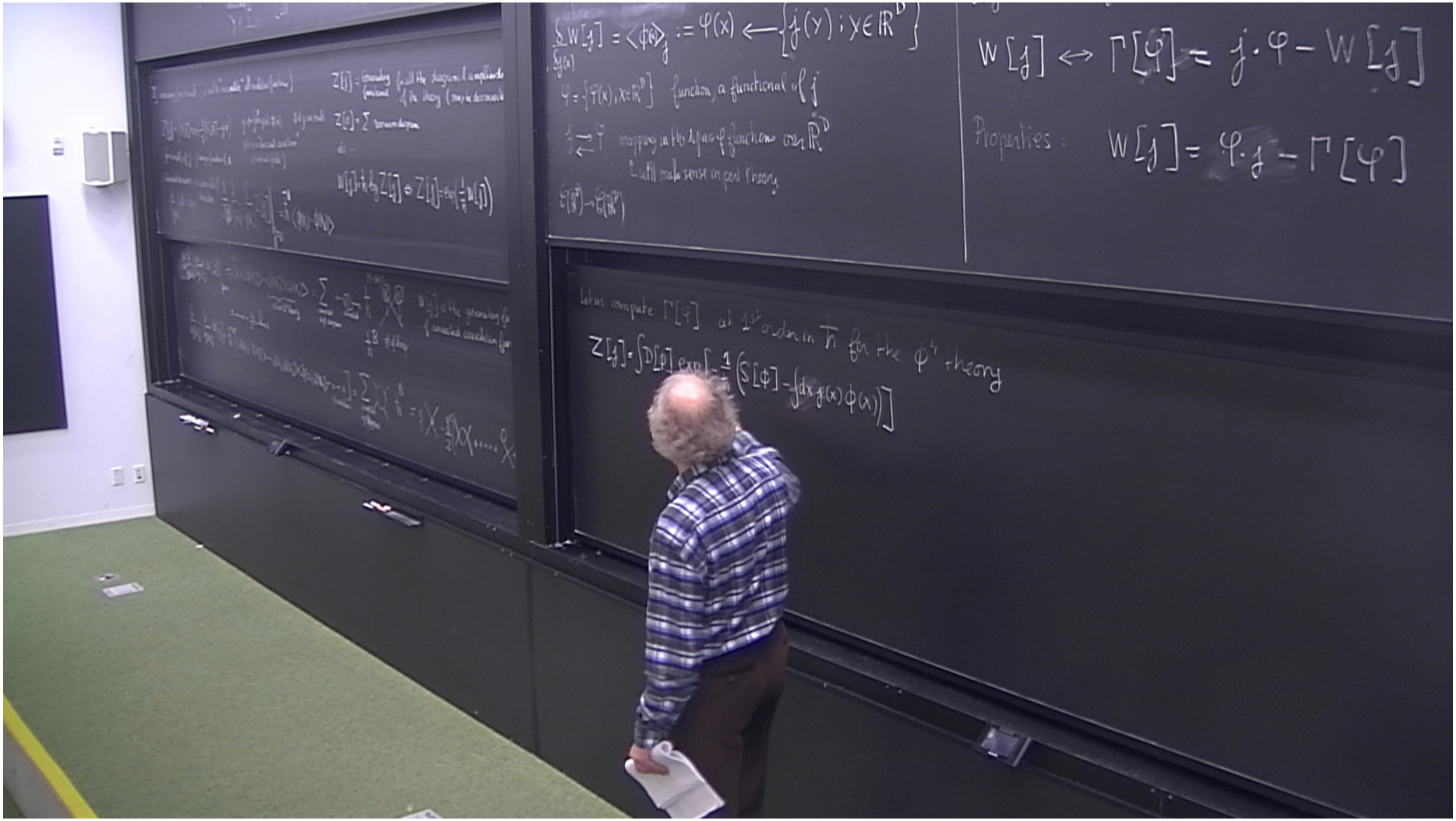
Effective action $\Gamma[\varphi]$ — φ will be a classical field $\varphi(x)$ function $\mathbb{R}^D \to \mathbb{R}$
(background field) or $M^{1,D-1}$

$$\frac{\delta}{\delta j(x)} W[j] = \langle \phi(x) \rangle_j := \varphi(x) \longleftarrow \{ j(y) \, ; \, y \in \mathbb{R}^D \}$$

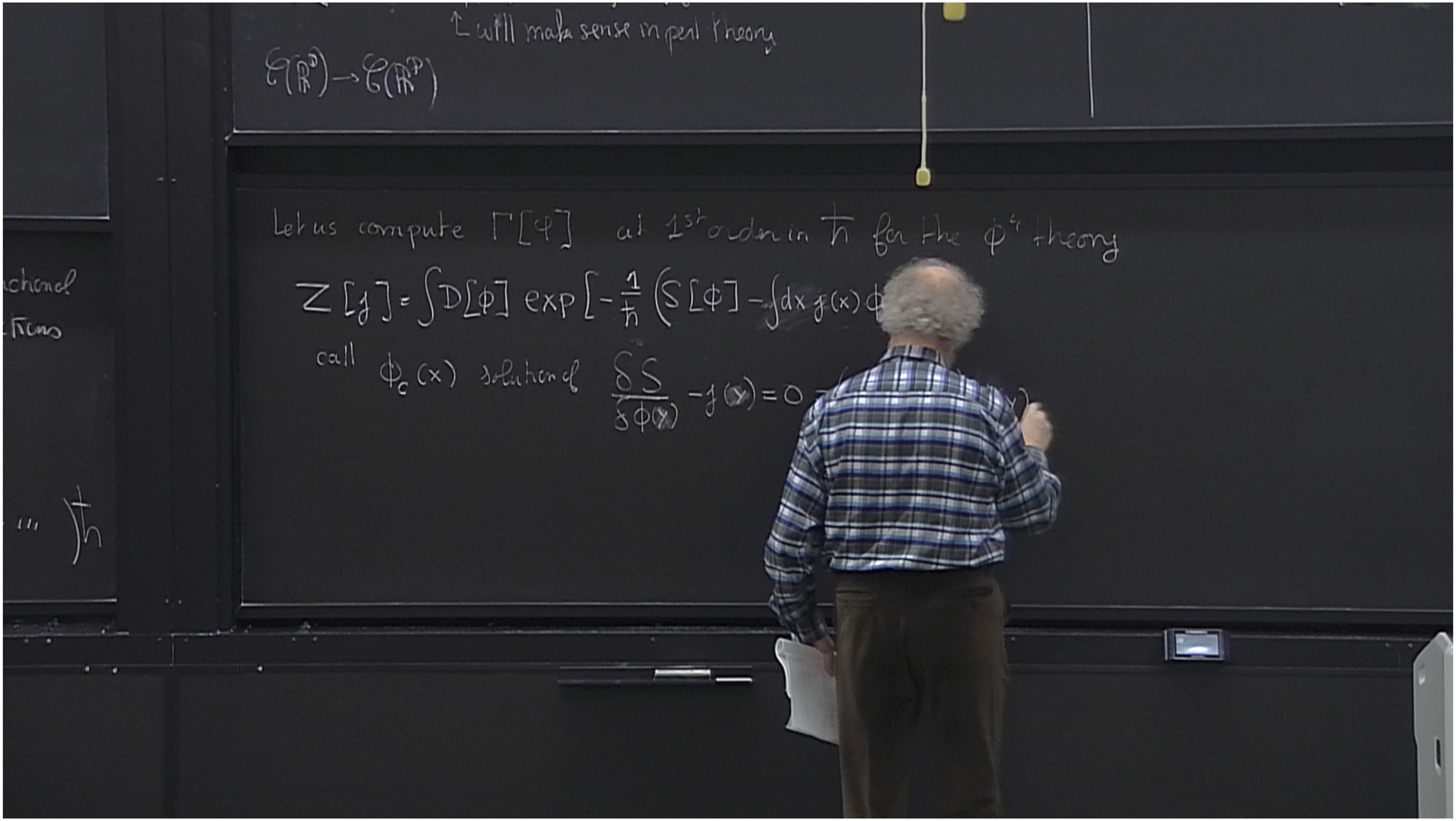
$\varphi = \{ \varphi(x), x \in \mathbb{R}^D \}$ function, a functional of j

$j \longrightarrow \varphi$ mapping in the space of functions over $\mathbb{R}^D$


W[j] ↔ Γ[φ] = j·φ − W[j]
Properties : W[j] = φ·j − Γ[φ]
function, a functional of j
mapping in the space of functions on ℝ^D
still make sense in pert theory
Let us compute Γ[φ] at 1st order in ħ for the φ^4 theory
Z[j] = ∫D[φ] exp[− 1/ħ S[φ] − ∫dx j(x) φ(x)]



$\frac{\delta}{\delta j(x)} W[j] = \langle \phi(x) \rangle_j := \varphi(x) \longleftarrow \{ j(y) ; y \in \mathbb{R}^D \}$
$\varphi = \{ \varphi(x), x \in \mathbb{R}^D \}$ function, a functional of j
$j \rightleftarrows \varphi$ mapping in the space of functions over $\mathbb{R}^D$
still make sense in pert theory
$W[j] \leftrightarrow \Gamma[\varphi] = j \cdot \varphi - W[j]$
Properties: $W[j] = \varphi \cdot j - \Gamma[\varphi]$
Let us compute $\Gamma[\varphi]$ at 1st order in $\hbar$ for the φ^4 theory
$Z[j] = \int D[\phi] \exp\left[-\frac{1}{\hbar} \left(S[\phi] - \int dx\, j(x) \phi(x) \right) \right]$


will make sense in pert theory
Let us compute Γ[φ] at 1st order in ħ for the φ⁴ theory
Z[j] = ∫D[φ] exp[− 1/ħ (S[φ] − ∫dx j(x) φ
call φc(x) solution of δS/δφ(y) − j(y) = 0

↑ will make sense in pert theory

$\mathcal{C}(\mathbb{R}^D) \to \mathcal{C}(\mathbb{R}^D)$

...nctional

...tions

... $\int \hbar$

Let us compute $\Gamma[\varphi]$ at 1^{st} order in $\hbar$ for the ϕ^4 theory

$$Z[j] = \int D[\phi] \exp\left[-\frac{1}{\hbar}\left(S[\phi] - \int dx\, j(x)\phi(x)\right)\right]$$

call ϕ_c solution of $\frac{\delta S}{\delta\phi(y)} - j(y) = 0 = \left(-\Delta_y + m^2\right)\phi_c(y) + \frac{g}{3!}\phi_c^3(y) - j(y) = 0$

ϕ_c is a functional of j, classical object; saddle point of the functional integral

$\phi = \phi_c +$

call ϕ_c solution of $\frac{\delta S}{\delta \phi(y)} - j(y) = 0 = \left(-\Delta_y + m^2\right)\phi_c(y) + \frac{g}{3!}\phi_c^3(y) - j(y) = 0$

ϕ_c is a functional of j. classical object ; saddle point of the functional integral

$\phi = \phi_c + \hbar^{1/2}\tilde{\phi}$, integrate over the $\tilde{\phi}$

$Z[j] = \exp\left(-\frac{1}{\hbar}\left(S[\phi_c] - j\cdot\phi_c\right.\right.$

$S''[\phi_c]$ = operator whose Kernel

$= \left[-\Delta_x + m^2 + \frac{g}{2}\phi_c^2\right]$

$$\left(S''[\phi_c]\cdot\psi\right)(x) = -\Delta_x\psi(x) + m^2\psi(x) + \frac{g}{2}\phi_c^2(x)\cdot\psi(x)$$

linear operator acting on functions ψ on $\mathbb{R}^D$

call ϕ_c solution of $\frac{\delta S}{\delta\phi(y)} - j(y) = 0 = \left(-\Delta_y + m^2\right)\phi_c(y) + \frac{g}{3!}\phi_c^3(y) - j(y) = 0$

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$\phi = \phi_c + \hbar^{1/2}\tilde{\phi}$, integrate over the $\tilde{\phi}$

$Z[j] = \exp\left[-\frac{1}{\hbar}\left(\right.\right.$

$S''[\phi_c] =$ operator

$= \left[-\Delta_x + m\right.$

$\frac{1}{\hbar}W[j]\big)$

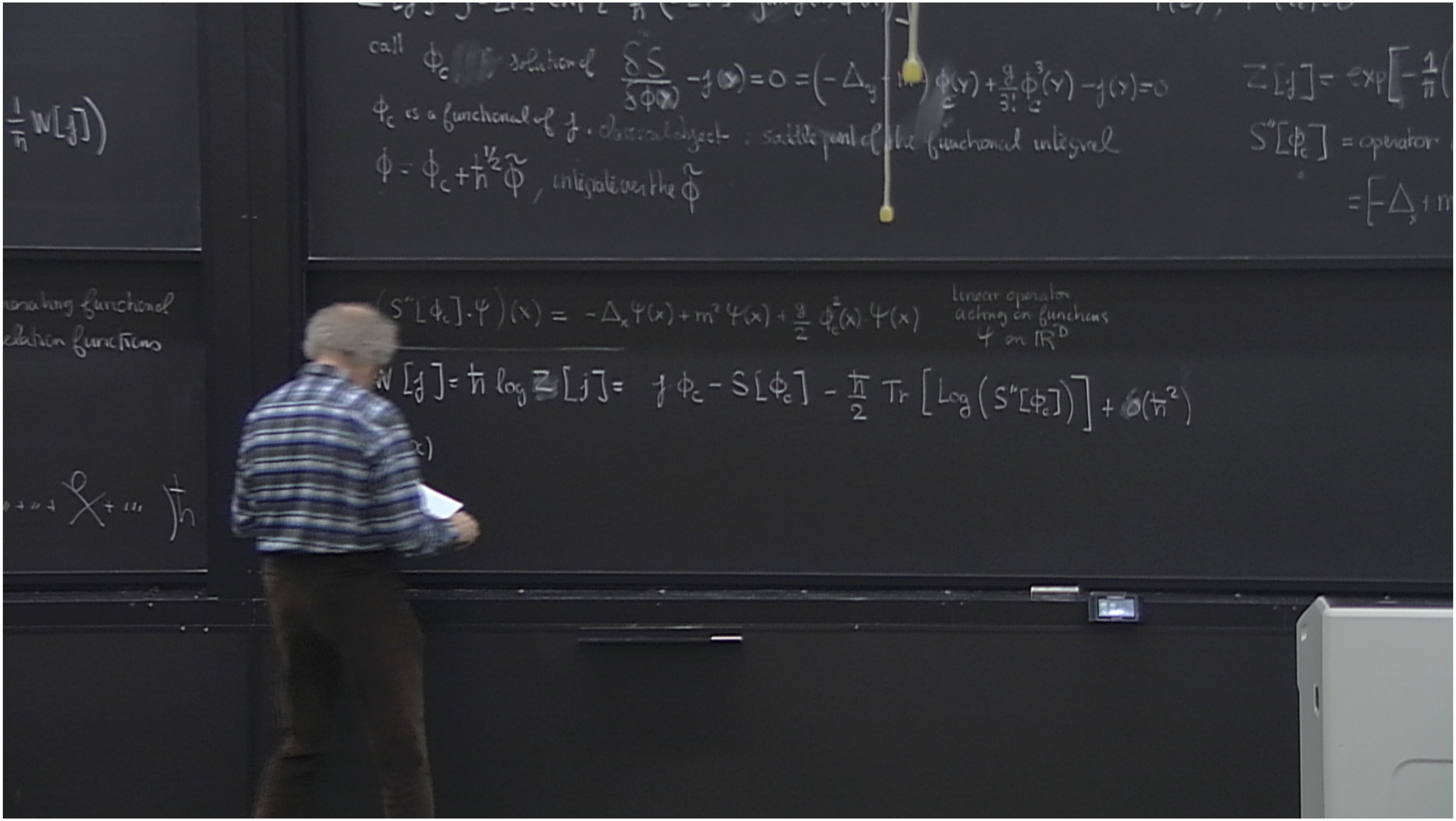
...rating functional

...elation functions

$\ldots + \ldots + \ldots + \ldots \big)\hbar$

$\left(S''[\phi_c]\cdot\psi\right)(x) = -\Delta_x\psi(x) + m^2\psi(x) + \frac{g}{2}\phi_c^2(x)\,\psi(x)$ linear operator acting on functions ψ on $\mathbb{R}^D$

$W[j] = \hbar\log Z[j] = j\,\phi_c - S[\phi_c] - \frac{\hbar}{2}\,\mathrm{Tr}\left[\mathrm{Log}\left(S''[\phi_c]\right)\right] + O(\hbar^2)$


call ϕ_c solution of $\frac{\delta S}{\delta\phi(x)} - j(x) = 0 = (-\Delta_y + m^2)\phi_c(y) + \frac{g}{3!}\phi_c^3(y) - j(y) = 0$
ϕ_c is a functional of j. classical object : saddle point of the functional integral
$\phi = \phi_c + \hbar^{1/2}\tilde{\phi}$, integrate over the $\tilde{\phi}$
$S''[\phi_c]$ = operator
$(S''[\phi_c]\cdot\varphi)(x) = -\Delta_x\varphi(x) + m^2\varphi(x) + \frac{g}{2}\phi_c^2(x)\cdot\varphi(x)$
linear operator acting on functions φ on $\mathbb{R}^D$
$W[j] = \hbar \log Z[j] = j\cdot\phi_c - S[\phi_c] - \frac{\hbar}{2}\,\mathrm{Tr}\left[\mathrm{Log}\left(S''[\phi_c]\right)\right] + O(\hbar^2)$

call ϕ_c solution of $\frac{\delta S}{\delta \phi(y)} - j(y) = 0 = (-\Delta_y + m^2)\phi_c(y) + \frac{g}{3!}\phi_c^3(y) - j(y) = 0$

ϕ_c is a functional of j. classical object : saddle point of the functional integral

$\phi = \phi_c + \hbar^{1/2}\tilde{\phi}$, integrate over the $\tilde{\phi}$

$Z[j] = \exp\left[-\frac{1}{\hbar}(\ldots\right.$

$S''[\phi_c]$ = operator

$= [-\Delta_x + m \ldots$

$\frac{1}{\hbar} W[j])$

...rating functional

...elation functions

$\ldots + \ldots + \ldots + \ldots)\hbar$

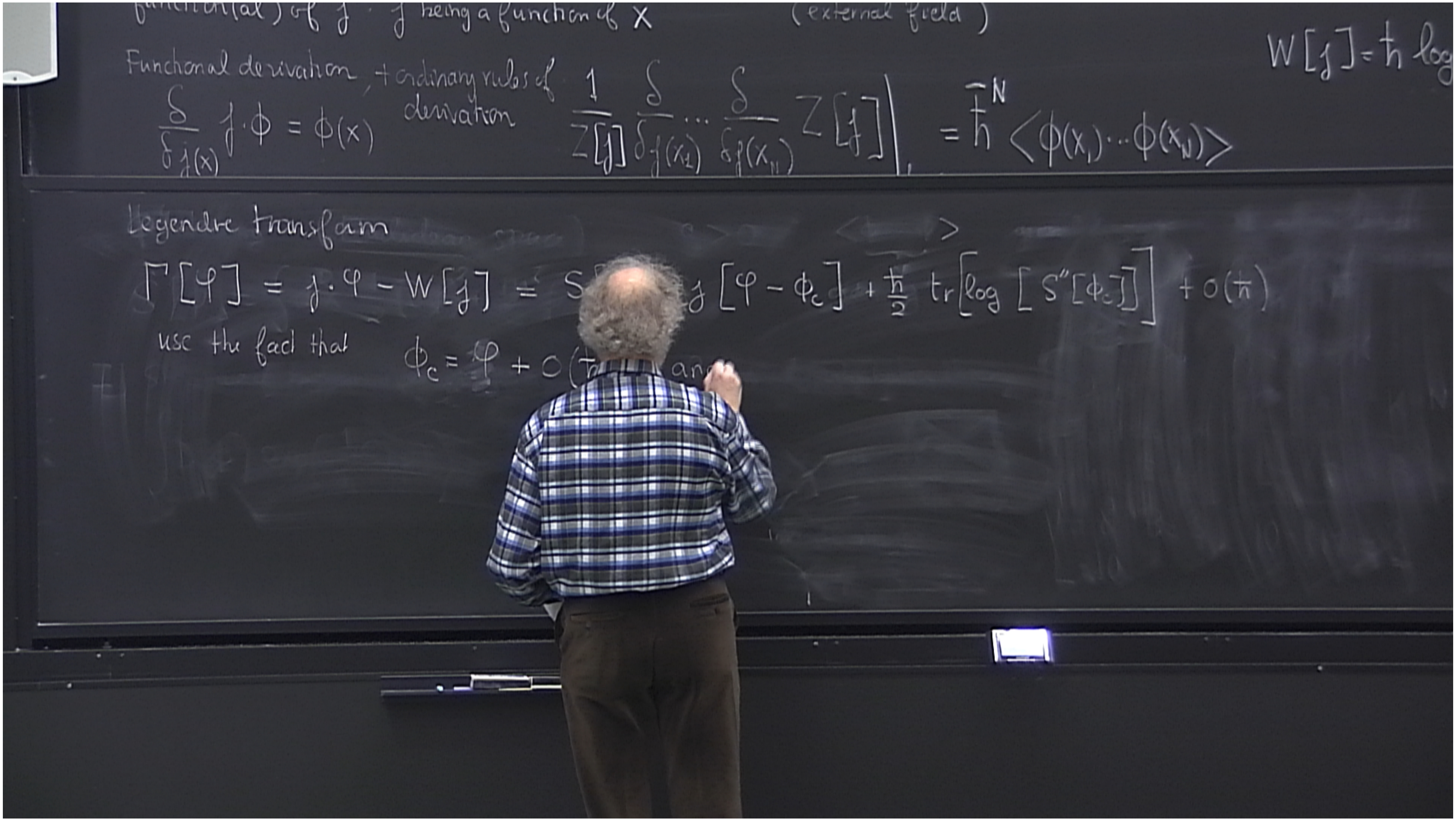
$(S''[\phi_c]\cdot\varphi)(x) = -\Delta_x \varphi(x) + m^2 \varphi(x) + \frac{g}{2}\phi_c^2(x)\varphi(x)$

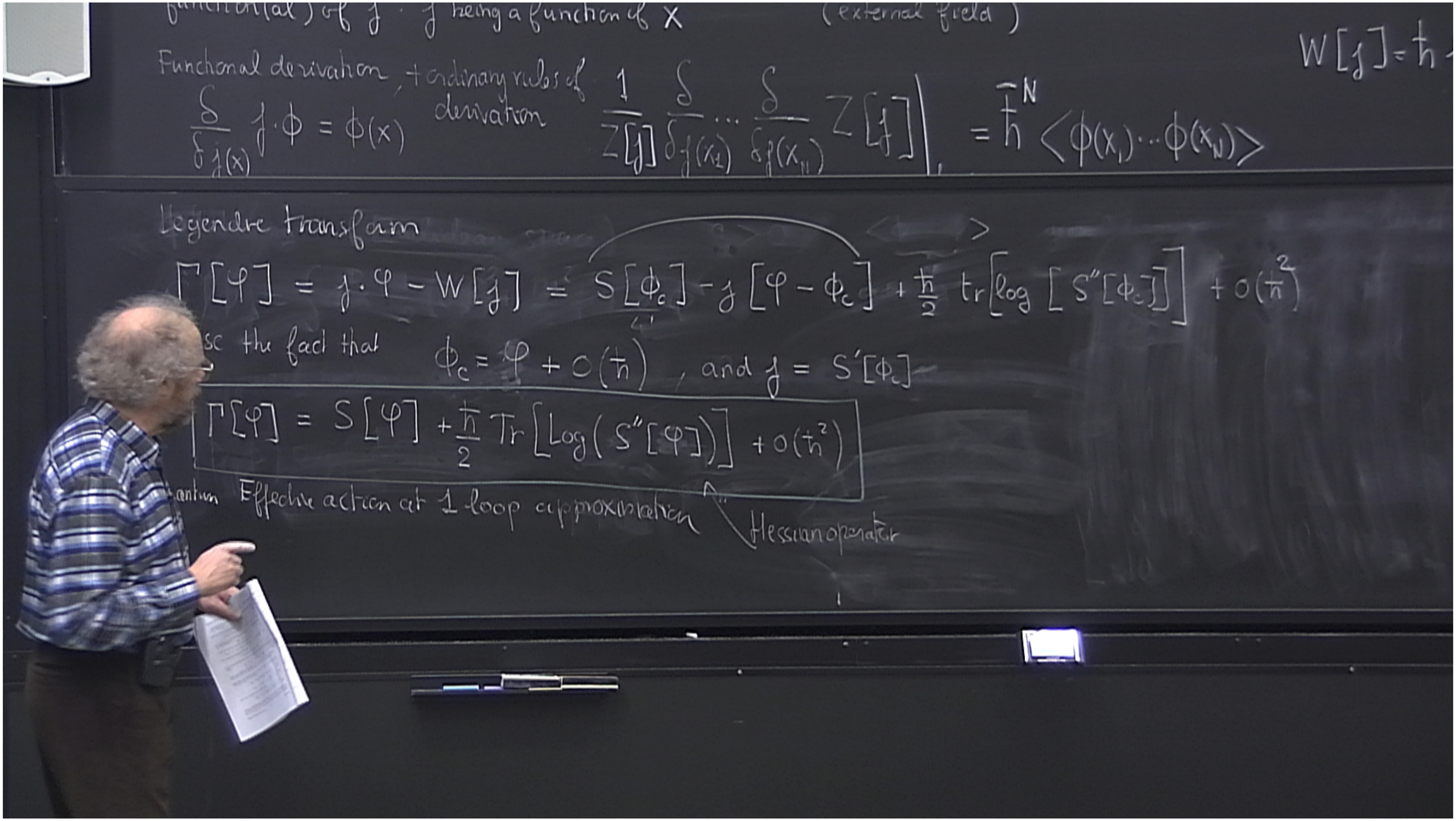
linear operator acting on functions φ on $\mathbb{R}^D$

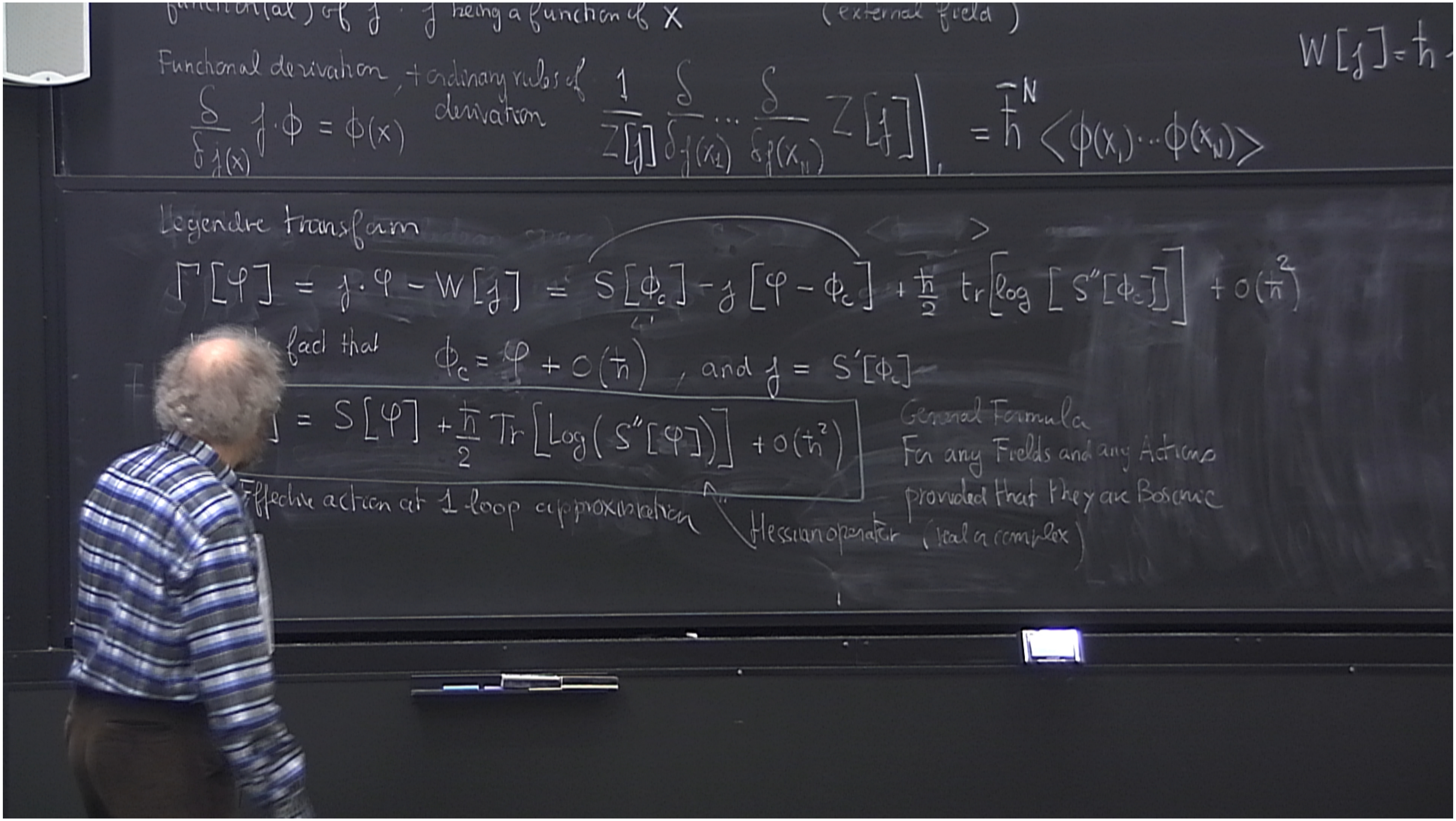
$$W[j] = \hbar \log Z[j] = j\cdot\phi_c - S[\phi_c] - \frac{\hbar}{2}\mathrm{Tr}\left[\mathrm{Log}\left(S''[\phi_c]\right)\right] + O(\hbar^2)$$

$$\varphi(x) = \frac{\delta W[j]}{\delta j(x)} = \phi_c(x) + \int d^D y \, \frac{\delta \phi_c(y)}{\delta j(x)} \cdot \left[\frac{\delta}{\delta \phi(y)}\left\{j\cdot\phi - S[\phi] - \frac{\hbar}{2}\mathrm{Tr}\left[\mathrm{Log}\left(S''[\phi]\right)\right]\right\}\right]_{\phi=\phi_c}$$

\varphi


functional) of j j being a function of x
(external field)
W[j] = ħ log
Functional derivation, + ordinary rules of derivation
δ/δj(x) j·φ = φ(x)
1/Z[j] δ/δj(x_1) … δ/δj(x_N) Z[j]| = ħ^N <φ(x_1)…φ(x_N)>
Legendre transform
Γ[φ] = j·φ − W[j] = S … j[φ − φ_c] + ħ/2 tr[log [S''[φ_c]]] + o(ħ)
use the fact that
φ_c = φ + o(ħ) and


functional of j . j being a function of x (external field)
$W[j] = \hbar$
Functional derivation, + ordinary rules of derivation
$\frac{\delta}{\delta j(x)} j \cdot \phi = \phi(x)$
$\frac{1}{Z[j]} \frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} Z[j] \Big| = \hbar^N \langle \phi(x_1) \cdots \phi(x_N) \rangle$
Legendre transform
$\Gamma[\varphi] = j \cdot \varphi - W[j] = S[\phi_c] - j[\varphi - \phi_c] + \frac{\hbar}{2} \, tr\left[\log\left[S''[\phi_c]\right]\right] + O(\hbar^2)$
use the fact that $\phi_c = \varphi + O(\hbar)$, and $j = S'[\phi_c]$
$\boxed{\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} Tr\left[Log\left(S''[\varphi]\right)\right] + O(\hbar^2)}$
Quantum Effective action at 1 loop approximation
Hessian operator


functional of j · j being a function of x (external field)
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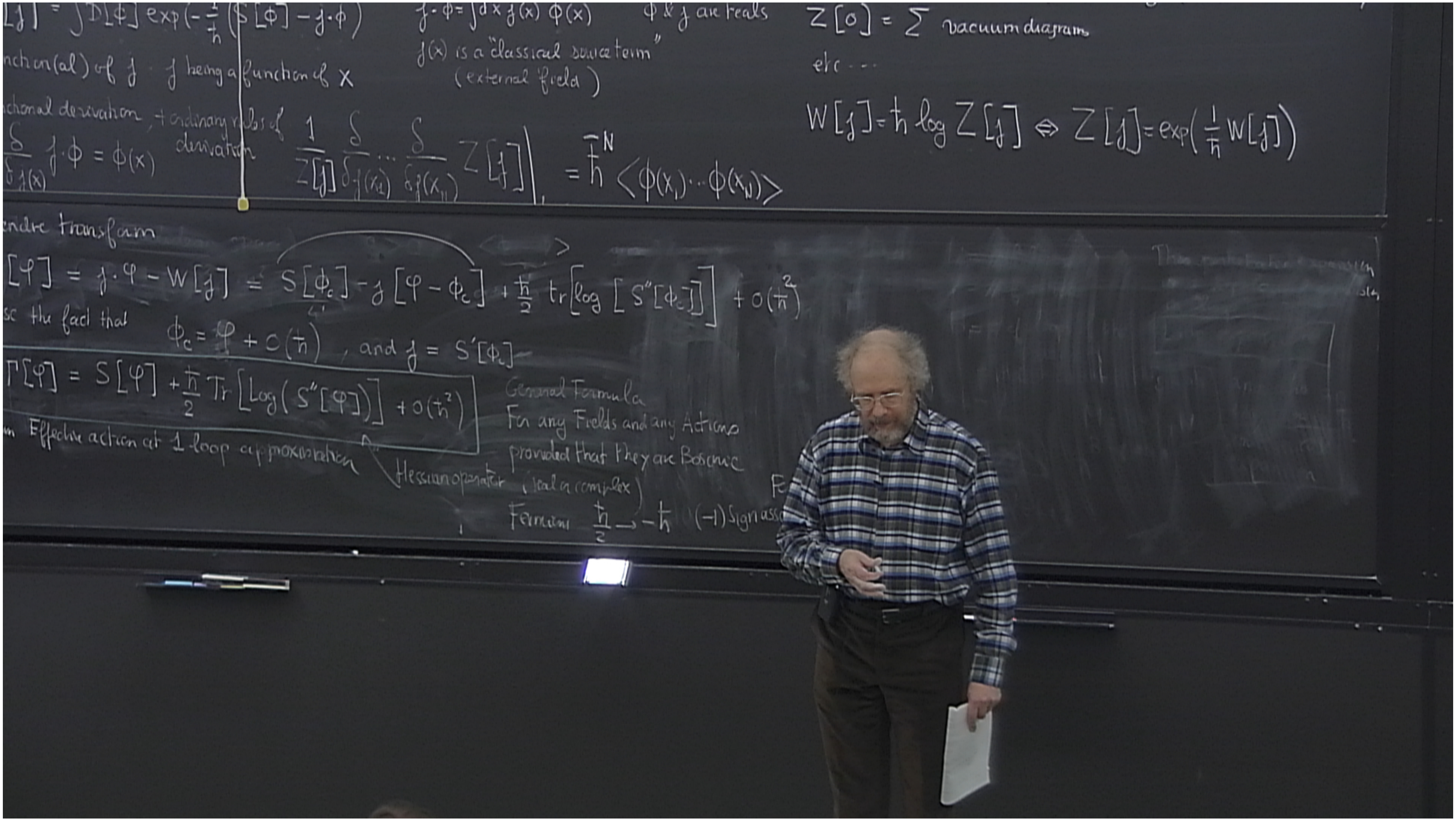
$\frac{\delta}{\delta j(x)} j \cdot \phi = \phi(x)$
$\frac{1}{Z[j]} \frac{\delta}{\delta j(x_1)} \cdots \frac{\delta}{\delta j(x_N)} Z[j] \Big| = \hbar^N \langle \phi(x_1) \cdots \phi(x_N) \rangle$
Legendre transform
$\Gamma[\varphi] = j \cdot \varphi - W[j] = S[\phi_c] - j[\varphi - \phi_c] + \frac{\hbar}{2} tr[\log[S''[\phi_c]]] + O(\hbar^2)$
fact that $\phi_c = \varphi + O(\hbar)$, and $j = S'[\phi_c]$
$] = S[\varphi] + \frac{\hbar}{2} Tr[Log(S''[\varphi])] + O(\hbar^2)$
Effective action at 1-loop approximation
Hessian operator
General Formula
For any Fields and any Actions
provided that they are Bosonic
(real or complex)


φ & j are reals
Z[0] = Σ vacuum diagrams
etc ...
j(x) is a "classical source term"
(external field)
W[j] = ħ log Z[j] ⇔ Z[j] = exp(1/ħ W[j])
functional derivation, + ordinary rules of derivation
Legendre transform
use the fact that
General Formula
For any Fields and any Actions
provided that they are Bosonic
(real a complex)
Fermions
Hessian operator
Effective action at 1 loop approximation