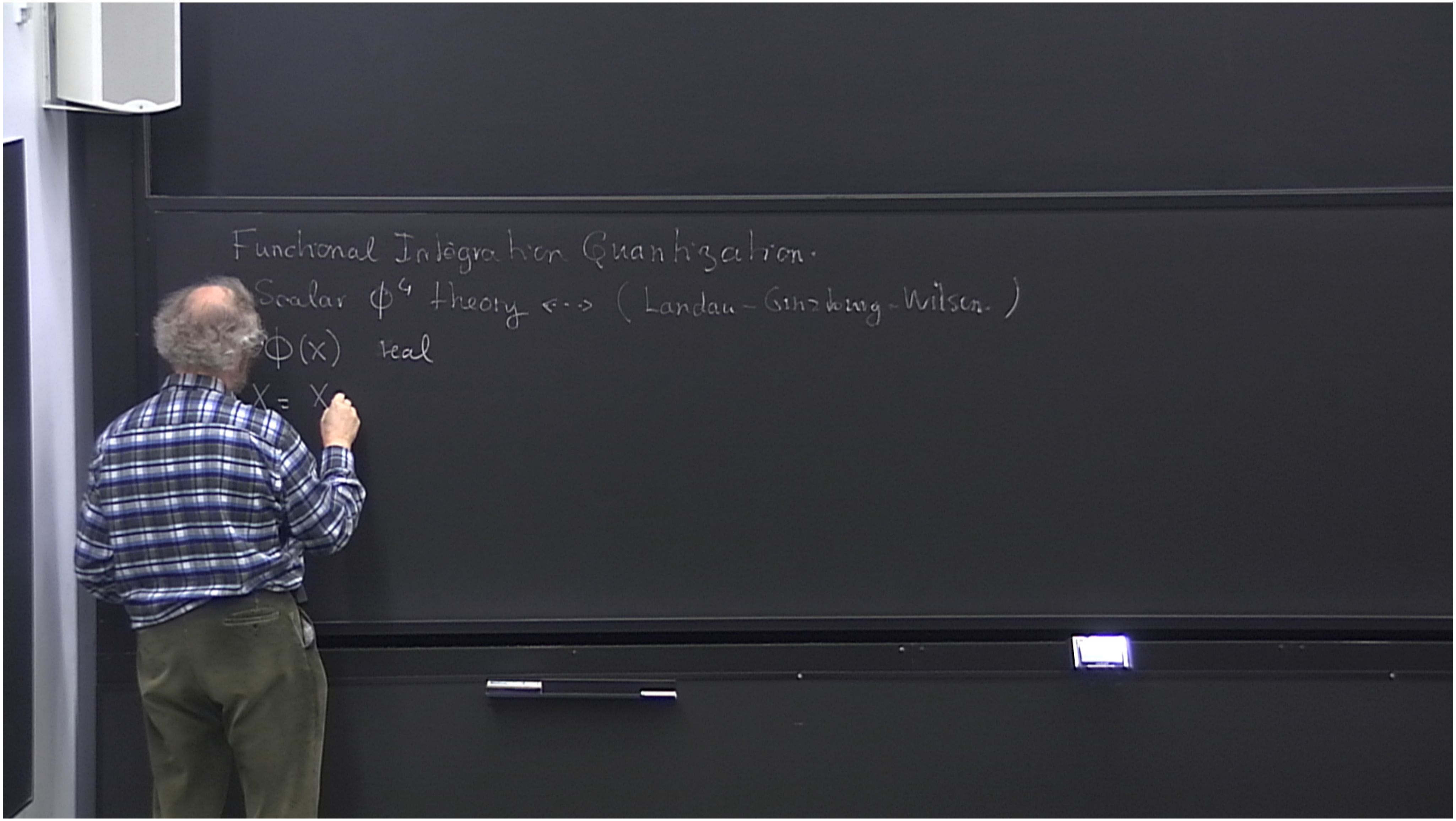
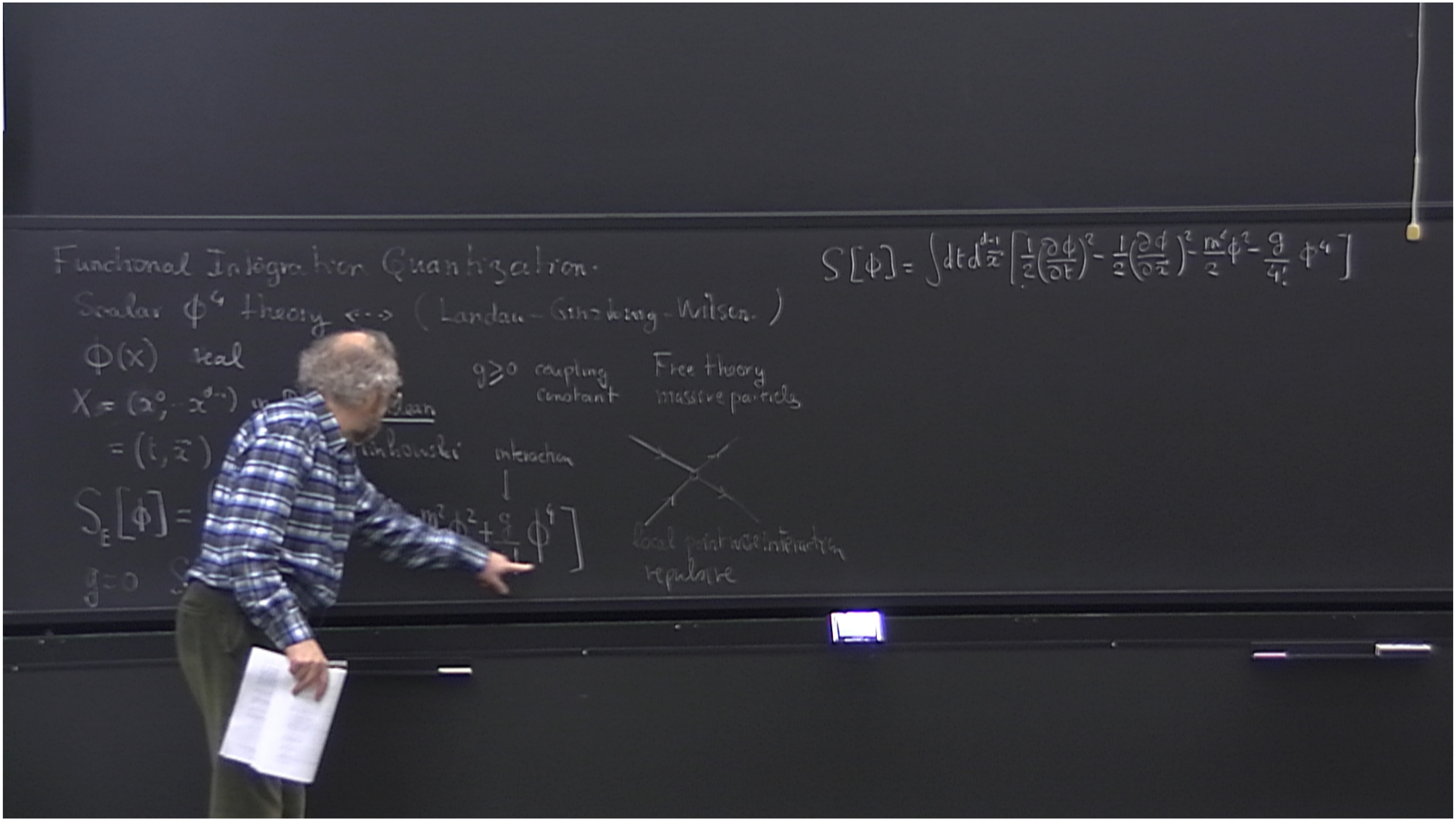
Title: 14/15 PSI - Quantum Field Theory II-Lecture 4

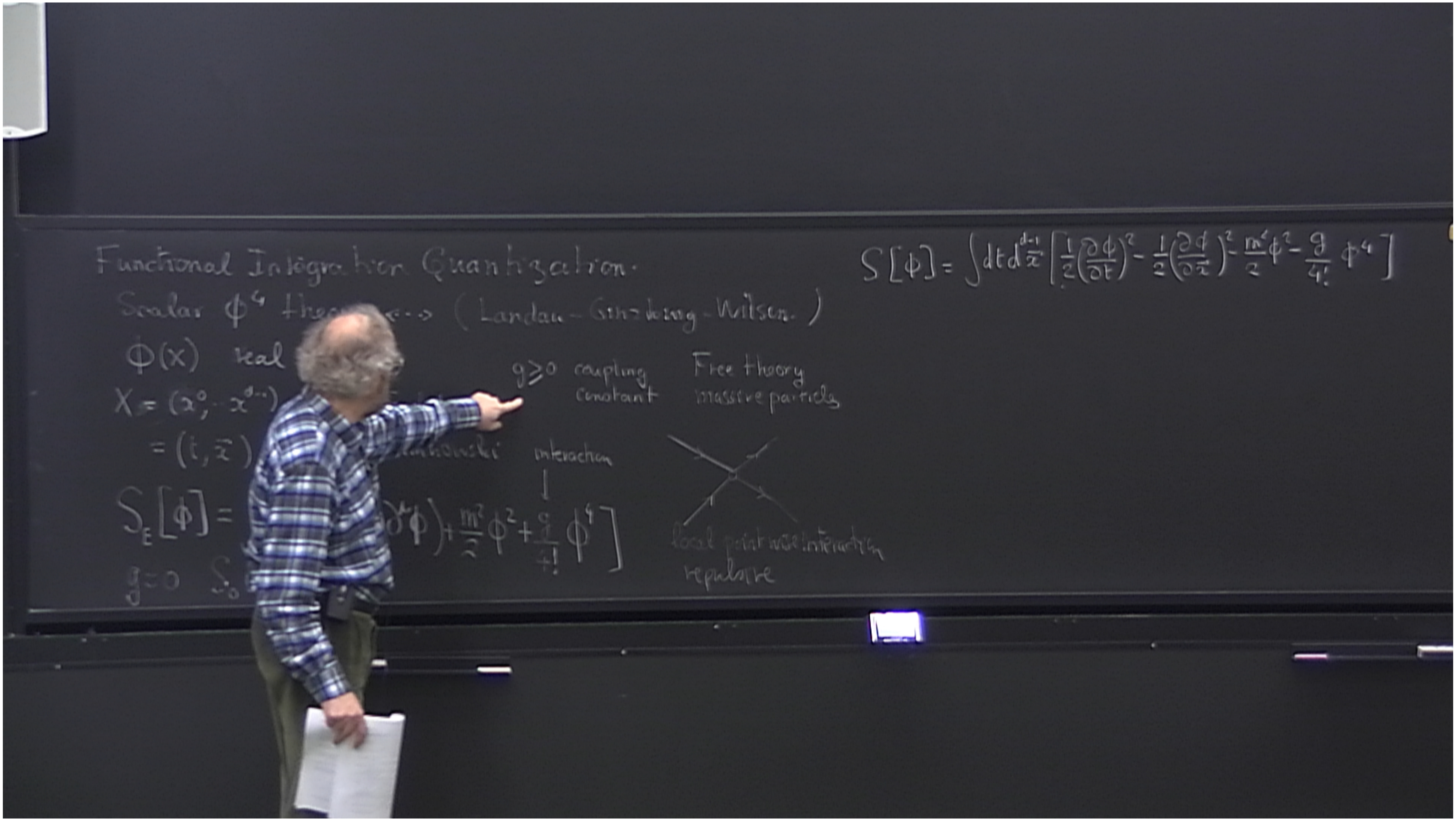
Date: Nov 13, 2014 09:00 AM

URL: http://pirsa.org/14110009

Abstract:


Functional Integration Quantization.
Scalar φ4 theory <--> (Landau-Ginzburg-Wilson)
φ(x) real
x = x


Functional Integration Quantization.
Scalar φ⁴ theory ↔ (Landau-Ginzburg-Wilson)
φ(x) real
= (t, x⃗)
g ≥ 0 coupling constant
Free theory massive particle
interaction
local pointwise interaction
repulsive
S[φ] = ∫dt d^{d-1}x [1/2 (∂φ/∂t)² − 1/2 (∂φ/∂x⃗)² − m²/2 φ² − g/4! φ⁴]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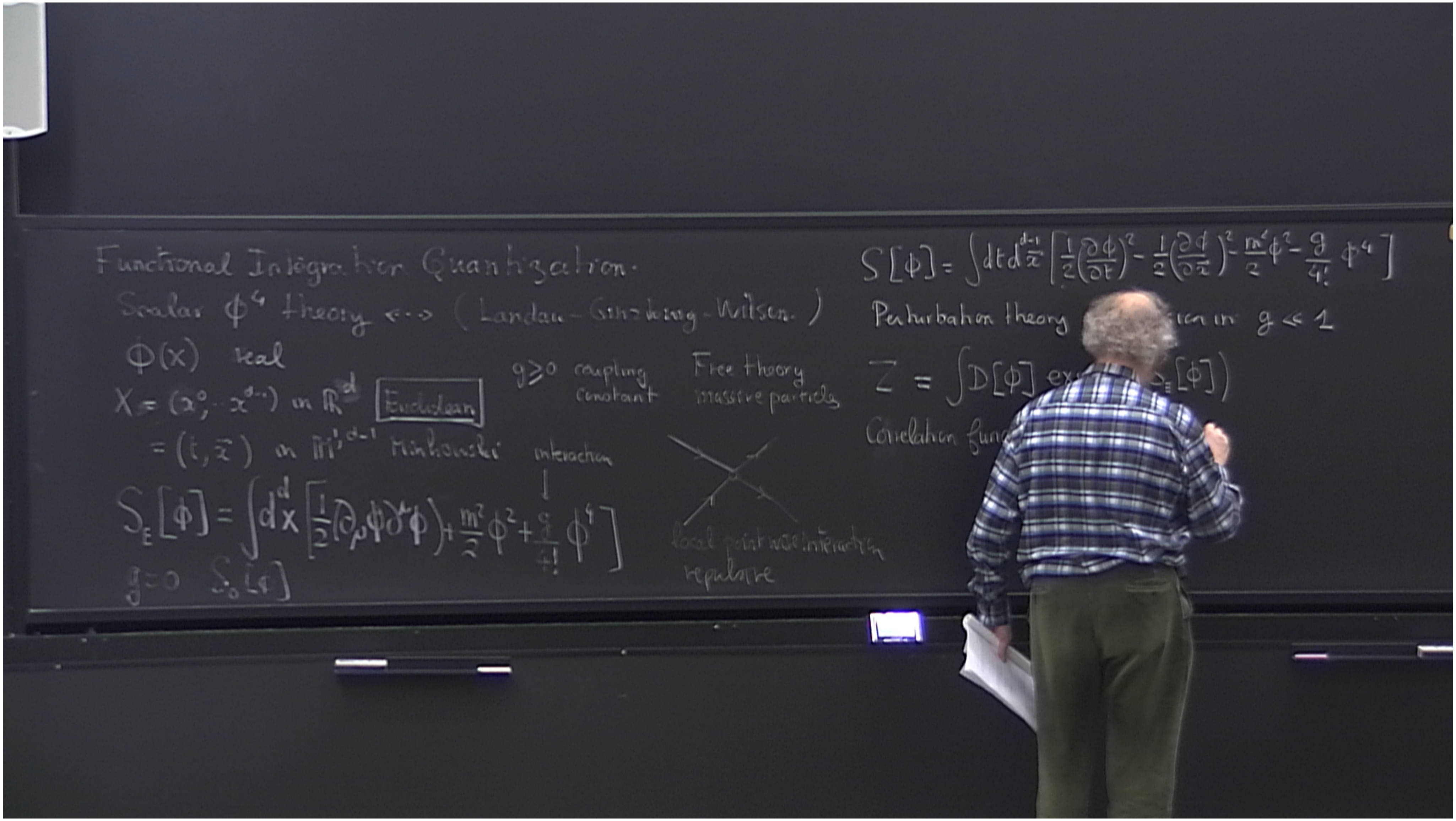
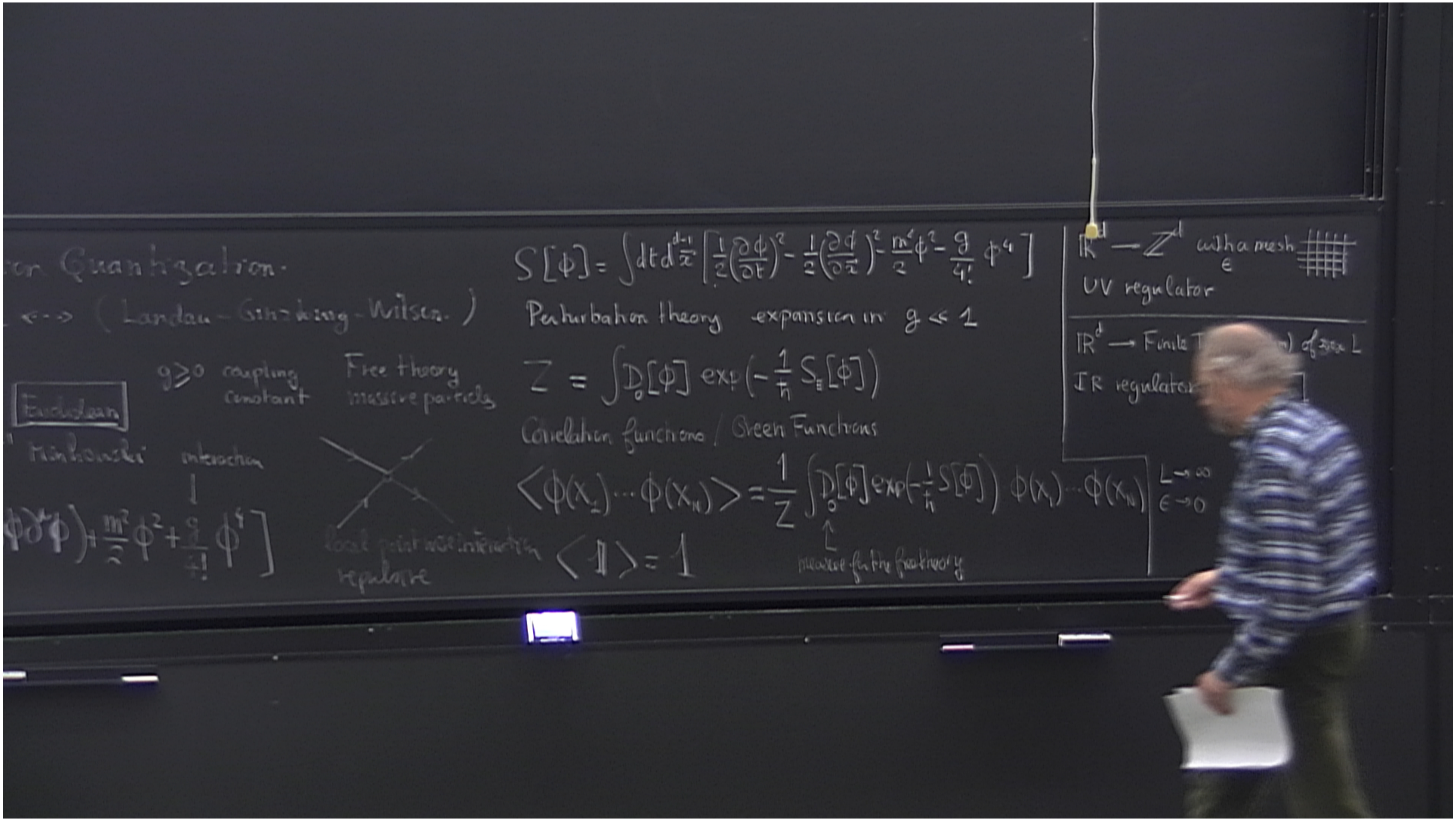
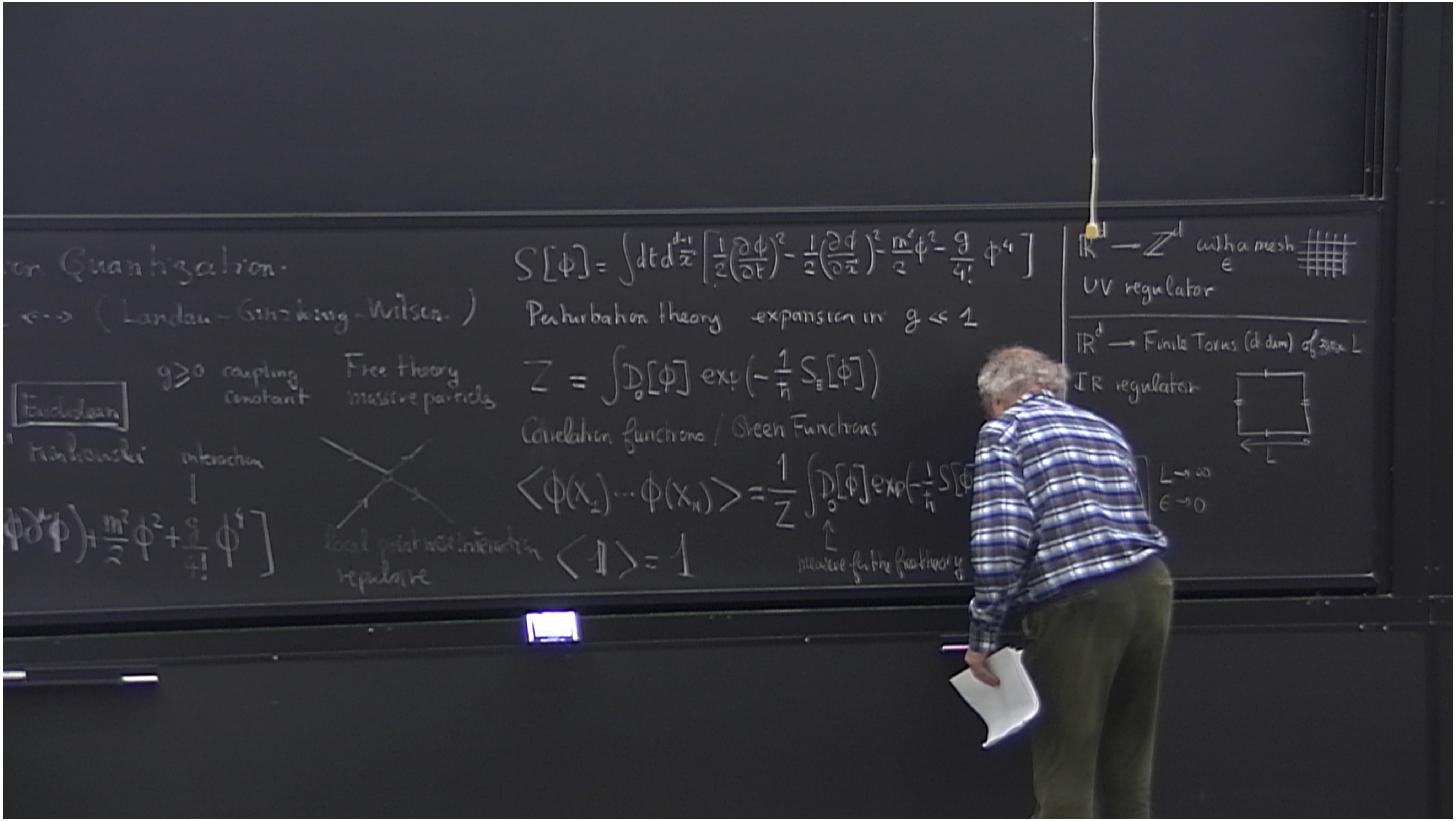
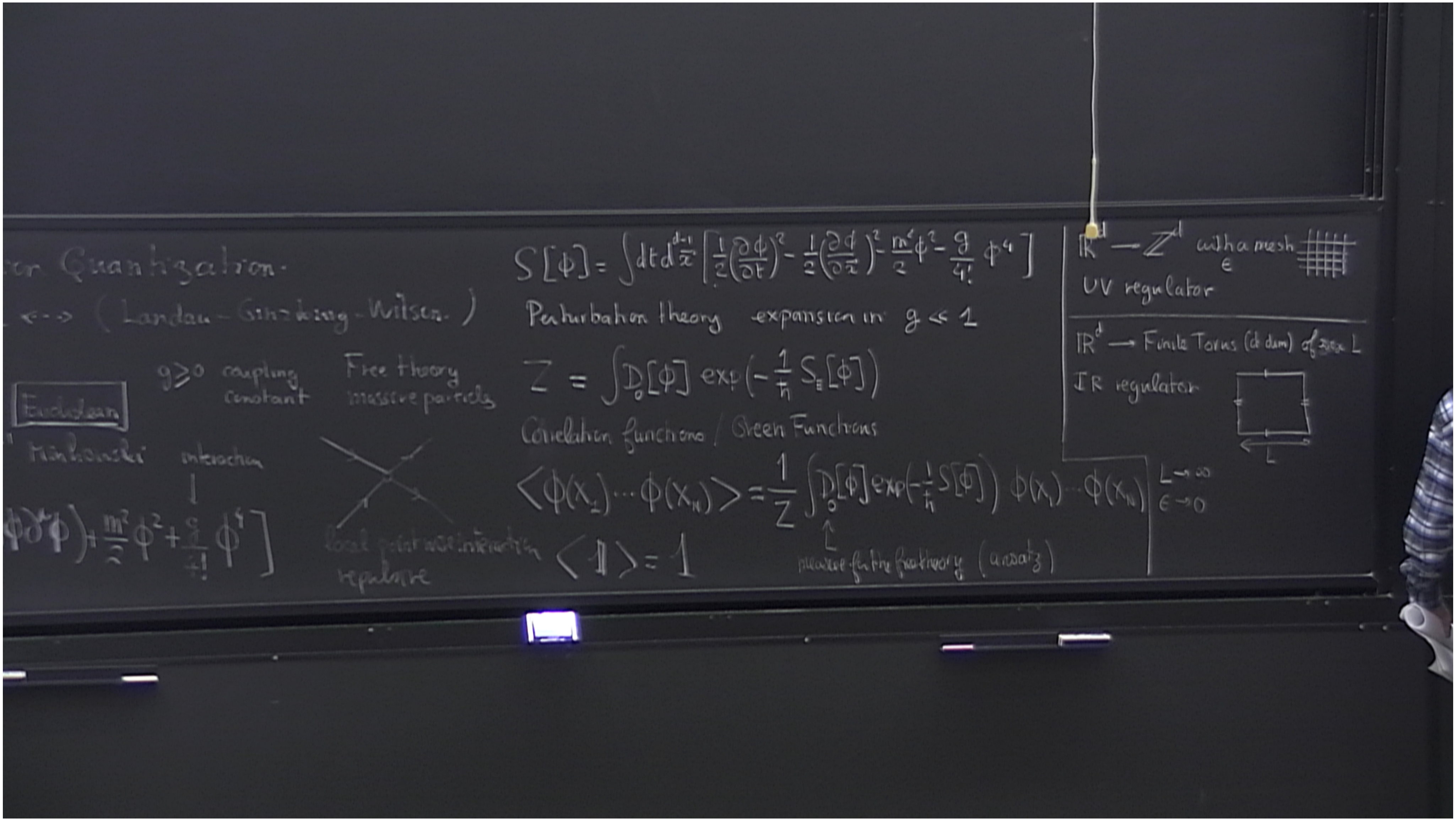
g=0

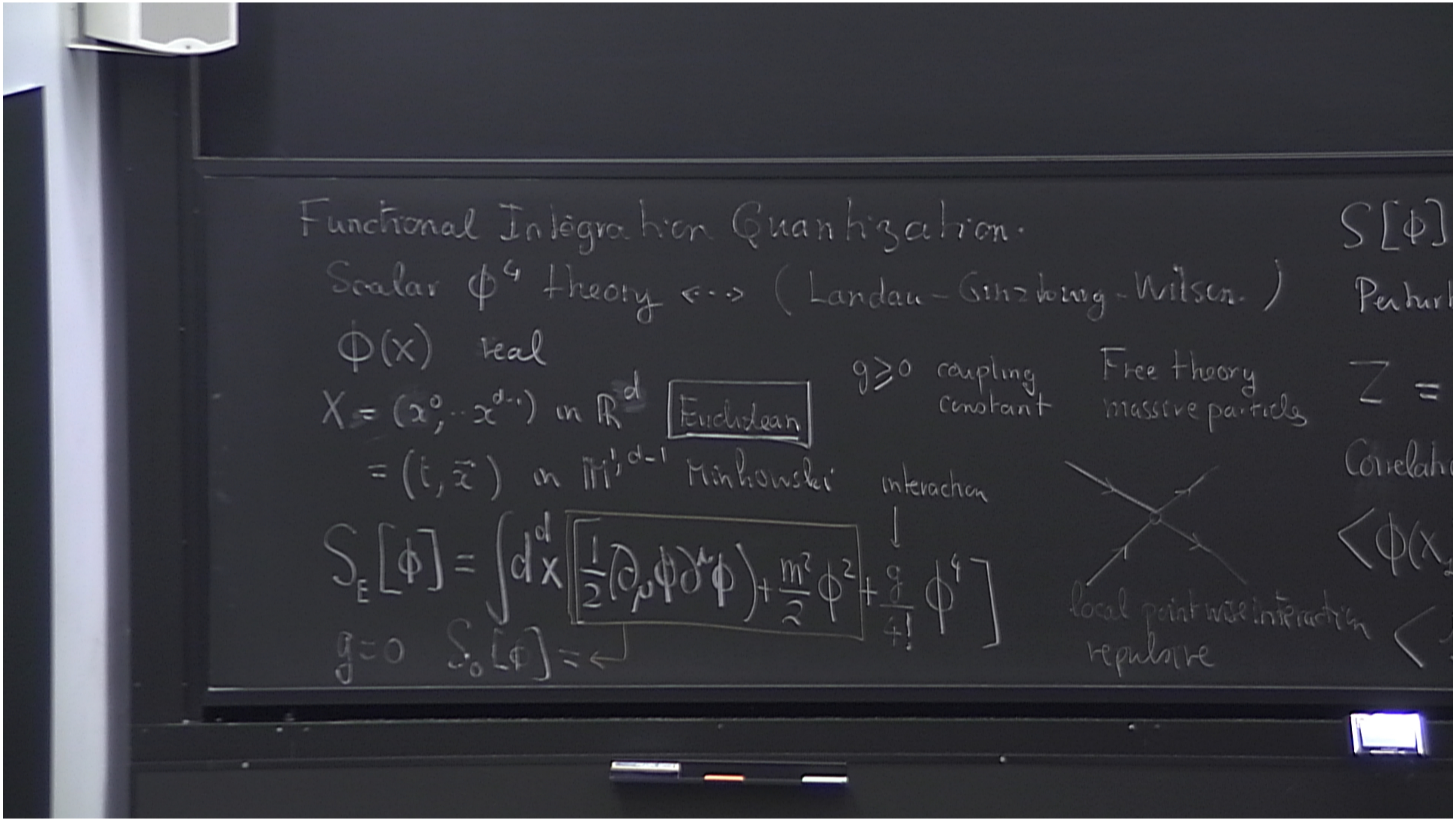

Functional Integration Quantization.
Scalar φ⁴ theory ⟷ (Landau - Ginzburg - Wilson)
φ(x) real
X = (x⁰, .. x^{d-1}) in ℝ^d
Euclidean
= (t, x⃗) in M^{1,d-1} Minkowski
g ≥ 0 coupling constant
Free theory massive particles
interaction
S_E[φ] = ∫d^d x [1/2 (∂_μ φ ∂^μ φ) + m²/2 φ² + g/4! φ⁴]
g = 0 S_0[φ]
local point wise interaction
repulsive
S[φ] = ∫dt d^{d-1}x [1/2 (∂φ/∂t)² - 1/2 (∂φ/∂x⃗)² - m²/2 φ² - g/4! φ⁴]
Perturbation theory
in g ≪ 1
Z = ∫D[φ] ex
[φ])
Correlation fun


Quantization.
(Landau - Ginzburg - Wilson)
g ≥ 0 coupling constant
Free theory massive particles
Euclidean
Minkowski
interaction
local point wise interaction
repulsive
S[φ] = ∫dt d^{d-1}x [1/2 (∂φ/∂t)^2 - 1/2 (∂φ/∂x)^2 - m^2/2 φ^2 - g/4! φ^4]
Perturbation theory expansion in g ≪ 1
Z = ∫D0[φ] exp(- 1/ħ S_E[φ])
Correlation functions / Green Functions
<φ(x1)···φ(xN)> = 1/Z ∫D0[φ] exp(- 1/ħ S[φ]) φ(x1)···φ(xN)
measure for the free theory
<1> = 1
L → ∞
ε → 0
R^d → Z^d with a mesh ε
UV regulator
IR regulator

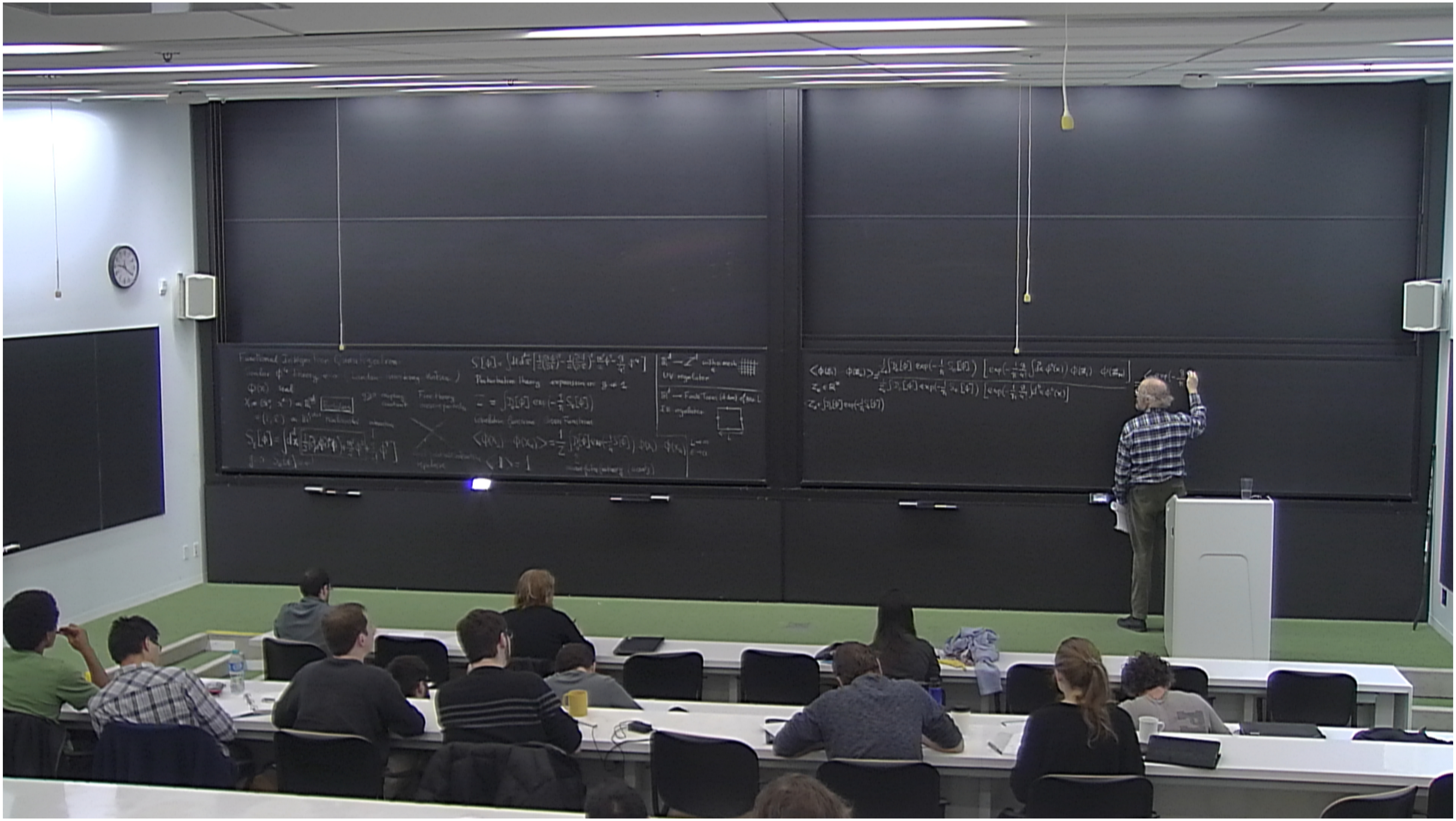

Quantization.
(Landau - Ginzburg - Wilson)
g ≥ 0 coupling constant
Free theory massive particle
Euclidean
Minkowski
interaction
local point wise interaction repulsive
Perturbation theory expansion in g ≪ 1
Correlation functions / Green Functions
< 1 > = 1
measure for the free theory
with a mesh ε
UV regulator
Finite Torus (d dim) of size L
IR regulator
L → ∞
ε → 0

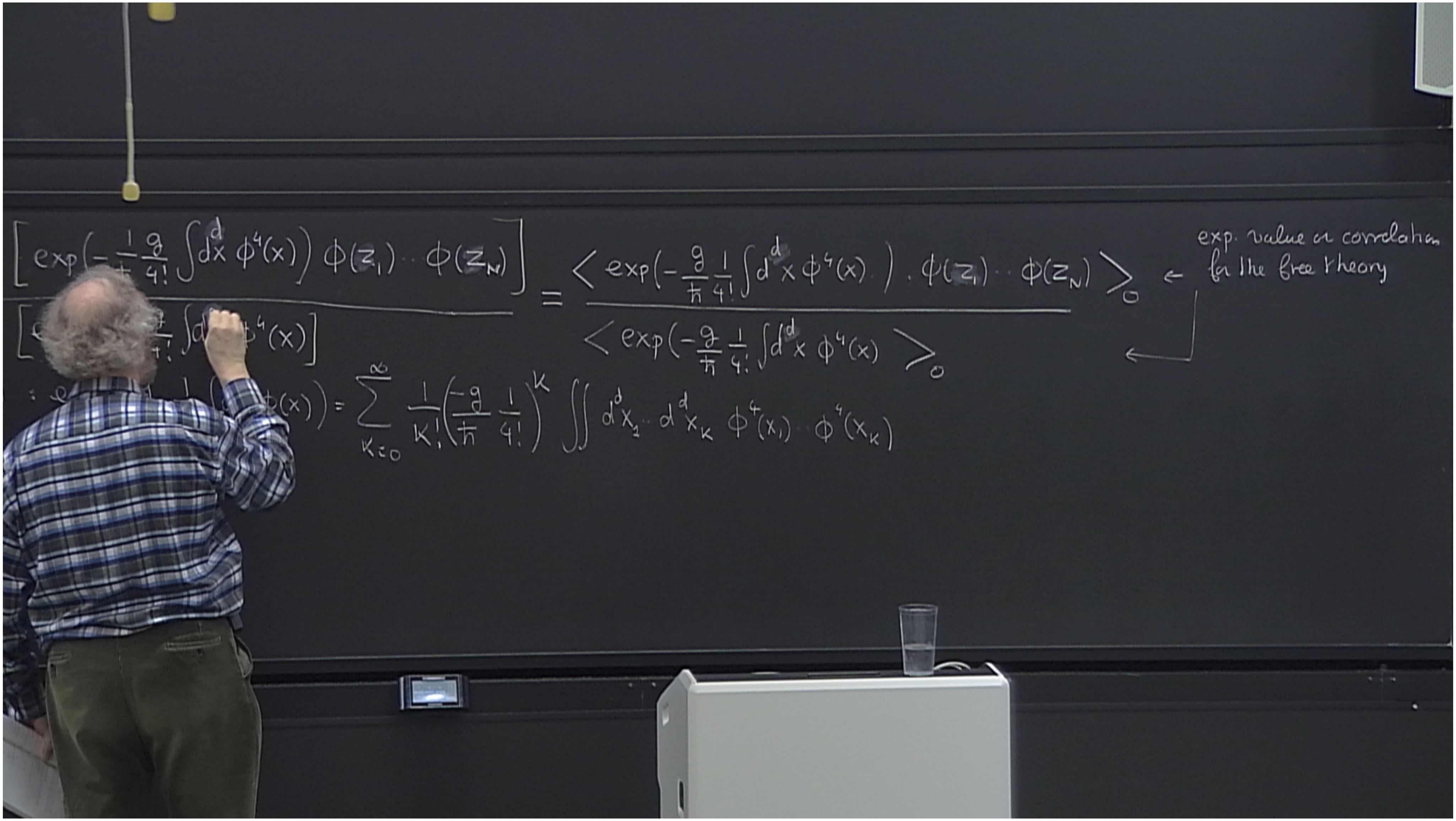


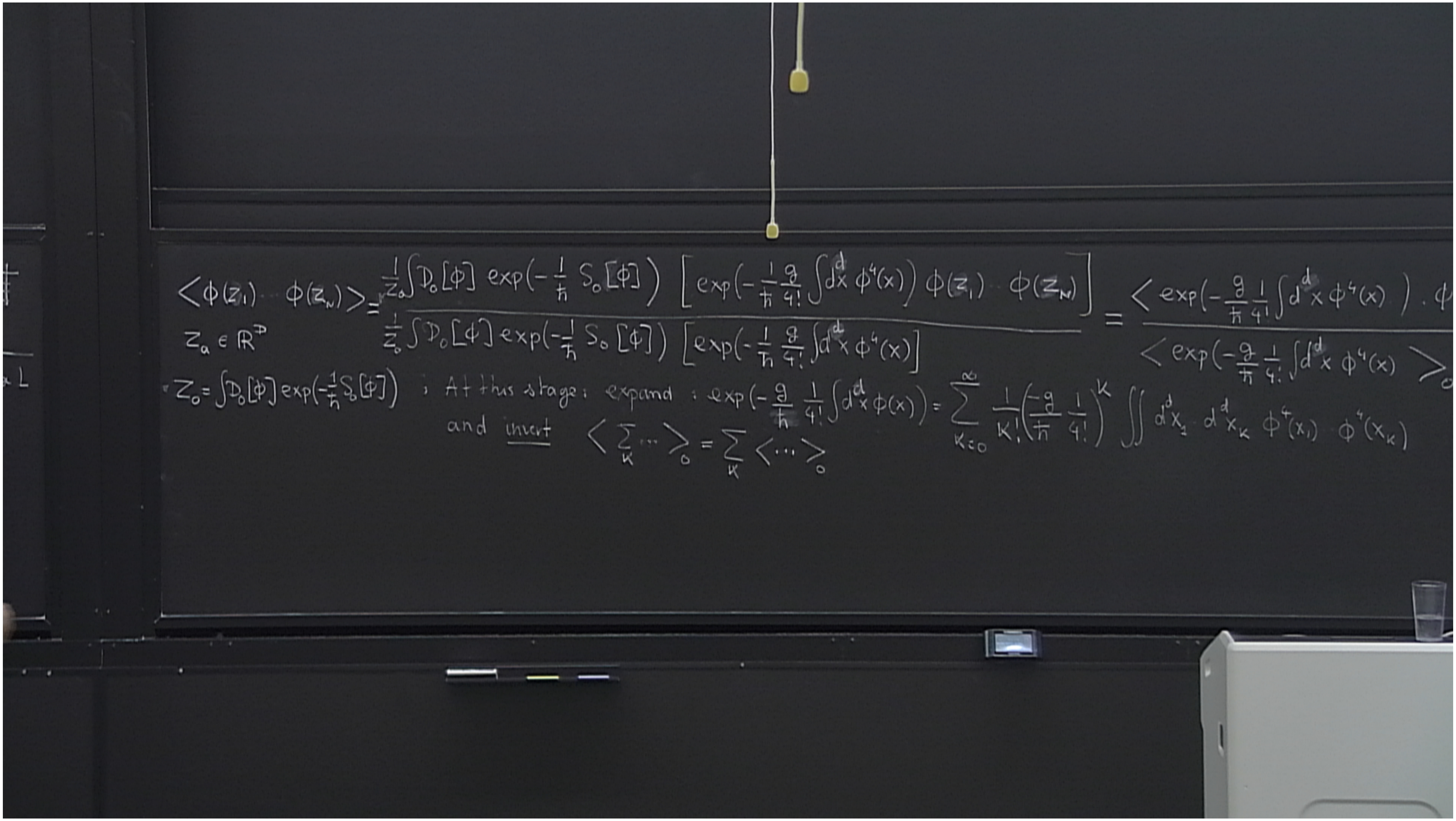
Quantization.
(Landau - Ginzburg - Wilson.)
g ≥ 0 coupling constant
Free theory massive particles
Euclidean
Minkowski
interaction
local point wise interaction
repulsive
S[φ] = ∫dt d^{d-1}x [1/2 (∂φ/∂t)² - 1/2 (∂φ/∂x)² - m²/2 φ² - g/4! φ⁴]
Perturbation theory expansion in g ≪ 1
Z = ∫D₀[φ] exp(- 1/ħ S_E[φ])
Correlation functions / Green Functions
<φ(x₁)···φ(x_N)> = 1/Z ∫D₀[φ] exp(- 1/ħ S[φ]) φ(x₁)···φ(x_N)
<1> = 1
measure for the free theory (ansatz)
IR^d → Z^d with a mesh ε
UV regulator
IR^d → Finite Torus (d dim) of size L
IR regulator
L
L → ∞
ε → 0

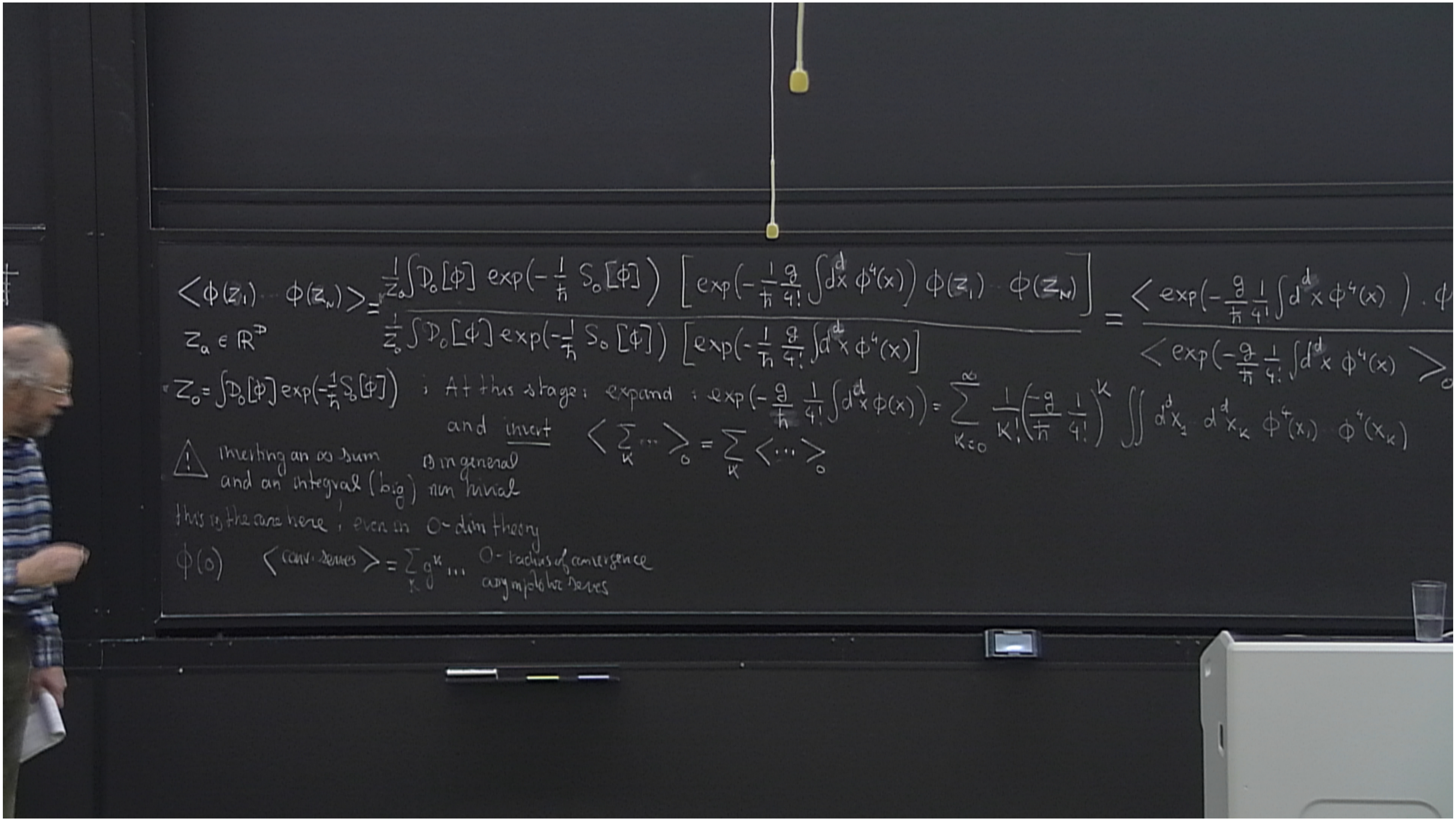


Functional Integration Quantization.
Scalar ϕ^4 theory <--> (Landau-Ginzburg-Wilson)
$\phi(x)$ real
$X = (x^0, \dots x^{d-1})$ in $\mathbb{R}^d$ Euclidean
$= (t, \vec{x})$ in $\mathbb{M}^{1,d-1}$ Minkowski
$g \geq 0$ coupling constant
Free theory massive particle
interaction
$S_E[\phi] = \int d^d x \left[\frac{1}{2}(\partial_\mu \phi \partial^\mu \phi) + \frac{m^2}{2}\phi^2 + \frac{g}{4!}\phi^4 \right]$
$g=0$ $S_0[\phi] =$
local point wise interaction
repulsive
$S[\phi]$
Perturb
$Z =$
Correlat
$\langle \phi(x$

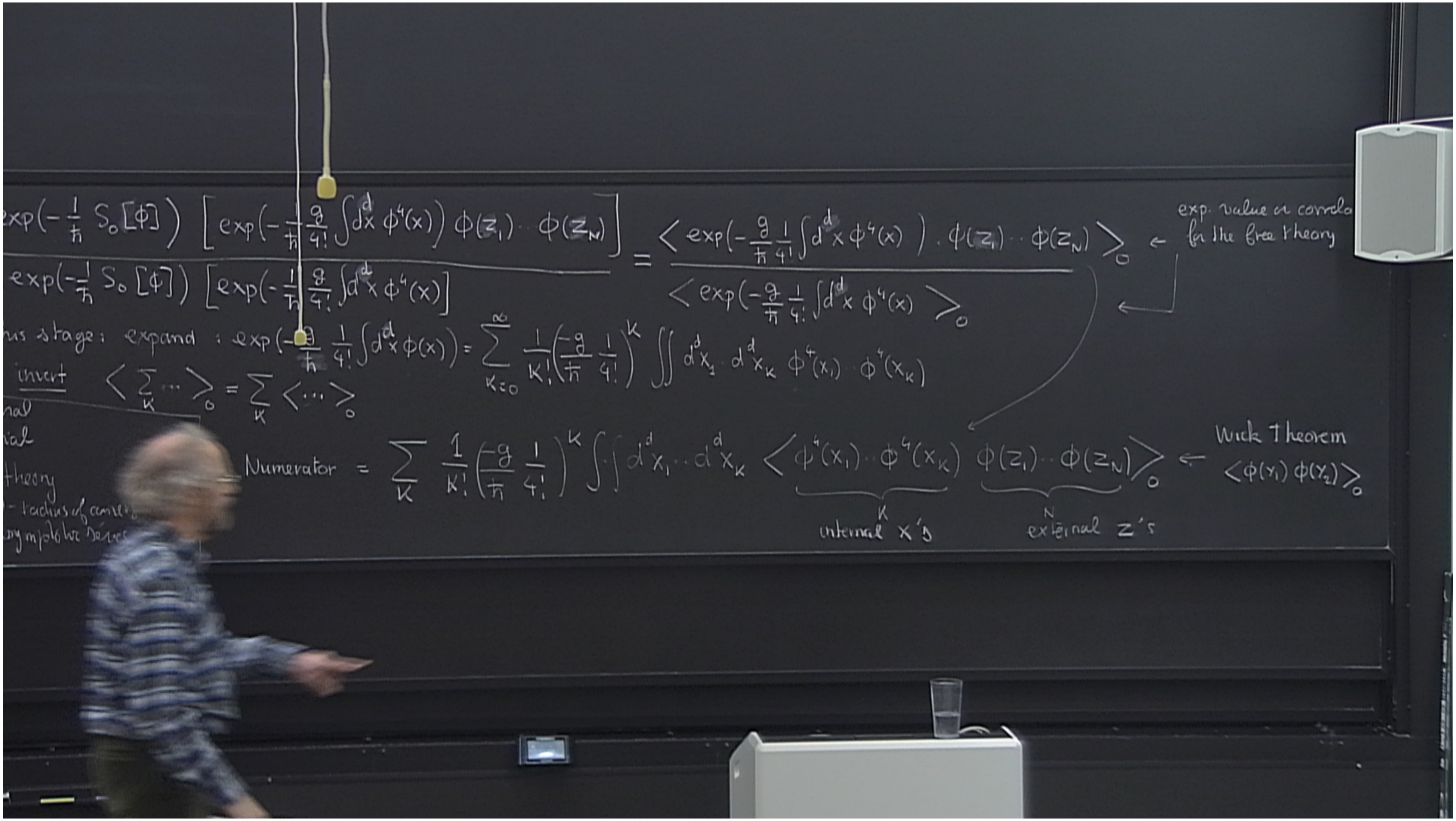


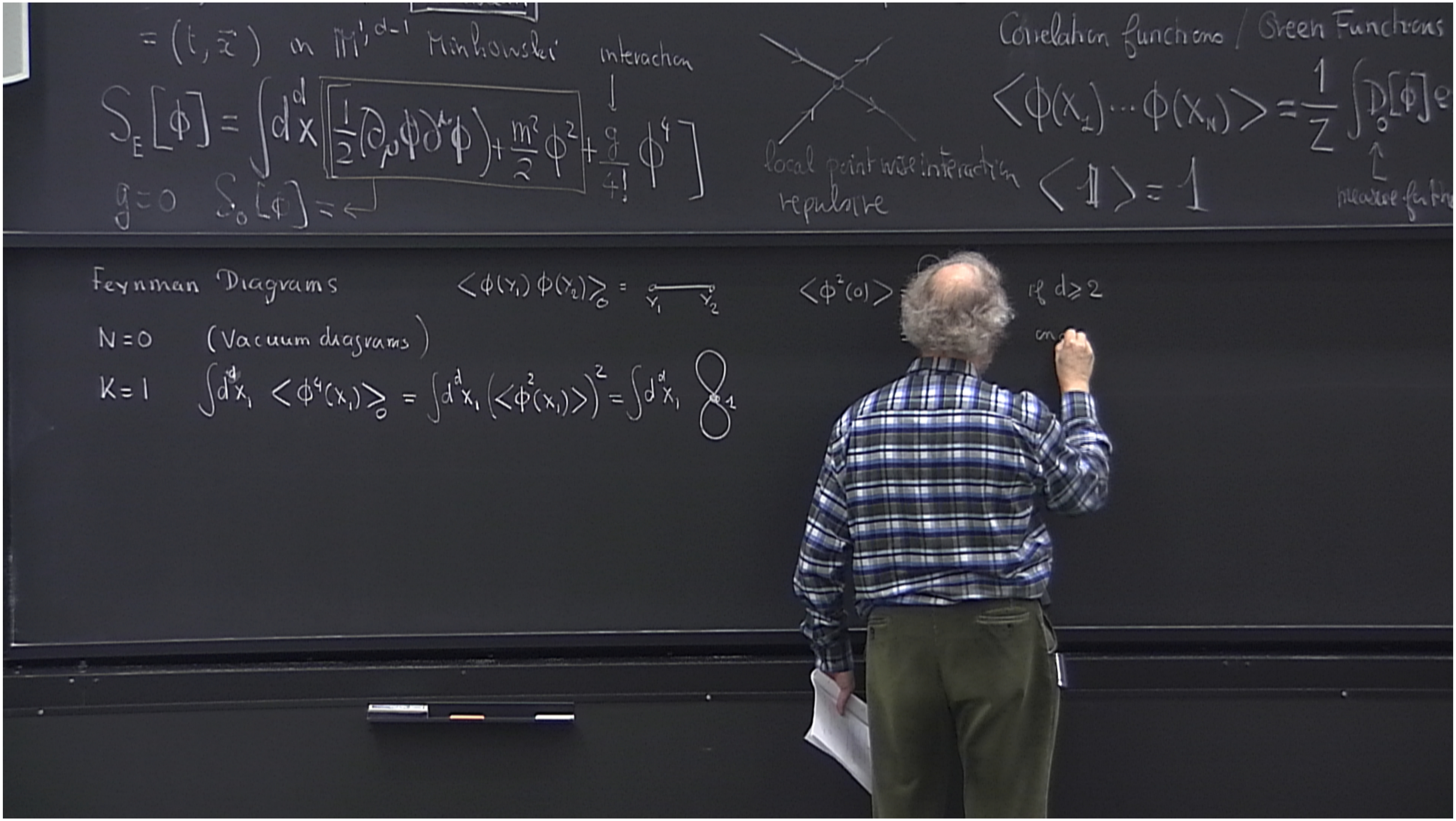

$$\frac{\left[\exp\left(-\frac{1}{\hbar}\frac{g}{4!}\int d^dx\, \phi^4(x)\right)\phi(z_1)\cdots\phi(z_N)\right]}{\left[\cdots\int d^dx\,\phi^4(x)\right]} = \frac{\left\langle \exp\left(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\,\phi^4(x)\right)\cdot\phi(z_1)\cdots\phi(z_N)\right\rangle_0}{\left\langle \exp\left(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\,\phi^4(x)\right)\right\rangle_0}$$
exp. value or correlation for the free theory
$$= \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{-g}{\hbar}\frac{1}{4!}\right)^k \iint d^dx_1\cdots d^dx_k\,\phi^4(x_1)\cdots\phi^4(x_k)$$

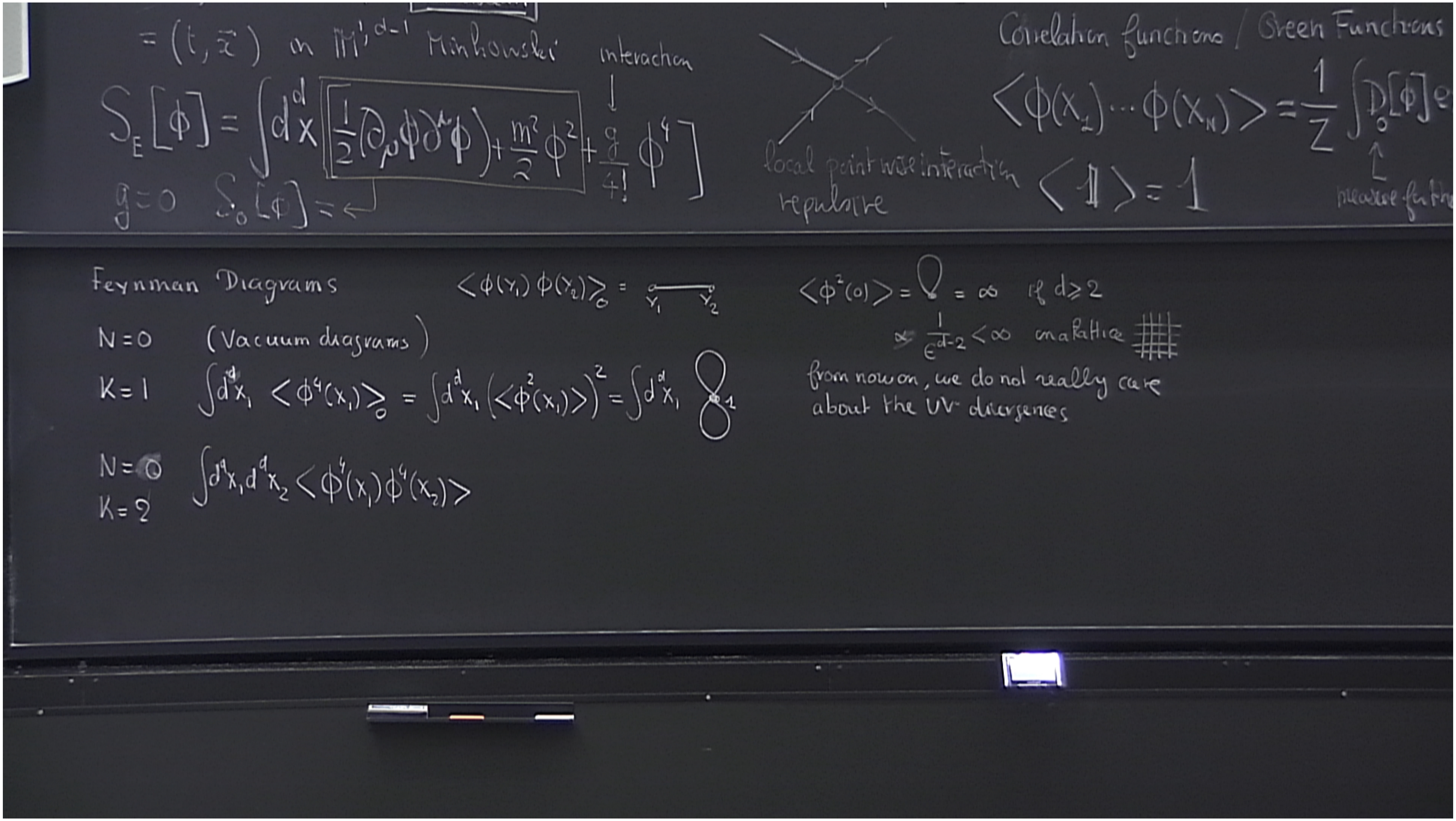

$\langle \phi(z_1) \cdots \phi(z_N) \rangle = \dfrac{\frac{1}{Z_0}\int \mathcal{D}_0[\phi]\, \exp\left(-\frac{1}{\hbar} S_0[\phi]\right) \left[\exp\left(-\frac{1}{\hbar}\frac{g}{4!}\int d^dx\, \phi^4(x)\right) \phi(z_1) \cdots \phi(z_N)\right]}{\frac{1}{Z_0}\int \mathcal{D}_0[\phi]\, \exp\left(-\frac{1}{\hbar} S_0[\phi]\right) \left[\exp\left(-\frac{1}{\hbar}\frac{g}{4!}\int d^dx\, \phi^4(x)\right)\right]} = \dfrac{\langle \exp\left(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi^4(x)\right) \cdot \phi}{\langle \exp\left(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi^4(x)\right) \rangle_0}$
$z_a \in \mathbb{R}^D$
$Z_0 = \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S_0[\phi]\right)$; At this stage: expand : $\exp\left(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi(x)\right) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{\hbar}\frac{1}{4!}\right)^k \int\int d^dx_1 \cdots d^dx_k\, \phi^4(x_1) \cdots \phi^4(x_k)$
and invert $\langle \sum_k \cdots \rangle_0 = \sum_k \langle \cdots \rangle_0$

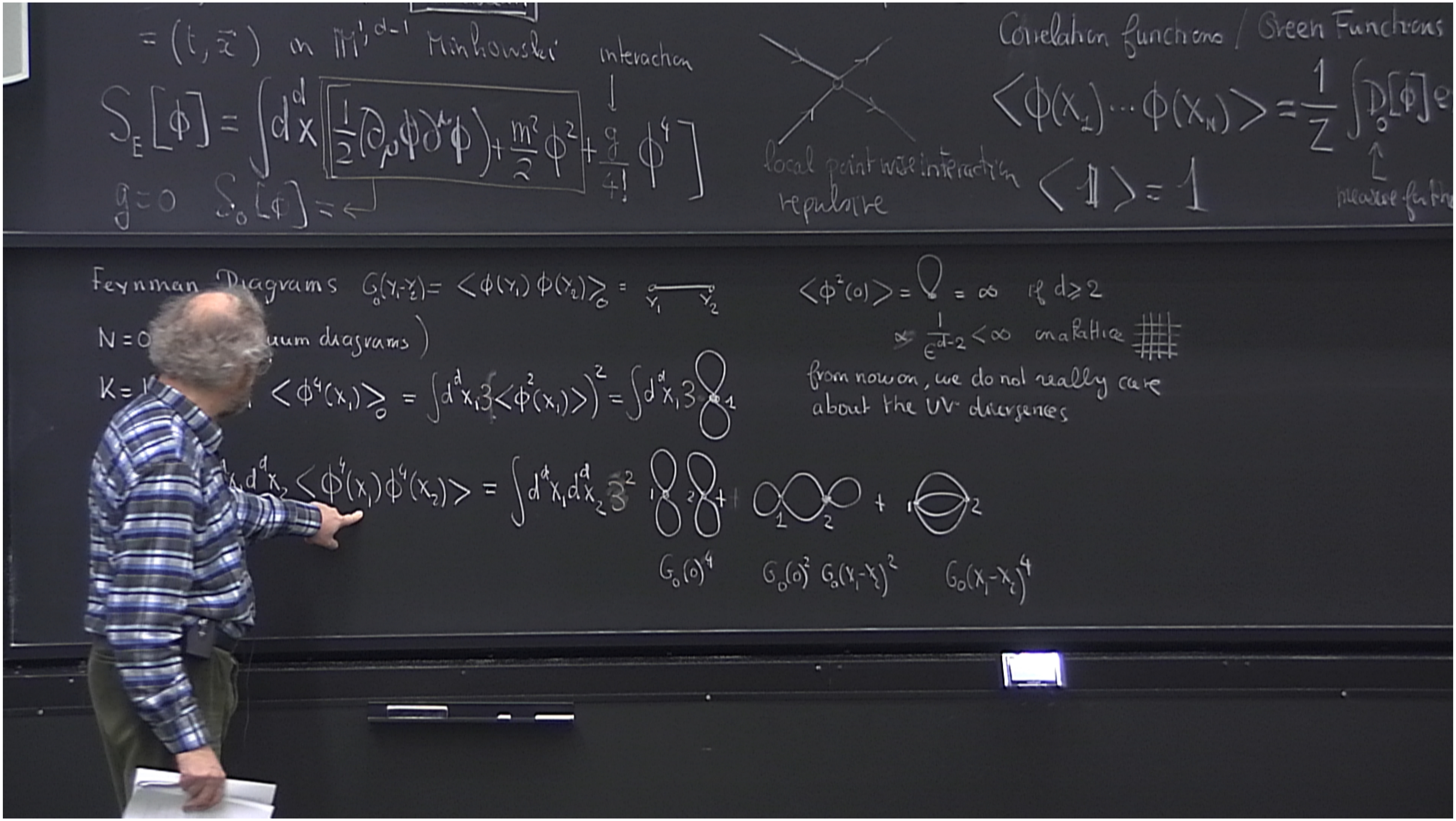


$\langle \phi(z_1) \cdots \phi(z_N) \rangle = \dfrac{\frac{1}{Z_0}\int D_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar}\frac{g}{4!}\int d^dx\, \phi^4(x))\, \phi(z_1) \cdots \phi(z_N)\right]}{\frac{1}{Z_0}\int D_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar}\frac{g}{4!}\int d^dx\, \phi^4(x))\right]} = \dfrac{\langle \exp(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi^4(x)) \cdot \phi \cdots}{\langle \exp(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi^4(x)) \rangle_0}$
$z_a \in \mathbb{R}^D$
$Z_0 = \int D_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$
At this stage: expand : $\exp(-\frac{g}{\hbar}\frac{1}{4!}\int d^dx\, \phi(x)) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(\frac{-g}{\hbar}\frac{1}{4!}\right)^k \iint d^dx_1 \cdots d^dx_k\, \phi^4(x_1) \cdots \phi^4(x_k)$
and invert $\langle \sum_k \cdots \rangle_0 = \sum_k \langle \cdots \rangle_0$
⚠ inverting an ∞ sum and an integral (big) is in general non trivial
this is the case here, even in 0-dim theory
$\phi(0)$ $\langle$ conv. series $\rangle = \sum_k g^k \cdots$ 0 - radius of convergence
asymptotic series


exp(− 1/ħ S₀[ϕ]) [exp(− 1/ħ g/4! ∫dᵈx ϕ⁴(x)) ϕ(z₁) ⋯ ϕ(z_N)] / exp(− 1/ħ S₀[ϕ]) [exp(− 1/ħ g/4! ∫dᵈx ϕ⁴(x))]
= ⟨exp(− g/ħ 1/4! ∫dᵈx ϕ⁴(x)) · ϕ(z₁) ⋯ ϕ(z_N)⟩₀ / ⟨exp(− g/ħ 1/4! ∫dᵈx ϕ⁴(x))⟩₀
exp. value or correla for the free theory
stage: expand : exp(− g/ħ 1/4! ∫dᵈx ϕ(x)) = Σ_{k=0}^∞ 1/k! (−g/ħ 1/4!)^k ∫∫ dᵈx₁ ⋯ dᵈx_k ϕ⁴(x₁) ⋯ ϕ⁴(x_k)
invert ⟨Σ_k ⋯⟩₀ = Σ_k ⟨⋯⟩₀
Numerator = Σ_k 1/k! (−g/ħ 1/4!)^k ∫∫ dᵈx₁ ⋯ dᵈx_k ⟨ϕ⁴(x₁) ⋯ ϕ⁴(x_k) ϕ(z₁) ⋯ ϕ(z_N)⟩₀
Wick Theorem
⟨ϕ(y₁) ϕ(y₂)⟩₀
k internal x's
N external z's
theory
radius of converg
asymptotic series


= (t, x) in M^{1,d-1} Minkowski
interaction
S_E[φ] = ∫d^d x [1/2 (∂_μ φ ∂^μ φ) + m²/2 φ² + g/4! φ⁴]
g=0 S_0[φ] =
local point wise interaction
repulsive
Correlation functions / Green Functions
⟨φ(x_1)⋯φ(x_N)⟩ = 1/Z ∫D_0[φ] e
⟨1⟩ = 1
Feynman Diagrams
⟨φ(y_1) φ(y_2)⟩_0 =
y_1 y_2
⟨φ²(0)⟩
if d≥2
N=0 (Vacuum diagrams)
K=1 ∫d^d x_1 ⟨φ⁴(x_1)⟩_0 = ∫d^d x_1 (⟨φ²(x_1)⟩)² = ∫d^d x_1
1


= (t, x⃗) in M^{1,d-1} Minkowski
interaction
S_E[φ] = ∫d^d x [1/2 (∂_μ φ ∂^μ φ) + m²/2 φ² + g/4! φ⁴]
g = 0 S_0[φ] =
local point wise interaction
repulsive
Correlation functions / Green Functions
⟨φ(x_1) ⋯ φ(x_N)⟩ = 1/Z ∫D_0[φ] e
⟨𝟙⟩ = 1
measure for the
Feynman Diagrams
⟨φ(y_1) φ(y_2)⟩_0 =
y_1 y_2
⟨φ²(0)⟩ = = ∞ if d ≥ 2
∝ 1/ε^{d-2} < ∞ on a lattice
N = 0 (Vacuum diagrams)
K = 1 ∫d^d x_1 ⟨φ⁴(x_1)⟩_0 = ∫d^d x_1 (⟨φ²(x_1)⟩)² = ∫d^d x_1
1
from now on, we do not really care
about the UV- divergences
N = 0 ∫d^d x_1 d^d x_2 ⟨φ⁴(x_1) φ⁴(x_2)⟩
K = 2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g=0 S_0[φ] =
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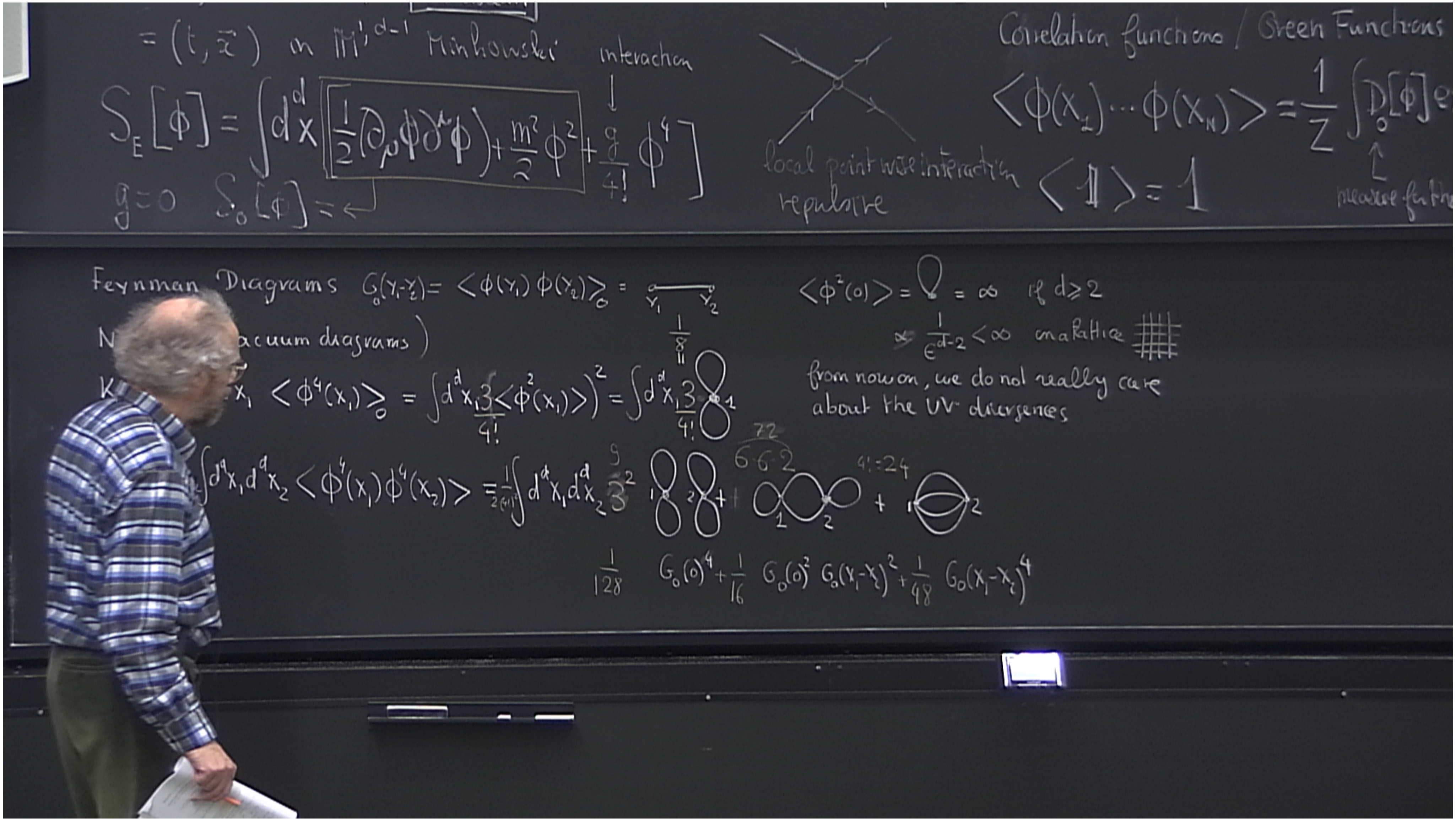
Correlation functions / Green Functions
<φ(x_1)···φ(x_N)> = 1/Z ∫D_0[φ]e
<1> = 1
measure for the
Feynman Diagrams G_0(y_1-y_2) = <φ(y_1) φ(y_2)>_0 =
y_1 y_2
N = 0
uum diagrams)
K = 1
<φ⁴(x_1)>_0 = ∫d^d x_1 3 <φ²(x_1)>² = ∫d^d x_1 3
<φ²(0)> = = ∞ if d≥2
1/ε^{d-2} < ∞ on a lattice
from now on, we do not really care
about the UV divergences
d^d x_2 <φ⁴(x_1) φ⁴(x_2)> = ∫d^d x_1 d^d x_2 3²
G_0(0)⁴
G_0(0)² G_0(x_1-x_2)²
G_0(x_1-x_2)⁴


= (t, x) in M^{1,d-1} Minkowski
interaction
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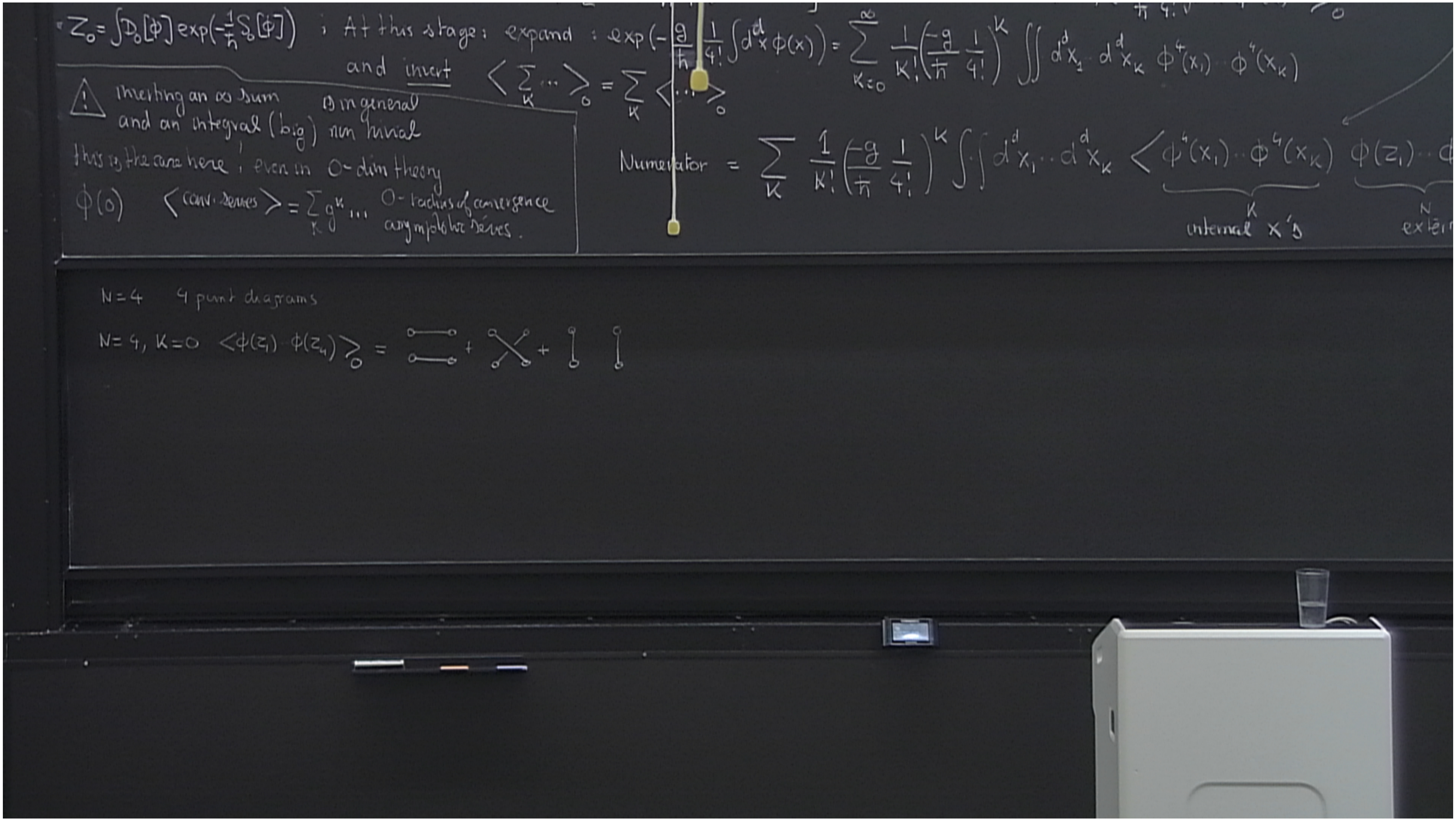
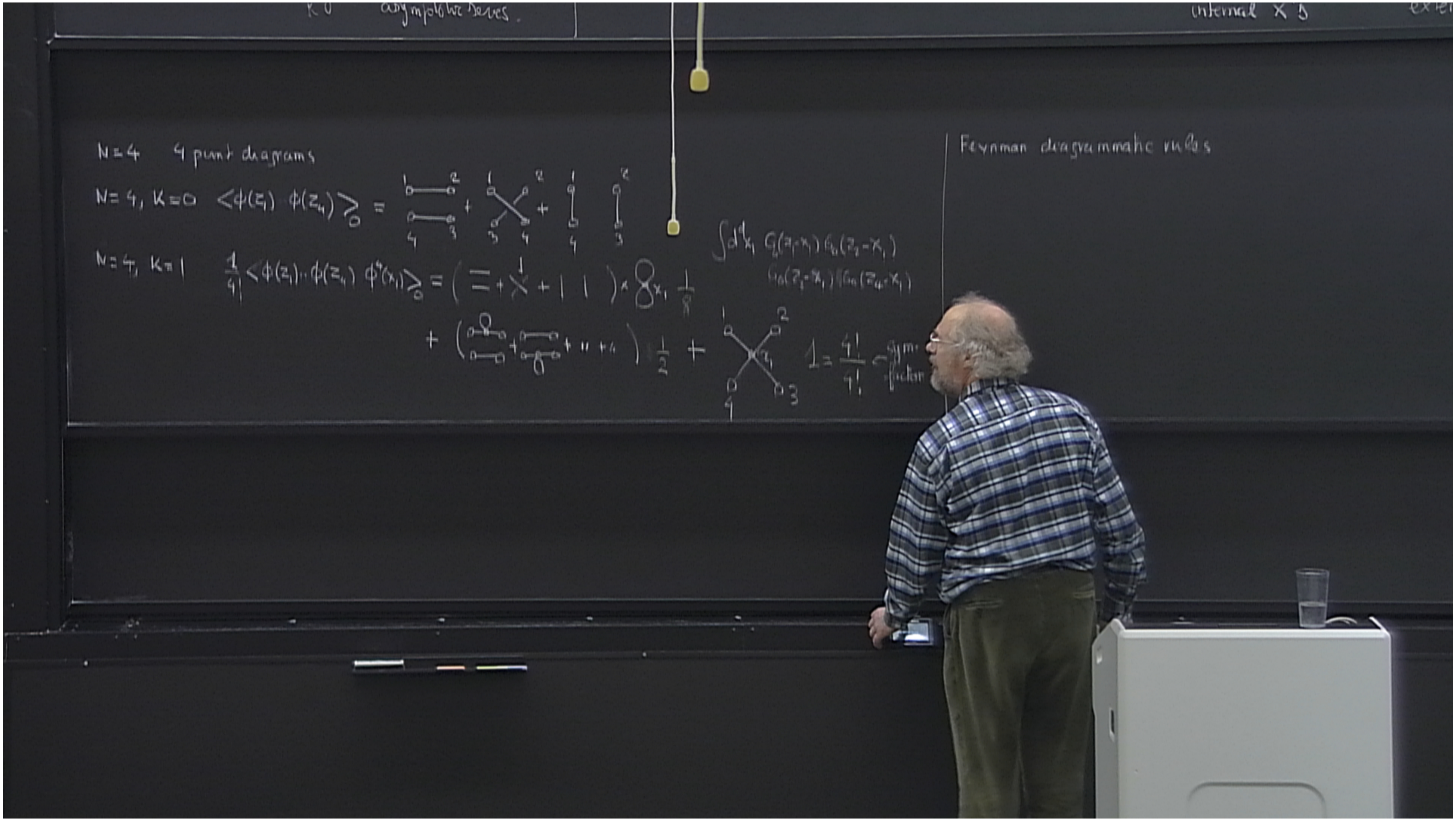
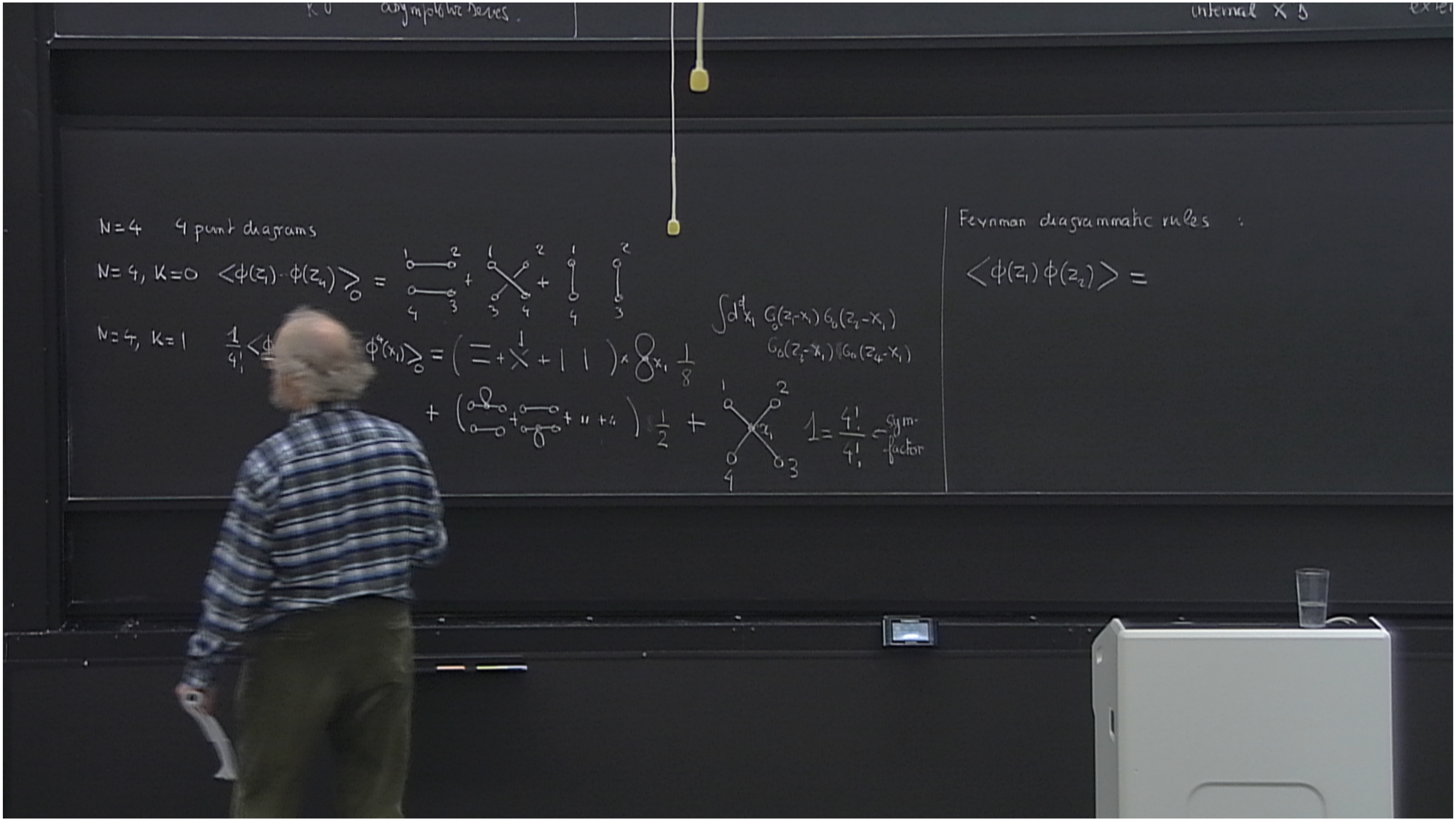
Correlation functions / Green Functions
<φ(x_1) ... φ(x_N)> = 1/Z ∫ D_0[φ] e
<1> = 1
measure for the
Feynman Diagrams G_0(y_1 - y_2) = <φ(y_1) φ(y_2)>_0 =
acuum diagrams)
<φ⁴(x_1)>_0 = ∫ d^d x_1 3/4! <φ²(x_1)>)² = ∫ d^d x_1 3/4!
∫ d^d x_1 d^d x_2 <φ⁴(x_1) φ⁴(x_2)>
<φ²(0)> = = ∞ if d ≥ 2
∝ 1/ε^{d-2} < ∞ on a lattice
from now on, we do not really care
about the UV-divergences
72
6·6·2
4! = 24
1/128 G_0(0)⁴ + 1/16 G_0(0)² G_0(x_1 - x_2)² + 1/48 G_0(x_1 - x_2)⁴

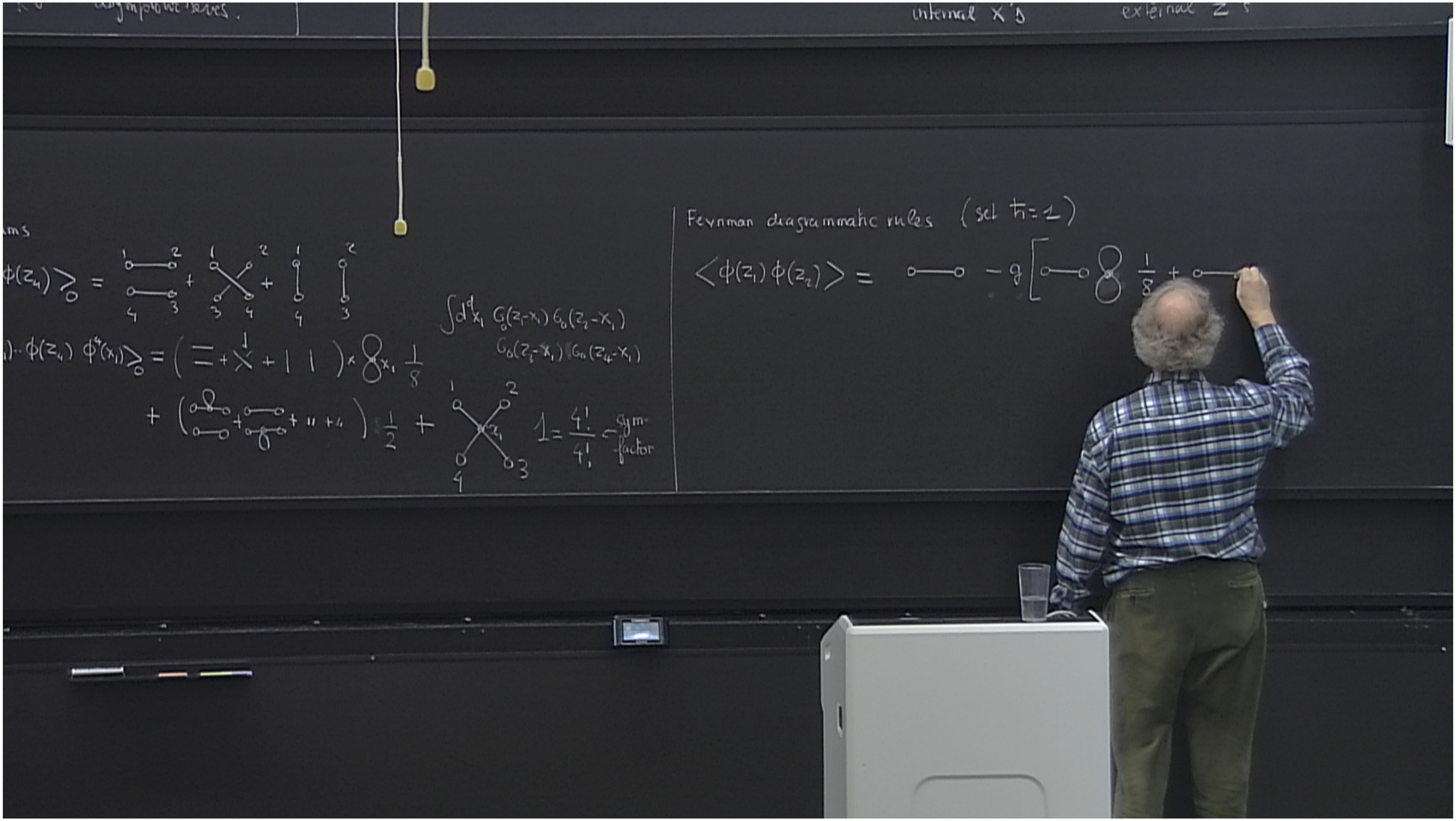


Z0 = ∫D0[φ] exp(-1/ħ S0[φ]) ; At this stage: expand : exp(-g/ħ 1/4! ∫d^dx φ(x)) = Σ_{K=0}^∞ 1/K! (-g/ħ 1/4!)^K ∫∫ d^dx1 .. d^dxK φ^4(x1) .. φ^4(xK)
and invert ⟨Σ_K ...⟩_0 = Σ_K ⟨...⟩_0
inverting an ∞ sum and an integral (big) is in general non trivial
this is the case here! even in 0-dim theory
φ(0) ⟨conv. series⟩ = Σ_K g^K ... 0 - radius of convergence asymptotic series.
Numerator = Σ_K 1/K! (-g/ħ 1/4!)^K ∫∫ d^dx1 .. d^dxK ⟨φ^4(x1) .. φ^4(xK) φ(z1) ..
internal x's
exter
N = 4 4 point diagrams
N = 4, K = 0 ⟨φ(z1) φ(z4)⟩_0 =


N=4 4 point diagrams
Feynman diagrammatic rules
internal



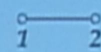
asymptotic series
internal x's
N=4 4 point diagrams
N=4, K=0 ⟨φ(z1)·φ(z4)⟩0 =
N=4, K=1
Feynman diagrammatic rules :
⟨φ(z1) φ(z2)⟩ =
sym-factor


internal x's
external z's
Feynman diagrammatic rules (set ℏ = 1)
sym-factor

2 points diagrams

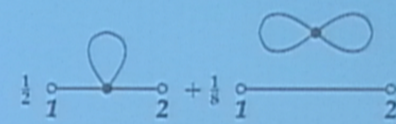
$N = 2,\ K = 0$

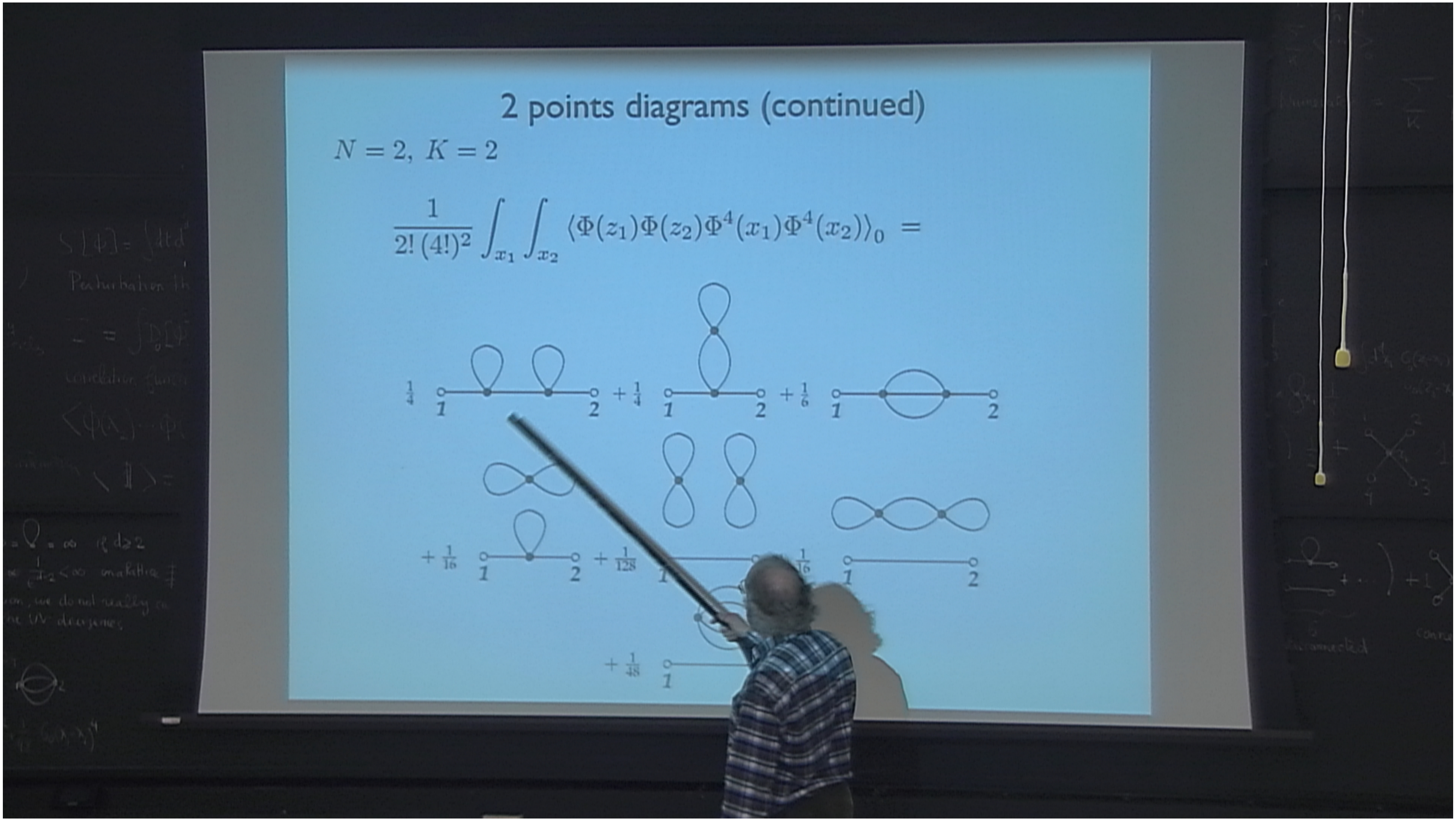
$$\langle \Phi(z_1)\Phi(z_2) \rangle_0 =$$

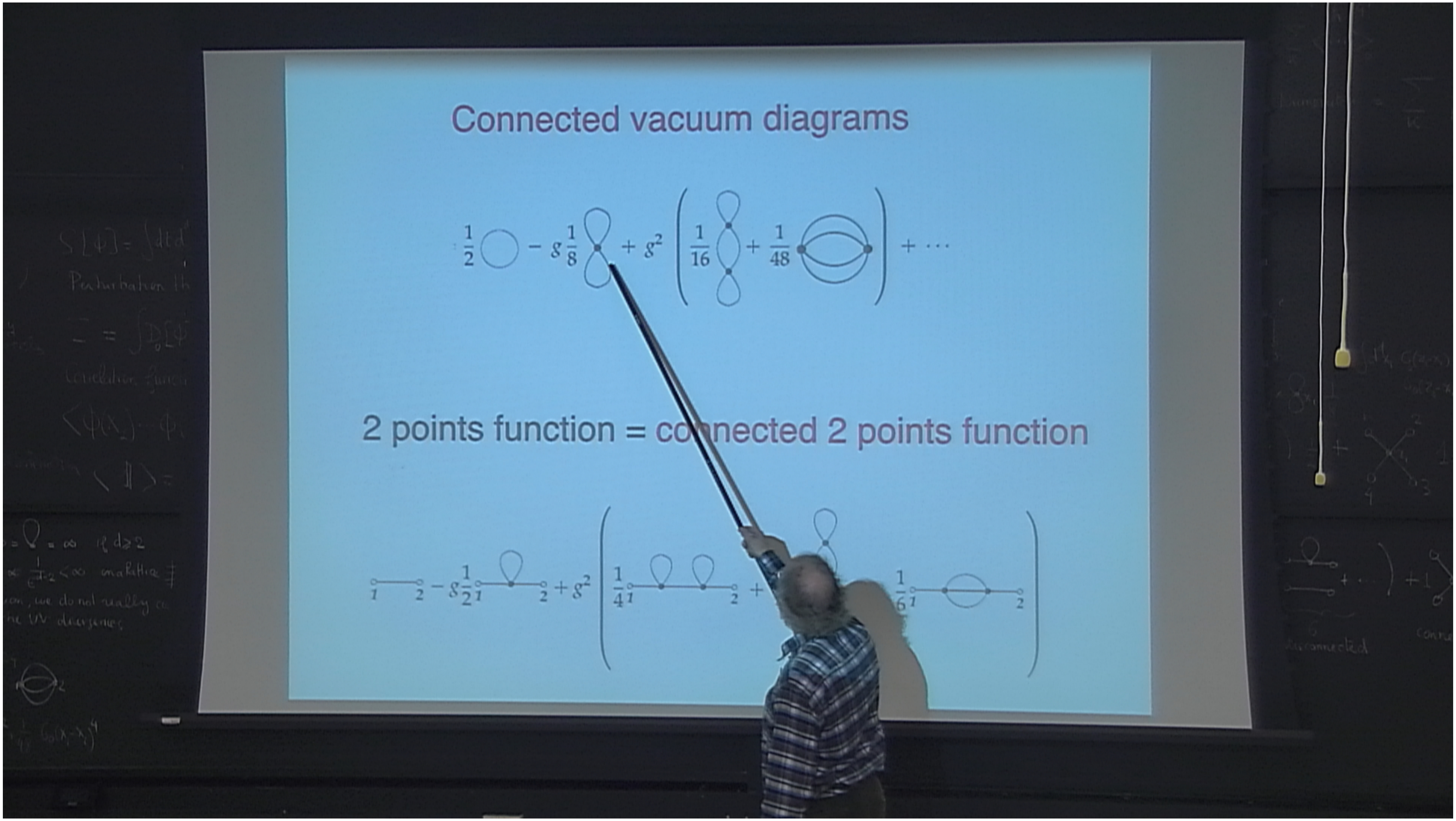


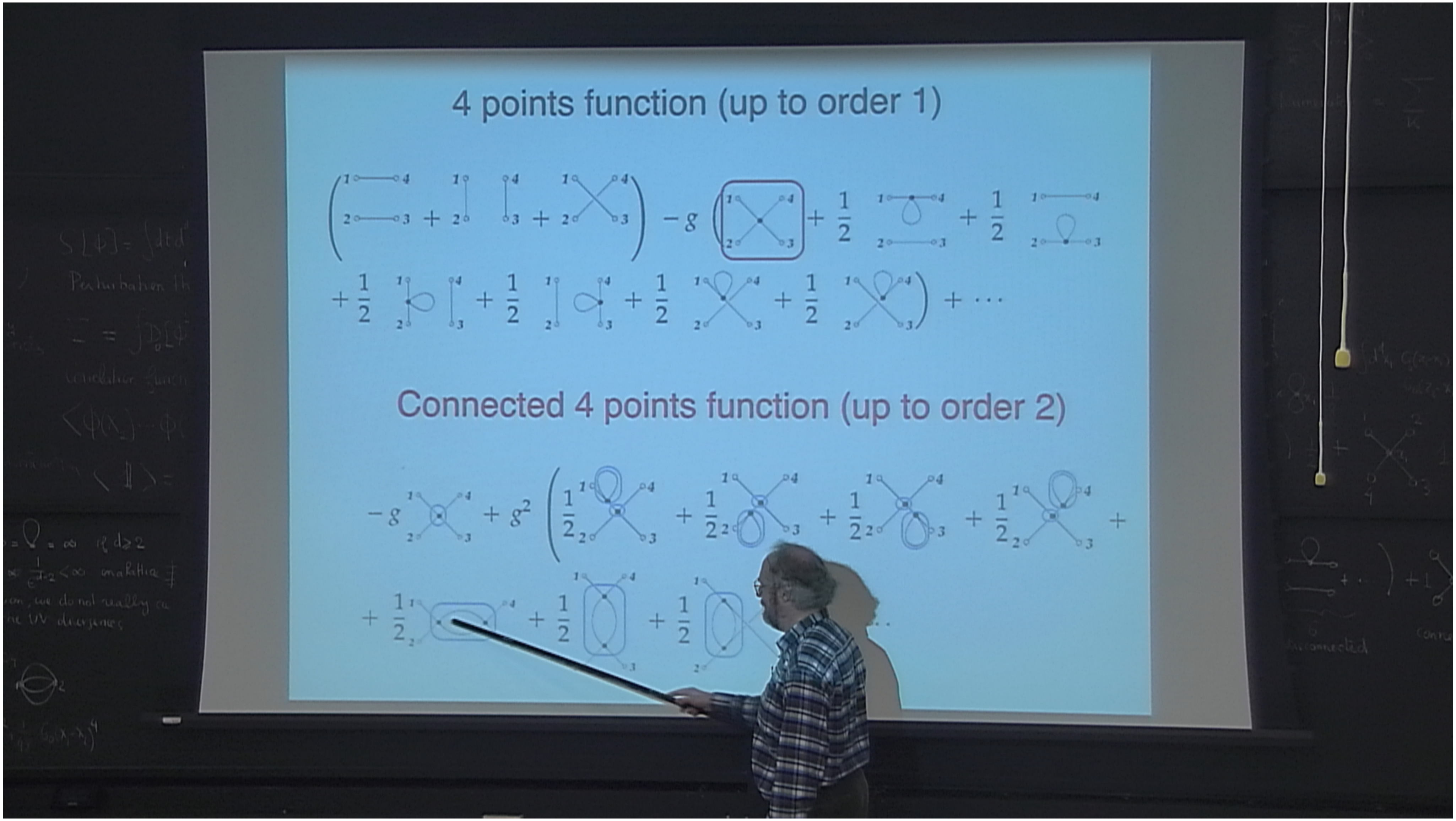
$N = 2,\ K = 1$

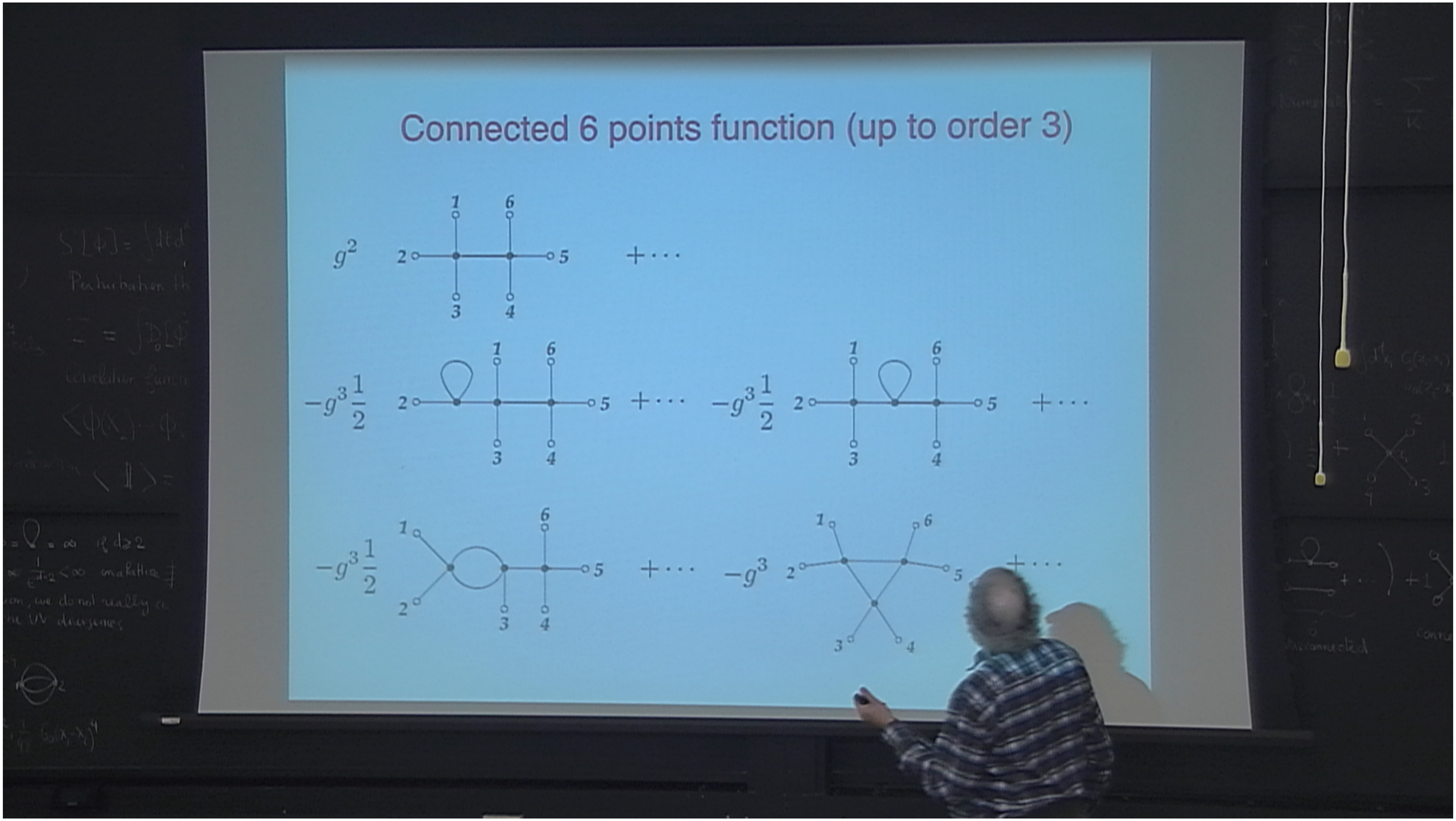
$$\frac{1}{4!}\int_{x_1} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1) \rangle_0 =$$

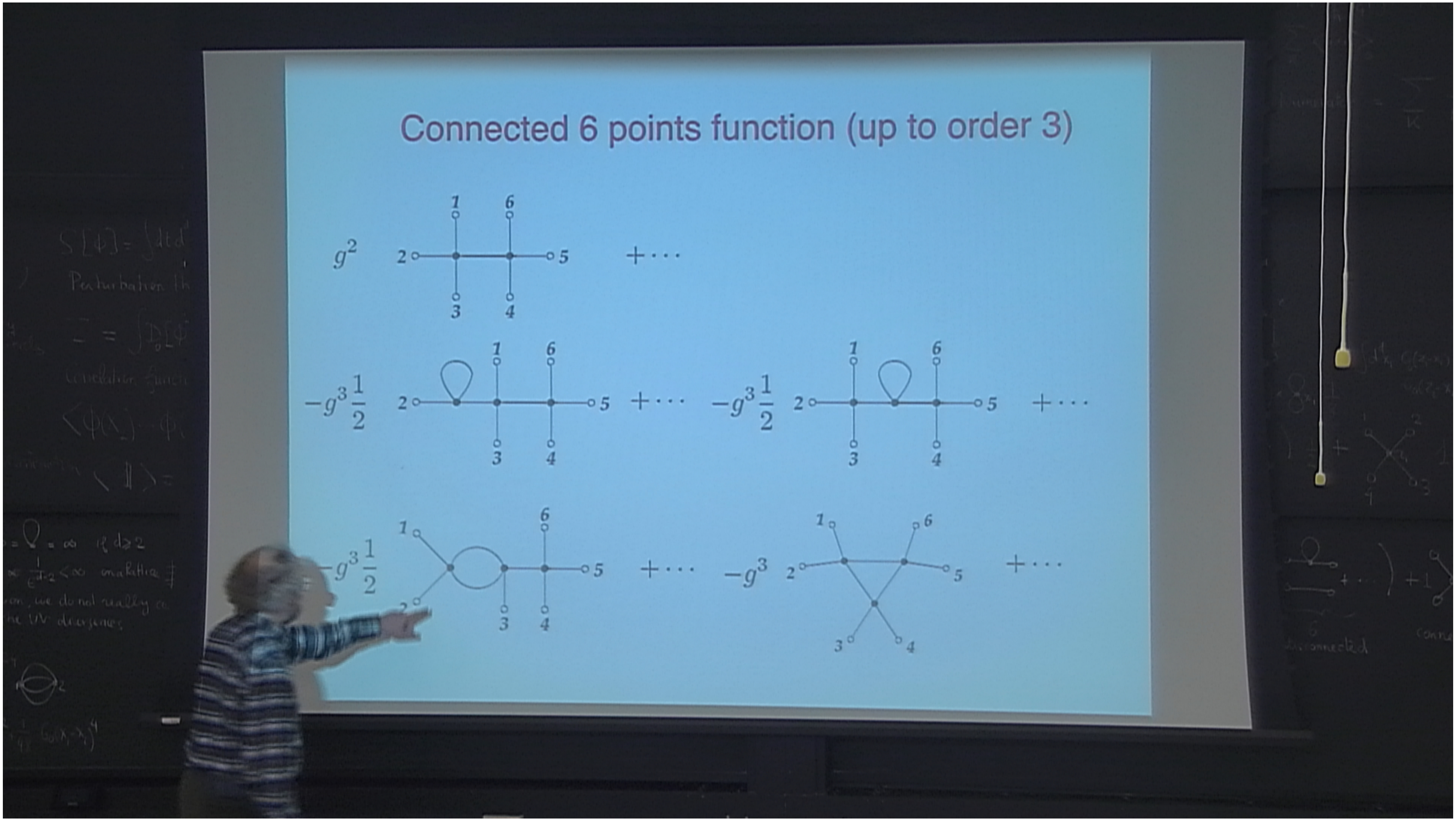



2 points diagrams (continued)
$N = 2,\ K = 2$
$\frac{1}{2!\,(4!)^2} \int_{x_1} \int_{x_2} \langle \Phi(z_1)\Phi(z_2)\Phi^4(x_1)\Phi^4(x_2) \rangle_0 =$
$\frac{1}{4}$
$+\frac{1}{4}$
$+\frac{1}{6}$
$+\frac{1}{16}$
$+\frac{1}{128}$
$\frac{1}{16}$
$+\frac{1}{48}$

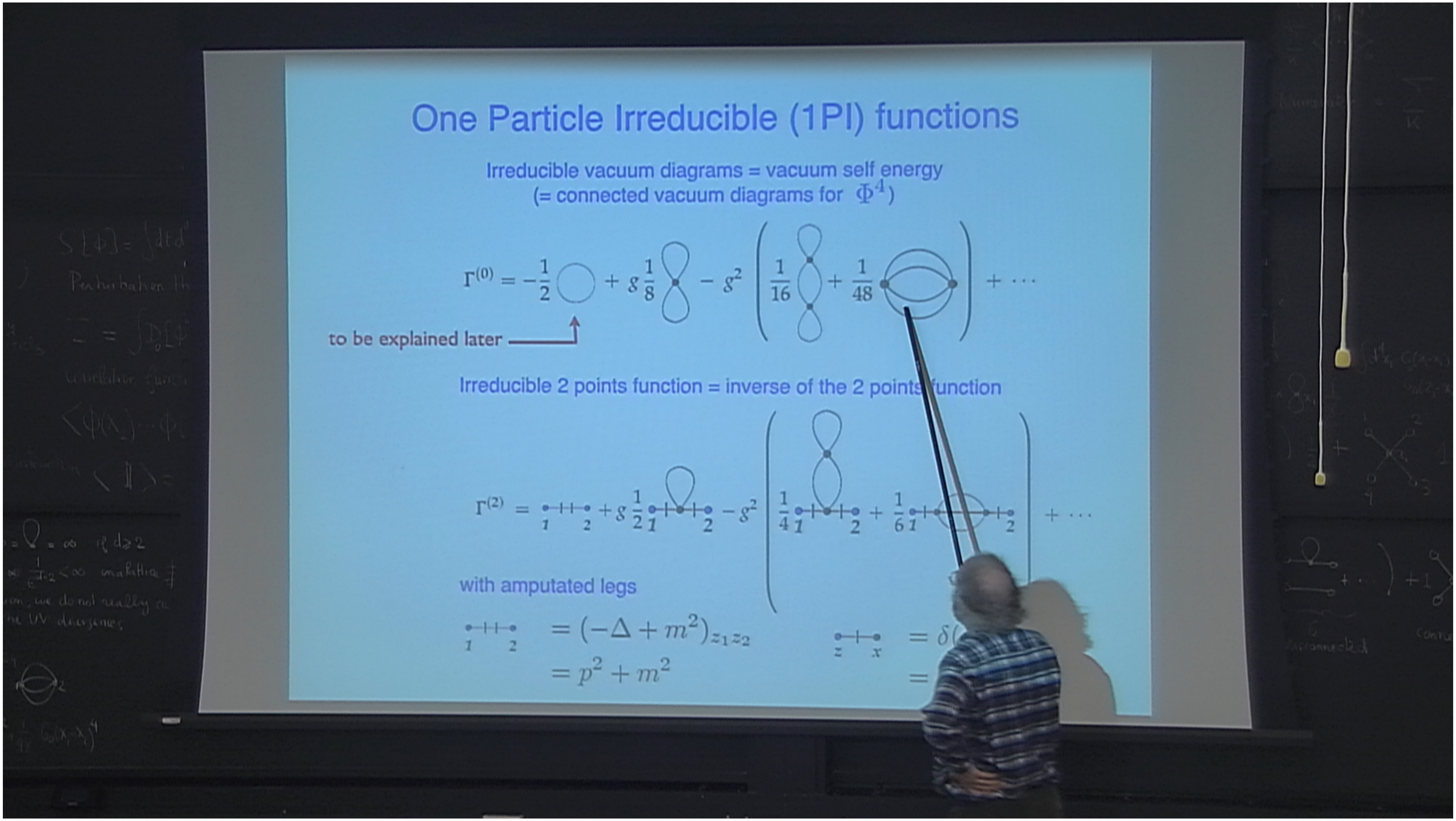

Connected vacuum diagrams
2 points function = connected 2 points function

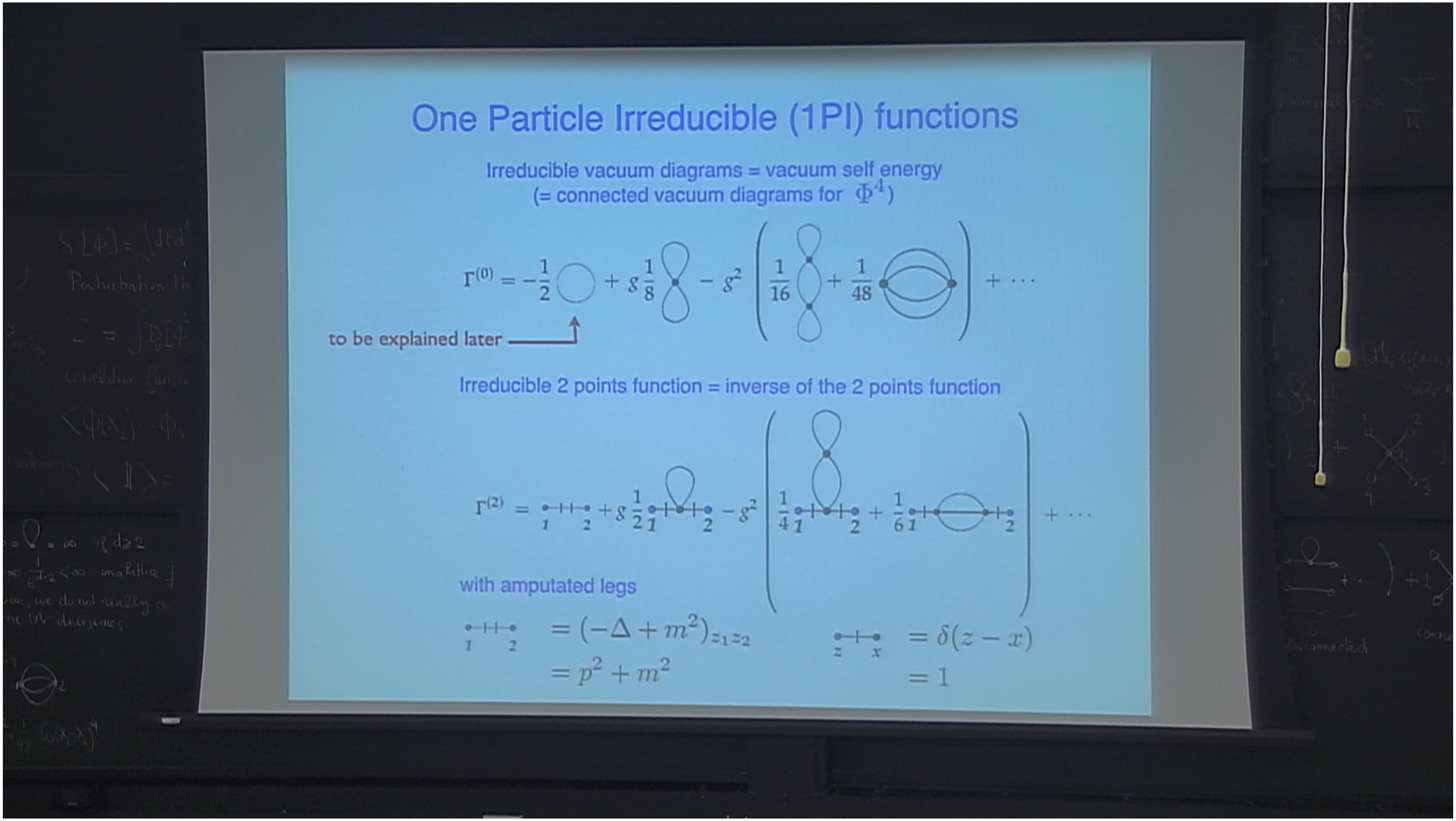

4 points function (up to order 1)
Connected 4 points function (up to order 2)


Connected 6 points function (up to order 3)
g^2
+ · · ·
−g^3 1/2
+ · · ·
−g^3 1/2
+ · · ·
−g^3 1/2
+ · · ·
−g^3
+ · · ·



Connected 6 points function (up to order 3)
g^2
$+\cdots$
$-g^3\frac{1}{2}$
$+\cdots$
$-g^3\frac{1}{2}$
$+\cdots$
$-g^3\frac{1}{2}$
$+\cdots$
$-g^3$
$+\cdots$


One Particle Irreducible (1PI) functions
Irreducible vacuum diagrams = vacuum self energy
(= connected vacuum diagrams for Φ^4)
$\Gamma^{(0)} = -\frac{1}{2}$ + $g\frac{1}{8}$ − g^2 ($\frac{1}{16}$ + $\frac{1}{48}$) + ⋯
to be explained later
Irreducible 2 points function = inverse of the 2 points function
$\Gamma^{(2)}$ = + $g\frac{1}{2}$ − g^2 ($\frac{1}{4}$ + $\frac{1}{6}$) + ⋯
with amputated legs
$= (-\Delta + m^2)_{z_1 z_2}$
$= p^2 + m^2$


One Particle Irreducible (1PI) functions
Irreducible vacuum diagrams = vacuum self energy
(= connected vacuum diagrams for Φ^4)
Γ^(0) = −1/2 + g 1/8 − g^2 (1/16 + 1/48) + ···
to be explained later
Irreducible 2 points function = inverse of the 2 points function
Γ^(2) = + g 1/2 − g^2 (1/4 + 1/6) + ···
with amputated legs
= (−Δ + m^2)_{z1 z2}
= p^2 + m^2
= δ(z − x)
= 1