

Title: 14/15 PSI - Quantum Field Theory II-Lecture 4

Date: Nov 13, 2014 09:00 AM

URL: <http://pirsa.org/14110009>

Abstract:

Functional Integration Quantization.

Scalar ϕ^4 theory \leftrightarrow (Landau-Ginzburg-Wilson.)

$\phi(x)$ real

$X = X$

Functional Integration Quantization

$$S[\phi] = \int dt d\vec{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \vec{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

Scalar ϕ^4 theory \leftrightarrow (Landau-Ginzburg-Wilson)

$\phi(x)$ real

$X = (x^0, \vec{x})$ in D dimensions

$= (t, \vec{x})$

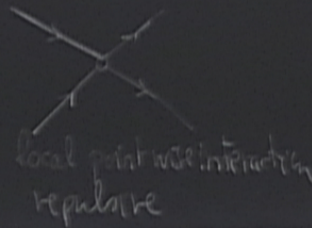
$g \geq 0$ coupling constant

Free theory massive particles

inhomogeneous interaction

$$S_E[\phi] = \int dt d\vec{x} \left[\frac{1}{2} \dot{\phi}^2 + \frac{1}{2} (\nabla \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0$ S



Functional Integration Quantization

Scalar ϕ^4 theory \leftrightarrow (Landau-Ginzburg-Wilson)

$\phi(x)$ real

$X = (x^0, \vec{x}^i)$

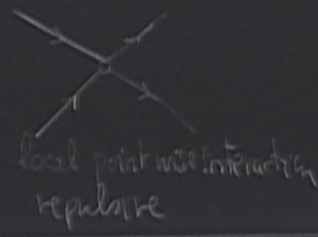
$= (t, \vec{x})$

$$S_E[\phi] = \int d^4x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

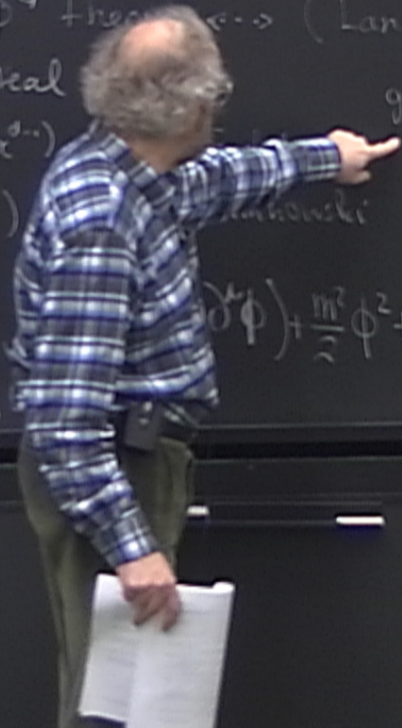
$g=0$ S_0

$g \geq 0$ coupling constant Free theory massive particles

renormalizable interaction



$$S[\phi] = \int dt d^3\vec{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \vec{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$



Functional Integration Quantization

Scalar ϕ^4 theory \leftrightarrow (Landau-Ginzburg-Wilson)

$\phi(x)$ real

$X = (x^0, \dots, x^{d-1})$ in \mathbb{R}^d Euclidean

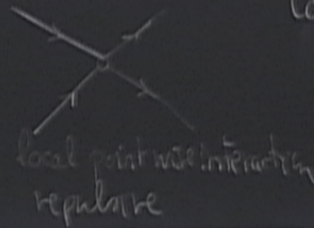
$= (t, \vec{x})$ in $M^{d-1,1}$ Minkowski interaction

$g \geq 0$ coupling constant

Free theory massive particles

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0 \rightarrow S_0[\phi]$

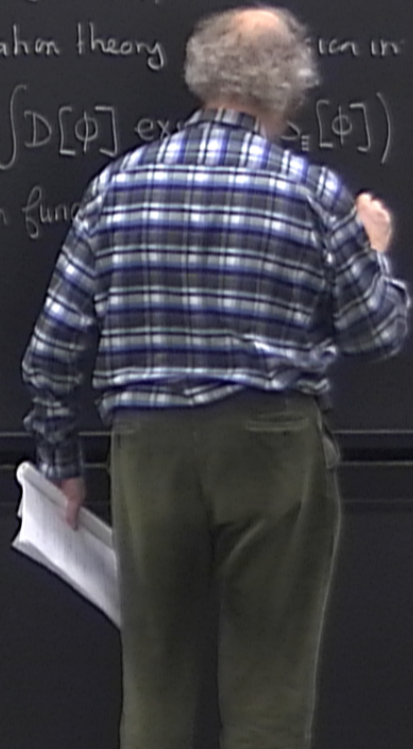


$$S[\phi] = \int dt d\vec{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \vec{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

Perturbation theory valid in $g \ll 1$

$$Z = \int D[\phi] \exp(i S[\phi])$$

Correlation functions



on Quantization.

↔ (Landau-Ginzburg-Wilson.)

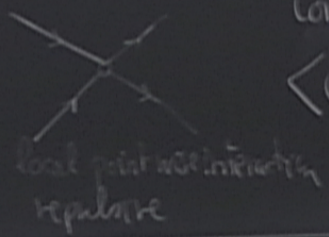
Euclidean

$g \geq 0$ coupling constant

Free theory massive particles

Polynomial interaction

$$\left[\phi \partial^2 \phi + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$



$$S[\phi] = \int dt d^{d-1} \bar{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \bar{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

Perturbation theory expansion in $g \ll 1$

$$Z = \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S_{\text{int}}[\phi]\right)$$

Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1) \dots \phi(x_N)$$

\uparrow
measure for the free theory

$L \rightarrow \infty$
 $\epsilon \rightarrow 0$

$$\langle 1 \rangle = 1$$

$\mathbb{R}^d \rightarrow \mathbb{Z}^d$ with a mesh ϵ
UV regulator

$\mathbb{R}^d \rightarrow$ Finite T () of size L
IR regulator

on Quantization.

↔ (Landau-Ginzburg-Wilson.)

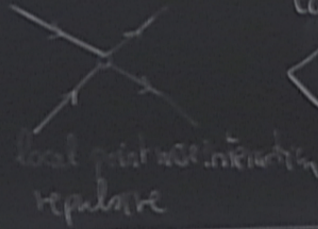
Euclidean

$g \geq 0$ coupling constant

Free theory massive particles

Polynomial interaction

$$\left[\phi \partial^2 \phi + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$



$$S[\phi] = \int dt d^d \vec{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \vec{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

Perturbation theory expansion in $g \ll 1$


$$Z = \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S_{\text{int}}[\phi]\right)$$

Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1) \dots \phi(x_N)$$

measure for the free theory

$$\langle 1 \rangle = 1$$

$\mathbb{R}^d \rightarrow \mathbb{Z}^d$ with a mesh 
UV regulator

$\mathbb{R}^d \rightarrow$ Finite Torus (d dim) of size L
IR regulator



$L \rightarrow \infty$
 $\epsilon \rightarrow 0$

on Quantization.

↔ (Landau-Ginzburg-Wilson.)

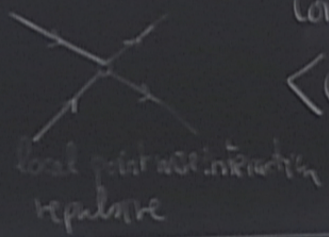
Euclidean

$g \geq 0$ coupling constant

Free theory massive particles

Minkowski interaction

$$\left[\phi \partial_t \phi + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$



$$S[\phi] = \int dt d^{d-1} \bar{x} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 - \frac{1}{2} \left(\frac{\partial \phi}{\partial \bar{x}} \right)^2 - \frac{m^2}{2} \phi^2 - \frac{g}{4!} \phi^4 \right]$$

Perturbation theory expansion in $g \ll 1$

$$Z = \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S_{\text{int}}[\phi]\right)$$

Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int \mathcal{D}_0[\phi] \exp\left(-\frac{1}{\hbar} S[\phi]\right) \phi(x_1) \dots \phi(x_N)$$

$$\langle 1 \rangle = 1$$

measure for the free theory (ansatz)

$\mathbb{R}^d \rightarrow \mathbb{Z}^d$ with a mesh ϵ

UV regulator

$\mathbb{R}^d \rightarrow$ Finite Torus (of dim) of size L

IR regulator



$L \rightarrow \infty$
 $\epsilon \rightarrow 0$

Functional Integration Quantization.

Scalar ϕ^4 theory \leftrightarrow (Landau-Ginzburg-Wilson.)

$\phi(x)$ real

$X = (x^0, \dots, x^{d-1})$ in \mathbb{R}^d Euclidean

$= (t, \vec{x})$ in $\mathbb{M}^{1,d-1}$ Minkowski

$g \geq 0$ coupling constant

Free theory massive particles

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0$ $S_0[\phi] = \leftarrow$

interaction



local pointwise interaction
repulsive

$S[\phi]$

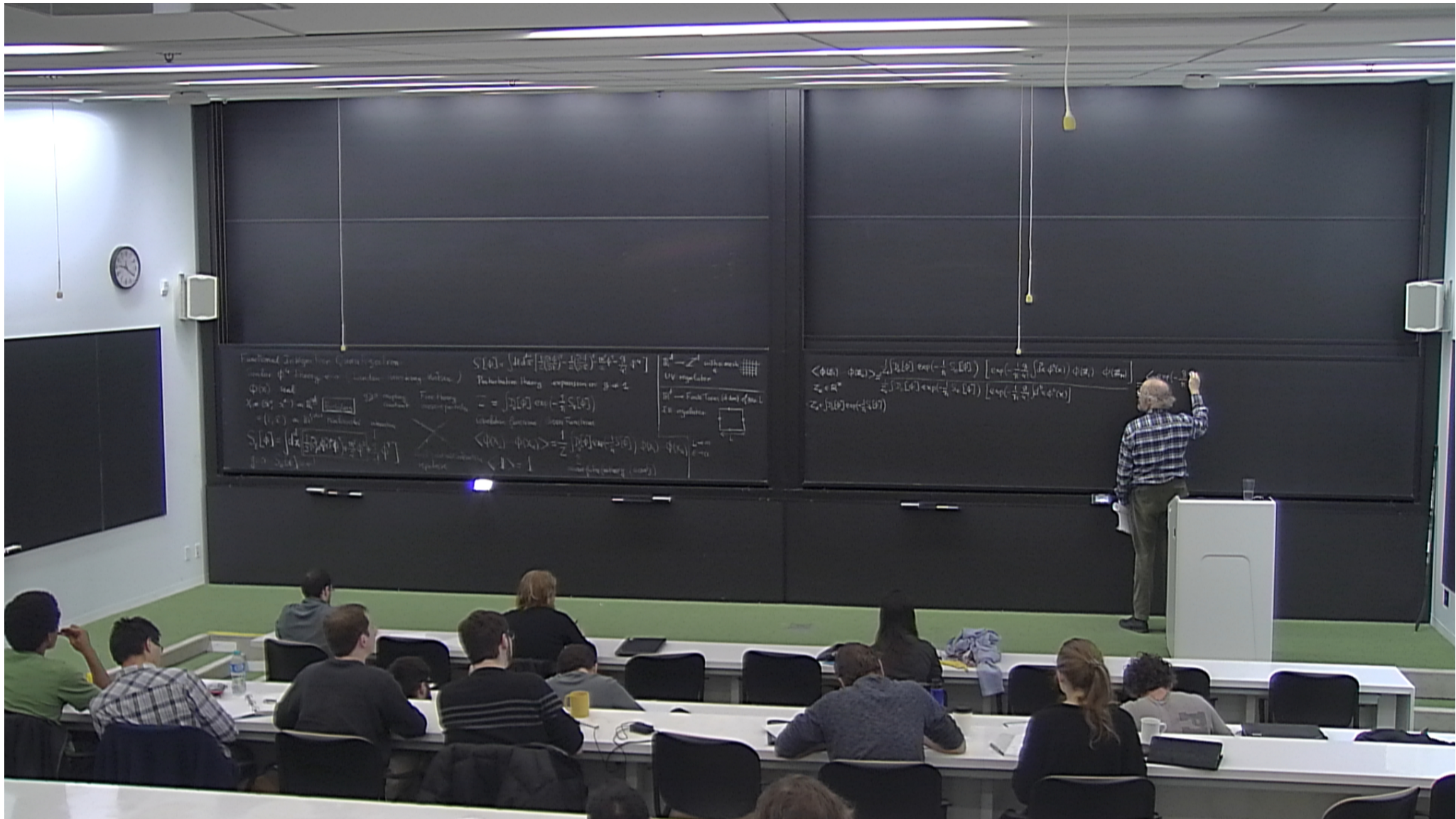
Perturb

$Z =$

Correlat

$\langle \phi(x)$

\langle



$$\left[\frac{\exp\left(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)\right) \phi(z_1) \cdots \phi(z_N)}{\exp\left(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)\right)} \right] = \frac{\langle \exp\left(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)\right) \cdot \phi(z_1) \cdots \phi(z_N) \rangle_0}{\langle \exp\left(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)\right) \rangle_0} \leftarrow \begin{array}{l} \text{exp. value or correlation} \\ \text{for the free theory} \end{array}$$

$$\exp\left(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)\right) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{-g}{\hbar} \frac{1}{4!}\right)^k \iint d^d x_1 \cdots d^d x_k \phi^4(x_1) \cdots \phi^4(x_k)$$

$$\langle \phi(z_1) \dots \phi(z_n) \rangle_{z_a \in \mathbb{R}^D} = \frac{\int_{z_0} \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \phi(z_1) \dots \phi(z_n) \right]}{\int_{z_0} \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \right]} = \frac{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \cdot \phi \rangle_0}{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \rangle_0}$$

$z_0 = \int \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$; At this stage: expand $\exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{g}{\hbar} \frac{1}{4!}\right)^k \int \int d^d x_1 \dots d^d x_k \phi^4(x_1) \dots \phi^4(x_k)$
 and invert $\langle \sum_k \dots \rangle_0 = \sum_k \langle \dots \rangle_0$

$$\langle \phi(z_1) \dots \phi(z_n) \rangle = \frac{\int_{Z_0} \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \phi(z_1) \dots \phi(z_n) \right]}{\int_{Z_0} \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \right]} = \frac{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \cdot \phi(z_1) \dots \phi(z_n) \rangle_0}{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \rangle_0}$$

$Z_0 = \int \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$; At this stage: expand $\exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{g}{\hbar} \frac{1}{4!}\right)^k \int \int d^d x_1 \dots d^d x_k \phi^4(x_1) \dots \phi^4(x_k)$
 and invert $\langle \sum_k \dots \rangle_0 = \sum_k \langle \dots \rangle_0$

! inserting an ∞ sum and an integral (big) is in general non trivial

this is the case here, even in 0-dim theory

$\phi(0) \langle \text{conv. series} \rangle = \sum_k g^k \dots$ 0-radius of convergence asymptotic series

$$\frac{\exp(-\frac{1}{\hbar} S_0[\Phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \phi(z_1) \cdots \phi(z_N) \right]}{\exp(-\frac{1}{\hbar} S_0[\Phi]) \left[\exp(-\frac{1}{\hbar} \frac{g}{4!} \int d^d x \phi^4(x)) \right]} = \frac{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \cdot \phi(z_1) \cdots \phi(z_N) \rangle_0}{\langle \exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) \rangle_0} \leftarrow \begin{array}{l} \text{exp. value or correla} \\ \text{for the free theory} \end{array}$$

this stage: expand: $\exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi^4(x)) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(-\frac{g}{\hbar} \frac{1}{4!} \right)^k \int d^d x_1 \cdots d^d x_k \phi^4(x_1) \cdots \phi^4(x_k)$

invert $\langle \sum_k \dots \rangle_0 = \sum_k \langle \dots \rangle_0$

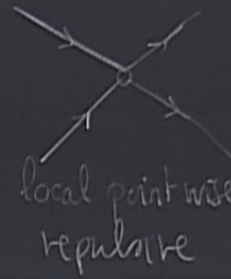
Numerator = $\sum_k \frac{1}{k!} \left(-\frac{g}{\hbar} \frac{1}{4!} \right)^k \int d^d x_1 \cdots d^d x_k \underbrace{\langle \phi^4(x_1) \cdots \phi^4(x_k) \rangle_0}_{\text{internal } x\text{'s}} \underbrace{\langle \phi(z_1) \cdots \phi(z_N) \rangle_0}_{\text{external } z\text{'s}}$

\leftarrow Wick Theorem $\langle \phi(x_1) \phi(x_2) \rangle_0$

$x = (t, \vec{x})$ in $M^{1,d-1}$ Minkowski interaction

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0 \quad S_0[\phi] = \leftarrow$



Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_n) \rangle = \frac{1}{Z} \int \mathcal{D}[\phi] e^{-S_E[\phi]}$$

$\langle 1 \rangle = 1$ ↑
measure factor

Feynman Diagrams

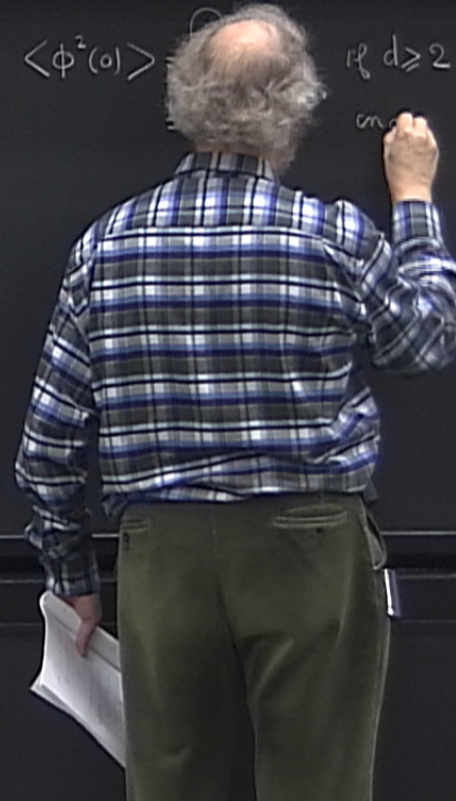
$$\langle \phi(x_1) \phi(x_2) \rangle_0 = \text{diagram of a line between } x_1 \text{ and } x_2$$

$$\langle \phi^2(x) \rangle$$

if $d \geq 2$

$N=0$ (Vacuum diagrams)

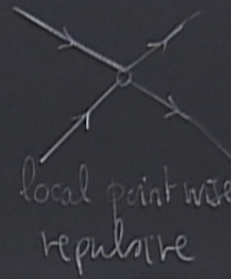
$$K=1 \quad \int d^d x_1 \langle \phi^4(x_1) \rangle_0 = \int d^d x_1 (\langle \phi^2(x_1) \rangle)^2 = \int d^d x_1 \text{diagram of two lines meeting at a point}$$



$= (t, \vec{x})$ in $M^{1,d-1}$ Minkowski interaction

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0$ $S_0[\phi] = \leftarrow$



Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int \mathcal{D}[\phi] e^{-S_E[\phi]}$$

$\langle 1 \rangle = 1$ (normalization factor)

Feynman Diagrams

$$\langle \phi(x_1) \phi(x_2) \rangle_0 = \text{diagram of a line between } x_1 \text{ and } x_2$$

$N=0$ (Vacuum diagrams)

$$K=1 \quad \int d^d x_1 \langle \phi^4(x_1) \rangle_0 = \int d^d x_1 (\langle \phi^2(x_1) \rangle)^2 = \int d^d x_1 \text{diagram of two lines meeting at a point}$$

$$N=0, K=2 \quad \int d^d x_1 d^d x_2 \langle \phi^4(x_1) \phi^4(x_2) \rangle$$

$$\langle \phi^2(0) \rangle = \text{diagram of a loop} = \infty \text{ if } d \geq 2$$

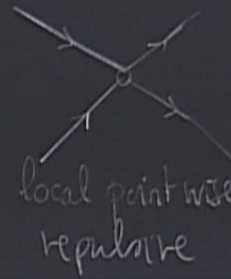
$$\propto \frac{1}{\epsilon^{d-2}} < \infty \text{ makes sense}$$

from now on, we do not really care about the UV divergences

(t, \vec{x}) in $M^{1,d-1}$ Minkowski interaction

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0 \quad S_0[\phi] = \leftarrow$



Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int_0^\infty [D\phi] e^{-S_E[\phi]}$$

$\langle 1 \rangle = 1$ (measure factor)

Feynman Diagrams $G_0(x_1, x_2) = \langle \phi(x_1) \phi(x_2) \rangle_0 = \overline{x_1} \overline{x_2}$

$N=0$ (sum diagrams)

$$K=1 \quad \langle \phi^4(x_1) \rangle_0 = \int d^d x_1 \langle \phi^2(x_1) \rangle_0^2 = \int d^d x_1 \text{ (diagram with two loops)}$$

$$\int d^d x_1 d^d x_2 \langle \phi^4(x_1) \phi^4(x_2) \rangle_0 = \int d^d x_1 d^d x_2 \left[\text{diagram 1} + \text{diagram 2} + \text{diagram 3} \right]$$

$G_0(0)^4 \quad G_0(0)^2 G_0(x_1-x_2)^2 \quad G_0(x_1-x_2)^4$

$$\langle \phi^2(0) \rangle = \text{loop} = \infty \quad \text{if } d \geq 2$$

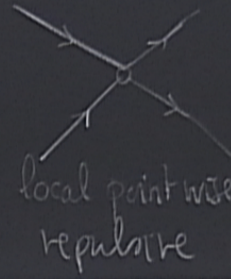
$$\propto \frac{1}{\epsilon^{d-2}} < \infty \quad \text{marginally } \#$$

from now on, we do not really care about the UV divergences

$= (t, \vec{x})$ in $M^{1,d-1}$ Minkowski interaction

$$S_E[\phi] = \int d^d x \left[\frac{1}{2} (\partial_\mu \phi)^2 + \frac{m^2}{2} \phi^2 + \frac{g}{4!} \phi^4 \right]$$

$g=0 \quad S_0[\phi] = \leftarrow$



Correlation functions / Green Functions

$$\langle \phi(x_1) \dots \phi(x_N) \rangle = \frac{1}{Z} \int_0^\infty [D\phi] e^{-S_E[\phi]}$$

$\langle 1 \rangle = 1$ ↑
measure factor

Feynman Diagrams $G_0(x_1, x_2) = \langle \phi(x_1) \phi(x_2) \rangle_0 = \overline{x_1} \overline{x_2}$

N vacuum diagrams)

$$\int d^d x_1 \langle \phi^4(x_1) \rangle_0 = \int d^d x_1 \frac{3}{4!} \langle \phi^2(x_1) \rangle_0^2 = \int d^d x_1 \frac{3}{4!} \overline{x_1} \overline{x_1}$$

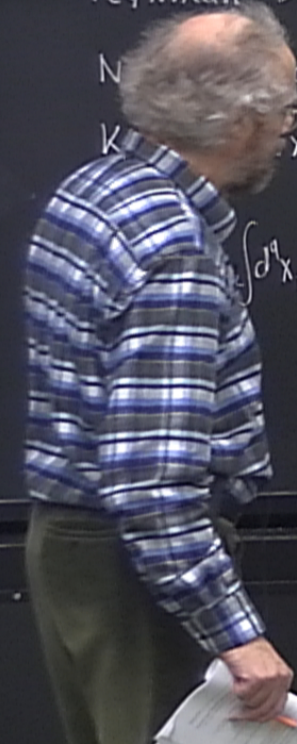
$$\int d^d x_1 d^d x_2 \langle \phi^4(x_1) \phi^4(x_2) \rangle_0 = \frac{1}{2^{(4!)}} \int d^d x_1 d^d x_2 \left[\overline{x_1} \overline{x_2} \overline{x_1} \overline{x_2} + \overline{x_1} \overline{x_2} \overline{x_1} \overline{x_2} + \overline{x_1} \overline{x_2} \overline{x_1} \overline{x_2} \right]$$

$$\frac{1}{128} G_0(0)^4 + \frac{1}{16} G_0(0)^2 G_0(x_1-x_2)^2 + \frac{1}{48} G_0(x_1-x_2)^4$$

$$\langle \phi^2(0) \rangle = \overline{0} = \infty \text{ if } d \geq 2$$

$$\propto \frac{1}{\epsilon^{d-2}} < \infty \text{ massive } \#$$

from now on, we do not really care about the UV divergences



$Z_0 = \int \mathcal{D}_0[\phi] \exp(-\frac{1}{\hbar} S_0[\phi])$; At this stage: expand : $\exp(-\frac{g}{\hbar} \frac{1}{4!} \int d^d x \phi(x)^4) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{-g}{\hbar} \frac{1}{4!}\right)^k \int \int d^d x_1 \dots d^d x_k \phi^4(x_1) \dots \phi^4(x_k)$
 and insert $\langle \sum_k \dots \rangle_0 = \sum_k \langle \dots \rangle_0$

$\phi(0) \langle \text{conv. series} \rangle = \sum_k g^k \dots$ 0 - radius of convergence any multiple series.

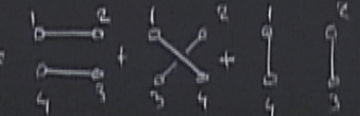
Numerator = $\sum_k \frac{1}{k!} \left(\frac{-g}{\hbar} \frac{1}{4!}\right)^k \int \int d^d x_1 \dots d^d x_k \langle \underbrace{\phi^4(x_1) \dots \phi^4(x_k)}_{\text{internal } k \text{ 's}} \underbrace{\phi(z_1) \dots \phi(z_N)}_{\text{external } N \text{ 's}} \rangle_0$

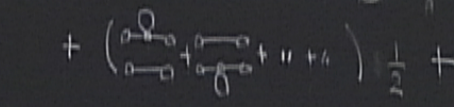
! inserting an ∞ sum and an integral (big) in general non trivial this is the case here, even in 0-dim theory

N=4 4 point diagrams

$N=4, K=0 \langle \phi(z_1) \phi(z_2) \phi(z_3) \phi(z_4) \rangle_0 =$

$N=4$ 4 point diagrams

$$N=4, K=0 \quad \langle \phi(z_1) \phi(z_2) \phi(z_3) \phi(z_4) \rangle_0 =$$


$$N=4, K=1 \quad \frac{1}{4!} \langle \phi(z_1) \phi(z_2) \phi(z_3) \phi(z_4) \rangle_0 =$$


$$\int d^4x_1 G_0(z_1-x_1) G_0(z_2-x_1) G_0(z_3-x_1) G_0(z_4-x_1)$$

$$1 = \frac{4!}{4!} \text{sym factor}$$

Feynman diagrammatic rules



N=4 4 point diagrams

$$N=4, K=0 \langle \phi(z_1) \phi(z_2) \rangle_0 = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4}$$

$$N=4, K=1 \frac{1}{4!} \langle \phi^4(x_1) \rangle_0 = \left(\text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \text{diagram 4} \right) \int \delta(x_1) \frac{1}{8}$$

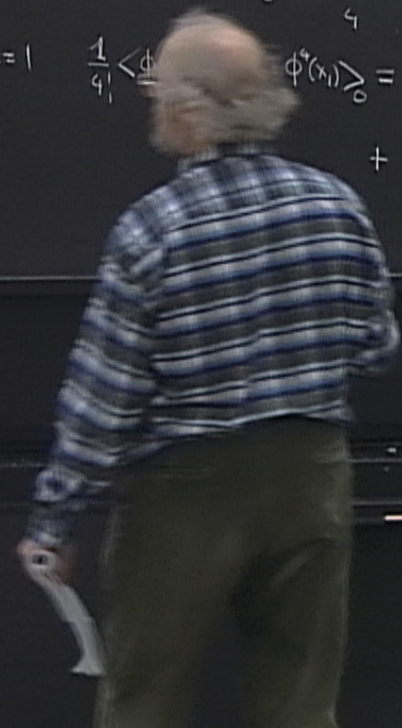
$$+ \left(\text{diagram 5} + \text{diagram 6} + \text{diagram 7} + \dots \right) \frac{1}{2} +$$

$$\int d^d x_1 G_0(z_1 - x_1) G_0(z_2 - x_1) G_0(z_3 - x_1) G_0(z_4 - x_1)$$

$$1 = \frac{4!}{4!} \leftarrow \text{Sym-factor}$$

Feynman diagrammatic rules :

$$\langle \phi(z_1) \phi(z_2) \rangle =$$



internal x 's external z 's

ms

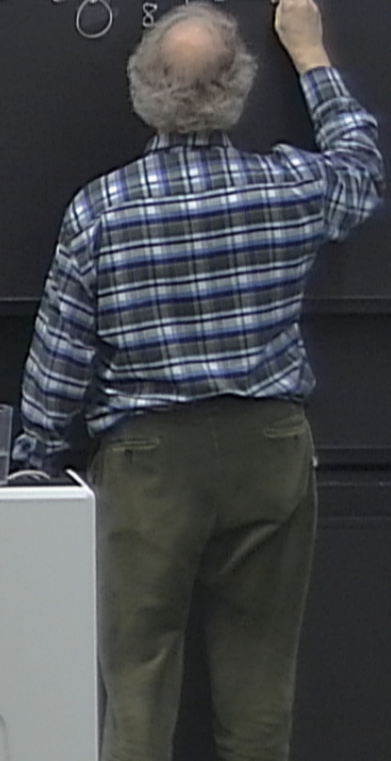
$$\langle \phi(z_4) \rangle_0 = \text{diagram 1} + \text{diagram 2} + \text{diagram 3}$$

$$\langle \phi(z_4) \phi^4(x_1) \rangle_0 = \left(\text{diagram 4} + \text{diagram 5} + \text{diagram 6} \right) \int d^d x_1 \frac{1}{8} G_0(z_4 - x_1) G_0(z_1 - x_1) G_0(z_2 - x_1) G_0(z_3 - x_1) G_0(z_4 - x_1)$$

$$+ \left(\text{diagram 7} + \text{diagram 8} + \dots \right) \frac{1}{2} + \text{diagram 9} \quad 1 = \frac{4!}{4!} \leftarrow \text{sym factor}$$

Feynman diagrammatic rules (set $\hbar=1$)

$$\langle \phi(z_1) \phi(z_2) \rangle = \text{diagram 1} - g \left[\text{diagram 2} \otimes \frac{1}{8} + \text{diagram 3} \right]$$



2 points diagrams

$$N = 2, K = 0$$

$$\langle \Phi(z_1) \Phi(z_2) \rangle_0 = \text{---} \text{---}$$

1 2

$$N = 2, K = 1$$

$$\frac{1}{4!} \int_{x_1} \langle \Phi(z_1) \Phi(z_2) \Phi^4(x_1) \rangle_0 = \frac{1}{2} \text{---} \text{---} + \frac{1}{2} \text{---} \text{---}$$

1 2 1 2

$$S[\Phi] = \int d^d x$$

Perturbation th

$$= \int D\Phi$$

correlation func

$$\langle \Phi(x) \rangle = \Phi$$

$$\langle 1 \rangle =$$

$$= Q = \infty \text{ if } d \geq 2$$

$$\propto \frac{1}{\epsilon^{d-2}} < \infty \text{ in } d=1$$

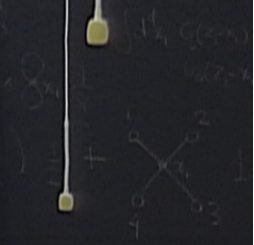
in, we do not really ca

the UV divergences



$$\frac{1}{(4\pi)^d} (\dots)$$

Handwritten notes on the right side of the slide.

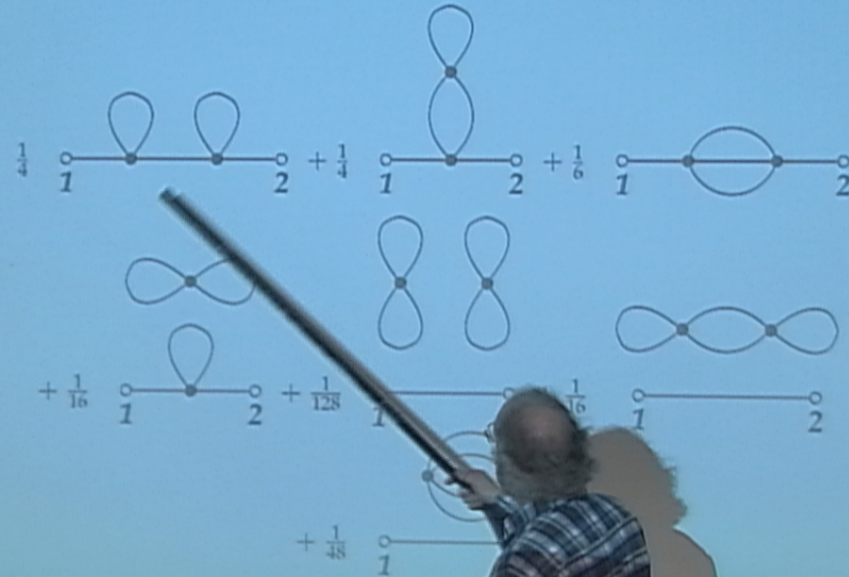


Handwritten notes on the right side of the slide, including the word 'connected'.

2 points diagrams (continued)

$$N = 2, K = 2$$

$$\frac{1}{2!(4!)^2} \int_{x_1} \int_{x_2} \langle \Phi(z_1) \Phi(z_2) \Phi^4(x_1) \Phi^4(x_2) \rangle_0 =$$



Connected vacuum diagrams

$$\frac{1}{2} \text{circle} - g \frac{1}{8} \text{figure-eight} + g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

2 points function = **connected** 2 points function

$$\text{line } 1-2 - g \frac{1}{2} \text{line with loop} + g^2 \left(\frac{1}{4} \text{line with two loops} + \frac{1}{6} \text{line with bubble} \right)$$

$S[\phi] = \int d^d x$
 Perturbation th
 $= \int \mathcal{D}\phi$
 correlation function
 $\langle \phi(x) \phi(y) \rangle$
 $\langle 1 \rangle =$
 $\langle \phi \rangle = 0$
 $\langle \phi^2 \rangle =$
 $\langle \phi^4 \rangle =$
 $\langle \phi^6 \rangle =$
 $\langle \phi^8 \rangle =$
 $\langle \phi^{10} \rangle =$
 $\langle \phi^{12} \rangle =$
 $\langle \phi^{14} \rangle =$
 $\langle \phi^{16} \rangle =$
 $\langle \phi^{18} \rangle =$
 $\langle \phi^{20} \rangle =$
 $\langle \phi^{22} \rangle =$
 $\langle \phi^{24} \rangle =$
 $\langle \phi^{26} \rangle =$
 $\langle \phi^{28} \rangle =$
 $\langle \phi^{30} \rangle =$
 $\langle \phi^{32} \rangle =$
 $\langle \phi^{34} \rangle =$
 $\langle \phi^{36} \rangle =$
 $\langle \phi^{38} \rangle =$
 $\langle \phi^{40} \rangle =$
 $\langle \phi^{42} \rangle =$
 $\langle \phi^{44} \rangle =$
 $\langle \phi^{46} \rangle =$
 $\langle \phi^{48} \rangle =$
 $\langle \phi^{50} \rangle =$
 $\langle \phi^{52} \rangle =$
 $\langle \phi^{54} \rangle =$
 $\langle \phi^{56} \rangle =$
 $\langle \phi^{58} \rangle =$
 $\langle \phi^{60} \rangle =$
 $\langle \phi^{62} \rangle =$
 $\langle \phi^{64} \rangle =$
 $\langle \phi^{66} \rangle =$
 $\langle \phi^{68} \rangle =$
 $\langle \phi^{70} \rangle =$
 $\langle \phi^{72} \rangle =$
 $\langle \phi^{74} \rangle =$
 $\langle \phi^{76} \rangle =$
 $\langle \phi^{78} \rangle =$
 $\langle \phi^{80} \rangle =$
 $\langle \phi^{82} \rangle =$
 $\langle \phi^{84} \rangle =$
 $\langle \phi^{86} \rangle =$
 $\langle \phi^{88} \rangle =$
 $\langle \phi^{90} \rangle =$
 $\langle \phi^{92} \rangle =$
 $\langle \phi^{94} \rangle =$
 $\langle \phi^{96} \rangle =$
 $\langle \phi^{98} \rangle =$
 $\langle \phi^{100} \rangle =$

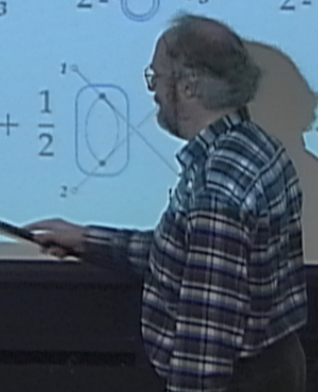
$\langle \phi(x) \phi(y) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \phi(d) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \phi(d) \phi(c) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \phi(d) \phi(c) \phi(b) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \phi(d) \phi(c) \phi(b) \phi(a) \rangle$
 $\langle \phi(x) \phi(y) \phi(z) \phi(w) \phi(v) \phi(u) \phi(t) \phi(s) \phi(r) \phi(q) \phi(p) \phi(o) \phi(n) \phi(m) \phi(l) \phi(k) \phi(j) \phi(i) \phi(h) \phi(g) \phi(f) \phi(e) \phi(d) \phi(c) \phi(b) \phi(a) \phi(0) \rangle$

4 points function (up to order 1)

$$\left(\begin{array}{c} 1 \text{---} 4 \\ 2 \text{---} 3 \end{array} + 2 \begin{array}{c} 1 \\ | \\ 3 \end{array} + 2 \begin{array}{c} 4 \\ | \\ 3 \end{array} + 2 \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} \right) - g \left(\begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} \right) + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ | \\ \text{loop} \\ | \\ 2 \text{---} 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ | \\ \text{loop} \\ | \\ 2 \text{---} 3 \end{array} \\
 + \frac{1}{2} \begin{array}{c} 1 \\ | \\ \text{loop} \\ | \\ 2 \text{---} 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \\ | \\ \text{loop} \\ | \\ 2 \text{---} 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} \Big) + \dots$$

Connected 4 points function (up to order 2)

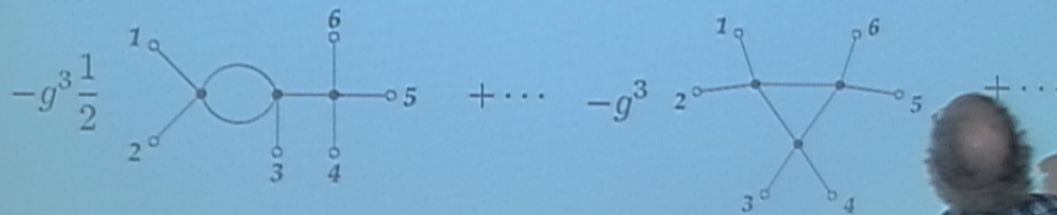
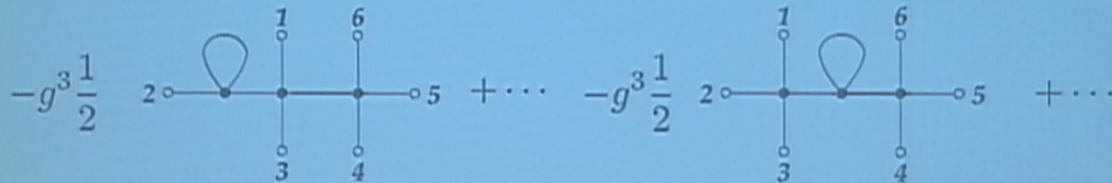
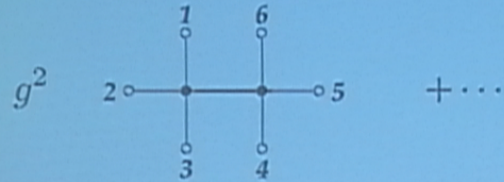
$$-g \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + g^2 \left(\frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \dots \right) \\
 + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \frac{1}{2} \begin{array}{c} 1 \text{---} 4 \\ \diagdown \quad \diagup \\ \text{loop} \\ \diagdown \quad \diagup \\ 2 \quad 3 \end{array} + \dots$$




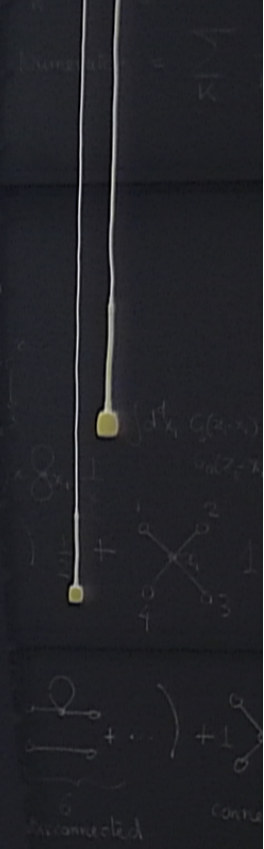
$S[\phi] = \int d^d x$
 Perturbation th
 $= \int \mathcal{D}\phi$
 correlation func
 $\langle \phi(x) \phi(y) \rangle$
 $\langle \mathbb{1} \rangle =$
 $\langle \mathbb{1} \rangle = \infty$ if $d \geq 2$
 $\propto \frac{1}{\epsilon^2} < \infty$ in d=2
 in, we do not really ca
 the UV divergences
 $\langle \mathbb{1} \rangle =$
 $\langle \mathbb{1} \rangle = \infty$ if $d \geq 2$

diagrams on the right blackboard showing various Feynman diagrams and some handwritten notes.

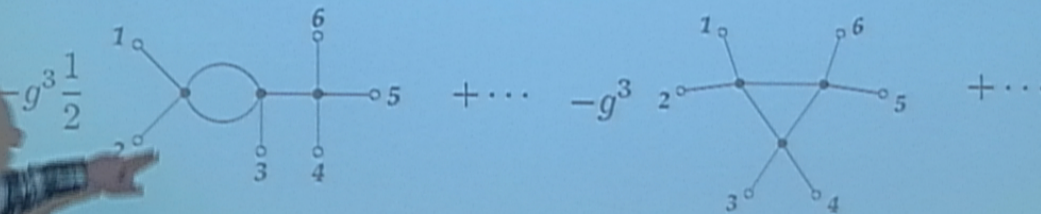
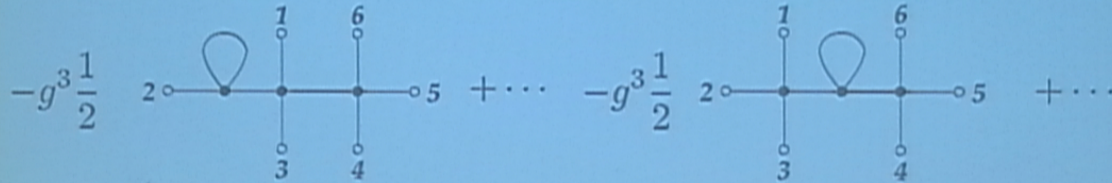
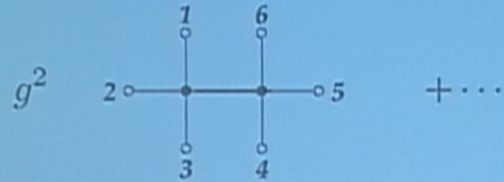
Connected 6 points function (up to order 3)



$S[\phi] = \int d^d x$
 Perturbation th
 $= \int D\phi [L$
 correlation func
 $\langle \phi(x) \phi(y) \rangle$
 $\langle 1 \rangle =$
 $Q = \infty$ if $d \geq 2$
 $\frac{1}{d-2} < \infty$ makes it
 fin, we do not really ca
 the UV divergences

 $\frac{1}{(d-2)^2}$

Diagrams


Connected 6 points function (up to order 3)



$S[\phi] = \int d^d x$
 Perturbation th
 $= \int d^d x$
 correlation func
 $\langle \phi(x) \phi(y) \rangle$
 $\langle \phi(x) \phi(y) \rangle =$
 $\langle \phi(x) \phi(y) \rangle = \infty$ if $d \geq 2$
 $\times \frac{1}{d-2} < \infty$ in arbitrary d
 in, we do not really ca
 the UV divergences
 $\langle \phi(x) \phi(y) \rangle$

$\langle \phi(x) \phi(y) \rangle =$
 $\langle \phi(x) \phi(y) \rangle =$
 $\langle \phi(x) \phi(y) \rangle =$
 disconnected
 connected

One Particle Irreducible (1PI) functions

Irreducible vacuum diagrams = vacuum self energy
 (= connected vacuum diagrams for Φ^4)

$$\Gamma^{(0)} = -\frac{1}{2} \text{circle} + g \frac{1}{8} \text{figure-eight} - g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

to be explained later \rightarrow

Irreducible 2 points function = inverse of the 2 points function

$$\Gamma^{(2)} = \text{propagator} + g \frac{1}{2} \text{self-energy} - g^2 \left(\frac{1}{4} \text{two-loop} + \frac{1}{6} \text{three-loop} \right) + \dots$$

with amputated legs

$$\text{propagator} = (-\Delta + m^2)_{z_1 z_2} = p^2 + m^2$$

$$\text{self-energy} = \delta(x) = \dots$$

$S[\Phi] = \int d^d x$
 Perturbation th
 $Z = \int D\Phi e^{iS[\Phi]}$
 correlation func
 $\langle \Phi(x) \rangle = \dots$
 $\langle \Phi(x) \Phi(y) \rangle = \dots$
 $d = 0 = \infty$ if $d \geq 2$
 $\propto \frac{1}{\epsilon} < \infty$ in finite d
 in, we do not really ca
 the UV divergences
 $\frac{1}{\epsilon} \ln(\lambda/\mu)$

Handwritten notes on the right blackboard, including diagrams and mathematical expressions.

One Particle Irreducible (1PI) functions

Irreducible vacuum diagrams = vacuum self energy
 (= connected vacuum diagrams for Φ^4)

$$\Gamma^{(0)} = -\frac{1}{2} \text{circle} + g \frac{1}{8} \text{figure-eight} - g^2 \left(\frac{1}{16} \text{two-loops} + \frac{1}{48} \text{three-loops} \right) + \dots$$

to be explained later \rightarrow

Irreducible 2 points function = inverse of the 2 points function

$$\Gamma^{(2)} = \text{propagator} + g \frac{1}{2} \text{self-energy} - g^2 \left(\frac{1}{4} \text{two-loops} + \frac{1}{6} \text{three-loops} \right) + \dots$$

with amputated legs

$$\begin{aligned} \text{propagator} &= (-\Delta + m^2)_{z_1 z_2} \\ &= p^2 + m^2 \end{aligned}$$

$$\begin{aligned} \text{self-energy} &= \delta(z - x) \\ &= 1 \end{aligned}$$

$$S[\phi] = \int d^d x$$

Perturbation th

$$Z = \int D\phi e^{iS[\phi]}$$

correlation func

$$\langle \phi(x) \phi(y) \rangle = \frac{\delta^2 Z}{\delta J(x) \delta J(y)}$$

$$\langle \phi(x) \phi(y) \phi(z) \rangle = \frac{\delta^3 Z}{\delta J(x) \delta J(y) \delta J(z)}$$

$$Q = \infty \text{ if } d \geq 2$$

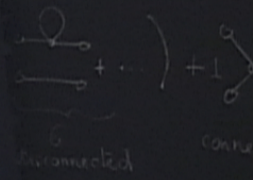
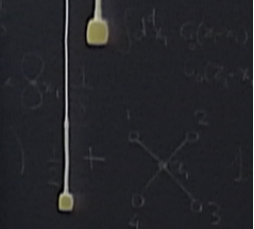
$$\propto \frac{1}{d-2} < \infty \text{ in } d=2$$

in, we do not really see

the UV divergences



$$\frac{1}{(4\pi)^d} \int d^d k$$



about the UV divergences

$$\int d^4x_1 d^4x_2 \langle \phi^4(x_1) \phi^4(x_2) \rangle = \frac{1}{2^{4!}} \int d^4x_1 d^4x_2 \left[\frac{72}{4!} \text{diagram 1} + \frac{72}{4!} \text{diagram 2} + \frac{4! \cdot 24}{4!} \text{diagram 3} \right]$$

$$\frac{1}{128} G_0(0)^4 + \frac{1}{16} G_0(0)^2 G_0(x_1-x_2)^2 + \frac{1}{48} G_0(x_1-x_2)^4$$

Feynman Amplitude or Integral $\frac{1}{8} \int G_0(\tau-z) G_0(0)^2 d\tau_1 \int G_0(\tau-x) G_0(\tau-x) G_0(0) d\tau_2$

$\frac{1}{2}$ - symmetry factors of the diagrams

combinatorics + Amplitudes (or integrals) $\langle \hat{\Phi}(p_i) \hat{\Phi}(p_n) \rangle$

external vertices	\sum_a	Fixed	Euclidean Space	Euclidean Space	momentum
internal vertices	x_a	$(-g) \frac{1}{\hbar}$	Position representation	Momentum representation	p_a
propagators	$y_1 \rightarrow y_2$	$G_0(y_1-y_2) \hbar$		vertex	q_2
integrate over the position of the vertices		$\int d^d x_i$		propagator	$\frac{1}{\hbar}$

