Title: How Quantum are the Cosmological Correlations?

Date: Nov 04, 2014 11:00 AM

URL: http://pirsa.org/14110003

Abstract: Cosmological perturbations are sourced by quantum fluctuations of the vacuum during inflation. In contrast, our observations of the Cosmic Microwave Background are classical. Can we test for the quantum origins of the perturbations? How much quantum information is lost when we make these observations? Have we totally screwed up by building PLANCK, and measured the correlations in the wrong basis and hence losing the primordial quantum information for good? I will talk about all these!

How Quantum are the cosmological perturbations?

Eugene A. Lim

arXiv 1410.5508

KING'S College LONDON

University of London

Perimeter Institute 2014

Preamble: Why do we care?

Primordial perturbations have a quantum origin : correlations are quantum.

Our observations are classical : we got a set of classical *probability distribution functions* pdf

e.g. CMB anisotropies power spectrum, the variance of gaussian pdf labeled by l is $P_l = \langle a_{lm} a^*_{lm} \rangle$

Preamble: Why do we care?

Primordial perturbations have a quantum origin : correlations are quantum.

Our observations are classical : we got a set of classical *probability distribution functions* pdf

e.g. CMB anisotropies power spectrum, the variance of gaussian pdf labeled by l is $P_l = \langle a_{lm} a^*_{lm} \rangle$

Is there a way to test for the quantum origin of perturbations?

How much of the primordial quantum information is accessible to us?

Preamble: Why do we care?

Example : 2 entangled qubits store 2 classical bits of info. Separate observations on qubits can recover at most 1 bit of info : "lost quantum information".

Preamble: Why do we care?

Example : 2 entangled qubits store 2 classical bits of info. Separate observations on qubits can recover at most 1 bit of info : "lost quantum information".

David Tong's Nightmare : The CMB has quantum correlations containing a Message from God but Humanity recklessly built Planck and made measurements which loses this information.

Preamble: Why do we care?

All is not lost! If a system has quantum correlations, then it doesn't obey classical correlation statistics -- we can check! (e.g. Bell's Inequality.)

CHSH inequality (Clauser-Horne-Shimony-Holt) :

Quantum information cannot be represented by a local joint prob. distribution function.

Preamble: Why do we care?

All is not lost! If a system has quantum correlations, then it doesn't obey classical correlation statistics -- we can check! (e.g. Bell's Inequality.)

CHSH inequality (Clauser-Horne-Shimony-Holt) :

Quantum information cannot be represented by a local joint prob. distribution function.

Entanglement is **not** *the only measure of quantumness!*

To construct such a statistic, we need to know the nature of the quantum correlations.

Outline

- How do we tell the difference between Quantum and Classical states?
- Starobinsky-Polarski-Kiefer decoherence model in inflation.
- *Quantum Discord* as a measure of "quantumness"
- Modeling joint primordial perturbations-environment bipartite system
- How Quantum are the perturbations?

Classical vs Quantum States

Classical States : described by *joint* probability distribution functions (pdf) of observables $P(x,p)$ and obey *Einstein Locality*.

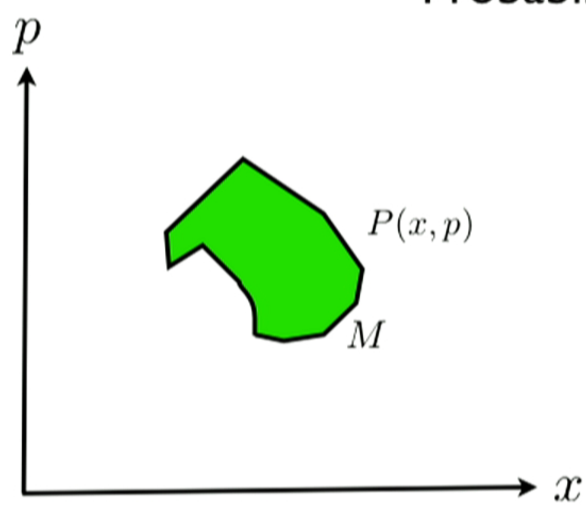
Einstein Locality : *If A and B are spacelike separated systems, then in a complete* ***description*** *of reality an action on A must not modify the* ***description*** *of B.*

Classical vs Quantum States

Classical States : described by *joint* probability distribution functions (pdf) of observables $P(x,p)$ and obey *Einstein Locality.*

Probability of finding particle in region M

$$\text{Prob} = \int_M dx\ dp\ P(x,p)$$



Phase space for single particle state (x,p)

Classical vs Quantum States

Quantum States : described by *density matrices* ρ

A state vector $|u_i\rangle$ describes a *pure state.*

Classical vs Quantum States

Quantum States : described by *density matrices* ρ

A state vector $|u_i\rangle$ describes a *pure state.*

We can "mix" different pure states to make a *mixed state.*

$\rho = \sum_i p_i |u_i\rangle\langle u_i|$ $\sum_i p_i = 1$ A *mixture* of pure states $|u_i\rangle$

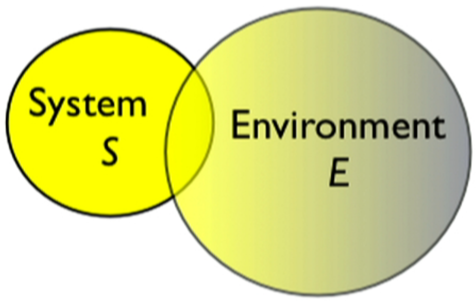
Note that there is an additional layer of "probability vector" p_i on top of the usual quantum prob. amplitude.

This makes things really complicated and much more interesting.

Classical vs Quantum States

Quantum States : described by *density matrices* ρ

Pure states $\rho = |u\rangle\langle u|$ can evolve into *mixed states* under *non-unitary* operations in Open systems.



Combined "Bipartite" $\rho_{S\mathcal{E}}$

Access only to S : $\rho_S = \mathrm{Tr}_{\mathcal{E}} \rho_{S\mathcal{E}}$

"Trace over unaccessible d.o.f."

"Ambiguity" = Von Neumann Entropy $S(\rho_S) = -\mathrm{Tr}(\rho_S \log \rho_S)$

(Zero for pure states.)

Classical vs Quantum States

What about Entanglement?

Given bipartite system, it is *separable* if

$$\rho = \sum_i p_i |u_i\rangle_S |e_i\rangle_{E\,E}\langle e_i|_S\langle u_i|$$

example $\rho = \frac{1}{2}|0_S\rangle|0_E\rangle\langle 0_S|\langle 0_E| + \frac{1}{2}|1_S\rangle|1_E\rangle\langle 1_S|\langle 1_E|$

Pure states :

separability = non-entanglement = classical pdf.

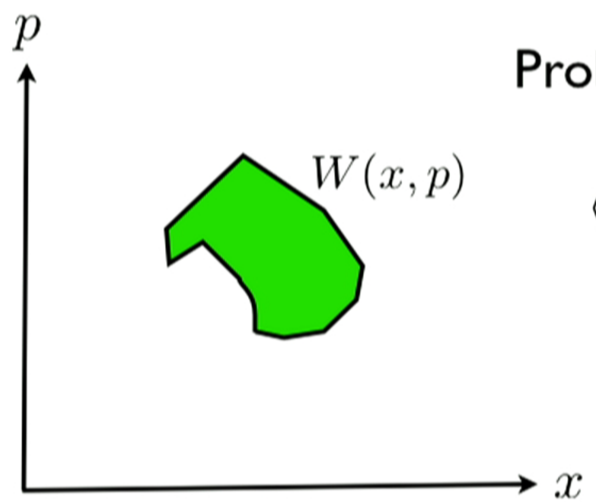
Mixed states :

separability = non-entanglement $\neq$ classical pdf

Classical vs Quantum States

Instead of density matrices --
Equivalent "quasi-pdf" picture : Wigner distribution

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{2ipy}\langle x - \frac{y}{2}|\rho|x + \frac{y}{2}\rangle$$



Phase space for single particle state (x,p)

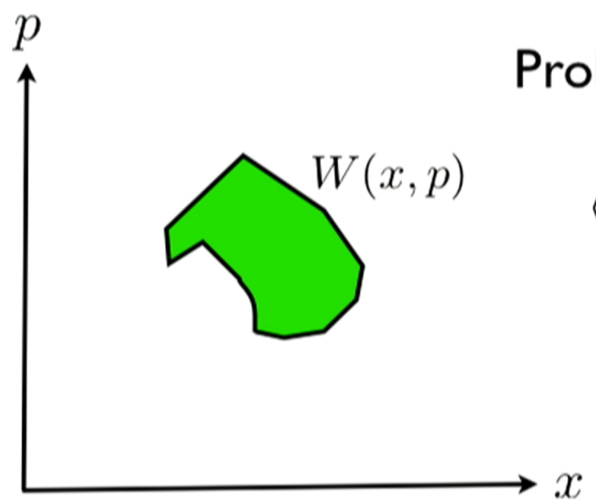
Prob density of x is then

$$\langle x|\rho|x\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\ W(x,p)$$

Classical vs Quantum States

Instead of density matrices --
Equivalent "quasi-pdf" picture : Wigner distribution

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{2ipy}\langle x - \frac{y}{2}|\rho|x + \frac{y}{2}\rangle$$



Phase space for single particle state (x,p)

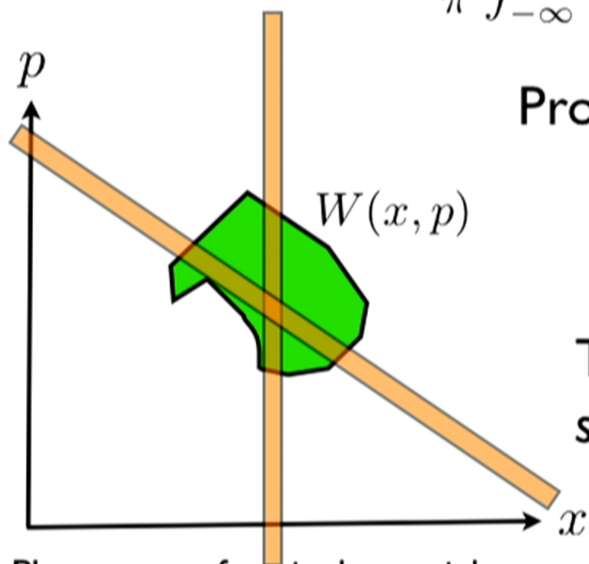
Prob density of x is then

$$\langle x|\rho|x\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\ W(x,p)$$

Classical vs Quantum States

Instead of density matrices --
Equivalent "quasi-pdf" picture : Wigner distribution

$$W(x,p) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{2ipy}\langle x-\frac{y}{2}|\rho|x+\frac{y}{2}\rangle$$



Phase space for single particle state (x,p)

Prob density of x is then

$$\langle x|\rho|x\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} dp\ W(x,p)$$

This is an integration over *an infinite strip of* p (uncertainty principle).

Bertrand and Bertrand's theorem : *any* infinite strip would do.

Classical vs Quantum States

All states are quantum.

They are considered "classical" if you can represent them as classical joint pdfs.

Is it Quantum? Is it Classical? Just check the statistics of the state!

Decoherence in a nutshell

Consider *pure* state $|S\rangle = \alpha|0\rangle + \beta|1\rangle$

Coherence = quantum phase of α and β preserved.

$$\rho = |S\rangle\langle S| = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Decoherence in a nutshell

Consider *pure* state $|S\rangle = \alpha|0\rangle + \beta|1\rangle$

Coherence = quantum phase of α and β preserved.

$$\rho = |S\rangle\langle S| = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Decoherence : couple *S* to environment *E*.

$$|S\rangle \otimes |E\rangle = (\alpha|0\rangle + \beta|1\rangle)|E\rangle \stackrel{couplings}{\longrightarrow} \alpha|0\rangle|E(0)\rangle + \beta|1\rangle|E(1)\rangle$$

If we have only access to *S*, then

$$\rho_S = \mathrm{Tr}_E \rho_{SE} = \begin{pmatrix} \rho_{00} & \rho_{01} \to 0 \\ \rho_{10} \to 0 & \rho_{11} \end{pmatrix} \to \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

Final matrix is *mixed* and phase info is lost.

Decoherence basis is crucial

Consider *pure* state $|S\rangle = \alpha|0\rangle + \beta|1\rangle$

Coherence = quantum phase of α and β preserved.

So the assumption: decoherence occurred in $\{|0\rangle, |1\rangle\}$ basis.

$$\rho = |S\rangle\langle S| = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Decoherence can occurs at rotated basis

$$\{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$

the classical pdf obtained from decoherence then depends on this basis

Decoherence : couple *S* to environment *E*.

$$|S\rangle \otimes |E\rangle = (\alpha|0\rangle + \beta|1\rangle)|E\rangle \xrightarrow{couplings} \alpha|0\rangle|E(0)\rangle + \beta|1\rangle|E(1)\rangle$$

If we have only access to *S*, then

$$\rho_S = \mathrm{Tr}_E\rho_{SE} = \begin{pmatrix} \rho_{00} & \rho_{01} \to 0 \\ \rho_{10} \to 0 & \rho_{11} \end{pmatrix} \to \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

Final matrix is *mixed* and phase info is lost.

Decoherence in a nutshell

Consider *pure* state $|S\rangle = \alpha|0\rangle + \beta|1\rangle$

Coherence = quantum phase of α and β preserved.

$$\rho = |S\rangle\langle S| = |\alpha|^2|0\rangle\langle 0| + |\beta|^2|1\rangle\langle 1| + \alpha\beta^*|0\rangle\langle 1| + \alpha^*\beta|1\rangle\langle 0|$$

$$= \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix}$$

Decoherence : couple *S* to environment *E*.

$$|S\rangle \otimes |E\rangle = (\alpha|0\rangle + \beta|1\rangle)|E\rangle \xrightarrow{couplings} \alpha|0\rangle|E(0)\rangle + \beta|1\rangle|E(1)\rangle$$

If we have only access to *S*, then

$$\rho_S = \mathrm{Tr}_E \rho_{SE} = \begin{pmatrix} \rho_{00} & \rho_{01} \to 0 \\ \rho_{10} \to 0 & \rho_{11} \end{pmatrix} \to \begin{pmatrix} |\alpha|^2 & 0 \\ 0 & |\beta|^2 \end{pmatrix}$$

Final matrix is *mixed* and phase info is lost.

Decoherence basis is crucial

Secret assumption : decoherence occurred in $\{|0\rangle, |1\rangle\}$ basis.

Decoherence can occurs at rotated basis

$$\{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$

the classical pdf obtained from decoherence then depends on this basis

Decoherence basis is crucial

Secret assumption : decoherence occurred in $\{|0\rangle, |1\rangle\}$ basis.

Decoherence can occurs at rotated basis

$$\{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$

the classical pdf obtained from decoherence then depends on this basis

Quantum nature of cosmological perturbations depends on how they interact with environment and how we measure them.

Decoherence basis is crucial

Secret assumption : decoherence occurred in $\{|0\rangle, |1\rangle\}$ basis.

Decoherence can occurs at rotated basis

$$\{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$

the classical pdf obtained from decoherence then depends on this basis

Quantum nature of cosmological perturbations depends on how they interact with environment and how we measure them.

What about inflation?

Starobinsky, Polarski (1998)
Grischuk, Sidorov (1990)

Single mode Hamiltonian for cosmological perturbations

$$\hat{H}_k = \frac{1}{2}\left(p_k^2 + k^2 y_k^2 + \frac{2a'}{a'} y_k p_k\right)$$

$$\delta\phi_k \equiv y_k \;,\; p_k = \frac{\partial L(y_k, y_k')}{\partial y_k'} = y_k' - a'/a y_k$$

$\hat{H}_k$ is a unitary evolution operator.

Schrodinger's equation of wave function $\psi(y,\eta)$ (drop k)

$$i\hbar \frac{\partial \psi(y,\eta)}{\partial \eta} = \hat{H}_k \psi(y,\eta).$$

with solution $\psi(y,\eta) = \left(\frac{2\Omega_R(\eta)}{\pi}\right)^{1/4} \exp(-(\Omega_R + i\Omega_I)y^2)$

for inflation background $\Omega_R \to k e^{-2r} \;,\; \Omega_I \to -k e^{-r}$

$$r = \#\text{efolds}$$

What about inflation?

Starobinsky, Polarski (1998)
Grischuk, Sidorov (1990)

Single mode Hamiltonian for cosmological perturbations

$$\hat{H}_k = \frac{1}{2}\left(p_k^2 + k^2 y_k^2 + \frac{2a'}{a'} y_k p_k\right)$$

$$\delta\phi_k \equiv y_k \ , \ p_k = \frac{\partial L(y_k, y_k')}{\partial y_k'} = y_k' - a'/a y_k$$

$\hat{H}_k$ is a unitary evolution operator.

Schrodinger's equation of wave function $\psi(y, \eta)$ (drop k)

$$i\hbar \frac{\partial \psi(y,\eta)}{\partial \eta} = \hat{H}_k \psi(y,\eta).$$

with solution $\psi(y,\eta) = \left(\frac{2\Omega_R(\eta)}{\pi}\right)^{1/4} \exp(-(\Omega_R + i\Omega_I)y^2)$

for inflation background $\Omega_R \to k e^{-2r}$, $\Omega_I \to -k e^{-r}$

$$r = \#\text{efolds}$$

Cosmological "Squeezed states"

Construct density matrix

$$\rho_S = |\psi\rangle\langle\psi| = \frac{2\Omega_R}{\pi} \exp\left[-(\frac{\Omega_R}{2}(y-y')^2 - \frac{\Omega_R}{2}(y+y')^2 - i\Omega_I(y^2 - y'^2)\right]$$

As $\Omega_R/k = e^{-2r} \ll 1 \Rightarrow$ off-diagonal terms get killed off

So far unitary evolution : *no decoherence* so still *pure state*.

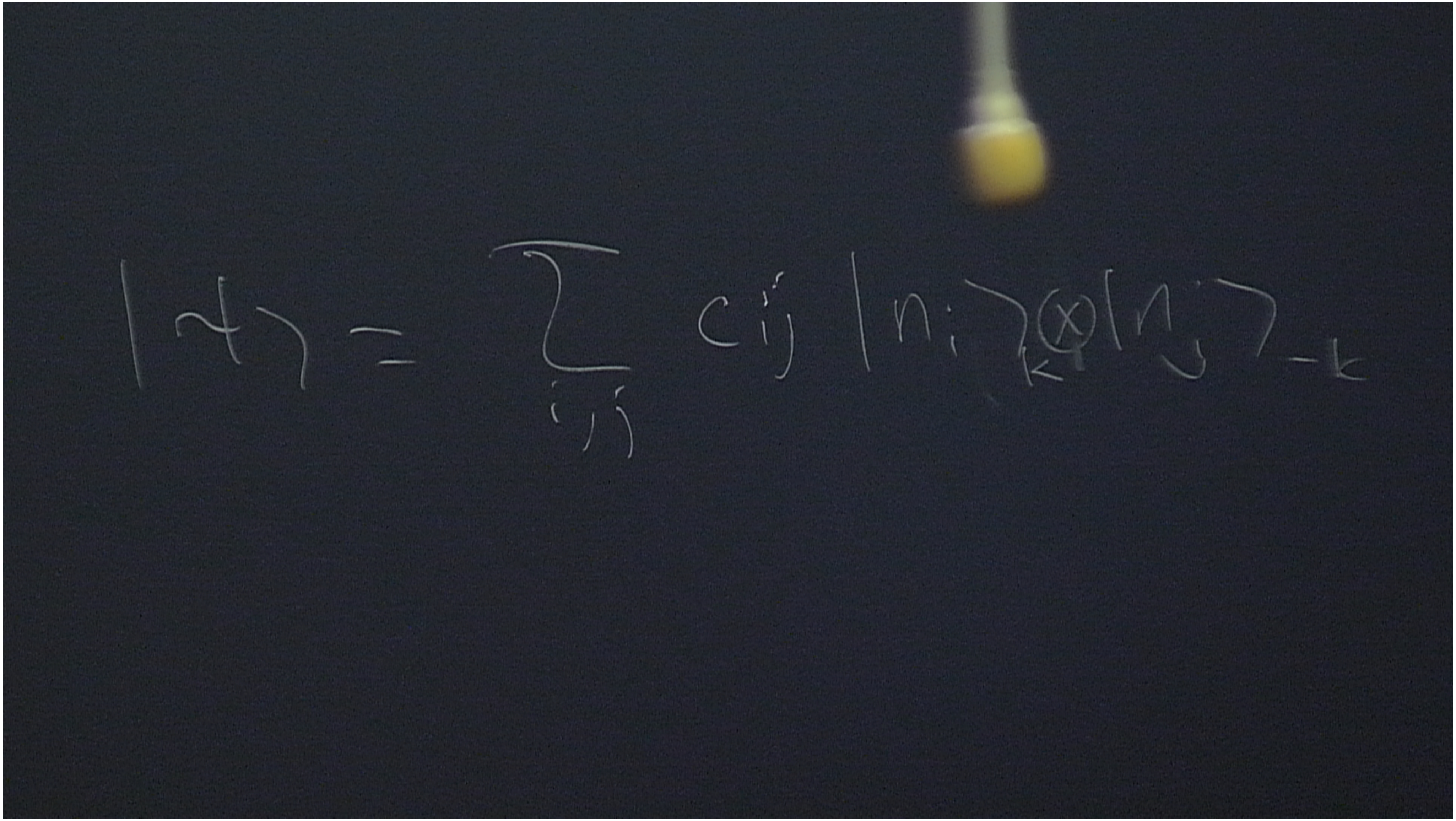
Cosmological "Squeezed states"

Construct density matrix

$$\rho_S = |\psi\rangle\langle\psi| = \frac{2\Omega_R}{\pi}\exp\left[-(\frac{\Omega_R}{2}(y-y')^2 - \frac{\Omega_R}{2}(y+y')^2 - i\Omega_I(y^2-y'^2)\right]$$

As $\Omega_R/k = e^{-2r} \ll 1 \Rightarrow$ off-diagonal terms get killed off

So far unitary evolution : *no decoherence* so still *pure state*.


$|\psi\rangle = \sum_{i,j} c_{ij} |n_i\rangle_k \otimes |n_j\rangle_{-k}$

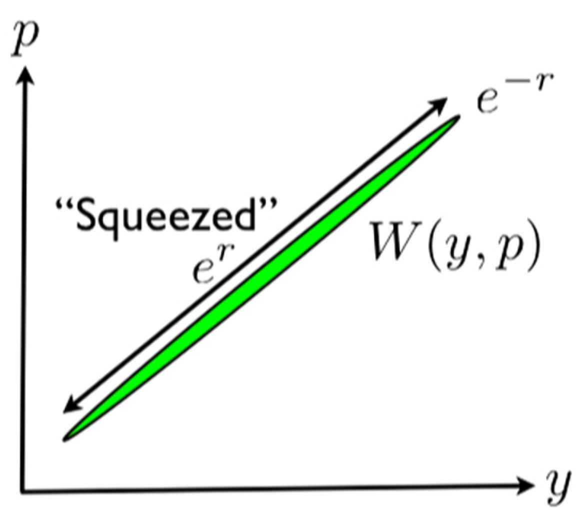
Cosmological "Squeezed states"

Construct density matrix

$$\rho_S = |\psi\rangle\langle\psi| = \frac{2\Omega_R}{\pi} \exp\left[-(\frac{\Omega_R}{2}(y-y')^2 - \frac{\Omega_R}{2}(y+y')^2 - i\Omega_I(y^2 - y'^2)\right]$$

As $\Omega_R/k = e^{-2r} \ll 1 \Rightarrow$ off-diagonal terms get killed off

So far unitary evolution : *no decoherence* so still *pure state.*

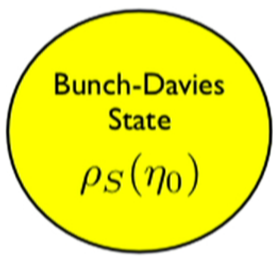


Wigner function is a *gaussian* with ellipsoid axes (e^{-r}, e^r)

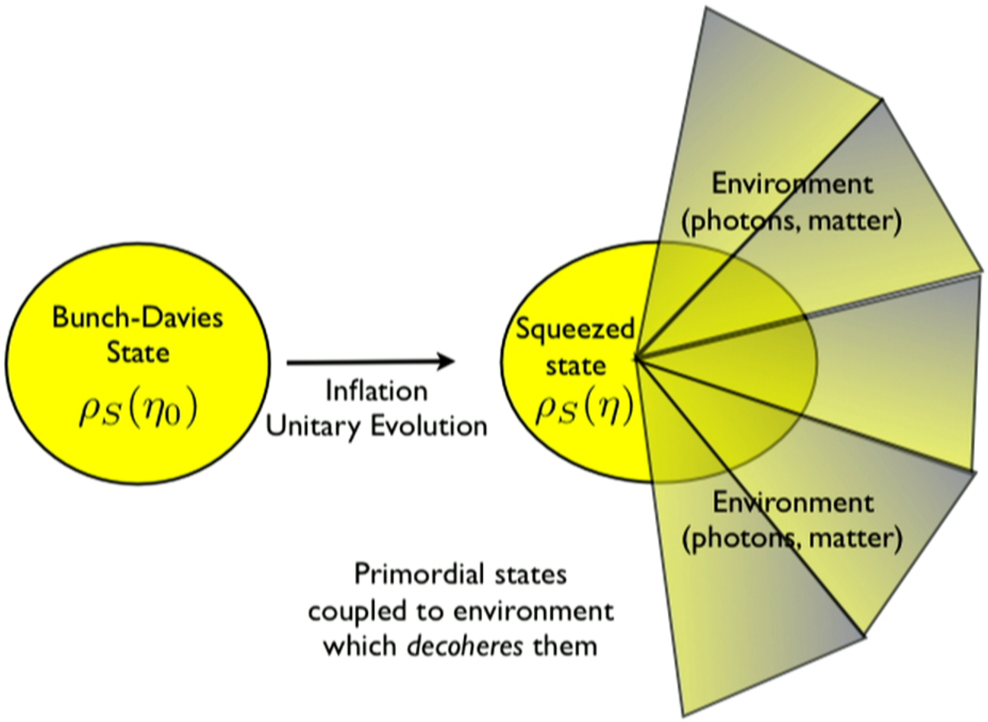
$$W(y,p) = \frac{1}{\pi} \exp\left[-\frac{1}{2}\mathbf{x}\sigma_S^{-1}\mathbf{x}^T\right]$$

$$\mathbf{x} = (x, p)$$

The environment as a quantum channel

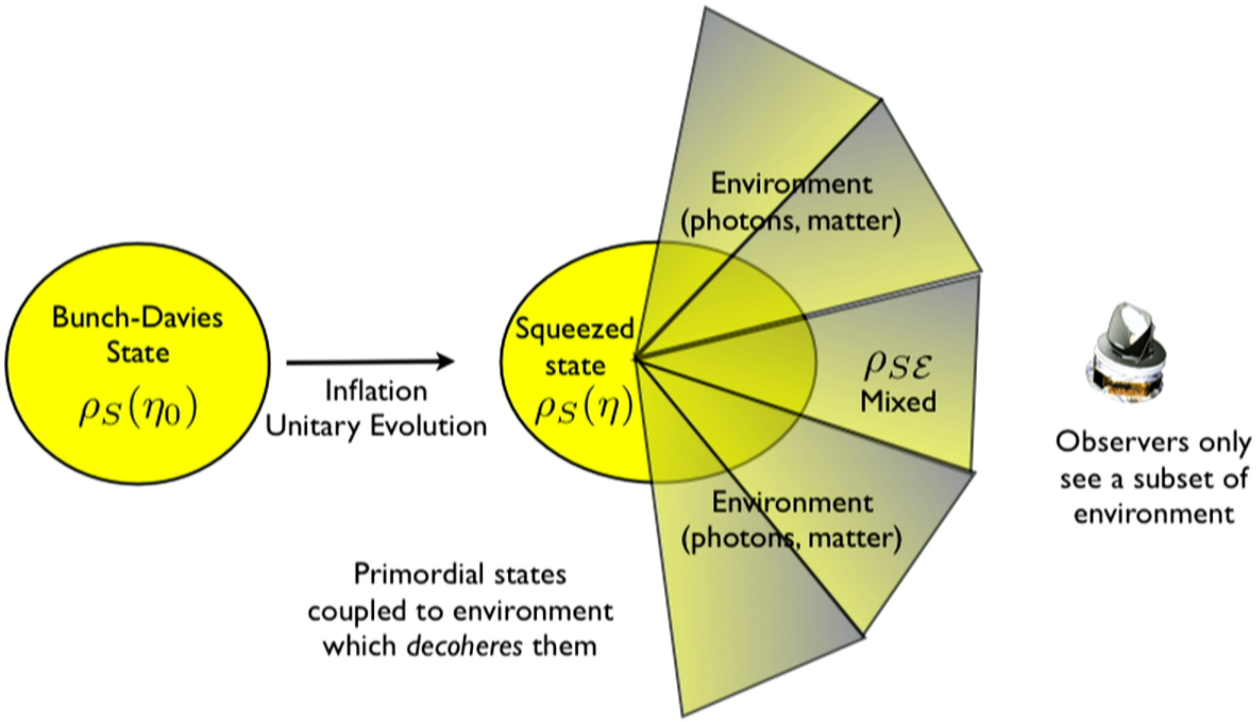


The environment as a quantum channel



Primordial states
coupled to environment
which *decoheres* them

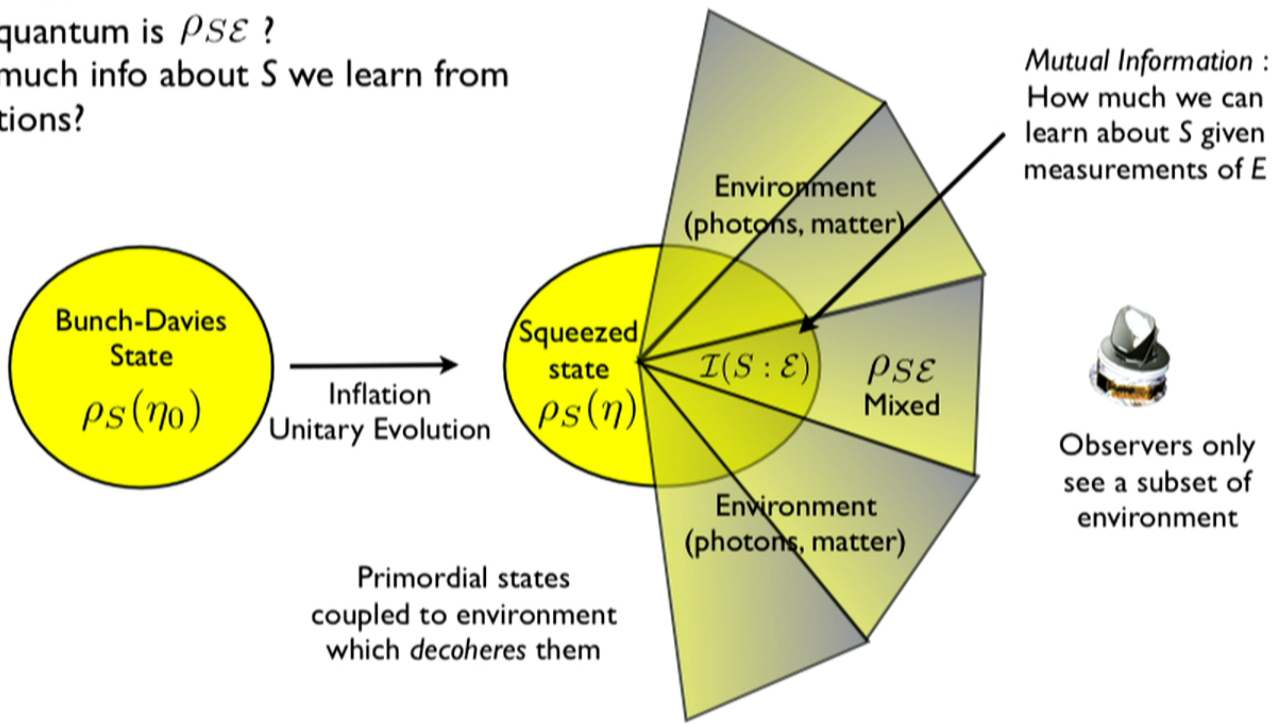
The environment as a quantum channel



The environment as a quantum channel

Want to know:

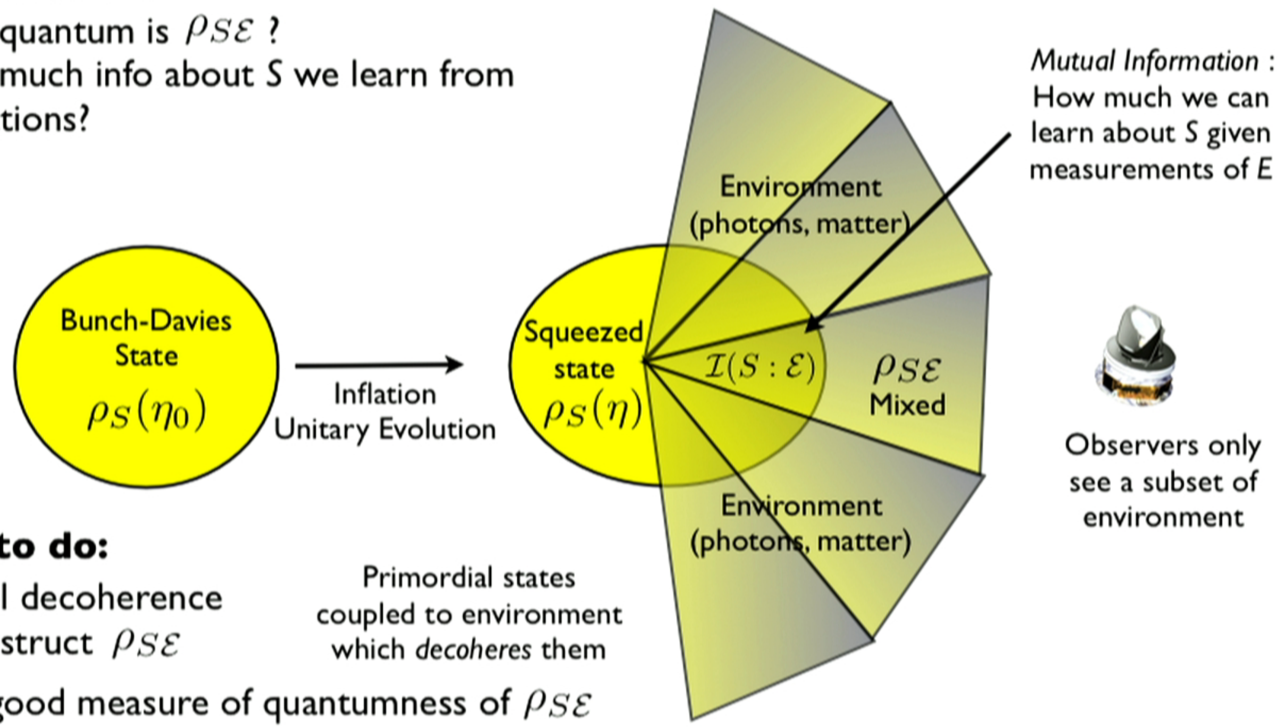
1. How quantum is $\rho_{S\mathcal{E}}$?
2. How much info about *S* we learn from observations?



The environment as a quantum channel

Want to know:

1. How quantum is $\rho_{S\mathcal{E}}$?
2. How much info about *S* we learn from observations?



Need to do:

1. Model decoherence and construct $\rho_{S\mathcal{E}}$
2. Find good measure of quantumness of $\rho_{S\mathcal{E}}$
3. Calculate $\mathcal{I}(S:\mathcal{E})$

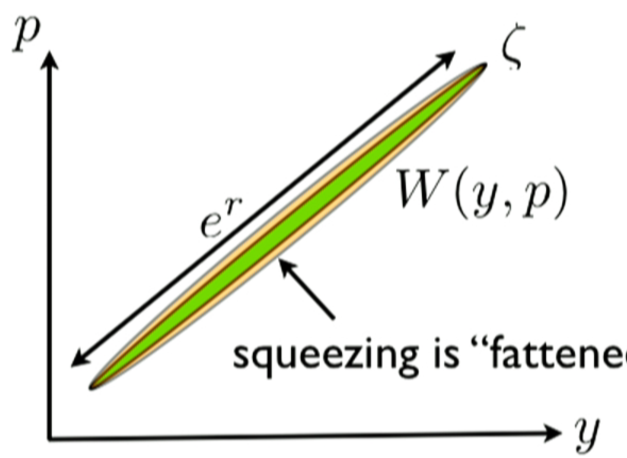
Starobinsky-Kiefer-Polarski decoherence ansatz (1998)

Assumption : environment scatters off primordial states large number of times. (Joos + Zeh, 1985)

Couple to environment ρ_{SE} , we add a decoherence term

$$\rho'_S = \mathrm{Tr}_E \rho_{SE} = \rho_S \times \exp[-\frac{\zeta}{2}(y-y')^2] \qquad \zeta \gg \Omega_R$$

New *mixed state* is still a gaussian but with axes (e^r, ζ)



Other models of decoherence : use high frequency modes as environment to low frequency "observed" modes.
(Burgess et. al. 2006, 2014)

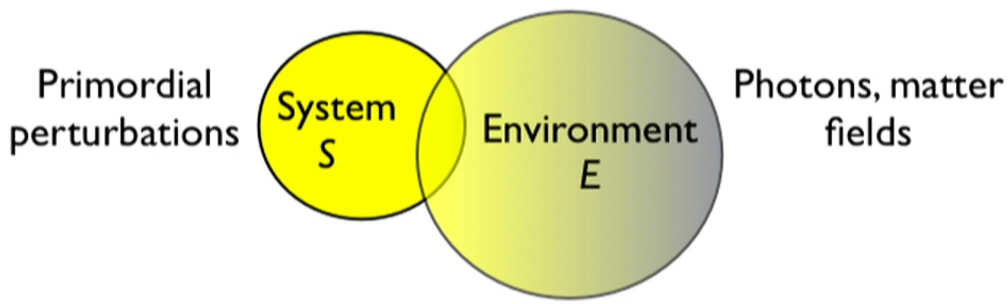
How to model system-environment?

Start with Starobinsky-Polarski-Kiefer (SPK) Gaussian ansatz

$$\rho'_S \rightarrow \quad W(y,p) = \frac{1}{\pi}\exp\left[-\frac{1}{2}\mathbf{x}\sigma_S^{-1}\mathbf{x}^T\right] \;,\;\; \sigma_S(\Omega_R, \Omega_I, \zeta)$$

We want to find a joint density matrix ρ_{SE} such that

$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$



How to model system-environment?

Start with Starobinsky-Polarski-Kiefer (SPK) Gaussian ansatz

$$\rho'_S \rightarrow \quad W(y,p) = \frac{1}{\pi} \exp\left[-\frac{1}{2} \mathbf{x} \sigma_S^{-1} \mathbf{x}^T \right] \ , \ \ \sigma_S(\Omega_R, \Omega_I, \zeta)$$

We want to find a joint density matrix ρ_{SE} such that

$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$

Not unique and continuous states are HARD. A way forward is to assume that ρ_{SE} is also Gaussian.

How to model system-environment?

Start with Starobinsky-Polarski-Kiefer (SPK) Gaussian ansatz

$$\rho'_S \to \quad W(y,p) = \frac{1}{\pi}\exp\left[-\frac{1}{2}\mathbf{x}\sigma_S^{-1}\mathbf{x}^T\right]\ , \ \ \sigma_S(\Omega_R, \Omega_I, \zeta)$$

We want to find a joint density matrix ρ_{SE} such that

$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$

Not unique and continuous states are HARD. A way forward is to assume that ρ_{SE} is also Gaussian.

These are not the droids you are looking for

How to model system-environment?

Start with Starobinsky-Polarski-Kiefer (SPK) Gaussian ansatz

$$\rho_S' \rightarrow \quad W(y,p) = \frac{1}{\pi}\exp\left[-\frac{1}{2}\mathbf{x}\sigma_S^{-1}\mathbf{x}^T\right]\ ,\ \ \sigma_S(\Omega_R,\Omega_I,\zeta)$$

We want to find a joint density matrix ρ_{SE} such that

$$\rho_S = \mathrm{Tr}_E(\rho_{SE})$$

Not unique and continuous states are HARD. A way forward is to assume that ρ_{SE} is also Gaussian.

Central Limit Theorem

How to model system-environment?

A unique *pure 2 mode Gaussian* ρ_{SE} can be constructed ("gaussian purification" of ρ_S).

$$W(y_1, y_2, p_1, p_2) = \frac{1}{\pi} \exp\left[-\frac{1}{2}\mathbf{x}\sigma_{SE}^{-1}\mathbf{x}^T\right] \quad \mathbf{x} = (y_1, p_1, y_2, p_2)$$

Parameterized around pure Gaussian state to get a mixed Gaussian state.

Many possibilities of parameterization. We choose a two parameter model to illustrate basis dependence of discord.

How to model system-environment?

Two parameter model $(\beta \geq 1, \tau)$

$$\sigma_{S\mathcal{E}} = \begin{pmatrix} \frac{1}{\lambda} & \frac{\alpha}{\lambda} & \sqrt{\frac{\xi}{\lambda}}\left(1+\frac{\xi}{\lambda}\right)^{-1/4} & 0 \\ \frac{\alpha}{\lambda} & \xi + \frac{\alpha^2}{\lambda} + \lambda & \alpha\sqrt{\frac{\xi}{\lambda}}\left(1+\frac{\xi}{\lambda}\right)^{-1/4} & -\sqrt{-1+\left(1+\frac{\xi}{\lambda}\right)}\,\tau^2(\lambda(\xi+\lambda))^{1/4} \\ \sqrt{\frac{\xi}{\lambda}}\left(1+\frac{\xi}{\lambda}\right)^{-1/4} & \alpha\sqrt{\frac{\xi}{\lambda}}\left(1+\frac{\xi}{\lambda}\right)^{-1/4} & \beta\sqrt{1+\frac{\xi}{\lambda}} & 0 \\ 0 & -\sqrt{-1+\left(1+\frac{\xi}{\lambda}\right)}\,\tau^2(\lambda(\xi+\lambda))^{1/4} & 0 & \beta\sqrt{1+\frac{\xi}{\lambda}} \end{pmatrix}$$

$$a = -\Omega_I/k \ , \ \lambda = \Omega_R/k \ , \ \xi = \zeta/k$$

Physicality of state: $\beta^2, \tau^2 \geq \left(1+\frac{\xi}{\lambda}\right)^{-1}$ and $\tau^2 \leq \beta$

State is pure when $\beta = \tau = 1$

State is also *entangled* for $\xi > 0$, i.e. SPK decoherence

SPK ansatz always generates entanglement with *E*

What is a good measure of "Quantumness"

Joint perturbations-environment state ρ_{SE} is *mixed*, so entanglement measures are insufficient.

More robust measure is *quantum discord* which captures quantumness beyond entanglement for mixed states.

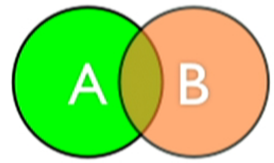
Ollivier and Zurek (2001)
Henderson and Vedral (2001)

Quantum Discord

Ollivier and Zurek (2001)
Henderson and Vedral (2001)

Classical Mutual Information

$$J(A:B) = H(A) - H(A|B)$$



A and B correlated, mutual info is how much we learn more about A when B is found out.

Classical pdf, Bayes Theorem $H(A|B) = H(A,B) - H(B)$

Get equivalent expression

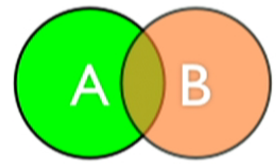
$$I(A:B) = H(A) + H(B) - H(A,B)$$

Quantum Discord

Ollivier and Zurek (2001)
Henderson and Vedral (2001)

Classical Mutual Information

$J(A:B) = H(A) - H(A|B)$



A and B correlated, mutual info is how much we learn more about A when B is found out.

Classical pdf, Bayes Theorem $H(A|B) = H(A,B) - H(B)$

Get equivalent expression

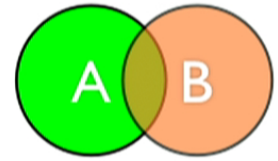
$$I(A:B) = H(A) + H(B) - H(A,B)$$

Quantum generalization: replace Shannon with Von Neumann $H(A) \to S(A)$

$$I(A:B) \to \mathcal{I}(A:B) = S(A) + S(B) - S(A,B)$$

$$J(A:B) \to \mathcal{J}(A:B) = S(A) - \overset{???}{S(A|B)}$$

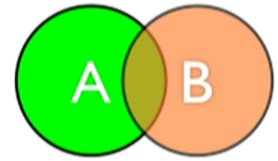
Quantum Discord



What is the quantum version of $S(A|B)$?

"Finding out B" = making measurement on B but quantum mechanically this will *disturb* A!

Quantum Discord



What is the quantum version of $S(A|B)$?

"Finding out B" = making measurement on B but quantum mechanically this will *disturb* A!

Ollivier and Zurek (2001) propose the following:

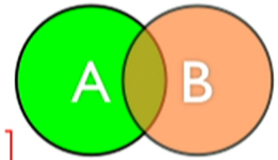
1. Given some basis of measurements (POVM) on B $\{\Pi_k^B\}$
2. Each Π_k^B measurement occurs with prob. P_k and

$$\rho_{AB} \to \frac{\rho_{AB}\Pi_k^B}{P_k} \; , \; \rho_{A|B=\Pi_k^B} = \mathrm{Tr}_B \frac{\rho_{AB}\Pi_k^B}{P_k}$$

3. Define $S(A|B=\{\Pi_k^B\}) = \sum_k P_k S(\rho_{A|B=\Pi_k^B})$
4. Quantum Discord is then

$$\delta(A:B)_{\Pi_k^B} = \mathcal{I}(A:B) - \mathcal{J}(A:B)_{\Pi_k^B} > 0$$

Quantum Discord



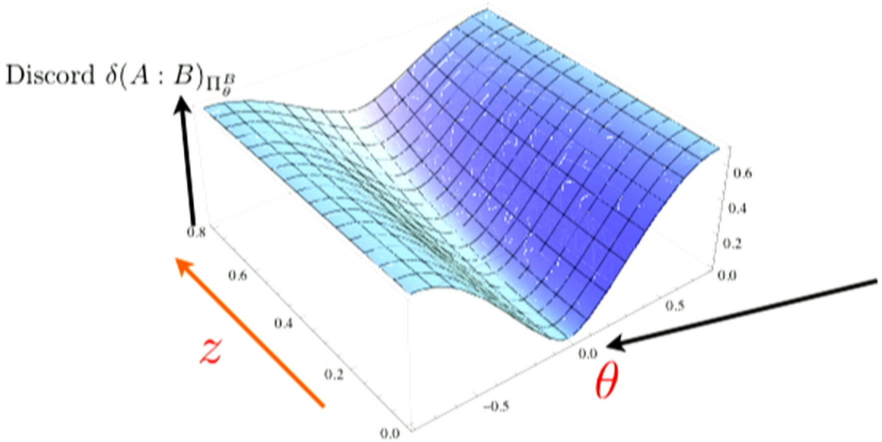
Consider mixed state parameterized by $z = [0,1]$

$$\rho_{AB} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right) + \frac{z}{2}\left(|00\rangle\langle 11| + |11\rangle\langle 00|\right)$$

When $z = 0$, B is aligned with A's pointer basis.

We choose measurement basis on system B to be parameterized by θ

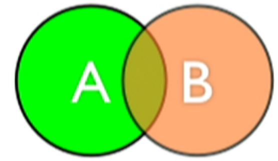
$$\Pi_\theta^B = \{\cos\theta|0\rangle + e^{i\phi}\sin\theta|1\rangle, -e^{-i\phi}\sin\theta|0\rangle - \cos\theta|1\rangle\}$$



at $z = 0$ discord is zero only if $\theta = 0$

i.e. the measurement basis is $\{|0\rangle, |1\rangle\}$

Quantum Discord



Some facts on Discord :

1. Zero discord $\delta(A:B)_{\Pi_k^B} = 0$ means decoherence occurred in "pointer basis" and no entanglement. Can define to be "Classical".

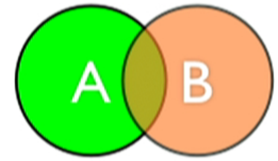
2. Mixed Separable state can have non-zero discord. *No entanglement* $\neq$ *no quantum correlations!*

3. Discord is basis dependent.

4. Recently shown a (particular) quantum circuit with discord (but zero entanglement) is better than a classical circuit.

Datta, Shaji, Caves (2007)

Quantum Discord



Some facts on Discord :

1. Zero discord $\delta(A:B)_{\Pi_k^B} = 0$ means decoherence occurred in "pointer basis" and no entanglement. Can define to be "Classical".

2. Mixed Separable state can have non-zero discord. *No entanglement $\neq$ no quantum correlations!*

3. Discord is basis dependent.

4. Recently shown a (particular) quantum circuit with discord (but zero entanglement) is better than a classical circuit.

Datta, Shaji, Caves (2007)

Cosmological Discord

Discord
$$\delta(S:\mathcal{E})_{\Pi_k^{\mathcal{E}}} = \mathcal{I}(S:\mathcal{E}) - \mathcal{J}(S:\mathcal{E})_{\Pi_k^{\mathcal{E}}}$$
$$= S(\mathcal{E}) - S(S,\mathcal{E}) + \sum_k P_k S(\rho_{S|\mathcal{E}=\Pi_k^{\mathcal{E}}})$$

Depends on choice of decoherence basis picked up by the environment

Again, in principle many different bases possible. Choose Gaussian POVMs generated by a Gaussian state with covariance matrix $\sigma_0 = R(\theta)\begin{pmatrix} \gamma & 0 \\ 0 & \frac{1}{\gamma} \end{pmatrix} R^T(\theta)$

γ is the squeezing and θ the *alignment* of the Gaussian with respect to the environment.

The environment choose (γ, θ) to decohere to.

Cosmological Discord

Case 1 : Pure State $\beta = \tau = 1$ bipartite ρ_{SE}

Discord of pure state = entanglement entropy

$$\delta(A:B) = \frac{1+\sqrt{1+\chi}}{2}\log\left(\frac{1+\sqrt{1+\chi}}{2}\right) - \frac{\sqrt{1+\chi}+1}{2}\log\left(\frac{\sqrt{1+\chi}-1}{2}\right)\ ,\ \chi = \frac{\zeta}{\Omega_R} \gg 1$$

$$= S(\mathrm{Tr}_{\mathcal{E}}\rho_{S\mathcal{E}})$$

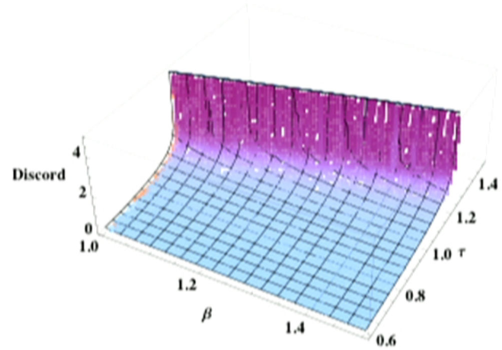
(Kiefer, Starobinsky and Polarski 1999)

Zero when $\zeta = 0$ so the non-decohered perturbations has classical statistics.

Unrealistic : it means we have access to the entire Universe.

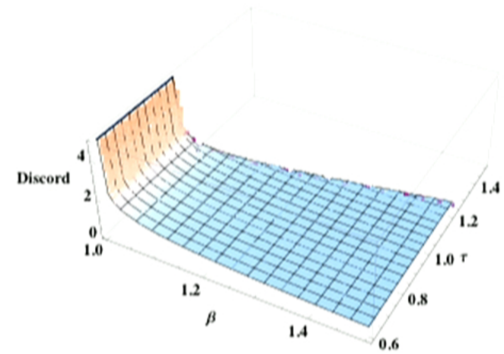
Cosmological Discord

Case 2 : Mixed State $\beta > 1$, $\tau^2 < \beta$ bipartite ρ_{SE}



$\gamma = 10^5$, $\theta = \pi/2$

measurement maximally unaligned with squeezed state



$\gamma = 10^5$, $\theta = 0$

measurement aligned with squeezed state

"Quantumness" of the cosmological correlations depend on the decoherence basis.

How much primordial info is accessible?

Conservation Equation

Zwolak + Zurek (2013)

$$\mathcal{I}(S:\mathcal{E}) = \delta(S:\mathcal{E})_{\Pi_k^{\mathcal{E}}} + \chi(P_k, \rho_{S|\Pi_k^{\mathcal{E}}})$$

Quantum Mutual Info.
How correlated are S and E

Quantum Discord

Holevo Information

upper bound of *classical* information P_k transmittable by a quantum channel constructed out of POVM $\{\rho_{S|\Pi_k^{\mathcal{E}}}\}$

Summary

- Unitary evolution in inflation drives primordial perturbations into classicality
- Primordial perturbations entangle with environment, and decohere. We observe environment to infer what the primordial perturbations are.
- This decoherence process "restores" the quantumness of the primordial perturbations (ironically) -- may allow Bell's inequality type tests.
- State is mixed, so robust measure of non-classicality is quantum discord.