Title: Decoherence tests without quantum theory

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Abstract: In quantum theory, people have thought for some while about the problem of how to estimate the decoherence of a quantum channel from classical data gained in measurements. Applications of these developments include security criteria for quantum key distribution and tests of decoherence models. In this talk, I will present some ideas for how to interpret the same classical data to make statements about decoherence in cases where nature is not necessarily described by quantum theory. This is work in progress in collaboration with many people.

Centre for Quantum Technologies

NUS National University of Singapore

WORK IN PROGRESS

Decoherence tests without quantum theory

Corsin Pfister (Speaker)

joint work with
Stephanie Wehner, Atul Mantri, Jędrzej Kaniewski,
Marco Tomamichel, Robin Schmucker,
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Outline of the talk

Warm-up & Motivation

Assume Quantum Theory

Beyond Quantum Theory

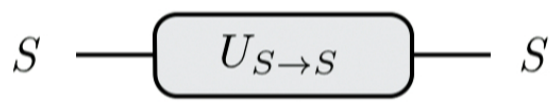
Outline of the talk

Warm-up & Motivation → Assume Quantum Theory → Beyond Quantum Theory

Warm-up & Motivation

A system S evolves as a…

- closed system: unitary evolution $\rho_S \mapsto U_{S\to S}\rho_S U_{S\to S}^\dagger$
- open system: interaction with environment E



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E E

S S

Warm-up & Motivation

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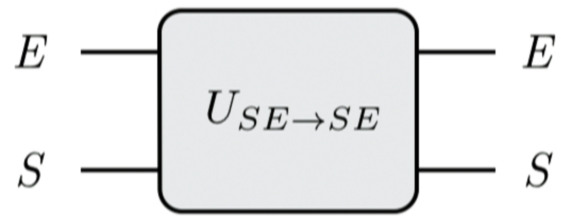
    - incorporating E : unitary



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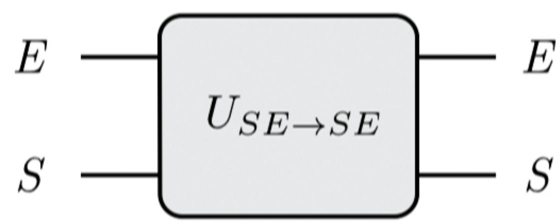

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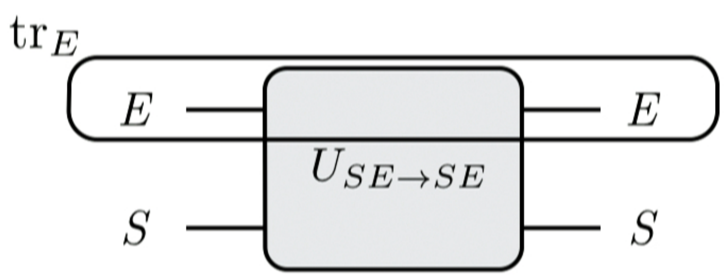

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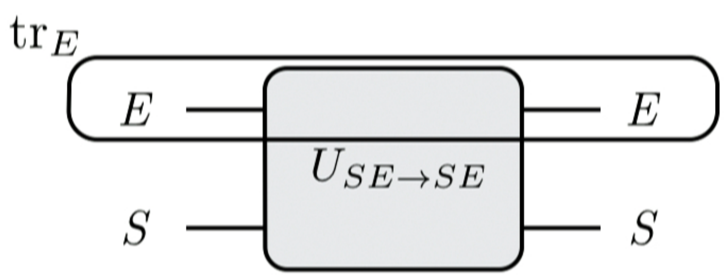

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What is decoherence?

Warm-up & Motivation

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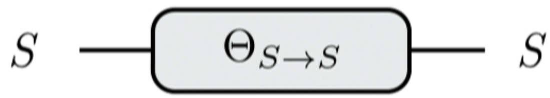

$\rho_S \mapsto \Theta_{S\to S}(\rho_S)$

Example: S spin-1/2 particle, ρ_S spin up in x direction, $\Theta_{S\to S}$ spin measurement in z direction

Warm-up & Motivation

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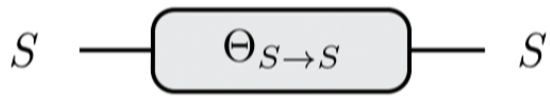

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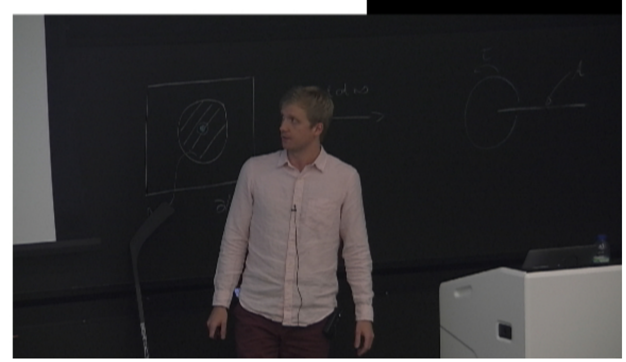

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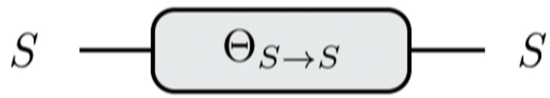

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Warm-up & Motivation

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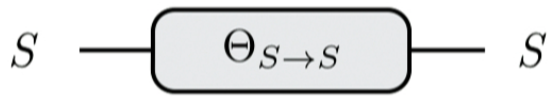

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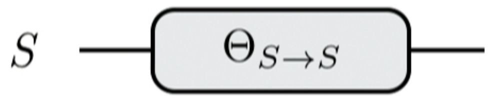

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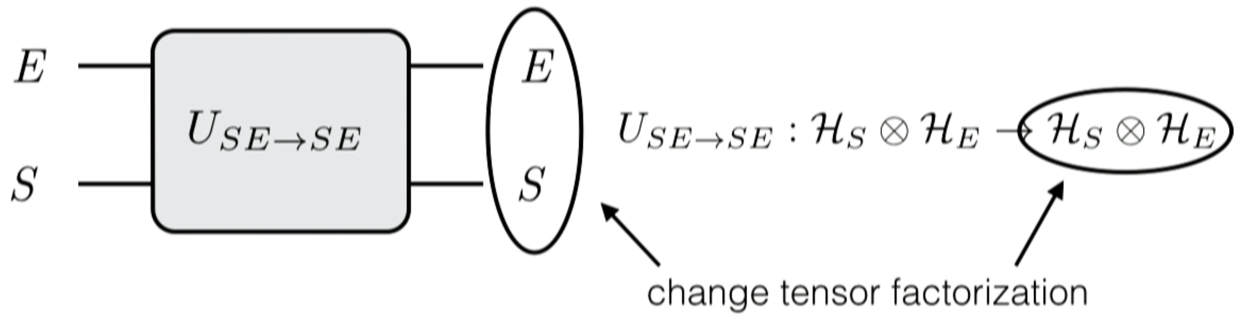
What about processes that change the type of system?

Warm-up & Motivation



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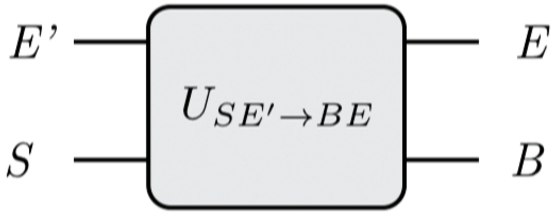
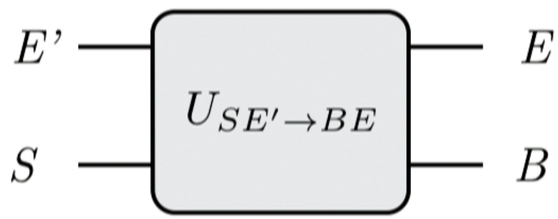


$$U_{SE' \to BE} : \mathcal{H}_S \otimes \mathcal{H}_{E'} \to \mathcal{H}_B \otimes \mathcal{H}_E$$

$$\mathcal{H}_S \otimes \mathcal{H}_{E'} \simeq \mathcal{H} \simeq \mathcal{H}_B \otimes \mathcal{H}_E$$

Warm-up & Motivation



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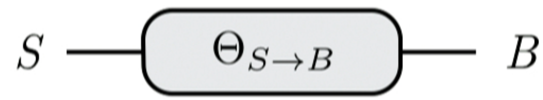
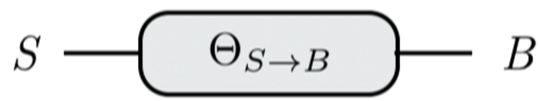
Ignoring the environment, we get:



Warm-up & Motivation

Motivating question: Performing measurements, what can we infer about the decoherence of the channel?



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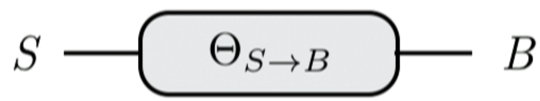

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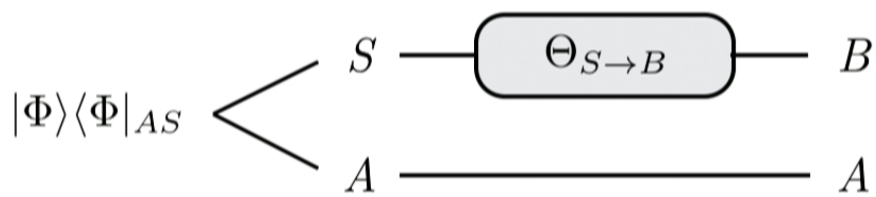

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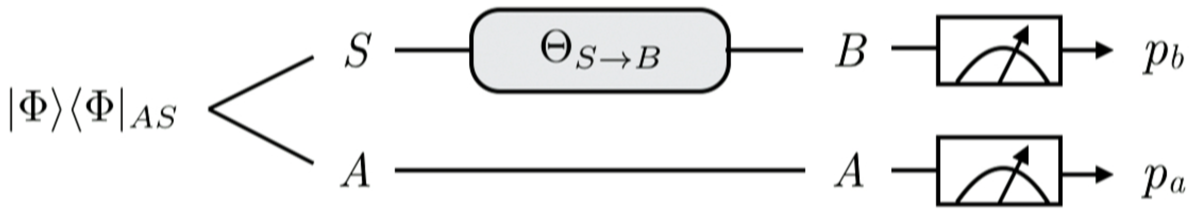

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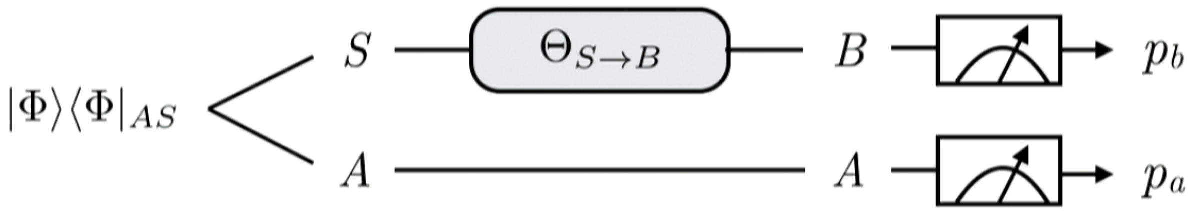
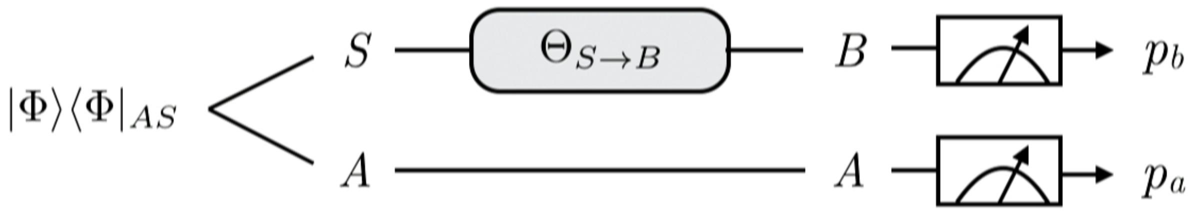


- Party A (Alice) prepares entangled state and sends one half through the channel to party B (Bob)
- Alice and Bob perform measurements and get statistics
- From the statistics, they infer how much correlation has been lost —> measure of decoherence

Warm-up & Motivation

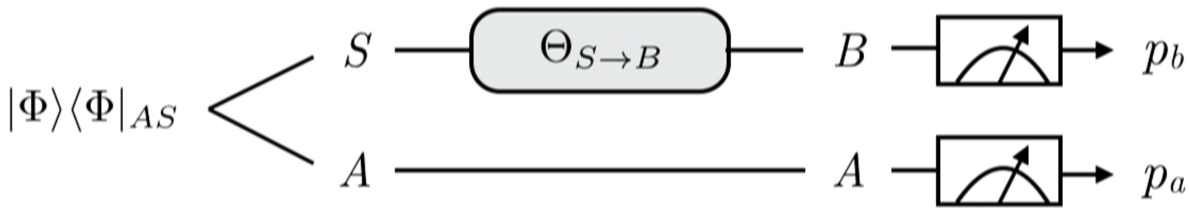
Motivating question: Performing measurements, what can we infer about the decoherence of the channel?



We look for a relation: "decoherence" ≤ f (statistics)

Warm-up & Motivation

Motivating question: Performing measurements, what can we infer about the decoherence of the channel?



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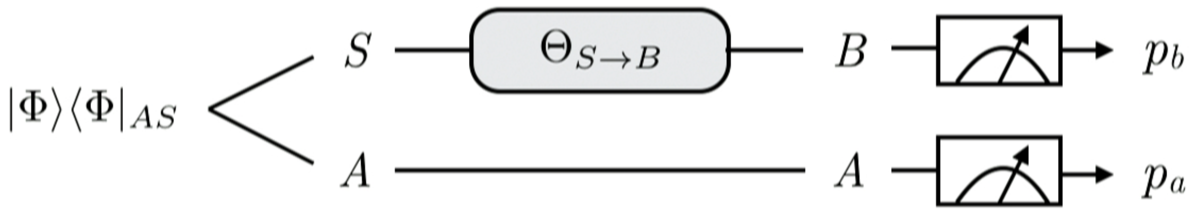
To be explained in this talk:

1. What is "decoherence" in quantum theory?

Warm-up & Motivation

Motivating question: Performing measurements, what can we infer about the decoherence of the channel?



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1. What is "decoherence" in quantum theory?
2. What is "decoherence" in a generalized probabilistic theory?

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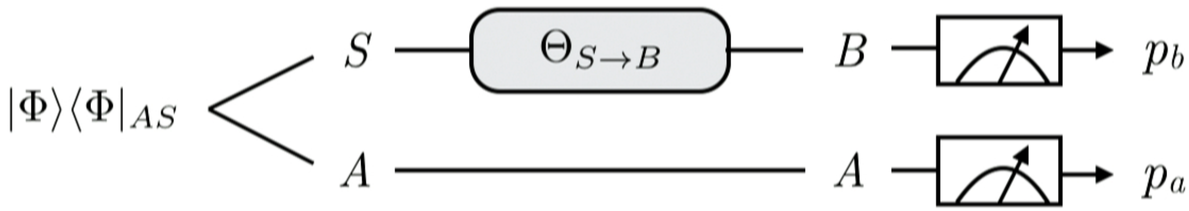

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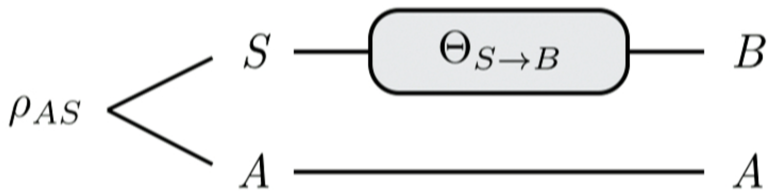
3. What is f (statistics) ?

Assume Quantum Theory

We are given a state ρ_S and a *channel* $\Theta_{S\to B}$.



Purify ρ_S to ρ_{AS}. How much of the correlation in ρ_{AS} is lost?

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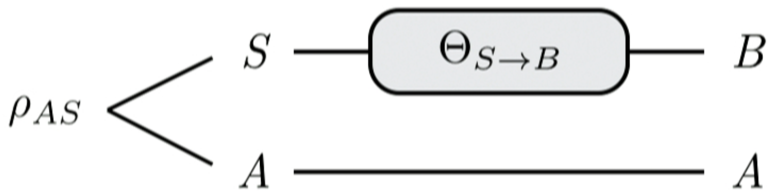

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Measure of decoherence: *Coherent Information* (Schumacher, Nielsen)

$$I(\rho_S, \Theta_{S\to B}) := H(B)_\rho - H(AB)_\rho = -H(A|B)_\rho$$

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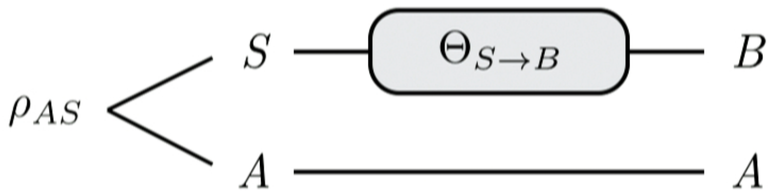

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Measure of decoherence: *Coherent Information* (Schumacher, Nielsen)

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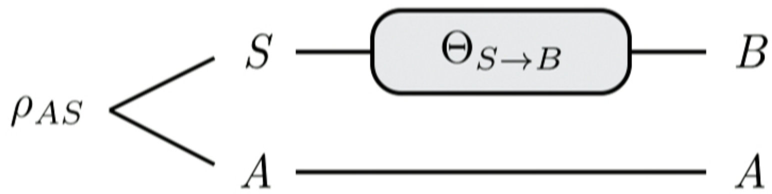
- Measure of "non-classicity" of $\rho_{AB} = \mathbb{I}_A \otimes \Theta_{S\to B}(\rho_{AS})$
- Related to the channel capacity (Lloyd-Shor-Devetak Theorem)

Assume Quantum Theory



Coherent Information: $I(\rho_S, \Theta_{S\to B}) := -H(A|B)_\rho$

Asymptotic quantity. For operational statements for finitely many uses of channel: *single-shot quantity*.

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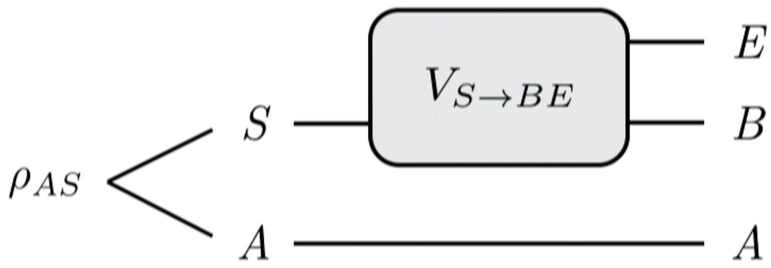

Coherent Information: $I(\rho_S, \Theta_{S\to B}) := -H(A|B)_\rho$

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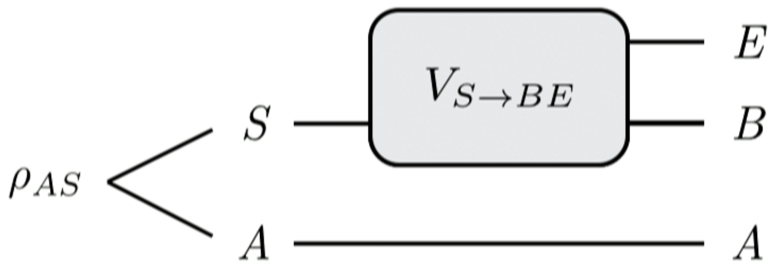
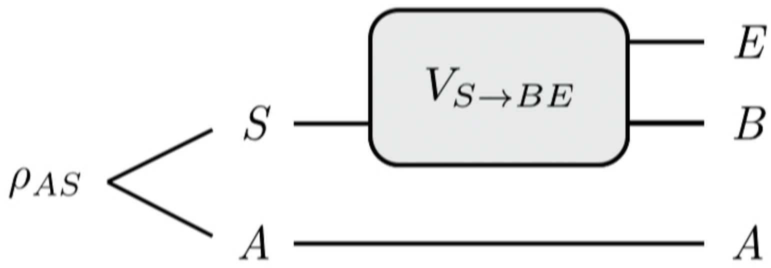
Consider *Stinespring dilation* $V_{S\to BE}$ of $\Theta_{S\to B}$

Assume Quantum Theory



$$H_{\min}(A|E)_\rho = \max_{\sigma_E} \sup\{\lambda \in \mathbb{R} \mid \rho_{AE} \leq 2^{-\lambda} \mathbb{I}_A \otimes \sigma_E\}$$

Assume Quantum Theory



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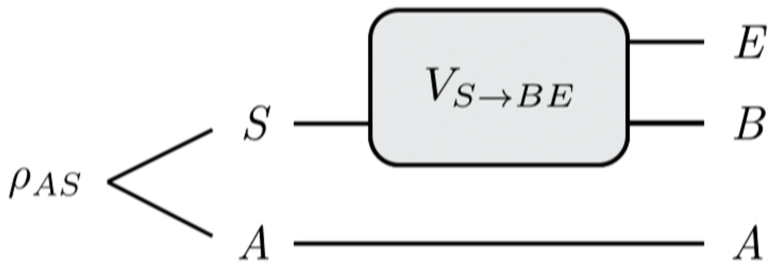
Assume Quantum Theory



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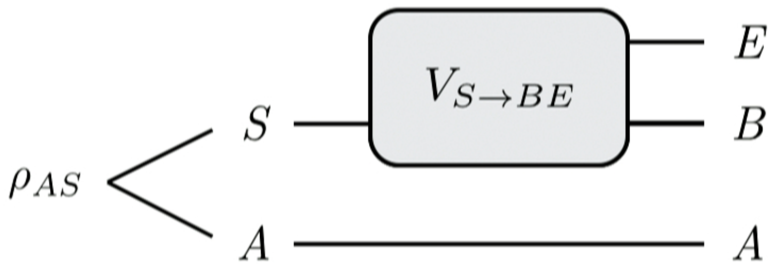
Then $H_{\min}(A|E)_\rho \sim$ number of EPR pairs that A and B can recover (Hayden et al., Berta et al.)

Contribution: $H_{\min}(A|E)_\rho$ can be estimated without iid assumption (resource problem, no decoherence estimation)

Assume Quantum Theory



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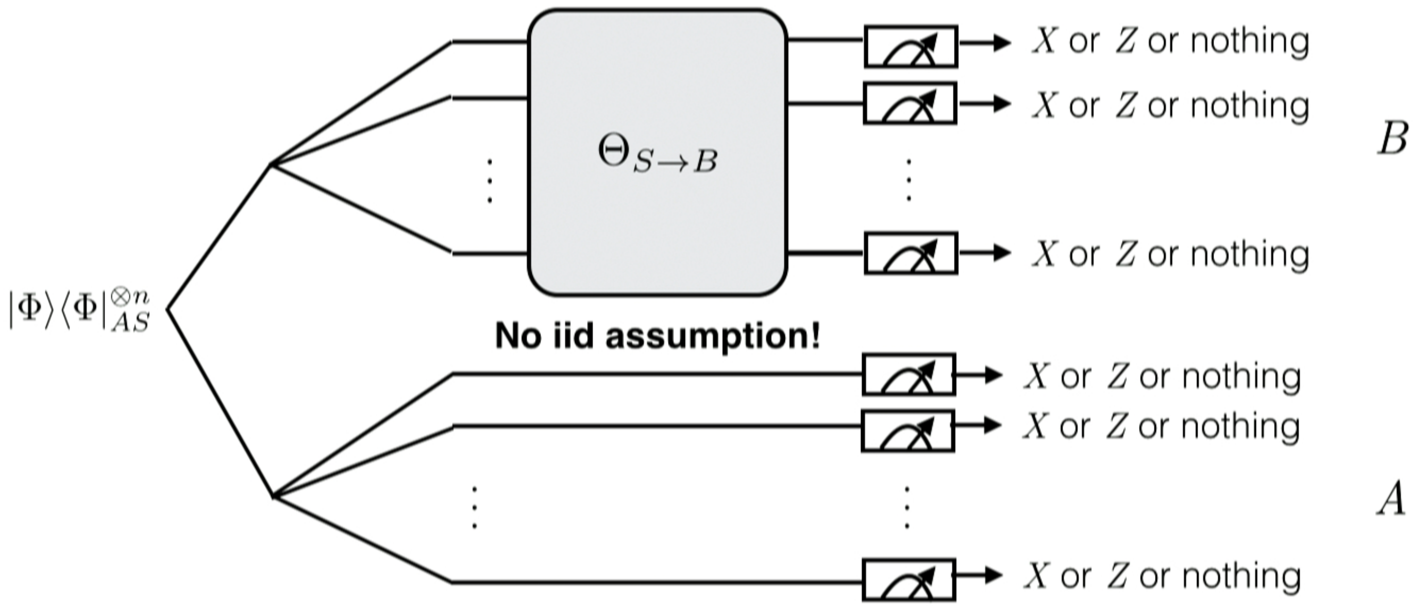


Both measure X: Determine $\delta_X = \frac{\#\text{ same result in } X}{\#\text{ different result in } X}$

Both measure Z: Determine $\delta_Z = \frac{\#\text{ same result in } Z}{\#\text{ different result in } Z}$

Conclude $H_{\min}(A|E)_{\rho'} \gtrsim n(1 - h(\delta_X) - h(\delta_Z))$, where ρ' unmeasured

Assume Quantum Theory

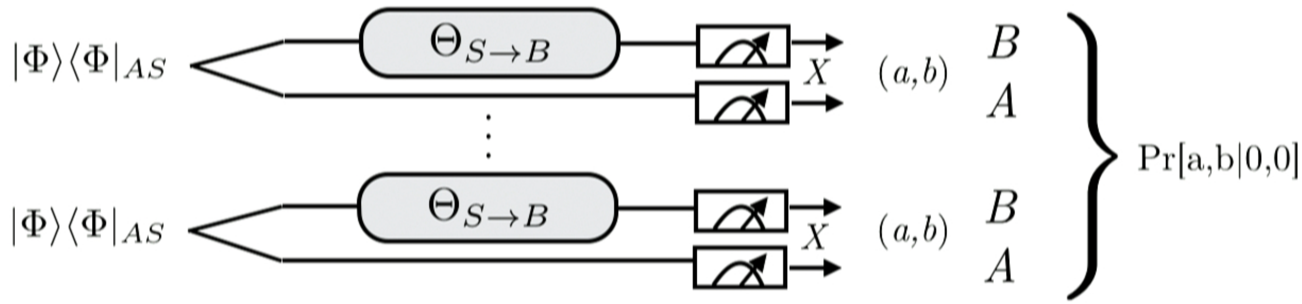
The test of the form $H_{\min}(A|E)_{\rho'} \gtrsim n(1 - h(\delta_X) - h(\delta_Z))$, where ρ' unmeasured, is…

- good for *resource estimation*
- unsuitable for channel decoherence analysis because it does not determine $H_{\min}(A|E)_{\rho}$ for the whole system that went through the channel

To estimate the channel decoherence, we *make the iid assumption.*

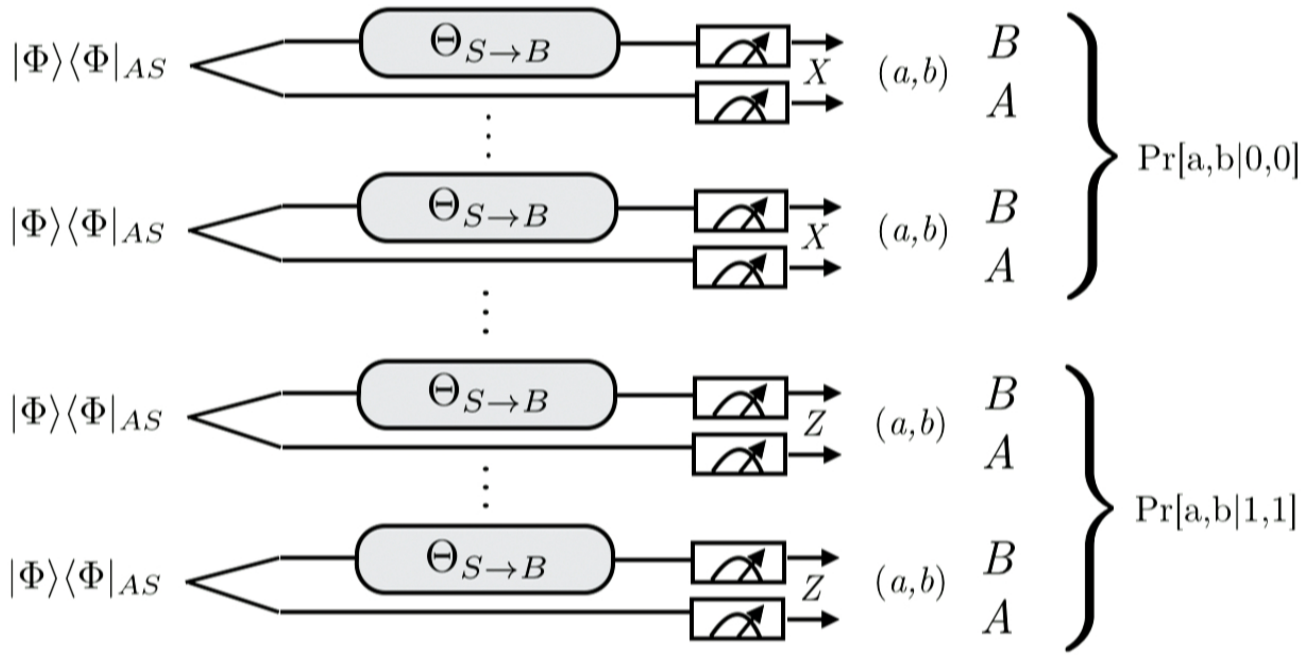
Assume Quantum Theory

Assume that channel is iid —> estimate probabilities



Assume Quantum Theory

Assume that channel is iid —> estimate probabilities



Assume Quantum Theory

Our channel decoherence estimation is of the form:

Assume that there are measurements such that

$$p_{AB}^{\mathrm{CHSH}} = \sum_{x,y} \sum_{\substack{a,b \\ a \oplus b = xy}} \Pr[a,b|x,y] \geq \lambda$$

Then

$$H_{\min}(A|E)_\rho \geq f(\lambda)$$

Why is this interesting?

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Why is this interesting?

A decoherence model may predict $H_{\min}(A|E)_\rho < \lambda$ —> ruled out

- For $f(\lambda)$ in quantum theory and proposed models to test: Stay tuned!

Beyond quantum theory

Beyond quantum theory ~ generalized probabilistic theory (GPT)

GPT ~ abstract state space

Definition: An *abstract state space* is a triple $(V,\ V^+,\ u)$, where

- V finite-dimensional real vector space
- V^+ cone in V
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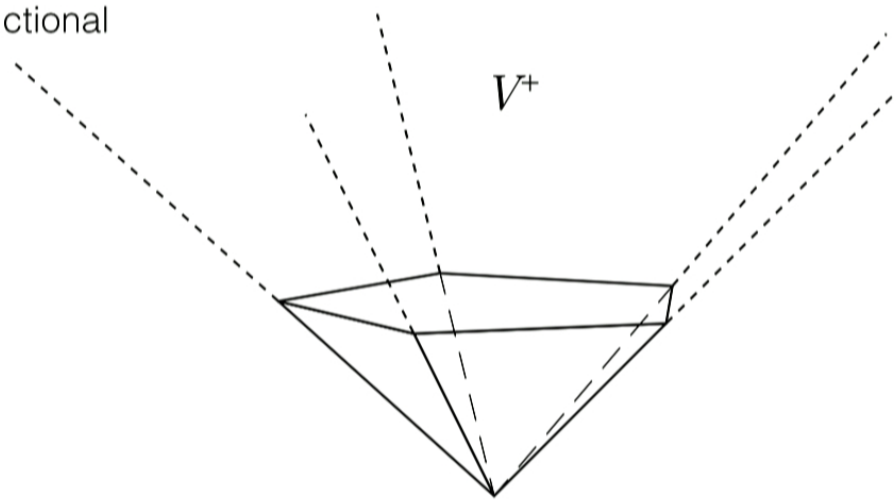

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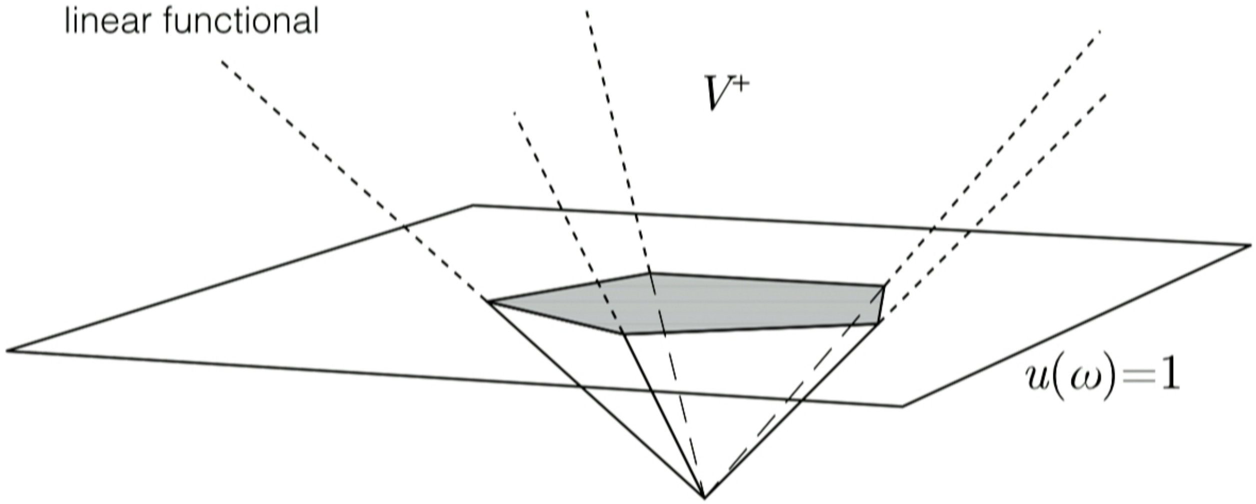
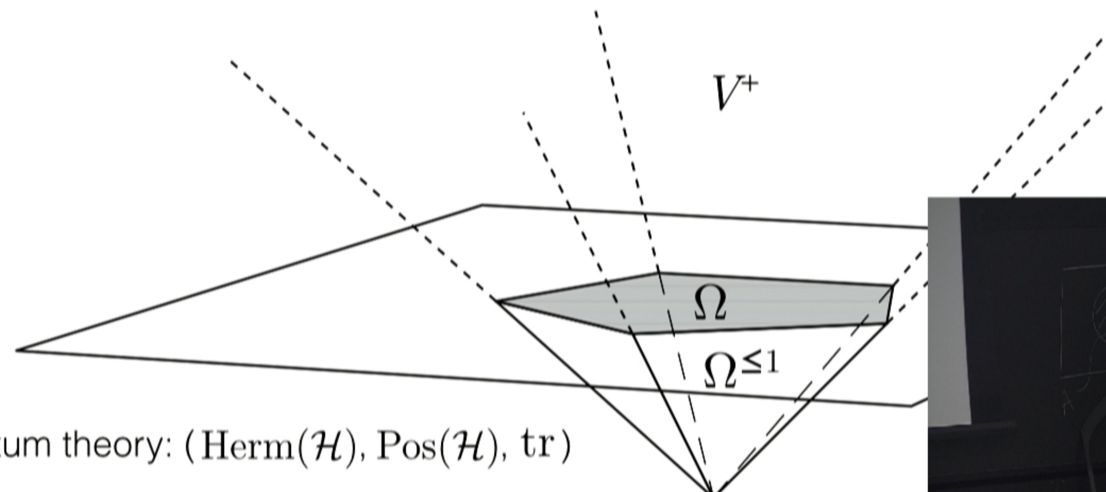


Beyond quantum theory

Measurements:

- set $\{f_1, ..., f_n\}$ of *effects*. $f_k(\omega)$: probability for outcome k



Quantum theory: $(\mathrm{Herm}(\mathcal{H}), \mathrm{Pos}(\mathcal{H}), \mathrm{tr})$

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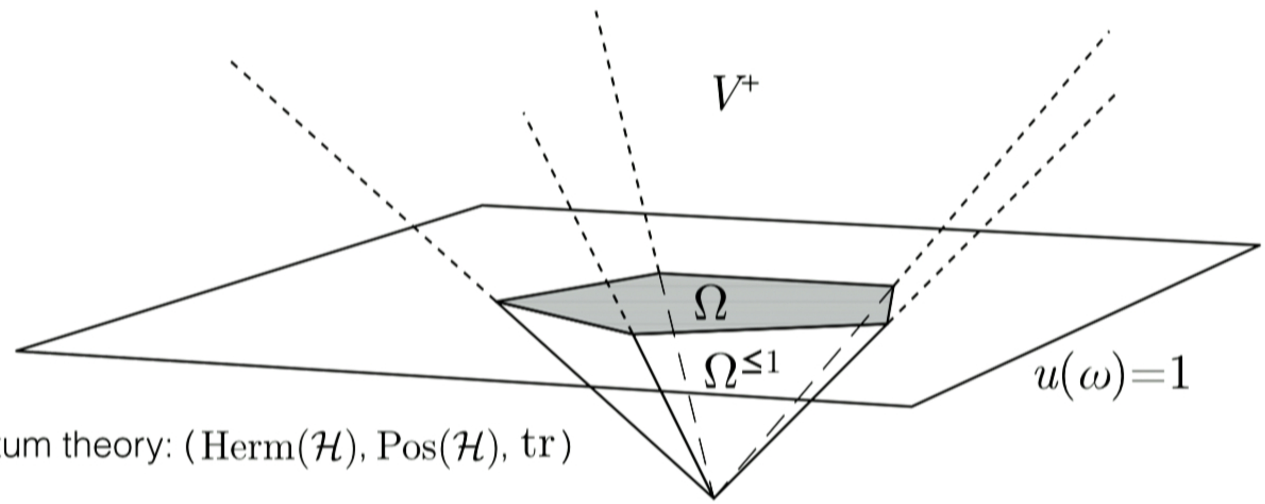

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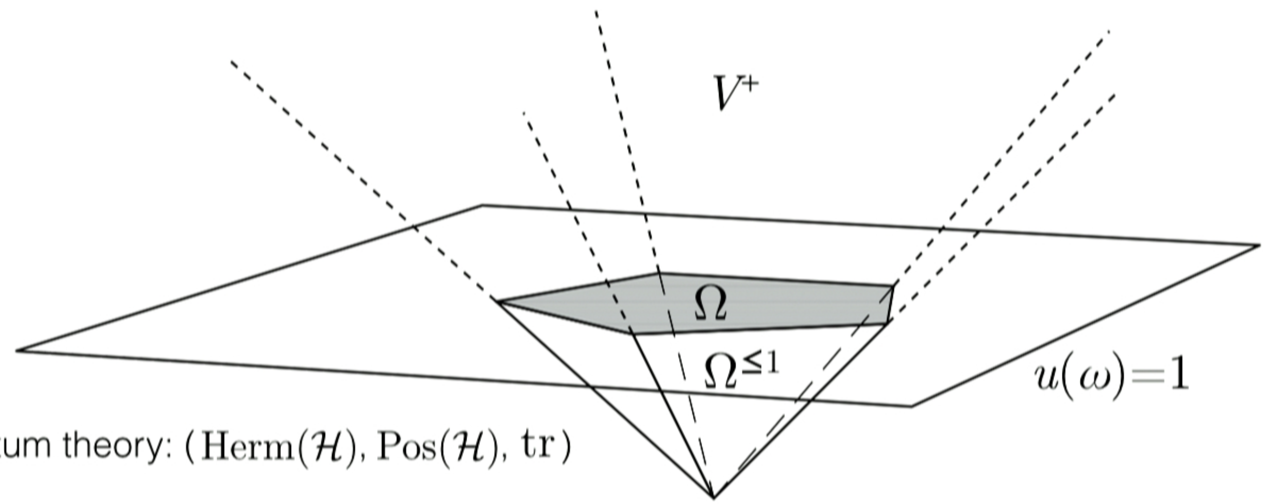
Beyond quantum theory

Measurements:

- set $\{f_1, ..., f_n\}$ of *effects*. $f_k(\omega)$: probability for outcome k
- effects: linear functionals f in V^* s.t. $0 \le f(\omega) \le 1$ for all ω in Ω
- this gives the set of effects $\mathcal{E}$
- measurement: set $\{f_1, ..., f_n\} \subseteq \mathcal{E}$ such that $\sum_k f_k(\omega) = 1$



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Beyond quantum theory

Tripartite scenarios in our framework:

- We do not specify a tensor product structure
- Make no assumption about how systems combine
- Instead: take overall state space $(V,\ V^+,\ u)$
- Specify parties via transformations they perform

Definition: A *tripartite scenario* is a quadruplet

$$S_{ABC} = ((V, V^+, u), \mathcal{T}_A, \mathcal{T}_B, \mathcal{T}_C)$$

where $(V,\ V^+,\ u)$ is an abstract state space and where

$$\mathcal{T}_A, \mathcal{T}_B, \mathcal{T}_C \subseteq \{E : V \to V \mid E(\Omega) \subseteq \Omega^{\leq}\}$$

such that transformations of different parties commute, i.e. $T_A T_B = T_B T_A$ for all $T_A \in \mathcal{T}_A, T_B \in \mathcal{T}_B$ etc.

Beyond quantum theory

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A *local instrument* is a finite set $\mathcal{I}_A \subseteq \mathcal{T}_A$ such that $\sum_{E_A \in \mathcal{I}_A} u(E_A(\omega)) = 1$ for all ω in Ω

Local instruments give rise to local measurements:
$\{u \circ E_A \mid E_A \in \mathcal{I}_A\}$ is a measurement, and
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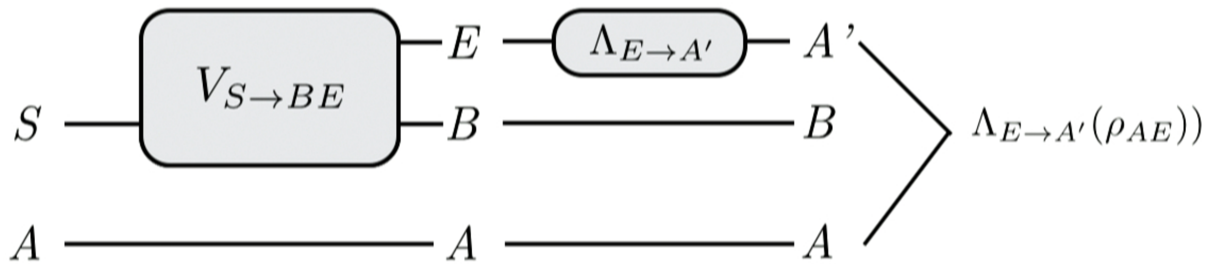
Beyond quantum theory

Want to find relation similar to $H_{\min}(A|E)_\rho \geq f(\lambda)$ for GPTs.

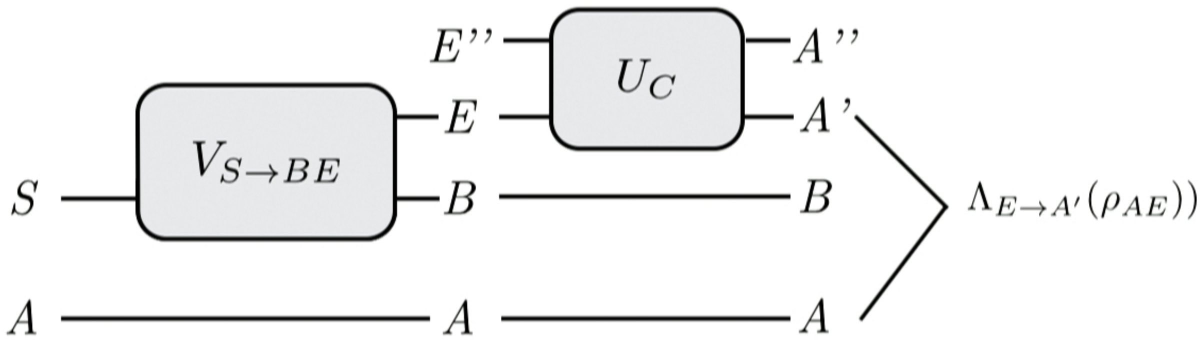
Problem: What is $H_{\min}(A|E)_\rho = \max_{\sigma_E} \sup\{\lambda \in \mathbb{R} \mid \rho_{AE} \leq 2^{-\lambda}\mathbb{I}_A \otimes \sigma_E\}$ in our framework?

Take a more inspiring expression:

$$H_{\min}(A|E)_\rho = -\log d_A \max_{\Lambda_{E\to A'}} F^2(\Phi_{AA'}, \mathbb{I}_A \otimes \Lambda_{E\to A'}(\rho_{AE}))$$



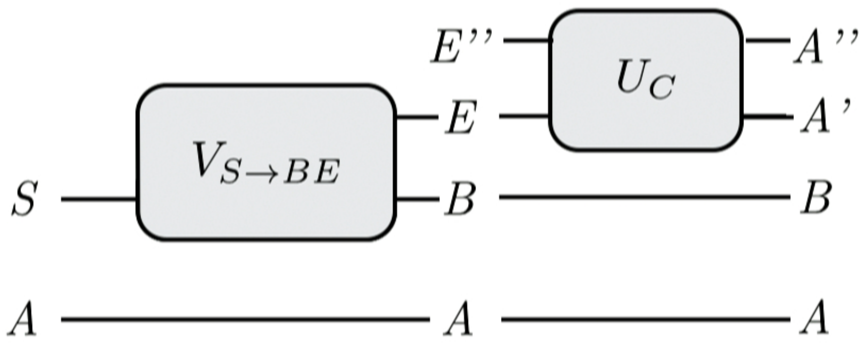
Beyond quantum theory



$$H_{\min}(A|E)_\rho = -\log d_A \max_{\Lambda_{E\to A'}} F^2(\Phi_{AA'}, \mathbb{I}_A \otimes \Lambda_{E\to A'}(\rho_{AE}))$$

- Reduced states & transformations between different parties undefined
- Solution: Purify!
- Get $H_{\min}(A|E)_\rho = -\log d_A \max_{U_C} \max_{\sigma_{BA''}} F^2(\Phi_{AA'} \otimes \sigma_{BA''}, (\mathbb{I}_{AB} \otimes U_C)\rho_{ABC}(\mathbb{I}_{AB} \otimes U_C^\dagger))$

Beyond quantum theory



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This inspires us to define our decoherence quantity:

$$\mathrm{Dec}(A|C)_\omega := -\log \sup_{T_C \in \mathcal{T}_C} \sup_{\psi \in \Psi_{AC}} F^2(\psi, T_C(\omega))$$

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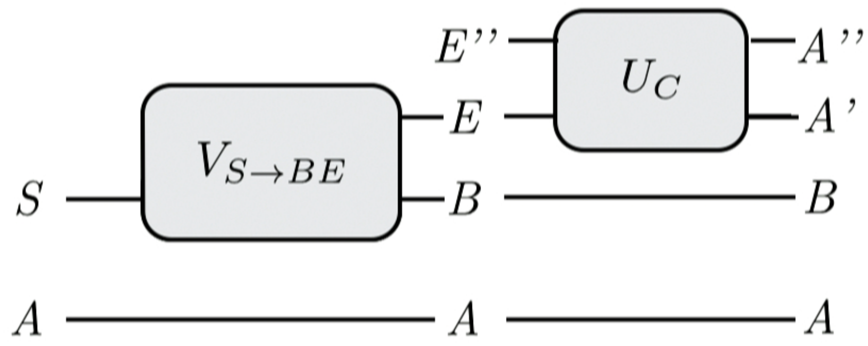

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where fidelity is lowest induced classical fidelity $F(\omega, \tau) := \inf_{M \in \mathcal{M}} F(\omega, \tau | M)$

Beyond quantum theory

$\psi \in \Psi_{AC}$: states with "maximal entanglement between A and C"

- We give two definitions: "maximally correlated", "maximally non-local"
- This leads to two versions of $\mathrm{Dec}(A|C)_\omega$

$$\Psi^{\mathrm{nl}}_{AC} := \left\{ \psi \in \Omega \;\middle|\; \begin{array}{l}\text{For every binary local instrument } I_A = \\ \{E^0_A, E^1_A\} \in \mathcal{I}_A, \text{ there is a binary local in-} \\ \text{strument } I_C = \{E^0_C, E^1_C\} \in \mathcal{I}_C \text{ such that} \\ uE^0_A E^0_C(\psi) + uE^1_A E^1_C(\psi) = 1. \end{array} \right\}$$

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Beyond quantum theory

$$\mathrm{Dec}(A|C)_\omega := -\log \sup_{T_C \in \mathcal{T}_C} \sup_{\psi \in \Psi_{AC}} F^2(\psi, T_C(\omega))$$

Finding a bound $\mathrm{Dec}(A|C)_\omega \geq f(\lambda)$:

- Assume that there are local instruments

$$\{E_A^{0|0}, E_A^{1|0}\}, \{E_A^{0|1}, E_A^{1|1}\} \in \mathcal{I}_A$$

$$\{E_B^{0|0}, E_B^{1|0}\}, \{E_B^{0|1}, E_B^{1|1}\} \in \mathcal{I}_B$$

such that

$$\frac{1}{4} \sum_{x,y} \sum_{\substack{a,b \\ a \oplus b = xy}} u E_A^{a|x} E_B^{b|y}(\omega) \geq \lambda$$

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- Use classical relations to bound $\mathrm{Dec}(A|C)_\omega$ by trace distance quantity
- Use fact that resulting distributions are non-signalling
- Solve resulting linear program using software

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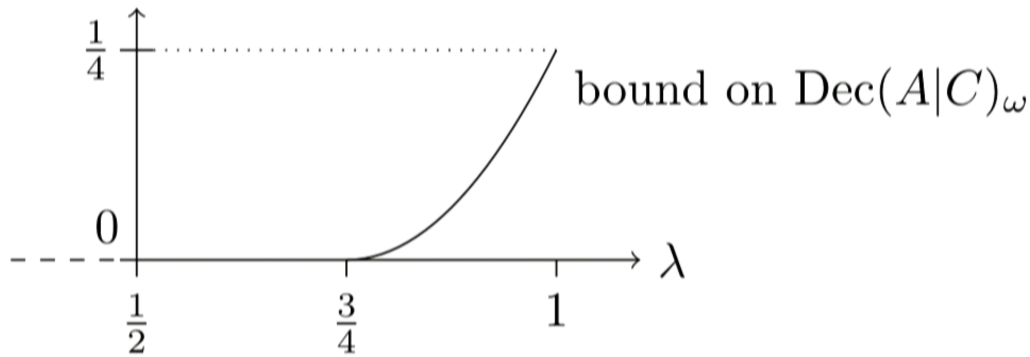

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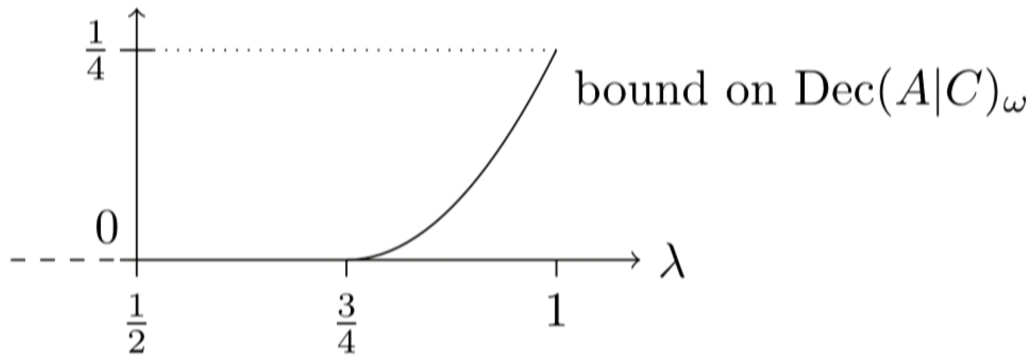

- Non-trivial for λ > 3/4 —> best you can hope for


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